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SWIRL-STABILIZED LEAN-PREMIXED FLAME COMBUSTION DYNAMICS: AN EXPERIMENTAL INVESTIGATION OF FLAME STABILIZATION, FLAME DYNAMICS AND COMBUSTION INSTABILITY CONTROL STRATEGIES

BY

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DISSERTATION

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Abstract

Though modern low-emission combustion strategies have been successful in abating the emission of pollutants in aircraft engines and power generation gas turbines, combustion instability remains one of the foremost technical challenges in the development of next generation lean premixed combustor technology. Combustion instability is the coupling between unsteady heat release and combustor acoustic modes where one amplifies the other in a feedback loop. This is a complex phenomenon which involves unsteady chemical kinetic, fluid mechanic and acoustic processes that can lead to unstable behavior and could be detrimental in ways ranging from faster part fatigue to catastrophic system failure. Understanding and controlling the onset and propagation of combustion instability is therefore critical to the development of clean and efficient combustion systems. Imaging of combustion radicals has been a cornerstone diagnostic for the field of combustion for the past two decades which allows for visualization of flame structure and behavior. However, resolving both temporal and spatial structures from image-based experimental data can be very challenging. Thus, understanding flame dynamics remains a demanding task and the difficulties often lie in the chaotic and non-linear behavior of the system of interest. To this end, this work investigates the flame dynamics of lean premixed swirl stabilized flames in two distinct configurations using a variety of high fidelity optical and laser diagnostic techniques in conjunction with advanced data / algorithm based post-processing tools.

The first part of this work is focused on establishing the effectiveness of microwave plasma discharges in improving combustor flame dynamics through minimizing heat release and pressure fluctuations. The effect of continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge on a swirl stabilized, lean premixed methane-air flame was investigated using quantitative OH planar laser induced fluorescence (PLIF), spectrally resolved
emission and acoustic pressure measurements. Proper Orthogonal Decomposition (POD) was used to post-process OH-PLIF images to extract information on flame dynamics that are usually lost through classical statistical approaches. Results show that direct plasma coupling accelerates combustion chemistry due to the non-thermal effects of plasma that lead to significantly improved combustor dynamics. Overall, this study demonstrates that microwave direct plasma coupling can drastically enhance dynamic flame stability of swirl stabilized flames especially at very lean operating conditions.

The second part of this work is focused on the development of a stable and efficient small-scale combustor architecture with comparable power density, performance and emission characteristics to that of existing large-scale burners with reduced susceptibility to extinction and externally imposed acoustic perturbations while maintaining high combustion efficiency and low emission levels under ultra-lean operating conditions. Prototype burner arrays were additively manufactured, and the combustion characteristics of the mesoscale burner array were studied using several conventional and optical diagnostic techniques. The burner array was specifically configured to enhance overall combustion stability, particularly under lean operating conditions, by promoting flame to flame interactions between the neighboring elements. Dynamic mode decomposition (DMD) analysis based on high speed OH-PLIF images was carried out to provide a quantitative measure of flame stability. Results show a marked improvement in combustion stability for a mesoscale burner array compared to a single swirl-stabilized flame with similar power output. Overall, this study shows promise for integration of mesoscale combustor arrays as a flexible and scalable technology in next generation propulsion and power generation systems.
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Chapter 1. Introduction

1.1. Background and Motivation

Combustion has played a fundamental role in the history and development of humans. The ability to control and utilize fire for warmth, cooking and protection from wild animals was in fact essential for survival to primitive human beings. With the evolution of civilization, humans gradually honed their skills to harness heat and thereby energy generated by combustion processes. In fact, the industrial revolution was heralded by the advent of the steam engines powered by combustion of wood and eventually cool in the late 18th and early 19th centuries. Since then, starting from ships and trains, mass transportation has always relied on combustion. From the internal combustion (IC) engines that powers millions of automobiles daily all over the world to the gas turbine engines that propel aircrafts across the skies to the massive rocket engines used for space explorations, the majority of modern day transportation systems that have been the linchpin of globalization rely on engines powered by combustion of either gaseous, liquid or solid fuel.

Furthermore, apart from transportation, today’s technologically advanced modern life style as we know it would never exist without our current expertise in controlling and utilizing combustion processes. In fact, almost 80% of the world’s energy needs are presently met through some form of combustion process. Also, as humanity continues its strides in technology and development, our energy needs are bound to rise in the future. Even, if we assume our per capita energy consumption to remain constant for the next thirty years, the total energy consumption of the world is still going to increase dramatically in that time. This is due to the expected increase in world’s population by 20% between now and 2050. Thus, our reliance upon the combustion of fossil fuels is not likely to decline any soon and combustion is likely to continue as a major
component in the energy portfolio for the near future.

Unfortunately, there are issues associated with the continuous burning of fossil fuels to satisfy our energy demands. Fossil fuels are a finite (limited) energy resource and hence are considered as non-renewable. Though according to recent estimate, our current coal, gas and oil reserves are expected to last for at least another fifty years, it is entirely realistic that the cost of fossil fuels that will inevitably be completely expended will most certainly increase beyond the realm of what can be considered as “affordable” in the current scenario leading to an international economic crisis. Furthermore, geopolitics and cost of fossil fuels are so closely interconnected, that any minor instability in the geopolitics without much warning can potentially jeopardize access to our primary energy reserves. This can in turn trigger a sharp increase in the cost of these fuels.

Even, if our access to fossil fuels is unimpeded for the next several decades, it is important to note that burning of fossil fuels in such massive scales has a major negative and potentially irreversible impact on our environment. As all fossil fuels are carbonaceous, copious amount of carbon dioxide (CO₂) and other harmful pollutants are released into the atmosphere when fossil fuels are burned. Over the last century, the burning of fossil fuels has increased the concentration of atmospheric CO₂ by over 50%. Substantial scientific evidence has shown that there is better than a 95% probability that major greenhouse gases like CO₂ have been directly linked to the observed climate change which has resulted in increasing global temperatures (global warming), extreme weather patterns and rising sea levels.

Beyond damaging our environment, burning of fossil fuels can cause serious health issues to humans and all forms of life. For example, nitrogen oxides (NOₓ) emissions, a byproduct of all fossil fuel combustion contributes to ground level ozone (smog) and acid rain which can cause and
exacerbate a variety of chronic respiratory ailments including bronchitis, asthma in humans. In addition to this, acidic precipitations (acid rain) increases the acidity of lakes and streams, which can be harmful to fishes and other aquatic life in addition to damaging trees and weakening forest ecosystems. Particulate matter (PM) or soot, polycyclic aromatic hydrocarbons (PAHs) and unburnt hydrocarbons (UHC) are produced as a result of incomplete combustion of fossil fuels. These emissions, in addition to be a major contributor for respiratory disorders, have also been established as potential carcinogenic agents. Thus, for our civilization to continue its progress and flourish in a world that is dependent on a continuous supply of energy, it is essential to develop energy systems that are affordable, efficient and harm the environment and life in general as little as possible.

To achieve this goal, there has been an increasing interest in using renewable energy sources such as solar, wind, hydroelectric and geothermal energy to meet at least some of our energy demands. However, there are several disadvantages associated with renewable energy sources. Renewable energy sources cannot provide a continuous source of energy and are very vulnerable to weather and other climatic conditions: clouds, days with low wind speed and seasonal droughts can reduce energy output from solar power plants, wind energy farms and hydroelectric power plants respectively. Also, it is difficult to efficiently throttle the energy output from renewable energy sources to satisfy ever fluctuating demands. In general, renewable energy is more expensive to produce and to use when compared to fossil fuels as they require large areas of land which are often located in remote areas it can be expensive to build power lines from the renewable energy sources to the cities that need the electricity. For example, dams that are essential to hydroelectric power plants can take up large portions of land and are essentially situated only in certain specific parts of the world. They can also have significant negative environmental impact
on the surrounding ecosystem. At present, renewable energy source are not particularly efficient and hence at this time do not offer an affordable alternative to burning of fossil fuels. Nuclear energy would be ideal in theory, however, existing geopolitical situation along with the general public’s negative view has hindered their development and will continue to hinder the development of nuclear power plants.

Moreover, the energy production to weight ratio (thrust to weight) for renewable energy sources is extremely lower when compared to modern gas turbine engines operating with liquid hydrocarbon fuels. Thus, even if a significant portion of our energy demands and ground-based transportation needs are met by using renewable energy sources, it is unlikely that our air transportation systems will be powered by anything other than engines relying on liquid hydrocarbon fuel combustion in the foreseeable future. Hence it is of paramount importance, that we are invested in developing combustion systems that are efficient, economical and clean in terms of emissions.

1.1.1. Gas Turbines

Today, gas turbines have become an important, widespread and a reliable device used in a multitude of applications ranging from power generation to transportation (aviation and marine propulsion). Gas turbines have become a dominant power generation technology used worldwide. A gas turbine is a heat engine that uses high temperature, high pressure gas as a working fluid to spin a turbine thereby generating power or thrust. The combustion of fuel with air is usually used to produce the needed temperatures and pressures to efficiently drive the turbine.

Some of the major factors driving the development of combustors used in gas turbine engines are:
• Improving thermal efficiency of the engine.
• Reducing pollutant emissions - primarily CO and NO\textsubscript{x}.
• Increasing operational flexibility.
• Ever increasing energy demand.
• High prices and strong price volatility

Since the passage of the Clean Air Act of 1970 which imposed stringent emission control norms, stricter government regulations regarding pollutant emission and in particular for oxides of nitrogen, continue to be enacted. To comply with these norms, the gas turbine industry is continuously seeking new ways to reduce NO\textsubscript{x} emissions.

Several solutions have been previously employed in gas turbine industry to reduce NO\textsubscript{x} emissions and meet Environmental Protection Agency (EPA) regulations. These techniques include catalytic combustion, rich-burn / quick-quench / lean-burn (RQL) combustion, fuel staging, water or steam injection and lean premixed combustion. Among these techniques, the RQL system is usually hampered by soot formation and incomplete mixing while catalytic combustion tends to be too expensive, but research on these technologies is still currently ongoing [1]. Currently, lean premixed combustion is garnering more interest of late and has become the most promising technology option for reducing NO\textsubscript{x} emissions. In lean premixed combustion systems, the flame temperature is reduced by the use of excess air (combustion occurs at lower-equivalence ratio) which results in a decrease in the formation of thermal (Zeldovich mechanism) for NO\textsubscript{x} [2, 3]. For example, General Electric (GE) in 2005 utilized a dry low NO\textsubscript{x} combustion system for their 12 MW class 10-2 gas turbine which reduced NO\textsubscript{x} emissions from 25 to 15 ppm. These emission levels are guaranteed over any operating condition for 50-100\% of load and ambient temperatures of -29\textdegree C to 38\textdegree C. The combustion chamber for this system incorporates four
GE5 premixed pilot burners and includes flame detection, flashback thermocouples, and a humming detection system for combustion instability control.

1.1.2. Lean Premixed Combustion

Older generations of gas turbine combustors were driven by diffusion type flames. The pollutant emissions particularly NO\textsubscript{x} could not be lowered sufficiently to abide by the emission regulations using the diffusion flame configuration. To address the issue, premixed combustion, where fuel and air are mixed in premixing sections prior to the combustion chamber was introduced to achieve significant reductions in NO\textsubscript{x} emissions. Figure 1.1 shows the effect of unpremixedness parameter $U$ on NO\textsubscript{x} emissions. $U = 0$, if fuel-air mixture is completely mixed and homogenous; $U > 0$, if fuel-air mixture is not completely premixed and $U = 1$ if no mixing has occurred. Furthermore, the fuel-air mixture is additionally kept fuel-lean to achieve low NO\textsubscript{x} emissions as shown in Figure 1.2 along with simultaneously decreased fuel consumption. This new technology is referred to as lean premixed combustion.

Though lean premixed combustion allows for pollutant emissions to be curtailed, it is more susceptible to flow field fluctuations than diffusion flames. Hence, combustors that employ lean premixed flames are also significantly more susceptible to thermoacoustic instability than those driven by diffusion flames. Thus, it is more likely that when lean premixed flames are involved, acoustic driving due to thermoacoustic coupling will exceed acoustic damping [4]. Such thermoacoustic instabilities are characterized by large amplitude oscillations of one or more natural acoustic modes of the combustor. The instabilities generally occur when the unsteady heat release from combustion couples with the natural acoustic modes of the combustor, resulting in self-excited oscillations. Combustion driven pressure oscillations can make the flame unstable,
Figure 1.1. NO\textsubscript{x} formation as a function of unpremixedness parameter of a fuel-air mixture [5].

Figure 1.2. NO production in a stirred reactor as function of the excess air ratio (λ = \phi^{-1}). λ > 1 characterizes a lean mixture [5].
reduce part life, and lead to structural damage and failure. This aspect of lean premixed systems has been hindering its development and use in gas turbines [6].

1.1.3. Combustion Instability

Combustion instabilities are a major problem in the design, development and operation of modern high performance propulsion systems [7]. Combustion instabilities refer to self-sustained oscillations at or near the acoustic frequency of the combustion chamber, which arise due to the closed-loop coupling between unsteady heat release and pressure fluctuations [8, 9] inside the combustion chamber. Combustion instabilities often manifest as large amplitude pressure oscillations that result in many undesirable effects leading to serious emission and performance degradation and catastrophic destruction of engine hardware [10]. The typically low frequency oscillations induce large mechanical vibrations in the system that often result in combustor failure. In unstable operation mode, enhanced heat transfer at combustor walls may lead to partial or total blow off [7-10]. The coupling between the heat release fluctuations and the acoustics was first identified by Lord Rayleigh [11] as follows:

If heat be periodically communicated to, and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged.
This is called the Rayleigh criterion, which is a necessary condition for the occurrence and sustenance of combustion instabilities. Mathematically, Rayleigh criterion can be expressed as:

$$\frac{1}{T}\int_{0}^{T} p'(t) \dot{q}'(t) \, dt > 0$$  \hspace{1cm} (1.1)

Where $p'(t)$ and $\dot{q}'(t)$ represent the pressure and heat release fluctuations respectively.

In a practical combustion system, where flames are typically confined within the combustion chamber, pressure fluctuations from unsteady flames will reflect from the combustor boundaries back to the burner. Flames, being susceptible to acoustic perturbations, respond to these direct acoustic fluctuations as well as to indirect hydrodynamic fluctuations generated by acoustic fluctuations at components of the burner and the flame holder. In this manner, a feedback loop can be established between the unsteady flame and acoustic fluctuations within the combustor. The basic nature of the feedback loop that drives the combustion instability phenomenon is shown in Figure 1.3.

*Figure 1.3. Combustion instability feedback loop showing the coupling between acoustics and combustion process along with Rayleigh criterion.*
When the amplification of acoustic energy at the flame during this feedback coupling exceeds the acoustic damping in the combustor, thermoacoustic instability will spontaneously occur. Acoustic fluctuations corresponding to the resonant modes of the duct are the least damped and consequently most likely to participate in the feedback coupling. The frequency of fluctuations in the flame associated with the feedback loop will accordingly correspond to the resonant modes. Thus, a constructive thermo-acoustic feedback loop leads to self-excited, self-sustained acoustic and heat release rate oscillations, known as thermoacoustic oscillations and the phenomenon is referred to as thermoacoustic instability.

![Diagram of combustion instabilities](image)

*Figure 1.4. Basic interactions leading to combustion instabilities [12].*

In general, combustion instabilities are driven by a variety of flow and combustion processes that can give rise to an unsteady heat release in a practical combustion system. A driving process generates a perturbation of the flow field while a feedback process couples this perturbation to the driving mechanism and produces the resonant interaction which may lead to combustion instability. The driving mechanism itself involves a wide variety of elementary
processes that are schematically outlined in Figure 1.4. The feedback generally relates the downstream flow to the upstream region and it is in most cases acoustic in nature. The coupling may also include convective modes like entropy waves.

1.1.4. Combustion Instability Control

Due to their potential harm to system performance, it is often necessary to find ways to reduce the magnitude of these oscillations in the course of developing a new combustion system. To suppress combustion instability, the feedback loop between the active acoustic modes and unsteady heat release should be interrupted. The methods that are generally employed to suppress or attenuate combustion instabilities can be divided into two classes [13]: passive and active control.

The first class of methods called passive methods require a certain physical understanding of the phenomenon and attempt to prevent combustion instabilities by modifying the combustion process to reduce its driving which involves modifications of the combustion systems, e.g., changing the injector design [14], changing the combustor geometry to prevent the excitation of unstable modes by the addition of baffles to the injector face in a liquid rocket [15], increasing the combustor’s damping by the addition of acoustic liners [16]. Unfortunately, passive control approaches have generally not been satisfactory due to lack of adequate understanding of the fundamental processes that drive the instability, which resulted in costly ‘solutions’ that were only applicable to a specific combustor design over a limited range of operating conditions [9].

In contrast to a passive solution approach, an active control mechanism minimizes combustion instability by continuously sensing and evaluating the state of the combustor and forcing it to perform in a desirable manner [13]. Due to easy implementation, most of the current
active control technology involves modulating the primary/secondary fuel injection e.g., shear layer mixing [17, 18], valves that oscillate air or fuel flow rate [19]. However, the limitations of such a control strategy make it impractical for many propulsion applications.

1.2. Scope of Research

Combustion instability is a complex phenomenon which involves unsteady chemical kinetic, fluid mechanic and acoustic processes that can lead to unstable behavior and be detrimental in ways ranging from faster part fatigue to catastrophic system failure. Understanding and controlling the onset and propagation of combustion instability is therefore critical to the development of clean and efficient combustion systems. Imaging of combustion radicals has been a cornerstone diagnostic for the field of combustion for the past two decades which allows for visualization of flame structure and behavior. However, resolving both temporal and spatial structures from image-based experimental data can be very challenging. Thus, understanding flame dynamics remains a demanding task and the difficulties often lie in the chaotic and non-linear behavior of the system of interest. To this end, this work investigates the flame dynamics of premixed swirl stabilized flames in two distinct configurations using a variety of high fidelity optical and laser diagnostics techniques in conjunction with advanced data / algorithm based post-processing tools.

The first part of this work is focused on establishing the effectiveness of microwave plasma discharges in improving combustor flame through minimizing heat release and pressure fluctuations. The effect of continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge on a swirl stabilized, premixed methane-air flame was investigated using quantitative OH planar laser induced fluorescence (PLIF), spectrally resolved emission and acoustic pressure measurements. Proper Orthogonal Decomposition (POD) was used to
post-process OH-PLIF data to extract information on flame dynamics that are usually lost through classical statistical approaches. Direct plasma coupling accelerated combustion chemistry due to the non-thermal effects of plasma that lead to significantly improved combustor dynamics. Overall, this study demonstrates that microwave direct plasma coupling can drastically enhance dynamic flame stability of swirl stabilized flames especially at very lean operating conditions.

The second part of this work is focused on development of a stable and efficient small-scale combustor architecture with comparable performance and emission characteristics to that of large-scale burners with reduced susceptibility to extinction and externally imposed perturbations while maintain high combustion efficiency and low emission levels under ultra-lean operating conditions. Prototype burner arrays were additively manufactured and the combustion characteristics of the mesoscale burner array were studied using several conventional and optical diagnostic techniques. The array was specifically configured to enhance overall combustion stability, particularly under lean operating conditions, by promoting flame to flame interactions between neighboring elements. Dynamic mode decomposition (DMD) analysis based on high speed OH-PLIF images was carried out to provide a quantitative measure of flame stability. Overall, this study shows promise for integration of mesoscale combustor arrays as a potentially flexible and scalable technology in next generation propulsion and power generation systems.
Chapter 2. Laser Diagnostics for Reactive Flows

2.1. Laser Induced Fluorescence (LIF)

Laser induced fluorescence (LIF) is a common, well-established method and a versatile tool for detection of chemical species or interrogation of a thermodynamic state in many science and engineering disciplines. LIF is highly selective by both species and quantum state. The absorption and emission of radiation is governed by quantum mechanics and is affected by collisional energy transfer which in turn allows for concentration, temperature, pressure density, velocity and reaction chemistry measurements [20]. In the field of combustion research, LIF has evolved to be the most predominant method for detection and measurement of minor species in flames and reactive flows [21-24]. Planar LIF imaging (PLIF) has matured into an effective tool for flow visualization and 2D imaging of concentration fields of chemical species and other thermodynamic variables [25, 26]. This allows for resolving the spatial structure of a reactive flow field. Chemical species like OH, CH and HCHO are particularly targeted in LIF as their presence marks the reaction zone, combustion products and the preheat zone respectively [27]. Other target species like NO are used for flow visualization by seeding the flow using the selected species [28]. In this chapter, the basic theory of LIF spectroscopy required interpretation of the LIF signal is presented.

2.2. Basic Theory of Laser Fluorescence

2.2.1. Quasi Two-Level System Model

A quasi two-level model is used to introduce the fundamental energy transfer processes involved in LIF essential to understanding the dynamics of LIF. LIF is fundamentally viewed as a two-step process: First, the target molecule is excited to a higher energy state by absorption of
energy from the resonant photons of the laser followed by molecules in the excited state relaxing back to the ground state through both radiative and non-radiative pathways. The LIF signal is result of radiative relaxation, by which the excess energy is released spontaneously in the form of fluorescent photons. The non-radiative relaxation pathways compete with LIF and their relative magnitude impacts greatly the quantitative interpretation of the signal.

For a typical diatomic molecule, LIF can be studied using a quasi two-level system as shown in Figure 2.1 with transitions occurring between two electronic energy states. The two-level model is a highly simplified representation of the actual physics associated with LIF process. However, it can be successfully used for developing a mathematical model based steady-state (non-transient) system analysis to understand LIF dynamics. The main mechanisms responsible for energy transfer between the two levels is also shown in Figure 2.1.

![Figure 2.1. Two level LIF model with relevant energy transfer processes.](image)

Absorption of radiation promotes the molecule or atom from the ground energy state \((E_1)\) to an excited energy state \((E_2)\). The excited molecules can be perturbed by the presence of a radiation field and can emit a resonant photon in the process, returning to the ground state. The
excited molecules can also relax to the ground state through collisions with other molecules through a non-radiative process referred to as quenching. The reverse process called collisional excitation occurs at a significantly slow rate when compared to other energy transfer processes and hence is typically neglected. Finally, the excited molecule can spontaneously emit radiation as it relaxes to the ground state through a process called fluorescence. The details regarding the energy transfer processes in the two-level model is discusses in detail in the following section.

2.2.2. Fundamental Energy Transfer Processes

2.2.2.1. Induced (Stimulated) Absorption

Stimulated absorption is an excitation mechanism directly coupled to the interaction between the laser and the targeted molecule. Absorption \( W_{12} \) is the event where energy from the incident photon of a laser beam is captured by the population in the ground energy state \( E_1 \) and added to the molecule’s internal energy and is thereby excited to an upper energy state \( E_2 \). The absorption rate \( W_{12} \) \( (s^{-1}) \) is directly proportional to the Einstein transition probability coefficient \( B_{12} \) \( (m^3/Js^2) \) and can be expressed as:

\[
W_{12} = \frac{B_{12}I_v}{c}
\]  

(2.1)

Where \( I_v \) \( (W/cm^2s^{-1}) \) is the incident laser irradiance per unit frequency interval (spectral irradiance) and \( c \) is the speed of light \( (m/s) \). A more rigorous model that accounts for the laser and absorption line shape is given by

\[
W_{12} = \frac{B_{12}}{c} \int I_v(v) \, g(v) \, dv
\]  

(2.2)

Where \( I_v(v) \) is the laser line shape and \( g(v) \) is the absorption line shape. The incident laser
irradiance can be decomposed as shown below.

\[
W_{12} = \frac{B_{12}}{c} I_0^0 \int g(\nu) L_\nu(\nu) \, d\nu = \frac{B_{12}}{c} I_0^0 \Gamma
\]  

(2.3)

Where \( I_0^0(\nu) \) is the normalized spectral irradiance and \( L_\nu(\nu) \) is the spectral distribution function. The convolution integral of the two distributions is the line overlap integral \( \Gamma \) and is a dimensionless number less than or equal to 1. Since the energy required to induce the absorption is generally provided by pulsed lasers which are spectrally broad when compared to the targeted absorption lines, it can be assumed that the laser intensity is constant over the spectral width of the absorption lines, which results in a value of 1 for the overlap integral and Eq. 2.2 is assumed to be valid as long as \( \nu \) is considered as the laser line center.

### 2.2.2.2. Induced (Stimulated) Emission

Stimulated emission is an excitation mechanism also directly coupled to the interaction between the laser and the targeted molecule. Stimulated emission \( (W_{21}) \) is the simultaneous event which induces a reverse, deactivation process which causes the excited molecules in the upper energy state \( (E_2) \) to relax back down to the ground energy state \( (E_1) \). The stimulated emission rate \( W_{21} \, (s^{-1}) \) is directly proportional to the Einstein transition probability coefficient \( B_{21} \, (m^3 / Js^2) \) and can be expressed as:

\[
W_{21} = \frac{B_{21} I_\nu}{c}
\]  

(2.4)

Stimulated emission and absorption processes are coupled by their corresponding Einstein transition probability coefficients and energy level degeneracies as shown below.

\[
g_1 B_{12} = g_2 B_{21}
\]  

(2.5)
Where \( g_1 \) and \( g_2 \) are the degeneracies of energy levels \( E_1 \) and \( E_2 \) respectively.

### 2.2.2.3. Spontaneous Emission

Spontaneous emission \((A_{21})\) is the event by which an excited molecule in the upper energy state \((E_2)\) spontaneously relaxes to the lower electronic energy states by emitting fluorescence. This radiative decay process constitutes the main mechanism for LIF signal production. The fluorescence rate is given by the Einstein coefficient \( A_{21} \) \((s^{-1})\) and the total fluorescence rate \( A \) is the sum of fluorescence rates over all individual transitions as given below.

\[
A = \sum_i A_{2i}
\]

(2.6)

Spontaneous emission and stimulates emission are related by the Einstein coefficients as shown below.

\[
\frac{A_{21}}{B_{21}} = \frac{8\pi\hbar^3}{c^3} = 8\pi\hbar\nu^3
\]

(2.7)

Where \( \hbar \) is the Planck’s constant, \( \nu \) and \( \bar{\nu} \) are the frequency and wavenumber corresponding to the transition \( E_2 \rightarrow E_1 \) respectively. This can be rewritten in terms of Einstein coefficient for absorption \( B_{12} \) as

\[
\frac{A_{21}}{B_{12}} = \frac{g_1}{g_2} \frac{8\pi\hbar^3}{c^3} = \frac{g_1}{g_2} 8\pi\hbar\nu^3
\]

(2.8)

### 2.2.2.4. Rotational and Vibrational Energy Transfer

In any molecule, the energy levels are fully described by a hierarchy of quantum numbers namely, electronic \((n)\), vibration \((\nu)\) and rotational \((J)\) quantum numbers which quantize the individual energy components namely electronic \( E_{\text{elec}}(n) \), vibrational \( E_{\text{vib}}(\nu) \) and rotational
$E_{\text{rot}}(J)$ energies respectively that constitute a particular energy level. The total internal energy $T(n, v, J)$ of the molecule in a specified energy level is the sum of these three individual components as shown below.

$$T(n, v, J) = E_{\text{elec}}(n) + E_{\text{vib}}(v) + E_{\text{rot}}(J)$$  \hspace{1cm} (2.9)

Figure 2.2. Excitation diagram. The molecule absorbs a photon equal to the energy gap between the upper and lower energy states. The electronic potential well consists of multiple vibrational levels which in turn consists of multiple rotational levels.
Transitions involving a change in the electronic state are called rovibronic, indicating the change in all three components. The excitation of the molecule by a narrowband laser, targeted to a single rovibronic transition is illustrated in Figure 2.2, which shows the hierarchy of energy levels and the excitation of a molecule through a specific rovibronic level based on the frequency of the narrowband laser. The two electronic states (A state: first excited upper electronic energy state, X state: ground electronic energy state) are indicated by the two potential energy diagrams. The vibrational levels are shown with horizontal lines in each electronic state and the rotational manifold attached to relevant vibrational state are magnified and shown in the inset.

While the excitation transition is between two specific rovibronic states defined by the set of quantum numbers (n, v and J), the subsequent spontaneous emission occurs over many such transitions. The excited state molecule may fall to a range of rotational and vibrational energy levels as illustrated in Figure 2.3, which results is fluorescence spectra over a range of wavelengths. Further, the molecules in specific rovibrational levels in the upper excited energy state can migrate to neighboring rotational or vibrational states by internal energy transfer processes through collision with other molecules before de-excitation (fluorescence) occurs. Rotational energy transfer (RET) is typically very fast and is particularly significant in LIF dynamics of short pulse nanosecond lasers. Thermal equilibrium is perturbed when the laser pumps population from specific rotational levels in the ground electronic energy state to those in the excited upper energy state. Rotational energy transfer is responsible for refilling or depleting the relevant energy levels in an effort to re-establish thermal equilibrium distributions. The rate of rotational energy transfer $Q_{\text{RET}}$ caused by collisions with all other species $j$ in the system is given by

$$Q_{\text{RET}} = \frac{P}{kT} \sum_{j} x_{j} v_{j} \sigma_{\text{RET},j}$$  \hspace{1cm} (2.10)
Where \( x_j \) and \( v_j \) are the mole fraction of the species ‘j’ and the relative velocity and the of the collisional particles respectively. The relative velocity of the colliding particles is given by

Figure 2.3. Fluorescence diagram. The molecule emits a photon equal to the energy gap between the upper and lower states but does not have to fall through the excitation transition. Excited molecules may fall to the ground vibrational state or the first vibrational level. An analogous process occurs with respect to the rotational transition. Furthermore, both the vibrational and rotational states may change before fluorescence occurs due to collisions. This is called vibrational energy transfer (VET) and rotational energy transfer (RET).
\[ v = \sqrt{\frac{8kT}{\pi \mu}} \], where \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) is the reduced mass of the system. The parameter \( \sigma_{\text{RET}, j} \) (cm\(^2\)) is the effective rotational cross section for collision with species ‘j’. The vibrational energy transfer (VET) is conceptually identical to the rotational energy transfer but for the fact that energy transfer occurs to neighboring vibrational levels. Rotational energy transfer and vibrational energy transfer become increasingly important at high pressures, due to the increase in collision rates.

### 2.2.2.5. Collisional De-excitation (Quenching)

Quenching \((Q_{21})\) is the event by which an excited molecule in the upper energy state \((E_2)\) can relax down to a lower electronic energy state \((E_1)\) by non-radiative transitions through collisions with other molecules. The overall rate of quenching \(Q_{21}(s^{-1})\) induced by collisions with all other species ‘j’ in the system is express as

\[
Q = \frac{P}{kT} \sum_j x_j v_j \sigma_{Q,j}
\]  

(2.11)

Rotational and vibrational energy transfer maintain the molecule in the upper electronic state, thereby retaining the possibility of fluorescence while quenching forces the molecule back down to the lower electronic state. Thus, quenching is a directly competing process to LIF. Also, from Eq. 2.11, it can be clearly seen that quenching is directly proportional to the pressure \(P\) and inversely proportional to temperature \(T\). Hence, the impact of quenching is significant in high pressure applications.

### 2.2.3. Other Energy Transfer Processes

In addition to the energy transfer processes described above, other energy transfer mechanisms can play significant roles in the analysis of LIF.
2.2.3.1. Predissociation

Predissociation is the process by which a molecule is excited into a rovibrational level of bonding state above the dissociation energy or onto a potential surface crossing to an anti-bonding state which results in the dissociation of the molecule as shown in Figure 2.4. The predissociation rate constant $P_{pd}$ is an intrinsic molecular property and specific to an excitation transition.

![Potential curves for ground, excited, and predissociative states of OH.](image-url)
2.2.3.2. Photoionization or photodissociation

Photoionization or photodissociation occurs when a molecule in the excited upper energy state absorbs additional photons to excite an electronic energy level to an unbound state, leaving the molecule ionized. The photo ionization rate constant $W_{2i}$ for energy level 2 can be expressed as,

$$W_{2i} = \frac{\sigma_{2i} I_i}{h\nu_i}$$

(2.12)

Where $\sigma_{2i}$ (cm$^2$) is the photoionization cross section from energy level 2 and $I_i$ is the irradiance of the photoionizing laser of frequency $\nu_i$.

2.2.3.3. Intersystem crossing

Intersystem crossing is a non-radiative process which involves the transition between two different electronic states having different spin multiplicities. As such a molecule from a single state can non-radiatively cross to a triplet state and vice versa. Intersystem crossing occurring between two overlapped potential surfaces can influence the dynamics of LIF.

2.2.3.4. Phosphorescence

Phosphorescence is the process in which the absorbed energy is slowly dissipated in the form of radiation. This process is relatively slow, because it is kinetically un-favored, since it involves transition that are normally forbidden.

Each one of the processes described earlier has an associated time scale which is shown in the Jablonski energy diagram in Figure 2.5.
2.2.4. LIF Equation Based on Two Level Model

The basic physics of excitation and de-excitation processes involved in LIF can be easily understood using a two-level model. The steady state (non-transient) rate analysis of the two-level model yields the LIF equation, which captures the key physics involved in the LIF process. The actual physics and distribution of energy levels for diatomic molecules are more accurately described by a quantum-mechanical density-matrix approach [29, 30]. However, the steady state rate analysis is sufficient to reflect most of the basic concepts involved with the added benefit of mathematical simplicity. In theory, this type of simplified modeling is appropriate for atomic
species and molecular systems with fully equilibrated or fully frozen rotational level manifolds. In practice, such models can be effectively used as a first approximation in actual LIF experiments. The basic transitions involved in the two-level model are shown in Figure 2.1. The stimulated absorption and stimulated emission rates are denoted by $W_{12}$ and $W_{21}$ respectively. The quenching and spontaneous emission rates are denoted by $Q_{21}$ and $A_{21}$ respectively.

The steady state rate analysis assumes a constant laser irradiance intensity $I_v^0$, which forces $W_{12}$ and $W_{21}$ to be constants based on Eq. 2.1 and Eq. 2.4 respectively. Now, based on steady state rate analysis of the two-level system, the rate of change of population of molecules for energy state 1 and 2 is given by,

$$\dot{n}_i = \frac{dn_i}{dt} = -n_i W_{12} + n_2 (W_{21} + Q_{21} + A_{21})$$

(2.13)

$$\dot{n}_2 = \frac{dn_2}{dt} = n_1 W_{12} - n_2 (W_{21} + Q_{21} + A_{21} + W_{2i} + P_{PD} + I_{IC})$$

Where $\dot{n}_i$ represents the rate of change of population in energy state ‘$i$’. Now, once the population reach an excites state through stimulated absorption ($W_{12}$), molecules can leave energy level 2 through spontaneous emission ($A_{21}$) or by the competing mechanisms of stimulated emission ($W_{2i}$), collisional quenching ($Q_{21}$), predissociation ($P$), photoionization ($W_{2i}$) and intersystem crossing ($I_{IC}$).

Now, typically photoionization only occurs by design and most excited state are not predissociative. If predissociation, photoionization and intersystem crossing are neglected, then there is only population exchange between states 1 and 2. And if the initial excited population is negligible, the total number density ($n^0$) is given by
\[ n^0 = n_1 + n_2 = n_1^0 \]  \hspace{1cm} (2.14)

Where \( n_1 \) and \( n_2 \) represent the population number densities of energy states 1 and 2 at any given time respectively and \( n_1^0 \) refers to the population of energy state 1 prior to laser excitation. Therefore, since the total population number density cannot change, the rate at which molecules are excited has to be equal to the rate at which molecules relax back to the ground state which results in a closed system described as,

\[ \dot{n}_1 + \dot{n}_2 = 0 \]  \hspace{1cm} (2.15)

Now solving Eq. 2.13, neglecting predissociation (P), photoionization (W_{2i}) and intersystem crossing (I_{ic}) along with the initial condition, at \( t = 0 \), \( n_1 = n_1^0 \), \( n_2 = 0 \), we have,

\[ n_1(t) = n_1^0 \left( \frac{W_{21} + Q_{21} + A_{21} + W_{12}e^{-(W_{21} + W_{21} + Q_{21} + A_{21})t}}{W_{12} + W_{21} + Q_{21} + A_{21}} \right) \]

\[ n_2(t) = n_1^0 W_{12} \left( 1 - e^{-(W_{21} + W_{21} + Q_{21} + A_{21})t} \right) \]  \hspace{1cm} (2.16)

At steady state \( (t = \infty) \), the population density of energy state 1 and 2 can be expressed as,

\[ n_1(t = \infty) = n_1^0 \left( \frac{W_{21} + Q_{21} + A_{21}}{W_{12} + W_{21} + Q_{21} + A_{21}} \right) \]

\[ n_2(t = \infty) = n_1^0 \left( \frac{W_{12}}{W_{12} + W_{21} + Q_{21} + A_{21}} \right) \]  \hspace{1cm} (2.17)

Thus, at steady state, the population density of energy state 2 can be expressed as,

\[ n_2(t = \infty) = (n_2)_{\text{steady state}} = n_1 \left( \frac{W_{12}}{W_{21} + Q_{21} + A_{21}} \right) \]  \hspace{1cm} (2.18)
2.2.5. LIF Regimes

Depending on the incident laser energy $I_v^0$, two limits of operation for LIF can be defined based on the steady state analysis. When the incident laser energy is sufficiently high $I_v^0 \geq I_v^{sat}$, stimulated absorption and emission balance and the population of the two energy states are determined by the ratio of the degeneracies in a two-level model, otherwise known as ‘saturation’. In this limit, the fluorescence signal is independent of laser intensity. The other mode of operation is the ‘weak excitation’, which occurs when the incident laser energy is below the threshold of saturation ($I_v^0 \leq I_v^{sat}$).

2.2.5.1. Weak Excitation (Linear LIF)

When $I_v^0$ is sufficiently below $I_v^{sat}$, the fluorescence signal is linearly proportional to the laser intensity. Generally, this ‘linear LIF’ is preferred since laser non-uniformities and attenuation effects pose practical problem in applying LIF in the saturated regime. In this regime, the induced emission from energy level 2 is much weaker than the sum of collisional and spontaneous decay processes i.e,

$$W_{21} = \frac{B_{21}I_v}{c} \ll A_{21} + Q_{21}$$  \hspace{1cm} (2.19)

Then $n_2 \ll n_1$, and $n_1^0 \approx n_1$. The population fraction in energy level 2 given by Eq. 2.18 can be rewritten as

$$n_2 = n_1^0 \frac{W_{12}}{Q_{21} + A_{21}}$$  \hspace{1cm} (2.20)

It has been experimentally shown that the LIF signal is proportional to the population in energy level 2, the rate of spontaneous emission $A_{21}$ and the solid angle of collection $\Omega$. Based on
this, the fluorescence signal $S_F$ (photons/s) can be written as,

$$S_F = n_2 VA_{21} \frac{\Omega}{4\pi} \quad (2.21)$$

Where $V$ is the volume of excited molecules given by $V = AL$, $A$ is the laser beam focus area, $L$ is the path length of the observed fluorescence and $\frac{\Omega}{4\pi}$ is referred to as the collection fraction. In standard practice, the fluorescence signal is quantified in terms of the LIF intensity $I_{\text{LIF}} (W/cm^2)$. The population density of energy level 2 is replaced by a function of the initial ground state population ($n^0_1$) using Eq. 2.20. For a more accurate model, the two-level model assumption is relaxed, and we assume that the lower energy level has a manifold of rotational levels in thermal equilibrium. The population in these lower energy levels maintain a Boltzmann distribution and the population of a specific rotational level can be written as $n^0_1f_b$, where $f_b$ represents the Boltzmann fraction of the individual energy level. Thus, the LIF intensity can be written as,

$$I_{\text{LIF}} = \left(n^0_1 f_b\right) (AL) \left(\frac{B_{21}}{c} \frac{\Gamma^0}{\Gamma} \frac{\Omega}{4\pi}\right) \left(\frac{A_{21}}{Q_{21} + A_{21}}\right) \quad (2.22)$$

Eq. 2.22 is known as the LIF equation and is fundamental for interpreting the LIF signal in experiments and in steady state computational simulations. The term $\frac{A_{21}}{Q_{21} + A_{21}}$ is called the fluorescence yield $\Phi$, which is the ratio of spontaneous emission rate (LIF) to all the excitation state de-excitation rates. It is a direct representation of how much of the excited population will fluoresce. Typically, the fluorescence yield is much less than 1. For other molecules where predissociation and intersystem crossing are large, the fluorescence yield can be expressed as
\[
\Phi = \frac{A_{21}}{Q_{21} + A_{21} + W_{2i} + P_{PD} + I_{IC}}
\]  

(2.23)

Where \( W_{2i} \), \( I_{IC} \) and \( P_{PD} \) are the rates of photoionization, intersystem crossing and predissociation respectively. Quenching is the dominant relaxation pathway for molecules that have absorbed laser energy. To make quantitative measurements in the linear regime, the quenching rate must be accounted for. This is a challenging task as the quenching rate constant is based on the energy exchange between colliding molecules. It is a function of both gas mixture composition and temperature and these two parameters are not trivial to measure and naturally have steep gradients in a reacting flow. For a laminar flame with well understood chemistry, quenching corrections can be determined accurately. However, other avenues that circumvent quenching rate corrections are usually preferred for unsteady and turbulent flames.

### 2.2.5.2. Saturation LIF

The difficulties in quantifying the quenching rate can be avoided by forcing a competing de-excitation process to occur at a rate sufficiently high to make the quenching rate negligible. Saturated LIF is a common strategy for avoiding the need for quenching corrections. This is opposite the linear regime, so that \( I_{sat}^0 \geq I_{sat}^{st} \). Under this condition, absorption and stimulated emission dominate population exchange. In a simple two-level model, saturation occurs when the induced emission is much larger than the collisional and spontaneous emission as given by

\[
W_{21} = \frac{B_{21} I_{sat}^0}{c} \gg A_{21} + Q_{21}
\]  

(2.24)

The population fraction in energy level 2 given by Eq. 2.18 can be rewritten as

\[
n_2 = n_1 \frac{W_{12}}{W_{21}} = n_1 \frac{B_{12}}{B_{21}} = n_1 \frac{g_2}{g_1}
\]  

(2.25)
Now using Eq. 2.14 and Eq. 2.25, the population fraction in energy level 1 can be expressed as

\[ n_1 = n^0_1 \left( \frac{g_1}{g_1 + g_2} \right) \]

(2.26)

Based on Eq. 2.23 and Eq. 2.24, the LIF intensity \( I_{\text{LIF}} \) for the saturation regime can be written as,

\[ I_{\text{LIF}} = \left( n^0_1 f_b \right) \left( \frac{g_1}{g_1 + g_2} \right) (AL) \left( \frac{\Omega}{4\pi} \right) A_{21} \]

(2.27)

Thus, from Eq. 2.27, it can be clearly seen that the fluorescence signal is independent of both laser irradiance and the quenching rate. In the saturation regime, the rates of laser absorption and stimulated emission become so large that they dominate the state to state energy transfer into and out of the directly pumped levels. Since quenching does not influence the fluorescence intensity, LIF in the saturation regime can potentially be less complicated with the added benefit of maximizing the fluorescence yield and therefore increasing detection sensitivity. However, complete saturation is difficult to achieve due mainly to the specific wavelength region of the absorption or the magnitude of the saturation intensity. Furthermore, the decrease of energy in outer edges of the laser beam and the temporal deviation of pulse to pulse fluctuations introduce further complications. Therefore, application of LIF techniques in the linear regime using laser energy below the saturation is generally recommended and constitutes the basic approach in this thesis. The criterion for the saturation spectral irradiance \( I^\text{sat}_\nu \) based on the two-level analysis can be expressed as

\[ I^\text{sat}_\nu = \left( \frac{A_{21} + Q_{21}}{B_{12} + B_{21}} \right) c \]

(2.28)
Chapter 3. Decomposition Techniques for Flame Dynamics

3.1. The Need for Decomposition Techniques to Understand Flame Dynamics

Advanced combustion systems designed to operate under lean, premixed conditions have great potential to substantially curtail emission and satisfy regulatory requirements [31]. Such systems, however, are more susceptible to combustion instability, which in turn can negatively impact performance and durability [32]. Understanding and controlling the onset and propagation of combustion instability is therefore critical to the development of clean and efficient combustion systems [33]. Numerous experimental [14, 34-37] and theoretical [38, 39] studies have made significant progress in understanding the key dynamics that affect combustion instability such as oscillatory heat release [40], acoustics [41], and equivalence ratio fluctuations [42]. In spite of these contributions, limited experimental data coupled with inadequate analysis techniques [43] have inhibited detailed combustion instability model development until the past few years. Coherent structures across multiple time and length scales can drive combustion instability. However, resolving both temporal and spatial structures from image-based experimental data can be very challenging. Thus, understanding flame dynamics remains a demanding task and the difficulties often lie in the chaotic and non-linear behavior of the system of interest.

Combustion instabilities arise due to the coupling of specific acoustic modes in the combustor configuration with the dynamics of the combustion heat release. When these phenomena occur in phase for a specific acoustic mode frequency, there is usually a constructive amplification of the mode, resulting in the growth of the mode and instability. Understanding the coupling of these phenomena is not trivial, especially for complex configurations involving
multiple physical interactions as it requires capturing a multi-scale phenomenon involving a large number of parameters [44]. These parameters can be captured as an ensemble of snapshots in a frozen state across 2D planes by advanced laser diagnostic techniques [45]. The physical variables captured could be major species or radical concentration as well as velocity vectors.

Planar measurements like OH-PLIF result in ensembles of 2D planes with each snapshot providing some insight about a typical turbulent flow field provided that the coherent structures are much stronger than small scales fluctuations. Instantaneous snapshots are often shown in the literature to illustrate, albeit qualitatively, the spatial arrangement of a dominant flow phenomenon. Such snapshots are clearly valuable for qualitative validation but are insufficient for quantitative comparisons to similar snapshot from a different system and can obscure less-dominant flow features that may be of importance, but which are overshadowed by the more-dominant structures. Hence, there is no significance associated with a particular snapshot and hence, it is customary to handle such data-sets by averaging the images and extracting ensemble averaged and root-mean-square (RMS) fields. The drawback of doing so is that a large fraction of the information contained in the snapshots is lost, especially the flame dynamics and particularly the flame interaction with large scale coherent structures. Hence it is safe to say that in terms of isolating the underlying fluid mechanisms, these statistical techniques are far inferior to the model-based techniques applied to numerically generated data.

Traditional data processing in combustion instability analysis involves the bandpass filtering of the signals around a frequency of interest. The frequencies of interest can be determined from a power spectral density (PSD) analysis of the signal. In this way, the correlation between acoustics and combustion can be explored within a certain frequency range. However, filtered results are sensitive to several factors like filter type, bandwidth, and sample quality.
While successive moments of the time statistics do represent a natural and intuitively meaningful technique for the validation of most quantities for engineering purposes and although power spectral density does allow for detailed analysis of the frequency content of a given time series, neither technique can provide an adequate description of the spatiotemporal nature of the system’s dynamics. One approach that has been used to attempt to solve this problem is phase-locked sampling, which has been successfully used to study flames with periodic instabilities [46]. This technique, however, requires an external forcing source that can be used to trigger the measurement, which prevents it from being applied to self-excited flames.

Hence to extract information regarding the flame dynamics from an ensemble of data, one needs a numerical microscope, in other words to perform a distillation enabling to sort out the information and only retain the fraction of interest in the form of a statistical quantity. The extraction of dynamical features by a global stability analysis has remained a tool that is nearly exclusively applied to numerical simulations. This is because the respective algorithms require the system matrix of the underlying flow in order to build a sequence of (artificial) flow fields upon which the convergence of the numerical method relies. In physical experiments this system matrix is not available, and whereas a subroutine call of the right-hand side is straightforward to accommodate in a numerical simulation, the same is not true for experiments. Rather, in experimental set-ups, the only input that is readily available are the flow fields themselves, either in form of particle image velocimetry (PIV) measurements or in form of visualizations of a passive tracer. Therefore, if coherent structures are to be identified from experimental data, algorithms need to be designed that rely on these measurements only. Thus, the decomposition of experimental measurements into temporally and spatially coherent structures is an important tool in the arsenal of any experimentalists, since the breakdown of a flow field into organized,
connected and large-scale fluid elements allows a more thorough analysis of complex fluid processes.

Few such techniques are available to do so including conditional averaging, wavelet analysis [47], Proper Orthogonal Decomposition (POD) [48, 49] and Dynamic Mode Decomposition (DMD) [43, 50]. Decomposition techniques, such as POD and DMD, allow many of these shortcomings to be overcome and allow for reproducible, statistically converged representations of the key flow features and their various magnitudes, time scales, and spatial distributions. These techniques use instantaneous realizations of the system with sufficient spatial and temporal range and resolution such that the relevant flow structures are sufficiently represented. Ensembles of these instantaneous snapshots can be decomposed in space and/or time to separate the various coherent modes from each other in a way that allows for a statistically representative, reproducible, and quantitative description of each. Because such decompositions rely only on the input data and require no underlying model or approximation, they are said to be agnostic to the source of the data, that is, the decomposition technique makes no distinction between data generated by a physical measurement or a numerical simulation. This makes these techniques well suited for the experimentally measured parameters presented in this work.

One primary advantage of the decomposition techniques over filtering is that they work with the entire data set with minimal information loss. Moreover, unlike filtering techniques, POD and DMD do not require prior knowledge or pre-analysis of the data to obtain the dominant frequencies. Another motivation is that decomposition techniques are capable of extracting dynamically significant structures from the flow field of interest. Each decomposed mode can be represented in terms of a spatial response and a temporal response, which provides detailed insight into the dynamics of acoustics and combustion. In our work, we seek to extend our understanding
of the combustion dynamics by employing advanced algorithm-based decomposition techniques to high fidelity experimental data. Specifically, we focus on the fundamental interactions between acoustic waves and unsteady combustion heat release and are interested in elucidating these effects at specific frequencies that may contribute to the generation of combustion instabilities.

3.2. Principle of Mathematical Model of Decomposition Techniques

The goal of any decomposition is to approximate a function \( z(x, t) \) over a specified domain as a finite sum of a temporal component \( a_k(t) \) and a spatial component \( \Phi_k(x) \),

\[
z(x, t) = \sum_{k=1}^{N} a_k(t) \Phi_k(x)
\]  

(3.1)

It is assumed that approximation becomes exact as \( N \) approaches infinity. Note that in Eq. 3.1, there is no fundamental difference between \( x \) and \( t \), but \( x \) is usually taken as the spatial coordinate and \( t \) as the temporal coordinate. The representation of Eq. 3.1 is not unique. For example, if the domain (either from experiment or computation) is a bounded interval \( X \) on the real line, then the functions \( \Phi_k(x) \) can be chosen as a Fourier series, Legendre polynomials, Chebyshev polynomials, and so on. Different choices of the space-dependent function \( \Phi_k(x) \) will result in different time-dependent functions \( a_k(t) \). The time-dependent functions can be periodic or nonperiodic, single-frequency dominated, or multifrequency dominated.

In the POD analysis, the spatial functions \( \Phi_k(x) \) are chosen to be orthogonal functions, i.e.,

\[
\int_{X} \Phi_{k_1}(x) \Phi_{k_2}(x) \, dx = \begin{cases} 1 & \text{if } k_1 = k_2 \\ 0 & \text{otherwise} \end{cases}
\]  

(3.2)

The orthogonality of \( \Phi_k \) means that \( a_k \) can be determined using only \( \Phi_k \), as opposed to all the \( \Phi \) functions,
\[ a_k(t) = \int_X z(x,t) \Phi_k(x) dx \] (3.3)

3.3. Proper Orthogonal Decomposition (POD)

Proper orthogonal decomposition (POD), is a data analysis technique originally proposed in 1901 by Karl Pearson [51] and first recognized by Lumley [52] for its value in analyzing turbulent flows. The statistical technique has been applied to data in many different fields that has been applied to a wide range of problems from pollutant dispersion [53] to reduced-order modeling [49] to machine vision [54] to neurology [55]. Owing in part to the diversity of its applications, POD is also referred to by a number of names, including: Principal Component Analysis (PCA), the Karhunen-Loève transformation (KLD), the method of empirical orthogonal functions, Singular Value Decomposition (SVD), eigenvalue decomposition, factor analysis, the Eckart-Young theorem, empirical component analysis, and the Hotelling transform, among others. The method is essentially a pattern recognition technique that seeks to approximate a dataset through a linear combination of a minimum number of orthogonal vectors [48]. While limited in aspects in its ability to discriminate between data classes, POD's proven application in detecting coherent structures in turbulent [48, 49, 56] and reacting flows [44, 57], together with its agnosticism toward the source of data, makes this technique particularly well suited to the comparison of simulated and measured combustion dynamic data.

POD can be used to develop reduced order models and investigate large-scale structures of a combustion system. POD decomposes a set of distributions or functions, the ensemble, into an optimal orthonormal set of eigenfunctions able to represent the distributions of the ensemble. These distributions are represented as a weighted expansion of the eigenfunctions. Subsets of these eigenfunctions can offer highly efficient representations of important variables in combustion.
systems. They are optimal in the sense that they contain the most information relative to any other basis set and allow one to capture the dominant features of a system using the fewest number of basis functions in an expansion. Examination of a few POD eigenfunctions quickly identifies the most important large-scale structures of the flow enabling the researcher to visualize the flow behavior and determine where more detailed investigation is desired.

Furthermore, the potential for a reduced order model utilizing these basis functions can be evaluated from their information content as determined from the associated eigenvalue spectrum derived from POD. The eigenvalue spectrum quantifies the average contribution of each eigenfunction in representing the distributions in the ensemble, and thus its relative importance, to the representation of the system properties. For a reduced order model utilizing an expansion in a set of basis functions, a measure of the potential for modeling is the number of basis functions necessary to capture the flow physics. Because the POD eigenfunctions provide an optimal basis set, the number of POD derived basis functions required will be the lowest of any basis set and POD analysis can be used as a best case to evaluate the potential for modeling.

### 3.3.1. Principle of Proper Orthogonal Decomposition (POD)

Proper Orthogonal Decomposition (POD) is a mathematical formulation used to obtain a modal decomposition of an ensemble series of measurements [48]. Without loss of generality, for the sake of simplicity, let us assume the quantity \( u(x, t) \) represents a scalar field. The POD technique proceeds by decomposing this field into a set of basis functions, namely spatial eigenmodes \( \Phi_n(x) \) and time coefficients \( a_n(t) \) such that any snapshot \( u(x, t) \) taken at time ‘t’ can be reconstructed as:

\[
u(x, t) = a_0\Phi_0(x) + \sum_{n=1}^{\infty} a_n(t) \Phi_n(x) \tag{3.4}\]
The zeroth eigenfunction or Mode 0 represents the mean field while the following modes represent the fluctuations i.e., capture the dynamics. In POD, the extracted basis functions are empirical because the eigenfunctions are computed from the structures in the original fields rather than prescribing them \textit{apriori} as in Fourier decomposition, where sinusoidal modes are chosen independently of the data. The basis in POD are chosen to maximize the quantity \cite{48, 49}:

\[
\frac{\langle (u, \Phi)^2 \rangle}{\norm{\Phi}^2}
\]

(3.5)

Where \( \langle \rceil \) denotes ensemble averaging, \( (a,b) \) is the inner product and \( \norm{\cdot} \) is the norm. It is necessary that any function of interest should be square integrable in the space \( L(\Omega_x) \), where \( \Omega_x \) is the physical domain under consideration i.e., the region of interest. The maximization represented in Eq. 3.5 is described as follows: the POD procedure seeks to decompose the ensemble data of the function \( u(x, t) \) onto a base that would maximize the variation content of the \( N \)-first modes for any integer \( N \) \cite{48, 49}. This maximization problem represented by Eq. 3.5 can be reduced to an eigenvalue problem as shown below.

\[
\langle u(x, t), u^T(x, t) \rangle \Phi(x) = \lambda \Phi(x)
\]

(3.6)

Where \( \langle u(x, t), u^T(x, t) \rangle \) is the auto correlation tensor and superscript \( T \) denotes the transpose of the field. The eigenvalues, \( \lambda \) are arranged in the descending order of magnitude \( \lambda_n > \lambda_{n+1} \) and their corresponding eigenfunctions \( \Phi(x) \) (modes) are normalized such that \( \langle \Phi_n, \Phi_n \rangle = 1 \). The eigenvalue \( \lambda_i \) characterizes the variance fraction content of the mode ‘i’ \cite{58} and the POD modes are optimal in capturing, on average the greatest possible fraction of variance for a projection onto a given number of modes \cite{48, 49, 54}. Hence the optimization of basis in \( L^2 \) space separates and ranks the eigenmodes according to the energy content \cite{58}. It is to be noted that when POD is applied to scalar fields, the square of the intensity has no physical meaning, but
the basis functions and coefficients are created just the same, so scalar POD can be used for extracting useful pattern features from scalar intensity [54].

POD analysis can be carried out using either the classical method [59] or the equivalent method of snapshots [60]. In the classical method, consider a physical variable \( u(x, t) \) with a finite number of field realizations \( N \), with each realization containing \( M \) grid points such that each realization has one value in each of the \( M \) grid points as shown below.

\[
\begin{bmatrix}
  u_{m1}^{(k)} \\
  u_{m2}^{(k)} \\
  \vdots \\
  u_{mM}^{(k)}
\end{bmatrix}
\quad \text{with } k = 1, 2, 3, \ldots, N
\]

(3.7)

\[
\mathbf{u}^{(k)}(x_m, t_k) = \mathbf{u}_m^k
\]

(3.8)

The discretization procedure leads to solving an eigenvalue problem for a matrix of dimensions \( M \times M \). For simplicity, let us consider that a typical OH concentration field obtained by means of PLIF has dimensions of 500 x 1024 pixels, resulting in number of spatial grid points, \( M = 500 \times 1024 = 512000 \). Thus, a direct resolution of the eigenvalue problem is computationally extensive and therefore not feasible. Hence, the eigenvalue problem at hand is transformed into an equivalent but less computationally extensive one as suggested by Sirovich [60].

**3.3.2. Singular Value Decomposition (SVD) in Proper Orthogonal Decomposition (POD) Analysis**

In practice, whole data sets or snapshots are arranged into a POD matrix first, for example, with each column containing the temporal data and each row containing the spatial data. Thus, if there are \( M \) rows of spatial data and \( N \) columns of temporal data, the POD matrix will be of size \( M \times N \).
Once we obtain the POD matrix \( A \), the SVD of \( A \) is

\[
A = U \Sigma V^T
\]  

(3.9)

Where \( U \) is an \( M \times M \) orthogonal matrix, \( V \) is an \( N \times N \) orthogonal matrix and \( \Sigma \) is an \( M \times N \) matrix with all elements zero except along the diagonal. The diagonal elements of \( \Sigma \) consist of \( N_p = \min(M, N) \) non-negative numbers \( \lambda_i \), which are the singular values of \( A \). The singular values are unique and are arranged in decreasing magnitude,

\[
\lambda_i > \lambda_{i+1}
\]  

(3.10)

In Eq. 3.9, let \( Q = U \Sigma \). Then \( A = QV^T \). The column matrix \( q_k \) can be interpreted as the spatial representation of the \( k \)th POD mode, while the column matrix \( v_k \) represents the temporal evolution of the \( k \)th POD mode. If the spatial mode \( q_k \) is obtained such that \( Q = U \Sigma \), the temporal mode \( v_k \) contains all normalized numbers and vice versa. Eq. 3.9 can also be rewritten as:

\[
A = u_k \lambda_k v_k^T
\]  

(3.11)

In Eq. 3.11, \( u_k \) is the \( k \)th column of \( U \) and \( \lambda_k \) corresponds to the \( k \)th singular value of matrix \( A \). The \( \lambda_k \) is defined as the mode power of the \( k \)th POD mode, and it indicates mathematically how representative the decomposed modes are compared with raw data, and physically it represents the level of information that one can extract in terms of the decomposed modes from the original data.

3.3.3. Method of Snapshots

The method of snapshots uses the linear relation between the eigenmodes and the ensemble of instantaneous samples (snapshots) to transform the \( M \times M \) eigenvalue problem and recast it into a lighter eigenvalue problem of size \( N \times N \), where \( N \) is the number of snapshots (typically,
M >> N). The modified eigenvalue problem is as follows:

\[
\begin{bmatrix}
  b^{(1,1)} & \cdots & b^{(1,N)} \\
  \vdots & \ddots & \vdots \\
  b^{(N,1)} & \cdots & b^{(N,N)}
\end{bmatrix}
\begin{bmatrix}
  c^{(1)} \\
  \vdots \\
  c^{(N)}
\end{bmatrix}
= \lambda
\begin{bmatrix}
  c^{(1)} \\
  \vdots \\
  c^{(N)}
\end{bmatrix}
\]  

(3.12)

Where \( b \) is the auto-correlation tensor computed using the data from the N snapshots shown below:

\[
b(N \times N) = \begin{bmatrix}
  u_1^1 & \cdots & u_M^1 \\
  \vdots & \ddots & \vdots \\
  u_N^1 & \cdots & u_M^N
\end{bmatrix}
\]  

(3.13)

Now Eq. 3.12 is a reduced eigenvalue problem of size N x N. The corresponding solution consists of N eigenvectors \( c_n \) along with N eigenvalues \( \lambda_n \). The POD modes are obtained as follows:

\[
\Phi_n(x) = \frac{1}{\lambda_n N} \sum_{k=0}^{N} c_n^{(k)} u^{(k)}(x)
\]  

(3.14)

The POD modes are a weighted average of the snapshots. The time coefficients of the POD modes can be computed by the projection of individual snapshots onto the POD modes as shown below:

\[
a^k(t_k) = \begin{bmatrix}
  a_1^k \\
  \vdots \\
  a_N^k
\end{bmatrix}
= [\Phi_1 \ \cdots \ \Phi_n \ \cdots \ \Phi_N]^T u^{(k)}
\]  

(3.15)

The time coefficient of the \( n^{\text{th}} \) mode and the \( k^{\text{th}} \) snapshot is computed as:

\[
a_n^k = \sum_{m=1}^{M} \Phi_n(x_m) u_m^k
\]  

(3.16)

Given that the quantity \( u(x, t) \) was sampled at a fixed time interval, which is otherwise not
necessary, the time coefficients $a_k$ can easily be used to extract the frequency content of the modes using a Fast Fourier Transform (FFT). A better frequency analysis may even be obtained for the time coefficients compared to using point data in the flow field, since the decomposition can work as a filter. The POD modes, in contrast to DMD modes, may contain multiple frequencies.

### 3.4. Dynamic Mode Decomposition (DMD)

Dynamic mode decomposition (DMD) is a recent data-based decomposition technique developed by Peter Schmid [43, 50] for extracting dynamic information from a series of snapshots sampled at a fixed time interval. The extracted flow structures are called as dynamic modes. The DMD algorithm is based on a variant of the Arnoldi algorithm suggested by Ruhe [61] to compute the DMD modes or approximate the Koopman modes for any field variable. The Koopman method decomposes the flow field into a series of so-called Koopman modes in which each mode is characterized by a frequency and a growth rate. These modes are determined from spectral analysis of the Koopman operator and separated by the frequency corresponding to each flow behavior. The dynamic information extracted from the snapshots through DMD is useful in identifying the coherent features of the fluid flow which is important in understanding fluid dynamical and transport processes. Dynamic modes are ranked according to their coherency by displaying the most coherent mode first. DMD can be applied to reactive flows (combustion chambers) to understand the underlying transition and instability mechanism. DMD takes in a time series of data and computes a set of dynamic modes, each of which are associated with a complex eigenvalue. The real part of the complex eigenvalue represents a growth or decay factor for the dynamic mode, while, the imaginary part of the complex eigenvalue gives the oscillation frequency of the dynamic mode.
DMD provides a means to decompose time-resolved data into modes, with each mode having a single characteristic frequency of oscillation with a growth or decay rate. DMD is based on the eigen decomposition of a best-fit linear operator that approximates the dynamics present in the data. As DMD is closely related to spectral analysis of Koopman operator [62], the DMD model analyzes a nonlinear system as a linear combination of the modes whose dynamics are governed by the eigenvalues. The Koopman operator is a linear but infinite-dimensional operator whose modes and eigenvalues capture the evolution of nonlinear dynamical systems [63]. Therefore, a nonlinear system could be described by superposition of dynamic modes whose dynamics are governed by eigenvalues.

Hence, DMD differs from dimensionality reduction methods such as POD, which although computes orthogonal modes, lacks temporal representation. POD modes are separated out based on their energy content while, DMD modes are separated out based on their frequency and rate of growth or decay. While POD is a statistical method, DMD is a stability analysis tool with connections to linear global modes. POD’s premise, rests on a hierarchical ranking of coherent structures based on their energy content. Mathematically, it uses an eigenvalue decomposition of a (commonly) time-averaged spatial correlation tensor computed from the snapshots. The reliance on second order flow statistics, however, does not directly capture the dynamics of the underlying coherent structures and thus limits the information that can be gained about fundamental dominant processes.

Two major drawbacks that are tacitly acknowledged by employing this method are associated with this technique. They are as follows:

1. The energy may not in all circumstances be the correct measure to rank the flow structures
2. Due to the choice of second-order statistics as a basis for the decomposition, valuable phase information is lost.

The first shortcoming has been widely recognized, and an explanation for the existence of dynamically highly relevant but zero-energy modes has been presented by Noack et al [64]. Choosing weight functions that put more emphasis on specific components of the flow field or more active regions of the flow can ameliorate the focus on the total kinetic perturbation energy. The second shortcoming is more difficult to overcome. The averaging process that produces second order statistics causes the loss of information that might be important when classifying the dynamic processes contained in the snapshots.

In many ways, DMD may be viewed as combining favorable aspects of both the POD and the discrete Fourier transform (DFT) [65, 66], resulting in spatio-temporal coherent structures identified purely from data. In the case of a linearized flow (i.e. a flow of small perturbation about a steady base flow), the extracted DMD modes are equivalent to the result of a global stability analysis; for a nonlinear flow, the results produce structures of a linear tangent approximation to the underlying flow and describe fluid elements that express the dominant dynamic behavior captured in the data sequence. Because DMD is rooted firmly in linear algebra, the method is highly extensible, spurring considerable algorithmic developments. Moreover, as DMD is purely a data-driven algorithm without the requirement for governing equations, it has been widely applied beyond fluid dynamics: in finance [67], video processing [68], epidemiology [69], robotics [70], and neuroscience [71]. As with many modal decomposition techniques, DMD is most often applied as a diagnostic to provide physical insight into a system. The use of DMD for future-state prediction, estimation, and control is generally more challenging and less common in the literature.

The main advantage of using this decomposition technique is that it is free from the system
matrix of the underlying flow and is solely dependent on the snapshots of the flow. In other decomposition methods, the averaging process to produce second order statistics causes loss of information that might be contained in the snapshots and important in the classification of the dynamic process. But in DMD, the coherent structures accurately describe the motion of the flow and is well suited for the analysis of systems with a large spectrum of time and length.

3.4.1. Arnoldi Algorithm in DMD Analysis

DMD analysis decomposes the data by frequency. To obtain single frequency dynamic modes, suppose the data set is represented as a sequence of snapshots,

\[ V_1^N = \{v_1, v_2, \ldots, v_N \} \quad (3.17) \]

Where \( v_i \) stands for the \( i \)th snapshot of the data. DMD assumes that a linear map exists between a snapshot and the next snapshot in the sequence; thus, if \( A \) represents the linear map, \( v_{i+1} = Av_i \). Therefore,

\[ V_1^N = \{v_i, Av_i, A^2v_i, \ldots, A^{N-1}v_i \} \quad (3.18) \]

Another assumption is that there exists a specific number \( N \), beyond which the vector \( v_N \) can be expressed as linear combination of the previous vectors,

\[ v_N = a_1v_1 + a_2v_2 + \ldots + a_{N-1}v_{N-1} \quad (3.19) \]

\[ v_i = V_1^{N-1}a + r \]

Hence,

\[ AV_1^{N-1} = V_2^N = V_1^{N-1}S + r_{N-1}^T \quad (3.20) \]

Where \( S \) is the companion type matrix,
Applying the eigenvalue decomposition for matrix $S$,

$$ S = T \Lambda T^{-1} \quad \text{(3.22)} $$

Where matrix $T$ is the eigenvector matrix of $S$. When sufficient number of snapshots are used, the eigenvalues of $S$ are representative of the eigenvalues of $A$, which contain the time evolution information of the flow field. Similarly, the $k^{th}$ dynamic mode corresponding to the frequency response can be constructed as,

$$ \Psi_k = V_i^{N-1} y_k \quad \text{(3.23)} $$

Where $y_k$ is the $k^{th}$ eigenvector of matrix $S$ in Eq. 3.22. The original data set can be decomposed into the form in Eq. 3.1,

$$ V_i^{N-1} = \sum_k \Psi_k y_k^T \quad \text{(3.24)} $$

Where $\Psi_k$ contains the dynamic spatial information and $y_k$ contains the temporal evolitional information.

### 3.4.2. Practical Implementations of DMD Analysis

As pointed out by Schmid [50], even though the previous Arnoldi algorithm is mathematically correct, in practical implementations, the companion matrix $S$ can be ill conditioned, especially when the datasets are contaminated with noise. Therefore, a more robust implementation is used for DMD analysis in the current studies. First, the snapshots are arranged as in Eq. 3.17 to get matrix $V_i^{N-1}$ and then SVD is applied,
\( \mathbf{V}^{N-1}_1 = \mathbf{W} \Sigma \mathbf{U}^T \) \hspace{1cm} (3.25)

Where matrices \( \mathbf{W} \) and \( \mathbf{U} \) are both orthogonal and contain spatial and temporal information, respectively. Substituting Eq. 3.25 into Eq. 3.20:

\[
\mathbf{A} \mathbf{V}^{N-1}_1 = \mathbf{V}^N_2 = \mathbf{A} \mathbf{W} \Sigma \mathbf{U}^T
\]

\[
\mathbf{A} \mathbf{W} = \mathbf{V}^N_2 \mathbf{U} \Sigma^{-1}
\]

\[
\mathbf{W}^T \mathbf{A} \mathbf{W} = \mathbf{W}^T \mathbf{V}^N_2 \mathbf{U} \Sigma^{-1} = \tilde{\mathbf{S}}
\] \hspace{1cm} (3.26)

It should also be noted from Eq. 3.20 that \( \mathbf{A} \mathbf{V}^{N-1}_1 = \mathbf{V}^N_2 = \mathbf{V}^{N-1}_1 \mathbf{S} \), so

\[
\mathbf{W}^T \mathbf{V}^{N-1}_1 \mathbf{S} \mathbf{U} \Sigma^{-1} = \tilde{\mathbf{S}}
\]

\[
\Rightarrow \mathbf{W}^T (\mathbf{W} \Sigma \mathbf{U}^T) \mathbf{S} \mathbf{U} \Sigma^{-1} = \tilde{\mathbf{S}}
\]

\[
\Rightarrow (\Sigma \mathbf{U}^T) \mathbf{S} (\Sigma \mathbf{U}^T)^{-1} = \tilde{\mathbf{S}}
\] \hspace{1cm} (3.27)

And it can be seen that \( \tilde{\mathbf{S}} \) is a similarity matrix to \( \mathbf{S} \) and they share the same eigenvalues. Therefore, eigenvalue decomposition is applied to \( \tilde{\mathbf{S}} \) instead of \( \mathbf{S} \),

\[
\tilde{\mathbf{S}} = \mathbf{T} \Lambda \mathbf{T}^{-1}
\] \hspace{1cm} (3.28)

In this way, the \( k^{\text{th}} \) spatial mode can be calculated as,

\[
\mathbf{\Psi}_k = \mathbf{W} \mathbf{y}_k
\] \hspace{1cm} (3.29)

Where \( \mathbf{y}_k \) is the \( k^{\text{th}} \) eigenvector of matrix \( \tilde{\mathbf{S}} \) in Eq. 3.28 and the temporal modes can be computed as,

\[
\mathbf{y}_k = \mathbf{U} \Sigma^{-1} \mathbf{y}_k
\] \hspace{1cm} (3.30)

Each DMD mode corresponds to a single frequency \( f_k \),

\[
2 \pi f_k = \log(\lambda_{k,i}) / \Delta t
\] \hspace{1cm} (3.31)
Where $\lambda_{k,i}$ is the imaginary part of the $k^{th}$ eigenvalue of matrix $\tilde{S}$ in Eq. 3.28 and $\Delta t$ is the time step in between two snapshots. Unlike POD, which produces modes in all real numbers, DMD modes consist of complex numbers, and usually each individual frequency is related to two modes, complex conjugate to each other. Therefore, the responses $V_k$ corresponding to the $k^{th}$ frequency can be reconstructed by taking the real part of DMD spatial and temporal modes’ product,

$$V_k = \text{Re}\{\Psi_k \Psi_k^T\}$$ (3.32)

### 3.5. POD and DMD Algorithm Validation

#### 3.5.1. Numerical Validation

An image $(q)$ of numeric functions is constructed to validate the POD and DMD results [72, 73]. The image superimposes a stationary field $(q_0)$ and three dominant dynamic structures $(q_1, q_2$ and $q_3)$ with varying characteristic temporal and spatial frequencies.

$$q = q_0 + q_1 + q_2 + q_3$$ (3.33)

$$q_0(x, y, t) = \exp\left(-\frac{y^2}{0.7}\right)$$ (3.34)

$$q_n(x, y, t) = \alpha_n(t) \sum_{m=-\infty}^{m=\infty} (-1)^m \exp\left[-\left(\frac{x-\beta_n - \gamma_n t}{d_n}\right)^2 + y^2\right]$$ (3.35)

$$f_n = \frac{\gamma_n}{2\beta_n}$$ (3.36)

Here, $d_n = a_n x + b_n$ is the diameter of the structure, $a_n$ and $b_n$ are constants, $\beta_n$ is the wavelength factor defined as the distance between two neighboring structures, $\gamma_n$ is the convection velocity of the structure in the streamwise direction and $\alpha_n(t)$ is the growth or decay factor. The dominant frequencies $f_n$ for $q_1, q_2$ and $q_3$ are 0.5 Hz, 1.64 Hz and 4.0 Hz, respectively. The value
of the coefficient $\alpha_n(t)$ determines variations in the convecting structures. $\alpha_n(t) = 1$ indicates a constant structure without growth or decay ($q_0$ and $q_1$ - no growth or decay), $\alpha_n(t) < 1$ and $\alpha_n(t) > 1$ correspond to decaying ($q_2$) and growing ($q_3$) structures, respectively. The numeric image with multi-dominant structure pattern is composed of three different dynamical structures ($n = 1, 2$ and $3$), each of which evolves with a certain velocity at different frequencies. The parameters for each function are listed in Table 3.1. The long-term behavior of $q_2$ and $q_3$ reach

Figure 3.1. Numerical images used for POD and DMD algorithm validation.
an asymptotically stable limit despite non-zero growth factors. Hence, stability analysis based on growth factors must be taken within the context of measurement duration and not for the lifetime of the system. 2000 superimposed images were generated at every 10 ms intervals. Instances of the stationary field \( q_0 \), the three different structures \( q_l \) through \( q_3 \) and the combined structure image \( q \) are shown in Figure 3.1.

<table>
<thead>
<tr>
<th>n</th>
<th>a_n</th>
<th>b_n</th>
<th>( \beta_n )</th>
<th>( \gamma_n )</th>
<th>f (Hz)</th>
<th>( \alpha_n(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0300</td>
<td>0.050</td>
<td>0.80</td>
<td>0.8</td>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.0150</td>
<td>0.035</td>
<td>0.55</td>
<td>1.8</td>
<td>1.64</td>
<td>( e^{-t/30} ) - 0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.0075</td>
<td>0.020</td>
<td>0.30</td>
<td>2.4</td>
<td>4.00</td>
<td>1 - ( e^{-t/20} ) + 0.2</td>
</tr>
</tbody>
</table>

*Table 3.1. Parameters for the numerical image.*

### 3.5.1.1. POD Analysis on the Numerical Image

POD analysis was carried out on these images, based on which the normalized eigenvalue and its cumulative distribution are shown in Figure 3.2. As the mode number increases, the eigenvalue rapidly decreases for the first 7 modes and then collapses to a negligible value. Correspondingly, the cumulative energy of the first 7 moves reaches 99.7% of the total and from there one the convergence speed considerably slows down as further eigenmodes are included. Thus, the first seven modes can give an accurate description of the stationary field \( q_0 \) and the three different structures \( q_l \) through \( q_3 \) that are present in the combined pattern.

The spatial structure of the dominant POD modes and the reconstructed snapshot of the numerical image based on the dominant POD modes are shown in Figure 3.3. Comparing the dominant POD modes to the structures which make up the numerical image shown in Figure 3.1, the POD modes shown generally exhibit structures corresponding to the stationary field \( q_0 \) and
the three different structures \((q_i \text{ through } q_j)\). However, a close comparative analysis of Figure 3.1 and Figure 3.3, shows significant distortions in the POD modes. These can be attributed to the contamination of the individual POD modes by other uncorrelated structures. This contamination due to the uncorrelated structures can be clearly seen by performing a Fast Fourier Transform (FFT) on the time coefficients of the dominant POD modes. Based on this, the resulting power spectra for the dominant POD modes i.e, POD Mode 0, 1, 3 and 5 are shown in Figure 3.4. The individual power spectra display a dominant peak corresponding to the frequency of the original structure that the POD mode represents. However, the individual power spectra also clearly show leakage of spectral information from the adjoining modes, which in turn shows up as distortions due to the presence of uncorrelated structures in the POD modes.

\[
\begin{align*}
\text{Normalized eigen value } \lambda_k &\approx 0.8 \\
\text{Cumulative eigen value } \Sigma \lambda_k &\approx 1.
\end{align*}
\]

*Figure 3.2. Normalized eigenvalue and its cumulative distribution for POD modes extracted from the numerical image.*
Figure 3.3. Spatial structure of the dominant POD modes which correspond to the coherent structures of the numerical image along with the reconstructed snapshot of the numerical image using the dominant POD modes.

Thus, for the multi-dominant structure pattern, although the general patterns of the POD modes match the corresponding predefined structures, the POD algorithm fails to cleanly and separately extract the desired dominant structures, which are unfortunately contaminated by the other uncorrelated structures. Again, this is closely associated with the spatial orthogonality.
enforced in the POD algorithm which does not enforce any constraint on the temporal (spectral) information.

![Power spectrum of POD mode time coefficients for the numerical image.](image)

**Figure 3.4.** Power spectrum of POD mode time coefficients for the numerical image.

### 3.5.1.2. DMD Analysis on the Numerical Image

Based on the DMD analysis carried out on these images, the real part of the first four temporal (spatial patterns) DMD modes are shown in Figure 3.5. The corresponding imaginary part of the DMD modes is phase-shifted by 90°. The extracted DMD modes are in excellent agreement with the numeric images as shown in Figure 3.1. As opposed to the POD analysis which showed obvious discrepancies between the spatial features of the POD modes shown in Figure 3.3
Figure 3.5. Spatial structure of dominant DMD modes which correspond to the coherent structures of the numerical image along with the reconstructed snapshot of the numerical image using the dominant DMD modes.

and the original structures shown in Figure 3.1, DMD modes show no such discrepancies. This remarkable difference stems from the fundamental approach of the POD and DMD algorithms. Using the second-order statistics as a basis for decomposition, the POD algorithm carries out the eigenvalues decomposition to produce a hierarchy of coherent structures based on
the time-averaged spatial correlation tensor. However, the DMD algorithm extracts the dominant features from the snapshot sequences by approximating the linear mapping between the snapshots. Schmid [50] concluded that the POD algorithm focuses on a representation based on the spatial orthogonality, whereas both the temporal orthogonality (frequencies) and the spatial orthogonality are enforced by DMD algorithm.

The eigenvalues \( \lambda_i \) of the DMD decomposition represent the temporal, dynamical behavior of their corresponding spatial eigenmodes \( \Psi_k \). Typically, the obtained eigenvalues \( \lambda_i \) lie in the vicinity of the unit circle \( |\lambda_i|=1 \) when plotted in a complex plane as shown in Figure 3.6. The unit circle itself represents stable or stationary flow structures, while the inside \( |\lambda_i|<1 \) represents dynamically decaying structures and the outside \( |\lambda_i|>1 \) represents dynamically growing structures.

To identify the temporal behavior, the eigenvalues \( \lambda_i \) (Ritz values) are transformed to an equivalent time signal eigenvalue \( \mu_i \) based on Eq. 3.37 also referred to as exponential eigenvalues.

\[
\lambda_i = \exp (\mu_i \Delta t)
\]  

(3.37)

The real part of the transformed exponential eigenvalues \( \sigma_i = \text{Re} (\mu_i) \) signifies the growth/decay rate. This parameter which determines the envelope of the variation in time can be also expressed in terms of the modulus of the eigenvalue \( \rho_i = |\lambda_i| \). The unit circle \( |\lambda_i|=1 \) shown in Figure 3.6 or the straight line \( \text{Re} (\mu_i)=0 \) shown in Figure 3.7 represents stable coherent structures. \( \text{Re} (\mu_i)>0 \) corresponds to a growing structure while \( \text{Re} (\mu_i)<0 \) corresponds to a decaying coherent structure. This concept of stability based on the eigenvalues \( \lambda_i \) or the
exponential eigenvalues $\mu_i$ is summarized in Table 3.2. The imaginary part of the transformed exponential eigenvalues $2\pi f_i = \omega_i = \text{Im} (\mu_i)$ represents the angular frequency $\omega_i$, based on which the frequency $f_i$ of the oscillations can be computed.

![Diagram of complex plane with labeled Ritz values](image)

**Figure 3.6.** Mapping of Ritz value based on DMD analysis of the numerical image.

This logarithmic mapping of Ritz values 0 through 3 based on Eq. 3.37 is shown in Figure 3.7, which is generally referred to as the DMD spectrum. Markers 0, 1, 3 and 5 accurately identify the frequency (0, 0.5, 1.64 and 2 Hz) and growth rate (zero, zero, decay and grow) corresponding to the structures $q_0$, $q_1$, $q_2$ and $q_3$ in the DMD spectrum as shown in Figure 3.7. The marker size is proportional to a measure of coherence of the associated modes (energy content of the spatial mode) and help rank the relevant structures ahead of noise contaminated ones. Growth rate reflects the temporal dynamic of the coherent structure and is only applicable to the
Table 3.2. Dynamic mode stability based on the eigenvalues and transformed exponential eigenvalues.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue $\lambda_i$</th>
<th>Exponential eigenvalue $\mu_i$</th>
<th>Spectrum plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growing</td>
<td>$\rho_i =</td>
<td>\lambda_i</td>
<td>&gt; 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Above the straight line $\text{Re} \ (\mu_i) = 0$</td>
</tr>
<tr>
<td>Stable</td>
<td>$\rho_i =</td>
<td>\lambda_i</td>
<td>= 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>On the straight line $\text{Re} \ (\mu_i) = 0$</td>
</tr>
<tr>
<td>Decaying</td>
<td>$\rho_i =</td>
<td>\lambda_i</td>
<td>&lt; 1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Below the straight line $\text{Re} \ (\mu_i) = 0$</td>
</tr>
</tbody>
</table>

total duration of the temporal analysis (20 seconds for this analysis). Therefore, a positive growth rate does not necessarily reflect the onset or propagation of instability and a negative growth does not necessarily mean that the coherent structure eventually dies out.

Figure 3.7. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of the numerical image.
3.5.2. Steady Laminar Flame

OH-PLIF images of a stable methane-air laminar premixed flame (Re ~ 645, \( \phi = 1.2 \)) captured at 60 Hz were used to validate the application of POD and DMD algorithm to experimental datasets.

The normalized eigenvalue and its cumulative distribution based on POD analysis of 200 OH-PLIF images of a steady laminar flame is shown in Figure 3.8. The POD analysis exhibits the typical outcome expected of a steady laminar flame. The POD mode corresponding to the time-averaged flame shape (mode 0) shown in Figure 3.9, is the most dominant mode and contains around 70% of the total energy. The eigenvalue then drops sharply as expected due to the minimal contributions of the fluctuating modes towards the total energy of the flame.

![Figure 3.8. Normalized eigen value and its cumulative distribution for POD modes extracted from a steady laminar flame.](image-url)
DMD analysis was also carried out on the steady laminar flame using the same set of 200 OH-PLIF images. The results of the DMD analysis are presented in Figure 3.10 and Figure 3.11. From Figure 3.10, it can be clearly seen that the eigenvalues $\lambda_i$ (Ritz values) tend to be well distributed on the unit circle in the complex plane. The corresponding DMD spectrum shown in Figure 3.11 exhibits the typical outcome expected of a steady laminar flame, where, all but one mode (marked as 0) have negative growth rates indicating a highly damped response. The steady state mode (0 Hz, denoted as 0) has a zero-growth rate and hence is considered to the stable.

The spatial structure corresponding to this DMD mode 0 is shown in Figure 3.12 which represents the time-averaged flame shape (similar to POD mode 0). It is interesting to note the presence of a group of modes clustered around 18 Hz, which is partly due to the characteristic frequency of the burner jet velocity: 0.4 m/s, burner exit diameter: 23 mm). The marker size is proportional to a measure of coherence of the associated modes (energy content of the spatial mode) and help rank the relevant structures ahead of noise-contaminated ones.
Figure 3.10. Mapping of Ritz value based on DMD analysis of a steady laminar flame.

Figure 3.11. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of a steady laminar flame.
Figure 3.12. Spatial structure of DMD mode 0 (average flame shape) for a steady laminar flame.
Chapter 4. Experimental Setup

4.1. Microwave Plasma Assisted Combustion

A novel microwave waveguide plasma system was developed to study plasma enhanced combustion in a realistic swirl stabilized combustion geometry using laser and optical diagnostics. The microwave plasma source (MPS) was designed to produce plasma discharge that is spatially coincident with a flame, referred to as direct coupling and with complete optical access. The plasma system allows for combustion power levels up to about 3 kW while microwave radiation can be generated in excess of 1 kW but also as low as 30 W.

4.1.1. Microwave Plasma System

The microwave plasma system shown in Figure 4.1 was developed in collaboration with ReCarbon Technologies to directly couple microwave energy into a flame by creating a plasma under atmospheric pressure that is spatially coherent with the reaction zone (direct coupling). The system consists of a WR284 waveguide (TE$_{10}$ mode) coupled to a 2.45 GHz radio frequency, water cooled magnetron head (National Electronics) powered by an Alter Power Systems SM 840E 2 kW power supply. A three-port circulator is used for controlling the incident power and redirecting the reflected power away from the magnetron head onto a water cooled dummy load. A directional coupler connected to a HP EPM-442A power meter was used to measure the incident and reflected powers. The power delivered to the plasma (coupled plasma power) was computed as the difference between the incident and reflected power levels. The coupled plasma power was maximized by impedance matching using a three-stub tuner (National Electronics) and a sliding short (Gerling).
4.1.2. Plasma Applicator

The plasma applicator shown in Figure 4.2 also serves as a combustion torch. The plasma applicator consists of an aluminum nozzle fitted with a solid tungsten electrode that protrudes into the WR284 waveguide. Microwave energy travels through the nozzle, acting as a coaxial guide, in the TEM mode. The internal geometry of the nozzle is specially optimized for plasma ignition by delivering a high electric field at the nozzle tip. A plasma discharge is initiated by adjusting the standing wave’s maximum electric field intensity position inside the waveguide using the sliding short. As the standing wave moves inside the waveguide, the electric field at the tungsten electrode tip also varies. The tungsten electrode is used to focus the microwaves at the exit of the nozzle creating a location of maximum electric field intensity, which when reaches the breakdown threshold, causes the air around the electrode tip to ionize thereby generating a plasma discharge above the surface of the nozzle that is completely accessible for optical diagnostics. The nozzle was fitted with a quartz glass insert as a dielectric barrier to prevent any arcing between the nozzle
and the electrode. The annular space between the quartz inset and the electrode allows for gas flow into the plasma discharge. A mixture of fuel and air enters the side of the nozzle through the premixed inlet ports and exits into the plasma discharge at the tip of the electrode. The electrode is held in place inside the nozzle using a Teflon insert that prevents the flow from entering the microwave cavity while allowing microwaves to travel upward through the nozzle.

Figure 4.2. Sectional view of plasma applicator.

4.1.3. Swirl Burner

The variable swirl burner shown in Figure 4.3 was mounted onto the plasma applicator to produce a swirl stabilized, plasma assisted flame configuration as depicted in Figure 4.4. The variable swirl burner consists of an axial premixed center jet of fuel and air introduced through the plasma applicator as described earlier. Four swirl inlet ports each measuring 2.5 mm in diameter placed tangentially on the circumference of a 35-mm diameter cylindrical burner supply swirl air into the burner. The resulting swirling motion created a lifted flame, swirl stabilized at a height of 5 to 25 mm from the nozzle exit. In this work, only air was injected through the swirl inlet ports; this dilutes the central premixed jet and hence lowers the overall operating equivalence
ratio of the burner. Both the axial (fuel and air) and tangential (air) flow rates were metered individually using three laminar flow, differential pressure, MKS mass flow controllers. This setup allowed for changing the relative proportion of the axial and tangential flow rates thereby controlling the operating swirl number of the burner. The combustor was fitted with a 1.5-inch side, 12-inch-long, square quartz glass enclosure to mimic realistic combustion geometries. It also
serves to maintain the swirling motion inside the combustor and to prevent the entrainment of outside air. This configuration facilitates the study of both swirl stabilized lifted flames and plasma assisted combustion on a single platform.

4.2. Mesoscale Combustion

The primary objective of the mesoscale burner array design is to achieve compact, well distributed, mutually supported flames that are less susceptible to extinction under lean operating conditions while achieving performance and emission characteristics comparable to that of existing large-scale burners. More importantly, the goal was to develop a mesoscale burner architecture that can be seamlessly scaled over a wide range of combustor outputs capable of powering large scale gas turbines to compact portable units without any performance degradation. In the current design, a combination of swirl and bluff body stabilization are used to induce sufficient product recirculation to sustain a stable flame.

4.2.1. Direct Metal Laser Sintering

All mesoscale burner array prototypes were fabricated by DMLS, a state of the art additive manufacturing process that constructs the mesoscale burner array prototypes one layer at a time by fusing GP1 stainless steel metal powder locally into solid parts. Each layer is usually 40 μm in thickness. The critical parameters that compare the competitiveness of DMLS with traditional machining processes are presented in Figure 4.5. From Figure 4.5, it can be clearly seen that DMLS is a net-shape (closeness to final form) process, accurately capable of creating complex parts with high accuracy, great surface quality and excellent mechanical properties within hours (no lead time)[74-76]. In addition, DMLS offers easier component miniaturization compared to conventional machining, making DMLS the optimal manufacturing technique for mesoscale
burner array production.

Figure 4.5. Comparison of competitiveness between traditional and additive manufacturing [76].
4.2.2. Mesoscale Burner Array

The key design aspects of the mesoscale burner array and the individual array elements are outlined in Figure 4.6 and Figure 4.7 respectively. Every mesoscale burner element is composed of a center (bluff) body surrounded by two tangential inlets. Premixed fuel air mixture is directed through the two-tangential inflow swirler inlets that create the swirl flow around the bluff body. The bluff body present at the center of each burner element also improves flame stability by creating a sufficiently large recirculation zone. The burner exit is also provided with a diverging quarl that causes flow divergence which improves both the size of the recirculation zone and the amount of gas recirculated due to flow divergence [77]. The recirculation zones created by the swirl flow results in rapid diffusion due to their inherently small length scale. This helps in reactant and product mixing, thereby enhancing heat recirculation which reduces heat loss, and improves emission performance as well as overall flame stability. The mesoscale burner array is designed such that the individual burner elements have a counter-rotating circulation pattern (Taylor-Green Vortex array) [78] as shown in Figure 4.8, such that any single flame can propagate over the entire array after ignition. However, it is to be noted that the counter rotating vortex pattern employed can cause cold air entrainment from the surroundings at the boundary burner elements that could result in pronounced edge effects at the nodes, particularly for smaller burner array. The initial 4x4 design is therefore a good platform for studying and isolating edge effects of larger mesoscale burner arrays.
Figure 4.6. Sectional view showing the schematic layout of mesoscale burner array design.

Figure 4.7. Schematic layout of a single individual mesoscale burner array element.
The actual combustor geometry was optimized based on preliminary performance, emission and combustion characterization of various 3D metal printed prototypes. The most optimal mesoscale burner array design parameters are presented in Table 4.1. A three-dimensional CAD image of the prototype mesoscale burner array manufactured as a single monolithic unit by 3D metal printing process is shown in Figure 4.9. The internal architecture reveals that the air enters the burner array in the axial direction through the inlet layer and is forced to follow the intricate flow channels of the swirl layer and enters each burner element through the two-tangential inlet swirlers. Each burner element is also provided with a diverging quarl at the exit and a conical bluff body at the center.

The 3D metal printed prototype of the mesoscale burner array is enclosed in a burner housing with fittings for supplying the required air fuel-mixture as shown in Figure 4.9. The burner housing consists of a 100 mm diameter, 75 mm long plenum chamber fitted with flow straighteners on the upstream end. The downstream end of the plenum chamber houses the mesoscale burner array or the single swirl burner and is fitted with a quartz glass tube, 100 mm long with a 100×100
mm square cross section, to provide complete optical access and to mimic realistic combustor geometries. The quartz glass tube also serves to maintain the swirling motion inside the combustor and to prevent the entrainment of outside air.

<table>
<thead>
<tr>
<th>Mesoscale burner element design parameters</th>
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<tbody>
<tr>
<td>Quarl included angle θ (deg)</td>
</tr>
<tr>
<td>Quarl length L_q (mm)</td>
</tr>
<tr>
<td>Quarl radius R_3 (mm)</td>
</tr>
<tr>
<td>Bluff body radius R_4 (mm)</td>
</tr>
<tr>
<td>Swirler length L (mm)</td>
</tr>
<tr>
<td>Cylindrical post radius R_1 (mm)</td>
</tr>
<tr>
<td>Cylindrical chamber radius R_2 (mm)</td>
</tr>
<tr>
<td>A_i (mm$^2$)</td>
</tr>
<tr>
<td>A_o (mm$^2$)</td>
</tr>
<tr>
<td>H_i (mm)</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Mesoscale burner overall dimensions</th>
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<tr>
<td>Array diameter D (mm)</td>
</tr>
<tr>
<td>Array thickness H (mm)</td>
</tr>
<tr>
<td>Center to Center distance (mm)</td>
</tr>
</tbody>
</table>

*Table 4.1. Design parameters for 4x4 mesoscale burner array (Design 3).*
4.2.3. Single Swirl Burner

To benchmark and highlight the improved flame stability characteristics of the mesoscale burner array under externally imposed acoustic forcing, a single swirl burner was designed. The single swirl burner consists of radial swirler made of eight curved vanes with a 60° vane angle, equally distributed around the circumference of a 70 mm diameter cylindrical hub as shown in Figure 4.10. The vanes are 12 mm thick, and the inlet passage is 25 mm deep axially. The geometric swirl number [79] is estimated to be 1.15. A 12-mm diameter, 45° conical bluff-body in centrally integrated in the swirler to enhance the swirling motion and flame stabilization. In order to have comparable power output under similar flow conditions, the exit area of the single swirl burner was designed to match the total flow area of the mesoscale burner array.
Figure 4.10. CAD image of the single swirl burner.
Chapter 5. Plasma Assisted Swirl Stabilized Combustion

The effect of continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge on a swirl stabilized, premixed methane-air flame was investigated using quantitative OH planar laser induced fluorescence (PLIF) and spectrally resolved emission. The plasma discharge was found to influence the dynamics of flame stabilization i.e., plasma assisted flames stabilized in the quiescent center body (CB) wake were relatively stable while swirl flames stabilized in the active inner shear layer (ISL) were prone to local extinction due to aerodynamic shear. At coupled plasma powers corresponding to less than 3% of the thermal power output, in addition to the improved flame stability, significant improvement in the lean blow out limit (~ 43%) and OH number density (~ 150%) was observed. The enhancements are shown to be non-equilibrium plasma effects and not predominantly Ohmic heating as significant equivalence ratio dependence of OH number density in plasma assisted flames was observed. Spectrographic measurements indicated nitrogen vibrational temperatures as high as 6100 K suggesting both vibrational and electronic excitation of nitrogen molecules in the presence of a plasma discharge. The activation of highly reactive species through vibrational-vibrational relaxation and direct impact dissociation accelerates the combustion chemistry. It is demonstrated that microwave direct plasma coupling can drastically enhance dynamic flame stability of swirl stabilized flames especially at very lean operating conditions.

5.1. Introduction

In order to comply with stringent emission regulations, modern day gas turbine combustors are made to operate at leaner equivalence ratios, which has significantly lowered NOx emissions due to lower flame temperatures. Under lean conditions, NOx production mechanism is strongly
influenced by local flame temperatures [80, 81]. In spite of reduced pollutant emissions, decreased maintenance cost, increased life time of components [77], even small perturbations in the combustor operating conditions can lead to strong combustion instabilities [12, 82] in lean premixed combustion which makes it highly susceptible to lean blow out. Lean blow out poses substantial problems for both aircraft and land-based turbine engines [83, 84]. Gas turbine flames are often stabilized using a swirling jet of reactants that produce high combustion intensities [79]. As a result of vortex breakdown and flow expansion, recirculation zones are created that transport hot combustion products back to the nozzle, thereby igniting the unburned gas resulting in flame stabilization. In addition to the aforementioned combustion instabilities, swirl flows are subject to hydrodynamic instabilities as well [85].

Therefore, given the importance of lean premixed, swirl stabilized combustion in practical applications, methods that extend the dynamic stable operating regime of such flames over leaner conditions is an area of active research. In this context, over the recent years, the use of electromagnetic radiation to affect combustion phenomena by coupling plasma discharges to combustion chemistry often referred to as plasma assisted combustion has gained in popularity [86-88]. Previous studies have revealed that there are multiple benefits associated with plasma assisted combustion including reduced ignition delay [89-91], increased flame speeds [92-94] and increased combustion stability limits [95-98]. Typically, near equilibrium plasmas produced by electric sparks and arcs are used for ignition as local heating of the gas (Ohmic heating) increases the rate of thermal dissociation [89-91] whereas non-equilibrium plasmas are suited for combustion control as they enhance the chemical reactivity of the system through dissociation and excitation of molecules by electron impact [97-100].

It is essential to improve the efficiency of plasma assisted combustion systems in order to
ascertain the actual impact of the energetic enhancement. Many of the aforementioned studies have been dedicated to the use of a high voltage nanosecond repetitive pulsed discharge system. However, direct plasma coupling using microwave radiation is considered to be particularly appealing due to its high energy efficiency, improved lifetime and reliable operation under extreme conditions [101-103]. Direct plasma coupling which involves generation of a spatially coincident microwave generated plasma with the reaction zone is highly efficient in achieving plasma induced flame enhancement even at very low plasma powers [101-103]. The high efficiency of a direct coupled microwave plasma system is a result of the complementing effect existing between the plasma and the flame i.e., the plasma discharge helps in enhancing the combustion process which in turn lowers the power required to sustain a stable plasma as a result of the associated heat release and generation of free electrons.

However, understanding about the complex interaction between non-equilibrium plasma discharge and flame chemistry is incomplete. The lack of knowledge about the specific mechanisms responsible for combustion enhancement has limited the capability of such plasma assisted combustion systems. In this work, we explore the possibility of employing a volumetric, direct coupled, microwave plasma discharge to a swirl stabilized flame in terms of its impact on lean combustion in a practical setting. Nevertheless, the focus of this work is to gain fundamental insight on how a non-equilibrium plasma discharge actually improves dynamic flame stability by studying the complex interaction involving three key factors: fluid physics, combustion chemistry and non-equilibrium plasma discharge. To this end, this work utilizes quantitative, spatially resolved OH concentration measurements coupled with spectrally resolved emission measurements in swirl stabilized as well as plasma assisted flame configurations over a broad range of operating conditions including Reynolds number, swirl flow rate, equivalence ratio and
coupled plasma power. Based on the results, a possible mechanism involving the “activation of inert nitrogen species” is proposed to explain the observed effects of improved flame stability and reduced lean blow off limit in plasma assisted swirl stabilized flames.

5.2. Experimental Setup

5.2.1. Microwave Plasma Assisted Combustion Setup

A tunable microwave waveguide is used to initiate and enhance combustion by coupling an atmospheric plasma discharge to a swirl stabilized premixed methane-air flame. The absorbed microwave power ranges from 35 to 120 W, generated from a continuous source operating at 2.45 GHz, whereas combustion power ranges from 600 to 3000 W. Plasma power is controlled by adjusting the reflected microwave power, measured at a dummy load attached to a circulator. To produce a premixed flame, a mixture of fuel and air enters the side of the nozzle through the premixed inlet ports and exits into the plasma discharge at the tip of the electrode. This configuration produces a direct-coupled plasma assisted combustion discharge. All data presented are from methane-air flames. The ignition procedure was to flow the fuel and air through the plasma torch and air through the swirl inlets, then ignite the flame by increasing the coupled plasma power to the nozzle tip, through adjustment of the three stub tuner, until breakdown and plasma formation thereby resulting in a plasma assisted flame. No additional ignition system was required. The plasma discharge was then turned off to sustain a swirl stabilized flame. For the plasma assisted combustion experiments, impedance matching was carried out so as to maintain the difference between the incident and coupled plasma power to less than 10 W thereby minimizing losses. The air and fuel flow rates were metered individually using three laminar flow, differential pressure, MKS mass flow controllers in standard liters per minute (SLPM), following
the IUPAC standard temperature and pressure as 273.15 K and 1 atm.

5.2.2. Diagnostics Setup

A schematic layout of the complete experimental setup is shown in Figure 5.1. A 750 mm focal length Acton 2750i spectrometer fitted with a Hamamatsu Photo Multiplier Tube (PMT) was used to analyze the emission spectrum in the range of 300 to 500 nm, using a 1200 g/mm grating with a blaze angle at 500 nm. The emission from the plasma was focused onto the entrance slit of the spectrometer using a 100 mm focal length, converging lens made of fused silica (UV). A constant 400 V bias was applied to the PMT in order compare the different emission spectra. The spectrometer was used as a monochromator by scanning through emissions wavelengths at 100 nm/min. The data from the PMT was acquired using a LabView interface at 3 kHz via NI DAQ system.

The OH Planar Laser Induced Fluorescence (OH-PLIF) system used the second harmonic output of a Spectra Physics Quanta Ray, Nd: YAG laser (532 nm, 500 mJ/pulse) operating at 10 Hz to pump a Sirah Precision Scan dye laser containing Rhodamine 6G dye dissolved in ethanol. The red dye laser beam was then frequency doubled using an integral β-barium-borate doubling crystal. The dye laser output (12 mJ/pulse, 0.1 cm$^{-1}$ spectral line width at 283 nm) was then tuned to excite the $Q_1(8)$ line of the $A^2Σ^+ \rightarrow X^2Π (1,0)$ band of OH at approximately 283.55 nm in air [101, 103-106]. The pulse energy was monitored digitally using a fast photodiode and an oscilloscope to ensure operation within the linear fluorescence regime. The laser was formed into a sheet of height of 50 mm and a thickness of approximately 0.3 mm (FWHM) in the probe region using a two-lens telescope. Fluorescence in the range of 310 nm was isolated from Rayleigh and surface scattering using an Asahi high transmission, narrow band pass filter (10 nm FWHM at
Figure 5.1. Schematic layout of the diagnostics setup used for 10 Hz OH-PLIF imaging and emission spectroscopy measurements.

310 nm) and collected at right angle (perpendicular to the laser sheet) onto the 1280 x 1024-pixel imaging array of a 10 Hz LaVision Imager Intense CCD camera coupled to a LaVision IRO image intensifier (gate = 100 ns) using a Sodern Cerco, 100 mm focal length, f/2.8 UV lens. The $Q_1(8)$ line was chosen for the OH-PLIF as the ground state population of $J_1 = 8$ state is relatively insensitive to temperature fluctuations. A LIFBASE simulation [107] showed that the population fraction varied by less than 10% for temperatures in the range of 1200 - 2300 K from the peak Boltzmann population fraction at around 1600 K. A flat flame burner was used for calibration of the laser wavelength and to correlate the signal intensity with the OH number density using
previously performed absorption measurements.

5.3. Results and Discussion

5.3.1. Flame Stabilization

Direct photographs of the two investigated flames, namely swirl stabilized flame and plasma assisted flame along with the flow field are shown in Figure 5.2. The two flames exhibit rather contrasting flame stabilization properties. The swirl flame is typically a lift off flame stabilized at a height of 15 mm from the nozzle exit. The swirl stabilized flame has well defined edges and emits a strong blue hue, typical of CH* chemiluminescence. On the other hand, the plasma assisted flame is strongly anchored to the tip of the electrode. The volumetric plasma discharge is visible as a purple cone 10 - 15 mm in height and slightly extending past the radial edge of the electrode. The plasma assisted flame is also considerably shorter in length around 15 - 20% when compared to the purely swirl stabilized flame.

The swirl burner creates an annular swirling jet where the flame is typically stabilized in the low velocity region of the shear layer or near the stagnation points [85]. In the present configuration, the tungsten electrode serves as the bluff center body that typically divides the flow field into four main regions [108-110]: (a) Outer Recirculation Zone (ORZ) (b) Center Body (CB) Wake and Inner Recirculation Zone (IRZ) (c) High velocity jet separating the IRZ and the ORZ (d) two annular shear layers on either side of the annular jet: Inner Shear Layer (ISL) between the jet and IRZ and the Outer Shear Layer (OSL) between the jet and the ORZ. In our case, the swirl flame was stabilized in the ISL [85, 111, 112] as opposed to plasma assisted flame that stabilized in the CB wake.

For all the flow conditions (main jet air) used in our current set up ranging from 10 SLPM
(Re ~ 2000) to 30 SLPM (Re ~ 6400), the swirl stabilized flame retained a similar flame structure. The Reynolds number is calculated at the exit of the burner based on the premixed flow rate before swirl air is introduced. This is in good agreement with the finding of Sheen et al. [113] who observed that the Reynolds number has a weak effect on the recirculation structure of swirling flows at Re > 500. However, the flame lift off distance exhibited a strong dependence on the swirl flow rate and the equivalence ratio as shown in Figure 5.3. The lift off distances were computed based on the calibrated OH-PLIF images. Higher swirl flow rates resulted in the merging of the CB wake with the IRZ and the moved the ORZ further upstream [113, 114]. This enabled the flame to stabilize closer to the nozzle exit resulting in a smaller lift off distance. Further increase in
equivalence ratio causes a progressive increase in the flame speed which resulted in decreased flame lift off distances. For leaner flames, there is a minimum swirl flow rate, below which the flame blows off as a result of decreased flame speed and weaker recirculation zones created by the swirling flow.

Figure 5.3. Variation of flame lift off distance with swirl flow rate for different main jet equivalence ratios. Main jet air flow rate is maintained constant at 10 SLPM (Re ~ 2000).

5.3.2. Lean Blow Off Limit

The lean blow off limits (LBO) of a swirl stabilized flame is dictated by the flame stabilization location [115]. The variation of LBO limit for a swirl stabilized flame is shown in Figure 5.4. The LBO limit was evaluated by decreasing the fuel flow rate for a fixed air flow rate (to minimize variation in the Reynolds number), until the flame blows off. The LBO increases almost linearly with the Reynolds number for a constant swirl. However, an increase in the
imposed swirl causes a significant lowering of the LBO, thereby allowing to stabilize and sustain much leaner flames at higher swirl intensities. For an axial air flow rate of 30 SLPM (Re ~ 6200) resulting in a bulk air velocity of around 33.1 ms⁻¹ at the nozzle exit, the LBO improved from 1.52 to 1.14 as the swirl flow rate was increased from 30 SLPM to 45 SLPM. This improvement in LBO by about 23% was also found to be nearly independent of the axial air flow rate.

![Figure 5.4. Variation of swirl stabilized flame lean blow off equivalence ratio with Reynolds number for different swirl flow rates. No plasma coupling.](image)

However such swirl stabilized flames operating close to their lean blow out limit are highly vulnerable even to the smallest perturbation in the combustor operating condition [84]. In order to improve the operational stability of such flames and to extend the LBO further, direct microwave plasma coupling was employed. The effect of direct plasma coupling on the LBO of a swirl stabilized, plasma assisted flame is shown in Figure 5.5. For 30 SLPM axial air flow rate, even for
the smallest coupled plasma power of 60 W which corresponds to less than 5% of the combustion power (1.65 kW), a drastic reduction of around 43% in the LBO was observed. The LBO was extended further by increasing the coupled plasma power but however this effect is quite trivial considering the significant increase in coupled plasma power (~ 100%).

![Graph showing variation of plasma assisted flame lean blow off equivalence ratio with Reynolds number for different coupled plasma powers. Swirl flow rate is maintained constant at 40 SLPM.](image)

Figure 5.5. Variation of plasma assisted flame lean blow off equivalence ratio with Reynolds number for different coupled plasma powers. Swirl flow rate is maintained constant at 40 SLPM.

The improved flame stability and extension of LBO is a direct result of the volumetric plasma discharge being coupled directly into the reaction zone of the flame. It is postulated that the plasma assisted flame still retains the typical flow pattern as described earlier due to the presence of the swirl flow. Once a volumetric plasma discharge was created, it quickly stabilized in the central wake region downstream of the solid tungsten electrode. This causes the flame to move from the ISL onto the tungsten electrode where it is predominantly stabilized in the
combined region of CB wake and IRZ. This configuration in a purely swirl stabilized flame is rather unstable as noted by Cheterev et al. [85] but however the presence of non-equilibrium plasma discharge in the CB wake enhances the combustion process through the process direct plasma coupling that strongly anchors the flame onto the electrode [101, 103, 106]. The active radical species created in plasma plume are also transported radially outwards due to the combined effect of the IRZ and the ISL that causes as the flame to branch out into the shear layer as seen in Figure 5.2.

5.3.3. OH Planar Laser Induced Fluorescence (OH-PLIF)

Single and ensemble averaged calibrated OH-PLIF images are shown for swirl stabilized flames and plasma assisted flames as a function of increasing swirl flow rate and coupled microwave plasma power are shown in Figure 5.6 and Figure 5.7 respectively. The ensemble averaged images were based on the 100 instantaneous images acquired for each data set. The main jet air flow rate was maintained at 10 SLPM at an equivalence ratio $\phi = 1.0$. This corresponds to a combustion power of around 650 W based on the lower heating value of methane (55 MJ/kg). Thus, the coupled plasma power to combustion power ratio is around 0.09 - 0.18 for microwave power levels ranging from 60 to 120 W.

Image post processing was carried out using an open source Java based software called ImageJ [116] developed by the National Institutes of Health. First a reference length was used to correlate the pixels to the actual length scale and thereby formulate a reference grid for the images. After background subtraction, the intensity of each pixel was correlated to the OH number density. Finally, after enhancing the brightness and contrast of these processed images, a custom lookup table was applied to enhance and visualize the flame features. The single shot images clearly indicate the differences between the dynamics of swirl stabilized flame and plasma assisted flame.
The swirl flame is clearly lifted off and stabilized in the ISL whereas the plasma assisted flame is anchored to the electrode and stabilized in the CB wake and IRZ. More interestingly, it can be seen that the swirl stabilized flame sheet becomes increasingly discontinuous as the swirl flow rate is increased. This is due to the aerodynamic flame stretch [117] caused by the shearing effect of the ISL which modifies the local temperature and burning rate [118].

![Image of OH number density for swirl stabilized flames for different swirl flow rates](image)

**Figure 5.6.** Images of instantaneous (Row 1) and averaged (Row 2) OH number density for swirl stabilized flames for different swirl flow rates. Main jet air flow rate is maintained constant at 10 SLPM (Re ~2000) at an equivalence ratio of φ = 1.0.
Figure 5.7. Images of instantaneous (Row 1) and averaged (Row 2) OH number density for plasma assisted flames for different coupled plasma powers. Main jet air flow rate and swirl flow rate are maintained constant at 10 SLPM (Re ~2000) and 40 SLPM respectively at an equivalence ratio of $\phi = 1.0$.

The discontinuities can be considered as localized extinctions where the flame stretch is too large. Isolated pockets of reacting mixture are also occasionally seen in the CB wake and IRZ at higher swirl flow rates possibly caused by shearing of premixed flamelets from the main flame front. This is in stark contrast when compared to the structure of the plasma assisted flame which appears more or less continuous with only slightly corrugated edges. This is due to the relatively
quiescent nature of the CB wake now merged with the IRZ where the flame stabilizes when compared to the much more active ISL. Increase in the coupled plasma power results in formation of a much larger flame volume. Further, the plasma assisted flame typically exhibits an elongated flame volume or bilobed structure that suggests initial plasma induced oxidation due to direct coupling followed by further continued oxidation downstream. The images also clearly indicate the divergence of the high velocity central jet around the IRZ and the presence of ORZ. This supports our earlier hypothesis that the plasma assisted flame still retains the typical flow pattern of the swirl stabilized flame however the location of flame stabilization differs between these two cases. Further, since the volumetric plasma discharge is directly coupled onto the reaction zone, it is capable continuously modifying and accelerating the reaction zone chemistry thereby resulting in more pronounced combustion enhancement as opposed to point discharges whose effect is rather localized and transient.

The variation of OH number density with the swirl flow rate and coupled plasma for various main jet equivalence ratios is presented in Figure 5.8 and Figure 5.9 respectively. The main jet air flow rate was maintained at 10 SLPM in both cases while the swirl flow rate was set to 40 SLPM in the plasma assisted flame. The data points were computed as the average OH number density in a 5 x 5 mm region (as shown in the inset) in the center of the reaction zone where the OH-PLIF signal was most intense. In the case of a swirl stabilized flame (Figure 5.8), it can be seen that though the OH number density increases with the swirl flow rate, it appears to plateau beyond a certain limit. This could indicate the onset of local flame extinction thereby leading to combustion instabilities due to excessive straining of the flame as discussed earlier. The OH number density also increases as the mixture is made progressively richer due to the dilution effect caused by the swirl flow which made the mixture approach stoichiometric conditions.
From Figure 5.9, it can be clearly seen that the plasma discharge has a very strong effect on the OH number density, increasing the OH number density by as much as 150% in certain cases (120 W coupled plasma power, $\phi = 0.9$). Also, unlike swirl stabilized flames, the OH number density in the case of plasma assisted flame increases continuously with the coupled plasma power. Doubling the plasma power increased the OH number density by as much as 30% in case of a lean mixture. It is to be noted that the effect of plasma discharge is more pronounced at leaner flame conditions, i.e., OH number density exhibits a decreasing trend with increasing equivalence ratios which is highly counter intuitive. Though this observed trend is highly consistent with the previously reported findings [101, 103, 119, 120] for a wide variety of flame configuration, there has been no conclusive justification offered to describe this behavior earlier.

*Figure 5.8. Variation of OH number density with swirl flow rate for different main jet equivalence ratios. Main jet air flow rate is maintained constant at 10 SLPM (Re ~ 2000). No plasma coupling. The inset image highlights the ROI used for data extraction.*
Figure 5.9. Variation of OH number density with coupled plasma power for different main jet equivalence ratios. Main jet air flow rate and swirl flow rate are maintained constant at 10 SLPM (Re ~2000) and 40 SLPM respectively. The inset image highlights the ROI used for data extraction.

Since the coupled plasma power was maintained constant over the range of equivalence ratios studied (constant Ohmic heating), it can be concluded that the observed increase in OH number density in the case of a plasma assisted flame is due to the new chemical reaction pathways among the radical species generated by the plasma discharge and not entirely due to Ohmic heating alone, i.e., non-thermal effects.

The comparison between the radial distributions of OH number density for a swirl stabilized flame and a plasma assisted flame measured at 10 mm above the tip of the tungsten electrode is shown in Figure 5.10. The data points were computed as the average OH number
density in a 3 mm wide region (as shown in the inset) at a height of 10 mm from the electrode tip. The OH number density exhibits two maxima for the swirl stabilized flame because of its lifted geometry. The asymmetries observed in the radial profile are due to the minor variation in the splitting of the swirl flow through the four tangentially placed swirl inlets which results in minor asymmetries in the flow field. The plasma assisted flame on the other hand exhibits a single maximum located along the centerline passing through the tip of the electrode. It is to be noted the plasma assisted flame exhibits an overall higher trend in the OH number density which continues to increase with further increase in the coupled plasma power.

Figure 5.10. Comparative radial distribution of OH number density for different coupled plasma powers measured at 10 mm above the electrode tip. Main jet air flow rate and swirl flow rate are maintained constant at 10 SLPM (Re ~2000) and 40 SLPM respectively at an equivalence ratio of $\phi=1.0$. The inset image highlights the ROI used for data extraction.
The comparison between the axial distributions of OH number density for a swirl stabilized flame and a plasma assisted flame measured along the centerline passing through the tip of the tungsten electrode is shown in Figure 5.11. The data points were computed as the average OH number density in a 3 mm wide region (as shown in the inset) along the centerline through the electrode tip. The OH number density peaks further downstream of the electrode for a swirl stabilized flame because of its lifted geometry as opposed to the plasma assisted flame. The bilobed structure of the plasma assisted flame shows two closely located peaks in the OH number density. The first peak is the result of direct coupling where the plasma is spatially coincident with the

![Figure 5.11. Comparative axial distribution of OH number density for different coupled plasma powers measured along the centerline. Main jet air flow rate and swirl flow rate are maintained constant at 10 SLPM (Re \( \sim 2000 \)) and 40 SLPM respectively at an equivalence ratio of \( \phi=1.0 \). The inset image highlights the ROI used for data extraction.](image-url)
reaction zone initiating oxidation and creating increased OH radical concentration. These radicals are further transported downstream where the combustion process is further completed resulting in the second peak as shown in Figure 5.11. Plasma assisted flame exhibits an overall higher trend in the OH number density which continues to increase with further increase in the coupled plasma power (around 100% increase at 120W).

5.3.4. Spectrally Resolved Emission Measurements

In order to probe further the role of non-thermal effects of the plasma discharge in improving the flame stability thereby enhancing the LBO and OH number density, spectrally resolved emission measurements of the reaction zone was carried out. The normalized emission spectrum (using maximum intensity at 60 W as baseline) from a plasma assisted flame for various coupled plasma powers ranging from 120 W to 60 W is shown in Figure 5.12. Most peaks of the emission spectrum can be traced to the second positive system of molecular nitrogen: various vibrational states in the $C^3\Pi_u \rightarrow B^3\Pi_g$ electronic transition. This well know band extending from 325 to 410 nm, typical of an arc discharge tube containing nitrogen or air at low pressure is responsible for the purple hue of the plasma assisted flames. Another dominant feature of the emission spectrum seen between 300 and 320 nm is the OH emission arising from the (0, 0) band of the $A^2\Sigma^+ \rightarrow X^2\Pi$ transition. Also traces of the CH radical emission arising from the (0, 0) band of the $A^2\Delta \rightarrow X^2\Pi$ transition can be seen around 430 nm. The emission intensity from an excited species is a measure of the relative concentration. From Figure 5.12, it is clear that even without performing the relevant temperature corrections, the emission intensities are generally higher for increased coupled plasma power. Species like
Figure 5.12. Normalized emission spectrum from the plasma assisted flame for different coupled plasma powers.
CH and OH have been shown to play very important roles in flame chemistry [121, 122]. In a plasma assisted flame, the increase in concentration of such chemically reactive species is attributed to the vibrational excitation of nitrogen molecules by direct electron impact [86] through the following set of reactions [123]:

\[
N_2\left(X^1\Sigma^+_g, \nu = 0\right) + e^- \rightarrow N_2\left(A^3\Sigma^+_u, \nu > 0\right) + e^- \text{ (6.0 eV threshold)} \quad (5.1)
\]

\[
N_2\left(X^1\Sigma^+_g, \nu = 0\right) + e^- \rightarrow N_2\left(B^3\Pi_g, \nu > 0\right) + e^- \text{ (7.2 eV threshold)} \quad (5.2)
\]

\[
N_2\left(X^1\Sigma^+_g, \nu = 0\right) + e^- \rightarrow N_2\left(C^3\Pi_u, \nu > 0\right) + e^- \text{ (10.8 eV threshold)} \quad (5.3)
\]

\[
N_2\left(X^1\Sigma^+_g, \nu = 0\right) + e^- \rightarrow N_2\left(a^1\Pi_u, \nu > 0\right) + e^- \text{ (8.2 eV threshold)} \quad (5.4)
\]

The electronically and vibrationally excited nitrogen molecules are very reactive compared to their ground state counterparts [124]. In order to obtain an estimate of the nitrogen vibrational temperatures, the second positive system of nitrogen molecule was simulated using PGOPHER [125] and compared with the experimental spectrum as shown in Figure 5.13. The vibrational and rotational temperatures estimated using the nitrogen spectrum were around 6100 K and 2800 K respectively. The rotational temperatures reported are in good agreement with earlier measurements carried out using OH [103] and CH [102] radicals in a similar direct microwave plasma assisted combustion setup. This clearly indicates that there is non-thermal equilibrium existing between the vibrational and rotational levels.

Subsequently a vibrationally excited nitrogen molecule can undergo Vibrational-Vibrational (V-V) relaxation by interacting with oxygen molecule thereby forming vibrationally excited oxygen molecules [86].

\[
N_2^\ast(\nu > 0) + O_2(\nu = 0) \rightarrow N_2^\ast(\nu - 1) + O_2^\ast(\nu = 1) \quad (5.5)
\]

Such vibrational excitation of oxygen molecules can accelerate direct electron impact
reactions leading to formation of oxygen radicals [86, 88].

\[ \text{O}_2^* (v > 0) + e^- \rightarrow e^- + O + O \]  
(5.6)

\[ \text{O}_2^* (v > 0) + e^- \rightarrow e^- + O + \text{O}^*(1D) \]  
(5.7)

\[ \text{O}_2^* (v > 0) + e^- \rightarrow 2e^- + O + \text{O}^- \]  
(5.8)

Quenching of \text{O}^*(1D) atoms in the metastable excited state and recombination of \text{O}^+ ions with electrons result in further oxygen radical generation. Also quenching of the excited electronic states of nitrogen by oxygen molecule can create oxygen radicals by direct nitrogen impact dissociation [88].

\[ \text{N}_2^* \left( \text{A}^3\Sigma_u^+, \text{B}^3\Pi_u, \text{C}^3\Pi_u, \text{a}^1\Sigma_u^- \right) + \text{O}_2 \rightarrow \text{N}_2 \left( \text{X}^1\Sigma_g^- \right) + \text{O} + \text{O} \]  
(5.9)

The formation of highly reactive oxygen radical results in various chain initiation and chain
branching reactions at lower temperatures that do not readily occur under typical flame conditions. Further formation of vibrationally excited oxygen molecules can accelerate reaction 5.13 which is considered to be one of the most important reactions in flame chemistry as its rate substantially determines the rate of CO oxidation and OH production which affects the combustion process as whole [126].

\[
\begin{align*}
\text{CH}_4 + O & \rightarrow \text{CH}_3 + \text{OH} & \text{(5.10)} \\
\text{CH}_4 + \text{OH} & \rightarrow \text{CH}_3 + \text{H}_2\text{O} & \text{(5.11)} \\
\text{CH}_3 + O & \rightarrow \text{CH}_2\text{O} + \text{H} & \text{(5.12)} \\
\text{H} + \text{O}_2^* & \rightarrow \text{OH} + \text{O} & \text{(5.13)}
\end{align*}
\]

Thus, the observed non-thermal effects of plasma discharge can be perceived as “activation of inert nitrogen molecules” which play a pivotal role in opening up new chemical pathways that are typically absent in a conventional flame where the nitrogen molecule functions only as an inert heat sink.

5.4. Conclusion

The effect of volumetric, direct coupled, continuous, atmospheric microwave plasma discharge on a swirl stabilized, premixed methane-air flame was investigated by employing optical diagnostic techniques. The variable swirl burner fitted onto a microwave plasma applicator serves to mimic realistic combustion geometries by offering a simple flexible platform to study both swirl stabilized and plasma assisted flames. Though both swirl stabilized and plasma assisted flames exhibited similar flow patterns, the dynamics of flame stabilization between these two flames were found to be entirely different. Swirl flames were lift off flames that stabilized in the active ISL. They were prone to local extinction due to the aerodynamic shearing effects associated with the ISL. While, plasma assisted flames on the other hand were much more stable as they were
stabilized in the relatively quiescent CB wake region behind the electrode. Also, such flames were strongly anchored to the electrode due to the presence of the plasma discharge.

In addition to increased flame stability, direct coupled, plasma assisted flames offered significant improvement in the LBO (~43%) and OH number density (~150%) for coupled plasma powers as low as 3% of the combustion output. Further the significant equivalence ratio dependence of OH number density in plasma assisted flames suggested that the influence of plasma discharge in swirl stabilized flame existed as complex non-thermal effects as opposed to mere Ohmic heating. Spectrographic measurements showed nitrogen vibrational temperatures as high as 6100 K indicating vibrational excitation of nitrogen molecules in addition to electronic excitation. V-V relaxation and direct nitrogen impact dissociation resulted in formation of highly reactive vibrationally excited oxygen molecules and oxygen radicals respectively which accelerated chemical reactions.

In conclusion, it has been shown that the non-thermal effect of “inert nitrogen excitation” can be successfully utilized in a swirl stabilized flame configuration by employing a volumetric, direct coupled, microwave plasma discharge which resulted in a significant improvement in the operating limits of the burner while enhancing flame stability especially at very lean operating conditions by using only a fraction of the combustion power.
Chapter 6. Plasma Assisted Swirl Stabilized Combustion Dynamics

The primary aim of this work is to establish the effectiveness of microwave plasma discharges to improve combustor flame dynamics and stability through minimizing heat release fluctuations. A continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge was applied to a swirl stabilized premixed methane-air flame to minimize combustion instabilities. Proper Orthogonal Decomposition (POD) is used to post-process data and extract information on flame dynamics that are usually lost through classical statistical approaches. POD analysis carried out on OH planar laser-induced fluorescence (PLIF) images reveal that even at coupled plasma powers corresponding to less than 5% of the thermal power output, significant improvement in mean energy content of flames (~23%) was observed. The corresponding decrease in heat release fluctuations resulted in improved combustor flame dynamics and flame stability, which was found to be in good agreement with acoustic pressure measurements. In the presence of plasma discharge, an effective decoupling between the flame oscillations and the fluid unsteadiness was established due to the differences in flame stabilization mechanisms resulting in up to 47% reduction in RMS pressure fluctuations. Thus, effective fluid-acoustic decoupling in addition to the accelerated combustion chemistry due to the non-thermal effects of plasma led to significantly improved combustor dynamics namely, decreased heat release and pressure fluctuations.

6.1. Introduction

Combustion instabilities are a major problem in the design of modern high-performance propulsion systems [7]. They often manifest as large amplitude pressure oscillations that result in many undesirable effects leading to serious drops in performance [10]. Combustion instabilities
refer to self-sustained combustion oscillations at or near the acoustic frequency of the combustion chamber, which are the result of the closed-loop coupling between unsteady heat release and pressure fluctuations [8, 9]. The typically low frequency oscillations induce large mechanical vibrations in the system that often result in combustor failure. In unstable operation mode, enhanced heat transfer at combustor walls may lead to partial or total blow off [7-10]. In particular, modern gas turbines are made to operate at increasingly ultra-lean conditions to lower NOx emissions. This causes increased susceptibility to lean blow-out, as even small perturbations in the operating conditions of lean mixtures can lead to strong combustion instabilities [12, 82]. Lean blow out poses substantial problems for both aircraft and land-based turbine engines [83, 84]. Due to the potential harm to system performance posed by combustion instabilities, it is often necessary to find ways to reduce the magnitude of these oscillations in the course of developing a new combustion system.

Quite recently, plasmas have been studied for use in combustion enhancement and control, often referred to as plasma assisted combustion [13]. To effectively employ plasma discharge for controlling combustion instability, it is necessary to understand the associated flame dynamics. This is not trivial as it requires capturing a multiscale phenomenon involving a large number of parameters [44]. These parameters (radical concentration, velocity field etc.) can be captured as an ensemble of snapshots in a frozen state across 2D planes by advanced laser diagnostic techniques [45]. Planar measurements like OH-PLIF result in ensembles of 2D planes with each snapshot providing some insight about a typical turbulent flow field. However, there is no significance associated with a particular snapshot and hence, it is customary to handle such data-sets by averaging the images and extracting ensemble averaged and root-mean-square (RMS) fields. The drawback of doing so is that a large fraction of the information contained in the
snapshots is lost, especially the flame dynamics and particularly the flame interaction with large scale coherent structures. To infer important information from ensemble averages, several techniques, such as conditional averaging, wavelet analysis [47] and Proper Orthogonal Decomposition (POD) [48, 49] are employed.

Though numerous articles have dealt with use of POD as a tool to understand turbulent flows [49, 127, 128], only a limited number of articles dealt with flames [129, 130]. However, none of these works have been focused on the flame dynamics of plasma assisted combustion (PAC). In this current work, we seek to extend our understanding of the combustion dynamics associated with plasma assisted flames and to gain fundamental insight into how plasma discharge improves combustor flame dynamics by correlating the heat release fluctuations, computed indirectly from OH radical concentrations (OH-PLIF measurements) with pressure fluctuations (acoustic measurements), so as to effectively utilize plasma for active combustion control. A comprehensive investigation over a broad range of operating conditions for a continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge applied to a swirl stabilized flame is presented. Based on these results, in addition to the earlier established non-thermal effects of plasma discharge through “inert nitrogen excitation” [104], a potential “effective fluid-acoustic decoupling mechanism” is proposed to be the primary reason for the observed improvement in flame dynamics i.e., decreased heat release and pressure fluctuations.

6.2. Experimental Setup

6.2.1. Microwave Plasma Assisted Combustion Setup

A tunable microwave waveguide is used to initiate and enhance combustion by coupling an atmospheric plasma discharge to a swirl stabilized premixed methane-air flame. The absorbed
microwave power ranges from 35 to 120 W, generated from a continuous source operating at 2.45 GHz, whereas combustion power ranges from 600 to 3000 W. Plasma power is controlled by adjusting the reflected microwave power, measured at a dummy load attached to a circulator. To produce a premixed flame, a mixture of fuel and air enters the side of the nozzle through the premixed inlet ports and exits into the plasma discharge at the tip of the electrode. This configuration produces a direct-coupled plasma assisted combustion discharge. All data presented are from methane-air flames. The ignition procedure was to flow the fuel and air through the plasma torch and air through the swirl inlets, then ignite the flame by increasing the coupled plasma power to the nozzle tip, through adjustment of the three stub tuner, until breakdown and plasma formation thereby resulting in a plasma assisted flame. No additional ignition system was required. The plasma discharge was then turned off to sustain a swirl stabilized flame. For the plasma assisted combustion experiments, impedance matching was carried out so as to maintain the difference between the incident and coupled plasma power to less than 10 W thereby minimizing losses. The air and fuel flow rates were metered individually using three laminar flow, differential pressure, MKS mass flow controllers in standard liters per minute (SLPM), following the IUPAC standard temperature and pressure as 273.15 K and 1 atm.

6.2.2. Diagnostics Setup

A schematic layout of the complete experimental setup is shown in Figure 6.1. Acoustic measurements were carried out using a PCB Model 377B26 High Temperature Probe microphone flush mounted to the top (open) end of the quartz tube. The raw signal was then conditioned using a PCB Model 480B21 ICP sensor signal conditioning unit. The acquired pressure signal is indicative of the acoustic perturbations. Further post-processing of the microphone data involved application of Fast Fourier Transform (FFT) to obtain the Power Spectrum Density (PSD) and the
Root Mean Squared (RMS) pressure fluctuations in order to identify and characterize any periodic phenomena present in the system.

![Figure 6.1. Schematic layout of the diagnostics setup used for 10 Hz OH-PLIF imaging and acoustic measurements.](image)

The OH Planar Laser Induced Fluorescence (OH-PLIF) system used the second harmonic output of a Spectra Physics Quanta Ray, Nd: YAG laser (532 nm, 500 mJ/pulse) operating at 10 Hz to pump a Sirah Precision Scan dye laser containing Rhodamine 6G dye dissolved in ethanol. The red dye laser beam was then frequency doubled using an integral β-barium-borate doubling crystal. The dye laser output (12 mJ/pulse, 0.1 cm⁻¹ spectral line width at 283 nm) was then tuned
to excite the $Q_1(8)$ line of the $A^2Σ^+ \rightarrow X^2Π (1,0)$ band of OH at approximately 283.55 nm in air [101, 103-106]. The pulse energy was monitored digitally using a fast photodiode and an oscilloscope to ensure operation within the linear fluorescence regime. The laser was formed into a sheet of height of 50 mm and a thickness of approximately 0.3 mm (FWHM) in the probe region using a two-lens telescope. Fluorescence in the range of 310 nm was isolated from Rayleigh and surface scattering using an Asahi high transmission, narrow band pass filter (10 nm FWHM at 310 nm) and collected at right angle (perpendicular to the laser sheet) onto the 1280 x 1024-pixel imaging array of a 10 Hz LaVision Imager Intense CCD camera coupled to a LaVision IRO image intensifier (gate = 100 ns) using a Sodern Cerco, 100 mm focal length, f/2.8 UV lens. The $Q_1(8)$ line was chosen for the OH-PLIF as the ground state population of $J_1 = 8$ state is relatively insensitive to temperature fluctuations. A LIFBASE simulation [107] showed that the population fraction varied by less than 10% for temperatures in the range of 1200 - 2300 K from the peak Boltzmann population fraction at around 1600 K. A flat flame burner was used for calibration of the laser wavelength and to correlate the signal intensity with the OH number density using previously performed absorption measurements.

### 6.2.3. Experimental Conditions

The experimental conditions along with details about the OH-PLIF and acoustic measurements setup are tabulated in Table 6.1.
### Experimental conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Details</th>
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<tbody>
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<td>Reynolds number Re</td>
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<td>Equivalence ratio ϕ</td>
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<tr>
<td>Swirl flow rate</td>
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<td>Main jet air flow rate</td>
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<td>Plasma power</td>
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### PLIF details

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<tr>
<td>POD domain</td>
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<tr>
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### Microphone details

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</tr>
<tr>
<td>Sensitivity</td>
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</tr>
</tbody>
</table>

*Table 6.1. Experimental conditions with OH-PLIF and Acoustic Measurement details summary.*
6.3. Results and Discussion

6.3.1. POD Convergence Analysis

The following analysis demonstrates the sensitivity of the POD to the ensemble data considered. The number of snapshots considered for the POD analysis is usually a critical parameter. A convergence analysis is performed by varying the number of snapshots used and by choosing two different sub-ensembles \( u_x(x,t) \) and \( u_y(x,t) \) composed of 150 snapshots each (ensemble A and B overlap by 50 samples). The results of the POD validation and convergence analysis are presented below. From Figure 6.2, it can be clearly seen that energy content of the first 10 dominant modes clearly converges within 100 snapshots. It should be stressed that the

![Figure 6.2. POD sensitivity / Convergence analysis: Variance distribution for ensembles of varying number of snapshots.](image)
rapid convergence of the POD results is due to the relatively low Reynolds number of this flow and performing POD analysis on turbulent flames may require many more samples [131].

The correlation coefficient $R(n)$ between the modes defined based on Eq. 6.1, computed from the ensemble data from two different 150 snapshots sub-ensembles $u_X(x,t)$ and $u_Y(x,t)$ is shown in Figure 6.3. Any inconsistencies between the modes computed from the two sub-ensembles would result in a sharp decrease in the value of the correlation coefficient discrepancies while ensemble-independent modes would lead to a correlation factor of $R(n) = 1$ [44, 131].

$$R(n) = \sqrt{\langle (\Phi_{n,X}(x), \Phi_{n,Y}(x)) \rangle}$$  \hspace{1cm} (6.1)

![Figure 6.3. POD sensitivity / Convergence analysis: Correlation coefficient between the modes from two different sub-ensembles.](image)
From Figure 6.3, it can be conclusively seen based on the value of the correlation factor that the first seven dominant modes (n < 7) have correlation factors very close to one and hence are ensemble independent; these modes are a feature of the flame and not of the ensemble chosen. However, in order to get better convergence, all the results presented are based on POD analysis carried out using the entire set of 200 snapshots. Figure 6.4 shows the reconstruction of a random snapshot by superimposing various number of dominant eigenmodes (10, 50, 100 and 200) obtained from POD analysis as opposed to the actual image. It is evident that the first 10 modes of the POD analysis are sufficient to capture the major features of the flow field thereby can be used as representative modes that portray the flame dynamics. As expected, the accuracy of reconstruction drastically improves as the number of modes considered is increased, and by using all 200 modes, the original snapshot can be fully reconstructed. It is also worth noting that the POD analysis presented here extracts non-trivial dynamics of a flame that otherwise would have been lost by use of classical averaging.

![Figure 6.4. POD sensitivity / Convergence analysis: Comparison of POD image reconstruction using varying number of modes and actual snapshot.](image)

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6.3.2. Summary of POD Analysis

The overall map of flame phenomenology is presented in Figure 6.5. From Figure 6.5, it can be seen that the lean blow off limit clearly demarcates the regions where flames can be sustained purely by swirl stabilization from flames that cannot be swirl stabilized and require plasma discharge for stabilization. The greyed region corresponds to experimental conditions that result in fairly unstable swirl stabilized flames near their extinction limit. Moving away from the lean blow off limit, as indicated by the direction of the arrows namely by increasing the swirl flow.

**Figure 6.5. Flame phenomenology map: Map of all experimental conditions highlighting the various regimes and characteristic test flame configurations C1, C2 and C3. Swirl flow rate: ◊ 20 SLPM, ○ 30 SLPM, □ 40 SLPM, △ 50 SLPM; Plasma power: black 0 W, red 35 W, blue 50 W, green 100 W, Violet 120 W; Open - Swirl flames, Dot - PAC flames). Main jet air flow rate maintained constant at 10 SLPM (Re ~ 2000).**
rate, the premixed equivalence ratio or a combination of both, resulted in enhanced flame stability. Based on the overall regime map, three particular configurations marked as C1, C2 and C3 are chosen. C1 is a fairly stable swirl stabilized flame, while C2 is an unstable flame close to the extinction limit and C3 cannot be swirl stabilized as it is below the lean blow off limit. The corresponding experimental conditions are summarized in Table 6.2. The effect of plasma discharge on these three distinct configurations and the corresponding changes in the pressure fluctuations measured by the microphone will be presented for discussion.

The overall summary of the POD analysis performed on all experimental conditions listed in Figure 6.5 is presented in Figure 6.6. It should be noted that since OH is a good indicator of heat release [132-135], all POD analysis performed on OH-PLIF data can be assumed to be a sound indicator of the energy content in these flames. The average energy content of the flames is computed as the variance of energy present in Mode 0 from the POD analysis as presented earlier.

From Figure 6.6, it can be clearly seen that flames that are close to the lean blow of limit have lower mean energy content (Mode 0) when compared to their corresponding stable counterparts. We note that the remainder of the energy \(1 - E(0)\) is present in other modes which correspond to heat release fluctuations. The mean energy content of swirl stabilized flames varies from ~ 70% to 90% depending on the stability of the flames. The effect of plasma on the stability of swirl stabilized flames under the same conditions can be clearly seen from the same figure. The addition of plasma is found to enhance the flame stability by improving the mean heat release (Mode 0) and therefore minimizing the heat release fluctuations (Mode > 0). It is important to note that the observed improvement in the mean heat release due to plasma addition is not merely due to the increased energy content of plasma assisted flames as the coupled plasma power corresponds to only a small fraction of the actual combustion power. The percent increase in the mean
energy content is found to be consistently much higher than the increase in combustion power due
to the addition of plasma discharge, which conclusively shows that the presence of plasma
modifies the flame dynamics of swirl stabilized flames. Plasma enhancement of the chosen
configurations is also highlighted in Figure 6.6 for better comprehension.

![Figure 6.6. Percentage of heat release present in Mode 0 (Mean) based on POD analysis of OH-PLIF data for all experimental conditions (Swirl flow rate: ◇ 20 SLPM, ○ 30 SLPM, □ 40 SLPM, △ 50 SLPM; Plasma power: black 0 W, red 35 W, blue 50 W, green 100 W, Violet 120 W; Open - Swirl flames, Dot - PAC flames). Main jet air flow rate maintained constant at 10 SLPM (Re ~ 2000).](image-url)
<table>
<thead>
<tr>
<th>Experimental Conditions</th>
<th>Configuration C1 &amp; C1-P</th>
<th>Configuration C2 &amp; C2-P</th>
<th>Configuration C3-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air flow rate</td>
<td>10 SLPM</td>
<td>10 SLPM</td>
<td>10 SLPM</td>
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<tr>
<td>Swirl flow rate</td>
<td>40 SLPM</td>
<td>20 SLPM</td>
<td>20 SLPM</td>
</tr>
<tr>
<td>Equivalence ratio</td>
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</tr>
<tr>
<td>Reynolds number</td>
<td>2020</td>
<td>2010</td>
<td>1990</td>
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<tr>
<td>Comments</td>
<td>Stable swirl flame</td>
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<td>Cannot be swirl stabilized</td>
</tr>
</tbody>
</table>

Table 6.2. Experimental conditions for three characteristic test configurations.

6.3.3. POD Mode Shapes and Flame Dynamics

The aim of applying POD analysis to OH-PLIF fields is to extract information about the flame dynamics. The mode shapes based on the POD analysis corresponding to the dominant eigenvalues for swirl stabilized flames and plasma assisted flames are shown in Figure 6.7 and Figure 6.8 respectively. The scale and the sign of the contour plots representing the mode shapes are arbitrary since the modes have been normalized and are multiplied by their corresponding time coefficients in the decomposition. It should be noted that Mode 0 represents the mean burning shape of the flame, while subsequent modes represent the fluctuations. The mode 0 of the swirl stabilized flames clearly demonstrate that they are lifted flames stabilized in the inner shear layer while the plasma assisted flames are anchored to the electrode as they tend to be stabilized in the center body wake. Higher modes show coherent structures that can be correlated with interesting flow features. These coherent structures are important for studying the inherent instability in the system. As the mode number increases, the wavenumber of this motion increases, and the frequency associated with the corresponding flow field (fluctuations) increases. In both cases, the
Figure 6.7. OH-PLIF POD modes 0 - 5 for plasma assisted flames.
Figure 6.8. OH-PLIF POD modes 0 - 5 for swirl stabilized flames.
mean flow field is followed by a pair of counter rotating vortices and the number of vortices increase with the mode number. Each of these modes and the corresponding coherent structures can be correlated to the heat release fluctuations that arise due to complex interactions between the flame front and the adjoining flow field established by swirling flow.

6.3.4. Effect of Equivalence Ratio and Swirl on Flame Dynamics

The effect of equivalence ratio and swirl flow rate on flame stability can be explained in terms of the detailed results of the POD analysis of OH-PLIF data presented in Figure 6.10 and Figure 6.9 respectively, both in terms of the mean energy content i.e., Mode 0 (top panel) and the heat release fluctuations for Modes 1 - 10 (bottom panel).

From Figure 6.10, it can be clearly seen that flames that are closer to the lean blow off limit marked in Figure 6.5 have much lower mean energy content (70.8% for $\phi = 0.85$) and are typically characterized by large heat release fluctuations as indicated by the energy content of the other dominant modes i.e., Modes 1 - 10, which make them unstable. On the other hands, flames that are far away from the lean blow off limit marked in Figure 6.5, have a much higher mean energy content (above 83% for $\phi = 1.0$ and higher) with reduced heat release fluctuations which make them comparatively more stable. The increase in the mean energy content of the flame and the corresponding decrease in heat release fluctuations can be explained in terms of the increased heat release as result of the additional combustion power output (increased fuel flow rate). Further, enrichment in the equivalence ratio also results in an increased flame speed which adds to the stability of the flame. However, a clear saturation effect can be seen as additional increase in the fuel flow rate results in negligible to almost no increase in the mean energy content.
Figure 6.9. Percentage of heat release based on POD analysis of OH-PLIF data present in Mode 0 (Mean) (top panel) Modes 1 - 10 (fluctuations) (bottom panel) for different equivalence ratios. No plasma coupling. Main jet air flow rate and swirl flow rate are maintained constant at 10 SLPM (Re ~ 2000) and 50 SLPM respectively.
Figure 6.10. Percentage of heat release based on POD analysis of OH-PLIF data present in Mode 0 (Mean) (top panel) Modes 1 - 10 (fluctuations) (bottom panel) for different swirl flow rates. No plasma coupling. Main jet air flow rate is maintained constant at 10 SLPM (Re ~ 2000) at an equivalence ratio of φ = 1.30.
From Figure 6.10, it can be clearly seen that the effect of swirl flow rate on overall flame stability is less pronounced when compared to that of equivalence ratio. Increasing the swirl flow rate from 20 SLPM to 50 SLPM increases the mean energy content of the flame from 86.4% to 89.6% with a corresponding decrease in the heat release fluctuations. Increase in the swirl flow rate strengthens the recirculation zone that results in improved flame stabilization leading to increased mean energy content and decreased heat release fluctuations. However, the improvement in the observed flame dynamics due to the increased swirl flow rate is more evident at leaner equivalence ratios as summarized by the mean energy content of the flames shown in Figure 6.6.

6.3.5. Effect of Plasma on Flame Dynamics

The detailed results from the POD analysis of OH-PLIF data carried out on the three characteristic flame configurations listed in Table 6.2 are presented in Figure 6.11 (configuration C1 and C1-P), Figure 6.12 (configuration C2 and C2-P) and Figure 6.13 (configuration C3-P), in terms of the mean energy content i.e., Mode 0 (top panel) and the heat release fluctuations for Modes 1 - 10 (bottom panel). From the top panels of Figure 6.11, Figure 6.12 and Figure 6.13, it can be clearly seen that in all the three configurations, the mean energy content of plasma assisted flames is clearly higher than the corresponding purely swirl stabilized flames.

In particular, the mode 0 energy content for the stable C1 flame is 79.1% while for the relatively unstable C2 flame, it is comparatively lower at 74.3%. We note that a C3 configuration cannot be swirl stabilized, so the corresponding mean energy content is not provided. Even with the minimum addition of plasma power name, 35 W, the mean energy content for both C1 and C2 configurations drastically increases to around 93%. This increase in the mean energy content of the flame is due to the stabilizing effect of the plasma and not due to higher energy content of the
Figure 6.11. Percentage of heat release based on POD analysis of OH-PLIF data present in Mode 0 (Mean) (top panel) Modes 1 - 10 (fluctuations) (bottom panel) for flame configurations C1 and C1-P.
Figure 6.12. Percentage of heat release based on POD analysis of OH-PLIF data present in Mode 0 (Mean) (top panel) Modes 1 - 10 (fluctuations) (bottom panel) for flame configurations C2 and C2-P.
Figure 6.13. Percentage of heat release based on POD analysis of OH-PLIF data present in Mode 0 (Mean) (top panel) Modes 1 - 10 (fluctuations) (bottom panel) for flame configuration C3-P.
plasma assisted flame, as the plasma power of 35 W corresponds to less than 5% of the combustion output. In addition to the improved mean energy content, the addition of plasma discharge results in decreased heat release fluctuations, as shown by the overall reduction in the energy content of the other dominant modes i.e., Modes 1 - 10 (bottom panels). Furthermore, it is to be noted, further addition in plasma power in both C1 and C2 configurations, results in negligible to almost no increase in the mean energy content. However, configuration C3 on the other hand, shows a considerable increase in the mean energy content of the flame from 73.5% to 83.5% as the plasma power is increased from 35 W to 120 W, although the saturating effect of the plasma on flame stability can be clearly seen. Though not shown here, higher order fluctuations (mode > 25 and beyond) were identically 0 for plasma assisted flames whereas for swirl stabilized flames higher order fluctuations cumulatively contributed around 10 - 12% of the total energy content of the flame.

The increase in mean energy content and the corresponding decrease in heat release fluctuations can be explained in terms of the differing flame dynamics arising due to the flame stabilization physics corresponding to swirl stabilized and plasma assisted flames. The presence of plasma discharge effectively decouples the flame chemistry from the fluid fluctuations, which results in decreased heat release fluctuations and improves the mean energy content. Thus, once this effective decoupling due to flame stabilization mechanism has been established, any increase in plasma power has no significant consequence as noted above. The details on the dynamics of flame stabilization in the presence or absence of plasma discharge is discussed in detail in the following section.

6.3.6. Reduction of Flame Dynamics

In previous work, we have reported on the differences in flame stabilization modes for the
swirl stabilized and plasma assisted cases [104]. To summarize, the swirl flame is clearly lifted off and stabilized in the inner shear layer (ISL) and is vigorously fluctuating. However, in the presence of the plasma discharge, the flame is strongly anchored to the electrode at the relatively quiescent center body wake (CB) with an observable decrease in the associated noise level. To quantify the differences in the sound pressure level between the swirl and plasma stabilized cases, acoustic measurements using a high temperature probe microphone were carried out. The raw microphone time signal and the corresponding power spectrum density for C1 and C1-P configuration is presented in Figure 6.14 and Figure 6.15 respectively. From Figure 6.15, it can be seen that there is a clear reduction in flame dynamics (acoustic oscillations) in the presence of the plasma discharge.

![Graph](image)

*Figure 6.14. Raw microphone time signal for flame configuration C1.*
Figure 6.15. Power spectrum showing the Root Mean Square pressure fluctuations for flame configurations C1 and C1-P.

Based on the observed frequency of the harmonics as indicated in Figure 6.15, an estimate for the average sound speed and the average temperature can be obtained using the standing wave formulation inside a closed tube as described below:

\[ f = \frac{c}{4} \sqrt{\left( \frac{n_x}{L_c} \right)^2 + \left( \frac{n_y}{B} \right)^2 + \left( \frac{n_z}{H} \right)^2} \]  \tag{6.2}

\[ c = \sqrt{\gamma RT} \]  \tag{6.3}

\[ L_c = L + 0.8 \sqrt{\frac{A}{\pi}} \]  \tag{6.4}

Where, \( f \) - Frequency of the harmonic for the standing wave inside the closed tube (Hz), \( c \) - Average speed of sound inside the quartz chamber (m/s), \( L, B, H \) - Dimensions of the quartz
chamber (m), \( n_x, n_y, n_z \) - Harmonic modes, \( L_{e} \) - Corrected length of the quartz chamber accounting for the end correction factor (m), \( A \) - Cross section of the quartz chamber (m\(^2\)), \( \gamma \) - Ratio of specific heat capacity, \( R \) - Specific gas constant (J/kg.K), \( T \) - Average temperature inside the quartz chamber (K).

The summary of this analysis for the data presented in Figure 6.15 has been tabulated in Table 6.3. It is important to understand that even though the theoretical analysis has been oversimplified (temperature is not constant throughout and acoustic waves can travel through the waveguide), it can be used to correlate and explain our experimental findings. From Figure 6.15

<table>
<thead>
<tr>
<th>Plasma power (W)</th>
<th>Frequency of first harmonic (Hz)</th>
<th>Average sound speed (m/s)</th>
<th>Average temperature based on sound speed (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>380</td>
<td>463</td>
<td>535</td>
</tr>
<tr>
<td>35</td>
<td>399</td>
<td>487</td>
<td>589</td>
</tr>
<tr>
<td>50</td>
<td>404</td>
<td>493</td>
<td>604</td>
</tr>
<tr>
<td>100</td>
<td>410</td>
<td>500</td>
<td>622</td>
</tr>
<tr>
<td>120</td>
<td>420</td>
<td>512</td>
<td>653</td>
</tr>
</tbody>
</table>

*Table 6.3. Summary of acoustic measurements for flame configurations C1 and C1-P.*

and Table 6.3, it can be seen that there is a strong shift in the fundamental frequency (and hence the corresponding harmonics) with the addition of plasma power. This shift in the fundamental frequency (~380 Hz no plasma as opposed to ~420 Hz, 120 W plasma) is due to significant increase in the burned gas temperature by ~ 22%. The increase in burned gas temperature is not merely due to the ohmic heating caused by plasma as the coupled plasma power corresponds to less than 5%
of the combustion output. Thus, the temperature increase is primarily due to the improved combustion as a result of the new reaction pathways opened up by the non-thermal effects of plasma i.e., inert nitrogen excitation [104].

Similar trends were observed for the other two cases as well, but the estimated mean temperature for C3-P (\(\sim 500 \text{ K}\)) was higher when compared to C1-P (\(\sim 487 \text{ K}\)) for the same plasma levels (35 W). This is highly counter-intuitive, as the latter case is expected to have higher mean temperature as it is operated at richer equivalence ratios (\(\phi = 1.00\)) when compared to the former (\(\phi = 0.85\)). This indicates that the effect of plasma discharge is more pronounced at leaner flame conditions, which further validates that the improved flame dynamics are due to the non-thermal effects of the plasma discharge rather than mere ohmic heating.

To further support the reduction in flame dynamics in the presence of plasma discharge, the overall RMS value of the pressure fluctuations (frequency integrated) for the three configurations is presented in Figure 6.16. It can be clearly seen that the plasma discharge results in a significant reduction in the pressure fluctuations in all the test cases. C1-P and C2-P showed an overall reduction of about 42% and 47%, respectively, in RMS pressure fluctuations when compared against their corresponding baseline cases, C1 and C2, even for the minimum plasma power addition of 35 W. This also supports the idea that non-thermal effects of the plasma discharge allow for stronger plasma-flame coupling at leaner equivalence ratios. It is also clear from test case C3-P that progressive increase in the plasma power actually over-energizes the flame, resulting in an undesirable increase in the RMS pressure fluctuations.

The reduction in flame oscillations in the presence of plasma can be explained in terms of the flame shape and flame location [97]. From the acoustic measurements and the POD analysis presented earlier, we have established that the presence of plasma discharge results in both
decreased heat release and pressure fluctuations. This indicates that in the presence of plasma discharge, the flame is comparatively steadier, likely due to the complete decoupling or minimal interaction between the combustion process and the inherent unsteadiness associated with the swirling flow field. This decoupling between the combustion process and the fluid unsteadiness can be explained by the flame stabilization location with and without the plasma discharge. As mentioned earlier, the swirl flames are stabilized in the ISL, which is prone to high levels of fluid unsteadiness. This unsteadiness is further enhanced as they are closely coupled with the acoustics of the combustion chamber, resulting in a feedback loop that results in a limit cycle oscillation, causing increased pressure fluctuations. All these factors cumulatively result in an unstable flame prone to acoustic and fluid perturbations. On the other hand, the plasma discharge stabilizes the

![Figure 6.16. Frequency averaged Root Mean Square pressure fluctuations for the three flame configurations for varying levels of plasma power.](image)
flame in the quiescent CB wake zone that is relatively free of unsteadiness, giving the flame a more robust response to any acoustic coupling. Thus, the presence of plasma discharge not only improves combustion due to its non-thermal effects but also improves the dynamics of flame stabilization by effectively decoupling the fluid unsteadiness from the flame oscillations i.e., the fluid-acoustic coupling is broken.

6.4. Conclusions

In this study, experimental findings that demonstrate the effectiveness of continuous, volumetric, direct coupled, non-equilibrium, atmospheric microwave plasma discharge for improving the flame dynamics in realistic combustor geometries are presented. A comprehensive procedure for extracting flame dynamics from high fidelity experimental data through a post-processing technique, Proper Orthogonal Decomposition (POD), has been demonstrated. The technique’s potential for extracting useful information about the flame dynamics from seemingly random samples that are typically lost by classical averaging has been demonstrated. In particular, POD applied to OH-PLIF images were shown to be insensitive to the choice of ensemble considered, which established that the modes extracted do indeed represent the features of the flame. Furthermore, the ensemble of statistical quantities obtained via POD analysis were used for characterizing the heat release fluctuation and thereby providing the basis for comparison of different flame configurations, revealing the effect of plasma discharge on flame dynamics and stability.

The POD analysis performed in this study established that even at coupled plasma powers corresponding to less than 5% of the thermal power output, there was significant reduction in the heat release fluctuations, leading to improved mean energy content (~23%) for the plasma assisted flames, which resulted in enhanced flame stability and improved combustor dynamics.
Nevertheless, a saturation limit exists, beyond which further addition of plasma had no effect in terms of improving the mean energy content of the flame. Furthermore, acoustic measurements indicated that the RMS pressure fluctuations decreased by up to 47% at minimal plasma levels; however, any further increase in the coupled plasma power “over-energized” the flame causing a slight increase in the pressure fluctuations. These findings were found to be in good agreement with the results of POD analysis and confirmed the presence of a saturation limit for control of combustor dynamics using microwave plasma discharge. In addition, significant improvement in burned gas temperature and better plasma coupling at leaner equivalence ratios confirmed that the improved combustion characteristics are due to non-thermal effects of the plasma that opened up new reaction pathways, resulting in accelerated chemical reactions.

In conclusion, it has been shown that plasma discharge in addition to its non-thermal effects, causes an “effective fluid-acoustic decoupling” i.e., flame oscillations are isolated from fluid unsteadiness through different flame stabilization mechanisms, which resulted in significant improvement in flame dynamics namely, decreased heat release and pressure fluctuations.
Chapter 7. Mesoscale Burner Array Combustion

Development of a stable and efficient small-scale combustor architecture with comparable performance emission characteristics to large-scale burners is presented. Furthermore, the proposed architecture reduced susceptibility to extinction, maintained high combustion efficiency and low emission levels under ultra-lean operating conditions for a wide range of combustion power outputs. Prototype burner arrays were additively manufactured and demonstrated with methane/air flames. The burner sustained lean flames ($\phi = 0.65$) independent of power output indicating good scalability. High combustion efficiencies (98%) were estimated using Gas Chromatography-Mass Spectrometry analysis of the exhaust gas. Combined unburned hydrocarbon (UHC) and carbon monoxide (CO) emission measurements were well below 0.1% by mass. Near-adiabatic flame temperatures with minimal spatial variations across the burner were observed due to enhanced flame interaction and reduced heat loss. Overall, this study successfully demonstrated the potential for a novel combustor architecture that can be scaled across a wide range of power outputs with minimal performance degradation.

7.1. Introduction

Over the past decade, there has been a growing interest within the combustion research community in the field of power generation at small scales. This recent interest has garnered wide motivation from the growing trends in the innovative approaches that have been developed to convert available energy into usable forms using micro and nanotechnologies, which can in turn contribute towards a sustainable energy development and address the ever-growing need for power generation in small scale applications [136, 137]. The progress made in micro-fabrication techniques has spurred exciting new opportunities for combustion applications, especially on
scales much smaller than previously explored [136-141]. The primary motivation for combustion power generation is the very high energy densities of hydrocarbon (HC) fuels [138-141] when compared to that of alternative energy carrier systems i.e, batteries. The potential advantage of hydrocarbon fuels over conventional batteries for use in miniaturized energy conversion systems, in terms of specific energy density is presented in Figure 7.1.

![Figure 7.1. Comparison of gravimetric and volumetric energy densities of various energy storage materials and technologies.](image)

Energy densities of HC fuel (on the order of 40 MJ/kg) are one or two orders of magnitude higher than that of commercially-available batteries as listed in Table 7.1. Even accounting for the rather inefficient ($\eta_c = 0.2$) conversion of HC fuel chemical energy to electrical energy, combustion can easily outperform state-of-the-art batteries (e.g., lithium batteries) for larger scale power systems [138, 139]. This promises that the overall power densities of the system will be
substantially larger than their macroscale counterparts, with the expectation that they operate with efficiencies that can compete with commercially available batteries [136, 140]. A successful research program in this area would lead to development of a widely-applicable distributed power source that would be capable of replacing batteries in micro-power generation systems for a variety of applications ranging from portable power sources to unmanned aerial vehicles (UAVs), satellite thrusters, and other military applications [136-138, 141].

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Energy Density (MJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane ($\eta_c = 0.2$)</td>
<td>10.9</td>
</tr>
<tr>
<td>Methanol ($\eta_c = 0.2$)</td>
<td>4.0</td>
</tr>
<tr>
<td>JP8 ($\eta_c = 0.2$)</td>
<td>9.2</td>
</tr>
<tr>
<td>Li-ion battery</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 7.1. Energy density of gaseous, liquid fuels and conventional battery with an assumed 20% efficiency of conversion of fuel chemical energy to electricity.

However, there are numerous problems that are inherent to developing a meso-scale combustion system, including: identification of promising energy conversion systems to convert the thermal energy into electricity, efficient heat recirculation and insulation of the hot end of the cycle from the cold one to achieve a decent thermodynamic efficiency [139]. Also, from a fundamental combustion perspective, flames at such increasingly small scales namely mesoscale (~ cm) and microscale (~ mm) flames are driven by completely different fluid physics and flame chemistry as opposed to their macroscale counterparts. Primarily, reduction in size, drastically increases the combustor surface-to-volume ratio, which is a rough indicator of the balance between heat loss and heat release rates. On a sufficiently small length scale, the high surface-to-volume ratio can lead to flame thermal quenching (extinction) as heat losses will be too high to sustain
stable combustion. Further, the absorption and destruction of active radicals generated during combustion which impact the flame stability is intensified as the combustor size decreases. Hence proper thermal and chemical stability management are required to establish stable combustion at such small length scales, which includes: careful burner design and optimization, a judicious selection of operating conditions and wall materials which ultimately determines the lower limit for the combustor dimensions [141].

In addition, the short residence times encountered in any small-scale combustor may prevent complete combustion of hydrocarbon fuel to end products, namely carbon dioxide (CO₂) and water (H₂O), that results in reduced heat release rates and pollutant formation such as CO and UHC. Since this combination of increased heat loss and radical quenching coupled with short residence time dictates the combustion process and the dynamic flame behavior, combustion at such small length scales is plagued by increased instabilities such as repetitive extinction and reignition [142-145], flame spinning [146], cellular flames [147] and flame streets [148] even at lower flow rates [137, 141, 143, 149-154]. Therefore, understanding the flame instability mechanisms (heat loss, surface interactions, and blowout) of micro and mesoscale combustion phenomena is critical to the successful optimization of small combustion system performance, efficiency, and durability.

However, our work takes a new approach to address the challenges of mesoscale combustors, by offering an array design that is focused on maximizing flame interaction between adjacent combustor nodes. This design incorporates swirl flows that have been shown to have a strong stabilizing effect on the flame due to its flow structure [79, 85]. Recirculation zones are created due to vortex breakdown and flow expansion that transport hot combustion products back upstream, mixing them with unburned reactants thereby resulting in improved flame stabilization
Most importantly, our approach offers a novel architecture that can be seamlessly scaled and adapted over a wide range of combustor outputs from those capable of powering large scale gas turbines to compact portable units to microscale systems without loss of efficiency or power to weight ratio in addition to improved heat loss management and flame stability characteristics.

The current work details several novel findings in the area of combustion energy conversion. This study demonstrates an advanced mesoscale burner array architecture that has been optimized for additive manufacturing processes. This innovative approach can deliver mesoscale combustors with enhanced energy efficiencies, that can match or potentially outperform existing large-scale burner designs in terms of both performance and emission characteristics with additional benefits of enhanced adaptability and scalability. The comprehensive experimental performance and emission characterization conducted on our optimal burner configuration for a wide range of operating conditions validates the ease of seamless combustor integration into multitude of power generation applications with minimal performance degradation. The data presented in this study is the first quantitative measurement of combustion efficiency and performance of a swirl-stabilized mesoscale combustor array and is expected to serve as a performance baseline for all future mesoscale combustor performance comparison. In conclusion, the potential for a compact, clean and efficient mesoscale swirl stabilized burner array architecture that can be scaled for a wide range of combustion power outputs, along with added ease of manufacture and integration for a multitude of applications without any serious performance degradation has been successfully demonstrated.
7.2. Experimental Setup

7.2.1. Mesoscale Burner Array

The mesoscale burner array consists of 16 individually swirl stabilized flames that are arranged in a counter rotating vortex pattern known as Taylor-Green vortex array in a 4x4 configuration. This arrangement allows for a single flame to propagate over the entire array after ignition. Each mesoscale burner array element consists of center bluff body surrounded by two tangential placed inlets for the premixed air-fuel mixture which causes the swirling motion. Thus, in this design, a combination of swirl and bluff body stabilization are used to induce sufficient product recirculation to achieve flame stabilization in the mesoscale burner array. The mesoscale burner array was metal 3D printed using Direct Metal Laser Sintering (DMLS), an advanced additive manufacturing technique. The mesoscale burner array is then enclosed in a conventionally machined burner housing equipped with fittings for supplying the required air fuel-mixture. All data presented are from premixed methane-air flames. The mesoscale burner array is capable of sustaining ultra-lean flames with combustion power output ranging from 150 to 3500 W. Air and fuel flow rates supplied to the burner array housing assembly were metered individually within ±1% using two laminar-flow, differential pressure, MKS mass flow controllers in standard liters per minute (SLPM), following the IUPAC standard temperature and pressure as 273.15 K and 1 atm. Reactant flow rates at the exit of the mass flow controllers were calibrated using a MesaLabs Definer 220 drum type gas flow calibrator. The reactants were fed at ambient conditions of 1 atm and 300 K to the mesoscale combustor.

7.2.2. Diagnostics Setup

A schematic layout of the complete experimental setup is shown in Figure 7.2. Temperature
measurements were carried out using Nordic Sensors type R (platinum - platinum 13%, rhodium) high temperature straight elements mineral insulated platinum thermocouple probe. The thermocouple setup was mounted on a translational stage so that it could be moved horizontally relative to the burner with a positioning accuracy of 0.1 mm. The temperature signal from the thermocouple was acquired using a LabView interface at 10 kHz via NI DAQ system which was cold junction compensated. The thermocouple data was then averaged for 100 seconds to compute the steady state temperature.

Figure 7.2. Schematic layout of the diagnostics setup used for mesoscale burner array performance and emission characterization.

For emission measurements, the primary diagnostic tool employed is an Agilent 6980N gas chromatograph (GC) equipped with an Agilent 5973N mass spectrograph (MS) detector. The exhaust gases were continuously fed into a loop volume and then injected into the GC by a gas
sampling valve. A HP-Plot Molesieve 5Å column was used to differentiate oxygen (O2) and CO, and an HP-Plot U was used to differentiate between O2/CO, CO2, nitric oxide (NO) and UHC. The operating pressure of the GC/MS system was maintained at 10⁻⁵ torr and helium (He) was used as the carrier gas for both columns. During emission measurements, a glass enclosure was used around the mesoscale burner array to prevent dilution of exhaust gases by ambient air. The sampling exhaust gas sampling probe was positioned ~ 25 mm above the burner surface, and axial profiles indicated that the flow was essentially non-reacting by this point. Time-resolved measurements indicated that a 60s delay before the commencement of data acquisition and gas sample injection was sufficient for to reach steady state sampling concentrations. A comparison between the mass spectrums obtained for each of the individual peaks from the total ion chromatogram (TIC) with the NIST database allowed for the correct identification of the gas species present. The peaks are then integrated to determine the mass fraction of each species present.

7.3. Results and Discussion

7.3.1. Flame Stabilization

Direct photographs of the three designs of 3D metal printed 4x4 mesoscale burner array in operation with methane air mixture under premixed configuration are shown in Figure 7.3. All three mesoscale burner array designs in Figure 7.3 generate compact, well distributed, and uniform flames under premixed lean operating conditions. There is minimal element to element variation between the individual flames in terms of flame shape and length for all the burner designs. These minor variations are due to differences in flow field around the individual burner elements caused by uncertainties between the bluff bodies and the diverging quarl associated with the
manufacturing process. The major differences between the three designs is the location of flame stabilization and the extent of flame to flame interaction. The individual flames in Designs 1 and 2 are almost isolated with minimal interaction as opposed to the stronger flame interaction observed in Design 3. This can be attributed to the recessed straight bluff bodies employed in first two designs when compared to the more conical bluff bodies that are flush with the burner exit plane in Design 3. This design change also creates a relatively stronger swirling motion which produces typical V shaped swirl stabilized flames ranging from 12-15 mm in length that are mutually supported by nearby burner elements due to strong flame interaction as visualized in Figure 7.4.

Figure 7.3. Photographs of the 3D metal printed 4x4 mesoscale burner array (a, b and c) operating with methane air mixture (d, e and f) in premixed configuration, respectively.
The typical flow field associated with swirl stabilization in a single burner element along with images of improved flame interaction (Design 3) is shown in Figure 7.4. Each burner element creates an annular swirling jet where the flame is typically stabilized in the low-velocity region of the shear layer or near the stagnation points [85]. The conical bluff center body (CB) typically divides the flow field into four main regions [108-110]: 1) outer recirculation zone (ORZ); 2) CB wake and inner recirculation zone (IRZ) which together constitute the central recirculation zone (CRZ); 3) high-velocity jet separating the IRZ and the ORZ; and 4) two annular shear layers, inner shear layer (ISL) between the jet and the IRZ, and the outer shear layer (OSL) between the jet and the ORZ, on either side of the annular jet. Swirling flows are prone to

![Flow field notation for a swirl stabilized flame: CB - center body, OSL - outer shear layer, ISL - inner shear layer, CRZ - Center Recirculation Zone, ORZ - Outer Recirculation Zone, PVC - Precessing Vortex Core. (b) Photographs showing the flame stabilization in a 3D metal printed 4x4 mesoscale burner array operating with methane air mixture in premixed configuration.](image)

Figure 7.4. (a) Flow field notation for a swirl stabilized flame: CB - center body, OSL - outer shear layer, ISL - inner shear layer, CRZ - Center Recirculation Zone, ORZ - Outer Recirculation Zone, PVC - Precessing Vortex Core. (b) Photographs showing the flame stabilization in a 3D metal printed 4x4 mesoscale burner array operating with methane air mixture in premixed configuration.
self-excited flow oscillations at higher flow rates and can form large scale helical coherent flow structures called the precessing vortex core (PVC) [156-161]. The PVC is characterized by a periodical off axis precession of the center of rotation. The individual mesoscale flames were strongly anchored to the center body stabilized by the combined effects of the ISL and CRZ [85, 111, 112, 156] as shown in Figure 7.4. This low speed region serves as continuous ignition source through recirculation of hot products. Also, the shearing effect due to outside air entrainment can be clearly seen along the boundary elements as they tend to appear slight elongated due to excessive strain. However, the flames supported by the center burner elements are more uniform indicating sufficient isolation from the any edge effects caused due to air entrainment.

7.3.2. Lean Blow-Off Limit

The flame stability characteristics of the mesoscale burner array under premixed operating conditions for three distinct burner designs are presented in Figure 7.5 and Figure 7.6 as a function of the power output per burner element and the exit Reynolds number, respectively. The LBO limit was evaluated by decreasing the fuel flow rate for a fixed air flow rate. As the fuel flow rate was gradually decreased, individual flame lift off was observed one by one until all the flames were completely lifted from the bluff body. The initiation or the onset of this sequential lift off phenomenon is defined as the first lift off while the condition where all flames were lifted off or extinguished is referred to as complete blow out (last lift off). The onset of the first lift off invariably sets in on the boundary burner elements as they were most prone to external disturbances such as rapid outside air entrainment and heat loss from the burner body to the relatively cold surroundings. Coupled with decreasing flame speed as a result of lower operating equivalence ratio, the combined effect ultimately initiates and propagates lift off across the mesoscale burner array. Conversely, when flow conditions were gradually transitioned from
blowout back to more stable region, propagation of flames over the entire array due to counter rotating vortices was observed. A single solitary flame always relit all the burner elements through cross burner flow interaction which demonstrated that the flames are mutually supported by one another. Furthermore, minimal variation was observed in the onset of first lift off and complete blow out during repeated trials carried out under similar experimental conditions. Hence, the LBO limits reported below have been estimated to show a less than 2% nominal deviation.

![Graph showing operational flame stability map as a function of burner power output for the 4x4 mesoscale burner array operating with methane air mixture in premixed configuration.](image)

**Figure 7.5.** Operational flame stability map as a function of burner power output for the 4x4 mesoscale burner array operating with methane air mixture in premixed configuration.

From Figure 7.5 and Figure 7.6, it is evident that there are drastic differences between the stable operating regimes of the three designs of the mesoscale burner arrays under investigation. These variations can be largely attributed to the differences in design between the three mesoscale arrays, which further exemplifies the need for optimal mesoscale burner design. Both Designs 1
and 2 exhibit a progressively increasing lean blow off as the burner output is increased. These designs can sustain a stable flame under lean operating conditions ($\phi < 1$) only when the power output per burner element was restricted to below 100 W. This in turn corresponds to a Reynolds number of limit of $Re < 250$ and $Re < 450$ for Design 1 and 2, respectively. Complying with modern day stringent emission and efficiency regulations, however, requires a practical limit on operating equivalence ratio to well below stoichiometric conditions ($\phi \sim 0.8$ or below), in effect restricting the power output of Design 1 and 2 to less than 50 W per burner element. This, in turn, drastically reduces mesoscale burner array scalability to match comparable existing macroscale burner power outputs.

![Operational flame stability map](image)

*Figure 7.6. Operational flame stability map (initiation of lift off) as a function of Reynolds number for 4x4 mesoscale burner array operating with methane air mixture in premixed configuration.*
Several design changes were implemented in Design 3 to improve the lean blow off characteristics and to ensure scalable high-power density based on the preliminary results. Design 3 was printed in a monolithic architecture to avoid sealing issues observed in the multi-layer architecture adopted in Design 1 and 2. Flame stabilization was further improved via non-recessed conical bluff bodies, which promote swirling motion and ensure sufficient recirculation between swirl and CB wake. Small recirculation zones observed in Design 1 and 2 is the likely cause of early blowouts indicated in Figure 7.5 and Figure 7.6. Also, the use of non-recessed bluff bodies ensured that flames stabilize at the burner element exit and promote flame to flame interaction between its immediate surrounding burner elements.

These design changes resulted in drastically improved mesoscale combustion performance characteristics as shown in Figure 7.5 and Figure 7.6. The LBO limit for Design 3 ($\phi = 0.65$) is found to be almost independent of the operating Reynolds number ($Re \sim 40$ to 600) over a wide range of power output per burner element (10 W to 200 W). The improved performance characteristics coupled with exceptional stability under ultra-lean operating conditions ensure that the optimized mesoscale burner array can be successfully scaled up to reach realistic combustion power outputs with no significant performance degradation. All further discussion and analysis presented is based on this optimized mesoscale burner array (Design 3).

### 7.3.3. Combustion Efficiency and Emission Measurements

In addition to LBO, another important performance characteristics of any burner is its hydrocarbon conversion efficiency. Investigating and characterizing exhaust gas composition under normal operating conditions is therefore a critical process in evaluating the mesoscale burner array. Major combustion products and light species ($N_2$, $O_2$, $CO_2$, $CO$, $CH_4$) were sampled downstream of the burner exit and measured using an Agilent 6980N gas chromatograph (GC)
equipped with an Agilent 5973N mass spectrograph (MS) detector.

A combination of HP-Plot U and HP-Molsieve 5Å columns were used to separate exhaust gas components. A typical total ion chromatogram (TIC) of the exhaust gas obtained from GCMS analysis using the two columns is shown in Figure 7.7. Based on the difference in retention times between the two columns, the individual exhaust gas components were successfully isolated.

![Normalized TIC spectrum of exhaust products](image)

**Figure 7.7.** Normalized TIC (Total Ion Chromatogram) spectrum of the exhaust products from the 4x4 mesoscale burner array operating with methane air mixture in premixed configuration. ($\phi = 0.7$, 55 W power output per burner element, 20 SLPM air flow rate).

Column-specific retention times for various gas species present in the exhaust gas stream are also indicated in Table 7.2. The exhaust gas species can be accurately identified by comparing the mass spectra obtained from peaks in the TIC against the NIST database. The peaks were then integrated to determine the mass fraction of each species present. All mass fraction values reported are on
dry basis as water vapor from the exhaust gas stream are removed before GCMS analysis. The emissions measurements were repeated at least three times, showing deviations between 0.5% and 5% of the quantified values. The related analysis and combustion efficiency calculation presented below are based on the averaged values that are reported here.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Column</th>
<th>Retention Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air / Carbon monoxide (CO) / Methane (CH₄)</td>
<td>HP-Plot U</td>
<td>1.711</td>
</tr>
<tr>
<td>Carbon dioxide (CO₂)</td>
<td>HP-Plot U</td>
<td>1.926</td>
</tr>
<tr>
<td>Argon (Ar)</td>
<td>HP-Molesieve 5Å</td>
<td>2.115</td>
</tr>
<tr>
<td>Oxygen (O₂)</td>
<td>HP-Molesieve 5Å</td>
<td>2.142</td>
</tr>
<tr>
<td>Nitrogen (N₂)</td>
<td>HP-Molesieve 5Å</td>
<td>3.516</td>
</tr>
<tr>
<td>Methane (CH₄)</td>
<td>HP-Molesieve 5Å</td>
<td>4.697</td>
</tr>
<tr>
<td>Carbon monoxide (CO)</td>
<td>HP-Molesieve 5Å</td>
<td>5.793</td>
</tr>
</tbody>
</table>

*Table 7.2. Column specific retention times for various gas species.*

For the baseline operation of lean CH₄-air mixture (ϕ = 0.7) and a net power output of 55 W per burner element, GCMS measurements of exhaust gas mass fraction in a dry sample yielded 80.7% N₂, 7.2% O₂, 11.7% CO₂ and 0.05% CH₄ and CO combined. Other minor species like H₂ and NO were well below the detection threshold of the GCMS apparatus. The corresponding composition of the exhaust gas mixture for complete combustion can be easily calculated as 80.9% N₂, 7.3% O₂ and 11.8% CO₂ from stoichiometry. The results of GC-MS combustion efficiency analysis are summarized in Table 7.3. The close agreement between the measured exhaust gas composition to the theoretical chemical balance indicate very efficient combustion process in the mesoscale burner array at the specified condition. Assuming the observed deficit of 0.1% in the measured mass fraction is combustible material and hence considered as loss, the combustion
efficiency of the mesoscale burner array is conservatively estimated to be in excess of 98%.

<table>
<thead>
<tr>
<th>Mass Fraction Y</th>
<th>GCMS Analysis (%)</th>
<th>Stoichiometry Analysis (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen (N₂)</td>
<td>80.7</td>
<td>80.9</td>
</tr>
<tr>
<td>Oxygen (O₂)</td>
<td>7.2</td>
<td>7.3</td>
</tr>
<tr>
<td>Carbon dioxide (CO₂)</td>
<td>11.7</td>
<td>11.8</td>
</tr>
<tr>
<td>Carbon Monoxide (CO) + Methane (CH₄)</td>
<td>0.05</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.3. Exhaust gas composition for lean methane-air mixture $\phi = 0.7$ and a net power output of 55 W per burner element

The CO₂/(CO+CH₄) mass ratio in the exhaust gas, a very good measure of the combustion efficiency [138, 139], is presented in Figure 7.8 along with the combined mass fraction of CH₄ and CO in the exhaust gas as a function of the power output per burner element for fixed equivalence ratio of $\phi = 0.7$. High values of CO₂/(CO+CH₄) ratio correspond to very efficient and clean combustion processes. Figure 7.8 clearly shows relatively large values (~200) of CO₂/(CO+CH₄) ratios under lean operating conditions over a wide range of burner power outputs for the mesoscale burner array, indicating clean and efficient combustion processes. In addition, extremely low combined CO and CH₄ mass fraction of less than 0.06% corroborate CO₂/(CO+CH₄) ratio measurements. Since the equivalence ratio is fixed, the power output of the burner (fuel flow rate) is inversely proportional to the residence time in the burner. Therefore, the combustion efficiency of the mesoscale burner array is largely independent of the residence time. This is attributed to the optimized burner design that enhances swirl stabilization, which in turn facilitates heat recirculation from the exhaust gas products back to the incoming reactants. The accelerated combustion chemistry prevents hydrocarbon conversion process from being kinetically limited.
Figure 7.8. Combined CH$_4$+CO mass fraction and CO$_2$/(CH$_4$+CO) ratio on a mass basis as a function of burner power output for the 4x4 mesoscale burner array operating with methane air mixture in premixed configuration (ϕ = 0.7).

Next, the mesoscale burner array combustion and emission characteristics were investigated under various equivalence ratio. The measured CO$_2$/(CO+CH$_4$) mass ratio and the combined mass fraction of CH$_4$ and CO in the exhaust gas is presented in Figure 7.9, as a function of the operating equivalence ratio. The fuel flow rate was gradually varied while the air flow rate was held constant to control the equivalence ratio ϕ without changing the Reynolds number (Re ~ 160). Based on the reported values of CO$_2$/(CO+CH$_4$) mass ratio and the combined mass fraction of CH$_4$ and CO in Figure 7.9, along with the analysis presented earlier, it is evident that the mesoscale burner array can be successfully operated at ultra-lean premixed conditions (0.6 < ϕ < 0.75) over a wide range of burner power output with high combustion efficiency (above 98%).
comparable to that of current generation macroscale burners.

![Combined CH₄+CO mass fraction and CO₂/(CH₄+CO) ratio on a mass basis as a function of equivalence ratio for the 4x4 mesoscale burner array operating with methane air mixture in premixed configuration (20 SLPM air flow rate).]

7.3.4. Temperature Measurements

7.3.4.1. Temperature Radiation Correction

All temperature values reported are corrected for radiation losses which can be significant due to the high gas temperature (≈1500 K) encountered. Conduction loss to the thermocouple extension wire can be neglected since the wires were aligned along isotherms in the flow field. Hence, convection-radiation energy balance [162] at the thermocouple bead was carried out to account for radiation losses as described below.
\[ T_g = T_b + \frac{\varepsilon_b \sigma b D b}{Nu_b k_g} (T_b^4 - T_\infty^4) \]  

(8.1)

\[ Nu_D = 2 + \left( 0.4 Re_D^{1/2} + 0.06 Re_D^{2/3} \right) Pr^{4/3} \left( \frac{\mu}{\mu_s} \right)^{1/4} \]

(8.2)

For \(0.71 < Pr < 380; 3.5 < Re_D < 7600; 1.0 < \left( \frac{\mu}{\mu_s} \right) < 3.2\)

Where, \(T_g\) - Corrected (actual) gas temperature (K), \(T_b\) - Thermocouple bead (measured) temperature (K), \(T_\infty\) - Ambient temperature (K), \(\varepsilon_b\) - Emissivity of the platinum bead, \(\sigma\) - Stefan-Boltzmann constant \((5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4})\), \(D_b\) - Bead diameter (m), \(k_g\) - Thermal conductivity of the gas at temperature \(T_g\) (Wm\(^{-1}\)K\(^{-1}\)), \(\mu\) - Dynamic viscosity of the gas at temperature \(T_g\) (Pa.s), \(\mu_s\) - Dynamic viscosity of the gas at temperature \(T_b\) (Pa.s), \(Nu_D\) - Nusselt number at the bead, \(Re_D\) - Reynolds number at the bead computed at temperature \(T_g\), \(Pr\) - Prandtl number at the bead computed at temperature \(T_g\).

The emissivity of platinum bead at temperatures higher than 1200 K can be expressed as follows [163]:

\[ \varepsilon_b = 1.108 \times 10^{-4} T_b + 0.014 \]

(8.3)

All the required transport properties were evaluated using a transport and chemical equilibrium solver CHEMKIN developed by Reaction Design [164]. Since the non-dimensional numbers \((Nu_D, Re_D, \text{and} Pr)\) are to be computed based on the actual gas temperature \(T_g\), an iterative procedure was adopted based on an initial guess for the temperature i.e., measured temperature \(T_b\), until the solution converged (within 1 K). For the typical operating conditions, radiation temperature corrections were in the range of 100 - 200 K. It is to be noted that due to the catalytic effects of platinum/rhodium surface in the chemically reactive region of the flame,
reaction rates are enhanced leading to a higher observed temperature than the actual value. However, in this study, catalytic effects were neglected, as all thermocouple temperature measurements were made well above the flame reaction zone.

7.3.4.2. Spatial Temperature Measurements

Spatially resolved temperature measurements were carried out to evaluate the performance of the mesoscale burner array in terms of heat loss and element to element variation within the array (combustion uniformity). The spatial temperature distribution of the mesoscale burner array under premixed conditions ($\phi = 0.7$, 55 W power output per burner element) measured at an axial distance, $z = 20$ mm from the exit plane of the burner is shown in Figure 7.10. Temperature measurements were carried out along three distinct transverse lines marked A, B and C as indicated. The relative location of these transverse sweeps A, B, C on the burner array along with the position markers for spatially resolved temperature measurement are shown in Figure 7.11. The adiabatic flame temperature estimated to be 1802 K using CHEMKIN under these conditions is also shown in Figure 7.10 for comparison. Based on the averaged thermocouple measurements, temperature fluctuations are estimated to be within 5-10 K (less than 0.5%) for the center burner elements and within 10-15 K (less than 1%) for the edge burner elements.

From Figure 7.10, it is clear that there is no significant variation in the spatial temperature distribution along the three transverse sweeps due to the presence of the counter rotating (Taylor Green) vortex pattern. However, all three transverse sweeps indicate a steep temperature gradient along the edges of the burner, most likely due to a combination of cold air entrainment from the surrounding air and heat loss from the burner boundary. It is to be noted that, despite this entrainment at the boundary burner elements which causes the trailing edge temperatures, the
Figure 7.10. Spatial temperature distribution at \( z = 20 \text{ mm} \) for 4x4 mesoscale burner array operating with methane air mixture in premixed configuration (\( \phi = 0.7 \), 55 W power output per burner element, 20 SLPM air flow rate).

Uniform temperature distribution profile remains relatively uniform throughout the center of the mesoscale burner array in close agreement to the indicated adiabatic flame temperature. Uniform temperature distribution

Figure 7.11. Spatial location of temperature measurements along the mesoscale burner array.
indicates that there is minimal element to element variation within the mesoscale burner array due to the optimal burner design. Furthermore, edge effects and heat loss from the burner have little effect for most part of the burner, with the exception of the boundary burner elements. Thus, it can be concluded that a 4x4 mesoscale burner array can well represent larger mesoscale burner arrays (scaled by adding more elements) as the center burner elements (2x2) are sufficiently isolated from relatively stronger edge effects. These results show that mesoscale burner arrays can be successfully scaled up to realistic combustion power outputs comparable to that of practical power systems without any performance deterioration.

7.4. Conclusion

A compact, clean, efficient and scalable mesoscale swirl stabilized burner array has been developed and manufactured using advanced additive manufacturing technique (DMLS). Challenges involved in mesoscale combustion were circumvented via design optimization such as use of combined swirl and bluff body stabilization, flow divergence, improved heat recirculation and increased mutual flame interaction. Use of counter rotating vortex pattern ensured flame propagation over the entire array after initial ignition.

Operation of the optimized mesoscale burner array under premixed conditions resulted in improved performance characteristics coupled with exceptional flame stability under ultra-lean conditions. Further, stable LBO limits ($\phi = 0.65$) over a wide range of operating conditions (0.1 kW to 3 kW total power output) were observed. Exhaust gas analysis indicated no gaseous pollutants except for some small amounts of UHC and CO (below 0.1%), corresponding to high levels of combustion efficiency (above 98%) despite the relatively short residence time and associated heat losses. Spatially resolved temperature measurements indicated minimal element to element variation with temperature distribution reaching up to adiabatic flame temperature levels.
with slight temperature drop at the boundary due to edge effects caused by surroundings air entrainment. Thus, it is evident that the mesoscale burner array can be successfully scaled up to realistic combustion power outputs without any performance deterioration.

In conclusion, the potential for an optimized scalable mesoscale burner array architecture with power density, performance and emission characteristics that can match and possibly outperform existing macroscale burner designs for current power generation systems has been successfully demonstrated. Easy and rapid manufacturing of such scalable mesoscale burner arrays by advanced additive manufacturing techniques (DMLS) offers added benefit of easy integration with direct energy conversion modules.
Chapter 8. Mesoscale Burner Array Combustion Dynamics

The primary aim of this work is to establish the effectiveness of enhanced flame to flame interaction in a mesoscale burner array in improving combustor flame dynamics and stability through minimizing heat release fluctuations. Flame phenomenology, flame interaction and flame stabilizing mechanism in the mesoscale burner array was characterized using multispectral (OH, CH and HCHO) PLIF imaging. Further, analysis of mesoscale flame dynamics of self-excited and acoustically excited flame oscillations based on advanced decomposition techniques using high speed imaging is presented. Dynamic mode decomposition (DMD) analysis based on high speed OH-PLIF images was carried out to provide a quantitative measure of flame stability. DMD can fully characterize and quantify dynamically relevant coherent structures in a complex flow field by projecting it onto a simplified dynamical system with significantly fewer degrees of freedom. Coherent structures are extracted from the experimental data sets and appropriately attributed to dominant flame dynamics without a priori knowledge of the reactive flow field. Acoustic forcing at different frequencies and amplitudes were induced using a speaker. Dominant spatial structures and their energy contents in the recirculation zone or shear layer were accurately resolved with DMD. In addition to the global frequency response spectrum, DMD can provide detailed descriptions of spectrally pure coherent features that can be systematically correlated to underlying physics that drives combustion instability and provide a consistent interpretation. The results show a marked improvement in combustion stability for a mesoscale burner array compared to a single swirl-stabilized flame with similar power output. The findings presented in this study enable a step forward for experimental community in flame oscillation and combustion instability analysis. The key insight regarding complex interactions between acoustics, fluid mechanics and combustion will ultimately serve a critical role in developing detailed combustion instability models.
8.1. Introduction

Advanced combustion systems designed to operate under lean, premixed conditions have great potential to substantially curtail emission and satisfy regulatory requirements [31]. Such systems, however, are more susceptible to combustion instability, which in turn can negatively impact performance and durability [32]. Understanding and controlling the onset and propagation of combustion instability is therefore critical to the development of clean and efficient combustion systems [33]. Numerous experimental [14, 34-37] and theoretical [38, 39] studies have made significant progress in understanding the key dynamics that affect combustion instability such as oscillatory heat release [40], acoustics [41], and equivalence ratio fluctuations[42].

In spite of these contributions, limited experimental data coupled with inadequate analysis techniques [43] have inhibited detailed combustion instability model development until the past few years. Coherent structures across multiple time and length scales can drive combustion instability. However, resolving both temporal and spatial structures from image-based experimental data can be very challenging. Global stability analysis techniques utilizing model equations such as the linearized Navier-Stokes equation or its variant, such as the Arnoldi method [165, 166], are limited to simulations as model equations cannot be trivially extracted from experimental data. The solution is to extract the dynamics of embedded coherent structures based on experimental data alone, which in many cases are a series of images. A common approach for extracting spatial structures (modes) is Proper Orthogonal Decomposition (POD) [48]. The POD algorithm ranks mutually orthogonal spatial modes by their energy content. However, this technique is ill-suited for combustion instability analysis for two reasons. First, energy content may not be the only appropriate measure of ranking contributions of each mode to the overall
combustion instability mechanism. Second, temporal dynamics cannot be resolved since mutually orthogonal spatial modes are spectrally contaminated.

One approach for analyzing coherent structures and their frequency response is through Dynamic Mode Decomposition (DMD) [50]. It does so by constructing a lower-dimension companion matrix from a series of images which form a Krylov sequence under linear-tangent approximation. The eigenvalues and eigenvectors of this newly constructed matrix approximate those of a nonlinear higher dimension system matrix governing the combustion phenomenon [65]. A detailed and rigorous mathematical description can be found in [43, 50] and is discussed in detail in the earlier chapter.

Successful DMD analysis is predicated on the proper selection of $\Delta t$ (time between images) and the number of images in the sequence. The former determines the range of frequencies that can be extracted while the latter determines the accuracy of the eigenvalue approximation. In addition, DMD is used to perform subdomain and spatial stability analysis. The former allows for analysis of localized regions within an experimental image with minimal effort since DMD does not rely on the system matrix nor the relevant boundary conditions. The latter enables analysis of spatially evolving dynamics by simply mapping one spatial location to the next using the same algorithm [43, 50].

The combustion community has only recently begun to adopt POD and DMD and as such only a small but growing number of studies have been published [167-173]. Consequently, very little is known about the significance of Ritz value, growth factor, and global energy norm (energy content) or correlated the dynamics modes to underlying physics that drive combustion instability phenomenon. In this current work, we seek to extend our understanding of the combustion dynamics associated with mesoscale burner arrays to gain fundamental insight into how flame
interaction in mesoscale burner arrays can improve combustor flame dynamics by employing advanced algorithm-based decomposition techniques to high fidelity experimental data. Specifically, we focus on the fundamental interactions between acoustic waves and unsteady combustion heat release and are interested in elucidating these effects at specific frequencies that may contribute to the generation of combustion instabilities. DMD results show very good agreement with the observed flame behavior and provide detailed quantitative characterization of key physical mechanisms based on few coherent modes that can adequately describe the overall behavior of mesoscale flame oscillation and stability. The analysis also shows a marked improvement in combustion stability for a mesoscale burner array compared to a single swirl-stabilized flame with similar power output.

8.2. Experimental Setup

8.2.1. Combustor Setup

8.2.1.1. Mesoscale Burner Array

The mesoscale burner array consists of 16 individually swirl stabilized flames that are arranged in a counter rotating vortex pattern known as Taylor-Green vortex array in a 4x4 configuration. This arrangement allows for a single flame to propagate over the entire array after ignition. Each mesoscale burner array element consists of center bluff body surrounded by two tangential placed inlets for the premixed air-fuel mixture which causes the swirling motion. Thus, in this design, a combination of swirl and bluff body stabilization are used to induce sufficient product recirculation to achieve flame stabilization in the mesoscale burner array. The mesoscale burner array was metal 3D printed using Direct Metal Laser Sintering (DMLS), an advanced additive manufacturing technique.
8.2.1.2. Single Swirl Burner

A single swirl burner was used to benchmark and highlight the improved flame stability characteristics of the mesoscale burner array under externally imposed acoustic forcing. The single swirl burner consists of radial swirler made of eight curved vanes with a $60^\circ$ vane angle, equally distributed around the circumference of a 70 mm diameter cylindrical hub. The vanes are 12 mm thick, and the inlet passage is 25 mm deep axially. The geometric swirl number [79] is estimated to be 1.15. A 12-mm diameter, $45^\circ$ conical bluff-body in centrally integrated in the swirler to enhance the swirling motion and flame stabilization. In order to have comparable power output under similar flow conditions, the exit area of the single swirl burner was designed to match the total flow area of the mesoscale burner array.

8.2.1.3. Burner Housing with Acoustic Forcing

The burner housing shown in Figure 8.1 was designed capable of accommodating either the mesoscale burner or the single swirl burner individually. This was done to minimize and match the effects of any combustor dynamics associated with the combustion chamber between the two burner configurations. This common burner housing ensured that the observed response to external acoustic forcing is solely due to the difference in flame stabilization mechanisms between the two burner configurations and not because of the difference in combustion chamber geometry. The burner housing consists of a 100 mm diameter, 75 mm long plenum chamber fitted with flow straighteners on the upstream end. The downstream end of the plenum chamber houses the mesoscale burner array or the single swirl burner and is fitted with a quartz glass tube, 100 mm long with a $100\times100$ mm square cross section, to provide complete optical access and to mimic
realistic combustor geometries. The quartz glass tube also serves to maintain the swirling motion inside the combustor and to prevent the entrainment of outside air.

Figure 8.1. CAD image of the burner housing with mesoscale burner array and single swirl burner along with acoustic forcing setup.

An 8 inch Dayton Audio subwoofer was also attached to the upstream end of the plenum chamber below the flow straighteners to acoustically excite the flow. The frequency of the acoustic perturbations was controlled by an Agilent waveform generator while the amplitude (sound pressure level) of the acoustic perturbations was controlled by a Pyle Pro 1400 W power amplifier. The amplitude of the waveform generator output was fixed to be 500 mV peak to peak, while the forcing frequency (single component sinusoidal variation) was varied between 0 Hz and 370 Hz. The burner housing is also equipped with fittings for supplying the required air fuel-mixture. All data presented are from premixed methane-air flames. Air and fuel flow rates supplied to the burner
array housing assembly were metered individually within ±1% using two laminar-flow, differential pressure, MKS mass flow controllers in standard liters per minute (SLPM), following the IUPAC standard temperature and pressure as 273.15 K and 1 atm. Reactant flow rates at the exit of the mass flow controllers were calibrated using a MesaLabs Definer 220 drum type gas flow calibrator. The reactants were fed at ambient conditions of 1 atm and 300 K to the combustor.

8.2.2. Diagnostics Setup

8.2.2.1. Multispectral PLIF Imaging Setup

A schematic layout of the simultaneous multispectral (HCHO/CH/OH) PLIF imaging setup is shown in Figure 8.2. The CH/OH Planar Laser Induced Fluorescence (CH/OH-PLIF) system used the second harmonic output of a Spectra Physics Quanta Ray, Nd: YAG laser (532 nm, 500 mJ/pulse) operating at 10 Hz to pump a Sirah Precision Scan dye laser containing DCM dye dissolved in ethanol. The red dye laser beam was then frequency doubled using an integral β-barium-borate doubling crystal. The dye laser output (12 mJ/pulse, 0.1 cm⁻¹ spectral line width at 310 nm) was then tuned to excite specific transitions based on the targeted species (CH, OH or combined CH-OH) as listed in Table 8.1. Due to the overlap between the $C^2\Sigma^+ \rightarrow X^2\Pi$ (0,0) band of CH and $A^2\Sigma^+ \rightarrow X^2\Pi$ (0,0) band of OH at around 310 nm, a judicious choice of excitation transitions and filtering schemes can be used to obtain CH and OH-PLIF images either simultaneously or separately with a single laser and camera configuration [174-176]. The pulse energy was monitored digitally using a fast photodiode and an oscilloscope. The laser was formed into a sheet of height of 50 mm and a thickness of approximately 0.3 mm (FWHM) in the probe region using a two-lens telescope. Fluorescence was isolated from background scattering and visible flame emission using two custom Semrock AFRL-0002 long-wave-pass filters and a Schott
Fluorescence signal collected at right angle (perpendicular to the laser sheet) using a 100 mm focal length, f/2.8 Sodern Cerco UV lens was focused onto the 1280 x 1024-pixel imaging array of a 10 Hz Andor iStar intensified (gate = 100 ns) CCD camera.

<table>
<thead>
<tr>
<th>Species</th>
<th>Transition</th>
<th>Wavelength (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OH</td>
<td>Q(_1) (12) line of A(^2\Sigma^+) → X(^2\Pi) (0,0) band</td>
<td>310.60</td>
</tr>
<tr>
<td>CH</td>
<td>R(_1) (13) + R(_2) (13) line of C(^2\Sigma^+) → X(^2\Pi) (0,0) band</td>
<td>310.69</td>
</tr>
<tr>
<td>Combined</td>
<td>R(_1) (8) + R(_2) (8) line of C(^2\Sigma^+) → X(^2\Pi) (0,0) band</td>
<td>311.96</td>
</tr>
<tr>
<td>CH-OH</td>
<td>Q(_1) (15) line of A(^2\Sigma^+) → X(^2\Pi) (0,0) band</td>
<td></td>
</tr>
<tr>
<td>HCHO</td>
<td>Multiple rotational lines of ( \tilde{A}<em>{1} A</em>{2} \rightarrow \tilde{X}<em>{1} A</em>{1} 4_{0}^1 ) band</td>
<td>355</td>
</tr>
</tbody>
</table>

Table 8.1. Targeted transitions and corresponding excitation wavelengths used for selected species in multispectral PLIF imaging.

The HCHO (formaldehyde) Planar Laser Induced Fluorescence (HCHO-PLIF) system used the third harmonic output of a Spectra Physics Quanta Ray, Nd: YAG laser (355 nm, 150 mJ/pulse) operating at 10 Hz. The 355 nm laser beam is capable of exciting multiple rotational levels within the \( \tilde{A}_{1} A_{2} \rightarrow \tilde{X}_{1} A_{1} 4_{0}^1 \) vibrational band of HCHO [135, 177-180]. For simultaneous multispectral PLIF imaging, the 355 nm laser beam (HCHO-PLIF) was made to be spatially coincident with 310 nm laser beam (CH/OH-PLIF) using a beam combiner (a short pass dielectric mirror which reflects 355 nm beam while allowing the 310 nm beam to pass through) before being expanded out into a laser sheet of height 50 mm and a thickness of approximately 0.3 mm (FWHM) in the probe region using the same set of sheet forming optics (two-lens telescope). To prevent cross talk between the two laser-based imaging systems, a temporal delay of 250 ns was set.
between the pulses of the 310 nm and 355 nm laser beams. Broadband fluorescence in the range of 375 to 425 nm was isolated from background scattering and flame emissions using an Edmund Optics high transmission, band-pass filter (50 nm FWHM at 400 nm). Fluorescence signal collected at right angle (perpendicular to the laser sheet) using a 100 mm focal length, f/2.8 Sodern Cerco UV lens was focused onto the 1280 x 1024-pixel imaging array of a 10 Hz LaVision Imager Intense CCD camera coupled to a LaVision IRO image intensifier (gate = 100 ns) mounted on the

Figure 8.2. Schematic layout of the diagnostics setup used for 10 Hz simultaneous multispectral (HCHO/CH/OH) PLIF imaging.
opposite side of the CH-OH imaging system.

8.2.2.2. High Speed OH Chemiluminescence Imaging Setup

For high speed OH chemiluminescence imaging, chemiluminescence signal from the mesoscale burner array was isolated using an Asahi high transmission, narrow band pass filter (10 nm FWHM at 310 nm). The chemiluminescence signal collected across a horizontal cross section (top view) of the mesoscale burner using a 100 mm focal length, f/2.8 Sodern Cerco UV lens was focused onto the 896 x 848-pixel imaging array of a high speed Photron SA-5 CMOS camera coupled to a Hadland image intensifier. The intensifier was gated to 10 μs at 10-kHz framing rates. Acoustic measurements were carried out using a PCB Model 377B26 High Temperature Probe microphone flush. The raw signal was then conditioned using a PCB Model 480B21 ICP sensor signal conditioning unit. The pressure signal from the microphone was acquired using a LabView interface via NI DAQ system at 16 kHz (16000 samples, 1 s sampling time). The microphone data was then averaged over 100 waveforms. The acquired pressure signal is indicative of the acoustic perturbations. Further post-processing of the microphone data involved application of Fast Fourier Transform (FFT) to obtain the Power Spectrum Density (PSD) and the Root Mean Squared (RMS) pressure fluctuations to identify and characterize any periodic phenomena present in the system.

8.2.2.3. High Speed OH-PLIF Imaging Setup

A schematic layout of the high-speed OH-PLIF setup is shown in Figure 8.3. The 10 kHz OH Planar Laser Induced Fluorescence (OH-PLIF) system used a Diode Pumped Solid State (DPSS), EdgeWave Inno lab IS12II-E, Nd:YAG laser, with a maximum output power of 60 W, at 532 nm (second harmonic). The 532 nm laser beam was then used to pump a tunable Sirah CREDO
dye laser containing DCM dye dissolved in ethanol. The red dye laser beam was then frequency doubled using an integral β-barium-borate doubling crystal, and the output pulse energy was roughly 0.2 mJ/pulse with a 0.1 cm-1 spectral line width at 310 nm. The dye laser was then tuned to excite the Q₁(12) line of the \( A^2\Sigma^+ \rightarrow X^2\Pi \) (0,0) band at approximately 310.60 nm in air.
Timing of the 100 ns intensifier gate for fluorescence collection was aided by using a fast photodiode and an oscilloscope to observe a (low energy) sampling of the laser pulse. The UV laser beam was formed into a sheet of thickness of approximately 120 µm (FWHM) in the probe region using a 25 mm plano-concave cylindrical lens and a 750 mm spherical plano-convex lens (two-lens telescope). Fluorescence was isolated from background scattering and visible flame emission using two custom Semrock AFRL-0002 long-wave-pass filters and a Schott UG-5 filter. Fluorescence signal collected at right angle (perpendicular to the laser sheet) using a 100 mm focal length, f/2.8 Sodern Cerco UV lens was focused onto the 896 x 848-pixel imaging array of a high speed Photron SA-Z CMOS camera coupled to a Lambert HI-CATT two-stage image intensifier.

8.3. Results and Discussion

8.3.1. Flame Phenomenology (OH-PLIF Imaging)

The flame phenomenology in the mesoscale burner array was characterized using OH-PLIF imaging. Image post processing was carried out using an open source Java based software called ImageJ [116] developed by the National Institutes of Health. After background subtraction and enhancement of brightness and contrast of the processed images, a custom lookup table was applied to enhance and visualize the flame features. Based on the operating equivalence ratio and the Reynolds number, a variety of flame structures are observed in the mesoscale burner array. These characteristic flame structures arise due to the differences in the flame stabilization and flame interaction mechanisms between the adjacent nodes of the array. Through OH-PLIF visualization, based on the observed flame structure, mesoscale flames can broadly be grouped into the following regimes namely: (a) V shaped flame (b) M shaped flame (c) Secondary flame (d) Merged flame. The different flame regimes along with their boundaries of operation in terms
of the equivalence ratio and Reynolds number is shown in Figure 8.4. Averaged OH-PLIF images showing the corresponding flame structures for these different regimes in a mesoscale burner array are shown in Figure 8.5.

Under typically preferred operating conditions of the mesoscale burner array, i.e., lean and ultra-lean premixed conditions, compact, well distributed, and uniform V shaped flames are observed. Each individual node of the burner array creates a typical swirl stabilized V shaped flame that is strongly anchored to the center bluff body and stabilized by the combined effects of the ISL (formed between the CRZ and the swirling jet) and CRZ [85, 111, 112, 156]. This low speed region serves as continuous ignition source through recirculation of hot products. It can be clearly from Figure 8.5, that the adjacent V shaped flames strongly interact however this interaction is limited to the end of the ISL further downstream of the bluff body.

![Figure 8.4. Flame phenomenology map of mesoscale burner array indicating the characteristic flame regimes in terms of operating Reynolds number and equivalence ratio.](image-url)
As the operating equivalence ratio is progressively increased, the V shaped flame starts to transition into a partial M shaped flame and at near stoichiometric conditions ($\phi = 1$), characteristic M shaped flames are observed as shown in Figure 8.6. In M shaped flames, the flame is stabilized with its root anchored in the CRZ while its tip protrudes into the ORZ or is strongly attached to the ORZ. The transition from V to M flame shape starts in the central flames that are typically isolated from edge effects and outside air entrainment due to the presence of the boundary elements. As the operating equivalence ratio is increased, there is an increase in the overall burning rate (flame speed) and preheating at the exit of the combustor [181, 182] which causes flashback.
Figure 8.6. Averaged OH PLIF images showing the V to M flame transition along with schematic flow fields surrounding two interacting mesoscales flames. CB - center body, OSL - outer shear layer, ISL - inner shear layer, CRZ - Center Recirculation Zone, ORZ - Outer Recirculation Zone.
of the V shaped flame into the low velocity region of the ORZ causing a V to M transition, where the flame front trapped by the large eddies of the ORZ propagates upstream into the OSL (formed between the ORZ and the swirling jet) resulting in an M shaped flame. Thus, as result of the increased flame speed, the flame is now anchored to both the central bluff body (CRZ) and the region between the adjacent nodes due to the presence of the ORZ. This improves flame interaction as the adjacent flames now interact at the base of the ORZ in addition to further downstream. It is to be noted that, the edge elements are still stabilized as V shaped flames on the outside as opposed to M shaped flames on the inside due to the flame interaction between adjacent nodes.

Further increase in the operating equivalence ratio to rich flame conditions, results in the formation of secondary edge flames which are essentially regions (highlighted in red in Figure 8.5) where the excess unburnt fuel burns in a predominantly partially premixed or diffusion mode as it comes in contact with the ambient air further downstream of the burner. Based on the OH-PLIF signal intensity, it can be seen that the secondary flame is usually much stronger than the primary flame that is stabilized at the exit of the burner as result of the swirling motion. Any further increase in the equivalence ratio completely collapses flame stabilization due to the swirling motion as the flame burns in a completely diffusion environment with no well-defined individual flame structures resulting in single merged primary flame as presented in Figure 8.5. Again, similar to the secondary flame regime, the primary flame is much weaker when compared to the secondary flame formed at the edges.

8.3.2. Flame Interaction (CH-PLIF Imaging)

CH-PLIF in particular is often employed as a flame front marker in laminar and turbulent flames, as CH is a short lived radical and its spatial distribution in the reaction zone corresponds well to the region of peak heat release and therefore can be used as a better flame sheet marker
than OH [183-187]. Ensemble averaged CH-PLIF images of vertical and horizontal sections (taken at 2 mm and 4 mm from the exit plane of the burner) of the mesoscale burner array are shown in Figure 8.7. Image post processing was carried out using an open source Java based software called ImageJ [116] developed by the National Institutes of Health. The median of the image was subtracted as a background. Then the image was smoothed by applying a 2-pixel median filter, and then binned by applying a 2x2 pixel averaging reducer in order to enhance the brightness and contrast, and finally a custom lookup table was applied to enhance and visualize the flame features.

**Figure 8.7.** Averaged CH PLIF images showing the flame interaction between individual elements of the mesoscale burner array.

The vertical section (top panel of Figure 8.7) shows the structure of the flame sheet in M shaped flames highlighting the flame to flame interaction between the adjacent nodes that was discussed in the previous section using OH-PLIF imaging. The horizontal section taken at 2 mm
from the exit plane of the burner (bottom left panel of Figure 8.7) shows the flame to flame interaction in the entire combustor array where the inner 2x2 nodes are completely shielded from any ambient perturbation by the surrounding flames. Local extinction in the center flames are mitigated by reignition induced by the surrounding flames, further adding to the combustion stability. The edge flames on the other hand show localized extinctions that are marked by the discontinuities in the CH profile. The Taylor Green vortex arrangement i.e., counter rotating vortex pattern can also be clearly seen from the horizontal section taken at 4 mm from the exit plane of the burner (bottom right panel of Figure 8.7).

### 8.3.3. Simultaneous Multispectral PLIF Imaging (HCHO/CH/OH)

Simultaneous multispectral PLIF imaging of selected radical species i.e, HCHO, CH and OH was carried out on the mesoscale burner to visualize the reaction layers of the flame. Since CH-PLIF imaging was carried out using the $C^2\Sigma^+ \rightarrow \chi^2\Pi \ (0,0)$ band around 310 nm [174-176], it has the additional advantage of capturing both OH-PLIF and CH-PLIF signals within the same image by exciting the overlapping the OH lines corresponding $A^2\Sigma^+ \rightarrow \chi^2\Pi \ (0,0)$ band in addition to the targeted CH lines. This allows for reaction layer structure visualization while also being able to clearly distinguish reactants from products. HCHO-PLIF was used for visualization of preheat layers as HCHO can be used as preheat zone marker in methane-air flames [179, 188]. Thus, simultaneous multispectral PLIF imaging of HCHO, CH and OH radicals allowed for visualization of the preheat zone, flame front (reaction zone) and post flame zone respectively [179, 188].

Ensemble averaged multispectral PLIF images of individual species and the overlaid PLIF image for the three species for the mesoscale burner array are shown in Figure 8.8. Image post
processing was carried out using an open source Java based software called ImageJ [116] developed by the National Institutes of Health. It is to be noted that since CH and OH-PLIF signals were captured in a single image, further post processing was necessary to isolate the CH signal. To extract the spatial CH distribution, a custom bandpass filter was applied to the image, followed by an edge detection scheme. In Figure 8.8, HCHO (panel A) is colored black to green, OH/CH (panel B) is colored black to red, CH (extracted from CH/OH - panel C) is colored black to blue and the region of spatial overlap between the three species is highlighted in violet in the overlaid image (panel D). From Figure 8.8, it can be clearly seen that HCHO and OH reside in the outer and inner regions of the flame front with CH in a narrow band along the flame front, overlapping both OH and HCHO.

Figure 8.8. Simultaneous multispectral PLIF (HCHO, CH and OH) imaging of mesoscale burner: Panel A - HCHO (green), Panel B - CH/OH (red), Panel C - extracted CH (blue) and Panel D - Overlaid image. The spatial overlap region between the three species is highlighted in violet in Panel D.
8.3.4. Self-Excited Mesoscale Burner Array Flame Dynamics

The mesoscale burner array exhibits self-excited flame oscillations under certain experimental conditions (usually when the flame undergoes a lean to rich transition) based on the air flow rate and the related acoustics of the chamber that amplifies these oscillations. One such experimental condition is listed in Table 8.2 along with a stable condition that is used as a baseline for the following analysis.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Air Flow Rate (SLPM)</th>
<th>CH\textsubscript{4} Flow Rate (SLPM)</th>
<th>Equivalence Ratio $\phi$</th>
<th>Reynolds Number</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>1.47</td>
<td>0.7</td>
<td>155</td>
<td>Stable flame, No audible acoustics</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>4.62</td>
<td>1.1</td>
<td>310</td>
<td>Oscillatory flame, Audible acoustics</td>
</tr>
</tbody>
</table>

*Table 8.2. Experimental conditions for the characteristic test configurations.*

8.3.4.1. Stable Mesoscale Flame

Case 1 listed in Table 8.2 is a stable mesoscale flame which exhibits no discernible oscillations with minimal combustor noise. POD analysis was carried out on 2000 images of high-speed OH chemiluminescence captured at 10 kHz, based on which the normalized eigen value and its cumulative distribution are shown in Figure 8.9. The POD analysis reveals the high stability characteristics of the mesoscale flame where the POD mode corresponding to the average is the most dominant and contains almost all the total energy and the eigen value drops sharply to almost zero in the first fluctuating mode. The mean burning shape of the flame indicated by POD Mode 0 is also shown in Figure 8.10. The higher POD modes lack any coherent structures indicating minimal fluctuations and hence are not shown here.
The results of the corresponding DMD analysis carried out on 2000 images of the stable mesoscale flame is shown in Figure 8.11. All 2000 images were analyzed, but only the most
relevant modes are shown in the DMD spectrum. As expected, the mode with the highest energy content and the most stable growth factor (closest to 0) is located at the mean frequency (0 Hz, denoted by A in the figure). The spatial structure of DMD mode A is shown in Figure 8.12 which

![DMD spectrum graph](image)

*Figure 8.11. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of a stable mesoscale flame.*

![Spatial structure of DMD mode A](image)

*Figure 8.12. Spatial structure of DMD mode A (average flame shape) for a stable mesoscale flame.*

represents the time-averaged flame position. All other modes exhibit very large negative growth rates and their contribution to the total energy content is negligible, but they all map onto or are
very close to the unit circle. Thus, the results of the DMD analysis are in excellent agreement with the results of POD analysis, in that the mean flame accounts for all the total energy present in this test condition. Also, other DMD modes reveal no coherent structures (similar to higher POD modes) confirming the that there are minimal fluctuations present in this particular configuration.

8.3.4.2. Oscillatory Mesoscale Flame

Unlike the stable mesoscale flame, case 2 listed in Table 8.2 is a self-excited condition, where the mesoscale flame exhibits oscillatory behavior coupled with a loud characteristic combustor noise. A small subset of high-speed OH chemiluminescence images under this test condition where, oscillatory behavior is observed is shown in Figure 8.13. The oscillatory behavior of the flame is clearly seen in the form of periodic changes observed in OH intensities. The red box in the figure marks the size and location of our interrogation window for subsequent FFT analysis of the image. Strong oscillatory behavior near the recirculation zone and the shear layer

![Figure 8.13. A selection of high speed OH Chemiluminescence snapshots of an oscillatory mesoscale flame. The window used for FFT analysis is shown in red.](image-url)
is observed as shown in the figure. POD and DMD analysis should not only provide similar findings, but also quantify these dynamics to serve as an effective combustion instability analysis technique.

POD analysis on a sequence of 2000 images of high-speed OH* chemiluminescence captured at 10 kHz, based on which the normalized eigen value and its cumulative distribution are shown in Figure 8.14. Unlike the earlier case, the energy content of the dominant mode for the oscillatory case accounts for only 33% of the total energy content. The fluctuating modes account for a significant portion of the total energy content as evidenced by the convergence level of the cumulative eigen values. Fluctuation can be visualized as higher POD modes shown in Figure 8.15.

![Figure 8.14. Normalized eigen value and its cumulative distribution for POD modes extracted from an oscillatory mesoscale flame.](image)

which show the presence of distinct coherent structures. The power spectrum of the POD mode time coefficients is shown in Figure 8.16. The power spectrum reveals the presence of multiple
frequencies (fundamental and subsequent harmonics) in each individual POD mode. This again is due to the spatially orthogonality imposed by the POD algorithm that allows for contamination (presence of uncorrelated structures) of spatial features with spectral information from multiple frequencies.

Figure 8.15. Spatial structure of POD modes 0, 1, 3, 5 and 7 for an oscillatory mesoscale flame.

Figure 8.16. Power spectrum of POD mode time coefficients for an oscillatory mesoscale flame.
The results of the corresponding DMD analysis carried out on the 2000 images of the oscillatory mesoscale flame is shown in Figure 8.17. Results show the presence of a pair of dynamics modes with positive growth rates (Marked as B) and other dynamics modes with significantly higher energy content (Marked as C, D and E). The spatial structure corresponding to the mean mode (A) along with four selected dynamic modes based on the small decay factor and significant energy contribution is shown in Figure 8.18. Also, the coherent structures are all concentrated in the recirculation zone and the shear layer thereby promoting oscillatory behavior. It is to be noted that the DMD modes (unlike POD modes) are temporally orthogonal (spatially orthogonal) but in general present spatial non-orthogonality.

![Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an oscillatory mesoscale flame.](image)

**Figure 8.17.** Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an oscillatory mesoscale flame.

![Spatial structure of DMD modes A, B, C, D and E for an oscillatory mesoscale flame.](image)

**Figure 8.18.** Spatial structure of DMD modes A, B, C, D and E for an oscillatory mesoscale flame.
As shown in Figure 8.13, the snapshots of OH chemiluminescence clearly reveal that case 2 presents a periodic oscillatory flame behavior. Hence subsequent FFT analysis of the region of interest (marked in red) was performed to spectrally resolve the oscillation and the resulting power spectrum is shown in Figure 8.19. It can be clearly seen that the dominant frequencies present in the power spectrum are in good agreement with the results of the POD and DMD analysis presented in Figure 8.16 and Figure 8.17 respectively. Further, for comparison the power spectrum showing the RMS pressure fluctuations based on FFT analysis of the microphone pressure signal is shown in Figure 8.20. The power spectrum reveals the presence of pressure fluctuations with a fundamental frequency of 350 Hz (and subsequent harmonics) that results in the characteristic loud combustor noise. This is in excellent agreement with the observed frequency of heat release.

![Power spectrum](image)

*Figure 8.19. Power spectrum based on FFT analysis of the marked region of interest (shown in red in Figure 8.13) for an oscillatory mesoscale flame.*
Figure 8.20. Power spectrum showing the Root Mean Square (RMS) pressure fluctuations for the oscillatory mesoscale flame.

Fluctuation captured by OH chemiluminescence analyzed using advanced decomposition techniques POD and DMD.

8.3.5. Single Swirl Burner Flame Dynamics (Forced Acoustics)

Flame stability in a single swirl burner under externally imposed acoustic forcing was studied using high-speed OH-PLIF (10 kHz) images and subsequent DMD analysis. Lean flames ($\phi = 0.97$) exhibited discernable oscillations but maintained attachment (no blowoffs or flashbacks) at all tested experimental conditions. The speaker attached to the burner housing was driven at multiple frequencies up to 370 Hz and at different amplitudes to study the effects of acoustic forcing on flame oscillation. DMD analysis was then carried out on these high-speed OH PLIF images to provide a quantitative measure of the flame stability. The results of the DMD analysis
on the single swirl burner will be then used to benchmark the mesoscale burner to highlight the improved flame stability due to increased flame interaction in the latter.

A small subset of high-speed OH-PLIF images without acoustic forcing is shown in the top row of Figure 8.21. It shows very stable flame characteristics, particularly at the recirculation zone (flame anchoring point) near the burner surface and small oscillations in the shear layer. The same flame, but acoustically forced at 50 Hz, is shown in the bottom row of Figure 8.21. Strong oscillatory behavior near the recirculation zone and the shear layer [189] is observed at the forcing frequency. DMD analysis should not only provide similar findings, but also quantify these dynamics to serve as an effective combustion instability analysis technique.

![Non-excited swirl flame](image)

**Figure 8.21.** A selection of high speed OH-PLIF images of a non-excited (top) and an acoustically forced swirl flame (bottom).

### 8.3.5.1. Non-Excited Swirl Flame

The 10 kHz OH-PLIF images allow for reliable DMD analysis of frequencies up to 5 kHz, which is more than enough to determine the effects of acoustic forcing in this study. DMD results
for a non-excited swirl flame is shown in Figure 8.22. All 2000 images were analyzed, but only the nine most relevant modes are shown in the DMD spectrum. As expected, the mode with the highest energy content and the most stable growth factor (closest to 0) is located at the mean frequency (0 Hz, denoted by A in Figure 8.22). Its spatial structure shown in Figure 8.23, much like that of laminar flame, represents time-averaged flame position. Small oscillations in the

![Figure 8.22. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of a non-excited swirl flame.](image1)

![Figure 8.23. Spatial structure of DMD modes A and B for a non-excited swirl flame.](image2)
shear layer, as observed in Figure 8.23, is mostly due to a pair of complex conjugate modes (denoted by B in Figure 8.22) near 110 Hz. Its spatial structures are concentrated in the outer shear layer further downstream of the recirculation zone. The weaker growth factor and lower energy content results in a minor contribution to the overall stability. A second mode at the same frequency produces similar spatial structures but provides an even smaller contribution.

8.3.5.2. Acoustically Forced Swirl Flame

Numerous acoustic forcing conditions were then applied to the swirl-stabilized flame. An instance of acoustic forcing at 210 Hz and high forcing amplitude is shown in Figure 8.24. DMD spectrum indicates that the three-relevant modes (mean, forcing frequency and its first harmonic) all have positive growth rates. The time-averaged spatial structure (mode A) as shown in Figure 8.25, is spread outwards due to the extremely oscillatory behavior of the flame. Also, coherent structures at 210 Hz (mode B) and 420 Hz (mode C) also shown in Figure 8.25 are all concentrated in the recirculation zone thereby promoting oscillatory behavior.

![Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an acoustically forced swirl flame at 210 Hz.](image)

Figure 8.24. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an acoustically forced swirl flame at 210 Hz.
A comparison of DMD spectra at 210 Hz for various forcing amplitudes for the swirl flame is shown in Figure 8.26. Results show that at lower amplitude, the coherent structures at the driving frequency and its harmonics have negative growth factor leading to a stable flame with small oscillations. As the forcing amplitude increases, modes quickly cross into positive growth rate leading to a highly oscillatory flame behavior. The onset of full blown combustion instability occurs at just above +9 dB, at which point the flame is no longer controllable and flashes back violently. Interestingly, the frequency response of some non-resonant modes is blue-shifted possibly to accommodate increased energy transfers from the highly excited modes.
Figure 8.26. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an acoustically forced swirl flame at various amplitudes and 210 Hz.
8.3.6. Mesoscale Burner Array Flame Dynamics (Forced Acoustics)

Flame stability in a mesoscale burner array under externally imposed acoustic forcing was studied using high-speed OH-PLIF (10 kHz) images and subsequent DMD analysis. Lean flame measurements ($\phi = 0.85$) exhibited no discernable oscillations at all tested experimental conditions. The speaker attached to the burner housing was driven at multiple frequencies up to 370 Hz and at different amplitudes to study the effects of acoustic forcing on flame oscillation. DMD analysis was then carried out on these high-speed OH PLIF images to provide a quantitative measure of the flame stability.

A small subset of high-speed OH-PLIF images without acoustic forcing is shown in the top row of Figure 8.21. The mesoscale burner array under no acoustic forcing exhibits the characteristic M shaped flame discussed earlier with very stable flame characteristics. The same

![Non-excited mesoscale flame](image)

![Swirl flame acoustically forced at 50 Hz](image)

*Figure 8.27. A selection of high speed OH-PLIF images of a non-excited (top) and an acoustically forced mesoscale flame (bottom).*
flame, but acoustically forced at 50 Hz, is shown in the bottom row of Figure 8.21. Strong oscillatory behavior observed at the recirculation zones and the shear layer [189] due to the forcing frequency causes the mesoscale burner array to transition between the V and M shaped flames.

8.3.6.1. Non-Excited Mesoscale Flame

The 10 kHz OH-PLIF images allow for reliable DMD analysis of frequencies up to 5 kHz, which is more than enough to determine the effects of acoustic forcing in this study. DMD results for a non-excited mesoscale flame is shown in Figure 8.28. All 2000 images were analyzed, but only the eleven most relevant modes are shown in the DMD spectrum. As expected, the mode with the highest energy content and the most stable growth factor (closest to 0) is located at the mean frequency (0 Hz, denoted by A in Figure 8.28). Its spatial structure shown in Figure 8.23. Spatial structure of DMD modes A and B for a non-excited swirl flame. Figure 8.29, much like that of laminar flame, represents time-averaged flame position. In contrast to the single swirl flame, even at relative leaner operating conditions (mesoscale flame at $\phi = 0.85$, single swirl flame at $\phi = 0.97$),

![Figure 8.28. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of a non-excited mesoscale flame.](image)

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the energy contribution of all the fluctuating modes for a mesoscale flame is relatively minimal as seen from the relative size of the markers and the very large negative growth factors. The marker size is proportional to a measure of coherence of the associated modes (energy content of the spatial mode) and help rank the relevant structures ahead of noise-contaminated ones.

![Mode A](image)

*Figure 8.29. Spatial structure of DMD mode A for a non-excited mesoscale flame.*

### 8.3.6.2. Acoustically Forced Mesoscale Flame

Numerous acoustic forcing conditions were then applied to the mesoscale burner array flame. An instance of acoustic forcing at 210 Hz and high forcing amplitude is shown in Figure 8.30. DMD spectrum indicates that the three-relevant modes (mean, forcing frequency and its first harmonic) all have positive growth rates. The time-averaged spatial structure (mode A) as shown in Figure 8.31, surprisingly exhibits the structure of a V shaped flame, though the non-excited mesoscale flame under similar conditions is M shaped. Again, it is to be noted that though the acoustic forcing conditions are similar, the DMD spectrum of the mesoscale flame (lower overall growth rates and lower energy content in the fluctuating modes), clearly shows that the mesoscale burner array is more resilient to acoustic perturbations though it is operated at leaner equivalence ratios than the single swirl burner. Also, coherent structures at 210 Hz (mode B) and 420 Hz (mode
C) also shown in Figure 8.31 are distributed further downstream of the flame stabilization region which is the reason for the observed improved flame stability of the mesoscale burner array under external acoustic forcing.

![Graph showing growth factor vs frequency](image)

*Figure 8.30. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an acoustically forced mesoscale flame at 210 Hz.*

![Spatial structure of DMD modes A, B and C](image)

*Figure 8.31. Spatial structure of DMD modes A, B and C for an acoustically forced mesoscale flame at 210 Hz.*

A comparison of DMD spectra at 210 Hz for various forcing amplitudes for the mesoscale burner array is shown in Figure 8.32. Though the overall results show a similar trend to that of the
Figure 8.32. Logarithmic mapping of Ritz values (DMD spectrum) based on DMD analysis of an acoustically forced mesoscale flame at various amplitudes and 210 Hz.
single swirl flame, the coherent structures exhibit a relatively lower energy content with comparatively smaller growth rates at all forcing amplitudes leading to a much stable flame with small oscillations. The onset of full blown combustion instability occurs also occurs at higher forcing amplitude of +12 dB, at which point the flame is no longer controllable and flashes back violently.

8.4. Conclusions

Flame phenomenology, flame interaction and flame stabilization mechanisms in a mesoscale burner array over a range of operating conditions was studied using multispectral PLIF imaging of major combustion radicals namely OH, CH and HCHCO. The mesoscale array was specifically configured to enhance overall combustion stability, particularly under lean operating conditions, by promoting flame to flame interactions between neighboring elements. A comprehensive comparison of DMD and POD as powerful analysis tools that can extract coherent structures from experimental datasets without recourse to an underlying model was carried out. This type of decomposition technique can help in identifying the dominant behavior of the flow field under investigation in terms of a few coherent structures and their temporal and spatial characteristics. Self-excited flame oscillations in the mesoscale burner array was studied using POD and DMD analysis of high-speed OH chemiluminescence images of mesoscale burner array.

DMD analysis based on high speed OH-PLIF images provided a quantitative measure of flame stability under externally forced acoustics perturbations. The results of the DMD analysis carried out on the single swirl burner was used to benchmark the mesoscale burner to highlight the improved flame stability due to increased flame interaction in the latter. DMD analysis accurately resolved dominant spatial structures present in the recirculation zone or shear layer along with
their corresponding growth rates and energy contents. The oscillatory flame behavior was accurately described by correlating the location of the coherent spatial structures of the relevant dynamic modes about the flame stabilization location. The results show marked improvement in combustion stability for the mesoscale burner while operating under much leaner conditions compared to a single swirl-stabilized flame with similar power output. The findings presented in this study enable a step forward for experimental community in flame oscillation and combustion instability analysis. The key insights regarding the complex interactions between acoustics, fluid mechanics and combustion will ultimately serve a critical role in developing detailed combustion instability models. In conclusion, the results show promise for integration of mesoscale combustor arrays as a potentially flexible and scalable technique in next generation propulsion and power generation systems.
Chapter 9. Conclusions and Future Work

The overall contributions of the work presented in this thesis are outlined in this chapter. Here, the results of this study are placed in context of the overall field of combustion instability research and the applications of this work towards improvement of low emissions combustor operation and design. Additionally, recommendations for further work are put forward for future consideration.

9.1. Microwave Plasma Assisted Combustion

In the first study, the effect of volumetric, direct coupled, continuous, atmospheric microwave plasma discharge on a swirl stabilized, premixed methane-air flame was investigated by employing high fidelity optical and laser diagnostic techniques. Experimental findings demonstrated the effectiveness of microwave plasma discharge in improving the flame dynamics in realistic combustor geometries. A variable swirl burner fitted onto a microwave plasma applicator served to mimic realistic combustion geometries by offering a simple flexible platform to study both swirl stabilized, and plasma assisted flames along with complete optical access for diagnostics.

Though both swirl stabilized and plasma assisted flames exhibited similar flow patterns, the dynamics of flame stabilization between these two flames were found to be entirely different. Swirl flames were lift off flames that stabilized in the active ISL. They were prone to local extinction due to the aerodynamic shearing effects associated with the ISL. While, plasma assisted flames on the other hand were much more stable as they were stabilized in the relatively quiescent CB wake region behind the electrode. Also, such flames were strongly anchored to the electrode due to the presence of the plasma discharge.
In addition to increased flame stability, direct coupled, plasma assisted flames offered significant improvement in the LBO (~43%) and OH number density (~150%) for coupled plasma powers as low as 3% of the combustion output. Further, the significant equivalence ratio dependence of OH number density in plasma assisted flames suggested that the influence of plasma discharge in swirl stabilized flame existed as complex non-thermal effects as opposed to mere Ohmic heating. Spectrographic measurements showed nitrogen vibrational temperatures as high as 6100 K indicating vibrational excitation of nitrogen molecules in addition to electronic excitation. V-V relaxation and direct nitrogen impact dissociation resulted in formation of highly reactive vibrationally excited oxygen molecules and oxygen radicals respectively which accelerated chemical reactions. Thus, it was shown that the non-thermal effect of “inert nitrogen excitation” can be successfully utilized in a swirl stabilized flame configuration by employing a volumetric, direct coupled, microwave plasma discharge which resulted in a significant improvement in the operating limits of the burner while enhancing flame stability especially at very lean operating conditions by using only a fraction of the combustion power.

Furthermore, a comprehensive procedure for extracting flame dynamics from high fidelity experimental data through a post-processing technique, Proper Orthogonal Decomposition (POD), was demonstrated. The technique’s potential for extracting useful information about the flame dynamics from seemingly random samples that are typically lost by classical averaging was demonstrated. Numerically-constructed images and laminar flame OH-PLIF images served to validate DMD analysis and its application on combustion phenomenon, respectively. In particular, POD applied to OH-PLIF images were shown to be insensitive to the choice of ensemble considered, which established that the modes extracted do indeed represent the features of the flame. Furthermore, the ensemble of statistical quantities obtained via POD analysis were used for
characterizing the heat release fluctuation and thereby providing the basis for comparison of different flame configurations, revealing the effect of plasma discharge on flame dynamics and stability.

The POD analysis performed in this study established that even at coupled plasma powers corresponding to less than 5% of the thermal power output, there was significant reduction in the heat release fluctuations, leading to improved mean energy content (~23%) for the plasma assisted flames, which resulted in enhanced flame stability and improved combustor dynamics. Nevertheless, a saturation limit exists, beyond which further addition of plasma had no effect in terms of improving the mean energy content of the flame. Furthermore, acoustic measurements indicated that the RMS pressure fluctuations decreased by up to 47% at minimal plasma levels; however, any further increase in the coupled plasma power “over-energized” the flame causing a slight increase in the pressure fluctuations. These findings were found to be in good agreement with the results of POD analysis and confirmed the presence of a saturation limit for control of combustor dynamics using microwave plasma discharge. In addition, significant improvement in burned gas temperature and better plasma coupling at leaner equivalence ratios confirmed that the improved combustion characteristics are due to non-thermal effects of the plasma that opened up new reaction pathways, resulting in accelerated chemical reactions.

In conclusion, it has been shown that microwave plasma discharge in addition to accelerating combustion chemistry through non-thermal effects, causes an “effective fluid-acoustic decoupling” i.e., flame oscillations are isolated from fluid unsteadiness through different flame stabilization mechanisms, which resulted in significant improvement in flame dynamics namely, decreased heat release and pressure fluctuations.
9.2. **Mesoscale Burner Array**

In the second study, the potential for a compact, clean, efficient and scalable mesoscale swirl stabilized burner array developed and manufactured using advanced additive manufacturing technique (DMLS) was demonstrated. Challenges involved in mesoscale combustion were circumvented via design optimization such as use of combined swirl and bluff body stabilization, flow divergence, improved heat recirculation and increased mutual flame interaction. Use of counter rotating vortex pattern ensured flame propagation over the entire array after initial ignition. Operation of the optimized mesoscale burner array under premixed conditions resulted in improved performance characteristics coupled with exceptional flame stability under ultra-lean conditions. Further, stable LBO limits ($\phi = 0.65$) over a wide range of operating conditions (0.1 kW to 3 kW total power output) were observed. Exhaust gas analysis indicated no gaseous pollutants except for some small amounts of UHC and CO (below 0.1%), corresponding to high levels of combustion efficiency (above 98%) despite the relatively short residence time and associated heat losses. Spatially resolved temperature measurements indicated minimal element to element variation with temperature distribution reaching up to adiabatic flame temperature levels with slight temperature drop at the boundary due to edge effects caused by surroundings air entrainment. Thus, it is evident that the mesoscale burner array can be successfully scaled up to realistic combustion power outputs without any performance deterioration.

Flame phenomenology, flame interaction and flame stabilization mechanisms in a mesoscale burner array over a range of operating conditions was studied using multispectral PLIF imaging of major combustion radicals namely OH, CH and HCHCO. The mesoscale array was specifically configured to enhance overall combustion stability, particularly under lean operating conditions, by promoting flame to flame interactions between neighboring elements.
A comprehensive comparison of DMD and POD as powerful analysis tools that can extract coherent structures from experimental datasets without recourse to an underlying model was carried out. This type of decomposition technique can help in identifying the dominant behavior of the flow field under investigation in terms of a few coherent structures and their temporal and spatial characteristics. Self-excited flame oscillations in the mesoscale burner array was studied using POD and DMD analysis of high-speed OH chemiluminescence images of mesoscale burner array.

DMD analysis based on high speed OH-PLIF images provided a quantitative measure of flame stability under externally forced acoustics perturbations. The results of the DMD analysis carried out on the single swirl burner was used to benchmark the mesoscale burner to highlight the improved flame stability due to increased flame interaction in the latter. DMD analysis accurately resolved dominant spatial structures present in the recirculation zone or shear layer along with their corresponding growth rates and energy contents. The oscillatory flame behavior was accurately described by correlating the location of the coherent spatial structures of the relevant dynamic modes about the flame stabilization location. The results show marked improvement in combustion stability for the mesoscale burner while operating under much leaner conditions compared to a single swirl-stabilized flame with similar power output. The findings presented in this study enable a step forward for experimental community in flame oscillation and combustion instability analysis. The key insights regarding the complex interactions between acoustics, fluid mechanics and combustion will ultimately serve a critical role in developing detailed combustion instability models.

In conclusion, the potential for an optimized scalable mesoscale burner array architecture with power density, performance and emission characteristics that can match and possibly
outperform existing macroscale burner designs with reduced susceptibility to extinction and externally imposed acoustic perturbations while maintaining high combustion efficiency and low emission levels under ultra-lean operating conditions for current power generation systems has been successfully demonstrated. Easy and rapid manufacturing of such scalable mesoscale burner arrays by advanced additive manufacturing techniques (DMLS) offers added benefit of easy integration with direct energy conversion modules. The results show promise for integration of mesoscale combustor arrays as a potentially flexible and scalable technology in next generation propulsion and power generation systems.

9.3. Recommendations for Future Work

Several recommendations for future work naturally follow the results of the current study. First, a more extensive characterization of the non-reactive and reactive flow field using PIV measurements with and without acoustic forcing is recommended. Simultaneous measurements of the velocity field and flame imaging can help establish a causal link between the velocity field fluctuations. Simultaneous high-speed PIV and PLIF measurements will help further our understanding of the flame response. Rayleigh index can also be constructed to visualize where in the combustion zone, heat release fluctuations are coupled with the pressure perturbations.

In addition, such simultaneous measurements would allow for the application of advanced decompositions techniques like Extended POD (EPOD) to reacting flows which can correlate variables representing the reaction zone (progress variable, species, temperature, etc.) with the velocity field, so that the interactions between flow and flame dynamics can be highlighted. Furthermore, the work presented in this thesis focused mostly on the use of decomposition techniques to study combustion flame dynamics and the effect of instability control strategies on flame dynamics. However, the modes obtained from these decomposition techniques can also be
used to develop reduced-order models, which can capture the dynamics of the flow with significantly lower computational cost. Such models can be useful in closed-loop combustion instability control implementation.


