ILLINOIS STATE WATER SURVEY
METEOROLOGIC LABORATORY
at the
University of Illinois
Urbana, Illinois

ANALYSIS OF 1952 RADAR AND RAINGAGE DATA

by

J. C. Neill

prepared as

RESEARCH REPORT No. 2

under

CONTRACT No. DA-36-039 SC-42446
with U. S. Army, Signal Corps Engineering Laboratories
Fort Monmouth, New Jersey
Department of the Army Project: 3-99-07-022
Signal Corps Project: 24-172B
July 1953

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SUMMARY

Estimating Areal-Mean Rainfall from Radar Data

Estimates of areal-mean storm rainfall were determined from 3-cm radar data for 9 storm periods over the Goose Creek network, which encompassed 96 square miles. These estimates were compared with areal-mean storm rainfall from the 50-gage network. Areal-mean storm rainfall estimates, which were computed from the radar data by using radar-rainfall equations developed by other investigators for 10-cm radar, were much less than those computed from the raingage network data. A comparison of the precipitation area obtained from both radar and raingage data indicated that it should be possible to develop a radar-rainfall relationship which would provide better quantitative rainfall estimates from the 3-cm data collected over the Goose Creek network.

Variance of Areal-Mean Storm Rainfall Estimates From Networks of Various Gage Densities

In quantitative radar-rainfall studies, some means is needed for evaluating the accuracy of areal-mean storm rainfall estimates which are computed from radar data. A systematic sampling study was made on the Goose Creek raingage network data to determine an estimate of the standard error to be expected in estimating mean-storm rainfall from different gage densities. These errors were determined for the purpose of establishing a standard for appraising the deviation between areal-mean storm rainfall values computed from radar data and those computed from the Goose Creek raingage network.

A chart was prepared to indicate the relation of the standard error of the estimates to storm size, as indicated by the network-mean rainfall, and to the number of gage observations included in the estimates. This chart indicates that the standard error increases as the storm size increases and that the standard error increases as the gage density decreases. For example, a mean rainfall of 0.50 inch over 96 square miles has expected standard errors of 0.118, 0.061, 0.024, and 0.020 inch for gage densities of 2, 4, 8, and 16 per 96 square miles, respectively. For a one-inch mean rainfall and the same gage densities, the errors are 0.156, 0.089, 0.050, and 0.028 inch, respectively.

Sampling Time-Interval Study

An analysis was made to obtain evidence which would indicate how frequently radar-rainfall observations must be recorded to achieve accurate estimates of areal-mean thunderstorm rainfall. A systematic sampling study was made with raingage data to determine the expected standard error of mean-rainfall estimates about the Goose Creek network-mean rainfall from one-minute network-mean rainfall samples. Samples were taken at every 2nd, 4th, 6th, 10th, 15th, 20th, and 25th minute during a storm period.

A chart was prepared which indicates the relationship of the standard error of the estimates to the storm size, expressed by the network-mean rainfall, and to the interval between one-minute samples. This chart shows that the standard error of the estimates increases as the storm size increases and as the number of minutes between one-minute observations increases. This chart may be used to determine the expected standard error when samples are taken every 2nd, 4th, 6th, 10th, 15th, 20th, and 25th minute. For example, the expected standard errors in the measurement of an areal-mean storm rainfall of 0.25 inch are 0.002, 0.005, 0.009, 0.016, 0.028, 0.040, and 0.054 inch for intervals of 2, 4, 6, 10, 15, 20, and 25 minutes, respectively. The errors for the same intervals for a mean rainfall of 0.50 inch are 0.005, 0.011, 0.019, 0.036, 0.061, 0.088, and 0.118 inch, respectively. Over the range of storm size studied the results indicate than an accuracy of 0.9, 2.2, 3.7, 7.0, 12.0, 17.5, and 23.1 per cent is obtained for sampling intervals of 2, 4, 6, 10, 15, 20, and 25 minutes, respectively, over the 96 square-mile network.

Rainfall Rate Frequency Study

When sampling for quantitative rainfall estimates with a limited number of radar receiver-sensitivity steps, the most reliable sample should be obtained by selecting the sensitivity settings in such a manner that the greatest number of observations are selected from those rates which contribute the greatest amount of water. The best sample should be obtained by using smaller increments of rate between sensitivity steps in the range of rates which produce the greatest proportion of the total amount of water.

One-minute rainfall amounts from 10 storms over the Goose Creek raingage network were assumed to be good approximations of rainfall rates. The frequency of these rates was tabulated and the percentage of total water contributed by each rate was determined. Curves relating percentage of cumulative water sampled to rainfall rate were prepared for each storm and for several storms combined. The combined curve was used to select an estimate of the best receiver-sensitivity settings for the stepping switch. The rainfall-rate settings suggested by this curve for a stepping switch of 10 steps were 0.10, 0.30, 0.50, 0.83, 1.15, 1.60, 2.25, 3.10, 4.70, and 11.00 inches per hour for steps 1 through 10, respectively.

ACKNOWLEDGEMENTS

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The author wishes to acknolwedge the assistance of Michael Spock, Jr., Howard Bell, and Lois Bivans who performed the major portion of the analytical work. Floyd A. Huff assisted in the planning of the analysis projects and with the editing of the manuscript. Principal research assistants in collection of data and analysis were James A. West and George D. Hack.

Credit is due Dr. Donald M. Swingle of the Signal Corps Engineering Laboratories and Dr. H. R. Byers of the University of Chicago, who reviewed the manuscript.

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INTRODUCTION

As a result of research during and immediately following World War II, radar was found to provide excellent indications of rainfall over large areas. On theoretical grounds it appeared possible to obtain quantitative information on the distribution of precipitation rates and amounts over such areas by properly calibrating radar equipment and by improving display of data. Therefore, the Illinois State Water Survey and others began preliminary research to investigate using radar for quantitative precipitation measurements.

During 1950 and 1951 the Illinois State Water Survey experimented with a modified AN/APS-15A radar set for measuring thunderstorm rainfall. An automatic receiver gain reduction device was constructed and installed on this radar set. A 35-mm scope camera photographed the PPI in synchronization with the gain reduction device. Simultaneous rainfall measurements were made with a dense raingage network. Under Signal Corps contract number DA-36-039 SC-4 2 446, the Water Survey radar-rainfall investigations were expanded in 1952 to investigate further the utility of radar for quantitative precipitation measurements.

In partial fulfillment of the Signal Corps contract, an extensive series of simultaneous measurements of radar-received-power and rainfall rate were made during 1952. These data were collected over a dense raingage network of 50 recording raingages in a 96 square-mile area. A detailed comparative analysis of the radar and raingage data was made to aid in evaluating the utility of radar for determining areal rainfall amounts. A rainfall rate frequency study was done as an aid to radar calibration and a sampling interval study was made as an aid in determining the frequency requirements of radar samples. This report summarizes the methods of data collection, analytical procedures, and results of analysis.

ESTIMATING AREAL MEAN RAINFALL FROM RADAR DATA

Radar Equipment and Installation

Early in May, 1952, a quonset-type building with an adjoining 47-foot tower was completed for the Illinois State Water Survey's Meteorologic Laboratory at the University of Illinois Airport (Figure 1). The main components of an AN/APS-15A*, 3-cm radar set, were installed in the building and radar antenna was installed on the tower.

This radar equipment was used to collect data from which mean-storm rainfall could be calculated and compared with that obtained over a concentrated raingage network provided for this purpose. Having the antenna installed on the 47-foot tower gave unrestricted scanning vision in the direction of the raingage network. The radar set was equipped with an automatic receiver-sensitivity stepping switch and timer[1]. This arrangement permitted the receiver sensitivity to be reduced through a selected number of steps during one-minute intervals. A 35-mm camera photographed the presentation on the PPI in synchronization with the stepping switch arrangement.

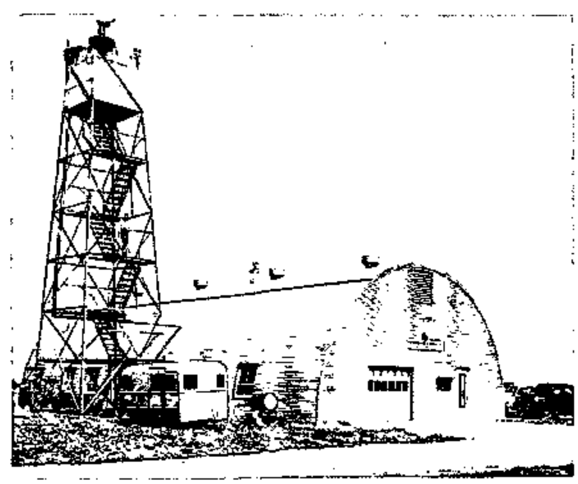

FIG. 1. ILLINOIS STATE WATER SURVEY METEOROLOGIC LABORATORY BUILDING AND RADAR ANTENNA TOWER. UNIVERSITY OF ILLINOIS AIRPORT.

Radar Operation

During the summer and fall of 1952, the APS-15 radar set was operated on a 24-hour schedule whenever rainfall was within the range of the radar set. Maximum range of the set was about 150 miles. Whenever precipitation echoes occurred over any part of the raingage network, the radar set was operated on 30-mile range to obtain detailed rainfall patterns. Sufficient receiver-sensitivity steps were used to delineate the rainfall cores. Thus, as the precipitation echoes

*Hereafter in this report the AN/APS-15A will be referred to as an APS-15 or as a 3-cm radar set.

[1] Rainfall-Radar Studies of 1951, Ill. State Water Survey Report of Investigation No. 19, May 1953, or Research Report No. 1, Under Contract No. DA-36-039 SC-42446, U.S. Army Signal Corps, 1953.

moved across the network, the camera automatically recorded their movement as well as the rainfall intensity zones within the storm area.

Raingage Network

Since the problem requires measuring rainfall with radar, which makes its observations in the atmosphere, a raingage network was installed on the ground to obtain data for comparison with rainfall measurements calculated from the radar data observed above the same area. This raingage network was installed on the Goose Creek watershed which is west-northwest of the radar set, and at a distance of 15 to 25 statute miles (Figure 2). The network covers an area of approximately 96 square statute miles, and comprises a total of 50 rain gages, spaced at intervals of about 1.5 miles along radii originating at the radar site. Each gage is a Bendix-Friez, Dual Traverse, Model 775-BS, recording raingage, and is equipped with a 12.648-inch diameter collector and a six-hour chart drive.

Raingage Network Operation

Raingage charts were replaced by new ones as soon as possible after the end of each rain. If another rain did not occur over the network within 24 to 48 hours, a new set of charts was often put on or the pen arm adjusted to a different recording level. This procedure prevented a broad ink line which results from several revolutions of the six-hour chart. If this line is allowed to form before rain begins or after rain ends, the beginning and ending of the rainfall record is often obscured.

Each raingage was serviced when the charts were changed. All gages were calibrated at the beginning of operations in May and at intervals during the summer and fall.

Analysis of Data

Preparation of Raingage Data. Total and one-minute rainfall amounts were obtained from the recording raingage cumulative traces. One-minute amounts were used in the preparation of one-minute isohyetal maps. Average rainfall over the network was computed from the total amounts for the 50 stations.

Conversion of Radar Film Record into Quantitative Rainfall Estimates. Each frame of the 35-mm film record, portraying the areal distribution of precipitation over the raingage network, was enlarged approximately 50 diameters and projected onto a base map of the raingage network. Outlines of precipitation patterns for each receiver sensitivity in a given series were traced on a single base map to obtain a one-minute isoecho contour map (Figure 3). These isoecho contours

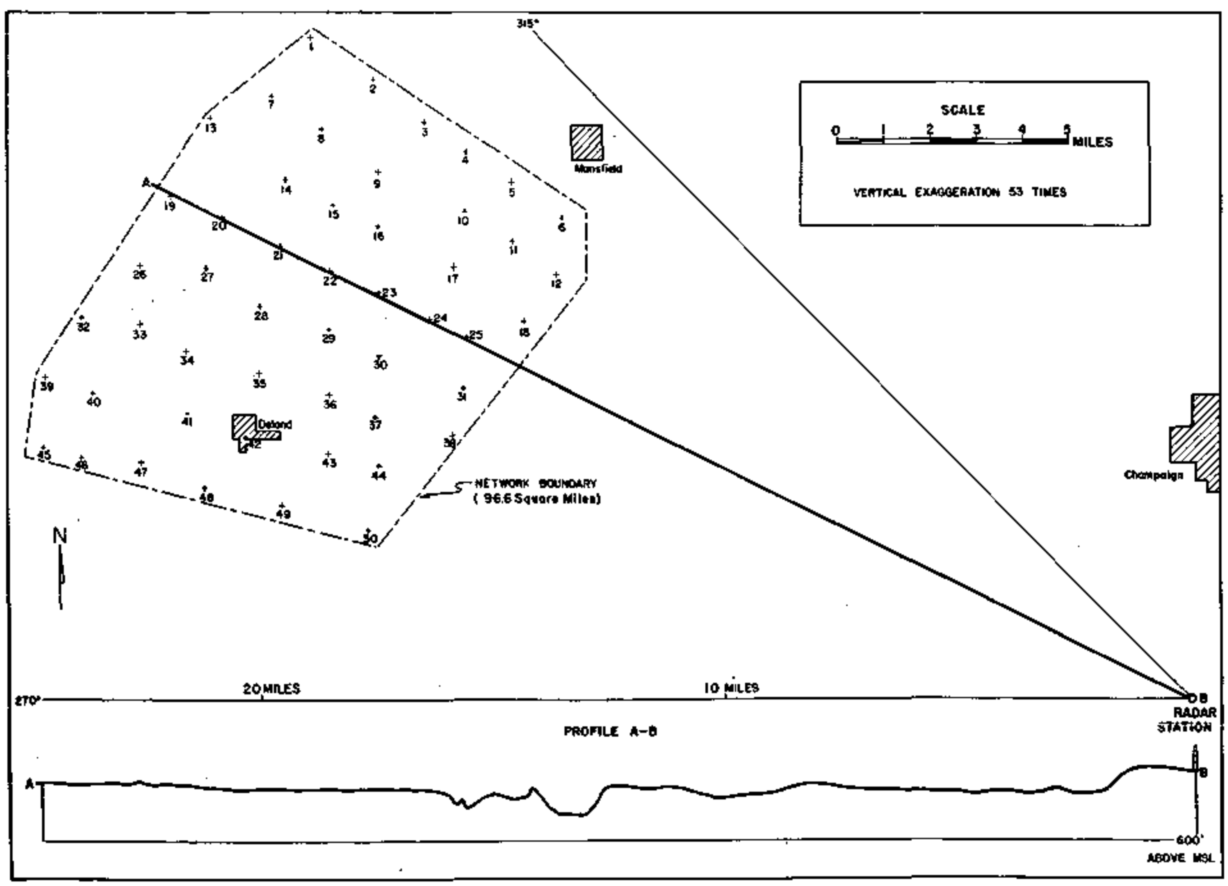

FIG. 2. TOPOGRAPHIC AND LOCATION RELATIONSHIPS OF THE RADAR STATION AND GOOSE CREEK NETWORK.

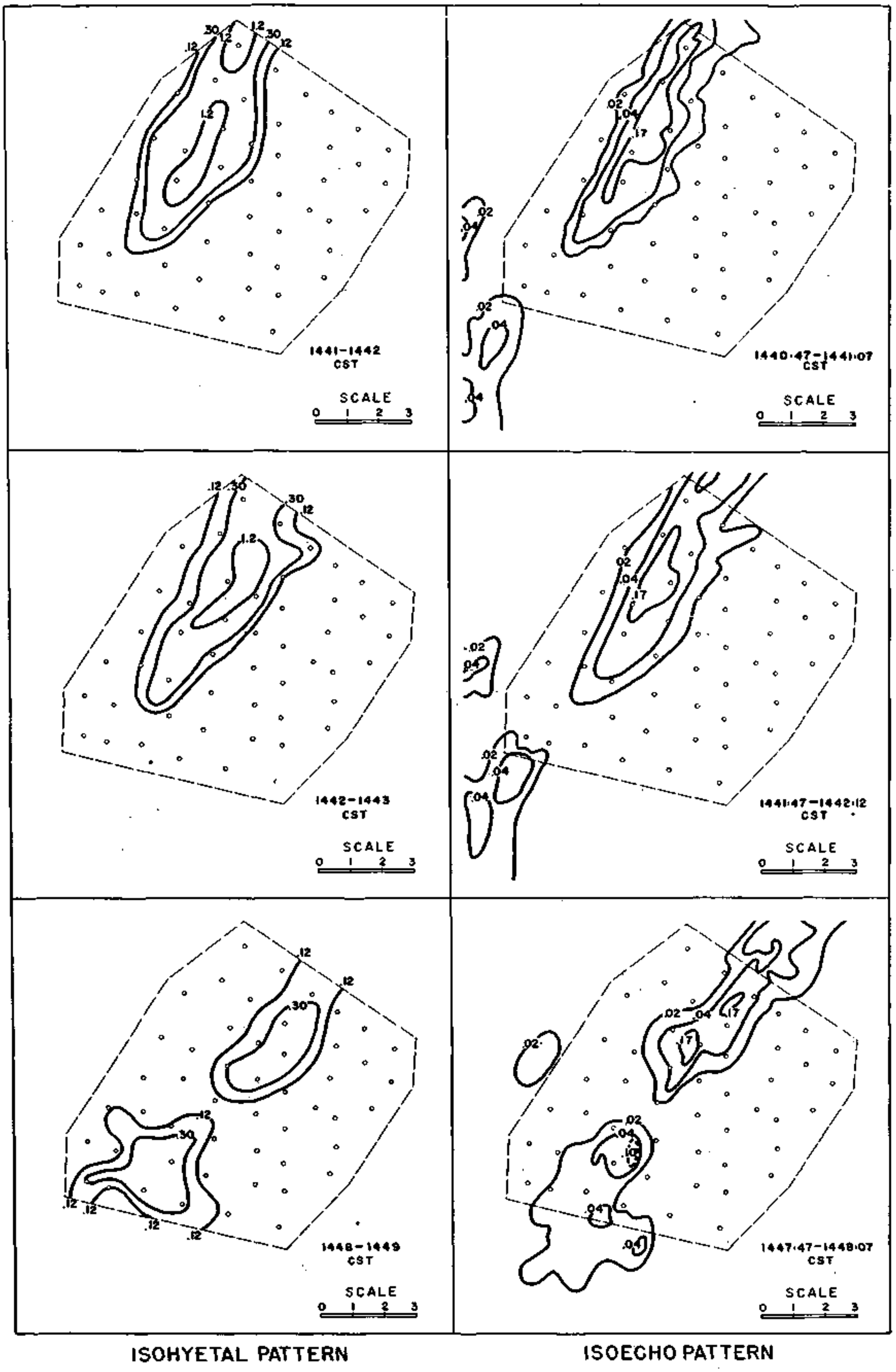

FIG. 3. COMPARATIVE RAINFALL MAPS. Rainfall in inches per hour for 1 September 1952. with scale in statute miles.

TABLE 1

RADAR-RAINGAGE SUMMARY OF NINE STORM PERIODS OVER THE
96 SQUARE-MILE GOOSE CREEK NETWORK, 1952

		Depth of Rainfall (Inches)				
		Raingage		Areal Mean		
Date	Time	Low	Max.	Raingage	Radar	Percent Difference
7-2	1551-1639	.05	.21	.10	.014	-86
8-15	0443-0640	.17	.42	.27	.033	-88
8-20	2037-2125	.08	.35	.17	.022	-88
9-1	1440-1500	.01	.10	.04	.004	-90
9-14	0833-0851	.00	.26	.06	.006	-90
9-14	0903-0915	.00	.11	.01	.002	-81
9-18	1352-1510	.08	1.35	.36	.050	-86
9-18	1540-1655	.20	1.07	.57	.011	-98
10-14	1354-1416	.04	.13	.09	.002	-98

were expressed in terms of rainfall intensity-through use of the formula

$$\text{Log}\ \frac{P_r R^2}{P_t} = 1.72 \log I - 10.895$$

where Pr is power received in watts, R is the range in nautical miles of the precipitation echo from the radar set, P_t is the power transmitted in watts and I is rainfall intensity in inches per hour. This equation is an adaptation of an empircial formula by Marshall, Langille, and Palmer[2] to the characteristics of the AN/APS-15A. The area enclosed by each rainfall intensity contour was obtained by planimetering.

It was assumed that the areas enclosed by the isoecho contours for each minute were representative sample rainfall areas for one-minute periods. The volume of rainfall represented by each one-minute isoecho contour map is equal to the sum of the products of the areas between adjacent isoecho lines and the "apparent" rainfall rates multiplied by a time factor of one minute. Rainfall rates for adjacent isoecho lines were determined by substituting the power values from the radar calibration data into the preceding formula. An average rate between adjacent isoecho lines was used in the rainfall computations. Total storm rainfall was then obtained by totaling the amounts for all the one-minute isoecho maps. These total and one-minute radar rainfall amounts were correlated with corresponding values from the Goose Creek Raingage network.

Total Areal-Mean Rainfall Comparisons. Areal-mean rainfall values computed for the Goose Creek network from radar and raingage data are presented in Table 1. With the exception of the 14 October storm, all of the storms included in this table were of the thunderstorm type. Rainfall rates over the network on 14 October were less variable than for the other eight rainfall periods. The low and maximum rainfall amounts recorded by any gage on the network are also included in Table 1 in order to give an estimate of the range in rainfall amounts over the network.

The radar areal-mean values were all much smaller than the corresponding raingage values. This result is in contrast to those reported in Research Report #1 under Contract No. DA-36-039 SC-42446 with U.S. Army, Signal Corps Engineering Laboratories and in the Illinois State Water Survey Report of Investigation No. 19.[1] Recently, a mathematical error was discovered in the formula used for computing rainfall intensity from radar return power. Use of the corrected formula resulted in radar areal-mean rainfall values which were of the order of magnitude of one-tenth of those computed with the previously used formula. The results were very similar when the formula was adjusted for the va Luc of

$$Z = 220\ R^{1.60}$$

reported by Marshall and Palmer[3] and to the value of

$$Z = 323\ R^{1.53}$$

reported by Wexler , where R is rainfall intensity, and $Z = \Sigma ND^6 \Delta D$, N being the number of drops of diameter D in an interval of diameter ΔD .

[2]Marshall, J. S., Langille, R. C., Palmer, W. Mek., "Measurement of Precipitation by Radar", J. Meteor. 4:186-192, 1947.

[3]Marshall, J. S. and Palmer, W. M., "The Distribution of Raindrops with Size", J. Meteor. 5:165-166, 1948.

[4]Wexler, R., "Rain Intensities by Radar", J. Meteor. 5:171-173, 1948.

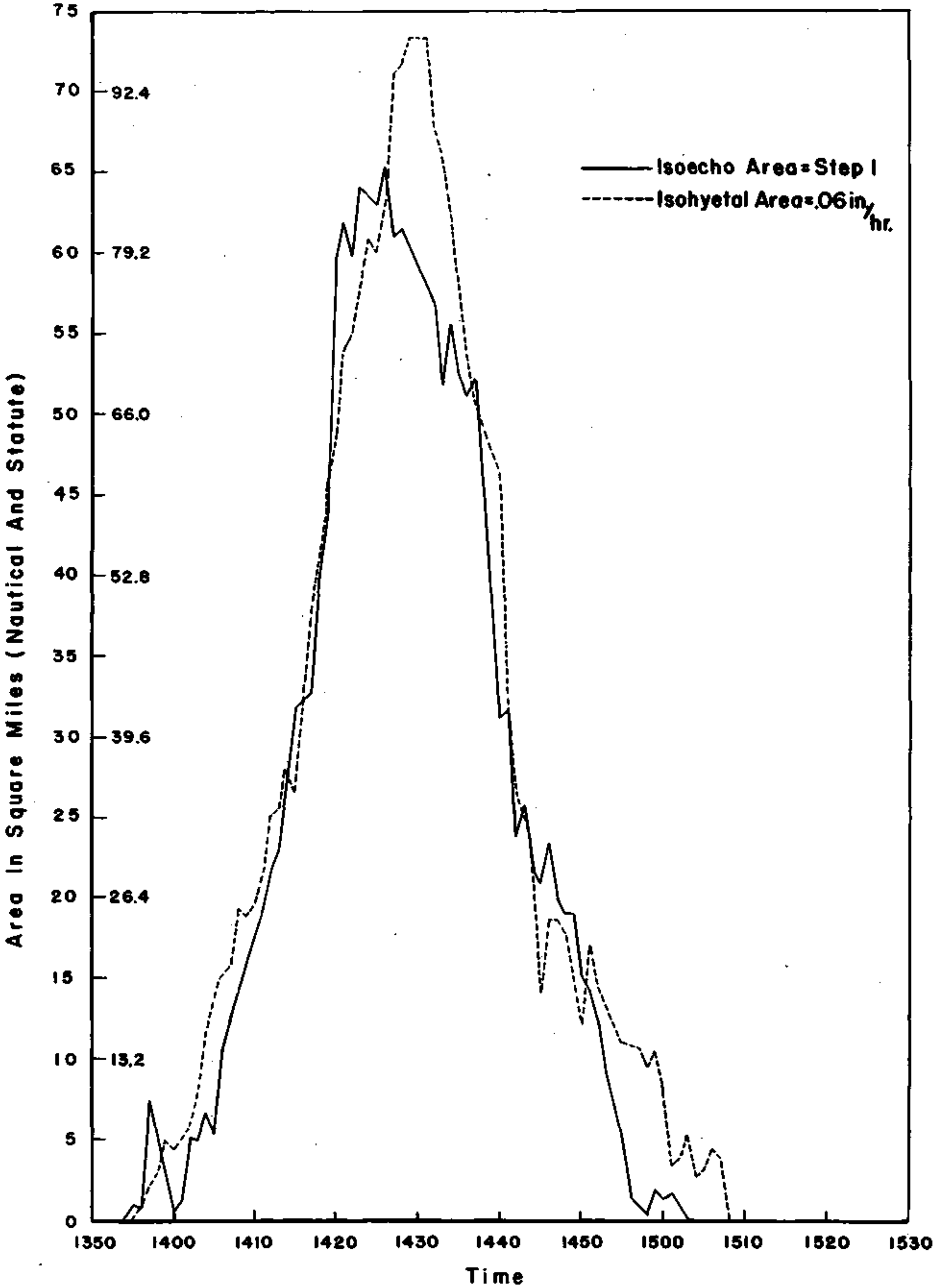

FIG. 4. COMPARISON OF THE PRECIPITATION AREA ENCLOSED BY RECEIVER SENSITIVITY STEP NO. 1 CONTOUR AND THE AREA ENCLOSED BY THE 0.06 IN. PER HOUR IOHYETAL. For 18 September 1952 storm over Goose Creek rain-gage network.

Radar-Raingage Areal Comparisons. Although the areal-mean rainfall yalues reported in the last section were very low in comparison with the raingage amounts, there is still considerable evidence which indicates that favorable quantitative rainfall values may be obtained from 3-cm radar data. Part of this evidence can be illustrated by a compa rison of isohyetal and isoecho maps of approximately the same time (Figure 3). One-minute isoecho maps, prepared from one series of receiver-sensitivity steps, were matched with one-minute isohyetal maps with approximately a one-minute time lag between the two maps. The time lag was necessary to allow the rain drops viewed by the radar to fall to the ground. Occasionally, a lag of two pr more minutes gave closer pattern comparisons. In general, for the sample maps shown in Figure 3, isohyetal and isoecho patterns compared favorably, although the rates computed for the isoecho contours are much too low. Minor differences in the isoecho and isohyetal patterns can be attributed to time variations between the 50 raingage clocks and the radar clock or errors of observation and interpretation of the data.

Another way of illustrating the ability of radar to outline areas of rainfall rate is shown in Figure 4. The broken line represents the area enclosed by the 0.06 in./hr isohyetal, as prepared from the one-minute rainfall amounts recorded by the Goose Creek raingage network during the passage of a squall line in advance of a cold front on 18 September 1952. The solid line is the area which was enclosed by the radar receiver-sensitivity step No. 1 contour. A good correlation between the step 1 contour and the 0.06 in./hr isohyetal is evident. For this storm, a value of 0.06 in./hr could be assigned to the step 1 contour with a fair degree of accuracy. The rate calculated with the radar rainfall equation was 0.03 in./hr at a range of 2 1 miles (mean range of Goose Creek network from the radar site).

Point rainfall rates during this storm period were as high as 11 in./hr. The highest network average rainfall rate computed from the raingage one-minute amounts was 1.26 in./hr which occurred at 14:26. The average rate over the network for the 40-minute period from 14:22 to 14:32 was approximately 1 in./hr. Attenuation loss due to rainfall does not appear to have been serious. This is a typical case where attenuation loss was limited to rainfall over the network, except for the last 15 minutes of the storm period when some rain echo was between the network and the radar site.

In the case of 3-cm radar, attenuation due to intervening raindrops may cause considerable differences between corresponding isoecho and isohyetal patterns, especially on the back side of a storm. Differences due to raindrop attenuation should have been small for the storm represented in Figure 3; since the echo areas were small, rainfall rates were relatively low and there was no precipitation between the network and the radar site. An example is shown in Figure 5 where attenuation due to intervening raindrops apparently caused a difference between the PPI patterns and the isohyetal patterns. In this example, a considerable area of relatively light rainfall was not detected by the radar. As indicated by the isoecho map, the radar beam had to pass through a broad band of precipitation on the forward edge of the rainfall zone.

Conclusion. Rainfall rates computed by using the formula

$$\log \frac{P_r R^2}{P_t} = 1.72 \log I - 10.895$$

were much less than surface rainfall rates recorded for the same time interval. However, results of precipitation area comparisons of radar and raingage network presentations from the Goose Creek network indicate that it should be possible to determine a relationship between 3-cm radar data and surface rainfall from which more accurate surface rainfall estimates can be made. An analysis has been started to determine and test the usefulness of such an analytical relationship.

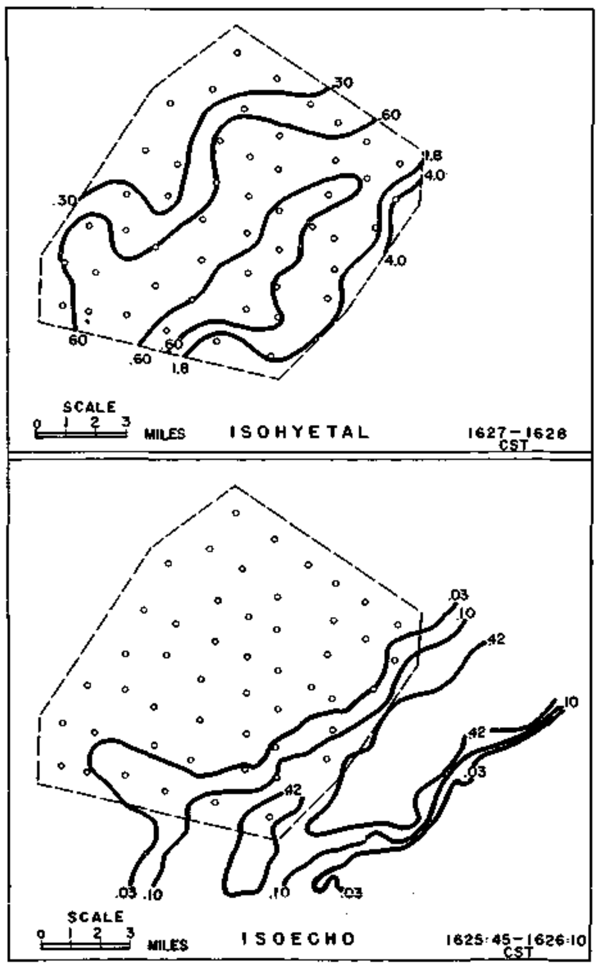

FIG. 5. ONE-MINUTE RAINFALL MAPS. Rainfall in inches per hour during 18 September 1952 storm.

VARIANCE OF AREAL-MEAN STORM RAINFALL ESTIMATES FROM NETWORKS OF VARIOUS GAGE DENSITIES

Introduction

In experimental estimation of rainfall with radar instrumentation, it is necessary to adopt some standard of rainfall measurement as a basis for judging the reliability of the radar estimates. Radar observations of precipitation are made in the volume of the beam at an altitude above the ground. However, precipitation measurements at ground level are of primary interest for most purposes. Before radar observations of rainfall amounts become practical a relationship to ground observations must be established. One possibility is to relate radar rainfall estimates for an area to a network of raingages on that area. One way of expressing the reliability of radar-rainfall estimates is in terms of the accuracy obtained with raingage networks of various gage densities.

During the 1952 thunderstorm season, the Water Survey operated an APS-15, 3-cm radar to obtain rainfall observations within the range of the equipment, especially for the 96 square-mile Goose Creek hydrologic network (Figure 2). A sampling scheme and procedure for analysis of raingage data were set up to obtain a measure of the variance of the estimates of areal-mean storm rainfall for various gage densities. The purpose of this study was to determine a measure of the deviation of estimates of areal mean rainfall obtained from different gage-density networks from the best estimate of the areal-mean rainfall, i.e., the mean calculated from the total population of gages used in the study. This measure of deviation for various gage densities from the best estimate of the true mean can be used as a standard for appraising the deviation of the radar areal-mean rainfall value from the best estimate of the true mean for the same storm. The investigation was limited to the development of relationships in shower-type precipitation.

Statistical Treatment of the Problems

The statistical treatment of the problem in this study was similar to that used by L. H. Madow[5] in a recent raingage density study in which the random start systematic sampling technique was used, and an equation of the form

$\sigma_{\bar{x}_n} = A\bar{P}^B n^C$ was fitted to the data. In this

equation, $\sigma_{\bar{x}_n}$ is the true standard deviation of the mean of a random start systematic sample of size n about $\bar{P}$, the best estimate of the population mean precipitation; n is the number of gages in a sample and A, B, and C are constants.

[5] Madow, L. H., Estimation of Mean Rainfall with Various Gage Densities in a Dense Raingage Network, Unpublished manuscript, Illinois State Water Survey, Urbana, Illinois, 1952.

There are two rather important differences between the Madow study and the one discussed in this section. Although the data for both studies came from two different networks of approximately the same size and topography, the Madow study was based on a total of 36 gages for one year and 24 for two other years of data; whereas, the study reported here was based on a total of 48 gages which were also more uniformily distributed over the network. Secondly, the raingage data in this study came from recording gages with 6-hour clocks, while the previous study was made on data from a gage network equipped with some recording gages with 7-day clocks and some stick gages. The recording gages with 6-hour chart drives made it possible to break the rainfall associated with any synoptic situation into individual rains. The storm period in the Madow study had to be, in general, a 24-hour period.

Sampling Procedure. A random start systematic sampling procedure was chosen for this study because it provides a plan for spreading the sample observations over the network and at the same time permits the use of data from all gages. A random sampling procedure allows the selection of gages in each sample to be entirely determined by chance. A. stratified random sampling plan provides a more consistently uniform spread of gages in each sample than that obtained by a purely random plan, but it allows the selection of gages to be determined more by chance than does the random start procedure. Sampling plans which involve the selection of centrally located gages in contiguous areas would have required the omission of a considerable number of observations from this study.

Secondly, rainfall observations in gage networks would not be obtained at random locations. Some plan for distributing gages in approximately a uniform manner would be used.

Madow[5] has pointed out that the purely random sampling variance is greater than the random start systematic variance for samples of the same size. Consequently, the radar would be given an unnecessary advantage if the accuracy of radar rainfall estimates were appraised by comparing them with random sampling errors. This is because the random start systematic sampling technique provides a more consistently uniform distribution of the observations over the network. In view of the above reasons, the random start systematic sampling procedure seemed like a satisfactory choice of sampling design.

Although there were 50 gages on the Goose Creek network, only 48 were used in this study. Records from two of the gages were omitted to have a larger number of prime factors which is convenient for subsampling. Gages number 24 and 36 (Figure 2) were omitted.

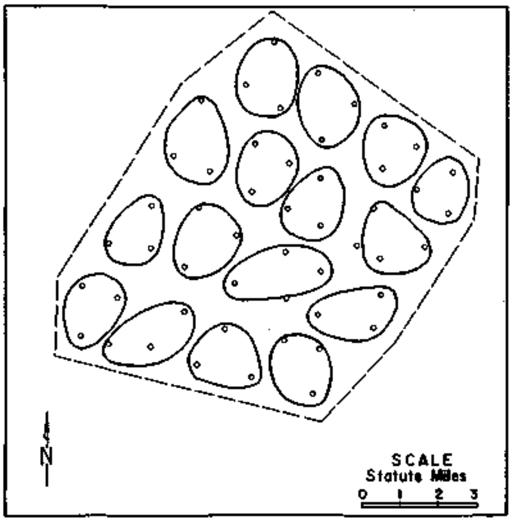

FIG. 6. GROUPS OF GAGES FROM WHICH 3 RANDOM START SYSTEMATIC SAMPLES OF 16 GAGES WERE SELECTED.

In order to estimate the random start systematic sampling variance, the possible samples (selection of rain gages) of various size had to be defined. Samples of size n = 2, 3, 4, 6, 8, 1Z, 16, and 24 gages were selected as illustrated in the following examples. Where n = 16, the 48 gages were divided into 16 groups of 3 gages (Figure 6). Division of the 48 gages into the 16 groups was done in a manner that provided as uniform a spread of the groups of 3 gages over the area as possible. According to the random start systematic sampling technique, a starting position is selected in one group of gages. One gage in approximately the same location is automatically designated in each of the other groups to complete the observations in each sample. Thus, there are three possible samples of size n = 16 which are treated as having equal probability of being an actual sample of 16 gages from the network. Sample sizes of 4, 6, 8, 12, and 24 were designated in a similar manner. A slight variation in this procedure was introduced for samples of size 2 and 3. For samples of size 2, for example, instead of pairing gages 6 and 31, 5 and 30 (Figure 7), gages 6 and 26, 5 and 27, etc. were paired in order to prevent these small samples from being extremely biased to one side of the network.

Data Used. Data for this study consisted of the rainfall records from 48 gages for 16 storms which occurred on the Goose Creek network during the months of July, August, and September 1952. In the place of missing values, estimates from an isohyetal map were used. All but two of the storms were associated with cold fronts and squall lines. Two storms on 16 July were associated with a warm front. The range in mean rainfall over the network was from 0.01 inch to 1.35 inches. Storms with less than 0.01 inch were eliminated, since some of the gage recordings were too small to read with any degree of accuracy.

For this study, the total rainfall associated with any synoptic situation was divided into individual storm totals. A stop in rainfall of 30 minutes or longer over the network was used as a standard for dividing the total rainfall into individual storm totals. This definition of the event to be studied was chosen so as to correspond closely with the storm event that would be of most interest for comparison with the radar record.

Analytical Procedure. The main problem in the analysis of the data was the estimation of the true variance, σ^2, of the estimated average precipitation, $\bar{x}_n$, about the true average precipitation, P, for the network area of 96 square miles. An estimate of σ^2 leads to an expected measure of the error involved in sampling with each different gage density. This error is the quantity which is used to judge the magnitude of the deviation between the areal-mean rainfall computed from radar data and that computed from the 48 gage recordings.

Discussions in statistical papers have shown that a variance can be broken-down into various components. The components of variance in this study may be designated as: $\sigma^2_{\bar{x}_n}$, sampling variance; $\sigma^2_{\bar{x}_{48}}$ variance of the mean of 48 gages about the true mean; and σ^2_0, variance which is due to errors of observation. If it is assumed that $\sigma^2_{\bar{x}_n}$, $\sigma^2_{\bar{x}_{48}}$ and σ^2_0 are independent, then $\sigma^2 = \sigma^2_{\bar{x}_n} + \sigma^2_{\bar{x}_{48}} + \sigma^2_0$.

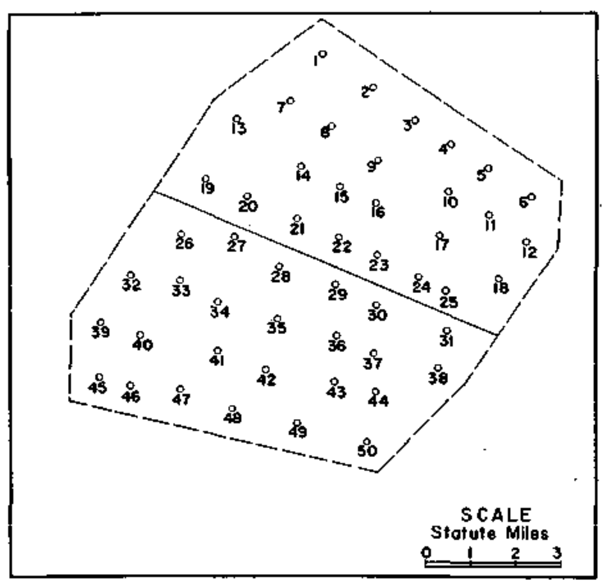

FIG. 7. TWO GROUPS OF GAGES FROM WHICH 24 RANDOM START SYSTEMATIC SAMPLES OF 2 GAGES WERE SELECTED.

For the purpose of computing an estimate of σ^2 it was assumed that σ_0^2 was zero. Although this is not entirely true, it is reasonable to expect that σ_0^2 is small, and can possibly be ignored in comparison with $\sigma^2_{\bar{x}_n}$ and $\sigma^2_{\bar{x}_{48}}$. Also, $\sigma^2_{\bar{x}_{48}}$ could not be estimated because the true network mean was unknown. Consequently, $\sigma^2_{\bar{x}_{48}}$ had to be ignored in the computations and the best available estimate, $\bar{P} = \frac{\sum_{j=1}^{48} p_j}{48}$, of the true mean, P, had to be used in the computations. The quantity p_j is the total rainfall recorded at the jth gage. Therefore, with σ_0^2 taken to be zero, estimation of σ^2 was accomplished by computing a best estimate of $\sigma^2_{\bar{x}_n}$ for samples of size n = 2, 3, 4, 6, 8, 12, 16, and 24. This computation consisted mainly of two steps. The first step was the determination of a group of quantities which were designated as $s^2_{\bar{x}_n}$. The $s^2_{\bar{x}_n}$ values represent the randomstart systematic sampling variance of $\bar{x}_n$ about $\bar{P}$. Each $s^2_{\bar{x}_n}$ is then a sample estimate of the population parameter, $\sigma^2_{\bar{x}_n}$, for a particular sample of size n.

In order to obtain a measure of the sampling variation in terms of the unit in which the rainfall data were measured, i. e., inches instead of (inches)^2, the standard error, $s_{\bar{x}_n}$ of the estimates, $\bar{x}_n$, about $\bar{P}$ was obtained by taking the square root of $s^2_{\bar{x}_n}$. The standard error is given by the expression

$$s_{\bar{x}_n} = \sqrt{\frac{\sum_{i=1}^{k} (\bar{x}_{n_i} - \bar{P})^2}{k}}$$

where k = the number of possible samples of size n. The values are tabulated in Table 2.

TABLE 2

OBSERVED STANDARD ERROR OF THE ESTIMATES OF MEAN RAINFALL FOR RANDOM START SYSTEMATIC SAMPLES BASED UPON VARIOUS NUMBERS OF RAINGAGES FOR 16 STORMS OVER GOOSE CREEK NETWORK DURING JULY, AUGUST, AND SEPTEMBER 1952

			$s_{\bar{x}_n}$ in Thousandths of an Inch of Rainfall							
Storm Date	Time	$\bar{P}$ (0. 00 inch)	Number of Gages in the Sample: 2	3	4	6	8	12	16	24
7-7	2342-0123	012	061	030	036	021	031	017	009	000
7-8	0434-0950	028	072	034	028	017	025	013	011	011
7-14	2000-2030	011	053	039	035	030	020	015	006	015
7-16	0517-0634	005	038	020	022	019	017	014	005	013
7-16	0733-0947	021	086	065	037	016	018	004	004	011
8-3	1057-1132	012	113	061	052	039	035	016	025	009
8-4	2119-2330	045	128	052	065	051	037	028	042	005
8-11	0515-0645	002	007	003	006	005	003	004	002	002
8-11	0830-1640	110	179	097	075	052	069	046	016	042
8-15	0234-0858	135	221	131	142	148	082	096	090	039
8-20	1128-1238	001	013	010	006	005	004	003	001	002
8-20	2030-2243	029	053	037	026	016	024	014	012	013
8-30	2210-0021	012	062	057	040	033	018	009	020	002
9-1	1433-2015	019	033	024	014	010	009	007	003	005
9-18	1400-1535	040	160	159	079	054	036	026	002	007
9-18	1542-1659	057	072	057	056	053	031	024	023	002

The second step in determining an estimate of sampling variation involved the fitting of a regression system to the $s_{\bar{x}_n}$ values. The regression lines provide values which are designated as the best estimates, $\hat{s}_{\bar{x}_n}$, of the standard error of $\bar{x}_n$ about $\bar{P}$. A discussion of the regression system follows.

It may be observed in Table 2 that the standard error of the estimates appear to increase in general as $\bar{P}$ increases, although there is considerable fluctuation in this upward trend. Undoubtedly, there are many other factors which contribute to the variability of the $s_{\bar{x}_n}$ values from storm to storm such as: (1) the meteorological factors causing the storm, (2) location of the storm core with respect to the center of the network, (3) duration of the storm, and (4) rate of rainfall. However, it is difficult to express (1) and (2) quantitatively and $\bar{P}$ is a function of (3) and (4). Also, it may be noted from Table 2 that the $s_{\bar{x}_n}$ values tend to increase as the number of observations in the sample decrease. It seemed reasonable to assume that $s_{\bar{x}_n}$ is a function of $\bar{P}$ and n and attempt to relate these factors by an equation of the form

$$\sigma_{\bar{x}_n} = A\bar{P}^B n^C$$

where A, B, and C are constants. It was also assumed that log $s_{\bar{x}_n}$ was distributed about log $\sigma_{\bar{x}_n}$ so that the variance of the random error in the $s_{\bar{x}_n}$ values for n in all cases was equal. This assumption was made so that the least squares method of curve fitting could be used without a complicated and time consuming system of weighting the $s_{\bar{x}_n}$ values because they were based on different k.

For convenience in computation, the above expression for $s_{\bar{x}_n}$ may be reduced to the linear form by taking the log of both sides and substituting a = log A. Thus, log $s_{\bar{x}_n} = a + B \log \bar{P} + C \log n$. The best estimate of log $\sigma_{\bar{x}_n}$ was obtained by determining a, B, and C by minimizing,

$$\sum_{n=1}^{8} \sum_{m=1}^{16} \left[\log s_{\bar{x}_n} - (a + B \log \bar{P}_m + C \log n)\right]^2$$

which is the sum of squares of deviation of observed log $s_{\bar{x}_n}$ from expected log $s_{\bar{x}_n}$, where n refers to the number of observations in the sample and m refers to the number of storms. Values for the constants A, B, and C as determined by the method of least squares were 0.2774, 0.531, and -0.824, respectively. When these values are substituted in the expression for $\sigma_{\bar{x}_n}$, the equation becomes

$$\hat{s}_{\bar{x}_n} = 0.2774\, \bar{P}^{0.531} n^{-0.824}$$

where $\hat{s}_{\bar{x}_n}$ and $\bar{P}$ are in inches of rainfall. By substituting values for $\bar{P}$ and n in this equation and solving for $\hat{s}_{\bar{x}_n}$, a set of points was determined for preparing the curves shown in Figure 8. These curves indicate the relationship between $\bar{P}$, n, and $\hat{s}_{\bar{x}_n}$; where $\hat{s}_{\bar{x}_n}$ is the best estimate of $\sigma_{\bar{x}_n}$, the true standard error of $\bar{x}_n$ about $\bar{P}$, that can be determined by the least squares technique of fitting $\sigma_{\bar{x}_n} = A\bar{P}^B n^C$ to the data.

A measure of the usefulness of the regression system for predicting was obtained by computing the correlation between the observed $s_{\bar{x}_n}$ values and the corresponding values which are predicted by the curves. This correlation index was +0.88. It indicates a relatively high degree of relationship between observed and predicted. However, an examination of the scatter of observed $s_{\bar{x}_n}$ values about the regression system indicated that a considerable amount of the total variance of the

TABLE 3

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum Squares	Mean Square	Variance Ratio	Variance Ratio Necessary for Significance at the 99% Level
Total	127	0.207			
Explained by Regression	3	0.156	0.052	130.0	3.96
Unexplained or Deviation from Regression	124	0.051	0.0004		

$s_{\bar{x}_n}$ values remained unexplained. A summary of the explained and unexplained variance is presented in Table 3. The mean squares in the third column were obtained by dividing the sum squares by their degrees of freedom. The variance ratio, obtained by comparing .052 with..0004, is considerably larger than that necessary for significance at the 99 per cent level. This indicates that a significant amount of the variance was explained by the regression system. However, the deviations between observed and expected values of s are still large enough to produce a relatively large unexplained sum of squares as shown in the last line of Table 3.

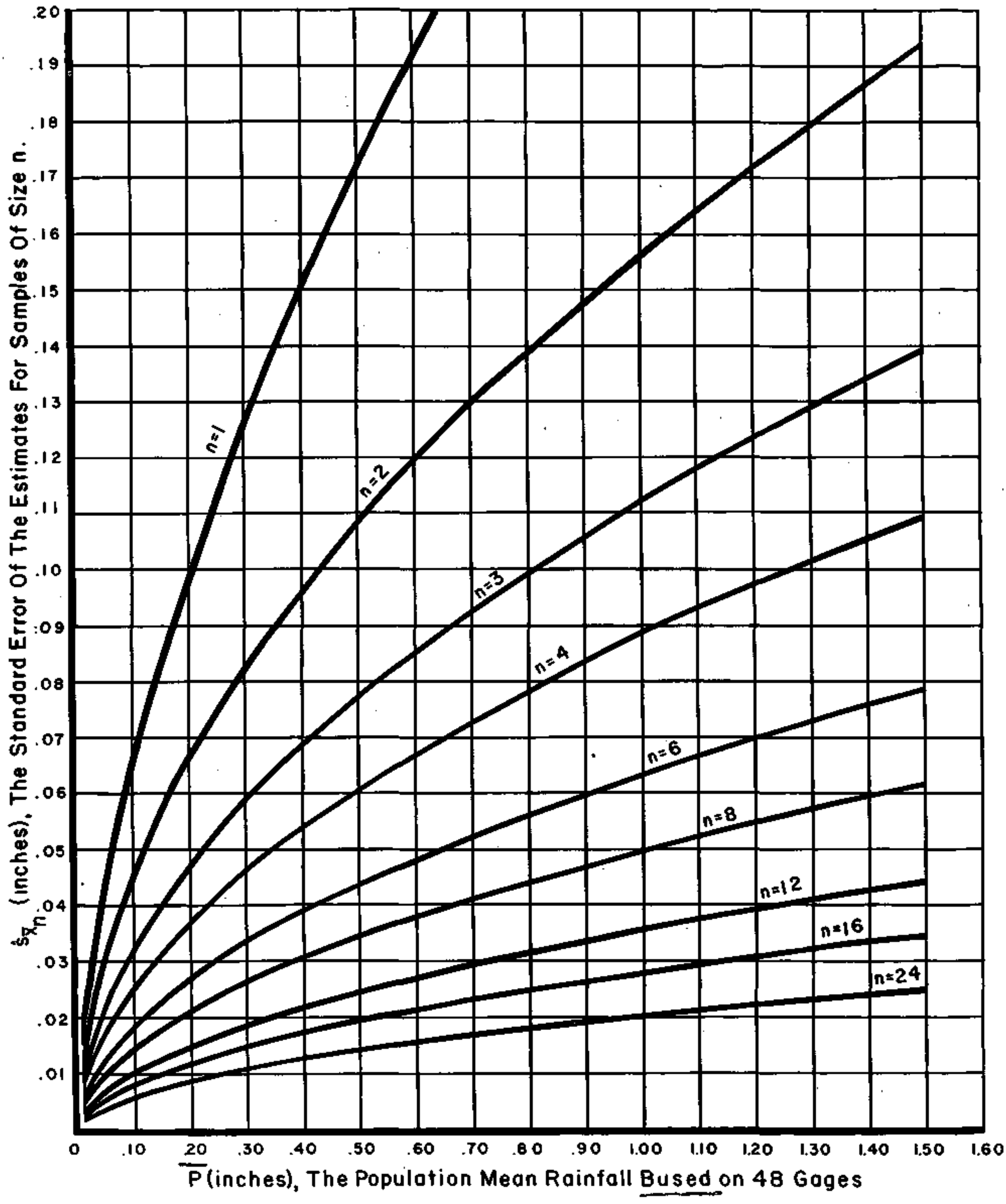

FIG. 8. VARIATION OF $\hat{s}_{\bar{x}_n}$ WITH $\bar{P}$ FOR SEVERAL SAMPLES OF SIZE n, FOR 16 STORMS OVER GOOSE CREEK NETWORK, 1952.

There are two possible explanations for the magnitude of the unexplained variation. Although the relationship of the standard errors of the estimates to $\bar{P}$ and n is very definite, there are undoubtedly factors in addition to $\bar{P}$ and n which contribute to the variance of the estimates. A better fitting regression system might be obtained by including additional factors in the formula for estimating $\sigma_{\bar{x}_n}$. However, a relationship of $\hat{s}_{\bar{x}_n}$ to $\bar{P}$ and n is the most convenient and useful for testing the reliability of the radar rainfall estimates. Secondly, the method of least squares may not be an adequate technique for fitting a regression system to the observed data in this problem. The observed $s_{\bar{x}_n}$ values are based on different numbers of degrees of freedom. Consequently, the assumption that the errors in the $s_{\bar{x}_n}$ values are homoscedastic, i.e., the assumption that log $s_{\bar{x}_n}$ varies about log $\sigma_{\bar{x}_n}$ independent of the values of $\bar{P}$ and n, may be incorrect. A better fitting regression system could probably be obtained especially with respect to n by the method of maximum likelihood if the values of A, B, and C found by least squares were taken as first approximations to be used in another method based upon the principle of maximum likelihood, and a new set of constants computed. The least squares method is a special case of the maximum likelihood principle. The maximum likelihood principle can be applied in such a way that the proper weighting factors for the $s_{\bar{x}_n}$ values are introduced to account for each being based on a different k.

The random start systematic sampling scheme does not produce a regression line for n = 1. It is at the point where n = 1 that the random start systematic manner of dividing the network into groups of contiguous gages becomes the same as selecting one observation at random. Variance of these estimates would be equivalent to the purely random sampling variance for samples of size n = 1. There is also a tendency for the random start systematic sampling variance to approach a random sampling variance for other small samples, because the spread of observations in each sample becomes less uniform over the area as n decreases.

The error curve (Figure 8) for samples of size 1 was determined from the standard deviations of random samples of size 1. This curve was based on the same storm data as were used in the random start systematic study. The resulting equation was $\hat{s}_r = 0.2598\,\bar{P}^{.597}$ where $\hat{s}_r$ is the best estimate of standard deviation of sample of size 1 and $\bar{P}$ is the mean storm rainfall based on 48 gages. The units of $\hat{s}_r$ and $\bar{P}$ are inches of rainfall.

As was previously mentioned the random sampling error curve for n = 1 may indicate errors which are somewhat larger than should be used in determining the accuracy of the radar rainfall estimates. Consequently, it may be reasoned that the radar estimates would be given an unnecessary advantage. The reason is that: using one gage in an area for obtaining estimates of rainfall it would logically be placed relatively close to the center of the area instead of at a random location. A centrally located gage would result in sampling errors which would be less than those from a randomly located gage. The same agrument may be advanced for the random start systematic sampling error curves for samples of size 2, 3, 4, and possibly 6 and 8 gages, since the tendency for the sampling variance of the random start systematic samples tends to approach the random sampling variance as n decreases. Again it is logical to obtain rainfall estimates from gages placed near the center of the areas they are sampling, i.e., approximately on a grid pattern. Since the contracting agency has recently expressed an interest in centrally located samples, analysis is being performed to determine whether a better set of error curves can be obtained for appraising the radar estimates, especially for the small sample sizes.

<u>Confidence Limits for the Network-Mean Storm Rainfall from 48 Gages</u>. An estimate of the mean storm rainfall, P, becomes more meaningful when some measure is made of the possible error in the estimate. It is, therefore, of interest to determine some interval about $\bar{x}_n$, with some measure of confidence that $\bar{P}$ is in that interval. The 95 per cent confidence interval is customarily chosen.

If the gage readings for a storm were normally distributed about $\bar{P}$ and a random sample of n gages had been used to obtain $\bar{x}_n$, a quantity

$$t = \frac{(\bar{x}_n - \bar{P})}{s_n/\sqrt{n}}$$

could be determined which has the t-distribution with n-1 degrees of freedom, where s_n is the standard deviation of the gage readings used in computing $\bar{x}_n$. From this formula for t, it is possible to find a number, say $t_{.05}$, such that the probability $[-t_{.05} < t < +t_{.05}] = .95$. It is then possible to convert the inequalities and obtain the probability $\left[\bar{x}_n - t_{.05}\frac{s_n}{\sqrt{n}} < \bar{P} < \bar{x}_n + t_{.05}\frac{s_n}{\sqrt{n}}\right] = .95$, the limits of which can be determined for any sample $\bar{x}_n$ to obtain a 95 per cent confidence interval for $\bar{P}$. However, systematic sampling was used instead of random sampling for samples of size 2 and greater. Systematic samples are expected, in general, to yield values of $\bar{x}_n$ which are closer to $\bar{P}$ than a random sample of the same size.

As an approximation, it was assumed that the quantity $t = \frac{(\bar{x}_n - \bar{P})}{\hat{s}_{\bar{x}_n}}$, where $\hat{s}_{\bar{x}_n} = 0.2774\bar{P}^{0.531} n^{-0.824}$ had a t-distributioh with 13 degrees of freedom The 13 degrees of freedom were based on the fact that 16 storms were used in computing $\hat{s}_{\bar{x}_n}$ and that 1 degree of freedom was used in computing each of the 3 constants, A, B, and C. The reasonableness of this approximation was tested by counting the percentage of $\bar{x}_n$ values which were included between $\bar{P} - t_{.05}\hat{s}_{\bar{x}_n}$ and $\bar{P} + t_{.05}\hat{s}_{\bar{x}_n}$, where $t_{.05}$ for 13 degrees of freedom is 2.160. This percentage would be 95 per cent if the assumption were correct. The actual count showed 93.5 per cent inside $\bar{P} \pm 2.160\hat{s}_{\bar{x}_n}$ and 6.5 per cent outside this interval. The number of $\bar{x}_n$ values which were inside and outside the confidence limits are tabulated for each storm and sample size in Table 4. In order to obtain confidence limits for samples of size 1, it was assumed that $t = \frac{p_j - \bar{P}}{\hat{s}_r}$ had a t-distribution with 14 degrees of freedom. The 14 degrees of freedom were based on the fact that 16 storms were used in computing $\hat{s}_r$ and that 2 degrees of freedom were used in computing the coefficient and the exponent of P. This assumption was checked by counting the percentage of p_j values which were included between $\bar{P} - t_{.05}\hat{s}_r$ and $\bar{P} + t_{.05}\hat{s}_r$, where $t_{.05}$ for 14 degrees of freedom is 2.145. The actual count was 95.3 per cent inside $\bar{P} \pm 2.145\hat{s}_r$ and 4.7 per cent outside this interval.

TABLE 4

NUMBER OF SAMPLE MEAN PRECIPITATION VALUES WHICH WERE INSIDE AND OUTSIDE. THE 95 PERCENT CONFIDENCE LIMITS FOR 16 STORMS OVER GOOSE CREEK NETWORK DURING JULY, AUGUST AND SEPTEMBER 1952

Storm Date	$\bar{P}$ (0.00) inch	n=24 In	n=24 Out	n=16 In	n=16 Out	n=12 In	n=12 Out	n=8 In	n=8 Out	n=6 In	n=6 Out	n=4 In	n=4 Out	n=3 In	n=3 Out	n=2 In	n=2 Out	Total In	Total Out
7-7	012	2	0	3	0	4	0	4	2	8	0	11	1	16	0	22	2	70	5
7-8	028	2	0	3	0	4	0	6	0	8	0	12	0	16	0	24	0	75	0
7-14	011	0	2	3	0	4	0	6	0	8	0	11	1	16	0	24	0	72	3
7-16	005	2	0	3	0	3	1	5	1	7	1	11	1	15	1	23	1	69	6
7-16	021	2	0	3	0	4	0	6	0	8	0	12	0	15	1	23	1	73	2
8-3	012	2	0	1	2	4	0	5	1	5	3	7	5	13	3	15	9	52	23
8-4	045	1	1	1	2	4	0	6	0	8	0	11	1	16	0	23	1	70	5
8-11	002	2	0	3	0	4	0	6	0	8	0	12	0	16	0	24	0	75	0
8-11	110	2	0	3	0	4	0	6	0	8	0	12	0	16	0	23	1	74	1
8-15	135	2	0	2	1	3	1	6	0	6	2	11	1	15	1	22	2	67	8
8-20	004	2	0	3	0	4	0	6	0	8	0	12	0	15	1	23	1	73	2
8-20	029	2	0	3	0	4	0	6	0	8	0	12	0	16	0	24	0	75	0
8-30	012	2	0	0	2	4	0	6	0	5	3	11	1	12	4	22	2	63	12
9-1	019	2	0	3	0	4	0	6	0	8	0	12	0	16	0	24	0	75	0
9-18	040	2	0	3	0	4	0	6	0	7	1	11	1	9	7	22	2	64	11
9-18	057	2	0	3	0	4	0	6	0	8	0	12	0	16	0	24	0	75	0
TOTAL		45	3	41	7	46	2	44	4	41	10	36	12	30	18	26	22	1122	78
		93.7		85.4		95.8		91.7		85.4		75.0		62.5		54.2		93.5	

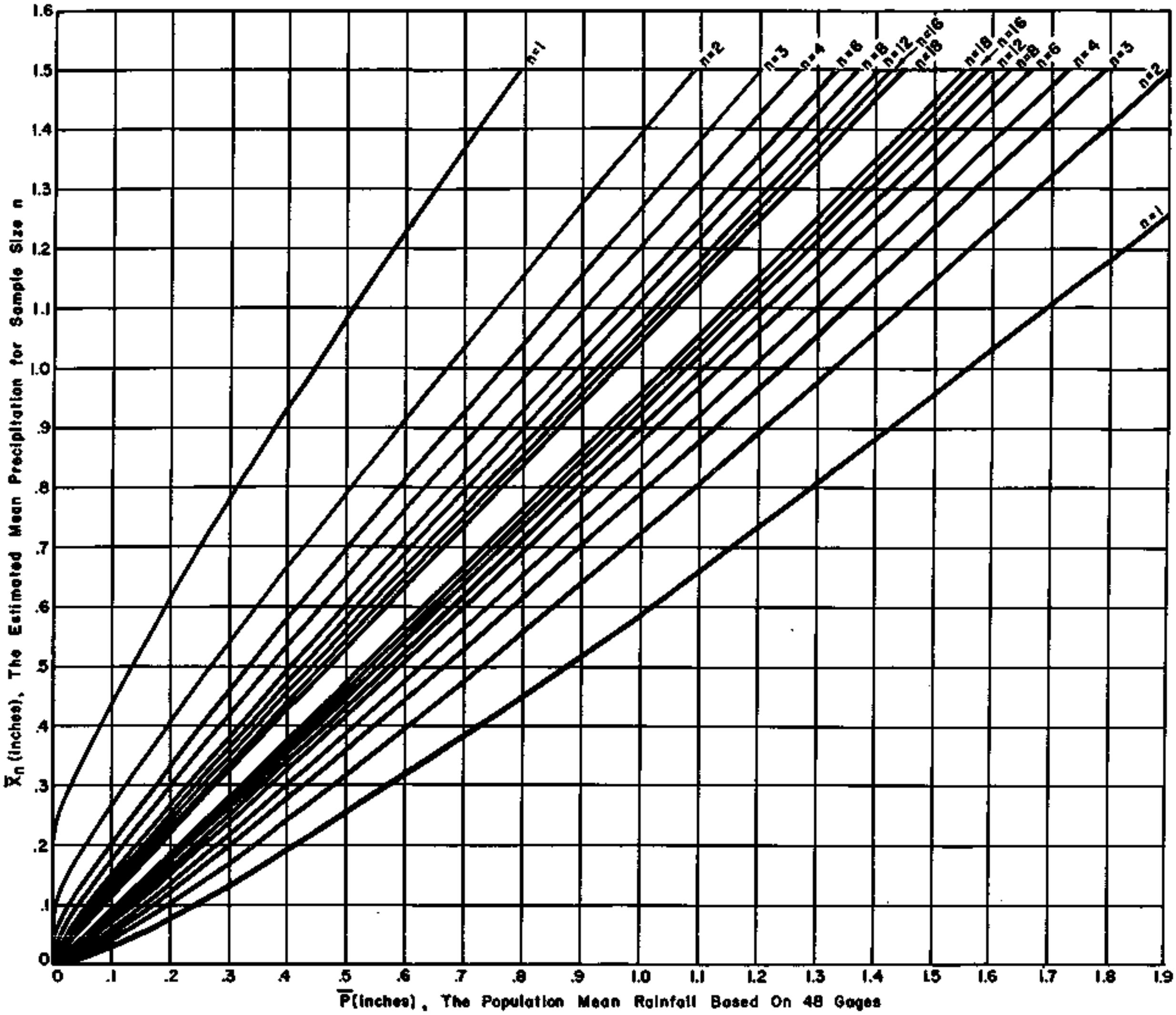

FIG. 9. CONFIDENCE BANDS FOR MEAN RAINFALL BASED ON ESTIMATED MEAN RAINFALL FOR SAMPLES OF SIZE n, GOOSE CREEK NETWORK, 1952.

Confidence bands are presented in Figure 9 for $\bar{P} \pm 2.145\hat{s}_r$ for samples of size 1 and for $\bar{P} \pm 2.160\hat{s}_{\bar{x}_n}$ for samples of size 2, 3, 4, 6, 8, 12, 16, and 24. The use of this chart may be illustrated by an example. If an $\bar{x}_n$ value of 1.00 inch of rainfall was obtained from a random start systematic sample of 8 gages, the confidence probability is .95 that $\bar{P}$ is between 0.89 inch and 1.11 inches. The same result can be obtained from Figure 8 by determining the interval 1.00 ± the product of (2.160) (.05), where .05 equals $\hat{s}_{\bar{x}_n}$ for 1.00 inch of rainfall and n = 8. Figure 9 is a convenience since upper and lower limits of the confidence band for $\bar{P}$ can be read directly and without calculations.

It should be noted that the confidence bands are for $\bar{P}$ instead of P. Confidence limits for P should be slightly larger than those for $\bar{P}$ since an estimate of $\sigma_{\bar{x}_{48}}$, the standard error of $\bar{P}$ about P was not included in $\hat{s}_{\bar{x}_n}$. However, $\sigma_{\bar{x}_{48}}$ could not be estimated since P was not known; therefore, the confidence limits for $\bar{P}$ are the best estimates available for confidence limits for P.

Introduction

In the present system of analysis, an estimate of areal-mean rainfall is obtained from a continuous series of radar observations during a storm period. Because this method is time-consuming, it might be desirable to eliminate some of the data, providing appreciable accuracy is not sacrificed. For example, work in compilation can be reduced by 20 per cent by eliminating every other minute of the data.

In designing an area rainfall integrator, it is pertinent to know how much data can be eliminated or omitted without reducing the accuracy of the results. Since a large amount of data is involved in area integration, an appreciable amount of time for printing by recording devices is required. If it is feasible to sample on an interval basis, the printing problem can be simplified. An area integrator developed at Massachusetts Institute of Technology records measurements at the same point every 15 or 30 minutes.

A time-interval sampling study was undertaken to obtain an estimate of the error involved in measuring areal-mean storm rainfall when a portion of the data is omitted at regular intervals. A discussion of the methods used and the results obtained is presented in this section.

Data Used

The problem is how to obtain a quantitative estimate of the error involved in mean-areal storm rainfall determinations from rainfall data recorded at different intervals during a storm. Quantitative radar-rainfall estimates of mean-areal storm rainfall are obtained from data records from successive series of receiver-sensitivity settings. Each series requires a time interval of one minute or less, depending on the number of sensitivity settings required to reduce the precipitation echo return to less than the threshold of minimum sensitivity. If it is assumed, as in the section on quantitative radar-rainfall measurements, that each series of data is a representative sample for a one-minute interval, then the mean-areal rainfall computed for each minute is a representative sample mean for the same interval of time'. In this study, the sampling unit is defined as a one-minute areal-mean rainfall accumulation. The number of sampling units in the population to be sampled depends upon the duration of the storm. A storm or rainfall event in this study is defined as the rainfall accumulation occurring between beginning and ending of rainfall over the Goose Creek network. As in the raingage density study, the rainfall associated with any synoptic situation was divided into individual storms, and cessation in rainfall of 30 minutes or longer was used as a standard for determining individual storm events.

Although the discussion thus far has been concerned with radar data, it should be noted at this point that raingage data instead of radar data were used in the computations which follow. One-minute, network-mean rainfall amounts computed from raingage data were used as sampling units instead of means from radar data for the following reasons: (1) During the 1952 thunderstorm season, mechanical and operational difficulties, in addition to a very dry season, prevented the collecting of a complete storm record by radar of a sufficient number of storms for this study. (2) Radar data is subject to attenuation whereas this factor can be eliminated by using raingage data. (3) The results from a study of this type should have application to radar sets in general. (4) One-minute network-mean rainfall amounts are good samples of what is desired from radar observations.

The sampling unit of data used in this study was the mean rainfall accumulation, $\bar{a}$, on the Goose Creek network during each one-minute interval of a storm where

$$\bar{a} = \frac{\sum_{i=1}^{[50-(M+R)]} a_i}{[50-(M+R)]} .$$

Each a_i is a one-minute accumulation at the ith gage. The number 50 denotes the total number of gages on the network. The letters M and R denote the number of gage records missing and the number of observations that could not be determined for a particular minute because of the character of the trace, respectively. Thus, for a storm of T minutes duration, there were T $\bar{a}$'s. The sum of the $\bar{a}$'s for T minutes is equal to the network-mean storm rainfall, $\overline{P}$, which represents the best estimate of the true network-mean rainfall. In succeeding discussion p_t will be used to denote estimate of $\overline{P}$ based on t minutes of data.

Statistical Formulation of the Problem

The main problem in the analysis of the data is to obtain an estimate of the true variance, σ^2, of the estimates of mean network rainfall, based on subsamples from a total of t $\bar{a}$'s. As was the case in the raingage density study, the true variance can be broken down into three components of variance: $\sigma^2_{\bar{p}_t}$, which is due to sampling; $\sigma^2_{\bar{p}}$, which is due to the variance of $\overline{P}$ about the true population mean, P, and σ^2_0, which includes all the errors of observation. Neither σ^2_0 or $\sigma^2_{\bar{p}}$ can be estimated from the data. It seems a plausible assumption to accept $\sigma^2_0=0$, since it is undoubtedly small in comparison with $\sigma^2_{\bar{p}_t}$. The value of $\sigma^2_{\bar{p}}$ cannot be estimated since the true mean rainfall is not known. Consequently, the problem of estimating σ^2 becomes one of computing an estimate of $\sigma^2_{\bar{p}_t}$, where the computation will actually yield an estimate of σ^2. In other words, the computation involved in estimating σ^2 is that of determining the sampling variance $s^2_{\bar{p}_t}$ which is the best estimate of $\sigma^2_{\bar{p}_t}$. It should be noted then that $s^2_{\bar{p}_t}$ does not include an estimate of $\sigma^2_{\bar{p}}$. This means that $s^2_{\bar{p}_t}$ may be an underestimate of σ^2

Sampling Scheme

A systematic time sampling scheme was devised for obtaining estimates of the mean precipitation, $\bar{P}$, over the raingage network. The sampling intervals which were chosen for this study were for every 2nd, 4th, 6th, 10th, 15th, 20th and 25th minute. For a sampling interval of 4 minutes, for example, there are four samples of a's as shown in Table 5, beginning at 1, 2, 3, and 4 minutes after the start of a storm. The sums of the $\bar{a}$'s in each sample were obtained and multiplied by a factor of t = 4, since each of these sums included only one-fourth of the storm rainfall. This method produced four estimates $(\bar{p}_t)$ of the mean precipitation $(\bar{P})$ over the network. The number of minutes in the time interval is equal to the number of different samples obtained, and is equal to the number which, when multiplied by the sums of sample $\bar{a}$'s, give the $\bar{p}_t$

In the example given above, t was divisible by the sampling interval. This circumstance did not always occur in the sampling procedure. When T was not divisible by the sampling interval, some of the rainfall estimates for any given sampling interval contained one or more minute observations which equalled zero. This circumstance introduced variability among the rainfall estimates. However, it is a source of error which would be present in rainfall estimates obtained by sampling with radar at an interval.

Determination of the Best Estimate of the Standard Error of Estimates of Network-Mean Storm Rainfall from Samples of Different Size

Calculation of Standard Error of Estimates of Network-Mean Rainfall from Sample Observations Taken at Selected Intervals. The quantity, $s_{\bar{p}_t}$ represents the standard error of the $\bar{p}_t$ values about $\bar{P}$ for a given sampling interval. As explained in the next section, the $\bar{p}_t$ values are used in obtaining $\hat{s}_{\bar{p}_t}$. where $\hat{s}_{\bar{p}_t}$ is the best estimate of $\sigma_{\bar{p}_t}$. Determining $s_{\bar{p}_t}$ instead of $s^2_{\bar{p}_t}$ provides a measure of variation in the original units of measurement, inches of rainfall.

The $\bar{p}_t$ values for all samples for each of the seven time intervals were calculated. The standard error of these estimates about $\bar{P}$ was determined by using the expression:

$$s_{\bar{p}_t} = \sqrt{\frac{\sum_{j=1}^{k} (\bar{p}_j - \bar{P})}{k}}$$

where $\bar{p}_j$ is the jth estimate of $\bar{P}$ for a particular interval, and k is the number of $\bar{p}_j$ values for each interval. The $s_{\bar{p}_t}$ values are tabulated in Table 6.

Analytical Relationship of Standard Error of Estimates of Network-Mean Storm Rainfall to the Sampling Interval and the Storm Size. An analytical relationship of the standard error of the estimates to the sampling interval was desired. An examination of the $s_{\bar{p}_t}$ values in Table 6 indicates that, in general, the error increases as the sampling interval increases. Undoubtedly there are several other factors which contribute considerably to the variance of the $s_{\bar{p}_t}$ values. It is felt that storm characteristics such as the average rate of rainfall, duration, and amount

TABLE 5

AN EXAMPLE OF THE SYSTEMATIC SAMPLES FOR A SAMPLING INTERVAL OF FOUR MINUTES

Minute Number	$\bar{a}$	Minute Number	$\bar{a}$	Minute Number	$\bar{a}$	Minute Number	$\bar{a}$
1	$\bar{a}_1$	2	$\bar{a}_2$	3	$\bar{a}_3$	4	$\bar{a}_4$
5	$\bar{a}_5$	6	$\bar{a}_6$	7	$\bar{a}_7$	8	$\bar{a}_8$
9	$\bar{a}_9$	10	$\bar{a}_{10}$	11	$\bar{a}_{11}$	12	$\bar{a}_{12}$
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
T-3	$\bar{a}_{T-3}$	T-2	$\bar{a}_{T-2}$	T-1	$\bar{a}_{T-1}$	T	$\bar{a}_T$

TABLE 6

STANDARD ERROR OF THE ESTIMATES OF MEAN-AREAL STORM RAINFALL FOR SYSTEMATIC SAMPLES OF SEVEN DIFFERENT TIME INTERVALS FOR EIGHT STORMS OVER GOOSE CREEK NETWORK DURING JULY, AUGUST, AND SEPTEMBER 1952

Storm Date	Duration (Min.)	$\bar{P}$ (in.)	$s_{\bar{p}_t}$ in inches of Rainfall — Sampling Interval (Minutes) 2	4	6	10	15	20	25
7-16	77	.046	.000	.001	.001	.002	.003	.004	.004
7-16	132	.200	.002	.007	.006	.009	.015	.043	.030
8-3	45	.110	.002	.005	.010	.018	.011	.027	----
8-4	131	.432	.005	.015	.007	.033	.030	.059	.066
8-20	133	.251	.002	.002	.007	.016	.012	.017	.028
9-14	44	.130	.000	.007	.009	.015	.014	.025	----
9-18	95	.417	.001	.010	.016	.030	.052	.060	.143
9-18	77	.571	.005	.011	.022	.067	.160	.248	.320

of rainfall are contributing factors in determining the variance of the errors of estimates.

Also, the variance of rainfall rate during the storm period is expected to contribute considerably to the variance of the estimates. In other words, it should be possible to obtain greater accuracy with a given sampling interval for a relatively steady rainfall rate during a storm than for a more variable rate.

A satisfactory quantitative measure of the rate variance with time over the raingage network is rather difficult to determine. Although this factor is expected to influence the error of the estimates, it was felt that it would be less useful in an analytical expression than a factor to represent the storm size. Mean-areal storm rainfall, $\bar{P}$, is a factor which represents storm size and is also a function of storm duration and average rainfall rate. An examination of the data in Table 6 indicates that, in general, the larger $s_{\bar{p}_t}$ values are associated with the larger $\bar{P}$ values.

In order to determine an equation which would express the relationship of $s_{\bar{p}_t}$ to the sampling interval, I, and mean rainfall, $\bar{P}$, it was decided to test the goodness-of-fit of an equation of the following form to the data.

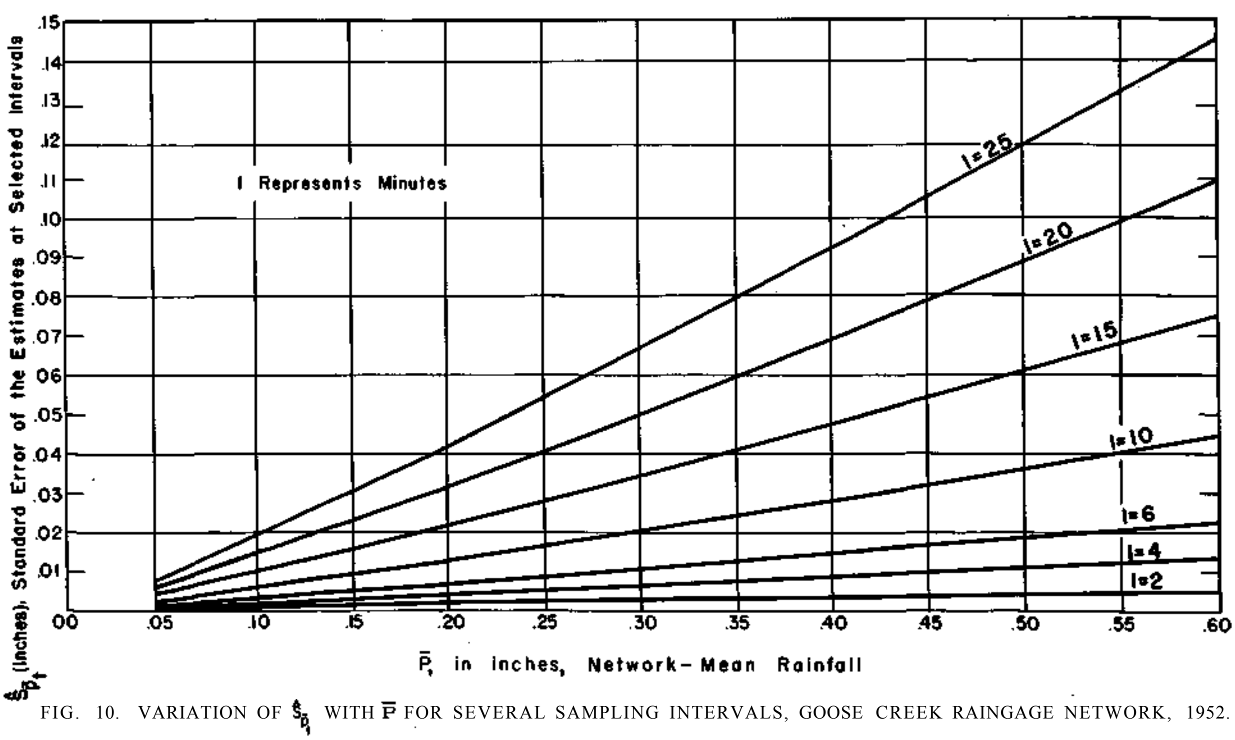

FIG. 10. VARIATION OF $s_{\bar{p}_t}$ WITH $\bar{P}$ FOR SEVERAL SAMPLING INTERVALS, GOOSE CREEK RAINGAGE NETWORK, 1952.

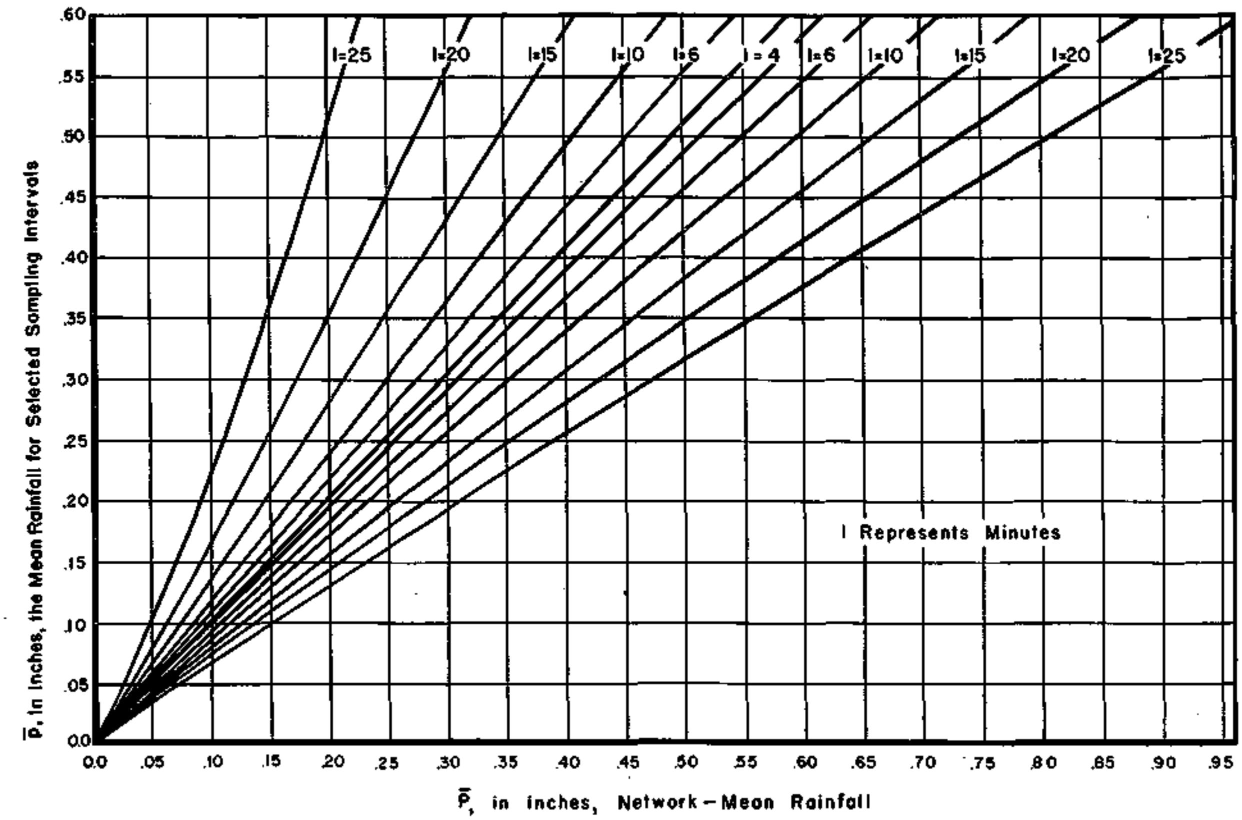

FIG. 11. CONFIDENCE BANDS FOR MEAN RAINFALL BASED ON ESTIMATES OF MEAN RAINFALL FOR SEVERAL SAMPLING INTERVALS, GOOSE CREEK RAINGAGE NETWORK, 1952.

$$\sigma_{\bar{p}_t} = K\bar{P}^{L}\, I^{M}$$

The least squares method was used to determine the constants K, L, and M. The resulting equation is

$$\hat{s}_{\bar{p}_t} = 0.004\bar{P}^{1.13}\, I^{1.29}$$

where $\hat{s}_{\bar{p}_t}$ and $\bar{P}$ are in inches of rainfall and I is in minutes. The quantity $\hat{s}_{\bar{p}_t}$ represents the best estimate of $\sigma_{\bar{p}_t}$. The correlation between the observed $s_{\bar{p}_t}$ values and $\hat{s}_{\bar{p}_t}$ values was 0.87. A graph of the relationship of $\hat{s}_{\bar{p}_t}$ to $\bar{P}$ and I is shown in Figure 10.

Confidence Bands for Network Mean Rainfall

Confidence limits for $\bar{P}$ were prepared for the 95 per cent level. As in the raingage density study, an approximation was assumed for t. The quantity $t = \frac{\bar{p}_t - \bar{P}}{\hat{s}_{\bar{p}_t}}$ was assumed to have a t-distribution with 5 degrees of freedom. The number of degrees of freedom in this study was based on 8 storms. One degree of freedom was used in computing each of the 3 constants K, L, and M. The reasonableness of this assumption was tested by determining the proportion of $\bar{p}_t$ values which were actually within the limits of $\bar{P} \pm t_{.05}\hat{s}_{\bar{p}_t}$. When the value of 2.571 is substituted for $t_{.05}$ for 5 degrees of freedom, the limits become $\bar{P} \pm 2.571\hat{s}_{\bar{p}_t}$. The proportion of $\bar{p}_t$ values within these limits was 94. The percentage would have been 95 if the assumption was correct.

The confidence bands, $\bar{P} \pm 2.571\hat{s}_{\bar{p}_t}$ are presented in Figure 11. The use of this chart can be explained by an example. If it is assumed that an accuracy of ±10 per cent is desired, an indication of the largest sampling interval that can be used, and still attain this accuracy, can be obtained from the chart. For an areal-mean rainfall of 0.25 inch, the chart indicates that 6 minutes is the longest interval that can be used and still obtain an estimate within the range of 0.25 ±0.025.

Introduction

Quantitative rainfall estimates are computed from repeated series of systematic radar sample observations of the rainfall-rate distribution over an area during a period of rainfall. Each sample of the rainfall-rate distribution is obtained by varying the receiver sensitivity in a stepwise fashion. A sample, as it is defined here, is an isoecho contour map such as those shown in Figure 3.

When sampling for quantitative rainfall estimates with a limited number of sensitivity steps (observations in the sample), the most reliable sample should be obtained by selecting the observations in a way so that the greatest number of observations are selected from those rates which contribute the greatest amount of the total water. The best sample can be obtained by using smaller increments of rate between sensitivity steps in the range of rates that produce the greatest amount of water.

This study was undertaken to determine an estimate of the amount of rainfall contributed by different rainfall rates during thunderstorm rainfall. The resulting information will be helpful in determining the best rainfall-rate setting for each step on a receiver-sensitivity stepping switch used in quantitative rainfall measurements.

Data Used

One-minute rainfall amounts for eight 1952 and two 1951 thunderstorms over the Goose Creek raingage network made up the basic data. The definition of a storm was the same as in the previous sections. The one-minute rainfall amounts were determined for the duration of each storm at 33 raingage locations for the two 1951 storms and at 50 raingage locations for the eight 1952 storms. A total of 5130 one-minute raingage amounts from the 1952 storms and 1579 one-minute amounts from the 1951 storms were included in the study.

The one-minute amounts were treated as rainfall rates since they are close approximations to rates. Rainfall amounts per minute ranged from 0.001 inch to 0.230 inch or 0.06 inch per hour to 13.8 inches per hour.

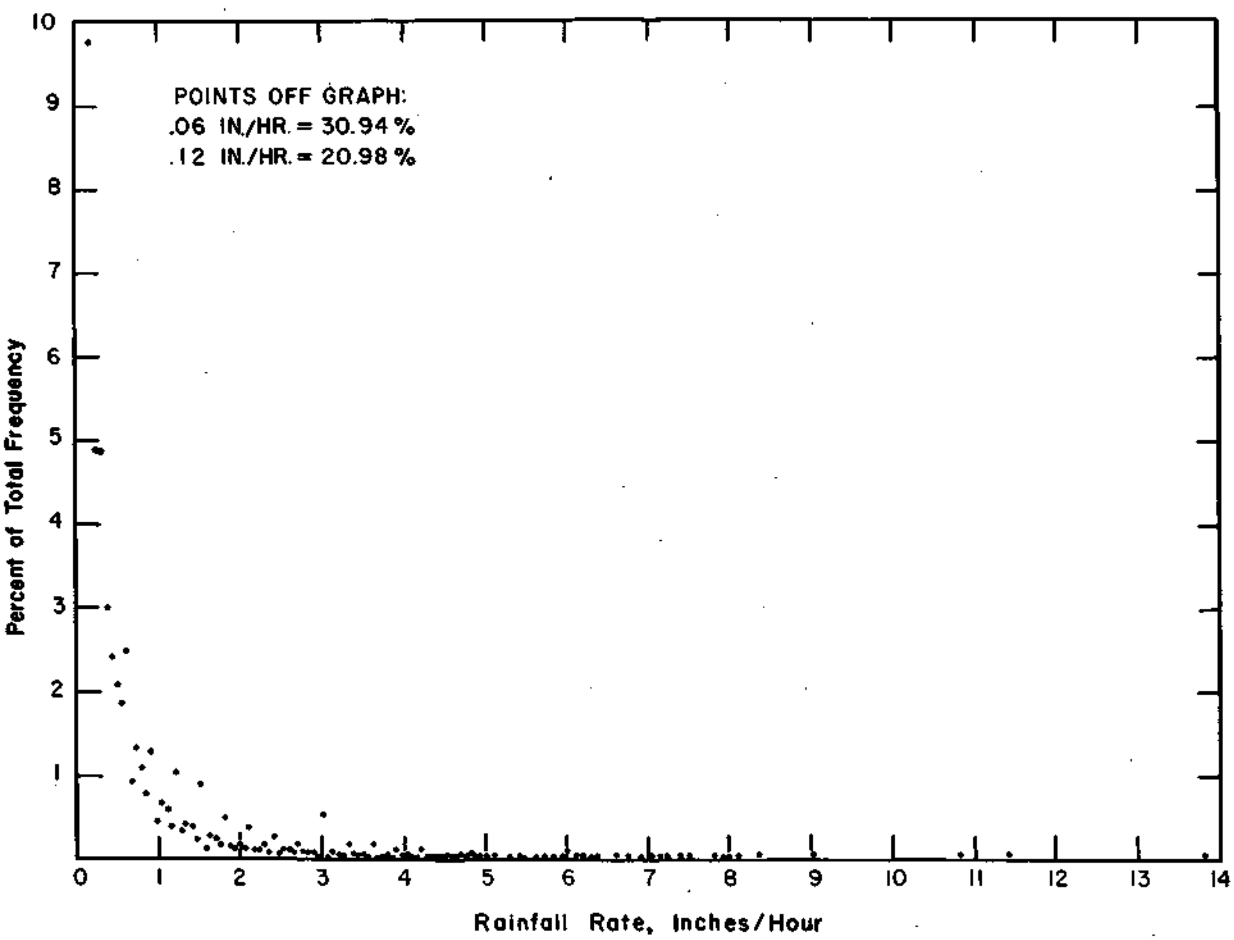

FIG. 12. PERCENT OF TOTAL FREQUENCY CONTRIBUTED BY VARIOUS RAINFALL RATES.

Procedure and Discussion

Rainfall Rate Frequency. The frequency of each rate during each of the ten storms was determined by counting the number of times each rate occurred at all the gages. Storm rate frequencies for each rate were totaled and each rate total expressed as a per cent of the total of all rates. The various rates and their percentages are presented in Figure 12. It is evident that the low rainfall rates occur more frequently than high rates. However, the frequency of occurrence of each rate is not the end product in the study. The proportion of the total rainfall which was contributed by the different rates is more important here. This aspect is discussed in the following paragraphs.

Per cent of Water Sampled Versus Rate. Since each rate is a one-minute accumulation of rainfall at a point (raingage location), the percentage of the total water collected in all gages which was contributed by each rate was determined as follows: (1) All the accumulations for each rate for all gages was obtained. These sums are equal to the amount of the network water sample contributed by each rate during a storm, (2) Each rate total was then expressed as a percentage of the total water collected, that is, sampled during a storm.

The per cent of accumulative water collected was plotted against rainfall rate. Figures 13 and 14 show examples which illustrate the range and differences in the appearance of the storm curves used in this study. These curves were prepared by drawing free hand lines through numerous points with very little scatter.

Raingage records are shown in Figures 15, 16, and 17 for the purpose of illustrating the type of point rainfall characteristics which are included in the curves of accumulative water sampled versus rainfall rate. The raingage record for 20 August 1952, Figure 16, has a short period of relatively high rates and a long period of low rates. Rainfall which occurs in this manner will produce a curve near the left hand extreme of Figure 13; whereas, the rainfall record of the 18 (B) September storm, Figure 6, is typical of the point rainfall records which produced the curve on the extreme right of Figure 14. The third raingage record, Figure 17, is typical of those which are included in the curve for 9 July 1951 in Figure 14.

All of the rate frequencies for each rate during the eight 1952 storms were totaled by rates and converted to the percentage of the total water which each rate contributed. From these percentages, a curve relating per cent of total ac-

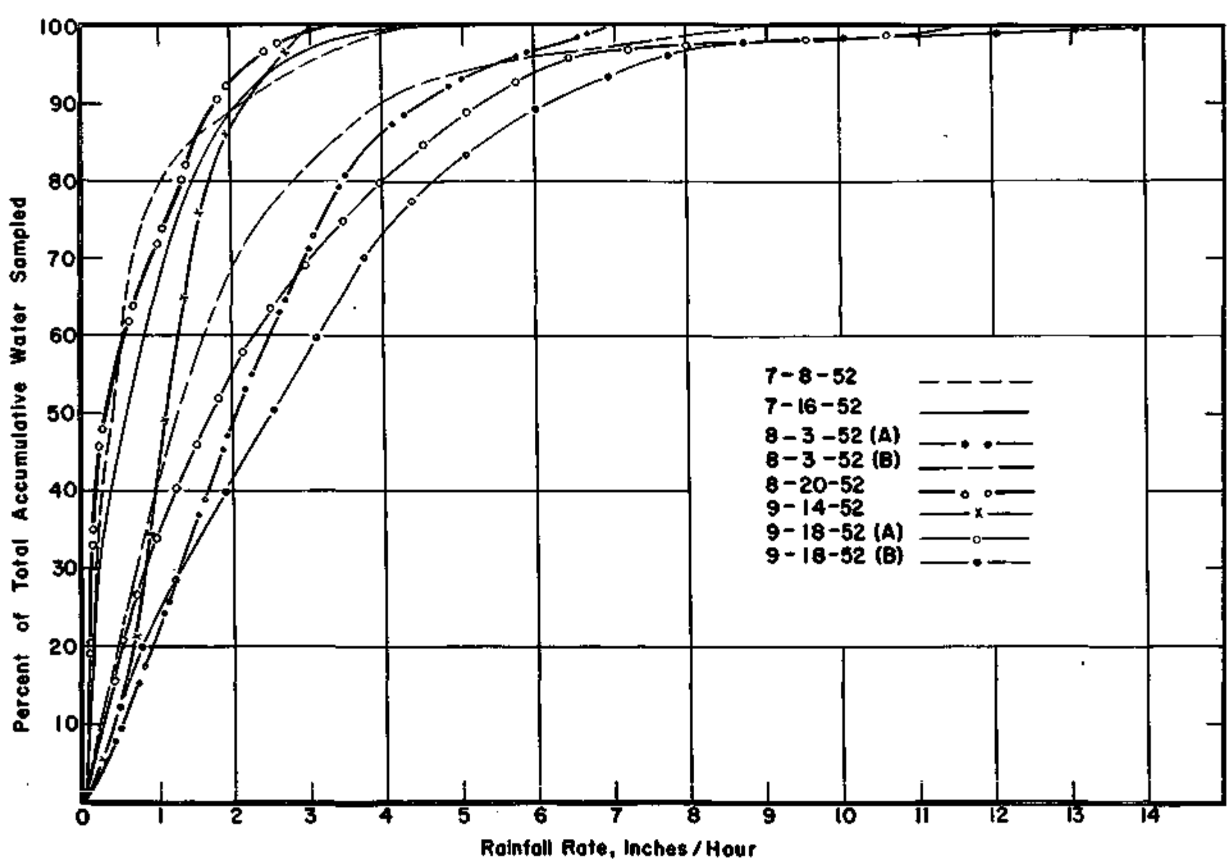

FIG. 13. RELATION BETWEEN RAINFALL RATE AND PERCENT OF TOTAL ACCUMULATIVE WATER SAMPLED FOR EIGHT 1952 STORMS OVER GOOSE CREEK RAINGAGE NETWORK.

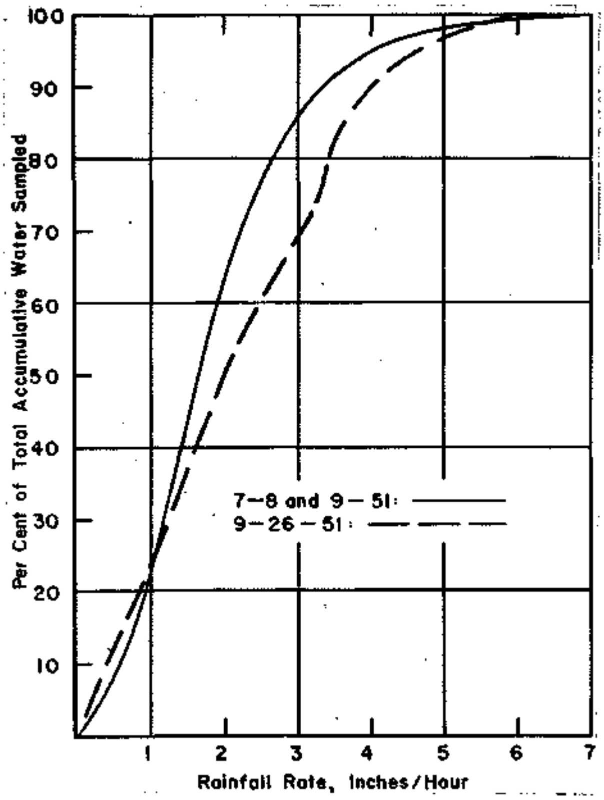

FIG. 14. RELATION BETWEEN RAINFALL RATE AND PERCENT OF TOTAL ACCUMULATIVE WATER SAMPLED FOR TWO 1951 STORMS OVER GOOSE CREEK RAINGAGE NETWORK.

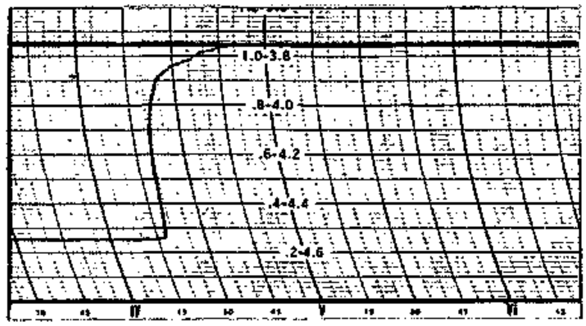

FIG. 16. RAINGAGE RECORD OF RAINFALL AT STATION 11 ON GOOSE CREEK NETWORK DURING THE 18 SEPTEMBER 1952 STORM.

cumulative water sampled to rainfall rate (solid line, Figure 18) was prepared. It is felt that this curve is a fair representation of each rate contribution for the 1952 thunderstorm season. The two 1951 storms were not included in Figure 18 because the number of gages and the size of the network were not the same as for 1952.

The two dashed curves in Figure 18 were prepared in order to indicate the extreme values around the curve for the per cent of total water sampled. The extreme lines do not represent any particular storm. They represent extreme values from all of the individual curves which are included in the solid line.

Receiver-Sensitivity Stepping Switch Settings. The solid-line curve in Figure 18 was used to indicate the best receiver-sensitivity settings for the APS-15 stepping switch. The lowest rainfall rate which can be detected by the APS-15 radar at a range of 15 to 25 miles (Goose Creek network) is approximately 0.1 inch per hour. A rate of 0.1 inch per hour, therefore, had to be accepted for step 1 on the stepping switch. A total of about 10 steps was desired. An upper rate limit for step 10 was chosen at the rate which would include about 99 per cent of the total water sampled. The rate at 99 per cent was chosen partially because it includes nearly all of the water sampled and partially for convenience. According to Figure 18, the step 1 rate of 0.1 inch per hour intersects the curve at about 9 per cent. When the step 10 rate is set at 99 per cent, it is convenient to divide the interval from step 1 to step 10 into 9 increments of 10 per cent each. According to this arrangement, rainfall accumulation contri-

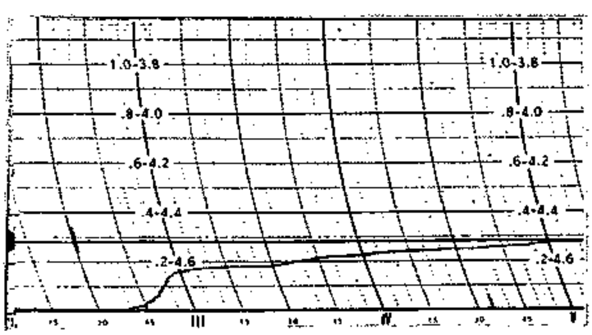

FIG. 15. RAINGAGE RECORD OF RAINFALL AT STATION 45 ON GOOSE CREEK NETWORK DURING THE 20 AUGUST 1952 STORM.

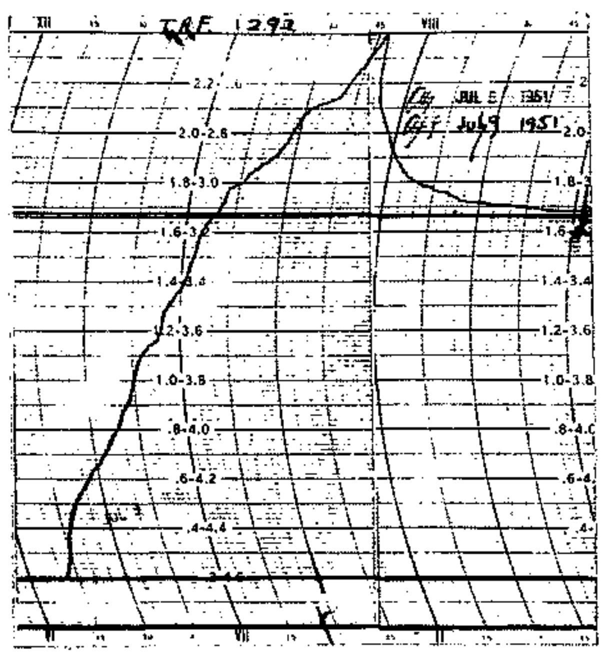

FIG. 17. RAINGAGE RECORD OF RAINFALL AT STATION 13 ON GOOSE CREEK NETWORK DURING THE 9 JULY 1951 STORM.

buted by rates less than 0.1 in./hr are not included in the radar rainfall measurement. Using 10 per cent increments, step 1 accounts for the water from 9 per cent up to, but not including, 19 per cent of the total water, etc. All rainfall rates of 11 in./hr and greater are recorded by step 10, or as 11 in./hr in the suggested arrangement. The rainfall rates suggested for each step by the rate versus per cent of accumulative water sampled curve are presented in Table 7. The suggested stepping switch values may be used as minimum or threshold values, as just described, or they can be used as means for the stepping switch values. If they are used as means, the stepping switch values would be centered about the suggested rate to be recorded.

TABLE 7

RAINFALL RATES VERSUS
RECEIVER SENSITIVITY SETTINGS

Step No.	Suggested Rate (in./hr)	Stepping Switch Settings (in./hr)
1	0.10	0.10
2	0.30	0.30
3	0.50	0.50
4	0.83	0.70
5	1.15	1.10
6	1.60	1.50
7	2.25	2.30
8	3.10	3.10
9	4.70	4.70
10	11.00	7.90
11	-----	9.50

Due to electronic limitations in adjusting the stepping switch to a certain series of step values, the actual step settings used on the APS-15 radar are not exactly the same as those suggested by the solid-line curve in Figure 18. However, the differences between the suggested and the actual settings are generally rather small. The actual settings are given in Table 7 for comparison with the suggested settings. It will be noted that an 11th step was added to aid in correlating settings with suggested settings.

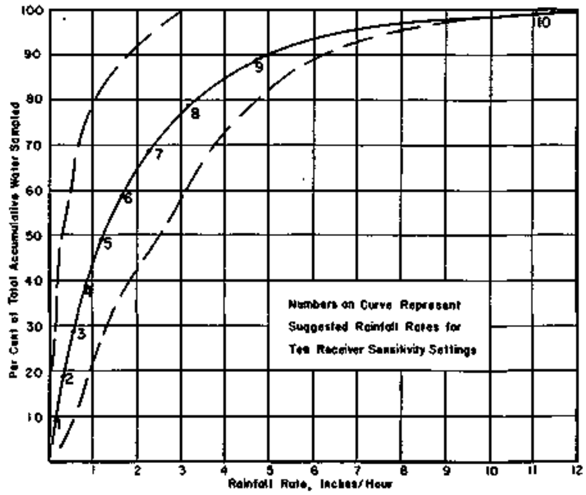

FIG. 18. RELATIONSHIP DETWEEN THE TOTAL FREQUENCY OF RAINFALL RATE AND PERCENT OF TOTAL ACCUMULATIVE WATER SAMPLED FOR EIGHT 1952 STORMS OVER GOOSE CREEK NETWORK.

It should be noted that the suggested and actual step settings are not recommended for individual storms. They are settings which are suggested for best results over a thunderstorm season. For greater accuracy in individual storm measurements, a different series of step settings is needed for each type of storm. This point can be illustrated by preparing a fictitious example from Figure 18. The left-hand, dashed curve is taken as a possible relationship in a specific storm. About 27 to 51 per cent of the accumulated rainfall (water) would be recorded on step 1. The rainfall from 51 per cent to 62.5 per cent would be recorded on step 2, etc. In this case, the accumulative water would not be sampled at equal percentage increments and, also, only seven steps would be used instead of the maximum number of steps. A similar example could be worked out for any storm curve of accumulative water sampled versus rainfall rate. The example used in this discussion was an extreme case. It would be difficult to provide for the most accurate radar measurements in each storm. Advance knowledge of the curve type representative of each storm and a stepping switch adjustment for each type of curve would be needed.