OF INVESTIGATION No. 29 1956

STATE OF ILLINOIS

William G. Stratton, Governor

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Quantitative

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Department of Registration and Education
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SUMMARY

Radar-Rainfall Empirical Equations

Published radar-rainfall equations were initially used in computing areal-mean storm rainfall depths from 3-cm radar observations. These depths were consistently very small in comparison with those computed from a dense network of raingages on a 100 square-mile area.

It was felt that a strictly empirical relationship between radar observations and raingage observations might prove to be more practical. A relationship was determined from one-minute areal-mean rainfall and one-minute radar data over the precipitation echo area within the raingage network. The resulting equation was

$$\log \frac{P_r R^2}{P_t} = 2.00 \log I - 11.641$$

where P_r is power received in watts, P_t is power transmitted in watts, R is range in nautical miles, and I is rainfall intensity in inches per hour.

Areal-mean rainfall depths for twenty-eight storm periods during 1951-1953 inclusive were computed with the use of the above equation. These depths for fifteen 1953 storms were compared with corresponding depths which were computed by using the Wexler[8] theoretical-empirical equation, adjusted for the parameters of the AN/APS-15A radar. The Wexler equation was

$$\log \frac{P_r R^2}{P_t} = 1.53 \log I - 10.932$$

For the 15 storms, a dense raingage network on 100 square miles accumulated an average areal depth of 4.02 inches. The Water Survey empirical equation produced an estimate of 2.95 inches and the Wexler relationship gave 0.82 inch for the same storm periods.

The ratio of the areal-mean rainfall depths computed from the Water Survey empirical equation to those computed from the dense network of raingages ranged from 0.16 to 4.20. An analysis of several meteorological factors was made in an attempt to account for the deviation of individual areal-mean radar-rainfall depths from the network averages. The meteorological factors analyzed were storm type, Showalter stability index, mean wind velocity from surface to 500 millibars, mean relative humidity for the surface to 700 mb layer, and the precipitable water for several layers in the atmosphere. No significant correlation was found between these factors and the deviations of the radar estimates from the network estimates of areal-mean depth.

Accuracy of Radar Areal-Mean Rainfall Estimates

Results of a sampling variance analysis were used to determine the accuracy of radar-indicated areal-mean rainfall depths for 28 storm periods. Fifty-four percent of these radar rainfall estimates were equal to or better than the accuracy which can be expected with one gage per 100 square miles. The other 46 percent were less accurate than one gage for 100 square miles.

Correlation of Surface Rainfall With PPI Echoes

A comparison of surface rainfall data with radar observations was made for 10 stations during 15 storms over the Goose Creek network during 1953. It was found that the total duration of rainfall for the 15 storms considerably exceeded the total duration of radar echoes over the 10 stations. Further investigation showed that no echo was present during 51 percent of the time rain was occurring at stations.

Great variability was obtained among radar-indicated storm mean rainfall estimates when plotted against actual values obtained. A limited study was made of the variability which may be introduced into radar areal mean rainfall estimates from precipitation attenuation effects using hypothetical storm data considered representative of Mid-west thunderstorms. Results indicated that attenuation may be an important source of variability (graphical scatter) in radar-indicated mean rainfall estimates.

A study was made of the frequency distribution of precipitation attenuation in shower-type rainfall across Goose Creek. One-minute rainfall data from eight 1953 storms were used in the analysis. Results showed that 50 percent of the rainfall occurred with attenuation exceeding 3.8 decibels per mile. These results indicate the importance of the attenuation factor in radar-rainfall estimates.

Overall results of the APS-15 evaluation indicate that this low-powered set is generally satisfactory for short range detection and tracking of storms and for quantitative estimates in light rain. When used in conjunction with synoptic and climatological data, the set should materially aid in determining areal rainfall rates in all types of storms. To increase the utility of such radar sets for quantitative precipitation estimates, it is recommended that further attention be given to:

(1) Development of prediction techniques using radar, climatology and synoptic weather data

(2) Refinement of empirical equation techniques which have proven inadequate to date

(3) Development of electronic techniques for precipitation attenuation correction

(4) Better definition of radar reflectivity-rainfall rate relations.

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Rainfall data were collected by James Harry, Anthony Sokol, Sidney Stutz, and William Grubbs under the direction of Douglas Jones.

Numerous staff members contributed in the evaluation of the data. Drafting was accomplished under the supervision of Stanley Changnon.

Credit is due D. M. Swingle and W. J. Richards of the Signal Corps Engineering Laboratories and H. R. Byers of the University of Chicago who reviewed the manuscript.

INTRODUCTION

Measurement of precipitation is an important phase of any water resources program. Knowledge of the intensity and areal distribution of precipitation is pertinent to the planning and execution of military operations, which may be severely hampered by floods and poor trafficability. Present raingaging techniques do not provide the accuracy needed by engineers and meteorologists in determining the intensity and areal distribution of storm rainfall. This is especially true for the shower-type precipitation which accounts for a major portion of the annual rainfall in Illinois and is responsible for flash floods on small watersheds.

During World War II, it was established that radar equipment could be used to locate and track areas of rainfall. This discovery led to investigations to determine the ability of radar to ascertain rainfall distribution over an area.

Realizing the inability of ordinary climatological networks to provide sufficiently detailed information on the distribution of thunderstorm rainfall, and the prohibitive costs involved in establishing satisfactory raingage networks for this purpose over large areas, the Illinois State Water Survey initiated an investigation in 1948 to determine the ability of radar to provide rainfall measurements needed by engineers and hydro-meteorologists. A war surplus AN/APS-15A*, 3-cm radar set, was purchased, and a network of 35 stick and 12 recording raingages was installed over an area of approximately 280 sq. miles in the vicinity of El Paso, Illinois.

Operations during the thunderstorm seasons of 1948 and 1949 emphasized the inability of ordinary climatological networks to accurately measure shower-type rainfall. During this time, however, radar had proven successful in detecting, tracking, and indicating the areal extent of precipitation in showers and thunderstorms. Since investigation by agencies of the Armed Forces and others indicated that radar could be adapted to the quantitative measurement of rainfall, it was decided to continue the radar program and concentrate efforts on the development of methods and techniques for quantitative determinations of rainfall.

The radar-rainfall program was expanded during 1952 under a contract with the U.S. Army Signal Corps. Results from investigations through 1952 have been reported in Research Reports 1 and 2 to the Signal Corps.[5,7]

Collection and analysis of APS-15 data, involving an extensive series of simultaneous measurements of radar received power and rainfall rate, were continued during the 1953-54 thunderstorm seasons. A 35-mm scope camera photographed the plan position indicator (PPI) in synchronization with an automatic

*Hereafter abbreviated to APS-15.

receiver gain reduction device to obtain a detailed record of echo distribution and intensity. The rainfall data were collected over a dense raingage network of 50 recording raingages in a 100 square-mile area.

The first portion of this report summarizes procedures and results of an analysis which was made in an attempt to obtain an empirical relation between radar received power and areal surface rainfall rate. A brief review of 1951-52 results which led to this undertaking is also given. Various analyses to ascertain the degree of correlation between rainfall data and radar observations with low-powered 3-cm radar are discussed.

RADAR-RAINFALL EMPIRICAL EQUATIONS

1951-52 Analysis

Quantitative estimates of rainfall rates and total rainfall amounts from radar observations of shower-type rainstorms were made during 1951 and 1952[5,7]. A modified APS-15 was used for making the radar observations of rain-storms over the 100 square-mile Goose Creek raingage network shown in Figure 1. Published radar-rainfall equations, derived from microwave scattering theory and an empirical relationship between received power and rainfall rate based upon limited drop size-distribution data, were adjusted to the characteristics of the APS-15. Using these equations, rainfall rates and areal-mean storm rainfall depths were computed from the radar observations.

The computed areal-mean storm rainfall amounts were consistently very low in comparison with those obtained from the dense network of raingages (Figure 1). These radar-indicated rainfall amounts are shown in Table 1 for comparison with the raingage network average. Also shown are the lowest (Min.) and the highest (Max.) storm rainfall measurements at any gage on the network.

The radar-rainfall values (column 6 in Table 1) were obtained from

$$\log \frac{P_r R^2}{P_t} = 1.72 \log I - 10.895 \qquad (1)$$

where P_r is power received in watts, P_t is power transmitted in watts, R is the range in nautical miles to the reflecting raindrops, and I is the surface rainfall intensity in inches per hour. This equation involved the use of the rainfall-reflectivity relation

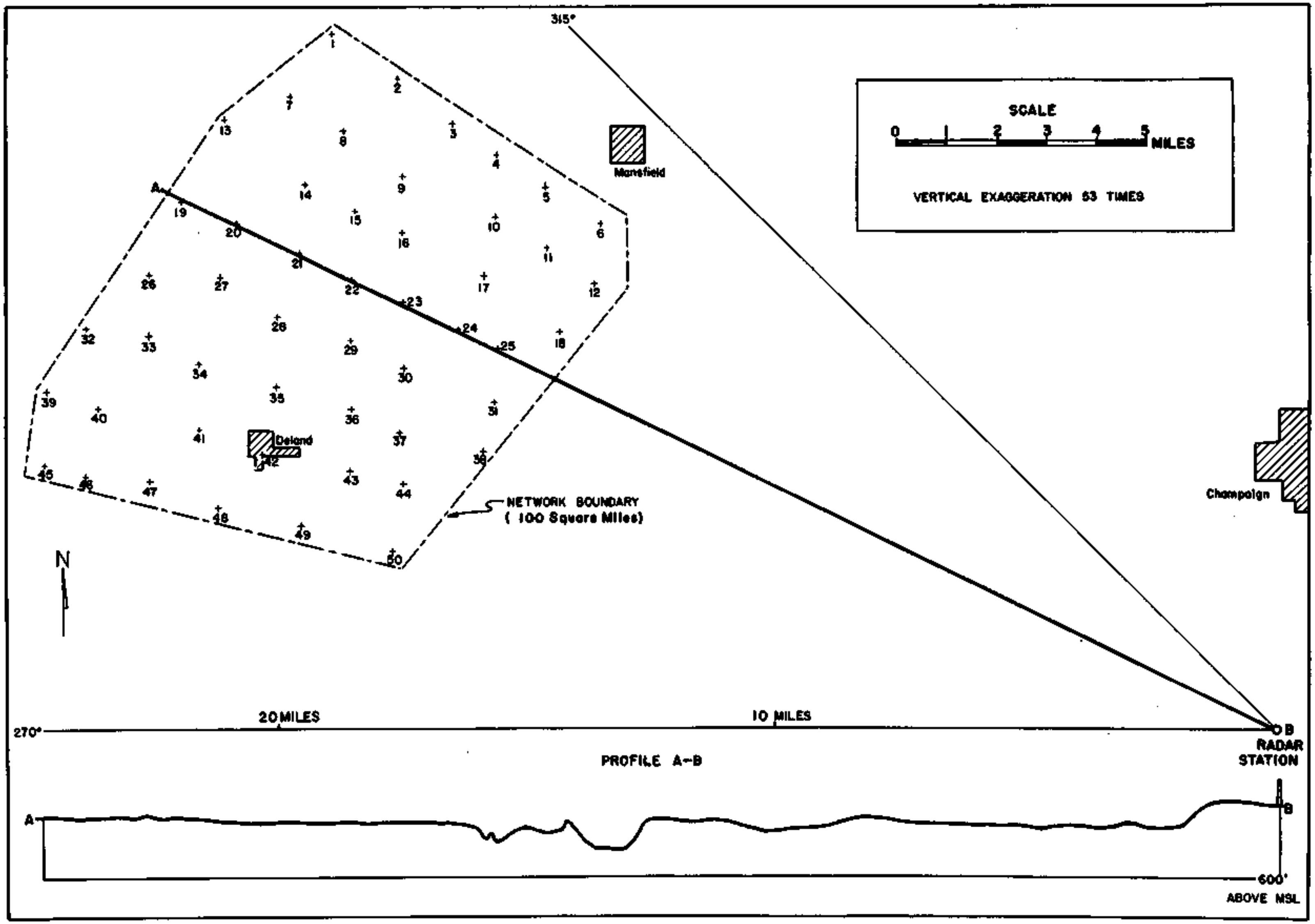

Fig.1. Topographic and Location Relationships of the Radar Station and Goose Creek Network

TABLE I

RADAR-RAINGAGE SUMMARY OF 13 STORM PERIODS OVER GOOSE CREEK NETWORK, 1951 and 1952

Depth of Rainfall (Inches)

Date	Time	Raingage Min.	Raingage Max.	Average Raingage	Average Radar	Ratio of Radar to Raingage
			1951			
7-22	2128-2155	.00	.32	.07	.010	.14
8-20	1715-1740	.09	.24	.12	.020	.17
9-12	1710-1754	.17	.79	.44	.020	.05
9-26	2010-2050	.26	.79	.44	.040	.10
			1952			
7- 2	1551-1639	.05	.21	.10	.014	.14
8-15	0443-0640	.17	.42	.27	.033	.12
8-20	2037-2125	.08	.35	.17	.022	.13
9- 1	1440-1500	.01	.10	.04	.004	.10
9-14	0833-0851	.00	.26	.06	.006	.10
9-14	0903-0915	.00	.11	.01	.002	.20
9-18	1352-1510	.08	1.35	.36	.050	.14
9-18	1540-1655	.20	1.07	.57	.011	.02
10-14	1354-1416	.04	.13	.09	.002	.02

$$Z = 190\,I^{1.72} \tag{2}$$

reported by Marshall, Langille, and Palmer[3], where I is surface rainfall intensity in mm per hr, and $Z = \sum_{i=1}^{n} (ND_i^6)$, N being the number of raindrops in a unit volume of air falling in a class interval of nominal diameter D_i.

Equation (1) was adjusted for two other Z values in order to determine the effect of different Z-I relationships on the computed radar-rainfall estimates. The value of

$$Z = 220\,I^{1.60} \tag{3}$$

reported by Marshall and Palmer[4] and the value of

$$Z = 323\,I^{1.53} \tag{4}$$

obtained by Wexler[8] were substituted. The computed areal-mean radar-rainfall; values were not significantly different from those" obtained by equation (1).

<u>Development of Empirical Relations From 1953 Data</u>

Since 3-cm radar observations may be subject to considerable attenuation by intervening raindrops, some of the difference between the radar amounts and the surface rainfall values can be attributed to this attenuation. However, it seems unlikely that all of the differences in the cases studied resulted from attenuation. It appears that other factors also contributed to the difference observed between the quantitative radar and raingage network observations.

Consequently, it was felt that a strictly empirical relationship between radar and raingage observations might be more practical. Radar-rainfall equations obtained by other investigators are essentially relationships between the ratio of the received power to the transmitted power and surface rainfall rate at a point. In the Water Survey study, a relationship between areal-mean rainfall rate and the average ratio of power returned to power transmitted over an area was determined, which reduces the effect of point matching errors.

An equation to estimate rainfall was obtained from a regression analysis relating areal-mean rainfall intensity, I, and the factor, $P_r R^2/P_t$, where P_r/P_t is the mean ratio of power received to power transmitted for an area and R is the range to the reflecting raindrops.

In order to obtain a mean-areal rainfall rate (I), it was assumed that one-minute accumulated rainfall amounts read from raingage charts were close approximations of the rainfall rate. The mean-areal rainfall rate is given by

$$I = \sum_{i=1}^{n_t} (60)\, a_i / n_t \tag{5}$$

where a. is the one-minute amount at the ith gage, n_t is the number of gages recording .001 inch or greater during the tth minute, and I is the mean-areal rain fall rate in inches per hour over the portion of the network where rain is falling. The factor 60 converts the one-minute rainfall amounts to rainfall rates in inches per hour.

The radar factor was calculated from isoecho contour maps drawn from the 35-mm film record obtained by photographing the PPI[5,7]. The 35-mm scope camera operated in synchronization with an automatic receiver-sensitivity stepping switch to obtain a record of the intensity and distribution of precipitation. The PPI presentation for as many as 10 gain steps could be photographed in one-minute. These isoecho contour maps were drawn on the Goose Creek base maps. A planimeter was used to obtain the area between adjacent contours.

The received power represented by each isoecho contour was obtained from a calibration of the radar. It was an established policy to calibrate the radar immediately before or after storms passed over the Goose Creek network. Since the calibration data provided threshold values of P_r/P_t for each isoecho contour, the arithmetical mean of adjacent P_r/P_t values was applied to the enclosed area to obtain a representative average.

The average radar received power value, X, represented by a single one-minute isoecho contour map, was obtained from the following expression

$$X = \sum_{j=i}^{s} \left(\frac{\overline{P}_r}{P_t}\right)_j \Delta A_j \, R^2 \Bigg/ \sum_{j=i}^{s} \Delta A_j \tag{6}$$

where P_r/P_t is the arithmetical average of the ratio of power received to power transmitted between adjacent receiver-sensitivity settings, j is the number of receiver-sensitivity settings (1 to 10) required to locate the echo core, ΔA_j is the increment of area between adjacent receiver-sensitivity contours on a one-minute isoecho contour map, R is the range in nautical miles between the radar set and the approximate center of the area enclosed by each contour.

Every other minute of data was omitted from the analysis in order to make the task less laborious. From a previous investigation, it was found that no great loss in accuracy would result when this procedure was followed[5].

Equations were determined from three sets of 1953 storm data. Pairs of one-minute mean values for the first set were obtained from a series of similar pairs of isoecho and isohyetal patterns when no precipitation was being detected between the radar site and the network. One-third or more of the network area was covered with echo and rainfall for each minute of data used. A time lag of thirty seconds to two minutes was allowed between paired radar and raingage one-minute values. This lag is the time allowed for the rain "observed" by the radar to reach the ground and to be recorded by the raingages. The need for a varying time lag is probably caused by differences in storm structure. A total of 71 observations from five storms were included in the first set of data.

Additional data from these five storms and data from a sixth storm were included with the observations of the first set to give a total of 95 observations. The 24 additional observations included data for cases when a portion of the echo over the network had moved past the network's forward boundary and was therefore located between the network and the radar. However, no other storms were between the network and the radar site in this set of data.

Additional data from the aforementioned six storms and data from four other storms were included in the third set of data. The additional observations included some which had previously been eliminated due to separate intervening storms between the radar set and the network, and some which presented an extremely poor isoecho and isohyetal pattern comparison. A total of 221 observations were included in this set of data.

Three expressions were tested for their "goodness of fit" to the observations of the first set of data. These expressions are

$$I = A X^{B} \tag{7}$$

$$I = A + B X \tag{8}$$

$$I = A + B X^{0.5} \tag{9}$$

where I is areal-mean rainfall intensity, X represents the areal-mean radar received power factor in equation (6), and A and B are constants. The least squares method of curve fitting was used to obtain estimates of the A and B constants. Log deviations were minimized for expression (7) and the absolute deviations were minimized for expression (8) and (9).

The empirical equations obtained with the 71 observations are listed in Table 2 along with a summary of their "goodness of fit". Sums of squares of deviations are in terms of absolute deviations. The results in Table 2 indicat.e that equation (12) fit the 71 points slightly better than the other equations.

TABLE 2

REGRESSION EQUATIONS AND A SUMMARY OF THEIR

GOODNESS OF FIT TO 71 OBSERVATIONS

Equation Number	Sum of Squares of Deviations from Regression	Degrees of Freedom	Error Variance	Correlation Factor
(10) $I = .054\ X^{.565}$	10.09	69	.146	.79
(11) $I = .425 + .003X$	10.46	69	.152	.78
(12) $I = -.130 + .094\ X^{0.5}$	9.29	69	.135	.81

The 71 points and the three equations are plotted in Figure 2. Linear scales were used to show the absolute deviations about these lines. The radar power factor was multiplied by a factor of 10^{14} for convenience.*

Expressions (7), (8), (9) and the expression

$$I = A + B\ X^{0.2} \tag{13}$$

were tested for their "goodness of fit" to the set of 95 observations. The four equations and a summary of their"goodness of fit" are presented in Table 3.

* To help avoid confusion between the linear scale for radar received power factor in Figures 2 and 3 and the log scale in Figures 4 and 7, it is noted for example that 1000×10^{14} is equivalent to 1.0×10^{-11}.

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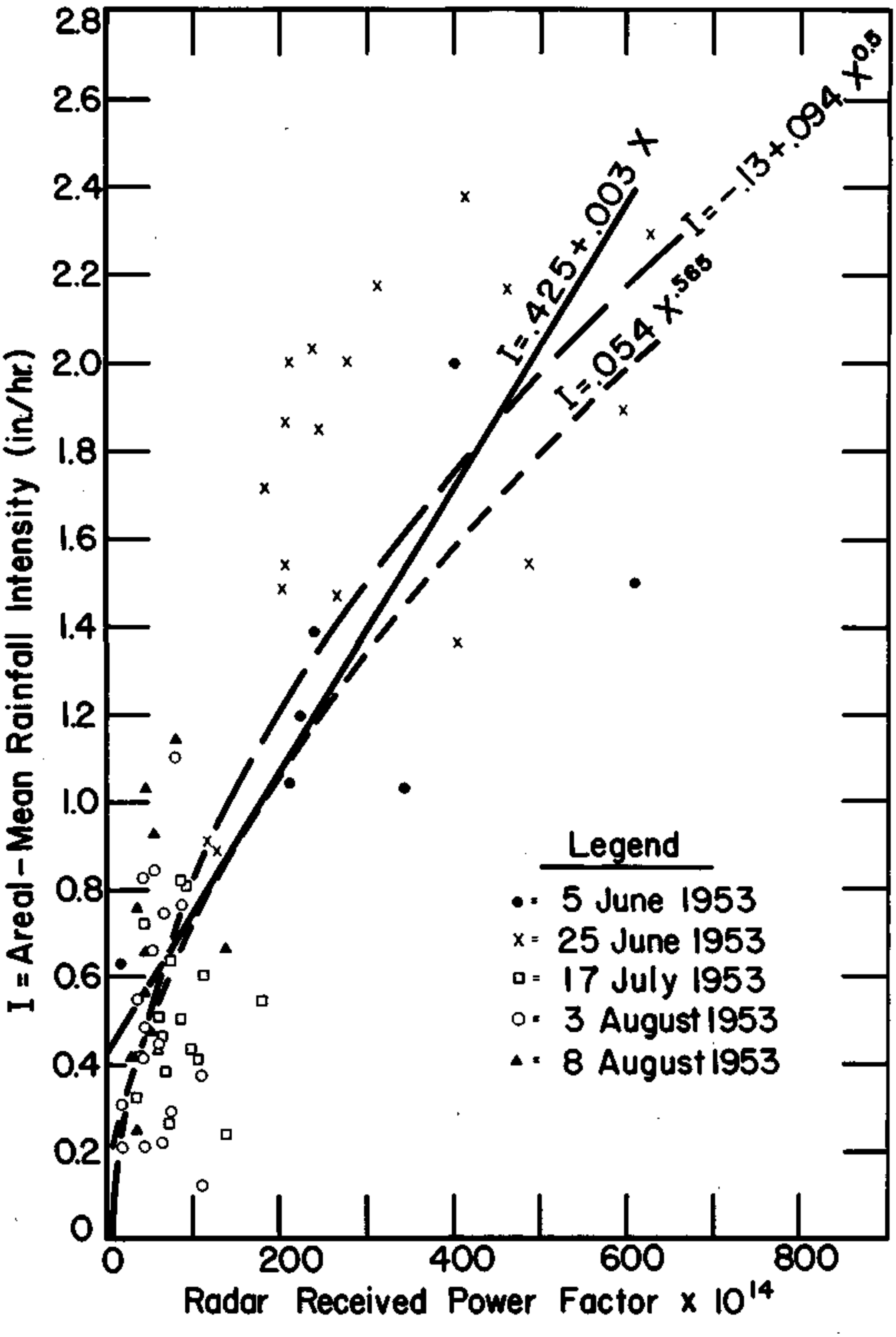


FIG.2 RELATIONSHIP BETWEEN RAINFALL INTENSITY AND RADAR RECEIVED POWER FACTOR

(NO INTERVENING PRECIPITATION BETWEEN RADAR AND RAINGAGE NETWORK)

TABLE 3

REGRESSION EQUATIONS AND A SUMMARY OF THEIR

GOODNESS OF FIT TO 95 OBSERVATIONS

Equation Number	Sum of Squares of Deviations from Regression	Degrees of Freedom	Error Variance	Correlation Factor
(14) $I = .61 + .001\ X$	21.23	93	.228	.57
(15) $I = .18 + .058\ X^{0.5}$	17.74	93	.191	.66
(16) $I = -1.06 + .739\ X^{0.2}$	15.84	93	.170	.70
(17) $I = .07\ X^{0.5}$	17.71	93	.190	.66

It is apparent from Table 3, that equation (14), the straight line, provided the poorest fit, equations (15) and (17) were equal in "goodness of fit" and equation (16) provided the best fit to the 95 points. However, the differences between the error mean squares and correlations for equations (15), (16), and (17) are small. The four empirical equations and the 95 observations are shown in Figure 3.

Only expression (7) was fitted to the 2Z1 observations. The resulting equation, expressed in logarithmic form, was

$$\log I = .329 \log X + 3.687. \qquad (18)$$

This equation had a correlation coefficient of 0.51. Equation (18) is shown in Figure 4 and the 221 points are plotted about the curve to indicate the scatter of data.

It is evident from the scatter of the observed data in Figure 4 and from the low correlation of 0. 51 that the least squares line or any other averaging line through these points, would be of very little use in estimating rainfall intensity. Additional data used in obtaining equation (18) included considerable data in which the isohyetal and isoecho patterns showed an extremely poor comparison. The reason for the poor pattern comparisons could not be determined in some cases. In other cases it appeared to be caused from attentuation, especially when rainfall occurred between the radar site and the raingage network.

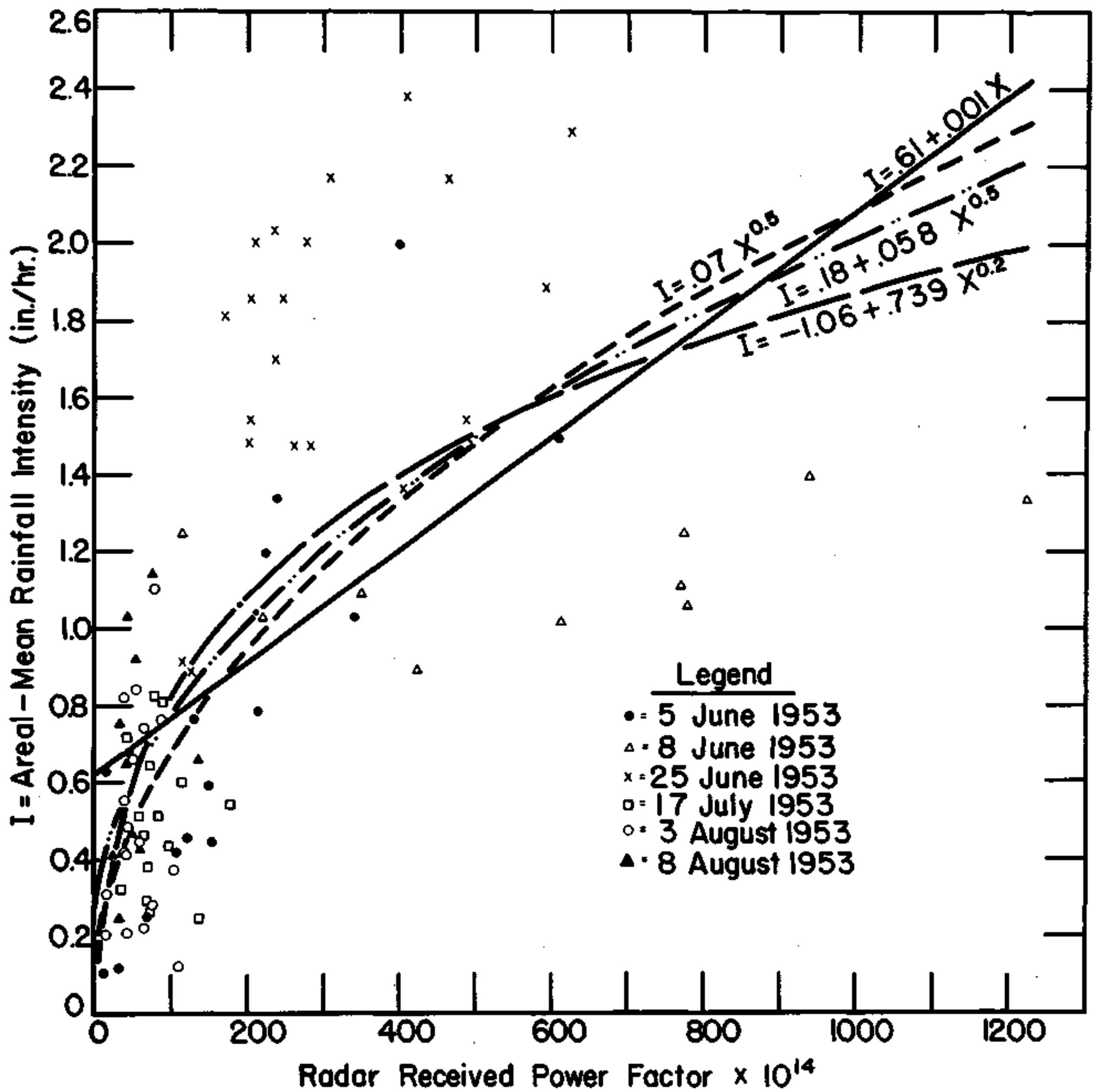


FIG.3 RELATIONSHIP BETWEEN RAINFALL INTENSITY AND RADAR RECEIVED POWER FACTOR

(INCLUDING PORTIONS OF DATA STORM WHICH WERE BETWEEN THE NETWORK AND THE RADAR BUT EXCLUDING CASES OF SEPARATE INTERVENING STORMS)

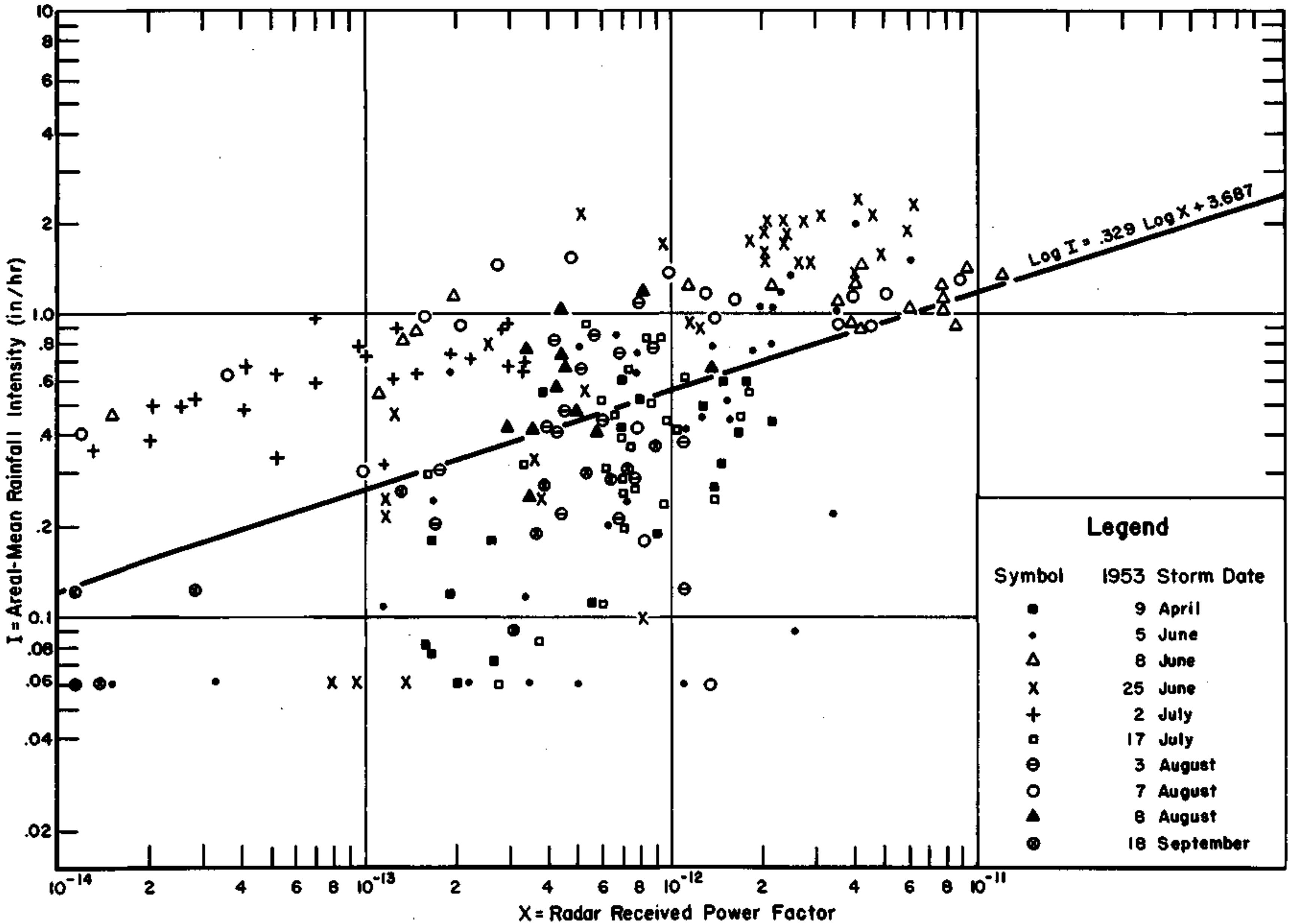


FIG.4 RELATIONSHIP BETWEEN RAINFALL INTENSITY AND RADAR RECEIVED POWER FACTOR

(NO RESTRICTIONS ON DATA DUE TO RAINFALL BETWEEN NETWORK AND RADAR)

Application of Empirical Equations

Several equations were available for testing with independent data. A comparison of the goodness of fit for each equation indicated only small differences in the efficiency of the equations in explaining the variation in the data from which they were obtained. Consequently, an equation of the form given in expression (7) was applied, since it is the form of radar-rainfall equation which has been discussed in previous reports on quantitative radar rainfall measurements. Also equation (17), based on the 95 observations, was used instead of equation (12), based on the 71 observations, since the 95 observations were considered more representative of the true population of radar observations.

In order to express equation (17) in the form which has been most commonly used, it was expressed in logarithmic form and the coefficient of log X was transferred to log I to obtain equation (19) in the following form

$$\log X = 2.00 \log I - 11.641 \tag{19}$$

Equation (19) was applied to data for the 13 storms which were summarized in Table 1, in order to check on the utility of the 1953 equation when applied to 1951 and 1952 radar data. The results are presented in Table 4, along with the previous results which were obtained by using equation (1). All 13 of the radar-indicated mean rainfall estimates were closer to the raingage network averages when equation (19) was applied. However, these estimates were consistently lower than the corresponding raingage estimates.

Equation (19) and theoretical-empirical equation (20) were also applied to the data of 15 storms in 1953. Equation (20) reported by Wexler[8] and adjusted for the APS-15 radar parameters is

$$\log \frac{P_r R^2}{P_t} = 1.53 \log I - 10.932 \tag{20}$$

The results are presented in Table 5.

From Table 5, it can be seen that equation (19) gave better estimates of the mean depth of rainfall for 1953 storms than the theoretical-empirical equation. Also it is apparent that the empirical equation did not consistently underestimate the rainfall as was the case in 1951 and 1952. A complete explanation has not been found for this. The same computational methods were used for all three years and there has been no reason to question the receiver sensitivity or power output measurements. Also, several attenuation checks of various nature were made in the equipment and no discrepancies were found. Equation (19) also produced the best estimate of the total precipitation for all storms combined.

TABLE 4

RADAR AND RAINGAGE MEAN RAINFALL COMPARISONS FROM 13 STORMS OVER GOOSE CREEK NETWORK IN 1951 AND 1952

Date	Time	Raingage Depth (in.)	Radar Depth (in) Eq. 19*	Radar Depth (in) Eq. 1**	Ratio of Radar to Raingage Eq. 19*	Ratio of Radar to Raingage Eq. 1**
			1951			
7-22	2128-2155	.07	.026	.010	.37	.14
8-20	1715-1740	.12	.104	.020	.87	.17
9-12	1710-1754	.44	.083	.020	.19	.05
9-26	2010-2050	.44	.166	.040	.38	.10
			1952			
7- 2	1551-1639	.10	.055	.014	.55	.14
8-15	0443-0640	.27	.112	.033	.41	.12
8-20	2037-2125	.17	.075	.022	.44	.13
9- 1	1440-1500	.04	.014	.004	.35	.10
9-14	0833-0851	.06	.018	.006	.30	.10
9-14	0903-0915	.01	.006	.002	.60	.20
9-18	1352-1510	.36	.152	.050	.42	.14
9-18	1540-1655	.57	.033	.011	.06	.02
10-14	1354-1416	.09	.009	.002	.10	.02

*Results from equation (19)

**Results from equation (1)

TABLE 5

RADAR AND RAINGAGE MEAN RAINFALL COMPARISONS FROM 15 1953 STORMS OVER GOOSE CREEK RAINGAGE NETWORK

Date	Raingage Depth (in) Min.	Avg.	Max.	Radar Avg. Depth (in) Eq. 19*	Eq. 20**	Ratio of Radar to Raingage Eq. 19*	Eq. 20**
4-9	.00	.05	.23	.21	.06	4.20	1.20
4-24	.09	.12	.17	.25	.06	2.08	.50
5-16	.04	.19	.48	.26	.06	1.37	.32
6-5***	.13	.35	.55	.42	.12	1.20	.34
6-8***	.28	.52	.73	.64	.22	1.23	.37
6-25***	.01	.73	1.39	.33	.10	.45	.14
7-2	.03	.40	.64	.07	.01	.18	.02
7-5	.64	.82	1.03	.13	.04	.16	.05
7-17***	.01	.15	.37	.18	.05	1.20	.33
8-3***	.00	.03	.23	.04	.01	1.33	.33
8-7	.21	.37	.55	.20	.05	1.54	.14
8-8***	.00	.08	.59	.06	.01	.75	.11
9-18	.00	.02	.07	.05	.01	2.50	.50
11-20	.00	.03	.09	.01	.002	.33	.07
11-22	.14	.16	.21	.10	.02	.62	.12
Total		4.02		2.95	.82		

* Results from equation (19)
** Results from Wexler's equation
*** Storms from which data were taken to obtain equation (19).

It will be noted in Table 5 that equation (19) produced some poor estimates. The 2 July and 5 July low estimates can be accounted for by excessive attenuation brought about by intervening rain. The 9 April storm was accompanied by hail, and a tornado developed from the same echo within 20 minutes after the storm cell passed over the raingage network. Unusually strong reflectivity from water-coated hail may have caused the overestimate.

A comparison of the rainfall rates indicated by the Wexler[8] equation and the Water Survey empirical equation is shown in Figure 5. The empirical equation indicates higher rates than the Wexler equation. However, the differences in indicated rates for the two equations decrease as the rates increase. The two equations do not intersect until a rainfall rate of approximately 30 inches per hour is reached.

It should be brought to the reader's attention that in order to obtain best results from equation (19) it should be applied in the same manner in which it was derived. That is, the mean ratio of the power received to power transmitted for all steps should be averaged over the echo area according to formula (6) before obtaining a mean rainfall rate for that area from equation (19). These one-minute rainfall rates were converted into depths and summed over all minutes for an estimate of the mean precipitation covered by the outermost isoecho contour. To obtain a mean rainfall value for the raingage network, the one-minute depths were converted to depths for the network area and summed over all minutes of the storm period. The above method can be stated in a formula as follows:

$$\overline{P} = \sum_{i=1}^{m} \left(\frac{I}{60} \left[\frac{\text{Area inclosed by gain step 1}}{\text{Area of the raingage network}} \right] \right)_i \qquad (21)$$

where $\overline{P}$ is mean storm rainfall over the network, I is an individual network rainfall rate per hour estimated from equation (19). The factor of 60 converts I to an average depth per minute, and m represents the number of minutes in the storm period. When this method of analysis is followed, very few estimations of I will come from extrapolated portions of the curve.

It will be noted that the preceding procedure is very laborious and time consuming. This was necessitated by the fact that gain step identity was lost in the method of obtaining equation (19). The loss of gain step identity while obtaining the equation was not originally expected to be a serious handicap in using the equation. It was expected that equation (19) would be useable for both point and areal relationships when written in the form

$$\log \frac{P_r R^2 / P_t}{P_t} = 2.00 \log I - 11.641 \qquad (22)$$

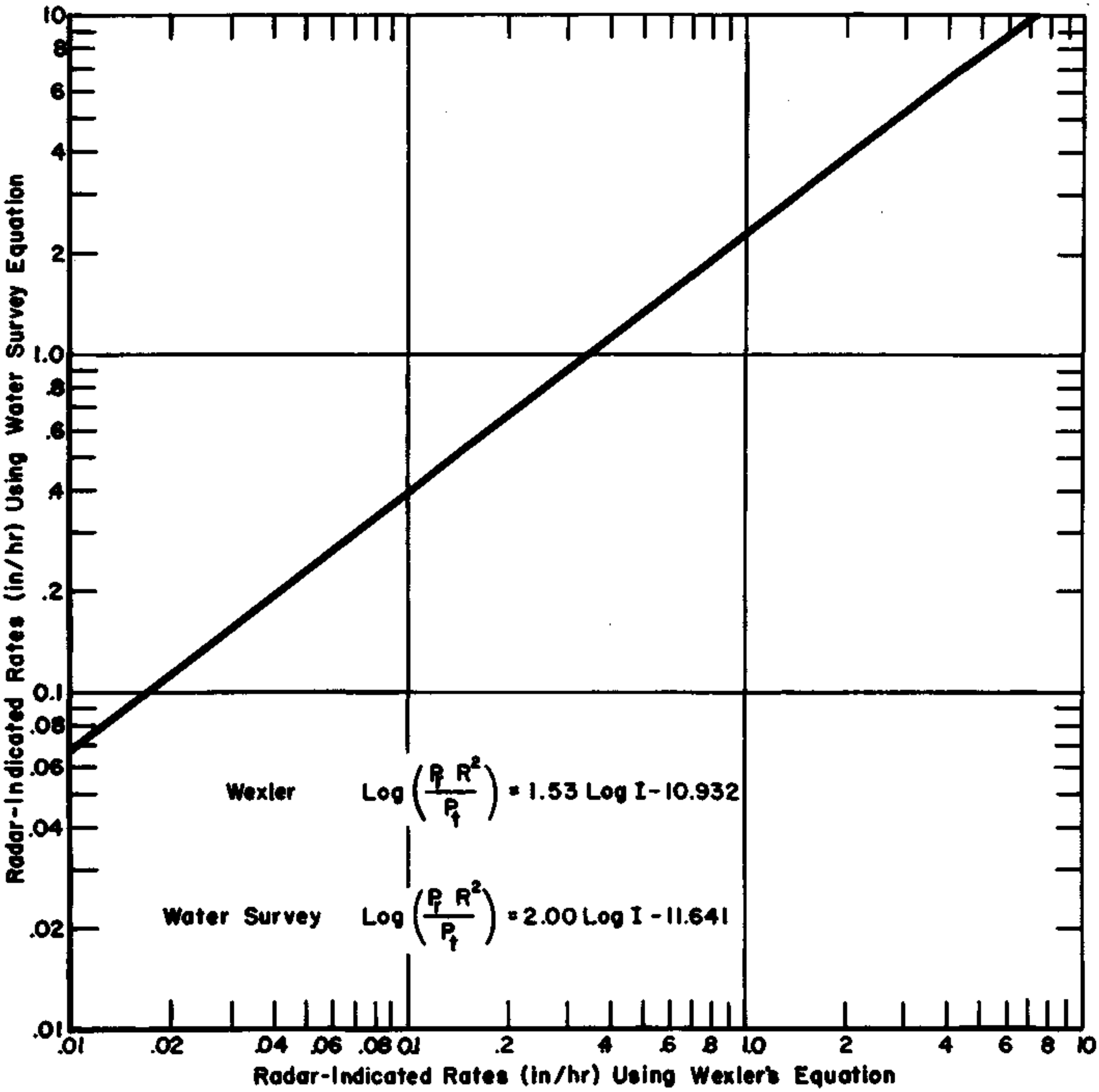


FIG. 5 COMPARISON OF RADAR-INDICATED RAINFALL RATES USING WEXLER AND WATER SURVEY EQUATIONS WITH APS-15

In equation (22) it was intended that individual gain step P_r/P_t values and computed I values be used as constants throughout the storm period. Assuming a mean range, R, for the network, the formula for areal-mean rainfall could then be written as

$$\overline{P} = \sum_{j=1}^{n} \left[\frac{(\Delta A_j)(I_j)}{60} \right] \Big/ \text{area of the Network} \tag{23}$$

where j represents the gain step contour; ΔA_j is obtained by summing the areas enclosed by the various gain step contours over all minutes of the storm period, and then getting the difference between sums for adjacent contours; I_j represents the average rainfall intensity per hour between adjacent contours, and is obtained by substituting an average of two P_r/P_t values for adjacent contours in equation (22), or from the graph of P_r/P_t, R, and I in Figure 6.

Using equation (23) instead of equation (21) reduces the computations tremendously. However, estimates of areal-mean rainfall were computed by both methods, and the less laborious method produced consistently smaller estimates of rainfall. Estimates from both methods for 10 storm periods are presented in Table 6. On the average, the proper method of using the empirical equation produced estimates which were 1. 32 times as large as the estimates from the other procedure. Applying the equation in the manner by which it was obtained generally produced better estimates of the individual network mean rainfall depths. While it is best to apply the equation in this manner, an average factor such as 1.32 could be applied to the estimates from equation (22) for climatological purposes where monthly or seasonal totals are of primary interest.

The way in which equation (22) is presented does not allow one to apply it to another radar. Equation (22) becomes more general when the radar constant K is removed from the term - 11.641, where

$$K = \frac{8}{9}\pi^5 \; A_P^2 \; \phi \, \theta \; \frac{1}{\lambda^6} \left(\frac{N_o^2 - 1}{N_o^2 + 1} \right)^2 h. \tag{24}$$

In expression (24) for K, the area of the antenna, (A_P), is .428 m^2; the vertical beam width, (ϕ), is 2. 5 degrees; the horizontal beam width, (θ), is 2.9 degrees; the wavelength (λ), is 3.2 cm; N_o is the refractive index of water; and the pulse length, (h), is 600 m. for the **APS-15** in use. Cons equently, K = 5.63×10^{10}. The units of K are m^{-1} . When K is removed from the constant in (10), the resulting expression becomes:

$$\log \frac{P_r R^2}{P_t K} = 2.00 \log I - 22.392 \tag{25}$$

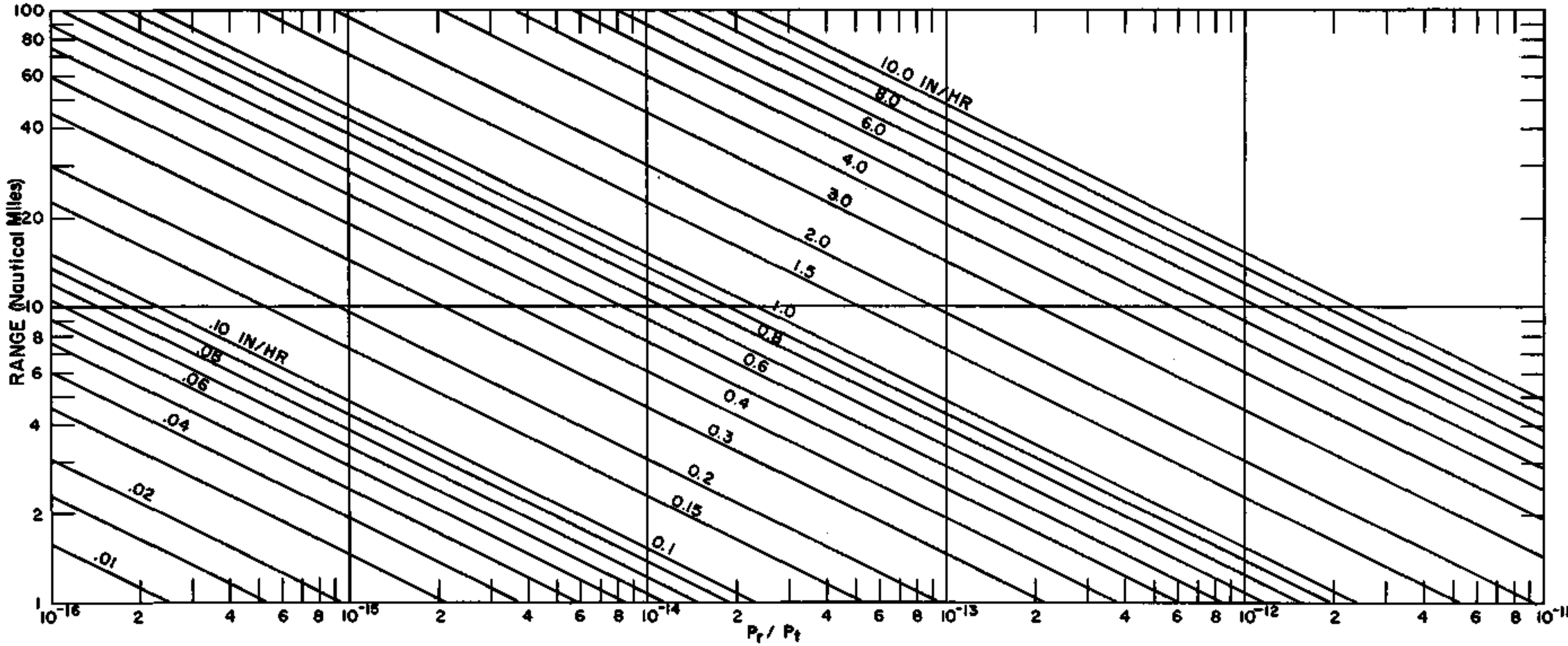


FIG. 6 ESTIMATED RAINFALL INTENSITIES FOR VARIOUS P_r/P_t AND RANGE VALUES

WATER SURVEY EMPIRICAL EQUATION, $\log \frac{P_r}{P_t} + 2 \log R = 2.00 \log I - 11.641$

TABLE 6

RADAR AND RAINGAGE MEAN RAINFALL

COMPARISONS OVER GOOSE CREEK NETWORK

Date (1953)	Raingage Depth (in.)	Radar Depth (in.) Eq. 21*	Eq. 23**	Ratio of Eq. 21 to Eq. 23
4-9	.05	.21	.18	1.17
6-5	.35	.42	.32	1.31
6-8	.52	.64	.48	1.33
6-25	.73	.33	.25	1.32
7-2	.40	.07	.06	1.17
7-17	.15	.18	.13	1.38
8-3	.03	.04	.03	1.33
8-7	.37	.20	.15	1.33
8-8	.08	.06	.05	1.20
9-18	.02	.05	.03	1.67
Total	2.70	2.20	1.68	Avg 1.32

*Depth obtained with equation (21)

**Depth obtained with equation (23)

where R is in nautical miles, I is in in/hr, P_r and P_t are in watts, and K is a radar constant for any 3-cm radar.

Confidence Bands

When a regression curve is used for estimation purposes, it is desirable to have an indication of the reliability of the estimated quantity.. Confidence bands for the 95 percent level were determined for equation (17). Equation (17) and these bands are shown on logarithmic scale in Figure 7. The inner-most bands (AB and CD) outline the 95 percent confidence region for the theoretical regression curve. These bands were obtained by adding and subtracting a factor of

$$\left(t_{.05\,(N-2)}\right)\sqrt{\left[s^2/N + s^2\,(\log X - \overline{\log X})\right] \Big/ \sum (\log X - \overline{\log X})^2} \qquad (26)$$

to the ordinates of equation (17), where N is the number of pairs of observations, s^2 is the variance of observed points about the empirical regression line, and $t_{.05(N-2)}$ is the t-value at the .05 probability level for N-2 degrees of freedom.

The quantity under the radical represents the variance about the locus of points represented by equation (17). The other confidence lines represent the 95 percent confidence limits for an individual areal mean rainfall intensity. These limits were obtained by adding and subtracting a factor

$$\left(t_{.05\,(N-2)}\right)\sqrt{\left[s^2 + s^2/N + s^2\,(\log X - \overline{\log X})\right] \Big/ \sum (\log X - \overline{\log X})^2} \qquad (27)$$

to the ordinates of equation (17).

Analysis of Meteorological Factors

An attempt was made to determine if certain meteorological factors could be found to account for the variance in the radar estimates of the mean precipitation. Surface and upper air maps and charts for each storm were studied for this purpose. The results are summarized in Table 7. The radar echoes associated with the synoptic conditions are shown in Figures 8 and 9. No significant correlation was found between the variance in the radar estimates and the meteorological factors investigated.

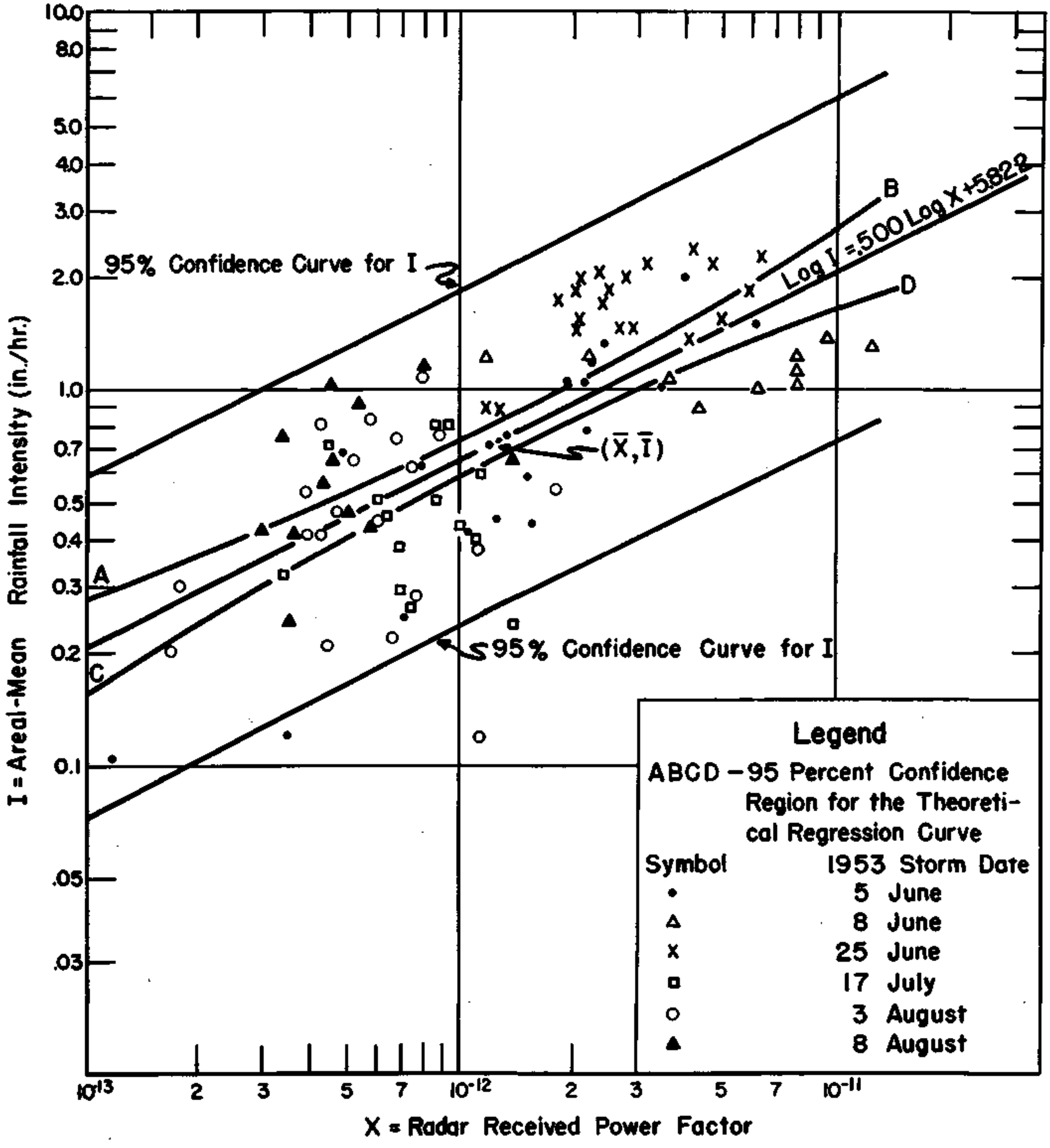


FIG. 7 RELATIONSHIP BETWEEN RAINFALL INTENSITY AND RADAR RECEIVED POWER FACTOR

(INCLUDING PORTIONS OF DATA STORMS WHICH WERE BETWEEN THE NETWORK AND THE RADAR BUT EXCLUDING CASES OF SEPARATE INTERVENING STORMS)

TABLE 7

RADAR AND METEOROLOGICAL DATA[1]

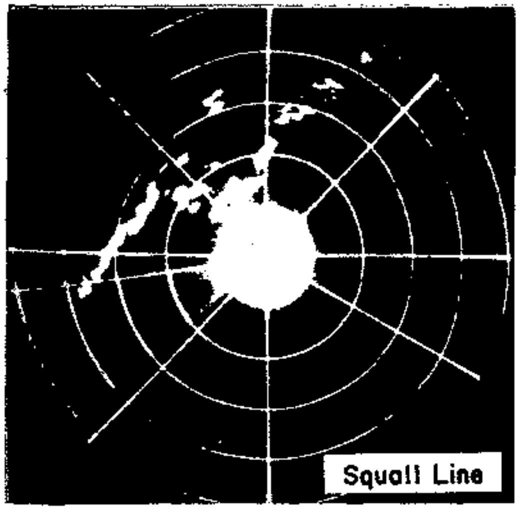
Storm Date 1953	Storm Duration (min.)	Average Depth (in) Raingage	Average Depth (in) Radar*	Rainfall Rate (in/hr.)	Precipitation Associated with**	SSI*** Nearest Rain Time	Mean Wind Velocity from Surface to 500 mb Speed (Knots)	Mean Wind Velocity from Surface to 500 mb Direction (Degrees)	Mean Relative Humidity % Surface to 700 mb	Precipitable Water Surface to 850 mb	Precipitable Water 850 mb to 700 mb	Precipitable Water 700 mb to 500 mb	Precipitable Water Surface to 500 mb
6-5	35	.35	.42	.52	Squall line	-1	32	222	62	1.2	0.7	0.5	2.4
6-8	29	.52	.64	1.08	Squall line	-2	16	188	68	0.7	0.6	0.5	1.9
6-25	35	.73	.33	1.20	Squall line	-5	28	242	66	1.1	0.8	0.6	2.5
7-17	27	.15	.18	.39	Unstable Air Mass	-2	31	145	92	0.9	0.7	0.5	2.1
8-3	21	.03	.04	.44	Unstable Air Mass	-1	13	212	68	1.2	0.7	0.5	2.4
8-8	12	.08	.06	.64	Unstable Air Mass (Post Frontal)	+1	10	325	80	0.7	0.4	0.3	1.4
4-9	19	.05	.21	.30	Formation of Squall line	-5	39	208	62	0.9	0.6	0.4	1.9
7-2	24	.40	.07	.62	Squall line	0	30	250	99	1.1	0.8	0.6	2.5
8-7	19	.37	.20	.89	Cold Front	0	44	275	75	0.9	0.6	0.4	1.9
9-18	13	.02	.05	.19	Cold Front	-5	28	198	59	1.0	0.8	0.4	2.2

1 The upper air soundings from Chanute Air Force Base, Rantoul, Ill. were used for upper air computations.

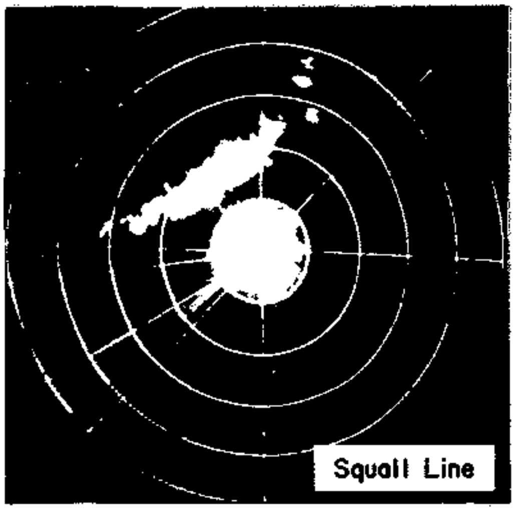
* Analysis performed in manner SWS Equation obtained.

** Frontal, squall line, and air mass classification based on WBAN Surface Synoptic Maps.

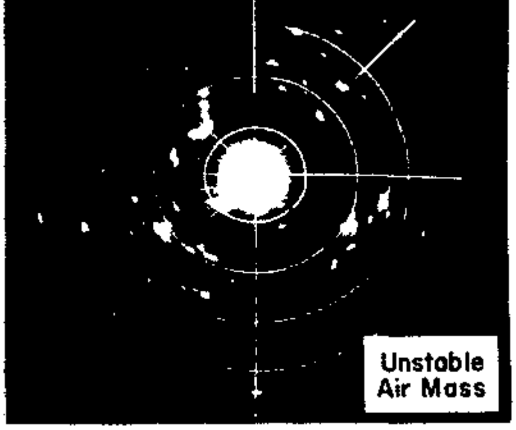
*** SSI-Showalter Stability index.



(1) 1536 on June 5, 1953

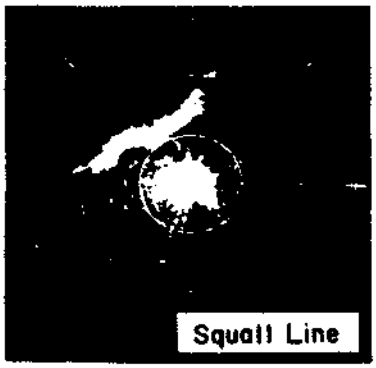


(2) 0443 on June 8, 1953

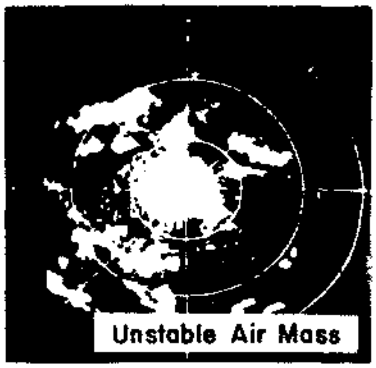


(3) 1329 on August 8, 1953

(a) 100 MILE RANGE, 20 MILE RANGE MARKERS



(1) 1616 on June 25, 1953

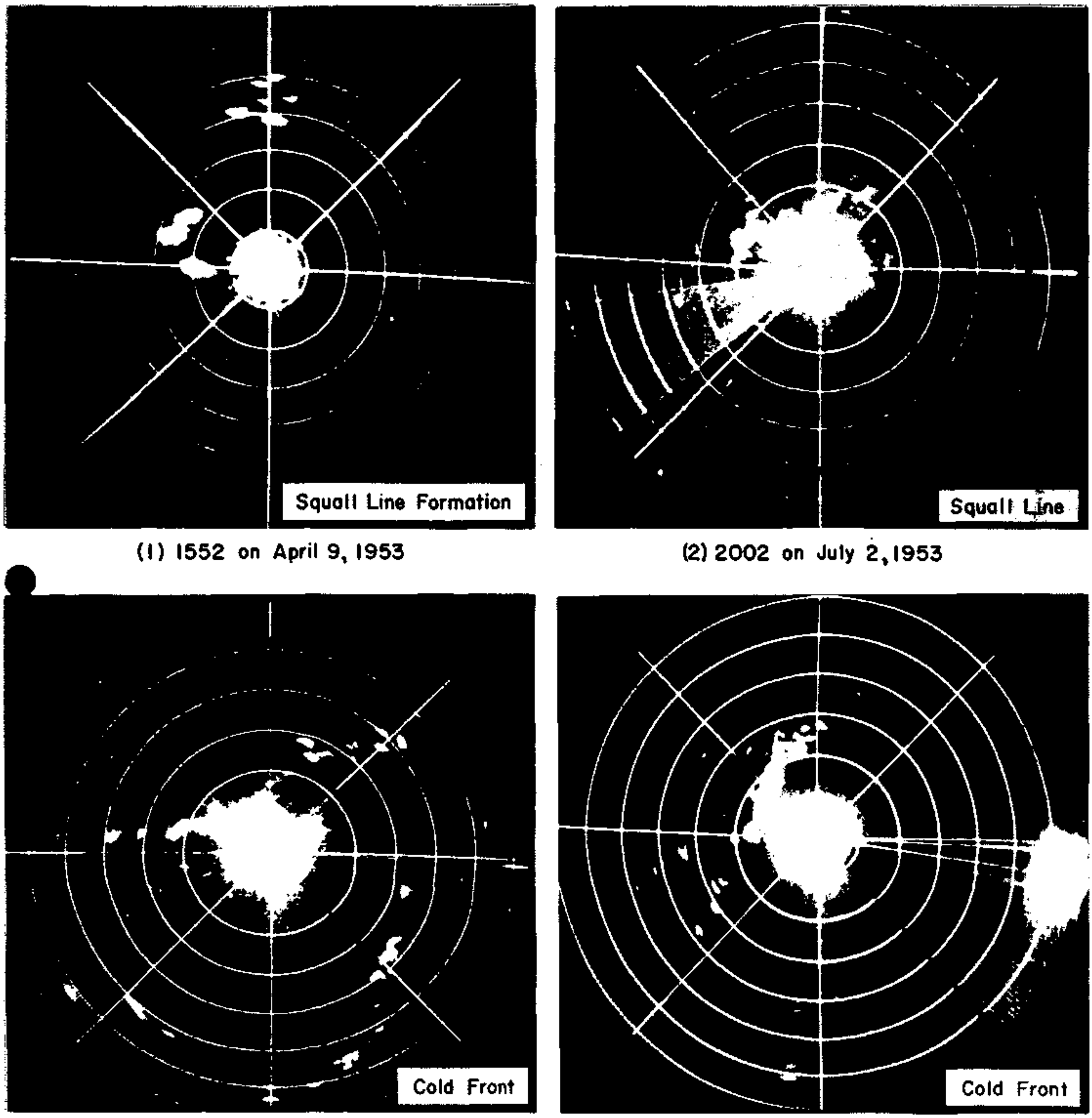


(2) 0112 on July 17, 1953

(3) 1334 on August 3, 1953

(b) 30 MILE RANGE, 10 MILE RANGE MARKERS

FIG. 8 PPI PHOTOGRAPHS FROM APS-15A RADAR



(1) 1552 on April 9, 1953 (2) 2002 on July 2, 1953

(3) 1657 on August 7, 1953 (4) 2250 on September 18, 1953

140 MILE RANGE, 20 MILE RANGE MARKERS

FIG. 9 PPI PHOTOGRAPHS FROM APS-15A RADAR

ACCURACY OF RADAR AREAL-MEAN RAINFALL ESTIMATES

In experimental estimation of rainfall with radar instrumentation, it is necessary to adopt some standard of rainfall measurement as a basis for judging the reliability of the radar estimates. Radar observations of precipitation are made in the volume of the beam at an altitude above the ground. However, precipitation measurements at ground level are of primary interest for most military and civilian purposes. Before radar observations of rainfall amounts become practical, a relationship to ground observations must be established. One possibility is to relate radar rainfall estimates for an area to a network of raingages on that area. A way of expressing the reliability of radar-rainfall estimates is in terms of the accuracy obtained with raingage networks of various gage densities.

Method of Evaluation

The reliability of sample mean rainfall estimates was studied, using data from a 20-gage network within a 100 square-mile area in Central Illinois. These data included the rainfall measurements of 198 storms which occurred during the months of May through September for a 6-year period on the Panther Creek Watershed.

The sampling plan which was selected for estimating the sampling error of areal mean rainfall was defined as the one "best" sub-sample for each gage density, that is, the approximately uniform or centered systematic sample. The equation for the sampling standard error, s, which resulted from this rainfall analysis is given by the following equation

$$s = .17\ P^{.45}\ n^{-.57} \tag{28}$$

where s is in inches; P, the areal mean rainfall, is in inches; and n is the number of gages per rainfall sample.

The graph in Figure 10 indicates the magnitude of the sampling error which can be expected with samples of different numbers of gages. These sampling errors, computed from equation (28), are estimates of the accuracy of areal-mean rainfall samples. The sampling errors may also be used as a standard for appraising the reliability of radar areal-mean rainfall amounts, i.e., the accuracy of areal-mean depths obtained by radar instrumentation can be expressed in terms of different gage densities.

Results of Evaluation

A comparison of the accuracy of radar areal-mean rainfall with the ac-

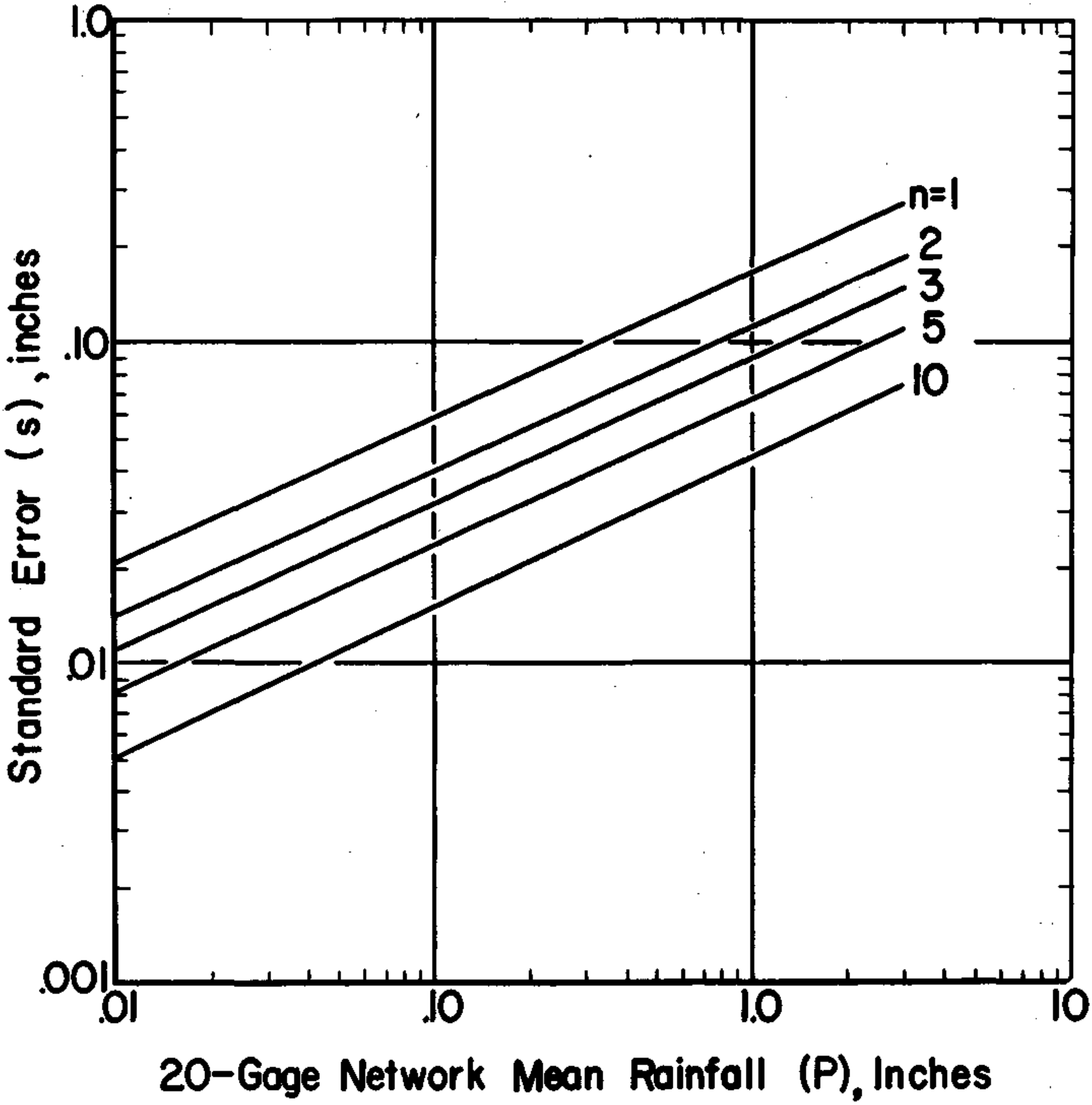


FIG.10 VARIATION OF s WITH P FOR A 100 SQUARE MILE AREA

TABLE 8

ACCURACY OF RADAR AREAL-MEAN STORM RAINFALL EXPRESSED IN TERMS OF RAINGAGE DENSITIES

Depth of Rainfall (in.)

Date	Time	Raingage Low	Raingage Max.	Average Gage	Average Radar	Radar and Gage Difference	Radar Accuracy Gages per 100 sq. mi.
				1951			
7-22	2128-2155	.00	.32	.07	.03	-.04	1
8-20	1715-1740	.09	.24	.12	.10	-.02	7
9-12	1710-1754	.17	.79	.44	.08	-.36	<1
9-26	2010-2050	.26	.79	.44	.17	-.27	<1
				1952			
7-2	1551-1639	.05	.21	.10	.06	-.04	2
8-15	0443-0640	.17	.42	.27	.11	-.16	<1
8-20	2037-2125	.08	.35	.17	.08	-.09	<1
9-1	1440-1500	.01	.10	.04	.01	-.03	2
9-14	0833-0851	.00	.26	.06	.02	-.04	1
9-14	0903-0915	.00	.11	.01	.01	00	20
9-18	1352-1510	.08	1.35	.36	.15	-.21	<1
9-18	1540-1655	.20	1.07	.57	.03	-.54	<1
10-14	1354-1416	.04	.13	.09	.01	-.08	<1
				1953			
4-9	1615-1654	.00	.23	.05	.21	+.16	<1
4-24	0134-0330	.09	.17	.12	.25	+.13	<1

TABLE 8 (Continued)

Date	Time	Raingage Low	Raingage Max.	Average Gage	Average Radar	Radar and Gage Difference	Radar Accuracy Gages per 100 sq. mi.
5-16	1530-1650	.04	.48	.19	.26	+.07	1
6-5*	1607-1727	.13	.55	.35	.42	+.07	2
6-8*	0514-0634	.28	.73	.52	.64	+.12	1
6-25*	1525-1644	.01	1.39	.73	.33	-.40	<1
7-2	0820-1006	.03	.64	.40	.07	-.33	<1
7-5	2226-2330	.64	1.03	.82	.13	-.69	<1
7-17*	0041-0140	.01	.37	.15	.18	+.03	5
8-3*	1322-1412	.00	.23	.03	.04	+.01	8
8-7	1703-1741	.21	.55	.37	.20	-.17	<1
8-8*	1346-1433	.00	.59	.08	.06	-.02	5
9-18	2307-2350	.00	.07	.02	.05	+.03	1
11-20	1233-1315	.00	.09	.03	.01	-.02	2
11-22	0155-0300	.14	.21	.16	.10	-.06	1

*Storms from which selected data were taken to obtain equation (17).

curacy of raingage sample means is presented in Table 8. Radar-indicated, areal-mean rainfall amounts in this table were determined with the use of equation (17). Fifteen of the 28 radar estimates were equal to or better than the expected accuracy of one gage per 100 square miles. Seven of the fifteen estimates, however, were for storms which contributed data to equation (17).

The results presented in Table 8 show relatively large errors in radar-rainfall estimates in many cases. Nevertheless, the results are encouraging when the density of most climatological networks is considered. In Illinois, for instance, the average raingage density is approximately one per 225 square miles. On many military operations there are no raingages and any estimate is better than none.

The trend for underestimates with the empirical equation in Table 8 is undoubtedly partially due to the equations inadequacy in compensating for attenuation. Although the equation was derived from attenuated data, only data in which no intervening storms occurred between the radar and the network storm were used. Consequently, when the equation is applied non-selectively to a group of storms (such as in Table 8), a trend for underestimates is to be expected. Actually the equation should only be used for cases of little or no intervening rainfall.

The noticeable trend in Table 8 for the degree of underestimation to become greater with increasing mean rainfall is also partially attributable to attenuation which increases with increasing rainfall rate. An investigation of the effect of minimizing logarithmic deviations instead of absolute deviations indicated that the foregoing trend can not be explained by the use of logarithms in equation (17).

The simple empirical techniques used in obtaining rainfall estimates to date have proven inadequate for general utilization. Numerous sources of error in estimates of areal mean rainfall by radar exist. Among these are attenuation, variations in drop-size distributions between and within storms, inadequate measurements of the average power received (P_r) for use in the radar-rainfall equation, and radar calibration errors.

Analysis of data collected with the raindrop camera during 1953-54 shows that there are significant differences between the rainfall-radar reflectivity (Z-R) relations in thunderstorms, rainshowers, and continuous rain, and that variations in Z-R relations between storms may be a source of 5 to 7 decibel errors[2]. Furthermore, observations with the drop camera have indicated that the majority of large drops within storms are nonspherical. These large drops contribute greatly to the reflectivity and attenuation from rain, and their nonsphericity is not considered in Z-R relations. Analytical results will be presented later which indicate the possible magnitude of errors attributable to attenuation.

CORRELATION OF SURFACE RAINFALL WITH PPI ECHOES

Comparison of Surface Rainfall Data With Radar Observations

Because of the difficulty experienced with radar estimates of rainfall during 1951-53, a limited investigation was undertaken to examine in more detail the degree of correlation between surface rainfall data and APS-15 radar observations. Data from 15 storms over Goose Creek during 1953, for which one-minute isoecho maps and one-minute raingage amounts were available, were used for this purpose. Because of the large amount of tabulation required for such a study, only 10 of the 50 raingage stations were used (Fig. 11). These were selected to give an approximately uniform sampling of the 96 square mile area. Three stations (12, 31, 50) were chosen along the forward edge of the network, four (4, 22, 35, 48) through the middle of the network, and three (1, 19, 39) along the rear edge.

The primary purpose of this study was to obtain an estimation of attenuation effects upon radar-indicated surface rainfall. As a first step, the total duration of echo over each station for the 15 storms combined was calculated, along with the total duration of rain at these same stations. Results of this analysis are shown as items 1 and 2 in Table 9. Next, to eliminate the effects of very light rainfall, the total duration of rail excluding traces was calculated (Item 3, Table 9). Traces were defined as rates less than .06 inch per hour in this study, since one-minute amounts on the raingage graphs could only be read to the nearest .001 inch. Total minutes of rain (excluding traces) was less than total minutes of echo for stations 31 and 50. Including traces, total minutes of rain considerably exceeded the total minutes of echo for all stations.

Considering that the study was largely restricted to warm season thunderstorms and rainshowers, it is believed that very few of the detection failures can be attributed to insufficient filling of the beam, whose top varies between 3000 feet and 4000 feet over the Goose Creek Network. Radar horizon effects should not be important, since the bottom of the beam is only about 350 feet above the ground over the network. Therefore, the major contributors to the duration differences observed between the raingages and the radar would appear to be the presence of appreciable attenuation and/or the occurrence of many light rainfall rates not detectable by the radar set. However, the general detection trend was maintained when traces were excluded, except for the three stations on the forward edge of the network. Because of the predominating SW-NE and NW-SE movement of rain cells across the network, average attenuation effects would be expected to be less severe on the forward edge of the network, which lies to the west of the radar station.

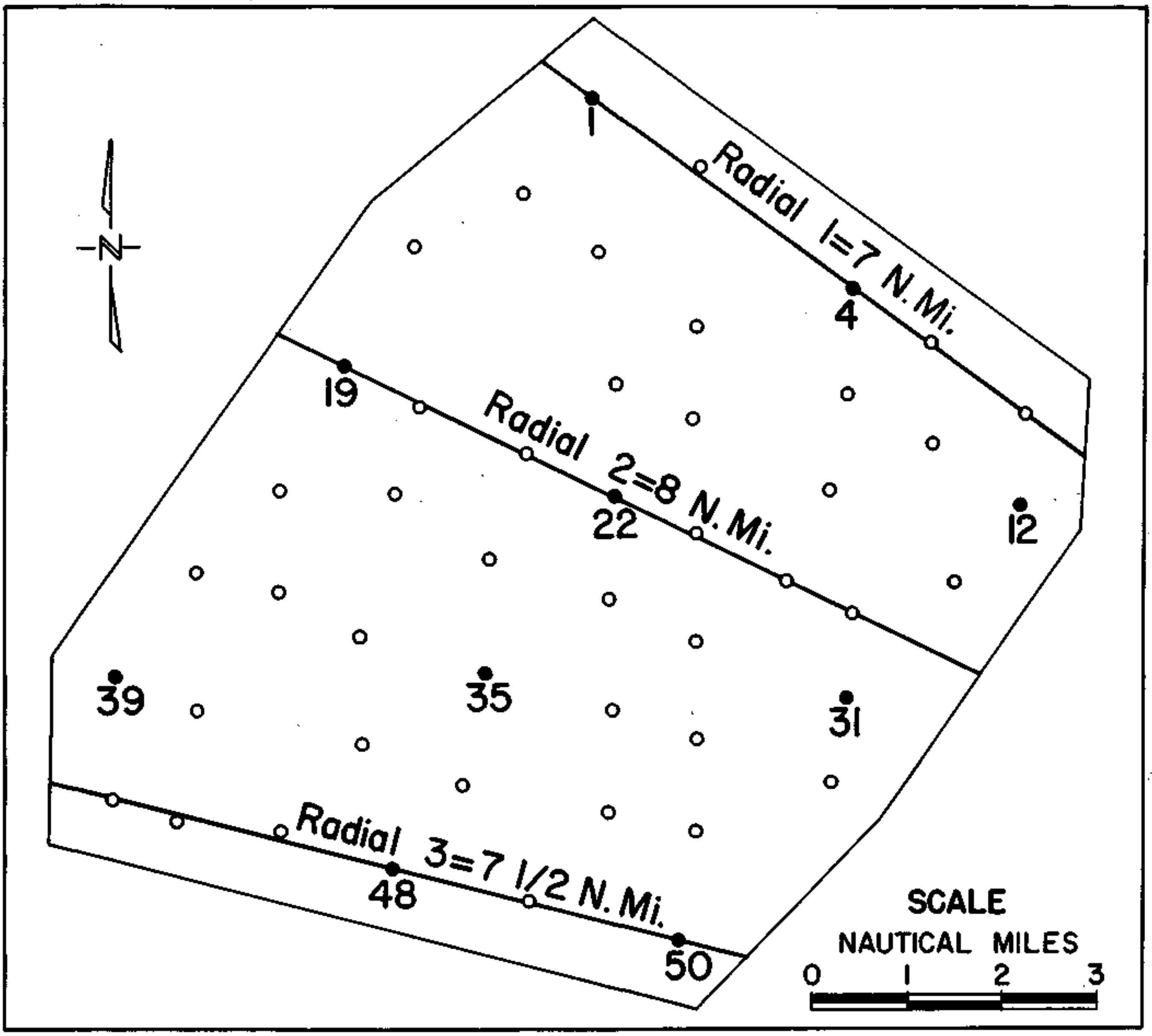


FIG.11 RADIALS USED IN FREQUENCY STUDY

TABLE 9

COMPARISON OF SURFACE RAINFALL DATA WITH RADAR OBSERVATIONS

(15 STORMS, SELECTED STATIONS, GOOSE CREEK, 1953)

	Station Number										
	12	31	50	4	22	35	48	1	19	39	Ave.
1. Total Minutes of Echo	432	455	416	401	464	419	320	328	337	271	
2. Total Minutes of Rain	560	578	509	706	685	622	621	708	663	472	
3. Total Minutes of Rain, Excluding traces	443	440	392	546	585	470	474	602	516	403	
4. Rain with Echo (min.)	342	316	311	326	386	330	263	269	268	194	
5. Rain without Echo (min.)	218	262	198	380	299	292	358	439	395	278	
6. Echo without Rain (min.)	90	139	105	75	78	89	57	59	69	77	
7. Measurable Rain without Echo (min.)-no traces	164	183	130	247	219	166	227	337	259	209	
8. Rain without Echo (%)	39	45	39	54	44	47	58	62	60	59	51
9. Echo without Rain (%)	21	31	25	19	17	21	18	18	20	28	22
10. Measurable Rain without Echo (%)	37	42	33	45	37	35	48	56	50	52	44
11. Total Rain (in.)	4.21	3.66	4.00	3.25	5.62	3.93	3.95	4.43	4.85	3.65	
12. Rain without Echo (in.)	1.05	1.16	0.92	1.24	0.85	0.83	1.39	1.42	1.67	1.38	
13. Rain without Echo (%)	25	32	23	29	15	20	35	32	34	38	28

To further define the relation between echo and rain occurrences, items 4-7 in Table 9 were calculated. Item 4 shows the total duration in minutes during which rain and radar echo coincided at each station. Item 5 shows the amount of time that rain was occurring at each station with no echo present over the station. Item 6 shows the amount of time that echo appeared over the stations, but was accompanied by no rain at the surface raingages. Item 7 shows the total duration of measurable rain (.06 in/hr or over) occurring in the absence of echo over each station.

In items 8-10, percentage values for item 5-7 have been tabulated. Based on rainfall duration, results of the tabulation show that 51 percent of all rain at the stations occurred in the absence of echo. Averages show that measurable rain occurred without echo about 44 percent of the time. On the other hand, about 22 percent of the time when echoes were observed over stations, no rain was occurring at the surface.

Items 11-13 in Table 9 show how the total depth of rainfall at each station was associated with radar echo occurrences. Combining all 10 stations, approximately 28 percent of the total rainfall depth occurred in the absence of echo. Considering the relatively high percentage of rain which was not observed, the difficulty of devising a reliable empirical radar-rainfall equation for the APS-15 is apparent.

In Table 10, storm rainfall durations recorded by the raingages and radar in each of the 15 storms are tabulated. Durations are 10-station averages for each storm. Raingage durations including traces are shown in column 2, while traces have been excluded in the raingage duration shown in column 3. Average rainfall rates in inches per hour for each storm, based upon the 10 stations, are shown in column 4. No outstanding trend with respect to storm duration and mean intensity is apparent from the table. Important factors such as the direction of cell movement across the network and the existence of other storm cells between the radar station and the network are active in determining the degree of attenuation observed at the radar station, and consequently affect the duration relations of Table 10.

In Table 11, a breakdown of echo occurrence in the absence of surface rainfall is given for each station. Results of the analysis indicate that on the average about 63 percent of the station echoes without surface rainfall occurred prior to the initiation of rain at the stations. That is, rain reached the ground within a few minutes after echoes were observed aloft over the stations. On the average only nine percent of echo without rain occurred after the rain had stopped at the stations. The analysis showed that about 28 percent of the echoes occurring in the absence of rain were not associated with rain at the station either before or after their appearance. In such cases, rain occurred some place on the network, but none fell from the echo as it passed over certain

TABLE 10

STORM RAINFALL DURATION RECORDED BY RAINGAGES AND RADAR
(15 STORMS, SELECTED STATIONS, GOOSE CREEK, 1953)

Date	Dr *	Dg*	Dm*	Im*
4-9	22	9	9	.27
4-24	77	80	54	.08
6-5	28	31	29	.70
6-8	29	54	46	.81
6-25	19	27	27	1.56
7-2	18	85	75	.35
7-5	19	43	38	1.22
7-16	52	94	76	.45
7-17	19	26	20	.42
8-3	8	4	4	.60
8-7	16	47	40	.57
8-8	13	40	13	.37
9-6	43	46	33	.13
9-18	11	12	10	.12
11-20	12	15	15	.16

*Dr, Dg, Dm, = Ave. rainfall duration (min.) for 10 stations from radar echoes, for raingages, and raingages with traces omitted; Im = Ave. rainfall intensity (in/hr) b ased on Dm.

stations. About 40 percent of the total rainless echo time occurred with the 9 April storm from which a tornado developed shortly after the storm left the network. Another 50 percent occurred on 6 September with a synoptic situation which appeared favorable for evaporation of falling rain.

TABLE 11

DISTRIBUTION OF RADAR ECHO OBSERVED WITH NO RAIN AT STATION
(15 STORMS, SELECTED STATIONS, GOOSE CREEK, 1953)

	Station Number										
	12	31	50	4	22	35	48	1	19	39	Ave.
Echo Total (min.)	90	139	105	75	78	89	57	59	69	77	
Percent Prior to Rain	60	45	51	63	88	54	68	100	94	45	63
Percent After Rain Stopped	10	20	7	0	12	11	4	0	0	11	9
Percent Echo With No Rain	30	35	42	37	0	35	28	0	6	44	28

Attenuation effects during rainfall over Goose Creek are illustrated in Figure 12, which is a sequence of four maps covering a 20-minute period during the storm of 5 July. Map 11A shows the forward edge of the storm as it crossed the raingage network toward the radar station. The isoecho line represents step 1, or maximum sensitivity. The rainfall rates shown on the map are one-minute amounts expressed in inches per hour. Rainfall rates in this storm were unusually heavy and do not represent average storm conditions. Map 12B, taken 10-minutes later, shows rainfall blanketing the entire network. Due to attenuation, the radar is not seeing the rainfall occurring in the northwest corner of the network as shown by the isoecho representing the rear edge of step 1. Map 12C, 5 minutes later, shows heavy rainfall rates still blanketing the entire network while the radar indicates the rear edge of the storm is about half way across the 100 square-mile area. Map 12D, representing conditions 5 minutes later, shows the radar-indicated rear edge of the storm at the forward edge of the network, whereas the raingages show relatively heavy rates still blanketing the entire 100 square-mile area. Some precipitation attenuation occurred from an intervening shower in addition to that contributed by the illustrated storm over the network.

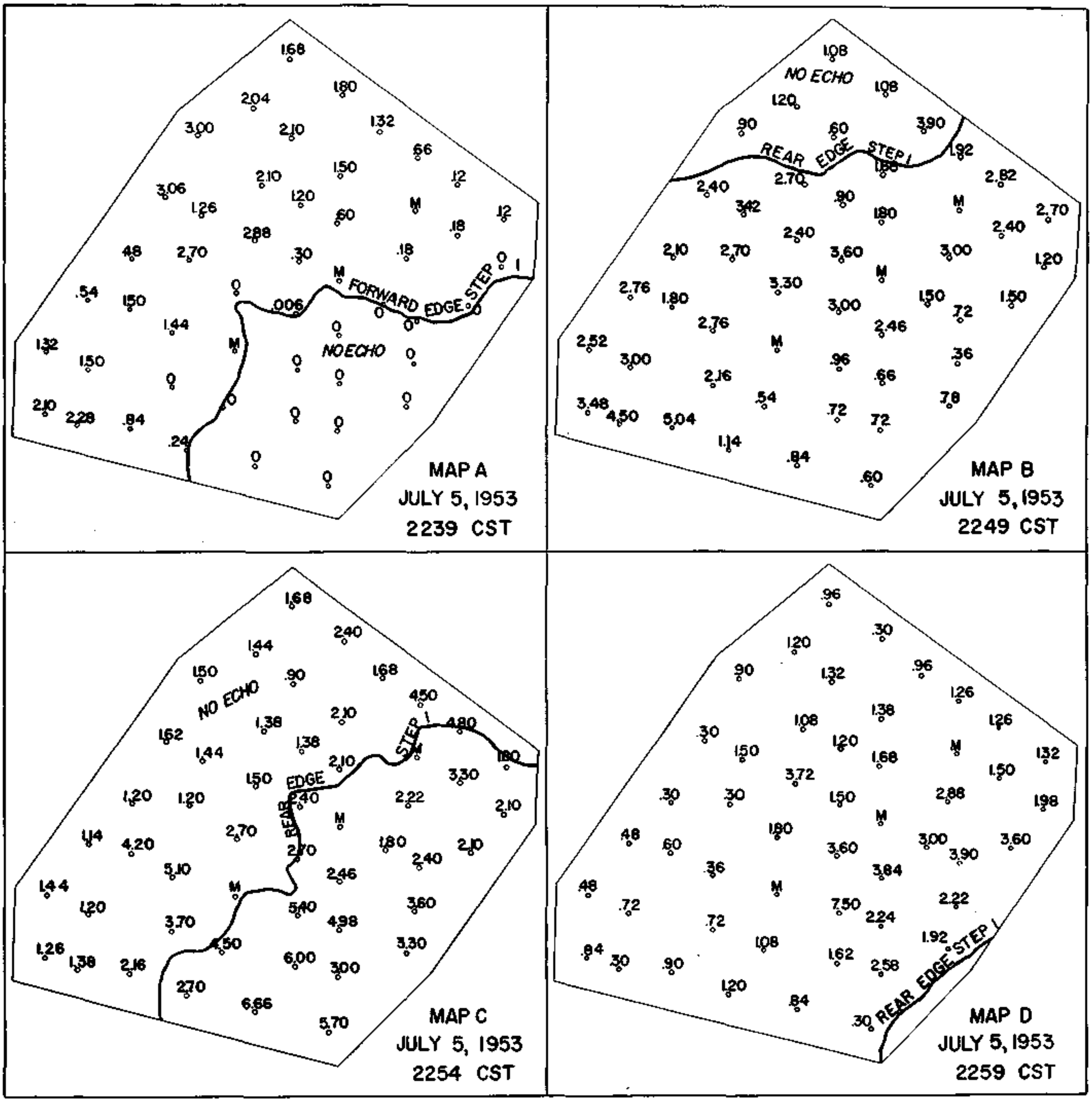


FIG.12 PRECIPITATION ATTENTUATION EFFECTS

Possible Variability in Radar Mean Rainfall Estimates Resulting From Attenuation

With the APS-15 radar during 1951-54, great variability was obtained among radar-indicated storm mean rainfall estimates when plotted against actual values obtained from the concentrated Goose Creek raingage network (Fig. 13). The radar-indicated rainfall estimates were obtained from equation (20). The results raised a question as to whether or not precipitation attenuation could have been a significant contributor to the observed graphical scatter. Available literature does not contain quantitative information on the effects of attenuation on areal mean rainfall estimates. Consequently, a study was made to obtain some information on the degree of variability which may be introduced in mean rainfall estimates by the attenuation factor.

Hypothetical data, considered representative of Mid-West thunderstorms, were used in a study to facilitate calculations. Areal storm mean rainfalls of 0.05, 0.10, 0.25, 0.50, 1.00, and 2. 00 inches were used for analysis purposes. While most storms will result in average storm depths under 0. 50 inch, average depths of one inch and greater are not rare. It was assumed that the area of interest was bounded on the west and east by radar range markers and on the north and south by radials emanating from the radar station. Storms were assumed to be moving from west to east across the given area towards the radar station. Further, the storms were assumed to be of uniform depth along radials from the radar when crossing the given area and to have a uniform rainfall gradient with depth (triangular profile). It was then arbitrarily decided to obtain six samples for each chosen mean rainfall. In the first three samples, the storm mean rainfall rate, depth, and speed were varied within reasonable limits to obtain the same actual mean rainfall in each case; however, different values of attenuation, and consequently, different radar-indicated mean rainfalls were obtained for these samples. In the fourth to sixth samples, intervening storms causing 5, 10, and 15 decibels of attenuation, respectively, were assumed between the radar and the storm of interest, which was taken as the second sample (second row) of each group in Table 12.

The data used in the study are summarized in Table 12. Radar-indicated mean rainfall values were obtained from the Wexler version of the radar-rainfall equation applied to the APS-15, and corrected for precipitation attenuation. The Wexler equation was used in preference to equation(19) in this study, since equation (19) was derived from attenuated data. The Robertson and King" attenuation coefficient, equal to approximately 3 db per nautical mile per inch per hour was applied to the data in calculating the radar-indicated mean rainfall values. It was assumed that the radar was capable of providing a rainfall rate observation at one-mile intervals throughout the storm.

In Figure 14, the scatter of the Goose Creek 1951-54 storms is compared with that obtained with the hypothetical examples. The writers do not intend to

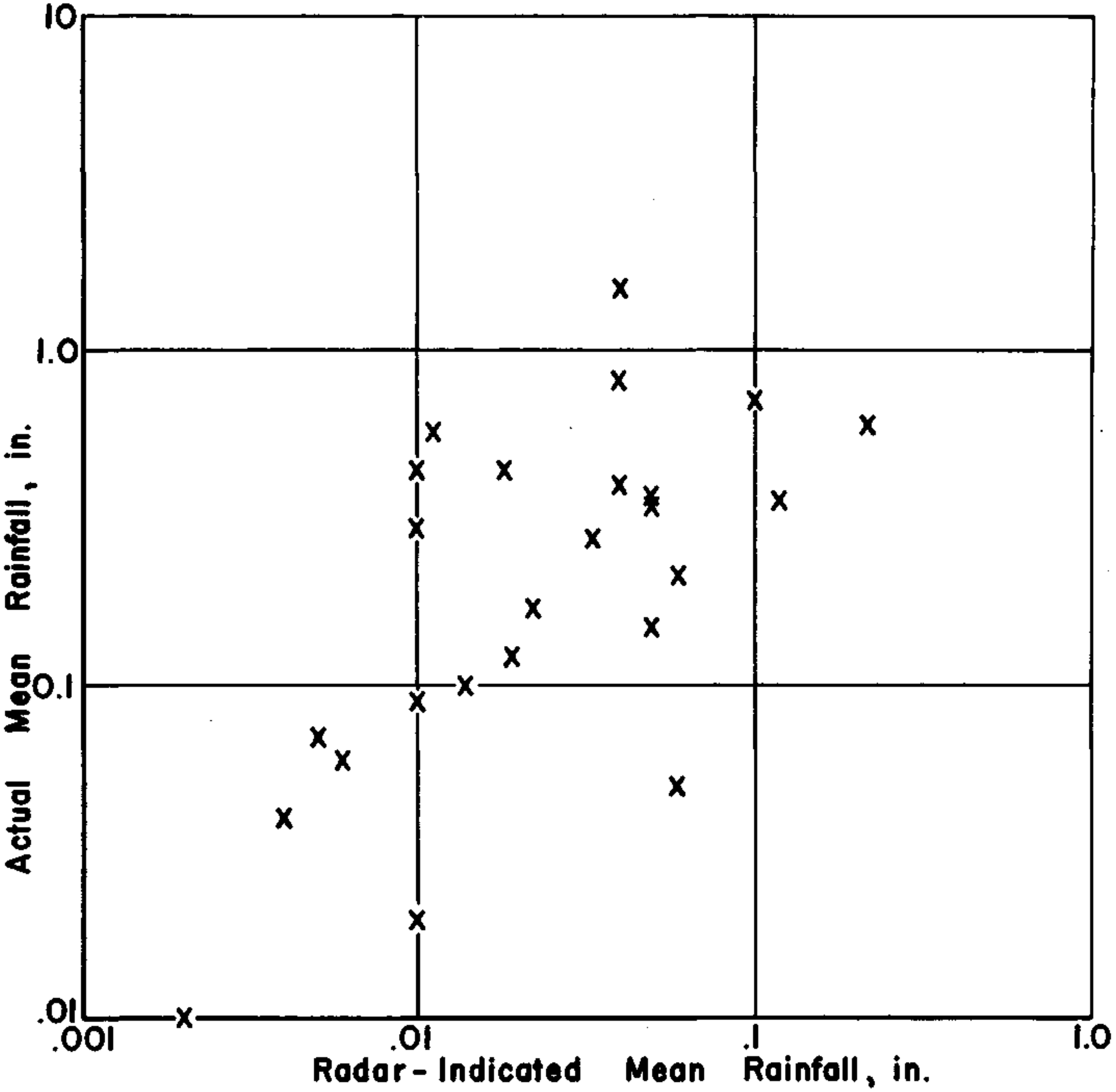


FIG.13 COMPARISON OF RADAR-INDICATED AND ACTUAL MEAN RAINFALL ON GOOSE CREEK, 1951 - 1954.

TABLE 12

VARIATION IN RADAR — INDICATED AREAL MEAN RAINFALL RESULTING FROM FLUCTUATING ATTENUATION

Ave. Rate (in/hr)	Storm Depth (N. Mi.)	Storm Speed (Knots)	Point Duration (min)	Attenuation from Intervening Storm (db)	Total Attenuation Through Rain (db)	Actual Areal Mean Rainfall (in.)	Radar Indicated Mean Rainfall (in.)
0.10	5	10	30	0	2	0.05	0.04
0.20	2	8	15	0	1	0.05	0.04
0.30	4	24	10	0	4	0.05	0.04
0.20	2	8	15	5	6	0.05	0.02
0.20	2	8	15	10	11	0.05	0.01
0.20	2	8	15	15	16	0.05	0.01
0.50	2	10	12	0	3	0.10	0.09
0.50	4	20	12	0	6	0.10	0.06
0.50	10	50	12	0	15	0.10	0.04
0.50	4	20	12	5	11	0.10	0.03
0.50	4	20	12	10	16	0.10	0.02
0.50	4	20	12	15	21	0.10	0.01
0.50	5	10	30	0	7	0.25	0.15
1.00	5	20	15	0	15	0.25	0.10
2.00	5	40	8	0	30	0.25	0.06
1.00	5	20	15	5	20	0.25	0.05
1.00	5	20	15	10	25	0.25	0.02
1.00	5	20	15	15	30	0.25	0.01
0.50	12	12	60	0	18	0.50	0.17
0.75	15	22	40	0	34	0.50	0.11
1.00	16	32	30	0	48	0.50	0.07
0.75	15	22	40	5	39	0.50	0.05
0.75	15	22	40	10	44	0.50	0.02
0.75	15	22	40	15	49	0.50	0.01
0.50	10	5	120	0	15	1.00	0.41
1.00	10	10	60	0	30	1.00	0.23
2.00	8	16	30	0	48	1.00	0.15
1.00	10	10	60	5	35	1.00	0.11
1.00	10	10	60	10	40	1.00	0.05
1.00	10	10	60	15	45	1.00	0.02
1.00	10	5	120	0	30	2.00	0.44
1.50	10	7	80	0	45	2.00	0.29
2.00	10	10	60	0	60	2.00	0.21
1.50	10	7	80	5	50	2.00	0.14
1.50	10	7	80	10	55	2.00	0.06
1.50	10	7	80	15	60	2.00	0.03

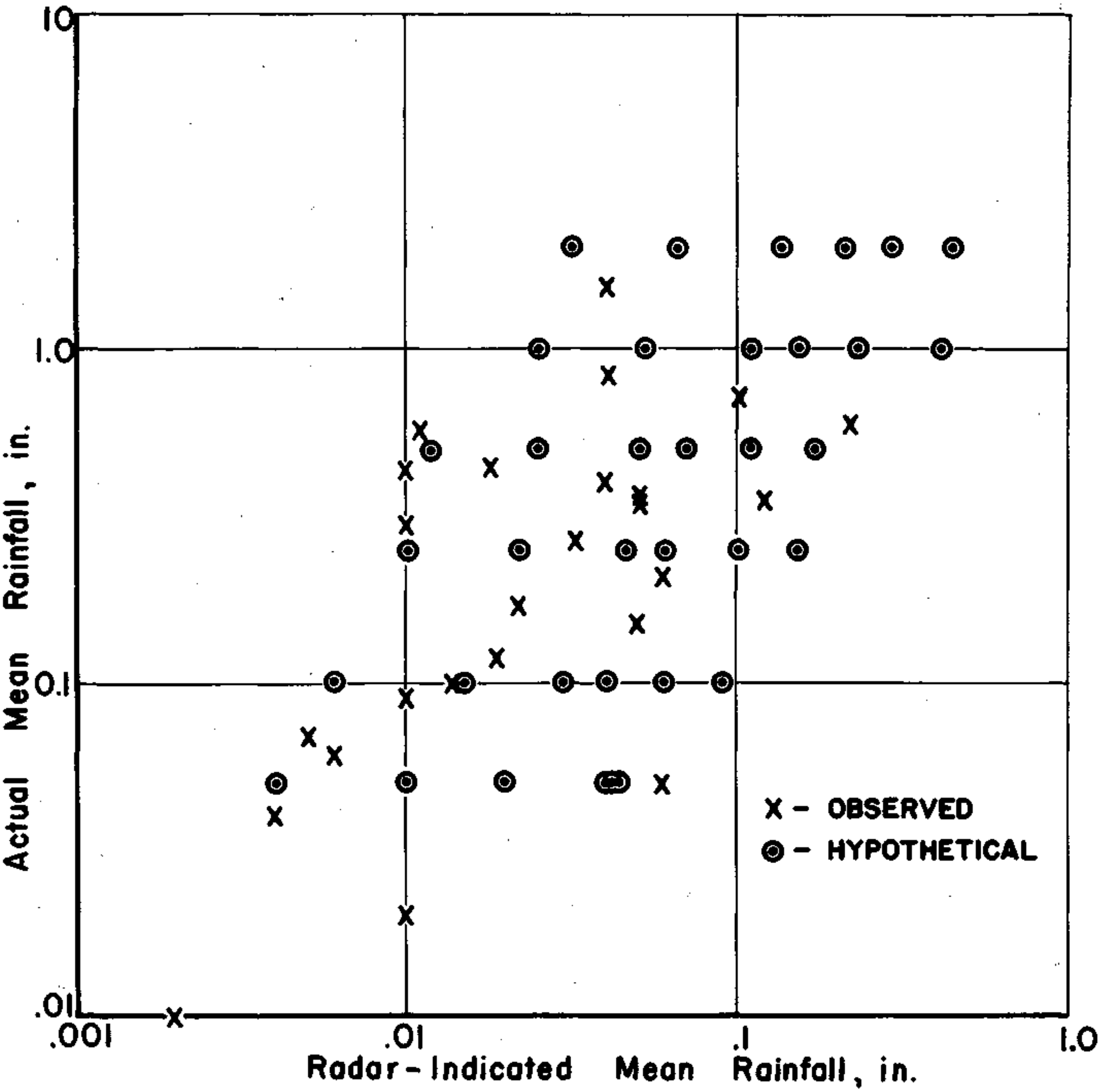


FIG.14 COMPARISON OF HYPOTHETICAL WITH OBSERVED SCATTER IN RADAR - INDICATED MEAN RAINFALL.

imply that all or most of the scatter in the 1951-54 storms was due to precipitation attenuation. However, it is intended to show that attenuation can be an important source of variability in radar-indicated mean rainfall values.

Table 13 shows the approximate number of decibels needed to equalize radar-raingage mean rainfall estimates for 14 storms on Goose Creek during 1953. The radar-indicated amounts were calculated using the Wexler[8] version of the radar-rainfall equation. Note the great variability in the decibel corrections needed for various storm sizes. This variability, of course, is due to various factors, among which is precipitation attenuation.

Frequency Distribution of Attenuation

A study was made of the frequency distribution of attenuation in shower-type rainfall across Goose Creek. Such information may be of value in establishing requirements for attenuation corrections in 3-cm radar sets.

One-minute rainfall data from eight storms of shower-type precipitation during 1953 were used in the study. Calculations of precipitation attenuation in storms by one-minute intervals were made along each of three radials across the network. The Robertson and King attenuation coefficient was again used in calculations. Location of the radials and their lengths is shown in Figure 11. Mean rainfall and duration of analyzed storms are given in Table 14.

Attenuation data for all radials and all storms were combined to provide an average frequency distribution of attenuation during rainfall. The result is shown in Figure 15 where magnitude of attenuation has been plotted against cumulative frequency in percent. Since the radials were of different length, attenuation has been expressed in decibels per mile rather than as total attenuation in combining the data. Note that 50 percent of the time it was raining over Goose Creek the attenuation was less than 0. 5 decibel per mile across the network. This analysis, of course, did not take into account the effect of rain between the radar station and Goose Creek. Results may be considered representative for a station located on the edge of the network, or perhaps in an area within a radius of 7 to 8 miles of a radar station during the thunderstorm season.

Next, the relation between mean rainfall and attenuation was investigated. The results are summarized in Table 15 where cumulative percent of total mean rainfall has been tabulated against the magnitude of attenuation for each radial, after combining data for the eight storms. In Figure 16, the three radials have been combined to obtain a mean relation. Again attenuation has been expressed in decibels per mile in combining the radials of unequal length. Attenuation distribution in specific storms is illustrated in Table 16.

TABLE 13

DECIBEL CORRECTIONS TO EQUALIZE RADAR-RAINGAGE ESTIMATES

(GOOSE CREEK, 1953)

Date	Mean Rainfall (in.) Raingages	Radar	Necessary db-addition
4-9	.05	.06	0
4-24	.12	.06	5
5-16	.21	.06	8
6-5	.35	.12	7
6-8	.60	.22	7
6-25	.71	.10	13
7-2	.44	.01	24
7-5	.82	.04	20
7-17	.15	.05	7
8-3	.03	.01	7
8-7	.37	.05	13
8-8	.09	.01	15
9-18	.02	.01	5
11-20	.02	.01	5

Figure 16 indicates that 50 percent of the rainfall over Goose Creek occurred with attenuation exceeding 3.8 decibels per mile. The median value of attenuation over the network during rain, computed from Figure 15, is approximately 0.35 decibel per mile. The foregoing statistics indicate that a large portion of the total rainfall occurs with relatively heavy rates since attenuation is approximately proportional to the rainfall rate. Figure 18 in Research Report No. 2 (under this contract) shows that 50 percent of the rainfall in eight storms over Goose Creek during 1952 occurred at rates exceeding 1.3 inches per hour Results of this study indicate the magnitude of the attenuation problem with 3-cm radar when using it for quantitative measurement of rainfall in Mid-Western thunderstorms.

TABLE 14

STORM DATA

Date 1953	Mean Rainfall (in.) Radial 1	Radial 2	Radial 3	Duration (min.) Radial 1	Radial 2	Radial 3
4/9	0.11	0.08	0.00	30	36	0
6/5	0.43	0.44	0.22	78	36	66
6/8	0.67	0.61	0.68	78	63	66
6/25	0.93	1.14	0.31	72	66	30
7/2	0.51	0.53	0.34	105	90	105
7/5	0.90	0.80	0.73	66	51	39
8/8	0.08	0.22	0.14	69	42	69
9/18	0.02	0.03	0.02	21	27	12

TABLE 15

RELATION BETWEEN MEAN RAINFALL AND ATTENUATION

Acc. Percent of Total Mean Rainfall	Attenuation (db) Equaled or Exceeded Radial 1 (7 mi.)	Radial 2 (8 mi.)	Radial 3 (7.5 mi.)
5	52	65	73
10	47	57	62
25	39	47	41
50	26	32	24
75	13	17	12
90	6	10	5

TABLE 16

DISTRIBUTION OF ATTENUATION IN STORMS ALONG RADIAL 1

Date 1953	Mean Rainfall (in.)	Attenuation (db) Equaled or Exceeded For Given Cumulative Percent of Total Mean Rainfall 25	50	75	90
4/9	0.11	15	11	7	3
6/5	0.43	36	24	12	5
6/8	0.67	31	23	15	8
6/25	0.93	41	34	28	22
7/2	0.51	17	11	7	4
7/5	0.90	51	42	26	17
8/8	0.08	9	4	2	1-
9/18	0.02	12	8	5	1

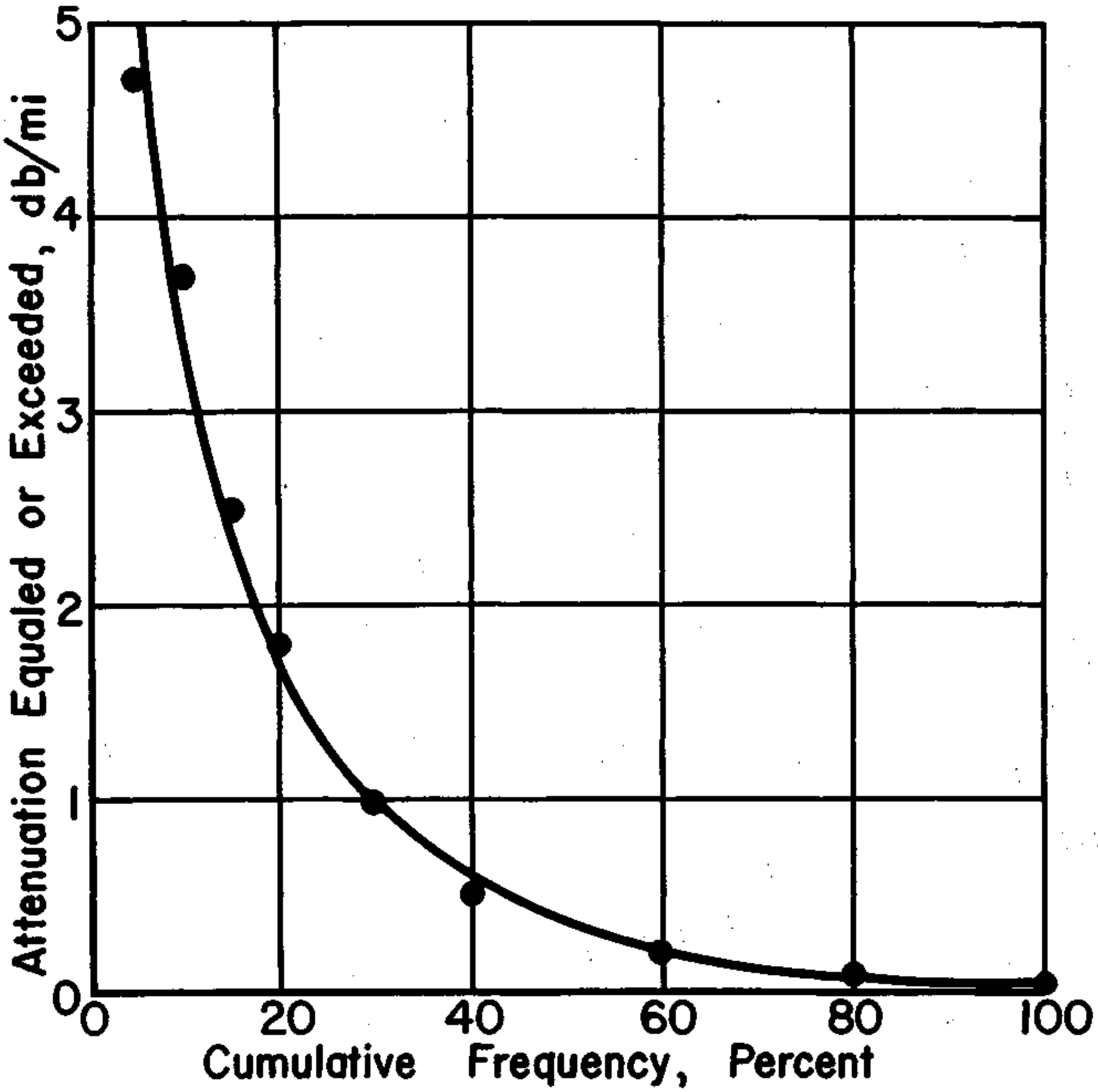


FIG.15 DISTRIBUTION OF ATTENUATION DURING RAIN (GOOSE CREEK, 8 STORMS, 1953)

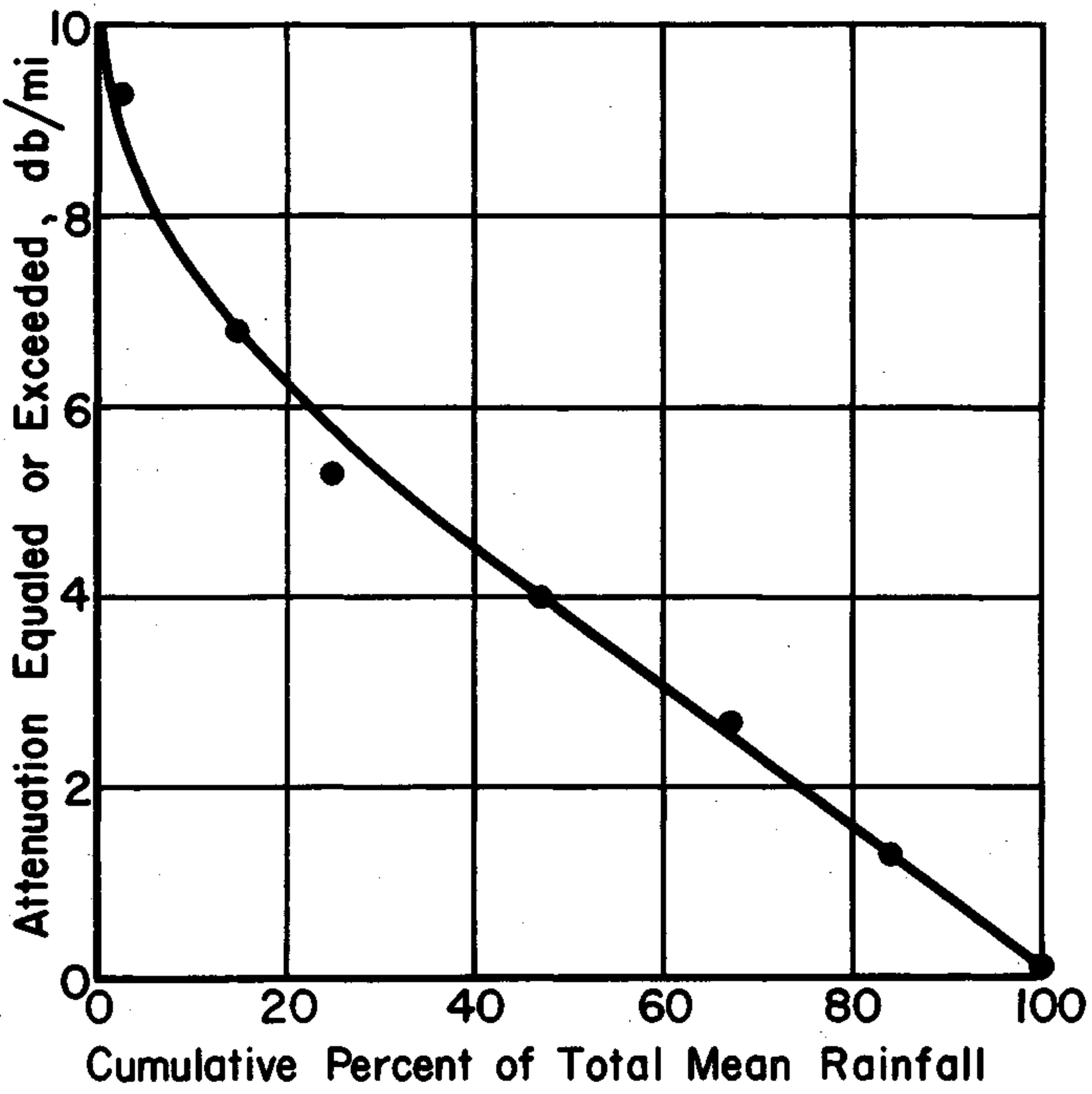


FIG.16 RELATION BETWEEN MEAN RAINFALL AND ATTENUATION (GOOSE CREEK, 8 STORMS, 1953)

CONCLUSIONS AND RECOMMENDATIONS

The low-powered APS-15 is generally satisfactory for short range detection and tracking of storms. Accurate quantitative rainfall measurements in moderate to heavy rainstorms have not been achieved with this low-powered set to date. However, for military operations it is believed that a competent radar meteorologist, using radar observations in conjunction with climatological and synoptic weather data, could determine the relative intensity of rainfall over an area more accurately than can be provided by synoptic forecasts alone. Consideration should be given to further development of prediction techniques using radar, climatology, and synoptic weather data.

Part of the difficulty encountered in making quantitative rainfall measurements with 3-cm radars is due to precipitation attenuation. Because of its magnitude and variability between and within shower-type storms, development of an empirical equation to adequately correct for attenuation effects has not been achieved. Testing of an empirical equation based on 1953 data indicates that the simple approach of developing an empirical equation from attenuated data is not adequate for attenuation compensation. Further efforts should be directed towards refining the empirical equation technique. It is recommended that consideration be given to the development of electronic techniques to compensate for precipitation attenuation at 3-cm wavelengths. Consideration should also be given to the use of 10-cm radar for quantitative precipitation analysis.

Results of the raindrop size-distribution study, which are treated in detail in Research Report No. 6 under this contract, indicate that relatively large variability among radar-rainfall estimates can arise from variations in drop size-distribution between and within storms. This study further indicates that there are appreciable differences between the Z-R relations in thunderstorms, rainshowers and continuous rain, and that non-spherical raindrops are quite common in shower-type rainfall. Considering the foregoing results, it is recommended that further effort be made toward better definition of radar reflectivity-rainfall rate relations, in order to develop better techniques and methods of application to quantitative precipitation measurements.

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