SEMICONDUCTOR LASER MODE ENGINEERING VIA WAVEGUIDE INDEX STRUCTURING

BY

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THESIS

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Semiconductor laser diodes are used in many applications, and their performance is often strongly influenced by the optical mode properties. In this work, dielectric waveguides and their guided modes have been simulated and analyzed as relevant to semiconductor diode lasers. The transverse refractive index structure of waveguides determines the modal field profiles and modal properties. By engineering the transverse index structure we show that it is possible to increase modal discrimination and impart modal selection. We further show that we can engineer the modal field profile itself for the purpose of engineering the properties of the laser beam, such as far-field brightness. Therefore waveguide engineering using index structuring may be beneficial to enhancing semiconductor laser performance.
I would like to thank my advisor, Professor Kent Choquette, and my fellow Photonic Devices Research Group members, Harshil Dave, Zihe Gao, Katie Lakomy, and Bradley Thompson, whose help and guidance have been invaluable.
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CHAPTER 1

INTRODUCTION

1.1 Motivation

Semiconductor diode lasers have many applications in today’s society, ranging from digital optical communication to optical sensing, materials processing and manufacturing, and high energy defense systems. Each application has its own specifications regarding the requisite properties and performance of diode lasers, such as requirements on power, power conversion efficiency, digital modulation speed, and spectral or spatial brightness. High power applications, such as are found in the manufacturing and defense industries, often require not only high output power and high efficiency but also, simultaneously, high brightness—either spectrally (concentrating power in a narrow spectral linewidth) or spatially (concentrating the mode power onto a compact spatial region) or ideally both. For example in some high power laser systems, semiconductor diode lasers are used as an efficient optical pump for a solid state laser, such as a fiber laser. In this application the diode laser is usually the source of a high power but relatively low brightness beam (due to multi-mode operation) while the fiber laser transforms the pump energy into a high brightness (i.e. single Gaussian mode) beam. In order to effectively and efficiently pump a fiber laser, it is imperative that the laser diode produce output only within the pump transition energy linewidth (thus requiring high spectral brightness) and that the pump beam efficiently couple into the core of the fiber laser (requiring a small far-field spot size and high spatial brightness).

One path to high brightness semiconductor diode devices is to control the laser modes, with a primary emphasis of limiting the number of modes that oscillate. High spectral brightness has been previously achieved by restrict-
ing the number of longitudinal modes, while high spatial brightness has most often been achieved by controlling the number of transverse modes. However, if the mode control also enables one to engineer the modes to optimize particular properties, then waveguide engineering provides two advantages for increasing laser diode brightness. Herein, it will be shown that spatial brightness can be improved not only by decreasing the number of oscillating transverse modes, but also by engineering the field profile of the modes themselves.

1.2 Prior work

Limiting the number of lasing modes can be as simple as making the laser cavity volume smaller. Indeed, the epitaxial structure of most diode lasers effectively limits the number of transverse modes in the direction of the laser epitaxy to a single fundamental mode. However, to increase high power operation the laser active region volume must be made larger (mainly in the longitudinal and lateral directions), so one must find ways of controlling the modes with larger laser dimensions. Past work has controlled the number of longitudinal modes by introducing refractive index gratings perpendicular to the direction of propagation [1], while lateral modes have been controlled also using gratings (parallel to the direction of propagation) [2, 3]. Past work has controlled the number of longitudinal and lateral modes simultaneously using two-dimensional index structures such as two-dimensional photonic crystals [4]. Selection of specific modes and profiles has been attempted using engineered mirrors [5, 6], phase structures [7], introducing mode dependent loss structures [8], as well as shaping the gain distribution [9].

Much of the prior work on mode control, both longitudinal and lateral, is based on the use of photonic crystals. A photonic crystal (PhC) is a dielectric structure with a periodic refractive index. The periodic refractive index of the PhC limits the propagation of light to those wavelengths that are commensurate with the periodicity permitted by the PhC [10]. While modes whose field is commensurate with the periodicity of the crystal will propagate within the PhC, deviation will lead to destructive interference that forbids propagation through the PhC. This leads to the two main approaches
for photonic crystal utilization: (i) band-edge devices that use a PhC within the cavity to define the lasing mode, and (ii) band-gap devices that confine the cavity by not supporting the lasing mode in the PhC that surrounds the cavity. Distributed feedback (DFB) structures are one-dimensional longitudinal gratings that operate as band-edge PhCs and have been shown to decrease the number of longitudinal modes, improving the spectral brightness [1]. Transverse Bragg resonance (TBR) structures are, like DFB structures, one-dimensional PhCs, but they are gratings in the transverse direction of a waveguide. TBR structures have been previously formulated as both a band-gap [2] and band-edge [3] device as a way of restricting the number of lasing transverse modes. Two-dimensional band-edge PhC devices have been shown to limit both the longitudinal and transverse modes simultaneously, leading to improved spectral and spatial brightness [4]. However, these prior efforts also led to low laser efficiency, as the mode control structures substantially contributed to increased optical cavity loss.

Limiting and selecting the transverse modes of broad-area diode lasers has also been achieved by other approaches. Loss structuring in the transverse direction, such as etching trenches within the cavity [8] or etching regions near the laser diode facet mirrors [5], makes the cavity losses dependent on the mode profile, allowing for discriminating against higher order modes. In a somewhat similar manner, transversely structuring the injection and gain profile [9] makes the modal gain dependent on the transverse mode profile, similarly favoring certain transverse modes over others. Changing the geometry of the facet mirrors [6], or adding phase structures to introduce a laterally varying phase delay into the cavity [7], can introduce diffraction losses that are dependent on the transverse profile of the mode at the facet, either increasing modal discrimination, imparting modal selection, or both.

1.3 Thesis scope

In this work, we consider the transverse refractive index structure of dielectric waveguides as it relates to transverse mode control and engineering of semiconductor diode lasers. The refractive index structures we consider are not necessarily photonic crystals as we will not require periodicity. In
this work we will not engineer the transverse gain or loss profile, and hence we will only consider the real part of the refractive indices.

Following is an outline of this thesis. In Chapter 2 we review the theory of dielectric waveguides and diode lasers as relevant to our analysis. We discuss waveguide and cavity modes, their properties, and the computational methods that we use to analyze them. In Chapter 3 we consider a selection of engineered waveguide structures and discuss their resulting modal properties. We show that waveguide engineering via a relatively simple refractive index profile can produce increased modal discrimination, increased transverse spectral splitting, and greater far-field brightness in comparison to simple waveguide structures of equal size. We show that waveguide engineering, even in the absence of cavity gain or loss engineering, can impart modal selection. Then we show that waveguide engineering could be a tool for creating lasers that support novel mode profiles (potentially replacing spatial light modulators for laser beam shaping and wavefront engineering). Finally, Chapter 4 summarizes our results and suggests future work.

1.4 Assumptions and conventions

Figure 1.1: Simple diode laser structure with the directions labeled. Blue layers are the cladding, orange the core, and the red layer is the active region. The laser beam is shown as green.

The focus of this thesis are the optical modes of edge-emitting semiconductor diode lasers (although the concepts are translatable to surface-emitting semiconductor diode lasers). The following conventions for the laser modes
will be used throughout this work. As depicted in Figure 1.1, the longitudi-

tudinal direction will refer to the direction of the propagation of the laser
beam (which is normal to the diode mirror facets). The epitaxial direction
is the direction of the epitaxial growth, and the lateral is the direction in
edge-emitting devices that is normal to both the longitudinal and epitaxial
directions. In previous literature, the “transverse” direction has referred at
times to the lateral and at times to the epitaxial direction. Herein “trans-
verse” refers to a direction that is perpendicular to the direction of propaga-
tion, which in edge-emitting devices could be either the lateral or epitaxial
direction. (As our focus is on lateral modes it will be generally synonymous
with the lateral direction, although much of this analysis could be straight-
forwardly applied to the epitaxial direction.)

We also make the following assumptions throughout our analysis. First,
the laser devices considered herein have longitudinally invariant index struc-
tures, meaning that the refractive index structure varies not in the longitudi-
nal direction, but only the transverse directions. Secondly, as the calculations
are one-dimensional and focused on the lateral structure, the index values
used for the waveguide structures can be considered to be the effective index
values of the guided mode of the epitaxially defined waveguide (the physical
implementation of index perturbations into the laser structure is thus ab-
stracted away). Third, in our analysis of modal discrimination and selection,
we will assume that the behavior is defined mainly by the modal confine-
ment factor. We will not take into consideration any spatially nonuniform
thermal or current injection effects, nor will we consider spatial or spectral
hole burning.
2.1 Theory overview

In this chapter, Section 2.2 will discuss the basics of dielectric waveguides and the calculation of their modes and Section 2.3 will explain the relevance of dielectric waveguides to semiconductor diode lasers and their modes. We then review modal qualities, including effective modal index (Section 2.4), modal confinement factor (Section 2.5), and the far-field “power-in-the-bucket” brightness (Section 2.6) along with the computational methods used in their determination. Finally, we discuss a potential method for engineering waveguides to support a desired mode in Section 2.7.

2.2 Dielectric waveguides

Dielectric waveguides are formed using layers with different refractive indices and exploit total-internal-reflection [11].

2.2.1 Dielectric waveguide theory

The transverse refractive index structure of edge-emitting semiconductor diode lasers generally forms a dielectric waveguide. Waveguides confine light in the transverse directions and guide it along the longitudinal propagation direction. The analysis of dielectric waveguides is governed by the Helmholtz equation,

$$\nabla^2 U + k^2 U = 0$$

(2.1)

where $U$ is the modal field and $k$ is the transverse propagation vector throughout the transverse domain. In practice, dielectric waveguide analysis focuses
on the interfaces between regions of different effective indices, and the boundary conditions these interfaces impose. While in free-space there are infinite solutions to the Helmholtz equation for a given frequency of light, the waveguide boundary conditions lead to a finite set of discrete solutions that are approximately localized within the waveguide. These discrete steady-state solutions to the waveguide Helmholtz equation are referred to as the modes of a waveguide. They are characterized by the transverse field \( U \) (that is invariant during propagation through a waveguide) and a modal effective refractive index \( n_{\text{eff}} \) (that relates to the longitudinal propagation of a mode). More generally and specific to the problem of waveguides, we write the Helmholtz equation in terms of the transverse index structure \( n \) as

\[
\nabla^2 U + (n^2 - n_{\text{eff}}^2)k_0^2 U = 0 \tag{2.2}
\]

where \( U \) is the modal field for light with a free-space wave-vector \( k_0 = \frac{2\pi}{\lambda_0} \) and a modal effective index \( n_{\text{eff}} \). However, mode solutions cannot always be found in an exact or even approximate analytical form [11, Chapter 7]. We thus use finite-difference computational methods to obtain approximate solutions to the waveguide modes, as discussed in Subsection 2.2.2.

![Figure 2.1: A simple dielectric slab waveguide and the field intensities of the three lowest order modes](image)

The simplest dielectric waveguide one can consider is the one-dimensional symmetric slab waveguide (shown in Figure 2.1), which takes the form of a core dielectric of index \( n_1 \) and thickness \( d \), sandwiched between semi-infinite layers of index \( n_2 \), where \( n_2 < n_1 \). We refer to the center higher index
layer as the core layer, and the outer layers as the cladding layers. In this simple example, we can identify two general categories of modes: guided modes where the field takes an exponential form in the cladding (so that the field attenuates to zero at the limits of infinity), and unbound radiation modes that do not attenuate to zero in one or more of the cladding layers but rather propagate energy out to infinity [11, Chapter 7]. Of these, only the guided modes are relevant to our discussion, as a laser cavity requires bound, confined modes.

An example of the index profile for a symmetric slab waveguide (with $n_1 = 3.00$ and $n_2 = 3.01$, arbitrarily chosen values that are on the order of a III-V compound semiconductor laser diode’s index values, with an index step on the order of a reasonable lateral confinement index step) and its first three (guided) modes (where the order is in decreasing modal effective index) is plotted in Figure 2.1. Throughout this work we depict the modes as intensity, which is the electric field magnitude squared. This would most closely match what could be experimentally measured. We observe that the modal field intensity approximates that of a sinusoidal wave within the core, but part of the mode extends slightly out of the core region. The higher order modes generally have more of their field extending beyond the core of the waveguide.

![Figure 2.1: The modal field intensity approximates a sinusoidal wave within the core.](image)

**Figure 2.1**: The number of guided modes vs. the waveguide width measured in units of wavelength

For high power lasers it is desirable to increase the laser volume, hence the waveguide width. For high brightness it is desirable to limit the number of
lasing modes and therefore to determine the relationship between waveguide width and the number of guided modes. Figure 2.2 shows the number of guided modes for slab waveguides of various widths (normalized to the free-space wavelength). We can see that the number of guided modes increases with width, and we can see that there is a core width below which only a single waveguide mode is supported (in Figure 2.2 this is \( \sim 2\lambda_0 \)). This creates a trade-off between the power (device width) and beam brightness (number of modes).

In this work, we start with the simple symmetric high index slab waveguide and introduce low index perturbations into the core region. Figure 2.3 shows the first three modes of the same waveguide as shown in Figure 2.1 but with a lower index perturbation introduced into the center of the waveguide. It is apparent in Figure 2.3 that the modal field profiles of the waveguide change after the introduction of the perturbation. As the modes and their structure are linked to the (engineerable) index structure of the waveguide, index perturbation provides an avenue to engineering the modes and their properties.

2.2.2 Dielectric waveguide modesolving

In order to calculate the modes of an arbitrary waveguide structure, we use a finite difference modesolver, which uses the finite difference method
(FDM) to determine the modes of a dielectric waveguide. The FDM modesolver uses a discrete index structure and wavelength to calculate the discrete field structure of the mode and the corresponding modal effective indices. We derive and implement a one-dimensional FDM modesolver by following Coldren’s two-dimensional derivation [11, appendix 17].

The foundation of the FDM modesolver is the discrete Helmholtz equation. We start with the Helmholtz waveguide equation (Equation 2.2) and rewrite for the one-dimensional case:

\[
\frac{d^2U(x)}{dx^2} + (n(x)^2 - n_{eff}^2)k_0^2U(x) = 0
\]  

We then discretize the equation onto a finite grid where \( x = i \cdot \Delta x \). The second-order derivative is approximated using the discrete central difference \( \left( \frac{d^2}{dx^2}f(x_i) \rightarrow \frac{f(x_i+\Delta x)-2f(x_i)+f(x_i-\Delta x)}{\Delta x^2} \right) \). Equation 2.3 becomes:

\[
\frac{U_{i+1} - 2U_i + U_{i-1}}{\Delta x^2} + (n_i^2 - n_{eff}^2)k_0^2U_i = 0
\]

\[
\frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta x \cdot k_0)^2} + n_i^2 \cdot U_i = n_{eff}^2 U_i
\]

where \( \Delta X = \Delta x \cdot k_0 \). At this point, we can reformulate this discrete scalar equation as a matrix equation:

\[
A \cdot U = n_{eff}^2 \cdot U
\]

where \( U \) is a column vector of discrete field values \( U_i \), and \( A \) is a matrix that takes the form:

\[
A = \begin{bmatrix}
a_1 & b & 0 & 0 & \cdots \\
b & a_2 & b & 0 & \cdots \\
0 & b & a_3 & b & \cdots \\
0 & 0 & b & a_4 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix}
\]

where \( a_i = n_i^2 - \frac{2}{\Delta X^2} \) and \( b = \frac{1}{\Delta X^2} \).
In this matrix formulation, the unknowns are the components of vector $U$ and the scalar $n_{\text{eff}}$. We observe that Equation 2.5 describes an eigenvalue problem, where the eigenvectors of $A$ are $U$ (the modal fields) and the eigenvalues are $n_{\text{eff}}^2$ (the square of the modal effective index).

The boundary conditions built into this formulation of the finite difference method require zero field values at the edges of the problem domain. While field of the modes generally have an infinite extent, the amplitude of the guided modes tends to become small (and negligible) after a few wavelengths into the cladding, thus this boundary condition is reasonable. However, when using this algorithm one must be careful to ensure that there is sufficient thickness of cladding included in the calculation domain so that the modes of interest would have negligible amplitude at the domain boundaries. Furthermore, in solving the eigenvalue problem there will exist many solutions for the modal fields and modal effective indices, not all of which correspond to confined and guided modes. It is necessary to distinguish the guided mode solutions (which will tend to have the highest modal effective indices, provided the waveguide index structure is reasonable) from the extraneous unbounded solutions.

A Julia [12] implementation of the finite-difference method waveguide modesolver is presented in Appendix A.

2.3 Laser cavities and modes

The fundamental requirements for laser operation are amplification of light (optical gain) combined with optical confinement/feedback. Semiconductor lasers contain an active region, usually in the form of a semiconductor quantum well heterostructure, to provide gain, while the index structure of the semiconductor layers forms a dielectric waveguide structure that confines light transverse to the direction of propagation. The semiconductor-air interfaces at the facets of the laser waveguide provide confinement and feedback in the longitudinal direction. In the steady-state lasing condition, the guided light must persist after a round through the cavity in terms of amplitude, phase, and transverse (mode) profile. The amplitude condition requires
that the gain encountered by the mode must perfectly balance any and all losses within the cavity, while the phase condition requires that after a round through the cavity the phase must match the initial condition [11, Section 2.5].

The phase condition requirement implies is that for a given cavity, the possible laser mode solutions (in terms of wavelength and field structure) are discrete, and given the finite bandwidth of the gain medium, the number of viable mode solutions is finite. In a longitudinally invariant cavity (that is, a cavity where the transverse index structure does not change along the longitudinal direction) we can consider cavity modes as combinations of two classes: longitudinal and transverse modes. In this form we find that the transverse modes of a semiconductor waveguide laser are waveguide modes, while the longitudinal modes can be considered Fabry-Pérot modes (whereby the index of refraction used is the modal effective index for a given transverse/waveguide mode). As this work focuses solely on the transverse index structuring and assumes longitudinal invariance in index structure, the longitudinal modes will not be discussed.

2.4 Modal effective index

Under the assumption that the dielectric structure of the laser cavity does not vary in the longitudinal direction, the modal effective index for the laser modes is equivalent to that of the waveguide modes. The modal effective index determines the effective cavity length needed to find the longitudinal wavelengths and their spectral separation:

\[
\delta \nu_{\text{longitudinal}} = \frac{c}{2n_{\text{effective}} L_{\text{cavity}}}
\]  

(2.7)

\[
\delta \lambda_{\text{longitudinal}} = \frac{\lambda_{\text{longitudinal}}^2}{2n_{\text{effective}} L_{\text{cavity}}}
\]  

(2.8)

Increased wavelength separation would be desirable, as it implies fewer possible lasing modes within the bandwidth of the gain medium. We could increase the wavelength spacing by decreasing either the effective index or the cavity length, neither of which is appropriate in this work.
While longitudinal gratings have provided a means for single longitudinal mode operation [1], larger active volume lasers (including broad-area devices that are desirable for high power operation) still have many transverse modes which affect both spectral brightness (via transverse mode spectral splitting) and spatial brightness (via irregular near-fields). While transverse spectral splitting can be derived using a rectangular cavity model, this model is not appropriate for the dielectric waveguides discussed in this work as the transverse modes cannot be adequately approximated as sinusoidal modes in a fixed sized waveguide (the modes tend to have neither sinusoidal shape, nor constant extent approximate to the waveguide core, nor constant effective modal refractive index). As such, we derive the transverse splitting based on the fact that the longitudinal mode-number, \( p = \frac{2L_{\text{cavity}}n_{\text{eff}}}{\lambda} \) (where \( L_{\text{cavity}} \) is the length of the cavity, and \( n_{\text{eff}} \) is the modal effective index), must be conserved across transverse modes (denoted by the transverse mode-number \( m \)) of the \( p^{th} \) longitudinal mode:

\[
\frac{p(m)}{\lambda_m} = \frac{2L_{\text{cavity}}n_{\text{eff}}(m) + 1}{\lambda_{m+1}}
\]

\[
\lambda_{m+1} = \frac{n_{\text{eff}}(m + 1)}{n_{\text{eff}}(m)} \lambda_m
\]

\[
\delta\lambda_{\text{transverse}} = \lambda_{m+1} - \lambda_m
\]

\[
= \frac{n_{\text{eff}}(m + 1)}{n_{\text{eff}}(m)} \lambda_m - \lambda_m
\]

\[
= \frac{n_{\text{eff}}(m + 1) - n_{\text{eff}}(m)}{n_{\text{eff}}(m)} \lambda_m
\]

\[
\therefore \delta\lambda_{\text{transverse}} = \frac{\Delta n_{\text{eff}}(m)}{n_{\text{eff}}(m)} \lambda_m
\]

Equation 2.10 shows that the transverse spectral splitting is proportional to the difference between modal effective indices. We thus could potentially improve the transverse mode discrimination by engineering the modal effective index difference between modes.

As the width of a slab waveguide increases, not only does the number of modes increase, as shown in Section 2.2, but the modes increase in modal effective index and become more closely spaced with regard to the effective
index, as shown in Figure 2.4. Indeed, if we plot the difference between the two highest effective index modes against the waveguide width, as in Figure 2.5, we can see that the difference quickly decreases and approaches the limit of zero as the waveguide width increases. As the lateral mode spacing is proportional to the difference in modal effective index, the wider waveguides will tend to have more lateral modes that are more closely spaced spectrally.

2.5 Modal confinement factor

More significant in our analysis is the modal confinement factor. The modal confinement factor ($\Gamma$) corresponds to the fraction of the mode intensity that
overlaps the active region of the device. Generally,

\[ \Gamma = \frac{\int_{\text{active}} |E|^2 dV}{\int_{\infty} |E|^2 dV} \]  

(2.11)

although since we assume that our devices are invariant in the direction of propagation, the integrals can be reduced to the two-dimensional surface integrals over a transverse slice of our device, or, as we focus on the lateral structure and its modes, a one-dimensional integral over the lateral cross-section.

2.5.1 Modal confinement factor theory

The significance of the confinement factor is its role in determining the threshold gain [11, Section 2.5] and, by extension, modal discrimination and selection. We find that:

\[ g_{th} = \frac{\alpha_m + \alpha_i}{\Gamma} \]  

(2.12)

\[ \Delta g_{th} = g_{th}(a) - g_{th}(b) \]
\[ = \frac{\alpha_m + \alpha_i}{\Gamma_a} - \frac{\alpha_m + \alpha_i}{\Gamma_b} \]
\[ = \frac{(\alpha_m + \alpha_i)(\Gamma_b - \Gamma_a)}{\Gamma_a \Gamma_b} \]

\[ : \Delta g_{th} = -\frac{(\alpha_m + \alpha_i)\Delta \Gamma}{\Gamma_a \Gamma_b} \]  

(2.13)

where \( \alpha_i \) and \( \alpha_m \) refer to the internal and mirror losses, respectively (assumed to be equal for all modes). The simplest laser model dictates that the mode that has the lowest threshold gain will reach lasing first, at which point gain clamping prevents other modes from lasing. However, in reality multiple modes simultaneously lase, especially when there is little difference between the modal threshold gain. We can see in Equation 2.13 that the difference in modal threshold gain is proportional to the difference in the modal confinement factors. As such, it is desirable to design devices where the desired lasing mode has a modal confinement factor that is much greater than that of all the other modes.
As a slab waveguide’s width is increased, a greater fraction of the mode field and power is contained within its core regions, as shown in Figure 2.6. If reduced threshold gain is desired, the higher confinement factor of a wider waveguide would provide a lower threshold gain, as Equation 2.12 shows. However, if improved modal discrimination is desired, the difference in the modal confinement factors becomes important, as evident in Equation 2.13.

While the confinement factor increases for all modes with increasing width (and as such the threshold gain decreases for all modes) as evident in Figure 2.6, the degree of modal discrimination is dependent on the relative rate of
change in the modal confinement factors. Figure 2.7 shows that the difference in the modal confinement factors quickly decreases towards zero as the waveguide width increases. This quickly diminishing difference in confinement factors is one factor behind the deterioration of the modal characteristics and beam quality as the laser waveguides, and thus the laser aperture, are made wider.

2.5.2 Modal confinement factor calculation

In order to calculate the confinement factor for discretized fields, we utilize finite integration, where $\int_{a}^{b} f(x)dx \to \sum_{i=c}^{d} f_i \cdot \Delta x$ for $c, d = [\frac{a}{\Delta x}], [\frac{b}{\Delta x}]$. Applying finite integration to Equation 2.11 we obtain the discrete modal confinement factor:

$$\Gamma = \frac{\sum_{i=c}^{d} |U_i|^2 \cdot \Delta x}{\sum_{i=1}^{N} |U_i|^2 \cdot \Delta x} = \frac{\sum_{i=c}^{d} |U_i|^2}{\sum_{i=1}^{N} |U_i|^2}$$ (2.14)

where we assume constant and uniform $\Delta x$, and where $c \leq i \leq d$ defines the bounds of the active region on the discrete grid. In this work the sum in Equation 2.14 is assumed to be the entire core region. (This assumption is reasonable for lateral modes, but for analyzing epitaxial modes the active region would be limited to the quantum well layers.)

The calculation of $\Gamma$ for a discretized structure and field, as implemented in Julia [12], is included in Appendix C.

2.6 Laser beam quality

The quality of a semiconductor laser diode can be quantified using different approaches.

2.6.1 Laser beam theory

Among the goals of this work is to show a new method for achieving improved beam qualities. There are several measures of beam quality of a laser, ranging from the $M^2$ factor to the beam parameter product (BPP).
These quantities are not generally interchangeable, nor are they necessarily good measures of laser beam performance for any given application [13]. As one of the requirements of a diode pump laser is to place as much of the output beam as possible onto the gain medium area that is being pumped, we use the “power-in-the-bucket” measure that is simple and pragmatic. We define the power-in-the-bucket brightness as the fraction of the power contained within a specified half-angle ($\theta_{HA}$), which can be calculated (for the one-dimensional case) using the following integral:

$$B_{PitB} = \frac{\int_{-\theta_{HA}}^{+\theta_{HA}} |E(\theta)|^2 d\theta}{\int_{-\pi/2}^{+\pi/2} |E(\theta)|^2 d\theta}$$

(2.15)

While the modal fields represent the fields throughout the laser waveguide and at the laser’s facets, the laser beam that is observed and characterized is the field at the output facet of the device after propagating some distance through free-space, which requires calculating a far-field beam profile from the (modal field) near-field profile [11, Appendix 3.4]. While there are various methods of calculating or approximating the far-field for a given near-field, the Fraunhofer method is a particularly simple and efficient method. For the one-dimensional propagation problem, we can derive the far-field $U_2$ given the near-field $U_1$ after propagating a distance $z$ as [14, Chapter 5]:

$$U_2(x) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{kz^2}{2}} \int U_1(\xi) e^{-j\frac{\lambda z}{2\pi} x \xi} d\xi$$

(2.16)

Note that Equation 2.16 is valid for $z \gg \max \frac{k\xi^2}{2}$. As we are concerned with the general far-field behavior (that is, we are not concerned with a specific distance $z$), this work assumes that this condition is always satisfied. Examples of the far-fields obtained from the near-fields of the simple slab waveguide depicted in Figure 2.1 using Equation 2.16 are shown in Figure 2.8.

The result of propagation in free-space is that a larger waveguide mode (such as one made by a wider device or a mode with a more uniform intensity) will tend to yield a smaller laser beam profile in the far-field. As such, the
Gaussian beam profile (or the Gaussian-like beam profile of the fundamental mode of a slab waveguide) is not necessarily optimal, nor is there a single beam profile that is optimal for all value of $\theta_HA$. Indeed, if we compare the brightness of the fundamental modes of two different waveguides, for example as shown in Figure 2.9, we can see that the relative performance of the two waveguide modes is dependent on the choice of $\theta_HA$. Since the simple waveguide will emit less power within angles $\lesssim 1.2$, we see that the engineered lateral index profiles can lead to improved performance.

2.6.2 Far-field brightness calculation

We assume that the laser’s near-field corresponds to the waveguide mode profile found by the modesolver. We then propagate the near-field into a far-field using Fraunhofer propagation to obtain a discrete field profile of the
Figure 2.9: Far-field $B_{PitB}$ by angle, for the fundamental modes of the waveguides shown in Figures 2.1 and 2.3

far-field on a plane. We follow Voelz’s derivation of the two-dimensional Fraunhofer propagation algorithm [14, chapter 5] and formulate the one-dimensional form. Fraunhofer propagation states that the field $U_1$, after propagating a distance $z$, will produce a field that takes the form $U_2$, which is described by the following equation:

$$U_2(x_2) = \frac{e^{jkz}}{j\lambda z} e^{\frac{j}{2\lambda z}x_2^2} \int U_1(x_1) e^{-\frac{j2\pi}{\lambda z}x_1x_2} dx_1$$

(2.17)

The integral constitutes a Fourier transform. This can be written in the discrete form:

$$\overline{U_2} = \frac{1}{j\lambda z} e^{\frac{j}{2\lambda z}\overline{x}_2^2} FFT(\overline{U_1})\Delta x_1$$

(2.18)

where $\overline{U_1}, \overline{U_2}$ are the vectors describing the discrete fields values, $\overline{x}_2$ is the vector of the far-field sample points (zero centered and spaced by $\Delta x_2 = \frac{\lambda z}{L_1}$, where $L_1$ is the length of the near-field being propagated) for the far-field, and $FFT(x)$ is the fast Fourier transform function with argument $x$. As we want the far-field, but not necessarily at a particular distance, we choose $z = \frac{L_1\Delta x_1}{\lambda}$, which is the critical sampling condition (for which $L_1 = L_2, \Delta x_1 = \Delta x_2$).

A Julia [12] implementation of the Fraunhoffer near-field to far-field propagator is included in Appendix B.

In order to calculate the power-in-the-bucket brightness, we need to determine the far-field in terms of the angle from normal, $\theta = arctan(\frac{x_2}{z})$. Similar
to the confinement factor calculations, once we determine the bounding indices, \(a\) and \(b\), of the far-field (such that \(U_2(a \leq i \leq b)\) corresponds to the far-field points for which \(|\theta| \leq |\theta_{HA}|\)), we can calculate

\[
B_{PitB} = \frac{\sum_{i=a}^{b} |U_2(i)|^2 \cdot \Delta x_2}{\sum_{i=1}^{N} |U_2(i)|^2 \cdot \Delta x_2} = \frac{\sum_{i=a}^{b} |U_2(i)|^2}{\sum_{i=1}^{N} |U_2(i)|^2}
\]

assuming a constant and uniform sampling spacing \(\Delta x_2\). The field \(U_1\) as returned from the FDM modesolver may not yield sufficient accuracy in the far-field after propagation for analysis, especially when the beam is compact or a small \(\theta_{HA}\) is chosen. To improve the far-field resolution, one can “zero-pad” (add additional zero-valued points to either end) the near-field \(U_1\) in order to increase the number of sampling points.

The calculation of \(B_{PitB}\) for a discretized near-field, as implemented in Julia [12], is included in Appendix C.

### 2.7 Waveguide structure from field structure

In this section we consider the inverse problem of finding a particular index profile that will support a preferred near-field mode profile.

#### 2.7.1 Waveguides from fields theory

The Helmholtz equation for a waveguide (Equation 2.2) is usually solved for a given index structure for the modal fields that are supported by the waveguide. However, it can also be solved for a given modal field to obtain a set of index structures that support that field that vary continuously in terms of \(n_{eff}\):

\[
\nabla^2 U + (n^2 - n_{eff}^2)k_0^2 U = 0
\]

\[
\frac{\nabla^2 U}{k_0^2 U} + (n^2 - n_{eff}^2) = 0
\]

\[
n_{eff}^2 - \frac{\nabla^2 U}{k_0^2 U} = n^2
\]

\[
\therefore n = \sqrt{n_{eff}^2 - \frac{\nabla^2 U}{k_0^2 U}}
\]

(2.20)
While Equation 2.21 is general, for there to be a realizable index structure \( n \) (that is, \( n \) should be finite and ideally bounded between finite \( n_{\text{min}}, n_{\text{max}} \)), we must impose constraints. In order to obtain a finite valued index structure, \( \nabla^2 U \) must be finite and bounded, and provided that \( \nabla^2 U \) is bounded, then \( n_{\text{eff}} \) can be chosen so as to make \( n_{\text{min}} \leq n \). Furthermore, if it is permitted to transversely scale the desired modal field \( U \) by a factor of \( w \) such that \( U_{\text{scaled}}(r) = U(w * r) \), then it is possible to obtain \( n \) such that \( n_{\text{min}} \leq n \leq n_{\text{max}} \). Indeed, we solve for \( n_{\text{eff}} \) and \( w \) for a bound \( n \):

\[
\begin{align*}
n_{\text{min}} &= \sqrt{n_{\text{eff}}^2 - w^2 \frac{\nabla^2 U}{k_0 U}} \\
n_{\text{max}} &= \sqrt{n_{\text{eff}}^2 - \min w^2 \frac{\nabla^2 U}{k_0 U}}
\end{align*}
\]  

(2.22)

where if we define \( X_{\text{max}} = \max \frac{\nabla^2 U}{k_0 U} \) and \( X_{\text{min}} = \min \frac{\nabla^2 U}{k_0 U} \), then we have the system

\[
\begin{align*}
    n_{\text{min}}^2 &= n_{\text{eff}}^2 - w^2 X_{\text{max}} \\
    n_{\text{max}}^2 &= n_{\text{eff}}^2 - w^2 X_{\text{min}}
\end{align*}
\]

(2.23)

which solves for \( n_{\text{eff}} \) and the scaling factor \( w \):

\[
\begin{align*}
n_{\text{eff}} &= \sqrt{n_{\text{max}}^2 X_{\text{max}} - n_{\text{min}}^2 X_{\text{min}}} \\
w &= \sqrt{\frac{n_{\text{max}}^2 - n_{\text{min}}^2}{X_{\text{max}} - X_{\text{min}}}}
\end{align*}
\]

(2.24, 2.25)

Many functions do not satisfy the constraint that \( \frac{\nabla^2 U}{U} \) be strictly finite and bound. Furthermore, the index profile as calculated by Equation 2.21 assumes an infinite extent and generally takes the form of continuous index variations (index gradients) that are not generally feasible in the fabrication of semiconductor waveguide lasers. Thus, we consider simplified and finite waveguide structures that support approximations of desired modal forms.

For physically implementable waveguide structures, it is desirable to obtain a finite index structure that resembles traditional dielectric waveguide structures (with core and clad regions) with some perturbation. Consider the
infinite structure \( n \) that supports a mode of field \( U \) that we have designed to be bounded by \( n_{\text{core,min}} \) and \( n_{\text{core,max}} \) (chosen to be the bounds of index values that could be implemented within the core region, and calculated using Equations 2.24 and 2.25). We then create a finite-width approximate index structure \( n_f \) of width \( w \),

\[
n_f = \begin{cases} 
  n & |x| \leq \frac{w}{2} \\
  n_{\text{clad}} & |x| > \frac{w}{2}
\end{cases}
\]  

(2.26)

for some cladding index value \( n_{\text{clad}} \). There is the question of when \( n_f \) is an acceptable approximation of \( n \). It is expected that \( w \) such that \( U(|x| > \frac{w}{2}) \approx 0 \) will likely yield acceptable results.

Finally, index gradients are often not easily achievable in laser fabrication. What would be desirable is a discrete index structure where the index values are constrained to 2 (or more) predetermined values (consider the set of possible index values \( n_s = [n_1, n_2, \ldots] \)). In its simplest form, we can consider calculating the analytical waveguide structure \( n \) for some bounds \( n_{\text{min}}, n_{\text{max}} \), and then forming the discrete structure \( n_d \) by rounding at each point to the nearest value within \( n_s \) such that:

\[
n_d(x) \in n_s \forall x
\]  

(2.27)

However, the extent to which this produces an effective approximation of the desired mode is largely dependent on \( U \), and it is not certain to what extent the product of this rounding process is optimal for the constraint in Equation 2.27.

### 2.7.2 Calculation of waveguide structures from field structures

As with waveguide modesolving in Subsection 2.2.2, we discretize the equation and use finite difference operators. Equation 2.21 becomes the discrete,
one-dimensional function:

\[ n(x) = \sqrt{n_{\text{eff}}^2 - \nabla^2 U(x) / k_0^2 U(x)} \]

\[ n(x) = \sqrt{n_{\text{eff}}^2 - \frac{\partial^2 U(x)}{\partial x^2} \cdot \frac{1}{k_0^2 U(x)}} \]  

(2.28)

\[ n_i = \sqrt{n_{\text{eff}}^2 - \frac{U_{i+1} - 2U_i + U_{i-1}}{U_i \Delta X^2}} \]

where, as before, \( \Delta X = \Delta x \cdot k_0 \). We can then write the equation for \( n_i \) as a matrix equation,

\[ n = \sqrt{n_{\text{eff}}^2 - (D \cdot U) \circ U} \]  

(2.29)

where \( n \) is a vector corresponding to the index structure, \( n_{\text{eff}} \) is the desired field’s modal effective index (the value may be defined by choosing the bounds of the index \( n \), as in Equation 2.24, or it may be a quantity subject to optimization once approximations to the form of \( n \) are made), \( U \) is a vector for the field values, and \( D \) is a matrix operator defined as

\[
D = \begin{bmatrix}
a & b & 0 & 0 & \ldots \\
b & a & b & 0 & \ldots \\
0 & b & a & b & \ldots \\
0 & 0 & b & a & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots 
\end{bmatrix}
\]  

(2.30)

where \( a = -\frac{2}{\Delta X^2} \) and \( b = \frac{1}{\Delta X^2} \). We note that \( \circ \) represents an element-wise division of two vectors.

While \( n \) as calculated here should yield an eigenmode \( U \) when analyzed using the modesolver described in Subsection 2.2.2, it does not necessarily constitute a physically realizable structure. Further manipulations may be necessary to obtain a reasonable index structure from the calculated \( n \). This could include neglecting the outer extents of \( n \) according to Equation 2.26 (or as the discrete vector \( n \) only covers a finite extent, this would be equivalent to appending sufficient lengths of \( [n_{\text{clad}}, n_{\text{clad}}, n_{\text{clad}}, \ldots] \) to the ends of \( n \)) in order to obtain a finite structure and clipping/rounding the values of \( n \) to match a range/set of realizable index values.
Calculation of a (finite) field supporting waveguide, with the option to binarize the core index structure, as implemented in Julia [12], is included in Appendix D.
CHAPTER 3

RESULTS

3.1 Methodology

In this chapter we aim to show the principle that index perturbations in a dielectric waveguide can be used to engineer the modes of the waveguide along the direction of propagation. This is primarily motivated by the desire to improve the lateral mode qualities of a broad-area edge-emitting semiconductor laser, as discussed in Chapter 1. The analysis in this work is wavelength invariant (that is, the structures are scaled to the wavelength) and we define our devices using a cladding index of 3 and a core index of 3.1, for a lateral confinement index step of $\Delta n = 0.1$. For the purposes of the modal confinement factor, the active region is assumed to overlap the entire core region of the waveguide. Index perturbations of $\Delta n = -0.01$ (the index in the perturbation region will be 3.09) will be added in the core regions of the devices in the form of lower index bands in the core, as a perturbation of such magnitude should be reasonably implementable using standard surface etches or regrowth-buried structuring. One could also consider introducing more complex index perturbations with multiple different index values, or perhaps even gradients of indices as perturbation. However in this work we only consider two index values in the core, as such a system would be simple to implement.

The results in this work are the result of calculations using the computations methods described in Chapter 2, implemented in the Julia programming language [12]. We have not presented a formal/analytical method for the design of these structured waveguides, so the structures presented here are the result of optimization algorithms [15] applied to the problem of determining the positions and widths of each of the index perturbation regions. As such,
the designs analyzed here are possibly (and likely) local optima, and even better performance may be possible.

3.2 Modal effective index and transverse mode spectral splitting

As discussed in Section 2.4, the modal effective indices are relevant in determining the transverse mode spectral splitting. Transverse mode spacing is not considered nearly as often as longitudinal mode spacing in edge-emitting diode lasers. Part of the reason is that the transverse mode spacing tends to be significantly smaller than the longitudinal mode spacing, and it has not been considered a critical parameter for improving the modal characteristics. However, as we will show, waveguide mode engineering provides a path to engineering the modal effective indices and thus the transverse mode spacing.

Consider the waveguide structure shown in Figure 3.1. The waveguide width is $30 \lambda_0$, as is the simple slab waveguide shown in Figure 2.1, but we have introduced low index perturbations into the core region that were intended to increase the difference in modal effective indices between the first two modes. We calculate and compare the modal effective indices for the simple slab waveguide and the optimized waveguides in Figure 3.2. We find
that the difference in the modal effective indices for the first two modes of the engineered waveguide is $4.05 \times 10^{-4}$, as compared to the simple waveguide’s $1.28 \times 10^{-4}$, which is greater than a 3x improvement. If we compare this result to the modal effective index differences shown in Figure 2.5, we find that this structure has a modal effective index difference in between those of a $16\lambda_0$ and a $17\lambda_0$ width simple slab waveguide. This suggests that such waveguide engineering could allow for significant increases in waveguide width (in this case nearly doubling the width) while retaining the same lateral mode spacing between the lowest order modes (as determined in Equation 2.10).

### 3.3 Modal discrimination via modal confinement factor

Consider the engineered waveguide structure shown in Figure 3.3. Figure 3.4 compares the modal confinement factors of this optimized waveguide to those of a simple slab waveguide of equal size. The difference in the modal confinement factor for the first two modes (which have the highest confinement factors) is approximately $3.84 \times 10^{-3}$. Relative to an unstructured waveguide of comparable size, this represents better than 7-fold improvement (up from $\Delta \Gamma = 5.4 \times 10^{-4}$). If we compare this result to the confinement factor differences shown in Figure 2.7, we find that this waveguide has a modal confinement factor difference in between those of a $15\lambda_0$ and a $16\lambda_0$ width simple slab waveguide. This suggests that such waveguide engineering could allow for significant increases in waveguide width (in this case nearly
In the simple symmetric slab waveguide the fundamental (Gaussian-like) mode is preferred, as it has the highest confinement factor and the lowest divergence. This is a property of the simple symmetric slab waveguide that is size-independent. However, waveguide engineering may provide a means to favor higher order modes. We thus seek an index design for which the mode with the highest modal confinement factor is not the fundamental, but rather a higher order mode.

3.4 Modal selection via modal confinement factor

In the simple symmetric slab waveguide the fundamental (Gaussian-like) mode is preferred, as it has the highest confinement factor and the lowest divergence. This is a property of the simple symmetric slab waveguide that is size-independent. However, waveguide engineering may provide a means to favor higher order modes. We thus seek an index design for which the mode with the highest modal confinement factor is not the fundamental, but rather a higher order mode.
Consider the waveguides shown in Figures 3.5, 3.6, and 3.7. As before, three waveguides are $30\lambda_0$ in width, but they have been designed so that the second, third, and fourth modes have the highest modal confinement factor. Figure 3.8 summarizes the modal confinement factors of these waveguides. These waveguides have higher order modes with $\Delta \Gamma = 20.9 \times 10^{-4}$, $2.31 \times 10^{-4}$, $7.77 \times 10^{-4}$ above the second highest confinement factor mode, respectively. These waveguides not only show modal selection, but in the first and third waveguide we have improved modal discrimination, as previously shown in Section 3.3 ([$\Delta \Gamma$ is increased by a factor of 3.90x and 1.45x respectively, relative to an unstructured waveguide of the same size].)
3.5 Far-field power-in-the-bucket brightness

The far-field power-in-the-bucket brightness as defined in Section 2.6 generally improves with increased laser aperture, as a wider near-field has a smaller divergence angle. Figure 3.9 shows that general trend for the fundamental mode of a symmetric slab waveguide (for an arbitrarily chosen $\theta_{HA} = 1.5$ deg). While this trend implies that greater brightness can be achieved by simply making larger apertures, such devices will tend to have degraded multi-mode properties (that is, greater near-field contributions from higher order modes), which in turn eliminate the brightness advantage of a wider near-field. Indeed, Figure 3.10 shows that the higher order modes have much lower brightnesses, so deteriorated modal characteristics will negatively impact the far-field power-in-the-bucket brightness.
Beyond improving the modal discrimination to mitigate the brightness loss due to higher order modes, one can also engineer the waveguide structure to improve the far-fields of the modes themselves. Consider the waveguide of $30\lambda_0$ width shown in Figure 3.11. The structure has been designed to improve the power-in-the-bucket brightness of the fundamental mode within an acceptance angle of $3\deg$ ($\theta_{HA} = 1.5\deg$). While an unstructured waveguide of comparable size has $B_{PitB} = 0.918$, the engineered structure has $B_{PitB} = 0.939$, which is a 2.3% improvement. This is not particularly impressive.

For smaller devices, there is more room for improvement. For a $20\lambda_0$ wide laser, the unstructured waveguide has $B_{PitB} = 0.757$. An optimized struc-
Figure 3.11: The first three modes of far-field brightness ($\theta_{HA} = 1.5\, \text{deg}$) optimized waveguide.

Figure 3.12: The first three modes of far-field brightness ($\theta_{HA} = 1.5\, \text{deg}$) optimized waveguide of width $20\lambda_0$.

ture of comparable size, shown in Figure 3.12, obtains $B_{PitB} = 0.807$, an improvement of 6.6%. Furthermore, the potential performance improvement is dependent on the specified $\theta_{HA}$. Figure 3.13 shows the brightness as a function of $\theta_{HA}$ for the unstructured and brightness optimized waveguides of width $30\lambda_0$ (shown in Figure 3.11). For this particular structure, it performs better than the simple slab waveguide of comparable size for the approximate range $0.2\, \text{deg} \leq \theta_{HA} \leq 1.6\, \text{deg}$. Different structured waveguides have different ranges of $\theta_{HA}$ over which they are superior to an unstructured waveguide, with some designs having multiple ranges.
Figure 3.13: The power-in-the-bucket brightness as a function of $\theta_{HA}$ for a pair of waveguides of width $30\lambda_0$

3.6 Mode profile engineering

Waveguide structure may also provide a means of engineering novel “designer” modes, that is, it may allow engineering a waveguide to obtain a desired mode profile. For example, consider the application of creating non-diffractive Bessel beams. As a Bessel function is unbound and infinite in energy, the (finite-energy) beams take the form of a Bessel-Gauss function [16],

$$U(r) = J_0(\beta r) \exp(-\left(\frac{r}{w_0}\right)^2)$$  \hspace{1cm} (3.1)

where $w_0$ and $\beta$ are parameters that determine the profile and propagation characteristics of a Bessel-Gauss beam (please refer to [16] for a full explanation of these parameters). While it may or may not be possible to create a physical dielectric waveguide that supports a Bessel-Gauss function (see the limitations in Section 2.7.1), one could try to make a waveguide that supports an approximation of a Bessel-Gauss function using waveguide engineering.

Consider the waveguide shown in Figure 3.14, obtained using an optimization algorithm similar to that used for the previously shown examples. A higher order mode of the waveguide is found to roughly resemble a Bessel-Gauss function.

Now we calculate an index structure according to Subsection 2.7.2 ($n_{eff} = 3.0095$, arbitrarily chosen out of a range of $n_{eff}$ values would produce similar
results) and apply cladding to create a waveguide of finite width. The resulting mode is plotted in Figure 3.15. The index structure shown in Figure 3.15 is not realizable, as it has numerous discontinuities, but we see that the mode matches the desired field nearly perfectly. In order to create a better approximation of a Bessel-Gauss mode, while still approximating a realizable structure, we proceed with the same calculations, but we binarize \( n(x) \) by rounding each point to the nearer value of either 3.009 or 3.01 before truncating and applying a cladding. In this case we have chosen \( n_{\text{eff}} = 3.0085 \), which appears to be near optimal. The resulting waveguide and mode are shown in Figure 3.16. The waveguide in Figure 3.16 appears to be a slightly better approximation of a Bessel-Gauss mode than that in Figure 3.14, and only involves single variable optimization (\( n_{\text{eff}} \)), as opposed to the multivariate optimization for Figure 3.14’s structure (for which the width of each
Figure 3.16: The 15th mode of a 100\(\lambda_0\) wide engineered waveguide, compared to a Bessel-Gauss function.

It is conceivable that if one desires a particular quality from a laser beam or mode that can be reduced to a desired mode profile (such as far-field brightness), the waveguide engineering methods discussed herein may provide a path to create a waveguide that would support said field, or an approximation thereof.
CHAPTER 4

CONCLUSION AND FUTURE WORK

4.1 Conclusion

In this work we have presented a computational analysis of one-dimensional dielectric waveguides and their properties, as relevant to semiconductor waveguide laser diodes. We have discussed the theory of dielectric waveguides and modes and explained the computational methods used to simulate dielectric waveguides and analyze their properties in Chapter 2. We have shown the effects of waveguide structure engineering on the waveguide modal and mode properties using examples presented in Chapter 3.

Section 3.2 shows that waveguide design can engineer the modal effective indices, which may provide a means to engineer the transverse mode spacing of a laser. Section 3.4 shows that waveguide structuring can also engineer the modal confinement factors to increase the modal discrimination, providing a path towards fewer transverse modes and improved modal performance for larger devices. Meanwhile, Section 3.4 takes confinement factor engineering further to show that structuring can not only improve modal discrimination, but also impart selection for a particular higher-order mode. Moving beyond engineering the modal characteristics of a waveguide, Section 3.5 shows that waveguide index structuring can be used to engineer the properties of the waveguide modes themselves, as shown by the improved far-field “power-in-the-bucket” brightness. Finally, we have also considered the inverse problem, and in Section 3.6 we see some promise in waveguide engineering as a path towards creating lasers that support novel and engineered mode profiles.
4.2 Future work

The natural path forward from this work is experimental verification of the core concepts put forth in this work. Preliminary two-dimensional mode calculations (analogous to the one-dimensional analysis in this work) support the idea that surface etched or regrowth-buried index structuring techniques could be used to implement waveguide mode engineering in edge-emitting diode lasers. It will likely also be useful, or perhaps even necessary, to expand this analysis to take into consideration electrical and/or thermal effects on the index structure and modal selection, as the simple waveguide structure and confinement factor based mode selection analysis used here may prove to be insufficient to accurately model the behavior of physical implementations of engineered waveguide laser devices.

Furthermore, the analysis in this work only studied real index waveguides and their modes. Recently both gain and loss have been shown as viable design process parameters for mode control in lasers [17, 18]. Complex index value analysis, taking into account the gain and loss structure in a device, may better model physical devices and be an avenue for engineering greater improvements in modal characteristics, and perhaps engineering the modal phases. Advanced analysis taking into consideration the thermal and gain characteristics of devices at different injection levels may show some possibility of dynamic mode engineering, whereby the lasing mode can be evolved, or switched, by varying the operating conditions.

The design process for the structures discussed in this work is almost entirely numeric and driven by numerical optimization algorithms. As such it can be computationally intensive while having no guarantees of producing optimal results. An analytical or semi-analytical process for engineering a finite and implementable waveguide to support a particular field, based on work in Section 2.7, may provide a foundation for solving directly for a waveguide structure that would maximize performance on certain metrics (such as those discussed in Sections 3.5 and 3.6), or else provide a basis for a simplified design optimization process.
APPENDIX A

1D FINITE DIFFERENCE METHOD

The calculations and simulations in this work were performed using the Julia programming language (https://julialang.org/). The code included in these appendices was written in Julia v1.0 and functioned at the time of writing. While later versions of Julia should be compatible with this code, future language changes or package updates may require some changes for correct function.

Figure A.1: Plot produced by the Julia code of Appendix A

```julia
# waveguide_modes.jl
using LinearAlgebra, Arpack

# waveguide_modes(index::Array[N, 1], lambda0, dx; n_modes=10) where N<:Number
Find the eigenmodes and their modal effective indices using the 1D Finite Difference Method.

function waveguide_modes(index::Array[N, 1], lambda0, dx; n_modes=10) where N<:Number
    # The spacing and wavelength should be greater than zero
    @assert dx>0 && lambda0>0

    # See Chapter 2 for a derivation/explanation of these values
    k0=2*pi/lambda0
    dX=k0*dx
    a=index.^2 - 2/dX^2
    b=1/dX^2

    # The matrix we create is sparse (has many zero entries), a property that allows the use of optimized linear algebra routines. We could
```

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use the sparse() function in the SparseArrays to create the matrix A as a sparse matrix. However, it is not only sparse but symmetric tridiagonal. This allows additional performance improvements by using the SymTridiagonal() function from the LinearAlgebra package.

```julia
A = SymTridiagonal(a, fill(b, length(a) - 1))
```

We can find the eigenvalues/eigenvectors of matrix A using two different methods. We can use the basic eigen() function, which is a direct solver, or we can use the eigs() function from the Arpack packages, which is an iterative solver. The direct solver finds all of the eigenvalues and eigenvectors, while the iterative solver only attempt to find a part of them. Furthermore, while eigs() works with any form of a matrix, eigen() requires either a full (non-sparse) matrix, or a well shaped one (such as SymTridiagonal). The result is that eigs() may be faster for cases where A is a large matrix or when only a few solutions (modes) are desired. This is especially relevant to 2D FEM calculations, where the matrix is not symmetric tridiagonal, and the full matrix form may not fit in RAM.

We have given eigs() a couple important arguments: 'which=LR' dictates that the solutions with the largest real eigenvalue should be found 'nev=n_modes' tells it to only find the first 'n_modes' solutions 'maxiter=100000' sets the maximum iterations for the solver to 100000. Having too small a number may cause the following error:

```
ERROR: ARPACKException: unspecified ARPACK error: 1
```

```julia
# n_effs, fields = eigen(A)
# n_effs, fields = eigs(A, which=LR, nev=n_modes, maxiter=100000)

# Select only the modes with effective indices greater than that of
# the minimal index
mode_select = minimum(real.(index.^2)) < real.(n_effs)

# We order the solutions in terms of decreasing (real) index
mode_order = sortperm(real.(n_effs), rev=true)

n_effs, fields = n_effs[mode_order], fields[:, mode_order]
```

We have given plotmode() a couple important arguments:

```
# which=LR' dictates that the solutions with the largest real
# eigenvalue should be found
'nev=n_modes' tells it to only find the first 'n_modes' solutions
'maxiter=100000' sets the maximum iterations for the solver to
```

```julia
# We create a dielectric waveguide structure of core index 3.01 and
# cladding index 3.0. The core is 20 wavelengths wide, and the spacing
# between the discrete points is a hundredth of a (free-space) wavelength.

lambda0 = 1.0
dx = 0.01
xrng = -25:dx:25
n = [abs(x) < 15 ? 3.01 : 3.0 for x in xrng]

# We find the first three modes using the finite difference method.

n_effs, fields = waveguide_modes(n, lambda0, dx, n_modes=3)

# We create a function to automate the plotting of a modal field on the
# waveguide structure. We plot the intensity (magnitude squared) of the
# field, normalized to the base and height of the index structure.

function plotmode(n, field)
    intensity = abs2.(field)
```

# fdm_1d.jl
include("waveguide_modes.jl")
using Plots, LaTeXStrings
pyplot()
\text{intensity} = \text{minimum}(n) + \text{intensity} / \text{maximum}(\text{intensity}) \ast \\
(\text{maximum}(n) - \text{minimum}(n))

\text{plot}(\text{xrng}, \\
[n, \text{intensity}], \\
\text{xlabel}=L"x \ [\lambda_0"]", \\
\text{ylabel}="\text{Real Refractive Index}";
\text{legend}=\text{false})
\text{end}

\# We plot the first three modes
\text{plt} = [\text{plotmode}(n, \text{fields}[:,: i]) \text{ for } i=1:3]
\text{plot}(\text{plt} ..., \text{layout}=(1, 3), \text{size}=(1200, 400))
\text{if @isdefined saveplots; savefig("appendix_a.png") end}
APPENDIX B

1D NEAR-FIELD TO FAR-FIELD PROPAGATION

The calculations and simulations in this work were performed using the Julia programming language (https://julialang.org/). The code included in these appendices was written in Julia v1.0 and functioned at the time of writing. While later versions of Julia should be compatible with this code, future language changes or package updates may require some changes for correct function.

This code is based on the Matlab propFF function for two-dimensional Fraunhofer propagation, as written in Computational Fourier Optics [14, chapter 5].

![Figure B.1: Plot produced by the Julia code of Appendix B](image)

```julia
# fraunhofer_propagator.jl
using FFTW

===

    fraunhofer_propagator(u1::Array(N, 1), dx1, lambda0, z) where
    N<:Number

Propagate a field using the Fraunhofer far-field approximation.

===

    function fraunhofer_propagator(u1::Array(N, 1), dx1, lambda0, z) where
    N<:Number
        # The inputs should be greater than zero
        @assert dx1>0 && lambda0>0 && z>0

    L1=dx1*length(u1)
    k0=2*pi/lambda0
```

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L2 = \lambda_0 \times \frac{z}{dx_1}

dx_2 = \lambda_0 \times \frac{z}{L_1}

x_2 = \text{LinRange}(-L_2/2, L_2/2, \text{length}(u_1))

c = 1/(im \times \lambda_0 + \pi) \times \exp(\frac{ik_0/(2z) \times x_2^2}{2})

u_2 = e^{-i \text{shift}(\text{fft}(\text{shift}(u_1)) \times dx_1)}

end

# farfield_1d.jl
include("waveguide_modes.jl")
include("fraunhofer_propagator.jl")
using Plots, LaTeXStrings
pyplot()

#=
Calculate the fields of a symmetric slab waveguide using
waveguide_modes() from Appendix A.

# lambda0 = 1.0
# dx = 0.01
# xrng = -25:dx:25
# n = [abs(x) < 15 ? 3.01 : 3.0 for x = xrng]
# n_effs, fields = waveguide_modes(n, lambda0, dx, n_modes=3)

# Zero-pad the near-fields to 7x the points
nfs = [zeros(size(fields) .* (3, 1)); fields; zeros(size(fields) .* (3, 1))]

# Propagate the near-fields
L1 = dx \times size(nfs)[1]
ffs = [fraunhofer_propagator(nfs[:, i], dx, lambda0, L1 + dx/lambda0) for i = 1:3]

# Crop the far-fields to the same size as the near-fields
xrng = ffs[1][2]
ffs = hcat((a => a[1])(ffs) ...)

# We create a function to automate the plotting of the far-field intensity (magnitude squared of the field).

function plotmode(u)
    plot(xrng,
        abs2.(u) / maximum(abs2.(u)),
        xlabel=L"x \ [\lambda_0]",
        ylabel="Far-field Intensity [a.u.]",
        legend=false,
        xlims=(-0.5, 0.5))
end

# We plot the far-fields for the first three modes
plt = [plotmode(ffs[:, i]) for i = 1:3]
plot(plt ... , layout=(1, 3), size=(1200, 400))
if @isdefined saveplots; savefig("appendix_b.png") end
The calculations and simulations in this work were performed using the Julia programming language (https://julialang.org/). The code included in these appendices was written in Julia v1.0 and functioned at the time of writing. While later versions of Julia should be compatible with this code, future language changes or package updates may require some changes for correct function.

```julia
# gamma.jl
using LinearAlgebra

function gamma(mode, active_region)
    # The mask defining the active region must be the size of mode
    @assert size(mode) == size(active_region)

    We make use of the fact that |x|^2 = x * conjugate(x) for complex x.
    The complex dot product x . x is a sum of the element-wise products
    of x and conjugate(x), which is effectively the sum of the
```

Figure C.1: Plot produced by the Julia code of Appendix C
magnitudes squared.

Given the mask vector `active_region` defines whether each point of 'mode' is in the active region (1) or not (0), an element-wise multiplication of the mode and active_region will produce the mode within the active region.

```julia
# Bpitb.jl
using LinearAlgebra

# Bpitb(x2, ff, z, thetaHA)
Find the confinement factor of a mode given the mask vector defining which parts of the mode are in the active region.

function Bpitb(x2, ff, z, thetaHA)
    # There should be an x value for every far field point
    @assert size(x2) == size(ff)
    # We require a positive half-angle
    @assert 0 < thetaHA

    # We create a mask vector that defines whether a point in the far-field 'ff' is within the half-angle 'thetaHA'
    theta_mask = abs.(atan.(x2, z)) .<= thetaHA

    # The actual discrete integral now look very similar to that for calculating the confinement factor in gamma()
    #
    real.(dot(theta_mask*ff, theta_mask*ff)/dot(ff, ff))
end

# misc.jl
include("waveguide_modes.jl")
include("fraunhofer_propagator.jl")
include("gamma.jl")
include("Bpitb.jl")

# Calculate the fields of a symmetric slab waveguide using waveguide_modes() from Appendix A.
lambda0 = 1.0
dx = 0.01
xrng = -20:dx:20
n = [abs(x)<10 ? 3.01 : 3.0 for x=xrng]
dxs = length(xrng)
ffs = [fraunhofer_propagator(nfs[i], dx, lambda0, z) for i=1:3]
xrng = ffs[1][2]
ffs = hcat((a->a[1])(ffs)...)

# We calculate the confinement factor of each mode

# And now we calculate the far-field power-in-the-bucket of each mode
brightnesses = [Bpitb(xrng, ffs[i], z, 1.5*pi/180) for i=1:3]
plot(scatter(gammas, xlab="Mode"),
ylabel="Confinelement Factor",
legend=false,
xticks=1:3),
scatter(brightnesses,
xlabel="Mode",
ylabel="PitB Brightness",
legend=false,
xticks=1:3),
size=(800, 400))
APPENDIX D

1D FIELD TO WAVEGUIDE CALCULATION

The calculations and simulations in this work were performed using the Julia programming language (https://julialang.org/). The code included in these appendices was written in Julia v1.0 and functioned at the time of writing. While later versions of Julia should be compatible with this code, future language changes or package updates may require some changes for correct function.

Figure D.1: Plot produced by the Julia code of Appendix D

```julia
# mode2waveguide.jl
using LinearAlgebra
include("waveguide_modes.jl")

function mode2waveguide(U::Array{N, 1}, dx0, lambda0, nmin, nmax, nclad; 
    binarize=false, normalize=true, neff=false, 
    n_modes=10) where N<:Number
    @assert nclad<nmin<nmax && !isequal(lambda0)
    Create a waveguide structure that should support a mode approximating a
given field profile U. If normalize=true, then it will transversely scale U and
choose neff such that the waveguide core will be bound by (nmin, nmax),
otherwise neff (or nmin if neff is not set) determines the target modal
effective index. If binarize=true then the core will not be allowed to
take continuous values but will be made to use the discrete values nmin
and nmax. nclad sets the index of the cladding. This function will return
the waveguide index n and sampling distance dx (which may or may not be
the given dx0), and will print out which mode of the waveguide most
closely matches U (and how similar it is).

function mode2waveguide(U::Array{N, 1}, dx0, lambda0, nmin, nmax, nclad; 
    binarize=false, normalize=true, neff=false, 
    n_modes=10) where N<:Number
    @assert nclad<nmin<nmax && !isequal(lambda0)
```

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# If no effective index specified, then assume the minimal index value if neff==false neff=nmin end

# Calculate the field supporting index structure, either transversely # scaling U to have the index fit some bounds, or else for the mode # to have a specified effective index
n, dx=normalize ?
U2n_bound(U, dx0, lambda0, nmin, nmax) :
U2n(U, dx0, lambda0, neff)

# If desired, binarize the index structure by rounding all values to # the nearest of (nmin, nmax)
 n=binarize ? a->snapto(a, [nmin, nmax]):(n) : n

# Remove the end points and apply the cladding to the index and field # to obtain a finite waveguide structure
n, U=lambda0, dx, n modes=n modes
 modes=waveguide_modes(n, lambda0, dx, n modes=n modes)

# Tell the bounds of the calculated index structure
 println("Waveguide bounds : ", extrema(n ) )

# Determine which mode is the closest to U and how similar it is
 mfac, mind=findmax ( abs . ( [ dot (U, fields [ : , i] ) for i=1:size (fields )[2] ] ) )
 println("Similarity factor of ", mfac, " for mode ", mind )

 n, dx end

---

U2n(U::Array[N, 1], dx, lambda0, neff) where N<:Number

Given the modal field U and modal effective index neff, calculate the mode supporting index structure.

---

function U2n(U::Array[N, 1], dx, lambda0, neff) where N<:Number
 @assert !any(iszero([[U; dx; lambda0]]))
 k0=2*pi/lambda0
 # Create the 1D finite difference Laplacian matrix operator
 D2=SymTridiagonal( fill(-2/dx^2, length(U) ) ,
 fill(1/dx^2, length(U)-1))
 sqrt.(neff^2 .- (D2*U) ./(k0^2 .* U) ) , dx
dx end

---

U2n_bound(U::Array[N, 1], dx, lambda0, nmin, nmax) where N<:Number

Given the modal field U, calculate the mode supporting index structure whose index values are bound by (nmin, nmax).

---

function U2n_bound(U::Array[N, 1], dx, lambda0, nmin, nmax) where N<:Number
 @assert !any(iszero([[U; dx; lambda0]])) & nmax>nmin>1
k0=2*pi/lambda0
 # Create the 1D finite difference Laplacian matrix operator
 D2=SymTridiagonal( fill(-2/dx^2, length(U) ) ,
 fill(1/dx^2, length(U)-1))
 # Find the bounds of \( \nabla^2 U/(k_0^2 U) \)
 D2U=diag.(D2*U) ./((k0^2 .- U) )
 Xmin, Xmax=extrema(D2U[2:end-1])
 # Using those bounds we can determine neff and the transverse mode # scaling factor w such that the index will have the desired bounds neff=sqrt((nmax^2*Xmax-nmin^2*Xmin)/(Xmax-Xmin))
 w=sqrt((nmax^2-nmin^2)/(Xmax-Xmin))
 # Calculate the index structure and the new scaled sampling distance

---
\[ n^2 = \sqrt{\text{neff}^2 - \text{w}^2} \times D^2 U \]

\[ \text{dx2} = \text{dx} \times \text{w} \]

\[ n^2, \text{dx2} \]

```python
# u2a_id.jl
include("mode2waveguide.jl")
using Plots, LaTeXStrings
pyplot() 

lambda0 = 1.0
nmin, nmax, nclad, neff = 3.009, 3.01, 3.0, 3.02
dx = 0.1
x = -15:dx:15
d = 9

# Calculate the field that we want a waveguide to support
Uf(x, w) = exp(-((x * w / d)^2) * (1 + cos(x * w)^2))
U = Uf(x, 1)
U = U / sqrt(sum(abs2(U)))

# Find the waveguide structure n that supports this field
n, dx2 = mode2waveguide(U, dx, lambda0, nmin, nmax, nclad;
    binarize=false, normalize=false,
    n_modes=5, neff=neff)

# As the n is larger than the U we gave (since cladding was added), we
# recalculate the field over the full length of n
w = dx2 / dx
x2 = LinRange(-length(n) * dx2 / 2, length(n) * dx2 / 2, length(n))
U2 = Uf(x2, w)
U2 = U2 / sqrt(sum(abs2(U2)))

# We calculate the modes of the waveguide
neffs, fields = waveguide_modes(n, lambda0, dx2, n_modes=1)

# We plot the engineered mode (#1) and plot it in comparison of the target
# field
yval = abs2([[U2 fields[:, 1]])
    yval = minimum(n) .+ [yval[:, 1] .+ (maximum(n) - minimum(n)) ./ maximum(yval[:, 1]) yval[:, 2] .+ (maximum(n) - minimum(n)) ./ maximum(yval[:, 2])]]
    yval = [n yval]
plot(x2, yval, 
    labels = ["Waveguide" "Target Mode" "Mode"],
    xlabel=LaTeXString("$\lambda_0$"), ylabel="Real Refractive Index", size=(1200, 400))

if @isdefined saveplots; savefig("appendix_d.png") end
```

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REFERENCES


