AN ACCURATE AND FLEXIBLE METHOD FOR ESTIMATING THE TRANSITION PROBABILITY OF A BOOLEAN FUNCTION

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   Motivated by the need to estimate power dissipation in VLSI circuits, attention has recently been focused on the estimation of transition probability of a Boolean function. In a clocked design, the transition probability is the average fraction of clock cycles in which the final value of a logic signal is different from its initial value. Some sources in the literature have proposed an accurate method for computing the transition probability that takes into account temporal correlation. That approach makes use of Binary Decision Diagrams (BDDs), but is rather restrictive because it limits the allowed orderings of the BDD input variables. In this paper, we present an alternative equally accurate method for estimating the transition probability, which is more flexible in that it places no restrictions on the BDD variable order, at the cost of introducing a number of new input variables. Thus the approach allows one to use existing optimization techniques to find a good variable order, leading to a potentially smaller BDD. Furthermore, the proposed technique can be applied at the logic diagram level, so that existing logic level techniques can be applied as well.
An Accurate and Flexible Method for Estimating the Transition Probability of a Boolean Function†

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1. Introduction

Motivated by the need to estimate power dissipation in VLSI circuits, attention has recently been focused on the estimation of transition probability of a Boolean function. In a clocked design, the transition probability is the average fraction of clock cycles in which the final value of a logic signal is different from its initial value. References [1, 2] propose an accurate method for computing the transition probability that takes into account temporal correlation. That approach makes use of Binary Decision Diagrams (BDDs), but is rather restrictive because it limits the allowed orderings of the BDD input variables. In this paper, we present an alternative equally accurate method for estimating the transition probability, which is more flexible in that it places no restrictions on the BDD variable order, at the cost of introducing a number of new input variables. Thus the approach allows one to use existing optimization techniques to find a good variable order, leading to a potentially smaller BDD. Furthermore, the proposed technique can be applied at the logic diagram level, so that existing logic level techniques can be applied as well.

2. Transition Probability Computation

Let $x_1, \ldots, x_n$ be the primary inputs to a combinational circuit, which may be part of a larger synchronous sequential design. We can represent the set of all possible vectors $x_1, x_2, \ldots, x_n$ by a random Boolean vector $x_1, x_2, \ldots, x_n$, where each $x_i$ is a Boolean random variable (RV). The probabilities of the random vector $x_1, x_2, \ldots, x_n$ correspond to the expected frequencies of occurrence of various input signal combinations.

If $y$ is the output node of a logic gate in the circuit, represented by the RV $y$, then its signal probability is defined as:

$$P(y) = P\{y = 1\}$$

where $P$ denotes the probability function. In a clocked design, we are interested in the values of the signals in two consecutive clock cycles. In general, if the RV $y$ denotes the random value of a Boolean signal $y$ in a certain clock cycle, then we denote its random value in the next clock cycle by $y'$. With this, we can define the transition probability of $y$ as:

$$P(y \oplus y') = P\{y \neq y'\} = P\{y \oplus y' = 1\} = P(y \oplus y')$$

(1)

where $\oplus$ denotes the Boolean exclusive-or (XOR) operation. Since $y$ depends on $x_1, \ldots, x_n$ and $y'$ depends on $x_1', \ldots, x_n'$, then the transition probability of $y$ is completely determined by the joint distribution of the $2n$ input RVs $(x_1, x_1', \ldots, x_n, x_n')$.

The transition probability represents the dependence or correlation between two consecutive values of a signal. This is commonly referred to as temporal correlation. Another form of correlation, called spatial correlation, has to do with the relationship between two signal values at two different circuit nodes in the same clock cycle. The joint distribution of the input RVs $(x_1, x_1', \ldots, x_n, x_n')$ contains information on both the temporal and spatial correlations between the circuit inputs. It is generally very difficult (computationally) to maintain the joint distribution, or even to use it if it is provided. In fact, it is even difficult to maintain

† This work was performed in 1996.
and use information on just the pairwise correlations between the signals, so that it is not uncommon to drop all correlations and assume independence.

In [1, 2], as in this paper, we will account only for the temporal correlations between consecutive input signal values, while ignoring spatial correlation between the circuit inputs. However, additional spatial correlations between internal circuit nodes, which may be generated due to to reconvergent fanout, will be accounted for. We will refer to this simplifying assumption as a spatial independence assumption.

Formally, the spatial (but not temporal) independence assumption can be expressed as follows. We assume that the groups of RVs \((x_i, x'_i)\), \(i = 1, \ldots, n\), are (mutually) independent. This means that the joint probability density function (pdf) of \((x_1, x'_1, \ldots, x_n, x'_n)\) is equal to the product of the pairwise pdfs, \(f_1(x_1, x'_1) \cdots f_n(x_n, x'_n)\). Thus, the joint distribution of \((x_1, x'_1, \ldots, x_n, x'_n)\) is completely determined by the \(n\) joint distributions of \((x_i, x'_i)\). Consequently, the transition probability of \(y\), \(P_t(y)\), is completely determined by the \(n\) joint distributions of \((x_i, x'_i)\).

Finally, for any two logic signals \(x\) and \(y\), when \(P(x) \neq 0\) we define the conditional probability of \(y\) given \(x\) as:

\[
P(y \mid x) = \frac{P\{y = 1 \mid x = 1\}}{P\{x = 1\}} = \frac{P(xy)}{P(x)}
\]

We will now make use of the following result.

**Proposition 1.** Let \(x\) and \(y\) be two Boolean variables, with corresponding RVs \(x\) and \(y\). Define two new independent Boolean variables \(a\) and \(b\), with corresponding RVs \(a\) and \(b\), such that \(P(a) = P(y \mid x)\) and \(P(b) = P(y \mid \bar{x})\). Then \((x, y)\) have the same joint distribution as \((a, z)\), where \(z\) is another Boolean variable, defined as:

\[
z = ax + bx\bar{x}
\]

**Proof:** It is enough to prove that \(P(x) = P(y)\) and \(P(xy) = P(xy)\), as follows:

\[
P(x) = P(ax) + P(bx) = P(a)P(x) + P(b)P(x)
\]

\[
= P(y \mid x)P(x) + P(y \mid \bar{x})P(\bar{x}) = P(yx) + P(yx) = P(y)
\]

and

\[
P(xy) = P(ax + bx\bar{x}) = P(ax)
\]

\[
= P(a)P(x) = P(y \mid x)P(x) = P(xy)
\]

Thus \((x, y)\) have the same joint distribution as \((a, z)\). \(\blacksquare\)

Therefore, if we replace \(x_1\) by \(x'_1 \triangleq a_1x_1 + b_1x_1\), the independent RVs \(a_i\) and \(b_i\) have the signal probabilities:

\[
P(a_i) = P(x'_i \mid x_i), \quad \text{and} \quad P(b_i) = P(x'_i \mid \bar{x}_i)
\]

then the value of \(P_t(y)\) would remain unchanged. As a result, \(P_t(y)\) can now be determined by the joint distribution of \((x_1, a_1, b_1, \ldots, x_n, a_n, b_n)\), all of which are independent random variables, due to the spatial independence assumption. This makes it possible to use a BDD to compute the transition probability.

**References**


