The ro-vibrational predissociation spectrum of He-H$_3^+$ has been recorded via excitation of the $\nu_2$ vibrational mode of the H$_3^+$ sub-unit in a cold 22-pole ion trap. The spectrum for the bare H$_3^+$ consists of only a few ro-vibrational lines each for the para and ortho nuclear spin configuration, respectively. Instead, the spectrum of the complex is very rich (several hundred lines) even at the low temperature (4 K) of the trap experiment. Part of this complexity is associated with the (almost) free internal rotation of H$_3^+$. The experimental results are compared to theoretical predictions of ro-vibrational spectra on the basis of ab-initio calculations of the He-H$_3^+$ potential energy surface. The energy levels result in transitions which agree in many cases with experimental results within a few wavenumbers. In particular the typical band structures of a P- and R-branch associated with an effective diatomic complex seen in the experimental and predicted spectrum help in assigning the rich spectrum. Moreover, an experimental energy term diagram is reconstructed from the observed transitions which can be compared to the rather accurate theoretical predictions. Despite of the floppiness of the complex rotational constants for the effective diatomic complex can be derived and match to the term diagram of a prolate, slightly asymmetric rotor. The influence of the Coriolis interaction resulting from the H$_3^+$ internal rotation in a rotating He-H$_3^+$ frame shall be discussed.