An algebraic approach based on the $U(4)$ algebra is proposed to describe 3D systems for effective potentials. Our approach is based on the addition of a scalar boson to the 3D harmonic oscillator space keeping the constraint of a total number of bosons $N$ constant, in similar form as in the vibron model. However instead of dealing directly with the dynamical symmetries we proceed to identify the coordinates and momenta in the algebraic space. This allows the kets associated with the different subgroup chains to be linked to energy, coordinate and momentum representations. This identification provides powerful tools to obtain the matrix representation of 3D Hamiltonians in a simple form through the use of the transformation brackets connecting the different bases. The exact energy and wave functions are obtained in the large $N$ limit. Because the relevance of the Coulomb interaction we consider as an example of our approach the analysis of the non relativistic Hydrogen atom as a first step to establish the ground to deal with multi-electron molecular systems.