MODELING WIRELESS ACOUSTIC POWER TRANSMISSION SYSTEMS

BY

JAE WON CHOI

THESIS

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Adviser:

Professor Andrew C. Singer

Abstract

In the presence of a barrier which is conductive and structurally prevents penetration of electromagnetic energy, acoustic wave transmission is often used for data communications and could be a viable option for wireless power transfer. Such a power transfer system design would need to incorporate the behavior of the entire acoustic channel, taking into account the properties of the acoustic transducers that create the acoustic signals, the propagation of the acoustic signals through the barrier, and the transducers and circuitry that transform the acoustic power back into electrical power. The thesis presents a model that translates the acoustic components of the system into a model that is suitable for analysis using the electrical components. Basic principles of acoustic physics and piezoelectric material properties will be discussed. Then an ABCD-parameter, two-port network representation is derived for a system compromising a piezoelectric transducer and a solid barrier. Such representations can be also be expressed in lumped-element circuits, which can be useful in designing the electrical end of the power transfer system. Using ABCD-parameter models, multiple acoustical and piezoelectric elements are cascaded and modeled as a single two-port network. Using parameter conversion of the two-port network, source and load impedance can be matched to maximize the power transfer.

Acknowledgments

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Contents

1 Introduction

Transferring electrical power through solid, RF-impenetrable barriers would traditionally require creating a hole through which wires can be fed and power sent electrically. However, such a solution is not feasible when the structural integrity is compromised by the penetration or the barrier requires complete isolation between two separate spaces. Barriers such as bulkheads of a ship, chemical tanks or even biological tissue are examples for which wired penetration is not a viable solution. To power electronic devices across these barriers, a wireless power transmission solution is needed.

Various wireless power transmission technologies have undergone enormous growth both in research and application. The most popular method among these is inductive RF coupling. Recent research has shown that the method is capable of transferring data over distances up to 2 m at high efficiency [1]. However, the inductive coupling method suffers from penetration losses through conductive media [2]. Although less extensively researched than inductive coupling, other methods for wireless power transfer include capacitive coupling and optical transfer. Capacitive coupling is used far less frequently because of the inverse proportionality of the capacitance with the distance. The optical power transfer also has greater transmission distance, but a substantial portion of the energy is lost in electrical-to-optical and optical-to-electrical conversion [3].

An alternative to the electromagnetic wireless power transmission methods described above is the use of wireless acoustic power transmission technology. An advantage of acoustic power transfer over electromagnetic transfer is its versatility over media through which acoustic waves travel more readily than electromagnetic waves, such as media that are highly conductive, like sea water, or metal barriers. Also, for media through which electromagnetic energy propagates only at the wavelengths such that the systems become prohibitively large (and therefore systems whose physical size scales with the wavelength of transmission), the transmitter and receiver could be designed in a much smaller form factor if substantially higher frequency acoustic waves (and therefore substantially smaller acoustic wavelengths and physical mechanisms) can be used. [1]. Because of these merits, acoustic power transfer is a suitable candidate for evaluation in applications such as power transmission through human tissue, metallic barriers, and salt water.

To achieve high power efficiency of the power transfer system, analytical approaches are essential. A common way to describe an electrical network is by using a two-port network model. In this thesis, we will discuss how the acoustic layer of a wireless acoustic power transfer system could be modeled as a two-port network. Also, using the mechanical properties of the material, we will derive an equivalent lumped-circuit representation of a solid barrier and a piezoelectric transducer.

The rest of the thesis is organized as follows. In chapter 2, we will derive ABCD parameters for an acoustic wave traveling through a solid barrier and the electromechanical conversion process within piezoelectric transducer. Chapter 3 describes an equivalent circuit model for models derived in the previous chapter. Chapter 4 will show how each component can be cascaded together to create an electromechanical channel for a power transfer system. Chapter 5 concludes the thesis with a summary and possible path for future research.

2 Electro-Mechanical Model for Piezoelectric and Solid Barriers

To characterize the mechanical properties of an acoustic wave, the following four equations are useful:

$$-\nabla p = \rho_0 \frac{\partial u}{\partial t} \tag{1}$$

$$u = \frac{\partial \xi}{\partial t} \tag{2}$$

$$s = -\nabla \xi \tag{3}$$

$$p = \rho_0 c^2 s \tag{4}$$

p, s, ξ, and u are stress, strain, particle displacement, and particle velocity, respectively. Also ρ_0, c and t are material density, propagation speed, and time. ∇ is a gradient operator across the 3-dimensional spatial axis ($x\ y\ z$). Equation (1) is the linear Euler's equation derived from Newton's 2nd law. Equations (2) and (3) are the definition of particle velocity and strain in terms of the particle displacement. Lastly, equation (4) is the linearized relationship between stress and strain, commonly known as linearized Hooke's law.

From equations (1), (2), (3), and (4), the classical wave equation is derived.

$$\frac{\partial^2 p(\bar{r}, t)}{\partial t^2} = c^2 \nabla^2 p(\bar{r}, t) \tag{5}$$

The stress is assumed to be a function of time t and space $\bar{r}$, where $\bar{r}$ is a vector in 3-dimensional space ($x\ y\ z$). For modeling acoustic power transfer through a solid barrier, we will be considering a planar wave in one-dimension x. Then the harmonic solution to the above equation can be given by the longitudinal wave traversing in x direction, shown as

$$p(x, t) = \left(Ae^{-jkx} + Be^{jkx}\right)e^{j\omega t} \tag{6}$$

where A and B are complex coefficients of a wave propagating forward $(+)$ and backward $(-)$, ω is the angular frequency in radians and k is the wave number,

$$k = \frac{\omega}{c} \tag{7}$$

The force f exhibited on area S on a plane perpendicular to the direction of the wave is given by

$$f = pS \tag{8}$$

However, the above model assumes the acoustic wave is propagating through a lossless medium. Any real acoustic transmission will have losses. These losses result from phenomena such as thermal, structural or/and chemical relaxation [4]. These phenomena represent changes in the state of the medium, away from equilibrium caused by propagating acoustic waves. As the medium returns to a state of equilibrium, the conversion of acoustic energy results in a loss, characterized by relaxation time τ. The classical wave equation with relaxation time τ becomes

$$\frac{\partial^2 p(\bar{r},t)}{\partial t^2} = c^2 \nabla^2 p(\bar{r},t) - \tau \frac{\partial}{\partial t} \nabla^2 p(\bar{r},t) \tag{9}$$

The solution of the equation for the harmonic planar wave case is similar to the classical wave equation. The solution is given by substituting k with new exponent $\tilde{k}$ in equation (6), given by

$$\tilde{k} = k_r - j\alpha = \frac{\omega}{c\sqrt{1 + j\omega\tau}} \tag{10}$$

where α and k are absorption and wavenumber based on the relaxation, and are dependent on frequency ω.

$$k_r = \frac{\omega}{\sqrt{2}c}\left[\frac{\sqrt{1 + (\omega\tau)^2} + 1}{1 + (\omega\tau)^2}\right]^{\frac{1}{2}} \tag{11}$$

$$\alpha = \frac{\omega}{\sqrt{2}c}\left[\frac{\sqrt{1+(\omega\tau)^2}-1}{1+(\omega\tau)^2}\right]^{\frac{1}{2}} \tag{12}$$

The addition of the absorption term gives rise to complex speed $\tilde{c}$ and phase speed c_p:

$$\tilde{c} = \frac{\omega}{\tilde{k}} \tag{13}$$

$$\mathrm{c_p} = \frac{\omega}{k_r} \tag{14}$$

We can now write p, f, and u for the lossy model:

$$p(x,t) = \left(Ae^{-j\tilde{k}x} + Be^{j\tilde{k}x}\right)e^{j\omega t} \tag{15}$$

$$f(x,t) = S\left(Ae^{-j\tilde{k}x} + Be^{j\tilde{k}x}\right)e^{j\omega t} \tag{16}$$

$$u(x,t) = \frac{1}{\rho_0\tilde{c}}\left(Ae^{-j\tilde{k}x} - Be^{j\tilde{k}x}\right)e^{j\omega t} \tag{17}$$

The specific acoustic impedance is given by

$$Z_s = \frac{\tilde{p}}{\tilde{u}} = \pm\rho_0\tilde{c} \tag{18}$$

where $\tilde{p}$ and $\tilde{u}$ are stress and particle velocity of the wave propagating in one direction respectively. The positive and negative signs of the specific acoustic impedance correspond to forward and backward propagating waves. The absolute value of the specific acoustic impedance is defined as the characteristic impedance Z_c,

$$Z_c = \rho_0\tilde{c} \tag{19}$$

Similar to the specific acoustic impedance, the mechanical acoustic impedance is the ratio of force to particle velocity traveling in one direction. For a planar wave propagating perpendicular to area S, the mechanical impedance can be expressed by the characteristic impedance Z_c,

$$Z_m = \frac{\tilde{f}}{\tilde{u}} = \pm SZ_c = \pm Z \tag{20}$$

Piezoelectric materials are capable of translating between electrical energy and mechanical energy. The linear model of the IEEE standard 176-1987 [5] is given by the following two constitutive equations:

$$p_{ij} = c_{ijkl}^{D} s_{kl} - h_{kij} D_k \tag{21}$$

$$E_i = -h_{ikl} s_{kl} + \beta_{ik}^{s} D_k \tag{22}$$

where p_{ij}, s_{kl}, E_i, and D_k are stress, strain, electric field, and electric displacement. The subscripts i, j, k, and l are one of the three coordinate axes x, y, and z, representing the measurement value projected onto the specified coordinate. There are three components to the electrical measurements indicating values in each coordinate. For mechanical measurements, there are nine different components. If two subscripts are the same, they represent a longitudinal wave, and otherwise, they represent a shear wave. The elastic compliance constants c_{ijkl}^{D} is defined as

$$c_{ijkl}^{D} = \left(\frac{\partial p_{ij}}{\partial s_{kl}}\right)_D \tag{23}$$

The superscript D on the left-hand side and the subscript D on the right-hand side suggest that c_{ijkl}^{D} is measured as a slope of stress and strain given that electrical displacement is held constant. The dielectric impermittivity constant β_{ik}^{s} is defined in a similar way,

$$\beta_{ik}^{s} = \left(\frac{\partial E_i}{\partial D_k}\right)_s \tag{24}$$

2.1 Wave Propagation Model of a Barrier to ABCD Parameters

To model wave propagation through a solid barrier, the planar wave propagation model is considered. The surface area of the barrier is assumed to be infinitely large, if it is large compared to the injection area of an acoustic wave.

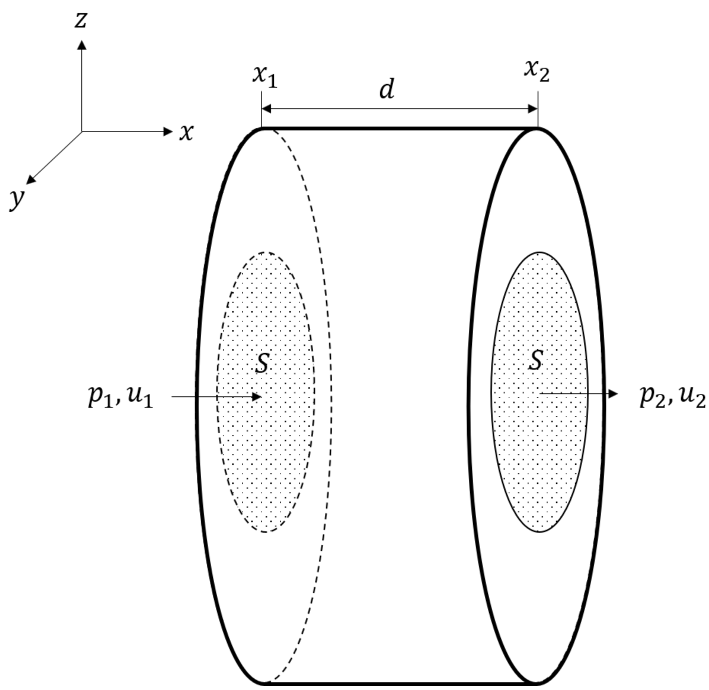

Figure 1. Planar acoustic wave through solid barrier

As in Figure 1, consider a bulk acoustic wave propagating through the barrier injected normal to the y-z plane from an attached source of cross-sectional area S. The barrier is assumed to have infinitely large surface and thickness of length d. At the boundary values x_1 and x_2, we have particle velocity

$$u_1(t) = u(x_1, t) = \frac{1}{\rho_0 \tilde{c}} \left(A e^{-j\tilde{k}x_1} - B e^{j\tilde{k}x_1} \right) e^{j\omega t} \tag{25}$$

$$u_2(t) = u(x_2, t) = \frac{1}{\rho_0 \tilde{c}} \left(A e^{-j\tilde{k}x_2} - B e^{j\tilde{k}x_2} \right) e^{j\omega t} \tag{26}$$

From the equations (25) and (26), A and B can be solved in terms of u_1 and u_2. Substituting into the expression of the force of the planar wave,

$$f(x,t) = \frac{S\rho_0 c}{j\sin(\tilde{k}d)}\left(u_1(t)\cos\left(\tilde{k}(x_2 - x)\right) - u_2(t)\cos\left(\tilde{k}(x_1 - x)\right)\right) \tag{27}$$

At the boundary x_1 and x_2,

$$f_1(t) = f(x_1,t) = \frac{S\rho_0 c}{j\sin(\tilde{k}d)}\left(u_1(t)\cos(\tilde{k}d) - u_2(t)\right) \tag{28}$$

$$f_2(t) = f(x_2,t) = \frac{S\rho_0 c}{j\sin(\tilde{k}d)}\left(u_1(t) - u_2(t)\cos(\tilde{k}d)\right) \tag{29}$$

Rearranging the terms, and substituting (20) in matrix form,

$$\begin{gathered}
\begin{bmatrix} f_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} A_{solid} & B_{solid} \\ C_{solid} & D_{solid} \end{bmatrix} \begin{bmatrix} f_2 \\ u_2 \end{bmatrix} \\
A_{solid} = \cos(\tilde{k}d) \\
B_{solid} = jZsin(\tilde{k}d) \\
C_{solid} = jZ^{-1}\sin(\tilde{k}d) \\
D_{solid} = \cos(\tilde{k}d)
\end{gathered} \tag{30}$$

Equation (30) is the ABCD-parameter two-port network representation of the acoustic wave propagating through a solid barrier. Forces and particle velocities at the boundary x_1 and x_2 are analogous to supply and receive voltages and currents in the two-port network system. Then we can observe that the ABCD-parameter model of acoustic wave propagation through a solid barrier is similar to the ABCD-parameter model of an electromagnetic wave through a transmission line [6].

2.2 Wave Propagation Model of a Piezoelectric Transducer

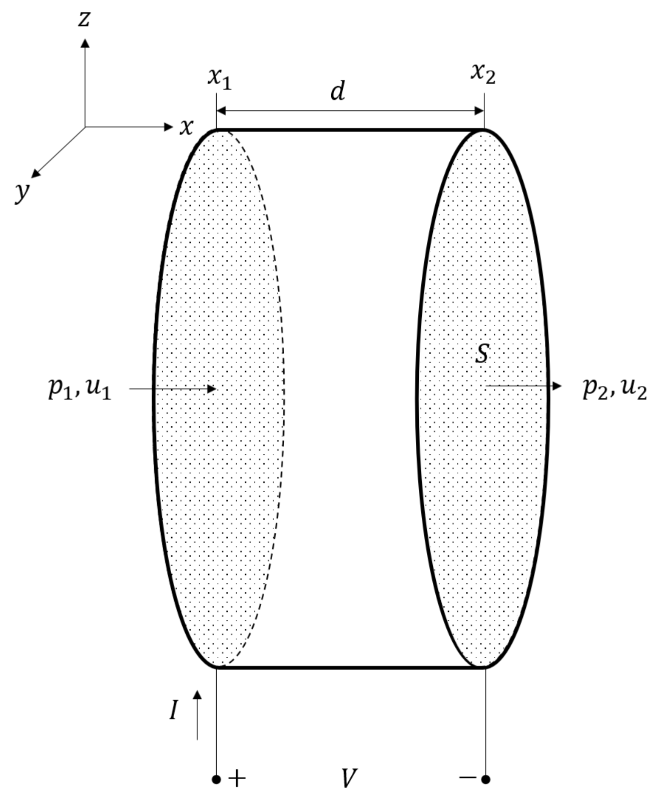

Figure 2 Piezoelectric transducer in thickness mode

The diagram of the piezoelectric source model, Figure 2, is similar to Figure 1, with the addition of electrodes on boundary surfaces at x_1 and x_2. To describe the behavior of the planar acoustic wave traveling in x direction with the electrical excitation across the same axis, the coupling behaviors of (21) and (22) are not needed. All of the subscripts describing the dimension, i, j, k and l, of the constitutive equations are equal to x. Then (21) and (22) reduce to a one-dimension model,

$$E = -h_x s + \beta_x^s D \tag{31}$$

$$p = c_x^D s - h_x D \tag{32}$$

In Figure 2, V represents the voltage difference between a positive node at x_1 and a negative node at x_2, and I is the current following into the system. Since the voltage is defined as the potential of an electric field, we have

$$V = -\int_{x_2}^{x_1} E\, dx \tag{33}$$

Since piezoelectric materials have electric displacement that is constant across the body, substituting (31) into (33) and deriving the strain s in terms of the particle velocity yields the following closed-form expression:

$$V = d\beta_x^s D + \frac{h_x}{j\omega}(u_1 - u_2) \tag{34}$$

Since D is a linear combination of p and s, it can be written as a harmonic waveform. Additionally, assuming that the electrical displacement is consistent across the piezoelectric material, the following expression for the current holds by Ampere's circuit law:

$$I = \iint_S \frac{\partial D}{\partial t} d\mathbf{S} = j\omega SD \tag{35}$$

Substituting (35) and following the definition of the zero-strain capacitance of the piezoelectric material, we have

$$C_0 = \frac{S}{\beta_x^s d} \tag{36}$$

The voltage is given by

$$V = \frac{1}{j\omega C_0} I + \frac{h_x}{j\omega}(u_1 - u_2) \tag{37}$$

The expression for the mechanical force f in the piezoelectric material is derived by multiplying both sides of (32) by S, and substituting (4) and (35) under the assumption that the wave propagating through the transducer is planar,

$$f(x,t)=\frac{Z}{j\sin(\tilde{k}d)}\left(u_1(t)\cos\left(\tilde{k}(x_2-x)\right)-u_2(t)\cos\left(\tilde{k}(x_1-x)\right)\right)-\frac{h_x}{j\omega}I \tag{38}$$

Piezoelectric transducers are usually designed with a backing layer. For the model described, we assume that the backing layer is adjacent to the piezoelectric source at boundary x_1. The behavior on this boundary can be modeled as a backing impedance,

$$Z_b=\frac{f_1}{u_1} \tag{39}$$

Since our interest is in the direction of x_2, behavior in x_1 is described from the point of the view of the transducer. Evaluating (38) at the boundary x_2, and applying (39) and (34), the following set of equations can be obtained:

$$V=\frac{1}{1+j\frac{Z_b}{Z}\sin(\tilde{k}d)-\cos(\tilde{k}d)}\left[\left(\frac{1}{C_0h_x}\cos(\tilde{k}d)+\left(\frac{h_x}{\omega Z}-j\frac{Z_b}{C_0h_xZ}\right)\sin(\tilde{k}d)\right)f_2\right.$$
$$\left.+\left(j\frac{2h_x}{\omega}-\frac{Z_b}{C_0h_x}\cos(\tilde{k}d)+\left(\frac{h_xZ_b}{\omega Z}-j\frac{Z}{C_0h_x}\right)\sin(\tilde{k}d)\right)u_2\right] \tag{40}$$

$$I=\frac{1}{1+j\frac{Z_b}{Z}\sin(\tilde{k}d)-\cos(\tilde{k}d)}\left[\left(\frac{j\omega}{h_x}\cos(\tilde{k}d)+\frac{\omega Z_b}{h_xZ}\sin(\tilde{k}d)\right)f_2\right.$$
$$\left.+\left(j\frac{\omega Z_b}{h_x}\cos(\tilde{k}d)-\frac{\omega Z}{h_x}\sin(\tilde{k}d)\right)u_2\right] \tag{41}$$

This expression can be rewritten in matrix form as an $ABCD$-parameter two-port network with an electrical port $[V\ I]^T$ and a mechanical port $[f_2\ u_2]^T$,

$$\begin{bmatrix}V\\I\end{bmatrix}=\begin{bmatrix}A_{piezo} & B_{piezo}\\C_{piezo} & D_{piezo}\end{bmatrix}\begin{bmatrix}f_2\\u_2\end{bmatrix} \tag{42}$$

$$A_{piezo} = \frac{1}{1 + j\frac{Z_b}{Z}\sin(\tilde{k}d) - \cos(\tilde{k}d)}\left(\frac{1}{C_0 h_x}\cos(\tilde{k}d) + \left(\frac{h_x}{\omega Z} - j\frac{Z_b}{C_0 h_x Z}\right)\sin(\tilde{k}d)\right)$$

$$B_{piezo} = \frac{1}{1 + j\frac{Z_b}{Z}\sin(\tilde{k}d) - \cos(\tilde{k}d)}\left(j\frac{2h_x}{\omega} - \frac{Z_b}{C_0 h_x}\cos(\tilde{k}d) + \left(\frac{h_x Z_b}{\omega Z} - j\frac{Z}{C_0 h_x}\right)\sin(\tilde{k}d)\right)$$

$$C_{piezo} = \frac{1}{1 + j\frac{Z_b}{Z}\sin(\tilde{k}d) - \cos(\tilde{k}d)}\left(\frac{j\omega}{h_x}\cos(\tilde{k}d) + \frac{\omega Z_b}{h_x Z}\sin(\tilde{k}d)\right) \tag{43}$$

$$D_{piezo} = \frac{1}{1 + j\frac{Z_b}{Z}\sin(\tilde{k}d) - \cos(\tilde{k}d)}\left(j\frac{\omega Z_b}{h_x}\cos(\tilde{k}d) - \frac{\omega Z}{h_x}\sin(\tilde{k}d)\right)$$

3 Equivalent Circuit Model

3.1 T-Section and π-Section Circuit Representations of an Acoustic Wave through Solid Barrier

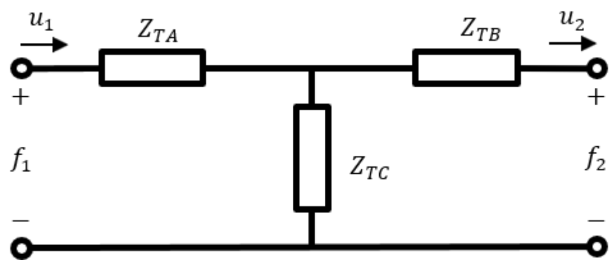

Figure 3 T-section circuit representation

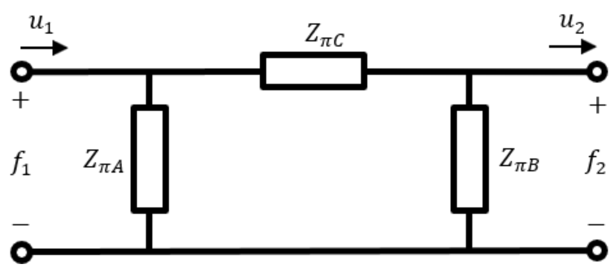

Figure 4 π-section circuit representation

In Chapter 2, we discussed how the wave acoustic behavior through a solid barrier can be expressed as an ABCD matrix. With the ABCD parameters, the model can be easily applied to T/π section electrical circuit structures, as shown in Figures 3 and 4. These two circuit structures are equivalent under the following transformation with a difference only in the convenience of calculation of the system values,

$$Z_{TA} = \frac{Z_{\pi A} + Z_{\pi C}}{Z_{\pi A} + Z_{\pi B} + Z_{\pi C}}$$
$$Z_{TB} = \frac{Z_{\pi B} + Z_{\pi C}}{Z_{\pi A} + Z_{\pi B} + Z_{\pi C}} \quad (44)$$

$$Z_{TC} = \frac{Z_{\pi B} + Z_{\pi C}}{Z_{\pi A} + Z_{\pi B} + Z_{\pi C}}$$

The relationships between ABCD parameters and the equivalent circuits are derived in Table 1.

Table 1 ABCD parameters of T and π section networks

Network	A	B	C	D
T-section	$1+\frac{Z_{TA}}{Z_{TC}}$	$\frac{Z_{TA}Z_{TB} + Z_{TA}Z_{TC} + Z_{TB}Z_{TC}}{Z_{TC}}$	$\frac{1}{Z_{TC}}$	$1+\frac{Z_{TB}}{Z_{TC}}$
π-section	$1+\frac{Z_{\pi C}}{Z_{\pi B}}$	$Z_{\pi C}$	$\frac{Z_{\pi A} + Z_{\pi B} + Z_{\pi C}}{Z_{\pi A}Z_{\pi B}}$	$1+\frac{Z_{\pi C}}{Z_{\pi A}}$

3.2 Equivalent Circuit Model for Piezoelectric Transducers

It was shown in [7] that 1-dimensional analysis of the electromechanical behavior of a piezoelectric transducer can be presented in an equivalent circuit that has one electrical port and two mechanical ports, as shown in Figure 5.

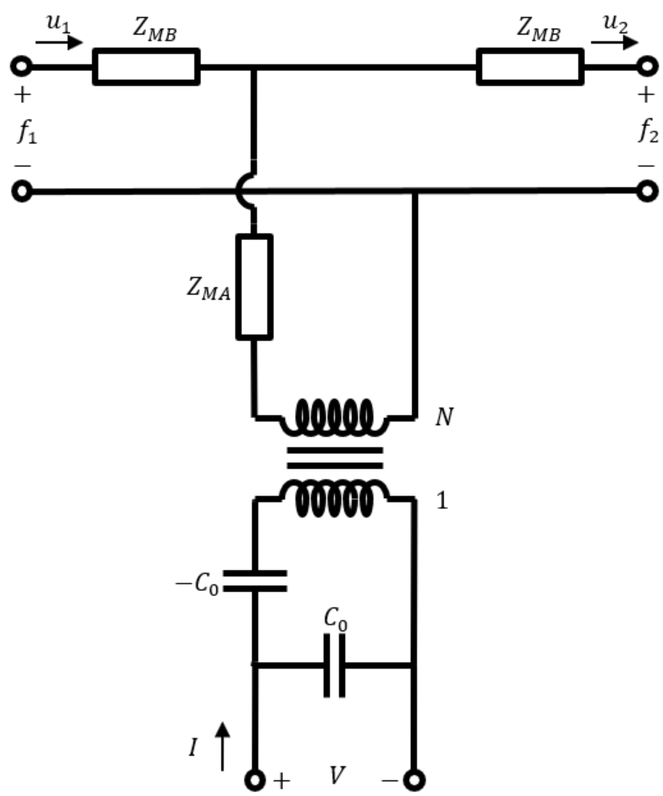

Figure 5 Mason's equivalent circuit

As in section 2.2, if one of the mechanical ports is connected by Z_b, the impedance looking into the backing layer, the Mason equivalent circuit can represent a two-port network model as in Figure 6.

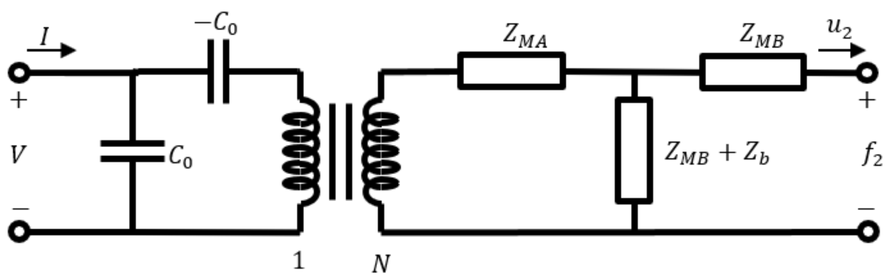

Figure 6 Mason's equivalent circuit with backing impedance Z_b

The parameter C_0 is the zero-strain piezoelectric capacitance (36). The transformer parameter N is given by

$$N = C_0 h_x \tag{45}$$

Applying equation (43) to the mason circuit model, the impedance parameters Z_{MA} and Z_{MB} are derived as

$$Z_{MA} = \frac{Z}{j\sin(\tilde{k}d)}$$
$$Z_{MB} = \frac{Z(1-\cos(\tilde{k}d))}{j\sin(\widetilde{kd})} \tag{46}$$

From the electrical port to the transformer component, elements can be rearranged to form a π section, as shown in Figure 7.

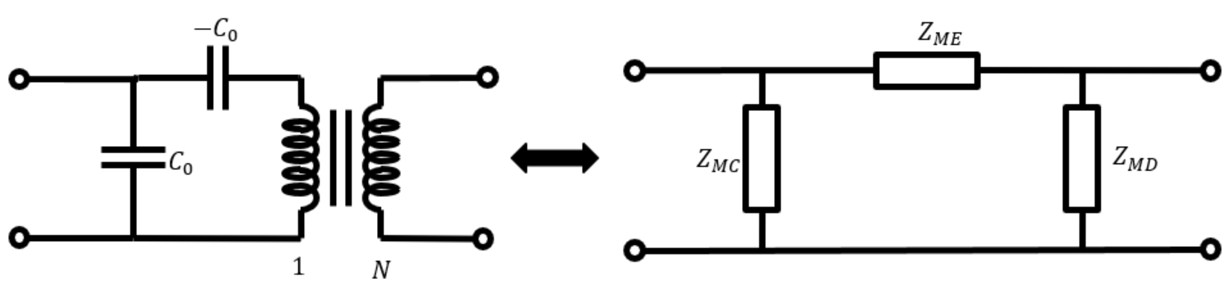

Figure 7 π section equivalent of the electrical port end of the piezoelectric transducer

The resulting impedance parameters are given by

$$Z_{MC} = Z_{ME} = \frac{N}{j\omega C_0} \tag{47}$$

$$Z_{MD} = \frac{N^2}{j\omega C_0(1 - N)}$$

Thus, given the physical parameters for the transducers and barriers, the acoustic channel can be modeled by cascading π sections and T sections.

4 Aggregated Modeling of the Acoustic Channel

4.1 S and Z Parameters and Impedance Matching

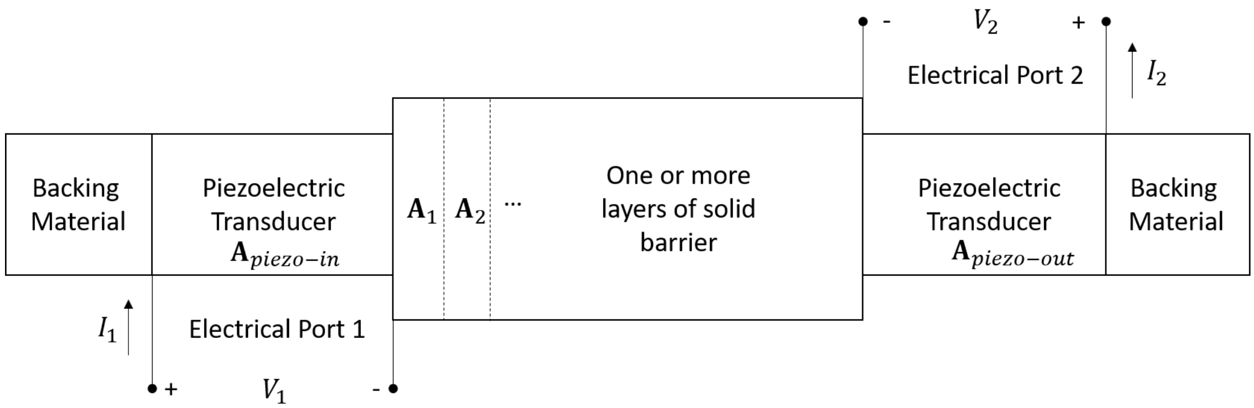

Figure 8 Acoustic channel through layers of solid barriers

By parameterizing each element in the acoustic channel by the ABCD matrix, all the elements can be cascaded into one ABCD matrix that characterizes the whole system. Taking Figure 8 as an example, the ABCD matrix for the whole channel, $\mathbf{A}$, can be calculated by cascading of each component as follows:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \mathbf{A}_{piezo-in}\mathbf{A}_1\mathbf{A}_2 \dots \mathbf{A}_{piezo-out} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{48}$$

ABCD parameters are useful in cascading multiple elements, but Z parameters and S parameters are more convenient in describing the aggregated response of the channel. The Z-parameter model is defined as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} \tag{49}$$

The S-parameter model is defined as

$$\begin{bmatrix} V_1^- \\ V_2^+ \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^- \end{bmatrix} \tag{50}$$

Superscript $(+)$ indicates the voltage wave propagating in the positive direction (into port 1/out of port 2), and $(-)$ indicates the voltage wave propagating in the negative direction (out of port 1/into port 2).

The parameter conversion is provided in [8], whereby

$$
\begin{aligned}
Z_{11} &= \frac{A}{C} \\
Z_{12} &= \frac{AD - BC}{C} \\
Z_{21} &= \frac{1}{C} \\
Z_{22} &= \frac{D}{C}
\end{aligned}
\tag{51}
$$

$$
\begin{aligned}
S_{11} &= \frac{AZ_{02} + B - CZ_{01}^{*}Z_{02} - DZ_{01}^{*}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\
S_{12} &= \frac{2(AD - BC)\sqrt{R_{01}R_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\
S_{21} &= \frac{2\sqrt{R_{01}R_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}} \\
S_{22} &= \frac{-AZ_{02}^{*} + B - CZ_{01}Z_{02}^{*} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}
\end{aligned}
\tag{52}
$$

with the values of Z_{01} and Z_{02} being the impedances looking out from port 1 and port 2 accordingly, defined as the source impedance and the load impedance. The values of R_{01} and R_{02} are the real parts of the impedance with the same subscript.

It is shown in [9] that these parameters can be used for the characterization and the optimization of the power transfer system. Once the parameters are computed, matching source and load impedances, Z_{m01} and Z_{m02} , can be found to maximize the power transfer, as in

$$
Z_{m01} = \frac{j(\Im\{Z_{12}Z_{21}\} - 2\Re\{Z_{22}\}\Im\{Z_{11}\}) + \sqrt{2\Re\{Z_{11}\}\Re\{Z_{22}\} - \Re\{Z_{12}Z_{21}\} - |Z_{12}Z_{21}|^{2}}}{2\Re\{Z_{22}\}} \tag{53}
$$

$$
Z_{m02} = \frac{j(\Im\{Z_{12}Z_{21}\} - 2\Re\{Z_{11}\}\Im\{Z_{22}\}) + \sqrt{2\Re\{Z_{11}\}\Re\{Z_{22}\} - \Re\{Z_{12}Z_{21}\} - |Z_{12}Z_{21}|^{2}}}{2\Re\{Z_{11}\}} \tag{54}
$$

Substituting Z_{m01} and Z_{m02} into Z_{01} and Z_{02} of the S-parameter expression will maximize the power transfer function defined as $|S_{21}|^2$.

4.2 Simulation of Aggregated Acoustic Channel

In this section we will compare the model suggested in chapter 2 with some actual measurements. It was shown in [10] that the measurements of the S-parameters of the acoustic channel of Figure 9 can be made.

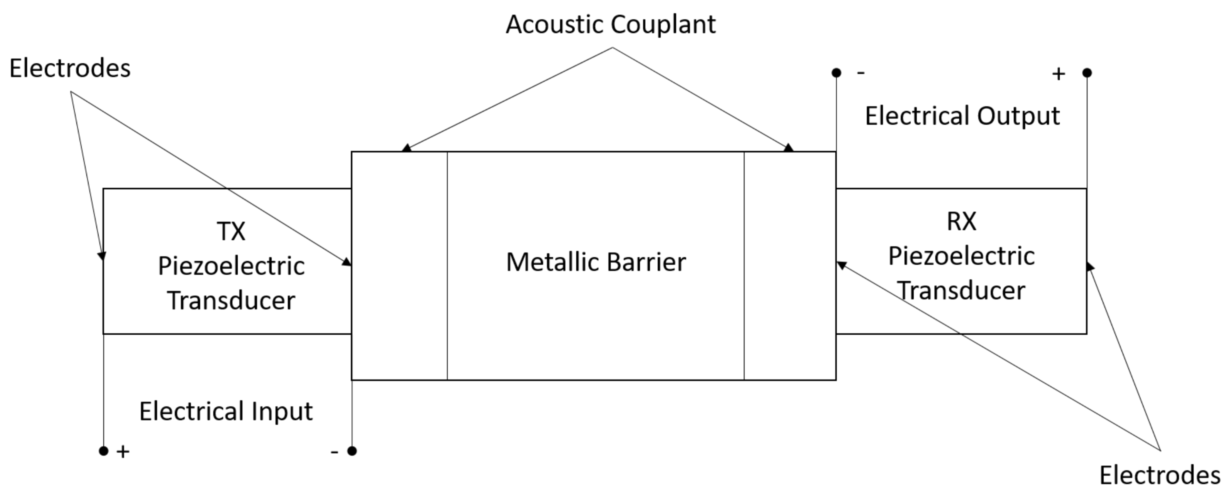

Figure 9 Cross-section of the acoustic-electrical layers adapted from [10]

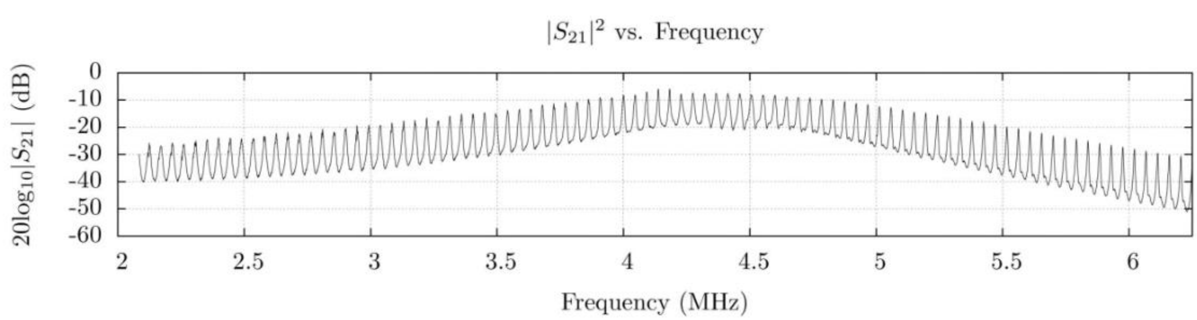

Figure 10 Measurement of the power transfer function [10]

Figure 10 shows the measurement of the S_{21} across the frequency range from 2.08 MHz to 6.25 MHz of the acoustic channel described in Figure 9 by [10]. Using the same model parameter, the aggregated acoustic channel is modeled with the following concatenation.

$$\mathbf{A}_{channel} = \mathbf{A}_{tx}\mathbf{A}_{couplant}\mathbf{A}_{barrier}\mathbf{A}_{couplant}\mathbf{A}_{rx} \quad (55)$$

$\mathbf{A}_{tx}$, $\mathbf{A}_{couplant}$, $\mathbf{A}_{barrier}$, and $\mathbf{A}_{rx}$ are $ABCD$ matrix of TX piezoelectric, acoustic couplant, metallic barrier and RX piezoelectric transducers in Figure 9 accordingly. Figure 11 compares the power transfer function $|S_{21}|^2$ of $\mathbf{A}_{channel}$ and the measurement from 3.4 MHz to 3.8 MHz. The model closely matches the actual measurement within the frequency range plotted in Figure 11. However, there are computational precision problems for the range outside the plotted frequencies which prevent further comparison.

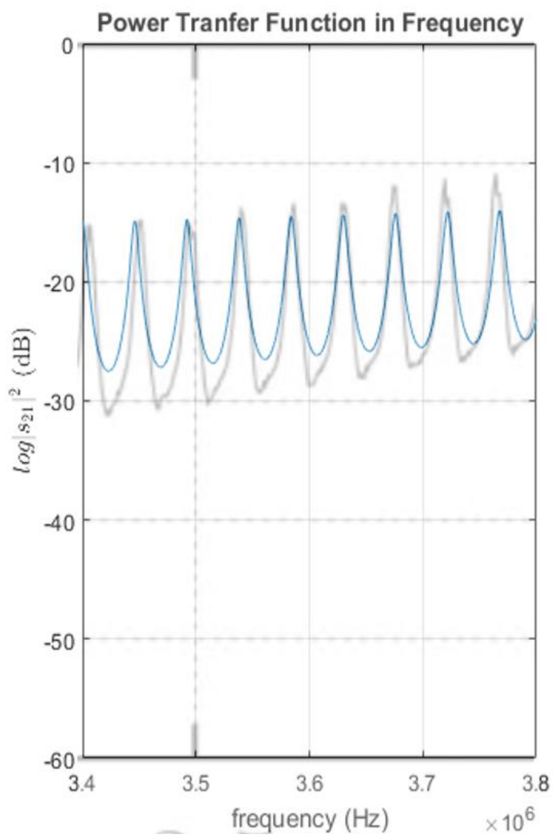

Figure 11 Power transfer function of the model (blue line) and the measurement [10] (black line)

5 Conclusion

In this work, we have studied the modeling of an acoustic wave that is convenient for electrical system design for use in wireless power transfer applications. Components of an acoustic channel are derived as ABCD parameters for ease of cascading. A lumped element circuit equivalent model of the acoustic power transfer channel based on the ABCD-parameter model is derived. Cascaded elements can be viewed as one two-port model, and its ABCD parameters can be converted into more useful parameters for designing a matching circuit for power transfer analysis to a given load impedance. Lastly, in this thesis, we verified the model with comparison to some actual measurements taken by the authors of [10]. From this analysis, we note that the simulation needs further work on stabilizing the numerical computation.

Further work is need on designing the power transmitter and receiver circuit on both the source and load end of the electromechanical layer by using the analysis model given in this thesis. One direction of further study could be to explore where the presence of feedback based on the given model could improve performance. Also, the current model only depicts constant attenuation over frequency and time. However, there are time- and frequency-dependent factors such as temperature and diffraction that play a role in determining the attenuation and losses of the acoustic wave. Lastly, the model presented simplifies the acoustic wave propagation equation to 1 dimension. More study is needed to determine how the shear wave generated by a thickness-mode piezoelectric transducer would affect use in a power transfer system.

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