DEVELOPING A DISTURBANCE SOURCE CHARACTERIZATION
TECHNIQUE FOR SMALL SATELLITE APPLICATIONS

BY

ADAM AUGUSTYNIAK

THESIS
Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Aerospace Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2020

Urbana, Illinois

Adviser:
Associate Professor Timothy Bretl
In this thesis we establish a framework with which to characterize candidate sources of disturbance for small satellite applications. By characterize we mean estimate disturbance source vibrational frequencies, and by candidate sources we mean sources previously determined with the ability to induce micro-vibrations. This framework centers on the operation of distributed sensors, and we present a set of components capable of performing a characterization effort of this nature. Our implementation of supervised learning enables us to predict actuator operational frequency values based on accelerometer readings. The standardized mean squared error (SMSE), a measure of error between the mean prediction and the true value, important for quantifying prediction performance, is shown to be a function of the Fourier transformation type used; and we conclude which considered Fourier transformation results in the lowest prediction errors. Furthermore, we analyze how different dataset sizes and sensor-actuator pairings affect the frequency predictions.
To my family for their love and support.
Engineering is not something one can do alone or in a vacuum, so I would like to acknowledge those who have had the largest influence on my graduate work. Firstly, I would like to thank my advisor, Professor Timothy Bretl, for his constant contributions. Discussions with Professor Bretl prompted me to reconsider not only how I should showcase my work but also what should be the focus of these efforts. Secondly, I would like to thank other past and present graduate students in the Bretl Research Group who enhanced my graduate experience through both friendly and academically-focused conversations: David Degenhardt, Xinke Deng, Alex Faustino, Felipe Figueroa, Dave Hanley, and Zhenghe Shangguan. In particular, I would like to thank Dave Hanley for all the advice he gave on how to improve my detailed work. Thirdly, I would like to thank Dr. David Carroll and Neil Hejmanowski of CU Aerospace for their recommendations on how to consider the commercialization aspect of developing a technology. Fourthly, I would like to thank my undergraduate research assistants who were essential for the realization of my experimental setup: Kevin Mienta, Ayush Nair, and Zihan (Hunter) Xu. Finally, I would like to thank my family for their constant support and words of encouragement. The material is based upon work supported by NASA under award No(s) 80NSSC18P2132 and 80NSSC20C0020. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Aeronautics and Space Administration.
# TABLE OF CONTENTS

**LIST OF SYMBOLS** ......................................................... vii

**CHAPTER 1 INTRODUCTION** .................................................. 1
  1.1 On-Orbit Demonstrated Distributed Sensing Networks .......... 3
  1.2 Outline ............................................................... 5

**CHAPTER 2 DISTURBANCE SOURCES CONSIDERATIONS** ............... 6
  2.1 Framework Mental Model ............................................. 6
  2.2 Case Studies with Applied Mental Model ......................... 6
  2.3 Proposed Work Mental Model ....................................... 10

**CHAPTER 3 EXPERIMENTAL SETUP AND PROCEDURE** .................... 14
  3.1 Introduction to the Experiments ................................... 16
  3.2 Structure Setup ..................................................... 17
  3.3 Actuator Setup ..................................................... 20
  3.4 Sensor Setup ........................................................ 20
  3.5 Experimental Procedure ............................................ 23
  3.6 Experiments’ Scope Summary ....................................... 25

**CHAPTER 4 ANALYSIS OF EXPERIMENTAL RESULTS** ..................... 27
  4.1 Pre-Processing of Data ............................................. 27
  4.2 Details on Polynomial Regression ................................. 29
  4.3 Polynomial Regression Results and Discussion .................. 38

**CHAPTER 5 GAUSSIAN PROCESS REGRESSION** .......................... 42
  5.1 Details on Gaussian Process Regression .......................... 43
  5.2 Gaussian Process Regression Results and Discussion .......... 52
  5.3 Gaussian Process Regression Coregionalization ................. 55

**CHAPTER 6 CONCLUSION** .................................................. 57

**REFERENCES** ................................................................... 59

**APPENDIX A RELEVANT MANUFACTURER’S PERFORMANCE DATA** ....... 64
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>data point count</td>
</tr>
<tr>
<td>$d$</td>
<td>polynomial degree</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>additive noise</td>
</tr>
<tr>
<td>$f(x_{*,i})$</td>
<td>the $i$th posterior prediction</td>
</tr>
<tr>
<td>$I$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$K$</td>
<td>$n \times n$ covariance matrix</td>
</tr>
<tr>
<td>$k_{*}$</td>
<td>covariance vector when there is only one test case</td>
</tr>
<tr>
<td>$k(x_{<em>}, x_{</em>})$</td>
<td>covariance function evaluated at $x_{*}$</td>
</tr>
<tr>
<td>$m(g(x))$</td>
<td>mean of some function $g(x)$</td>
</tr>
<tr>
<td>$m_{i}$ and $m_{*,i}$</td>
<td>the $i$th median input training (and testing) value</td>
</tr>
<tr>
<td>$n$ and $n_{*}$</td>
<td>number of training (and testing) experiments</td>
</tr>
<tr>
<td>$p_{i}$</td>
<td>the $i$th polynomial coefficient</td>
</tr>
<tr>
<td>$\sum_{p}$</td>
<td>prior covariance matrix</td>
</tr>
<tr>
<td>$\sigma_{n}^{2}$</td>
<td>noise variance</td>
</tr>
<tr>
<td>$\sigma_{y}^{2}$ and $\sigma_{y,*}^{2}$</td>
<td>training (and testing) target variance</td>
</tr>
<tr>
<td>$\sigma_{*}^{2}$</td>
<td>predictive variance</td>
</tr>
<tr>
<td>$w$</td>
<td>vector weights</td>
</tr>
<tr>
<td>$x_{i}$ and $x_{*,i}$</td>
<td>the $i$th training (and testing) input</td>
</tr>
<tr>
<td>$X$ and $X_{*}$</td>
<td>matrix of training (and testing) inputs</td>
</tr>
<tr>
<td>$y_{i}$ and $y_{*,i}$</td>
<td>the $i$th training (and testing) target</td>
</tr>
</tbody>
</table>
Spacecraft operate in a microgravity environment, where the forces and torques of disturbance sources degrade pointing stability. Still, spacecraft developers expect capable systems, able to satisfy pointing stability requirements. To cope with pointing inaccuracies, some reoccurring methodologies are employed. Prior to launch, one can: replace components having high levels of vibration \(^1\), stiffen the structure to change the attenuation and vibration path \(^2\), actively isolate disturbance sources \(^3\) or passively isolate disturbance sources \(^4\), and/or isolate sensors directly \(^5\). If unexpected levels of high frequency pointing error (jitter) develop on-orbit, additional processes exist for managing this error. Methods implemented for previous spacecraft missions include: the detection and correction of jitter through imagery \(^6\), limitation of operational speeds/ranges of components \(^8\), and the tuning of controllers \(^9\). Consequently, an area of interest for future missions is the incorporation of distributed sensing networks for ascertaining a better comprehension of on-orbit performance.

Recently, the National Aeronautics and Space Administration (NASA) published a micro-vibration study \(^1\), authored by Cornelius Dennehy and Oscar S. Alvarez-Salazar, which introduced the micro-vibration field; provided example spacecraft and their experiences with micro-vibration and/or jitter during design, test, and operation; and presented recommendations for future missions. The study augmented the material with a forward-looking section where the authors outlined ways the micro-vibration problem could be better addressed given technological advancements. In the “The Road Ahead” Section, the case was made that a distributed sensing system, composed of accelerometers or laser-based metrology sensors, could improve sensing capabilities on-orbit. A distributed sensing system could compare on-orbit performance to that of the requirements, plus a distributed sensing system could determine the motion of vehicle subsystems relative to each
other. Previously, when trying to discover the cause of and mitigate the effects from micro-vibrations with the Hubble Space Telescope (HST), engineers were unable to accurately determine the vehicle’s mode shapes due to a limited number of sensors [10], reinforcing the Dennehy and Alvarez-Salazar proposed suggestion for a spacecraft distributed sensing network.

With the miniaturization of sensor technology enabling smaller and lighter systems, spacecraft developers are turning to smaller, more cost effective spacecraft options. Often, these miniaturized spacecraft are referred to simply as ‘small satellites’, and some small satellites are even constructed for compliance with a small, 10 cm cube form factor [11]. Accordingly, the design and operation of these small satellites differs from that of their larger counterparts, and as such, a new community formed to further advance the small satellite field. Since 1987 Utah State University has hosted the SmallSat Conference, a venue dedicated to small satellites and their technologies. When searching through the SmallSat Conference proceedings, a general trend can be observed. As seen in Figure 1.1, four terms related to this thesis have been analyzed: Jitter, Stability, Vibration, and Disturbance. Overall, the number of sources referencing the respective search terms follows an upwards trend. The upwards trend coupled with the argument for distributed sensing presented in the previous paragraph suggests the implementation of a distributed sensing network for use in small satellites is growing in significance and associated publication opportunities likewise are growing.

The contributions of this thesis are twofold. Firstly, we created a functional distributed accelerometer sensor network, introducing a pipeline for distributed sensing in small satellites on-orbit. And secondly, we show how to implement and quantify the performance of supervised learning for spacecraft disturbance source characterization, something previously with limited publications. See Section 1.1 for the summaries of three other on-orbit distributed sensing systems. With our functional prototype we:

1. Operate actuators to induce micro-vibrations on a test article
2. Collect specific force measurements with distributed accelerometers
3. Implement Fourier transforms on saved data
4. Create regression models
5. Predict actuator operational frequencies

6. Quantify the goodness of fit for the presented models

Figure 1.1: Number of Small Satellite Conference sources mentioning relevant terminology. We plotted the number of sources from 1987 through 2019 using the search terms Jitter, Stability, Vibration, and Disturbance. Additionally, we searched for any combination of the preceding search terms within a respective year and plotted that total number of terms with orange, triangle markers. A line of fit for this combinatorial search term dataset has been computed and plotted as well, and the total number of publications having any of the four search terms increases over time following the form \( y = 1.48x - 2910 \), with \( R^2 = 0.503 \), indicating there is a moderately positive linear relationship \[12\].

1.1 On-Orbit Demonstrated Distributed Sensing Networks

In this section three cases of distributed sensing systems realized for on-orbit testing are provided. The first two cases did not utilize any learning methods, as these cases of distributed sensing systems compared disturbance data prelaunch to postlaunch. The third case, on the other hand, did use learning methods/pattern recognition to analyze disturbances.
The Japan Aerospace Exploration Agency (JAXA) equipped their spacecraft Optical Inter-orbit Communications Engineering Test Satellite (OICETS) with three, tri-axial accelerometers for measuring micro-vibrations with respect to laser communication pointing requirements. In short, comparisons of OICETS power spectral density (PSD) data between tests on the ground and on-orbit were analyzed for understanding the realized performance, and these on-orbit PSD results followed the same trends but with lower magnitudes than the worst case ground tests [13].

The National Centre for Space Studies (CNES) equipped their spacecraft Satellite Probatoire de l’Observation de la Terre (SPOT) 4 with ten distributed, pendular accelerometers; one piezoelectric accelerometer; and one mini-shaker, where the shaker enabled the determination of frequency response functions (FRFs) of the vehicle both on the ground and on-orbit. In a comparison of the SPOT 4 FRF data, low frequency harmonics from the tests conducted prior to launch were not observed on-orbit [14].

The International Space Station (ISS), used by NASA and others to conduct scientific experiments on-orbit, was equipped with its own version of a distributed sensing system. The Space Acceleration Measurement System-II (SAMS-II) was a system designed for measuring acceleration values at payload locations within the ISS for aiding researchers. In particular, the SAMS-II utilized Kohonen’s Self-Organizing Feature Mapping, Learning Vector Quantization, Back-Propagation Neural Networks, and Fuzzy Logic to detect disturbances, inform researchers of the disturbance sources, and inform researchers on the confidence of the measurements. From this supervised learning approach, researchers were able to relate experimental results to the environmental conditions during the experiments. Two limitations to SAMS-II, however, were 1.) the lag time between measurements and their classifications and 2.) the mass and size of the system. Specifically, the SAMS-II hardware was allocated 46 kg for the Control Unit, 5 kg for each of the Remote Triaxial Sensor (RTS) Electronics Enclosures, and 1.5 kg for each of the RTS Sensor Enclosures [15, 16, 17].

Technological advancements from electronics miniaturization, increases in computational performance, and improvements in algorithm development are favorable for implementing a distributed sensing network capable of source identification and micro-vibration characterization in vehicles smaller than the ISS. Our work aims to leverage these advancements for establishing a
framework to integrate disturbance source characterization through supervised learning in small satellites.

1.2 Outline

In Chapter 2, we define the conceptual terminology frequently used throughout this thesis and relate each term to on-orbit manifestations of the concepts. Additionally, we provide details on what facet of the micro-vibration/jitter campaign we aim to improve. In Chapter 3, we depict the prototype constructed for furthering the disturbance source characterization framework, describe our experimental methodology, and outline how we limited the scope of our experimental work. In Chapter 4, we explain how we processed our accelerometer specific force measurements and subsequently created and implemented polynomial regressions for predicting the operational frequency values of our testing case experiments. And similarly, in Chapter 5 we explain how the processed accelerometer data was implemented in Gaussian process regressions for predicting the operational frequency values of our testing case experiments.
CHAPTER 2

DISTURBANCE SOURCES
CONSIDERATIONS

2.1 Framework Mental Model

To ensure clarity with terminology throughout this thesis, we base our vocabulary on the vocabulary established by Dennehy and Alvarez-Salazar [1]. Disturbance sources result as undesirable byproducts of non-stationary systems. Reaction wheels, cryocoolers, and drive mechanisms are three example, internal disturbance sources; and in particular, a mass imbalance in rotating componentry could be the root cause for a disturbance. Disturbance sources result in dynamic interactions, or interactions between the spacecraft subsystems due to mechanical coupling. Likewise, the dynamic interactions result in micro-vibrations, where micro-vibrations are oscillatory accelerations/specific forces, measurable through accelerometers. Finally, line-of-sight jitter or simply jitter results as a consequence of micro-vibrations, with jitter being the high frequency angular motion [18], quantified as pointing error. This error, shown as an outcome in Figure 2.1, results in the degradation of image quality with telescope optics; so minimizing jitter is integral with satisfying sensitive optical sensor requirements. While in the selected publication systems responsible for perturbations are referred to as disturbance sources, micro-vibration sources, and jitter sources, we solely refer to these sources as disturbance sources.

2.2 Case Studies with Applied Mental Model

This section contains two prominent NASA missions to give examples of disturbance effects with on-orbit vehicles, and each example mission will be linked to the mental model terminology defined in the previous section. Here,
Figure 2.1: Disturbance model for high frequency spacecraft perturbations. With high frequency disturbances, the body can no longer be assumed as rigid, meaning the sensor pointing error is no longer a combination of attitude error and alignment error. For this reason, the attitude error and pointing error must be evaluated separately [19]. Disturbance sources affect the structural dynamics of a spacecraft, resulting in dynamic interactions leading to micro-vibrations. These vibrations perturb sensors and optics, producing attitude and pointing errors. Attitude control and/or pointing control can reduce the errors, though the control also can cause a feedback loop. With our work, we aim to characterize the disturbance sources after measuring the micro-vibrations throughout a test article.

we chose to highlight particular missions cited in the Dennehy and Alvarez-Salazar publication [1] in which a functional distributed sensing network would have been beneficial, as will be illustrated in Section 2.3.

The Hubble Space Telescope, launched in April 1990 to an approximate 600 km circular orbit, functions as an astronomical telescope. This NASA spacecraft was designed to maintain a pointing stability of 0.0070 arc-seconds over a 24 hr period. After launch the HST Pointing Control System (PCS) was unable to satisfy the preceding pointing requirement, as disturbances perturbed the system’s stability, reaching pointing errors around 0.10 arc-seconds. Orbital day and night terminator crossings produced these high levels of pointing error, resulting from the Solar Array (SA) booms experiencing out-of-plane deflections and the SA mechanisms releasing stored thermal/mechanical energy. To modify the PCS for mitigating jitter, a dynamic model was created with On-orbit Transfer Function Tests which computed a transfer function of output gyroscope angular rate from input reaction wheel
torque. Through the tests the modal gain factor, frequency, and damping related to the disturbances were characterized for use in updating the vehicle’s control laws. Furthermore, the HST SAs underwent a redesign, with the changes being implemented during the service mission of December 1993. The combined control and hardware modifications resulted in HST being able to maintain the original 0.0070 arc-seconds pointing requirement for 95% of the orbit and 0.012 arc-seconds for 100% of the orbit [9, 10, 20].

- **Disturbance Source:** The on-orbit realized disturbance sources perturbing the HST were the SA Storable Tubular Extendible Member booms, spreader bars, pulleys, cable/pulley mechanisms, drums, and clutch-type brakes. These systems facilitated movement and likewise energy transfer in the form of dynamic interactions [20].

- **Dynamic Interactions:** The disturbance sources dynamically interacted with the HST solar arrays’ natural vibration modes, amplifying orbital day and night crossings’ effects on detector stability [10].

- **Micro-vibration:** N/A. No comprehensive micro-vibration analysis was possible on-orbit due to limitations in sensors. Additionally, guidance sensors were limited to less than 20 Hz [21].

- **Jitter:** Implementation of fast Fourier transforms (FFTs) on High Speed Photometer sensor data found that significant portions of HST jitter from the micro-vibrations transpired below 5 Hz, though additional, smaller modes were found between 15-30 Hz and at 61 Hz [21]. The allowable upper limit on jitter prelaunch was 0.0070 arc-seconds, however, an approximate jitter level of 0.10 arc-seconds was reached on-orbit until modifications in hardware and software enabled jitter levels of 0.012 arc-seconds or below for the entire orbit [9, 20].

- **Disturbance Identification:** The SAs were determined as the disturbance sources responsible for the unanticipated levels of jitter through the process of elimination in that the micro-vibration frequencies computed from on-orbit data prevailed closest to the solar array predicted fundamental bending mode frequencies. Furthermore, an observed beating phenomenon with harmonic oscillations indicated there were either two disturbances with closely spaced frequencies or two closely
spaced structural modes. Given there existed two SAs, this phenomenon reinforced the conclusion that the arrays were the disturbance sources. Finally, because of the arrays’ flexibility, the logical argument was made that the effects from orbital transition thermal gradients would disturb this system the most.

The Solar Dynamics Observatory (SDO), launched in February 2010 to a geosynchronous orbit, functions as a NASA research platform for Sun studies. This mission centers on the operation of three payloads: the Atmospheric Imaging Assembly (AIA), the Helioseismic and Magnetic Imager (HMI), and the Extreme Ultraviolet Variability Experiment, with the HMI detector requiring the most stringent pointing stability at 0.094 arc-seconds. Despite considerable baseline jitter modeling and analysis, several uncertainties lingered from the lack in structural finite element modeling above 50 Hz and the lack in instrument stabilization performance characterization. Although component-level disturbance tests were conducted, the reaction wheels for attitude control and the High Gain Antenna (HGA) assemblies for tracking and communicating with a ground station both raised concerns for meeting the sensor pointing stability requirements. Given limitations in time; money; personnel; and ability to risk damaging flight hardware, SDO was launched with these uncertainties, citing on-orbit contingency plans if deemed necessary. After testing on-orbit, the initially imposed reaction wheel operational limit of ±400 RMP was raised to ±800 RMP while still meeting stability requirements, whereas operation of the HGAs resulted in HMI jitter above 0.12 arc-seconds. To combat this jitter, two mitigation techniques were employed with respect to the HGA gimbals: stagger stepping and the No Step Request (NSR). Stagger stepping refers to only operating one set of gimbals at a time, to avoid any constructive behavior of multiple source interactions. Furthermore, during times when the AIA or HMI have open shutters, the NSR enables a request for delaying gimbal actuation to reduce jitter during these critical times. With stagger stepping and NSR, the jitter at the HMI was minimized below 0.050 arc-seconds while imaging, thus maintaining an acceptable pointing stability level.

- Disturbance Source: The on-orbit realized disturbance sources perturbing the SDO were the High Gain Antenna arrays, particularly the gimbals (stepper motors) with integrated harmonic drive gearboxes.
This system facilitated movement and likewise energy transfer in the form of dynamic interactions [8, 23].

- Dynamic Interactions: The disturbance sources dynamically interacted with each other, where the ringdown (residual actuation during the time span required to power down) of one gimbal set constructively interacted with the successive stepping motion of the other gimbal set [23].

- Micro-vibration: N/A. No comprehensive micro-vibration analysis was possible on-orbit due to limitations in sensors. Additionally, the Attitude Control System flight software was limited to 5 Hz, meaning high frequency oscillations above this threshold were unobservable with this system [23].

- Jitter: On-orbit, PSD computations as measured at the HMI transpired between 50-70 Hz [8]. Although the allowable upper limit on jitter was 0.094 arc-seconds, an approximate maximum magnitude of 0.12 arc-seconds was reached until modifications in software enabled jitter levels to remain below 0.050 arc-seconds while imaging [23].

- Disturbance Identification: The HGAs were determined as the disturbance sources responsible for the unanticipated levels of jitter through the systematic testing of different operating conditions. Prior to launch, tests and modeling predicted anticipated disturbance sources which could produce levels of jitter exceeding requirements. The reaction wheels could operate on-orbit at speeds above originally expected without exceeding jitter requirements, indicating this was not an unsatisfactory disturbance source during nominal operating conditions. Modifications in the actuation of antenna gimbals, however, proved to lower the excessive jitter levels of the system, demonstrating that the HGAs generated the high levels of jitter as considered feasible prelaunch [22, 23].

2.3 Proposed Work Mental Model

While mission requirements dictate the level of design, modeling, and analysis conducted for a jitter assessment, and these vary mission by mission,
generally the process recommended for determining stability performance predictions follows the format seen in Figure 2.2. Like other spacecraft assessments, tests pertinent to jitter determination initiate with component-level characterization efforts and eventually are conducted with the integrated components. On-orbit, the jitter can be characterized for the overall vehicle, but current methods do not support a reversal of the testing process where components can be recharacterized or disturbance sources are directly identified. Furthermore, with a limited number of sensors on-board (if no distributed sensor network is in place), the details on micro-vibrations from dynamic interactions throughout the structure are unobservable.

Figure 2.2: Recommended jitter testing methodology, initiating with individual component tests and analysis and advancing to system level tests and analysis [1]. With true in-flight/on-orbit jitter performance being unknown prior to launch, the characterization effort can incorrectly describe the response of the vehicle. On-orbit modifications can then become necessary to address jitter performance concerns, though usually only system level performance can be quantified once on-orbit. Our work intends to identify and characterize component-level disturbances on-orbit, something not currently standard practice.

For a myriad of reasons, the on-orbit stability performance may not match that of the predictions. Computational resources are limited meaning every detail cannot be modeled. Restrictions in time and money may reduce testing prelaunch. Dynamic interactions from the space environment can be overlooked. Components can be damaged during launch or degrade with use over time. Whatever the case may be, the ability to observe the micro-
vibrational effects of disturbance sources not only at the payload(s) of importance but also at individual subsystems would be advantageous for managing disturbances and adapting to the situation at hand. The Hubble Space Telescope and the Solar Dynamics Observatory, both featured in Section 2.2, required on-orbit modifications for some of the mentioned reasons to meet requirements, though for both missions information was lacking on how the micro-vibrations propagated throughout the vehicles. Cause and effect analyses were needed to deduce the largest sources of disturbance for the HST and SDO vehicles, where individual systems were tested to see how their operation affected stability. With additional, distributed sensors strategically placed near possible disturbance sources, the vehicles’ mode shapes could have been determined. Also, extra sensors could have aided with the hypotheses of which sources caused the unexpected jitter, based on the on-orbit operational performance. Complementing additional sensors, a supervised learning approach, similar to the one implemented for the ISS SAMS-II, could have aided in realizing micro-vibrational monitoring and disturbance source identification for the HST and SDO vehicles.

When considering the implementation of a supervised learning approach for examining spacecraft disturbance sources, several approaches could be pursued. Firstly, the disturbance source training data for the supervised learning could originate exclusively from real-world tests. As seen in Figure 2.2, Bus + Payload Tests should be conducted in the jitter characterization campaign for missions with stringent jitter requirements. These tests could capture data before launch, when testing personnel have time to create comprehensive datasets. Effects from the operation of reaction wheels, cryocoolers, and other actuators could be examined. On the other hand, disturbance source training data could be created through dynamic simulation software such as with the Adams Multibody Dynamics Simulation Solution. Finally, a third option for creating the supervised learning disturbance source training data is to combine real world disturbance source data with simulation data. As previously stated, creating a model that captures the dynamics of a complex spacecraft is challenging, so that is a disadvantage of utilizing simulations as the learning data. Conversely, disturbance source testing with spacecraft hardware is risky, especially considering the price tag associated with some missions. Comparatively, the simulations pose no threat to a vehicle’s integrity, so many tests can be conducted without worry. Further-
more, these simulations could be conducted after launch, giving engineers
the ability to improve their work as they understand the vehicle better. An
advantage to the hardware tests is with this method it would be easier to
add actuators or other disturbance source components for perturbing the
vehicle in addition to the nominal perturbations from the operation of the
aforementioned spacecraft components; and in this way, one could create
learning case data outside of what is notionally expected. Given the ease of
entry for testing with hardware over that of creating a simulation model, we
have elected to develop a supervised learning model based on the tests with
hardware experimental data. The rest of the thesis aims to document the
process in which we went from four sets of specific force measurements to
that of predicting the actuation frequencies of our hardware.

The previous paragraph introduced how the process for disturbance source
colorization through distributed sensing and supervised learning is not
standardized or completely obvious. The questions we thus focused on and
aim to answer are as follow:

1. What hardware is necessary to create a proof-of-concept disturbance
source characterization experimental setup?

2. How do you evaluate collected sets of specific force measurements for
creating a regression model capable of predicting disturbance source
frequency values?

3. What comes next? In what way can one improve the prediction results?

To quantify the goodness of our disturbance source actuation frequency
predictions throughout this thesis, we computed performance metrics often
associated with supervised learning. The standardized mean squared error, a
metric for the “squared residual between the mean prediction and the target
value” \[^{24}\], was the first performance metric we calculated; and the mean
standardized log loss (MSLL), the “negative log probability of the target
under the model” \[^{24}\], was the second performance metric we calculated,
where the target was our experiments’ actuation frequencies. Details on the
experimental hardware and data processing will be presented in Chapters 3
and 4, respectively.
CHAPTER 3  
EXPERIMENTAL SETUP AND PROCEDURE

As introduced in Section 2.3, regularly complete, system level characterization tests are conducted on spacecraft with stringent pointing error requirements. In the literature, details on the European Space Agency (ESA) and NASA’s Solar and Heliospheric Observatory spacecraft analyses, testing, and flight results can be found [25]. In this publication, the authors report on the micro-vibration tests conducted prior to launch. For simulating the boundary conditions of space, free-free, the vehicle was isolated with soft slings. Additionally, approximately 40 distributed accelerometers were temporarily installed near sensitive payloads for this characterization effort. Reaction wheels, the Coronal Diagnostic Spectrometer supports, and the Solar Ultraviolet Measurements of Emitted Radiation scan/focus mechanism were all operated to study the effects each disturbance source had on the jitter at the payload instruments.

With disturbances interfacing to sensors by way of spacecraft structures, stability requirements are defined at the disturbance source locations as well as at sensor/receiver location(s). As a result, micro-vibrational budgets can be allocated at each of these disturbance source locations [26]. To compute performance and satisfy stability requirements, Hardware-in-the-Loop tests are conducted prelaunch with additional sensors, and a Finite Element Model (FEM) compliments the hardware tests as simulated predictions [25]. However, 1-g gravity effects and test facility environmental effects influence Hardware-in-the-Loop tests’ results; and FEM’s results’ accuracy is degraded by the simulated frequency, where FEM’s accuracy ordinarily decreases as the frequency is increased [1]. Thus, reassessing disturbance source micro-vibrations on-orbit would facilitate a systematic way to constantly monitor if systems are within their bounding micro-vibrational limits. Additional sensors would aid in early pointing performance degradation detection and subsequently provide better observability, enabling improvements in the hy-
potheses of disturbance sources requiring a redefining of operational limits to reduce vibrations. See Figure 3.1 for an overview on how a distributed sensing system would outperform the current sensing methods with regards to managing disturbance sources and jitter once on-orbit. While HST and SDO were able to redefine their concept of operations to minimize jitter without distributed sensors [4, 20, 22, 23], a distributed sensing network, such as the one in development for this thesis, would have enabled for the micro-vibrations at disturbance sources to be observed and compared against their requirements, reducing the number of trial and error tests conducted for determining the sources causing unexpected levels of jitter.

Figure 3.1: Simple illustration of two differing methods for jitter detection and mitigation. Typically, the only disturbance metric monitored autonomously is jitter, due to limitations in sensors. If jitter requirements are not satisfied, human response becomes necessary for conducting tests to determine the sources violating the requirements. With a distributed sensing network offering latency and observability advantages, the observation of micro-vibrations at disturbance sources allows for comparisons of realized micro-vibrations against the requirements, acting as an early warning system for degradation in performance, and thus distributed sensing reduces the number of candidate sources an operator must inspect to determine the cause of unexpected jitter levels.

Fixing distributing sensors in a spacecraft would enable for the disturbance source micro-vibrations to be measured on-orbit, and we hypothesize that supervised learning enables predictions of micro-vibration frequencies and amplitudes for these on-orbit spacecraft. Ultimately, the end product of our
work will predict at predetermined disturbance source locations, and we per-
form the first step in realizing the on-orbit disturbance source micro-vibration
analysis tool by utilizing a mock small satellite and predicting disturbance
source frequencies by way of learning data produced with the simplified ex-
perimental setup (see Section 3.6 for a summary of the experimental scope).

We constructed an experimental platform composed of hardware neces-
sary for a proof-of-concept disturbance source characterization system—a
structure, disturbance sources, and sensors—imitating key systems on real
spacecraft pertinent to disturbance characterization, and we subsequently
performed systematic experiments in fixed, narrow intervals within a prede-
fined frequency range similar to known spacecraft disturbance frequencies.
Then we generated a learning dataset to map the measurements to the re-
spective disturbance source actuation frequency targets. With this dataset,
implementing supervised learning methods such as Gaussian process regres-
sion (GPR) for identifying the operational frequencies of disturbance sources
becomes possible.

3.1 Introduction to the Experiments

Typically, the first few modes of vibration (fundamental modes) occur in the
10-100 Hz regime [26]. Notably, while disturbance characterizing the SDO
reaction wheels at a speed of 850 RPM, the maximum axial force occurred
around 75 Hz, and the maximum radial forces occurred around 48 Hz and
76 Hz [27]. Similarly, the HST Reaction Wheel Assemblies at a speed of 800
RPM produced the maximum force with high-frequency harmonics around 80
Hz [28]. Taking into consideration the preceding information, we decided to
mimic fundamental modes resembling that of these reaction wheel assemblies
for our experiments, as reaction wheels are a reoccurring disturbance source
in many spacecraft. Details on these experiments will be presented in Section
3.5.

The construction of a mock small satellite system approximately the size
of a 3U CubeSat [11] enabled us to conduct experiments and iterate on our
development of the disturbance source characterization process. What we
refer to as the ‘test article’ has been created with three main systems: the
structure system, the actuation system, and the sensor system. For this
work, we employed four actuators and four sets of accelerometers, where the actuators were used to emulate reaction wheel peak frequency values. The simplified representation of the experimental setup can be viewed in Figure 3.2, and the entire test article experimental setup can be viewed in Figure 3.3. In the following sections, details on the hardware systems and the experimental methods will be provided.

![Figure 3.2: Simplified representation of experimental setup. Two differently sized disturbance source actuators were installed on the test article, and the locations of the two sizes are shown by the circles' sizes in the image. The orange arrows represent the direction of actuator mounting and of actuation, where one of each actuator size was actuated for the x-axis and the y-axis. Each actuator was accompanied by a tri-axial accelerometer, and in this way we formed a distributed sensing system. All accelerometers collected specific force measurements for all experiments, independent of which actuator was operated.](image)

### 3.2 Structure Setup

The structure for this project was created with a mentality of modularity, and as such, the structure has an outer rail system for mounting hardware and notches in the inner area for incorporating adapter plates. The structure seen in Figure 3.4 was designed with proportions similar to that of a 3U CubeSat [11] as previously mentioned, and the mass was compliant with the specified
Figure 3.3: Complete experimental setup, barring desktop workstation necessary for data saving. Three out of four disturbance source actuators and accelerometers can be seen in this view, with the accelerometers being mounted on black brackets directly above the metal actuators. The Arduino board for data collection was mounted on the orange plate partially seen in the upper one-third of the test article, and the Arduino boards used for controlling the actuators were situated on the tabletop. Additional hardware such as the amplification boards and associated circuitry was placed on the tabletop also.
mass limit of this spacecraft class as well.

Figure 3.4: Isometric view of Siemens NX computer-aided design (CAD) model for the test article structure. The 3D printed structure enabled for multiple actuator and sensor configurations to be examined throughout the project’s duration.

The combined mass of the test article with mounted componentry as suspended and seen in Figure 3.3 was 2.0 kg, half of the maximum allowable total mass for a 3U CubeSat [11]. The suspension system which upheld and isolated the structure consisted of four soft, elastic bands, mimicking the soft slings used with flight model spacecraft [25]. When setting up the complete experimental system, the test article was leveled within $\pm2.5^\circ$ of horizontal for the x-axis and y-axis by way of a digital level.
3.3 Actuator Setup

The actuation system required a collection of components to accomplish the task of inducing a micro-vibrational disturbance upon the test article structure. Following preliminary experiments, the Adafruit Large and Medium Surface Transducers, which we refer to as the ‘actuators’ throughout the duration of this thesis, were not being supplied enough power from the Arduino MEGA 2560 boards to yield distinct peaks in Fourier transform analyses. As a result, audio 2.5W Class D Amplifiers were incorporated into the design, to compliment these permanent magnet coil speaker actuators. With the inclusion of the amplifier boards, the power consumption at each disturbance source actuator was increased from approximately 0.01 W to 0.10 W, though these values were not consistent among experiments as the actuator output varied with differing frequencies (see Appendix A).

As seen in Figure 3.5, the amplifier boards utilized power from a power source—a battery bank—to amplify the signal provided by Arduino boards. Save for the actuators themselves and the necessary wiring, all actuation system hardware resided on the tabletop as seen in Figure 3.3, as this hardware would not need to fit into a flight model spacecraft for launch into space. The powered signal wiring which went from the amplifier boards to the actuators was flexible to minimize effects from these wires on the experimental results. Details of the experiments will be provided in Section 3.5, and detailed photos for the actuation system components can be seen in Figure 3.6.

3.4 Sensor Setup

Central to this project was the distributed sensing system, as this system was responsible for the detection and recording of the micro-vibrational disturbances. Consequently, this system was constituted of a few key electronics: the accelerometers, the data acquisition system, and the data storage system.

The selected accelerometers for this study were the Analog Devices ADXL355 accelerometers, specifically the EVAL-ADXL355Z accelerometers, evaluation boards with mounting holes and solderable breadboard holes. We installed four of these evaluation boards on the test article, and these accelerome-
Figure 3.5: Simplified representation of the disturbance source actuator setup. The signal from an Arduino board, reprogrammed for each of the experiments, was amplified by an audio amplifier board which was powered from a battery bank. The amplified signal was transmitted to the actuators for inducing the disturbance on the test article.

ter boards communicated with the data acquisition system through the \text{I}^{2}\text{C} protocol. These sensors were selected as they are tri-axial, digital accelerometers with \( \pm 2g \), \( \pm 4g \), or \( \pm 8g \) outputs. Given the magnitude of disturbances we induced on the test article, we restricted our sensor outputs to the \( \pm 2g \) range, while keeping the high pass filter set to off and the low pass filter set to 1000 Hz, the default settings\cite{29}. With the selected data acquisition system, we sampled above 180 Hz with each of the four sensors, or approximately 725 total samples per second. As described by the Nyquist–Shannon sampling theorem, the sample collection rate is sufficient if the sample collection rate is at least twice that of the sample rate of interest. Therefore, we could effectively capture disturbance source frequencies of up to 90 Hz for each sensor with this iteration of the test setup, which was the actuator operational frequency upper limit we set for our training and testing case experiments.

These days, many data acquisition systems options exist. Predominate current data acquisition options include microcontrollers, microprocessors, field programmable gate arrays (FPGAs), application-specific integrated circuits (ASICs), and dedicated devices such as the National Instruments Mul-
Figure 3.6: Close-ups of main components necessary for the disturbance source actuation system. (a) Adafruit Mono Class D Audio Amplifier (b) Arduino MEGA 2560 (c) Adafruit Large Surface Transducer (d) Adafruit Medium Surface Transducer. The test article system had four amplifiers, two Arduino MEGA boards, two large surface transducers/actuators, and two medium surface transducers/actuators.

tifunction I/O products. As a flight version of this distributed sensing system for small satellites would necessitate a minimization of power and size, microprocessors and dedicated devices were not considered past the concept exploration stage of the project. Microcontrollers, FPGAs, and ASICs all were then still viable options, as these three data acquisition systems are repeatedly employed in spacecraft systems. However, due to the barriers to entry with FPGAs and ASICs, these systems were not preferred for use either at this time, and thus we decided to utilize a microcontroller as our data acquisition system. In particular, we wanted to find a preexisting microcontroller evaluation board for purchase, similar to how the EVAL-ADXL355Z
accelerometer board was selected. The Arduino MKR ZERO board was chosen for its Microchip Technology SAMD21 microcontroller, where this 32-bit microcontroller has a clock speed of 48 MHz, a flash memory of 256 KB, and 32 KB of SRAM. When conducting trade studies on the computation power of current Arduino boards, the SAMD21 was the most powerful microcontroller available for purchase, hence the selection of the MKR ZERO with its SAMD21 microcontroller. See Figure 3.7 for detailed photos on the selected accelerometer and Arduino board with its microcontroller.

![Image of Analog Devices EVAL-ADXL355Z and Arduino MKR ZERO](image)

Figure 3.7: Close-ups of main components necessary for the distributed sensing system. (a) Analog Devices EVAL-ADXL355Z (b) Arduino MKR ZERO. The test article system had four of the Analog Devices accelerometers and one Arduino MKR board, where the former measured specific force values and the latter collected and prepared these values for storage.

While trying to maximize the experiment sample collection rate, we explored a couple different methods for storing data. Storage with a microSD was considered, though the nontrivial time to read-write with this method was unacceptable for our needs. In the end, our method of transferring and storing data was through serial printing over a USB cable to a 64-bit Windows 10 desktop computer in conjunction with the software CoolTerm.

3.5 Experimental Procedure

In total for this project we conducted 68 training case experiments and 64 testing case experiments. Training case experiments refers to the experi-
ments conducted for training/fitting the model, while testing case experiments refers to experiments conducted for tuning model parameters or coefficients and for evaluating said model’s fit. All experiments for both types of cases had the same number of specific force measurements. Specifically, all experiments were programmed with each tri-axial accelerometer collecting 62,500 samples, totaling 250,000 samples between the four distributed sensors. Given the number of samples and the collection data rate of ~180 Hz (per sensor), each experiment required around 5 minutes to complete, excluding setup time.

In developing the details to our experimental procedure, we examined a component-level disturbance characterization testing method for a mission with flight heritage. In particular, we wanted to imitate, at a fundamental level, how the mission’s reaction wheels were characterized, since reaction wheels were the disturbance source we most closely represented with our selected experimental actuator operational frequencies. Notably, in order to characterize disturbances prior to launch, ground validation tests were completed with respect to NASA’s SDO vehicle. Component-level disturbance tests were conducted on SDO’s reaction wheels, with two types of tests being conducted. Firstly, the team tested the reaction wheels with a ramp up acceleration rate of 0.1 RPM/sec or 1.0 RPM/sec depending on the wheel speed. Secondly, the team completed constant/dwell speed tests, where they held a constant wheel speed for 2-5 minutes to determine the associated jitter at payloads of interest [27], as the ability to maintain a constant wheel speed is important on-orbit.

Considering the testing methodology of the preceding real world mission coupled with our plan for implementing a supervised learning approach with distinct frequency targets for each experiment, we operated the disturbance source actuators at a constant frequency for each respective experiment for marginally more than 5 minutes. The time was pushed to the upper limit of the SDO testing time frame to maximize available data since the experiments’ time could be trimmed later if so desired. The project’s selected actuators, the speaker-like surface transducers, enabled for the straightforward use of the Arduino function `tone()` for commanding the frequency of actuation. Our experimental operational frequency values were chosen based on the SDO noted key reaction wheel frequencies of ~50 Hz and ~75 Hz as introduced in Section 3.1.
All training or testing experiments were conducted within the 50 Hz to 90 Hz range. For any selected frequency within this range, we conducted four learning case experiments, one experiment with each of the four actuators. Our 132 experiments, 68 training case experiments and 64 testing case experiments, resulted in each actuator being operated thirty-three times. Training case experiments were conducted at frequency values of \( f = \{50, 52.5, 55, 57.5, \ldots, 90\} \) in units of Hz, and testing case experiments were conducted at frequency values of \( f = \{51.25, 53.75, 56.25, 58.75, \ldots, 88.75\} \) in units of Hz. These frequency values were selected since they allowed unique testing and training frequency values. Results using these experiments and the objective performance metrics will be discussed in Chapter 4.

3.6 Experiments’ Scope Summary

In this section we summarize the scope of experimental work undertaken for this thesis. In Table 3.1 the approach we took for creating a distributed sensing network for disturbance source characterization and the approach one should take for creating a sensing and actuating system more closely resembling a spacecraft flight-like system is presented. In the NASA Systems Engineering Handbook, NASA overviews how a Technology Maturity Assessment is used to determine the maturity of a technology based on a Technology Readiness Level (TRL) from 1-9, with 1 being “basic principles observed and reported” and 9 being “actual system ‘flight proven’ through successful mission operations” [31]. Also in Table 3.1 we estimated the TRL we demonstrated with this work.

Time, money, and expertise factors influenced our scope of work, and while we estimated that this work was at a TRL of 4 (“component and/or breadboard validation in laboratory environment” [31]), there is more work one can do, even within the TRL 4 rating, to improve the capabilities and the degree of reality to which a system mimics our envisioned flight version of the technology. Therefore, our future work will target these identified next steps.
Table 3.1: Scope for experiments conducted. Listed are the systems relevant to our experiments with how we realized said systems, how to increase the maturity level of this technology, and what Technology Readiness Levels are associated to each system or method.

<table>
<thead>
<tr>
<th>Realized System or Method</th>
<th>Current TRL</th>
<th>Improved System or Method</th>
<th>TRL with Next Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actuation</td>
<td>4</td>
<td>Actuation</td>
<td>4</td>
</tr>
<tr>
<td>Control of disturbance source frequency but not force</td>
<td></td>
<td>Control of disturbance source frequency and force</td>
<td></td>
</tr>
<tr>
<td>Actuation</td>
<td>4</td>
<td>Actuation</td>
<td>4</td>
</tr>
<tr>
<td>Linearly reciprocating surface transducers</td>
<td></td>
<td>Reaction wheels or other actual spacecraft hardware</td>
<td></td>
</tr>
<tr>
<td>Actuation</td>
<td>4</td>
<td>Actuation</td>
<td>4</td>
</tr>
<tr>
<td>One disturbance source actuator in operation per experiment</td>
<td></td>
<td>Multiple disturbance source actuators in operation per experiment</td>
<td></td>
</tr>
<tr>
<td>Computation</td>
<td>4</td>
<td>Computation</td>
<td>4</td>
</tr>
<tr>
<td>Desktop computer for data saving, processing, and prediction</td>
<td></td>
<td>On-board data processing and prediction</td>
<td></td>
</tr>
<tr>
<td>Hardware Location</td>
<td>4</td>
<td>Hardware Location</td>
<td>4</td>
</tr>
<tr>
<td>Accelerometers adjacent to actuators, with fixed locations for all experiments</td>
<td></td>
<td>Varying distances between accelerometers and disturbance source actuators</td>
<td></td>
</tr>
<tr>
<td>Sensor</td>
<td>4</td>
<td>Sensor</td>
<td>4</td>
</tr>
<tr>
<td>Sequential sensor data collection</td>
<td></td>
<td>Simultaneous data collection from multiple sensors</td>
<td></td>
</tr>
<tr>
<td>Structure</td>
<td>4</td>
<td>Structure</td>
<td>4</td>
</tr>
<tr>
<td>3D printed</td>
<td></td>
<td>Metal</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 4

ANALYSIS OF EXPERIMENTAL RESULTS

To develop a disturbance source characterization technique for small satellite applications, we formulated a process for exploiting features in our experimental results. The experimental specific force measurements (see Section 3.5 for experimental procedure details) were evaluated following the process shown in Figure 4.1. In the end, we employed two types of regression—polynomial regression and Gaussian process regression—to estimate statistical relationships between accelerometer specific force magnitude and frequency values to the disturbance source actuation frequency target values. The polynomial regressions served as a baseline for comparing to a supervised machine learning approach and to aid in the creation of the supervised machine learning regressions as well. The supervised machine learning approach we employed, Gaussian process regression, offers advantages over the typical polynomial regressions, as the former regression type enables for the computing of predictive covariance [24].

To create the baseline analysis of experimental results, we regressed with low dimensional features from the datasets created with the four distributed accelerometers located on our test article. Our primary objective with these analyses was to determine the relationship between actuator operational frequency values and accelerometer readings, and the end result of each analysis was the prediction of the actuator operational frequency for each testing case experiment. The performance metric SMSE, introduced in Section 2.3, was used to quantify the error of these predictions.

4.1 Pre-Processing of Data

Collect Accelerometer Measurements:

The first step to our disturbance source characterization technique was
Figure 4.1: Process for disturbance source characterization using distributed sensors, which started with conducting experiments and collecting the accelerometer specific force measurements. We then utilized three algorithms, with the initial algorithm performing frequency analysis on the accelerometer data and saving the respective frequency analysis results as new files, for importing into subsequent analyses. The second algorithm sorted in descending order the specific force magnitudes for each respective experiment. The magnitude and associated frequency values were paired, and a reduced set of pairs was saved. Finally, in the third set of algorithms we implemented regressions, predicted the actuation frequencies of the testing case experiments, and computed performance metric values.

to conduct experiments and create datasets of accelerometer specific force readings. Per our scope of work, we conducted 17 training case experiments, \( n \), for each of the four actuators and 16 testing case experiments, \( n_a \), for each of the four actuators; and in turn, this equated to 132 experiments being conducted with a single actuator in operation at a time.

**Perform Frequency Analysis:**

Fourier transforms, mathematical tools for analyzing waveforms, are used to decompose time-based measurements into their frequency-based representations. Our accelerometers measured specific force in units of g (gravity) as a function of time. With Fourier transforms, however, we were able analyze the specific force measurements as a function of frequency. Specifically, we used the discrete-time Fourier transform (DTFT), power spectral density analysis, and a Lomb-Scargle periodogram (PLOMB) algorithm, where the first two methods assumed even sampling of data and the last method accounted
for unevenly-sampled data \[32, 33\]. These three frequency analysis methods were conducted independently, for all experiments, and used as a parameter in our polynomial regressions. In Appendix \(B\) can be viewed resulting plots from these frequency analysis methods, and in Section \(4.3\) we compare the results of the three frequency analysis methods to make conclusions on the uniform sampling assumption for the DTFT and PSD methods.

**Sort by Specific Force Maximum Magnitudes:**

Given the limitations of even modern spacecraft flight random-access memory (RAM), with size on the order of megabytes \[34\], we designed our disturbance source characterization technique considering the size of our experiment files. With the Fourier transformed results initially taking up ~5-15 MB per experiment depending on the transform used, we further processed the results to reduce the number of relevant data points and thus the size of the files, since the supervised learning requires all relevant processed experimental files to be accessed at once unlike with the pre-processing computation of the Fourier transformations which can be computed for individual experiments alone to limit computational costs. To reduce file size, we sorted all specific force magnitude values in descending order, while retaining the associated frequency values for each magnitude value (since the specific force magnitude is a function of frequency). Next, we set a minimum keep threshold of 10 Hz for frequency values, since fundamental modes typically occur above this frequency \[26\]. With the sorted dataset and frequency floor established, we saved the highest 100 pairs of specific force magnitudes and their frequency values, resulting in a new file size of ~15 KB per experiment. The 100 magnitude and frequency pairs acted as another parameter to vary in our regression, and the logic for multiple pairs of data is this additional information could minimize effects from outlier data while still being low dimensional in nature.

See Figure \(4.2\) for a summary of our algorithm development to predict unknown target values through polynomial regressions.

### 4.2 Details on Polynomial Regression

To predict the disturbance source actuation frequencies of our testing case experiments, we first computed the estimated polynomial regression coeffi-
Algorithm 1:
**Input:** Accelerometer specific force, as a function of time
- Perform Fourier transformation
**Output:** Accelerometer specific force, as a function of frequency

Algorithm 2:
**Input:** Accelerometer specific force, as a function of frequency
- Sort specific force magnitude values in descending order, keeping paired associated frequency values
- Eliminate pairs below minimum frequency of interest
- Reduce number of pairs to desired size
**Output:** Paired peak specific force magnitude and frequency values

Algorithm 3:
**Input:** Paired peak specific force magnitude and frequency values. Target actuator operational frequency values, \( y/y \).
- Compute median value from each experiment’s magnitude and frequency distributions, \( n/m \).
- Compute training experiments’ polynomial coefficients, \( p \).
- Predict testing experiments’ target actuation frequency values, \( f(x) \).
- Compute SMSE
**Output:** Standardized mean squared error

Figure 4.2: High level overview of disturbance source characterization algorithms, with regards to the polynomial regressions. We used three separate algorithms so the full processing was not necessary every time we iterated on our methods of actuator frequency target prediction.

We used \( p \), through the ordinary least squares estimation of the training case experimental data following the form

\[
\begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{pmatrix} =
\begin{pmatrix}
m_1^d & m_1^{d-1} & \cdots & 1 \\
m_2^d & m_2^{d-1} & \cdots & 1 \\
\vdots & \vdots & \ddots & \vdots \\
m_n^d & m_n^{d-1} & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
p_1 \\
p_2 \\
\vdots \\
p_{d+1}
\end{pmatrix} +
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_n
\end{pmatrix}.
\]  

(4.1)

Then, with the computed polynomial coefficients, \( p \), and the testing case experimental data we predicted testing case target actuation frequencies, \( f(x_\ast) \), based on the equation
\[
\begin{pmatrix}
  f(x_{s,1}) \\
  f(x_{s,2}) \\
  \vdots \\
  f(x_{s,n_s})
\end{pmatrix}
= 
\begin{pmatrix}
  m_{s,1}^d & m_{s,1}^{d-1} & \cdots & 1 \\
  m_{s,2}^d & m_{s,2}^{d-1} & \cdots & 1 \\
  \vdots & \vdots & \ddots & \vdots \\
  m_{s,n_s}^d & m_{s,n_s}^{d-1} & \cdots & 1
\end{pmatrix}
\begin{pmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_{d+1}
\end{pmatrix}.
\]

Equation (4.1) and Equation (4.2) were employed multiple times for our work, so for each usage we will explicitly state what the variables represent and which sensor-actuator pairings were considered.

In this section we show two scenarios for regression, and by this we mean that we considered and predicted with different sets of accelerometer experimental data. Our baseline polynomial regression results informed us on how to construct the Gaussian process regression models, which will be detailed in Chapter 5. For the polynomial regressions, we varied different parameters to determine which selection of parameters resulted in the best target prediction performance, and seen in Figure 4.3 are the parameters we varied. For this work, all possible parameter combinations were analyzed.

<table>
<thead>
<tr>
<th>Frequency Analysis</th>
<th>Feature of Importance</th>
<th>Polynomial Degree—d</th>
<th>Data Point Count—c</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTFT</td>
<td>Magnitude</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PSD</td>
<td>Frequency</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>PLOMB</td>
<td></td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 4.3: Parameters varied in our polynomial regressions. Frequency Analysis refers to the type of Fourier transform performed to process the accelerometer specific force data. Feature of Importance refers to which aspect of the frequency analysis was evaluated. Polynomial Degree is the degree/order of regression used for establishing polynomial coefficients. And Data Point Count is the number of points considered from each experiment in the polynomial regression, where the first point was the maximum magnitude or associated frequency, the second point was the second highest magnitude or associated frequency, with this trend continuing through the first \( c \) number of data points.
In particular, with these polynomial regressions we wanted to answer the following questions:

1. How well can we predict the actuation frequency given only the data from co-located accelerometers (i.e. accelerometers mounted at the same location as the actuator of interest)?

2. How well can we predict the actuation frequency if the system only had a single accelerometer instead of the distributed set of four accelerometers?

Additionally, we wanted to see if the Fourier transform choice influenced the prediction results and how well we could discern which actuator was in use. The specific force magnitude values versus the associated frequency values, the polynomial order, and the count or number of data points per experiment associated to that regression were parameters we varied for this study also. In the end, conclusions on the features of importance and the number of data points were made in this part of the study, and subsequently we integrated the established best results for these two parameters into our GPR models.

To quantify the goodness of our predictions, we calculated the standardized mean squared error based on the method outlined by Carl Edward Rasmussen and Christopher K. I. Williams in their book Gaussian Processes for Machine Learning [24]. First, we computed the mean squared error (MSE), which is the squared residual between the mean prediction and the target. Since the MSE is a function of the target values’ scale, after computing the MSE we standardized it, thus computing the SMSE, by normalizing the MSE with the variance of the target values, described by

\[ \text{SMSE} = \frac{m \left( y^* - \bar{f}(x^*) \right)^2}{\sigma_{y^*}^2}. \] (4.3)

With the standardization of the mean squared error, an SMSE value of ~1 equates to the trivial model guessing the mean of the targets, and the goal when predicting is to minimize SMSE, aiming to approach a value of 0 [24].
Results with one actuator per experiment:

The most straightforward regression to conduct is for experiments operating a single actuator at a time, with predictions only considering the data from the accelerometer co-located to the actuator in use, and so these single sensor, single actuator regressions were the first regressions we conducted. To determine the polynomial coefficients, \( p \), we performed polynomial regressions based on Equation (4.1), with \( d \) being the polynomial degree, \( m \) being the median of the first \( c \) accelerometer specific force magnitude or frequency data points for each of the \( n \) training case experiments, and \( y \) being the programmed actuator operational frequencies. See Figure 4.4 for an overview of the code used to perform the polynomial regressions and predict the SMSE performance metric.

In total, four independent regressions were conducted for each of our parameter options: the first regression was with Accelerometer 1 x-axis experimental data when Actuator 1 was actuated in the \( \pm x \)-axis direction, the second regression was with Accelerometer 2 y-axis experimental data when Actuator 2 was actuated in the \( \pm y \)-axis direction, the third regression was with Accelerometer 3 x-axis experimental data when Actuator 3 was actuated in the \( \pm x \)-axis direction, and the fourth regression was with Accelerometer 4 y-axis experimental data when Actuator 4 was actuated in the \( \pm y \)-axis direction. Subsequently, with the same accelerometer and actuator pairings, we predicted the target actuator operational frequencies (or the unknown target values), \( f(x_\ast) \), with Equation (4.2), with \( d \) being the polynomial degree, \( m_\ast \) being the median of the first \( c \) accelerometer specific force magnitude or frequency data points for each of the \( n_\ast \) testing case experiments, and \( p \) being the polynomial coefficients we previously computed. With the concatenation of the four sets of predicted target values, \( f(x_\ast) \), and the concatenation of the four sets of the testing target values, \( y_\ast \), we were able quantify the goodness of these predictions by computing SMSE with Equation (4.3). The best/lowest SMSE for each Fourier transform type and accelerometer feature was found, and these values; the associated polynomial regression order, \( d \); and number of data points, \( c \), can be seen in Table 4.1. Overall, the lowest SMSE was 0.0401 with the PLOMB Fourier transform type for the accelerometer frequency feature, and a full discussion for the co-located results will be provided in Section 4.3.
Figure 4.4: Overview of code for polynomial regressions for the co-located scenarios. This code loops through the various parameters shown in Figure 4.3, and with each loop we performed a polynomial regression and subsequently quantified the performance with the SMSE metric. NOTE: “exp” is short for experiment and “act” is short for actuator.
Table 4.1: Best SMSE for single actuator experiments considering only co-located accelerometer data. The associated polynomial regression order and number of points to these best/lowest SMSE values are shown and fixed as the values for use in the second regression of this section.

<table>
<thead>
<tr>
<th>Type</th>
<th>Feature</th>
<th>Polynomial Degree</th>
<th>Count</th>
<th>Best SMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTFT</td>
<td>Magnitude</td>
<td>2</td>
<td>50</td>
<td>0.2744</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>2</td>
<td>5</td>
<td>0.2912</td>
</tr>
<tr>
<td>PSD</td>
<td>Magnitude</td>
<td>2</td>
<td>100</td>
<td>0.5226</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>2</td>
<td>10</td>
<td>0.2502</td>
</tr>
<tr>
<td>PLOMB</td>
<td>Magnitude</td>
<td>2</td>
<td>25</td>
<td>0.5098</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>2</td>
<td>3</td>
<td>0.0401</td>
</tr>
</tbody>
</table>

Results with one actuator per experiment, if there was only one accelerometer:

Additionally, we examined how well we could predict had there been only a single accelerometer for all experiments instead of the co-located accelerometers at every disturbance source actuator, based on the code overviewed in Figure 4.3. Similar to the polynomial regressions with co-located accelerometers and actuators, these single actuator experiments necessitated multiple polynomial regressions with the first set of regressions determining the polynomial coefficients, \( p \), and the second set of regressions predicting the target values, \( f(\mathbf{x}_x) \). For each type and feature’s regression we used the results from Table 4.1 for the polynomial order, \( d \), and the data point count, \( c \). With the median training case accelerometer specific force magnitude or frequency values, \( m \), and Equation (4.1) the polynomial coefficients were computed, and subsequently the median testing case accelerometer specific force magnitude or frequency values, \( m_x \), were regressed with (see Equation (4.2)) to predict the actuator unknown operational frequency values. In total, for each sensor, four independent regressions were determined for finding the polynomial coefficient and then the predicted target value: the first regression was with the accelerometer x-axis experimental data when Actuator 1 was actuated in the ±x-axis direction, the second regression was with the accelerometer y-axis experimental data when Actuator 2 was actuated in the ±y-axis direction, the third regression was with the accelerometer x-axis experimental data when Actuator 3 was actuated in the ±x-axis direction, and the fourth regression was with the accelerometer y-axis experimental data when Ac-
tuator 4 was actuated in the ±y-axis direction. This process was repeated for each of the four accelerometers. The SMSE results for a hypothetical single sensor system were computed with Equation (4.3) and are shown in Table 4.2. Overall, the lowest SMSE value was 0.0520, with the PLOMB Fourier transform type and the accelerometer frequency feature minimizing the SMSE, similar to the co-located results; and a full discussion for the single sensor results will be provided in Section 4.3.

Table 4.2: Best SMSE for single accelerometer (accel) analyses, independently determining the resulting performance had there only been one sensor for all of the training and testing experiments.

<table>
<thead>
<tr>
<th>Type</th>
<th>Feature</th>
<th>Accel 1</th>
<th>Accel 2</th>
<th>Accel 3</th>
<th>Accel 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTFT</td>
<td>Magnitude</td>
<td>0.5671</td>
<td>0.4671</td>
<td>0.4439</td>
<td>0.5357</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.4515</td>
<td>0.2750</td>
<td>0.3711</td>
<td>0.3833</td>
</tr>
<tr>
<td>PSD</td>
<td>Magnitude</td>
<td>0.8575</td>
<td>0.7335</td>
<td>0.5717</td>
<td>0.6626</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.4725</td>
<td>0.2573</td>
<td>0.4328</td>
<td>0.3574</td>
</tr>
<tr>
<td>PLOMB</td>
<td>Magnitude</td>
<td>2.0367</td>
<td>0.6465</td>
<td>0.4455</td>
<td>29.4346</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.2346</td>
<td>0.3923</td>
<td>0.5210</td>
<td>0.0520</td>
</tr>
</tbody>
</table>

Predicting which actuator was operated:

We assumed for the preceding regression cases that we knew which actuator was operated, since typical system identification tests—like those used with the HST and SDO missions—would require a spacecraft operator to cycle on and off differing systems for these tests, resulting in information about which systems are operational to be known. In contrast, we wanted to examine with what accuracy we could predict which actuator was in use with no prior assumptions favoring any candidate disturbance source actuator over another; and we predicted which actuator source was in operation based on specific force readings from all four accelerometers. The combined number of correctly identified training and testing case experiments for each sensor and Fourier transform type can be seen in Table 4.3. These predictions were computed based on the accelerometer with the maximum median specific force magnitude, \( m \) or \( m_* \), in the direction of actuation for the number of points, \( c \), determined in Table 4.1 which minimized the SMSE for each Fourier transformation type. We were able to correctly predict Actuator 4 was the actuator in operation for 32/33 experiments with the PLOMB method, and a full discussion for the identification results will be provided in Section 4.3.
Figure 4.5: Overview of code for polynomial regressions for the all sensor-actuator pairings. This code loops through the different parameters while incorporating results from the co-located scenarios which yielded the lowest SMSE for each frequency analysis and feature of importance, and with each loop we performed a polynomial regression and subsequently quantified the performance with the SMSE metric. NOTE: “exp” is short for experiment, “act” is short for actuator, and “accel” is short for accelerometer.
Table 4.3: Number of training and testing experiments where the specific force magnitudes as a function of frequency were maximized at the actuation location.

<table>
<thead>
<tr>
<th>Type</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTFT</td>
<td>23/33</td>
<td>22/33</td>
<td>14/33</td>
<td>28/33</td>
</tr>
<tr>
<td>PSD</td>
<td>26/33</td>
<td>23/33</td>
<td>11/33</td>
<td>28/33</td>
</tr>
<tr>
<td>PLOMB</td>
<td>19/33</td>
<td>24/33</td>
<td>17/33</td>
<td>32/33</td>
</tr>
</tbody>
</table>

4.3 Polynomial Regression Results and Discussion

**Frequency analysis method:**

The specific force measurements for each accelerometer were collected every 0.005 s to 0.006 s. Given the $\frac{1}{1000}$th of a second difference between the minimum and maximum time between samples, we initially considered the time between collected samples to be sufficiently uniform and assumed the mean sample rate as the constant sample rate for computing the DTFT and PSD. On the other hand, we employed a PLOMB algorithm accounting for nonuniform sampling to test this hypothesis. As seen in Table 4.1, the PLOMB frequency feature outperformed the DTFT and PSD frequency features, achieving lower SMSE values for polynomial regressions with co-located sensor-actuator pairs. Furthermore, the PLOMB frequency feature produced the lowest two SMSE values when predicting based on single sensor regressions, as seen in Table 4.2.

Since all experiments were conducted with a single actuator, nominally one would expect a direct relationship between the frequency where the peak magnitude was measured and the actuation frequency. Some observations confirmed visually about the data trends are as follows:

- The DTFT and PSD frequency results had numerous outliers in data at lower frequencies for co-located Accelerometers 1 and 3 with Actuators 1 and 3, respectively, and additionally throughout the experimental frequency range for the Accelerometer 4 with Actuator 4 (see Figures B.1 and B.2).

- The PLOMB frequency results most closely followed the expected trend lines, where two sensor-actuator pairs had no noticeable outliers in data and the other two sensor-actuator pairs only had a small number of outliers in data (see Figure B.3).
• For all Fourier methods, the specific force frequency median values followed a trend better than any single point peak per experiment could, meaning the median value acted adequately as a simple method for outlier rejection (see Figures B.1-B.3).

In the end, we concluded one cannot assume even sampling with the Microchip Technology SAMD21 microcontroller or comparable models, so all our Gaussian process regressions were conducted with the PLOMB dataset alone. Additionally, we recommend algorithms for nonuniform datasets are given preference over algorithms which do assume uniformity of data. NOTE: other methods accounting for uneven sampling besides PLOMB exist and should be considered, such as the nonuniform fast Fourier transform \cite{35, 36}.

**Feature of importance:**

Our polynomial regressions in Section 4.2 demonstrated, as one might expect, that the accelerometer specific force frequency better models the actuator operational frequency as compared to the accelerometer specific force magnitude modeling the actuator operational frequency, though both features predicted the co-located actuator frequency with an SMSE value below 1.0. See Figures B.4-B.6 for plots of the accelerometer specific force magnitude data, where a large spread in data points can be observed. Based on our scope of work (see Table 3.1), since we did not actively control the input force which the actuators imparted to the test article structure, it was not possible to have target specific force magnitude values for regressions, so actively controlling the force magnitude is something for future consideration as it will aid in the identification of which actuators are being operated. In Chapter 5 in our GPR models we will again regress with magnitude, frequency, and also with both magnitude and frequency at the same time, though this work will differ from the previous polynomial regressions as we will no longer consider the median value alone for each experiment. Instead, the GPR regressions will be constructed with vectors of the \( c \) number of data points per experiment.

**Number of data points:**

In Appendix B, we tabulated the SMSE for the three frequency analysis methods while using the magnitude and the frequency features of our datasets. In Tables B.1-B.3 can be viewed the SMSE for the accelerometer specific force frequency analysis, and in particular the lowest SMSE was
found using: 5 data points for the DTFT method, 10 data points for the PSD method, and 3 data points for the PLOMB method. And at the same time, however, the DTFT and PSD had similar SMSE values using only 3 data points instead of the 5 and 10 data points, respectively (for the same polynomial degree). As a result, all Gaussian process regressions were initially modeled with the first 3 data points, though we reassessed this conclusion in parameter choice by performing the GPR with 1, 5, 10, 25, 50, and 100 data points also.

**Number of accelerometers:**

We predicted actuation frequency values best when the polynomial regressions were computed exclusively with data from co-located sensor-actuator pairs, as seen in Table 4.1. Due to the amplification or attenuation of microvibrations throughout the test article, attempting to model and predict with a single sensor for all experiments produced higher errors than when the co-located sensor-actuator scheme was employed. See Table 4.2 for the results of these single sensor regressions, where each accelerometer independently predicted for the 64 testing case experiments, $n_x$, based on the 68 training case experiments, $n$, independent of which actuator was operated (see Figures C.1 and C.2 for an overlaying of all training and testing data from the four sensors for each actuator’s experiments). This result, therefore, further motivates our argument that a distributed sensing network will aid in better micro-vibration observation capabilities.

**Operated actuator:**

With the processed accelerometer specific force data, we were interested in seeing if the highest median specific force magnitude values in the frequency domain occurred at the location of actuation, as this could provide a basis for an assumption on location if it was unknown. Based on our results with this comparison method, however, the prediction of location is also a function of location itself. Accelerometer 4 measured the highest specific force values when Actuator 4 was operated ~85% of the time, while Accelerometer 3 predicted the correct actuator only ~40% of the time with the same methodology (see Table 4.3). This result adds to the motivation for supervised learning, since the supervised machine learning method GPR can be used for non-linear mapping [24] and pattern discovery [37]. In the future, with a further developed experimental setup, it will be possible for us to have control over
actuator force magnitude; and in turn, this force magnitude control should enable for the creation of more descriptive models, as we can have additional data for varying magnitude and frequency independently. We hypothesize with this intended future change in experimental setup, we will be able to more accurately determine which actuator is in operation over the current method based on a $max()$ function.
CHAPTER 5

GAUSSIAN PROCESS REGRESSION

To add capability, in terms of quantifying uncertainty, to our disturbance source characterization technique, we employed Gaussian process regressions, based on our findings in Chapter 4. The polynomial regressions presented in Chapter 4 were important for determining which Fourier transform method should be favored and how many data points from each experiment should be analyzed to produce the lowest prediction errors. Results exhibited in Table 4.1 point towards the DTFT and PSD methods being inadequate given the uniform sampling assumption they are derived with, while the PLOMB method, accounting for unevenly-sampled data, yielded low errors when predicting actuator frequency based on co-located accelerometer frequency measurements. For this reason, we only regress with the PLOMB transformed data for the remaining work. Additionally, we found and stated in Section 4.3 that 3 data points per experiment was the ideal number to minimize SMSE with frequency feature predictions, given our established choices of 1, 3, 5, 10, 25, 50, and 100 data points; and in this chapter we will reevaluate that claim with our Gaussian process regressions.

With the GPR, we again regressed with low dimensional features from the datasets created with the four distributed accelerometers located on our test article. Our primary objective still with these analyses was to determine the relationship between actuator operational frequency values and accelerometer readings, and the end result of each analysis was the prediction of the actuator operational frequency for each testing case experiment, as was the case with the polynomial regressions also. The performance metrics SMSE and MSLL, introduced in Section 2.3, were used to quantify the error and uncertainty of these predictions.
5.1 Details on Gaussian Process Regression

Similar to first order polynomial regressions, the formulation of the Gaussian process regression starts with the standard linear regression model

\[ y = x^\top w + \varepsilon, \quad (5.1) \]

where \( x \) is an input vector, \( w \) is vector weights, \( y \) is the target value, and \( \varepsilon \) is the independent, identically distributed Gaussian noise following the form \( \varepsilon \sim \mathcal{N}(0, \sigma_n^2) \).

The Gaussian process regression differs from the polynomial regression, however, in that the former is a type of Bayesian analysis, with a prior of \( p(w) \sim \mathcal{N}(0, \Sigma_p) \), likelihood of \( p(y|X, w) = \mathcal{N}(X^\top w, \sigma_n^2 I) \), and a marginal likelihood of \( p(y|X) = \int p(y|X, w)p(w)dw \). The posterior thus is described by the equation

\[ p(w|X, y) \sim \mathcal{N} \left( \frac{1}{\sigma_n^2} A^{-1} X y, A^{-1} \right), \quad (5.2) \]

where \( A = \sigma_n^{-2} XX^\top + \Sigma_p^{-1} \).

Finally, based on posterior probabilities, we can average over all possible parameter values to predict testing case target values with the equation

\[ p(f_*|x_*, X, y) = \mathcal{N} \left( \frac{1}{\sigma_n^2} x_*^\top A^{-1} X y, x_*^\top A^{-1} x_* \right), \quad (5.3) \]

and the detailed derivation of Equation (5.3) along with the preceding equations in this section can be examined in Rasmussen and Williams’s book [24].

To quantify the goodness of our Gaussian process regression predictions, we computed the SMSE and MSLL, performance metrics for the error and uncertainty of the predictions, respectively. The SMSE was previously defined by Equation (4.3), while the computation of the MSLL starts with the formulation of the negative log likelihood (NLL) function by
\[ NLL = \frac{1}{2} \log (2\pi \sigma^2_y) + \frac{(y_* - \bar{f}(x_*))^2}{2\sigma^2_y}, \quad (5.4) \]

where the predictive variance, \( \sigma_*^2 \), is computed through the equation

\[ \sigma_*^2 = \mathbb{V}(f_*) + \sigma_n^2, \quad (5.5) \]

and where the variance \( \mathbb{V}[f_*] \) is computed through the equation

\[ \mathbb{V}[f_*] = k(x_*, x_*) - k_*^\top (K + \sigma_n^2 I)^{-1} k_* . \quad (5.6) \]

Similar to SMSE, a standardized version of the MSE, we standardized the NLL function. To standardize the NLL function, we subtracted the loss for the trivial model from the negative log likelihood function. Next, we computed the mean of that standardized log loss function, resulting in the development of the mean standardized log loss function. This final equation for our MSLL calculations is described as

\[ MSLL = m \left( NLL - \frac{1}{2} \log (2\pi \sigma^2_y) - \frac{(y_* - \bar{y})^2}{2\sigma_y} \right) . \quad (5.7) \]

With the mean and standardization applied to the negative log likelihood function, an MSLL value of \(~0\) is expected for simple methods, while more negative values are indicative of better methods \[24\].

In particular, with our Gaussian process regressions we wanted to answer the following questions:

1. How well can we predict the actuation frequency given only the data from co-located accelerometers?

2. How well can we predict the actuation frequency if the system only had a single accelerometer instead of the distributed set of four accelerometers?
3. How well can we predict the actuation frequency using all four sets of accelerometer data for all experiments?

The pre-processing work completed for the polynomial regressions and detailed in Section 4.1 was utilized again for the construction the GPR models. In Figure 4.1, it can be seen that the first three processes are identical, so these steps did not need to be repeated for the GPR work. Refer back to Section 4.1 for details on the collection of accelerometer measurements, performing of the frequency analysis, and the sorting by specific force magnitudes. For the GPR work, we utilized an open source software, GPy. The Sheffield machine learning group developed GPy as a Python based Gaussian Process framework able to solve Gaussian process regressions. We selected GPy over other GPR packages partly as GPy has been noted for yielding better fitting results compared to other GPR packages [38]. With this code base, we were able to focus our efforts on the implementation of these regressions as compared to developing them from the ground up. See Figure 5.1 for a summary of our algorithm development to predict unknown target values through Gaussian process regressions, with the Algorithm 3 being where GPy was implemented.

Results with one actuator per experiment, for co-located pairings:

See the following subsection, as co-located sensor-actuator pairings are a special type of one sensor configuration, where the sensor and the actuator in operation are located together. So for co-located results, we only considered one-fourth of the total experiments for each sensor, as we only considered the co-located sensor-actuator scenarios. The results for this situation are given in Table 5.1 and the lowest SMSE and MSLL results were found with the combined accelerometer specific force magnitude and frequency features, with the SMSE values ranging between 0.0012 and 0.0019 and the MSLL values ranging between -3.2655 and -3.1037. See the following subsection for how these results were found, and see Section 5.2 for a full discussion of the results for the GPR models based on the co-located experiments.

Results with one actuator per experiment, if there was only one sensor:

For our GPR models, we again predicted our actuators’ operational frequencies, or the target values, based on the co-located accelerometer’s read-
Figure 5.1: High level overview of disturbance source characterization algorithms, with regards to the Gaussian process regressions. We used three separate algorithms so the full processing was not necessary every time we iterated on our methods of actuator frequency target prediction. Additionally, we also tested each of the sixteen sensor-actuator pairings, where four of the pairings repeated the co-located scenarios. With GPy, for our regressions we had to create matrices for the training inputs, \( X \), testing inputs, \( X_\ast \), training targets, \( y \), and testing targets, \( y_\ast \). Additionally, we had to choose initialization values for the variance and length scale, select a model, and select an optimization method. The creation of the inputs and targets can be seen in Figure 5.2; and an important distinction between the polynomial regressions and the Gaussian process regressions is in the former we only used the accelerometer specific force magnitude or frequency features individually, while in the latter we use the features individually and together in a larger dataset. Our other selected algorithm parameters for GPR were as follows:

**Algorithm 1:**
- **Input:** Accelerometer specific force, as a function of time
- **Output:** Accelerometer specific force, as a function of frequency

**Algorithm 2:**
- **Input:** Accelerometer specific force, as a function of frequency
- **Output:** Paired peak specific force magnitude and frequency values

**Algorithm 3:**
- **Input:** Paired peak specific force magnitude and frequency values. Target actuator operational frequency values, \( y/y_\ast \).
- **Output:** Standardized mean squared error and mean standardized log loss
• variance $= [1 \times 10^{-2}, 1 \times 10^{-1}, 1 \times 10^{0}, 1 \times 10^{1}, 1 \times 10^{2}]$
• lengthscale $= [1 \times 10^{-2}, 1 \times 10^{-1}, 1 \times 10^{0}, 1 \times 10^{1}, 1 \times 10^{2}]$
• model $= \text{GPy.kern.RBF()}$
• optimizer $= \text{`bfg’ (Broyden–Fletcher–Goldfarb–Shanno)}$

For the Gaussian process regressions, we initialized with the listed variance values and length scales, to give the solver the best chance at converging on a solution with minimal error and uncertainty. For each of the sensor-actuator pairings, our Algorithm 3 (see Figure 5.2) predicted the target values, $f(x_*)$, and the predictive variance, $\sigma^2_*$. With Equation (4.3) and Equation (5.7) we were able to compute the SMSE and MSLL, respectively, for each of the initializations; and we have reported the results which had a minimization of the aforementioned performance metrics. It should be noted, for each regression, we only considered a single axis per experiment, the $\pm x$-axis for Actuators 1 and 3 experiments and the $\pm y$-axis for Actuators 2 and 4 experiments, since these were the directions we actuated in for the respective actuators. Table 5.1 shows the results for the co-located scenarios, while in Tables 5.2–5.5 are the results had we hypothetically only created a system with a single sensor for each of the actuator’s experiments. Since there are sixteen SMSE and MSLL values for each of the three feature options, we will reserve the full discussion of the results for Section 5.2, but overall, it can be seen that the results with accelerometers and actuators which were not co-located (i.e. the numbers do not match) are less consistent across the different actuators’ experiments compared to the co-located experimental results of Table 5.1.

Table 5.1: PLOMB SMSE and MSLL results, using 3 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>0.6540</td>
<td>0.3853</td>
<td>0.9405</td>
<td>0.9821</td>
</tr>
<tr>
<td>SMSE</td>
<td>Frequency</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0044</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0015</td>
<td>0.0012</td>
<td>0.0014</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>Magnitude</td>
<td>-0.1477</td>
<td>-0.4434</td>
<td>0.5505</td>
<td>-0.0078</td>
</tr>
</tbody>
</table>
Figure 5.2: Overview of GPR code for all the single sensor, single actuator pairings. This code loops while incorporating the selection of $c = 3$ and the selection of the PLOMB feature from the polynomial regressions, since these parameter choices minimized SMSE. With each loop of this code we performed a Gaussian process regression and subsequently quantified the performance with the SMSE and MSLL metrics. NOTE: “exp” is short for experiment, “act” is short for actuator, and “accel” is short for accelerometer.
Table 5.2: PLOMB SMSE and MSLL results, using 3 data points considering only Accelerometer 1 data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.6540</td>
<td>0.8335</td>
<td>0.9604</td>
<td>0.7898</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.7989</td>
<td>0.0115</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0015</td>
<td>0.0014</td>
<td>0.7999</td>
<td>0.0021</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.1477</td>
<td>-0.0859</td>
<td>-0.0539</td>
<td>-0.1693</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-3.1982</td>
<td>-3.2280</td>
<td>-1.1307</td>
<td>-3.0665</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-3.1795</td>
<td>-3.2188</td>
<td>-0.1350</td>
<td>-2.5549</td>
</tr>
</tbody>
</table>

Table 5.3: PLOMB SMSE and MSLL results, using 3 data points considering only Accelerometer 2 data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.6211</td>
<td>0.3853</td>
<td>0.8828</td>
<td>0.9907</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0479</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.2568</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0529</td>
<td>0.0012</td>
<td>0.0016</td>
<td>0.2568</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.5096</td>
<td>-0.4434</td>
<td>-0.1081</td>
<td>0.0014</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.5022</td>
<td>-3.1618</td>
<td>-2.9252</td>
<td>-0.5985</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.9990</td>
<td>-3.2655</td>
<td>-2.8964</td>
<td>-0.5985</td>
</tr>
</tbody>
</table>

Table 5.4: PLOMB SMSE and MSLL results, using 3 data points considering only Accelerometer 3 data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.9772</td>
<td>0.4323</td>
<td>0.9405</td>
<td>0.8867</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.4860</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.1300</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.2944</td>
<td>0.0014</td>
<td>0.0014</td>
<td>0.0988</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.0114</td>
<td>-0.4311</td>
<td>0.5505</td>
<td>-0.0578</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>9.0030</td>
<td>-3.1684</td>
<td>-3.1887</td>
<td>-1.3334</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-1.7943</td>
<td>-3.2199</td>
<td>-3.2063</td>
<td>-1.3292</td>
</tr>
</tbody>
</table>

Results with one actuator per experiment, based on the all four accelerometers’ readings:

Additionally, we had an interest in seeing if the combined data from four accelerometers for each experiment as compared to the use of a single accelerometer would improve the prediction performance. Like with the single
Table 5.5: PLOMB SMSE and MSLL results, using 3 data points considering only Accelerometer 4 data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>1.1152</td>
<td>0.8091</td>
<td>0.7634</td>
<td>0.9821</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0586</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.0044</td>
</tr>
<tr>
<td></td>
<td>SMSE</td>
<td>0.0586</td>
<td>0.0014</td>
<td>0.0015</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0731</td>
<td>-0.0981</td>
<td>-0.1320</td>
<td>-0.0078</td>
</tr>
<tr>
<td></td>
<td>MSLL</td>
<td>-0.9004</td>
<td>-3.1337</td>
<td>-3.1837</td>
<td>-3.0477</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.9003</td>
<td>-3.2325</td>
<td>-3.1700</td>
<td>-3.1037</td>
</tr>
</tbody>
</table>

sensor, single actuator pairings, we had to create matrices for the training inputs, \(X\), testing inputs, \(X_\ast\), training targets, \(y\), and testing targets, \(y_\ast\). Also, we had to choose initialization values, a model, and an optimization method. The creation of the inputs and targets can be seen in Figure 5.3, and we employed the following parameters again in this regression model:

- variance \(= [1 \times 10^{-2}, 1 \times 10^{-1}, 1 \times 10^0, 1 \times 10^1, 1 \times 10^2]\)
- lengthscale \(= [1 \times 10^{-2}, 1 \times 10^{-1}, 1 \times 10^0, 1 \times 10^1, 1 \times 10^2]\)
- model \(= \text{GPy.kern.RBF()}\)
- optimizer \(= \text{‘bfg’ (Broyden–Fletcher–Goldfarb–Shanno)}\)

As explained with the single sensor, single actuator pairings, we performed our regressions, computed the SMSE and MSLL with Equation (4.3) and Equation (5.7) for each initialization, and reported the best results for each actuator in Table 5.6. Since Actuators 1 and 3 were operated in the \(\pm x\) direction, we utilized the x-axis specific force features from Accelerometers 1-4 for these regressions. Likewise, with Actuators 2 and 4 being operated in the \(\pm y\) direction, we utilized the y-axis specific force features from Accelerometers 1-4 for these regressions. Again, like with the not co-located single sensor, single actuator results, these results with the data from all four sensors are not consistent across the different actuators’ experiments, and further remarks on these results with the four sensor system will be provided in Section 5.2.
Figure 5.3: Overview of GPR code which utilizes values from all sensors independent of which actuator was operated. This code loops while incorporating the selection of $c = 3$ and the selection of the PLOMB feature from the polynomial regressions, since these parameter choices minimized SMSE. With each loop of this code we performed a Gaussian process regression and subsequently quantified the performance with the SMSE and MSLL metrics. 

NOTE: “exp” is short for experiment, “act” is short for actuator, and “accel” is short for accelerometer.

```plaintext
Algorithm 3:
Foi = Feature of Importance = [accel. specific force freq., accel. specific force mag.]
c = Data Point Count
num = Number of Experiments = [n, n-]

for act = 1:4
  for train_test = 1:2
    for exp = 1: num(train_test)
      if FoI == 1 and 2
        X_{train_test(exp)} = [FoI[1] train_test] [exp, act] (1:x, Accel 1), FoI[1] [train_test] [exp, act] (1:x, Accel 2),
                      FoI[1] [train_test] [exp, act] (1:x, Accel 3), FoI[1] [train_test] [exp, act] (1:x, Accel 4),
                      FoI[2] [train_test] [exp, act] (1:x, Accel 1), FoI[2] [train_test] [exp, act] (1:x, Accel 2),
                      FoI[2] [train_test] [exp, act] (1:x, Accel 3), FoI[2] [train_test] [exp, act] (1:x, Accel 4)]
      else
        X_{train_test(exp)} = [FoI[1] [train_test] [exp, act] (1:x, Accel 1), FoI[1] [train_test] [exp, act] (1:x, Accel 2),
                      FoI[1] [train_test] [exp, act] (1:x, Accel 3), FoI[1] [train_test] [exp, act] (1:x, Accel 4)]
      end
      y_{train_test(exp, act)} = y_{y-(exp, act)}
    end
  end
  create kernel, select model, and optimize - with X, y, and GPy functions
  predict $f(x)$, $\sigma^2$ with X, model, and GPy functions
  compute SMSE with $f(x)$, $y$, and Equation (4.3)
  compute MSLL with $f(x)$, $y$, $\sigma^2$, and Equation (5.7)
end
```
Table 5.6: PLOMB SMSE and MSLL results, using 3 data points considering all four sets of accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.5871</td>
<td>0.3853</td>
<td>0.6365</td>
<td>0.7164</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0160</td>
<td>0.0015</td>
<td>0.0017</td>
<td>0.1567</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0579</td>
<td>0.0011</td>
<td>0.0018</td>
<td>0.9963</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.4972</td>
<td>-0.4433</td>
<td>-0.4434</td>
<td>-0.1698</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.3119</td>
<td>-3.2089</td>
<td>-2.8525</td>
<td>-0.0102</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.9187</td>
<td>-3.3472</td>
<td>-2.8095</td>
<td>-0.0016</td>
</tr>
</tbody>
</table>

5.2 Gaussian Process Regression Results and Discussion

**Feature of importance:**

The predictions for actuator target frequency values with our GPR models minimized SMSE and MSLL, as seen in Table 5.1, when we regressed with the co-located sensor-actuator accelerometer specific force frequency feature as compared to the co-located sensor-actuator accelerometer specific force magnitude feature, matching the results of the polynomial regressions (see Section 4.3). Notably, the predictions with magnitude values were better with the polynomial regressions than with the Gaussian process regressions, though as we previously indicated, the magnitude values alone are not good indicators of actuator frequency. Nevertheless, the GPR predictions with frequency values outperformed the predictions with frequency for polynomial regressions, which aligns well with our hypothesis that creation of GPR models, a supervised learning method in the area of machine learning, is a proper approach to take for developing a disturbance source characterization technique.

New with our GPR models as compared to the polynomial regression models, we regressed with the accelerometer specific force magnitude and frequency features jointly for predicting the actuator target frequency values, and all SMSE values with this combined set either matched or exceeded the performance of the predictions with the accelerometer specific force frequency values. Additionally, three out of the four computed MSLL values were found to be better with the magnitude and frequency joined datasets as
compared to the frequency datasets alone. See Table 5.1 for tabulation of the aforementioned results. In future work, it would be worthwhile to include more features such as the phase of the Fourier transforms in our regressions to further improve results.

**Number of data points:**

Due to our conclusions with the polynomial regressions (see Table 4.1), we initially selected \( c = 3 \) data points for the PLOMB-based Gaussian process regressions. At the same time, we retested all of the co-located sensor-actuator pairings in our GPR model for the set \( c = \{1, 5, 10, 25, 50, 100\} \), and these results can be seen in Tables D.1-D.6. Overall, the regressions with \( c = 3 \) data points predicted with the most consistent results across the sensor-actuator pairings (see Table 5.1). Furthermore, it can be seen that the some of the sensors have a considerable degradation in prediction performance with differing amount of data points, notably Accelerometer 4 reached an SMSE value of 0.4038 with \( c = 100 \) as compared to the SMSE value of 0.0044 with \( c = 3 \), both for the frequency feature. With our results, we conclude again with our current hardware setup and algorithm development that the selection of \( c = 3 \) is the best choice for minimizing error and uncertainty given our proposed set of data points.

**Number of accelerometers:**

In addition to the co-located scenarios, we also considered two other use cases for our sensors and actuators in their current configuration. Again, as was done with polynomial regressions, we predicted target actuation frequencies if hypothetically we could operate one sensor for all experiments. New to this GPR work, we also predicted using the sensor data from all four sensors for all experiments.

**Only one accelerometer:** In Tables 5.2-5.5 can be seen the prediction metric results for the sixteen combinations of sensor-actuator pairings, with four of the combinations being repeated from the co-located scenarios (see Table 5.1). In short, the predictions based on regressions using sensor measurements located at a distance from the actuation location hardly performed better than the co-located situation if ever. Accelerometer 2 predicted the Actuator 1 operation frequency with the specific force magnitude feature at an SMSE value of 0.6211 as compared to the SMSE of 0.6540 for the co-located Accelerometer 1 Actuator 1 pairing. Additionally, Accelerometer 1
was able to predict for Actuator 2 using the specific force frequency feature with an MSLL value of -3.2280 compared to the -3.1618 of the co-located prediction for Actuator 2. On the other hand, many predictions were worse for the not co-located scenarios. Accelerometer 1 predicted for Actuator 3 with an SMSE value of ~0.80 while employing the accelerometer frequency or accelerometer magnitude & frequency data as compared to the ~0.0015 Accelerometer 3 was capable of, and the MSLL performance for the prediction of Actuator 3 degraded from around -3.2 to around -0.13 for Accelerometers 3 and 1, respectively. It is inconclusive at this time the complete rationale for this drop in performance, though we speculate the movement of wires and the tolerances in the 3D printed parts are influencing the measurements, so we will have to address this in future work as well. Nevertheless, since no system is manufactured perfectly or with zero degrees of freedom, co-locating sensors to disturbance sources of interest is our recommendation to most effectively measure a source’s micro-vibrations.

All four accelerometers: In Table 5.6 can be seen the prediction metric results if measurements from all four sensors were examined for every actuators’ experiments. In short, all results for the predictions of actuator operational frequency values based on the accelerometers’ specific force magnitude values found lower SMSE and MSLL values, improving the results over the single sensor scenarios with the same feature choice. However, the predictions with accelerometer frequency or accelerometer magnitude frequency data all had higher or equivalent SMSE values when utilizing all four sets of sensor data as compared to the co-located sensor data alone, and additionally, most of the MSLL values were worse with the data from four sensors as compared to the co-located sensor alone. When viewing Figures C.2 and C.1, we can see how the measurements for each accelerometer’s specific force magnitude and frequency vary for the different actuators. As we noted, the frequency value feature did not see benefit from considering more data beyond that from the co-located accelerometer, though interestingly in Figure C.1 it can be seen that experiments for Actuator 2 did not have any results obviously off the expected direct relationship trend line between the output accelerometer frequency and the input of actuator frequency. Though, with Actuators 1, 3, and 4 there was a nontrivial amount of experiments which resulted in outlying data. As previously mentioned, a robust outlier rejection method is something we have noted for integration in our future work; and we believe, if
integrated properly, we will be able to predict more accurately with multiple sensors over a single sensor once we are able to eliminate the discrepancies which can occur with differing datasets.

5.3 Gaussian Process Regression Coregionalization

Modern supervised learning continues to advance, and supervised learning methods exist for handling multiple output scenarios [24]. Since our end goal of this work and the subsequent efforts is to be able to identify and characterize spacecraft disturbance sources violating micro-vibrational requirements, we will have to advance beyond analyses considering only single input, single output use cases (i.e. we will need to monitor and analyze multiple disturbance sources at a time for the technology to be employed on-orbit as intended). In particular, we have identified coregionalized models as a potential solution for considering multiple outputs, where with the coregionalized models, the outputs are linear combinations of independent functions [39].

Continuing our research with the GPy software introduced in Section 5.1, we employed the \texttt{GPy.kern.Coregionalize()} kernel selection for our coregionalized Gaussian process regressions. While we utilized the same training and testing experimental data for all regressions, new to this section is the consideration of all actuators for all experiments at once instead of independently studying each actuator on its own. What this means is we took sensor measurements from all accelerometers, noted the target actuator and the operational frequency, and then performed the regressions. Ideally, we wanted to predict something looking like \([51.25, 0, 0, 0]\) Hz had the first actuator been operated at 51.25 Hz while the remaining actuators were not in use. Our preliminary results with the coregionalized model can be seen in Table 5.7.

Based on our current method of coregionalization, we are unable to predict with less error than the trivial mode when using the frequency feature alone, as seen by the SMSE values of 1.0 for all four actuators. However, for the magnitude feature or magnitude & frequency feature we are able to outperform the trivial model for all actuators. Intuitively, one would speculate that the frequency data alone would not inform much about a location of actuation, since the frequency of the maximum measured specific force nominally
Table 5.7: PLOMB SMSE and MSLL results, using 3 data points considering all possible accelerometers and actuators, coregionalized.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Actuator 1</th>
<th>Actuator 2</th>
<th>Actuator 3</th>
<th>Actuator 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>0.9030</td>
<td>0.4701</td>
<td>0.9951</td>
<td>0.9755</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.8106</td>
<td>0.3535</td>
<td>0.9817</td>
<td>0.9674</td>
</tr>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>-0.1570</td>
<td>-0.5564</td>
<td>-0.0024</td>
<td>-0.0103</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0002</td>
<td>0.0250</td>
<td>0.0053</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-0.2500</td>
<td>-0.7466</td>
<td>-0.0082</td>
<td>-0.0133</td>
</tr>
</tbody>
</table>

should be somewhat consistent across sensors for a given experiment, so the prediction with frequency alone result is not disheartening. Furthermore, with the combined magnitude and frequency data minimizing SMSE and MSLL for each actuator, we validate our claim that the inclusion of different features is necessary for disturbance source identification and characterization, and we will continue to refine our models to include specific force magnitude targets, once the test hardware has been modified, in future work.
CHAPTER 6

CONCLUSION

In this thesis, we identified the PLOMB Fourier transformation and its suitability for the disturbance source regression problem. A PLOMB algorithm was suitable for this problem because the number of outliers in the resulting dataset were fewer compared to other Fourier transformation types. In future work, we can better mitigate the influence of outliers by explicitly incorporating outlier rejection in the Gaussian process regression algorithm. In fact, there already exist several methods that addressed outlier rejection in prior literature on Gaussian process regression [40]. Additionally, we showed that the number of selected data points utilized in the regression influenced the predictions and corresponding performance metrics. When trying to predict actuator target operational frequency values, both the polynomial and Gaussian process regressions for co-located sensor-actuator scenarios had a minimization of standardized mean squared error when we incorporated the three frequency values paired to the peak three magnitude values. Finally, with this work we showed that predictions using data from accelerometers co-located to disturbance source actuators resulted in less errors and less uncertainty than when we predicted with accelerometers not co-located to the actuator, reinforcing the motivation for a distributed sensing system for spacecraft applications.

In keeping with the scope of the proposed work, we placed limitations on the disturbance source characterization effort. Firstly, our structure was fabricated with 3D printed polymers so we could iterate often and keep the design modular with respect to introducing new components and moving others around as we developed the system from the ground up. As such, the response of a plastic structure differs from that of a metal, spacecraft structure. Secondly, we limited the operation of the actuators in terms of the number actuated at a given time and operational frequencies which we experimented at. Thirdly, all data processing was conducted offline due to
the computational limitations of an Arduino board.

Accordingly, our next steps will be to build upon the work shown in this thesis to further the technology and address the current limitations. Principle investigations into metal structure requirements for this distributed sensing disturbance source system have begun. Moreover, alternative computer and sensor options are being evaluated. Increases in computation power should enable faster collection data rates and additional sensors to be incorporated into the system. With more sensors, we could experiment with alternative configurations against that of the current square configuration. Furthermore, the inclusion of more sensors will enable us to observe micro-vibration mode shapes throughout a structure better, and we can study how the distance from a disturbance source affects the characterization predictions. A final limitation to address is the time necessary for analyses. Again, a more powerful computer architecture will be necessary, and this will be essential for on-orbit disturbance source studies in real-time.
REFERENCES


APPENDIX A

RELEVANT MANUFACTURER’S PERFORMANCE DATA

In this appendix we show the performance data for the two different two sizes of actuators we operated for our experiments.

Figure A.1: Performance data provided in the large and medium actuator datasheets [41, 42]. Since these disturbance source actuators are surface transducers, a type of speaker which enable any surface to function as a speaker cone, the metric used to quantify the performance is in units of pressure. As can be seen, in the area of interest for this study, 50-90 Hz, the pressure values as a function of frequency have an upwards trend meaning the induced disturbing force was not consistent among our experiments. This data was extracted through the tool WebPlotDigitizer [43] for this adaptation.
APPENDIX B

POLYNOMIAL REGRESSION PLOTTING AND TABULATING

In this appendix we show the low dimensional co-located datasets and regression results. Each of the six plots was created with the number of data points which minimized the SMSE for that feature and Fourier transform method, and each of the six tables shows the SMSE for each considered number of data points, $c$, and each polynomial degree, $d$, for our polynomial analyses.

Figure B.1: Accelerometer specific force frequency values based on the DTFT method, as a function of actuator frequency, with data point count $c = 5$ and polynomial regression degree $d = 2$.

Table B.1: SMSE for regressions with the specific force frequency feature and the DTFT method, as a function of data count and polynomial degree.

|     | $c = 1$ | $c = 3$ | $c = 5$ | $c = 10$ | $c = 25$ | $c = 50$ | $c = 100$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 1$</td>
<td>0.5058</td>
<td>0.5841</td>
<td>0.4441</td>
<td>0.4486</td>
<td>0.4422</td>
<td>0.4510</td>
<td>0.4733</td>
</tr>
<tr>
<td>$d = 2$</td>
<td>1.1690</td>
<td>0.3012</td>
<td>0.2912</td>
<td>0.3376</td>
<td>0.3276</td>
<td>0.3107</td>
<td>0.3079</td>
</tr>
</tbody>
</table>
Figure B.2: Accelerometer specific force frequency values based on the PSD method, as a function of actuator frequency, with data point count $c = 10$ and polynomial regression degree $d = 2$.

Table B.2: SMSE for regressions with the specific force frequency feature and the PSD method, as a function of data count and polynomial degree.

<table>
<thead>
<tr>
<th></th>
<th>$c = 1$</th>
<th>$c = 3$</th>
<th>$c = 5$</th>
<th>$c = 10$</th>
<th>$c = 25$</th>
<th>$c = 50$</th>
<th>$c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 1$</td>
<td>0.5059</td>
<td>0.4969</td>
<td>0.4550</td>
<td>0.3875</td>
<td>0.4398</td>
<td>0.4600</td>
<td>0.4871</td>
</tr>
<tr>
<td>$d = 2$</td>
<td>1.1835</td>
<td>0.3222</td>
<td>0.3794</td>
<td>0.2502</td>
<td>0.2907</td>
<td>0.3117</td>
<td>0.3050</td>
</tr>
</tbody>
</table>
Figure B.3: Accelerometer specific force frequency values based on the PLOMB method, as a function of actuator frequency, with data point count \( c = 3 \) and polynomial regression degree \( d = 2 \).

Table B.3: SMSE for regressions with the specific force frequency feature and the PLOMB method, as a function of data count and polynomial degree.

<table>
<thead>
<tr>
<th>( c )</th>
<th>( d = 1 )</th>
<th>( d = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1436</td>
<td>0.0765</td>
</tr>
<tr>
<td>3</td>
<td>0.0596</td>
<td>0.0401</td>
</tr>
<tr>
<td>5</td>
<td>0.1359</td>
<td>0.0683</td>
</tr>
<tr>
<td>10</td>
<td>0.1658</td>
<td>0.1183</td>
</tr>
<tr>
<td>25</td>
<td>0.2489</td>
<td>0.3828</td>
</tr>
<tr>
<td>50</td>
<td>0.4348</td>
<td>0.3321</td>
</tr>
<tr>
<td>100</td>
<td>0.5512</td>
<td>0.3481</td>
</tr>
</tbody>
</table>
Figure B.4: Accelerometer specific force magnitude values based on the DTFT method, as a function of actuator frequency, with data point count $c = 50$ and polynomial regression degree $d = 2$.

Table B.4: SMSE for regressions with the specific force magnitude feature and the DTFT method, as a function of data count and polynomial degree.

<table>
<thead>
<tr>
<th></th>
<th>$c = 1$</th>
<th>$c = 3$</th>
<th>$c = 5$</th>
<th>$c = 10$</th>
<th>$c = 25$</th>
<th>$c = 50$</th>
<th>$c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 1$</td>
<td>0.4818</td>
<td>0.4584</td>
<td>0.4601</td>
<td>0.4504</td>
<td>0.4560</td>
<td>0.4489</td>
<td>0.4534</td>
</tr>
<tr>
<td>$d = 2$</td>
<td>0.3691</td>
<td>0.3348</td>
<td>0.3225</td>
<td>0.3015</td>
<td>0.2860</td>
<td>0.2744</td>
<td>0.2929</td>
</tr>
</tbody>
</table>
Figure B.5: Accelerometer specific force magnitude values based on the PSD method, as a function of actuator frequency, with data point count $c = 100$ and polynomial regression degree $d = 2$.

Table B.5: SMSE for regressions with the specific force magnitude feature and the PSD method, as a function of data count and polynomial degree.

<table>
<thead>
<tr>
<th></th>
<th>$c = 1$</th>
<th>$c = 3$</th>
<th>$c = 5$</th>
<th>$c = 10$</th>
<th>$c = 25$</th>
<th>$c = 50$</th>
<th>$c = 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 1$</td>
<td>0.6856</td>
<td>0.6528</td>
<td>0.6420</td>
<td>0.6227</td>
<td>0.6294</td>
<td>0.6331</td>
<td>0.6232</td>
</tr>
<tr>
<td>$d = 2$</td>
<td>1.2357</td>
<td>0.8226</td>
<td>0.7738</td>
<td>0.6247</td>
<td>0.6294</td>
<td>0.6331</td>
<td>0.6232</td>
</tr>
</tbody>
</table>
Figure B.6: Accelerometer specific force magnitude values based on the PLOMB method, as a function of actuator frequency, with data point count \( c = 25 \) and polynomial regression degree \( d = 2 \).

Table B.6: SMSE for regressions with the specific force magnitude feature and the PLOMB method, as a function of data count and polynomial degree.

<table>
<thead>
<tr>
<th></th>
<th>( c = 1 )</th>
<th>( c = 3 )</th>
<th>( c = 5 )</th>
<th>( c = 10 )</th>
<th>( c = 25 )</th>
<th>( c = 50 )</th>
<th>( c = 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d = 1 )</td>
<td>0.8733</td>
<td>0.8402</td>
<td>0.8478</td>
<td>0.6614</td>
<td>0.7217</td>
<td>0.8660</td>
<td>0.6593</td>
</tr>
<tr>
<td>( d = 2 )</td>
<td>43.5502</td>
<td>47.0879</td>
<td>52.3324</td>
<td>0.7428</td>
<td>0.5098</td>
<td>1.1880</td>
<td>0.5838</td>
</tr>
</tbody>
</table>
APPENDIX C

PLOTTING SUPERIMPOSED CO-LOCATED AND NOT CO-LOCATED DATASETS

In this appendix we show the low dimensional accelerometer readings for all experiments. Each of the subplots displays the training and testing case experimental data from each of the four accelerometers in the direction that the actuator was operated in (±x for Actuators 1 and 3 and ±y for Actuators 2 and 4).

![Graphs showing accelerometer specific force frequency values based on the PLOMB method, with c = 3 data points per experiment.](image)

Figure C.1: Accelerometer specific force frequency values based on the PLOMB method, with \( c = 3 \) data points per experiment. All sensors’ measurements are displayed for each actuator’s experiments (i.e. considering more than the co-located scenarios shown in Figures B.1-B.6).
Figure C.2: Accelerometer specific force magnitude values based on the PLOMB method, with $c = 3$ data points per experiment. All sensors’ measurements are displayed for each actuator’s experiments (i.e. considering more than the co-located scenarios shown in Figures B.1-B.6).
In this appendix we show the SMSE and MSLL performance metrics computed for the co-located scenario Gaussian process regressions with the PLOMB low dimensional accelerometer data. We hypothesized that \( c = 3 \) data points was the ideal number of points for minimizing prediction error based on our polynomial regressions, and in this appendix we show the results for all other considered number of data points besides \( c = 3 \), as the tabulated results with \( c = 3 \) can be found in Section 5.1 with Table 5.1.

Table D.1: PLOMB SMSE and MSLL results, using 1 data point considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.7305</td>
<td>0.4809</td>
<td>0.8411</td>
<td>0.9214</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0977</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.1335</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0921</td>
<td>0.0013</td>
<td>0.0014</td>
<td>0.2353</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.4610</td>
<td>-0.3363</td>
<td>-0.0869</td>
<td>-0.0673</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-0.6871</td>
<td>-3.1538</td>
<td>-3.2079</td>
<td>-2.3328</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-0.7064</td>
<td>-3.2602</td>
<td>-3.2218</td>
<td>1.9944</td>
</tr>
</tbody>
</table>

Table D.2: PLOMB SMSE and MSLL results, using 5 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.5903</td>
<td>0.7116</td>
<td>0.7906</td>
<td>0.8282</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0022</td>
<td>0.0012</td>
<td>0.0015</td>
<td>0.1808</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0015</td>
<td>0.0012</td>
<td>0.0015</td>
<td>0.1661</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.6537</td>
<td>-0.1504</td>
<td>-0.2047</td>
<td>-0.1653</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.9996</td>
<td>-2.7306</td>
<td>-3.1755</td>
<td>-2.2127</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-3.0393</td>
<td>-3.3008</td>
<td>-3.1929</td>
<td>-1.2724</td>
</tr>
</tbody>
</table>
Table D.3: PLOMB SMSE and MSLL results, using 10 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>0.4793</td>
<td>0.2222</td>
<td>0.5850</td>
<td>0.8282</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.1289</td>
<td>0.0015</td>
<td>0.0015</td>
<td>0.1871</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.1671</td>
<td>0.0098</td>
<td>0.0014</td>
<td>0.1142</td>
</tr>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>-0.7372</td>
<td>-0.1652</td>
<td>-0.2702</td>
<td>-0.1654</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-1.3528</td>
<td>-3.1278</td>
<td>-2.7152</td>
<td>-2.1397</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.5273</td>
<td>-2.9830</td>
<td>-3.2100</td>
<td>-2.3805</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.6283</td>
<td>-0.2635</td>
<td>3.0829</td>
<td>-0.0390</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.7068</td>
<td>-2.8243</td>
<td>-2.9937</td>
<td>-2.7561</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.8938</td>
<td>-2.9958</td>
<td>-2.9315</td>
<td>-2.5271</td>
</tr>
</tbody>
</table>

Table D.4: PLOMB SMSE and MSLL results, using 25 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>0.5405</td>
<td>0.2229</td>
<td>0.8511</td>
<td>0.9346</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0027</td>
<td>0.1037</td>
<td>0.0030</td>
<td>0.1072</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0021</td>
<td>0.1049</td>
<td>0.0017</td>
<td>0.5051</td>
</tr>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>-0.6283</td>
<td>-0.2635</td>
<td>3.0829</td>
<td>-0.0390</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.7068</td>
<td>-2.8243</td>
<td>-2.9937</td>
<td>-2.7561</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.8938</td>
<td>-2.9958</td>
<td>-2.9315</td>
<td>-2.5271</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.6200</td>
<td>-0.1178</td>
<td>-0.4327</td>
<td>-0.0380</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.6905</td>
<td>-2.5532</td>
<td>-2.7885</td>
<td>-1.1817</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.8970</td>
<td>-2.8293</td>
<td>-2.7462</td>
<td>-2.2936</td>
</tr>
</tbody>
</table>

Table D.5: PLOMB SMSE and MSLL results, using 50 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Magnitude</td>
<td>0.5349</td>
<td>0.2243</td>
<td>0.4957</td>
<td>0.9354</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0032</td>
<td>0.0121</td>
<td>0.0050</td>
<td>0.1560</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0045</td>
<td>0.0027</td>
<td>0.0052</td>
<td>0.0106</td>
</tr>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>-0.6200</td>
<td>-0.1178</td>
<td>-0.4327</td>
<td>-0.0380</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.6905</td>
<td>-2.5532</td>
<td>-2.7885</td>
<td>-1.1817</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.8970</td>
<td>-2.8293</td>
<td>-2.7462</td>
<td>-2.2936</td>
</tr>
</tbody>
</table>
Table D.6: PLOMB SMSE and MSLL results, using 100 data points considering only co-located accelerometer data.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Feature</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Sensor 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMSE</td>
<td>Magnitude</td>
<td>0.4934</td>
<td>0.2402</td>
<td>0.5840</td>
<td>0.9437</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>0.0047</td>
<td>0.2413</td>
<td>0.0015</td>
<td>0.4038</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>0.0337</td>
<td>0.0159</td>
<td>0.0264</td>
<td>0.0021</td>
</tr>
<tr>
<td>MSLL</td>
<td>Magnitude</td>
<td>-0.5011</td>
<td>-0.0721</td>
<td>-0.4150</td>
<td>-0.0298</td>
</tr>
<tr>
<td></td>
<td>Frequency</td>
<td>-2.5361</td>
<td>-2.1450</td>
<td>-3.1566</td>
<td>-0.9928</td>
</tr>
<tr>
<td></td>
<td>Mag &amp; Freq</td>
<td>-2.8966</td>
<td>-1.8537</td>
<td>-1.3257</td>
<td>-2.7810</td>
</tr>
</tbody>
</table>