THE COLD ATOM TOOLBOX IN MOMENTUM SPACE

BY

FANGZHAO ALEX AN

DISSETATION

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Doctoral Committee:

Professor Brian DeMarco, Chair
Assistant Professor Bryce Gadway, Director of Research
Professor James Eckstein
Assistant Professor Barry Bradlyn
Abstract

The many weird properties of quantum mechanics at the very small scale have led to surprising and useful discoveries that manifest at the macroscopic level, like the quantum Hall effect and high temperature superconductivity. Yet trying to understand the origin of correlated behavior from interacting quantum systems via classical simulation requires an infeasible level of computing power. Instead, we can use an easily tunable, clean quantum system as a quantum simulation of a more unwieldy system, building the same model to study the same physics, but in a more controlled environment. Our field of cold neutral atoms in optical lattices has seen success over many years as a platform for quantum simulation of various lattice models from condensed matter physics. The recent (2015) implementation of lattices not in position, but in a synthetic dimension by coupling individual quantum states with lasers has led to a more bottom-up approach to engineering lattice models.

In this thesis, we present our “momentum-space lattice” technique in which we use individual laser frequencies to couple the momentum states of a $^{87}$Rb Bose–Einstein condensate, in order to create lattices with site-by-site and link-by-link precision. This technique is simple to implement experimentally, requiring just two additional common optical components (acousto-optic modulators) compared to a real-space lattice, yet is incredibly versatile. Using momentum-space lattices, we have studied the physics of artificial magnetic fields, disorder and pseudodisorder-induced localization, and atomic interactions across eight works described in this thesis. More interesting are the not-so-well-known effects that arise in the interplay among these three components of topology, disorder, and interactions, and we have made headway towards studying physics in this regime.

To be more specific, in our studies we have generated an artificial magnetic field for
neutral atoms, and directly observed the resulting chiral currents in both a square ladder and zigzag lattice geometry. We have further monitored the quantum walk behavior of atoms under disordered and pseudodisordered lattices, observing a transition to localization under a quasiperiodic potential. We have been able to introduce a tunable energy dependence to this localization transition (single-particle mobility edge) in two ways: with the addition of more tunneling pathways, and by modifying the form of the potential. Finally, we have studied the effects of nonlinear inter-atomic interactions in the momentum-space lattice, observing self-trapping in a double well system as well as on a full lattice, showing a skewed current-phase relationship in an analog to Josephson junction arrays, and investigating an interaction-induced shift in localization behavior under pseudodisorder.

In constructing the momentum-space lattice apparatus, Eric, Bryce, and I have created a promising new platform for Hamiltonian engineering. The studies described here not only show off the capabilities of the technique, but also realize new models, reveal new physics, and provide a new perspective complementary to both real-space lattice techniques and real materials. We have observed topological edge states more directly than previous works and engineered precise lattice parameter variations unavailable to other techniques, and yet the best is still to come. With our ongoing experimental upgrades comes access to the regime of strong inter-particle interactions, which promises more challenging yet more rewarding experiments.
For my parents
Acknowledgments

Graduate school has been harder than I expected, but not in the way I expected. It has taken great physical and mental endurance to keep pace in the marathon that is grad school, and so, to all of the people who have helped me in any capacity through the past six years in Urbana-Champaign, I can’t thank you enough.

I want to first acknowledge my committee for helping me through my prelim and my defense, and for reading through this tome of work: Jim Eckstein, Barry Bradlyn, Brian DeMarco, and my advisor, Bryce Gadway.

I consider myself incredibly lucky to have been one of Bryce’s first students. While I never picked up his work ethic, Bryce has taught me a lot about how to do research, and has supported and guided me to many papers, presentations, and overseas conferences. Like he predicted in his first email to me, I’ve gained invaluable hands-on experience starting in a brand new lab, and I’ve developed a lot of skills that will hopefully take me far. What he didn’t mention was how useful it was to have a labmate starting at the same time.

My colleague and friend Eric Meier is an excellent experimentalist, and has patched many holes I’ve left in the experiment, set up things I didn’t want to touch, and written papers that go over my head. It is some very strange stroke of luck that he and I started at the same time, worked so well together, and clicked on day one, bonding over Steve Brule videos and Smash Mouth coming and bombing at the Urbana Corn Festival. When I presented my first poster in grad school, people remarked on how quickly we had gotten a Bose–Einstein condensate (1 year). But it was so easy to see why, since time flies when you’re having fun.

And of course, the rest of the Gadway lab has become good friends to me, all with unique quirks and different backgrounds. Michael’s outgoing nature, Shraddha’s enthusiasm, and Jackson’s quick wit have driven some great conversations over the years. I wish Sai and Shraddha the best of luck in continuing the momentum-space lattice experiment, and wish
I could’ve stayed just a bit longer to see some experiments in the exciting regime of many-body physics. I want to thank all of the people who have made up the Gadway Lab: Shraddha Agrawal, Ritika Anandwade, Jackson Ang’ong’a, Cait Battle-McDonald, Michael Castle, Andrew Eberlein, Micheal Highman, Samantha Lapp, Zejun Liu, Hannah Manetsch, Gilad Margalit, Eric J. Meier, Kinfung “Frankie Fung” Ngan, Yuhao Pan, Myeonggon Park, Sai Naga Manoj Paladugu, Derek Ping, Yaashnaa Singhal, Hannah Ullberg, Zhenyu Wei, Garrett Williams, Teng Xiao, Lingfeng Yan, and Autumn Zender.

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Chapter 1

Introduction

“Let’s start what we finished.”

-Sam Lapp

In one sentence, my graduate work can be summarized as: making cool lattice models with cold atoms in a momentum-space lattice. On one hand, the experimental system is solidly in the camp of atomic, molecular, and optical physics, with a lot of details about lasers, optics, and atomic energy levels. On the other hand, the physics that we study often draws from condensed matter physics, from Anderson localization in disordered systems to topological edge states of the quantum Hall model. It is in this joining of 3rd floor Loomis Lab (AMO), the Materials Research Lab (CM experiment), and the Engineering Sciences Building (CM theory) that I have spent the last 6 years, both mentally and physically.

The general background for my work is the idea of quantum simulation. Quantum systems containing many interacting particles with myriad correlations are too cumbersome to fully model with classical computing. Hence, we follow the idea of and obligatorily mention Richard Feynman [1], who in 1982 envisioned a well-controlled, highly-tunable quantum system that could be used to mimick and study a more experimentally-challenging quantum system. Such “analog quantum simulation” involves creating the Hamiltonian of the more unwieldy quantum system in a cleaner, more tunable environment, allowing an AMO graduate student to probe and study the same physics.

Cold atoms in optical lattices have seen great success as a platform for analog quantum simulation of condensed matter physics, and continue to press onward, including in the lab next door to ours. Despite the dirty floor of our lab, the optical lattices that we create are remarkably clean compared to real materials, free of defects and thermal effects. Parameters like lattice depth, atomic interactions, and disordered potentials are distilled down to easily tunable inputs on a computer screen, and an entire experiment takes only seconds to run.
My work at UIUC has focused on extending this general technique to cover time-resolved, site-resolved control of every parameter of a given lattice, using a new “momentum-space lattice” approach.

Momentum-space lattices (MSLs) use quantum states carrying different momenta as the lattice “sites”, which are linked with laser-driven transitions to emulate tunneling between lattice sites. This is in analogy to real-space optical lattices, where the lattice sites are instead potential wells separated in position. More generally, this approach of connecting quantum states not in position space but in some other “synthetic dimension” is called “synthetic lattices” [2, 3], and the community has made and proposed a variety of synthetic lattices for atoms, molecules, and photons: atomic momentum states (our work) [4, 5], internal hyperfine [6, 7] and electronic states [8, 9], molecular states [10, 11], harmonic oscillator levels from shaking lattices [12], and a variety of photonic modes.

The main benefit of MSLs, and synthetic lattices in general, is the precise control of individual laser-addressed transitions. While real-space optical lattices offer highly tunable parameters that affect the entire system, it is difficult to control individual lattice sites or tunneling links. With MSLs, transitions between all lattice sites are addressed separately with individually-tunable laser fields, harnessing all the benefits of precise control over laser power and laser frequency to offer link-by-link, site-by-site microscopic engineering of all tunneling amplitudes, tunneling phases, and site energy terms in the lattice. These parameters can also be tuned dynamically, allowing for everything between slow adiabatic ramps to instantaneous jumps. Finally, this microscopic control is matched by microscopic, site-by-site measurement, allowing for direct readout of all lattice site populations with a single image. This site-resolved control and site-resolved detection have allowed us to explore non-Hermitian loss [13], various forms of disorder and quasiperiodic variations on tunneling and site energy terms [14–17], topological models in one dimension (SSH, $t_1-t_2$ model) [16, 18] and artificial magnetic fields (synthetic gauge fields) in two dimensions [15, 19], and many-body physics via interactions within momentum space [20, 21]. These studies have outlined the toolbox of momentum-space lattices, which we hope to use in future experiments operating in the strongly correlated regime. We note that in some sense, this approach is no longer analog quantum simulation in the vein of real-space lattices, but rather a bottom-up...
“Hamiltonian engineering” of lattice systems, building up lattices piece by piece.

I set up the MSL apparatus with Dr. Eric J. Meier (UIUC ’19) and Dr. Bryce Gadway (Stony Brook ’12) over the course of my first year at UIUC, starting from an empty optical table in September 2014 to getting a BEC and eventually seeing the first signs of atoms diffracting from the lattice on September 25, 2015. From 2015 to late 2018, Eric and I planned, implemented, and ran many experiments using the MSL. The early experiments were highly collaborative efforts between me and Eric (and, of course, Bryce), but the later ones quickly became mostly independent projects, with a bit of “I’ll help you run the apparatus and take data this week if you do the same for me next week.”

In late 2018, we depleted our dispenser source of $^{87}$Rb atoms, and I have devoted my time since then to switching the entire apparatus to a separate atomic species, $^{39}$K. This switch was motivated primarily by the broad, easily accessible Feshbach resonance of $^{39}$K, a tool that would allow us to tune the atomic scattering length (and thus interaction strength between atoms) by varying an external magnetic field. Our potassium setup is very similar to our rubidium one, but the different internal energy level structure has required a complete overhaul of the laser system in favor of different laser frequencies and more beam paths. Additionally, because we needed to break vacuum to insert a new atomic source cell and a new science cell, we had to remove most of the optics surrounding the chamber in order to re-bake the entire system. These changes and additional information are available in the final chapter of this thesis.

1.1 Three pillars of MSL studies

The studies that Eric led focused mainly on the Su-Schrieffer-Heeger (SSH) model [22, 23], a one-dimensional lattice model with alternating tunneling strengths that supports a topological phase with a signature edge mode. Despite its simplicity, the SSH model was difficult to probe microscopically in both real materials and with cold atom and optical lattice techniques. Specifically, the edge state was never directly observed in cold atoms until our work with the MSL [18] (see also concurrent work [24]). Eric later observed the effects of disorder on the topological properties of the system, which he probed using a bulk
measure of topology [16] (see also related work in photonics [25]).

On the other hand, my studies have not had such a convenient (thesis-wise) narrative. Instead, I have spent my time meandering between disparate experimental topics, producing six studies probing, individually and in combination, three main capabilities of the MSL system: topology by working towards 2D lattices with synthetic gauge fields for the study of quantum Hall physics, localization physics related to disorder and quasiperiodic lattice models, and atomic interactions in momentum space. Figure 1.1 summarizes all of the experiments that both me (in black) and Eric (in red) have performed using the MSL, sorted into these three main categories.

My first three studies probed separately artificial gauge fields (Chap. 3), disordered and quasiperiodic lattice parameters (Chap. 4), and interactions (Chap. 5), and the detailed background for each of these three topics are included in their respective chapters. These studies have expanded the “toolbox” of the MSL technique, showing off features that would be difficult or impossible to realize with real-space optical lattice techniques. Ultimately,
though, the goal of my research has been to combine these disparate components, seeking to study the interplay among disorder, nontrivial topology, and many-body physics. To this end, some of my more recent works have populated the overlap regions in the Venn diagram, studying the effects of artificial gauge fields (Chap. 6) and interactions (Chap. 8) on a localization transition under a quasiperiodic lattice model.

1.2 Outline

This thesis covers all of my first-author experiments in chronological (publishing) order. Each chapter covering an experiment I have performed will include the full text of the associated paper with some modifications for readability, along with additional background, experimental details, and commentary on the work. I will not discuss the experiments on the MSL that I did not lead, although I did contribute in taking data, discussing experimental procedures, and writing/editing the manuscript for these works. Thus I will not include Eric’s study realizing the topological Su-Schrieffer-Heeger model [18] and his later work adding disorder to this model [16], our exploration of wave chaos in the kicked top model [26], and Eric’s latest work on counter-diabatic driving, a shortcut to adiabaticity [27].

This manuscript is organized as follows:

- **Chapter 2: The Momentum-Space Lattice Apparatus** [4, 5]
  I start by discussing the experimental apparatus, first detailing the various atomic physics techniques of laser cooling and trapping $^{87}$Rb down to a Bose–Einstein condensate. Next I describe the momentum-space lattice technique: how the lattice offers control over tight-binding lattice Hamiltonians, how we read out the atomic populations, how we perform simulations of lattice dynamics, and how a typical experiment is run. Finally, I discuss three simple experiments that we performed in Ref. [5] demonstrating the capabilities of the technique.

- **Chapter 3: 2D Flux Ladders: Towards Hofstadter Lattices** [19]
  This chapter discusses our work observing chiral edge states on two-leg ladder systems ($2 \times 5$ sites) with tunable synthetic gauge fields. I give a brief background and history of artificial magnetic fields/synthetic gauge fields in cold atoms, and describe our approach to generating and controlling the “synthetic flux.” I discuss results showing chiral currents on a lattice with uniform flux, and atomic population reflecting from a discontinuous jump in the flux.
• **Chapter 4: Transport under Disorder and Quasiperiodicity [14]**
  This chapter presents transport measurements under three different forms of parameter variation: static random tunneling phases (ballistic spreading, same quantum random walk as no disorder), dynamically-varying random tunneling phases (diffusive spreading, like a classical random walk), and static site-energy landscapes following the quasiperiodic Aubry-André model (no spreading, Anderson localization). I describe in detail this Aubry-André model, which is relevant for Chapters 6 and 8.

• **Chapter 5: Interactions in the Momentum-Space Lattice [20]**
  This chapter shows our first explorations of finite-ranged atomic interactions in the momentum-space lattice. I describe the mechanism for these interactions, and present two simple experiments probing the effects of interactions on a double well: Bragg spectroscopy and sweeps which show an asymmetric response indicative of hysteresis. Finally, I propose two possible experiments: showing many-body effects on a zigzag lattice and generating squeezed states in a double well.

• **Chapter 6: Mobility Edges on a Zigzag Lattice [15]**
  This chapter presents our work examining the effect of artificial magnetic fields (see Chap. 3) on the localization transition under the quasiperiodic Aubry-André model (see Chap. 4). This was performed on a “zigzag” lattice geometry constructed with nearest-neighbor and next-nearest-neighbor links. We show that the extra connectivity leads to the appearance of a single-particle mobility edge (energy-dependent localization), and further show that the position of this mobility edge can be tuned via the applied magnetic flux.

• **Chapter 7: Losers [13]**
  This chapter presents our simulation work on generating local “loss” on a lattice site in the momentum-space lattice by either coupling to a lossy internal state, or by weakly linking the site with a reservoir of strongly-coupled sites. We implement this latter method experimentally and observe the expected loss rates. This project was led by Samantha Lapp and Jackson Ang’onga who provided simulation results, while I managed the experimental side.

• **Chapter 8: The Generalized Aubry André Model [17]**
  This chapter discusses our work realizing in experiment a generalized version of the Aubry-André model, as proposed by Ref. [28]. By monitoring the localization behavior of the extremal eigenstates of the system, we infer the existence of the predicted single-particle mobility edge in this model, and tune it via a single parameter that modifies the shape of the quasiperiodic potential. Atomic interactions affect the results in a nontrivial way, and our data match the interacting eigenstate behavior predicted from imaginary-time propagation of a mean-field model (Gross–Pitaevskii equation). This project was done in collaboration with Karmela Padavić of the Smitha Vishveshwara group (numerical/theory support), and with S. Ganeshan and J. Pixley who proposed the original model.
• **Chapter 9: Interacting Atoms on a Synthetic Josephson Junction Array [21]**

This chapter details our latest work on interacting atoms in 1D momentum-space lattices (21 sites). We discuss three related experiments: 1) self-trapping in a flat lattice, 2) asymmetric Bloch oscillations in a tilted lattice, and 3) a skewed current-phase relationship in a Josephson junction array. For all three experiments, we tuned the ratio of interactions to tunneling by changing the tunneling rate. This study received much simulation support from theory collaborators: Bhuvanesh Sundar and Kaden Hazzard calculated local density approximations for the Bloch oscillations study as well as evaluated the effective loss rates that would result from mode-changing collisions. Junpeng Hou and Chuanwei Zhang ran real-space Gross–Pitaevskii simulations for the wave packet study, with additional support from Xi-Wang Luo.

• **Chapter 10: Future Work**

This final chapter discusses the switch from rubidium-87 to potassium-39, including already implemented changes as well as planned improvements for the future. I also discuss an “atomic STM” study that can be performed using the changes to the apparatus, and list out some other future projects.

**Appendix A: Optics Tips and Tricks** is a primer to aligning lasers and working with optics, some of which is specific to the hardware in the Gadway lab.
Chapter 2

The Momentum-Space Lattice Apparatus

“Uh... why don’t we have a MOT?”

Eric Meier, FAA (many times)

This chapter describes the experimental apparatus in three sections: the technical aspects of creating a Bose–Einstein condensate (BEC) out of rubidium-87 atoms, details about the momentum-space lattice (MSL) itself, and the typical procedure to run an experiment on the MSL. Because so much of my work setting up the apparatus was performed alongside Eric J. Meier, much of this chapter is similar to the corresponding sections in his thesis [29]. To avoid a word-for-word reproduction, here I try to give a shorter, broader description of the system and omit the more minute experimental details, as Eric has covered these topics thoroughly in his thesis.

A large portion of this chapter is derived from our first experimental paper in Ref. [5] and its supplemental text.

2.1 Making a Bose–Einstein condensate of rubidium-87 atoms

To reach a BEC, we must both cool and condense a cloud of (bosonic) atoms, such that the spatial extent of each atom becomes comparable to the distance between the atoms. At this point, the bosonic atoms begin to macroscopically populate the same (single-particle) ground state, forming a BEC. This can be quantified by comparing the thermal de Broglie wavelength \( \lambda_{dB} = \frac{h}{\sqrt{2\pi mk_B T}} \) to the mean interparticle distance \( n^{-1/3} \), where \( n \) is the atomic density, \( m \) is the atomic mass, and \( T \) is the temperature. The figure of merit is then the “phase-space density” \( n\lambda_{dB}^3 \), and the transition to BEC has been proven to occur at \( n\lambda_{dB}^3 \geq 2.612 \) for any trapping potential [30]. Thus our goal is to both cool the atoms and increase the atomic density.
In this section, I discuss the experimental techniques of laser cooling, trapping, and manipulation that we employ to go from a hot atomic vapor to a BEC in 20 seconds. Contained in these 20 seconds are months to years of work from me, Eric, and Bryce, built upon decades of Nobel prize-winning research. Because these techniques (especially for $^{87}$Rb) are so widely known and well documented in helpful resources like Refs. [31, 32], I will describe them only briefly here. The roadmap to BEC is in three main cooling steps: laser cooling to the Doppler limit with a magneto-optical trap, laser cooling below the Doppler limit with optical molasses, and evaporative cooling down to a BEC in a crossed optical dipole trap. Along the way, we also increase the atomic density via compressed MOT and dark SPOT techniques, and optically pump the atoms to the same hyperfine sublevel.

This section is organized as follows. I first outline each step of the experimental sequence in Sec. 2.1.1. Then in Sec. 2.1.2, I present a picture of the actual apparatus and describe some of the hardware. In the rest of this section, I go into more detail about the steps of the sequence, including the properties of $^{87}$Rb and various laser cooling techniques.

### 2.1.1 The experimental sequence

1. **Source**: Atomic dispenser source is run to get a hot atomic vapor in the source cell.

2. **2D magneto-optical trap (MOT)**: Atoms are trapped in a 2D MOT within the source cell, forming a line of atoms along the direction of the vacuum chamber.

3. **Push beams**: Red- and blue-detuned beams push the line of atoms in the 2D MOT from the source cell through the vacuum chamber into the science cell.

4. **3D MOT**: Three pairs of beams catch the pushed atoms in a 3D MOT, and cool them down to mK temperatures. The bulk of the cycle time is spent here waiting for the MOT to load with atoms from the source side (10-15s).

5. **Increased MOT density**: Two techniques (compressed MOT and dark SPOT) are used to increase the density of atoms in the MOT in preparation for loading into the optical traps.

6. **Sub-Doppler cooling (optical molasses)**: Turning off the magnetic fields for the 3D MOT subjects the atoms to an optical molasses (polarization gradient cooling) which cools below the Doppler limit to $\mu$K temperatures.

7. **Optical pumping**: Under a magnetic field, atoms are optically pumped to a single Zeeman sublevel ($|F, m_F\rangle = |2, 2\rangle$). A magnetic field gradient is applied to levitate the atoms and get rid of any atoms remaining in other magnetic sublevels.
8. **Optical dipole trap (ODT):** 3–4 optical dipole trapping beams are applied to trap the atoms.

9. **Evaporation:** The power in the ODT beams are ramped down, and atoms cool down to a BEC (2-3s).

   After this procedure, we have a BEC of about $10^5$ rubidium-87 atoms. Depending on the day, this can be accompanied by some small thermal fraction of atoms, which we can get rid of by tweaking the evaporation trajectory or compensate for by fitting them in data analysis. To run an experiment, there are a few additional steps after the above list:

10. **Ramp up OT2 power:** Hold atoms almost entirely in one ODT beam which doubles as the lattice beam (which we refer to as “OT2”), and ramp up its power.

11. **Apply lattice:** Turn on lattice acousto-optic modulators (AOMs) that write the lattice frequencies onto a laser beam (retro beam of OT2) and allow this multifrequency beam to propagate back towards the atoms.

12. **Time-of-flight absorption imaging:** The trapping beams are turned off, and the atoms fall due to gravity in time-of-flight. Atoms in different momentum states separate spatially, and a resonant beam is shined onto the atoms, which cast a shadow on the camera.

   The camera sends the picture to our main computer, which automatically saves the data which we then import, process, and analyze in Mathematica. The absorption imaging procedure is a destructive measurement, so to take experimental data we simply run the entire cycle repeatedly, spending around 20 seconds each time.

### 2.1.2 The apparatus

Figure 2.1 shows a labeled picture of our apparatus taken in late 2018. The apparatus is a 2D magneto-optical trap (MOT) to 3D MOT configuration, which means we capture $^{87}$Rb atoms in a 2D trap in the “source” glass cell, and push them to a separate “science” glass cell where they are trapped in three dimensions and subjected to various stages of laser cooling down to a BEC. The two cells are separated by a pair of differential pumping tubes which are small enough (0.5 cm in diameter) to provide a high differential pressure between both sides, isolating the higher pressure source chamber with its many hot atoms flying about
from the science side. The entire system is put under vacuum to minimize heating from various gas particles in the air, and this is maintained by two ion pumps which ionize and remove any stray atoms or gases in the chamber, allowing us to reach ultra-high vacuums of $10^{-10} \sim 10^{-11}$ Torr.\footnote{The pressure on the science side pump is typically an order of magnitude lower than the one on the source side, as the two pumps are separated by a differential pumping tube.}

Not pictured here is the laser system. We use two Toptica DL100 external cavity diode lasers (continuous wave, CW) tuned to the cycling and repump transitions of $^{87}$Rb (roughly 780 nm), and use various acousto-optic modulators (AOMs) to shift the frequencies for the 2D and 3D MOT beams, the push beams, imaging light, etc. The cycling laser is locked using a polarization spectroscopy setup with a $^{87}$Rb vapor cell, and the repump laser is offset locked 6.5 GHz from the cycling laser by beating both lasers on a photodiode and manipulating the signal electronically (see Eric’s thesis for more details on this [29]). For the optical trapping beams, we use two high power, off-resonant Ytterbium fiber lasers from IPG Photonics: a broad (1 nm spectral width) 10 W laser at 1070 nm (YLR-10-1070-LP) and a narrow (150 kHz) 20 W laser at 1064 nm (YLR-20-1064-LP-SF). The broad 1070 nm...
laser is used for one of the three ODT beams, and the narrower 1064 nm laser is used for the other two beams, one of which performs double duty as the lattice beam as well (the horizontal beam in the photo). The setup for the lattice is at the top left corner of the photo, and we describe it in more detail below in Sec. 2.2.3. The rest of the photo shows a lot of optics surrounding the vacuum chamber which serve to shape and direct the various laser beams towards the atoms in the two glass cells.

As touched on above, we use AOMs to shift and control the frequency and power in each of the various laser beams. As described in Sec. A.6, an AOM holds a small crystal which vibrates at some input radio frequency (RF),\(^2\) and Bragg diffracts incoming laser light. For most of the beams, we generate the input RF signals with voltage-controlled oscillators (VCOs), and vary the amplitude and frequency of these signals with variable-voltage attenuators. Most of these are linked to our control software via a National Instruments card, and we can send digital signals to switch on and off the RF to the AOMs (turning on/off the beam), and send analog signals to control the frequency and amplitude of the RF (controlling the frequency and power in the beam). For the ODT beams, we use AOMs only as switches and never need to vary the frequency, so instead of the relatively noisy VCOs, we generate these RF signals with Wenzel crystal oscillators (SC Sprinter 501-09133). For the lattice AOMs, we need to send in RF signals with multiple precisely controlled frequencies, so we use a Keysight 33612A arbitrary waveform generator (AWG) which we have affectionately named “Newbo” (for being the new “Dumbo”). The exact RF signal we send to this AWG is generated in Mathematica and exported to the AWG by USB (see Secs. 2.2.3 and 2.5).

Finally, I want to briefly mention our software. To run the experiment, we use the Cicero Word Generator control software which was written by Aviv Keshet of Wolfgang Ketterle’s lab specifically to run cold atom experiments [33]. This program has a nice graphical interface which shows the values of various digital signals (0 or 5 V TTL), analog signals (0 to 10 V continuous\(^3\)), and serial commands (code used for communication with the AWG, for

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\(^2\)We use a variety of AOMs mostly from Gooch and Housego, and most of them operate around 80 MHz or 180 MHz. For larger frequency shifts, we direct light through an AOM twice in a double-pass “cat’s eye” configuration, as detailed in Sec. A.7.

\(^3\)This is set by our analog isolator boards which only handle positive voltages. Cicero and the connected National Instruments card can both output \(-10\) V to \(10\) V.
example) for each timestep of the entire experimental sequence. This software is connected to a field-programmable gate array (FPGA), which talks with National Instruments hardware, which in turn sends the signals to various hardware in our experiment: the AWG, shutters, RF control boxes (for AOMs), magnetic field coil control circuits, etc.

For data acquisition, we have an ANDOR NEO camera (hidden behind an ion pump in the photo) which is connected directly to a computer, and we use the corresponding ANDOR SOLIS software to automatically save the images upon a trigger signal from Cicero. Data analysis and numerical simulations are done in Mathematica, which is... slow, but easy to pick up and offers great data visualization.

2.1.3 Rubidium-87 level structure

The atomic species we used for our experiments from 2014 to 2019 was rubidium-87 ($^{87}$Rb), a bosonic isotope of an alkali atom. Because the specific atomic species does not matter for our momentum-space lattice studies, $^{87}$Rb was chosen for ease of use with its nicely closed cycling transition, easily accessible laser wavelengths, good vapor pressure, etc. It also has a long history to draw from: $^{87}$Rb was the first atomic gas to be Bose-condensed [34], and many groups have made $^{87}$Rb BECs in the 20+ years since. As such, the properties of $^{87}$Rb are very well documented and compiled in great resources like Daniel Steck’s “Rubidium D Line data” [35].

As an alkali atom with one valence electron, $^{87}$Rb has a simple level structure. For laser cooling, we focus on the so-called D$_2$ line of $^{87}$Rb shown in Fig. 2.2, which is the transition from the ground level $5^2$S$_{1/2}$ to the excited level $5^2$P$_{3/2}$ (which is the higher energy of two fine structure levels). The spectroscopic notation here lays out all of the electronic quantum numbers of each level as $n^{2S+1}L_J$, where $n$ is the principal quantum number, $S$ is the spin quantum number, $L = S, P, D, F...$ is confusing shorthand for the orbital angular momentum quantum number $L = 0, 1, 2, 3...$, and $J$ is the total electronic angular momentum ($J = L + S$).$^4$ Both of these states are split further by the hyperfine interaction between the electronic angular momentum $\mathbf{J}$ and the nuclear spin $\mathbf{I}$ ($I = 3/2$ for $^{87}$Rb), leading to states

$^4$These letters are all uppercase to be consistent with multi-electron cases, where these quantum numbers are sums over the respective quantum numbers of each electron, i.e. $\mathbf{L} = \sum_i \ell_i$. 

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Figure 2.2: **Level structure of the $^{87}$Rb D$_2$ line.** Hyperfine structure and relevant laser transitions are labeled. Adapted from Fig. 3 of Ref. [35] and inspired by Fig. 3.3 of Ref. [29].

of the total angular momentum of the atom $F = J + I$ as shown in the figure.$^5$

The main “cycling” transition goes from $F = 2$ (in $5^2$S$_{1/2}$) to $F' = 3$ (in $5^2$P$_{3/2}$), and is

$^5$The prime in $F'$ is shorthand for the hyperfine states of the excited level $5^2$P$_{3/2}$. 
nearly closed, meaning that atoms in the excited state have very little probability to decay to any state other than the $F = 2$ starting state. Thus by addressing this transition, atoms can “cycle” through these transition many times, absorbing and emitting many photons before falling out. On the off-chance that an atom does fall out of the cycle and gets stuck in the $F = 1$ state, we can address the repump transition from $F = 1$ to $F' = 2$, from which the atom may go back to $F = 1$ and get re-pumped or it can fall back into the cycling transition by landing in $F = 2$. The resonant cycling transition is used for absorption imaging, and by detuning from resonance, both the cycling and repump transitions are used for cooling and trapping during the MOT stage. The depump transition between $F = 2$ and $F' = 2$ is used in conjunction with repump for optical pumping, as discussed below.

We make a minor note here that Fig. 2.2 follows the common atomic physics practice of writing energy differences in terms of wavelengths (for large energy differences) and frequencies (for smaller energy differences). More generally in this thesis, we often report energies in units of frequency to better compare to our MSL setup, and hopefully these all include explicit $h$ and $\hbar$ terms.

2.1.4 Laser cooling and trapping

**Magneto-optical traps (MOTs)**

In general, laser cooling of atoms works if an atom emits a photon with higher energy than the one it absorbs. One way to do this is via the Doppler effect [36, 37]: we apply a red-detuned cycling beam onto a moving atom, meaning that the beam frequency is $\sim 10 \text{ MHz}$ lower than the cycling transition frequency. Then, atoms which move toward the beam see a Doppler shifted frequency, bringing the light closer to resonance and thus increasing the rate of absorption from the beam. The atom then spontaneously emits the photon in some random direction, carrying away some of the kinetic energy of the atom. This leads to a velocity-dependent damping force which is reminiscent of wading through molasses, hence the name optical molasses [38].

A magneto-optical trap (MOT) utilizes this technique with red-detuned beams and thus cooling in multiple directions. Furthermore, a MOT also incorporates a position-dependent
force to trap the atoms in space via the Zeeman effect. As shown in the photo of our apparatus (Fig. 2.1), both of our glass cells are surrounded by magnetic field coils which provide magnetic field gradients along the directions of the MOT.\(^6\) Here, the gradients shift the \(m_F\) sublevels of the hyperfine states, such that the light is resonant at some distance from the trap center. Atoms that stray from the trap center then absorb photons from an incoming beam, giving a force back towards the center. We note that the beams must have the proper (circular, \(\sigma^\pm\)) polarization to address the appropriate transitions to Zeeman-shifted states.

Put together, the MOT both traps the atoms and cools them down to milliKelvin temperatures.

**Increasing atomic density in the MOT: CMOT, DSPOT**

In preparation for loading into the optical trap, we wish to increase the density of atoms trapped within the MOT. We have done this with two techniques: a compressed MOT and a dark spontaneous-force optical trap (dark SPOT).

The dark SPOT technique \([39, 40]\), developed by the Ketterle/Pritchard group with sodium, holds atoms in a dark state, for us the \(F = 1\) state in Fig. 2.2. This is done by applying a repump beam shaped like a donut, such that atoms in the center do not experience repump light and thus eventually fall out of the cycling transition and into \(F = 1\). The point here is to both avoid loss from collisions between ground state and excited state atoms, and to skirt the issue of atoms reabsorbing emitted photons from other atoms, which leads to an outward radiation pressure that competes with the trapping force and places a limit on the atomic density. In experiment, we create a donut shaped beam of repump light by simply placing into the repump beam path a glass slide with a small circle of black tape, and then direct this onto the atoms from two orthogonal directions to create a “boxy shell thing” (the words of Eric Meier). As one might expect, this method is quite crude, and would benefit with more precise beam-shaping using a digital micromirror device (DMD).

The compressed MOT technique was developed by the Cornell group with \(^{87}\text{Rb}\) \([41]\), and

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\(^6\)The coils, wound in an anti-Helmholtz configuration, provide roughly 10 G/cm of field gradient (higher for the 3D MOT, lower for 2D). We also have Helmholtz coils which provide a magnetic field to shift the field center.
simply increases the magnetic field gradient to compress the MOT and increase the atomic density, in parallel with some other tunings of laser powers and frequencies.

In experiment, we have flip-flopped back and forth between one or both of these techniques, initially using both to generate a BEC, but ultimately omitting the DSPOT step in the final experimental sequence as it had little impact on the BEC atom number.

**Sub-Doppler cooling: Molasses/Polarization gradient cooling**

Under Doppler cooling, atoms absorb photons with opposing momentum and spontaneously emit photons in random directions, leading to a random walk. By equating the heating rate of this walk with the cooling rate, one can find a “Doppler limit” on the temperature. Historically, sub-Doppler cooling [38, 42, 43] bypassed this limit accidentally.\(^7\) The technique is simply taking the MOT and switching off the magnetic field gradient to get only the molasses discussed above, which should provide only Doppler cooling.

However, it turns out that the beam polarizations, which are opposite for pairs of counter-propagating MOT beams, lead to a polarization gradient which can further cool the atoms. Under a “lin \perp lin” configuration where two beams have orthogonal linear polarizations, this cooling can be explained by an analogy to the greek myth of Sisyphus, the soul in Hades who is doomed to repeatedly push a boulder up a hill, only for it to fall on him and roll back down the hill. In cold atoms, the atoms experience an elliptical polarization gradient which shifts the internal energy levels such that atoms spend their kinetic energy climbing up a potential hill, only to get excited to a higher state and fall back down to the bottom of another hill to climb.

Unfortunately, this commonly studied picture is not our situation, which is instead a \(\sigma^+\text{-}\sigma^-\) configuration, leading to a rotating linear polarization which does not give a hilly potential landscape [43]. Instead, here the atoms’ motion affects their orientation, leading to an imbalance in absorption from the two beams and an imbalance in the radiation pressure, resulting in cooling.

\(^7\)For Doppler cooling, the heating rate is dependent on the rate of spontaneous emission, and thus dependent on the natural linewidth of the transition (\(\Gamma \sim 2\pi \times 6\) MHz for our cycling transition). To bypass the Doppler limit, sub-Doppler cooling uses not spontaneous emission, but rather coherent scattering of photons from one beam into another.
Optical pumping

We next optically pump all of the atoms to a single Zeeman sublevel $|F = 2, m_F = 2\rangle$ (in the $5^2\text{S}_{1/2}$ level) using the repump and depump transitions along one beam path. We apply a small magnetic field to set the quantization axis along this beam path, such that by tuning the polarization of the beam to be $\sigma^+$, we only address $\Delta m_F = m'_F - m_F = +1$ transitions from $F = 1, 2$ to $F' = 2$. Thus all of the various $m_F$ sublevels of the ground hyperfine states $F = 1, 2$ are addressed by either a repump or depump transition except the target state $|F = 2, m_F = 2\rangle$, as there is no $|F' = 2, m'_F = 3\rangle$ state. By keeping these beams on for a few milliseconds, all of the atoms end up in $|F = 2, m_F = 2\rangle$.

To ensure that no other Zeeman sublevels are present, we apply a magnetic field gradient tuned to levitate only $|F = 2, m_F = 2\rangle$, so atoms in any other Zeeman level fall out.

2.1.5 Optical trapping, evaporation, and Bose–Einstein condensation

From here, we apply three to four very far red detuned, high power optical dipole trapping (ODT, or OT) beams to optically trap the atoms by the ac Stark effect.\(^8\) The oscillating electric field of the laser field induces a dipole moment in the atom, ultimately resulting in an ac Stark shift (or light shift) of the energy levels that scales with intensity. For a red-detuned beam, the shift is negative, such that atoms see a potential minimum at most intense part of the beam, thus getting trapped not only radially, but also more weakly along the direction of the beam at its focus. To make a tighter trap, we apply multiple OT beams (with different polarizations to prevent interference) and spend a long time overlapping the positions and focuses of all beams. While the MOT stage ends with several billion atoms, this crossed dipole trap can only hold a few million atoms at $\mu$K temperatures, which is sufficient for our MSL purposes.

Finally, the last stage of the process is all-optical evaporative cooling. The idea is to get rid of the hottest, fastest atoms from the trap, let the atoms rethermalize (collide and reform a thermal/Maxwell–Boltzmann velocity distribution), and once again cut off the high

\(^8\)As described in Sec. 2.1.2, these beams have wavelengths of 1064 nm and 1070 nm compared to the 780 nm $D_2$ line, and are about 1 W in power (compared to $\sim$30 mW for a MOT beam).
velocity tail of the new distribution. In our case, this is done by simply lowering the power in the OT beams, simultaneously spilling hot atoms out of the shrinking trap while the remaining atoms rethermalize. By tweaking the evaporation trajectory of how the different beam powers are lowered, we can achieve a BEC of $10^5$ atoms in 2.5 s of evaporation.

The onset of degeneracy is quite striking, and as is tradition with BEC experiments, in Fig. 2.3 we show absorption image data taken on that momentous day in our lab of the BEC popping out from a thermal distribution. These three pictures are taken for different evaporation trajectories ending at lower and lower OT power until the BEC appears.

2.2 The momentum-space lattice

A real-space optical lattice can be made by shining onto the atoms a pair of counterpropagating beams that are high power and highly red-detuned (like our optical trapping beams), or by reflecting one such beam off of a mirror. This creates a standing wave of light, and by the ac Stark effect, atoms see a periodic potential, or a lattice. Such an implementation can offer great insight into lattice models, but is hampered by its global tunability, i.e., the lattice parameters can only be tuned for the entire lattice. There are more microscopic approaches which may, for example, create a potential barrier at one site, but these are
Figure 2.4: Engineering the momentum-space lattice. (a) The $^{87}$Rb Bose–Einstein condensate (BEC) is subjected to counter-propagating lattice laser beams with wavelength $\lambda = 1064$ nm (frequency $\omega^+ = c/2\pi \lambda$, wavenumber $k = 2\pi/\lambda$), where Laser 2 carries many frequency tones (shifted from $\omega^+$ by 10~100 kHz). (b) Dispersion relation showing momentum states coupled with two-photon Bragg transitions. Yellow transition frequency $\omega_{0,1}$ has been detuned from resonance to demonstrate lattice site energy control. (c) Effective tight-binding lattice model generated by the couplings. Adapted from Ref. [14] with the authors’ permission.

generally quite difficult.$^9$ A more bottom-up approach might offer independently tunable parameters at the individual lattice site level.

The momentum-space lattice (MSL) technique [4, 5] is a type of “synthetic” optical lattice [2, 3, 6, 7] for cold atoms which seeks to deliver this bottom-up approach. Here, the lattice sites are not potential wells separated in position, but a set of quantum states in a “synthetic” dimension – for us, momentum states. These states are coupled with laser frequencies, such that the very precise control of laser powers, phases, and frequencies can be harnessed and mapped to precise control of the corresponding lattice parameters: tunneling amplitudes, tunneling phases, and site energies, respectively. This allows us to engineer lattices with site-resolved control, enabling all of the studies in this thesis, most of which are very difficult to realize with real-space optical lattices.

$^9$Though I must point out here the usefulness of techniques like optical tweezer arrays (fully tunable lattices built site-by-site), quantum gas microscopes (single-site detection), and potentials generated by digital micromirror devices (individual site-level control).
2.2.1 Overview

We begin with the $^{87}\text{Rb BEC}$ at rest, trapped primarily in one optical trapping beam with wavelength $\lambda = 1064$ nm (frequency $\omega^+ = c/2\pi\lambda$, wavenumber $k = 2\pi/\lambda$). After going through the atoms, this beam travels to two acousto-optic modulators, which write onto the beam a spectrum of frequencies $\sum_n \omega_n^-$ (more detail below in Sec. 2.2.3). This multifrequency beam then comes back along the same path, hitting the atoms from the other side as shown in Fig. 2.4(a).

We make the assumption that the atoms stay near the center of the trapping potential and thus take on a free particle dispersion as shown in Fig. 2.4(b), with energy $E = p^2/2M_{\text{Rb}}$, where $p$ is the momentum. We consider only the momentum states achievable through transitions driven by the two beams in Fig. 2.4(a), such that atoms can only change their momentum by absorbing a photon from one beam and emitting into the other. Thus, the momentum becomes quantized into twice the photon recoil momentum, giving available momentum states $p_n = 2n\hbar k$ with energies $E_n = p_n^2/2M_{\text{Rb}} = 4n^2E_R$ (for photon recoil energy $E_R = \hbar^2k^2/2M_{\text{Rb}}$), denoted by the black circles in panel (b).

To create a lattice, we couple these states by addressing two-photon Bragg transitions, tuning the frequencies in Laser 2 such that the frequency differences between Laser 1 and each tone in Laser 2 matches the energy differences between adjacent momentum states. For example, to couple states $n = 0$ and $n = 1$, we apply one frequency tone $\omega_0^-$ to Laser 2 such that the difference between this and the frequency of Laser 1 matches energy difference between the states: $\omega^+ - \omega_0^- = (E_1 - E_0)/\hbar = 8.2$ kHz. Put another way: under these two lasers, an atom may undergo a two-photon Bragg process from $n = 0$ to $n = 1$ via a far-off excited internal state $|e\rangle$, ultimately undergoing Rabi oscillations between $n = 0$ and $n = 1$ at a rate depending on the detuning from $|e\rangle$.\(^{10}\) Likewise, we can apply another frequency $\omega_{-1}^-$ to Laser 2 to couple states $n = -1$ and $n = 0$, where $\omega^+ - \omega_{-1}^- = (E_0 - E_{-1})/\hbar = -8.2$ kHz. Because the frequency difference between any two states $n$ and $n + 1$ is unique, we can expand this and couple a full lattice of states.

As detailed in the next section and sketched in Fig. 2.4(c), this coupling scheme allows\(^{10}\)

---

\(^{10}\)The math for this two-photon Bragg transition (between momentum states) is the same as for two-photon Raman transitions (between internal states), which are more easily Google-able.
us to realize effective tight-binding models of the form

\[ H \approx \sum_n -t_n \left( e^{i\phi_n} c_{n+1}^\dagger c_n + \text{h.c.} \right) + \sum_n \varepsilon_n c_n^\dagger c_n, \]  

(2.1)

where \( n \) indexes the lattice sites (momentum states \( p_n = 2n \hbar k \)), and \( c_n^\dagger \) (\( c_n \)) is the creation (annihilation) operator at site \( n \). We have independent, dynamical control over all tunneling amplitudes \( t_n \), tunneling phases \( \phi_n \), and lattice site energies \( \varepsilon_n \) by way of the relative amplitude, phase, and detuning from Bragg resonance of the corresponding frequency tone in Laser 2.

In the following sections I will describe the MSL approach in more detail, show the remarkably simple experimental setup, discuss measurement, and present three simple experiments from Ref. [5] demonstrating some capabilities of the technique.

### 2.2.2 Math

In this section, I go through the math of how the two-photon Bragg coupling scheme discussed in Sec. 2.2.1 leads to an effective tight-binding model of the form Eq. (2.1). This treatment can also be found in the supplemental materials of Refs. [5, 16].

Figure 2.5 is the same as Fig. 2.4(a-b) featuring a few more labeled variables. The electric fields of the beams can be written as \( \vec{E}^+ \cos (kx - \omega^+ + \phi^+) \) for the single frequency beam and \( \sum_n \vec{E}_n^- \cos (-kx - \omega_n^- + \phi_n^-) \) for the counterpropagating multifrequency beam. These laser fields induce a dipole moment \( e \vec{r} \) in the atom, and we can consider the resulting light-atom interaction \( e \vec{r} \cdot \vec{E} \) as a perturbation of the free-particle Hamiltonian of the atoms (again, ignoring the trap). However, as this is worked out in references like Appendix E of Atomic Physics by Chris Foot [31], we skip to the result: The resonant two-photon Bragg process between momentum states \( |n\rangle \) and \( |n+1\rangle \) with lasers far detuned by \( \Delta \) from an excited internal level \( |e\rangle \) results in Rabi oscillations between \( |n\rangle \) and \( |n+1\rangle \), with an effective Rabi rate given below in Eq. (2.5).

We write out the full Hamiltonian as

\[ H = H_0 + V(\tau), \]  

(2.2)
Figure 2.5: The **momentum-space lattice scheme**. (a) The oscillating electric fields of the single frequency beam (left) and multifrequency beam (right) are shown. (b) Dispersion relation of atoms in the ground internal state $|g\rangle$ showing momentum states coupled with two-photon Bragg transitions detuned by $\Delta$ from an excited state $|e\rangle$. Adapted from Fig. S1 of Ref. [5] with the second author’s permission.

where $H_0 = \sum_n E_n |\psi_n\rangle\langle\psi_n|$ is the free particle kinetic energy, $E_n = p_n^2/2M_{\text{Rb}}$ and the perturbative light-atom interaction term from the Bragg lasers, as explained above, couples momentum states $n$ and $n + 1$ in the form

$$V(\tau) = \sum_n (\chi_n(\tau) |\psi_n\rangle\langle\psi_{n+1}| + \chi^*_n(\tau) |\psi_{n+1}\rangle\langle\psi_n|),$$

for time $\tau$ and a two-photon coupling term between states $n$ and $n + 1$

$$\chi_n(\tau) = \sum_j \hbar \tilde{\Omega}_j e^{i\phi_j} e^{-i\tilde{\omega}_j \tau}.\quad (2.4)$$

Here, the difference in the phases and frequencies of the two lasers are encoded in $\tilde{\phi}_j = \phi^+ - \phi^-_j$ and $\tilde{\omega}_j = \omega^+ - \omega^-_j$. The strength of the coupling, or the effective two-photon Rabi rate, goes as

$$\tilde{\Omega}_j = \frac{\Omega_{j,e} \Omega_{e,j+1}}{2\Delta},$$

23
where $\Omega_{j,e}$ and $\Omega_{e,j+1}$ are the single-photon Rabi couplings of the two momentum states $|j\rangle$ and $|j+1\rangle$ to the excited state, and can be written as $\Omega_{j,e} = \langle j|e \vec{r} \cdot \vec{E}^+|e\rangle/\hbar$.

To remove the diagonal kinetic energy terms from $H_0$, we move to the interaction picture, writing the Hamiltonian as

$$H_{\text{int}}(\tau) = e^{iH_0\tau/\hbar}V(\tau)e^{-iH_0\tau/\hbar}$$

$$= \sum_n \left( \chi_n(\tau)e^{i(E_n-E_{n+1})\tau/\hbar}|\psi_n\rangle\langle\psi_{n+1}| + \chi^*_n(\tau)e^{-i(E_n-E_{n+1})\tau/\hbar}|\psi_{n+1}\rangle\langle\psi_n| \right)$$

where in the last step we have written the coupling in this interaction picture as

$$\tilde{\chi}_n(\tau) = \sum_j \hbar\tilde{\Omega}_je^{i\tilde{\omega}_j e^{-i(\omega_{\text{res}}^n-\tilde{\omega}_j)}\tau},$$

where $\hbar\omega_{\text{res}}^n = (E_n - E_{n+1})$ is the energy difference between adjacent momentum states $n$ and $n+1$ (as shown in Fig. 2.5(b)), or the resonance condition for the Bragg transition.

The laser frequencies $\tilde{\omega}_j$ that we apply to couple states $j$ and $j+1$ are in general detuned by some amount $\xi_j$ from the Bragg resonance condition $\omega_j^{\text{res}}$, so we write $\tilde{\omega}_j = \omega_j^{\text{res}} + \xi_j$ (this is sketched in Fig. 2.7(a) below). This lets us rewrite Eq. (2.9) as

$$\tilde{\chi}_n(\tau) = \sum_j \hbar\tilde{\Omega}_je^{i\tilde{\omega}_j e^{-i(\omega_{\text{res}}^n-\omega_j^{\text{res}}-\xi_j)}\tau}. $$

This coupling $\tilde{\chi}_n(\tau)$ and Eq. (2.8) make up the “full” Hamiltonian, where all of the frequencies $\tilde{\omega}_j$ are applied to all Bragg transitions. This reflects the experimental situation of both the single-frequency beam and the multi-frequency beam addressing all of the trapped atoms, regardless of their momentum.

However, only one of the multiple frequencies ($j = n$) is intended to address any given Bragg transition, and all of the other frequencies are further off-resonant and rapidly oscillating. As such, we can take a rotating wave approximation and drop all off-resonant terms.
\[ j \neq n \] to end up with only one frequency per Bragg transition:

\[
\tilde{\chi}_n(\tau) = \ldots + \hbar \tilde{\Omega}_{n-1} e^{i\phi_{n-1}} e^{-i(\omega_{n}^{\text{res}}-\omega_{n-1}^{\text{res}}-\xi_{n-1})\tau} + \hbar \tilde{\Omega}_{n} e^{i\phi_{n}} e^{i\xi_{n}\tau} \\
+ \hbar \tilde{\Omega}_{n+1} e^{i\phi_{n+1}} e^{-i(\omega_{n+1}^{\text{res}}-\omega_{n}^{\text{res}}-\xi_{n+1})\tau} + \ldots
\]

\approx \hbar \tilde{\Omega}_{n} e^{i\phi_{n}} e^{i\xi_{n}\tau}.

(2.11)

(2.12)

Such an approximation is valid in the limit where the coupling strength \( \hbar \tilde{\Omega}_{n} \) is smaller than the slowest oscillation frequency, \( \omega_{n}^{\text{res}} - \omega_{n+1}^{\text{res}} - \xi_{n+1} \). Disregarding the applied detuning \( \xi \), the frequency difference between adjacent Bragg resonances \( \omega_{n}^{\text{res}} - \omega_{n+1}^{\text{res}} = 8E_{R} \approx 16.2 \text{ kHz} \) is actually constant due to the quadratic dispersion, thus placing a limit of \( \hbar \tilde{\Omega}_{n} < < 16.2 \text{ kHz} \).

If the coupling becomes too strong, then off-resonant frequencies \( \tilde{\omega}_{j} \) for \( j \neq n \) may affect and even drive the wrong transitions. We have found that a practical limit is \( \hbar \tilde{\Omega}_{n} = 1.3 \text{ kHz} \), above which we begin to see significant off-resonant effects. As we will mention in our discussion of some early results in Sec. 2.6, it turns out that the equal spacing \( 8E_{R} \) between Bragg resonances leads to off-resonant effects in our observed population dynamics occurring at a period corresponding to the frequency spacing.

Finally, we can absorb the weak time-dependence due to the \( \xi_{n} \) terms as diagonal site energies \( \varepsilon_{n} \), where \( \hbar \xi_{n} = \varepsilon_{n} - \varepsilon_{n+1} \) (see Sec. 2.3.4 for a discussion of an ambiguity not mentioned here). This is done by redefining the states as \( |\tilde{\psi}_{n}\rangle = e^{i\varepsilon_{n}\tau/\hbar} |\psi_{n}\rangle \), allowing us to rewrite the Hamiltonian of Eq. (2.8) as

\[
H \approx \sum_{n} \hbar \tilde{\Omega}_{n} \left( e^{i\phi_{n}} |\tilde{\psi}_{n}\rangle \langle \tilde{\psi}_{n+1}| + \text{h.c.} \right) + \sum_{n} \varepsilon_{n} |\tilde{\psi}_{n}\rangle \langle \tilde{\psi}_{n}|.
\]

(2.13)

But because we like second-quantized Hamiltonians, we typically write this as

\[
H \approx \sum_{n} -t_{n} \left( e^{i\phi_{n}} c_{n+1}^{\dagger} c_{n} + \text{h.c.} \right) + \sum_{n} \varepsilon_{n} c_{n}^{\dagger} c_{n},
\]

(2.14)

which recovers Eq. (2.1) as promised! Here, we have defined the tunneling to be \( -t_{n} = \hbar \tilde{\Omega}_{n} \) and rewritten the phase \( \tilde{\phi}_{n} \to \phi_{n} \) to be in line with Eq. (2.1).
2.2.3 Experimental setup

The experimental setup for the MSL is shown in Fig. 2.6, and is very simple. The single-frequency beam passes through the atoms, and reaches this lattice setup (shown in red). It passes through two acousto-optic modulators (AOMs), getting a frequency shift of +80 MHz from the first AOM and \(-80 \text{ MHz} + \sum_n \bar{\omega}_n / 2\pi\) from the second AOM for a total shift of just the desired Bragg frequencies \(\sum_n \bar{\omega}_n\) (where again, \(\bar{\omega}_n = \omega^+ - \omega^-\) is the frequency difference between the single frequency beam and the \(n^{th}\) component of the multifrequency beam). We need to use two AOMs since the Bragg frequencies are on the order of kHz, and AOMs can only operate in the MHz range.

Both AOMs are connected to a two-channel arbitrary waveform generator (AWG), where one channel supplies an 80 MHz sine wave and the second channel outputs an arbitrary waveform (which we call an “arb”) containing the Bragg frequencies. We create “arbs” in a Mathematica notebook by adding together sine waves with the appropriate frequencies, amplitudes, and phases that we want in the lattice, and then sampling this function at a sampling rate of 500–600 MSamples/s. We export the resulting list of numbers with some preamble including the sampling rate, which sets the timescale for the arb, and transfer it over to the AWG by a USB flash drive.
During the experiment, our Cicero control software sends a trigger pulse to the AWG, allowing it to output both channels to the two AOMs. Through Cicero, we can control the duration of the experiment (how long we apply the frequency tones) as well as the output voltage from the AOMs to control the overall scaling of the resulting tunneling amplitudes. All other lattice parameters must be changed when creating an arb, but we can generate many arbs with different lattice parameters, and switch between them automatically via serial communication with the AWG.

2.2.4 Measurement

Measurement in our system is done with the standard cold atom technique of time-of-flight absorption imaging. The trapping potential (the OT2 beam) is turned off, and atoms fall for a duration of “time-of-flight” in the direction of gravity. This maps the momentum distribution of the atoms to real space, as atoms in different momentum states separate spatially, allowing them to be distinguished and imaged separately.

To image the atoms, we shine on them a beam that is nearly resonant with the cycling transition. Atoms absorb photons from this beam, casting a shadow and lowering the intensity of the beam, which is then directed onto a camera and measured. The amount of absorption can be described by Beer’s Law,

\[ I_{\text{out}} = I_{\text{in}} e^{-n\sigma} = I_0 e^{-OD}, \]

(2.15)

where \( n \) is the column density of the atoms along the imaging direction and \( \sigma \) is the absorption cross section (related to the linewidth of the transition, detuning of the beam, etc.). Our observable is the optical density \( OD = n\sigma \), which can be related to the total number of atoms.

In experiment, we take three successive images: a signal image, a reference image, and a background image. The signal image shows the atomic shadow in the beam, and we compare this to a reference image taken after all of the atoms have been blasted away. Then, we turn off the imaging beam and take a background image to account for any stray background
light, noise from the camera, etc. The OD can be calculated as

$$\text{OD} = \ln \frac{I_{\text{ref}} - I_{\text{bg}}}{I_{\text{sig}} - I_{\text{bg}}},$$

(2.16)

for the measured intensity of the signal (sig), reference (ref), and background (bg) shots.

To get the atomic population, we perform gaussian fits on the OD images, and relate the area under the gaussian to the total atom number. This is useful in characterizing the BEC and the efficiency of laser cooling.

For MSL experiments, we only care about the normalized lattice site populations, so we perform a multi-gaussian fit on the many populated lattice sites, and divide by the sum. Specifically, if the state of the atoms in the lattice is $|\psi\rangle = \sum_n c_n |n\rangle$ for lattice sites $|n\rangle$ with momentum $p_n = 2n\hbar k$, this measurement gives the normalized values $P_n = |c_n|^2$.

### 2.3 More details on the MSL: extensions and limitations

#### 2.3.1 Some commentary

The MSL technique was developed by our lab (and has since spread to other groups! [44]), and offers a very different approach compared to real-space optical lattices. First, because the lattice sites are momentum states and the BEC starts at rest, every experiment begins with all population in a single lattice site $n = 0$. Thus for most lattice systems, we primarily study out-of-equilibrium dynamics. We can alternatively load some lattice eigenstates through an adiabatic process (Chaps. 6 and 8, and several of Eric’s studies [18, 26, 27]) – however, this is highly challenging for states spread out over many lattice sites. This can also be viewed as a feature, though, as this out-of-equilibrium regime is typically harder to access in real-space systems. Second, our technique allows for control at the individual lattice site level, and allows for dynamical control of all lattice parameters. This enables us to create lattices with simple features that are tough to achieve in real-space systems, including hard-wall boundary conditions [5], alternating tunneling links [16, 18], specific site energy configurations [14, 17], artificial magnetic fields from control over tunneling phases [15, 19], and more.
2.3.2 Extensions to the MSL

In this chapter, I have described how to create a momentum-space lattice in one-dimension with nearest-neighbor couplings. With the control over the tunneling amplitudes, tunneling phases, and lattice site energies, this MSL technique can be used to study a whole host of interesting physics, as shown by the introductory studies discussed above. However, in the course of my graduate work, I have implemented and/or studied additional capabilities of the MSL technique.

First and foremost is the presence of nonlinear atomic interactions, which we discuss later in Chaps. 5 and 9. In short, we initially believed that the long-ranged interactions in the MSL stemming from short-ranged contact interactions in real space would lead to equal interaction energies for atoms in all momentum states/lattice sites. It turns out that there exists a mode-dependence to the interactions, making them look like an effectively on-site attraction (when ignoring many effects like the harmonic trap, superfluid screening, etc.). We have begun to explore the effects of these interactions, yet have quickly found the need to access a Feshbach resonance to tune the scattering length and thus the interaction strength... and $^{87}$Rb is not the best choice for such a task. As we detail in Chap. 10, we are currently switching atomic species to $^{39}$K to harness its more forgiving Feshbach resonance (broader resonance at a lower magnetic field) in order to study interacting physics.

I have also implemented the MSL in two dimensions by adding a second lattice beam with a different wavelength $\lambda = 781.5$ nm. As described in Chap. 3, this allowed us to realize a quantum Hall-like system, mimicking the effect of a magnetic field on electrons on a 2D lattice. Unfortunately, this was the only study in which we used this second lattice – as I explain later, there were many issues with this setup, stemming from both the denser frequency spectrum and our lack of phase stabilization between the two lattice beams. In our future setup with $^{39}$K, we are planning to implement higher MSL dimensions in a different way: by orienting three beams in a “T” or “Y” shape in space.

Finally, as discussed in Chap. 6, I have implemented longer-ranged tunneling terms by addressing higher order Bragg transitions, e.g. a four-photon transition between $n = 0$ and $n = 2$. This also had its issues stemming from significant off-resonant effects, but these can...
be addressed by appropriately detuning the beams ahead of time, as shown by Refs. [45, 46].

2.3.3 Limitations: off-resonant effects and spatial decoherence

One limitation of the MSL that we have already discussed is the effect of off-resonant frequencies. This places a cap on the tunneling amplitudes we can achieve: \( t << 8E_R \approx h \times 16 \text{ kHz} \) (empirically, \( t < h \times 1.3 \text{ kHz} \)) for a uniform 1D lattice. The limit above is sort of a best-case scenario, as in many cases we would like to detune some of these frequencies, leading to even smaller spacings between frequency teeth and a lower cap on the tunneling. When working in higher dimensions, whether with another laser (Chap. 3) or with higher-order tunnelings (Chap. 6), the frequency spectrum becomes more densely populated as well, leading to both significant off-resonant effects and lower working tunneling amplitudes. Finally, even when working with tunneling values near 800–1000 Hz, we see jagged dynamics due to the uniform spacing between adjacent frequencies, as touched on in Sec. 2.6.1.

A second limitation is spatial decoherence: atoms in momentum states eventually fly apart and lose spatial overlap, preventing us from driving transitions between states. This sets a limit on the timescales we can achieve in experiment, and is dependent on the tunneling and what/how momentum states are occupied – for example, some of our interacting studies feature population self-trapped to the \( n = 0 \) site, so we can measure out to much longer evolution times in that case. This problem is especially problematic for low tunneling rates (and thus long tunneling times), which means we need to apply the lattice for longer evolution times while the states separate in space more. This also hampers out ability to adiabatically load eigenstates of lattices, as such a process depends on slow parameter ramps. For all tunneling values, we typically cannot see dynamics past \(~10\) tunneling times. For an example of this behavior, see Sec. 2.6.3 (and basically every chapter).

There are more limitations to the MSL technique specific to the studies we perform, so we will discuss these in their respective chapters.
2.3.4 Implementation of site energies

In our implementation of lattice site energies, we detune by $\xi = \omega^+ - \omega_j^- - \omega_{j\text{res}}^+$ from Bragg resonance the lasers driving a Bragg transition between sites $j$ and $j + 1$, leading to a difference in energy between the resulting lattice sites $j$ and $j + 1$ of $\xi$ (see Fig. 2.7(a)). However, there is some ambiguity here: if we detune above resonance, does lattice site $j$ or $j + 1$ have a higher energy?

For our single-particle experiments (in the absence of interactions), it doesn’t actually matter which site has more energy, as long as everything is consistent. But atomic interactions in our system can be treated as an effective on-site attraction, leading to drastically different behavior for atoms in a lattice site at high energy vs. atoms in a lattice site at low energy. The interactions can be thought of as dragging down the local chemical potential of a lattice site, such that for a low energy site, the interactions bring the effective chemical potential further away from the energies of nearby sites, and thus promoting self-trapping and localization. In contrast, dragging down the chemical potential of a high energy site brings it closer in energy to other lattice sites, increasing the rate of tunneling out of the site and thus promoting delocalization. We see this effect in all of our interactions studies, but most notably in our work on the generalized Aubry-André model in Chap. 8, where interactions have a significant impact on the localization physics of a low energy state and a high energy state.
Thus, when studying interacting physics we must resolve this ambiguity in site energies... and what better way to do so than by employing interactions themselves? In the course of our study on interactions in a double well in Chap. 5, we determined that a detuning \( \xi = \omega^+ - \omega^- - \omega^\text{res} \) leads to a site energy difference of \( \varepsilon_j - \varepsilon_{j+1} = \hbar \xi \). This is best explained pictorially, as shown in Fig. 2.7.

### 2.4 Simulations

In conducting experiments, we run numerical simulations to plan out the experiment, predict what we expect to see, and try to explain what we observed. These simulations take one of two forms:

1. **“Ideal” simulations** of the tight-binding Hamiltonian of Eq. (2.14) where each applied frequency addresses exactly one Bragg transition.

2. **“Full” simulations** that consider the effects of all frequencies on all Bragg transitions (no rotating wave approximation), simulating the Hamiltonian of Eq. (2.8) with couplings given by Eq. (2.10).

For single-particle studies (in the absence of atomic interactions), we use Mathematica to numerically solve the time-dependent Schrödinger equation on the appropriate Hamiltonian matrix (Eq. (2.14) for an ideal simulation and Eq. (2.8) for a full one). We call Mathematica’s `NDSolve` function on the matrix, supplying also an initial condition of population entirely on one lattice site (\( \mid \psi_{\text{init}} \rangle = \mid n = 0 \rangle \)). We do not explicitly specify the numerical method used by `NDSolve` (though it is an option), and it switches methods for efficiency depending on the situation. Also, it is impossible to find what method Mathematica chooses for `NDSolve` after the fact for multi-step numerical methods. That said, as Urbana-Champaign residents, we trust Wolfram with their numerical outputs, which have agreed quite well with our experimental data in many studies.

In the presence of atomic interactions, we do the same process, but solving the Gross-Pitaevskii equation instead, assuming some mean-field interaction. This process is described later in Sec. 5.1.4 after we discuss the form of the interactions in the MSL.
2.5  Typical MSL experiment procedures

This section is dedicated to describing the workflow of a typical experiment. The four main components of the entire experiment are:

1. The apparatus
2. The arbitrary waveform generator (AWG, nicknamed “Newbo”)
3. Cicero+Atticus: the control software
4. The main desktop computer (nicknamed “Cleopatra”, replaced in 2019 with two computers “KMOM” and “KPOP”), which contains...
   (a) Image collection (ANDOR SOLIS software, physical cable to ANDOR NEO camera)
   (b) Data analysis (Mathematica)
   (c) Arbitrary waveform generation (Mathematica)
   (d) Simulation code (Mathematica/Python)

Typically, an experiment begins with simple tight-binding simulations of some lattice model we wish to explore, as described in the previous section. By including all Bragg frequencies in a “full” simulation, we are able to predict with very good accuracy the results we eventually see in experiment. It turns out that much of the same code to set up the Hamiltonian for a “full” simulation can be used to generate arbitrary waveforms as well (without the NDSolve step).

We generate “arbs” to control the lattice as described in Sec. 2.2.3. In addition to the “data arb” containing the frequencies that make up the lattice we want to study, we also generate a corresponding “calibration arb” consisting of one tunneling link between sites \( n = 0 \) and \( n = 1 \), scaled to be the same amplitude as a tunneling link in the data arb.

We use this to calibrate the tunneling amplitudes, which is set not only by the relative amplitudes of the frequencies within the arb, but also by the peak-to-peak voltage of the overall waveform, which is set externally through Cicero. In such a calibration, we apply the one frequency tooth for some duration, and vary that duration to observe full Rabi oscillations between the two sites. We fit the dynamics to obtain a Rabi frequency (and thus tunneling amplitude), and feed back on the voltage of the waveform until we achieve the
desired tunneling amplitude. Because the power and alignment of the lattice beam on the
atoms can drift day-to-day and even hour by hour, we perform these two-site Rabi oscillation
calibrations several times a day, and always before and after every data run.

Then, we run the data arb. The full experimental cycle from hot atomic gas to BEC
takes about 20 seconds, and we apply the lattice for evolution times typically less than
1 ms. After each “shot” or experimental cycle, we can vary a parameter and take another
shot.\textsuperscript{11} Depending on what we wish to study about the lattice, we can vary the duration
that the arb is applied (the evolution time under the lattice), the overall tunneling by scaling
the voltage of the arb, or lattice parameters like site energies or some tunneling phase by
selecting multiple arbs in succession. We can also manipulate the atoms via Cicero, such
as holding them in a tighter or looser trap by adjusting the powers in the optical trapping
beams, or by waiting for some time to lower the atom number. Regardless of the parameters
we vary, we typically take data for a maximum of around 3 hours (around 500 shots) before
taking another calibration run. Usually the value of the tunneling does not drift too much
(easily $< 10\%$), but in some of our later works under a leaky vacuum and dying atomic
source, we had to check the tunneling much more often.

At the end of each shot, the camera is triggered and we take an absorption image
(Sec. 2.2.4), which is automatically saved to file on the computer. At the end of a data
run, we can import the files and perform a multi-Gaussian fit to the data to read out the
normalized population in each lattice site. We then can calculate observables like average
position, spread of the population distribution, etc. and perform further data analysis and
visualization in Mathematica.

One important benefit of our MSL approach is how easy it is to study a “new system”: by
simply writing a new arb with different frequencies, amplitudes, and phases, we can switch
on the fly between studying loss in a 1D lattice (Chap. 7) to artificial gauge fields on a zigzag
lattice (Chap 6). Not only is this useful for making new experiments, it is also practically
helpful, as we have switched between my experiment on Monday to Eric’s study on Tuesday,
and so on.

\textsuperscript{11}Often a shot in the dark, sometimes accompanied by a shot of something stronger.
2.6 A few simple demonstrations

Here I’ll present three simple experiments demonstrating some of the capabilities of the MSL. This material was covered in our first experimental paper in Ref. [5].

2.6.1 Continuous-time quantum walks

For our very first experiment on the MSL, we engineer a straightforward 1D lattice of \( N = 21 \) sites with uniform tunnelings \( t \) and no site energy shifts. Population begins at rest in the \( n = 0 \) state, and we “quench” on the lattice beams, quickly flashing on the lattice frequencies and light onto the atoms. Then, this system follows the simple tight-binding Hamiltonian

\[
H = -t \sum_n \left( c_{n+1}^\dagger c_n + c_n^\dagger c_{n+1} \right). \tag{2.17}
\]

The site populations can be measured with time-of-flight absorption imaging, as shown in Fig. 2.8(a) for the initial state at time \( \tau = 0 \) and after 1.9 tunneling times (experimental data on left, “ideal” numerical simulation of Eq. (2.17) on right). We take these optical density images and integrate along the vertical dimension to produce a 1D line of data. By taking many images at different evolution times under the lattice, we stitch together many of these 1D cuts to visualize the entire dynamics at once, as shown in Fig. 2.8(b). For our system size of 21 sites, population does not reach the edge of the lattice until 5 tunneling times.

We extract the normalized populations with a multi-Gaussian fit, and plot the populations of three central sites \( n = 0,1,2 \) as black circles, open green circles, and red squares, respectively, in Fig. 2.8(c). Overlaid on top of the data are an ideal simulation of Eq. (2.17) (dashed, transparent curves) and a full simulation accounting for all applied frequencies (solid curves). As foretold at the end of Sec. 2.2.2, here we see the effects of off-resonant frequencies on the dynamics, showing up as jagged jumps in both the data and the full simulation. More specifically, because the lattice is uniform, the frequencies we apply to the atoms are equally spaced by \( \omega_{n+1}^{\text{res}} - \omega_n^{\text{res}} = 8E_R \approx 16.2 \text{ kHz} \). All of the off-resonant frequencies contribute in some constructive fashion, leading to jumps in the data occurring at this
Figure 2.8: Continuous-time quantum random walks on momentum-space lattices. (a) Time-of-flight absorption (optical density, OD) images of lattice site populations for evolution times \( \tau = 0 \) and \( 1.9 \ h/t \) on a \( N = 21 \) site uniform 1D lattice, shown for experimental data (left) and “full” numerical simulation (right). (b) Full dynamics for experiment and simulation, where each row of pixels is an OD image like in panel (a), but integrated along the vertical dimension. (c) Fitted site populations for lattice sites \( n = 0, 1, 2 \) in black, green and red, respectively. Full and ideal simulations are shown as solid and transparent dashed curves, respectively. (d) Full dynamics of atoms in an \( N = 5 \) site lattice. (e) Spread of population distribution on an \( N = 21 \) site lattice (black) and an \( N = 5 \) site lattice (red), with corresponding full numerical simulation results (solid curves). Adapted from Fig. 2.3 of Ref [29], which was in turn adapted from Fig. 1 of Ref. [5].

same frequency. The good news is that such jumps, which are also visible in the integrated OD plot of Fig. 2.8(b), are mostly cosmetic as the data still follows the ideal simulation.

Because the couplings are all equal, the atoms on this lattice undergo a continuous-time quantum walk (CTQW) which displays many of the hallmark features associated with a discrete-time quantum walk [47], as realized in experiments with single atoms [48], ions [49], and photons [50, 51]. Both cases differ drastically from a classical random walk, where the distribution of the random walkers at long times is Gaussian and peaked in the center. As shown in our data, in the quantum version, interference between the different paths leads to a markedly different probability distribution peaked at the edges. This difference in behavior can be characterized by the standard deviation of the distribution, which goes as \( \sigma_n(\tau) \propto \tau^{\beta} \) for evolution time \( \tau \). The diffusive spreading of a classical random walk results in \( \beta = 1/2 \), whereas the ballistic spreading of a quantum walk gives \( \beta = 1 \).

We plot the spread of the population distribution \( \sigma_n \) in Fig. 2.8(e) as black dots, again with a full simulation overlaid on top. As expected, both the data and the simulation show a linear increase, indicating ballistic spreading. Plotted here also is the spread on an \( N = 5 \)
site lattice (red), with dynamics shown in Fig. 2.8(d). By simply not coupling sites beyond
$n = \pm 2$, we are able to easily implement hard-wall boundary conditions, showing population
reaching the edge of the lattice around 1.5 tunneling times and coming back to the center.

2.6.2 Bloch oscillations

As an extension to the flat 1D lattice explored in the previous section, here we add a constant
potential tilt $\hbar \xi$ to the lattice, mimicking the effect of a uniform force on a real-space lattice.
This is done by simply detuning all of the applied frequencies from Bragg resonance by $\xi$, leading to the Hamiltonian

$$H = -t \sum_n \left( c_{n+1}^\dagger c_n + c_n^\dagger c_{n+1} \right) + \sum_n \varepsilon_n c_n^\dagger c_n,$$  \hspace{1cm} (2.18)

where $\varepsilon_{n+1} = \varepsilon_n + \hbar \xi$, and we again use a system size of 21 sites. We note that because
we can only control 20 frequencies, we can only set the difference in adjacent site energies,
generating the various site energies up to an overall constant. This lattice is sketched out in
Fig. 2.9(a).

While a classical particle on a potential slope would accelerate indefinitely down the
hill, a quantum particle in a tilted periodic potential instead undergoes oscillatory mo-
tion. These Bloch oscillations were predicted nearly a century ago [52, 53], and over the
past several decades have been experimentally studied in electronic systems [54], with cold
atoms in optical lattices [55], in optics [56], and even in the rotational excitations of N$_2$
molecules [57]. The absence of dissipation in our system prohibits transport of our initially
localized wavepackets in the tilted potential, so instead we expect periodic spreading and
refocusing in momentum-space, as recently observed using cold atom microscopy [58].

We observe such oscillatory behavior in the dynamics, shown for $t = 0.33E_R$ and $\hbar \xi \approx$ $E_R$ in Fig. 2.9(b). The atomic populations undergo periodic cycles of delocalization and
refocusing at the original position, with a characteristic Bloch frequency given simply by
$\omega_B = \xi$. Fig. 2.9(c) plots the spread of the population distribution $\sigma_n$ versus time (in units
of tunneling time $\hbar/t$) under constant tunneling $t = 0.33E_R$ and three different values of
2.6.3 Tunneling echo

In this third experiment, we focus on tunneling phases, which can be used to generate artificial magnetic fluxes for neutral atoms in higher dimensions (which we demonstrate in Chap. 3). In 1D with nearest-neighbor couplings, any static pattern of inhomogeneous tunneling phases (i.e., any static gauge field) is of no consequence with respect to either equilibrium density distributions or site occupation dynamics. This results from the fact that these phases can simply be “gauged away” via local transformations.

Here, we demonstrate not only the MSL’s capability to control tunneling phases, but also
its ability to vary lattice parameters in time. Specifically, we demonstrate the reversal of population dynamics by periodically inverting the phase on all tunneling links, $\phi \rightarrow \phi + \pi$. For a dispersive lattice with uniform tunneling amplitudes $t$, this phase inversion can be thought of as band inversion $t \rightarrow -t$, leading to a complete reversal of dynamics such as in the case of light propagation in negative index materials. More directly, this can be thought of as the higher-spin version of a rotary spin echo sequence [59].

Figure 2.10(a) shows population dynamics (integrated OD image) on a 21-site 1D lattice with uniform tunnelings $t \approx 0.3E_R$. Population undergoes a quantum walk as in Fig. 2.8 until we suddenly invert the phases at the red line near 325 $\mu$s, after which the dynamics run in reverse. The fitted populations for sites $n = 0, 1, 2$ (black, green, red) are shown in Fig. 2.10(b-d) for faster and faster rates of phase inversion (denoted by the vertical gray lines). We can see that for very fast rates of phase inversion (with respect to the tunneling rate $t/\hbar$), transport is inhibited and population remains largely in the central momentum order.

The absence of a perfect reversal of dynamics in Fig. 2.10 is a consequence of the
main practical limitation expected for this experimental scheme – the loss of spatial mode-matching between the different momentum states. As our trapped sample of atoms initially have some finite coherence length, spatial separation between the differing momentum states \( n \) will lead to the loss of coherent momentum-space “tunneling dynamics” as driven by two-photon Bragg transitions. We expect that this technical limitation, which may be largely mitigated by working with extended samples of atoms, at the moment presents the greatest source of decoherence in the presented studies.
Chapter 3

2D Flux Ladders: Towards Hofstadter Lattices

“There’s no bulk”

-Conference audience

Momentum-space lattices are well-equipped to realize artificial magnetic fields in two dimensions, simulating the quantum Hall behavior of electrons on a 2D lattice under a magnetic field. In fact, studying these quantum Hall systems was one of the motivations for creating synthetic lattices in the first place (in addition to simulating extra dimensions) [3]. In this chapter, I briefly describe the physics of quantum Hall systems, discuss how we implement an artificial magnetic field for neutral atoms, and present experimental results working towards full 2D quantum Hall systems.

This chapter is adapted from the text of Ref. [19], with additional background and notes.

3.1 Background

3.1.1 Quantum Hall Systems

A topologically nontrivial system cannot be fully described by a local measurement. Instead, a global measurement must be made to capture the behavior of the full system. The classic example is the donut: Imagine an ant living on the outside rim of the donut. It may make a local measurement of its surroundings without seeing the hole in the center, and determine that it lives on a disc, or a sphere. To fully capture the topology of the system, it needs to traverse the whole donut and see the hole, eating with glee.

Quantum Hall systems exhibit nontrivial topology much like the donut, exhibiting drastically different behavior in different regions of the system. This topology arises in two-dimensional electron systems under an external magnetic field. The underlying microscopic
topology gives rise to striking macroscopic quantum effects \[60\], resulting in a Hall conductance that is quantized as “Hall plateaus” with increasing applied magnetic field. The topology of the lattice is robust to (weak) disorder, such that even real materials dirty with defects can offer very precise measurement of Hall conductance and resistance. In fact, the measurement of Hall resistance helps form the basis for the SI unit of current, with a miniscule error of $\delta R/R \sim 2 \times 10^{-11}$ \[61\].

Microscopically, the quantum Hall system is the paradigmatic example of a topological insulator, where the bulk of the system is insulating and the surface is conducting. In the bulk of the lattice, electrons undergo cyclotron orbits under the magnetic field, leaving them localized to a few sites. Along the edge of the lattice, where the system meets another material or the vacuum, electrons cannot complete full orbits and instead undergo “skipping orbits”, traveling along the edge of the system. These “chiral edge states” or “chiral currents” are linked to the quantized conductance and are a hallmark of the topology of the quantum Hall system. However, direct observation of these microscopic chiral edge states has been challenging in real materials.

Some more notes on the quantum Hall model: In addition to the macroscopic quantized conductance and the microscopic topology and edge states, this system also exhibits a fractal energy spectrum called Hofstadter’s butterfly, which we plan to net in the future (see Sec. 10.2). Finally, with particle interactions, this picture changes to give the fractional quantum Hall effect. How this effect occurs microscopically, and why plateaus occur at specific magnetic field values, is an open question in physics.

Under a magnetic field $\mathbf{B} = B\hat{z} = \nabla \times \mathbf{A}$, this system obeys the Harper-Hofstadter Hamiltonian,

$$H = -t_x \sum_{n,m} \left( c_{n+1,m}^\dagger c_{n,m} + \text{h.c.} \right) - t_y \sum_{n,m} \left( c_{n,m+1}^\dagger c_{n,m} e^{i\phi_{n,m}} + \text{h.c.} \right),$$

where $t_{x,y}$ denotes tunneling in the $x$ and $y$ directions. The phase term $\phi_{n,m} = \frac{e}{\hbar} \int_{r_{n+1,m}}^{r_{n,m}} \mathbf{A} \cdot d\ell$ denotes the Aharanov-Bohm phase that a particle picks up along a tunneling link $n \to n+1$. For simplicity, we write this as $\phi_{n,m} = 2\pi \alpha n$ under the Landau gauge, where $\alpha$ is the ratio
Due to the flux from the magnetic field $B$, a particle wavefunction picks up an Aharonov-Bohm phase $2\pi\alpha = \frac{ea^2 B}{\hbar}$ as it circles a 4-site plaquette on the lattice. This flux and phase can be encoded into the phases on the tunneling links.

$$\alpha = \frac{\Phi}{\Phi_0} = \frac{Bd^2}{\hbar/e} = \frac{ed^2 B}{\hbar}, \quad (3.2)$$

where $d$ is the distance between adjacent lattice sites and $e$ is the electron charge. Figure 3.1 shows the physical picture on one 4-site plaquette of the lattice (an elementary cell). Only the vertical tunneling links get a phase dependent on the horizontal position, whereas horizontal links have no tunneling phase. A particle that traverses a full loop about this plaquette gets a phase of $2\pi\alpha = \frac{ed^2 B}{\hbar}$, a value that is gauge invariant.

In electron systems, this phase is generated with an applied magnetic field, and leads to the topological behavior. However, cold atoms are charge neutral, and thus do not respond to a magnetic field in the same way. Instead, the fluxes and phases are created directly, effectively making an artificial magnetic field (synthetic gauge field) for neutral atoms.

### 3.1.2 Synthetic gauge fields with real-space optical lattices

Since it seems like a hassle to generate artificial magnetic fields for neutral atoms, one important question to address is: why do it at all? As always, the usual arguments of quantum simulation apply: cold atom systems are very pure and highly tunable, allowing for more controllable experiments with topological systems. To pursue some strongly correlated physics like the fractional quantum Hall effect, the easy control over atomic scattering length (via a Feshbach resonance) makes cold atoms an appealing platform as well. The microscopic
Figure 3.2: **A scheme to generate artificial magnetic fields in real-space optical lattices with Raman-assisted tunneling.** (a) Atoms in the lattice are subjected to a tilt $\Delta$ (between adjacent sites) which inhibits tunneling along the horizontal axis, but two Raman lasers (red and blue) address a two-photon transition between adjacent sites. (b) The 2D picture. Tunneling phases from the Raman beams are imprinted onto the atomic wavefunctions, generating an effective magnetic flux $\phi_y$. Adapted from Ref. [72].

detection of atomic gases can allow for detection of microscopic phenomena which are hard to see in real materials, like directly observing chiral edge states. Finally, by focusing on generating only the flux terms, cold atom techniques have been able to generate enormous effective magnetic fields, well above any realistic magnetic field values for electron systems (a flux of $\pi/2$ corresponds to fields higher than 1000T). These benefits, coupled with the rise of synthetic lattice techniques like the MSL, have made topology a thriving topic of study in the cold atom community [62].

In cold atoms, early approaches saw success creating artificial magnetic fields in bulk atomic gases (in the absence of a lattice), either by rotation [63] or bulk Raman addressing [64–66]. Real-space lattice techniques utilizing lattice modulation [67–69] and laser addressing [70–72] have proven capable of reaching the regimes of large effective magnetic fields and strong spin-orbit coupling.

In works like Ref. [71, 72], the idea was to write phases onto the tunneling links with laser-addressed transitions between sites in real-space lattices, as shown in Fig. 3.2. These groups applied a potential tilt $\Delta$ to their lattices, strong enough to inhibit tunneling between
nearest-neighbor lattice sites along the horizontal axis. Then, they applied a pair of lasers (frequencies $\omega_1, \omega_2$) to address two-photon Raman transitions between adjacent sites, such that the frequency difference matched the energy difference between adjacent sites: $\omega_1 - \omega_2 = \Delta / \hbar$. This allowed atoms to again tunnel between sites, but via this Raman transition. Further, these two Raman lasers are angled such that their interference pattern produces a spatially-varying phase, effectively controlling the phases on tunneling links. By changing the angle between these Raman lasers and the angle of these lasers with respect to the lattice, the effective magnetic flux can be controlled.

This Raman-assisted tunneling scheme has realized the Harper-Hofstadter Hamiltonian of Eq. (3.1) [71, 72], measured cyclotron orbits within the lattice, created staggered fluxes [71], demonstrated the quantum spin Hall effect [73], indirectly observed chiral currents [74], and more. Still, nontrivial heating remains an issue for lattice-based schemes [75, 76].

### 3.1.3 Synthetic gauge fields with synthetic lattices

The use of atomic internal states as synthetic dimensions [2, 3, 6, 7, 77] has emerged as a recent alternative strategy that may obviate some sources of heating. Two simultaneous works realized this system with one real-space dimension and one synthetic (hyperfine state) dimension, realizing 2D systems with fixed artificial magnetic flux and observing the resulting chiral edge currents using bosonic [6] and fermionic atoms [7]. The key idea here is the control over tunneling phases via field-driven transitions. By simply writing phases onto the beams controlling individual transitions, synthetic lattice approaches can quite easily generate the required phase pattern of Fig. 3.1.

Our approach with momentum-space lattices uses a fully synthetic 2D lattice, with synthetic dimensions along both axes. This offers two key benefits: our complete control over tunneling phases gives a fully tunable synthetic flux, from the full range of $[-\pi, \pi]$, and our site-resolved measurement allows us to directly image chiral currents. We directly image chiral atomic currents induced by a homogeneous flux, and observe magnetic reflection of atoms from a step-like jump of an effective magnetic vector potential generated by an inhomogeneous flux. These advances in the creation of artificial gauge fields, combined with the
Figure 3.3: The two-leg flux ladder. (A) Two sets of lattice laser fields (with wave numbers $k_1$ and $k_2$) addressing transitions between atomic momentum states of a Bose-Einstein condensate (BEC). (B) Free-particle dispersion relation showing momentum states on the $m = 0$ (white circles) and $m = 1$ (gray circles) legs, labeled by $(m, n)$ with momentum $p = 2\hbar (mk_1 + nk_2)$. Short red and tall blue arrows denote transitions controlled by $k_1$ and $k_2$ wave number lattices, respectively. Inset: 2D lattice representation, with links addressed by the $k_1$ (red, vertical) and $k_2$ (blue, horizontal) wave number beams. The recoil energy is given by $E_{R,2} = \hbar^2 k^2 / 2M_{\text{Rb}}$. (C) Time-of-flight image of atoms in momentum orders $(m, n)$. (D) Image from (C) rearranged to show the 2D lattice. This figure and (C) show absorption images using the normalized OD scale at the right. (E) Schematic of a two-leg ladder with applied tunneling phases $\phi_i$ on each link of the $m = 1$ leg, resulting in fluxes $\phi_i$ around each four-site plaquette.

available control of all tunneling terms and site energies, should greatly expand the variety of topological systems open to investigation through cold atom simulation.

3.2 Momentum-space lattices in 2D

For this work, we extended our MSL technique described in Chap. 2 to two dimensions by adding a second pair of lattice beams with wavelength $\lambda_1 = 781.5$ nm (wavenumber $k_1 = 2\pi / \lambda_1$). This is in addition to our existing lattice beams of wavelength $\lambda_2 = 1064$ nm (wavenumber $k_2 = 2\pi / \lambda_2$). As shown in Fig. 3.3A, the two lattices (solid red and dashed blue) are aligned to be co-propagating, and the wavelengths are chosen to be incommensurate. That is, one pair of lattice beams drives transitions between momentum states $p_m = 2\hbar mk_1$, and the other pair accesses states $p_n = 2\hbar nk_2$. By choosing incommensurate wavelengths
(irrational $k_1/k_2$), these two separate momentum-space lattices should never overlap except at $p = 0$. We can thus consider the two lattices to be separate degrees of freedom, or effectively two separate dimensions. The overall 2D lattice is then comprised of states $p_{m,n} = 2\hbar(mk_1 + nk_2)$ which are accessed by applying $m$ and $n$ two-photon Bragg transitions from the $k_1$ and $k_2$ lasers, respectively.

In this study, we create two-leg ladder lattices of 2 by 5 sites, using the $k_1$ laser for the shorter dimension and the $k_2$ laser for the longer one. Figure 3.3B shows where the states of the effective two-leg ladder lie on the atoms’ free-particle dispersion relation (1D momentum distribution), and how they map onto the resultant 2D lattice with site indices $(m, n)$. To be more illustrative, some example absorption image data is shown in Figs. 3.3C-D. Lattice site populations in our typical absorption images (Fig. 3.3C) are spread out in momentum in one dimension, and it is often difficult to tell where each momentum order is (especially when there is little to no population in an order). We use the initial at-rest condensate for reference ($p = 0$) and label all of the momentum orders with their appropriate indices $(m, n)$, rearranging the data into a 2D picture like in Fig. 3.3D. To generate the synthetic fluxes, we simply place tunneling phases $\phi_n$ on all of the links of the bottom leg of the ladder, as shown in Fig. 3.3E. This implementation allows us to individually control the flux through each plaquette in the lattice, allowing us to either create homogeneous flux to mimic a uniform magnetic field or generate an arbitrary staggered flux pattern. We can even ramp this flux in time to mimic changing the applied magnetic field.

There are several complications to working with 2D MSLs using these two lattices, most of which stem from our lack of phase stabilization between the two lattices, and from the more compact frequency spectrum. These issues are discussed in more detail later in this chapter, in Sec. 3.5.2. In general, these experimental challenges lead to messy results in the data, and further limit our lattices to a small geometry of 2 $\times$ 5 sites. All of the sites in this small lattice geometry live on its edge, so there is no real “bulk” to the system. Thus, while we hesitate to call this a “quantum Hall” system in full, it still displays an analogous chiral current behavior under a synthetic gauge field.
Figure 3.4: **Shearing in the flux ladder.** (A) Schematic showing atoms undergoing clockwise shear (arrows) for positive flux $\phi$, corresponding to an effective magnetic field $B$ directed out of the page. Red filled-in circles represent the initial state. (B) Shearing dynamics for $\phi = -\pi/2$ (top, blue) and $\phi = +\pi/2$ (bottom, red). Dashed and solid curves represent numerical simulation results based on Eq. 3.3 and a more complete model taking into account off-resonant transitions, respectively, both scaled and offset to match the data. Dashed vertical lines indicate the time when the data for (C) and (D) were taken. (C) Site populations for $\phi = -\pi/2$ (left, blue) and $\phi = +\pi/2$ (right, red). Color scale used is the same as in Fig. 3.3B. (D) Shearing vs. applied flux. Solid line represents results from a simulation of the more complete model. Measurements for (C) and (D) were taken after 500 $\mu$s ($\sim 1.06 \, \hbar/t$), indicated by dashed vertical lines in (B). The calibrated tunneling rates for (B) and (D) are slightly different, so this time translates into different tunneling times for the two. All error bars denote one standard error.

### 3.3 Shearing in the homogeneous flux ladder

We begin by directly mimicking a magnetic vector potential in the Landau gauge, $\hat{A} = (0, Bx, 0)$, through homogeneous tunneling phases on the 2×5-site ladder. This gives rise to a uniform effective magnetic field as shown in Fig. 3.4A. The dynamics of our cold atoms are effectively governed by the Hamiltonian

$$
\hat{H} = -t_x \sum_n \left( \hat{c}^\dagger_{1,n} \hat{c}_{0,n} + \text{h.c.} \right) - t_y \sum_{m,n} \left( e^{i\phi_{m,n}} \hat{c}^\dagger_{m,n+1} \hat{c}_{m,n} + \text{h.c.} \right),
$$

(3.3)
which resembles the Harper-Hofstadter Hamiltonian of Eq. (3.1), and where \( \hat{c}_{m,n} \) (\( \hat{c}_{m,n}^\dagger \)) is the bosonic annihilation (creation) operator for the state with indices \((m,n)\).

In terms of the effective magnetic field \( B \), the engineered tunneling phases along \( y \) are given by \( \phi_{m,n} = -m\phi \), where the flux associated with a closed loop around an individual four-site plaquette is given by \( \phi = 2\pi e d^2 B / h \), as in Eq. (3.2). Again, \( d \) is the effective spacing between synthetic lattice sites, \( q \) is the effective charge of the particles, and \( h \) is Planck’s constant. Here, and in the remainder of this work, we employ homogeneous tunneling strengths and engineer hard-wall system boundaries through the direct control of all tunneling magnitudes.

To probe the influence of our tunable field \( B \) on these “charged” particles, we observe their nonequilibrium response to a sudden quench of the effective field. In particular, we study the response of atoms initially prepared in a symmetric superposition of occupation on sites \((0,0)\) and \((1,0)\). This initialization is done with a simple square \( \pi/2 \) pulse: we turn on only one tunneling link \( m = 0 \rightarrow m = 1 \) for the exact amount of time necessary to transfer half of the population.

Due to the lack of interior lattice sites, this two-leg ladder geometry does not host the same bulk localization and conductance at the boundary typical of the integer quantum Hall effect. However, as depicted in Fig. 3.4A, the applied fluxes do lead to anisotropically conducting chiral currents, or a “shearing” of the initial symmetric state along the \( m = 0 \) and \( m = 1 \) legs. We define this shearing to be

\[
\text{shearing} \equiv \langle n \rangle_0 - \langle n \rangle_1, \tag{3.4}
\]

where \( \langle n \rangle_{0(1)} \) is the average site index along the \( m = 0 \) (\( m = 1 \)) leg. In general, application of a positive flux \( \phi \) will induce a clockwise chiral current and a positive shear, as shown in Fig. 3.4A. A sign reversal of the flux should result in a reversal of the shearing direction, and for fluxes of zero or \( \pm \pi \) we expect only symmetric spreading of the initial state along the \( y \) direction. While other experiments performed around the time of this study [6, 7, 74] observed evidence for chiral currents on similar two- and three-leg flux ladders, our use of a fully synthetic lattice allows us to engineer arbitrary fluxes, and furthermore enables direct
observation of all site populations and shearing dynamics at the site-resolved level.

Figure 3.4B shows the observed shearing dynamics for applied fluxes $\phi = -\pi/2$ (top, blue) and $\phi = +\pi/2$ (bottom, red). Initially, all of the population resides in the middle sites, and thus should give zero shear (see Sec. 3.5.1 regarding the small initial nonzero shear). The atoms then follow the general trend described above: positive flux causes atoms to move clockwise around the ladder, and negative flux leads to motion in the opposite direction. Due to the finite system size, the value of the shearing does not continue to grow ad infinitum, but saturates and decreases as the atoms reach the ends of the ladder and move between the two legs. Figure 3.4C shows the population distributions 500 $\mu$s after a quench (dashed vertical lines in Fig. 3.4B) for fluxes $\phi = \pm \pi/2$. A clear distinction between the cases of positive and negative flux can be seen at this time, corresponding to the case of near-maximum shear. For longer times, as seen in Fig. 3.4B, the data tends to deviate from the simple theory simulations. The dashed lines are the predictions of Eq. 3.3 for a tunneling rate $t/\hbar = 2\pi \times 338$ Hz, which exceeds the experimentally calibrated tunneling rates of Fig. 3.4B and Fig. 3.4D by $\sim 25\%$ and $\sim 31\%$, respectively (see Sec. 3.5.2 for more). Solid curves represent a more detailed model that includes the influence of off-resonant Bragg transitions [4], but which still ignores the influences of atomic interactions, finite condensate size, and effective decoherence due to both the phase instability of the Bragg lasers and the physical separation of wavepackets with different momenta.

Figure 3.4D displays the measured shearing after 500 $\mu$s for the full range of applied flux values, demonstrating our wide control over homogeneous effective fields. While for $\phi = 0$ almost no shear is measured (corresponding to symmetric spreading along $y$), maximal shearing magnitudes are observed for flux values near $\pm \pi/2$. The data are in excellent qualitative agreement with the theory curve, which has been scaled by a factor of 0.45 to account for reductions of shearing due to decoherence and other influences. The majority of deviations from the idealized dynamics of Eq. 3.3, including the small, non-zero shear for zero flux, are reproduced by the theory accounting for residual off-resonant Bragg couplings [4]. Our complete control of flux values is a necessary step towards measurement of the Hofstadter spectrum in cold atoms [70].
3.4 Reflection from a magnetic defect

As a second study, we engineer inhomogeneous artificial gauge fields for cold atoms, studying the transport of atomic wavepackets incident upon a sharp dislocation of the effective magnetic field. As shown in Fig. 3.5A, we engineer a step-like jump of the magnetic vector potential $\hat{A}$ by fixing the flux in the left-most plaquette to zero while retaining a tunable homogeneous flux $\phi$ in the remaining plaquettes. Without any initialization procedure, we begin with all of the population in the corner of the flux-free region on the zero momentum site $(0, 0)$ (colored in red). By switching our couplings along $y$ to the range $n = 0$ to $n = 4$, we shift the lattice such that atoms with zero momentum naturally start on the corner site. We then quench on tunneling and the full flux distribution and track the dynamics of the atomic distributions, monitoring the percentage of atoms that transmit through the step-like flux boundary, escaping the left-most four-site plaquette.

We probe the full range of $\phi$, as shown in Fig. 3.5B, directly measuring the transmitted
fraction of atoms after an evolution time of 1500 $\mu$s ($\sim 2.94 \hbar/t$). The tunneling rate $t/\hbar = 2\pi \times 311(14)$ Hz has been determined by calibrations to 2-site Rabi oscillations. A clear trend is observed: maximum transmission near $\phi = 0$ where the step in the vector potential vanishes, and maximum reflection for flux dislocations of $\pm \pi$. This is in good qualitative agreement with the predictions of the idealized tight-binding Hamiltonian of Eq. 3.3, shown as the green solid line in Fig. 3.5B. We note that this behavior is purely due to the presence of a flux boundary in this two-dimensional system, since no corresponding reflection is observed in one-dimensional chains with a step-like variation in tunneling phase.

While the idealized predictions of Eq. 3.3 expect full transmission for $\phi = 0$ (and roughly 40% for $\phi = \pm \pi$), we observe reduced dynamics in the data, which we attribute to experimental sources of decoherence and dephasing that may be ameliorated in future investigations (see Sec. 3.5.2 for more). Moreover we find that a sizable fraction of the atoms in our initial condensate (site $(0, -2)$) does not participate in the Bragg laser-driven dynamics. This owes to the wide momentum spread of our finite-sized condensate compared to the sharp spectral selectivity of our weak coupling fields (with tunneling time of $\hbar/t = 511(22) \mu$s). To account for these deviations (detailed in Sec. 3.5.2), we scale the predicted transmission curve by a factor of 0.48 with no extra offsets. This scaling better matches the lessened transmission near $\phi = 0$, but diverges from the data for larger values of flux where atoms should reflect off the boundary, regardless of effects that hinder transmission.

We additionally investigate the full dynamics for the cases of homogeneous zero flux ($\phi = 0$) and maximally inhomogeneous flux ($\phi = \pi$), as shown in Figs. 3.5C-D. In both cases, we compare the complete momentum-state distributions to those predicted by Eq. 3.3, and extract the percentages of reflected and transmitted atoms. The calibrated tunneling rate for these data ($t/\hbar = 2\pi \times 344(21)$ Hz) differs from the varying flux data discussed above. The normalized integrated optical density (OD) plots for the $\phi = 0$ case in Fig. 3.5C show a significant percentage of the population leaving the four left-most sites (denoted by white markers) and entering the right-most sites (shaded gray markers). The number of transmitted atoms at times exceeds the number that remain in the four left-most sites, as shown in the reflected and transmitted population dynamics at bottom. These data agree quite well qualitatively with the theory predictions (with the same scaling as in Fig. 3.5B).
The observations of significant transmission for $\phi = 0$ are contrasted by our measurements for $\phi = \pi$, shown in Fig. 3.5D. Here, in the upper OD plots, good qualitative agreement is found between the measured population dynamics and the unscaled theory predictions, with population first leaving and then returning to the initial site (left-most red marker). While the $\phi = 0$ case showed limited transmission, here the populations clearly display reflection from the boundary. At the bottom, we see that the number of atoms staying in the four left-most sites always significantly exceeds the number of transmitted atoms. The theory curves have been scaled down to correct for the limited transmission near $\phi = 0$, so in this case of maximal reflection, the scaling causes an underestimate of the transmitted fraction. This observation of reflection from a flux boundary, without any variation in the underlying potential energy landscape, is a purely quantum mechanical effect, in analogy to previous observations of quantum reflection [78].

Our capabilities to directly engineer artificial homogeneous and inhomogeneous gauge fields and to directly image site populations in a synthetic lattice are extremely promising for future realizations of myriad model systems relevant to topology and transport. These include 2D models of localization at topological interfaces [79], in disordered quantum Hall systems, and in random gauge fields [80]. While our results are predominantly driven by single-particle physics, the condensate atoms in our momentum-space lattice have a very long-ranged (nearly all-to-all) interaction energy, allowing for a straightforward extension to studies of interacting topological fluids. This could be accomplished through either Feshbach-enhanced scattering properties, longer interrogation and coherence times, or by mapping to other forms of discrete motional eigenstates (trapped states instead of plane-wave momentum states) with a more local interaction [81] or internal spin states [2, 3].
Figure 3.6: **Nonzero shearing from atomic interactions.** Time-of-flight images of shearing experiment after an evolution time of 0 µs (top) and 500 µs (bottom) showing nonzero calculated initial shearing, due to the presence of a faint \(s\)-wave scattering halo in the upper image.

### 3.5 Experimental details and challenges

#### 3.5.1 Nonzero initial shearing

The plots of shearing dynamics in Fig. 3.4B exhibit sizeable, nonzero shearing at zero evolution time. This is likely non-physical, or unrelated to the physics of the lattice, since at zero time the lasers coupling sites along each leg (in the \(n\) direction) have not been turned on. Instead, this measured shear is likely the result of interaction effects of the atoms that occur during measurement as they fall in time-of-flight. The TOF image at zero time is shown in Fig. 3.6 (top), along with a TOF image taken at later a evolution time (bottom, same as Fig. 3.3C). After initialization, the atoms are in a superposition of the two \(n = 0\) sites on each leg of the ladder, populating only the modes \((0,0)\) and \((1,0)\). Taking a TOF image at this point causes the two orders of the condensate to separate according to their momenta. Initially overlapped in space, the two orders fly through each other such that \(s\)-wave scattering leads to the pairwise redistribution of atoms into a spherical shell of momentum states about the center of mass, creating an “\(s\)-wave halo”.

While this atomic “halo” signal is not very impressive in the 2D images shown in Fig. 3.6, it can have nontrivial consequences when we consider fits to the 1D momentum distributions of the atoms, which we obtain by integrating TOF images over the vertical direction. In particular, atomic population from this \(s\)-wave halo overlaps with momentum orders that should in principle not be populated, in this case the lattice sites \((1, -1)\) and \((0, 1)\). Because
we apply the same fit function and parameters (ten lattice sites) to all of our data, when we fit the initial two-mode distribution, we obtain some unintentional nonzero population in these two sites, as can be seen for site \((1, -1)\) in Fig. 3.6. As population in either of these sites contributes to a measured positive (clockwise) shear, this interaction-driven effect leads to a nonzero, positive shear signal at short times. At longer evolution times, population spreads out over more momentum orders, so that the influence of this nonlinear scattering process becomes less severe. To avoid these minor complications in future experiments, one could suppress the atomic interactions to zero during time-of-flight.

In addition to atoms scattering from \(s\)-wave collisions, occasional atoms from the tail of the \((1, 0)\) state’s momentum distribution are fit as part of the nearby \((0, 1)\) state, as the two orders are closely spaced with respect to the expanded size of the condensate momentum orders. The same happens to the adjacent \((0, 0)\) and \((1, -1)\) orders.

### 3.5.2 Challenges of working in 2D

Here we briefly describe some technical challenges associated with the extension of our previously studied technique to two synthetic dimensions, leading to deviations between the experimental data and theory predictions at longer evolution times.

**Condensed frequency spectrum and reduced tunneling rates**

In particular, the present scheme using an effective 2D manifold of momentum states requires us to operate at a range of tunneling times that are much longer than those employed in previous 1D studies [5, 18]. This restriction to long tunneling times stems from the more condensed spectrum of Bragg frequencies required to generate our 2D synthetic lattice. Figure 3.7A illustrates the mapping between the momentum states and the sites of the effective 2D synthetic lattice, and Fig. 3.7B shows the relevant Doppler shifts related to the Bragg resonances enabling tunneling between the legs (red) and along the legs (blue) of the ladder. This spectrum of frequency components, each of which need to be separately spectroscopically addressed, is much denser than the 1D case.

As mentioned in Chap. 2, to faithfully resolve individual resonances and avoid off-resonant
coupling to nearby transitions, we usually operate in the regime where the two-photon Rabi rates driven by any given frequency tooth is much smaller than the spacing between the teeth. Because the spacing here is much smaller than in the 1D case, we need to significantly reduce the tunneling rates and tunneling times by a factor of $\sim 4–5$ compared to those used in our studies of 1D lattice systems [5, 18].

This requirement of reduced tunneling bandwidths alone would not necessarily present any challenges, but it reduces the number of coherent tunneling events we can observe if our system features fixed timescales associated with decoherence or dephasing. Presently, one major limitation of our system is the natural loss of coherent momentum-space dynamics as the momentum states of our condensate separate and lose spatial overlap. While some effort has been put into decreasing the trapping frequencies of our optical trap along one axis to $\sim 2\pi \times 10$ Hz, thus increasing the size of our condensate and its coherence length at low temperatures, such considerations should still presently limit us in 2D to the observation of coherent dynamics over $\lesssim 4 \ h/t$. This natural source of decoherence can be ameliorated in
future experiments by working with larger condensates with lower momentum-spread, or by working with trapped spatial eigenstates as suggested in Ref. [81].

Finally, we note that the aforementioned complications associated with these longer tunneling times make it difficult to directly calibrate tunneling rates via 2-mode Rabi oscillations, as discussed in [5].

**Finite momentum spread of the condensate**

Even ignoring this loss of near-field interference, these reduced tunneling rates still have a noticeable effect at short evolution times, as shown in Fig. 3.5. Due to the finite size of the trapped condensates, our atoms feature a spread of momenta along the direction of imparted momentum. Because the resonance frequency of each Bragg transition depends on the initial state of the atoms, this leads to a spread of relevant Bragg resonance conditions that can exceed the spectral bandwidth of the applied Bragg laser “pulses” [82]. Thus, the expected dynamics of atoms in coherently-coupled momentum orders is not realized for all atoms in the finite-sized condensate, and in particular a significant portion of atoms can be seen persisting in the zeroth momentum order (site (0,0)) in Figs. 3.5C-D. We expect that this source of deviation from the expected transmission and reflection dynamics for the inhomogeneous flux lattice could also be changed by working with larger, more spectrally narrow condensates.

**No phase stabilization between different lattice lasers**

Further, this first realization of a fully synthetic 2D lattice of states, coupled by two separate co-propagating pairs of Bragg laser fields, was achieved even without active phase stabilization of the relevant laser paths, which each contain multiple active elements (AOMs). We should thus expect to be sensitive to vibrations, thermal drifts, and other uncontrolled variations in the effective path lengths, resulting in a loss of phase coherence of the driven Bragg transitions. All of this noise is essentially common mode, *i.e.* shared by all relevant Bragg transitions along a given direction in the synthetic lattice, and should not affect the flux per plaquette when considered at any instance of time. However, temporal variation of the rel-
Figure 3.8: Phase instability. (A) Rearranged time-of-flight images of the magnetic reflection experiment for φ = 0.1 π (left) and φ = π (right), taken after 1500 µs evolution time (∼2.94 ℏ/τ). From top to bottom: average of 20 individual images, a single image with maximum transmission, and simulated results. (B) Transmitted fraction of atoms in the magnetic reflection experiment vs. applied flux for the average over 20 trials (green circles, same as Fig. 3B) and the single trial with maximum transmission at each flux value (open gray squares). Simulation results have been scaled to fit the mean (solid green curve) and max (dashed gray curve) data.

In particular, when phase drifts of order π occur on timescales comparable to or shorter than the tunneling time, the tunneling between adjacent sites may be suppressed. More explicitly, if all tunneling phases along a particular lattice direction increase linearly with time (as may happen if a driven AOM crystal is heating up during an experiment), an effective electric field along that direction is induced [5]. We believe that large shot-to-shot variations of the experimentally measured dynamics result from this technical phase instability, which can in future experiments be addressed by active stabilization of the lasers’ relative phase [69].
To illustrate the magnitude of the shot-to-shot variations in the experiment, and the influence that this had on the observed departures from the expected coherent dynamics, Fig. 3.8A compares images of the magnetic reflection experiment (as in Fig. 3.5) for $\phi = 0.1\pi$ (left) and $\phi = \pi$ (right). At the top are time-of-flight images averaged over 20 experimental shots represented on the synthetic 2D lattice (Mean), the same images used to generate the transmitted fraction data as a function of $\phi$ in Fig. 3.5B. In the middle we have selected the experimental run exhibiting the largest fraction of transmitted atoms (Max), which we believe in many cases may simply be the run with the least uncontrolled phase variation over the relevant time of evolution. Both of these cases are compared to the theoretically predicted population distribution for fully coherent dynamics. In general, the Max data tends to have better agreement with the theory. More telling is the fact that when large disagreements are seen between theory and the Mean data, a significant fraction of atoms has not left the initial condensate order. For further comparison, Fig. 3.8B shows the experimentally measured transmitted fraction, as in Fig. 3.5B, for both Mean (green circles) and Max (open gray squares) data. While the Mean data fluctuate somewhat wildly around $\phi = 0$, the Max data show better agreement with the transmission closer to the expected 100% and depict a more consistent trend.

3.6 More notes and comments

Here I’ll give a few more notes that didn’t make it into the publication.

3.6.1 Limits on lattice size

Our choice of lattice wavelengths was a bit poor. It turns out that $\lambda_1 = 781.5$ nm and $\lambda_2 = 1064$ nm are unfortunately close enough that $4\lambda_1 \approx 3\lambda_2$, so that lattice sites $m = 3$ and $n = 4$ are effectively overlapped and degenerate. This places a hard limit on our lattice dimensions, limiting severely the size of one dimension: $2 \times N$ or $M \times 3$, for any $N$ or $M$. Of course, we can make larger lattices and simply avoid population moving to the affected lattice sites.
3.6.2 Other 2D lattices

This isn’t the only way to construct 2D lattices in momentum space. As I’ll discuss in Chap. 6, we have also made a 2D zigzag geometry using higher-order tunneling terms.

And for separate lattice lasers, we can also aim them at the atoms along different spatial dimensions. For our future $^{39}$K setup, we plan to implement three lattice beams in either a “T” shape to create a square lattice in 2D or a “Y” shape for access to square, triangular, hexagonal, and Kagome lattice geometries [83]. This avoids many of the complications discussed above in Sec. 3.5.2, specifically the compressed frequency spectrum which is an inherent issue that can’t be dodged. However, if we implement all three lattice beams with the same wavelength, we run into the issue of degenerate transition frequencies, where one frequency may resonantly address multiple links in the lattice. This can still give rise to interesting physics but may limit the values of applied flux that we can access.

3.6.3 Floquet topological insulator

We also implemented a “Floquet” topological insulator [84] on a $5 \times 4$ site lattice as one of our earliest studies on a 2D system (note: this geometry avoided the degenerate lattice site issue I just mentioned by only turning on a few tunneling links at a time). This experiment didn’t implement any artificial magnetic field or flux. Instead, as shown in Fig. 3.9A-B, we simply started in the top center site $(m,n) = (0,0)$ and turned on the link to $(1,0)$. After population transferred over, we turned off that link and turned on the link to $(2,0)$, repeating this process clockwise around the lattice. This somewhat trivial experiment can realize a topological insulator, and more involved works using a similar periodic (Floquet) drive have studied topology in 1D and 2D [85–87].

Our data, shown in Fig. 3.9, gives the expected result: population follows the links around the edge until it eventually dies out from the various sources of decoherence discussed in the previous section. It is a bit surprising to me now that we were able to pump atoms around the corner of this lattice, even without phase stabilization between the two lattices. On a related note, Eric also did a similar topological charge pump with a 1D topological model (Su-Schrieffer-Heeger), and saw similar topologically-protected population transfer. Both
Figure 3.9: **Floquet topological insulator.** (A) Periodic driving of tunneling links gives rise to both localized bulk states and conducting chiral edge states, leading to a dynamic (Floquet) topological insulator. Adapted from Fig. 2 of Ref. [84]. (B) Our experiment: atomic population was pumped clockwise around the edge of a 5 × 4 site lattice, starting in the top center site (0, 0). Population is shown here after 13.5 tunneling times. (C) Individual lattice site populations showing charge pumping to site (2, −2).

His work and this work were somewhat deservedly shoved out of the final publications, and in this case we have joked for many years that it was going to be forever unpublished... until now.
Chapter 4

Transport under Disorder and Quasiperiodicity

With the precise control over the momentum-space lattice, we have the unique ability to generate exact values for any lattice parameter we wish: tunneling amplitudes, tunneling phases, and site energies. In this work [14], we implemented three different forms of disordered and quasiperiodic parameter variations, and studied the drastically different responses in the resulting atomic transport properties:

1. Random static tunneling phases
   → Ballistic transport: quantum random walk
2. Random, dynamically-varying (annealed) tunneling phases
   → Diffusive transport: classical random walk
3. Quasiperiodic site energies (Aubry-André model)
   → Arrested transport: Anderson localization

This chapter is adapted from the text of Ref. [14], with additional background in Sec.4.1.

4.1 Background

Disorder is unavoidable in crystalline materials, showing up as lattice defects like interstitials or vacancies. Thus it is important to consider the effects of disorder on purely crystalline effects, to better simulate real materials. At the same time, strong(er) disorder gives rise to interesting phenomena that have been the subject of research for decades, with the centerpiece being Anderson localization, or arrested transport, in disordered materials [88]. Over the past two decades, dilute atomic gases have become a fertile testing ground for the study of localization phenomena in disordered quantum systems [89]. They have allowed for some of the earliest and most comprehensive studies of Anderson localization of quantum particles [90–96], strongly interacting disordered matter [97–102], and many-body
localization [103–106]. Still, the emulation of many types of disorder relevant to real systems - e.g., crystal strain and dislocation, site vacancies, interstitial and substitutional defects, magnetic disorder, and thermal phonons - will require types of control that go beyond traditional methods based on static disorder potentials [98].

The technique of synthetic dimensions and momentum-space lattices offers a prime way to implement specifically tailored, dynamical realizations of disorder and lattice parameters that would otherwise be difficult to study. However, current studies based on internal states [6, 7, 77, 107, 108] have been limited to a small number of sites along the synthetic dimension, inhibiting the study of quantum localization in the presence of disorder. In this chapter and its related publication [14], we employ the MSL technique to engineer tailored and dynamical disorder in synthetic dimensions. Our approach introduces several key advances to cold atom studies of disorder: the achievement of pure off-diagonal tunneling disorder, the dynamical variation of disorder, and site-resolved detection of populations in a disordered system. For the case of tunneling disorder, we examine the scenario in which only the phase of tunneling is disordered. As expected for a one-dimensional system with only nearest-neighbor tunneling, these random tunneling phases are of zero consequence when applied in a static manner. When this phase disorder fluctuates on time scales comparable to intersite tunneling, however, we observe a crossover from ballistic to diffusive transport [109]. We compare to the case of static quasiperiodic site-energies, observing Anderson localization at the site-resolved level.

4.1.1 Disordered transport

This work focuses on atomic transport in a lattice with different forms of disorder and lattice parameter variations. In the absence of disorder, atoms placed on a 1D lattice undergo a discrete quantum random walk, as we showed in Chap. 2 and in Ref. [5]. On the other hand, this transport gets exponentially suppressed in a disordered medium, and the atoms experience Anderson localization [88]. This is straightforward to understand with potential disorder: under strong disorder, the energy difference between adjacent sites may be large, inhibiting tunneling between the sites and trapping the particles to some localization length
of sites. Alternatively, this can be viewed as destructive interference between possible paths out of a region, leading to localization.

While Anderson originally proposed this idea in regards to electrons in a crystal lattice under random disorder, this phenomenon is general to all wave physics. Under truly random disorder, Anderson localization in one- and two-dimensional systems occurs for any infinitesimal amount of disorder (and in 3D, it gets a little more involved). A strikingly simple realization of Anderson localization can be seen with light: by stacking transparent glass plates of randomly varying thickness and shining some light through, the transmission of light gets exponentially suppressed with additional plates. This can be rephrased as reflection from a stack of transparent plates, resulting in a truly impressive paper title: “transparent mirrors” [110]. However, when it comes to real crystal lattices, the many interactions between electrons and with phonons have made direct observation of Anderson localization difficult. Only recently (2016) have signatures of disorder-driven localization been observed in a 3D single crystal [111], and, to my knowledge, observations have been lacking in lower dimensions.

The highly versatile environment of cold atoms seems ideal to study such localization/disordered transport physics, since disorder should be yet another highly tunable parameter. Indeed, Anderson localization was realized in cold atoms as early as 2008, when two simultaneous works realized the phenomenon in one-dimensional BEC systems, generating the disorder with two different techniques. One group (Ref. [93]) created a “speckle potential” by shining and scattering an off-resonant low power laser beam through a rough piece of glass, creating a random intensity pattern that was directed onto a 1D BEC [112]. The other group (Ref. [92]) instead placed their BEC in a 1D lattice, and overlaid a second lattice with an incommensurate wavelength as the “disorder” potential. The resulting quasiperiodic site energies follow the Aubry-André model, which we discuss in the section below.
4.1.2 The quasiperiodic Aubry-André model

The Aubry-André (AA) model [113] is simple to create in real-space optical lattice setups, yet features surprising depth in localization physics and in its relation to topology. This model is central to my work, not only in this chapter (arrested transport under AA), but in several other parts of this thesis. We extend this model with higher-order tunnelings to include a synthetic gauge field in Chap. 6, realize the generalized version of the model in Chap. 8, and propose to measure the Hofstadter butterfly energy spectrum of the model in Sec. 10.2. In this section I spend some time describing various aspects of this model.

This model can be expressed in the tight-binding limit as

\[ \hat{H} = -t \sum_n \left( \hat{c}_{n+1} \hat{c}_n + \hat{c}_n \hat{c}_{n+1} \right) + \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n, \tag{4.1} \]

where the lattice site energies are shifted according to

\[ \varepsilon_n = \Delta \cos (2\pi bn + \phi), \tag{4.2} \]

where \( n \) is the site index of the lattice, \( \Delta \) and \( \phi \) are the amplitude and phase of the potential, and the periodicity of the potential \( b \) can be any irrational number to be incommensurate with the underlying lattice. In both the original cold atom Anderson localization study [92] and in my works, we have chosen the (irrational) golden ratio conjugate \( b = \sqrt{5} - 1 \). Although the model is a sinusoid with some periodicity, the resulting potential shifts to the lattice sites are unique and can be considered a form of disorder. To reflect the differences between this correlated model of disorder and truly random, completely uncorrelated disorder, this model is often called quasi-periodic, pseudorandom, and pseudodisorder. Figure 4.1 is a bit more illustrative: under uncorrelated random disorder, the lattice site energies have a uniform distribution of values. Under the sinusoidal pattern of the AA model, the site energies are preferentially selected from more extremal values.

As mentioned above, any amount of random disorder (in 1D) leads to Anderson localization. But when the system is under the quasiperiodic AA model of pseudodisorder, all of the eigenstates stay delocalized as plane waves under small disorder strengths \( \Delta \), resulting in
ballistic dynamics. With increasing $\Delta$, the system exhibits a delocalization-localization transition (metal-insulator transition) at a critical value of disorder amplitude, $\Delta_c = 2t$. Above this critical amplitude, all of the eigenstates become exponentially localized: lattice site populations go as $P_n \propto e^{-|n|/\xi}$, for lattice site index $n$ and localization length $\xi = \frac{1}{\ln(\Delta/2t)}$. At the critical value $\Delta_c$, the eigenstates obey a power law scaling and dynamics become diffusive [114].

Under the bare AA model, the localization transition occurs at the same critical disorder value for all eigenstates regardless of energy, so the system does not support any energy-dependent mobility edge. In this work and the first study of Chap. 6, we show this energy-independent localization transition, and observe the dynamics in each regime. In later chapters, we open up a mobility edge in the system by adding in effects from more tunneling pathways (Chap. 6), atomic interactions (Chap. 8), and higher frequency harmonics on top of Eq. (4.2) (Chap. 8).

While not relevant for our works on localization, it is important to point out that the AA model, also called the Harper or Aubry-André-Harper (AAH) model, is a dimensionally-reduced version of the 2D Harper-Hofstadter Hamiltonian of Eq. (3.1), where a Fourier transform has been applied along one spatial dimension (see Sec. 10.3.2). This connection has been exploited to realize topological states and topological charge pumps in quasicrystals [115]. Not surprisingly, the two models share the same energy spectrum, the fractal
Hofstadter butterfly. We plan to measure this butterfly spectrum via the 1D AA model, after completing some ongoing improvements to our system, including control over atomic interactions (Feshbach resonance). This planned work is discussed in more detail in Sec. 10.3.

4.2 Ballistic transport under static tunneling phases

For all experiments performed in this chapter, we start with population restricted to a single lattice site and suddenly turn on the Bragg laser fields, quenching on the general MSL Hamiltonian

$$\hat{H}(\tau) \approx -t \sum_n (e^{i\varphi_n(\tau)} \hat{c}_{n+1}^\dagger \hat{c}_n + \text{h.c.}) + \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n,$$

where $\tau$ is the time variable, $t$ is the (homogeneous) tunneling energy, and $\hat{c}_n$ ($\hat{c}_n^\dagger$) is the annihilation (creation) operator for the momentum state with index $n$ (momentum $p_n = 2n\hbar k$). In this study, we focus on control over tunneling phases $\varphi_n$ and site energies $\varepsilon_n$ on a 21-site lattice.

In this section, we explore disorder arising purely in the phase of nearest-neighbor tunneling elements. In higher dimensions, such disordered tunneling phases would give rise to random flux patterns that mimic the physics of charged particles in a random magnetic field [80, 116, 117], similar to how we implemented the flux ladders in Chap. 3. In 1D, however, the absence of closed tunneling paths renders any static arrangement of tunneling phases inconsequential to the dynamical and equilibrium properties of the particle density. Time-varying phases, however, can have a nontrivial influence on the system’s dynamical evolution.

We first examine the case of no disorder, with all site-energies set to zero and uniform, static tunneling phases $\varphi_n(\tau) = \varphi$. Figure 4.2(a) shows the evolution of the 1D momentum distribution, obtained from time-of-flight images integrated along the axis normal to the imparted momentum, displaying ballistic expansion characteristic of a continuous-time quantum walk. For times before the atoms hit the open boundaries of the 21-site lattice, we find good qualitative agreement between the observed momentum distributions and the expected form $P_n = |J_n(\vartheta)|^2$, where $J_n$ is the Bessel function of order $n$ and $\vartheta = 2\tau t/\hbar$. Fig-
Figure 4.2: Atomic quantum walks in regular and disordered momentum-space lattices. (a-d) Nonequilibrium quantum walk dynamics of 1D atomic momentum distributions vs. evolution time for the cases of (a) uniform tunneling, (b) random static tunneling phases, (c) random, dynamically-varying tunneling phases characterized by an effective temperature $k_B T/t = 0.66(1)$, and (d) quasiperiodic site energies for $\Delta/t = 5.9(1)$. (e-h) Integrated 1D momentum distributions (populations in arbitrary units; symmetrized about zero momentum) for the same cases as in (a-d), after evolution times $\tau = (2.96(2)\hbar/t, 2.51(2)\hbar/t, 3.80(3)\hbar/t$, and an average over the range $5.1(1)$ to $6.4(1)\hbar/t$ for (e-h). For (e) and (f), we compare to quantum random walk distributions of the form $P_n \propto |J_n(2\tau t/\hbar)|^2$, for (g) we compare to a Gaussian distribution $P_n \propto e^{-n^2/2\sigma_n^2}$ for $\sigma_n = \sqrt{2\tau t/\hbar}$, and for (h) we compare to an exponential distribution $P_n \propto e^{-|n|/\xi}$. (i) Annealed disorder realized with tunneling phases $\varphi(\tau)$ that vary dynamically with time $\tau$. Phases contain $N = 50$ frequency components $\omega$ that sample an ohmic spectrum $S(\omega)$, shown here peaked at effective temperature $k_B T/t = 1$. (j) Transport under quasiperiodic Aubry-André site energies following the form $\varepsilon_n = \Delta \cos(2\pi b n + \phi)$ of an incommensurate cosine potential (dashed line). As in (h), 1D momentum distributions are shown for varying pseud disorder strengths $\Delta/t$. 
Figure 4.2(e) shows the (symmetrized) momentum profile at time $\tau = 2.96(2)\hbar/t$ along with the Bessel function distribution for $\vartheta = 5.4$. The discrepancy between the measured evolution time and the argument of the Bessel distribution stems from the uncertainty in the measured tunneling time $\hbar/t$, which is dependent on local laser intensity and prone to variations.

In comparison, Fig. 4.2(b) shows the case of zero site energies and static, random tunneling phases $\varphi_n \in [0, 2\pi)$. The dynamics are nearly identical to the case of uniform tunneling phases. This is consistent with the expectation that any pattern of static tunneling phases in 1D is irrelevant for the dynamics of the effective tight-binding model realized by our controlled laser coupling, since these phases can be gauged away with local transformations. For this case, Fig. 4.2(f) shows the (symmetrized) momentum profile at $\tau = 2.52(2)\hbar/t$ along with the Bessel function distribution for $\vartheta = 5.35$.

### 4.3 Diffusive transport under annealed disorder

While static phase disorder has little impact on the quantum random walk dynamics, we may generally expect that controlled random phase jumps or even pseudorandom variations of the phases should inhibit coherent transport, mimicking random phase shifts induced through interaction with a thermal environment. To probe such behavior, we engineer annealed, or dynamically varying, disorder [118–120] of the tunneling phases and study its influence through the atoms’ nonequilibrium dynamics following a tunneling quench. We implement dynamical phase disorder by composing each tunneling phase $\varphi_n$ from a broad spectrum of oscillatory terms with randomly-defined phases $\theta_{n,i}$ but well-defined frequencies $\omega_i$, the weights of which are derived from an ohmic bath distribution. Specifically, the dynamical tunneling phases take the form

$$\varphi_n(\tau) = 4\pi \sum_{i=1}^{N} S(\omega_i) \cos(\omega_i \tau + \theta_{n,i}) / \sum_{i=1}^{N} S(\omega_i),$$

(4.4)

where $S(\omega) = (\hbar\omega/k_B T)\exp[-(\hbar\omega/k_B T)]$, the $\theta_{n,i}$ are randomly chosen from $[0, 2\pi)$, and $T$ is an artificial temperature scale that sets the range of the frequency distribution. In this discrete formulation of $\varphi_n(\tau)$, we include $N = 50$ frequencies ranging between zero and
The frequency spectrum and dynamics for one tunneling phase $\varphi_n(\tau)$ are shown in Fig. 4.2(i) for the case of $k_B T/t = 1$.

Figure 4.2(c) displays the population dynamics in the presence of this dynamical disorder, characterized by an effective temperature $k_B T/t = 0.66(1)$ and averaged over three independent realizations of the disorder using different phase distributions $\theta_{n,i}$. We note that the population spreads asymmetrically because we do not average over a large range of $\theta_{n,i}$ distributions. The dynamics no longer feature ballistically separating wavepackets, instead displaying a broad, slowly spreading distribution peaked near zero momentum. A clear deviation of the (symmetrized) momentum distribution from the form $P_n = |J_n(\vartheta)|^2$ describing the previous quantum walk dynamics can be seen in Fig. 4.2(g). Instead, this more diffusive behavior is better described by a Gaussian distribution characterized by a width $\sigma_n = \sqrt{2\pi t/\hbar}$. We find excellent agreement with a Gaussian distribution at our measured evolution time of $\tau = 3.80(3)\hbar/t$, consistent with spreading governed by an effectively classical or thermal random walk.

### 4.4 Localization under Aubry-André site-energy pseudodisorder

While no influence of static tunneling phase disorder is expected in 1D, the effect of static site-energy pseudodisorder is dramatically different. Here, with homogeneous static tunneling terms, we explore the influence of pseudorandom variations of the site energies governed by the Aubry-André model [92, 97, 100, 104]. With an irrational periodicity $b = (\sqrt{5} - 1)/2$, the site energies $\varepsilon_n = \Delta \cos(2\pi bn + \phi)$ do not repeat, and are governed by a pseudorandom distribution. For an infinite system, this diagonal Aubry-André model features a metal-insulator transition at the critical disorder strength $\Delta_c = 2t$. The expansion dynamics for the strong disorder case $\Delta/t = 5.9(1)$ are shown in Fig. 4.2(d), with population largely restricted to the initial, central momentum order. The exponentially localized distribution of site populations (symmetrized and averaged over all profiles in the range $\tau = 5.1(1)$ to $6.4(1)\hbar/t$) is shown in Fig. 4.2(h), along with an exponential distribution with decay length $\xi = 0.6$ lattice sites. The theoretically predicted localization length can be described by $1/\xi = \ln(\Delta/2t)$ [121], giving a value of $\xi = 0.9$ lattice sites that deviates from the
value we fit from the short-time dynamics. Analogous population distributions (again sym-
metrized and averaged over the same time range) are shown for the cases of weaker disorder
[Δ/t = 0.98(1), 1.96(3), 3.05(4), 4.02(9)] in Fig. 4.2(j). Because atoms in different lattice sites
(momentum states) eventually separate spatially, we have a limited experimental timescale
to observe localization. Close to the critical point, we cannot accurately describe the popula-
tion distributions with localization lengths, though they still exhibit an apparent transition
to exponential localization for Δ/t ≥ 2.

4.5 Comparison of expansion dynamics

For all of the explored cases, we study these expansion dynamics in greater detail in Fig. 4.3.
Figure 4.3(a) examines the momentum-width (σ_p) dynamics of the atomic distributions for
the cases of static and dynamic random phase disorder. For static phase disorder, we observe
a roughly linear increase of σ_p until population reflects from the open system boundaries,
while dynamical phase disorder leads to sub-ballistic expansion. In particular, for time τ
measured in units of ℏ/t and momentum-width σ_p in units of the site separation 2ℏk,
these two cases agree well with the displayed theory curves for ballistic and diffusive expansion,
having the forms σ_p = √2τ and σ_p = √2τ, respectively (with the latter curve shifted by
0.35ℏ/t). To explore these two different expansions more quantitatively, we fit the momen-
tum variance V_p ≡ σ_p^2 to a power-law V_p(τ) = ατ^γ [122], performing a linear fit to variance
dynamics on a double logarithmic scale as shown in Fig. 4.3(c). The fit-determined expansion
exponents γ for the cases of static and dynamically disordered tunneling phases are
2.05(2) and 1.27(2), respectively. These values are roughly consistent with a coherent, quan-
tum random walk for the case of static tunneling phases (γ = 2) and an incoherent, nearly
diffusive random walk for the case of dynamical phase disorder (γ = 1).

The observed transport dynamics cross over from ballistic to diffusive as the effective
thermal energy scale k_B T approaches the coherent tunneling energy t, matching our expec-
tation that randomly-varying tunneling phases can mimic the random dephasing induced by
a thermal environment. We note that similar classical random walk behavior has been seen
previously for both atoms and photons, due to irreversible decoherence [48, 51, 123, 124]
and dissipation [125, 126], and thermal excitations [127]. However, this observation is based on reversible engineered noise of a Hamiltonian parameter. These observations of a thermal random walk suggest that annealed disorder may provide a means of mimicking thermal fluctuations and studying thermodynamical properties [118] of simulated models using atomic momentum-space lattices, and by extension other nonequilibrium experimental platforms such as photonic simulators.

We also analyze the full expansion dynamics for the case of static quasiperiodic site energies in Figs. 4.3(b,d). For homogeneous static tunnelings and zero disorder ($\Delta/t = 0$), we observe momentum-width dynamics similar to the case of static random tunneling phases, but with one distinct difference: while $\sigma_p$ features a linear increase for random static phases,
it increases in a step-wise fashion for uniform tunneling phases [5].

Because our underlying implementation applies a comb of 20 discrete, equally-spaced frequencies to the atoms, each Bragg transition is addressed not only by one on-resonant frequency, but also by 19 other frequencies in an off-resonant fashion. These off-resonant couplings add up constructively to generate jumps in the dynamics with a frequency that exactly matches the spacing between frequency teeth. By introducing random tunneling phases onto the teeth, this constructive behavior is suppressed, resulting in smoother dynamics. We note that the expected smooth behavior emerges in the limit where the tunneling is far smaller than the spacing between frequency teeth, though due to dephasing concerns we cannot work at such low tunneling rates.

Evolution of the momentum-width ($\sigma_p$) for the site-energy pseudodisorder cases of $\Delta/t = 0.98(1), 2.47(3), 5.9(1)$ are also shown in Fig. 4.3(b). We observe the reduction of expansion dynamics with increasing disorder, with nearly arrested dynamics in the strong disorder limit. More quantitatively, fits of the variance dynamics as shown in Fig. 4.3(d) reveal sub-ballistic, nearly diffusive expansion for intermediate disorder [$\gamma = 1.00(2)$ for $\Delta/t = 0.98(1)$], giving way to a nearly vanishing expansion exponent for strong disorder [$\gamma = 0.12(6)$ for $\Delta/t = 5.9(1)$].

The extracted expansion exponents for all of the explored cases are summarized in Fig. 4.3(e). For static site-energy pseudodisorder (red circles), while longer expansion times than those explored ($\tau \lesssim 6.3\hbar/t$) would better distinguish insulating behavior from sub-ballistic and sub-diffusive expansion, a clear trend towards arrested transport ($\gamma \sim 0$) is found for $\Delta/t \gg 1$. Numerical simulation (red curve) verifies this qualitative trend, but reaches a finite value of $\gamma$ due to our fits taking into account transient dynamics at short times (compared to the localization time). The deviation from this simulation curve can possibly be attributed to the same off-resonant tunneling terms that give rise to the step-like behavior in Fig. 4.3(b). Combined with the observation of exponential localization of the site populations in Fig. 4.2(h) and Fig. 4.2(j), these observations are consistent with a crossover in our 21-site system from metallic behavior to quantum localization for $\Delta/t \gtrsim 2$.

Our observations of a crossover from ballistic expansion ($\gamma \sim 2$) to nearly diffusive transport ($\gamma \sim 1$) for randomly fluctuating tunneling phase disorder are also summarized in
Fig. 4.3(e). In the experimentally-accessible regime of low to moderate effective thermal energies \( k_B T/t \lesssim 1 \), our experimental data points (blue squares) match up well with numerical simulation (open black circles). For the magnitude of tunneling energy used in these experiments, we are restricted from exploring higher effective temperatures \( k_B T/t \gtrsim 1 \), as rapid variations of the tunneling phases introduce spurious spectral components of the Bragg laser fields that could drive undesired transitions. Simulations in this high-temperature regime suggest that the expansion exponent should rise back up for increasing temperatures, saturating to a value \( \gamma \sim 2 \). This results from the fact that the time-averaged phase effectively vanishes when the time scale of pseudorandom phase variations is much shorter than the tunneling time.

The demonstrated levels of local and time-dependent control over tunneling elements and site energies in our synthetic momentum-space lattice have allowed us to perform explorations of annealed disorder in an atomic system. Such an approach based on synthetic dimensions should enable myriad future explorations of engineered Floquet dynamics \cite{128–131} and unconventional disordered lattices \cite{132, 133}. Furthermore, the realization of designer disorder in a system that supports nonlinear atomic interactions \cite{20, 134} should permit us to explore aspects of many-body localization \cite{135}.

4.6 Experimental details

4.6.1 Interaction effects

Mean field interactions in this system cause shifts in the Bragg resonance frequencies from the single-particle resonances. By directly measuring this shift to be \( 2\pi \times 430(40) \) Hz \cite{20}, we find a peak mean-field energy of \( \mu_0 = gn_0 = 2\pi \times 760(70) \) Hz, relating to the peak atomic density \( n_0 \approx 10^{14} \text{ cm}^{-3} \) at the center of our harmonic trap \cite{82}. Here, \( g = 4\pi \hbar^2 a/M_{\text{Rb}} \) for \( M_{\text{Rb}} \) the mass of rubidium and \( a \) the scattering length.
4.6.2 Calibrated tunneling times

The tunneling times for all data were calibrated using two-site Rabi oscillations. These times are: $\hbar/t = 111.6(7) \mu s$ for the clean, non-disordered data (Fig. 4.2(a,e)), $\hbar/t = 115.3(9) \mu s$ for the random static tunneling phases data (Fig. 4.2(b,f)), $\hbar/t = 126.4(9) \mu s$ for the annealed disorder data (Fig. 4.2(c,g) and Fig. 4.3(a,c)), and $\hbar/t = 158(7) \mu s$ averaged over all of the Aubry-André model data (Fig. 4.2(d,h) and Fig. 4.3(b,d)).
Chapter 5

Interactions in the Momentum-Space Lattice

“What if you set two equal to zero?”

-Jackson Ang’ong’a

One could argue that “quantum simulation” requires the presence of inter-particle interactions which can lead to correlations between particles, the key ingredient that makes quantum systems so complex and difficult to simulate on a classical computer. In our lab’s quest to realize and simulate lattice models, it is then natural to study interactions and correlated behavior.

However, up until late 2017 or so, our MSL apparatus was used solely for engineering single-particle, non-interacting Hamiltonians [5, 14, 18, 19]. We were unsure if interactions would play a meaningful role in momentum space: namely, because atomic interactions are typically very short ranged in real space, one should expect that in momentum space they would be nearly infinitely long ranged. If uniform, these all-to-all interactions would be independent of momentum order/lattice site, preventing us from observing any correlated dynamics. Thankfully, this is not the case. Because the atoms are indistinguishable bosons occupying various spatial modes (plane-wave-like momentum states), there is an additional exchange energy term between two atoms occupying distinct momentum orders. This leads to an effective finite-ranged attractive interaction (for real-space repulsion), which allows for interesting correlated behavior.

In this chapter and the paper that it adapts [20], we explain nonlinear atomic interactions in momentum space in the context of MSL experiments, show experiments on a momentum-space double well demonstrating the presence of interactions, and propose two future experiments: solitons in a zigzag momentum-space lattice, and squeezing in a momentum-space double well. Later in Chap. 9, we extend this work to larger, 21-site lattices to examine self-trapping, Bloch oscillations, and wavepacket spreading.
5.1 Interactions in momentum space

5.1.1 Overview

In real-space ultracold atomic experiments, correlated physics has largely been driven by two-body “contact” interactions [136, 137], which are derived from van der Waals interactions. For dilute gases of atoms with collisions at low energy (primarily scattering in the s-wave regime), these contact interactions can be treated as a delta-function-like interatomic potential,

\[ V_{\text{contact}} = g \sum_{i,j} \delta(\vec{r}_i - \vec{r}_j) = \frac{4\pi \hbar^2 a}{m} \sum_{i,j} \delta(\vec{r}_i - \vec{r}_j), \]  

(5.1)

where \( m \) is the mass of the atoms, \( a \) is their scattering length, and \( \vec{r}_i \) is the position of atom \( i \).

This short-ranged real-space interaction is then nearly infinite ranged in momentum space (reciprocal space). In the 1D MSL, where collisions are, to a good approximation, mode preserving (atoms that collide stay within the lattice), the resulting all-to-all interactions would appear incapable of driving correlated behavior.

However, due to bosonic statistics (indistinguishable atoms), there is an additional exchange energy that allows for a differential part to the mode-dependent interactions that is finite-ranged in momentum space: that is, intra-mode and inter-mode interactions between two atoms will take different values. If we write the overall Hamiltonian as \( H = H_{\text{sp}} + H_{\text{int}} \), we can write the interacting Hamiltonian as a sum over all two-particle collisions,

\[ H_{\text{int}} = \frac{u}{2} \sum_{i,j,k,l} c_i^\dagger c_j^\dagger c_k c_l, \]  

(5.2)

where \( i, j, k, l \) label the momentum orders of the atoms (\( k \) and \( l \) collide, resulting in \( i \) and \( j \)), and \( u = U/N \), for a total number of atoms \( N \) and a characteristic interaction energy scale given by \( U = g\rho = \frac{4\pi \hbar^2 a}{m} \rho \) for some atomic density \( \rho \) (note \( U \) would simply be the mean-field energy for all atoms occupying one momentum state). For two identical atoms in the same momentum order \( i \) that occupy the Fock/number state \( |2_i\rangle \), we can apply this
interacting Hamiltonian to get

\[ \langle 2_i | \frac{u}{2} \left( c_i^\dagger c_i^\dagger c_i^\dagger c_i \right) | 2_i \rangle = u. \]  

(5.3)

But, for two identical atoms (bosons) in different, distinguishable momentum orders \( i \) and \( j \) (state \( |1_i, 1_j\rangle \)), the symmetrization of the two-body wave function adds an exchange interaction term in addition to the direct term \([138–141]\):

\[ \langle 1_i, 1_j | \frac{u}{2} \left( c_i^\dagger c_j^\dagger c_i c_j + c_i^\dagger c_j^\dagger c_j c_i + c_j^\dagger c_i^\dagger c_i c_j + c_j^\dagger c_i^\dagger c_j c_i \right) | 1_i, 1_j \rangle = 2u. \]  

(5.4)

If the scattering length \( a \) is positive (as with rubidium-87 at zero field), then these interactions are repulsive \((U > 0)\). Because two atoms in different momentum orders are more repulsive than two atoms in the same momentum order, we can, under certain conditions (if the total population in the defined modes remains fixed), subtract off an overall constant \(2u\) to obtain an an effective attraction of \(-u\) for atoms occupying the same momentum order/lattice site. We note that this effective attraction in momentum space resulting from the combination of mode-dependent collisions and bosonic quantum statistics, results in several hallmark features of bosonic quantum fluids: the Bogoliubov quasiparticle dispersion (described below in Sec. 5.1.3), distinct transport properties of heat and sound \([142]\), and enhanced condensation/reduction of the critical temperature \(T_c\) for weakly repulsive gases \([143]\). To note, the fact that the inter-mode interactions are more repulsive than the intra-mode interactions also means that atoms in different momentum orders should be immiscible and should self-segregate and form spatial domains when not strongly coupled (e.g., similar to the observations of Ref. \([144]\)).

The existence of finite-ranged interactions in momentum space allows us to probe more complex, correlated physics using the MSL. Perhaps the most straightforward experiment is observing self-trapping in the strong interaction regime \(U/t \gg 1\), where \( t \) is the tunneling strength. In this regime, atoms feel such a strong attractive interaction to other atoms in the same lattice site that they cannot tunnel to other sites. We show this behavior with a double well in Sec. 5.2.2 (Ref. \([20]\)) and with a 1D lattice in a later work (Chap. 9, Ref \([21]\)).
5.1.2 Tuning MSL interactions

While our exploratory work with interactions in this chapter does not deal with tuning the interaction strength $U$, in general we would like this to be a highly controllable knob in experiments, just as with disorder, tunneling terms, and other Hamiltonian terms. In this section, I briefly discuss three ways we can tune $U$, one of which we use later in Chap. 9.

The interaction strength $U$ depends on both the atomic density $\rho$ and the interaction parameter $g = 4\pi \hbar^2 a/m$. As in real-space experiments, the interaction can be tuned directly by changing the scattering length $a$ with a Feshbach resonance [145]. A Feshbach resonance occurs when the total energy of two colliding atoms becomes degenerate with the energy of a molecular bound state. The atoms experience a discontinuity in their scattering length, and can couple to a loosely-bound “Feshbach molecule” state. Typically for bulk gases, this results in greatly enhanced atomic loss, as these Feshbach molecules can collide with a third body (atom). In such an enhanced “three-body loss” process, the Feshbach molecule would decay to a more deeply bound state, while the third atom can fly off, carrying away the excess energy. If the initial atomic state and molecular state have different magnetic moments (different $m_F$ Zeeman sublevels), the energy difference between the two states can be tuned by applying a magnetic field. For such a magnetically-controllable Feshbach resonance, the scattering length of the atoms (which can be viewed as being dressed by the molecular state in the vicinity of the resonance) can be tuned drastically by simply tuning the strength of a magnetic field. This is by far the most versatile and controllable method for tuning $U$, and we plan to exploit the technique in future works. We have discussed the idealized picture when coupling exists between the atomic and molecular channels, but the appearance of such inter-channel coupling is not quite regular, and not all atomic species permit easily accessible and broad Feshbach resonances. Unfortunately, this was the case for our original atomic species rubidium-87 (which is used for all of the experiments in this thesis) which does not have such a nice Feshbach resonance at low fields, prompting us to switch to potassium-39, which boasts a broad Feshbach resonance near 400 G.

It is also possible to tune the collective interaction term $U = N u = gN/V = g\rho$ (where $V$ is the effective trapping volume of the single-particle eigenstates) by changing the atomic
density, such as by modifying either the final trapping potential or the final number of condensed atoms for a given trap. An interacting Bose gas filling the harmonic trap will end up in a Thomas-Fermi distribution, which leads to different atomic densities at different spatial locations within the trap, resulting in inhomogeneous interaction terms $g\rho(\vec{r})$. We can have some control over the overall density distribution of the atoms by tightening and loosening the trap, but in practice this only allows us to access a small range of $U$ values. We note that the inhomogeneous density in such a trap will, in effect, smear out some potential interaction-driven effects that are highly dependent on the value of $U$, an example of which we discuss in Sec. 10.2. A future improvement that we have begun to implement is the addition of a blue-detuned “box” trapping potential, which instead of attractively trapping in a harmonic trap, uses repulsive forces to trap atoms in a region with uniform potential, resulting in a homogeneous atomic density. Moreover, by dynamically changing the box volume, the atomic density may be varied directly. As one last consideration for the tuning of interactions by the underlying potential: lattices may be introduced along the directions transverse to the Bragg lasers, such that the effective trapping volume can be modified by confinement of the bosonic gas into 2D “pancakes” or 1D “tubes.” This direct modification of $V$ could result in a sizable enhancement of the per-pair interaction term $u$.

Finally, in synthetic lattice studies, the onsite attractive interaction competes with the tunneling term, making $U/t$ the relevant parameter (potentially along with other tunable parameters, such as site-energy terms $\Delta$). Thus we can tune the effective interaction strength in a lattice by changing the tunneling strength $t$, ideally allowing us to reach a regime in which interactions dominate over the kinetic terms, $U/t \gg 1$. In practice, small $t$ values lead to very long tunneling times, and thus very long evolution times under the lattice. With enough time, atoms in different momentum orders can separate spatially (i.e., leaving the near-field regime) and lose spatial coherence. Such spatial separation, or more generally any momentum mode-dependent spatial dynamics, leads to a loss of coherence between the different momentum orders. This is an inherent limit of our system that prevents us from monitoring long-time dynamics for large $U/t$ values by reducing the tunneling $t$. However, this method is the easiest way to tune interactions, and we use it in three separate lattice experiments in Chap. 9, where we were able to reliably measure out to 2-3 tunneling times.
for $U/t \approx 9 \ (t \approx 100 - 150 \text{ Hz})$.

### 5.1.3 Bogoliubov picture

The microscopic treatment described above in Sec. 5.1.1 requires that we sum over all individual pairs of colliding atoms, and becomes intractable for a system containing many modes and more than even a few hundred atoms. Later below, we make the simplifying assumption to ignore correlations and consider the system in terms of $c$ numbers describing the macroscopic population and phase of different condensate momentum modes using a Gross–Pitaevskii approach. Here, to gain a bit more insight on the problem before making such an assumption, we look at the experimentally-relevant scenario in which all population begins in one state (i.e., initially at rest) and is weakly transferred to other momentum states. This limiting case of initial weak excitations of one macroscopically populated momentum state maps onto the well-studied scenario of Bogoliubov excitations [82, 138, 146]. We consider this restricted scenario as a simple description of how superfluid screening can influence the distinguishability of momentum states, and thus the range of the effective attractive interaction. The more general description of the range of interactions for many macroscopically populated momentum states is nontrivial, but when the total density or total interaction strength is sufficiently low such that the effects of screening are negligible, then an effectively site-local attraction is recovered.

Considering the Bogoliubov picture of weak excitations, we assume the atoms have a uniform number density $\rho_N$ relating to a homogeneous mean-field energy $U = g \rho_N$, for interaction parameter $g = 4\pi\hbar^2a/m$ and $s$-wave scattering length $a$. While repulsive interactions raise the energy of $p = 0$ condensate atoms by $U$, high-momentum excitations ($E_p^0 \gg 2U$) experience an interaction energy shift of roughly $2U$ due to both direct and exchange interactions with the $p = 0$ condensate. For a general momentum $p$, the Bogoliubov quasiparticle excitations have an energy $E_p = U + \sqrt{E_p^0(E_p^0 + 2U)}$. Figure 5.1 depicts this modified dispersion, along with the form of the effective interaction-dependent shifts to the MSL site energies, which relate to the difference in energy between the final state and the initial $p = 0$ state. The interaction has an effective range in momentum space
Figure 5.1: The Bogoliubov picture. (a) The Bogoliubov dispersion $E_p$ of a homogeneous gas with weak repulsive interactions and a mean-field energy $U/E_R = 4$ (blue dashed line, recoil energy $E_R$) overlaid onto the non-interacting dispersion $E_p^0$ (red solid line). (b) Effective momentum-space lattice site energies (with a common shift of $U$ removed and renormalized to $U$) experienced by weakly-coupled excitations of a macroscopically populated $p = 0$ condensate, shown for $U/E_R = 0.1$, 1, and 4 (solid red, dash-dotted purple, and dashed blue lines, respectively). (c) Cartoon of site energies shifted by interactions with a $p = 0$ condensate for $U/E_R = 0$ and $U/E_R = 4$.

which increases for increasing $U$ (Figs. 5.1(b,c)), reflecting the influence of screening on the distinguishability (from the parent condensate) of the states with nonzero momentum. In the limit of low interaction strengths ($2U \ll 4E_R$), these states become fully distinguishable and the interaction is effectively site local. We note that, in principle, the interactions with the zero-momentum superfluid are expected to vanish for excitations with extremely large momenta ($p \gg \hbar/a$) due to energy-dependent corrections to the $s$-wave scattering, but such extreme conditions are beyond current experiments.

5.1.4 Simulating MSL interactions

To perform simulations incorporating interactions, we consider a simplified description of the exact interacting system, which in general is highly non-trivial and depends on the total density and exact distribution of all site populations. First, we assume that all momentum states occupy the same spatial mode, ignoring effects of spatial separation. Next, we assume that the momentum states are fully distinguishable quantum states and that the interactions are mode-conserving, an assumption which is only approximately true for our experimental conditions. As described above, this makes the inter-mode interaction twice as strong as the
intra-mode interaction, leading to an effectively site-local interaction. Finally, given that
the number of atoms in experiment vastly exceeds the number of sites, we ignore quantum
fluctuations and simply represent the condensate wave function by appropriately normalized
complex amplitudes $\phi_n$ for the various discrete momentum states [147]. Additionally, to
capture the inhomogeneous density distribution caused by the harmonic trap [82], we use
a local density approximation (LDA), taking a weighted average of simulation curves with
different homogeneous mean-field energies $U$ ranging from 0 to a peak mean-field energy $U_0$
(see Sec. 5.5.2 for details on the LDA).1

Under these conditions, the influence of momentum-space interactions may be captured
by the Gross-Pitaevskii equation,

$$i\hbar \dot{\phi}_n = \sum_m H_{sp}^{mn} \phi_m + U|\phi_n|^2 \phi_n + \sum_{m \neq n} 2U|\phi_m|^2 \phi_n$$

$$= \sum_m H_{sp}^{mn} \phi_m + U [2 - |\phi_n|^2] \phi_n,$$

where we have subtracted an overall constant of $2U$ in the final step, and assuming the
normalization condition $\sum_n |\phi_n|^2 = 1$. Here, $U = g\rho = \frac{4\pi\hbar^2 a}{m}\rho$ and $H_{sp}^{mn}$ is the matrix
element between states $p_m$ and $p_n$ of the usual MSL single-particle Hamiltonian $H_{sp}$,

$$H_{sp} \approx -\sum_{n,\alpha} t_{n,\alpha} (e^{i\varphi_{n,\alpha}} \hat{c}_{n+\alpha}^\dagger \hat{c}_n + \text{h.c.}) + \sum_n \varepsilon_n \hat{c}_n^\dagger \hat{c}_n.$$  

The form of Eq. 5.6 hints at the effectively attractive, mode-local momentum-space inter-
action. We note that here we consider only mode-preserving collisions, as they are strongly
dominant over mode-changing collisions which would populate orders outside of the synthetic
lattice (as discussed in Sec. 5.5.1).

1As a sidenote, for other experiments we typically do not use a LDA and instead make the gross sim-
plication of homogeneous density. The small system size in this study (2 sites) allows us to employ the
LDA and numerically simulate in a reasonable amount of time, but larger, more complex systems featuring
dynamically varying parameters are more challenging.
5.2 The interacting momentum-space double well

5.2.1 Bragg spectroscopy

We can directly probe the interaction energy shifts of the momentum states through Bragg spectroscopy [82, 138, 146]. For our laser wavelength of 1064 nm, the (non-interacting) first-order Bragg resonance which couples momentum states \( p = 0 \) and \( p = +2\hbar k \) should occur when the frequency difference between the two beams matches \( 4E_R/h \approx 2\pi \times 8.1 \text{ kHz} \). We probe this by applying two lasers with a frequency difference \( \omega_1 - \omega_2 = 4E_R/h + \Delta/h \), where \( \Delta/h \) is the detuning from Bragg resonance. We turn on these lasers for a 400 \( \mu s \) square pulse, long enough to transfer some small amount of population, and change \( \Delta \) to map out the shift due to interactions. As expected, the data in Fig. 5.2(a) show a sizeable shift of 1.31(3) kHz for this transition. We measured a shift of 0.98(6) kHz for the \( p = 0 \) to \( p = -2\hbar k \) transition, and overlay both shifted transitions onto the Bogoliubov dispersion in Fig. 5.2(b). The different shifts to the \( \pm 1 \) transitions are consistent with a small but nonzero initial momentum \(-0.018\hbar k\) of the condensate atoms, and we have shifted (in \( p \), horizontal axis) the plotted Bogoliubov dispersion to account for this.

5.2.2 Hysteretic sweeps

As a first experimental study, we explore the influence of momentum-space interactions on population dynamics in a coupled double well. We initialize all of the population in the left well (\( p = 0 \) state), with a large initial energy bias \( \Delta_i \) inhibiting tunneling between the wells. This bias \( |\Delta_i|/\hbar = 2\pi \times 8 \text{ kHz} \) is chosen to greatly exceed the tunneling energy \( t/h \approx 2\pi \times 390 \text{ Hz} \). As depicted in Fig. 5.2(c), the bias is linearly swept through zero (single particle resonance) to a final value \( \Delta_f = -\Delta_i \) over 1 ms, similar to the methods of Ref. [148]. We consider both a positive sweep (\( \Delta_i < 0 \) to \( \Delta_f > 0 \)) and a negative sweep (vice versa).

In the absence of interactions, the dashed curves in Fig. 5.2(d) show that the amount of population transferred is roughly independent of the sweep direction, with a slight difference stemming from initial condensate momentum of \(-0.018\hbar k\). (see Sec. 5.5.2 for details on the simulation procedure). In contrast, the presence of site-dependent, attractive interactions
Figure 5.2: Interaction effects in a momentum-space double well. (a) Bragg spectroscopy of the $0 \rightarrow 1$ transition showing an interaction-driven shift (dashed line) of 1.31(3) kHz from single particle resonance (solid line). The momentum distributions, relating to the integrated optical density after 18 ms time of flight, show population transferred to the state $p = 2\hbar k$ by a 400 µs-long Bragg pulse vs. detuning from single-particle resonance $\Delta$. (b) Measured shifts for both $0 \rightarrow \pm 1$ transitions overlaid onto the Bogoliubov dispersion (dashed blue, shifted by initial condensate momentum $-0.018\hbar k$) with single-particle dispersion for comparison (solid black). (c) Experimental protocol for double well sweeps. Population begins in left well ($L$) and transfers to the right well ($R$) as the imbalance $\Delta$ (detuning from single-particle resonance) is swept linearly across 0 (dashed line) in the positive (left, blue arrow) and negative (right, red arrow) directions over 1 ms. (d) Population in the right well $P_R$ plotted vs. time $\tau$ (lower horizontal axis) and vs. the ratio of the bias to the initial bias $\Delta/\Delta_i$ (upper horizontal axis). Positive (left, blue squares) and negative (right, red dots) sweeps are shown with single-particle predictions (dashed gray curves) and predictions (Sec. 5.5.2) taking into account the inhomogeneous density distribution with an average mean-field energy $U/\hbar \approx 2\pi \times 1.81$ kHz (solid black curves). (e) Adiabatic energy levels (I and II) of the non-interacting double well vs. $\Delta$. Cartoon insets depict the population distributions for large $|\Delta/\hbar|$. (f) Population projection of the adiabatic levels in (e) onto the right well vs. $\Delta$. (g,h) Energy levels and population projections as in (e,f), but with an added homogeneous mean-field energy of $U/\hbar \approx 2\pi \times 1.81$ kHz. Gray arrows A and B on the negative sweep denote forced tunneling pathways as the population transfer overshoots due to self-trapping. Error bars in (b) and (d) denote one standard error of the mean.

causes a highly asymmetric, direction-dependent response in the population dynamics. Comparing the positive sweep data to the single particle theory, we find that population begins to transfer earlier and more population is transferred at the ramp’s end. For the negative sweep data, interactions cause self-trapping, leading to slightly lower, delayed population transfer.

The simulated dynamics for the double-well case, where $H^{\text{sp}} = \Delta(\tau) \hat{c}_1^\dagger \hat{c}_1 - t(\hat{c}_0^\dagger \hat{c}_1 + \hat{c}_1^\dagger \hat{c}_0$) for time $\tau$, are shown as solid curves in Fig. 5.2(d). These simulations reproduce the observed direction-dependent response, while the lack of oscillatory behavior in the data can
be attributed to spatial decoherence between momentum orders. We performed a combined fit of the data from Figs. 5.2(a-d) to obtain values for tunneling energy $t/\hbar \approx 2\pi \times 390$ Hz, initial condensate momentum $-0.018\hbar k$, and a peak mean-field energy $U_0/\hbar \approx 2\pi \times 3.17$ kHz of our inhomogeneous density distribution, with an average mean-field energy of $U/\hbar \approx 2\pi \times 1.81$ kHz (see Sec. 5.5.2 for details on the simulation and fitting procedure).

To better understand these direction-dependent results, we consider the adiabatic energy levels of this coupled two-level system and their projections onto the measured well populations (see Sec. 5.5.3 for details on calculating these energy levels). Without interactions, the Bragg-driven “tunneling” leads to an avoided crossing of the adiabatic energy levels (Fig. 5.2(e)), allowing for complete transfer between wells in the limit of an infinitely slow $\Delta$ sweep (Fig. 5.2(f)), independent of the sweep direction. In contrast, adding interactions introduces a swallow-tail-like loop structure with metastable branches (Fig. 5.2(g)) [149–151]. For a slow positive sweep (starting on the top branch), the avoided crossing is maintained, leading to full population transfer as seen in Fig. 5.2(h). For a slow negative sweep (starting on the lower branch), atoms are forced to tunnel between the energy branches (path B), relating to self-trapping in the initial left well as seen in Fig. 5.2(h). For finite sweep times as in our experiment, a combination of forced tunneling along pathways A and B leads to a transfer efficiency that depends on the sweep direction, but with less extreme of a distinction as compared an infinitely slow ramp.

We note that this type of swallow-tail structure is generic to systems with strong nonlinear interactions, relating to the breakdown of adiabaticity and the possibility of hysteretic response [150–153].

5.3 Proposal: Solitons in a zigzag lattice

Here we explore the influence of interactions on the particle dynamics in a zigzag lattice (Fig. 5.3), where artificial fluxes play a nontrivial role [15, 154]. We later created this lattice geometry (Chap. 6), but did not explore the high interaction strengths necessary to observe the many-body effects presented here.

We consider a zigzag lattice with uniform nearest and next-nearest neighbor tunneling
Figure 5.3: **Interaction effects in a zigzag flux lattice.** (a) Cartoon depiction of atoms initialized at the central site \((n = 0, \text{gray})\) of a zigzag lattice with uniform magnetic flux \(\varphi\). Nearest- (solid black) and next-nearest (dashed red) neighbor tunneling links have uniform amplitude \(t\). Shaded yellow region indicates two-site unit cell. (b) Site population distributions for evolution times \(\tau = \{12.5, 25, 50, 100\} \hbar/t\), shown for several combinations of interaction-to-tunneling ratios and flux values on a 401-site lattice: solid black denotes \((U/t, \varphi) = (0, \pi/6)\), rightmost solid blue for \((7.2, \pi/6)\), solid purple for \((7.2, 0)\), and leftmost solid orange for \((7.2, -\pi/6)\). (c) Average site position \(\langle n \rangle\) (red dashed line; left vertical axis with linear scale) and population in the most-populated site \(P_{n, \text{max}}\) (blue solid line; right vertical axis with logarithmic scale) versus \(U/t\). Simulations are shown for \(\varphi = \pi/6\) after an evolution time \(\tau = 65 \hbar/t\) on an 801-site lattice, with population never reaching the boundaries. Shaded gray region indicates chiral soliton stability.

t_{n,1} = t_{n,2} \equiv t\) and a uniform magnetic flux \(\varphi\) (Fig. 5.3(a)). We initialize population to a single, central site \((n = 0)\) and simulate dynamics following a tunneling quench. The normalized site populations \(P_n\) at various evolution times \(\tau\) are shown in Fig. 5.3(b). For a positive flux value of \(\pi/6\), dramatically different behavior is found for zero \((U/t = 0)\), moderate \((U/t = 7.2)\), and strong \((U/t = 12)\) interactions. Without interactions, chiral currents are present, but with a rapid dispersive spreading of the atomic distribution. Moderate interactions stabilize the distribution, leading to soliton- or breather-like states [155]. For
very strong interactions, the atoms remain localized at \( n = 0 \) due to self-trapping.

To gain more insight into the general behavior for nonzero fluxes and varied interactions, we plot in Fig. 5.3(c) the average site position \( \langle n \rangle \) and the population in the most-populated site \( P_n^{\text{max}} \) versus the interaction-to-tunneling energy ratio \( U/t \), for a flux \( \pi/6 \) following a duration \( \tau = 65 \hbar/t \). For weaker interaction strengths \( U/t \lesssim 5 \), the initially localized wavepacket becomes highly delocalized (low \( P_n^{\text{max}} \)), while on average the population moves in a chiral fashion, reflecting an underlying spin-momentum locking of the flux lattice model [15]. In the intermediate regime \( (5 \lesssim U/t \lesssim 7.5) \), dynamics relating to chiral solitons can be found. The jump in the value of \( P_n^{\text{max}} \) to a finite, nearly fixed value relates to self-stabilization against wavepacket spreading. This self-stabilization can also be seen in Fig. 5.3(b), where the size of the chiral soliton wavepackets (blue and orange curves) remain nearly fixed, even as they propagate throughout the system. With increasing \( U/t \), the solitons become more and more “massive” and travel less far (lower average site position \( \langle n \rangle \)) for the fixed evolution time \( (65 \hbar/t) \). This eventually gives way to full self-trapping and the inhibition of chiral currents for \( U/t \gtrsim 7.5 \), with population localized to the initial site.

The chiral behavior observed for weak interactions stems from the presence of spin-momentum locking in the single-particle band structure, where an effective “spin” degree of freedom relates to the two sites of the zigzag lattice unit cell [15, 154] (shaded in Fig. 5.3(a)). The emergence of non-dispersing chiral solitons can be understood in terms of interaction-driven hybridization [156] of the two energy bands in the system. Stability is found as the interaction energy \( U \) starts to exceed the width of the lower energy band \( (4t) \), and complete self-trapping ensues when the interactions dominate over the combined band width \( (6t) \). The collective chiral behavior of the atoms under intermediate interactions is of fundamental interest to understanding how emergent behavior can arise from the interplay of interactions and synthetic gauge fields in kinetically frustrated systems [154, 157–160].

In addition to this novel behavior predicted to occur in momentum-space flux lattices [15], the ability to engineer arbitrary forms of disorder in MSLs should enable studies on the interplay of long-ranged momentum-space interactions and disorder-driven localization [135, 161]. Furthermore, while we have presently considered the influence of mode-preserving collisions (relevant to 1D, free-space elastic scattering) relating to effectively local nonlinearities in
MSLs, other interesting types of correlated phenomena may result from mode-changing collisions \[67, 134, 147, 162–164\], which would relate to correlated pair-hopping processes in MSLs.

5.4 Proposal: Interaction-driven squeezing in a double well

The momentum-space interactions described in this chapter may also allow the study of symmetry breaking \[165\] and two-mode entanglement \[166\] in momentum-space double wells. In particular, the generation of squeezed momentum states with the effectively mode-local interactions could lead to practical advances in inertial sensing, as we will describe in this section. Beyond double wells \[148, 152, 165, 167–171\], MSLs offer unique capabilities for engineering multiply-connected lattice geometries.

The role of quantum fluctuations of the momentum-space double well site populations has been ignored in the previous sections, yet significant quantum correlations should be expected to develop in such a system as a result of interactions. In particular, the momentum-space interaction shown to lead to the onset of nonlinear self-trapping in Fig. 5.2 can also be harnessed to generate squeezing of the momentum-space distribution. Specifically, a generic two-mode quantum system featuring mode-dependent interactions can be mapped onto an effective collective spin Hamiltonian featuring a one-axis twisting term that can generate squeezed collective spin states \[166, 172–174\].

We consider a system of particles distributed between two momentum modes (labeled mode 0 and mode 1) with a fixed total particle number \(N = N_0 + N_1\). In the previous sections, when simulating the experimental data for the case of \(N \sim 10^5\) atoms and only two sites, we ignored quantum fluctuations of the atomic populations. This assumed that for a given site \(n\) with particle number \(N_n\), \(\langle \hat{c}_n \rangle \sim \langle \hat{c}_n^\dagger \rangle \sim \sqrt{N_n}\), where \(\hat{c}_n\) (\(\hat{c}_n^\dagger\)) is the annihilation (creation) operator for the mode \(n\). This assumption allowed us to describe the system by a greatly simplified multimode Gross-Pitaevskii equation \[147\]. More generally, following the treatment of Ref. \[166\] and as in Sec. 5.1.1, we may include the effect of quantum fluctuations by considering instead a second quantized Hamiltonian \(H = H_{\text{sp}} + H_{\text{int}}\) that describes two-body interactions of atoms in the same and different modes \(H_{\text{int}}\), as well as the engineered
Figure 5.4: **Squeezing in a momentum-space double well.** (a) Visualization of many-particle (N = 100) spin states |Ψ⟩ through their overlap with different coherent spin states |⟨θ, ϕ|Ψ⟩|^2. Shown are the cases of an initial coherent spin state |π/2⟩,π⟩ (upper plot), and the transformed state after evolution under H_{sq} for a time κτ_{sq}/ℏ = 0.0173π (lower plot). (b) Squeezing along the \( \hat{z} \)-axis, \( ξ_z \), for different evolution times κτ_{sq}/ℏ = {0.1, 0.25, 0.5, 0.75, 1} \times 0.0173π (solid lines, with colors varying from blue to red) and for different angles of rotation \( ϕ_{rot} \) of the final distribution about \( \hat{J}_x \).

Synthetic lattice tunneling and site energies (\( H_{sp} \)). For the double well, these contributions may be written as

\[
H_{sp} = ε_0 \hat{c}_0^\dagger \hat{c}_0 + ε_1 \hat{c}_1^\dagger \hat{c}_1 - t(\hat{c}_0^\dagger \hat{c}_1 + \hat{c}_1^\dagger \hat{c}_0)
\]

and

\[
H_{int} = \frac{u_{00}}{2}(\hat{c}_0^\dagger \hat{c}_0 \hat{c}_0^\dagger \hat{c}_0) + \frac{u_{11}}{2}(\hat{c}_1^\dagger \hat{c}_1 \hat{c}_1^\dagger \hat{c}_1) + u_{01}(\hat{c}_0^\dagger \hat{c}_0 \hat{c}_1^\dagger \hat{c}_1).
\]

Here, \( t \) is the tunneling energy, which we assume to be real, and \( ε_0 \) and \( ε_1 \) are the site energies (with energy bias \( Δ = ε_1 - ε_0 \)), which may be controlled through the Bragg frequency detuning. The interaction coefficients are given by \( u_{00} = u_{11} = U/N \) (for the assumed uniform mean-field energy \( U \) and total particle number \( N \)) and \( u_{01} = 2U/N \), with the difference arising from the added exchange interactions [147].

Analysis of this interacting two-mode system is simplified by considering the atomic distribution in terms of a collective spin. Each atom is imbued with an effective spin-1/2 degree of freedom relating to the two momentum modes that may be occupied. The total state of the system may be considered as an effectively large, collective spin with maximum length \( N/2 \). One may define effective angular momentum operators relating to the coherences and macroscopic occupations \( N_0 \) and \( N_1 \) of two possible momentum orders (ignoring thermal

90
and quantum depletion), given by
\[
\hat{J}_x = \frac{(\hat{c}_0^\dagger \hat{c}_1 + \hat{c}_1^\dagger \hat{c}_0)}{2} \quad (5.10)
\]
\[
\hat{J}_y = \frac{\imath(\hat{c}_0^\dagger \hat{c}_1 - \hat{c}_1^\dagger \hat{c}_0)}{2} \quad (5.11)
\]
\[
\hat{J}_z = \frac{(\hat{c}_1^\dagger \hat{c}_1 - \hat{c}_0^\dagger \hat{c}_0)}{2} \quad (5.12)
\]

In this modified description in terms of a collective spin, we will consider a basis of Dicke states \(|j, m\rangle\) having total spin \(j = N/2\) and \(\hat{z}\)-projection \(m = (N_1 - N_0)/2\), describing the collective two-mode number states. Uncorrelated coherent spin states (CSSs) of the form
\[
|\theta, \varphi\rangle = \sum_{m=-j}^{j} f_m^j(\theta)e^{-\imath(j+m)\varphi}|j, m\rangle, \quad (5.13)
\]
for \(f_m^j(\theta) = (j+m)\imath^{m-j} \cos(\theta/2)^j \sin(\theta/2)^j\), result from a global rotation of spin-polarized states \(|j, j\rangle\) about the spin vector \(\hat{n}_\varphi = \cos(\varphi)\hat{J}_x + \sin(\varphi)\hat{J}_y\) by an angle \(\theta\) [175]. In this description in terms of a collective spin, the momentum-space interaction gives rise to an effective nonlinear squeezing Hamiltonian \(H_{sq} = \kappa \hat{J}_z^2\) (after removing the contribution from mode-independent interactions), where \(\kappa = (u_{00} + u_{11})/2 - u_{01} = -U/N\) [166].

When combined with single particle manipulations of the effective spin degree of freedom through control of \(H_{sp}\), the interactions can be used to generate correlations and entanglement in the double well system. We examine the case of how interactions modify an initially prepared CSS \(|\pi/2, \pi\rangle\) aligned along \(\hat{J}_x\). For short evolution times, the nonlinear Hamiltonian \(H_{sq}\) leads to a “shearing” of such coherent states. This is depicted in Fig. 5.4(a), for the initial CSS (upper plot) and the sheared, non-classical squeezed state after a time \(\kappa\tau_{sq}/\hbar = 0.0173\pi\) (lower plot), through the overlap of these states with CSSs of varying \(\theta\) and \(\varphi\) values. For ease of calculation, dynamics are shown for the case of only \(N = 100\) atoms (\(j = 50\)).

Figure 5.4(b) shows, for sheared distributions relating to various evolution times \(\tau_{sq}\), the \(\hat{z}\)-axis squeezing parameter \(\xi_z = 2j \frac{(\Delta \hat{J}_z^2)}{j^2 - (\langle \hat{J}_z \rangle)^2}\) as a function of rotation angle \(\varphi_{rot}\) about the \(\hat{J}_x\) spin axis. For typical experimental parameter values (\(N = 10^5\) atoms, \(U/\hbar = 2\pi \times 1.5\) kHz),
an optimal squeezing of $\xi^\text{min}_x \approx (3j^2)^{-1/3}$, relating to $-33$ dB (after appropriate rotation of the sheared distribution about the $\hat{J}_x$ spin axis), would be expected after a total duration $\tau_{\text{sq}} \approx 1.13j^{1/3}(\hbar/U) \approx 6.6 \text{ ms}$ [166]. The same squeezing could be achieved in a shorter time through a temporary modification of the atomic scattering length, such as through a magnetically tunable Feshbach resonance. Shorter squeezing timescales would help to mitigate effects owing to spatial separation of the momentum orders, which would in practice degrade and limit the available squeezing. Additionally, refocusing $\pi$ (echo) pulses or twist-and-turn squeezing schemes [176] can be used to maintain spatial overlap of the momentum states. In practice, refocusing or twist-and-turn schemes, that mitigate the issue of spatial separation, could also allow for operation at smaller scattering lengths. Somewhat counter-intuitively, this may be the most ideal regime to work in, to enhance the ratio of “good” (mode-conserving) to “bad” (mode-changing) collisions, as discussed shortly.

5.5 More experimental details

5.5.1 Elastic scattering loss

In general, the interactions between atoms in different momentum modes are not restricted to preserve the populations within these modes, for example scattering into $4\pi$ steradians and generating $s$-wave halos. This elastic scattering “loss” (from the MSL sites) can be an issue when populating many momentum modes, but for the experiments shown here (neighboring modes), the loss is mostly negligible.

More quantitatively, we follow the treatment in Ref. [162] and compare the strength of the elastic scattering loss (i.e. scattering into states outside the MSL) to the strength of the mean-field interaction. Since both terms come from two-body elastic collisions, they share the same linear dependence on density (i.e. with mode-changing and mode-preserving terms scaling as $n\nu\sigma$ and $2gn$, respectively). Thus we can compare the interaction parameter $2g/\hbar = 8\pi\hbar a_{\text{Rb}}/m$ to the coefficient of the loss term between wavepackets $i$ and $j$,

$$\nu\sigma = 8\pi\hbar |\mathbf{k}_i - \mathbf{k}_j| a_{\text{Rb}}^2/m,$$

(5.14)
where \( v \) is the relative velocity between the wavepackets and \( a_{\text{Rb}} \approx 100a_0 \) is the rubidium background scattering length.

The ratio of loss to interaction strength is then

\[
R = \frac{v\sigma}{2g/h} = 2|k_i - k_j| a_{\text{Rb}},
\]

which for interactions between atoms in neighboring momentum states \( p = 0 \) and \( p = 2\hbar k \) (as in experiments shown in this work) gives a small value of \( R \approx 0.12 \). This ratio increases to 1 for interactions between atoms 8 lattice sites apart (\( p = 0 \) and \( p = 16\hbar k \)). So while these losses are negligible for MSL experiments probing only a few populated orders (which would be relevant, \( e.g. \), for probing the effects of interactions on localization phenomena), they can become significant for larger systems.

In order to explore correlated phenomena with larger MSLs, we can reduce the effects of elastic scattering loss in several ways. First, we can reduce the relative loss by directly lowering the scattering length \( a_{\text{Rb}} \), as through a Fano-Feshbach resonance. To maintain the same interaction strength, we can proportionally increase the atomic density, or alternatively decrease the tunneling to work at the same interaction strength relative to other relevant energy scales (\( i.e. \) same \( U/t \)).

Related to the change of \( a_{\text{Rb}} \), which reduces the ratio of “bad” (mode-changing) to “good” (mode-conserving) collisions for modes \( R = 2|2k\Delta n| a_{\text{Rb}} \) (with \( \Delta n \) the difference in mode/site index), one could also decrease the momentum imparted by the Bragg transitions, \( i.e. \) reduce \( 2\hbar k \). So long as the effects of screening were still relatively minor, \( i.e. \) so long as \( 2k \) was still much greater than \( 1/\xi \), with \( \xi \) the healing length, then a description of site-local attraction would still hold, but with reduced elastic scattering loss.

Another method would be to geometrically and energetically restrict how the atoms can scatter, by confining the atoms to one or two real-space dimensions. In our case, by applying real-space lattices transverse to our MSL, we can make the atoms experience a free-space dispersion only along the direction of the MSL. For a pair of atoms confined to a 1D geometry (\( e.g. \), loading into a 2D lattice resulting in “tubes”) with different momentum, momentum and energy conservation require that the collision be mode preserving. That is, the two
allowed scattering processes of transmission and reflection (in the center-of-mass frame) give
the same result of one atom in each of the starting momentum modes, thus preserving the
mode populations. We can consider this system to be 1D when the collision energy is much
less than the harmonic oscillator spacing in the transverse direction, assuming an effective
harmonic trapping potential imposed by transverse confinement.

5.5.2 Fitting and simulation procedure

We determine the tunneling energy \( t \), average mean-field shift \( U \), and initial condensate
momentum \( p_i \) by performing a combined fit of all experimental data in Figs. 5.2(a-d) to
numerical simulations. Specifically, we minimize the combined residuals of the two Bragg
spectroscopy data sets for the \( 0 \rightarrow \pm 1 \) transitions and the two (positive and negative) sweep
data sets as a function of the simulation parameters \( t, U, \) and \( p_i \).

For each data set, we generated simulation curves by solving the nonlinear Schrödinger
equation of Eq. (5.7) with appropriate detuning/well imbalance \( \Delta \) (fixed \( \Delta \) for the Bragg
data and swept \( \Delta \) for the sweep data). The simulations assume a given tunneling energy \( t \),
initial condensate momentum \( p_i \) (relating to an added intersite bias), and peak mean-field
energy \( U_0 \). In the spirit of a local density approximation, we take into account the inhomoge-
neous density and inhomogeneous mean-field energy of our trapped atomic condensates
by assuming a Thomas-Fermi density distribution and taking a population-weighted sum of
simulation curves based on a homogeneous mean-field energy \( U \), ranging from 0 to \( U_0 \).

We took the residual sum of squares for each data set, and weighted each of these by the
number of data points in the respective data set to give equal weighting to all data sets before
summing up all of the values to get one fitting metric. We then minimized this weighted sum
with respect to the input values \( t, U_0, \) and \( p_i \), obtaining tunneling energy \( t/\hbar \approx 2\pi \times 390 \text{ Hz} \),
average mean-field energy \( U/\hbar \approx 2\pi \times 1810 \text{ Hz} \) relating to an inhomogeneous distribution
with peak mean-field energy \( U_0/\hbar \approx 2\pi \times 3170 \text{ Hz} \), and an initial condensate momentum
\( p_i \approx -0.018 \hbar k \).
5.5.3 Calculation of adiabatic energy levels

Even ignoring quantum fluctuations and nonclassical correlations, the nonlinear momentum-space interactions can result in interesting behavior of the two-mode condensate field. Here, we briefly describe the calculations (following the work of Ref. [149]) used to determine the adiabatic energy levels and their model projections, as shown in Fig. 5.2. The identification of cusps, or swallow-tails, in the adiabatic energy level structure provides insight into the mechanism underlying the observed dynamical phenomenon of nonlinear self-trapping.

We approximate the state of our two-mode Bose–Einstein condensate (with possible single particle states represented in terms of superpositions of the left ($|L\rangle \equiv |p/2\hbar k = 0\rangle$) and right well ($|R\rangle \equiv |p/2\hbar k = 1\rangle$) orbitals) in terms of the field $|\psi(\tau)\rangle = \sqrt{N}[\phi_L(\tau)|L\rangle + \phi_R(\tau)|R\rangle]$, where $\tau$ is the time variable, and the complex amplitudes $\phi_L$ and $\phi_R$ relate to populations $N_L = N|\phi_L|^2$ and $N_R = N|\phi_R|^2$ in the left and right well. As described in Sec. 5.1.4, the nonlinear Schrödinger equation describing the evolution of the condensate wave function results in the two coupled differential equations

$$i\hbar \dot{\phi}_L = U[2 - |\phi_L|^2]\phi_L - t\phi_R$$  \hspace{1cm} (5.16)

$$i\hbar \dot{\phi}_R = U[2 - |\phi_L|^2]\phi_R - t\phi_L + \Delta(\tau)\phi_R$$,  \hspace{1cm} (5.17)

where $U$ is the homogeneous mean-field energy relating to direct interactions, $t$ is the real-valued tunneling energy, and $\Delta$ is the site-to-site energy bias. In experiment, the bias is swept in time as $\Delta(\tau) = \Delta_i - 2\Delta_i(\tau/T)$ for a sweep duration $T = 1$ ms.

Intuition into the dynamical response of this system is gained by first considering the steady-state solutions to these coupled equations for various conditions of $U$, $t$, and $\Delta$. We determine the adiabatic energy levels relating to steady state solutions of the form $|\psi(\tau)\rangle = e^{-i\mu\tau}|\psi(0)\rangle$ by solving

$$\mu\phi_L = U(2 - |\phi_L|^2)|\phi_L| - t\phi_R$$ \hspace{1cm} (5.18)

$$\mu\phi_R = \Delta + U(2 - |\phi_R|^2)|\phi_R| - t\phi_L$$, \hspace{1cm} (5.19)

where $\mu$ is the chemical potential.
Solution of these coupled equations determines the adiabatic eigenstate solutions, their energy values, and their projections onto the two available momentum modes (wells). For a given combination of $U$, $t$, and $\Delta$ values, between two and four solutions with distinct $\mu$ and $|\phi_L|$ combinations (and with $|\phi_R|^2 = 1 - |\phi_L|^2$ fixing $|\phi_R|$) can be found, with the presence of more than two solutions relating to cusp or swallow-tail structures in the energy band structures. The projected probabilities onto the left and right well states $|L\rangle$ and $|R\rangle$ are given by

$$|\phi_L|^2 = 1 - |\phi_R|^2 = \frac{\Delta + U - \mu}{\Delta + 2(U - \mu)}.$$  \hfill (5.20)

The eigenstate solutions, having different $\mu$ values in general, are expressed in terms of these projected probabilities as $|I\rangle = \sqrt{N} (|\phi_L||L\rangle - |\phi_R||R\rangle)$ (the ground state) and $|II\rangle = \sqrt{N} (|\phi_L||L\rangle + |\phi_R||R\rangle)$ (the excited state). The relevant $\mu$ values may be determined by solving the determinant

$$\left| \begin{array}{cc} U(2 - |\phi_L|^2) - \mu & -t \\ -t & \Delta + U(2 - |\phi_R|^2) - \mu \end{array} \right| = 0$$  \hfill (5.21)

in combination with solution of Eq. 5.18 and Eq. 5.19. We note that in Fig. 5.2, these $\mu$ values are labeled as the energies $E$ of the adiabatic energy levels.

### 5.5.4 Alternate description of sweep direction-dependent behavior

In Sec. 5.2.2, we primarily describe the direction-dependent behavior of the positive and negative sweep data in terms of the adiabatic energy levels. Here we present an alternative description of the behavior based on site energy shifts. If we consider, as in the experimental limit, the interactions to be site-local, we can think of the effective attraction as a negative shift of the site energy, leading to a negative shift of the site energy of the initially-populated left well.

For the positive sweep, this decreases the initial site energy difference between the wells, and leads to an effective resonance condition for the initial transfer of population at negative single-particle bias values, i.e. at earlier sweep times compared to the non-interacting case. This earlier transfer of population is evident in both the data and the right-well population.
projection (Fig. 5.2(d,f,h)).

For the negative sweep, the negative energy shift to the left well increases the initial site energy difference, leading to an effective resonance condition for initial population transfer occurring at negative single-particle bias values, i.e. at later sweep times compared to the non-interacting case. This is evidenced in the lower, later population transfer in the data and well projections.
Chapter 6

Mobility Edges on a Zigzag Lattice

In this chapter and the related work (Ref. [15]), we implement next-nearest-neighbor tunneling links to create a zigzag lattice, and study how a synthetic gauge field affects the localization transition in the Aubry-André model. By realizing and monitoring the lowest and highest energy eigenstates of the system, we find that their localization properties are drastically different, and change as we vary the applied magnetic flux, effectively tuning the band structure of the lattice. From this we infer the existence of a tunable single-particle mobility edge in the system, an energy-dependent localization transition in the absence of atomic interactions.

This work is the first step our lab made to realizing the “overlap” regions of the big picture in Fig. 1.1, combining two key ingredients from previous work: synthetic gauge fields in MSLs (Chap. 3) and the quasiperiodic Aubry-André model of pseudodisorder (Chap. 4). More generally, this study realized several “firsts” for cold atom experiments: the first zigzag lattice geometry, the first explorations of the combined influence of artificial gauge fields and engineered pseudodisorder, and the first direct evidence for energy-dependent localization in a lower-dimensional system.

6.1 Introduction

6.1.1 Single-particle mobility edges

As discussed in Sec. 4.1, in the absence of interactions, Anderson localization is the generic fate of quantum states in disordered lower-dimensional ($d \leq 2$) systems featuring static, random potential energy landscapes and short-ranged tunneling [88, 177]. In higher dimensions, the increasing density of states with increasing energy ensures the possibility of delocalization. The exploration of an energy-dependent localization transition, i.e., a mobility edge,
has been undertaken in atomic gases (here at UIUC, even!) [94, 96] in three dimensions through the precise control over disorder and atomic state energies. Cold atom techniques in principle also allow for the exploration of such physics in lower-dimensional systems, where mobility edges can be introduced by correlations in the applied disorder or modified lattice connectivities (e.g., through long-range tunneling).

Despite the exquisite control over cold atom systems and the observations of Anderson localization in one dimension (1D) over a decade ago, for both nearly random disorder [93] and correlated pseudodisorder [92], single-particle mobility edges (SPMEs) in lower dimensions have gone unexplored. The reasons for this are technical – it is quite difficult to modify lattice connectivities, and the varieties of engineered disorder that have been explored in experiment have been either practically random, generated via speckle [93, 94, 96, 178] with short-range correlations due to diffraction, or of the Aubry-André form of correlated disorder which, due to a peculiar fine-tuning, does not admit a SPME [92, 97, 104, 113]. While Aubry-André pseudodisorder allows for a localization transition in 1D, the fine-tuning of the cosine-distributed site energies and the cosine nearest-neighbor band dispersion results in an energy-independent metal-insulator transition, and thus the absence of a SPME. By deviating from this fine-tuned condition, either by modifying the band dispersion [179] or by modifying the form of the pseudodisorder [28], one can, in principle, controllably introduce a SPME in such a system.

6.1.2 This work

In this work, we seek to generate a SPME in a 1D lattice with a quasiperiodic Aubry-André potential by introducing longer-ranged tunneling. Specifically, we use our MSL system to independently engineer controllable nearest-neighbor (NN) and next-nearest-neighbor (NNN) tunneling terms (Fig. 6.1(a)). The combination of NN and NNN tunneling pathways results in closed tunneling loops that can support a nontrivial flux (Fig. 6.1(b)), which we control directly through the complex phase of the various tunneling terms, much like our work in Chap. 3. This system realizes an effective zigzag chain with a tunable magnetic flux [180]. With the combination of controlled pseudodisorder and tunable flux, we perform the first
We observe direct evidence for a flux-dependent SPME in this system, through measurement of the localization properties of the extremal energy eigenstates. In addition to the SPME that results from multi-ranged hopping, we observe asymmetric (with applied flux) localization behavior of the system’s lowest-energy and highest-energy eigenstates, caused by the presence of effectively attractive interparticle interactions in the lattice of momentum states [20]. The influence of interactions is even more strongly evident in the case of the 1D AA with only NN tunneling, where a drastic shift in the localization transition is observed between low- and high-energy eigenstates, corresponding to a mobility edge driven purely by inter-particle interactions.

### 6.2 The zigzag lattice

#### 6.2.1 Engineering the lattice

To engineer a zigzag chain, we need to engineer a 1D lattice with both NN and NNN tunneling terms. We follow the usual MSL approach, but address not only first-order Bragg transitions, but also second-order Bragg transitions, as depicted in Fig. 6.1(c). As usual,
we apply a set of counter-propagating lattice laser beams with wavelength $\lambda = 1064$ nm, wavenumber $k = 2\pi/\lambda$, and frequency $\omega^+ = c/2\pi\lambda$, allowing for quantized momentum transfer to the atoms in units of $\pm 2\hbar k$. By applying a frequency to the laser beam satisfying the two-photon Bragg condition between $p = 0$ and $p = 2\hbar k$ ($\omega^+ - \omega_{0,1}$), we can create a NN tunneling link between the two sites (solid black arrows in Fig. 6.1(c)). This process involves virtually absorbing one photon ($\omega^+$) from the non-shifted, single frequency beam and emitting one photon ($\omega_{0,1}$) into the shifted, multi-frequency beam. Likewise, we can address a four-photon, second-order Bragg process between $p = 0$ and $p = 4\hbar k$ by imprinting the appropriate frequency/energy difference onto the laser ($\omega^+ - \omega_{0,2}$), thus generating a NNN tunneling link (dashed red arrows). Again, this is virtually absorbing two photons from the single frequency beam ($\omega^+$) and emitting two photons into the multi-frequency beam ($\omega_{0,2}$). Because each of the spectral tones associated with a given NN or NNN tunneling term is unique (sort of... see Sec. 6.2.3), we are able to individually control each of the tunneling links in our synthetic lattice.

6.2.2 NNN parameter control

Control of the effective lattice parameters for these four-photon NNN processes requires slightly different considerations compared to the two-photon NN terms. For example, to enable a NNN tunneling phase of $\pi$, we apply half this phase ($\pi/2$, relative to incoming field) to the corresponding frequency component $\omega_{0,2}$. A similar consideration holds for the relationship between site energy and frequency detuning from resonance. More generally, these differences can be summarized in the relationships between the tunneling terms for NN ($t_{nn}e^{i\varphi_{nn}}$) and NNN ($t_{nnn}e^{i\varphi_{nnn}}$) processes. Taking into account the field strengths (assumed to be real) of the incoming beam ($\Omega_I$) and a particular frequency component of the multi-frequency beam ($\Omega_R$), the phases of these same fields ($\phi_I$ and $\phi_R$), the large single-photon detuning from atomic resonance $\Delta$ (relating to roughly 100 THz for our laser wavelength $\lambda = 1064$ nm), and the recoil energy $E_R = \hbar^2/2M_{Rb}\lambda^2$, these terms are given at resonance...
as:

\[ t_{nn} e^{i\phi_{nn}} = \frac{\Omega_I \Omega_R}{\Delta} e^{i(\phi_R - \phi_I)} \quad \text{and} \]
\[ t_{nnn} e^{i\phi_{nnn}} = \frac{\Omega_I^2 \Omega_R^2}{\Delta^2 (4E_R)} e^{i(2\phi_R - 2\phi_I)} , \] (6.1)

where the \(4E_R\) in Eq. (6.2) comes from the detuning from the NN resonance. This all assumes that the NNN tunneling links are made with two photons of the same frequency...

### 6.2.3 Other NNN contributions

Unfortunately, there is a caveat: we discovered later in our work that the frequencies associated with NNN tunnelings are not unique. In retrospect, this is quite obvious: the transition from \( p = 0 \) to \( p = 4\hbar k \) can be made by emitting two photons of the same frequency \( \omega_{0,2} \), which is separate and unique from all NN tunneling frequencies,

\[
(E_{p=4\hbar k} - E_{p=0}) / \hbar = 2 \left( \omega^+ - \omega_{0,2} \right),
\]

or it can be made by absorbing two photons of different frequencies. Inconveniently, these are already present in the system for the two NN processes from \( p = 0 \) to \( p = 2\hbar k \) and \( p = 2\hbar k \) to \( p = 4\hbar k \):

\[
(E_{p=4\hbar k} - E_{p=0}) / \hbar = \omega^+ - \omega_{1,2} + \omega^+ - \omega_{0,1}.
\]

This alternate pathway has a much smaller contribution because the frequencies are further off-resonant from the first-order resonance, but they still affect the resultant tunneling rates. So we could control the NNN tunneling parameters to some degree, but very small NNN tunneling amplitudes were difficult to achieve due to these ever-present alternate contributions. We attempted to cancel out these contributions by introducing time-dependent (echo) tunneling phases that would cause the unwanted contributions to vanish, however, we never ended up successfully implementing this in experiment. We ended up measuring the NNN tunneling strengths empirically, though two very recent works (2020) on MSLs with NNN
links by the Bo Yan group have shown that these tunneling terms and unwanted corrections to the site energies can be systematically calculated, controlled, and compensated by applying appropriate modifications to the frequency components [45, 46].

6.3 Homogeneous gauge field studies

As a first experiment in this study, we demonstrate our control of a homogeneous synthetic gauge field in the zigzag lattice. We directly impose a synthetic magnetic flux $\phi$ on every three-site plaquette using engineered tunneling phases. Because the plaquettes alternate pointing up and down, to generate a homogeneous positive flux $\phi$ we need to impose an alternating sign on the NNN tunneling phases, as shown in Fig. 6.1(a,b). The effective tight-binding Hamiltonian describing the 21-site zigzag lattice is then given by

$$\hat{H} = -t \sum_{n=-10}^{9} (\hat{c}_{n+1}^{\dagger} \hat{c}_{n} + \text{h.c.}) - t' \sum_{n=-10}^{8} (e^{i(-1)^{n+1}\phi} \hat{c}_{n+2}^{\dagger} \hat{c}_{n} + \text{h.c.}),$$

where $t$ ($t'$) is the NN (NNN) tunneling energy and $\hat{c}_{n}^{\dagger}$ ($\hat{c}_{n}$) is the creation (annihilation) operator at site $n$.

The synthetic gauge field, which can lead to the breaking of time-reversal symmetry, allows us to engineer an analog of spin-momentum locking in the zigzag lattice [6, 7, 19, 74, 107, 108, 156]. We consider the upper and lower rows of the lattice as an effective spin degree of freedom with (pseudo)spins $\sigma = 1$ and $-1$, respectively (Fig. 6.2(a)). Under conditions of broken time-reversal symmetry ($\phi \neq 0, \pm \pi$) we expect to observe chiral trajectories for atoms “polarized” on one row of the lattice. The band structure (shown for the tunneling ratio $t'/t = 0.62$ used in experiment) of the lattice shows this correlation between the sign of the group velocity and the (colored) spin/row degree of freedom. The two bands here reflect the two-site unit cell of the lattice, highlighted in yellow boxes (see Sec. 6.6.3 for more on band structure calculations).

To explore this spin-momentum locking in experiment, we initialize atoms on the lower row at site 0 and quench on the tunnel couplings according to Eq. (6.3). With zero applied flux, the population delocalizes across the lattice symmetrically, as shown in the top middle
optical density (OD) image of Fig. 6.2(b). For positive flux $\phi/\pi = +0.5$ (right panel), population initially in site 0 moves towards lattice site 2, corresponding to counter-clockwise chiral motion. Under a negative flux $\phi/\pi = -0.5$ (left panel), population moves in a clockwise fashion to lattice site $-2$. These observed chiral flows for $\phi = \pm \pi/2$ are clear signatures of spin-momentum locking.

By tuning the applied flux, we map out the entire range of chiral behavior, as shown in Fig. 6.2(b), bottom. Here we plot the population imbalance between lattice sites 2 and
such that a positive (negative) value of imbalance indicates counter-clockwise (clockwise) motion. The data agree qualitatively with an ideal simulation of the experiment using only Eq. (6.3) (dashed curve), but agree more closely with a full simulation of the system parameters (solid curve) which considers the exact form of atomic coupling to the many laser frequency components, accounting for off-resonant Bragg couplings (Sec. 6.6.2).

We are also able to directly observe the fully site-resolved chiral dynamics of initially localized atomic wave packets, as shown in Fig. 6.2(c,d). For positive flux, we see that atomic population moves counter-clockwise from site 0 to site 2, and further on to sites 4 and 6, remaining confined to the bottom row. Because the initial state (site 0) does not project entirely onto states with positive group velocity, a portion of the population stays near the center plaquette and oscillates between site 0 and sites ±1. Off-resonant Bragg coupling causes deviations from the ideal simulation (right), but these major qualitative features remain present in both the data (left) and full simulation (middle). For the case of negative applied flux, we observe the opposite chiral behavior, demonstrating that the nature of the spin-momentum locking can be controlled by the applied synthetic flux.

6.4 Localization studies

Localization phenomena in disordered quantum systems depend intimately on the properties of applied disorder and on the connectivity between regions of similar energy. For random potential disorder in three dimensions, a localization-delocalization transition is assured for states with energies beyond a critical value due to an increasing density of states. For a given disorder strength, a mobility edge, or energy-dependent localization transition, is found in such a system [94, 96]. In lower dimensions, for truly random potential disorder, all energy states remain localized in the thermodynamic limit even for arbitrarily small strengths of disorder [177].

Considering instead the influence of correlated pseudodisorder, one finds that the localization physics is strongly modified, with delocalization and mobility edges permitted even in lower dimensions. One form of quasiperiodic pseudodisorder that has been of interest to experimental studies with both light [181] and atoms [92] is that described by the diagonal
AA model. Interest in this model has stemmed in part from its intriguing localization phenomenology and connections to the Hofstadter lattice model [113, 121, 182]. Experimental interest in this form of disorder has also been driven by the relative ease of its realization through the overlap of two incommensurate optical lattices [92].

The AA model of pseudodisorder has interesting properties in the context of SPMEs. The highly correlated disorder allows for the possibility of a metallic, delocalized states in lower dimensions. However, a subtlety arises due to a correspondence between the distribution of pseudodisorder – characterized by quasiperiodic, cosine-distributed site energies – and the cosine dispersion in a NN-coupled 1D lattice. This fine tuning results in a metal-insulator transition that occurs at the same critical disorder value (in units of the tunneling energy) for all energy eigenstates, and thus the absence of a mobility edge. By moving away from this fine-tuned scenario in any number of ways – by introducing longer-range hopping [179], by modifying the pseudodisorder correlations [28], or by adding nonlinear interactions [97, 98, 100, 104, 161] – a SPME can be introduced into the AA model.

The addition of longer-range tunneling, as in our zigzag lattice, allows for the band dispersion to be modified from its simple cosineoidal form. For a flux of \( \phi = \pm \pi/2 \), as shown in Fig. 6.2(a), increasing the tunneling ratio \( t'/t \) from zero leads to a deformation of the low-energy band structure from quadratic, to quartic, to forming a double-well structure [67, 144, 183], with a symmetric modification of the band energies at high energy. The high ground state degeneracy of the quartic band in this system and of flat bands in similar multi-range hopping models has attracted great interest [154, 159, 184]. Such systems promise interesting localization properties under disorder [179], and the inherent high single-particle degeneracy allows for the study of emergent physics driven by interactions [154, 159, 184, 185]. For all other flux values (\( \phi \neq \pm \pi/2 \)) the dispersion of the bands at low and high energies is asymmetric, and this system permits the localization properties of the extremal energy eigenstates to be tuned through modification of the effective mass at low and high energies.

Here, we study the localization properties under the AA model on a 1D lattice and on the multi-range hopping zigzag lattice, observing evidence for an interaction-induced mobility edge as well as the emergence of a flux-dependent SPME.
Figure 6.3: Localization in the 1D AA model with NN tunneling. (a) Quasiperiodic lattice site energies under the AA model, $\varepsilon_n = \Delta \cos(2\pi \beta n + \varphi)$, for periodicity $\beta = (\sqrt{5} - 1)/2$ and amplitude $\Delta$. (b) Measured population outside the central three lattice sites ($P_{\text{out}}$) vs disorder-to-tunneling ratio $\Delta/t$, averaged over four values of the AA phase $\varphi/\pi = \{0.96, 0.64, 1.35, 1.88\}$. These $\varphi$ values correspond to the energy eigenstates $\{|\psi_0\rangle, |\psi_7\rangle, |\psi_{17}\rangle, |\psi_{18}\rangle\}$, where $|\psi_0\rangle$ is the ground state and $|\psi_{20}\rangle$ is the highest excited state. Dashed and solid curves represent ideal and full simulations of experimental parameters, respectively. Arrow indicated experimental ramp from $(\Delta/t)_i = \infty$ to some final disorder value. Inset: Integrated OD image showing site populations vs. disorder strength, also averaged over the four eigenstates. (c) Integrated OD images for the individual eigenstates, labeled with the relevant AA phase value. (d) $P_{\text{out}}$ vs disorder strength for eigenstates $|\psi_0\rangle$ (red circles) and $|\psi_{18}\rangle$ (blue squares). Gray circles are the averaged data of (c). Dashed red, dotted blue, and solid gray curves represent full simulation results including interactions of strength $U/\hbar \approx 2\pi \times 500$ Hz, for $|\psi_0\rangle$, $|\psi_{18}\rangle$, and the averaged data, respectively. Inset: Cartoon depicting lattice site energies for initial sites with low (left, red) and high (right, blue) energy. Dashed lines show the effect of attractive interactions on the initially-populated central site. Vertical lines in (b,d) indicate the critical disorder $(\Delta/t)_c = 2$ for an infinite system without interactions. Error bars in (b,d) denote one standard error of the mean. OD images in (b,c) are plotted with the color scale in Fig. 6.2(b).

6.4.1 1D Aubry-André localization transition

We first examine the localization properties of the one-dimensional AA model, or the $t'/t = 0$ limit of the zigzag lattice. This is similar to the studies we performed on the system in Chap. 4, but here we also discuss the role of atomic interactions.

Figure 6.3(a) shows this model’s pseudodisordered distribution of site energies $\varepsilon_n = \Delta \cos(2\pi \beta n + \varphi)$, for irrational periodicity $\beta = (\sqrt{5} - 1)/2$ and a given value of the phase de-
gree of freedom $\varphi$. Under this model, all energy eigenstates experience a transition from delocalized metallic states to localized insulating states at the same critical disorder, $(\Delta/t)_c = 2$, for an infinite system size. To probe the crossover in our finite 21-site system, we initialize various energy eigenstates and explore their localization properties as a function of $\Delta/t$.

The experiment begins with population at site 0 (the BEC at rest) with all tunnelings turned off. In this initial limit of infinite disorder $(\Delta/t)_{i=\infty}$, all eigenstates are trivially localized to individual sites of the lattice, with a vanishing localization length. We can initialize our atoms in a particular energy eigenstate of the system through choice of $\varphi$, as the eigenstates and eigenstate energies are solely determined by the site energies in this $t=0$ limit. We then linearly ramp the magnitude of the tunneling energy to a final value over 1 ms, and probe the localization properties of the prepared eigenstate as a function of $\Delta/t$.

The near-adiabatic quench of $t$ to its final strength $t/\hbar = 2\pi \times 1013(9)$ Hz (corresponding to a tunneling time of $\hbar/t = 157(1)$ $\mu$s, determined through two-site Rabi oscillations) is slow enough to largely remain within the prepared eigenstate. In each experiment, the disorder strength is fixed to a given value $\Delta$, such that the tunneling ramp (always to the same $t$ value) can be seen as traversing in parameter space from $\Delta/t = \infty$ to the chosen final value (shown as an arrow in Fig. 6.3(b)). We expect that for final values with $\Delta/t > (\Delta/t)_c$, the population should largely remain localized to the initial site, whereas for $\Delta/t < (\Delta/t)_c$ we should see population begin to delocalize across the lattice.

In Fig. 6.3(b), we plot the measured population outside the central three sites $P_{\text{out}}$, averaged over four realizations of the AA phase $\varphi/\pi = \{0.96, 0.64, 1.35, 1.88\}$ corresponding to energy eigenstates $\{\rvert \psi_0 \rangle, \rvert \psi_7 \rangle, \rvert \psi_{18} \rangle\}$, where $\rvert \psi_0 \rangle$ is the ground state and $\rvert \psi_{20} \rangle$ is the highest excited state. As expected, the measured delocalized fraction is almost entirely absent for large disorder, and grows steeply for $\Delta/t < (\Delta/t)_c$. We find excellent agreement between our $\varphi$-averaged measurements and numerical simulation results based on our experimental ramp (dashed curve, idealized simulations ignoring off-resonant Bragg couplings) in Fig. 6.3(b), suggesting the observation of a localization crossover that is broadened due to finite-size effects as well as the finite ramp duration. This same behavior can also be seen in the integrated optical density data, shown in the inset, which directly shows the averaged site populations for each final disorder value $\Delta/t$. For large disorder, population remains
localized to the initial site, while the metallic regime shows population spreading out to sites $n = \pm 7$.

The data for individual energy eigenstates is also shown, both as integrated optical density images in Fig. 6.3(c) and the $P_{\text{out}}$ observable in Fig. 6.3(d). While all four data runs show localization crossovers, their positions in terms of a critical disorder-to-tunneling ratio $(\Delta/t)_c$ differ according to the state energies. Visually, the ground state $|\psi_0\rangle$ appears to localize for smaller disorders than the intermediate energy eigenstates, with the highly excited state $|\psi_{18}\rangle$ requiring the largest critical disorder strength for localization. While some of the broadening of the transition observed in Fig. 6.3(b) can be attributed to effects of finite size and finite ramp durations, to a large degree it is explained by this averaging over unique localization transitions of different energy eigenstates.

The difference in localization properties for different energy eigenstates runs counter to our expectations of an energy-independent transition for the NN-coupled AA model, but can be explained by the presence of nonlinear atomic interactions in our momentum-space lattice [20]. In particular, the interactions between indistinguishable bosons in momentum space are effectively attractive and site-local, in the sense that direct interactions are present for collisions between two atoms occupying any pair of momentum modes, while exchange interactions are present only when two identical bosons occupy distinguishable modes [138, 147]. Thus, while the momentum-space interactions are physically long-ranged and repulsive, they give rise to an effective local attraction. For atoms initially prepared at the site with lowest energy, attractive interactions can be seen to bring atoms further away from tunneling resonance with other sites (Fig. 6.3(d), inset). Thus, such a state should remain localized even when the disorder drops below the single-particle critical value. In contrast, for atoms prepared at the highest energy site, attractive interactions effectively lower the total site energy and bring the atoms closer to tunneling resonance with the unoccupied lower-energy sites of the lattice (Fig. 6.3(d), inset). Then, by filling the high-energy sites with attractively-interacting bosons, the disorder potential can be effectively smoothed out at high energies by atomic interactions [161].

This behavior for our effectively attractive momentum-space interactions is exactly the opposite of that found for real-space repulsive interactions, the influence of which has pre-
viously been studied on ground state localization properties of the AA model [161]. The simulation curves in Fig. 6.3(d) take into account the effective attractive interactions present in our system at an approximate, mean-field level (also ignoring the inhomogeneous atomic density and neglecting off-site contributions of the effective attraction, which arise due to partial indistinguishability of atoms in different momentum states resulting from superfluid screening, as described in Sec. 6.6.4). The simulations assume a mean-field interaction based on our condensate’s central mean-field energy $U_0/\hbar \approx 2\pi \times 860$ Hz (as measured through Bragg spectroscopy), which is of the order of the single-particle tunneling energy $t/\hbar = 2\pi \times 1013(9)$ Hz. To account for the inhomogeneous density distribution, we take a weighted average over homogeneous mean-field energies ranging from 0 to the peak mean-field energy $U_0$ to get an average mean-field energy of $U/\hbar \approx 2\pi \times 500$ Hz. We then use this average value as a homogeneous mean field energy in our simulations. These simplified simulation curves already reproduce well the observed shifts of the localization transitions for the low- ($|\psi_0\rangle$) and high-energy ($|\psi_{18}\rangle$) states. These direct observations of interaction-induced localization and delocalization for low and high-energy states, respectively, are indicative of a many-body mobility edge. Such measurements are enabled by our ability to approximately prepare arbitrary eigenstates in our synthetic lattice.

### 6.4.2 Localization studies in zigzag chains

With the addition of longer-range tunneling, the energy-independent transition of the simple 1D AA model begins to depend critically on the eigenstate energy even at the single-particle level. By tuning the NNN tunneling strength and the artificial flux in our effective zigzag chains, we can introduce a tunable SPME through band structure engineering. While in the demonstration of control over flux and the observation of spin-momentum-locking in Fig. 6.2 we employed a tunneling ratio of $t'/t \approx 0.6$, here we work at a smaller value of $t'/t \approx 1/4$. Under this condition, a maximal difference in the band dispersion at low and high energies appears for flux values of 0 and $\pi$, where a quartic dispersion appears at high and low energies, respectively.

To probe the mobility edge, we prepare the two extremal energy eigenstates of the system,
Figure 6.4: Flux-dependent localization transition in the AA model with multi-range tunneling.
(a) Top: Ground state (GS) localization behavior for varying values of disorder $\Delta/t$ and flux $\phi/\pi$, in a zigzag lattice with NN tunneling $t/h = 493(2)$ Hz and NN to NNN tunneling ratio $t'/t = 0.247(4)$. Colors indicate the width (standard deviation) of the site population distribution $\sigma_n$ (inset color scale in (d)), interpolated from the sampled data points (small solid black circles). The empirically-determined transition disorder strength between delocalized and localized regions is shown for the data (white circles), a non-interacting simulation of the experiment (dashed black line), and a simulation including attractive interactions of strength $U/h \approx 2\pi \times 500$ Hz (dotted black line). Bottom: Localization properties as visualized by the integrated 1D density patterns for roughly 0 and $\pi$ flux. The 1D atomic distributions are interpolated from integrated optical density (OD) images for $\phi/\pi = 0.05$ (left) and 0.95 (right), shown as a function of $\Delta/t$. Horizontal lines show the empirically-determined localization transition point for data (solid red), non-interacting simulation (dashed gray), and simulation including interactions (dotted gray). (b) Cuts as a function of $\phi$ showing site populations interpolated from integrated OD images, shown for the GS (top) and the highest excited state (ES, bottom). These integrated OD plots are averaged from data taken in the ranges $2.75 \leq \Delta/t \leq 3$ and $3.2 \leq \Delta/t \leq 3.5$, respectively, as indicated by the shaded regions. (c) Band structure diagrams for the zigzag lattice with applied flux $\phi/\pi = 0$ (left) and $\phi/\pi = 1$ (right). As in Fig. 6.2(a), color represents spin polarization. Dashed black curves represent the folded band structure for $t'/t = 0$. (d) ES localization behavior for varying disorder and flux values, with the same format as in (a). Bottom: Localization properties of the ES as a function of $\Delta/t$, also with the same format as in (a). Error bars in (a,d) denote one standard error of the mean. OD images in (a,b,d) are plotted with the color scale in Fig. 6.2(b).

the ground state (GS) and the highest excited state (ES), and compare their localization properties. As in the 1D study, our experiment begins with all atomic population prepared at site 0 with all tunnelings turned off, i.e., in the infinite-disorder limit of the system ($\Delta/t = \Delta/t' = \infty$) where all energy eigenstates are localized to individual sites of the
lattice. To initialize the atoms in a particular energy eigenstate of the system, we simply vary the AA phase: \( \varphi = 0 \) for the GS and \( \varphi = \pi \) for the ES.

In short, we track how the prepared eigenstate evolves as the parameters of the Hamiltonian, given by

\[
\hat{H}(V) \approx \sum_{n=-10}^{10} \varepsilon'_n(V) \hat{c}^\dagger_n \hat{c}_n - t \sum_{n=-10}^{9} \left( \hat{c}^\dagger_{n+1} \hat{c}_n + \text{h.c.} \right) - t' \sum_{n=-10}^{8} \left( e^{i(-1)^{n+1} \phi} \hat{c}^\dagger_{n+2} \hat{c}_n + \text{h.c.} \right),
\]

are smoothly and slowly varied to some final desired conditions of \( \Delta/t \) for fixed tunneling ratio \( t'/t \) and fixed flux \( \phi \). To help ensure adiabaticity over a large part of the parameter ramp, we added an extra potential offset of strength \( V \) at the initial site \( n = 0 \), such that the modified site energies are given by \( \varepsilon'_n(V) = \Delta \cos(2\pi \beta n + \varphi) - V \delta_{n,0} \). By setting \( V > 0 \) (\( V < 0 \)) for the GS (ES), we further separate the initial eigenstate from the rest of the spectrum by a potential well (hill). Starting from the initial limit of \( V/t = \infty \) and \( \Delta/t = \infty \), we perform a near-adiabatic quench to approximately load our desired eigenstate by linearly ramping up both tunneling terms (\( t \) and \( t' \)) over 2 ms while also smoothly removing the potential well by ramping \( V \) to zero, as described in more detail in Sec. 6.6.5.

We perform this procedure over parameter ranges \( 1 \leq \Delta/t \leq 4.25 \) and \( 0 \leq \phi/\pi \leq 1 \), mapping out the localization behavior of the GS and the ES in Fig. 6.4(a,d). We plot the standard deviation of the population distribution in the lattice, \( \sigma_n \) (\( i.e. \), the momentum standard deviation \( \sigma_p \) normalized to the spacing between sites of \( 2\hbar k \)), where the values are resampled from the actual \( (\Delta/t, \phi/\pi) \) points where data were taken (small black dots). The \( \Delta/t \) values of the data have variations and uncertainties stemming from variations and measured uncertainties in calibrated tunneling rates for the experimental runs, with an overall averaged NN tunneling rate \( t/\hbar = 493(2) \) Hz and tunneling ratio \( t'/t = 0.247(4) \).

For the ground state in Fig. 6.4(a), we see that the region of metallic, delocalized states (red region, corresponding to states with large \( \sigma_n \)) extends out to larger \( \Delta/t \) values when the applied flux is near zero than for the case of an applied \( \pi \) flux. This can also be seen in the integrated optical density images at bottom: sites as far as \( n = \pm 2 \) remain populated even
at large disorder $\Delta/t \sim 3.5$ at small flux $\phi/\pi = 0.05$ (left), while for large flux $\phi/\pi = 0.95$ (right) population fully localizes for $\Delta/t > 3$. The top panel of Fig. 6.4(b) highlights that for a fixed disorder-to-tunneling ratio of $\Delta/t \sim 2.9$, the GS can be driven from metallic to insulating by changing only the flux.

In the absence of flux, the shift of the GS localization transition to larger disorder values as compared to the $t' = 0$ case is intuitive: simply adding longer-range tunneling increases the connectivity of the lattice, increasing the dispersion at low energy, and enhancing delocalization. As non-zero flux is added, however, the GS localization transition shifts towards smaller critical disorder values. This effect is perhaps surprising when considering effects such as the suppression of weak localization by broken time-reversal symmetry, as observed recently in measurements of coherent back scattering [186]. However, in the context of our zigzag flux chains, this flux-enhanced localization of the GS is easy to interpret. The shift of the GS localization transition towards smaller $(\Delta/t)_c$ is driven by a flattening of the low-energy band dispersion, owing to kinetic frustration of the different tunneling pathways. The system is maximally frustrated at $\phi = \pi$ for $t'/t = 1/4$, corresponding to a nearly flat, quartic low-energy dispersion (Fig. 6.4(c), right). Under these conditions, the states at low energy become heavy (large effective mass) and easier to localize in the presence of disorder.

In considering the flux-dependent localization properties of the highest energy eigenstate, a similar argument holds, but with the opposite trend with applied flux. The high energy states of the band structure are maximally dispersive for $\phi = \pi$, becoming flatter for decreasing flux, with a quartic band appearing for zero flux. This modified band structure leads to the measured dependence of the ES localization properties following the near-adiabatic quench to final $\Delta/t$ values for different flux values (Fig. 6.4(d)). The flux-dependence of the localization transition is also seen in striking fashion in the integrated OD images at the bottom of Fig. 6.4(d): while the low flux panel clearly shows a transition from localized to delocalized behavior at $\Delta/t \sim 2.3$, in the high flux panel the site populations remain delocalized for all investigated disorder values.

For both states, we empirically estimate the approximate “critical” disorder strength (normalized to $t$) relating to the metal-insulator transition by finding the $\Delta/t$ value at which $\sigma_n$ equals 0.68 lattice sites. This estimate is determined for each flux value of the data, and
Figure 6.5: **Flux-dependent mobility edge.** Empirically determined critical disorder-to-tunneling strength ratios marking localization transition for ground state (GS, red circles) and highest excited state (ES, blue squares), as shown separately in Fig. 6.4(a,d). Non-interacting and interacting simulations \((U/\hbar = 2\pi \times 500 \text{ Hz used in the latter})\) are shown as dashed and dotted lines, respectively. For flux values where no critical disorder values are plotted, atomic population was determined to be delocalized (based on the set threshold value of the standard deviation) over the full range of disorder strengths. Vertical gray line at \(\phi/\pi = 0.5\) denotes flux value at which the GS and ES curves should cross in the absence of interactions and any off-resonant coupling terms. Error bars denote one standard error of the mean.

The extracted critical disorder strengths are shown as white circles in Fig. 6.4(a,d). We can compare these experimentally-extracted points to the predicted threshold values of disorder, based on numerical simulations of our experimental ramp protocol. These single-particle predictions are shown as dashed lines in Fig. 6.4(a,d), and show the same qualitative trend as the experimental points for both the GS and ES.

To better contrast the localization behavior of the GS and ES, we additionally plot both the experimentally determined transition points and the theory predictions for both extremal eigenstates together in Fig. 6.5. With the two datasets overlaid, one can more clearly see the direct evidence for a flux-dependent SPME. While this sampling of the two extremal eigenstates does not determine the critical energy at which delocalization occurs for given values of \(\Delta/t\) and \(\phi\), it does provide the first direct experimental evidence for a SPME in lower dimensions.

The behavior of the transition \(\Delta/t\) values for the GS and ES are nearly opposite to one another. For flux values near zero, the disorder strength needed to localize the GS exceeds that of the ES by nearly \(t\), due to kinetic frustration of the high energy states.
The situation reverses for flux values near $\pi$: the GS becomes localized at lower disorder strengths $\Delta/t \sim 2.3$, and the ES remains delocalized even up to the highest disorder value used in experiment $\Delta/t \sim 4.25$. This apparent asymmetry, \textit{i.e.}, that a larger magnitude of shift between the GS and ES transition points is found for flux values near $\pi$ than for flux values near 0, is in disagreement with the single-particle prediction. Moreover, at the single-particle level the flux-dependence of the GS and ES localization properties should essentially be mirror images of one another (dashed lines, with a slight asymmetry resulting from effects due to off-resonant driving), such that their transitions points should cross very near to $\phi/\pi = 0.5$ (vertical gray line in Fig. 6.5). However, the apparent crossing point is offset to lower flux values by nearly $0.1\pi$. As in the previously discussed case of the 1D AA model with only NN interactions (Fig. 6.3(c,d)), the nonlinear interactions present in our atomic system are largely responsible for this asymmetry observed between the localization properties of our low and high energy eigenstates. We note that any effects on the localization behavior due to trap confinement, which causes our atomic wave packets to have a finite extent in real space, should have similar impacts on the observed ground state and excited state behavior. This is not consistent with the qualitatively disparate behavior that we observe in experiment.

As described earlier in the context of the NN-coupled AA model, we can approximately capture the influence of the momentum-space interactions in this system by including a site-local mean-field attraction in a multi-site nonlinear Schrodinger equation (Sec. 6.6.4), with an interaction energy that is determined independently by calibration via Bragg spectroscopy. Including these interactions (dotted lines, also shown in Fig. 6.4(a,d)), the transition lines get shifted to lower (GS) and higher (ES) disorder values, so that they cross at lower flux values. The interacting simulation results better capture the localization properties of the ES, which was shifted to significantly higher disorder strengths than was predicted at the single-particle level. It also qualitatively captures the shift of the crossing of the critical disorder curves in Fig. 6.5 to lower flux values, although it predicts a slightly larger shift than seen in experiment. In the future, by studying fluctuations of the atomic number distribution and inter-site correlations in our synthetic lattice, or by more closely studying fine features of the localization properties, this simulation platform may enable unique
explorations into the physics of interacting disordered/quasiperiodic systems, in particular related to the physics of many-body localization. It also offers a unique platform to study the interplay of disorder/quasiperiodicity, artificial gauge fields, and interactions.

6.5 Conclusions

This work represents the first direct observation of a single-particle mobility edge in lower dimensions, which is enabled by the unique ability to stably prepare atoms in any energy eigenstate and explore their localization properties in a system with precisely controlled pseudodisorder and tunable artificial gauge fields. The multi-ranged tunneling zigzag model that we have engineered displays many more topological properties which could be interesting to probe in experiment, such as a fractal energy spectrum and topological flat bands [180]. We also present the first cold atom evidence for a many-body mobility edge through direct eigenstate preparation, studied through a shift of the localization properties of low- and high-energy eigenstates in the 1D AA model that arise due to many-body interactions. We note that previous studies have utilized other techniques to observe a many-body mobility edge [104]. These interaction shifts are also observed in the localization transitions of a multi-range hopping AA model that admits a flux-dependent SPME, leading to the interplay of single-particle and many-body shifts of the localization transition for states at different energies.

This work also constitutes the first cold atom study combining synthetic gauge fields and pseudodisorder, and its extension to fully two-dimensional lattices beyond coupled chains promises to pave the way towards studies of disordered quantum Hall systems. In particular, by moving to a larger system containing bulk lattice sites, a robustness of the observed chiral-propagating modes to disorder (similar to the robustness to disorder observed recently for the bulk winding of chiral symmetric wires [16]) should be readily observable.

Following the arXiv posting of this paper, two related works were also posted, exploring the SPMEs (Ref. [187]) that appear naturally due to NNN tunneling in weak real-space optical lattices under AA pseudodisorder [188] and exploring the interplay of disorder and artificial gauge fields in kicked rotor systems (Ref. [189]).
6.6 Additional notes

6.6.1 Tunneling times and tunneling ratios

Here we present the exact NN tunneling times $\hbar/t$ and NNN to NN tunneling ratios $t'/t$ for each individual data set shown in the main text. For the variable flux data of Fig. 6.2(b), $\hbar/t = 172(1) \, \mu s$ and $t'/t = 0.633(3)$. For the dynamics under $\phi/\pi = 0.5$ of Fig. 6.2(c), $\hbar/t = 182(1) \, \mu s$ and $t'/t = 0.605(5)$. For the dynamics under $\phi/\pi = -0.5$ of Fig. 6.2(d), $\hbar/t = 174(2) \, \mu s$ and $t'/t = 0.628(6)$. The tunneling ratio used to make the band structures of Fig. 6.2(a) was an average of these three values: $t'/t = 0.622(3)$. For the 1D Aubry-André data of Fig. 6.3, $\hbar/t = 157(1) \, \mu s$. For the multi-range hopping Aubry-André data of Fig. 6.4 and Fig. 6.5, $\hbar/t = 323(1) \, \mu s$ and $t'/t = 0.247(4)$, averaged over all data points taken.

6.6.2 Off-resonant excitations

While we seek to address individual Bragg transitions with single frequency components, in practice we apply the full spectrum of frequencies to the condensate, as shown in Fig. 6.6(b). Thus each transition is not only addressed by one resonant frequency, but also feels the effects of all of the other non-resonant frequency components. For a lattice with only NN tunnelings, the frequency components are equally spaced at $8E_R/\hbar = 8 \times \hbar k^2/2M_{Rb} \approx 2\pi \times 16.2 \, \text{kHz}$. Adding NNN links halves the spacing of applied frequency components to $4E_R/\hbar \approx 2\pi \times 8.1 \, \text{kHz}$.

On a lattice with only NN tunnelings, the off-resonant couplings result in step-like intervals in the dynamics (Fig. 6.6(c)). For this data (from Ref. [14]), atomic population undergoes a continuous-time quantum walk on a 21-site lattice engineered with 20 equally-spaced frequency teeth. Due to the equal spacing of the frequency teeth, the off-resonant effects add up constructively. This is evident in the period $T$ between these steps, which corresponds exactly to the spacing between adjacent frequency teeth, $T = h/8E_R$. We have shown in a previous study [14] that adding random tunneling phases onto equally-spaced frequency teeth suppresses these steps, resulting in smoother dynamics.

We note that the magnitude of the tunneling rate plays a significant role in the magnitude
Figure 6.6: Effects of off-resonant excitations. (a) Dispersion relation and depiction of Bragg transition pathways for NNs (solid grey) and NNNs (dashed red). Energy is shown in units of recoil energy, \( E_R = \hbar^2 k^2 / 2M_{Rb} \). (b) Frequency spectrum (w.r.t. the incident beam frequency) used to create a 21-site lattice with 20 NN and 19 NNN links. (c) Dynamics of the width of the population distribution, \( \sigma_n \) (in units of synthetic lattice spacing), for a lattice with \( t'/t = 0 \). (d) Dynamics of \( \sigma_n \) for a lattice with \( t'/t = 0.628(6) \) and applied flux \( \phi/\pi = -0.5 \) (same data as in Fig. 6.2(d)). Dashed and solid curves in (c,d) represent results from an ideal simulation of the experiment and a full simulation accounting for off-resonant coupling, respectively.

of off-resonant effects. If the tunneling rate is comparable to or greater than the frequency spacing between the first-order Bragg resonances (\( t/h \gtrsim 8E_R/h \)), then each frequency tooth may address multiple transitions. The extreme limit of non-resonant addressing is often encountered in cold atom experiments, e.g., when a deep stationary potential (two interfering fields with equal frequency) that is suddenly turned on results in Kapitza–Dirac diffraction [190] of atomic matter waves. With respect to synthetic lattices, this can be viewed as a system with constant NN tunneling terms in the presence of a quadratic potential, set simply by the single-particle dispersion relation. This leads to expansion dynamics in momentum space at short times up until population reaches outer regions where the site (momentum state) energy roughly equals the effective tunneling bandwidth [191].

On the other hand, in the limit where the tunneling energy is much smaller than the
energy spacing between relevant Bragg resonances (i.e., in the limit that the rotating wave approximation is valid), each transition will be ideally addressed by only a single spectral component, and the step-like behavior in Fig. 6.6(c) (data, solid curve) should approach the ideal smooth behavior (dashed curve). Intuitively, this occurs as the number of steps per tunneling period gets very large, or in other words as the tunneling time $\hbar/t$ gets much larger than the step period $T$, such that the dynamics are spread out over many, many steps. For the data in Fig. 6.6(c), we are in the intermediate regime (with a tunneling time $\hbar/t = 111.6(7) \mu s$, corresponding to a tunneling rate of $t/\hbar = 2\pi \times 1425(9) \text{ Hz}$), where off-resonant coupling primarily results in the observed dynamics with small step-like behavior.

For the zigzag lattice, we introduce NNN frequency teeth that halve the spacing to $4E_R/\hbar \approx 2\pi \times 8.1 \text{ kHz}$ (addition of dashed red peaks in Fig. 6.6(b)). This results in longer steps of exactly twice the duration, i.e., $T' = 2T$ (Fig. 6.6(d)) due to off-resonant first-order Bragg processes. The smaller structure on top of these long steps, with a spacing of $T/2$ relating to a frequency spacing of $16E_R/\hbar$, is due to off-resonant second-order Bragg processes. For this data, we used a tunneling ratio $t'/t = 0.628(6)$ and NN tunneling time $\hbar/t = 174(2) \mu s$, corresponding to a tunneling rate $t/\hbar = 2\pi \times 917(9) \text{ Hz}$.

The “full” simulations in Fig. 6.6(c,d) (solid curves) and in the main text account for both resonant and off-resonant driving on every Bragg transition, and thus retain these step-like features. The “ideal” simulations in Fig. 6.6(c,d) (dashed curves) and in the main text ignore off-resonant effects and consider only the smooth behavior of the idealized tight-binding Hamiltonians.

As mentioned in Sec. 6.2.3, two recent works (2020) by the Bo Yan group have also worked with NNN transitions in a momentum-space lattice [45, 46]. In their works, they more rigorously examined off-resonant contributions and were able to correct for any light shifts by shifting their driving frequencies beforehand. This method should be followed for all further studies, not only those including NNN transitions.
6.6.3 Band structure calculations

The band diagrams shown in Fig. 6.2(a) and Fig. 6.4(c) were calculated using the same method as described in Sec. II of Ref. [154]. While the studies there focused on a semisynthetic lattice with one synthetic dimension and one real-space dimension, the same physics hold for our fully synthetic momentum-space lattice.

We consider the zigzag lattice in the absence of any applied disorder. We can take the rows of the lattice to be an effective spin degree of freedom, with a two-site unit cell comprised of one spin up ($\sigma = +1$) site from the top row and one spin down ($\sigma = -1$) site from the bottom row. We generate the spinful dispersion by calculating the $2 \times 2$ Hamiltonian introduced in Ref. [154] at each value of quasimomentum. In terms of the creation and annihilation operators at spin $\sigma$ and quasimomentum $\tilde{q}$ ($\hat{c}_{1,\tilde{q}}^\dagger$ and $\hat{c}_{\sigma,\tilde{q}}$), the Hamiltonian is given by

$$\hat{H} = \sum_{\tilde{q}} \left( \begin{array}{cc} \hat{c}_{1,\tilde{q}}^\dagger & \hat{c}_{-1,\tilde{q}}^\dagger \\ \end{array} \right) \left( \begin{array}{cc} h_{11} & h_{12} \\ h_{21} & h_{22} \\ \end{array} \right) \left( \begin{array}{c} \hat{c}_{1,\tilde{q}} \\ \hat{c}_{-1,\tilde{q}} \\ \end{array} \right), \quad (6.5)$$

where $h_{jj} = -2t' \cos [\tilde{q}d + \phi (-1)^j]$ and $h_{12} = h_{21} = 2t \cos[\tilde{q}d/2]$ for lattice spacing $d = 4\hbar k$ (not $2\hbar k$ due to the two-site unit cell). By diagonalizing this Hamiltonian for every value of $\tilde{q} \in [-\pi/d, \pi/d]$, we generate the double-band dispersions. We note that for $t'/t = 0$ (dashed black curves in Fig. 6.2(a) and Fig. 6.4(c)), the dispersion relation is cosinusoidal, but folded back at the edges of the Brillouin zone due to the two-site unit cell.

The spin magnetization $\langle \sigma \rangle$ is simply the projection of the quasimomentum eigenvectors derived from Eq. (6.5) onto the rows of the lattice. We take the difference between the projections onto the upper row and the lower row such that a positive (negative) $\langle \sigma \rangle$ corresponds to population on the upper (lower) row.

\footnote{We note that because our lattice is in momentum space, the quasimomentum $\tilde{q}$ should really be called “quasiposition,” though we stay with the conventional term “quasimomentum.”}
6.6.4 Influence of interactions

As mentioned in the main text, atomic interactions show effects on the localization properties of both the 1D Aubry-André data in Fig. 6.3 and the longer-range Aubry-André data in Fig. 6.4 and Fig. 6.5. Interactions in momentum-space lattices are described in detail in Ref. [20], where we show that effects like self-trapping can be observed when the mean-field energy becomes large compared to the tunneling.

Atoms in a particular momentum state experience an added positive self energy due to repulsive cold collisions, i.e., mean-field interactions. In addition, atoms overlapped in space but occupying distinct spatial eigenstates (i.e., distinguishable plane-wave momentum states) experience both a direct interaction as well as an added exchange energy, resulting in twice as large a repulsive energy [138, 147]. For a fixed total density, this situation where atoms occupying the same momentum state have a weaker repulsive interaction energy may be recast as an effectively site-local attraction with a scale set by the mean-field interaction energy $U$. In reality, for an interacting degenerate Bose gas, superfluid screening can make distinct plane wave states partially indistinguishable, resulting in some off-site contribution to the effective attraction (although this vanishes as $U$ becomes much less than $2E_R$).

As discussed in Ref. [20], these interactions shift the Bragg resonance frequencies away from the single particle resonances. Under typical experimental conditions, we measured this shift to be $\sim 300$ Hz, relating to a peak mean-field energy $U_0 = gn_0 \approx 2\pi\hbar \times 860$ Hz at the center of the harmonic trap, and an homogeneous mean-field energy $U/\hbar \approx 2\pi \times 500$ Hz averaged over the entire trap (the measured shift is distinct from the average $U$ value due to a combination of the aforementioned screening effects and the long duration of the Bragg pulses used in this determination). The central atomic density is $n_0 \approx 10^{14}$ cm$^{-3}$ and $g = 4\pi\hbar^2a/M_{Rb}$, where $a$ is the scattering length [82].

We incorporate this mean-field energy $U$ by considering an attractive interaction that depends on the population of atoms at each site [20], resulting in the curves shown in Fig. 6.3(d) and the dotted theory curves in Fig. 6.4. Specifically, the evolved state at the end of the tunneling-ramps described in the main text are found by solving a time-dependent multi-site nonlinear Schrödinger equation that includes a local attractive self-nonlinearity $-U|\psi_n|^2$, ...
where the $\psi_n$ are $c$-numbers (with normalization $\sum_n |\psi_n|^2 = 1$) relating to the atomic field terms at each site. This approach is approximate in a number of ways: it ignores quantum fluctuations, ignores off-site contributions to the effective atomic interaction, ignores energy-dependent corrections to the collisional scattering cross-section, ignores spatial variations in the atomic density in our trapped sample, ignores effects such as the loss of spatial overlap of the momentum wavepackets, and explicitly restricts the collisions to be mode-preserving (ignoring both $s$-wave collisions that may scatter atoms out from the considered set of 21 modes into many “halos” of additional states, as well as mode-changing collisions within our defined set of states, that would be energetically suppressed in the absence of our drive fields, but may be effectively enabled through higher-order, Bragg-mediated processes). We point out that a fuller quantum treatment of the problem (still restricted to being mode-preserving, still ignoring spatial variations of the density, still ignoring loss of spatial overlap of momentum states, and still ignoring energy-dependent corrections to the scattering cross-section) would instead include an interaction term $(U/N)\sum_{n,n'} c_{n'}^\dagger c_n^\dagger c_n c_{n'}$ in the effective tight-binding Hamiltonians of the main text. Even for our modest 21-mode system, the time-dependence of this problem would become intractable for particle numbers well below the $\sim 10^5$ used in experiment.

### 6.6.5 Ramp procedure

For the localization studies of the zigzag lattice of Fig. 6.4 and 6.5, we slowly load into the extremal eigenstates (ground state or highest excited state) of the Hamiltonian we wish to explore (Eq. (6.4)). We begin, as described in the main text, by preparing with high fidelity the ground state of the system in the zero tunneling ($t = t' = 0$) limit by shifting the Aubry-André site energy distribution such that the initial site has the lowest energy ($\varphi = \pi$ for $\Delta > 0$). To ensure that there is a relatively large energy gap from this initially populated ground state to all other eigenstates even after finite tunneling is introduced, we add an effective potential well of depth $V$ at the central site.\footnote{For the most excited state, we set the initial site to have the highest energy ($\varphi = 0$) and impose a potential hill of height $V$ at the initial site $n = 0$.} Then, over the course of 2 ms, we smoothly vary the system parameters until we reach the desired Hamiltonian. If these
ramps are quasistatic adiabatic, such that the energy associated with the ramp rate is much smaller than the smallest energy gap encountered, then this procedure should prepare the desired ground state with high fidelity.

First, we describe the different ramps used in experiment. The depth of the potential well at site $n = 0$ is ramped from $V$ to zero over $2$ ms (for comparison, the NN tunneling time for this experiment was $\hbar/t = 304(4)$ µs). Over the same $2$ ms duration, we also ramp both the NN and NNN tunneling amplitudes linearly from zero to their final magnitudes $t$ and $t'$, respectively. Over the course of this ramp we preserve the flux distribution, imposed by fixed tunneling phases. We additionally preserve the ratio of $t'/t$ by ramping the field strengths of the first- and second-order Bragg spectral components ($\propto \tau$ and $\propto \sqrt{\tau}$) according to their distinct scalings with the applied field strengths (Eqs. (6.1) and (6.2)).

One complication arises from the small spacing between applied spectral components (frequency teeth), as mentioned in the “off-resonant excitations” section above and shown in Fig. 6.6(b). To implement the pseudorandom site energy shifts, we detune the frequencies away from Bragg resonances, thus moving some frequency teeth closer together and some further apart. We restrict the maximum absolute detuning applied to any one frequency tooth to be less than half the spacing between adjacent teeth, or else it would then be more closely associated with a different Bragg resonance. While it would be preferable to not work
at such extreme detunings, proportionally smaller tunneling strengths would be required to obtain the same final values of $\Delta/t$. With the presence of our localized potential well, this restriction of the maximum detuning imposes the condition $(2\Delta + V)/\hbar < 2\pi \times 4056 \text{ Hz}$, where the spacing between relevant first- and second-order Bragg transition frequencies is $4E_R/\hbar \approx 2\pi \times 8111 \text{ Hz}$. This means that there is some trade-off between the depth of our initial well and the initial disorder strength. We choose to ramp up the disorder from $\Delta_0 = 0.2\Delta_f$ to a final disorder value $\Delta_f$ (for a range $1.0 \leq \Delta_f/t \leq 4.25$), and use the condition above to dictate the initial well depth $V$.

These ramps can be summarized in a plot of the eigenstate energies as a function of time during the ramps, shown in Fig. 6.7 for loading the ground state. At time $\tau = 0$, we begin with all tunnelings turned off, such that the states are separated based on their eigenstate energies. The initial potential well of depth $V$ further separates the initially populated ground state (dashed red curve) at site $n = 0$ from the rest of the spectrum. As the ramp progresses, none of the states cross in energy, and by following the ground state (excited state) in a nearly adiabatic quasistatic fashion, we prepare a state that is close to the ground state (excited state) of the full Hamiltonian. Then, by measuring the population distribution relating to this state in the synthetic lattice, we determine whether it remains localized or becomes delocalized for various $\Delta/t$ values.

We note that the procedure for the 1D Aubry-André studies of Fig. 6.3 is similar to the one described here, but for $t' = 0$, $\phi = 0$, and $V = 0$ (no additional well/hill) throughout. Also, we look not just at the extremal states, but instead probe various eigenstates.
Chapter 7

Losers

“Lasers are usually operated as oscillators rather than amplifiers but the acronym with ‘o’ instead of ‘a’ is not so good.”

-Chris Foot, Atomic Physics, 1st ed.

In this chapter and the paper it adapts [13], we investigate with numerical simulations two ways to introduce “loss” onto individual lattice sites in our MSL system (individual “losers”), and experimentally demonstrate one of the two methods. The idea is straightforward: we can either directly remove atoms from the lattice, or keep them in the lattice, but move them to a region of the lattice from which there is no return. The first method couples our MSL to a separate spin state, to which we can apply a resonant laser field to remove atoms. The second method weakly connects a lattice site to a reservoir of strongly coupled sites, such that atoms that make it over the weak link into the reservoir are quickly shuttled deeper into the reservoir. Both methods realize “effective” loss in our MSL, and we implemented the reservoir scheme experimentally.

This work was led by Samantha Lapp (UIUC ’19, undergrad) and Jackson Ang’ong’a, both of whom ran the numerical simulations and helped plan the experimental portion. My role was to implement the simulation work in experiment, and take data in three regimes of “loss”. As such, in this chapter I will only go into more detail about the experimental portion of the work (Sec. 7.5.2), though the full text of the paper is presented below with minor changes. Finally, while I have made ultracold atoms at ∼100nK temperatures, it pales in comparison to the bitter cold of that one night that we spent working on this paper.
7.1 Introduction

Hermitian Hamiltonians, which describe closed quantum systems, feature unitary time evolution and a spectrum of real energy eigenvalues. However, real world systems are coupled to their surroundings, and must be described with non-Hermitian Hamiltonians and their complex eigenvalues, including effects from dissipation and loss. The influences of non-Hermiticity and loss on quantum systems have garnered recent interest in several areas. Researchers have sought to generalize powerful techniques such as optical pumping and dark-state cooling to a many-body context [192–194]. Additionally, there has been interest in how processes like loss and gain can influence and enrich the topological properties of lattice systems [195–204] and various types of single-particle and many-body localization phenomena in disordered systems [88, 205–208]. The ability to engineer dissipation and artificial environments in cold atom systems has offered a new window into hallmark phenomena associated with quantum electrodynamics [209, 210]. The use of correlated loss, as well as classical noise [211], has been envisioned as a way of realizing effective constraints on Hamiltonian dynamics for the purpose of stabilizing many-body phases or dynamics of interest [212], or for giving rise to unique quantum phases [213, 214]. Finally, the detailed study of correlated loss can also be used to probe particle densities or lattice filling factors [215–217] as well as magnetic ordering [218].

Some experimental challenges remain, however, when engineering non-Hermitian Hamiltonians. How does one, for example, introduce tunable loss on individual site positions within a lattice without affecting nearby sites? And how does one relate features such as engineered topology and disorder with controlled particle loss? In this work, we show that synthetic lattices present a natural platform for engineering tight-binding lattice models with controllable local loss, and demonstrate in experiment one method for achieving loss.

In a synthetic lattice [2, 3, 6, 7, 10, 77, 107, 108, 219–221], the parametric coupling of discrete quantum states of a particle mimics tunneling along an effective dimension. By additionally coupling these states to an auxiliary reservoir, loss terms can be introduced organically in synthetic systems for the purpose of studying band topology [16, 18], localization physics [14, 15], and nonlinear atomic interactions [20]. Such capabilities enabled
by synthetic lattice systems will complement powerful existing techniques for engineering controlled local loss in real-space lattices [222–226].

Here, we discuss two mechanisms by which tunable, site-dependent loss can be engineered in synthetic lattices. Both methods rely on the controlled coupling of the individual sites of a synthetic lattice system to an auxiliary set of quantum states. The first connects the auxiliary system to a lossy atomic excited state, relying on an explicit form of dissipation by spontaneous emission. In the second approach, a large reservoir of unoccupied states acts as the auxiliary system, yielding an effective form of loss without explicit dissipation. We discuss implementations in the specific context of one-dimensional (1D) synthetic lattices based on linear atomic momentum states; however, these approaches are generalizable to higher dimensions and other experimental platforms.

### 7.2 Engineered loss in synthetic lattices involving multi-level atoms

In MSL experiments to date [5, 14–16, 18–20], all atoms occupy the same hyperfine state, and pairs of properly detuned Bragg laser beams are applied to change the atoms’ linear momentum state while leaving the internal state unchanged [82, 227]. By including many pairs of Bragg lasers, each addressing a unique two-photon Bragg resonance, many discrete linear momentum states (separated by two photon momenta) can be resonantly coupled to form a synthetic lattice of momentum states. A natural way to incorporate local loss into this system would be to use momentum-selective Raman-Bragg transitions [228, 229] to change the internal state of population in specific momentum orders (lattice sites), and then remove population from the resulting internal state with resonant light. To be clear, here we are making a distinction between Bragg transitions (momentum change only, as in MSLs) and Raman-Bragg transitions (both momentum and internal state change).

#### 7.2.1 The scheme

We first focus on generating loss using such a scheme in the simplest system made of only two states coupled with Raman-Bragg laser fields. We consider the two ground state hyperfine
manifolds as they appear for alkali atoms (ground state electron angular momentum $J = S = 1/2$), with hyperfine quantum numbers $F = I \pm 1/2$ for nuclear spin $I$. We further restrict the states to have the same magnetic moment, such as the $|F, m_F\rangle = |F, 0\rangle$ clock states for bosonic isotopes. At low fields, this choice helps to avoid sensitivity of the Raman-Bragg transition to variations of the magnetic field strength. We explicitly consider a pair of $|F, m_F\rangle$ clock states $|\uparrow\rangle \equiv |2, 0\rangle$ and $|\downarrow\rangle \equiv |1, 0\rangle$, relevant for species such as $^{23}$Na, $^{39}$K, $^{41}$K, and $^{87}$Rb. As depicted in Fig. 7.1(a), we set $|\downarrow\rangle$ as the stable internal state, and $|\uparrow\rangle$ as the “lossy” internal state, from which atoms may be effectively removed from the system by applying resonant internal removal light. This removal manifests as atom loss both from the physical trap as well as from the momentum-space lattice.

In addition to these two ground hyperfine states, we also implicitly assume that $|\uparrow\rangle$ can be selectively coupled to a lossy excited state $|e\rangle$ by a one-photon optical transition. This assumption is valid for all of the alkalis, where the frequency separations of the ground hyperfine manifolds, $E_{\uparrow e} = E_{\uparrow} - E_{\downarrow}$ (of order hundreds of MHz to several GHz), greatly exceed the excited state loss rates $\Gamma_e$ (of order several MHz for the low-lying excited states accessible via $D_1$ or $D_2$ transitions). By utilizing optical cycling transitions, many photon momenta may be quickly imparted to atoms in $|\uparrow\rangle$, leading to an effective loss coefficient $\Gamma_{\uparrow}$ (of order tens to hundreds of kHz), tunable through the intensity, frequency, or stroboscopic control of the cycling light.
We consider atoms initialized in $|\downarrow\rangle$ with roughly zero momentum. By using a pair of counter-propagating Raman-Bragg fields along the $\hat{x}$-axis separated in frequency by $\Delta f = (E_{\uparrow\downarrow} + 4E_{\text{RB}}^{\text{rec}})/\hbar$ (for Planck’s constant $\hbar$) the initial state can be coherently coupled to atoms in $|\uparrow\rangle$ and moving with momentum $+2\hbar k_{\text{RB}}$ in the $+\hat{x}$ direction. Here, we assume that the higher frequency field travels in the $+\hat{x}$ direction. The recoil energy is given by $E_{\text{rec}}^{\text{RB}} = \hbar^2 k_{\text{RB}}^2/2M$, where $M$ is the mass of the atom and $k_{\text{RB}}$ is the wave vector of the Raman-Bragg lasers. We further assume that the light fields have roughly equivalent wavelengths $\lambda_{\text{RB}}$ and wavevectors $k_{\text{RB}} = 2\pi/\lambda_{\text{RB}}$, where an optimal choice of this wavelength would be at a so-called “tune-out wavelength” [230], e.g., between the D$_1$ and D$_2$ transition wavelengths for alkali atoms. Such a choice, while not completely necessary, helps to avoid unwanted light shifts from the Raman-Bragg lasers on the two chosen internal states. The change in the energy of the light field as a photon is virtually absorbed from one beam and emitted, in a stimulated fashion, into the other beam accounts for the change in energy of the atom in going from $|\downarrow\rangle_p = 0\rangle$ to $|\uparrow\rangle |p = +2\hbar k_{\text{RB}}\rangle$, i.e. $E_{\uparrow\downarrow} + 4E_{\text{rec}}^{\text{RB}}$. We let $t'$ represent the two-photon Raman-Bragg coupling strength between these two states. As we find below, momentum-selectivity of such Raman-Bragg transitions will limit $t'/\hbar$ to be of order ten kHz or less in typical experiments, where this scale is set by the frequency spacing between neighboring Raman-Bragg resonances, which in turn is directly related to the recoil energy associated with the Raman-Bragg lasers, $E_{\text{rec}}^{\text{RB}}$.

Population begins in $|\downarrow\rangle |p = 0\rangle$ with weak Raman-Bragg coupling rate, i.e. $\Gamma_{\uparrow} \gg t'$. The rapid loss due to the application of resonant light in $|\uparrow\rangle |p = 2\hbar k_{\text{RB}}\rangle$ prevents population from coherently building up in this state. In this limit, the dynamics of this system can be effectively mapped to a single-state, $|\downarrow\rangle |p = 0\rangle$, with an effective loss coefficient $\Gamma_{\downarrow,p=0} \approx (t')^2/\Gamma_{\uparrow}$ as shown in Fig. 7.1(a). The scaling of $\Gamma_{\downarrow,p=0}$ with $\Gamma_{\uparrow}$ and $t'$ reflects the quantum Zeno effect [231, 232], where enhanced dissipation actually reduces loss in a system by effectively decoupling stable and unstable subspaces. Of great importance to our stated goal of engineering designer loss in a synthetic lattice, this effective loss coefficient for $|\downarrow\rangle |p = 0\rangle$ is tunable through $t'$.

The effective, tunable loss introduced above for the single $|\downarrow\rangle$ momentum state can be extended to a large array of linear momentum states, $|\downarrow\rangle |j\rangle$, by driving a set of two-photon
Bragg transitions. We note that these transitions differ from the Raman-Bragg transitions by both preserving the internal state and by imparting different amounts of momentum. These Bragg transitions connect linear momentum states $p_j = 2j\hbar k_B$, quantized in units of the photon recoil momentum $2\hbar k_B$, where $k_B$ is the wavevector of the Bragg fields. Associated with this Bragg wavevector is the Bragg recoil energy $E^B_{\text{rec}} = \hbar^2 k_B^2 / 2M$.

Raman-Bragg transitions then couple atoms between the two spin states in a momentum-dependent fashion: for transitions from an initial state $|\downarrow\rangle_j$ with momentum $p_j = 2j\hbar k_B$ to a final state $|\uparrow\rangle_j$ with momentum $p_j + 2\hbar k_{RB}$, the resonant Raman-Bragg condition involves an energy change of $E_{\uparrow\downarrow} + (2\hbar^2 / M)(2jk_Bk_{RB} + k_{RB}^2)$. The explicit $j$-dependence of this Raman-Bragg resonance condition allows for a local, momentum- or site-dependent coupling of $|\downarrow\rangle$ atoms to the lossy state, $|\uparrow\rangle$. To ensure momentum selectivity, the rate of individual Raman-Bragg transition couplings should be less than ten kHz for typical conditions, i.e. much less than $4\sqrt{E^B_{\text{rec}} E^{RB}_{\text{rec}}}$ to avoid off-resonant driving. By simultaneously driving many Raman-Bragg transitions, many sites can independently be coupled to the “lossy” manifold, $|\uparrow\rangle$, at different rates $t_j'$. Assuming momentum-independent loss at a rate $\Gamma_{\uparrow}$ from $|\uparrow\rangle$, this allows for an effectively tunable, site-dependent loss in the synthetic lattice of momentum states, $|\downarrow\rangle_j$.

The engineering of controllable local dissipation can be combined naturally with the ability to engineer an effective “tunneling” between the $|\downarrow\rangle_j$ sites as shown in Fig. 7.1(b). As long as all of the individual Raman-Bragg coupling rates $t_j'$ are much lower than the loss coefficient $\Gamma_{\uparrow}$, each site will experience a tunable loss coefficient $\Gamma_{\downarrow,j} \approx (t_j')^2 / \Gamma_{\uparrow}$. This scheme can easily be implemented in the context of momentum-space lattices [4, 5].

### 7.2.2 Numerical simulations

We now compare the dynamics under two lossy Hamiltonians: one “full” version that features a loss term $\Gamma_{\uparrow}$ acting only on $|\uparrow\rangle$, and an “effective” Hamiltonian that includes only $|\downarrow\rangle$, but with effective, site-dependent loss rates $\Gamma_{\downarrow,j}$. The “full” model, which implements loss by
exposing $|\uparrow\rangle$ to cycling light, is given by
\begin{equation}
H_{\text{full}} = -\sum_{\sigma} \sum_{j} \left( t_{j\sigma,j+1} \hat{c}_{\sigma,j} + \text{h.c.} \right) - \sum_{j} \left( t'_{j\uparrow,j} \hat{c}_{\uparrow,j} + \text{h.c.} \right) + i\Gamma_{\uparrow} \sum_{j} \hat{c}_{\uparrow,j}^{\dagger} \hat{c}_{\uparrow,j} .
\end{equation}

Here, the $\hat{c}_{\sigma,j}^{\dagger}$ ($\hat{c}_{\sigma,j}$) terms create (annihilate) a particle in the internal state $\sigma$ ($|\downarrow\rangle$ or $|\uparrow\rangle$). Implicitly, the index $j$ relates to linear momentum states with $p = 2j\hbar k_{B}$ along the $\hat{x}$ direction for $|\downarrow\rangle$ and momentum $p = 2j\hbar k_{B} + 2\hbar k_{RB}$ for $|\uparrow\rangle$. Without loss of generality, we also assume all Bragg ($t_{j}$) and Raman-Bragg ($t'_{j}$) “tunneling” terms to be purely real.

The effective model that describes the dynamics purely in $|\downarrow\rangle$, is given by
\begin{equation}
H_{\text{eff}} = -\sum_{j} \left( t_{j} \hat{c}_{j,j+1}^{\dagger} \hat{c}_{j} + \text{h.c.} \right) + i \sum_{j} \Gamma_{j} \hat{c}_{j}^{\dagger} \hat{c}_{j} .
\end{equation}

Here, the $\Gamma_{j}$ terms are equivalent to the $\Gamma_{\downarrow,j}$ terms described above, as the description is purely in terms of $|\downarrow\rangle$ atoms.

In Fig. 7.2 we directly compare the dynamics under the full model Eq. (7.1) with those under the effective model Eq. (7.2). These should be essentially equivalent when beginning in $|\downarrow\rangle$ and for cases with $t'_{j} \ll \Gamma_{\uparrow}$ for all $j$. We note that the precise value of the ratio $t'_{j}/t_{j}$ has little impact on the agreement between the full model and effective model simulations. In our numerical simulations, we consider a situation in which an effective loss appears only at one site of a synthetic lattice [233], and investigate the dynamics that result from the case of population initially localized at the leftmost site (beginning purely in $|\downarrow\rangle$ for Eq. (7.1)).

For the full model we consider two 1D synthetic lattices each consisting of 10 sites with nearest-neighbor coupling $t/h = 0.5$ kHz as shown in Fig. 7.2(a), top. The two lattices, representing $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively, are coupled at site 4 via the Raman-Bragg coupling scheme where $t'_{4}/h = 2$ kHz. Additionally, global uniform loss is included in $|\uparrow\rangle$ with strength $\Gamma_{\uparrow}/h = 10$ kHz.

Population is initially localized on site $j = 0$ and then allowed to evolve. As shown in Fig. 7.2(a), middle, population initially coherently transfers out of site 0 but is partially reflected at site 4. The transmitted fraction of the population continues until it encounters the edge of the lattice at site 9 and reflects back. Unlike reflection at site 9, every time
Figure 7.2: **Partial reflection from a lossy “barrier” in a synthetic lattice.** (a) A lossy barrier implemented by coupling to an auxiliary set of lossy states (“full” model). Top, depiction of the model, with red arrows showing loss out of $|\uparrow\rangle$. Both 1D lattices ($|\uparrow\rangle$ and $|\downarrow\rangle$) have uniform tunneling $t/h = 0.5$ kHz, and the lossy barrier at site $j = 4$ connects the two spin states with tunneling $t'/h = 2$ kHz. The $|\uparrow\rangle$ sites feature a global uniform loss of $\Gamma_{\uparrow}/h = 10$ kHz. Middle, numerical simulation of the evolution of population initialized at site $j = 0$ on the $|\downarrow\rangle$ lattice. Bottom, total population in $|\downarrow\rangle$ (solid blue curve) and $|\uparrow\rangle$ (dashed red curve). Jumps in population coincide with population hitting the lossy barrier at site 4. (b) The effective model of a lossy barrier with a direct loss term. Top, depiction of the effective model, with red arrow showing local loss at site 4, $\Gamma_{\downarrow,4}/h = 0.4$ kHz. Tunneling rates are again $t/h = 0.5$ kHz. Middle, numerical simulation of the evolution of population initialized at site $j = 0$ on the 1D lattice. Bottom, total population in the 1D lattice as a function of time. Jumps in population again coincide with population hitting the lossy barrier at site 4.

Population reflects from site 4, the total population in $|\downarrow\rangle$ drastically decreases leading to a step-like profile as shown by the solid blue curve in Fig. 7.2(a), bottom. This population reduction is due to the fact that reflection at site 4 is also accompanied by transfer of population to $|\uparrow\rangle$. The transferred population is, however, quickly lost from the system due to the strong global loss in $|\uparrow\rangle$. The total population in $|\uparrow\rangle$ therefore briefly builds up whenever there is reflection at site 4 as shown by the dashed red curve. To note, although the influence of Bragg coupling between the $|\uparrow\rangle$ momentum orders is included in the full model
simulation, it has essentially no impact on the dynamics as the loss rate $\Gamma_\uparrow$ is momentum-independent and much larger than all $t'_j$ terms.

In the regime where $t'_j \ll \Gamma_\uparrow$ we expect the full model to map onto an effective model with an effective local loss at site 4 given by $\Gamma_\downarrow,4 \approx (t'_4)^2/\Gamma_\uparrow$. For this effective model we consider a 1D lattice consisting of 10 sites as shown in Fig. 7.2(b), top. The nearest-neighbor tunneling is given by $t/h = 0.5$ kHz while the effective local loss is given by $\Gamma_\downarrow,4/h = 0.4$ kHz. Population is initially localized at site 0 and is then allowed to evolve as shown in Fig. 7.2(b), middle. The dynamics agree nearly identically with the full model, showing parts of the population reflecting from and transmitting through the lossy “barrier” at site 4. As shown in Fig. 7.2(b), bottom the total population in the lattice drastically decays whenever population is reflected at site 4, leading to a step-like profile akin to the case of the full model. In the $t'_j \ll \Gamma_\uparrow$ limit, we have found good agreement between the full-model and effective-model simulations of Fig. 7.2, confirming the protocol for implementing local loss through coupling to an auxiliary, lossy set of states.

We note that while we have only described how dissipation may be engineered into 1D synthetic lattices with nearest-neighbor tunneling terms, this scheme naturally extends to situations with longer-range hopping terms [15] or higher-dimensional lattices [4, 19].

7.3 “Loss” without dissipation: reversible coupling to a large reservoir of states

In the previous scenario, we invoked a natural form of dissipation from atomic physics experiments - spontaneous emission - to create a controlled, effective loss in a synthetic lattice. This scheme involved two key elements: first, we assumed that the states in the “lossy” subspace ($|\uparrow\rangle_j$) could be strongly coupled to a near continuum of states (many different final momentum values after multiple absorption-spontaneous emission cycling events), such that the probability of returning to the initial state was essentially zero. Second, we assumed that the coupling rate between the stable and lossy subspaces was much smaller than the spontaneous emission loss rate from the lossy states. This assumption ensured that popu-
lation would not coherently build up in the lossy subspace, but would instead be lost from the system.

We note that the above description does not involve any explicit particle loss, and could be fully captured by a description involving a continuum of momentum states and the two internal states, $|\downarrow\rangle$ and $|\uparrow\rangle$. However, true dissipation does in fact enter through the loss of phase coherence during spontaneous emission (considering the information loss to the emitted light fields to be irreversible). In Fig. 7.3 we investigate whether genuine dissipation is actually necessary to engineer an effective form of loss in synthetic lattices, or whether weak coupling to a large, empty reservoir of states is sufficient [234]. We find evidence for the latter, at least in terms of providing an effective dissipation over some timescale set by the size of the engineered reservoir. We show, both in simulation and in experiment, how a tunable and local effective loss can be engineered into synthetic lattices even without any true form of dissipation.

For simplicity, we consider the case of a two-site synthetic lattice, a double well with sites $|L\rangle$ and $|R\rangle$ coherently coupled with an inter-well tunneling rate $t_{sys}$. By forming this double well from two sites of a larger 1D array, the left and right wells may be coupled, separately, to auxiliary sets of lattice sites which can form the large reservoir of initially unoccupied states. Here, we restrict ourselves to the scenario in which only the right well experiences coupling to additional states at a rate $t_{link}$. While there is only a single link from $|R\rangle$ to the left boundary of the reservoir, we can create a situation in which $|R\rangle$ is effectively irreversibly (on some timescale) coupled to a near continuum of states. We consider that the reservoir consists of $N \gg 1$ sites, with a large nearest-neighbor coupling rate $t_{res}$ that is greater than $t_{link}$. Considering only the reservoir, it will feature a band of delocalized eigenstates with a small energy spacing $\sim 4t_{sys}/N$. For sufficiently large $t_{link}$, the right well simultaneously couples to many unoccupied states of the reservoir, whose time-dependent superposition represents a wavepacket that propagates away from the interface and into the reservoir. For a sufficiently large reservoir, the revival time of this superposition state, relating to the time it takes to reflect from the right end of the reservoir and return to the system-reservoir interface, can be longer than the time of relevant system dynamics.

These dynamics can be viewed purely in terms of the effective loss induced at $|R\rangle$, with
Figure 7.3: Loss without dissipation. (a) Implementation of a double well with local loss using a reservoir of unoccupied states. A double well with tunneling $t_{\text{sys}}$ is coupled via $t_{\text{link}}$ to a reservoir of states ($N = 29$ states in experiment) with tunneling $t_{\text{res}}$, mimicking an effective loss rate $\Gamma_{\text{eff}} \sim t_{\text{link}}^2/t_{\text{res}}$. (b) Sketch of experimental implementation of (a). Atomic momentum states of the Bose–Einstein condensate (BEC) are connected via two-photon Bragg transitions. These are addressed by the left laser beam (frequency $f_0$) and frequency components of the right laser beam, $f_{\text{sys}}$, $f_{\text{link}}$, and $f_{\text{res}}$, corresponding to tunnelings $t_{\text{sys}}$, $t_{\text{link}}$, and $t_{\text{res}}$, respectively. (c) Population dynamics in |L⟩ (filled blue circles) and |R⟩ (open red circles) under small effective loss $\Gamma_{\text{eff}}/t_{\text{sys}} = 0.16$. Solid and dashed curves are simulations of the exact two-site model with loss and of the reservoir scheme, respectively, and the dotted black curve shows exponential decay corresponding to $\Gamma_{\text{eff}}/t_{\text{sys}} = 0.16$. Simulation curves are plotted with tunnelings ($t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 0.4, 1) \times 888$ Hz. (d) Population dynamics under medium effective loss $\Gamma_{\text{eff}}/t_{\text{sys}} = 1$. Simulation curves are plotted with tunnelings ($t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 1.25, 1.56) \times 612$ Hz. (e) Population dynamics under large effective loss $\Gamma_{\text{eff}}/t_{\text{sys}} = 4$. Simulation curves are plotted with tunnelings ($t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 4, 4) \times 179$ Hz. All error bars denote one standard error of the mean.

Associated loss coefficient $\Gamma_{\text{eff}} = t_{\text{link}}^2/t_{\text{res}}$. Figure 7.3(a) depicts this mapping between the full reservoir system and the effective model of a double-well system with tunable loss introduced at one well. We validate this picture of effective loss by comparing numerical simulations of both the full dynamics (with tunneling coefficients $t_{\text{sys}}, t_{\text{link}},$ and $t_{\text{res}}$) and the effective dynamics (with equivalent inter-well tunneling coefficient $t_{\text{sys}}$ and effective right well loss rate $\Gamma_{\text{eff}} = t_{\text{link}}^2/t_{\text{res}}$).

Moreover, we experimentally validate this protocol by realizing a tunable effective loss from one well of a synthetic double well of momentum states [4, 5]. Figure 7.3(b) shows the setup, which, as usual, begins with a BEC of $\sim 10^5$ atoms held in a single beam (wavelength $\lambda_0 = 1064$ nm and frequency $f_0 = c/\lambda_0$), which doubles as the Bragg lattice beam...
\( (k_B = 2\pi/\lambda_0) \). We write onto the lattice beam the appropriate frequencies to create the corresponding tunnelings \( t_{\text{sys}}, t_{\text{link}}, \) and \( t_{\text{res}} \). In both experiment and simulation, we initialize all population in the left well, and monitor the population of both wells over time. We use a reservoir of \( N = 29 \) lattice sites, sufficiently large such that no population returns to the system from the reservoir. As shown in Fig. 7.3(c-e), we investigate three regimes: small effective loss \( (\Gamma_{\text{eff}}/t_{\text{sys}} = 0.16) \), intermediate effective loss \( (\Gamma_{\text{eff}}/t_{\text{sys}} = 1) \), and large effective loss \( (\Gamma_{\text{eff}}/t_{\text{sys}} = 4) \). We overlay the experimental data with results from numerical simulations of the full system including the reservoir (solid curves) and the effective double well system (dashed curves).

We obtain the experimental tunneling rates by fitting to the data an exact simulation of the momentum-space lattice experiment that accounts for possible off-resonant effects due to the experimental implementation. This fit procedure finds the appropriate tunneling rates by varying only one free parameter: an overall scaling of the three tunnelings, giving \( (t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 0.4, 1) \times 888 \) Hz for the small loss regime, \( (t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 1.25, 1.56) \times 612 \) Hz for the intermediate loss regime, and \( (t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 4, 4) \times 179 \) Hz for the large loss regime. We then use these fitted tunneling rates to generate both displayed simulation curves of the reservoir model and of the effective loss system it mimics. We note that these tunneling rates roughly match those independently measured through simpler two-site Rabi oscillations: \( t_{\text{sys}}/\hbar = \{976(6), 773(5), 274(1)\} \) Hz for the small, intermediate, and large loss data, respectively.

Under small effective loss (Fig. 7.3(c)), the tunnelling between the wells \( (t_{\text{sys}}) \) is larger than the transfer out of the system \( (t_{\text{link}}) \), such that population transfer between the wells dominates over population transfer into the reservoir. A small fraction of the population is, therefore, lost from the system every full period of oscillation, leading to oscillations that are damped over time. To highlight this damping, we show an exponential decay curve \( e^{-\Gamma_{\text{eff}}t} \) (dotted black curve) based on the expected loss rate, \( \Gamma_{\text{eff}}/t_{\text{sys}} \approx t_{\text{link}}^2/t_{\text{res}}t_{\text{sys}} = 0.16 \). Because population spends roughly equal time in the left and right wells, the resulting envelope should decay at one-half the loss rate. However, because we are plotting populations rather than amplitudes, this loss rate should be doubled, resulting in an overall loss rate of \( \Gamma_{\text{eff}} \). In the intermediate regime shown in Fig. 7.3(d), \( t_{\text{link}} \) is slightly larger than \( t_{\text{sys}} \). Population
briefly builds up in the right well before rapidly tunneling out of the system into the reservoir. Under large loss (Fig. 7.3(e)), we observe that population transfer out of the left well is vastly reduced compared to the intermediate case. After 5 tunneling times \( (5\hbar/t_{\text{sys}}) \), ~60\% of the population still remains in the left well for the large loss case, in contrast to the intermediate loss case where population is entirely within the reservoir. We attribute this difference to the quantum Zeno effect, where strong coupling to a lossy environment actually limits population decay. We also observe that under strong loss, negligible population builds up in the right well as the inter-well coupling is much smaller than the coupling to the reservoir. We note that under our chosen tunneling strengths \( (t_{\text{sys}}, t_{\text{link}}, t_{\text{res}})/\hbar = (1, 4, 4) \times 179 \text{ Hz} \), this scenario is equivalent to a single site with strong loss out of the system.

For all regimes, the simulation of the effective model matches closely with the exact simulation considering an effective loss of the form \( \Gamma_{\text{eff}} = t_{\text{link}}^2/t_{\text{res}} \). Furthermore, the data also shows close agreement with both simulations, with small discrepancies arising due to off-resonant effects in our implementation of the momentum-space lattice [5]. We have shown in both experiment and simulation that effective loss can be easily implemented in a non-dissipative system by coupling a small subset of states, representing the system, to a reservoir consisting of the rest of the states. We further confirm the validity of this scheme by demonstrating the quantum Zeno effect where tunneling out of the left well is reduced for strong loss rates.

Here, for the case of engineering “loss” through reversible coupling to a large reservoir of states, we have only explicitly discussed the scenario in which tunable loss appears at the boundaries of a 1D system (specifically, a two-site double well) embedded within a larger 1D lattice. By simple extension to 2D synthetic lattices [4, 19], this approach can also allow for the inclusion of tunable loss at every site of a 1D synthetic lattice. Generally speaking, extensions to higher dimensions are also possible by embedding the system of choice in an even higher-dimensional system.
7.4 Conclusion

Synthetic lattices allow for engineering Hamiltonians with spectroscopic precision, and have proven to be a versatile platform for exploring the physics of topological and disordered systems. Here, we have introduced the idea of engineering locally-controlled dissipation in these types of systems, discussing two experimental approaches to introducing site-tunable dissipation. We have experimentally demonstrated one of the approaches, based on introducing an effective “loss” by coupling individual sites to a large, empty reservoir of states, and found good agreement between experiment and the expected non-Hermitian Hamiltonian dynamics.

While there are a number of interesting avenues of research related to combining engineered dissipation with engineered topology or disorder, much of the most compelling directions of study relate to the role of loss in interacting quantum systems. Strong nonlinearities arise naturally in atomic gases, making them a promising system in which to explore such physics (as compared to photonic systems, in which loss is straightforward to engineer but strong interactions are harder to come by). We note that the non-Hermitian Hamiltonian of Eq. 7.2 is not suitable for describing the general behavior of an interacting atomic system. Because atom-atom interactions can give rise to correlations between particles in different momentum orders, the selective loss of atoms to an environment will result in decoherence. Such effects would not be captured by the described non-Hermitian Hamiltonian, but should rather be described through a master equation approach.

However, as previous experiments on synthetic lattices of momentum states [15, 20] have taken place in a regime in which a mean-field description of the interactions does a suitable job of describing the observed dynamics (as the population per momentum state is typically much greater than one), the effective non-Hermitian processes we’ve described could be incorporated into a nonlinear Schrödinger equation with interaction terms to describe the interplay of interactions and loss at the mean-field level.
7.5 Additional comments

7.5.1 Transition rate into a tight-binding reservoir

In this section, contributed by Jackson, we use Fermi’s golden rule to analytically calculate the loss rate for the lattice-reservoir scheme. We consider the simplest case, in which the “system” consists of just one lattice site, and the reservoir consists of a 1D tight-binding lattice with \( N \) sites. This is the same as Fig. 7.3 (a), but without site \(| L \rangle\). The system is weakly coupled to the first site in the reservoir (\( t_{\text{link}} \)) while the sites in the reservoir are strongly coupled (\( t_{\text{res}} \)), such that \( t_{\text{res}} \gg t_{\text{link}} \).

To calculate the transition rate from the system into the reservoir, we use Fermi’s golden rule,

\[
\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \psi_f | V' | \psi_i \rangle|^2 \rho(E_f) \text{sinc}^2 \left( \frac{E_f}{\tau_\pi} \right),
\]

(7.3)

where \( \langle \psi_f | V' | \psi_i \rangle \) is the matrix element between an initial state, \(| \psi_i \rangle\), and a final state, \(| \psi_f \rangle\). The density of states of the reservoir at the final state energy, \( E_f \), is \( \rho(E_f) \). The initial state is assumed to be at zero energy. To account for the fact that the initial state can be off-resonantly coupled to states in the reservoir, to within some energy bandwidth set by the coupling strength \( t_{\text{link}} \), we also include a sinc-shaped response function. This specific choice is motivated by the fact that the coupling to the reservoir is assumed to turn on to a constant value in a step-wise fashion. We effectively fix the energy bandwidth to be of order the link tunneling bandwidth by selecting a relevant timescale \( \tau_\pi \) such that \( \tau_\pi t_{\text{link}}/\hbar = \pi \).

The dispersion relation of the tight-binding reservoir is \( E(k) = 2t_{\text{res}} \cos(ka) \), where \( t_{\text{res}} \) is the tunneling strength in the lattice, \( a \) is the lattice constant, and \( k = \frac{2\pi m}{(N+1)a} \), where \( m \) labels the eigenvalues. The eigenstates are expressed as linear combinations of single-mode states (here, we use \( s \) to denote atomic \( s \)-orbitals, however these represent generic single-mode states)

\[
|\psi_k \rangle = \sqrt{\frac{2}{N+1}} \sum_j \sin(kja) |\psi_s^j \rangle.
\]

(7.4)

Here, \( |\psi_s^j(x) \rangle = |\psi_s(x - ja) \rangle \) corresponds to an atomic wavefunction localized at site \( j \). The initial state we take for Eq. (7.3) is a site labeled 0, \(| \psi^0 \rangle\), considered to be external to
the reservoir, which however can be directly coupled to the first site of the reservoir with a tunneling coefficient $t_{\text{link}}$. The final state we will take to be an eigenstate of the tight-binding reservoir, $|\psi_k\rangle$. In general, the overlap of the energy eigenstates of the reservoir with a particular reservoir site $j$ is given by $\langle \psi_j^s | \psi_k \rangle = \sqrt{\frac{2}{N+1}} \sin(ka_j)$. The coupling between the system and the first site in the reservoir is $\langle \psi_1^s | V^\prime | \psi_0 \rangle = t_{\text{link}}$. The transition matrix element is, therefore,

$$\left| \langle \psi_k | V^\prime | \psi_0 \rangle \right|^2 = \left| \langle \psi_k | \psi_1^s \rangle \langle \psi_1^s | V^\prime | \psi_0 \rangle \right|^2$$

$$= \frac{2}{N+1} \sin^2(ka) \times t_{\text{link}}^2.$$  \hspace{1cm} (7.5)

The density of states for the reservoir (1D lattice) is given by

$$\rho(E) = \frac{\partial N}{\partial E(k)} = \frac{\partial N}{\partial k} \frac{\partial E(k)}{\partial k}$$

$$= \frac{(N + 1)a}{\pi} \left| \frac{\partial E(k)}{\partial k} \right|^{-1}$$

$$= \frac{(N + 1)}{2\pi t_{\text{res}}} \frac{1}{\sqrt{1 - \cos^2(ka)}}.$$  \hspace{1cm} (7.7)

where $\partial N/\partial k = 2 \times (N + 1)/(2\pi/a)$, where the factor of 2 accounts for the degeneracy of the eigenstates at $k$ and $-k$. Since we consider a scalar bosonic particle, no spin degeneracies are taken into account. Putting this together, Fermi’s golden rule (Eq. (7.3)) gives the loss rate of population associated with a final state $|\psi_f\rangle$ in the reservoir as

$$\Gamma_{i \to f} = \frac{2\pi}{\hbar} \frac{t_{\text{link}}^2}{(N + 1)} \frac{2(N + 1)}{2\pi t_{\text{res}} \sqrt{1 - \cos^2(ka)}} \sin^2 \left( \pi E_f / t_{\text{link}} \right)$$

$$= \frac{1}{\hbar} \frac{2t_{\text{link}}^2}{t_{\text{res}}} \sin(ka) \sin^2 \left( \pi E_f / t_{\text{link}} \right).$$  \hspace{1cm} (7.10)

To obtain the total loss rate into the reservoir, we sum the loss rates associated with all possible final states. Thus, taking a continuum limit, we perform an integral over all final
states as

\[
\Gamma_{\text{total}} = \frac{1}{\hbar} \frac{2 t_{\text{link}}^2}{t_{\text{res}}} \times \frac{1}{\pi t_{\text{link}}} \int_{-2 t_{\text{res}}}^{2 t_{\text{res}}} dE_f \sqrt{1 - \left(\frac{E_f}{2 t_{\text{res}}}\right)^2} \sin^2 \left(\pi \frac{E_f}{t_{\text{link}}}\right) \\
= \frac{1}{\hbar} \frac{2 t_{\text{link}}^2}{t_{\text{res}}} \times F \left(\frac{4 \pi t_{\text{res}}}{t_{\text{link}}}\right).
\]

We evaluate the function \( F(\alpha) \) for \( \alpha = 4 \pi t_{\text{res}}/t_{\text{link}} \) and obtain an analytical form

\[
F(\alpha) = J_1(\alpha) \left(\frac{\pi}{2} \alpha H_0(\alpha) - 1\right) + \frac{1}{\alpha} J_0(\alpha) \left(1 + \alpha^2 - \frac{\pi \alpha^2}{2} H_1(\alpha)\right),
\]

where \( H_n(\alpha) \) and \( J_n(\alpha) \) are the Struve function and the Bessel function of the \( n \)th order, respectively. For smaller values of \( \frac{t_{\text{link}}}{t_{\text{res}}} \) (larger values of \( \alpha \)), \( F(\alpha) \) approaches 1. In the limit \( t_{\text{res}} \gg t_{\text{link}} \), the transition rate from the system into the reservoir is, therefore, well approximated by \( \Gamma_{\text{total}} = \frac{1}{\hbar} \frac{2 t_{\text{link}}^2}{t_{\text{res}}} \).

### 7.5.2 Additional experimental details

First, I want to note that when we took the data for this work (October 2018), the \(^{87}\text{Rb}\) apparatus was already on its last legs. Each day, we were driving the applied current to the atomic dispenser sources higher and higher each day, trying to overcome a leaky vacuum. Yet only two weeks after this data was taken, we took our final data on the apparatus and called it quits. Thus while I wrote in the notebook that the atom number was surprisingly stable when we took this data, I cannot help but think that there were some huge number fluctuations coming from the erratic source.

This could offer some explanation for the tunneling mismatch that we see in the data of Fig. 7.3(c-e): the tunneling rate measured from calibration runs is higher than the tunneling rate calculated from a fit to the observed dynamics. Specifically, our calibrated tunnelings were measured to be \( t_{\text{sys}}/\hbar = \{976(6), 773(5), 274(1)\} \) Hz while the fitted tunnelings were \( t_{\text{sys}}/\hbar = \{888, 612, 179\} \) Hz, for the small (c), intermediate (d), and large loss regimes (e), respectively. If there was a sudden shift in the atom number between a calibration run and the data run, this could offer some explanation for this discrepancy.
However, more likely is the effects of off-resonant frequency teeth. In two of the three loss regimes (small and large loss), one of the tunneling links was relatively weak \((t/h < 400 \text{ Hz})\). In most of the MSL experiments to date, we have worked with tunneling rates of \(t/h \approx 600\sim 1200 \text{ Hz}\), which is well below the spacing between adjacent teeth. In contrast, here we are faced with a challenge: not only do we have one “weak” link below what we normally work with, but we also have many “strong” tunneling links that make up the reservoir. Thus the off-resonant effects from the strong tunneling links begin to compete with the weak, yet resonant link. If the relevant link was made too weak, we began to observe tunneling into the wrong side of the lattice, where there were no links (as in, to the left of site \(|L\rangle\) in Fig. 7.3(a)).

The proper solution to such a problem would be to rigorously go through and calculate all of the light shifts/contributions from the off-resonant frequencies, but unfortunately we never did so (though a separate group later did a thorough job of it \([45, 46]\)). Instead, we took informed guesses to solve this problem.

From our disorder work of Chap. 4, we knew that the effects from off-resonant links could add up constructively to give jagged behavior in the dynamics. In that work, we found that by placing random, static tunneling phases on all of the links, we were able to diminish much of those jagged dynamics by disrupting this constructive interference (Fig. 4.2(b,f)). Following that same reasoning, in this work we placed onto all of the links staggered tunneling phases \((0, \pi, \ldots)\). Next, we mirrored the tunneling links of the reservoir on the opposite side of the system (included links to the left of \(|L\rangle\) in Fig. 7.3(a)). The reasoning was that we had never seen such strong off-resonant population transfer when population began in the center of the lattice, so we believed that the shifts from a bunch of strong tunneling links to one side of the system could be counteracted by placing the same strong tunneling links on the other side. This worked. Finally, we picked the tunneling strengths and their ratios to be “just right”, where we were able to observe the expected loss behavior, but where the off-resonant effects didn’t completely wash out the dynamics. Putting all three of these together, we were able to probe loss in all three regimes, but with a hit to the tunneling amplitude.
Chapter 8

The Generalized Aubry André Model

In this chapter and the related paper [17], we realize the “generalized Aubry-André” (GAA) model of 1D pseudodisorder, as proposed and detailed by Ref. [28]. This study builds off of our previous work on the normal Aubry-André model in Chaps. 4 and 6 and the Aubry-André model with next-nearest-neighbor tunneling in Chap. 6. While the normal Aubry-André model has an energy-independent delocalization-localization transition, the generalized version exhibits a single-particle mobility edge (SPME) where eigenstates localize depending on their energy. We implement the GAA model experimentally on the MSL system, and probe the localization properties of its lowest and highest energy eigenstates, observing the presence of the SPME. We also examine the role of atomic interactions, which serves to shift the mobility edge.

This work got local (UIUC) theory support from Karmela Padavić-Callaghan and Suraj Hegde, and their advisor Smitha Vishveshwara. In addition, we received input from two authors of the original GAA paper, Jed Pixley and Sriram Ganeshan. I ran the experimental portion of the work with help from Eric, and Karmela led the simulation side with some of my input.

8.1 Background

8.1.1 Anderson localization and the Aubry-André model

As described in the background sections of Chaps. 4 and 6, Anderson localization occurs for 1D systems under any amount of random disorder. Such localization is a perfect subject for study using cold atoms, since electronic systems feature electron-electron and electron-phonon interactions which prevent direct observation. Some of the first observations of Anderson localization with cold atoms [92] as well as light [235] have used a correlated...
form of pseudodisorder, the Aubry-André (AA) model [113, 236–238]. The AA model is a straightforward choice for cold atoms, as it is relatively simple to implement using real-space optical lattices - by overlaying two lattices with incommensurate periodicities - yet it features great depth in localization and topological physics.

To recap, the diagonal Aubry-André model features sinusoidal site energies on a 1D lattice with the usual tight-binding Hamiltonian

$$H = -J \sum_n (c_{n+1}^\dagger c_n + \text{h.c.}) + \sum_n \varepsilon_n c_n^\dagger c_n$$  \hspace{1cm} (8.1)$$

with site energies

$$\varepsilon_n = \Delta \cos (2\pi bn + \phi),$$  \hspace{1cm} (8.2)$$

where the periodicity $b$ is chosen to be incommensurate with the underlying lattice. While this generally doesn’t affect the localization physics, we choose the golden ratio conjugate $b = (\sqrt{5} - 1)/2$ to stay consistent with our previous works [14, 15] and other cold atom studies [92]. As explained in previous chapters, this model features an energy-independent transition from delocalization at low disorder/potential amplitude $\Delta$ to localization at high disorder, occurring at the same critical disorder strength $\Delta/J = 2$ for all eigenstates. This stems from the fine-tuning of the cosine nearest-neighbor band dispersion as well as the cosine-distributed site energies, as shown in the $\alpha = 0$ panel of Fig. 8.1(a). The energy-independent nature of the transition stands in contrast with not only generic Anderson localization for disordered systems having dimension $d \leq 2$, but also more general forms of correlated pseudodisorder that support mobility edges [239, 240].

By disrupting the fine-tuned condition of the AA model, whether by modifying band dispersion or tweaking the form of the potential (Eq. (8.2)), this localization transition can be made to be energy-dependent. That is, for a given disorder strength, there is a certain energy called the single-particle mobility edge (SPME) that separates localized eigenstates and delocalized eigenstates. Such a SPME has been demonstrated in two separate experiments from 2018: our earlier work with the zigzag lattice (Chap. 6, Ref. [15]), and the Bloch group with bichromatic lattices [187]. Both works “broke” the energy independence of the
transition by introducing tunneling terms beyond nearest neighbor. Our work [15] directly engineered next-nearest-neighbor tunneling terms with control that was mostly independent from the nearest-neighbor terms, generating a mobility edge like in previous theory [179, 241, 242]. We also introduced a synthetic gauge field which allowed us to tune the position of the mobility edge. On the other hand, it was long known that bichromatic lattice realizations of the AA model naturally feature tunneling terms beyond nearest-neighbor which could give rise to a SPME [188, 242, 243], and this was shown by the Bloch group in experiment [187].

8.1.2 The generalized Aubry-André model

In this work, we engineer a SPME not by changing the tunneling terms, but by deforming the cosine modulation of the site energies. While this can be done in many ways, here we specifically realize the “generalized Aubry-André” (GAA) model introduced in Ref. [28], a simple model which allows for analytical results on mobility edges. The GAA model adds higher harmonics to the cosine AA site energies, and can be seen as dual to a model with algebraically-decaying hopping terms beyond nearest neighbor [244]. While this “generalized Aubry-André model” would be nearly impossible to realize in real space lattice experiments, approximations to the infinite sum of harmonics were suggested, requiring just a few sets of interfering lasers. Here, we implement the exact GAA model, taking advantage of the full spectroscopic control that our MSL technique offers over individual lattice site energies.

Again, we study a 1D lattice obeying the tight-binding Hamiltonian of Eq. (8.1) with tunnelings $J$ (real, positive) and GAA site energies

$$
\varepsilon_n = \Delta \frac{\cos(2\pi nb + \phi)}{1 - \alpha \cos(2\pi nb + \phi)}, \quad (8.3)
$$

where like for AA we choose $b = (\sqrt{5} - 1)/2$, and the parameter $\alpha \in (-1, 1)$ controls the shape of the potential and resulting distribution of site energies, as shown in Fig. 8.1(a). At $\alpha = 0$, Eq. (8.3) reduces to the normal AA form of Eq. (8.2), again displaying both a cosine dispersion and a cosine distribution of site energies, leading to the energy-independent transition. For more negative values of $\alpha$, the potential shows deeper, narrower dips con-
Figure 8.1: The generalized Aubry-André model. (a) The generalized Aubry-André potential and lattice site energies of Eq. (8.3) shown for tuning parameter values $\alpha = -0.5, 0, 0.5$, with corresponding distributions of lattice site energies. (b) Calculated eigenenergies and participation ratios (PRs) vs. $\alpha$ for a non-interacting model just below the critical disorder strength at $\Delta/J = 1.8$ ($N = 51$ sites). Away from $\alpha = 0$, eigenstates localize at different energies, forming a mobility edge. Dashed black lines show analytically predicted energy values of the SPME following Eq. (8.4).

taining fewer lattice sites and broader peaks with more sites, leading to a distribution of site energies peaked at high energies. For positive $\alpha > 0$, the opposite is true: sharp peaks with broader, shallower dips result in a distribution of site energies skewed towards low energies. As we explain in the next section, this causes the transition to become energy-dependent, so that a SPME separates localized eigenstates from delocalized ones with an energy value that follows the simple analytical relation [28]

$$\alpha E = \text{sgn}(\Delta) \left( |2J| - |\Delta| \right), \quad (8.4)$$

where since we work with positive values only ($\Delta > 0, J > 0$), this simplifies to $\alpha E = 2J - \Delta$.

Here we note a few small differences between our forms of Eqs. (8.3) and (8.4) and those presented in the original theory paper of Ref. [28]. While we set $\Delta > 0$ and $J > 0$, the original theory paper uses negative values of $\Delta_{\text{theory}}$, leading to the form of Eq. (8.4) and some mirroring between our figures. Also, the amplitude of the GAA pseudodisorder that we use is half of the value used in the original theory paper, such that the transition for the regular AA model occurs at $\Delta/J = 2$ and theirs occurs at $\Delta_{\text{theory}}/J = 1$. This slightly changes both Eqs. (8.3) and (8.4). Finally, while we choose $b = (\sqrt{5} - 1)/2$, Ref. [28] uses the inverse of this value, $b_{\text{theory}} = 1/b$. While this does have some effect, especially considering the small system sizes in experiment ($N = 21$ sites), it does not significantly affect the results.
we present here, as the important point is that $b$ is irrational. I also want to make a small note here that both choices of $b$ lead to similar-looking energy spectra split into three bands.

### 8.1.3 Localization properties of the GAA model

To probe the SPME of the GAA model, we determine the localization properties of its eigenstates, and quantify localization with the participation ratio (PR),

$$\text{PR} = \frac{1}{\sum_n P_n^2},$$  \hspace{1cm} (8.5)

where $P_n = |\psi_n|^2$ is the measured (normalized) atomic population in lattice site $n$, where $\psi_n = \langle n | \psi \rangle$ for atomic wavefunction $|\psi\rangle$. For a system of size $N$, this PR observable takes on values ranging from $N$ for fully extended states to 1 for fully localized states. This is simple to see: a delocalized state spread equally over all sites $|\psi_{\text{deloc}}\rangle = \sum_n |n\rangle \sqrt{N}$ when plugged into Eq. (8.5) gives an PR of $N$, and a state fully localized to a single lattice site $|\psi_{\text{loc}}\rangle = |0\rangle$ gives an PR of 1. For the results in this work, we report the PR normalized to the system size $N$, in order to account for the difference in system size between experiment ($N = 21$ sites) and some numerical simulations where larger system sizes allow for better visualization of the physics (though $N = 21$ for simulations directly comparing to data).

Figure 8.1(b) plots all of the eigenstates of the GAA model for a disorder amplitude $\Delta/J = 1.8$, just below the localization transition for normal AA (calculated for 51 sites). The colors represent $\text{PR}/N$, such that red denotes a localized state (small PR) and blue denotes a delocalized state (large PR). For $\alpha = 0$, all of the eigenstates are delocalized as expected of the energy-independent transition.\footnote{Looking closely, there are a few states between the three main bands that remain localized for all $\alpha$. These correspond to topological “edge states” since the AA model is a dimensionally-reduced version of the Harper-Hofstadter Hamiltonian described in Chap. 3. We hope to take advantage of this and measure these edge states in future work, described in Sec. 10.2.} For $\alpha \neq 0$, there is a clear difference between localized and delocalized states, separated by the mobility edge of Eq. (8.4) shown as a dashed black line. By changing $\alpha$, we can tune and invert the mobility edge, selecting whether low or high energy eigenstates localize at this disorder.

To gain some insight into why this occurs, we again look at the distribution of site
energies for different $\alpha$ in Fig. 8.1(a), first focusing on the $\alpha < 0$ case. As we described earlier, the shape of the GAA potential under negative $\alpha$ results in a site energy distribution skewed towards high energies. Adding tunneling to this results in the same observations for the eigenstates (albeit split into three bands under our choice of $b$): under negative $\alpha$, there are more eigenstates near high energies than there are near low energies. So under the GAA pseudodisorder, an eigenstate at high energy has many lattice sites (and states) close in energy that it may overlap with, making it harder to localize. The opposite is true for low energy eigenstates: because there are fewer states/sites around, there is a tendency towards localization. Thus the eigenstates behave differently under disorder, opening up a SPME. The localization behavior is opposite under $\alpha > 0$: there are more eigenstates at low energies compared to high energies, promoting localization for high energy states and delocalization for low energy states. Thus the parameter $\alpha$ not only tunes the shape of the potential, but also the resulting position of the SPME.

In this study, we realize the GAA model experimentally using the momentum-space lattice technique. To probe the SPME, we adiabatically prepare the ground state and highest excited state of the GAA model, in an experimental procedure similar to that used in Sec. 6.4.2. We measure the opposing localization properties (PR) of these two states for a wide range of parameters ($\Delta/J, \alpha$) and infer the existence of the SPME. Finally, we observe significant shifts in the observed critical disorder strengths due to nonlinear atomic interactions, and use

8.2 Probing localization properties of the generalized Aubry-André model eigenstates

We implement the GAA model of Eq. (8.3) using the momentum-space lattice technique for parameter ranges $0 < \Delta/J \lesssim 6$ and $-0.5 \leq \alpha \leq 0.5$. Because we encode these site energies as frequency detunings from Bragg resonance for each tunneling link, and because the GAA site energies diverge as $\alpha$ approaches $\pm 1$, we are limited in the parameters we can realize. Specifically, the site energies must be small enough that the detunings are much smaller than
the frequency spacing between adjacent frequency teeth, or else one frequency component may off-resonantly drive the wrong Bragg transition.

We adiabatically prepare the lowest (ground state, GS) and highest energy eigenstates (highest excited state, ES) of the GAA system. We begin with all atomic population in the central lattice site \( p = 0 \) with all tunneling links set to 0 and lattice site energies following the GAA model. We choose the GAA phase to be \( \phi = \pi \) (0), forcing the initial lattice site to have the lowest (highest) energy, and thus beginning in the ground (highest excited) state of the zero tunneling system. We then ramp up the tunneling linearly from 0 to a final value of \( J/h = 625 \) Hz over 0.75 ms, and hold at \( J \) for 1.25 ms. If this procedure is adiabatic, population that began in the lowest (highest) energy eigenstate of the zero tunneling lattice should end up populating the lowest (highest) energy eigenstate of the full Hamiltonian with nonzero \( J \). We can then run this procedure while varying the parameters \( \Delta \) and \( \alpha \) to realize different potential landscapes. This procedure works for all \( \alpha \), as while the site energy distribution changes (Fig. 8.1(a)), a phase of \( \phi = \pi \) (0) still yields the ground state (highest excited state).

This tunneling ramp can be alternatively viewed as tuning the system from the limit of infinite pseudodisorder (the initial state at \( J = 0 \) is like \( \Delta/J = \infty \)) to some final \( \Delta/J \) ratio, as shown in Figs 8.2(a,b). As such, we expect this procedure to be robust in the insulating regime, where there is poor overlap and thus weak coupling between the system’s localized eigenstates. However, for \( \Delta/J \) near the delocalization transition and in the metallic regime, adiabaticity with respect to the small finite-size energy gaps will be significantly challenged. Thus, while this procedure may not fully capture eigenstate properties in the metallic regime, we expect that it is well-suited for determining the delocalization transition for a given eigenstate. As an example, in Fig. 8.2(a) we show this procedure performed on the highest excited state of the normal AA model \( \alpha = 0 \), demonstrating extended delocalization below the critical disorder strength \( (\Delta/J_f)_c = 2 \) and localization above.

Here I want to give a few miscellaneous notes on this procedure: 1. Ideally, this procedure should be able to prepare any eigenstate by choosing the appropriate \( \phi \), but in experiment the ramp duration is just too short compared to the timescale associated with the spacings between eigenstates, dashing all hope of adiabaticity. 2. When compared to the similar
Figure 8.2: Probing the localization transition by adiabatic Hamiltonian evolution. (a) Cartoon of effective experimental sequence (arrows). Population initially localized at $\Delta/J = \infty$ is adiabatically loaded into an eigenstate of the GAA model at some final disorder strength $\Delta/J$. $\Delta$ is held fixed while the tunneling is ramped from 0 to $J = h \times 625$ Hz over 0.75 ms, and then held for 1.25 ms before imaging. Bottom: Lattice site populations of the highest excited state in the delocalized regime ($\Delta/J = 0.9$), near the transition ($\Delta/J = 2.1$), and in the localized regime ($\Delta/J = 4.2$) of the $\alpha = 0$ case. (b) Numerically-calculated participation ratios (PR) overlaid on the eigenenergies of the GAA model for $\alpha = -0.5$, $\phi = \pi$, and a system size of 201 sites. Higher energy states localize at larger disorder strengths than low-energy states, highlighting the presence of the SPME of Eq. (8.4) (dashed black line). Our experimental procedure probes the localization properties of the ground state (bottom curve) for $\phi = \pi$ and the highest excited state (top curve) for $\phi = 0$. (c) PR/N vs. $\Delta/J$ for the ground (open blue circles) and highest excited states (yellow diamonds) under $\alpha = -0.5, 0, 0.5$, showing evidence for a SPME tunable via $\alpha$. Numerical Gross-Pitaevskii results assume a homogeneous mean-field energy $U/J = 0.48$ ($U/h = 300$ Hz). Solid curves include the exact tunneling ramp used in experiment and dashed lines indicate eigenstates calculated with imaginary time propagation.

procedure we used to realize the eigenstates of the zigzag lattice (Sec. 6.4.2), here we need only ramp $J$ and no other parameters. 3. To better prepare eigenstates given our experimental limitations on evolution time, we could employ some shortcut to adiabaticity, like the counter-diabatic driving scheme that Eric has explored on the MSL system [27].

Although it would be preferable to measure the localization properties of all eigenstates to fully characterize the SPME, to show the presence of a mobility edge it is sufficient to probe only the lowest (ground state, denoted “GS”) and highest energy eigenstates (highest excited state, denoted “ES”). Concretely, the numerically-calculated PR values of the eigenstates
for $\alpha = -0.5$ shown in Fig. 8.2(b) show a clear energy dependence, in agreement with the prediction of Eq. (8.4). The highest energy eigenstate localizes near $\Delta/J = 3$, whereas the ground state localizes near $\Delta/J = 1$, in the absence of interactions. Figure 8.2(c, top) shows this clear energy-dependent localization in experiment, for the same $\alpha = -0.5$. As expected from simulation, the measured PR of the highest energy state begins to rise above $1/N$ near $\Delta/J = 3$, while the ground state experiences the transition near $\Delta/J = 1$. From this clear separation of the localization transitions for these two states, we can infer the existence of an intervening mobility edge.

For $\alpha = +0.5$ in Fig. 8.2(c, bottom), our experimental data show an inversion of the mobility edge: the excited state localizes at a weaker pseudodisorder amplitude than the ground state. This inversion is due to a symmetry of the Hamiltonian that exchanges the lowest and the highest energy state as $\alpha \rightarrow -\alpha$. Looking at Eq. (8.3), we see that $\varepsilon$ can be kept constant by flipping the sign of $\alpha$ and additionally shifting $\phi \rightarrow \phi - \pi$, which also switches the minimum of the GAA potential with the maximum, thus exchanging the ground and highest excited states of the system. So, we expect the behavior of the GS (ES) under $\alpha > 0$ ($\alpha < 0$) to match the ES (GS) under $\alpha < 0$ ($\alpha > 0$). Ultimately, this results in a mobility edge that can be tuned and inverted simply by changing $\alpha$.

### 8.3 Interaction effects

The $\alpha = 0$ case relates to the standard AA model, so we would expect all energy eigenstates to undergo a delocalization transition at the same value of $\Delta/J = 2$. However, as shown in Fig. 8.2(c, center), we observe that the transition in fact splits for the lowest and highest energy eigenstates, signaling a mobility edge driven by atomic interactions in our momentum-space lattice. We have discussed this same effect before as part of our Zigzag lattice study in Sec. 6.4.1, and here we go over it again briefly and present numerical calculations of the interacting eigenstates in Sec. 8.3.2.

As described in detail in Chap. 5, the interactions between atoms in our momentum-space lattice can be considered an effectively site-local attraction. These attractive interactions affect the localization properties of low and high energy states differently. For localized eigen-
states at low energy, the attractive interactions further promote localization by self-trapping, shifting the localization transition to lower disorder strengths, similar to the instability of bosonic ground states under real-space attraction \[245\] In contrast, for eigenstates at high energy the attractive interactions dominate and effectively screen the GAA pseudodisorder, leading to enhanced delocalization in the same manner that repulsive interactions can promote delocalization of ground state bosonic gases under disorder \[161, 245\] The effect of interactions can also be viewed as an additional chemical potential shift to the initial site where all population is located, promoting self-trapping for the ground state by bringing the effective site energy further away from other states. For the highest excited state, though, dragging down the local chemical potential brings the energy closer to nearby states, promoting delocalization. In either case, our data match this intuition: enhanced localization for the ground state and enhanced delocalization for the highest excited state.

8.3.1 Numerically simulating the interacting system

To better understand how the interactions affect not only the $\alpha = 0$ case but all $\alpha$, we numerically simulate the system by assuming a homogeneous mean-field energy and solving the Gross-Pitaevskii equation,

\[
-i\hbar \dot{\psi}_n = \sum_m H_{mn} \psi_m + U \left(2 - |\psi_n|^2\right) \psi_n, \tag{8.6}
\]

where $H_{mn}^\text{sp}$ is the matrix element between states $p_m$ and $p_n$ of the single particle Hamiltonian from Eq. (8.1) and $U$ is the interaction strength (see Sec. 5.1.4 for more detail).

In this work, we perform two different simulations that both assume a homogeneous mean-field interaction energy of $U/J = 0.48$ ($U/h = 300$ Hz). The first, shown as solid curves in Fig. 8.2(c), solves the G-P equation including the tunneling ramp we use in experiment to calculate the exact population dynamics we should expect. This is the standard simulation technique that we use for interacting systems, used in many other chapters in this thesis, and we utilize it for not only the solid curves of Fig. 8.2(c), but also in Fig. 8.1(b) and Fig. 8.2(a-b) (color plot of (a)). While this simulation can reveal flaws in experimental
implementation, it cannot tell us if the procedure loads the desired eigenstates. Thus, we perform a second simulation (dashed curves) to numerically approximate the eigenstates (GS, ES) of the interacting Hamiltonian via imaginary time propagation.

### 8.3.2 Interacting eigenstates via imaginary time propagation

Our theory collaborator Karmela Padavić-Callaghan spearheaded this simulation work, and more details can be found in Sec. 11.4.1 of her thesis [246], but we still give a description of the method here. For a Hamiltonian $H$, its eigenstates can be propagated in time $t$ by

$$\psi_n(t) = e^{-iHt/\hbar}\psi_n(0) = e^{-iE_nt/\hbar}\psi_n(0),$$  \hspace{1cm} (8.7) 

for real eigenenergies $E_n$. Here, we simply substitute real time $t$ for imaginary time $\tau$ ($\tau = it/\hbar$), resulting in exponential decay instead of a phase term,

$$\psi_n(\tau) = e^{-\tau H}\psi_n(0) = e^{-\tau E_n\psi_n(0)}.$$  \hspace{1cm} (8.8) 

The idea, then, is to begin with some initial “guess” wavefunction, which in general is a superposition of all the eigenstates of the system, and propagate forward in imaginary time:

$$\psi_{\text{guess}}(0) = \sum_n C_n\psi_n(0)$$  \hspace{1cm} (8.9) 

$$\psi_{\text{guess}}(\tau) = e^{-\tau H}\psi_{\text{guess}}(0) = \sum_n C_n e^{-\tau E_n}\psi_n(0)$$  \hspace{1cm} (8.10) 

$$\approx C_0 e^{-\tau E_0}\psi_0(0).$$  \hspace{1cm} (8.11) 

The eigenenergies $E_n$ dictate how quickly each eigenstate exponentially decays/grows: high, positive energy eigenstates exponentially decay, while low, negative energy states grow exponentially. If we run this for long enough $\tau$, then every state except the lowest negative energy state (ground state, $\psi_0$) becomes exponentially suppressed, and the end result is directly proportional to the ground state as shown by Eq. (8.12). Similarly, we can change
Figure 8.3: **High energy eigenstates of the interacting Hamiltonian.** Lattice site energy distributions of the GAA model, for $\alpha = -0.5, 0, +0.5$. Interactions of strength $U$ allow the excited states to be comprised of lattice sites within $U$ in energy (shaded gray box).

$\tau \to -\tau$ to exponentially suppress all states except the highest positive energy state (ES). By running this procedure on the G-P equation of Eq. (8.6), we can find the expected forms of the two extremal eigenstates of the interacting Hamiltonian.

We overlay results from both simulations in Fig. 8.2(c), finding relatively good qualitative agreement with the experimental data, especially in where each eigenstate experiences the transition. In particular, we note that without interactions, $\alpha = -0.5$ should be the exact opposite of $\alpha = +0.5$: the localization behavior of the ground and excited states should swap. Here, we see that interactions promote localization for the ground state and delocalization for the excited state, for all $\alpha$. The argument here is the same for the $\alpha = 0$ case: interactions screen the effect of the pseudodisorder for high energy states, while promoting self-trapping for low energy states.

However, we notice a few points of disagreement between data and simulation:

- Imaginary time simulation: the high PR value of the excited state at large disorder for $\alpha = -0.5$.

This is because the interacting eigenstate in this regime is localized not to one lattice site, but localized to several separated sites. The ramp procedure is not equipped to handle such a state, since the initial state we prepare is entirely within one lattice site. Under strong disorder there is no overlap between this initial state and the other lattice sites, so our measured PR is lower than that of the actual eigenstate.

As for why the eigenstate is split among several sites, we go back to two previous arguments made in this chapter: screening by interactions (causing delocalization) and the skewed lattice site distributions for $\alpha \neq 0$. As sketched in Fig. 8.3, in the presence
of interactions with strength $U$, the highest excited state can be a superposition of the lattice sites around $\sim U$ away in energy. For $\alpha = -0.5$, there are many of these high energy sites, leading to a highest excited state that is extended over several lattice sites and an PR above $1/N$ (in fact, it looks like about 3 sites, as the PR is roughly $3/N$).

There are fewer high energy sites for $\alpha = 0$ and fewer still for $\alpha = +0.5$, resulting in lower PR for those two simulation curves at high disorder strengths (a few sites for $\alpha = 0$, and just one site for $\alpha = +0.5$).

Note that this effect is really only relevant for the highest excited state, where interactions screen the effect of disorder. For the ground state, the interactions promote self-trapping to further localize the state in a single site.

- Imaginary time simulation: Simulation curves go far higher than both data and ramp simulation near $\Delta/J = 0$.

  This shows the failing of the ramp procedure in the delocalized regime. Because the initial state is in only one lattice site, it is relatively easy to adiabatically load into a localized state that occupies that single site. But to adiabatically load a state delocalized across the entire lattice is much harder.

- Ramp simulation: a peak in PR at low disorder strengths, which doesn’t match data.

  This happens in simulation likely because the population hits the edge of the finite, 21-site lattice and returns. It is also an indicator of how non-adiabatic this procedure is for the delocalized regime.

  Why this doesn’t happen in experiment is a tougher question, and we don’t have a very good answer for this. It is possible that only a small fraction of atoms travels out to the edges of the lattice in simulation, which is washed out in experimental noise.

### 8.4 Full localization diagram of the GAA model

Figure 8.4 provides a comprehensive picture for the localization properties of the low- and high-energy states over the full range of experimentally-realizable $\alpha$ and $\Delta/J$ values, again
Figure 8.4: Localization diagrams of the ground and excited states. (a,b) Localization diagrams depicting PR vs. $\Delta/J$ and $\alpha$ for the (a) ground state (GS) and (b) highest excited state (ES). Filled circles and open diamonds mark the “critical” disorder for each $\alpha$ column, calculated by determining where the PR goes below a threshold $\text{PR}_{\text{thres}} = 0.19$. The few data points above $\Delta/J > 6$ are not shown for presentation clarity. (c) Critical disorder values for the GS and ES (filled circles and open diamonds), overlaid on interacting eigenstate simulation results showing the difference in PR between the ground and highest excited states (in color, calculated assuming homogeneous mean-field interaction $U/J = 0.48$, $U/h = 300$ Hz). The critical disorder “lines” do not coincide, showing a mobility edge, and they cross away from $\alpha = 0$, indicating a shift due to interactions.

using the same adiabatic ramp procedure described above. The color denotes the experimental PR values as a function of $\Delta/J$ and $\alpha$, linearly interpolated between measurements taken at parameter values corresponding to the small black circles. This presentation of the data for all $|\alpha| < 0.5$ clearly shows the smooth tunability of the mobility edge. Moreover, we see plainly the effect of interactions: we expect the plots for the ground state (Fig. 8.4(a)) and the highest excited state (Fig. 8.4(b)) to be exactly mirrored without interactions. Yet in this interacting system, the ground state is more localized than the excited state for all $\alpha$, as explained in the previous section.

To see this more directly, we calculate for both states the position of the localization transition (i.e., a critical disorder strength) at every value of $\alpha$, taking the disorder value at which the PR crosses a threshold value set to 0.19 (more explanation in Sec. 8.5.2). Figure 8.4(c) shows these critical disorder values (black circles for GS, white diamonds for ES) overlaid on top of the difference in PR between the interacting eigenstates, calculated

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2This is done to neatly generate the plot on a rectangular grid, as the data were taken with unequal spacing in $\Delta/J$ due to fluctuations in tunneling strength.
with imaginary time propagation \((U/J = 0.48, \ U/h = 300 \text{ Hz})\). First, the fact that the two (GS, ES) lines do not coincide indicates that the localization transition is energy-dependent, and thus there must be a mobility edge. Next, we see that the lines are not flat and that they cross, allowing for both situations where only the ground state is insulating and where only the excited state is insulating, and indicating that the mobility edge is parameter-tunable. Finally, the two lines do not cross at \(\alpha = 0\), showing the effect of atomic interactions: namely, that they localize the ground state and delocalize the excited state for all \(\alpha\).

Using our momentum-space lattice technique, we have successfully realized the generalized Aubry-André model in experiment. By way of an adiabatic parameter ramp, we load and probe the localization properties of the ground and highest excited states, finding qualitative agreement with numerically-calculated eigenstates of the interacting system. We observe the expected parameter-tuned single-particle mobility edge of the model, and generate a full localization diagram of the two measured states for a wide range of disorder parameters in Fig. 8.4. Interaction effects serve to shift the two eigenstates in opposite directions, and we find good qualitative agreement with numerically-calculated eigenstates of the interacting model. However, our work here does not fully characterize the form of the SPME in Eq. (8.4), which would require identifying the energies of the eigenstates directly through spectroscopy or other methods. We have a proposed technique (atomic “STM”) described in more detail in Sec. 10.2 which may allow us to measure eigenenergies of arbitrary 1D lattices. We plan to use this technique to further explore the connection between the quasiperiodicity of the 1D Aubry-André model and the topology of the 2D Harper-Hofstadter model, and hope to map out the Hofstadter butterfly energy spectrum of the Aubry-André model.

8.5 More experimental and numerical details

8.5.1 Ramp procedure

To realize the ground state of the generalized Aubry-André model, we begin in the ground state of the zero tunneling system. Because atomic population starts at rest, we always begin in the \(p = 0\) lattice site. To ensure that this is the ground state, we choose the phase
of the GAA potential such that the central site has the lowest energy, or $\phi = \pi$. We then linearly ramp the tunneling up to some final value $J_f$ over 0.75 ms, slowly enough that the atoms populate the instantaneous ground state of the system for the whole ramp. At the end of the ramp, we hold at $J_f$ for 1.25 ms and then take an image. To realize the highest excited state, we perform the same procedure, choosing $\phi = 0$ so that the initial site has the highest energy. Ideally, any eigenstate can be prepared by choosing the appropriate $\phi$, but the ramp duration is too short compared to the spacings between eigenstates with intermediate energy, leading to poor overlap with the eigenstate. This procedure could be shortened considerably by implementing counter-diabatic driving [27].

This procedure also works for $\alpha \neq 0$, as while the site energy distribution changes (Fig. 8.1(a)), a phase of $\phi = \pi(0)$ still yields the ground state (highest excited state).

8.5.2 Calculating critical $\Delta/J$ values

In Fig. 8.4, we estimate the critical $\Delta/J$ value at which the localization transition occurs for each $\alpha$ value. Ideally, the realized eigenstates should be exponentially localized above this value, and delocalized below this value. However, due to imperfect eigenstate preparation, especially in the delocalized regime, we do not see any sharp localization transition. Thus to estimate the transition value, we resort to finding where the value of the PR crosses some threshold value, $\text{PR}_{\text{thres}} = 0.19$. This value must be larger than the minimum allowed value $1/N = 1/21$, but is made a little larger to accommodate fluctuations in the data and any localized states that may be spread out over more than 1 site. Specifically, for each value of $\alpha$, we take a pairwise moving average to smooth out more jagged sections in the PR vs. $\Delta/J$ data, linearly interpolate between the points, and find the value of $\Delta/J$ where that interpolated curve crosses $\text{PR}_{\text{thres}}$. While we do not expect this procedure to match exactly the localization transition point, we believe it to be a good estimate that explains the key features of our model, especially when plotted as in Fig. 8.4(c).
8.5.3 Full localization diagrams of the numerically calculated eigenstates of the interacting model

Figure 8.5 shows numerically-calculated localization diagrams like Fig. 8.4 for the eigenstates of the interacting model, assuming a homogeneous mean-field interaction energy $U/J = 0.48$ ($U/h = 300$ Hz). The circles and diamonds indicate the critical disorder values of the experiment, while the dashed lines show the critical disorder values of the eigenstates themselves, calculated in the same manner as the data (Sec. 8.5.2). The colors of the ES plot at high $\Delta/J$ show the issue we discussed in Sec. 8.3.2: the eigenstates may be localized, but they occupy more than one site, leading to a localized regime that is not as deep a red as the same region in the GS plot.

8.5.4 Simulation details and parameters

As detailed in Sec. 8.3, we performed two types of numerical simulations of the interacting model in this work: solving the G-P equation (Eq. (8.6)) including the parameter ramp, and propagating a wavefunction in imaginary time to approximate the lowest and highest energy wavefunctions of the same G-P Hamiltonian. We also performed simulations of the
single-particle Hamiltonian. Here we list the parameters used for all simulations.

For the imaginary time propagation results, we guessed an initial wavefunction that was spread over all eigenstates equally: \( \psi_{\text{guess}}(0) = \sum_n \frac{1}{\sqrt{N}} \psi_n(0) \). This ensured that there was at least some overlap with the target eigenstate (GS or ES). As a very minor technical detail: we ran these simulations for some duration of imaginary time by looping on a function several times, taking the output \( \psi_{\text{guess}}(\tau) \) and reentering it as a guess wavefunction. This is the same as running it all at once, but is less computationally demanding due to Mathematica’s quirks. The values we used for this simulation in various parts of this study are:

- Fig. 8.2(c) (dashed lines): 21 sites, 100 loops of 2ms each
- Fig. 8.4(c), Fig. 8.5 (same simulation): 21 sites, 100 loops of 2ms each
- Fig. 8.6(b): 51 sites, 100 loops of 3ms each

The parameter ramp simulations of the interacting model used a system size of \( N = 21 \) sites to match experiment, and were only presented in Fig. 8.2(c) (solid curves).

For our simulations of the single-particle Hamiltonian, we varied the system size to better depict and highlight the energy bands:

- Fig. 8.1(b), Fig. 8.6(a) (same plot): 51 sites
- Fig. 8.2(a): 21 sites
- Fig. 8.2(b): 201 sites

### 8.5.5 Interaction effects on energy bands

Figure 8.6(a) shows the localization behavior of the eigenstates overlaid on their energy spectrum \( E \) vs. \( \alpha \) (same as Fig. 8.1(b)). Using imaginary time propagation, we plot the same diagram in Fig. 8.6(b) for the two extremal eigenstates of the interacting Hamiltonian, for both the noninteracting case (\( U = 0 \)) and assuming a homogeneous mean-field interaction \( U/J = 0.48 \) (\( U/h = 300 \) Hz). The interactions drag down the energy of both the ground state (bottom pair of curves) and the highest excited state (top pair of curves). Their
localization properties change as expected: the ground state becomes more localized (more red here) while the highest excited state becomes more delocalized (more blue). Hopefully in future work we can probe this energy spectrum directly, in order to measure the curve of the mobility edge under interactions.
In this chapter and the related paper to come [21], we expand our previous exploratory work on atomic interactions in momentum space (Chap. 5) to full lattices. Here we perform some of the first experiments on many-body physics within a synthetic lattice. The regime that our momentum-space lattice technique explores - namely, an interacting system far out of equilibrium (i.e., initial population in only one or a few lattice sites) - offers direct insight into the physics of self-trapping and even acts as a cold-atom analog of an array of Josephson junctions. Here, we perform three straightforward experiments on 1D momentum-space lattices with atomic interactions:

1. Self-trapping in a 1D lattice

2. Asymmetric Bloch oscillations in a tilted 1D lattice

3. Nonsinusoidal current-phase relationship in wave packet dynamics

While I was able to run simulations for the first experiment, the latter two included numerics which were much more involved, and we were helped out immensely by our collaborators from two institutions. Bhuvanesh Sundar and Kaden Hazzard of Rice University provided the numerics for the Bloch oscillations dataset, running simulations including a local density approximation of the atoms in our harmonic trap. Junpeng Hou, Xi-Wang Luo, and Chuanwei Zhang of UT Dallas provided theory support for the Josephson junction array dataset, running real-space Gross-Pitaevskii simulations. Finally, I planned and ran the experimental portion of this work, again with technical assistance from Eric J. Meier.
Figure 9.1: *Synthetic momentum state lattices with mode-dependent interactions.* A prettier version of the usual MSL setup figure. (a) Two-photon Bragg transitions individually couple atomic momentum states along a “synthetic” dimension. (b) The engineered synthetic lattice features inter-site tunnelings $J_i$, lattice site energies $\varepsilon_i$, and mode-dependent interactions: atoms in the same site have a repulsive interaction $u$, and atoms in different sites have a repulsive interaction $2u$.

### 9.1 Background

This study contains three separate experiments on interacting atoms in our momentum-space lattice. Hence, the backdrop to this work is the form of interactions in momentum space, which was discussed in detail in Chap. 5. In this section I will walk through an abridged version of that discussion and introduce the equations which we will refer to later.

For cold atoms in real space, interactions are normally considered as short-ranged contact interactions (Van der Waals) with an interaction strength

$$U = g\rho_N = \frac{4\pi\hbar^2 a}{M}\rho_N,$$

where $\rho_N$ is the atomic density (for $^{87}\text{Rb}$), $M = M_{\text{Rb}}$ is the atomic mass, and $a$ is the $s$-wave scattering length, which takes a value of $\sim 100a_0$ for the various hyperfine states of $^{87}\text{Rb}$. In momentum space, this interaction becomes long ranged and naively we would expect homogeneous all-to-all interactions... but due to boson statistics, there is a mode dependence that lends finite range to the interaction. While two atoms occupying the same momentum state experience one factor of this interaction strength $u = U/N$, two atoms in
different, distinguishable momentum states experience an added exchange energy term for a total of \(2u\). For our atomic species, the scattering length is positive and thus the interaction is repulsive. So by subtracting an overall constant of \(2u\), we can consider these interactions as an effective on-site attraction of \(-u\).

In the following experiments, we study atoms on 1D lattices obeying the usual tight-binding Hamiltonian

\[
H^{\text{sp}} = -J \sum_n \left( c^\dagger_{n+1} c_n + \text{h.c.} \right) + \sum_n \varepsilon_n c^\dagger_n c_n ,
\]

(9.2)

where \(\varepsilon_n\) is the energy of lattice site \(n\) and \(J\) is the nearest-neighbor tunneling strength which in general can carry a tunneling phase, and we exploit this control to study how relative phases can drive atomic currents in Sec. 9.4. To treat interactions in our system in a manageable form, we assume that the momentum states are fully distinguishable, such that the interaction is fully site-local.\(^1\) Then, if we take \(\psi_n\) to be the normalized probability amplitude for lattice site \(n\), we can plug in the single-particle Hamiltonian above into a Gross-Pitaevskii functional,

\[
i\hbar \dot{\psi}_n = \sum_m H^{\text{sp}}_{mn} \psi_m + U \left[ 2 - |\psi_n|^2 \right] \psi_n ,
\]

(9.3)

where we assume some mean-field interaction energy \(U\) (Eq. (9.1)). We note that this simple treatment ignores many relevant effects like inhomogeneous density and dynamics of the real-space distribution, and we (our theory collaborators, really) employ more refined simulations later in this section for Figs. 9.3 and 9.5.

As described more in Sec. 5.1.2, the interaction strength can be tuned directly via a Feshbach resonance, but our choice of atomic species (\(^{87}\)Rb) makes this difficult. But the experiments here are performed on lattices, such that the interesting physics comes from the competition between the self-trapping of the attractive interactions and the tunneling. Thus the relevant quantity for our studies is \(U/J\), and instead of directly tuning \(U\), we

\(^1\)This further ignores the influence of real-space screening, which gives an effective off-site range to the attraction.
tune $J$, probing at equivalent timescales in units of the tunneling time $\hbar/J$. This is simple to do, but has some drawbacks, most notably that a very low tunneling $J$ leads to very long tunneling times. For the subsequent long evolution times necessary to probe these low tunneling values, atoms in different momentum modes begin to drift apart in space and lose coherence. However, we find that we can still see dynamics out to 4 tunneling times for tunnelings as low as $J \approx 100$ Hz, or interaction strengths as high as $U/J \approx 9$. This is helped by the fact that under strong interactions, population is self-trapped at these large $U/J$ values, such that population mostly remains in one momentum mode, and the loss of spatial coherence between momentum modes is not very relevant.

In this work, we present numerical results from three different simulations of the G-P equation. As a rough approximation of the interacting physics, we can assume a completely homogeneous mean-field interaction energy, as we do in Sec. 9.2. This is simply numerically solving Eq. 9.3 with a constant $U$, as we have done in previous studies in Chaps. 6 and 8.

For the Bloch oscillations study in Sec. 9.3, we present results of simulations run by Bhuvanesh Sundar and Kaden Hazzard of Rice University. These simulations include a local density approximation (LDA) which seeks to factor in the inhomogeneous atomic density and thus inhomogeneous interaction strengths of the atoms within their harmonic trapping potential. These simulations integrate over the many different $U$ values at each point of the Thomas-Fermi profile of the atomic cloud. This is similar to our own LDA simulations of the double well system described in Sec. 5.5.2.

Finally, the wavepacket study in Sec. 9.4 presents results of a real-space G-P simulation run by Junpeng Hou and Chuanwei Zhang of UT Dallas. In this process, starting from the ground state of the trapped gas with interactions (determined by imaginary-time evolution similar to the process described in Sec. 8.3.2), the influence of the trap and interactions are taken into account throughout the duration of lattice driving. This simulation considers dynamics along one dimension (with the transverse degrees of freedom effectively integrated out). The motivation for these real-space simulations was related to our initial state in this experiment being asymmetric in momentum space (a superposition of the $p = 0$ and $p = -2\hbar k$ states), and our concern that the trap would have a nontrivial influence on the dynamics when probing on timescales comparable to the trapping period. Based on the
Figure 9.2: Macroscopic quantum self-trapping in an array of laser-coupled momentum states. (a) A cartoon of the system: self-trapping occurs when strong attractive interactions $U$ exceed the tunneling bandwidth $4J$. (b,c) Population dynamics for high (1281 Hz) and low (93 Hz) tunneling, with atoms initialized to site 0. Left: Data from integrated optical density images after 18 ms time of flight, averaged over 5 experimental realizations. Right: Numerical Gross–Pitaevskii simulations. (d) Integrated OD images taken at 1.5 tunneling times vs. tunneling $J$. (e) Standard deviation of the atomic distributions from (d), shown with Gross–Pitaevskii calculations of the tight-binding model (solid blue line) and of the full Bragg frequency spectrum (dashed red line). The expected point of self-trapping $U = 4J$ is shown with a vertical gray line. Data for (d,e) are averaged over 20 experimental realizations. Numerical Gross-Pitaevskii simulations in (b,c,e) assume a homogeneous mean-field energy of $U/h = 520$ Hz.

excellent agreement of the theory with the experimental results, this suspicion seems to have been justified.

9.2 Self-trapping in a 1D lattice

We first study the simplest situation: a 1D lattice of $N = 21$ sites with uniform tunnelings $J$ and no potential shifts, with population initialized entirely in the $p = 0$ lattice site. For evolution times before population reaches the edges, this system should behave like a uniform 1D lattice with a band structure of the form $E(q) = -2J \cosqd$ with bandwidth of $4J$, 166
where $q$ is quasimomentum and $d = 2\hbar k$ is the lattice spacing (see below in Sec. 9.6.1 for a simple derivation of this). Because the interactions are effectively on-site and attractive, we expect that they effectively drag down the local chemical potential at this initial site, as shown by the cartoon in Fig. 9.2(a). When the interaction energy exceeds the tunneling bandwidth, $U > 4J$, we expect that dynamics should completely stop and the population in $p = 0$ be completely self-trapped, as the shift in chemical potential exceeds the tunneling bandwidth.

Dynamics out to 4 tunneling times are shown in the weakly interacting (strong tunneling, Fig. 9.2(b)) and strongly interacting (weak tunneling, Fig. 9.2(c)) regimes. The top of each plot shows population initialized to the central lattice site, beginning from rest with zero momentum. Under weak interactions ($U/J \approx 0.43$), the atoms undergo ballistic spreading in the expected continuous-time quantum random walk [5, 14]. By reducing the tunneling bandwidth, we enter a regime of much stronger interactions ($U/J \approx 5.91$), under which almost all of the population is self-trapped in the initial lattice site. These dynamics data agree well with GP simulations which assume a uniform atomic density and mean-field energy of $U/h = 520$ Hz, and which also take into account the exact experimental parameters (i.e., all off-resonant contributions from all applied Bragg frequencies).

In Fig. 9.2(d), we isolate the effect of interactions by plotting populations at the same normalized evolution time (1.5 tunneling times) for all $J$. Thus in the absence of interactions, the population distributions should be identical for all tunneling values. For large $J$, the population spreads out ballistically as expected of a quantum walk (in contrast, a classical random walk would show diffusive spreading). As the tunneling rate decreases and interactions begin to dominate, more and more population remains in the initial site until all of the atoms are self-trapped. By plotting the width of the distribution at each $J$ value in Fig. 9.2(e), we see that the transition is rather sharp, and matches our intuition, turning over roughly where the interaction strength matches the tunneling bandwidth, $U = 4J$.

The data also agree quite well with a simple G-P simulation assuming a homogeneous mean-field energy of $U/h = 520$ Hz (dotted red line). This is a “full” simulation of our experimental parameters, applying the exact Bragg frequency tones that we use in experiment to account for any off-resonant effects. As described in Chap. 2 and evident in the data and
simulation for Fig. 9.2(b), this leads to jagged step-like behavior that occurs at a timescale corresponding to the frequency separation between adjacent frequency components. Such an effect naturally shows up in the standard deviation data of panel (e) as well, and can be contrasted with the smooth blue curve which represents results of an “ideal” GP simulation of Eq. (9.2), i.e., without accounting for off-resonant effects.

We further note that the interaction energy \( (U/h = 520 \, \text{Hz}) \) used for all of the simulation results in Fig. 9.2 is derived from a fit of the GP simulation to the data in Fig. 9.2(e). This is done because the interaction strength varied slightly between when the different datasets were taken (due to changes in atom number and trap shape), and independent verifications of \( U \) were not performed (though independent tunneling calibrations were performed). So in all of the studies in this chapter, we take the interaction energy as a free parameter that we determine by comparing to numerical simulations.

### 9.3 Asymmetric Bloch oscillations in a tilted lattice

In this second study, we add a linear potential shift to the lattice, mimicking a constant force applied to a real-space lattice. While a classical particle on such a tilted potential would continue to accelerate down the hill, quantum particles in a periodic potential under a uniform force undergo periodic Bloch oscillations [52, 53]. Such Bloch oscillations are highly challenging to observe in crystals, as electrons scatter on a timescale much faster than the oscillations.

As with many topics in this thesis, Bloch oscillations are a great subject for study with the clean systems of cold atoms in optical lattices. Indeed, this phenomenon was successfully realized in experiments with real-space optical lattices as early as 1996 [55], and even in our own system in our first experimental demonstration of MSLs [5]. More recent work has observed oscillations of higher bands of the lattice, resulting in huge amplitudes observable in both momentum space and position space [247]. One work more relevant to our interacting study here is Ref. [58], in which the researchers observed Bloch oscillations of two bosonic atoms initialized to the same lattice site for a range of repulsive interactions, finding that the oscillation frequency doubled under strong interactions as the two atoms moved as a
Figure 9.3: **Bloch oscillations of an interacting fluids in a synthetic lattice.** (a) The physical system: Atoms with attractive interactions $U$ on a lattice with a constant potential gradient of $\Delta = 1.5J$ for all tunneling values $J$. (b) Lattice site population dynamics for a range of tunneling values $J/h = (1270, 594, 299, 104)$ Hz. [Experimental data, Numerical simulation: Left, Right] (c) Population imbalance vs. time as a function of interaction strength. Data are offset for clarity by 0.4 along the vertical axis. (d) Asymptotic population imbalance, with the dotted theory line representing the (LDA) GP expectation for a mean interaction strength of $U/h = 900$ Hz (peak interaction strength 1575 Hz). Data for (b-d) is averaged over 5 experimental realizations.

Here, we study Bloch oscillations in the limit of having a large number of atoms all starting off at a central lattice site, and with an effective attractive, local interaction between atoms in momentum space. As depicted in Fig. 9.3(a), we apply a constant energy shift $\Delta$ between adjacent lattice sites by simply detuning all applied frequency components by $\Delta$. Like in the previous section, we flash on the lattice frequencies for some evolution time, drop the trapping potential for 18 ms time-of-flight, and measure populations with absorption imaging. We measure population dynamics out to 8 tunneling times for a range of tunneling values $J$ listed in Sec. 9.6.2 (only 4 tunneling times for the lowest tunneling value).
isolate the effects of interactions, we scale the tilt of the lattice to the tunneling, $\Delta = 1.5J$.

For high tunneling values (top panels in Fig. 9.3(b)), the effects of interactions are weak and population undergoes ordinary Bloch oscillations occurring at the frequency associated with the tilt, $\Delta = 1.5J$. Population spreads out symmetrically from the initial site until they reach a site with an energy difference equal to the tunneling bandwidth $4J$ ($4J/\Delta = 2.7$ sites), and turn back. Under strong interactions (bottom panels), atomic population becomes completely self-trapped in the initial site. The simple picture (Fig. 9.3(a)) is again of attractive interactions dragging down the local chemical potential of the initial lattice site. Self-trapping occurs when this chemical potential shift exceeds the sum of the tunneling bandwidth and the added tilt.

For intermediate interaction strengths (middle panels), the oscillations skew to the left side, becoming highly asymmetric. Similar asymmetric Bloch oscillations have been observed in nonlinear photonic lattices [248], but in our system such asymmetry is driven by atomic interactions. Interactions lower the local chemical potential, bringing it closer to the neighboring site on the left $p/2\hbar k = -1$ and further from the one on the right, $p/2\hbar k = +1$, leading to a preference to transfer to the left side of the lattice.

These data qualitatively match numerical GP simulations (right column) which, as mentioned above, use a local density approximation to account for the inhomogeneous density distribution of the atoms in the harmonic trap, integrating over the full trap with a mean interaction strength $U/\hbar = 900$ Hz (corresponding to a peak interaction strength $U/\hbar = 1575$ Hz). While the GP simulations show repeating oscillations for all tunneling values, the oscillations in the low tunneling datasets damp out quickly. We expect atoms in different momentum states to lose spatial coherence, as can be seen fairly clearly for the $-2$ state in the 299 Hz data (discussed more below in Sec. 9.6.5). This issue is exacerbated by the long tunneling times for low tunneling values, limiting us from observing any long time behavior. We hope to avoid this issue in future studies by switching atomic species to $^{39}$K and accessing its Feshbach resonance, which will allow us to work at large tunneling values and change the scattering length instead.

To quantify the asymmetry, we plot in Fig. 9.3(c) the imbalance between population on
the left and right sides of the lattice,

\[ \text{imbalance} = \sum_{i<0} P_i - \sum_{i>0} P_i, \]  

(9.4)

where \( P_i \) is the normalized population in site \( i \). This imbalance increases drastically as tunneling decreases, but drops down again once the population becomes fully self-trapped. Figure 9.3(d) shows the imbalance averaged over all evolution times past 2 tunneling times, in order to exclude the effect of the initially balanced distribution. The same trend appears in this averaged data, with self-trapping eventually dominating over the enhanced asymmetric tunneling to the left.

### 9.4 Skewed current-phase relationship in wavepacket dynamics

In this third experiment, we study the dynamics of a wavepacket with some initial tunable phase gradient under strong interactions. More specifically, the initial state is given by equal population in lattice sites \( p/2\hbar k = 0 \) and \(-1\) with some phase difference \( \phi \), as shown in Fig. 9.4(a). For interpretation purposes in the noninteracting limit, we treat this system as a wavepacket and consider its dynamics in terms of the band structure of the lattice. This is related to the well-studied topic of cold atom realizations of Josephson junctions and Josephson junction arrays.

#### 9.4.1 Cold atom Josephson junction arrays

A solid state Josephson junction is comprised of two superconducting plates separated by a potential barrier that weakly couples the two wavefunctions (e.g., a thin insulator). When a voltage, current, or phase difference is applied across the junction, population can tunnel across the barrier and result in a measured supercurrent. By tuning the phase between the two wavefunctions, this supercurrent can vary in some sinusoidal pattern. For an ideal Josephson junction, like one realized by parametric coupling between two states, this current-phase relationship (CPR) should be purely sinusoidal. More generally, the form of the CPR reveals information about the system and how transport occurs, and can become skewed.
and nonsinusoidal due to details of the bound states within the junction and the junction “transparency,” among other reasons [249].

Josephson junctions are a natural subject of study with cold atoms, and has been an active field of research for over two decades [167, 169, 250–256] (related work with polariton condensates [257]). A single junction can be readily mimicked with two superfluids separated by a potential barrier, which can be performed experimentally by shining an off-resonant “sheet” laser beam onto a BEC, or by simply turning on one Bragg frequency in our lattice. The two parameters characterizing a junction are also tunable: the population and phase difference across the barrier. In cold atom Josephson junctions, inter-atomic interactions play a critical role, and can lead to regimes of macroscopic quantum self-trapping (MQST) which do not appear in superconducting junctions [167].

Another easy system to realize in experiment is an array of Josephson junctions, which in cold atoms is simply an array of superfluids, or a superfluid in a 1D lattice. The asymmetric Bloch oscillations in the previous section can be thought of as an extension of the ac Josephson effect to such a junction array. In contrast, in solid state systems it is typically infeasible to achieve sufficiently large voltage biases between adjacent junctions so as to realize Bloch oscillations, though oscillations of the phase degree of freedom have been predicted [258, 259], explored [260, 261], and realized [262, 263]. We note that in this language of junction arrays, the Bloch oscillations work of the previous Section was performed with a very specific initial state: all population on one site of one junction. It would be interesting to extend the Bloch oscillations work to start with different initial population imbalances, and with phase control.

In this section, we examine the current-phase relationship of a Josephson junction array (a 1D lattice with no potential shifts), with atoms initialized into a state with equal population on both sides of a central junction with full control over the phase difference. We show that our particular form of attractive, on-site interactions causes the sinusoidal CPR to skew as the population becomes self-trapped, and that such self-trapping in the system is dependent on the applied phase difference.
9.4.2 Wavepacket dynamics

As mentioned above, here we initialize the system in a superposition state in which population is delocalized equally over two adjacent lattice sites, with a controlled relative phase $\phi$ (Fig. 9.4(a)). This is done by way of a square $\pi/2$ pulse of a single Bragg frequency, cutting off population transfer midway through a Rabi oscillation (see Sec. 9.6.3 for details on initialization procedure). We then apply the lattice beams and quench on the Hamiltonian, allowing the initial two-site “wavepacket” to evolve for two tunneling times ($\Delta t = 2\hbar/J$), after which we image the resulting population distribution by standard time-of-flight methods. The two observables that we measure are the average position $\bar{n}$ and spread $\sigma_n$ of the population distribution at this time as a function of the applied $\phi$. In general, this measurement of how the atomic “current” moves in the lattice depending on the phase can be thought of as CPR for the array, and we seek to show how stronger and stronger interactions can affect this CPR.

Figure 9.4: Wavepacket dynamics under interactions. (a) Top: Experimental setup of a 1D lattice with an initial state of equal population on sites $n = -1$ and 0 with a phase difference $\phi$. Middle: Our initial state mimics a wavepacket with some phase gradient is a rough approximation of a state with quasimomentum centered around $q = \phi/\pi d$. Bottom: Long time ($\Delta t$) dynamics of the wavepacket on the array. The position $\bar{n}$ reveals the group velocity ($\bar{n} = v_g\Delta t$, $v_g \sim \partial E/\partial q$), while the size of the wavepacket relates to the group velocity dispersion (GVD $\sim \partial^2 E/\partial q^2$). This picture works when an effective band picture is valid, but breaks down in the regime of MQST. (b) Sketches of band structure ($E(q)$), group velocity ($v_g \sim \partial E/\partial \phi$), and group velocity dispersion (GVD $\sim \partial^2 E/\partial \phi^2$) for a non-interacting 1D lattice system. (c-d) Color plots showing fitted site populations vs. initial relative phase $\phi$, taken after 2 tunneling times in the regime of (c) strong tunneling ($J/h = 1253$ Hz) and (d) weak tunneling ($J/h = 157$ Hz). Data are averaged over 10 experimental realizations, and were taken in a $2\pi$ range of $\phi$ values, then shifted to 0 to remove constant shifts from tunneling phases and mapped onto the extended zone scheme (see Sec. 9.6.3).
However, in the absence of interactions, we also connect the observables to some properties of the band structure of the lattice. The initial state can be seen as a very gross approximation of a quasimomentum state of the lattice, projecting onto a range of quasimomentum states centered around a value \( qd = \phi/\pi \) that is dependent on the applied phase. The duration of free evolution then maps from quasimomentum space to position space (position within the lattice), such that the position \( \bar{n} \) and spread \( \sigma_n \) after this time can be related to derivatives of the band structure, as shown by the sketches in Fig. 9.4(b). Specifically, the average position \( \bar{n} \) is directly proportional to the group velocity \( v_g = \bar{n}/\Delta t \), which is the first derivative of the band structure: \( v_g \equiv \partial E/\partial q \). Similarly, the spread of the wavepacket after some time \( \sigma_n \) relates to the group velocity dispersion, which is related to the second derivative of the band structure: \( \text{GVD} \sim \partial^2 E/\partial q^2 \).\(^2\) We note that under interactions, this simple interpretation of the observables in terms of the band structure fails.

Figure 9.4(c) shows an example of the fitted site populations from our data, taken for one tunneling value \( J/h = 1253(12) \) Hz in the strong tunneling regime after a duration of 2 tunneling times. Each horizontal line here is a different initial \( \phi \) value, and averaged over 10 individual shots. As expected of different initial “quasimomentum” states, the results for different \( \phi \) \( (q) \) show population flowing with different group velocities and spreading at different rates. To relate this plot with the band structure pictures, we focus on the \( \phi/\pi = 0.5 \) point (denoted by a yellow arrow), where the wavepacket has the furthest position (largest group velocity) and the smallest spread. Looking at the band structure \( E(q) \) in panel (b), this relative phase of \( \phi/\pi = 0.5 \) corresponds to the yellow point halfway to the edge of the band, \( q = \pi/2d \). Because this point on the band structure has maximal slope, the resulting group velocity is a maximum as well. This point also has the least variation in the slope (everything is close to linear), such that a spread of \( q \) values has little spread in group velocity, leading to a small group velocity dispersion and thus we see a tight distribution with little spread. This comparison gives us confidence that our initial state, an admittedly rough approximation of a wavepacket with some quasimomentum \( q \), provides the expected results of such a state.

\(^2\)Because our observable \( \sigma_n \) has to be positive, it is more like \( \sigma_n \sim |\partial^2 E/\partial q^2| \).
Figure 9.5: The skewed current-phase relationship of a synthetic Josephson array. (a) Average position $\bar{n}$ (related to group velocity) vs. $\phi$ for several values of tunneling. The sinusoidal current-phase relationship skews with increasing interaction strength, until total self-trapping. Circles are experimental data and lines are results from real-space GP simulations, with a mean interaction strength $U/h = 1568$ Hz and trap frequency 60 Hz. Data and simulations are offset by 2 for clarity. (b) Average distribution width $\sigma_n$ (related to group velocity dispersion, GVD) vs. $\phi$ for several values of tunneling. Under weak interactions, there is an asymmetry in GVD values at $\phi/\pi = \pm 1$ and $\phi = 0$, relating to higher band curvature at $\phi/\pi = \pm 1$. With increasing interactions, $\phi$-dependent self-trapping occurs, with initial states near $\phi/\pi = 0$ experiencing enhanced self-trapping. Data are averaged over 10 experimental realizations, and were taken in a $2\pi$ range of $\phi$ values, then shifted to 0 to remove constant shifts from tunneling phases and mapped onto the extended zone scheme (see Sec. 9.6.3).

9.4.3 Adding interactions

The extracted position and spread data are shown in Fig. 9.5. Under weak interactions (red data, same as Fig. 9.4(c)), we see the expected forms of the first and second derivatives of the band structure for $\bar{n}$ and $\sigma_n$, respectively. The average position reveals a close-to-sinusoidal relationship between the population current and the applied phase, as expected of an “ideal” Josephson junction array in the absence of interactions. The spread shows the expected double dip behavior expected of the GVD, though we note that the central peak at $\phi/\pi = 0$ is lower than the peaks at the edge of the band $\phi/\pi = \pm 1$. This asymmetry means that the band has higher curvature near the band edge compared to the center, revealing
effects even from weak interaction strengths.

With decreasing tunneling strength and thus increasing effective interaction strength $U/J$, the current-phase relationship in panel (a) becomes more and more skewed. In contrast to solid state junctions, where transport and the CPR may be affected by many different causes, here the skewing is caused solely by the attractive interactions. With strong interactions, the system enters a self-trapped state with very little dynamics. This can also be seen in the site populations shown in Fig. 9.4(d).

More information about this self-trapping can be read from the GVD plot in Fig. 9.5(b), where we observe two effects with increasing interactions: the central peak at $\phi/\pi = 0$ disappears, and the data shift towards negative $\phi$ values. The first indicates self-trapping that is dependent on the phase, as initial states with $\phi/\pi$ near 0 begin to show tighter distributions compared to states that begin near the band edge $\phi/\pi = \pm 1$. However, as reflected in the CPR data, for strong enough interactions, initial states with any $\phi$ become self-trapped. The shift towards $-\phi$ is more of a technical detail, stemming from our initial wavepacket being biased towards $-p$. That is, in real space the wavepacket starts with some momentum going in the $-p$ direction, and slows down as the harmonic potential grows in that direction.\(^3\)

All of our results match up very well with a real-space GP simulation performed by our collaborators at UT Dallas. This simulation accounts for our initial wavepacket bias towards $-p$, and also takes into account the inhomogeneous density distribution of the atoms, leading to very good agreement with all datasets.

We also ran simpler GP simulations assuming a *homogeneous* mean-field energy of $U/h = 1500$ Hz, as shown in Fig. 9.6. We first note the lack of a “shift” towards $-\phi$, as this simulation neglects effects of the real-space distribution, and the overall poor quantitative agreement with the data, especially in the $\sigma_n$ plot of panel (b). However, even this naive simulation captures the key qualitative features that we observe: the skewed CPR in Fig. 9.6(a) and the $\phi$-dependent self-trapping evident in the disappearance of the peak at $\phi = 0$ in Fig. 9.6(b). We must point out, however, that there is one very interesting facet

\(^3\)In hindsight, this could have been checked in experiment by redoing some of this data with a superposition of lattice sites 0 and +1, which should bias the data to $+\phi$
of this system that we were unable to explore experimentally: current reversal. For a small range of large interaction strengths right above the self-trapping regime, the current-phase relationship should actually invert, and atomic current should flow in the opposite direction. This behavior is shown clearly in the naive GP simulations of the bottom $J/h = 157$ Hz data in panel (a), though it is washed out by the inhomogeneous spread of interaction strengths present in our harmonic trap.

9.5 Conclusions and future work

In this work, we have further expanded the scope of synthetic lattices to include significant nonlinear interactions. This extension of our work in Chap. 5 allowed for direct observation of self-trapping on a flat lattice, asymmetric Bloch oscillations on a tilted lattice, and
both a skewed current-phase relationship and phase-dependent self-trapping on a Josephson junction array.

The extreme control of MSLs and synthetic lattices can offer new insights into the field of cold atom junctions and related fields like atom interferometry. Our study here was somewhat hampered by our long tunneling times and harmonic trap, both of which we are currently working to address, by accessing a Feshbach resonance to tune $U$ directly and implementing a blue-detuned box trap for homogeneous densities (see Chap. 10). With these improvements, this current study could be extended to realize current reversal on a junction array and explore more regimes of the phase diagram for Josephson junctions and junction arrays (varying the population imbalance and relative phase, both with and without a tilt). Related work could be to squeeze spin states in a double well or a larger system, and apply interactions to a topological system like the 1D Su-Schrieffer-Heeger model or a 2D Quantum Hall system like in Chap. 3.

9.6 Experimental details and other notes

9.6.1 1D lattice band structure

In all three experiments, we work with a simple 1D lattice with a cosine band structure. Because the form of this band structure is integral to our interpretation of the experiments, particularly in Sec. 9.4, here we go through a simple derivation.

The Hamiltonian for a 1D lattice is simply Eq. (9.2) without any lattice site energy shifts, and we can write it as

$$H = -J \sum_n (|n\rangle\langle n+1| + |n+1\rangle\langle n|).$$

Then, using the time-independent Schrödinger equation $E |\psi\rangle = H |\psi\rangle$ for some general
state $|\psi\rangle = \sum_n \psi_n |n\rangle$ for normalized coefficients $\psi_n = \langle n|\psi\rangle$, we get

$$E \sum_n \psi_n |n\rangle = -J \sum_n (\psi_{n+1} |n\rangle + \psi_n |n + 1\rangle)$$

$$E\psi_m = -J (\psi_{m+1} + \psi_{m-1}),$$

where we have applied $\langle m|$ to both sides of the equation. If we take $d = 2\hbar k$ to be the unit cell lattice spacing, then we can take the Bloch ansatz and assume the lattice is $d$ periodic, or $2\pi/d$ periodic in quasimomentum (reciprocal space): $q = q + 2\pi/d$.\textsuperscript{4} Thus we assume

$$\psi_m = e^{iqa}/\sqrt{N},$$

for $\psi_{m+1} = e^{iqd}\psi_m$, simplifying the above equation to get the form of the band structure,

$$E(q) = -2J \cos (qd),$$

which features a bandwidth of $4J$. It is important to note that each of our lattice sites supports only one quantum state, so there is only one band. This is in stark contrast to real-space optical lattices, which can support higher energy states in each site, leading to higher bands.

### 9.6.2 Tunneling calibrations and tunneling values

For all experimental data runs, we calibrated the value of tunneling by independently measuring two-site Rabi oscillations. That is, we turn on one tunneling link for various evolution times, and fit the resulting Rabi oscillations to get the effective amplitude of the tunneling link. We then generate the frequencies for a lattice experiment under the same experimental conditions (voltages, laser powers), scaling the tunneling amplitudes according to the calibration. We run the calibration immediately before and after each experimental data run, and average the two calibrated tunneling values to account for drifts over time.

Here we list the exact calibrated tunneling values and their corresponding error for all of the data presented in the three experiments, as the errors are not directly shown in the figures for clarity of presentation.

\textsuperscript{4}In some sense, this should be called “quasiposition” since our lattice itself begins in momentum space.
• Fig. 9.2(b-c) (dynamics): \( J/h = [1281(5), 93.3(2)] \) Hz

• Fig. 9.2(d-e): \( J/h = [1250(9), 1000(7), 900(7), 700(5), 600(4), 500(4), 400(3), 350(3), 300(2), 250(2), 200(1), 150(1), 125(1), 100(1), 75(1), 50.0(4)] \) Hz

• Fig. 9.3: \( J/h = [1270(4), 793(3), 594(2), 397(1), 299(1), 192(1), 103.7(4)] \) Hz

• Fig. 9.4(c) and Fig. 9.5: \( J/h = [1253(12), 583(6), 402(3), 257(3), 157(1)] \) Hz

### 9.6.3 Initialization for CPR data

For the data in Fig. 9.4(c) and Fig. 9.5, we initialized the system with equal populations in lattice sites 0 and \(-1\) with some phase. We sent a square \( \pi/2 \) pulse to the \( 0 \rightarrow -1 \) tunneling link by turning on the link for one fourth of a Rabi period, and applied some phase within the range \( \phi/\pi = [-1, 1] \). The atoms pick up an additional, constant phase during this initial tunneling process, which we accounted for by shifting all results by \(-0.85\pi\), leaving us with shifted phases in the range \( \phi/\pi = [-0.95, 0.95] \). We then mapped the data onto the extended zone scheme (copying \( \phi/\pi = [0.05, 0.95] \) to \( \phi/\pi = [-1.95, -1.05] \) and copying \( \phi/\pi = [-0.95, -0.05] \) to \( \phi/\pi = [1.05, 1.95] \)).

### 9.6.4 \( s \)-wave halos

As previously discussed in Sec. 3.5.1, during our time-of-flight imaging procedure, atoms in different momentum states fly through each other, and can scatter into a full \( 4\pi \) steradians. These \( s \)-wave halos can be seen in our data as population between lattice sites, like the blue smear between sites 0 and \( \pm 1 \) in Fig. 9.3(b) (though this is also a signature of some thermal fraction of atoms not in a BEC). This leads to some loss in our fitted site populations and the amount of loss depends on the distribution of atoms - population split between two sites would have a lot more scattering than population wholly on one site, for example. Luckily, though the effect is visible in the data, it doesn’t seem to significantly impact the resulting observables.

We also note that we ran simulations of the Bloch oscillation dataset that factored in loss, and found little effect, choosing to show the 0 loss simulation results instead (specifically, I
ran the simulations that Bhuvanesh wrote).

9.6.5 Momentum states spreading in real space

Atoms in momentum states naturally move in real space and eventually lose spatial overlap, leading to decoherence and placing a limit on our experimental timescales. This issue is worsened by the low tunneling rates that we use to achieve high $U/t$ ratios, which leads to long tunneling times and thus long evolution times for these states to drift. This effect can be seen quite clearly in the $J/h = 299$ Hz experimental data in Fig. 9.3(b) (left column, third row), where atoms in the $n = -2$ lattice site drift leftwards as time increases. In this plot, we explore dynamics out to 8 tunneling times (4.25 ms) of evolution time under the lattice, but the last third (or even half) of this range shows little change in the populations as the system loses coherence.

Yet even before the system totally decoheres, the drifting momentum states complicates our fitting procedure. For typical (non-drifting) data, we fit the lattice site populations with a multi-Gaussian function which has the position of each lattice site bounded to within $\sim 10$ pixels of a “guess” position. For each dataset here, we note the lattice site positions at both short times and long times, and linearly interpolate between them as starting guesses to the fit function. Because atoms are constantly tunneling in and out of these drifting momentum states, we do not expect the states to “move” at their respective momenta - for example, the $n = -2$ state we mention above shows some jittery motion rather than a smooth line. Still, this linear function seems to work well for the fits, although we must perform the procedure independently for different datasets as the states respond differently.
Chapter 10

Future Work

This section details the steps needed to construct the new $^{39}$K apparatus (Spring 2019–Present), and details a future experiment planned for this new setup.

10.1 Switching to $^{39}$K

In this section, I will give a brief description of the motivation, current status, and plans of switching atomic species from $^{87}$Rb to $^{39}$K. For a more thorough description of the switch, including detail on the level structure of $^{39}$K, the Feshbach resonance of $^{39}$K, and the hardware we need for the switch (in particular the MOT and Feshbach magnetic field coils), see Sec 3.3 of Eric’s thesis [29]. I also want to point out the great resource that is Tobias Tiecke’s “Properties of Potassium” document [264].

10.1.1 Motivation and some history

Our switch to $^{39}$K from $^{87}$Rb was motivated by the tunability of potassium’s atomic scattering length via its more accessible Feshbach resonance, which would enable us to directly vary the strength of atomic interactions in our lattice. At the same time, we had several ideas of other improvements to the apparatus, including a uniform trapping potential (for a uniform atomic density), imaging from multiple directions, and a better attempt at lattices in multiple dimensions. However, switching atomic species is the same as destroying and completely rebuilding an entire experiment, and we would have held out for a few more years... but our hands were forced by the total and utter depletion of our rubidium source.

The $^{87}$Rb apparatus had three atomic dispenser tubes as its atomic source, and throughout fall 2018, we depleted one, then another, and finally the very last dispenser until our
beloved $^{87}$Rb apparatus breathed its last in December 2018.\(^1\) We were still optimistic as Bryce had the foresight to include three potassium dispenser tubes in the same source cell, and Jackson’s neighboring experiment with potassium already had all of the correct laser frequencies. So, we directed some $^{39}$K light into our source chamber, and saw... nothing. At that point in spring 2019, we had been suspecting a leak in the vacuum chamber for months (which we could not fix by spraying Vacseal in various nooks and crannies), as we were constantly seeing high pressure readings on our smaller 25 L/s ion pump (the one near the source cell). So, we decided to break vacuum and replace the source cell with one containing 6 potassium dispensers, replace the small ion pump, and replace the awkwardly small science cell with a larger one for better optical access. Because the vacuum chamber was filled with rubidium atoms from years of running experiments, and because these atoms could react violently with air, the vacuum chamber needed to be opened and immediately pumped with argon gas (inert). Luckily, I was away on travel while Eric and Bryce led this scary process of swapping the vacuum parts (July 2019), and it seemed to go smoothly.

10.1.2 Potassium-39

Laser cooling $^{39}$K down to BEC is a bit trickier than the $^{87}$Rb process described in Sec. 2.1 for three main reasons:

1. **Lower vapor pressure.** We need to heat the source cell with heater strips to about 50°C to have enough potassium to trap in a 2D MOT. Easily dealt with, but also annoying since it makes detecting $^{39}$K much harder than $^{87}$Rb (which is as bright as the sun in comparison).

2. **Dense hyperfine structure.** This makes sub-Doppler cooling on the D\(_2\) line much more challenging, and people often use a combination of D\(_2\) molasses and D\(_1\) “gray molasses” to achieve sub-Doppler temperatures [265, 266].\(^2\)

\(^1\)Near the end, we were putting upwards of 5–6 A of current through multiple tubes, heating to such a high temperature that we could actually see them glowing by eye. Brian DeMarco commented that under such conditions, all of the atoms would be blasted off in minutes.

\(^2\)The gray molasses technique, first realized in 1995 [267, 268], is again polarization gradient cooling, but with 3 levels and alternating between dark and bright states (hence, “gray”). For more details on the technique in $^{39}$K specifically, see Refs. [265, 266].
3. **Negative scattering length.** Under zero field, $^{39}$K has a scattering length of roughly $-30a_0$ (compared to $100a_0$ for $^{87}$Rb), and such attractive interactions in a BEC leads to collapse [269, 270]. Thus we need to tune the scattering length to be positive (repulsive interactions) via the Feshbach resonance before the final evaporation to BEC.

More generally, switching atomic species to $^{39}$K means we must change the entire laser system, as both the $^{39}$K D$_2$ line (near 767 nm) and the D$_1$ line (near 770 nm) are far from our old laser wavelengths near 780 nm, and cannot be reached with acousto- and electro-optic modulators. This requires tearing down the existing $^{87}$Rb laser system and tuning one of the lasers down by 13 nm to the D$_2$ line (and in the process discovering that the diode is broken, leading to more headaches). Our plan here is to use one laser for both the cycling and repump transitions of the D$_2$ line (detuning of about 434 MHz) and a separate laser for the cycling and repump transitions of the D$_1$ line (detuning of about 420 MHz). Much of the D$_2$ laser system was set up well in advance of the switch over summer 2018 (Samantha Lapp and Hannah Manetsch) and fall 2018 (Zejun Liu), and it was finalized later in 2019. For the D$_1$ laser, we tapped into our local resources: a separate experiment in our lab (headed by Jackson Ang’ong’a) uses $^{39}$K as well, and has an already set up D$_1$ laser. We are planning to share this single D$_1$ laser between the two experiments, with individual power/frequency control set by acousto- and electro-optic modulators.

### 10.1.3 Changes to the apparatus

Here I will detail the changes we have made/plan to make to the apparatus. The changes necessary to make a $^{39}$K BEC and run MSL experiments are as follows:

- **Swap vacuum parts:** Swap source cell, science cell, and small ion pump.

- **D$_2$ laser system:** Build a new laser system at $^{39}$K D$_2$ wavelengths (767 nm).

- **D$_1$ laser system:** Build a laser system for D$_1$ sub-Doppler cooling, branching off from Jackson’s laser.

- **Feshbach field coils:** Build water-cooled high current Feshbach coils.
• **Re-build all optics going towards vacuum chamber:** Optics are torn down to re-bake the vacuum chamber. New layout and planning required.

However, we also wanted many “optional” changes to the apparatus that would ideally make our lives easier and enable more capabilities with the MSL. These changes are as follows:

• **New MOT field coils:** Improved MOT field coils and coil holders, with more thought given to coil mounting and allowing for maximal optical access. Sort of necessary, as the MOT coils and Feshbach coils must fit well in one space around the science cell.

• **Imaging lines from multiple directions:** Imaging solely from one direction led to very difficult alignment of the optical trapping beams. Imaging along multiple directions would help a LOT.

• **Digital micromirror device (DMD) & blue-detuned beam:** A DMD can shape a laser to project an arbitrary potential (in space), useful for the donut mode of the DSPOT. Paired with a green laser, this can also create repulsive optical traps.

• **A “sheet” beam:** By driving an acousto-optic deflector (basically an AOM) at a rate faster than the atoms can respond, we can create a time-averaged sheet of light as a flat trap for the atoms (also called a “painted potential”). Can also be done with cylindrical lenses.

• **New lattice setup:** Instead of one beam and its reflection as the lattice beams, we plan to implement three beams in either a “T” or “Y” setup. This requires active phase stabilization among all beams.

• **Microscopy?** Two high numerical aperture objectives can be placed above/below the cell for high resolution imaging in real space. Possibly useful, even for momentum-space lattices.

• **Various electronics:** We want to swap out many old/dying electronics like frequency source boxes (for AOM rf inputs), magnetic field switch and current control circuits, etc.
10.1.4 Progress so far

Starting from summer 2018 through early 2019, we set up the D$_2$ laser system, built various electronics, and cleaned the new vacuum parts. In late 2018 through early 2019, Eric designed, built, and tested the coil holders and coils for the 2D MOT, 3D MOT (small gradient coils and large shim field coils), and Feshbach fields. In July 2019, we (sans me) completed the swap of the vacuum parts and began baking and pumping down the vacuum system. In fall 2019, we fired off the $^{39}$K dispensers and saw fluorescence to much delight, eventually forming a 2D MOT and pushing atoms to the science cell to trap in a 3D MOT. The 2D MOT setup is much the same as the previous $^{87}$Rb iteration of the experiment, but because we want additional beam lines on the science cell side, the 3D MOT setup has been changed significantly. Simultaneously, we worked on building up the laser system for the optical trapping beams.

Due to COVID-19, this has been the state of the apparatus for a few months, and we have been working on simulations and theses. Once lab work starts up again, the next steps are, in chronological order:

1. **Cicero**: For the $^{39}$K experiment, we have so far NOT used our control software, as the 2D MOT and 3D MOT do not require precise timings. Going forward, we must return to our good friend Cicero.

2. **Imaging**: We have used fluorescence imaging so far with a small ThorCam camera. The absorption imaging line must be set up with the more heavy-duty Andor camera.

3. **Sub-Doppler cooling**: D$_1$ laser system needs to be set up. Also may need to look into setting up the DMD for DSPOT, and look into compressed MOT.

4. **Optical trapping**: We plan for two OT beams (1064 nm and 1070 nm) to run along each of the two horizontal MOT beam paths (a total of 4 beams), with small ThorCam cameras at the end of each line for OT alignment.

5. **Feshbach coils**: The water cooling on the Feshbach coils have not yet been tested. Must also test effect on scattering length, as we need to shift to positive scattering lengths in preparation for evaporation.
6. **Evaporative cooling**: A lot of probing in the dark here (tweaking evaporation trajectories).

7. **Uniform potential trap**: Set up sheet beam for red-detuned trap with flat potential. Eventually this may be replaced with a blue-detuned “box” trap using a green laser.

8. **Lattice beams**: We plan to implement 3 separate lattice beams in either a “T” or “Y” configuration [83], and these must be phase stabilized. The phase stabilization can be done in parallel with any of the other steps.

### 10.2 Atomic STM

This section details an experiment that we attempted to perform on the old $^{87}$Rb setup, but found it was hindered by the presence of atomic interactions, which can be tuned out on the new $^{39}$K setup using a Feshbach resonance.

The idea here is a cold atom-optical lattice analogue of scanning tunneling microscopy (STM), using a probe (lattice site 0) weakly coupled to a system (lattice sites 1+) to measure the energy spectrum of the system. By tuning the energy difference between the probe and the system and measuring the loss rate of population into the system, we should be able to map out the eigenenergies of arbitrary 1D lattices. However, in taking data on the $^{87}$Rb setup, we found that the nonuniform atomic interactions in our harmonic trap smeared out any signal for all but the most basic systems. Thus we hope to perform this experiment in full on the new $^{39}$K setup, where a uniform trapping potential and accessing the Feshbach resonance to tune the scattering length to 0 should both allow for this atomic STM technique to work.

This project was the focus of my prelim exam, and one primary goal of this project was to measure the Hofstadter butterfly energy spectrum of the Aubry-André model. In this section, I will describe the technique, show preliminary data that we took on the $^{87}$Rb setup for a double well, a triple well, and a flat 1D lattice, and present our proposed measurements of the Hofstadter butterfly spectrum.
10.2.1 Overview and motivation

As mentioned in Sec. 2.3.1, the experiments we have performed on the momentum-space lattice fall generally into one of two categories:

1. Out-of-equilibrium dynamics (quenching on the lattice)

2. Adiabatic ramps into eigenstates (slow parameter ramps)

We have been limited to these two types of measurements because our atoms necessarily begin with zero momentum, forcing each experiment to start with all population in a single lattice site. For many systems (i.e., a simple 1D lattice), this highly localized state does not resemble the ground state or any eigenstate, and is thus far out of equilibrium. Thus we can suddenly apply the laser frequencies and observe dynamics in this out-of-equilibrium regime.

Ideally, we would like to prepare arbitrary initial states before turning on the full lattice, but the experimental timescales are limited to less than 10 tunneling times because atoms in different momentum orders fly apart and lose coherence (see Sec. 2.3.3). So while we have initialized the atoms in states spread out over a few lattice sites (sites 0 and −1 in Sec. 9.4, and sites 0 and +2 in Ref. [18]), we cannot, for example, load the ground state of a 21-site 1D lattice.

However, eigenstate behavior is crucial to the study of lattices and concepts like topology. So in studies like Ref. [18], Chap. 6, and Chap. 8, we have adiabatically loaded into eigenstates from our initial state of all atoms in one site, usually by ramping up lattice tunnelings from 0 to their final value. By varying the lattice parameters slowly enough, this should adiabatically load atoms into the desired eigenstate of the final system. While we have seen success in realizing some states far separated from the rest of their respective spectra (states living in a band gap [18], or states at the edge of a band like the ground state and highest excited state [15, 17]), we have been unable to measure others that are more closely spaced. This is again due to the same timescale limitation: if we could ramp our lattice parameters infinitely slowly, we should be able to adiabatically load into any eigenstate. But because we can only ramp for so long before the atoms lose coherence, we are forced to change our parameters non-adiabatically, and thus populate other states beside the desired one. Eric
Figure 10.1: Atomic STM. (a) In scanning tunneling microscopy (STM), a probe tip is brought close to a sample (system). Under an applied voltage, electrons may tunnel across the gap (vacuum) into the system, provided there are available eigenstates at the corresponding energy and position. This results in a measurable current. Image adapted from Hoffman Lab website [271]. (b) In atomic STM, all atoms begin within one “probe” lattice site ($n = 0$), which is weakly coupled ($t_{inj}$) to a 1D lattice system (sites $n > 0$). By varying the energy $E_{inj}$ of the probe lattice site, atoms can tunnel across the weak link into the system, provided there are eigenstates at the corresponding energy and position (i.e., the eigenstate must overlap with site 1). This results in a measurable population loss rate out of the probe site.

has recently conducted a study [27] on counter-diabatic driving of lattice parameters to load eigenstates in a much shorter duration, and has shown the technique on the states of a particle in a box.

The two measurements of dynamics and eigenstate preparation, even without limitations to the experimental timescales, do not reveal the full information about a lattice system. Here, we propose an “atomic STM” technique that supplements these measurement techniques by measuring the eigenenergies of a system.

### 10.2.2 The atomic STM technique

In regular scanning tunneling microscopy (STM), a probe tip is brought close to a sample, and a voltage is applied across the gap, as shown in Fig. 10.1(a). Electrons may tunnel across the gap (which is vacuum) into the sample if there are available states at the corresponding
energy at the position of the probe, leading to a measurable current proportional to the integral of the local density of states of the sample. By tuning the voltage and thus energy difference between the probe and the sample, and then by scanning across the entire sample, this technique can map out the full density of states.

In our proposed “atomic STM” technique shown in Fig. 10.1(b), we mimic most of these aspects. Atoms all begin in one “probe” lattice site (site \( n = 0 \)), and are coupled by a weak link \( t_{\text{inj}} \) to a 1D lattice system (sites \( n > 0 \) with some characteristic tunneling \( t )\).\(^3\) In the weak coupling limit \( (t_{\text{inj}} \ll t) \), atoms in the injection site couple not just to the leftmost site of the system (lattice site 1), but to the eigenstate wavefunctions of the system. That is, atoms that tunnel from the probe site into the system populate whatever available eigenstates \( |\psi_i \rangle \) that overlap with lattice site 1. If these eigenstates are delocalized across the lattice, the atoms move out into the lattice, which is made large enough that the atoms will not return to the probe site. The “loss” rate \( \Gamma \) of atoms into the system is then a measure of the local density of states at site \( n = 1 \), but scaled by \( t_{\text{inj}} \) and by the overlap of the eigenstate wavefunctions \( \psi \) with the edge of the system:

\[
\Gamma \sim \text{DOS}(n = 1) \\
\sim t_{\text{inj}} \sum_i |\langle n = 1 |\psi_i \rangle|^2 \delta (E_{\text{inj}} - E_i),
\]

where \( |\psi_i \rangle \) and \( E_i \) are the eigenstates and eigenenergies of the system.

So, the population of atoms in the initial probe site at some energy \( E_{\text{inj}} \) can be used to measure if the system has any states at that energy. By varying \( E_{\text{inj}} \), we can map out the energy spectrum of the system by simply measuring the population in the injection site. We note that while this technique can probe delocalized states or states localized to the left edge of the system, it fails to capture any states that do not overlap with site 1, important for systems with many localized states. We can, of course, inject into the other edge of the system, but this still does not fulfill the “scanning” part of STM: we can only probe the ends of a 1D system.\(^4\) If we add a second laser to introduce a second dimension (as in Ref. [19]),

\(^3\)The “inj” stands for “inject”, as we are injecting atomic population into the system.

\(^4\)Maybe this 1D version of the technique should be called atomic TM?
we can then implement a true scanning technique, allowing us to inject atoms into every site of the system.

The loss features are broadened by the nonzero coupling strength $t_{\text{inj}}$, so in experiment, we strive to make $t_{\text{inj}}$ far smaller than any spacing in the system's energy spectrum. The more pressing issue is atomic interactions. Because interactions in our system behave as an on-site attraction that drags down the local chemical potential, we expect interactions of strength $U$ to shift loss features by $U$. However, atoms have nonuniform density in our harmonic trap, leading to nonuniform interactions and a spread of interaction-induced shifts to the loss features. Thus any states closely spaced in energy get broadened and the measured loss features get smeared together. As we see below, this occurs for systems as simple as a triple well, which has just three states.

### 10.2.3 Preliminary data on the $^{87}\text{Rb}$ apparatus

Back in October 2018, we implemented atomic STM with the $^{87}\text{Rb}$ apparatus, probing the energy spectra of three simple systems: a double well, a triple well, and a 1D lattice with 26 sites.
Double well

First, we examined the simplest possible system: a double well (Fig. 10.2). For this experiment, we set $t = 1598(7)$ Hz and $t_{\text{inj}} = 0.025t \approx 40$ Hz, and swept $E_{\text{inj}}$ across a range $-5$ kHz to 5 kHz. As shown in Fig. 10.2(b) for $E_{\text{inj}} = 0$, we monitored the population in the injection site over an evolution time that ranged from 0 to 80 ms, and extracted a “loss time” from an exponential decay fit to these dynamics. We note that this data and the experimental setup is very similar to the “strong loss” regime that we studied in Chap. 7, in particular similar to the data of Fig. 7.3(e), albeit with a much weaker $t_{\text{inj}}$ here.

By extracting these loss times for all $E_{\text{inj}}$, we map out the energy spectrum of the double well in Fig. 10.2(c). There are two clear loss features at $-t$ and $+t$, corresponding to the two states of the double well: the lower energy symmetric state and the higher energy antisymmetric state, respectively. While we expect the dips to be broadened on the order of $t_{\text{inj}} = 0.025t$, these dips are an order of magnitude wider, indicating the effect of the inhomogeneous interactions.

Triple well

After taking the double well data, we were fairly discouraged, as even for the simplest possible system with just two states, the broadening of each state due to inhomogeneous interactions
was huge. Thus when our triple well data of Fig. 10.3(a) looked nearly unrecognizable, it wasn’t much of a surprise. For this data, we tried to minimize the effects from interactions by increasing the tunneling to \( t = 2040(2) \) Hz \( (t_{\text{inj}} = 0.025t \approx 51 \) Hz\), thus separating the states as much as possible. However, even under such high tunnelings we observed significant loss for all applied \( E_{\text{inj}} \), observing only wisps of the expected features, and not at the expected energies \( E = 0, \pm \sqrt{2}t \).

**1D lattice**

One system that we could still probe even under such conditions, though, was a flat 1D lattice with 26 sites. Because this system supports 26 states, we expect to see one large loss feature for the entire band of states, with a width corresponding to the bandwidth of the system, \( 4t \) (see Sec. 9.6.1 for a simple derivation). In Fig. 10.3(b) we show our data, which shows quite roughly the correct bandwidth. Here, we plot the population in the probe site after 20 ms of evolution time, and find it is also a good observable for this experiment.

The tunneling values used here were relatively weak, with \( t/h = 492(10) \) Hz and \( t_{\text{inj}}/h = 0.1t/h \approx 50 \) Hz. Under such weak tunneling, the interaction effects become more prominent, and we see an overall shift of the band by an average \( U \sim 1.5t \) away from \( E_{\text{inj}} = 0 \), though we note that the inhomogeneous interactions also broaden the loss feature.

**A minor experimental note: waveform duration**

As a note for future experiments... one additional technical hiccup is the limit on the duration of the waveform that controls the Bragg frequencies of the lattice. The onboard memory of our arbitrary waveform generator (AWG, model Keysight 33612A\(^5\)) can only store waveforms of \( \leq 3 \times 10^6 \) points, which we run at a minimum of \( \sim 500 \) MSamples/s.\(^6\) Thus we are forced to use waveforms less than roughly 10 ms long. For these atomic STM studies with such small \( t_{\text{inj}} \) values, we need to measure the injection/loss rate into the system

\(^5\)This is affectionately called “Newbo” for being the NEW “Dumbo,” our old AWG.

\(^6\)This is not the Nyquist frequency for our \( \sim 80 \) MHz frequencies, but rather an empirically-determined value: frequencies smaller than \( \sim 450 \) MSamples/s led to incomplete Rabi oscillations, with total suppression below \( \sim 400 \) MSamples/s. Though this should be checked (it’s in an old lab notebook), because I’m recalling these numbers from memory due to COVID quarantine.
after long evolution times (80 ms for the data in Fig. 10.2).

To reach evolution times longer than the length of the waveform, we simply repeat the entire waveform many times until the final evolution time. Because the phase and amplitude isn’t the same at the beginning and end of the multifrequency waveform, this introduces a discontinuous jump between the end of the waveform and the beginning of the next cycle. While this has minimal effect on the frequencies within the beam, it still results in sudden jumps in the applied laser power and resulting population dynamics, and is visible in Fig. 10.2(b), particularly at 5 ms and 20 ms. Two things can be done to lessen this effect: change the way we normalize the waveform, and change $\cos$ to $\sin$.

Our generated waveforms are normalized by the AWG to their maximum value, which is then set to 5 Volts (which is changeable, but we don’t touch it). However, because we often change our waveforms for calibration (one resonant link for Rabi oscillations) vs. data taking (many resonant links), or change the amplitude of the tunneling links, the maximum value of the sum of the many frequencies over the waveform duration is always changing. To keep this constant through an experiment, I append onto the end of the waveform two points (positive and negative) which are larger than all other values, setting the scale for the entire waveform. These end normalization points are never meant to be reached (we turn off the drive before this is through), since they cause a huge spike in the laser power. However, if we need to repeat waveforms, we cannot include a huge spike every few milliseconds. To properly normalize the waveforms, I added an appropriate number of very far off-resonant frequencies (addressing the $n = 49$ to $n = 50$ transition, for example) with appropriate amplitudes, such that the sum of all the amplitudes of the individual frequencies would be the same. To be specific, we generate waveforms by summing together many $\cos(2\pi \omega_n t)$ in Mathematica, so that the very first point is the maximum value of the entire waveform. Of course, this fails when any of the tunneling links is out of phase (i.e., when we want a tunneling phase), but in that case, I would create the waveform and find its maximum and minimum values, then set the amplitudes of the far-off-resonant frequencies appropriately. By implementing this off-resonant frequencies approach, I was able to lessen the drastic jumps in the loss dynamics.

However, because we use $\cos$ instead of $\sin$ to represent the frequencies, this again
introduces a sudden blast of laser power as the waveform repeats and experiences the first, maximum value. Ideally, we would want to switch all our cosines to sines to avoid this problem in the future. However, for these data we didn’t switch because we had already begun taking data and wanted to be consistent.

The long-term fix is to simply buy a new waveform generator with much larger onboard memory and more functionality regarding arbitrary waveforms. This would bypass the issue entirely, but I haven’t looked into the available options. Also, I want to point out that the manual for our current AWG says it only can hold arbitrary waveforms with up to $1 \times 10^6$ points, but that is clearly wrong since we’ve been able to store, run, and see dynamics for waveforms up to three times that length.

10.3 Atomic STM: Measuring the Hofstadter spectrum with the 1D Aubry-André model

With the upgrades to the apparatus, we should bypass the issue of inhomogeneous interactions and be able to measure energy spectra of more complex lattices. And what better system to study than the one model that I have written about in Chaps. 3, 4, 6, and 8? I am referring to both the quasiperiodic 1D Aubry-André model and the 2D Harper-Hofstadter model describing the quantum Hall effect. It turns out that the 1D Aubry-André model is a dimensionally-reduced version of the 2D Harper-Hofstadter Hamiltonian, and displays the same striking fractal energy spectrum of Hofstadter’s butterfly.

10.3.1 Hofstadter’s butterfly

We first forget about the Aubry-André model and focus on the 2D case of electrons on a lattice under a magnetic field $B$, the situation which leads to the quantum Hall effect described in Sec. 3.1.1. This system features interplay between the lattice and the magnetic field, both of which have characteristic length scales: lattice spacing $a$ (and corresponding frequency $2\pi\hbar/ma^2$) vs. the magnetic length $\sqrt{\hbar/eB}$ for cyclotron frequency $eB/m$. The energy spectrum of the system gets split into both Bloch bands and Landau levels, but
because the ratio between these two (frequencies, length scales) may be incommensurate, the spectrum takes on the beautiful butterfly-like fractal pattern shown in Fig. 10.4(a) [182]. The horizontal axis here is the ratio of the two characteristic frequencies, $\alpha = e a^2 B / h$, which is also (not coincidentally) the ratio between the magnetic flux and the elementary flux quantum, $\alpha = \Phi / \Phi_0$. Plotted here is a sort of density of states, where each eigenstate is a dot, such that a band of states appears as a vertical line on the plot. From here, we can see that for rational values of $\alpha = p/q$ (integers $p$ and $q$), the spectrum splits into $q$ bands.

However, as we have explored in Chap. 3, the quantum Hall model is topological, and the topology can be reflected in the energy spectrum. Fig. 10.4(b) shows the butterfly, but with colors corresponding to the Hall conductance [272]. These butterfly spectra were calculated on a torus, so there are no boundary states in the gaps between bands. In our MSL system with open boundary conditions, we should be able to additionally observe edge states crossing between bands.

In the original system of electrons on a 2D lattice under a magnetic field, the butterfly spectrum is impossibly difficult to measure. Due to the small lattice spacing of real crystals (on the order of 2 Å), huge magnetic fields would be required to reach a comparable magnetic length (and thus non-negligible $\alpha$). However, recent work using bilayer graphene has effectively made larger lattice spacings by using Moiré superlattices, allowing for the
observation of the butterfly [273–275]. The butterfly has also been caught with microwave waveguides [276] and superconducting qubits [277], but a cold atom realization has been lacking.

10.3.2 Dimensionally reducing to 1D Aubry-André

Our approach follows the path of the superconducting qubit study [277] by measuring the spectrum not of the 2D model, but of the dimensionally-reduced 1D Aubry-André model. As explained in Sec. 3.1.1, the 2D Harper-Hofstadter Hamiltonian can be written as

$$\hat{H}_{2D} = -t_x \sum_{n,m} (c_{n,m+1}^\dagger c_{n,m} + \text{h.c.}) - t_y \sum_{n,m} (c_{n,m+1}^\dagger c_{n,m} e^{i2\pi \alpha n} + \text{h.c.}),$$  \hspace{1cm} (10.3)$$

where we have written Eq. (3.1) in the Landau gauge. Here, $t_x$ and $t_y$ denote tunnelling in the $x$ and $y$ directions, and an electron that tunnels around a 4-site plaquette picks up a phase $2\pi \alpha = ea^2B/\hbar$ as shown in Fig. 3.1.

We map this to a 1D model by assuming periodic boundary conditions in the $y$ direction, and taking a quantum fourier transform along that direction,

$$c_{n,m}^\dagger = \sum_k e^{-ikm} c_{n,k}^\dagger,$$

such that $\hat{H}_{2D} = \sum_k \hat{H}_k$ gets us the 1D Hamiltonian

$$\hat{H}_k = -2t_y \sum_n \cos (2\pi \alpha n + k) c_n^\dagger c_n - t_x \sum_n (c_{n,k}^\dagger c_{n+1,k} + \text{h.c.})$$  \hspace{1cm} (10.5)$$

$$\hat{H}_{\text{AHH}} = \sum_n \Delta \cos (2\pi bn + \phi) c_n^\dagger c_n - t \sum_n (c_n^\dagger c_{n+1} + \text{h.c.})$$  \hspace{1cm} (10.6)$$

where in the last step we have dropped the $k$ subscripts, and made various redefinitions of variables to recover the Aubry-André-Harper Hamiltonian! The two important definitions here are $b = \alpha$ and $\phi = k$, allowing us not only to probe a wide range of $\alpha$ values to see the butterfly, but also letting us probe the band structure $E(k)$ by simply varying the Aubry-André phase $\phi$.  

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Figure 10.5: Hofstadter's butterfly from atomic STM of the Aubry-André model. (a) Cartoon of the setup: atoms in a probe site are weakly coupled to the Aubry-André system with amplitude $\Delta$ and variable periodicity $b$. (b) Numerical simulations of a single atomic STM run at $\Delta/t = 1.8$ and $b = 1/3$, showing the expected three loss features corresponding to the three bands. (c) Full butterfly spectrum, run for many $b$ from 0 to 0.5, at $\Delta/t = 2.5$. Minute details for small $b \lesssim 0.2$ cannot be easily distinguished, but large scale structure is evident.

We have explored this model, shown in Fig. 10.5(a), in depth in Chaps. 4, 6, and 8, but have always focused on its localization properties under an incommensurate periodicity $b$. Here, we vary $b$ and examine the resulting fractal spectrum.

10.3.3 Simulation results: the butterfly

Fig. 10.5(b) shows numerical simulations of one run of atomic STM on the Aubry-André model, taken for $\Delta/t = 1.8$ and $b = \alpha = 1/3$. As expected, there are three loss features corresponding to the three bands of the system under such a value of $b$.

We note that, as previously studied, the Aubry-André model features a transition from delocalization to localization at $\Delta/t = 2$. Because atomic STM in 1D can only measure states with overlap at site $n = 1$, we cannot work in the deeply localized regime $\Delta/t \gg 2$. Additionally, we found empirically that working near the critical point $\Delta/t = 2$ offers the best numerical results.

In Fig. 10.5(c) we plot the full butterfly spectrum by taking many such atomic STM runs for different $b$ at $\Delta/t = 2.5$. Here, each vertical slice is one scan like in panel (b), and the colors represent the probe population after some long time: lighter/bluer regions correspond
Figure 10.6: **Edge states and band structure of the Aubry-André model.** (a) Numerical simulations showing “band structure” of the Aubry-André model by varying $\phi$ with atomic STM, for parameters $\Delta = 1.8t$, $t_{\text{inj}} = 0.05t$, and $b = 1/3$, averaged over probing at both the left (L, as in Fig. 10.5) and right (R) sides of the system. Edge states live in the gaps between with boundary states labeled with corresponding injection direction. (b) Same simulations as (a), but taking the difference between probing at the left and right sides of the system to isolate the boundary states.

to higher population loss, showing where the energy states lie. Overall, the results uncover the characteristic fractal butterfly pattern, and we note that while here we only explicitly show half the pattern, it is mirrored across $b = 0.5$ to give the full butterfly.

For some more insight into the results, we can look at $b = 1/2$, for which the site energy of every other site is the same. This leads to a two-band spectrum, which shows up as two blue regions along the vertical cut at $b = 1/2$. Similarly, there are three bands at $b = 1/3$ (as shown in panel (b) as well), four bands at $b = 1/4$, and so on. For smaller and smaller values of $b$, the energy spectrum becomes more and more dense as the number of energy bands increases. In experiment, it is unlikely that we will be able to distinguish the many bands in this region, and will likely fail to capture the fine detail for $b \lesssim 0.2$.

Finally, we note that this plot was taken for a phase $\phi$ that varied with time, similar to integrating over all $\phi$. This enabled us to probe all of the states, as at some specific $\phi$ some states may not have much overlap with the one site $n = 1$ we are probing.
10.3.4 Simulation results: band structure and boundary states

The butterfly spectrum of Fig. 10.5 is a sort of integrated band structure that averages over all $k (\phi)$. We can also look at the band structure itself by varying $\phi$ directly. Furthermore, since the system has open boundary conditions on both ends, we should be able to see the topological states that live at these boundaries. By injecting into both the left (as in Fig. 10.5) and right sides of the system, we expect to measure something like Fig. 10.6, showing boundary states that exist in the gaps between energy bands.

This simulation was run with $b = 1/3$ to give three energy bands, and the plotted values are averaged over injection from both ends. Probing from the left edge of the system gives only the states marked “L” and the same with states on the right edge (“R”). In panel (a) we plot the average of these two to show the full band structure, and in panel (b) we plot the difference to show only the boundary states. To supplement this measurement, we hope to extract information about the topological indices of the various bands, possibly in collaboration with a theory group. This scheme should complement the butterfly spectrum of Fig. 10.5(c) by providing information about the topology of different regions in the pattern.

Taking this topological state measurement should be straightforward, but could be made easier experimentally by measuring in the localized regime of the AAH model. As touched on before, the AAH model exhibits delocalized eigenstates for $\Delta/t < 2$, and localized eigenstates for $\Delta/t > 2$. We performed the numerical simulations in Fig. 10.6 in the weakly delocalized regime ($\Delta = 1.8t$) in order to capture the full spectrum of states, as in this regime there are more eigenstates with spatial overlap at the edge for us to probe. However, it may be easier to take data in the localized regime, where most of the states do not overlap with the edge of the system. We would probe only the topological states at live at the edge and the few other eigenstates with overlap there, shrinking the width of the measured bands significantly and allowing us to distinguish the topological states more clearly.

10.3.5 Future future work

Beyond these goals, future work could focus on deforming the butterfly spectrum by adding additional terms to the AAH model such as longer-ranged tunneling as in Chap. 6 or by
modifying the potential landscape as in Chap. 8. Perhaps more interesting would be to explore the effect of atomic interactions on the butterfly spectrum and on the boundary states, as the combination of topology and interactions is not well understood. This could be done by both implementing a uniform trapping potential to minimize the effects of the inhomogeneous atomic density, and then increasing interactions via a Feshbach resonance.

10.4 Future projects

Here I will very briefly list future projects that we can run on the $^{39}$K MSL apparatus:

- **Atomic STM**: Described above in Sec. 10.2. Atomic STM should allow us to measure the Hofstadter butterfly spectrum, and moreover look at the topology of the model, all in 1D! This is the project I am personally most excited about, and it seems feasible on the $^{39}$K apparatus.

- **Solitons on a zigzag lattice**: Described in Sec. 5.3. There is a regime of moderately strong interactions ($5 \lesssim U/t \lesssim 7.5$) under which wavepackets should travel without dispersing. This should be possible if we are able to cancel out the detrimental off-resonant effects associated with longer-ranged tunnelings in the MSL (see Sec. 6.6.2).

- **Squeezing in a double well**: Described in Sec. 5.4. By considering atoms in a double well as spin-1/2 particles, interactions take the form of a one-axis twist which can shear and squeeze the distribution on the Bloch sphere. Such a squeezed distribution may allow for enhanced inertial sensing.

This project is currently led by Sai N. M. Paladugu, and has gone beyond this simple scope described here.

- **Current reversal in Josephson junction arrays**: Touched on in Sec. 9.4.3. Though we were able to measure a skewed current-phase relationship and other interesting properties, the simple system features a complete reversal of atomic current at strong interaction strengths (but not strong enough to cause full self-trapping). While we predicted this behavior in simple GP simulations assuming a uniform mean-field energy
(Fig. 9.6), we were unable to measure it experimentally due to the inhomogeneous interaction strengths in the harmonic trap. This should be observable with a uniform trapping potential.

- **2D lattices with three lattice beams in a “Y” shape**: Touched on in Sec. 10.1. Although we have been able to perform experiments in 2D with two lasers (Chap. 3), such a scheme only allowed us access to small lattice geometries (Sec. 3.6.1) and incurred significant off-resonant effects among other challenges (Sec. 3.5.2). Phase stabilizing the two different lasers would have helped significantly.

For our new $^{39}$K apparatus, we plan to orient three beams with the same wavelength (1064 nm) in a “Y” shape (or alternatively, a “T” shape). These beams would be phase stabilized, and each would carry separate frequency components such that atoms would experience Bragg transitions using two of the three beams (absorbing from one and emitting into another). By utilizing only one pair of these beams, we can still create arbitrary 1D lattices as we have done in most of this thesis, but utilizing all three beams allows for 2D lattices of much larger size, and more lattice geometries. The “Y” shape orientation allows us to tune between square (rhomboidal), triangular, hexagonal, and Kagome lattice geometries, and study the physics of flat bands in the latter two cases.

On the other hand, because the lattice beams are the same wavelength, the Bragg transition frequencies are no longer unique. This scheme, then, sacrifices our full independent control over lattice parameters, in particular sacrificing the abilities to create hard-wall boundary conditions and arbitrary synthetic fluxes in 2D.

This project is currently led by Shraddha Agrawal [83].

- **Interactions with topological models**: This is a more general, long-term goal of the lab. Going back to the Venn diagram of Fig. 1.1, we see that the overlap between interactions and topology is conspicuously empty. We hope to fill this gap by studying the 1D Su-Schrieffer-Heeger model under interactions (building upon Eric’s work with the clean [18] and disordered models [16]), and the quantum Hall model under interactions, including the butterfly spectrum as mentioned in Sec. 10.3.5.
Appendix A

Optics Tips and Tricks

This appendix is meant to be a primer to working with lasers and optics, and specifically ways to align laser beams into and with various optics. These practical skills are crucial to building an atomic physics experiment, but they are taught almost exclusively in labs through direct mentoring by more experienced grad students and advisors, and honed through hours of slaving over the setup. As one of Bryce’s first students, I didn’t have older grad students or postdocs to ask and Bryce wasn’t always around to consult. Sometimes I felt like some outside resource would have helped immensely when I was floundering around, and hopefully this will provide that resource to someone else.\(^1\) Unfortunately, I have finite time and this appendix misses many important aspects of optics, like working with lenses, waveplates, beamsplitters, and much more.

To make this appendix easier to explain, I’ll define the axes of a laser system: the \(\hat{z}\) axis points along the direction of laser propagation, so the \(\hat{x}-\hat{y}\) plane is perpendicular to that (cross-section of the laser beam).

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I. Laser basics: don’t look at the beam
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VII. Aligning a tapered amplifier
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\(^1\)The caveat, though, is that the audience of the appendix of a thesis like this is infinitesimally small, and it’s very likely that this has already been done... and I couldn’t find it.
A.2 Laser basics: don’t look at the beam

Laser safety can be summed up into one rule: **never look at a laser** (don’t shine a laser into an eye). This seems like the most obvious thing, but I and other people very often accidentally skirt this number one rule, because there are so many ways eyes can contact laser beams. Some sub-rules that cover most of the safety are:

- **Wear laser goggles** (of the right wavelength/optical density).
- **Don’t put optics at eye level. Don’t put eyes at laser level.**
- **Keep track of (block) stray reflections.** Look for stray reflections (using an IR camera for near-IR beams) and block high powered ones. Build an enclosure.
- **Be careful when placing optics.** Don’t angle optics when you place them. Put them straight down, so that a beam can’t deflect vertically into an eye.
- **Look away from the laser/table** when bending down to pick things off the ground.

A.3 Aligning a beam to a straight line

This is a basic technique for steering laser beams using the core concept of laser alignment: **“near” and “far” points.** Some applications include making a beam parallel to the table as a first step for coupling into an AOM/fiber, or directing a trapping beam along an exact path through the center of a cell. This is documented pretty well by various optics companies like EdmundOptics: [https://www.edmundoptics.com/resources/application-notes/lasers/simplifying-laser-alignment/](https://www.edmundoptics.com/resources/application-notes/lasers/simplifying-laser-alignment/).

A (collimated) laser beam propagating along a straight line can be steered to any angle and position with two mirrors on kinematic mounts that can tilt the mirrors vertically and horizontally (Newport Suprema is #1). The setup to steer a beam is shown in Fig. A.1: a laser hits two mirrors on 2-axis mounts (labeled near and far), and we measure the position of the beam at two corresponding reference points (also labeled near and far) after the mirrors.
Figure A.1: **Setup to align a laser.** Two “near” and “far” mirrors direct the position and angle of an incident beam, which can be measured on targets placed at two “near/far” positions after the mirrors. Inset: an example of a stable laser beam target.

The near and far mirrors control the position and angle of the resultant laser beam, respectively. To best measure the angle of the beam, we look far downstream at the “far” reference point as we adjust the “far” mirror. And to look at the beam position, we look as close as possible to the mirrors, hence the choice of the “near” mirror to align to the “near” point. Then, the alignment procedure is quite simple: **use the near mirror to align to a target at the near point, then use the far mirror to a target the far point, and iterate until the beam is aligned to both targets.**

Some notes on this procedure:

- It is important to identify the correct “near” and “far” mirrors and reference points, as if this procedure is performed with the wrong pairs of mirrors/reference points, the beam will become more and more misaligned.

- Place the mirrors at $45^\circ$ to the beam to get the most range of motion.

- The near reference point should be as close to the mirrors as possible, and the far reference point as far downstream as possible.

- The **target** needs to stably define the desired position of the beam. Vertically, this is easy: just tape a piece of paper to a long post and make a dot at the desired height. Horizontally, it’s a bit more challenging, but it can be done by taping a marked paper
onto a post attached to a clamp, and using the holes on the optical table to define two positions (see Fig. A.1, inset).

To be a little more descriptive about the procedure, we can take the limit where the distance between the two mirrors is infinite. Then, by tilting the “near” mirror, it effectively changes only the position of the beam by the time it reaches the second mirror. And the “far” mirror changes only the angle of the resulting beam. To match these two degrees of freedom to our desired line, we look at the angle of the beam far downstream at the far reference point, and look at the position of the beam at the near reference point. In reality, though, the angle and position are coupled together so we need to continually iterate between the two mirrors and reference points.

A.4 Aligning a beam on top of another beam

Aligning two beams on top of each other is the same as aligning a beam to a straight line, since one beam defines the line for the other. However, it may be challenging to identify the near/far points when the beams are counterpropagating. In the next section, I’ll discuss coupling a laser beam into an optical fiber, but the first step of that process is to shine a fiber pen back through the optical fiber. The incoming laser must be aligned to counter-propagate with the fiber pen output from the fiber, thus allowing the laser to enter the fiber.

As shown in Fig. A.2, these setups are usually compact. Choosing the usual near/far
reference points after the two mirrors (as in the previous section) would only give an inch or
two of room between the points, making the alignment very tedious with lots of iteration.
Instead, as shown in Fig. A.2(a), we can choose reference points on either side of the mirrors.

The “near” reference point is chosen like usual, and we can align the incoming red beam
to the fiber pen spot at this point (black vertical line). For the far point, however, we can
instead align the orange fiber pen output to the incoming red beam in front of the “near”
mirror (gray vertical line). This is like choosing the “near” point if we look at the situation
in reverse (aligning the orange to the red). Iterating between these two points with their
 corresponding mirrors eventually overlaps the beams.

A.5 Fiber coupling

We use optical fibers extensively to split the laser system into an input beam generation side
and an output science side. This way, drifts in laser alignment (due to heating up mirrors,
bumping optics, etc.) on the input side do not affect the highly sensitive alignment of beams
onto the atoms on the output side, only changing the overall beam power. However, this
means that there is a lot of coupling laser beams into optical fibers.

To start, we need to gather the equipment. As with any beam steering, we need two
mirrors on 2-axis kinematic mounts, and of course need an optical fiber. We buy (pre-
clad) polarization-maintaining FC/APC to FC/APC fibers of our desired wavelength from
Thorlabs (e.g., P3-780PM-FC-5). We need a fiber adapter for this (SM1FCA), an aspheric
(no spherical aberrations) lens with a small focal length (A375TM), and a z-axis mount that
can control the distance between the lens and fiber adapter (SM1Z). Along with some cage
hardware, the typical fiber coupling setup looks like Fig. A.2(b).

The idea is to match the mode of the laser beam with the mode that the fiber accepts,
meaning that the angle and position of the laser must be exactly right (the two mirrors),
and the asphere must be positioned exactly to focus the light into the fiber (z-axis mount).
As a first pass, we roughly place the mirrors such that the laser hits the center of the fiber
coupler straight on (not at a large angle).

Since our goal is to mode match, we look at the mode of the fiber by shining light back
through the fiber. We hook up a “fiber pen” (visual fault locator like OVF-1), a low powered handheld laser with a fiber input, to the fiber in reverse. We roughly get the position of the asphere correct by moving the asphere and twisting the z knob on the z-axis mount until the fiber pen output is collimated. Next, we align this fiber pen output with the incoming laser beam that we want to fiber couple, as described in the previous section. This rough alignment ensures that at least some of the incoming laser is matched to the fiber mode, giving some light through the fiber.

For fine alignment, we look at the power going through the fiber as we “walk” corresponding pairs of knobs (horizontal/horizontal, vertical/vertical, z-knob/horizontal or z-knob/vertical) on the mirror mounts. We are trying to find a global maximum in the output power in the 2D parameter space of the two knobs (call them 1 and 2). In practice, this means maximizing the power with knob 1 at different values of knob 2, following the procedure:

- Maximize power by twisting knob 1.
- Twist knob 2 a little.
- Maximize power by twisting knob 1.
- If the power here is larger than at the previous knob 2 position, repeat steps 2-3. If it’s smaller, twist knob 2 in the other direction and repeat step 3. Practice can make this go very quickly.

Eventually, this gets you pretty close to the maximum power with these two knobs. We do this with pairs of horizontal knobs and pairs of vertical knobs, and with the z knob and some of the other knobs. By iterating on many pairs of knobs, eventually we get to the global maximum of the 5D (!!!) parameter space.

For collimated, gaussian beams, we typically get around 70-80% fiber coupling efficiency (output power from fiber/input power). For more non-ideal beams (e.g., a weird mode from a tapered amplifier), 50% is pretty good. Sometimes long-focal-length cylindrical and spherical lenses placed before the asphere can increase the efficiency with better mode matching.
Figure A.3: **AOM inner workings.** A transducer shakes a crystal within the AOM at the specified radio frequency (RF), generating a sound wave. Input laser light undergoes Bragg diffraction into several orders, where the diffraction efficiency is controlled by the input beam alignment (Bragg angle) and the RF power. Tuning the frequency changes the diffraction angle. Adapted from RP Photonics [278].

**A.6 Aligning an acousto-optic modulator**

In an acousto-optic modulator (AOM), a crystal vibrates at some input radio frequency (RF), Bragg diffracting incoming laser light. By tuning the power and frequency of the input RF (with a variable voltage attenuator), we can use AOMs to tune laser powers and frequencies to the kHz level. The output power from an AOM is primarily in a few orders (Bragg regime), and we typically choose the $+1$ (a frequency shift of $+1 \times \text{input RF}$) or $−1$ order for highest diffraction efficiency.

As shown in Fig. A.3, which side the RF connects to the AOM determines where the orders come out. A piezoelectric transducer inside the case converts the RF into a sound wave traveling from the RF connection to the other side of the AOM (leftwards in the figure). By momentum conservation, the diffracted order which gains a $+1$ kick must then diffract in that same direction (leftwards).

Maximizing the power in the desired diffraction order is usually straightforward:

- Mount an AOM to a 2-axis mount and fix it at the height of the incoming beam.
- Attach the AOM to some RF source, and *then*, checking the RF power against the maximum allowed power for the AOM, turn on the RF source. Tune to the desired frequency.
• Make the beam parallel to the optical table (see Section A.3), and direct it into the AOM. While it is better to have two mirrors in front of the AOM, we regularly only use one since the alignment is not that sensitive.

• Placing a card (Thorlabs:VRC5) behind the AOM, move the AOM around side to side (transverse to the beam), rotating it at each position, until the power in the desired order looks maximized. Clamp down the AOM.

• Adjust the input RF power to maximize the diffraction efficiency.

• (Optional) Place a half-wave plate in front of the AOM and vary its angle to maximize the diffraction efficiency.

This coarse alignment is sometimes sufficient, since our eyes are very good at measuring how bright a beam looks on a card. For fine alignment, measure the power in the desired order while adjusting the horizontal/vertical tilt of the AOM mount and the incoming beam alignment.

A.7 Aligning a double pass AOM

Tuning the frequency of an AOM changes the angle of the output beams, changing the alignment of the laser downstream and ruining fiber coupling. To get around this, we can use a double-pass AOM setup, also called a cat’s eye configuration, which allows the frequency of the resulting beam to be tuned without losing power/diffraction efficiency [279].

This setup, shown in Fig. A.4, bounces the diffracted +1 order back into the AOM to get twice the frequency change for a net output +2. Two passes through a quarter-wave plate (λ/4) changes the polarization of the retroreflected beam, allowing us to separate the input beam and the output beam with a polarizing beam splitter (PBS). The key here is a lens placed one focal length away from the AOM (specifically, where the RF is injected into the crystal) and one focal length away from a mirror, in a “cat’s eye” configuration.

The diffraction orders (three are shown here: 0 and ±1) come out of the AOM at an angle θ which depends on the frequency input to the AOM. The lens placed f away catches these
orders, making them go straight. We block off the undesired orders with an iris. However, because each individual order was collimated to begin with, the lens causes the beams to focus. We place a mirror at the focus of this lens ($f$ away), such that the +1 order hitting the mirror retraces its path, expanding back to the lens where it is once again collimated. By adjusting the tilt of the mirror, we can direct the reflected beam to go back into the AOM and diffract again. I’ve shown again the $-1$, 0, and +1 orders of the second pass, where the beam that gets another +1 kick is now overall shifted by +2 orders and retraces the path of the input beam (green, downward arrow). By tuning the angle of the quarter-wave plate, we can change the polarization of the +2 output to be orthogonal to the input polarization,
reflecting from the PBS towards a fiber coupling setup.

To set this up, first construct a single-pass AOM setup as described in the previous section, but with a half-wave plate and PBS in front to catch the double-pass output. Next, place a convex lens \((f = 50 \text{ mm})\) around \(f\) away from the \textit{RF input} of the AOM (shown in the figure with a dashed line). This is where the RF is injected and shakes the crystal, and where the diffracted orders are generated. To be more precise, we want to move the lens longitudinally until the orders stop spreading apart from each other. Place a card downstream and, looking at the separation \textit{between the centers} of the orders, move the lens until the separation matches the separation immediately after the lens. Since the beams are not collimated, this may be challenging to measure far downstream. Make sure to center the 0 order on the lens!

Make room for the quarter-wave plate, but don’t put it in yet. The iris should be very well centered on the desired order (+1 in the figure, but can also be −1), such that no light goes through if we switch off the RF power to the AOM. The mirror should be placed at the focus of the beam, and tilted until the beam goes back through the iris and through the AOM, producing diffraction on the other side. Place the quarter-wave plate in and rotate it until the output beam is fully reflected by the PBS. Determine which order is the +2 (vs. +1 or +3) by varying the frequency input to the AOM. If the lens was positioned correctly \((f\) from the AOM), \textbf{the +2 order should be stationary}. Maximize the output power by tuning the mirror mount knobs, and the AOM mount knobs: the AOM alignment for highest single-pass diffraction efficiency may be different from the alignment for highest double-pass efficiency. Finally, a half-wave plate can be placed immediately before the AOM to improve the signal. Iterate between the angles of this half-wave plate and the quarter-wave plate.

A.8 Aligning a tapered amplifier

Tapered amplifiers (TAs) house some lasing medium that, when powered, can amplify some incoming 30 mW of light into 1 W or more. These are indispensable in our laser system, but they are notoriously hard to align... and output some BAD looking modes. Aside from the few Thorlabs TAs that we bought before they were discontinued, our TAs are all designed
Figure A.5: A fully-aligned tapered amplifier. Two aspheric lenses focus the input seed beam and collimate the output. On the card is the desired output of a bright line inside the larger TA mode. Not shown are a half-wave plate before the TA to change the input polarization, a Faraday isolator after the output, and cylindrical lenses to shape the elliptical output.

In-house: we buy the chips online (Eagleyard 765 nm 1.5 W TA), but the housing is done by the folks down at the MRL machine shop.

Before aligning a TA, we must take great care not to destroy it. Make sure no reflections go back into the TA on the output side: use Faraday isolators and skew all optics a little (don’t put them directly perpendicular to the beam). Do not drive the TA at high current without any seed light going in, and do not seed the TA without supplying some current to the chip.

So, the TA chip must be wired up to both a current and temperature controller, with the current controller set to some nonzero value (typically $\sim 500 \text{ mA}$). Then, let roughly 30-35 mW of input seeding light into the TA. The TA, shown in Fig. A.5, has a chip/lasing medium that must be focused into with an aspheric lens, and whose output is collimated with another aspheric lens. Not shown here is a half-wave plate which adjusts the (linear) polarization of the input light. The TA outputs a mode which is low power and spatially large, even without any seeding/input light. The goal here is to see the input light “seed” the TA, appearing as a bright line/dot within this larger TA mode (Fig. A.5).

To do so, we need to very finely couple the input light into the TA with two mirrors. After
roughly mounting the two lenses into the holder, we align the two mirrors to go in a straight line through the center of the two lenses (following Sec. A.3). This is challenging and must be done very precisely, ideally with a knife-edge card (a card cut in half). Simultaneously, we should monitor the output of the TA, looking for the telltale bright line. After the initial seeding is complete, we can iterate on the longitudinal ($z$) position of the input asphere and the input polarization (vs. the input beam alignment) until the output power is maximized.

To shape the output beam, we need several (3) cylindrical lenses while looking at the TA output far, far downstream. Because the TA outputs are often line-shaped (elliptical), the horizontal and vertical axes of the output spread out at different rates. The output asphere on the TA should be adjusted until the beam is collimated in one axis far downstream, while an external cylindrical lens should collimate the other axis. To make the beam look gaussian again, we then use a cylindrical telescope (two cylindrical lenses) to shrink the longer dimension of the beam to the size of the shorter dimension.

This can all be modeled using various gaussian beam propagation programs (matrices in Mathematica, Zemax, etc.) to determine exact lens focal lengths and positions, after measuring beam widths at different locations. However, the TA output may be absolutely terrible: it may look like one elliptical beam, but comprised of many, many individual “lines” which all spread at different rates. This scenario has happened when we seed one TA with the output of another TA, leading to nonsensical results when we place the expected lenses in the expected locations. In this case, we place the lenses and shape the beam along the two axes empirically.

A.9 Cleaning optics

Optics (lenses, mirrors, beam splitters...) are often covered in dust and occasional fingerprints, and it’s important to keep them clean to prevent stray reflections and keep up performance. There are many techniques that are documented online through optics companies, but this is what I do:

1. FIRST, use compressed air to blow off any large debris/dust from the optic.
2. Use methanol/solvent with lens tissue to clean the optic. (Newport says 60% acetone + 40% methanol)

3. Repeat with methanol if it’s still dirty.

There are different ways to use methanol to clean an optic:

1. **Drop and Drag**: Lay the lens tissue on top of the optic, put a few drops of methanol on top, and drag the wet part of the tissue across the entire optic. This works well if the optic is unmounted and the surface is flat. If any residue remains, you’ve put on too much methanol.

2. **Brush**: Wad up some lens tissue and grab it with forceps, without touching the part of the tissue that will contact the optic. Drop some methanol on the tissue. Wipe from the center towards the outside, and don’t touch the optic twice with the same part of the tissue. This way, you don’t drag dust across the entire optic and create a scratch.

Ideally, this should all be done above some lens tissue and directly above a table so the optic doesn’t get dirty or damaged if you happen to drop it. Gloves are recommended, but not by me.

### A.10 Tightening posts

Optics placed in mounts must be extremely secure, and failing to tighten one part of one mount can cause instability in alignment, laser power, and fiber coupling efficiency. The rule of thumb is: **tighten with an allen wrench or a long lever arm**, because a hex screwdriver can’t tighten nearly enough. This should apply to all connections except thumb screws and a few other uncommon cases.

As an example, Fig. A.6 shows a mirror in a mount, with many connections that should all be tightened with allen wrenches. In order, the connections to tighten are:

1. 1/2” post to optics holder.

2. Mirror inside optics holder. (1” retaining ring)
3. $\frac{1}{2}''$ post to 90 degree angle adapter. (x2)

4. $\frac{1}{2}''$ post holder to post holder base. (try using a lever arm in the thumb screw hole for leverage)

5. $\frac{1}{2}''$ post holder to $\frac{1}{2}''$ post. (thumb screws are fragile - don’t tighten too much)

6. Post holder base to table with fork clamp.

In very small spaces, I sometimes resort to the forbidden technique: a vise-grip around an Allen wrench can give more leverage, but at the risk of flying apart and damaging optics.
References


