Channel-Relay Price Pair: Towards Arbitrating Incentives in Wireless Ad hoc Networks

Yuan Xue, Baochun Li, Klara Nahrstedt

Abstract

Cooperation in wireless ad hoc networks has two-fold implications. First, each wireless node does not excessively and greedily inject traffic to the shared wireless channel. Second, intermediate nodes voluntarily relay traffic for upstream nodes towards the destination at the cost of its own private resource. Such an assumption supports almost all existing research when it comes to protocol design in ad hoc networks. We believe that without appropriate incentive mechanisms, the nodes are inherently selfish (unwilling to contribute its private resource to relay traffic) and greedy (unfairly sharing the wireless channel). In this paper, we present a price pair mechanism to arbitrate resource allocation and to provide incentives simultaneously such that cooperation is promoted and the desired global optimal network operating point is reached by convergence with a fully decentralized self-optimizing algorithm. Such desired network-wide global optimum is characterized with the concept of Nash bargaining solution, which not only provides the Pareto optimal point for the network, but is also consistent with the fairness axioms of game theory. We simulate the price pair mechanism and report encouraging results to support and validate our theoretical claims.

I. INTRODUCTION

Nodes in wireless ad hoc networks not only share the wireless channel in the same local neighborhood, but also relay traffic so that destinations multiple hops away may be reached. In almost all previous work related to wireless ad hoc networks, the following two fundamental assumptions are made. First, nodes do not excessively inject traffic to the locally shared wireless channel. Second, intermediate nodes voluntarily relay traffic for upstream nodes towards the destination. In this paper, we believe that such assumptions may not hold in realistic scenarios, at least not without appropriate incentive-based mechanisms. In fact, they behave in quite the contrary fashion: they are both greedy when it comes to sharing public resource (wireless channel), and selfish when it comes to contributing private resource (such as battery energy). In other words, the network may fail to function at all in realistic scenarios once neither assumption holds.

The only way to solve these problems is to design appropriate incentive mechanisms to not only encourage cooperative behavior of selfish nodes, but also curb unfair and excessive resource usage when sharing a common resource pool, such as a shared channel. Such designs of incentives should optimize towards a clearly specified objective, which is a desired optimal operating point of the wireless network. At such an optimal point, resources are shared fairly, and the levels of cooperation are adequate for all necessary data communications and network functions. This paper exactly targets this critical issue in multi-hop wireless networks.

Our original contributions are two-fold. First, we clearly characterize the desired network-wide optimal operating point using a game theoretic framework, based on the concept of Nash Bargaining Solution (NBS). NBS naturally

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encapsulates two favorable properties: (1) Pareto efficiency in terms of resource usage; and (2) a set of fairness axioms with respect to resource allocations. Using this framework, the problem of finding the desired globally optimal operating point may be formulated as a non-linear optimization problem. Second, we propose a decentralized algorithm that uses a price pair mechanism to arbitrate incentives. With a pair of prices, localized self-optimization by individual nodes naturally converges to globally optimal network operating points. Within the price pair, the channel price regulates greedy usage of the shared wireless channel, while the relay price encourages traffic relaying. Effectively, our price pair mechanism transforms non-cooperative behavior in wireless ad hoc networks to a cooperative game, whose optimal operating points demonstrate more advantageous properties than the usual Nash Equilibrium in typical non-cooperative environments.

The essence of our paper is to integrate the mechanisms that use pricing as signals to (1) fairly allocate resources; and (2) adequately incentivize cooperative behavior. Though there exists previous work towards either one of these objectives, we are not aware of existing work that integrates both prices into a coherent framework. Such integration becomes more complicated if we consider the unique channel contention characteristics in wireless ad hoc networks, where the traffic flows contend in multiple contention cliques. Considerations of such unique complications in ad hoc networks are beyond all of the existing work in the area of pricing or incentives.

The remainder of this paper is organized as follows. Sec. II presents some preliminaries before formal treatment of this topic. Sec. III defines the desired network operating points using the concept of Nash Bargaining Solution. We present the distributed algorithm in Sec. IV. We show simulation results in Sec. V, present related work in Sec. VI and finally conclude the paper in Sec. VII.

II. Network Model and Resource Constraints

A. Characterizing wireless ad hoc networks

We consider a wireless ad hoc network which consists of a set of nodes $\mathcal{N} = \{1, 2, ..., N\}$. In this network, only nodes that are within the transmission range of each other can communicate directly and form a wireless link. We model such a network as a bidirectional graph $G_N = (\mathcal{N}, \mathcal{L})$, where $\mathcal{L} = \{1, 2, ..., L\}$ is the set of wireless links.

In such a network, a wireless node $i \in \mathcal{N}$ may establish an end-to-end flow, or simply flow, $f_i$ with rate $x_i$ to another node. Flow $f_i$ is assumed to be elastic: it requires a minimum rate of $x_i^m$ and a maximum rate of $x_i^M$, i.e., $x_i^m \leq x_i \leq x_i^M$. In general, $f_i$ flows through multiple hops in the network, passing a set of wireless links. We use this set of wireless links to represent $f_i$, i.e., $f_i \subset \mathcal{L}$. We denote the set of relaying nodes for flow $f_i$ as $R(f_i)$, and the destination of $f_i$ as $D(f_i)$. For simplicity of exposition, we further define $H(f_i) = R(f_i) \cup \{D(f_i), i\}$ as the set of nodes $f_i$ traverses, and $K(f_i) = H(f_i) - \{i\}$. A single-hop data transmission along a particular wireless link is referred to as a subflow, and is a part of a flow. Several subflows from different flows along the same wireless link form an aggregated subflow.

In such a network, nodes compete for two types of resources: shared wireless channel and individual nodes’ relaying cost (such as energy). The availability of these resources constrains the solution space of resource allocations. We proceed to analyze the characteristics of both types of resources.

B. Shared wireless channel: location-dependent contention

The shared-medium multi-hop nature of wireless ad hoc networks presents unique characteristics of location-dependent contention and spatial reuse of spectrum. Compared with wireline networks where flows contend only
at the router with other simultaneous flows through the same router (contention in the time domain), the unique characteristics of multi-hop wireless networks show that, flows also compete for shared channel bandwidth if they are within the transmission ranges of each other (contention in the spatial domain).

In particular, two subflows contend with each other if either the source or destination of one subflow is within the transmission range of the source or destination of the other. The locality of wireless transmissions implies that the degree of contention for the shared medium is location-dependent. On the other hand, two subflows that are geographically far away have the potential to transmit simultaneously, reusing the wireless channel.

We now formulate the resource constraints that reflect the unique characteristics of wireless ad hoc networks. First, let us consider a set of mutually contending subflows. In this set, only one subflow can transmit at a time. Intuitively, the aggregated rate of all subflows in this set can not exceed the channel capacity. Formally, we consider a subflow contention graph. In this graph, each vertex corresponds to an aggregated subflow in the original network. Each edge in the graph denotes that two aggregated subflows which correspond to the two vertices, contend with each other. Formally, let $V_C = \cup_{i \in N} f_i \subseteq \mathcal{L}$ be the set of aggregated subflows in network $G_N$, then a bidirectional graph $G_C = (V_C, E_C)$ is a subflow contention graph of network $G_N$.

In a graph, a complete subgraph is referred to as a clique. A maximal clique is defined as a clique that is not contained in any other cliques. In a subflow contention graph, the vertices in a maximal clique represent a maximal set of mutually contending subflows. Intuitively, each maximal clique in a subflow contention graph represents a “maximal distinct contention region”, since at most one subflow in the clique can transmit at any time and adding any other subflows into this clique will introduce the possibility of simultaneous transmissions. For simplicity, we use the set of vertices in a clique to represent the clique, and denote it as $q$. Furthermore, we denote the set of all maximal cliques in a subflow contention graph as $Q$. We illustrate above concepts with an example in Sec. II-D.

We proceed to consider the problem of allocating rates to wireless links. We claim that a rate allocation $y = (y_l, l \in \mathcal{L})$ is feasible, if there exists a collision-free transmission schedule that allocates $y_l$ to $l$. We now formalize the condition implied by such a feasible rate allocation.

**Lemma 1.** If a rate allocation $y = (y_l, l \in \mathcal{L})$ is feasible, then the following condition is satisfied:

$$\forall q \in Q, \sum_{l \in q} y_l \leq C$$  \hspace{1cm} (1)

where $C$ is the channel capacity.

Eq. (1) gives an upper bound on the rate allocations to the wireless links. In practice, however, such a bound may not be tight, especially with carrier-sensing-multiple-access-based wireless networks (such as IEEE 802.11). In this case, we introduce $C_q$, the achievable channel capacity at a clique $q$. More formally, if $\sum_{l \in q} y_l \leq C_q$ then $y = (y_l, l \in \mathcal{L})$ is feasible. To this end, we observe that each maximal clique may be regarded as an independent channel resource unit with capacity $C_q$. It motivates the use of the maximal clique as a basic resource unit for pricing in wireless ad hoc networks, as compared to the notion of a link in wireline networks.

We now proceed to consider resource constraints on rate allocations among flows. To facilitate discussions, we define a clique-flow matrix $R = \{R_{qi}\}$, where $R_{qi} = |q \cap f_i|$ represents the number of subflows that flow $f$ has

---

1 If we assume that the interference range is greater than the transmission range, the contention model can be straightforwardly extended.

2 Note that maximal clique has a different definition from maximum clique of a graph, which is the maximal clique with the largest number of vertices.

3 The measurement of $C_q$ has been studied in both theoretical and practical settings in [1], [2], [3], [4].
in the clique $q$. If we treat a maximal clique as an independent resource, then the clique-flow matrix $R$ represents the “resource usage pattern” of each flow. Let the vector $C = (C_q, q \in Q)$ be the vector of achievable channel capacities in each of the cliques. Constraints with respect to rate allocations to end-to-end flows in wireless ad hoc networks are presented in the following theorem.

**Proposition 1.** In a wireless ad hoc network $G_N = (\mathcal{N}, \mathcal{L})$, there exists a feasible rate allocation $x = (x_i, i \in \mathcal{N})$, if and only if $Rx \leq C$.

**Proof:** It is obvious that $Rx \leq C \iff \forall q \in Q, \sum_{i \in \mathcal{N}} R_q x_i \leq C_q$. By the definition of $R$, we have $\sum_{i \in \mathcal{N}} R_q x_i = \sum_{i \in q} y_i$. The result follows naturally from Lemma 1 and its following discussions. \hfill \square

### C. Costs of Relays

Relaying traffic for upstream nodes apparently incurs cost, since localized resources need to be consumed, such as energy, CPU cycle, and memory space. Without loss of generality, we use energy levels on each node as an example to characterize such costs of relays. Given a minimum expected lifetime in the network, each node $j$ has a budget on its energy consumption rate, denoted as $E_j$. Here, we consider two types of energy consumption related to packet transmission: (1) $e^r$ as the energy consumed for receiving a unit flow; (2) $e^s$ as the energy consumed for transmitting a unit flow. Then the energy consumption at node $j$ is $x_j e^s + \sum_{i: j \in R(f_i)} x_i (e^r + e^s) + \sum_{i: j = D(f_i)} x_i e^r$. As the energy consumption rate can not exceed the energy budget, we have the following relation:

\[
x_j e^s + \sum_{i: j \in R(f_i)} x_i (e^r + e^s) + \sum_{i: j = D(f_i)} x_i e^r \leq E_j
\]

We now proceed to define a $N \times N$ matrix $B$ as follows.

\[
B_{ji} = \begin{cases} 
  e^s & \text{if } j = i \\
  e^r + e^s & \text{if node } j \text{ forwards packets for flow } f_i, \\
  e^r & \text{i.e., } j \in R(f_i) \\
  0 & \text{if node } j \text{ is the destination of flow } f_i, \\
  & \text{i.e., } j = D(f_i) \\
\end{cases}
\]

$B$ specifies the relaying relation among nodes in the ad hoc network. To summarize, the local constraint on energy can be formalized as follows:

\[
B : x \leq E
\]

where $E = (E_j, j \in \mathcal{N})$ is the energy consumption budget vector.

Table I shows a summary of important notations introduced in this section.

### D. Example

Here we illustrate the above concepts using an example. The network topology and the flows in the example are shown in Fig. 1(a). The corresponding subflow contention graph is shown in Fig. 1(b). In this example, there are three maximal cliques in the contention graph: $q_1 = \{\{1, 2\}, \{3, 2\}, \{3, 4\}, \{3, 6\}\}$, $q_2 = \{\{3, 2\}, \{3, 4\}, \{4, 5\}, \{3, 6\}\}$ and $q_3 = \{\{3, 2\}, \{3, 4\}, \{3, 6\}, \{6, 7\}\}$. Let us use $y_{ij}$ to denote the aggregated rate of all subflows along node $i$ and $j$. For example, $y_{12} = x_1 + x_2 + x_7$, $y_{23} = x_1 + x_3 + x_7$, $y_{36} = x_6 + x_7$. In each clique, the aggregated rate can not exceed the channel capacity, i.e.,
### Mathematical Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$i, j \in \mathcal{N} = {1, \ldots, N}$</td>
<td>Network nodes</td>
</tr>
<tr>
<td>$f_i, R(f_i), D(f_i)$</td>
<td>The flow from source node $i$, $f_i$’s relaying nodes and destination</td>
</tr>
<tr>
<td>$H(f_i)$</td>
<td>$R(f_i) \cup {D(f_i), i}$</td>
</tr>
<tr>
<td>$K(f_i)$</td>
<td>$H(f_i) - {i}$</td>
</tr>
<tr>
<td>$l \in \mathcal{L} = {1, \ldots, L}$</td>
<td>Wireless links</td>
</tr>
<tr>
<td>$q \in \mathcal{Q} = {1, \ldots, Q}$</td>
<td>Maximal cliques</td>
</tr>
<tr>
<td>$\mathbf{x} = (x_i, i \in \mathcal{N})$</td>
<td>Rate allocation vector</td>
</tr>
<tr>
<td>$\mathbf{x}^m, \mathbf{x}^M$</td>
<td>Minimum (or maximum) rate vector</td>
</tr>
<tr>
<td>$\mathbf{C} = (C_q, q \in \mathcal{Q})$</td>
<td>Clique capacity vector</td>
</tr>
<tr>
<td>$\mathbf{E} = (E_{ji}, j \in \mathcal{N})$</td>
<td>Energy budget vector</td>
</tr>
<tr>
<td>$\mathbf{A} = (A_{qi})_{Q \times N}$</td>
<td>Channel constraint matrix</td>
</tr>
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![Network topology](attachment:image1.png)  
(a) Network topology  
(b) Subflow contention graph

Fig. 1. Example of wireless ad hoc network and its subflow contention graph

\[
y_{12} + y_{32} + y_{34} + y_{36} \leq C_1 \quad (5)
\]
\[
y_{32} + y_{34} + y_{45} + y_{36} \leq C_2 \quad (6)
\]
\[
y_{32} + y_{34} + y_{36} + y_{67} \leq C_3 \quad (7)
\]

When it comes to end-to-end flow rate allocation, the resource constraint imposed by shared wireless channel is as follows:

\[
\begin{pmatrix}
3 & 1 & 1 & 1 & 1 & 2 & 3 \\
3 & 0 & 1 & 1 & 2 & 3 & 2 \\
2 & 0 & 1 & 1 & 1 & 2 & 3
\end{pmatrix}
\mathbf{x} \leq \mathbf{C}.
\]

In this example, let the energy consumed for receiving $e^r = 1$, for transmitting $e^s = 2$. Then the energy constraint is as follows:
III. PROBLEM FORMULATION

In this section, we characterize the desired network-wide optimal operating point using a game theoretic framework, based on the concept of Nash Bargaining Solution (NBS). NBS naturally encapsulates two favorable properties: (1) Pareto efficiency in terms of resource usage; and (2) a set of fairness axioms with respect to resource allocations. Using this framework, the problem of finding the desired globally optimal operating point may be formulated as a non-linear optimization problem. We show how such a global optimization problem may be decomposed into localized greedy optimization problems via a price pair.

A. Nash Bargaining Solution: a game theoretical formulation

We present the basic concepts and results of Nash bargaining solutions (NBS) from game theory [5] and show how it can characterize and formulate the desired network operation point, towards which our price pair mechanism should converge.

The basic setting of the problem is as follows: The set of nodes \( N \) in the wireless ad hoc network \( G_N \) constitutes a set of players in the game. They compete for the use of a fixed amount of resources (wireless channel and costs of relays such as energy). The rate allocation \( x = (x_i, i \in N) \) is the utility vector of all players in the game. Let \( S \subset \mathbb{R}^N \) be the set of all feasible utility vectors. We assume that \( S \) is a non-empty convex closed and bounded set. Further, we denote the initial agreement point of the game as \( x^* \), which is the guaranteed utilities of players without any cooperation in order to enter the game.

In such a problem setting, a bargaining problem is any \( (S, x^*) \) where \( \{x \in S | x \geq x^* \} \neq \emptyset \). In other words, a bargaining problem is actually a set of rate allocations \( x \) that is acceptable to all players. Let \( \mathcal{B} = \{(S, x^*)\} \) denote the set of all bargaining problems. It then follows that a bargaining solution is any function \( \varphi : \mathcal{B} \rightarrow \mathbb{R}^N \), so that \( \forall (S, x^*) \in \mathcal{B}, \varphi(S, x^*) \in S \). A bargaining solution actually specifies finding a rate allocation within all acceptable allocations. A natural question about the bargaining solution is how such a function \( \varphi \) is to be defined. There are two reasonable properties desired for a bargaining solution: (1) efficient use of resources; and (2) fair allocation among all players. These two conditions are precisely encapsulated by the concept of Nash Bargaining Solution defined as follows.

**Definition 1 (Nash Bargaining Solution).** A bargaining solution \( \varphi : \mathcal{B} \rightarrow \mathbb{R}^N \) is a Nash Bargaining Solution (NBS), if \( \bar{x} = \varphi(S, x^*) \) satisfies the Nash Axioms A1-A6.

A1 (Individual rationality) \( \bar{x} \geq x^* \);
A2 (Feasibility) \( \bar{x} \in S \);
A3 (Pareto optimality) If \( \forall x \in S, x \geq \bar{x} \), then \( x = \bar{x} \);
A4 (Independence of irrelevant alternatives) If $\bar{x} \in T \subset S$ and $\bar{x} = \varphi(S, x^*)$, then $\bar{x} = \varphi(T, x^*)$;

A5 (Independence of linear transformation) Let $T$ be obtained from $S$ by the linear transformation $\phi(x)$ with

$$\phi(x)_i = a_ix_i + b_i, a_i, b_i > 0, i = 1, 2, ..., N. \quad (8)$$

Then if $\varphi(S, x^*) = \bar{x}$, then $\varphi(T, \phi(x^*)) = \phi(\bar{x})$;

A6 (Symmetry) Suppose $S$ is symmetric with respect to a subset $J \subseteq \{1, 2, ..., N\}$ of indices, then $\varphi$ is symmetry, which means that if $x \in S$ and for $i, j \in J, x_i^* = x_j^*$, then $\bar{x}_i = \bar{x}_j$.

The above axioms encapsulate both the concept of Pareto optimality (A3) and the concept of fairness (A4-A6). Pareto optimality means that there is no other point which gives strict superior utility for all the players simultaneously. The definition of Pareto optimality reflects the condition of efficient use of resources, where there are no “idle” resources in the network. Axioms A4-A6 represent the axioms of fairness. The independence of irrelevant alternatives axiom (A4) states that, if a bargaining solution $\bar{x}$ to a problem on an enlarged feasible set $S$ can be found on a restricted domain $T$, then it is also the bargaining solution to the problem on the restricted set. The independence of linear transformation axiom (A5) states that the bargaining solution is scale invariant, i.e., the bargaining solution is unchanged if the utility is changed using a positive linear transformation. The symmetry axiom (A6) states that the bargaining point does not depend on the specific labels, i.e., players with the same initial points and objectives will realize the same utility.

The existing work in game theory [5] and its application in wireline communication networks [6] establish the following results for NBS.

**Proposition 2.** There exists a unique function $\varphi$ defined on all bargaining problems $(S, x^*)$ that satisfies axioms A1-A6, i.e., a unique Nash Bargaining Solution (NBS) exists. Moreover, the unique solution $\bar{x}$ is a unique vector that solves the following maximization problems:

$$N_1 : \max_{\bar{x}} \prod_{i \in N} (\bar{x}_i - x_i^*); \quad (9)$$

Equivalently,

$$N_2 : \max_{\bar{x}} \sum_{i \in N} \ln(\bar{x}_i - x_i^*). \quad (10)$$

![Fig. 2. Example of Nash bargaining solution](image)

Above concepts are illustrated in Fig. 2, where the utility space of a two-player game is plotted. In the example, two players are competing for a unit resource. The initial agreement point $x^* = (0, 0)$, and the NBS $\bar{x} = (1/2, 1/2)$. 
B. Network operating points: optimal and fair rate allocations

We assume that each player involved in the game can be guaranteed with its minimum rate vector $x^m$. Thus, the minimum rate vector $x^m$ can be regarded as an initial agreement point of the game $x^*$. From the discussions in Sec. III-A, it is clear that the NBS formulates the desired network operation point, which is fair to all nodes while efficient from the network’s point of view. Thus, the problem of finding the globally optimal resource allocation is transformed to solving the NBS of its corresponding game, which is the solution of the following nonlinear optimization problem by Proposition 2.

\[
P : \quad \text{maximize} \quad \sum_{i \in N} \ln(x_i - x^m_i) \tag{11}
\]

subject to

\[
A \cdot x \leq C \tag{12}
\]

\[
B \cdot x \leq E \tag{13}
\]

over

\[
x^m \leq x \leq x^M \tag{14}
\]

The objective function in Eq. (11) of the optimization problem corresponds to the optimization problem whose solution is NBS. The constraints of the optimization problem Eq. (12) and Eq. (13) are the constraints from the shared wireless channel and costs of relays, respectively, as discussed in Sec. II.

Note that the objective function $\sum_{i \in N} \ln(x_i - x^m_i)$ is strictly concave. In addition, the feasible region of the optimization problem is non-empty, convex, and compact. By non-linear optimization theory, there exists a maximizing value of argument $x$ for the above optimization problem. Let us consider the Lagrangian form of the optimization problem $P$:

\[
L(x, \mu^\alpha, \mu^\beta) = \sum_{i \in N} \ln(x_i - x^m_i) + \mu^\alpha(C - Ax) + \mu^\beta(E - Bx) \tag{15}
\]

where $\mu^\alpha = (\mu^\alpha_q, q \in Q)$, $\mu^\beta = (\mu^\beta_j, j \in N)$ are two vectors of Lagrange multipliers. The first-order Kuhn-Tucker conditions are necessary and sufficient for optimality of problem $P$. Thus, for $i \in N$, the following conditions hold:

\[
\frac{1}{x_i - x^m_i} - \sum_{q \in Q} \mu^\alpha_q A_{qi} - \sum_{j \in N} \mu^\beta_j B_{ji} = 0 \tag{16}
\]

\[
\mu^\alpha(C - Ax) = 0; \mu^\alpha \geq 0 \tag{17}
\]

\[
\mu^\beta(E - Bx) = 0; \mu^\beta \geq 0 \tag{18}
\]

\[
x^m \leq x \leq x^M \tag{19}
\]

In the Lagrangian form shown in Eq. (15), the Lagrange multipliers $\mu^\alpha_q$ can be regarded as the implied cost of unit flow accessing the channel at the maximal clique $q$. In other words, $\mu^\alpha_q$ is the shadow price of clique $q$, called channel price. This price corresponds to the shared channel resource constraint. The Lagrange multipliers $\mu^\beta_j$ can be regarded as the implied relay cost of unit flow at node $j$. In other words, $\mu^\beta_j$ is the shadow price of relay at node $j$, called relay price. This pair of prices $(\mu^\alpha, \mu^\beta)$ will be used as incentives so that localized self-optimizing decision can implement the global optimum. In particular, the price $\mu^\alpha$ will signal a “charge” (contrary to incentives) to the
shared channel usage and regulate the greedy behavior, while the price $\mu^\beta$ will provide incentives to traffic relays at intermediate nodes and regulate the selfish behavior of wireless nodes.

Let us denote

$$\lambda^\alpha_i = \sum_{q \in Q} \mu^\alpha_q A_{qi}$$  \hspace{1cm} (20)
$$\lambda^\beta_i = \sum_{j \in \mathcal{N}} \mu^\beta_j B_{ji}$$  \hspace{1cm} (21)

Clearly,

$$\lambda^\alpha_i = \sum_{q \in f_i \cap q \neq \emptyset} \mu^\alpha_q A_{qi}$$  \hspace{1cm} (22)
$$\lambda^\beta_i = \sum_{j \in H(f_i)} \mu^\beta_j B_{ji}$$  \hspace{1cm} (23)

$$\lambda^\alpha_i = \mu^\alpha_i e^s + \sum_{j \in R(f_i)} \mu^\beta_j (e^s + e^r) + \mu^\beta_{D(f_i)} e^r$$  \hspace{1cm} (25)

Then $\lambda^\alpha_i$ and $\lambda^\beta_i$ are the prices for node $i$, which is the source of flow $f_i$, for accessing shared channels and relay services, respectively. For channel usage, node $i$ needs to pay for all the maximal cliques that it traverses. For each clique, the price to pay is the product of the number of wireless links that $f_i$ traverses in this clique and the shadow price of this clique as in Eq. (22). Alternatively, the price of flow $f_i$ is the aggregated price of all its subflows. For each subflow, its price is the aggregated price of the maximal cliques that it belongs to as in Eq. (23). Note that such a pricing policy for end-to-end flows is fundamentally different from the pricing models in wireline networks, where a flow’s price is the aggregation of the link prices which it traverses. Such difference is rooted at the nature of the shared medium — the interference and spatial reuse of wireless channel in an ad hoc network. For traffic relay services, node $i$ needs to pay for the relay costs of all relaying nodes of $f_i$, including itself and the destination. Using flow $f_1$ as an example, the channel price model is illustrated in Fig. 3(a) and the relay price model is illustrated in Fig. 3(b). The channel price for $f_1$ is $\lambda^\alpha_1 = 3\mu^\alpha_1 + 3\mu^\alpha_2 + 2\mu^\alpha_3$; and the relay price $\lambda^\beta_1 = 2\mu^\beta_1 + 3\mu^\beta_2 + 3\mu^\beta_3 + 3\mu^\beta_4 + \mu^\beta_5$.

![Fig. 3. An example of the price pair mechanism](image)

To summarize, we have the following results with respect to the globally optimal network operating point.
Theorem 1. There exists a unique solution to problem \( P \) (i.e., unique NBS), which is characterized as follows: There exist two vectors \( \mathbf{\mu}^\alpha = (\mu^\alpha_q, q \in Q) \), \( \mathbf{\mu}^\beta = (\mu^\beta_j, j \in N) \) such that

\[
x_i = \left[ \frac{1}{\lambda_i^\alpha + \lambda_i^\beta} + x^m_i \right] x^M_i, \text{ for } i \in N
\]

where \([z]^b = \max\{\min\{z, b\}, a\}\).

C. Local strategies: self-optimizing decisions

With the understanding of the global optimal point, we now study how this point can be achieved in a distributed manner by localized self-optimizing decisions at each individual node. The key to this goal is to use the pair of prices \( (\mathbf{\mu}^\alpha, \mathbf{\mu}^\beta) \) as signals to coordinate the distributed decisions.

First, we study the conditions that the prices need to satisfy in order to incentivize local node decisions so that they could implement the global network optimum. Let us consider the following problems:

Channel(\( \mathbf{A}, \mathbf{C}; \mathbf{\mu}^\alpha \)):

\[
\begin{align*}
\text{maximize} & \quad \sum_{i \in N} \lambda_i^\alpha x_i \\
\text{subject to} & \quad \mathbf{A}x \leq \mathbf{C} \\
\text{over} & \quad x^m \leq x \leq x^M
\end{align*}
\]

where \( \lambda_i^\alpha \) is a function of \( \mathbf{\mu}^\alpha \), as defined in Eq. (20). Problem Channel(\( \mathbf{A}, \mathbf{C}; \mathbf{\mu}^\alpha \)) maximizes the total revenue of channel based on charging \( \lambda_i^\alpha \) per unit of bandwidth to user \( i \) subject to the channel capacity constraint.

Relay(\( \mathbf{B}, \mathbf{E}; \mathbf{\mu}^\beta \)):

\[
\begin{align*}
\text{maximize} & \quad \sum_{i \in N} \lambda_i^\beta x_i \\
\text{subject to} & \quad \mathbf{B}x \leq \mathbf{E} \\
\text{over} & \quad x^m \leq x \leq x^M
\end{align*}
\]

where \( \lambda_i^\beta \) is a function of \( \mathbf{\mu}^\beta \), as defined in Eq. (21). Problem Relay(\( \mathbf{B}, \mathbf{E}; \mathbf{\mu}^\beta \)) maximizes the total relay revenue of all nodes subject to the relay cost constraint at each individual node.

Now let us consider a local self-optimizing decision at each node \( i \) which corresponds to the following problem: Node\(_i\)(\( \lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta \)):

\[
\begin{align*}
\text{maximize} & \quad \ln(x_i - x^m_i) - \lambda_i^\alpha x_i - \lambda_i^\beta x_i + \mu_i^\beta E_i \\
\text{over} & \quad x^m_i \leq x_i \leq x^M_i
\end{align*}
\]

The relationship between localized node decision and the global optimal network operating point is then given as follows.
Theorem 2: There exist vectors $\mu^\alpha$, $\mu^\beta$ and $x$ such that
1) $x_i$ is the unique solution to $\text{Node}_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)$;
2) $x$ solves $\text{Channel}(A, C; \mu^\alpha)$;
3) $x$ solves $\text{Relay}(B, E; \mu^\beta)$;
Then $x$ also solves problem $P$.

The proof of this theorem is given in the Appendix A. Let us denote
$$
\Phi(x_i) = \ln(x_i - x_i^m) - \lambda_i^\alpha x_i - \lambda_i^\beta x_i + \mu_i^\beta E_i
$$
(37)
Now we show that $\Phi(x_i)$ reflects the “net benefit” of node $i$ and problem $\text{Node}_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)$ maximizes the node $i$’s net benefit. This claim is based on the following two observations. First, node $i$ does not pay its relay cost to itself from a local point of view, while the flow relay price $\lambda_i^\beta$, which is defined from a global point of view, contains the price of relay for itself. Thus from a local point of view, the cost of node $i$ is:
$$
\lambda_i^\alpha x_i + \sum_{j \in K(f_i)} B_{ji} = (\lambda_i^\alpha + \lambda_i^\beta) x_i - \mu_i^\beta e^s x_i
$$
(38)
And the revenue of node $i$ is:
$$
\sum_{j:i \in K(f_i)} \mu_i^\beta B_{ij} x_j = \mu_i^\beta \sum_{j:i \in H(f_j)} B_{ij} x_j - \mu_i^\beta e^s x_i
$$
(39)
Second, from Theorem 2 we have
$$
\mu_i^\beta = 0, \text{ if } \sum_{j:i \in H(f_j)} B_{ij} x_j < E_i
$$
(40)
$$
\mu_i^\beta > 0, \text{ if } \sum_{j:i \in H(f_j)} B_{ij} x_j = E_i
$$
(41)
Thus it is clear that $\Phi(x_i)$ reflects the “net benefit” of node $i$, which is the difference between utility, revenue and cost.

IV. ALGORITHM

Although the global problem $P$ can be mathematically solvable in a centralized fashion, it is impractical for realistic operations in wireless ad hoc networks. In this section, we present a distributed iterative algorithm for price calculation and resource arbitration. We show that the iterative algorithm converges to the global optimum, and maximizes the local net benefit at each node simultaneously.

A. Dual Problem

In order to achieve a distributed solution, we first look at the dual problem of $P$ as follows.
$$
D : \min_{\mu^\alpha, \mu^\beta \geq 0} D(\mu^\alpha, \mu^\beta)
$$
(42)
where

\[
D(\mu^\alpha, \mu^\beta) = \max_{x} \sum_{i \in \mathcal{N}} \left( \ln(x_i - x_i^m) - (\lambda_i^\alpha + \lambda_i^\beta)x_i + \mu_i^\beta E_i \right)
\]

\[
+ \sum_{q \in \mathcal{Q}} \mu_q^\alpha C_q
\]

Note that \(\Phi(x_i)\) is node \(i\)'s “net benefit”. By the separation nature of Lagrangian form, maximizing \(L(x, \mu^\alpha, \mu^\beta)\) can be decomposed into separately maximizing \(\Phi(x_i)\) for each node \(i \in \mathcal{N}\) (Sec. 3.4.2 in [7]). We now have

\[
D(\mu^\alpha, \mu^\beta) = \sum_{i \in \mathcal{N}} \max_{x_i^m \leq x_i \leq x_i^M} \left\{ \Phi(x_i) \right\} + \sum_{q \in \mathcal{Q}} \mu_q^\alpha c_q
\]

(43)

As \(\Phi(\cdot)\) is strictly concave and twice continuously differentiable, a unique maximizer of \(\Phi(x_i)\) exists when

\[
\frac{d\Phi(x_i)}{dx_i} = \frac{1}{x_i - x_i^m} - (\lambda_i^\alpha + \lambda_i^\beta) = 0
\]

We define the maximizer as follows:

\[
x_i(\lambda_i^\alpha, \lambda_i^\beta) = \arg\max_{x_i^m \leq x_i \leq x_i^M} \left\{ \Phi(x_i) \right\}
\]

(44)

Note that \(x_i(\lambda_i^\alpha, \lambda_i^\beta)\) is usually called demand function, which reflects the optimal rate for node \(i\) with channel price as \(\lambda_i^\alpha\) and relay price as \(\lambda_i^\beta\).

B. Distributed Algorithm

We solve the dual problem \(D\) using the gradient projection method [7]. In this method, \(\mu^\alpha\) and \(\mu^\beta\) are adjusted in the opposite direction to the gradient \(\nabla D(\mu^\alpha, \mu^\beta)\):

\[
\mu_q^\alpha(t + 1) = [\mu_q^\alpha(t) - \gamma \frac{\partial D(\mu^\alpha(t), \mu^\beta(t))}{\partial \mu_q^\alpha}]^+
\]

(45)

\[
\mu_j^\beta(t + 1) = [\mu_j^\beta(t) - \gamma \frac{\partial D(\mu^\alpha(t), \mu^\beta(t))}{\partial \mu_j^\beta}]^+
\]

(46)

where \(\gamma\) is the stepsize. \(D(\mu^\alpha, \mu^\beta)\) is continuously differentiable since \(\ln(\cdot)\) is strictly concave [7]. Thus, it follows that

\[
\frac{\partial D(\mu^\alpha, \mu^\beta)}{\partial \mu_q^\alpha} = C_q - \sum_{i: f_i \cap q \neq \emptyset} x_i(\lambda_i^\alpha, \lambda_i^\beta) A_{qi}
\]

(47)

\[
\frac{\partial D(\mu^\alpha, \mu^\beta)}{\partial \mu_j^\beta} = E_j - \sum_{i: j \in H(f_i)} x_i(\lambda_i^\alpha, \lambda_i^\beta) B_{ji}
\]

(48)

Substituting Eq. (47) into (45) and (48) into (46), we have
\[ \mu^\alpha_q(t + 1) = \left( \mu^\alpha_q(t) + \gamma \left( \sum_{i:f_i \cap q \neq \emptyset} x_i(\lambda^\alpha_i(t), \lambda^\alpha_i(t)) A_{qi} - C_q \right) \right)^+ \]

\[ \mu^\beta_j(t + 1) = \left( \mu^\beta_j(t) + \gamma \left( \sum_{i \in H(f_i)} x_i(\lambda^\beta_i(t), \lambda^\beta_i(t)) B_{ji} - E_j \right) \right)^+ \]

Eq. (49) and Eq. (50) reflect the law of supply and demand. If the demand for bandwidth at clique \( q \) exceeds its supply \( C_q \), the channel constraint is violated. Thus, the channel price \( \mu^\alpha_q \) is raised. Otherwise, \( \mu^\alpha_q \) is reduced. Similarly, in Eq. (50), if the demand for energy at node \( j \) exceeds its budget \( E_j \), the energy constraint is violated. Thus, the relay price \( \mu^\beta_j \) is raised. Otherwise, \( \mu^\beta_j \) is reduced.

We summarize our algorithm in Table II, where clique \( q \) and node \( i \) are deemed as entities capable of computing and communicating.

**Clique Price Update** (by clique \( q \)): At times \( t = 1, 2, \ldots \)
1. Receive rates \( x_i(t) \) from all flows \( f_i \) where \( f_i \cap q \neq \emptyset \)
2. Update price
   \[ \mu^\alpha_q(t + 1) = \left( \mu^\alpha_q(t) + \gamma \left( \sum_{i:f_i \cap q \neq \emptyset} x_i(t) A_{qi} - C_q \right) \right)^+ \]
3. Send \( \mu^\alpha_q(t + 1) \) to all flows \( f_i \) where \( f_i \cap q \neq \emptyset \)

**Relay Price Update** (by node \( j \)): At times \( t = 1, 2, \ldots \)
1. Receive rates \( x_i(t) \) from all flows \( f_i \) where \( j \in H(f_i) \)
2. Update price
   \[ \mu^\beta_j(t + 1) = \left( \mu^\beta_j(t) + \gamma \left( \sum_{i \in H(f_i)} x_i(t) B_{ji} - E_j \right) \right)^+ \]
3. Send \( \mu^\beta_j(t + 1) \) to all flows \( f_i \) where \( j \in H(f_i) \)

**Rate Update** (by node \( i \)): At times \( t = 1, 2, \ldots \)
1. Receive channel prices \( \mu^\alpha_q(t) \) from \( q \) where \( f_i \cap q \neq \emptyset \)
2. Receive relay prices \( \mu^\beta_j(t) \) from \( j \) where \( j \in H(f_i) \)
3. Calculate
   \[ \lambda^\alpha_i = \sum_{q:f_i \cap q \neq \emptyset} \mu^\alpha_q A_{qi} \]
   \[ \lambda^\beta_i = \sum_{j \in H(f_i)} \mu^\beta_j B_{ji} \]
4. Adjust rate
   \[ x_i(t + 1) = x_i(\lambda^\alpha_i, \lambda^\beta_i) \]
5. Send \( x_i(t + 1) \) to corresponding cliques.

<table>
<thead>
<tr>
<th>TABLE II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributed algorithm</td>
</tr>
</tbody>
</table>

We now show the property of this distributed iterative algorithm. Let us define \( Y(i) = \sum_q A_{qi} + \sum_j B_{ji} \), and \( \bar{Y} = \max_{i \in N} Y(i) \). Further, we define \( U(q) = \sum_{i \in N} A_{qi} \) and \( \bar{U} = \max_{q \in Q} U(q) \); \( V(j) = \sum_{i \in N} B_{ji} \) and \( \bar{V} = \max_{j \in N} V(j) \); \( \bar{Z} = \max \{ \bar{U}, \bar{V} \} \). Let \( \kappa_i = (x^M_i - x^m_i)^2 \) and \( \bar{\kappa} = \max_{i \in N} \kappa_i \).

**Theorem 3** (Global convergence and optimality). Suppose \( 0 < \gamma < 2 / \bar{r} \bar{Y} \bar{Z} \). Starting from any initial rates \( x^m \leq x(0) \leq x^M \), and prices \( \mu^\alpha(0) \geq 0 \) and \( \mu^\beta(0) \geq 0 \), every accumulation point \( (x^*, \mu^{\alpha*}, \mu^{\beta*}) \) of the sequence \( (x(t), \mu^{\alpha}(t), \mu^{\beta}(t)) \) generated by the algorithm in Table II is primal-dual optimal.
The reader is referred to Appendix B for a detailed proof. Though there exists a unique maximizer \( x^* \) to the problem \( P \), there may be multiple dual optimal prices, since only the flow price is constrained at optimality according to \( U_i'(x_j^*) = \lambda_i^* + \lambda_i^{\beta*} \). Theorem 3 does not guarantee convergence to a unique vector \((x^*, \mu_\alpha^*, \mu_\beta^*)\), though any convergent subsequence leads to the optimal rate allocation \( x^* \).

### C. Implementation Issues

In the above iterative algorithm, a maximal clique is regarded as a network element that can carry out certain network functions. In particular, it assumes that a maximal clique \( q \) can perform the following tasks for price calculation and resource allocation: (1) obtain the aggregated bandwidth demand within the clique \( q \); (2) calculate the per-clique shadow price \( \mu_q^\alpha \); and (3) notify the price \( \mu_q^\alpha \) to the flows that pass through it. However, a maximal clique is only a concept defined based on subflow contention graph. To deploy the algorithm in an actual ad hoc network, the above tasks of a maximal clique need to be carried out by the nodes that constitute the clique in a distributed fashion. For the implementation details, readers are referred to Appendix C.

The price pair based self-optimizing algorithm needs the support from a virtual credit system to enforce the pricing signal. As this paper mainly focuses on how to calculate the incentive price, the design of such a virtual credit system is beyond the scope of this paper. Interesting readers are referred to [8] [9] for these issues.

### V. Simulation Results

We illustrate our price pair mechanism and the distributed algorithm via simulation. The network used in the simulation has 20 nodes deployed on a 600×600 square meter region as shown in Fig. 4. Such a network has 10 cliques that represent maximal contending regions in the wireless channel as shown in Table IV. Each node sets up a connection with a randomly selected destination and delivers traffic via shortest path routing. The flows and their routes are shown in Table III. We use uniform achievable channel capacities \( C_q = 1 \text{Mbps} \) for all cliques \( q \) in the simulation. The default initial energy level for all nodes is 432 Joules. The transmission energy consumption is \( 2 \times 10^{-7} \text{J/bit} \) and the receiving energy consumption is \( 10^{-7} \text{J/bit} \). The network is expected to last 2 hours with an energy budget of \( 0.06 \text{J/sec} \) for each node. The minimum and maximum rate requirement of flows are \( x_i^{m} = 0 \text{Mbps} \) and \( x_i^{M} = 1 \text{Mbps} \) for all flows \( f_i \).

#### A. Convergence

We first show the convergence behavior of our iterative algorithm. In the simulation, the initial values are \( x_i(0) = 0 \text{ Kbps} \), \( \mu_q^\alpha(0) = \mu_j^{\beta}(0) = 0.01 \) for all \( i, q, j \). The step size that ensures the convergence is set to \( \gamma = 10^{-6} \). As shown in Fig. 5, the algorithm converges to a global network equilibrium within about 300 iterations. At the equilibrium point, the optimal resource allocation and prices are listed in Table V. We observe that node 1, 10, 11 and 17 are the bottleneck relaying nodes and clique 1 is the bottleneck channel resource. They, therefore, have positive prices. We have also experimented with different sets of initial values for the rates and prices, and the results have been coherent with our illustrated example.

#### B. Impact of resource capacity

We further illustrate the roles and interactions of the channel and relay prices, by repeating the simulation with different values of energy budget \( E \) and channel capacity \( C \). In the experiment, the energy budgets of two bottleneck
Fig. 4. Random topology used in the simulation

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<tr>
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<td>6 → 0 → 12 → 14</td>
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<td>14</td>
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<tr>
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<td>18 → 0 → 6</td>
</tr>
<tr>
<td>19</td>
<td>19 → 6 → 0 → 11 → 1 → 4</td>
</tr>
</tbody>
</table>

TABLE III

Flows and their routes
nodes 1 and 11 are changed to $E_1 = E_{11} = 0.1$J/sec. The convergence behavior and the equilibrium results are shown in Fig. 6 and Table VI respectively. We observe that the relay prices at node 1 and 11 have been decreasing, since more energy is available at these two nodes, causing less competition with respect to energy. At the same time, we also observe that the channel price has increased, since the decreasing relay cost increases the rate demand and causes more competition for the channel.

We now show two special cases where the role of one price in the price pair can implicitly be assumed by the other. In the first case, $E_i = 0.04$J/sec for all $i$. The result is plotted in Fig. 7. From the figure, we observe that the channel prices eventually converge to 0, with the increase of relay prices. It shows that in this scenario, the energy constraints are the dominant factors in the network. Thus, when the relay prices give adequate incentives to these energy constrained nodes to relay appropriate amount of traffic, the wireless channel is naturally shared among all nodes in a fair manner. In the second case, we show a scenario where the shared wireless channel capacity is the dominant factor. In this experiment, the energy budget is set to be sufficiently large: $E_i = 0.2$J/sec for all $i$. As shown in Fig. 8, we can observe that the channel prices play a dominant role in arbitrating resource contention among different nodes.
<table>
<thead>
<tr>
<th>Node/Flow ID</th>
<th>Flow rate (Kbps)</th>
<th>Relay price</th>
<th>Clique ID</th>
<th>Channel price</th>
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**TABLE V**

**EQUILIBRIUM RATES AND PRICES**

(a) Flow Rates $x_i$ ($i \in \mathcal{N}$)

Fig. 6. Convergence when more energy is available at node 11 and 1.

(a) Flow Rates $x_i$ ($i \in \mathcal{N}$)

(b) Channel Price $\mu_q^x$ ($q \in \mathcal{Q}$)

(c) Relay Price $\mu_j^\beta$ ($j \in \mathcal{N}$)

Fig. 7. Energy is dominant factor
<table>
<thead>
<tr>
<th>Node/Flow ID</th>
<th>Flow rate (Kbps)</th>
<th>Relay price</th>
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**TABLE VI**

Equilibrium rates and prices when more energy is available at node 11 and 1.

![Fig. 8. Channel is dominant factor](image)

(a) Flow Rates $x_i$ ($i \in \mathcal{N}$)  
(b) Channel Price $\mu^\alpha_q$ ($q \in \mathcal{Q}$)  
(c) Relay Price $\mu^\beta_j$ ($j \in \mathcal{N}$)

C. Network performance with incentives

We now show the performance of the network as the price pair is used as incentives. In particular, we compare the case where all nodes follow the incentive signals with the case where there exist nodes that deviate from the signals. In the latter case, the node with ID 3 deviates from the price pair signal by increasing its transmission rate by 60 Kbps. Table VII shows the results of such a comparison, including the following performance metrics:

- **Throughput** – the average transmission rate of a flow over the network operation time;
- **Cost** – the total cost (price $\times$ rate) over the network operation time, i.e., $\text{cost}_i = \sum_t (\lambda^\alpha_i(t) + \lambda^\beta_i(t)) \times x_i(t)$;
- **Revenue** – the total revenue of a node from relaying, i.e., $\text{revenue}_i = \sum_t \mu^\beta_i(t) \times \sum_j x_i(t)B_{ij}$;
- **Remaining energy** – the remaining energy at the end of network operation.
From the results, we have the following observations. First, from a single node point of view, though node 3 has a higher throughput when deviating from the incentive signals, it is also charged with a much higher cost, which leads to a decrease of net benefit (utility from throughput + revenue - cost) by 20233.8. Second, from the point of view of the entire network, the total utility of the network decreases from 78.27 to 77.31; the total remaining energy decreases from 3839.91J to 3701.55J. We can draw the conclusion that, a node that deviates from the price-pair incentive signal does not only cause the entire network to operate at a suboptimal point, but also decreases the net benefit of itself – which is unacceptable for a rational node.

The main difference that distinguishes our work from the existing works is rooted in the unique characteristics of resource models of ad hoc network. First, the allocation mechanism needs to consider two types of resources, namely, the shared channel and the private resource such as energy. Second, due to the location dependent contention of resource models of ad hoc network. First, the allocation mechanism needs to consider two types of resources, namely, the shared channel and the private resource such as energy. Second, due to the location dependent contention sharing among multiple users, e.g., [6][13].

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TABLE VII

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TABLE VII

Comparison of Network Performance

To summarize our results of performance evaluations, when using price pairs as incentives, the localized distributed algorithm at each node correctly converges to network operating points that are optimal, for the benefits of both individual nodes and the entire network.

VI. RELATED WORK

The problem of optimal and fair resource allocation has been extensively studied in the context of wireline networks. Among these works, pricing has been shown to be an effective approach to achieve distributed solution for flow control [10][11] and service differentiation [12]. Simultaneously, game theory is applied to model resource sharing among multiple users, e.g., [6][13].

The main difference that distinguishes our work from the existing works is rooted in the unique characteristics of resource models of ad hoc network. First, the allocation mechanism needs to consider two types of resources, namely, the shared channel and the private resource such as energy. Second, due to the location dependent contention and spatial reuse of the shared channel resource, the channel price is associated with a maximal clique in the subflow contention graph, rather than a wireline link. This presents a different pricing policy for end-to-end flows.

Incentives in wireless networks have stimulated much research interests. (e.g., in the context of ad hoc networks [8][14][15] and in wireless LAN [16]). In particular, the works in [8] [9] present virtual credit based mechanisms
to stimulate cooperation in ad hoc networks, where virtual credits (so called nuglets) are awarded for packet forwarding. Some approaches [17] [18] use a reputation based mechanism where selfish or misbehaving nodes are identified, isolated or punished. Our work distinguishes from the existing works in that, it does not only promote cooperation in packet forwarding, more importantly, it studies at what level of cooperation the network operates at its optimal point, and how to achieve such cooperation using pricing as incentives. Auction-based approach [19] has also been employed for incentives in ad hoc networks. The essential idea of this approach is that each relay node operates as a market, the users of the network put bids for their packets and will be charged accordingly, when the packets are forwarded. As an important approach of mechanism design, auction has been shown to have several important properties such as truthful bidding, social welfare maximization in single market. However, directly applying this approach to ad hoc network environments, where relaying nodes constitute multiple networked markets, these properties no longer hold. For example, user pay-off and social welfare are no longer maximized. And how much they deviate from the optimum is not clear. Our approach instead can be shown to achieve both user self optimization and global social welfare optimum (i.e., Nash Bargaining Solution).

Noncooperative game theory has been used to model the relaying behavior among nodes in ad hoc networks in [14] [15]. By designing appropriate game strategies and analyzing the Nash Equilibrium of the corresponding relaying game, these works show the existence of a network operating point where node cooperation is promoted. In our work, Nash bargaining solution is used to characterize the global network operating point, which usually demonstrates more advantageous properties, such as Pareto optimality and fairness, than the usual Nash Equilibrium in a noncooperative game.

There are also previous works that address the issue of resource allocation [20] and use a price-based approach [21]. However, the ad hoc network models in these works do not consider the shared nature of the wireless channel, and thus their solutions are not able to capture the unique issues in wireless ad hoc networks. Moreover, the price-based distributed algorithm presented in [20] only converges to a network optimum when its utility function takes certain a special form, and such a utility function does not satisfy the fairness axiom.

VII. CONCLUDING REMARKS

This paper presents a price pair mechanism that both regulates greedy behaviors and incentivizes selfish users in ad hoc networks. A pair of prices is the centerpiece of this mechanism: (1) the channel price that reflects the unique characteristics of location dependent contention in ad hoc networks, and regulates the usage of shared wireless channel; (2) the relay price that gives incentives to reach the adequate level of cooperation with respect to traffic relay. By using such a price pair as a signal, the decentralized self-optimizing decisions at each individual node converges to the global network optimal operation point. Simulation results are also presented to validate our theoretical claims. We believe that we have reached an important milestone that integrates pricing and incentives strategies for channel access and traffic relays into the same coherent framework, which is a problem that has not been addressed previously before this work, especially when the objective is to converge to a desirable optimal operating point for the entire wireless ad hoc network.

VIII. APPENDIX

A. Proof of Theorem 2
Proof. As the objective function Eq. (35) of Node\(_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)\) is strict concave, the problem Node\(_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)\) has a unique solution given by,

\[
x_i = \left[ \frac{1}{\lambda_i^\alpha + \lambda_i^\beta} + x_i^m \right] x_i^m
\]  

(51)

As the Lagrangian form for the Channel\((\mathbf{A}, \mathbf{C}; \mu^\alpha)\) problem is

\[
L(x, \mu^\alpha, p^\alpha) = \sum_{i \in \mathcal{N}} \lambda_i^\alpha x_i + p^\alpha (C - Ax)
\]

(52)

at the optimum of \(L(x, \mu^\alpha, p^\alpha)\) the following conditions hold:

\[
(\mu^\alpha - p^\alpha)A x = 0
\]

(53)

\[
p^\alpha (C - Ax) = 0
\]

(54)

Similarly, as the Lagrangian form for the Relay\((\mathbf{B}, \mathbf{E}; \mu^\beta)\) problem is

\[
L(x, \mu^\beta, p^\beta) = \sum_{i \in \mathcal{N}} \lambda_i^\beta x_i + p^\beta (E - Bx)
\]

(55)

at the optimum of \(L(x, \mu^\beta, p^\beta)\) the following conditions hold:

\[
(\mu^\beta - p^\beta)B x = 0
\]

(56)

\[
p^\beta (E - Bx) = 0
\]

(57)

The triple \((\mu^\alpha, \mu^\beta, x)\) which satisfies conditions Eq. (27), and Eq. (28) identifies a tuple \((p^\alpha, x)\) that satisfies condition Eq. (53) and Eq. (54) with \(p^\alpha = \mu^\alpha\). Thus it identifies a solution for Channel\((\mathbf{A}, \mathbf{C}; \mu^\alpha)\) problem. Similarly, the triple \((\mu^\alpha, \mu^\beta, x)\) also identifies a tuple \((p^\beta, x)\) that satisfies condition Eq. (56) and Eq. (57) with \(p^\beta = \mu^\beta\), Thus it identifies a solution for Relay\((\mathbf{A}, \mathbf{C}; \mu^\beta)\) problem. Moreover, note that Eq. (26) is the same as Eq. (51), which means it is also a solution to Node\(_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)\).

Conversely, we can construct a triple \((\mu^\alpha, \mu^\beta, x)\) with \(\mu^\alpha = p^\alpha\) and \(\mu^\beta = p^\beta\). Thus if \(x_i\) solves Node\(_i(\lambda_i^\alpha, \lambda_i^\beta, \mu_i^\beta)\), Channel\((\mathbf{A}, \mathbf{C}; \mu^\alpha)\) and Relay\((\mathbf{B}, \mathbf{E}; \mu^\beta)\), then it satisfies Eq. (51), Eq. (53), Eq. (53), Eq. (56), and Eq. (57), which lead to Eq. (26), Eq. (27), and Eq. (28), thus identifying a solution to \(P\).

\[\square\]

B. Proof of Theorem 3

Proof. First, the dual objective function \(D(\mu)\) is convex, lower bounded, and continuously differentiable.

For any price vector \((\mu^\alpha, \mu^\beta)\) define \(\psi_i(\mu^\alpha, \mu^\beta)\) as

\[
\psi_i(\mu^\alpha, \mu^\beta) = \begin{cases} 
(x_i(\lambda_i^\alpha, \lambda_i^\beta) - x_i^m)^2 & \text{if} \ \frac{1}{x_i} \leq \lambda_i^\alpha + \lambda_i^\beta \leq \frac{1}{x_i} \\
0 & \text{otherwise}
\end{cases}
\]

where \(\lambda_i^\alpha\) and \(\lambda_i^\beta\) are defined as in (20), (21) and \(x_i\) is defined as in (44).

Now we define \(\mathbf{H}(\mu^\alpha, \mu^\beta) = \text{diag}(\psi_i(\mu^\alpha, \mu^\beta), i \in \mathcal{N})\) be a \(|\mathcal{N}| \times |\mathcal{N}|\) diagonal matrix with diagonal elements
\( \psi_i(\mu^\alpha, \mu^\beta) \). Note that \( 0 \leq \psi_i(\mu^\alpha, \mu^\beta) \leq \kappa_i \).

Let \( \frac{\partial x_i}{\partial \mu_i^\alpha}(\mu^\alpha, \mu^\beta) \) denote the \(|N| \times |Q|\) Jacobian matrix whose \((i, q)\) element is \((\frac{\partial x_i(\mu^\alpha, \mu^\beta)}{\partial \mu_i^\alpha})\), where

\[
\frac{\partial x_i(\mu^\alpha, \mu^\beta)}{\partial \mu_i^\alpha} = \begin{cases} -A_{iq}(x_i(\lambda_i^\alpha, \lambda_i^\beta) - x^m_i)^2 & \text{if } \frac{1}{x_i^m} \leq \lambda_i^\alpha + \lambda_i^\beta \leq \frac{1}{x_i^m} \\ 0 & \text{otherwise} \end{cases}
\]

Let \( \frac{\partial x_i}{\partial \mu_i^\beta}(\mu^\alpha, \mu^\beta) \) denote the \(|N| \times |N|\) Jacobian matrix whose \((i, j)\) element is \((\frac{\partial x_i(\mu^\alpha, \mu^\beta)}{\partial \mu_i^\beta})\), where

\[
\frac{\partial x_i(\mu^\alpha, \mu^\beta)}{\partial \mu_i^\beta} = \begin{cases} -B_{ij}(x_i(\lambda_i^\alpha, \lambda_i^\beta) - x^m_i)^2 & \text{if } \frac{1}{x_i^m} \leq \lambda_i^\alpha + \lambda_i^\beta \leq \frac{1}{x_i^m} \\ 0 & \text{otherwise} \end{cases}
\]

From (47), (48) we have,

\[
\nabla_{\mu^\alpha} D(\mu^\alpha, \mu^\beta) = C - Ax(\lambda_i^\alpha, \lambda_i^\beta) = (60)
\]

\[
\nabla_{\mu^\beta} D(\mu^\alpha, \mu^\beta) = E - Bx(\lambda_i^\alpha, \lambda_i^\beta) = (61)
\]

Using (58), we have

\[
\begin{bmatrix} \frac{\partial x}{\partial \mu_i^\alpha} \end{bmatrix}(\mu^\alpha, \mu^\beta) = -H(\mu^\alpha, \mu^\beta)A^T
\]

\[
\begin{bmatrix} \frac{\partial x}{\partial \mu_i^\beta} \end{bmatrix}(\mu^\alpha, \mu^\beta) = -H(\mu^\alpha, \mu^\beta)B^T
\]

Thus the Hessian of \( D \) is given as follows.

\[
\nabla_2^\alpha \mu^\alpha D(\mu^\alpha, \mu^\beta) = AH(\mu^\alpha, \mu^\beta)A^T
\]

\[
\nabla_2^\alpha \mu^\beta D(\mu^\alpha, \mu^\beta) = BH(\mu^\alpha, \mu^\beta)B^T
\]

\[
\nabla_2^\beta \mu^\alpha D(\mu^\alpha, \mu^\beta) = AH(\mu^\alpha, \mu^\beta)B^T
\]

\[
\nabla_2^\beta \mu^\beta D(\mu^\alpha, \mu^\beta) = BH(\mu^\alpha, \mu^\beta)A^T
\]

Now we show that \( \nabla D \) is Lipschitz with

\[
||\nabla D(\nu) - \nabla D(\mu)||_2 \leq \bar{\kappa}\bar{Y}\bar{Z}||\nu - \mu||_2
\]

for all \( \mu, \nu \geq 0 \).

First, given any \( \mu, \nu \geq 0 \), using Taylor theorem we have

\[
\nabla D(\nu) - \nabla D(\mu) = \nabla^2 D(\omega)(\nu - \mu) = (69)
\]

for some \( \omega = t\mu + (1 - t)\nu \geq 0, \ t \in [0, 1] \).
Hence,

$$\|\nabla D(\nu) - \nabla D(\mu)\|_2 \leq \|\nabla^2 D(\omega)\|_2 \cdot \|\nu - \mu\|_2$$  \hfill (70)

Now we show that $$\|\nabla^2 D(\omega)\|_2 \leq \kappa \bar{Y} \bar{Z}.$$ First

$$\|\nabla^2 D(\omega)\|_2^2 \leq \|\nabla^2 D(\omega)\|_\infty \cdot \|\nabla^2 D(\omega)\|_1$$  \hfill (71)

Since $$\nabla^2 D(\omega)$$ is symmetric, we have

$$\|\nabla^2 D(\omega)\|_\infty = \|\nabla^2 D(\omega)\|_1$$  \hfill (72)

Hence,

$$\|\nabla^2 D(\omega)\|_2 \leq \|\nabla^2 D(\omega)\|_\infty = \max_r \sum_{r'} [\nabla^2 D(\omega)]_{rr'}$$  \hfill (73)

Actually,

$$[\nabla^2 D(\omega)]_{rr'} = \left\{ \begin{array}{ll}
\sum_i \psi_i(\omega) A_{ri} A_{ri} & \text{if } r, r' \in [0, Q - 1] \\
\sum_i \psi_i(\omega) A_{ri} B_{(r'-Q)i} & \text{if } r \in [0, Q - 1], \quad r' \in [Q, Q + N - 1] \\
\sum_i \psi_i(\omega) B_{(r-Q)i} A_{ri} & \text{if } r \in [Q, Q + N - 1], \quad r' \in [0, N - 1] \\
\sum_i \psi_i(\omega) B_{(r-Q)i} B_{(r'-Q)i} & \text{if } r, r' \in [Q, Q + N - 1] \end{array} \right\}
$$

Now we have,

$$\sum_{r'} [\nabla^2 D(\omega)]_{rr'} = \left\{ \begin{array}{ll}
\sum_i \left[ \psi_i(\omega) A_{ri} \left( \sum_{r'=0}^{Q-1} A_{ri} \right) \right] & \text{if } r \in [0, Q - 1] \\
\sum_i \left[ \psi_i(\omega) B_{(r-L)i} \left( \sum_{r'=0}^{Q-1} A_{ri} \right) \right] & \text{if } r \in \left[ Q, Q + N - 1 \right] \end{array} \right\}
$$

As $$Y(i) = \sum_q A_{qi} + \sum_j B_{ji},$$ and $$\bar{Y} = \max_{i \in \mathcal{N}} Y(i),$$ we have

$$\sum_{r'} [\nabla^2 D(\omega)]_{rr'} \leq \left\{ \begin{array}{ll}
\bar{Y} \sum_i \psi_i(\omega) A_{ri} & \text{if } r \in [0, Q - 1] \\
\bar{Y} \sum_i \psi_i(\omega) B_{(r-L)i} & \text{if } r \in [Q, Q + N - 1] \end{array} \right\}$$  \hfill (74)

Also because $$U(q) = \sum_{i \in \mathcal{N}} A_{qi}$$ and $$\bar{U} = \max_{q \in \mathcal{Q}} U(q); V(j) = \sum_{i \in \mathcal{N}} B_{ji}$$ and $$\bar{V} = \max_{j \in \mathcal{N}} V(j); \bar{Z} = \max \{\bar{U}, \bar{V}\}; \bar{\kappa} = \max_{i \in \mathcal{N}} \kappa_i,$$ we have

$$\max_r \sum_{r'} [\nabla^2 D(\omega)]_{rr'} \leq \bar{\kappa} \bar{Y} \bar{Z}. \hfill (75)$$

Since the dual objective function $$D$$ is lower bounded and $$\nabla D$$ is Lipschitz, then any accumulation point $$(\mu^\alpha, \mu^\beta)$$ of the sequence $$(\mu^\alpha(t), \mu^\beta(t))$$ generated by the gradient projection algorithm for the dual problem is dual optimal. Provided that $$0 < \gamma < 2/\bar{\kappa} \bar{Y} \bar{Z},$$ then let $$(\mu^\alpha(t), \mu^\beta(t))$$ be a subsequence converging to $$(\mu^\alpha, \mu^\beta).$$ Note that
\(x(\mu^\alpha, \mu^\beta)\) is continuous. Thus, the subsequence \(\{x(t)\}\) converges to the primal optimal rate \(x^*\).

\[\square\]

\section*{C. Implementation Details}

The algorithm treats maximal cliques as entities that are able to perform the communication and computation tasks. Obviously, these tasks need to be performed by the network nodes that constitute the maximal clique. As a starting point, a decentralized algorithm to construct maximal cliques is required. Here we present a decentralized maximal clique construction algorithm that explores the characteristics of subflow contention graphs. In this algorithm, the network topology is decomposed into overlapping subgraphs, and maximal cliques are constructed based only on local topological information within each of the subgraphs. Since only wireless links that are geographically close to each other will form an edge in the wireless link contention graph, the communication and computational overhead is significantly reduced.

To facilitate discussions, we introduce the following terms. The \textit{wireless link neighbor set} \(\mathbb{LN}(l)\) of a wireless link \(l \in L\) is defined as \(\mathbb{LN}(l) = \{l' | l \cap l' \neq \emptyset, l' \in L\}\). Similarly, the \textit{wireless link k-neighbor set} \(\mathbb{LN}^k(l)\) of \(l\) is defined by induction: (1) \(\mathbb{LN}^1(l) = \mathbb{LN}(l)\); and (2) \(\mathbb{LN}^k(l) = \mathbb{LN}(\mathbb{LN}^{k-1}(l))\) for \(k > 1\). For \(l \in V_C\), we further define \(\mathbb{SN}^k(l) = \mathbb{LN}^k(l) \cap V_C\).

Let graph \(G_C[V_C(l)]\) be an induced subgraph of \(G_C\) with \(V_C(l) = \mathbb{SN}^2(l) \subseteq V_C\). Then we observe that \(G_C[V_C(l)]\) contains sufficient and necessary topological information to construct \(Q(l)\) – all maximal cliques that contain wireless link \(l\). \(Q(l)\) can be acquired by applying the Bierstone algorithm [22] on graph \(G_C[V_C(l)]\). The algorithm of the decentralized maximal clique construction is summarized in Table VIII.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
\textbf{Clique Construction:} & \\
3 & Construct \(\mathbb{LN}^3(l)\) and \(\mathbb{SN}^2(l)\) \\
4 & Construct graph \(G_C[V_C(l)]\) \\
5 & Use Bierstone algorithm to get all maximal \\
 & cliques that contain wireless link \(l\) \\
 & \(Q(l) = \text{Bierstone}(G_C[V_C(l)])\) \\
\hline
\end{tabular}
\caption{Decentralized clique construction}
\label{table:clique}
\end{table}

\section*{References}


