A role-based access control type system for boxed ambients.*

Adriana Compagnoni, Stevens Institute of Technology, †
Elsa L. Gunter, University of Illinois, Urbana - Champaign, ‡
Philippe Bidinger, Stevens Institute of Technology, §

Abstract

Our society is increasingly moving towards richer forms of information exchange where mobility of processes and devices plays a prominent role. This tendency has prompted the academic community to study the security problems arising from such mobile environments, and in particular, the security policies regulating who can access the information in question.

In this paper we describe a calculus for mobile processes and propose a mechanism for specifying access privileges based on a combination of the identity of the users seeking access, their credentials, and the location from which they seek it, within a reconfigurable nested structure.

We define BACI_R, a boxed ambient calculus extended with a Distributed Role-Based Access Control mechanism where each ambient controls its own access policy. A process in BACI_R is associated with an owner and a set of activated roles that grant permissions for mobility and communication. The calculus includes primitives to activate and deactivate roles. The behavior of these primitives is determined by the process's owner, its current location and its currently activated roles. We consider two forms of security violations that our type system prevents: 1) attempting to move into an ambient without having the authorizing roles granting entry activated and 2) trying to use a communication port without having the roles required for access activated. We accomplish 1) and 2) by giving a static type system, an untyped transition semantics, and a typed transition semantics. We then show that a well-typed program never violates the dynamic security checks.

1 Introduction

The exchange of information by electronic means in a mobile environment has become part of everyday life, with cellphones, PDA’s, and laptop computers accessing remote information and transmitting signals and data. An increasingly mobile workforce needs to be able to access corporate information while at work, from home, and on the road. This tendency has led the academic community to study the security problems arising from this ever increasing mobility.

One aspect of secure computing is the control of who gains access to which shared and sensitive computing resources. Role-Based Access Control (RBAC) is a standardized methodology for defining security policies and for giving privileges to users, based on using roles as an abstraction representing a set of activities to be performed. Access is fundamentally controlled by roles. On one side, each user of a system is associated with a set of roles. On the other side, each role is associated with a set of permissions (access privileges to existing resources). Some roles may be mutually exclusive, and others may be deactivated leaving the user with only a subset of the full set of roles with which she is associated. Therefore, in simple RBAC, a user is granted an access privilege to a resource if one of her activated roles has that privilege. This factorization of access control simplifies the administration of the security policy by allowing the systems administrator to separately decide which resources a given role needs in order to successfully operate, and what roles to assign to each user. It also allows for the choice of authentication method to be handled separately. How to enrich RBAC by adding orderings and other forms of structure on the roles and the privileges is an active

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†abc@cs.stevens.edu
‡elsa@cs.uiuc.edu
§pbidinge@cs.stevens.edu
area of research. They all share in common the separation of concerns given by the introduction of roles. RBAC, however, is not concerned with the authentication of users. Whether the user claiming to be Bob is indeed Bob precedes the application of access control, and is beyond the scope of this work.

Mobility adds a new dimension to RBAC, since the services available to a given user also depend on the location of the user, agreements between parties, and the technology underlying the connection. For example, without roaming agreements in place, a cell-phone may be rendered useless beyond the scope of its provider’s network. Furthermore, whether a user’s connection is wireless, wired, secure, or insecure also conditions the available services. For example, an administrator on an insecure wireless connection may be denied access to sensitive information. In a distributed environment the policies regulating access control may be distributed among several parties, and each principal may only have partial knowledge of the overall security policy. In a mobile environment, different domains will have different access policies and when users (and potentially programs) migrate from domain to domain the access policy governing them will change with their enclosing domain.

Role-based access control is currently a popular mechanism for governing the access to databases, files, executable programs and other computational resources. In networking there is another kind of access control that is done by packet filtering. A given router may be configured to drop all SMTP or HTTP packets denying access to certain services of a domain from outside that domain. Here, there is no notion of user and role, but only IP domain and packet type. However, it can be beneficial to have a finer-grain access control that is aware of roles and network domains. Consider the following example:

The University of Wizbrau is equipped with intelligent buildings, and students carry their laptops with them to class. While in the classroom, students have only limited Internet access and they are not allowed to use e-mail, instant messenger, or visit general websites. However, these activities are allowed when done from the student lounge instead. Since the instructor of the course needs a greater access to resources than the students, those activities locally disabled to the students are available to the instructor. For example, during a lecture, the instructor may consult her e-mail to address a question raised by a student in an e-mail message.

The restrictions placed on users in this environment need to be sensitive to both the location of the user (classroom versus lounge) as well as the role (student versus instructor). Such fine-grained control is not readily handled by either packet filtering or RBAC.

In this paper we design a formal language featuring formal notions for resource, access, computation, communication, location and mobility. The starting point of our design is a mobile ambient calculus in the style of [10], where principals and locations are modeled by ambients. This formal language further includes a type system based on roles and localized role-based access control. We further show that well-types programs in this system don’t attempt unauthorized access to resources.

1.1 Background on Ambient Calculi

In Cardelli and Gordon’s Mobile Ambients (MA) [11], ambients represent nested computational environments containing data and live computation. In a nutshell, ambients are administrative units forming a dynamic hierarchy, where an ambient can move up and down the hierarchy by moving into a sibling or a parent ambient. Furthermore, a mobile ambient is a communicating entity that can exchange information with parents and children. MA are capable of moving under the influence of the process they enclose and can dissolve their perimeter with an open operation. Mobile Ambients provide a direct characterization of computational processes as well as computational devices.

Boxed Ambients (BA) [6] evolved from MA, by removing the ability of an ambient to dissolve its boundary. In BA, an ambient is a “box” that cannot be opened. This notion of closed ambient provides a complete encapsulation of the agents they contain. To enable the communication lost by disabling the open operation, ambients are equipped with communication channels to exchange information with adjacent ambients (parent and children ambients).

Both in MA and BA, ambient mobility is commanded by processes inside the ambient. The commands for mobility are called capabilities. The capabilities tell an ambient to open or move inside or outside another ambient. Unrestricted mobility, however, can lead to undesired interferences between two concurrent processes. To address this concern, control over capabilities was first introduced in Safe Ambients [21] and
later used in New Boxed Ambients (NBA) in the form of co-capabilities. A capability can be exercised only in the presence of a matching co-capability. Hence, in order to enter an ambient using the ins capability, that ambient must contain a matching in co-capability authorizing that access; similarly for exiting using the out capability.

Boxed Ambients with Communication Interfaces (BACI) introduced the notion of local views. In this calculus, each ambient has an associated communication port and a local view. The communication port is used for sending and receiving messages to and from other ambient, and the local view represents the communication types that are used by the processes enclosed inside the ambient. BACI is flexible enough to allow an ambient to communicate with different parents using different types. However, this flexibility came with the price of a rather complex syntax and some run-time type checking required to guarantee type safety. BACIv2 further enhanced communication mechanisms and mobility control by introducing multiple communication ports, access control lists, and port hiding.

Motivated by our earlier work on BACI, we define a typed boxed ambient calculus called BACIR extended with a Distributed Role-Based Access Control mechanism where each ambient controls its own access policy. Following the style of BACI, our new calculus distinguishes between names of ambients and names of communication ports. Ambients are used for mobility and ports are used for communication, either locally within the main process of an ambient or between a parent and a child. This distinction is instrumental in defining our RBAC mechanism, since it provides for a finer grain in the security policy. Each kind of ambient (as determined by its name) controls its own access policy by specifying which roles a user may activate for it, and which roles are sufficient to allow another ambient to enter it. Similarly, each kind of ambient specifies for the ports it generates which roles can read from it and which roles can write to it. The idea behind grouping ambients by name is that the name should indicate the general task to be performed, and all ambients of the same name should be uniform in the way they interact with other ambients.

An ambient in BACIR is associated with an owner and a set of activated roles that grant permissions for mobility and communication. The calculus includes primitives to activate and deactivate roles. The behavior of these primitives is determined by the owner of the ambient, its current location, and its currently activated roles. In order for an ambient to activate a role, the security policy has to allow the owner of the ambient to do so. Moreover, deactivating roles should not remove the roles authorizing the ambient to be in its current location.

We consider two forms of security violations that our type system prevents: 1) attempting to move into an ambient without having the authorizing roles granting entry activated and 2) trying to use a communication port without having the roles required for access activated. We accomplish 1) and 2) by giving a static type system, an untyped transition semantics, and a typed transition semantics in Section 4. We then show that a well-typed program never violates the dynamic security checks in Theorem 4.3.

This technical report is a revision of the extended abstract that appeared in the proceedings of the international symposium Trustworthy Global Computing, 2005. The work here differs from that in the abstract in the following ways: The grammar of our language, and correspondingly the type system and transition rules, have been simplified by the removal of cases that were semantically equivalent to other constructs. We have also added the details of how to encode the example of the student versus the instructor in the classroom in our calculus, and described how the type system prevents the unwanted network actions from taking place.

2 Syntax of BACIR

Based on our earlier work on BACI, we define BACIR, a boxed ambient calculus with a Distributed Role-Based Access Control mechanism, where the location of an ambient conditions its privileges. The intuitive idea is that to accommodate security checking an ambient is associated with its owner and with a set of roles that are currently activated. This set of roles can be changed by activation and deactivation primitives. Whether a role can be activated or deactivated depends on the location of the ambient and its owner. This control is made explicit in the type system where the type of an ambient has a set of roles authorizing the entrance of ambients. Going back to the example, the professor can send mail because she can activate the faculty_mail role, while the students can only activate the student_mail role, which is not enough to qualify to send mail in the classroom.
### Basic types

- $\tau ::= \text{amb}(\rho, \sigma)$ ambient type
- $\text{cap}(\rho, \sigma)$ capability type

### Communication types

- $\sigma ::= \text{shh}$ no exchange
- $(\rho_r, \rho_w, \tau)$ exchange tuple

### Locations:

- $\eta ::= \uparrow c$ parent port
- $\downarrow c$ child port
- $\star$ local

### Actions:

- $\pi ::= C(c)$ capability
- $K(c)$ co-capability
- $\text{activate}(r)$ activate role $r$
- $\text{deactivate}(r)$ deactivate role $r$
- $(x_1, \ldots, x_k)^n$ input
- $(M_1, \ldots, M_k)^n$ output

### Capabilities:

- $C ::= \text{id}$ capability variable
- $\text{in } m$ enter
- $\text{out } m$ exit
- $C_1.C_2$ path

### Co-Capabilities:

- in allow enter
- out allow exit

### Messages:

- $M, N ::= m$ ambient name
- $C$ capability

### Processes:

- $P ::= 0$ nil process
- $P_1 | P_2$ composition
- $\nu(n: \tau)P$ restriction
- $!P$ replication
- $m_n[P]_{\rho}$ ambient

### Table 1: Syntax of BACI$_R$

<table>
<thead>
<tr>
<th>Basic types</th>
<th>Capabilities:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau ::= \text{amb}(\rho, \sigma)$ ambient type</td>
<td>$C ::= \text{id}$ capability variable</td>
</tr>
<tr>
<td>$\text{cap}(\rho, \sigma)$ capability type</td>
<td>$\text{in } m$ enter</td>
</tr>
<tr>
<td>$(\rho_r, \rho_w, \tau)$ exchange tuple</td>
<td>$\text{out } m$ exit</td>
</tr>
<tr>
<td>$C_1.C_2$ path</td>
<td></td>
</tr>
</tbody>
</table>

In order to define the syntax of BACI$_R$ we use the following disjoint categories of identifiers:

- User Names: $u, v \in \text{Users}$
- Roles: $r \in \text{Roles}$
- Port Names: $c, c' \in \mathcal{C}$
- Ambient Names: $n, m \in \text{Amb}$
- Capability Variables: $i \in \text{CapVar}$
- Message Identifiers: $x \in \text{Amb} \cup \text{CapVar}$

We assume a fixed set Users of users, a fixed set Roles of roles, and a fixed function UserPolicy associating each ambient, user and set of currently activated roles with a set of roles that may become activated for the given ambient. The syntax of BACI$_R$ is presented in Table 1.

BACI$_R$ may be seem as an extension of the $\pi$-calculus. Processes and Actions are the two main syntactic categories. We add to the processes of the $\pi$-calculus that of ambients. Further, we modify restriction to apply only to ambient names, which are distinct from port names (a.k.a. channel names). The $\pi$-calculus actions of input and output are modified to provide the location of the communication port (parent, child, or local). We add to these actions those for activation and deactivation of roles and those for requesting and granting permission for movement of ambients (capabilities and co-capabilities). Capabilities and co-capabilities, in addition to controlling the movement of ambients, introduce port names and provide their scope. Capabilities include paths (sequences of movement requests) to allow directions to be passed to an ambient directing it to a specific location. All the capabilities in the path except the last involve no communication and generate no port.

We introduce the usual notion of process equivalence through the structural congruence generated by alpha conversion, associativity and commutativity of parallel composition with 0 for identity, and the rules given in Table 2. The rules for replication and restriction are fairly standard. We add a rule for allowing restriction to pass through an ambient, provided that ambient is not the one whose name is being restricted. (The functions fn and fp give the free ambient names and the free port names, respectively, in a process.) The last rule tells us that to follow a path is the same as to follow it in pieces. This makes sense because ambients can only enter one other ambient at a time. In Section 4, we will see that the operational semantics respects process equivalence.
3 Types for Security

Attempting to enter an ambient without an authorizing role activated is a security violation. Trying to use a communication port without having activated at least one of the required roles to access the port is also a security violation. In this section we define a type system such that well-typed processes can compute without committing security violations. The type of a process is a set of roles sufficient for it to compute without security violations. In particular, the type of an ambient name is the set of roles needed for mobility and communication. The syntax of types can be found in Table 1. Basic types describe the kind of data to be communicated over a port, either ambient name or capability, together with the roles sufficient for communication. The communication type includes the sets of roles that the process may at any time in its computation safely deactivate, granting read and write access to a port. In this presentation of the calculus, we allow only ambient names and capabilities to be passed over ports, but not port names. Allowing the communication of computational types such as the integers does not affect the results here.

In Tables 2, 3 and 4 let Γ be a typing environment mapping message identifiers to basic types and port names to communication types. We further require that Γ(m) be an ambient type for ambient names, and Γ(i) be a capability type for capability variables (the two sorts of message identifiers). The typing judgment for a process is of the form Γ, ρhere, ρdeac, m, u ⊢ P : ρact, where Γ is the typing environment for free message identifiers and port names, m is the assumed surrounding ambient, u is the current user (owner of m), ρhere is the set of roles sufficient for authorizing m to be in its current location (the entrance policy for the ambient containing m), ρdeac is the set of roles that the process may at any time in its computation safely deactivate, and ρact is the set of “currently active” roles. The judgments for the other syntactic categories are similar.

Typing of ambients and capabilities is mostly as would be expected. One should note that when typing these, one may use a more restrictive set of roles than is allowed by the typing environment. Thus, when typing a path, we may use the intersection of all the roles in of the ambients in the path. However, we should note the only the communication type of the right-most component in a path is used for the communication type of the whole path. To type local communication, we use the ambient assumed as the surrounding ambient, and we want no restrictions on reading or writing. However, it is important that we maintain the restrictions on the types of data transmitted. We could violate the security policy if we omitted the type checks on messages locally communicated, because we potentially could send a capability with one security policy, but receive it with a different one.

The rules for typing actions and processes appear in Table 4 and Table 5 respectively. Actions are the...
**Table 4: Well-typed Actions**

<table>
<thead>
<tr>
<th>Capabilities:</th>
<th>Co-Capabilities:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma \vdash C : \text{cap}(\rho_{in}, \sigma)$</td>
<td>$\Gamma \vdash m : \text{amb}(\rho_{in}, \sigma)$</td>
</tr>
<tr>
<td>$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash C(c) : (\Gamma + (c : \sigma), \rho_{in}, \rho_{act})$</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash K(c) : (\Gamma + (c : \sigma), \rho_{here}, \rho_{act})$</td>
</tr>
</tbody>
</table>

**Activation:**

$r \in \text{UserPolicy}(u, \rho_{act})$

$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash \text{activate}(r) : (\Gamma, \rho_{here}, \rho_{act} \cup \{r\})$

**Deactivation:**

$r \in \rho_{deact}$

$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash \text{deactivate}(r) : (\Gamma, \rho_{here}, \rho_{act} - \{r\})$

**Input:**

$m \notin \{x_1, \ldots, x_k\}$

$\Gamma, m \vdash \eta : (\rho_r, \rho_w, \tau)$

$(\rho_{act} - \rho_{deact}) \cap \rho_r \neq \emptyset$

$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash (x_1, \ldots, x_k)^\eta : (\Gamma + \sum_{i=1}^{k}\{x_i : \tau\}, \rho_{here}, \rho_{act})$

**Output:**

$\Gamma, m \vdash \eta : (\rho_r, \rho_w, \tau)$

$\Gamma \vdash M_i : \tau \quad i = 1, \ldots, k$

$(\rho_{act} - \rho_{deact}) \cap \rho_w \neq \emptyset$

$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash \langle M_1, \ldots, M_k \rangle^\eta : (\Gamma, \rho_{here}, \rho_{act})$

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<table>
<thead>
<tr>
<th>Table 5: Well-typed Processes</th>
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</table>

<table>
<thead>
<tr>
<th>Nil:</th>
<th>Composition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash 0 : \rho_{act}$</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash P_1 : \rho_{act}$ $\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash P_2 : \rho_{act}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Restriction:</th>
<th>Replication:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma + (m' : \tau), \rho_{here}, \rho_{deact}, m, u \vdash P : \rho_{act}$</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash P : \rho_{act}$</td>
</tr>
<tr>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash \nu(m' : \tau)P : \rho_{act}$</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash !P : \rho_{act}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prefixing:</th>
<th>Ambient:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash \pi : (\Gamma', \rho'<em>{here}, \rho'</em>{act})$</td>
<td>$\Gamma, m : \text{amb}(\rho_{in}, \sigma)$ $\Gamma, \rho_{in}, \rho'<em>{deact}, m, v \vdash P : \rho</em>{m}$</td>
</tr>
<tr>
<td>$\Gamma', \rho'<em>{here}, \rho</em>{deact}, m, u \vdash P : \rho'_{act}$</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash m [P]@ \rho_{m} : \rho_{act}$</td>
</tr>
</tbody>
</table>
4 Operational Semantics

Our goal in defining the static type system given in Section 3 is to enable us to prove that if a process type checks with a given set of roles, then it will never attempt an action that is not authorized to perform when executed in a state where all the roles in the set have previously been activated. To this end, we define two transition semantics for our language, one with dynamic security checks and one without. For the untyped semantics, we have a form of subject reduction. We also have that, if a process type checks, then it reduces to another process in the untyped transition system if and only if it reduces to that process in the typed transition system.

4.1 Untyped Transition Semantics

The untyped transition semantics for BACI_R is given in Table 6. It is worth noting that almost all the reduction rules explicitly mention a context containing an ambient, except for the rule for LOCAL communication. The rules for ambient movement (ENTER and EXIT) are the most complicated. For an ambient to ENTER another, the two ambients must be directly in parallel with each other, the first ambient must contain a process requesting entrance to the second, and the second ambient must have a process allowing
users, (Subject Reduction) Let $\Gamma$ be a typing environment. If $\Gamma, \rho_{\text{here}}, \rho_{\text{act}}, m, u \vdash P_1 : \rho_{\text{act}}$ and $P_1 \Rightarrow P_2$, then there exists a typing environment $\Gamma' \supseteq \Gamma$ with $\Gamma', \rho_{\text{here}}, \rho_{\text{act}}, m, u \vdash P_2 : \rho_{\text{act}}$. Moreover, if $m_u[P_1]@\rho_{\text{act}} \Rightarrow m_u[P_3]@\rho'_{\text{act}}$, then $m = n$ and $u = v$ and there exists $\Gamma'' \supseteq \Gamma$ such that $\Gamma'', \rho_{\text{here}}, \rho_{\text{act}}, n, v \vdash P_3 : \rho'_{\text{act}}$.

4.2 Typed Transition Semantics

In this section we briefly sketch the transition semantics with runtime type checks (e.g. security checks).

The rules of the semantics are found in Tables 7–10. These rules augment the processes to be evaluated with a typing context in which the evaluation is to take place. This context is comprises a typing environment, $\Gamma$, a set of roles, $\rho_{\text{here}}$, and a basic type, $\tau$. As usual, the typing environment supplies us with the types for free ambient names and ports occurring in our process. The set of roles tells which roles are sufficient to authorize the process’s current location. The basic type is the type of a message that can be locally communicated at top level. We do not need read and write policies, because there are no security checks on local communication. The typed reduction relation transforms a process and its context into a new process in a new context. If we ignore the context, including the premises concerning it, then we get the untyped system in the previous section. The typing environment, role set and basic type are the extra information we need to carry to do dynamic security checks.

Since the reductions on the processes are the same as in the untyped transition semantics, we will focus on the security checks and the transformations to the typing environment and basic type. Activation and deactivation are relative to an enclosing ambient and serve to change that ambient’s set of active roles. For activation, we must check that the user of the ambient together with the currently active roles are allowed to activate the role. For deactivation, we need to check that deactivating the role will still leave some other role that is sufficient to authorize the ambient’s current location.

When one ambient enters another, we need to know that the entering ambient has an appropriate role activated authorizing it to enter, and we need to establish a shared communication port for sending and

<table>
<thead>
<tr>
<th>ACTIVATE:</th>
<th>$r \in \text{UserPolicy}(u, \rho)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[(\text{activate}(r)P)</td>
<td>R]@\rho \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[P</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DEACTIVATE:</th>
<th>$(\rho - {r}) \cap \rho_{\text{here}} \neq \emptyset$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[(\text{deactivate}(r)P)</td>
<td>R]@\rho \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[P</td>
</tr>
</tbody>
</table>

Table 7: Roles
Enter:

\[
\Gamma(n) = \text{amb}(\rho_m, \tau) \quad \rho_m \cap \rho_n \neq \emptyset \quad e'' \not\in \text{dom}(\Gamma).
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright m_u[n_u m(n(c)).P_1 \mid P_2]@\rho_m \mid n_u m(n(c')).P_3 \mid P_4]@\rho_n \longrightarrow
\]

\[
(\Gamma + (e'': \tau), \rho_{here}, \tau) \triangleright n_u[m_u[P_1\{c := c''\} \mid P_2]@\rho_m \mid P_3\{c' := c''\} \mid P_4]@\rho_n
\]

Exit:

\[
\Gamma(p) = \text{amb}(\rho_m, \tau) \quad \rho_m \cap \rho_n \neq \emptyset \quad e'' \not\in \text{dom}(\Gamma)
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright p_w[n_v[m_v[\text{out}(p(c)).P_1 \mid P_2]@\rho_m \mid P_3]@\rho_p \mid \text{out}(c')P_4 \mid P_5]@\rho_p \longrightarrow
\]

\[
(\Gamma + (e'': \tau), \rho_{here}, \tau) \triangleright p_w[n_v[P_3]@\rho_n \mid m_u[P_1\{c := c''\}]@\rho_m \mid P_3\{c' := c''\} \mid P_5]@\rho_p
\]

Table 8: Mobility

LOCAL:

\[
(\Gamma, \rho_{here}, \tau) \triangleright \langle M_1, \ldots, M_k \rangle^*, P \mid (x_1, \ldots, x_k)^*, Q \longrightarrow (\Gamma, \rho_{here}, \tau) \triangleright P \mid Q\{x_i := M_i \mid i = 1 \ldots k\}
\]

TO CHILD (|):

\[
\Gamma(c) = (\rho_r, \rho_m, \tau') \quad \rho_m \cap \rho_n \neq \emptyset \quad \rho_n \cap \rho_r \neq \emptyset \quad \Gamma \vdash M_i : \tau' \quad i = 1, \ldots, k
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright m_u[(M_1, \ldots, M_k)^i c_i. P_1 \mid n_v[(x_1, \ldots, x_k)^i c_i. P_2 \mid R_1]@\rho_m \mid R_2]@\rho_m \longrightarrow
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright m_u[P_1 \mid n_v[P_2\{x_i := M_i \mid i = 1 \ldots k\} \mid R_1]@\rho_m \mid R_2]@\rho_m
\]

TO PARENT (|):

\[
\Gamma(c) = (\rho_r, \rho_m, \tau') \quad \rho_m \cap \rho_n \neq \emptyset \quad \rho_n \cap \rho_r \neq \emptyset \quad \Gamma \vdash M_i : \tau' \quad i = 1, \ldots, k
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright n_u[m_u[(M_1, \ldots, M_k)^i c_i. P_1 \mid R_1]@\rho_m \mid (x_1, \ldots, x_k)^i c_i. P_2 \mid R_2]@\rho_m \longrightarrow
\]

\[
(\Gamma, \rho_{here}, \tau) \triangleright n_u[m_u[P_1 \mid R_1]@\rho_m \mid P_2\{x_i := M_i \mid i = 1 \ldots k\} \mid R_2]@\rho_m
\]

Table 9: Communication

Receiving messages of a type specified by the host ambient. The new communication port needs to be added to the typing environment. (See Enter in Table 8) The location of the encompassing process hasn’t changed, so \(\rho_{here}\) remains the same. The side conditions for the rule for Exit are comparable to those for Enter.

There are three kinds of communication: local communication between top-level subprocesses, sending a message from a parent to a child and sending a message from a child to a parent. For local communication, we only need to check that the type of the messages being sent is of the type specified for local communication (by \(\tau\)) and that the number of messages sent is the same as those received. For trans-generational communication, in addition to checking the number of messages as before, we need to check that the writing ambient has write access to the port and the reading ambient has read access, and that the type of all messages sent is the type specified in the type of the port as given by the typing environment.

The structural rules for our transition semantics tell us how and when we can descend through structures. None of the structural rules impose any security checks in and of themselves. The rules for recursion and composition use the same environment to security check the premises as they use in their conclusions. Restriction uses a type environment augmented by the type assignment for the restricted ambient name for reducing the body of the restriction. For descending through ambients, the typing environment is the same in the premise as in the conclusion, but here we need to change the type for the local communication to that of the basic type in the communication policy of the ambient, and we need to change the authorizing roles to the entrance policy of the outer ambient.

The next theorem gives us that the typed transition semantics is a refinement of the untyped transition semantics.
There are five roles available, instructor in the class, a student in the class, and the systems administrator for the classrooms in the university. There is more limited than that of the instructor. Let the example outlined in the introduction of the university with classrooms where students’ access to the internet is more limited than that of the instructor. We need to combine mobility with localized checking of access authorization to local resources. Recall the typed transition semantics, we can still perform security checks even in a situation where static rules are predicated on static access to the information as to which roles are granted access to which resources. With the typed transition semantics, we can still perform security checks even in a situation where static type checking. It is worth noting that this semantics is of value in its own right. Theorem 4.3 tells us that, if a process type checks, then to evaluate it you can omit all runtime checks. A side-effect of this is that if a process type checks, there is no runtime significance to activation and deactivation, and they could be removed after type-checking as an optimization.

<table>
<thead>
<tr>
<th>COMPOSITION:</th>
<th>RESTRICTION:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$(\Gamma, \rho_{\text{here}}, \tau) \triangleright P_1 \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright P_2$$</td>
<td>$$(\Gamma + { m : \tau }, \rho_{\text{here}}, \tau) \triangleright P \longrightarrow (\Gamma + { m : \tau }, \rho_{\text{here}}, \tau) \triangleright R$$</td>
</tr>
<tr>
<td>$$(\Gamma, \rho_{\text{here}}, \tau) \triangleright P_1 \mid R \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright P_2 \mid R$$</td>
<td>$$(\Gamma, \rho_{\text{here}}, \tau) \triangleright \nu(m : \tau)P \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright \nu(m : \tau)R$$</td>
</tr>
</tbody>
</table>

| AMBIENTS: | $$(\Gamma(m) = \text{amb}(\rho_{\text{in}}, (\rho_r, \rho_u, \tau')))$$ |
| | $$(\Gamma, \rho_{\text{in}}, \tau') \triangleright P \longrightarrow (\Gamma, \rho_{\text{in}}, \tau') \triangleright R$$ |
| | $$(\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[P]@\rho \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright m_u[R]@\rho$$ |
| STRUCTURAL EQUIVALENCE: | $P_1 \equiv P_2$ |
| | $$(\Gamma, \rho_{\text{here}}, \tau) \triangleright P_2 \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright R_2 \quad R_2 \equiv R_1$$ |
| | $$(\Gamma, \rho_{\text{here}}, \tau) \triangleright P_1 \longrightarrow (\Gamma, \rho_{\text{here}}, \tau) \triangleright R_1$$ |

Table 10: Structural Rules

Theorem 4.2 Let $P$ and $P'$ be processes, $\Gamma$ and $\Gamma'$ be typing environments, $\rho_{\text{here}}$ and $\rho'_{\text{here}}$ be sets of roles, and $\tau$ and $\tau'$ be basic types. If $$(\Gamma, \rho_{\text{here}}, \tau) \triangleright P \longrightarrow (\Gamma', \rho'_{\text{here}}, \tau') \triangleright P'$$, then $\rho_{\text{here}} = \rho'_{\text{here}}, \tau = \tau', \Gamma \subseteq \Gamma'$, and $P \Rightarrow P'$.

Theorem 4.3 Let $P$ be a process that type checks with role set $\rho$ using typing environment $\Gamma$, authorizing role set $\rho_{\text{here}}$, ambient $m$, such that $\Gamma(m) = \text{amb}(\rho_{\text{in}}, (\rho_r, \rho_u, \tau))$, and user $u$ (e.g. $\Gamma$, $\rho_{\text{here}}$, $\rho_{\text{deact}}$, $m$, $u \vdash P : \rho_{\text{act}}$). If $P \Rightarrow P'$ for some process $P'$, then $$(\Gamma, \rho_{\text{in}}, \tau) \triangleright P \longrightarrow (\Gamma', \rho_{\text{in}}, \tau) \triangleright P'$$, for some $\Gamma'$ such that $\Gamma \subseteq \Gamma'$.

The typed transition semantics developed in this section was primarily introduced as a vehicle to formalize the benefit of static type checking. It is worth noting that this semantics is of value in its own right. The static rules are predicated on static access to the information as to which roles are granted access to which resources. With the typed transition semantics, we can still perform security checks even in a situation where the control policy is only known at runtime.

5 Example: The University of Wizbrau

To see the utility of the calculus discussed in this paper, let us see how we could use it to express an example where we need to combine mobility with localized checking of access authorization to local resources. Recall the example outlined in the introduction of the university with classrooms where students’ access to the internet is more limited than that of the instructor. Let $\{\text{ProfSue}, \text{Dan}, \text{Chuck}\}$ be users representing the instructor in the class, a student in the class, and the systems administrator for the classrooms in the university. There are five roles available, student, student_mail, instructor, faculty_mail and sys_admin. To send mail, one should use an ambient named mail. To use the automated class response system, one should use an ambient named answer. The laptops of the students and the instructor will be represented by ambients named laptop, the classroom ambient will be named classroom and the student lounge ambient will be named lounge. The classroom and the student lounge are part of the Univ ambient.

The user policy and a partial initial typing environment are given in Table 11. For this example, the user policy doesn’t depend on the currently active policies, and they are omitted in the table. For brevity, we will also omit communication types from the typing environment description.

The basic program for both the classroom and the student lounge is the same. It allows other ambients to come and go, provided they have the right roles activated. For those entering, no communication is done. For those exiting, a path to a router is provided. The program for the instructor is to enter the classroom.
and send some mail, in parallel with some other work. The student enters the classroom, answers some questions, but also wants to send some mail. Here we can examine some options that fail, together with one that works. The formal processes are listed below. To assist in examining the possibilities for Dan, his process will be parametrized by the the role activated for the mail ambient and the ambient to which the mail ambient tries to exit.

Table 11: Wizbrau Security Policy

<table>
<thead>
<tr>
<th>Ambient</th>
<th>User</th>
<th>Roles</th>
<th>Ambient</th>
<th>User</th>
<th>Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td>laptop</td>
<td>ProfSue</td>
<td>{instructor}</td>
<td>mail</td>
<td>ProfSue</td>
<td>{faculty_mail}</td>
</tr>
<tr>
<td>Dan</td>
<td></td>
<td>{student}</td>
<td>Dan</td>
<td></td>
<td>{student_mail}</td>
</tr>
<tr>
<td>Chuck</td>
<td></td>
<td>{}</td>
<td>Chuck</td>
<td></td>
<td>{}</td>
</tr>
<tr>
<td>classroom</td>
<td>ProfSue</td>
<td>{}</td>
<td>answer</td>
<td>ProfSue</td>
<td>{instructor}</td>
</tr>
<tr>
<td>lounge</td>
<td>Dan</td>
<td>{}</td>
<td>Dan</td>
<td></td>
<td>{student}</td>
</tr>
<tr>
<td>Univ</td>
<td>Chuck</td>
<td>{sys_admin}</td>
<td></td>
<td>Chuck</td>
<td>{}</td>
</tr>
</tbody>
</table>

$$\Gamma(\text{classroom}) = \text{amb}\{{\text{student, instructor, faculty_mail}}, .\}$$

$$\Gamma(\text{Univ}) = \Gamma(\text{lounge}) = \text{amb}\{{\text{student, student_mail, instructor, faculty_mail}}, .\}$$

6 Related Work

For a variety of calculi for mobile and distributed systems that have emerged in the last years, access control was one of the primary concerns. The proposed access control mechanisms range from simple ones that use
of co-actions allowing or denying all access to a particular location (and the resources it contains) to more refined ones that use different approaches: credentials to authorize the access, restricted groups, Mandatory Access Control mechanisms to constraint unauthorized access, and even “membranes” that specify security policies for controlling the access to a particular location.

The work most closely related to our study of RBAC for an ambient calculus is [4]. The authors define a distributed π-calculus (D-π) based on [19] with primitives to activate and deactivate roles. However, there is no notion of an individual privilege being disabled or enabled depending on the current location, and the domain topology is static: domains cannot move. In [20] Hennessy and Riely introduce a type system for a distributed version of the π-calculus for restricting the access of processes to resources based on the current location of the process. In this work, again the domain topology is static, and there is no direct connection to RBAC.

At the Symposium on Trustworthy Global Computing 2005 (TGC 2005), during his invited address, Matthew Hennessy presented a calculus for RBAC based on D-π. Unlike our system, his calculus has dependent types to avoid dynamic typechecks of the security policy.

The work of RBAC in [24, 25] does not deal with the implementation of an RBAC mechanism in a given calculus as is the case in [4]. Instead they define a calculus to describe an RBAC security policy and how to answer queries to the security policy.

Various groups have developed methods for guaranteeing that specifications of RBAC systems are consistent. In [27], Schaad and Moffett discuss the application of formal methods for the development of specifications of a conflict-free role-based system. In [1] a formal language for the specification of role-based authorization constraints, including prohibition, is introduced. Bertino et al. [2] develop a logical framework for reasoning about access control models in general, including RBAC models.

7 Conclusions and Future Work

We defined $\text{BACI}_R$, a boxed ambients calculus with Distributed Role-Based Access Control, where the privileges associated to processes change during computation and are determined by their location, their owners, the roles they have activated, and the security policy. The distributed nature of the RBAC mechanism comes from the fact that each ambient controls the security policy authorizing the entrance of ambients and each port specifies the security policy controlling the reading and writing privileges.

Our type system prevents two forms of security violations, those consisting of attempting to enter an ambient without proper authorization, and those consisting of trying to read or write from ports without the corresponding permissions. These security violations are controlled using roles, that can be dynamically activated and deactivated. The type system prevents security violating actions by those processes not vested with the required authorizing roles.

Our main contribution is the design of the first ambient calculus with a distributed RBAC mechanism where the location of a process conditions its mobility and its ability to communicate with other processes. Our main result in Theorem 4.3 shows that a well-typed program never violates the dynamic security checks.

Although the classroom example in the introduction and Section 5 is focused on Internet networking for a sense of location and communication, our Distributed RBAC mechanism should be applicable to other settings such as those arising from mobile telecommunications.

The area remains full of open and challenging problems. An interesting aspect to consider is the notion of trust in such a way that the access control policy governing the users’ requests will further depend on whether the user is in a trusted or untrusted domain. Furthermore, RBAC can be enriched by placing order structures on roles (role hierarchies), constraints on roles such as mutual exclusion (no user may activate two given roles at the same time), combination of roles (two given roles have to be activated at the same time), and composition of roles (users having a given role are given another role). Defining type systems to address these richer notions of RBAC is the subject of our ongoing and future research.
8 Acknowledgments

We are grateful to Pablo Garralda, Healfdeane Goguen, and Mariangiola Dezani for illuminating discussions and comments on earlier drafts. We also appreciate Kaijun Tan for introducing us to the idea of combining RBAC with calculi for concurrency.

A Auxiliary Lemmas

In section 2 we saw that processes make use of names $n, m$, variables $i$, and ports $c, c'$. We use $x$ for message identifiers, which can be either variables or names. We define in Figures 1, 2 and 3 the free names, free variables, and free ports of a process. The free identifiers are defined as $fmi(P) = fn(P) \cup fv(P)$. We also defined the names, variables, identifiers, and ports bound by a prefix, $bn(\pi)$, $bv(\pi)$, $bmi(\pi)$ and $bp(\pi)$ respectively, where $bmi(\pi) = bn(\pi) \cup bv(\pi)$.

Free Names of a Process

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$fn(\emptyset)$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$fn(P_1 \mid P_2)$</td>
<td>$fn(P_1) \cup fn(P_2)$</td>
</tr>
<tr>
<td>$fn(\nu(n:\tau)P)$</td>
<td>$fn(P) \setminus {n}$</td>
</tr>
<tr>
<td>$fn(!P)$</td>
<td>$fn(P)$</td>
</tr>
<tr>
<td>$fn(m[u][\rho])$</td>
<td>$fn(P) \cup {m}$</td>
</tr>
</tbody>
</table>

Free Names of an Action

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$fn(C(c))$</td>
<td>$names(C)$</td>
</tr>
<tr>
<td>$fn(K(c))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$fn(activate(\langle r \rangle))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$fn(deactivate(\langle r \rangle))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$fn((x_1, \ldots, x_k)^\eta)$</td>
<td>$names(x_1) \cup \ldots \cup names(x_k)$</td>
</tr>
<tr>
<td>$fn(\langle M_1, \ldots, M_k \rangle)^\eta)$</td>
<td>$names(M_1) \cup \ldots \cup names(M_k)$</td>
</tr>
</tbody>
</table>

Binding Names of an Action

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bn(C(c))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$bn(K(c))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$bn(activate(\langle r \rangle))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$bn(deactivate(\langle r \rangle))$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$bn((x_1, \ldots, x_k)^\eta)$</td>
<td>$names(x_1) \cup \ldots \cup names(x_k)$</td>
</tr>
</tbody>
</table>

Names of a Capability

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$names(i)$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$names(in \ m)$</td>
<td>${m}$</td>
</tr>
<tr>
<td>$names(out \ m)$</td>
<td>${m}$</td>
</tr>
<tr>
<td>$names(C_1.C_2)$</td>
<td>$names(C_1) \cup names(C_2)$</td>
</tr>
</tbody>
</table>

Names of a Message

<table>
<thead>
<tr>
<th>Expression</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$names(m)$</td>
<td>${m}$</td>
</tr>
</tbody>
</table>

$names(C) = names(C)$

Figure 1: Free Names and Binding Names

The type system in section 3 is defined with the following judgments.

<table>
<thead>
<tr>
<th>Category</th>
<th>Judgment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capabilities</td>
<td>$\Gamma \vdash C : (\rho_m, \sigma)$</td>
</tr>
<tr>
<td>Locations</td>
<td>$\Gamma, m \vdash \eta : \sigma$</td>
</tr>
<tr>
<td>Messages</td>
<td>$\Gamma \vdash M : \tau$</td>
</tr>
<tr>
<td>Actions</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, \rho_{act}, m, u \vdash \pi : (\Gamma', \rho_{here}, \rho_{act})$</td>
</tr>
<tr>
<td>Processes</td>
<td>$\Gamma, \rho_{here}, \rho_{deact}, m, u \vdash P : \rho_{act}$</td>
</tr>
</tbody>
</table>
Free Variables of a Process
\[ \text{fv}(0) = {} \]
\[ \text{fv}(P_1 | P_2) = \text{fv}(P_1) \cup \text{fv}(P_2) \]
\[ \text{fv}(\nu(n: \tau)P) = \text{fv}(P) \]
\[ \text{fv}(!P) = \text{fv}(P) \]
\[ \text{fv}(\pi.P) = \text{fv}(P) \setminus \text{bv}(\pi) \]
\[ \text{fv}(m_u[P]@\rho) = \text{fv}(P) \]

Binding Variables of an Action
\[ \text{bv}(C(c)) = {} \]
\[ \text{bv}(K(c)) = {} \]
\[ \text{bv}(\text{activate}(r)) = {} \]
\[ \text{bv}(\text{deactivate}(r)) = {} \]
\[ \text{bv}((x_1, \ldots, x_k)^\eta) = \text{vars}(x_1) \cup \ldots \cup \text{vars}(x_k) \]
\[ \text{bn}((M_1, \ldots, M_k)^\eta) = {} \]

Variables of a Message
\[ \text{fv}(n) = {} \]
\[ \text{fv}(C) = \text{vars}(C) \]

Free Variables of an Actions
\[ \text{fv}(C(c)) = \text{fv}(C) \]
\[ \text{fv}(K(c)) = {} \]
\[ \text{fv}(\text{activate}(r)) = {} \]
\[ \text{fv}(\text{deactivate}(r)) = {} \]
\[ \text{fv}((x_1, \ldots, x_k)^\eta) = \text{vars}(x_1) \cup \ldots \cup \text{vars}(x_k) \]
\[ \text{fv}(Q.C) = \text{fv}(Q) \cup \text{fv}(C) \]

Variables of a Capability
\[ \text{names}(i) = \{i\} \]
\[ \text{names}(\text{in } m) = {} \]
\[ \text{names}(\text{out } m) = {} \]
\[ \text{names}(C_1, C_2) = \text{names}(C_1) \cup \text{names}(C_2) \]

Free Ports in Processes
\[ \text{fp}(0) = {} \]
\[ \text{fp}(P_1 | P_2) = \text{fp}(P_1) \cup \text{fp}(P_2) \]
\[ \text{fp}(\nu(n: \tau)P) = \text{fp}(P) \]
\[ \text{fp}(!P) = \text{fp}(P) \]
\[ \text{fp}(\pi.P) = \text{fp}(P) \setminus \text{fp}(\pi) \]
\[ \text{fp}(m_u[P]@\rho) = \text{fp}(P) \]

Ports in Locations
\[ \text{ports}(\uparrow c) = \{c\} \]
\[ \text{ports}(\downarrow c) = \{c\} \]
\[ \text{ports}(\star) = {} \]

Free Ports in Actions
\[ \text{fp}(C(c)) = {} \]
\[ \text{fp}(K(c)) = {} \]
\[ \text{fp}(\text{activate}(r)) = {} \]
\[ \text{fp}(\text{deactivate}(r)) = {} \]
\[ \text{fp}((x_1, \ldots, x_k)^\eta) = \text{fp}(\eta) \]
\[ \text{fp}((M_1, \ldots, M_k)^\eta) = \text{fp}(\eta) \]

Binding Ports in Actions
\[ \text{bp}(C(c)) = \{c\} \]
\[ \text{bp}(K(c)) = \{c\} \]
\[ \text{bp}(\text{activate}(r)) = {} \]
\[ \text{bp}(\text{deactivate}(r)) = {} \]
\[ \text{bp}((x_1, \ldots, x_k)^\eta) = \text{bp}(\eta) \]
\[ \text{bp}((M_1, \ldots, M_k)^\eta) = \text{bp}(\eta) \]

Figure 2: Free Variables and Binding variables

Figure 3: Free ports
Lemma 1 (Action Judgments) If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma', \rho'_{\text{here}}, \rho'_{\text{act}})$, then we have that $\text{bmi}(\pi) \cup \text{bp}(\pi) = \text{dom}(\Gamma') \setminus \text{dom}(\Gamma)$.

Proof 1 By case analysis of the rules for well-typed actions, see Table 4.

Lemma 2 (Variable Weakening)

1. If $\Gamma \vdash m : \tau' \text{ and } x \neq m$ then $\Gamma + (x : \tau) \vdash m : \tau'$

2. If $\Gamma \vdash C : (\rho_{\text{in}}, \sigma) \text{ and } x \notin \text{fmi}(C)$ then $\Gamma + (x : \tau) \vdash C : (\rho_{\text{in}}, \sigma)$

3. If $\Gamma, m \vdash \eta : \sigma \text{ and } x \neq m$ then $\Gamma + (x : \tau), m \vdash \eta : \sigma$

4. If $\Gamma \vdash M : \tau' \text{ and } x \notin \text{fmi}(M)$ then $\Gamma + (x : \tau) \vdash M : \tau'$

5. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma', \rho_{\text{here}}, \rho_{\text{act}})$ and $x \notin \{m\} \cup \text{fmi}(\pi)$, then if $x \in \text{bmi}(\pi)$ then $\Gamma + (x : \tau), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (x : \tau), \rho_{\text{here}}, \rho_{\text{act}})$ and otherwise $\Gamma + (x : \tau), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (x : \tau), \rho_{\text{here}}, \rho_{\text{act}})$

6. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}} \text{ and } x \notin \{m\} \cup \text{fmi}(P)$ then $\Gamma + (x : \tau), \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$

Lemma 3 (Port Weakening)

1. If $\Gamma \vdash m : \tau \Gamma + (c : \sigma) \vdash m : \sigma$

2. If $\Gamma \vdash C : (\rho_{\text{in}}, \sigma') \text{ then } \Gamma + (c : \sigma) \vdash C : (\rho_{\text{in}}, \sigma')$

3. If $\Gamma, m \vdash \eta : \sigma' \text{ then } \Gamma + (c : \sigma), m \vdash \eta : \sigma'$

4. If $\Gamma \vdash M : \tau \text{ then } \Gamma + (x : \tau) \vdash M : \tau'$

5. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma', \rho_{\text{here}}, \rho_{\text{act}})$ and $c \notin \text{fp}(\pi)$, then if $c \in \pi(\pi)$ then $\Gamma + (c : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (c : \sigma), \rho_{\text{here}}, \rho_{\text{act}})$ and otherwise $\Gamma + (c : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (c : \sigma), \rho_{\text{here}}, \rho_{\text{act}})$

6. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}} \text{ and } c \notin \text{fmi}(P)$ then $\Gamma + (c : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$

Lemma 4 (Variable Strengthening)

1. If $\Gamma + (x : \tau) \vdash C : (\rho_{\text{in}}, \sigma)$ and $x \notin \text{fmi}(C)$ then $\Gamma \vdash C : (\rho_{\text{in}}, \sigma)$

2. If $\Gamma + (x : \tau), m \vdash \eta : \sigma$ and $x \neq m$ then $\Gamma, m \vdash \eta : \sigma$

3. If $\Gamma + (x : \tau) \vdash M : \tau$ and $x \notin \text{fmi}(M)$ then $\Gamma \vdash M : \tau$
4. If $\Gamma + (x : \tau), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (x : \tau), \rho_{\text{here}}, \rho_{\text{act}})$ and $x \notin fmi(\pi) \cup \{m\}$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma', \rho_{\text{here}}, \rho_{\text{act}})$

5. If $\Gamma + (x : \tau), \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$ and $x \notin fmi(P) \cup \{m\}$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$

**Lemma 5 (Port Strengthening)**

1. If $\Gamma + (c : \sigma) \vdash C : (\rho_{\text{in}}, \sigma')$ then $\Gamma \vdash C : (\rho_{\text{in}}, \sigma')$

2. If $\Gamma + (c : \sigma), m \vdash \eta : \sigma'$ then $\Gamma, m \vdash \eta : \sigma'$

3. If $\Gamma + (c : \sigma) \vdash M : \tau$ then $\Gamma \vdash M : \tau$

4. If $\Gamma + (c : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma' + (c : \sigma), \rho_{\text{here}}, \rho_{\text{act}})$ and $c \notin fp(\pi)$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma', \rho_{\text{here}}, \rho_{\text{act}})$

5. If $\Gamma + (c : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$ and $c \notin fp(P)$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$

**Lemma 6 (Port Substitution)** If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$ with $\Gamma(c) = \sigma$, and $c' \notin fp(P)$ then $\Gamma + (c' : \sigma), \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P\{c'/c\} : \rho_{\text{act}}$

**Lemma 7 (Message Substitution)** Let $x$ be a variable, and $M$ be a message we wish to substitute for $x$. Let $\Gamma$ and $\Gamma'$ be two environments such that for all port names $c$, we have $\Gamma(c) = \Gamma'(c)$, and for all variables $y \neq x$, we have $\Gamma(y) = \Gamma'(y)$. For all cases, we suppose $\Gamma \vdash M : \tau$.

1. If $\Gamma \vdash C : (\rho_{\text{in}}, \sigma)$ then $\Gamma' \vdash C\{x := M\} : (\rho_{\text{in}}, \sigma)$

2. If $\Gamma \vdash M' : \tau$ then $\Gamma' \vdash M'\{x := M\} : \tau$

3. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m, u \vdash \pi : (\Gamma''', \rho_{\text{here}}, \rho_{\text{act}})$ then,

   (a) if $x \in bmi(\pi)$ then $\Gamma', \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m\{x := M\}, u \vdash \pi\{x := M\} : (\Gamma''', \rho_{\text{here}}, \rho_{\text{act}})$

   (b) else $\Gamma', \rho_{\text{here}}, \rho_{\text{deact}}, \rho_{\text{act}}, m\{x := M\}, u \vdash \pi\{x := M\} : (\Gamma'' + (x : \Gamma'(x)), \rho_{\text{here}}, \rho_{\text{act}})$

4. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$

   then $\Gamma', \rho_{\text{here}}, \rho_{\text{deact}}, m\{x := M\}, u \vdash P\{x := M\} : \rho_{\text{act}}$

**Lemma 8 (Preservation of Structural Congruence)** If $P \equiv Q$ then,

1. If $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$ and there exist $\Gamma', \rho'_{\text{here}}, \rho'_{{\text{deact}}}, \rho'_{{\text{act}}}$ such that $\Gamma', \rho'_{\text{here}}, \rho'_{{\text{deact}}, m, u \vdash Q : \rho'_{{\text{act}}}}$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash Q : \rho_{\text{act}}$,
2. and if $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash Q : \rho_{\text{act}}$ and there exist $\Gamma', \rho'_{\text{here}}, \rho'_{\text{deact}}$ and $\rho'_{\text{act}}$ such that $\Gamma', \rho'_{\text{here}}, \rho'_{\text{deact}}, m, u \vdash P : \rho'_{\text{act}}$ then $\Gamma, \rho_{\text{here}}, \rho_{\text{deact}}, m, u \vdash P : \rho_{\text{act}}$.

References


