FLUID-STRUCTURE INTERACTION OF A CANTILEVERED PLATE IN SUPersonic SEPARATED FLOW

BY

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THESIS

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ABSTRACT

The study of fluid-structure interaction (FSI) is one of ever-increasing importance as high-speed aircraft continue to reach higher Mach numbers. Research into high-speed FSI is especially critical with respect to control surfaces that may become compliant at these high Mach numbers. In collaboration with similar efforts at The Ohio State University and the University of Tennessee Space Institute, recent studies at the University of Illinois at Urbana-Champaign (UIUC) have focused on the FSI of a cantilevered plate geometry in Mach 2 flow. The geometry of interest, representative of a compliant control surface, consists of an overhanging plate that extends past the edge of a backward-facing step to create a separated region in the supersonic flow. This setup allows for the study of recirculation effects and unsteady pressure forcing on the cantilevered plate in the absence of shock-boundary layer interactions that would be present if the plate were inclined to the flow. The height of the cavity has been chosen such that it corresponds to the height of a flap of equal length deflected at 20 degrees. Both rigid and compliant variations of the cantilevered plate have been studied in order to capture the fluid response with and without structural deformation. These geometries have been studied at fully started wind tunnel conditions, as well as during tunnel startup when the unsteady flow elicits a highly dynamic response from the flexible plate. High-speed schlieren visualization, high-frequency pressure transducer measurements, oil flow visualization and planar particle image velocimetry (PIV) measurements have been used to characterize the flow around both geometries. Stereo digital image correlation (sDIC) was used to capture time-resolved surface deflection measurements for the flexible plate. The data show the presence of distinct frequencies in the pressure fluctuations beneath the rigid plate and a very strong correlation
between cavity pressure fluctuations and plate deflection in the case of the flexible plate. PIV velocity measurements show similar results between the rigid and compliant geometries in the flow outside the recirculation region but reveal several differences in the cavity flowfields, though some of these could be linked to three-dimensional facility effects. Spectral analysis of the sDIC results revealed several distinct frequencies present in the dynamic response of the flexible plate, and modal decomposition showed that the strongest of these frequency peaks always corresponded to the same four mode shapes.
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### List of Symbols

- $C_f$ skin friction coefficient
- $f$ frequency
- $h$ flexible cantilevered plate thickness
- $H$ undeflected cantilevered plate step height
- $H$ boundary layer shape factor
- $L$ cantilevered plate length
- $P_{cavity}$ static pressure in cavity region beneath cantilevered plate
- $P_{freestream}$ static pressure in incoming freestream flow
- $P_0$ stagnation chamber pressure
- $\Delta P$ pressure differential across cantilevered plate ($P_{freestream} - P_{cavity}$)
- $Re_{xx}, Re_{yy}$ Reynolds normal stresses
- $Re_{xy}$ Reynolds shear stress
- $St$ Strouhal number ($f H/U_\infty$)
- $U_\infty$ incoming freestream velocity
- $u$ streamwise ($x$) component of velocity
- $v$ transverse ($y$) component of velocity
- $|V|$ velocity magnitude
- $x$ streamwise position relative to the cantilevered plate tip
- $y$ transverse position relative to the cantilevered plate surface
- $\delta$ $0.99 U_\infty$ boundary layer thickness
- $\delta^*$ boundary layer displacement thickness
- $\Theta$ boundary layer momentum thickness
CHAPTER 1: INTRODUCTION

1.1 Background and Motivation

The large dynamic pressures associated with high-speed flight and the substantial aerodynamic forces that result present a significant risk to deformable vehicle components that may experience aeroelastic deformation under these loads. These deformations, in turn, can cause an unanticipated aerodynamic response that could jeopardize the stability or controllability of the vehicle as a whole. Understanding these fluid-structure interactions (FSIs) in high-speed flow is of critical importance for designing successful supersonic and hypersonic flight systems, especially with regard to control surfaces, the structural integrity and aerodynamic performance of which are vital to the performance of the vehicle. Additionally, the coupling between nonlinear structural and aerodynamic responses makes this problem an extremely difficult one to model computationally. High-speed FSI has been the subject of many past and recent research efforts only some of which, however, focus on deformable control surfaces.

One common configuration in high-speed FSI research is the flat, compliant panel with supersonic flow above it, similar to a skin panel of a high-speed vehicle. This setup has been widely studied for decades and continues to be the focus of many recent investigations. Much of the early experimental work on flexible panels focused on linear panel flutter and was published as technical reports by the U.S. Air Force and NASA [1]. As experimental facilities and diagnostic techniques have advanced, more complex investigations have been carried out. Casper et al. [2], for example, studied the effects of intermittent turbulent spots on the dynamic response of a compliant panel embedded in a slender cone in hypersonic flow. Ravichandran et al. [3] controlled the cavity pressure behind their flexible panel geometry to investigate the influence of
cavity pressure on the motion of the panel. Ravichandran et al. [3] also investigated the response of their panel to impingement by an oblique shock wave, another common focus of FSI research that uses the unsteady interaction of the shock wave with the boundary layer as a forcing mechanism for the flexible panel rather than a turbulent boundary layer.

Many theoretical and numerical investigations of flexible panels have also been carried out since this configuration first became of interest in the early days of high-speed flight. Dowell [4] used Von Karman’s large deflection plate theory and quasi-static aerodynamic theory to reproduce the limit-cycle oscillations originally seen only in early panel flutter experiments [5]. More recently, Ostoich et al. [6] used direct numerical simulation to model a turbulent boundary layer over a fluttering panel and investigated the effects of the panel deformation on the fluid response in the boundary layer.

While unsteady shock-boundary layer interactions (SBLI) can play an important role in the FSI of a flexible panel, they are also an important feature in the flowfield of a compression ramp in supersonic flow. This geometry is another configuration of interest within the field of FSI as it represents flow in front of a deflected trailing-edge control surface. Sullivan et al. [7] used high-fidelity simulations to investigate a 2D compression ramp in Mach 6 flow with a compliant panel imbedded in the ramp. These simulations were conducted in conjunction with complementary experiments carried out by Whalen under the same conditions [8]. Pandey et al. [9] conducted similar experiments on a flexible compression ramp at a variety of hypersonic Mach numbers and wedge angles, though the compression ramp in this case was placed on a longitudinally sliced part of a cone, increasing the complexity of the problem. Rather than using a compliant ramp, Pham et al. [10] placed a compliant surface of urethane rubber upstream of the
compression ramp to investigate the possibility of unsteady shock load mitigation in the compression corner.

Much of the early work that occurred in high-speed FSI was driven by the original push for supersonic and hypersonic flight during the mid to late 20th century. This research often focused on the aeroelastic instabilities and failure criteria for specific lifting and control surfaces for high-speed vehicles such as the North American X-15 and the Space Shuttle [5]. Lauten et al. [11], for example, conducted tests specifically to identify the flutter point of the proposed horizontal tail design for the X-15 and examined the validity of theoretical models used to calculate the flutter speed.

Since these early investigations, FSI research into compliant control surfaces has expanded to leverage improved experimental and computational capabilities, with joint computational-experimental investigations becoming increasingly common [12] as renewed interest in hypersonic flight accentuates the need for this work. This type of collaboration ensures maximum similarity between experimental and computational tests, providing the numerical efforts with highly comparable validation data while simultaneously allowing the computational results to support experimental findings. Currao [12] conducted an experimental and computational investigation of the FSI of a cantilevered plate in hypersonic flow. The geometry, similar to the one discussed in the present work, consisted of a flexible cantilevered plate mounted to a rigid support with a sharp leading edge and inclined at 20 degrees to a Mach 5.8 freestream. The experiment was simulated using both a low-fidelity piston theory model as well as coupled commercial CFD and FEM solvers from Ansys. It was found that these inviscid two-dimensional simulations matched the experimental system response reasonably well except
for the region near the trailing edge of the cantilever, suggesting that three-dimensional and viscous effects did not play significant roles in the FSI of this particular setup.

Bhattrai et al. [14] conducted experimental and numerical studies of rigid and compliant trailing edge flaps in hypersonic flow. These geometries, much more closely resembling a true wing-flap assembly than most other cantilevered plate or beam models studied, were tested in the same facility used by Currao [12] as part of a larger campaign to develop closed-loop control methods for an aeroelastic hypersonic control surface. While the numerical results were found to agree well with the experimental measurements, non-ideal startup effects created difficulty in obtaining accurate measurements in some scenarios that led to the conclusion that a reevaluation of the test setup was necessary.

In a fully numerical investigation, McHugh et al. [13] developed and tested a new computational model for inextensible beams and plates that, coupled with classical piston theory, was used to capture the flutter and limit cycle behavior of a cantilevered plate in supersonic flow. The dynamics of this geometry, representative of a trailing edge control surface on a high-speed vehicle, are typically modelled using a standard inextensible beam model that has been previously shown only to be effective before the onset of flutter [13].

While several investigations of cantilevered plate and control surface geometries in high-speed flows have been carried out, few have considered the effects of the separated downstream flow on the FSI of a compliant control surface. These effects can be important in the dynamics of a stalled control surface or in the performance of a control surface like the speed brake on the vertical stabilizer of the X-15, shown in Figure 1. The present work aims to better understand the interactions between a cantilevered control surface and the separated flow behind it, as well as the impacts of structural compliance on the flowfield as a whole.
1.2 Present Work

A collaborative effort between The Ohio State University (OSU), the University of Tennessee Space Institute, and the University of Illinois at Urbana-Champaign (UIUC) has focused on studying the aerodynamic response to structural deformations of a test article immersed in supersonic separated flow. The experimental studies discussed in this work were conducted at UIUC and investigated the FSI of a cantilevered plate in Mach 2 flow as part of an extended test campaign with the goal of characterizing the flow field around a geometry representative of a compliant control surface of a high-speed vehicle. The geometry of interest in this study was the first in a series of three configurations of increasing complexity and was meant to act as a baseline case to which the following configurations could be compared.

Ultimately, the goal of this progression of experimental configurations, shown in Figure 2, was to study a more realistic configuration reminiscent of a control surface on a hypersonic
vehicle like the one shown in Figure 1. This series of test geometries was devised so that the important phenomena could be combined and studied incrementally rather than testing a single, highly complicated geometry. Configuration 1, meant as a building block towards Configurations 2 and 3 and the configuration on which this paper focuses, incorporated the cantilevered plate geometry and recirculation region without creating a complex shock-boundary layer interaction. Configuration 2 would add this additional complexity by inclining the cantilevered plate to the flow, and in Configuration 3, an actuator or support rod would be placed beneath the inclined cantilever, which presumably would alter the structural modes of the flexible cantilever. At the time of this writing, the project has been redirected and investigations of Configurations 2 and 3 will not be carried out. A schematic of the general flow phenomena present in these investigations is also shown in Figure 2.

Figure 2. Progression of Test Configurations for Control Surface FSI Studies
Two versions of the Configuration 1 geometry were studied herein, one with a compliant plate and one with a rigid one, to obtain a proper assessment of the impacts of structural compliance. These two geometries were also studied under multiple conditions: the startup flowfield, where transient startup waves in the test section led to substantial deformations in the flexible geometry, and at fully started conditions, where the incoming flow was fully supersonic and more stable than during startup.

A variety of flow diagnostics were used to study both aerodynamic and structural phenomena for the geometry of interest. Schlieren photography, particle image velocimetry (PIV), oil flow visualization and high-frequency pressure transducers were used to gather velocity and pressure data around both the rigid and flexible cantilevered plates and assess the fluid response both with and without structural deformation. Stereo digital image correlation (sDIC) was used to obtain time-resolved surface deflection measurements as the flexible cantilevered plate deformed under aerodynamic loading. Together, these measurements aimed to build an understanding of the implications of structural compliance for the aerodynamics around a cantilevered plate with significant downstream separation in a supersonic flow, the latter effects having not been substantively addressed in the past to the author’s knowledge.

The remainder of this thesis is organized as follows. Chapter 2 describes the facility and experimental techniques used to study this geometry. Chapters 3 and 4 discuss the results obtained under startup conditions and fully started conditions, respectively. Within these two chapters, measurements from the rigid cantilevered plate are compared to those from the flexible cantilevered plate to highlight the effects of structural compliance. Chapter 5 summarizes this series of experiments and the important conclusions that can be drawn from them, as well as discusses further investigations that could build on this work.
CHAPTER 2: EXPERIMENTAL METHODOLOGY

2.1 Wind Tunnel Facility

The experiments discussed here were carried out in the Large Rectangular Supersonic Tunnel (LRST) in the Gas Dynamics Lab at UIUC. The LRST is a supersonic blowdown wind tunnel, designed and built at the University of Illinois in 2010 for supersonic micro-vortex generator experiments. Details on the design and assembly of the LRST are discussed by Chang [15]. It should be noted that since its initial setup in Aeronautics Lab A at UIUC, the LRST was taken down and moved to storage for several years. After being selected as the ideal wind tunnel for this series of FSI experiments, the LRST was then removed from storage, disassembled, thoroughly cleaned, and reassembled in the Gas Dynamics Lab. Thus, the high-pressure air supply and exhaust system are slightly different than those described by Chang [15] in the original use of the LRST. This particular wind tunnel was chosen over other existing facilities because the dimensions of the test section left greater flexibility in the design of the deflected cantilevered plate geometry that will succeed the configuration discussed in this study. More specifically, an inclined cantilevered plate will present less of a blockage in the LRST than it would in the other facilities considered. The laboratory setup of the LRST is shown in Figure 3.

Figure 3. Lab Setup of the LRST
The LRST is suspended from an I-beam via five turnbuckles along the tunnel centerline. High-pressure air is supplied from a tank farm maintained at 145 psia to a schedule 40, class 250 pipe cross that acts as a stagnation chamber by manually opening a gate valve upstream of the stagnation chamber. Air enters the stagnation chamber from the top and is deflected toward the rear of the chamber in order to facilitate mixing and achieve stagnation-like conditions. It is then passed through flow conditioning elements consisting of a honeycomb flow straightener and wire screen before passing through a planar Mach 2 converging-diverging nozzle and entering the rectangular test section. Flow from the test section is then diffused into a round, double-walled, insulated silencing duct before being exhausted to the atmosphere outside of the building. A cutaway of the LRST is shown in Figure 4.

![Figure 4. Cross-Section View of the LRST](image)

The entrance to the test section of the LRST measures 5 x 5 inches and sits about 24 inches downstream from the throat of the nozzle. Optical access is provided by identical side windows with effective viewing areas measuring 10 x 7 inches and a top window measuring 12.5 x 1 inches. The side windows are placed in removable inserts that can be reoriented to shift the viewing area further downstream, if needed. The original nozzle-half assembly of the LRST test section was designed to be reconfigurable to allow for interchanging test geometries. In order to accommodate the cantilevered plate test geometry for this study, a new inner piece for this test section wall was designed and machined from aluminum 7075. This piece acts as the bottom
wall of the test section and is 0.7 inches thinner than the original part to leave room for the depth of the cavity beneath the cantilevered plate. It also features a 5-degree diffusing section along the final 8.5 inches to mate with the 5-degree diffuser extension described in Chang [15]. Two versions of this test section wall were designed and fabricated, one featuring a row of pressure tap holes along the centerline and one featuring a window insert. Only the model with pressure taps was ultimately used in experiments.

![Figure 5. CAD Models of Test Section Bottom Wall with A) Window and B) Pressure Taps](image)

The LRST was operated at a stagnation pressure of 40 psia to ensure that the tunnel would remain fully started, a condition that is met at a stagnation pressure of around 37 psia. The test conditions achieved at this stagnation pressure for a typical run are tabulated below in Table 1. The stagnation temperature cited in this table represents the conditions under which the boundary layer was measured. Due to the exterior location of the supply air, this stagnation temperature along with the freestream velocity, varied seasonally and was typically observed to range from 270 K to 295 K.
Table 1. Standard Operating Conditions for the LRST

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<table>
<thead>
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<tr>
<td>Stagnation Pressure (psia)</td>
<td>40.26</td>
</tr>
<tr>
<td>Test Section Static Pressure (psia)</td>
<td>4.37</td>
</tr>
<tr>
<td>Stagnation Temperature (K)</td>
<td>276.98</td>
</tr>
<tr>
<td>Mach Number (calculated from P/P₀)</td>
<td>2.11</td>
</tr>
<tr>
<td>Nozzle Wall Boundary Layer Thickness (mm)</td>
<td>12.83</td>
</tr>
</tbody>
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Stagnation and test section static pressures were monitored by a Pressure Systems Inc. Netscanner Model 98RK, fitted with eight Model 9816 modules, only three of which were actually used to measure pressures for these experiments. The module used for stagnation pressure readings has a differential range of 100 psid while the other two modules used for static pressure measurements in the test section have a differential range of 15 psid, all featuring a full-scale accuracy of ±0.05% as reported by the manufacturer. The pressure transducers in these modules were calibrated prior to the beginning of experiments using a multi-point calibration as described in Reedy [16], Appendix C. A zero-point calibration was also used to correct for thermal drift and changes in ambient pressure each day that data were collected. A Setra Model 370 Digital Pressure Gauge with a manufacturer-reported accuracy of ±0.02% was used to measure the atmospheric pressure. Stagnation pressure was measured from a single pressure tap inside the stagnation chamber while the test section static pressure was measured from a pressure tap on the upstream end of the top window insert near the entrance to the test section and upstream of any flow effects from the test geometry. Thirteen pressure taps along the centerline of the bottom wall of the test section also measured the pressure distribution downstream of the cantilevered plate; these results will be discussed later.

Stagnation temperature measurements were collected via an Omega type-J thermocouple with a manufacturer estimated uncertainty of ±2.2 K mounted in the wall of the stagnation
chamber. These signals were passed through an Omega temperature controller to a National Instruments USB 6009 DAQ. The thermocouple was originally calibrated using a two-point calibration in which the thermocouple was submerged in boiling water and an ice bath and the respective output voltages were recorded.

Both pressure and temperature signals were read into a LabVIEW 2018 virtual instrument (VI) originally created for use with the supersonic axisymmetric base flow tunnel that previously used the test stand now occupied by the LRST. Since the LRST continues to use the same piping setup and pressure transducer modules as the base flow tunnel, the VI was easily adapted for use with the LRST. This included eliminating some of the pressure channels that were no longer needed and adding functionality to collect high-frequency pressure measurements from two Kulite pressure transducers. The front panel view of the LabView VI is shown in Figure 6.

![Figure 6. LabView Front Panel Used to Operate the LRST](image-url)
The LabVIEW VI reads pressure and temperature measurements at 5 Hz and displays them in real time such that the wind tunnel conditions can be safely and accurately monitored while conducting experiments. To operate the LRST, one simply has to run the LabVIEW VI and manually open the gate valve separating the high-pressure air supply from the stagnation chamber until the stagnation pressure reaches the operating pressure of 40 psia as indicated by the VI front panel. A ValTek Mark I automatic control valve installed between the main gate valve and the stagnation chamber was originally intended to more accurately control the stagnation pressure; however, this valve no longer functions properly and instead acts as a backup to the main gate valve. This automatic control valve must also be opened prior to operating the LRST.

2.2 Test Geometry

Two versions of the cantilevered plate geometry, one rigid and one flexible, were studied in this series of experiments. A third Plexiglas model was also used exclusively for boundary layer PIV measurements. Both rigid and flexible models feature the same 2-inch overhanging plate length and total plate height above the tunnel bottom wall of 0.7 inches but differ in material and plate thickness. The 0.7-inch drop-off between the downstream end of the cantilevered plate and the test section bottom wall was chosen to correspond to the height of a 2-inch flap deflected at 20°. This ensures that the separation region for the inclined plate configuration mentioned previously will be of similar dimensions to the recirculation cavity present in this geometry. Top and bottom views of the flexible cantilevered plate geometry are shown in Figure 7.
Both rigid and flexible geometries also feature a counterbored hole in the rear wall of the cavity designed to house a custom plug containing a Kulite pressure transducer for high-speed pressure measurements. A small rectangular cut out in the back of the model provides access to this hole from beneath for wiring or tubing. In order to reduce stress concentrations at the corner of the cavity beneath the cantilevered plate, both models also feature a 0.04-inch radius fillet where the bottom surface of the plate meets the cavity wall. All models were secured into the test section via four 3/8-inch bolts accessed through the bottom wall of the test section.

2.2.1 Rigid model

The rigid cantilevered plate model was machined from stainless steel and features a 1/8-inch-thick plate, resulting in a cavity height of 0.575 inches. This plate thickness was chosen based off of the estimated pressure loading calculated from experimental and computational data for flow over blunt bodies and backward-facing steps of similar dimensions to the cantilevered plate. Using the estimated pressure differential across the plate, the structural responses of various plate thicknesses were simulated in Ansys Workbench in order to approximate the plate
deflection at operational conditions. These analyses showed that a stainless-steel plate with a thickness of 1/8 inch would exhibit sufficiently negligible deflections to be considered rigid. This rigidity was ultimately confirmed via high-speed schlieren video which revealed no visible deformation of the rigid plate at any point.

A Plexiglas model was used to physically represent the rigid cantilevered plate geometry during boundary layer PIV measurements. The use of Plexiglas kept laser reflections to a minimum, making it possible to image substantially closer to the surface of the model. Since measurements were only obtained on the top surface of the model in the case of boundary layer PIV, the Plexiglas version did not feature a cantilevered plate or cavity but was simply a rectangular prism with external dimensions matching that of the rigid plate model. The acquisition of boundary layer measurements using the Plexiglas block are discussed further in section 2.3.2.

2.2.2 Flexible Model

The flexible cantilevered plate model was machined from Aluminum 7075 and featured a substantially thinner cantilevered plate at 0.04 inches, creating a cavity height of 0.66 inches. As with the rigid plate, the material choice and plate thickness were based off FEA predictions for plate deflection under the expected aerodynamic loading. In this case, the pressure differential measured across the rigid plate geometry was used in the structural simulations, making them substantially more accurate than the estimated deflections for the rigid plate. While this measured pressure differential only represented the loading on the plate at fully started conditions, the difficulty of predicting the response of the flexible plate geometry due to the unsteady loading experienced during tunnel startup limited the extent of the structural simulations. It was decided that a predicted steady-state deflection of -0.1 inches for the chosen
material and plate thickness left a wide enough margin for larger deflections that might occur during startup.

In order to prevent friction between the sidewalls of the test section and the moving cantilevered plate, a 0.02-inch gap, also shown in Figure 7, was cut into each edge of the flexible plate geometry. While this gap eliminated unwanted contact between the side walls and the flexible plate, it also introduced a potential source of minor leakage between the low-pressure cavity beneath the plate and the freestream flow above. Despite this small difference between the boundary conditions of the two cantilevered plate geometries, the flow fields remained largely the same and, perhaps most importantly, the dynamic response of the flexible plate remained as close to a true cantilever as possible.

2.3 Flow Diagnostics

2.3.1 Schlieren Photography

Schlieren photography was used to visualize the flow around both cantilevered plate geometries in order to gain an understanding of the phenomena involved in both the startup and fully started flows. A Z-type configuration similar to the schematic shown in Figure 8 was used. For these experiments, a THOR Labs LED provided a point light source which was then collimated by 12-inch concave mirrors with focal lengths of approximately 100 inches. A vertically oriented knife edge at the focal point of the second mirror allowed for the visualization of the density gradients in the test section which were imaged by a Photron SA5 high-speed camera with a Nikon Nikkor 70-300 mm zoom lens.
For the vast majority of the high-speed videos captured, exposures of 1 μs were used to capture flow features as close to instantaneously as possible. For the rigid plate geometry, schlieren frames were captured at 100,000 frames per second while for the flexible plate, the rate was lowered to only 10,000 frames per second in order to increase the recording time and capture the entire startup sequence. In the case of the rigid plate which did not deform measurably during startup, this was not a concern.

Plate deflection measurements for the flexible plate were extracted from the high-speed schlieren videos by analyzing the pixel values and tracking the location of the bottom edge of the cantilevered plate with respect to the bottom of the image. Using the known length of the cantilevered plate as a reference to establish the physical length associated with each pixel and aligning the bottom of the image with the test section bottom wall, it was possible to measure the instantaneous height of the oscillating plate in each frame. These measurements could then be
compared against data collected synchronously with the schlieren video. Sub-pixel resolution was achieved by expanding each image using bicubic interpolation.

One instance of these synchronous measurements was the stereo digital image correlation (sDIC) experiments discussed in section 2.3.4. In this case, a Photron SA-Z high-speed camera in place of the SA-5 cameras used previously, was used to capture schlieren images in conjunction with the simultaneous sDIC measurements. This schlieren setup was identical to the original, discussed previously, with the exception of the camera model as well as the inclusion of a planar mirror between the knife edge and camera that allowed for greater flexibility in the placement of the much larger Photron SA-Z. The Photron Fastcam Viewer Software was used to synchronize the schlieren and sDIC cameras at 7000 fps rather than 10000 fps, as the Photron SA-5 cameras that were used to capture sDIC images can only record up to 7000 fps while utilizing the entire camera sensor.

2.3.2 Planar PIV

Planar particle image velocimetry (PIV) was used to measure the velocity field around the cantilevered plate geometries. Two different PIV setups were used during this series of experiments. In the first, used to measure in the boundary layer velocity profile entering the test section, the laser sheet entered the test section via the top window and illuminated the flow near the downstream end of the cantilevered plate as shown in Figure 9A. In the second setup, used to measure the entire velocity field around the cantilevered plate, the laser sheet again entered through the top window, though at a location further downstream in the test section. It was then redirected upstream towards the cantilevered plate by a 1.57-inch right-angle prism made of N-BK7 glass mounted into the bottom wall of the test section. A schematic of this setup is shown in Figure 9B.
Figure 9. Experimental Setups for A) Boundary Layer and B) Full-Field PIV

In both PIV setups, a double-pulsed New Wave Solo Nd:YAG laser with an output wavelength of 532 nm was used to illuminate seed particles within the region of interest. The laser beam was directed through plano-convex spherical and cylindrical lenses to create a laser sheet less than 1 mm thick. The laser, timing for which was controlled by a delay generator, was operated at an output power of around 40 mJ with a pulse separation of 1 μs as measured by a photodiode and oscilloscope.

Seed particles with a manufacturer-reported diameter of 0.2-0.3 μm (and also as measured in situ in the previously mentioned base flow tunnel) were introduced into the piping upstream of the stagnation chamber by a Concept Smoke Systems ViCount 1300 smoke generator. A nitrogen tank provided around 35 psig to the smoke generator to supply a steady, sufficiently thick stream of smoke into the piping just upstream of the stagnation chamber without forcing large amounts of oil droplets in as well.

Due to the depth of the recirculation region beneath the cantilevered plate, it was found that insufficient amounts of seed particles were entrained into this region of the flow from the free stream. To supplement the level of seeding in the cavity and obtain accurate velocity
measurements in this region for the full-field PIV setup, additional smoke particles were piped into the cavity through the 0.17-inch hole in the rear wall of the cavity originally designed to hold a Kulite pressure transducer. A 20-inch length of 0.156-inch (4 mm) plastic tubing was secured into this hole with RTV and connected via a small valve and a second short length of tubing to a 5 L plastic container filled with smoke particles. The configuration of the main seeder as well as the cavity seed particle supply are shown in Figure 10.

![Figure 10. Particle Seeding Setup for the A) Freestream Flow and B) Cavity Region](image)

Prior to full-field PIV runs, this container was filled with smoke from the same ViCount smoke generator and attached to the tubing system. Since even a small amount of mass flow into the cavity could easily disturb the relatively quiescent flow field, great care was taken to adjust the mass flow control valve such that enough smoke could be injected into the cavity to obtain acceptable data, while also keeping the mass flow rate sufficiently low for the effect on the cavity pressure to be negligible. Through careful trial and error, the optimal valve position was found by simultaneously monitoring both the cavity pressure via pressure taps in the bottom wall and the density of cavity seeding via the PIV camera. It was found that with the proper valve
position, an acceptable density of seed particles could indeed be introduced without affecting the cavity pressure.

A PCO 2000 CCD camera with 4 GB of RAM and a Nikon Nikkor 60 mm lens with an aperture set at f/4 were used to capture PIV images. The maximum resolution of 2048 x 2048 pixels was cut down to regions of interest of 1800 x 600 and 1000 x 1190 pixels for the full-field and boundary layer PIV, respectively, to allow for faster image acquisition. Given the cameras’ distance from the measurement location in each case, these equate to fields of view of 1.46 x 4.29 inches and 0.68 x 0.85 inches, respectively. Full-field images were acquired at 3.75 Hz while boundary layer images were acquired at 4 Hz, both with 1 μs exposures. The PCO camera was controlled by the same delay generator as the laser to facilitate overlap of the camera exposure and laser pulses.

All raw PIV images were processed using the LaVision DaVis 8.4.0 software. To address inconsistencies in the backgrounds of these raw images which were caused by varying reflections from the opposing sidewall or the test geometry itself, PIV images were first preprocessed in DaVis. First, a sliding Gaussian average was subtracted from each image to accentuate the seed particles from the background. Next, all pixel counts were multiplied by a constant to strengthen intensity peaks, then lowered by a constant to reduce the noise floor to near zero. The constants used in preprocessing varied as necessary depending on the strength of the reflections present in the raw images. Preprocessed image pairs were then cross correlated over several passes starting with two passes with interrogation windows of 64 x 64 pixels and 50 percent overlap followed by four passes with interrogation windows of 32 x 32 and 75 percent overlap. Finally, the resulting vector fields were then filtered for erroneous vectors through post-processing. This included eliminating vectors that fell outside of a reasonable range, differed
from their neighbors by more than 2 standard deviations, or had a Q-ratios of less than 1.5. Additionally, the series of post-processed vector fields were reviewed and any image pairs that featured exceptionally poor data, usually due to oil buildup on the windows, which were not entirely filtered out in the post-processing step, were manually eliminated from the dataset.

To calibrate boundary layer PIV images, a 0-58 LaVision calibration plate was placed in the test section such that the face visible to the camera aligned with the plane of the laser sheet and its downstream edge was flush with the downstream edge of the cantilevered plate. Consistent placement of the calibration plate was ensured by measuring the location of the laser sheet relative to the sidewall using laser burn paper and assembling a series of spacers to place the calibration plate at the proper distance from the sidewall. This ensured that distances specified by the calibration plate matched true distances within the imaging plane as closely as possible. Since multiple runs are required to assemble data sets of several thousand image pairs necessary for PIV, calibration images were obtained prior to each run to ensure that the camera had not moved since the last run.

Due to the geometry of the flow field around the cantilevered plate and the dimensions of the LaVision calibration plate, it was impossible to obtain a calibration over more than half of the field-of-view with this calibration plate when collecting full-field PIV data. To obtain an accurate calibration across the entire image for these measurements, a custom calibration plate was created from a 3D-printed structure and a laser printed paper calibration plate which was adhered to the PLA support by double-sided tape. This custom calibration plate, shown in Figure 11, featured dots 0.05 inches in diameter spaced 0.25 inches apart and made it possible to achieve a calibration both above the cantilevered plate as well as in the cavity region beneath.
The custom plate provided a calibration fit with an RMS value of 0.332 pixels, which was actually slightly better than the 0.349-pixel RMS of the calibration fit using the LaVision calibration plate for boundary layer PIV. For a 4-megapixel camera like the one used, RMS values of less than 0.4 pixels are considered to provide exceedingly good calibrations [17]. A 3rd order polynomial calibration model was used for both PIV setups.

Uncertainty for the PIV measurements can be estimated from uncertainties calculated by Favale [18] for a supersonic base flow experiment that featured a flowfield both qualitatively and quantitatively similar to the present work and that used much of the same equipment and processing techniques. Favale used the method described by Lazar et al. [19] that quantifies four sources of error in planar PIV: equipment, particle lag, processing error, and sampling uncertainty. Average equipment error, which takes into account factors such as uncertainties in calibration, image distortion from the lens, and uncertainties in the laser and timing unit, was calculated to be 0.3% $U_\infty$ in the freestream, and only 0.1% $U_\infty$ in the recirculation region. Error due to particle lag, driven by the inertia of the smoke particles, was the largest source of error, especially in regions of large velocity gradients such as the shear layer. Average particle lag uncertainty was found by Favale to be approximately 1% of $U_\infty$ in the freestream and recirculation region and typically less than 3% of $U_\infty$ in the shear layer. Processing uncertainty, caused by small errors in the processing of raw PIV images, was also highest in regions of high...
acceleration but was found to typically be less than 2% of $U_\infty$. Due to the computational expense of estimating processing error, Favale calculated the uncertainties only 50 of his 3000 image pairs. The sampling error, which represents the total uncertainty in the mean, was estimated based off the average and standard deviation of each image pair and the total uncertainty of the first three contributors to instantaneous error. This total uncertainty was found to be typically below 3% of $U_\infty$ in the shear layer and 0.5% of $U_\infty$ in the freestream for the $u$-velocity, and generally less than 2% of $U_\infty$ for the $v$-velocity.

2.3.3 Oil Flow Visualization

In this series of experiments, oil flow visualization was used study the surface flow patterns along the bottom wall of the test section. A mixture of 10W30 motor oil and fluorescent dye was applied in a thin layer onto matte black contact paper adhered to the bottom wall. Two 20W, 365 nm fluorescent black lights, one on each side of the test section, were used to fluoresce the dye. The oil mixture, as it appears under blacklight illumination and prior to being disturbed by the flow, is shown in Figure 12 with the top surface of the rigid plate highlighted in blue for clarity.

![Figure 12. Undisturbed Fluorescent Oil Prior to Run](image)
A Nikon D7500 DSLR camera mounted on a tripod captured video of the oil flow throughout the run at 30 fps with maximum exposure. One of the main challenges to obtaining quality images was maintaining sufficient depth of field to visualize the entire region of interest while also maintaining a large enough aperture to capture sufficiently bright images. To compensate, the ISO sensitivity was set close to the maximum value at 51200.

2.3.4 Digital Image Correlation

Stereo digital image correlation was used to measure the time-resolved surface deflections of the flexible cantilevered plate while subjected to aerodynamic loading. The experimental setup, shown in Figure 13, consisted of two Photron SA5 high-speed cameras each mounted at a standoff distance of around 18 inches from the plate surface itself and at an angle of approximately 51 degrees from the plate surface normal, resulting in a relatively large camera stereo angle of 102 degrees. Camera locations and angles were sharply limited by optical access limitations to the cantilevered plate surface from above, as the cameras could only achieve a minimum angle of around 48 degrees from the surface normal before the field of view became obstructed by the edge of the window frame. The large camera angle made it difficult to achieve a sufficient depth of field to properly visualize the entire span of the plate while also maintaining a sufficiently large aperture. To accommodate this setup, LaVision Scheimpflug mounts were placed on each high-speed camera, allowing the lens angle to be adjusted such that the full plate was brought into focus without severely limiting the aperture size and creating excessively dark images. With this setup, the Nikon Nikkor 60 mm lenses were mounted at an angle of around 41 degrees from the surface normal, 10 degrees offset from the camera angle. After several tests it was found that f/4 was the largest possible aperture setting with a depth of field large enough.
such that the cantilevered plate remained in focus even at its peak displacement of almost 0.3 inches.

![LED and High-Speed Camera Setup for sDIC Experiments](image)

Figure 13. LED and High-Speed Camera Setup for sDIC Experiments

The main limitation to both aperture setting as well as camera exposure was the intensity of light used to illuminate the cantilevered plate surface. Illumination was provided through the top window of the test section by two Thor Labs LEDs as shown in Figure 13. This setup provided sufficient illumination to operate the high-speed cameras at a shutter speed of 50 μs without blurring the speckling pattern of the plate as it oscillates rapidly.

The speckling pattern on the surface of the cantilevered plate consisted of two layers: a layer of Rust-Oleum Ultra Cover Flat White Primer spray paint underneath a pattern of dots created by a Rust-Oleum Black Flat Protective Enamel spray paint and supplemented with a black Sharpie marker. The flat white primer was applied in as thin of a layer as possible to avoid affecting the bending properties of the flexible plate. The speckling pattern was applied to the surface by firing the black spray paint over the plate surface such that the heaviest drops would fall onto the white primer creating sufficiently large speckles, while the majority of the smaller
droplets would simply pass over. After several passes with the spray paint can, a thin permanent marker was then used to add speckles as necessary in areas with insufficient speckling. Small fiducial markers added to the fixed end of the cantilevered plate with the same permanent marker indicate the upstream edge of the area of interest. The speckling pattern of the cantilevered plate as seen by the two sDIC cameras can be seen in Figure 14.

![Figure 14. Speckled Cantilevered Plate Surface as Seen by the sDIC Cameras](image)

It should be noted that the original flexible cantilevered plate geometry failed at the base of the cantilever the first time that it was subjected to aerodynamic loading with the primer and speckling pattern applied. While the aluminum plate had already undergone thousands of deflection cycles over more than a year of testing, it is possible that an excessively thick coat of primer could have changed the elastic properties of the plate enough to cause a failure of the material.

Both calibration and processing of the sDIC data were carried out using Correlated Solutions’ VIC3D 2010 software. The sDIC camera system was calibrated using the bundle
adjustment method and a standard calibration plate from Correlated Solutions with a 9 x 12 grid of dots spaced 6 mm apart. One hundred five image pairs were captured by the camera system as the calibration plate was moved through a variety of angles within the test section and run through the VIC3D calibration algorithm. Roughly half of these images were ultimately rejected from the calibration due to either the software’s inability to locate the calibration dots, or to poor residual scores that indicate high calibration error for individual images. The calibration plate is shown within the test section in Figure 15. While the standard stereo calibration in the VIC3D software typically requires a correction for the distortion introduced by Scheimpflug imaging, if was found that with the moderate Scheimpflug angle present in the sDIC setup, a calibration with an acceptable score of 0.044 could be obtained without any additional corrections. While the relatively thick side windows of the test section also added bias to the stereo calibration, this was not considered an issue because the main quantities of interest were the frequencies present in the out-of-plane deflections of the cantilevered plate and not the magnitude of the deflections themselves.

![Correlated Solutions Calibration Plate Viewed from High-Speed Camera](image)

Figure 15. Correlated Solutions Calibration Plate Viewed from High-Speed Camera
The two high-speed cameras, synchronized by the Photron Fastcam Viewer 3 software via TTL trigger signals output by the “master” camera, acquired sDIC images at 7000 fps under both startup and fully started conditions. These images were then loaded into the VIC3D software and processed using a subset size of 17 pixels and a step size of 7 pixels, resulting in an overlap of around 40%. Gaussian weighting of the pixels in each subset window was used as it provides the best balance between spatial resolution and displacement resolution [18]. An 8-tap spline function was applied in order to achieve the maximum sub-pixel accuracy via gray value interpolation, and the default “normalized squared differences” correlation criterion was used to ensure the best combination of processing flexibility and deflection results. Single images of the flexible plate under quiescent conditions from each camera were used as references from which instantaneous deflections were measured. After processing, a coordinate system fit to the planar surface of the undeflected plate was applied to each set of deformation measurements via the VIC3D software.

As mentioned in section 2.3.1, simultaneous schlieren video was captured during acquisition of the sDIC data. By analyzing the individual schlieren frames and extracting the position of the cantilevered plate, it was possible to validate the performance of both the schlieren and sDIC systems in measuring plate deflection. During tunnel startup when the flexible cantilevered plate begins to oscillate under aerodynamic loading, it was found that the schlieren and sDIC measurements for plate deflection differed by an average of less than one percent of the amplitude of oscillation. While the difference was significantly higher in the fully started flow at almost 15% of the oscillation amplitude, this sizable percentage error was largely due to the extremely small deflection amplitudes observed at fully started conditions and the relative difficulty of measuring these deflections accurately from the schlieren video given the
limited spatial resolution of the camera. Figure 16 displays the close agreement of plate deflection measurements garnered from schlieren and sDIC data over the first 500 frames captured, as well as the excellent synchronization of the measurements.

![Figure 16. Comparison of Schlieren and sDIC Measurements of Plate Deflection During Tunnel Startup](image)

Similar experimental techniques and processing procedures were used by Hortensius [21], who estimated the out-of-plane measurement accuracy and resolution for his own sDIC system. A linear traverse of the test article through the measurement region at known displacements showed an agreement of within 1% between these known displacements and those measured via sDIC. Out-of-plane displacement resolution was estimated from the standard deviation of 25 stationary sDIC images to be 1.1 μm. Based on the stated out-of-plane measurement accuracy of sDIC (1/50,000 multiplied by the camera field of view) [21], the accuracy of the sDIC setup in the present work is approximately 2.4 μm based on the 122 mm field of view.

2.3.5 Pressure Measurements

Pressures were measured at several locations within the test section and wind tunnel facility by a variety of instrumentation. As mentioned previously, a Pressure Systems Inc.
Netscanner Model 98RK was used to monitor stagnation chamber pressure, test section static pressure, and the bottom wall static pressure distribution with an acquisition frequency of 5 Hz set via the LabView interface. Incoming test section static pressure was measured from a pressure tap at the upstream end of the top window insert, while the 13 bottom wall pressure taps were spaced 1 inch apart along the centerline beginning 1/4 inch from the rear wall of the cavity beneath the cantilevered plate. Pressure taps consisted of a 0.040-inch hole and a short length of steel tubing secured with epoxy and were connected to the Netscanner modules via lengths of 1/16-inch diameter plastic tubing. This plastic tubing was relatively long and thus damped out any high-frequency pressure fluctuations.

High-frequency pressure measurements were captured above and beneath the cantilevered plate by two wall-mounted Kulite XCQ-062-10A pressure transducers with a rated pressure of 10 psia, maximum pressure of 20 psia, and sensitivity of around 9 mV/psia. These pressure transducers were secured into custom aluminum plugs and mounted securely into the test section, one in the sidewall 1 inch above the plate and 0.59 inches upstream from its tip, and one centered in the rear wall of the cavity beneath the cantilevered plate 0.3 inches above the bottom wall. A cutaway of a pressure transducer mounted within a plug as well as their two locations within the test section are shown in Figure 17.

As is seen in the figure, the Kulite pressure transducers were held in place by a spacer made from rubber tubing with the screen of the transducer flush with the face of the plug and sealed with RTV from behind. Threads at the end of the plug allowed each plug to be secured by a nut from the opposite side of the sidewall or cantilevered plate model. A 1/16-inch (number 008) O-ring placed immediately beneath the head of the pressure transducer plug prevented leakage into or out of the test section. Two solid plugs were also machined such that either of the
holes designed for the pressure transducers could be sealed during experiments such as PIV or oil flow visualization that involved oils that could damage the instrumentation.

Figure 17. Pressure Transducer Cutaway (A) and Locations Within Test Section (B)

Each of the Kulite transducers was powered by a Vishay Model 2311 signal conditioning amplifier with an excitation of 10 V DC and amplification factor of 10. The pressure signals, acquired continuously at 50 kHz, were then filtered through a Krohn-Hite Model 3342 analog filter with a cutoff frequency of 12.5 kHz. A National Instruments USB 6366 data acquisition device read these filtered signals into LabView.

The Kulite pressure transducers were calibrated in situ using a custom rig designed to apply a sweep of pressures to each transducer individually. These pressures were read simultaneously by the Netscanner such that a calibration scale could be established for each transducer within its operating range, which was then input into the LabView program. A full description of the calibration procedure is given in Appendix B.

To study the highly unsteady flow field around the flexible plate during tunnel startup, Kulite pressure measurements were obtained synchronously with high-speed schlieren video. Synchronization of these measurements was achieved via the data acquisition clock that controls
sampling of the Kulite signals from within the LabView VI. By using the exposure signal output by the schlieren camera as the data acquisition clock rather than the default internal clock, it was possible to ensure that pressure measurements were read simultaneously with the acquisition of schlieren frames. This method of synchronization was verified by using the same National Instruments USB 6366 DAQ used to read the Kulite measurements to read a periodic reference signal output by a delay generator. Using an oscilloscope and varying the timing of the sample clock signal used by LabView to trigger measurements, it was found that an overlap of only 10 ns (the smallest increment achieved by the delay generator) between the reference signal and the sample clock signal was sufficient for the DAQ to measure the reference signal. This indicated that the lag between a sample clock signal and the actual acquisition of a measurement by the DAQ was less than 10 ns, an acceptable margin when recording at the 10 kHz sampling frequency used in the experiments.
CHAPTER 3: THE STARTUP FLOWFIELD

The startup flowfield can be generally defined as the flow around the cantilevered plate geometry that is observed at any point between the “flow off” condition and the fully started condition when the shear layer attaches to the bottom wind tunnel wall. While this definition encompasses a wide range of upstream stagnation pressures and flow conditions, the main area of interest within the startup sequence is the period during which transient waves pass through the test section and elicit a highly dynamic response from the compliant cantilevered plate. This encompasses a range of tunnel stagnation pressures from around 20 to 30 psia. The flowfield present under these conditions is characterized by the high degree of unsteadiness caused by the passage of these transient waves and is amplified by the dynamic response of the cantilevered plate in the case of the compliant test geometry. While both rigid and flexible geometries were studied under these startup conditions, much of the focus was placed on the flexible version as the rigid plate was not observed to deform measurably.

Like most supersonic wind tunnels, the LRST passes through several stages as the stagnation pressure, $P_0$, increases, and the pressure ratio across the facility changes. Figure 18 shows the progression of the test section flowfield around the flexible geometry through several of these stages and demonstrates how shock waves move through the tunnel as it approaches fully started conditions. In the figure, the final image depicts the test section flowfield once the tunnel has become fully started. The experiments discussed in this chapter focused mainly on parts of the startup sequence before the first standing shock waves, likely reflections of the main starting shock wave, entered the test section at a stagnation pressure of approximately 28 psia. At the conditions studied, the flexible plate exhibited a highly unsteady response, the dynamics of
which were of great interest. Once the first standing waves were pushed into the test section, the
dependence of both the fluid and structural response on the position of these shock waves and the
unpredictability of the movement of these waves made these conditions both difficult to study
and to reproduce exactly from run-to-run.

Figure 18. Schlieren Visualization of the Progression of Test Section Conditions During Tunnel
Startup

3.1 Rigid Cantilevered Plate Results

3.1.1 Schlieren Photography

A time-averaged schlieren photo of the startup flow around the rigid cantilevered plate is
shown in Figure 19 and displays the prevalence of waves within the test section as the flow
transitions from subsonic to supersonic. In the image shown, the shear layer, visualized in white
due to the horizontal knife edge, formed at the downstream tip of the cantilevered plate where
the high-velocity free stream flow met the relatively quiescent cavity flow. As the stagnation
pressure rose from atmospheric pressure to the operating conditions, this shear layer remained
free and extended horizontally out from the end of the cantilevered plate. This condition is
indicative of the insufficient stagnation pressure required to force the shear layer down towards the bottom wall. As will be discussed later, the cavity pressure becomes sufficiently low for shear layer reattachment to the bottom wall at a stagnation pressure of approximately 37 psia at which point the tunnel is considered to be fully started. For reference, the fully started flowfield for the flexible plate, which is similar to that of the rigid plate, is shown in Figure 18 (lower right).

![Schlieren Image of Rigid Plate During Startup](image)

Figure 19. Schlieren Image of Rigid Plate During Startup

3.1.2 Static Pressure Measurements

Measurements of freestream and cavity static pressure taken by the two Kulite pressure transducers discussed in section 2.3.5 captured the evolution of the test section pressures as the tunnel stagnation pressure rose to operating conditions and further revealed the unsteady nature of the flow during tunnel startup. The static pressure measurements, shown in Figure 20, were collected by the Kulite transducers at 50 kHz and averaged over 0.2 second intervals to correspond to the stagnation pressure measurements, collected at 5 Hz. These results indicate
that the freestream and cavity static pressure decreased in unison as the stagnation pressure increased to around 22 psia at which point a standing shock wave had reached the exit of the nozzle and sat upstream of the test section. At this condition, the test section flow had reached approximately Mach 0.6 and the static pressures began to deviate from each other. At a stagnation pressure of 26 psia the pressure differential across the plate began to oscillate sharply as a series of starting shocks is pushed through the test section. Once the last of these waves passed the freestream pressure transducer, the freestream static pressure dropped sharply, followed by an even larger drop in the cavity static pressure as the shear layer attached to the bottom wall. Soon after, at a stagnation pressure of \( P_0 = 37.4 \) psia, the pressure differential across the plate inverted (\( P_{\text{cavity}} < P_{\text{freestream}} \)) and the tunnel transitioned to the fully started state.

Figure 20. Static Pressure Measurements for the Rigid Plate Geometry During Startup

While in the case of the rigid cantilevered plate the strong pressure fluctuations experienced during this startup procedure elicited no structural response from the geometry, this unsteady loading resulted in a highly dynamic response in the case of the flexible plate, as will be discussed in the following section.
Averaging of the Kulite data shown in Figure 20 enhances the general trends in the startup pressures; however, it also hides the true unsteadiness of the pressure fluctuations experienced by the cantilevered plate. The unaveraged (i.e., time-resolved) 50 kHz pressure measurements, shown in Figure 21, demonstrate exactly how unsteady the startup pressures are, exhibiting fluctuations as large as 5 psid in amplitude.

![Figure 21. Instantaneous Pressure Measurements During Tunnel Startup](image)

Though seemingly random at first glance, spectral analysis of the cavity pressure signal revealed distinctive peaks in the frequency content of these pressure fluctuations. Some of those frequency peaks were seen to change in both frequency and amplitude as the rising stagnation pressure changed the dynamics of the flow in the test section, while others remained constant. The evolution of the cavity pressure frequency spectrum is shown in Figure 22 over a range of tunnel stagnation pressures. The top and bottom frequency spectra of each pair shown here correspond to the frequency content of the freestream and cavity static pressure signals, respectively.
An important pattern present in these plots is the prevalence of a distinct frequency peak in the cavity pressure fluctuations at around 3 kHz. As can be seen in the final frequency spectrum corresponding to a stagnation pressure of 40.6 psia, this 3 kHz peak was the dominant frequency present in the cavity pressure at fully started conditions. Its appearance throughout the startup process suggests that its source may be more closely tied to the cantilevered plate geometry itself than to the supersonic flow conditions. This frequency peak will be discussed further in section 4.1.2.

A second notable frequency peak that appeared in many of the freestream pressure spectra occurred at approximately 400 Hz. As will be discussed later in this chapter, this is approximately the same frequency at which the flexible cantilever plate was observed to oscillate during portions of the startup process as well as at fully started conditions. While the source of this freestream pressure fluctuation and its role in the flexible plate’s response are not clear, its proximity in frequency to the plate’s oscillatory motion makes it important to acknowledge.

Figure 22. Evolution of Kulite Pressure Frequency Content During Startup
3.2 Flexible Cantilevered Plate Results

As mentioned above, the highly unsteady nature of the startup flowfield elicited a strong dynamic response from the flexible cantilevered plate which deformed significantly under the aerodynamic loading imposed by the startup flow. The aerodynamic effects of this deformation, in turn, resulted in a substantially different instantaneous flowfield at certain stagnation pressures during startup. While it is difficult to definitively classify this dynamic response as the result of aeroelastic instability or simply a driven response to the unsteady flow, the series of pressure and surface measurements discussed in this section were obtained with the goal of clarifying the dynamics at work and helping to identify the phenomena involved.

3.2.1 Static Pressure Measurements

Averaged static pressure measurements collected from the same Kulite pressure transducers as those used for the rigid plate reveal extremely similar trends in the evolution of static pressures above and below the flexible plate as those seen for the rigid plate. These measurements were again collected at 50 kHz and averaged over 0.2 second intervals to correspond to the 5 Hz stagnation pressure measurements. Shown in Figure 23 along with the static pressure measurements for the rigid case, the time-averaged pressures around the flexible plate indicate that while the structural compliance of the cantilevered may have resulted in much different instantaneous flowfields, the time-averaged trends of the pressure field were nearly identical. One major difference that was clearly observed for the flexible plate, however, was the fact that the fully started condition was reached at a significantly lower stagnation pressure with the flexible geometry. This condition, indicated by a sharp jump in pressure differential, required
a stagnation pressure of only 35.7 psia with the flexible plate, almost 2 psi lower than the pressure required to start the tunnel with the rigid geometry.

Figure 23. Static Pressure Measurements for Flexible and Rigid Plates During Startup

This difference is most likely attributed not only to the compliance of the plate, but also to the small gaps left between the sidewalls and the edges of the flexible plate. As discussed previously, these gaps were necessary to prevent friction between the moving plate and the test section walls, but also introduced a source of leakage between the freestream and cavity regions. Thus, it is possible that the presence of these gaps artificially lowered the pressure in the cavity for the flexible plate, allowing the shear layer to attach to the bottom wall earlier than in the case of the rigid plate. The result of this leakage can also be seen in the pressure differential measurements across the flexible plate that, for stagnation pressures exceeding approximately 31 psia, were observed to be of lower magnitude than the same measurements for the rigid plate.

3.2.2 Simultaneous Schlieren – Pressure Transducer Measurements

High-speed schlieren video obtained synchronously with high-frequency pressure transducer measurements provides insight into the conditions to which the flexible cantilever is
subjected at various times during the startup sequence. Additionally, analyzing the pixel values from the schlieren images makes it possible to obtain an approximate measurement for the deflection of the cantilevered plate tip in each frame. As mentioned previously, pressure measurements could be obtained by two Kulite pressure transducers, one in the back wall of the cavity region and one in the side wall above the plate. To capture simultaneous schlieren images, however, the removeable window insert into which the sidewall pressure transducer is mounted was reoriented to provide optical access to the cantilevered plate. Thus, while it is ultimately the pressure differential across the plate that drives the plate motion, only cavity static pressure could be measured simultaneously with schlieren video. At an upstream stagnation pressure of around 25 psia, approximately 69 percent of the necessary pressure to bring the wind tunnel to fully started conditions, the test section flowfield was observed to be the most unsteady and the flexible plate’s response the most significant.

Figure 24. Simultaneous Schlieren-Pressure Measurements Showing Flexible Plate Oscillation
Figure 24 displays a series of simultaneous schlieren-pressure transducer measurements encompassing one plate oscillation during this highly dynamic period. The oscillations seen at this stage reached as much as 7.9 plate thicknesses and represent the largest plate deflections observed at any point during wind tunnel operation.

As can be seen in the figure, cavity static pressure and plate position tracked each other very closely as the flexible plate moved. Unsurprisingly, the frequency spectra for both the cavity pressure as well as the plate position displayed significant peaks at 400 Hz, the frequency at which they both oscillated. These frequency spectra, seen below in Figure 25, are substantially different than those observed for the rigid plate cavity static pressure measurements discussed previously.

![Frequency Spectra](image)

Figure 25. Frequency Spectra of A) Cavity Static Pressure and B) Plate Displacement During Flexible Plate Oscillations \( (P_0 = 25.7 \text{ psia}) \)
Where in the rigid case there were multiple frequencies present in the static pressure fluctuations, the ability of the flexible cantilevered plate to deform resulted in a single dominant frequency driven by the highly regular sinusoidal response of the plate. The second frequency peak in the cavity pressure signal is simply a harmonic of the first main peak and is not representative of a separate phenomenon. It is once again worthwhile to note that as discussed in section 3.1.2, a 400 Hz fluctuation was measured in the freestream static pressure above the rigid cantilevered plate during the startup process. It is impossible to directly link this freestream pressure fluctuation with the frequency of oscillation observed for the flexible plate; however, the reasons for these coinciding frequencies may warrant further investigation.

Computing the cross-correlation of the cavity pressure and plate displacement over the range of stagnation pressures in which this oscillatory behavior was observed and across several different startup runs revealed that the cavity static pressure signal initially led the deflection of the plate by approximately 0.2 ms. Once the stagnation pressure had risen sufficiently for the first starting shocks to be pushed into the test section, causing the plate’s motion to stabilize, the cross-correlation showed that the cavity pressure lagged the deflection of the flexible plate by approximately 0.2 ms. This evolution of the cross-correlation between the two signals suggests that the flexible plate’s motion may have been initially driven by pressure fluctuations in the cavity region, and as those fluctuations diminished with the increase in stagnation pressure, the continued motion of the plate instead drove the changes in cavity static pressure that continued to be observed.

3.2.3 Simultaneous Schlieren – sDIC

While analysis of the high-speed schlieren video of the flexible plate made it possible to obtain approximate measurements of plate deflection under various conditions, these
measurements were severely limited both in their spatial resolution, as well as by their inability to properly capture spanwise twisting of the plate. sDIC allows one to measure out-of-plane deflections at any point across the plate surface at much higher spatial resolution than schlieren video, providing a significantly more accurate description of the plate’s dynamic response. Once again, high-speed schlieren video was captured synchronously with the sDIC measurements to provide insight into the instantaneous flowfield in addition to the sDIC deflections. An example of this sDIC-schlieren data captured during tunnel startup is shown in Figure 26. This particular frame, captured at a stagnation pressure of $P_0 = 22.1$ psia, represents the first significant response from the flexible plate during the startup process.

Figure 26. Deflection Contours and Schlieren Snapshot Captured During Tunnel Startup
The low-amplitude movement of the flexible plate measured here highlights the superior abilities of sDIC to capture spanwise twisting of the cantilevered plate that is otherwise extremely difficult to observe using only schlieren imaging. Plate deflection measurements captured via sDIC, like those extracted from schlieren images, are referenced to the undeflected plate height.

By analyzing the time-resolved instantaneous deflection measurements at various spanwise locations along the downstream edge of the cantilevered plate, it was possible to extract the frequency content of the oscillating plate deflections at various stagnation pressures during the startup sequence. The frequency spectra for the two main points of interest, corresponding to stagnation pressures of 22.1 and 26.9 psia, at which the flexible plate undergoes low-amplitude and high-amplitude oscillations, respectively, are shown in Figure 27. The flexible plate’s low-amplitude response, the same response captured in Figure 26 above, exhibited four distinct frequency peaks. Each of these first four frequency peaks lies within 3 percent of an experimentally observed natural vibration frequency of the cantilevered plate. These natural vibration frequencies, observed at 303 Hz, 389 Hz, 661 Hz, and 1159 Hz, were measured in a separate test via sDIC in which the flexible plate was set in motion by an initial impulse and the ensuing free vibration was recorded. The close correspondence of frequencies suggests that in the early phases of startup during which these data were obtained, the plate’s motion was largely governed by its natural vibration characteristics with the flow acting as a source of excitation, but not playing a significant role in the dynamic response of the plate.

It should also be noted once again that the original flexible geometry failed prior to beginning sDIC experiments, thus the one used for the sDIC experiments was a newly machined version of the original. Though the two test articles were dimensionally identical, the frequencies observed in the new plate’s response were slightly lower than those discussed in section 3.1.2.
Figure 27. Frequency Spectra for Plate Displacement During A) Low-Amplitude and B) High-Amplitude Oscillations

The second frequency spectrum, representing the flexible plate response at a stagnation pressure of 26.9 psia, displays a significant change from the low amplitude response. At this point during startup, plate oscillations were of their largest amplitude at any point during startup. The main frequency peak, representing the first bending mode of the cantilevered plate, had shifted more than 60 Hz higher to 375 Hz from the low-amplitude response of 311 Hz, nearly
overlapping the second frequency peak which remained at around 390 Hz, approximately the same from earlier in the startup sequence.

3.2.4 Modal Decomposition of sDIC Data

Using proper orthogonal decomposition (POD) and Dynamic Mode Decomposition (DMD) [22][23] to analyze the time-resolved displacement data for the plate surface, it was possible to extract both the energy-dominant mode shapes present in the motion of the cantilever as well as the specific mode shapes corresponding to each frequency peak observed. For the early startup condition, the mode shapes obtained via DMD that correspond to the first four peaks observed in the frequency spectrum are shown in Figure 28. It was found these four DMD modes were the same shapes as the first four most energy-dominant modes obtained via POD, which contained 98 percent of the total kinetic energy of the plate motion. This correspondence of mode shapes obtained via the two decompositions indicates that the dominant frequencies present in the flexible plate’s motion corresponded exactly to the highest-energy mode shapes. The first two modes, containing the vast majority of the kinetic energy, represent the first bending and twisting modes of a canonical cantilevered plate under free vibration [24]. The third and fourth mode shapes present in the low-amplitude startup response deviate from what is expected of a standard cantilevered plate, exhibiting spanwise bending mode shapes rather than any additional twisting modes. These spanwise bending modes differ from twisting modes or streamwise bending modes in that the additional nodes of the mode shapes appear in the spanwise direction rather than the streamwise direction. It is possible that the appearance of these mode shapes occurs due to the specific aspect ratio of the cantilevered plate design, which is 2.5 time longer along the spanwise axis than the streamwise axis [12]. These mode shapes do, however, constitute a very small percentage of the overall system energy and match those
observed in the natural vibration test conducted on this geometry in both shape and frequency. The spurious or missing points near the edges of the surfaces were caused by poor data at the edges of the field of view or towards the downstream end of the cantilevered plate. These extraneous data points were generally caused by poor illumination towards the edges of the plate, or by poor focus as the tip of the deforming flexible plate moved too far from the focus plane of the cameras.

Figure 28. Four Most Dominant Mode Shapes Present in the Low-Amplitude Response During Startup ($P_0 = 22.1$ psia)

As with the frequency content of the cantilevered plate’s response, the respective energies of these mode shapes were seen to change as the tunnel stagnation pressure rose and the test section flowfield changed. Later during startup at a stagnation pressure of 26.9 psia, not only
did the main frequency peak shift, but most of the energy became contained in only the first two plate modes. Despite the changes in frequency and energy distribution, the mode shapes present in this higher-amplitude response, shown in Figure 29, remained the same bending and twisting modes that were observed earlier in startup.

![Mode Shapes](image)

Figure 29. Dominant Mode Shapes in High-Amplitude Plate Response ($P_0 = 26.9$ psia)

It should be noted that at this point during the startup sequence, almost all of the energy of the plate’s motion was contained in a single frequency peak, thus the single mode shape obtained via DMD at the dominant frequency was a linear combination of the dominant POD modes [23]. By subtracting the first and second POD mode shapes from the DMD result, it was possible to obtain the shapes shown in Figure 29 for modes 2 and 1, respectively, and confirm the presence of both of these mode shapes at the same frequency. Due to the substantial oscillatory bending of the cantilevered plate during this high-amplitude response, it is somewhat unsurprising that almost all of the kinetic energy was contained in the first bending mode.
CHAPTER 4: THE FULLY STARTED FLOWFIELD

The LRST is considered to become fully started when the shear layer, which during startup leaves the tip of the cantilevered plate horizontally, attaches to the bottom wind tunnel wall, driven downward by the low cavity static pressure. As was mentioned in the preceding chapter, this condition is reached at stagnation pressures of 37.4 and 35.7 psia for the rigid and flexible plates, respectively. By the time this condition has been met, the starting shock waves have been pushed through the test section and the incoming flow is uniform and fully supersonic (except for the boundary layer on the surface of the plate). Because of this, the flowfield at fully started conditions is generally much steadier than the startup flow, and the cantilevered plate’s response is less dynamic. As will be discussed in the following sections, the effect of structural compliance of the plate is much less significant at fully started conditions than during the startup sequence, though not entirely absent.

4.1 Rigid Cantilevered Plate Results

4.1.1 Schlieren Photography

The fully started flowfield around the rigid cantilevered plate, captured via schlieren photography as shown in Figure 30, is largely unchanging in time due to the steady, uniform incoming flow and the structurally rigid plate. The main flow features observed for this cantilevered plate geometry are the fully turbulent incoming boundary layer, the expansion fan seen in white at the tip of the cantilever, the shear layer, angled downwards and attaching to the bottom wall approximately $2.9H$ downstream of the end of the cantilever, and the reattachment shock, emanating from the point of shear layer reattachment. For this geometry, the step height
(H) is defined as the vertical distance from the top surface of the cantilevered plate to the bottom wall of the test section.

![Rigid Plate Image](image)

Figure 30. Instantaneous Schlieren Image of Rigid Plate Under Fully Started Conditions

An important detail of the flowfield in this schlieren image is that the flow in the cavity region can be seen to be relatively inactive. While schlieren photography only captures density gradients and can generally only be used to qualitatively assess the flow, the uniformity of the schlieren image in the cavity region is indicative of the relatively quiescent flow beneath the cantilever that will be discussed in greater detail in section 4.1.3.

4.1.2 Pressure Measurements

Static pressure taps along the centerline of the bottom wall of the test section measured the pressure distribution along the wall at fully started conditions. These measurements, shown in Figure 31, show the distinct relatively constant low-pressure region present in the cavity region beneath the rigid plate. The sharp jump in bottom-wall static pressure at approximately 5.7H from the rear wall of the cavity indicates the point of shear layer reattachment and the formation of the reattachment shock. The streamwise positioning of this transition in the pressure
distribution measurements corresponds well to what is observed in the schlieren image shown above.

![Graph showing static pressure distribution](image)

Figure 31. Static Pressure Distribution Along Bottom Wall at Fully Started Conditions

The additional smaller jump in pressure at approximately $13H$ is likely the location at which a weak wave generated by the seam between the wind tunnel top wall and the top window insert impinges on the bottom wall of the test section. While seams like this one were smoothed over with putty, they could not be eliminated entirely and the weak waves generated by them can be seen faintly in the schlieren image above.

The same Kulite pressure transducers that captured high-frequency pressure fluctuations during the unsteady startup sequence were also useful in characterizing the flow at fully started conditions. Figure 32 shows the frequency spectrum of the fluctuations in cavity pressure once the tunnel has become fully started. Here, the Strouhal Number is based on the 0.7-inch (17.8 mm) step height of the plate and a nominal freestream velocity of 512 m/s. While the
fluctuations themselves were less than 0.3 psia in magnitude, several distinct frequencies were present.

Three of these frequencies appear to correspond to the three commonly observed instability modes in the flow over a backward-facing step, a commonly studied geometry similar to the rigid cantilevered plate geometry only without a cavity. The lowest frequency is typically driven by a “flapping” of the shear layer around the location of reattachment and was observed by Driver et al. [25] to occur at $St = 0.03$ and by Horchler et al. [26] at $St = 0.052$. While the lowest frequency peak in the rigid plate cavity pressure fluctuations was observed at $St = 0.0625$ (1.8 kHz), slightly higher than both Driver et al. [25] and Horchler et al. [26], this difference may be attributed to the large cavity region present in this geometry that differentiates from a traditional backward-facing step or by the fact that no visible “flapping” was observed in the high-speed schlieren video, suggesting that the shear layer dynamics were affected by the much larger recirculation region beneath the rigid plate.
The second and most dominant frequency mode downstream of a backward-facing step is generally the “wake” mode, representative of vortex shedding similar to the wake behind a bluff body. Driver et al. [25] and Horchler et al. [26] observed this mode at $St = 0.1$ and $St = 0.15$, respectively, aligning closely with the dominant frequency peak in the cavity pressure fluctuations measured beneath the rigid plate at $St = 0.1$ (3 kHz). This 3 kHz peak was also present in an inviscid simulation carried out at The Ohio State University, indicating that it is driven by a strong, inviscid phenomenon.

The third and highest frequency typically observed in backward-facing step flows is the shear layer mode that arises from the Kelvin-Helmholtz instability seen in free shear layers [27]. This shear layer mode has a characteristic frequency of $St_\Theta = 0.013$ where $St_\Theta$ is the Strouhal number normalized using the momentum thickness of the boundary layer at the point of shear layer separation rather than the step height. This mode was measured by Hasan [28] at $St_\Theta = 0.012$ for a backward-facing step. Based on the compressible momentum thickness calculated via the Modified Wall Wake profile that was fit to the measured incoming boundary layer and is discussed in section 4.1.3, the frequency peak observed at 8 kHz in the rigid plate cavity pressure fluctuations corresponds to $St_\Theta = 0.013$, indicative of the shear layer mode.

The source of the frequency peak observed at $St = 0.19$ (5.5 kHz) remains unclear. Other potential sources for the observed frequency peaks that have both been investigated and eliminated as possibilities were pipe modes and Rossiter modes [30].

4.1.3 Particle Image Velocimetry

Planar PIV measurements allowed for a more quantitative characterization of the fully started flowfield than schlieren visualization, providing insight into both the incoming turbulent boundary layer as well as the flow within the cavity region. As discussed in section 2.3.2, a
Plexiglas block was used in place of the actual stainless steel rigid test geometry and was fabricated specifically for boundary layer PIV measurements to decrease laser reflections and improve imaging. The boundary layer was measured at a streamwise location 5 mm upstream from the tip of the Plexiglas geometry due to difficulties obtaining consistent data directly at the downstream edge. The measured boundary layer velocity profile, averaged from approximately 3000 images and shown in Figure 33, was found to reach 99% of the freestream velocity at a height of $\delta = 12.83$ mm. The Modified Wall Wake Velocity Profile developed by Sun and Childs [31] was fit to the experimental data and used to obtain additional quantities of interest such as skin friction coefficient, displacement and momentum thicknesses, and the shape factor. This velocity-profile fit, as well as the incompressible integral quantities of interest, are shown in Figure 33A and Figure 33B in outer and inner coordinates, respectively.

![Figure 33. Modified Wall Wake Profile Fit to Measured Boundary Layer Velocity Profile in A) Outer Coordinates and B) Inner Coordinates](image-url)
Accuracy of the experimentally measured velocity profile close to the surface was improved by masking the pixels closest to the wall while processing the raw images. This ensured that reflections of the laser sheet from the surface did not interfere with the velocity calculations and was found to substantially improve the fit of the Modified Wall Wake Profile to the experimental data. Reynolds stresses near the surface were also measured and are shown in Figure 34. These Reynolds stresses were found to be slightly higher than commonly cited Morkovin-scaled Reynolds stress measurements [32], [33]; however the reasons for this discrepancy are currently unclear.

![Figure 34. Measured Boundary Layer Reynolds Stress Profiles](image)

Full velocity field PIV measurements, averaged over approximately 1500 images, are shown in Figure 35 with the cavity flowfield plotted a second time on a different scale to highlight the changes in velocity within this region. In these plots, the $x$ and $y$ axes indicate the locations of the bottom wall and rear cavity wall, respectively. Due to reflections from the laser sheet, it was impossible to collect data up to these boundaries without risking damage to the
camera sensor. The edges of the cantilevered plate were masked as well for the same reason. Out of convenience, the coordinate origin was placed at the downstream tip of the cantilevered plate.

Figure 35. Velocity Field Surrounding Rigid Plate (A) and Within Cavity (B) at Fully Started Conditions

The velocity measurements in the upper flow, the high-velocity flow above the cantilevered plate and shear layer, do not provide any additional information about the flow characteristics that was not already observed (and expected) via high-speed schlieren video, other than providing quantitative results for the velocities over the cantilevered plate and through the expansion fan. The cavity flowfield measurements, however, provide valuable information about the flow beneath the shear layer and cantilevered plate that was not available from the schlieren images. The velocity data clearly show the recirculation patterns of the flow entrained into this region by the shear layer, specifically the presence of two counter-rotating vortices at either end of this region. The bottom plot highlighting the cavity flow data also reveals just how relatively quiescent the cavity flowfield is; apart from the flow recirculating directly beneath the
shear layer, the flow within the cavity can be seen to be extremely low velocity compared to the surrounding flow. This observation is supported by the lack of density variations seen in this same region in the schlieren images.

An important detail to note in the velocity data shown in Figure 35 is the prevalence of velocity vectors directed towards the closed end of the cavity region and the relative lack of vectors in the positive $x$-direction. Since only one end of this region is open, conservation of mass suggests that there must be a non-trivial amount of three dimensionality to this flow. The most likely scenario is that upon reaching the rear wall of the cavity, a portion of the flow disperses towards either of the sidewalls and conserves mass via a spanwise recirculation unseen by these planar centerline measurements. This three-dimensionality will be discussed further in the following section.

4.1.4 Oil Flow Visualization

Oil flow visualization of the flow patterns on the bottom wall of the test section allowed for a more comprehensive understanding of the three-dimensional effects closer to the test section sidewalls that planar PIV could not capture. These flow patterns, shown in Figure 36, were indicative of the spanwise recirculation that was hypothesized in the previous section. As can be seen in the figure, the influence from the sidewalls extended approximately halfway to the centerline of the test section. Although the oil flow shows that there still existed a region of approximate two dimensionality near the center of the test section, the spanwise recirculation suggested by the PIV results indicated that even at the centerline, the three-dimensional influence of the sidewalls was felt.
Though it is difficult to discern due to the shadow of the plate and from only a single surface-flow frame, the relatively undisturbed oil in the region directly beneath the plate and behind the strongest recirculation vortex is yet another sign as to the relative quiescence of the flow in the cavity region. This has important implications in the flexible plate case where the structural response of the plate is highly dependent on the unsteadiness of the flow both above and below the cantilever.

4.2 Flexible Cantilevered Plate Results

4.2.1 Schlieren Photography

Similar to the rigid plate geometry, the flowfield around the flexible cantilevered plate at fully started conditions is relatively stable compared to the highly dynamic startup flow. In the schlieren image shown in Figure 37, below, the flow can be seen to be largely the same as with the rigid geometry with the exception of a mean plate deflection. High-speed schlieren video reveals that in reality, the flexible plate oscillates very slightly around this mean deflected height,
but at an amplitude of less than 30 percent of the plate thickness such that the general flow pattern remains largely unaffected.

Figure 37. Schlieren Image of Flexible Plate at Fully Started Conditions

While the turbulent boundary layer, expansion fan, shear layer, and reattachment shock seen in the rigid plate’s fully started outer flowfield were all still present, the reattachment shock downstream of the flexible plate differed slightly, appearing to coalesce from a series of weaker waves emanating from the region around the point of shear layer reattachment. It is likely that the slight deflection of the compliant cantilever allowed for a marginally shallower shear layer angle and a more gradual reattachment to the bottom wall that ultimately resulted in a reattachment shock wave that formed less abruptly than in the rigid case. An additional weak expansion wave at the upstream end of the deflected plate that was absent in the undeflected rigid plate case was also present in this flowfield.

4.2.2 Pressure Measurements

As could be expected from the similarities between the fully started flowfields with the two geometries illustrated via schlieren, the centerline mean pressure distribution along the bottom wall with the flexible plate is strikingly similar to that of the rigid case. This pressure
distribution, measured via the same pressure taps discussed for the rigid geometry, is shown in Figure 38 alongside the rigid geometry pressure distribution for comparison. The data in the plot show that downstream of $8H$ the pressure distribution for the flexible plate was almost identical to that of the rigid plate. Upstream of this point, however, the pressures deviated from those measured in the rigid case and displayed a less dramatic pressure change across the point of shear layer reattachment. As was discussed in section 3.2.1, the gaps between the test section side walls and the edges of the flexible cantilevered plate likely allowed for a small amount of leakage between the cavity region and the freestream flow. This leakage may have been the reason for the differences seen in the first four pressure taps as it allowed air from the freestream upper flow to leak into the low-pressure cavity region. The smaller pressure change across the reattachment point may be related to the gradual shear layer reattachment and shock formation seen in the schlieren image of the flexible plate flowfield discussed previously.

![Figure 38. Average Static Pressure Distribution Along Bottom Wall at Fully Started Conditions for Rigid and Flexible Plates](image)

Figure 38. Average Static Pressure Distribution Along Bottom Wall at Fully Started Conditions for Rigid and Flexible Plates
The effects of structural compliance are somewhat more clearly seen in the cavity pressure spectrum for the flexible plate at fully started conditions. This spectrum is shown in Figure 38 and illustrates the two important contributors to the cavity static pressure fluctuations in the case of the flexible plate. The Strouhal number is once again based on a freestream velocity of 512 m/s and the 0.7-inch step height of the undeflected plate. Similar to the rigid case, frequency peaks at $St = 0.1$ (3 kHz) and $St = 0.15$ (5.5 kHz) were observed, indicating that the same flow phenomena that led to these peaks in the rigid plate flowfield are also present for the flexible plate. The sharp peak at $St = 0.014$ (400 Hz) corresponded to the frequency at which the flexible plate was observed to oscillate in the high-speed schlieren video under fully started conditions. The strength of this frequency peak relative to those at $St = 0.1$ and $St = 0.15$ shows that though the response of the flexible plate at fully started conditions was very slight, its effect on the flowfield is nontrivial.

![Figure 39. RMS Averaged Cavity Pressure Frequency Spectrum at Fully Started Conditions](image-url)
4.2.3 Particle Image Velocimetry

PIV results for the flexible plate at fully started conditions, shown in Figure 40, reinforced the key takeaways from the schlieren and pressure results. Qualitatively, the flowfield was generally the same as with the rigid cantilevered plate with several minor exceptions. While instantaneous differences generated by the slight oscillation of the flexible plate were generally not present in this time-averaged flowfield, the mean deflection of the plate could be seen to change the angle of the shear layer slightly, as it allowed the flow to leave the tip of the cantilever at a height closer to the bottom wall, while still reattaching at the same streamwise location. This change in shear layer angle could also be seen to flatten the recirculation vortex present beneath the shear layer.

Perhaps the most notable changes to the flowfield that occurred with the introduction of the flexible geometry appear in the cavity region beneath the plate. Here, whereas in the rigid plate case a pair of counterrotating vortices of approximately equal size formed, only the recirculation vortex immediately beneath the shear layer appeared, with the second vortex at the back of the cavity region either lying outside the field of view or being absent entirely. The velocity of the flow towards the rear of the cavity region beneath the flexible plate was also substantially higher than that for the rigid plate, exhibiting differences of more than 100 m/s between the two flowfields in some locations. Similar to the rigid plate PIV results discussed in section 4.1.3, the streamlines beneath the flexible plate indicate that the flow in the cavity region predominantly moves towards the closed end. This once again suggests that spanwise movement of the flow might occur in order to conserve mass within this partially closed region.
While it is likely that the deflection of the flexible plate at these conditions contributed somewhat to the differences seen between the two cases, it is important to consider the three-dimensional effects discussed in section 4.1.3 for the rigid plate and how the narrow gaps between the tunnel sidewalls and cantilevered plate edges may have exacerbated some of these effects for the flexible plate. As will be discussed in the following section, three-dimensional effects, in addition to the mean plate deflection, were a likely contributor to the differences observed in the cavity flowfields of the rigid and flexible plate geometries.

4.2.4 Oil Flow Visualization

With the introduction of small gaps between the sidewalls of the test section and the edges of the cantilevered plate into the flexible version of the test geometry, an understanding of the spanwise flow effects at work is even more important than in the rigid case. Surface flow
visualization via fluorescent oil applied to the bottom wall of the test section revealed, as it did for the rigid case, the extent of these three-dimensional effects and provided some understanding of the two-dimensional PIV results discussed in the previous section.

Oil flow results for the flexible plate, shown in Figure 41, displayed spanwise recirculation similar to what was observed in the case of the rigid plate. These spanwise vortices, however, appeared to extend nearly all the way to the centerline of the test section, indicating that the three-dimensional effects present in the flexible plate flowfield were more substantial than those around the rigid plate. As a result, when compared to the rigid plate surface flow, the region of apparently two-dimensional flow near the centerline of the test section was narrower in the case of the flexible plate geometry. Additionally, the movement of oil beneath the flexible plate indicated that the flow in the cavity moved at a relatively higher velocity towards the rear of the cavity than for the rigid plate, a conclusion supported by the PIV results discussed in the previous section and a result much different than the relatively quiescent cavity flow observed in the rigid plate oil flow results.

Figure 41. Bottom-Wall Surface Flow Patterns for Flexible Plate at Fully Started Conditions
While this increase in spanwise effects should be expected when the potential leakage effects of the gaps on either side of the flexible cantilever are considered, the fact that these spanwise effects extended farther towards the centerline than that observed in the rigid plate oil flow results makes it difficult to distinguish the effects of leakage around the flexible cantilevered plate from the effects of structural compliance. The large spanwise vortices seen with the flexible plate do, however, seem to easily explain the high-velocity flow towards the rear of the cavity. This, in turn, may also explain the absence of a second vortex at the rear of the cavity if this high-velocity flow dispersed in the spanwise direction upon reaching the rear cavity wall rather than forming a vortex as it did in the case of the rigid plate. The convenience of these explanations may indicate that the spanwise effects driven by the gaps between the sidewalls and the flexible plate did, in fact, play a larger role in the cavity flow than the effects driven by the mean plate deflection.

4.2.5 Digital Image Correlation

Once again, sDIC allowed for accurate deflection measurements across the entire surface of the flexible plate at a recording frequency of 7 kHz. At fully started conditions where the oscillations of the flexible plate were very small in amplitude and tracking the plate’s position via high-speed schlieren provided insufficient spatial resolution, sDIC was a valuable tool in characterizing the flexible plate’s dynamic response. It should once again be noted that the cantilevered plate geometry used in these sDIC experiments was a newly machined test article that exhibited very slightly different vibration characteristics than the original geometry that failed.

The instantaneous deflection of the flexible plate at fully started conditions is shown in Figure 42. The deflection contours shown in the sDIC results below approximately reflect the
average state of the flexible cantilever under these conditions as the tip of the plate is observed to
oscillate only $0.3h$ about the mean deflection of $2h$ (where $h$ is the plate thickness of 0.04 inches)
as has been stated previously. A small degree of spanwise asymmetry can be seen in the
deflection measurements towards the tip of the cantilever and, in addition to the three-di-
dimensional effects discussed in the previous section, is yet another indication of potential
sources of uncertainty introduced by the physical test geometry. It is most likely that the
machining of this aluminum plate had introduced structural imperfections that allowed one side
of the cantilevered geometry to deflect more than the other. Apart from these sDIC results,
however, this asymmetry does not appear to have influenced any other measurements.

![Out-of-Plane Deflection Contours](image)

Figure 42. Out-of-Plane Deflection Contours for Fully Started Conditions Captured via sDIC

Analysis of the instantaneous deflection data once again revealed the underlying
frequencies present in the oscillatory response of the flexible plate. The frequency spectrum for
the movement of three different points along the downstream edge of the plate at fully started
conditions is shown below in Figure 43. Under these flow conditions, two dominant frequency
peaks were observed at similar frequencies as the main frequency peaks observed in the plate’s
motion during tunnel startup, but once again shifted slightly with the change in flow conditions. Relative to the high-amplitude response recorded during startup, the most dominant frequency shifted downward from 375 Hz to 366 Hz while the second frequency shifted upward from 385 Hz to 427 Hz.

![Figure 43. Frequency Spectrum of Plate Tip Deflection at Fully Started Conditions](image)

4.2.6 Modal Analysis of sDIC Data

As with the sDIC results recorded for the startup flowfield, it was possible to extract the mode shapes corresponding to these two frequency peaks using DMD and assess their corresponding contributions to the total deflection energy using POD. Once again, it was found that the first two most energy-dominant modes were, in fact, the mode shapes corresponding to the first two frequency peaks shown above. These mode shapes, shown in Figure 44, were found to once again be the first bending and twisting modes of a cantilevered plate with essentially all of the kinetic energy contained in the first bending mode. These results reflect similar findings for the flexible plate’s behavior in the startup flow discussed in section 3.2.3.
Overall, these sDIC data along with the data collected at startup showed that while the response of the flexible plate can change in amplitude, frequency, and energy distribution between modes as the tunnel stagnation pressure rises and the flow conditions change, the underlying characteristics remain similar. The dominant mode shapes present in the plate’s motion did not change, nor did the ordering of their frequencies or contributions to the total energy. The first bending mode, always the most dominant in terms of energy, always occurred at the lowest frequency and the first twisting mode, consistently slightly higher in frequency, was always the second most dominant mode. The modes were also consistently observed at frequencies no more than 25 percent away from their naturally observed natural vibration frequencies. Thus, while the dynamics of the flexible plate are certainly tied to the flow conditions to which the plate is exposed, they are also firmly rooted in the intrinsic properties of the plate itself.
CHAPTER 5: CONCLUSION

5.1 Summary of Work

The fluid-structure interaction of a cantilevered plate in Mach 2 separated flow was studied in this investigation. Two cantilevered plate geometries, one rigid and one flexible, were studied during the wind tunnel startup process during which the unsteadiness of the startup flow elicited a highly dynamic response from the flexible plate, as well as at fully started wind tunnel conditions at which the incoming flow was supersonic and relatively uniform apart from the turbulent boundary layer. High-speed schlieren video, planar PIV, oil flow visualization, and pressure measurements were used to characterize the flow around both cantilevered plates, while sDIC was used to capture the oscillatory deflection response of the flexible plate. Frequency analysis of the pressure fluctuations within the recirculation region beneath the cantilevered plate provided insight into the high-frequency dynamics of this cavity region while frequency analysis, and POD and DMD modal analyses of the time-resolved surface displacement data captured via sDIC allowed a deeper understanding of the motion of the flexible cantilevered plate.

Simultaneous schlieren-pressure measurements obtained during startup showed that cavity static pressure and plate displacement tracked each other very closely as the flexible plate oscillated under the unsteady loading of the startup flow. Analysis of the cross correlation of these signals suggested that pressure fluctuations were the initial driver of plate motion, but that they later became driven by the plate’s oscillations as the pressure fluctuations along with those oscillations died out at higher stagnation pressures.

sDIC data revealed distinct frequencies present in the dynamic response of the flexible plate during startup. At lower upstream stagnation pressures when the flexible plate’s motion...
was still relatively minimal, these frequencies aligned closely with the plate’s experimentally measured natural vibration frequencies, and DMD revealed that the mode shapes were the same as well. POD showed that the majority of the kinetic energy of the flexible plate’s response was contained in the first two mode shapes, the first bending and twisting modes of a typical cantilevered plate. As the stagnation pressure continued to rise through the startup process and the amplitude of the flexible plate’s oscillations increased to almost 8 plate thicknesses, these frequencies were observed to shift while the mode shapes remained the same. POD showed that nearly all of the energy at this point was contained in only the first bending mode.

Schlieren and PIV measurements of the full flowfield at fully started conditions showed that only minor differences occurred between the rigid and flexible plate geometries due to the relatively small oscillations of the flexible plate. While the mean plate deflection appeared to result in a more gradual expansion of the flow and formation of the reattachment shock, little else changed in this outer flow above the shear layer. Larger differences were observed between the cavity flows of the two geometries with the flexible-plate cavity flowfield apparently lacking a second recirculation vortex near the cavity back wall and containing much higher velocities than in the rigid plate case, as much as 100 m/s higher at some locations. High-frequency pressure transducer measurements showed that the small oscillatory response of the plate at fully started conditions also played a role in the cavity static pressure fluctuations.

Narrow clearance cutouts designed into both edges of the flexible cantilevered plate geometry may have increased the three-dimensional effects present in the flexible plate flowfield, and it was difficult to isolate changes in cavity dynamics caused by the plate’s mean deflection from those caused by this three-dimensionality. Oil flow visualization showed definitively that three-dimensional flow effects were more substantial with the flexible plate
geometry, likely due to these same gaps. Mean static pressure tap measurements along the bottom wall of the test section also showed little variation between the two geometries at fully started conditions with a marginally higher cavity static pressure potentially caused by the same gaps between the edges of the plate and the test section side walls that could have allowed some leakage between the freestream and the cavity region.

sDIC data for the flexible plate at fully started conditions showed a continuation of the trends seen during startup; two main frequency peaks were present, corresponding to the same two first plate modes, but shifted from where they occurred during startup. Despite the substantially lower amplitude oscillations of only 0.3 plate thicknesses, POD showed a further increase in the energy contained in the first bending mode.

5.2 Conclusions

This collection of results revealed several important conclusions about the nature of high-speed FSI for a compliant cantilevered plate. The startup data made it clear that structural compliance in the cantilevered geometry makes a critical difference in the plate’s response to unsteady pressure loading compared to a completely rigid surface. Control surfaces on a high-speed vehicle could be subjected to highly unsteady conditions like those seen during tunnel startup during vehicle maneuvers or while transitioning from subsonic to supersonic flight. The data also suggested that while the plate’s response is partially determined by its natural vibration characteristics, the frequencies present and the distribution of energy between mode shapes can vary depending on the flow conditions.

Data collected at fully started conditions showed that structural compliance does not play as important of a role in the supersonic flowfield when the incoming flow is largely steady. The
dynamics of the separated flow behind the cantilevered plate appeared to be influenced by vortex shedding and shear layer dynamics, similar to the flow behind a bluff body. Differences were observed in the details of the cavity flowfield with the introduction of the flexible plate including an additional pressure fluctuation frequency driven by the plate motion. Small but necessary gaps left between the edges of the flexible late and the test section side walls may have introduced three-dimensional effects that could have also played a role in the differences observed.

5.3 Recommendations for Future Work

Several limitations were imposed by the nature of wind tunnel testing that presented complications in this investigation. As mentioned previously, narrow slots machined into the sides of the flexible cantilevered plate were necessary to allow the plate to move freely without interference from the wind tunnel sidewalls, but also potentially contributed to the three-dimensional effects and some leakage observed in the flowfield around the flexible plate. While excluding these gaps may have alleviated some of these effects, allowing more accurate comparisons between the cavity flowfields for the rigid and flexible plates, doing so would have compromised the structural response of the plate and risked damage to the facility, making it unreasonable for an experimental study. These restrictions, however, are not necessarily present for a numerical simulation. A computational investigation on this same geometry currently underway at OSU will study the flexible plate with no gaps present, potentially revealing additional information about the idealized flexible plate flowfield.

Currao [12] noted that the aspect ratio of a cantilevered plate has important implications in the dominant mode shapes, potentially affecting the three-dimensional nature of the plate motion, and thus the flowfield. For the plate under consideration in this study, which had a
greater spanwise length than streamwise length, these effects may have played a role in the third and fourth mode shapes observed for the flexible plate, which were bending modes in the spanwise direction rather than bending or twisting along the streamwise axis. An investigation into the effects of plate aspect ratio on both the plate dynamics as well as the three-dimensionality of the flow could reveal important information on this relationship, and the changes in plate length and thus cavity depth may also provide greater insight into the behavior of the recirculation region.

Finally, the nature of a supersonic wind tunnel makes the startup flow a difficult one to study. Separated flow and shock waves upstream of the test section result in an unsteady incoming flow that is challenging to characterize and reproduce between runs. While the dynamic response of the flexible plate was observed to remain similar across startup runs, it was difficult to ensure that the incoming flow was always the same. An independent investigation into the effects of unsteadiness in the incoming flow on the response of the flexible cantilevered plate could potentially reveal important information about the nature of the plate’s response and how its motion depends on the incoming flow.
References


Appendix A: Part Drawings for FSI-Specific LRST Parts

Flexible Plate Test Geometry

University of Illinois
at Urbana - Champaign

DRAWN BY: Griffin Boian
DATE: 5/16/2019

MATERIAL: Aluminum 7075

QUALITY: 3/16 UNC - 2B \( \neq \) 450 4X

UNITS: INCHES

SCALE 1/2: 1

sheet 1 of 1
Appendix B: Calibration Procedure for Kulite Pressure Transducers

The following is a procedure for establishing a linear calibration scale for the Kulite XCQ-062-10A pressure transducers in their respective measurement locations within the LRST using the Pressure Systems Inc. Netscanner and the Edwards E2M2 vacuum pump. To ensure accuracy in the calibration of the Kulites, the Netscanner’s calibration should be up to date and should be zeroed prior to this procedure.

1. Secure the Kulite pressure transducer being calibrated into its location within the test section and mount the calibration rig as shown in Figure 45.
   - Ensure that the rig is aligned properly such that the O-ring creates a tight seal against the face of the pressure transducer plug.
   - In the case of the side-wall transducer, spacers between the window insert and calibration rig may be necessary to maintain proper alignment.

Figure 45. Kulite Calibration Setup for Cavity Transducer (left) and Side Wall Transducer (right)
2. Using sufficiently stiff tubing to withstand the low pressures created by the vacuum pump, connect the calibration rig, vacuum pump, Netscanner, and an N₂ tank via a dial regulator. Figure 46 shows a diagram of the pressure lines necessary for calibration.

- Note: it will be easiest to connect to a Netscanner channel already read by the LabView interface.

- If possible, use a second regulator on the N₂ tank to limit the output pressure to less than 20 psi to ensure that the supply to the pressure transducer will not exceed its design limit.

![Figure 46. Illustration of Tubing System for Calibration](image)

3. Run the LabView code and initiate data acquisition by both the Netscanner as well as the Kulites.

- There should be no scale applied to the Kulite signal in LabView. The program should be recording only the voltages output by the signal conditioning amplifier.

- For simplicity, the Kulites should be set to record at the same frequency as the Netscanner.
4. Remove the red intake cap on the vacuum pump and turn it on. The measured pressure should drop to below 1 psia.
   - The original setup for this experiment included a ball valve between the N₂ tank and dial regulator. It may be necessary to close this valve completely to achieve maximum vacuum.

5. Use the dial regulator to slowly raise the pressure to 10 psia, the upper limit of the operating range of the Kulites. Do not approach the 20-psi burst pressure.

6. Stop recording in LabView and save the data.

7. Plot the output voltages of the Kulite being calibrated against the pressures measured by the Netscanner and apply a linear fit to the data between 0 and 10 psia.
   - If the data do not appear to be linear, it is likely that there is an issue in the calibration system. This may be a leak in the tubing or a data acquisition error.

8. Apply a custom scaling in LabView for the pressure transducer using the same slope and y-intercept as the linear curve fit.

9. Repeat steps 1-8 for the second Kulite pressure transducer if necessary.