Topology Control for Maintaining Network Connectivity and Maximizing Network Capacity Under the Physical Model

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Abstract—In this paper we study the issue of topology control under the physical Signal-to-Interference-Noise-Ratio (SINR) model, with the objective of maximizing network capacity. We show that existing graph-model-based topology control captures interference inadequately under the physical SINR model, and as a result, the interference in the topology thus induced is high and the network capacity attained is low. Towards bridging this gap, we propose a centralized approach, called Spatial Reuse Maximizer (MaxSR), that combines a power control algorithm T4P with a topology control algorithm P4T. T4P optimizes the assignment of transmit power given a fixed topology, where by optimality we mean that the transmit power is so assigned that it minimizes the average interference degree (defined as the number of interfering nodes that may interfere with the ongoing transmission on a link) in the topology. P4T, on the other hand, constructs, based on the power assignment made in T4P, a new topology by deriving a spanning tree that gives the minimal interference degree. By alternately invoking the two algorithms, the power assignment quickly converges to an operational point that maximizes the network capacity. We formally prove the convergence of MaxSR. We also show via simulation that the topology induced by MaxSR outperforms that derived from existing topology control algorithms by 50%-110% in terms of maximizing the network capacity.

I. INTRODUCTION

Topology control and management — how to determine the transmit power of each node so as to maintain network connectivity, mitigate interference, improve spatial reuse, while consuming the minimum possible power — is one of the most important issues in wireless multi-hop networks [1]. Instead of transmitting using the maximum possible power, wireless nodes collaboratively determine their transmit power and define the topology by the neighbor relation under certain criteria.

A common notion of neighbors adopted in most topology control algorithms [2], [3], [4], [5], [6], perhaps except those in [7], [8], is that two nodes are considered neighbors and a wireless link exists between them in the corresponding communication graph, if their distance is within the transmission range (as determined by the transmit power, the path loss model, and the receiver sensitivity). Algorithms that adopt this notion are collectively called graph-model-based topology control. Under this notion, topology control aims to keep the node degree in the communication graph low, subject to the network connectivity requirement. This is based on the common assertion that a low node degree usually implies low interference.

We claim that this assertion no longer holds under the physical Signal-to-Interference-Noise-Ratio (SINR) model. This is because under the physical model, whether the interference — the sum of all the signals of concurrent, competing transmissions received at the receiver — affects the transmission activity of interest depends on the SINR at the receiver, which in turn depends on the transmit power of all the transmitters and their relative positions to the receiver of interest. The node degree under the graph model, however, does not adequately capture interference. In particular, a transmission of interest may fail because other concurrent transmissions cause the SINR at the receiver to fall below the minimal SINR required for the receiver to decode the symbols correctly. This could occur even if competing transmitters are outside the transmission range of the receiver.

There are two undesirable consequences as a result of the inadequacy of graph-model-based topology control under the physical model. First, because the node degree does not capture interference adequately, the interference in the resulting topology may be high, rendering low network capacity. Second, a wireless link that exists in the communication graph may not in practice exist under the physical model, because of high interference (and consequently low SINR). As a result, the network connectivity may not even be sustained.

In this paper, first we formally argue that a node with a small node degree in the communication graph may suffer from high interference. Then, we define the interference graph that faithfully captures interference under the physical model. An interesting question is whether or not there exists a power assignment that enables the communication graph of the topology to represent its interference graph as well. We formally prove that such a power assignment exists only if the topology satisfies a certain criterion. Unfortunately, most of the topologies generated by existing graph-model-based topology control do not satisfy this criterion.

In order to mitigate interference, improve network capacity,
while maintaining network connectivity, we propose a centralized approach, called Spatial Reuse Maximizer (MaxSR), that consists of two component algorithms: T4P and P4T. Conceptually, given the topology induced by certain topology control algorithm, each node may, instead of using the minimal possible power to reach its farthest neighbor (as defined in the communication graph), increase its transmit power in order to increase the SINR at the receiver and better tolerate interference. On the other hand, if every node transmits with high power, it contributes more to the interference as perceived by other nodes. MaxSR seeks to strike a balance between increasing the SINR and controlling the interference as perceived by others to an acceptable level. Specifically, T4P optimizes assignment of the transmit power given a fixed topology, where by optimality we mean that the transmit power is so assigned that it minimizes the average interference degree (defined as the number of nodes that will interfere with transmission on a link), and (ii) P4T constructs, based on the power assignment made in T4P, a new topology by deriving a spanning tree that gives the minimal interference degree. By alternately invoking the two algorithms, the power assignment quickly converges to an operational point that maximizes network capacity. We formally prove the convergence of MaxSR, and show via simulation that the topology induced by MaxSR outperforms that derived from existing topology control algorithms by 50-110% in terms of maximizing network capacity.

The remainder of the paper is organized as follows. We first introduce in Section II the notation and the assumptions made throughout this paper. Then we formally argue that a small node degree does not necessarily imply low interference in Section III. Following that, we investigate in Section IV the issue of whether or not a feasible power assignment exists that enables the communication graph to represent the interference graph as well. After obtaining a negative answer, we devise in Section V a new topology control algorithm, called MaxSR, that alternatively invokes T4P and P4T until the power assignment converges to an optimal operational point. We also formally prove its convergence there. We present in Section VI simulation results. Finally, we provide an overview of related work in Section VII, and conclude the paper in Section VIII with a list of future research agendas.

II. PHYSICAL INTERFERENCE MODEL

In this section, we first give the notation used and the assumptions made throughout the paper. Then we explicitly define interference under the physical model.

A. Notation and Assumptions

We envision a wireless network as a set of nodes \( V \) located in the Euclidean plane. All nodes are stationary or have low mobility. Let \((X, Y)\) denote the Euclidean coordinates, \( v \in V \) the shorthand of \((v(x, y))\), \( x \in X \) and \( y \in Y \), and \( d_{ij} = d(v_i, v_j) \) the Euclidean distance between two nodes \( v_i \) and \( v_j \). Every node \( v_i \) is configured with a transmit power \( p_t(i) \) and \( P_t \) denotes the transmit power assignment \( \{p_t(1), p_t(2), ..., p_t(n)\} \), where \( n = |V| \).

The large-scale path loss model is used to describe how signals attenuate along the transmission path. Let \( g_{ij} \) be the channel gain from node \( v_i \) to node \( v_j \) (which is usually assumed to be a constant independent of the distance), then the received power can be expressed as

\[
p_r(i, j) = \frac{g_{ij} \cdot p_t(i)}{d_{ij}^\alpha},
\]

where \( \alpha \) is the path loss exponent. The value of \( \alpha \) typically ranges between 2 and 4, depending on which propagation model is used (e.g. \( \alpha = 2 \) for the free space model and \( \alpha = 4 \) for the two-ray ground model).

Whether a transmission succeeds or not is determined by two factors: namely the receive sensitivity and the signal to interference and noise ratio (SINR). Specifically, let \( RX_{\text{min}} \) be the threshold for the receiver to decode the received signal correctly, and \( \beta \) the SINR threshold. A signal can be successfully received and decoded only if the following two constraints are satisfied:

\[
p_t(i, j) = \frac{g_{ij} \cdot p_t(i)}{d_{ij}^\alpha} \geq RX_{\text{min}}, \quad (1)
\]

and

\[
SINR_{i,j} = \frac{g_{ij} \cdot p_t(i) \cdot d_{ij}^{-\alpha}}{N + I_j} \geq \beta, \quad (2)
\]

where \( N \) denotes the noise power, and \( I_j \) the interference perceived at receiver \( v_j \) and contributed by other concurrent transmissions. We will elaborate on \( I_j \) in Section II-B. Eq. (1) also defines the minimal power required to reach a receiver at a distance of \( d_{i,j} \) away. In this paper, we assume that all nodes are homogeneous, i.e., they have the same maximum power level \( P_{\text{max}} \), SINR threshold \( \beta \), and receiver sensitivity \( RX_{\text{min}} \).

**Definition 1.** A link \((i, j)\) is said to exist (i.e., node \( v_i \) can send packets to node \( v_j \) that is \( d_{i,j} \) away, without consideration of interference) if and only if

\[
p_t(i) \geq \frac{d_{i,j}^\alpha RX_{\text{min}}}{g_{ij}}.
\]

We also define an edge as a bi-directional link. That is, an edge \( e_{ij} \) exists if and only if \( p_t(i) \geq d_{i,j}^\alpha RX_{\text{min}}/g_{ij} \) and \( p_t(j) \geq d_{i,j}^\alpha RX_{\text{min}}/g_{ji} \).

Given all the definitions, the communication graph of a network is represented by a graph \( G = (V, E) \), where \( E \) is a set of undirected edges. Note that following the definition of an edge given in **Definition 1**, \( E \) is actually determined by the power assignment \( P_t \). In other words, given a power assignment \( P_t \), \( E \) is induced according to **Definition 1**. This is the graphic model used in conventional topology control. Note that the same model is also used in [9] [4] and [2].

B. Interference Model

As mentioned in Section I, mitigating interference is one of the major objectives of topology control. However, most existing topology control algorithms characterize interference with the node degree, and argue that a low node degree implies
low interference. While this is an appropriate assumption under the graphic model, this may not be valid under the physical model. Before delving into the analysis, we first define interference under the physical model.

Recall that in Section II-A, the constraint in Eq. (1) is used to define the existence of a communication link. We now use Eq. (2) to define the interference in terms of the interference degree.

**Definition 2. Interfering node:** A node $v_k \in V$ is said to be an interfering node for link $(v_i, v_j)$ if

$$\frac{p_t(i)d_{i,j}^{-\alpha}}{N + p_t(k)d_{k,j}^{-\alpha}} < \beta.$$ (3)

The physical meaning of the above definition is that if node $v_k$ transmits with power $p_t(k)$, then the transmission on link $(v_i, v_j)$ can not proceed simultaneously, i.e., the receiver $v_j$ is unable to decode the received signal due to the violation of the SINR constraint. The transmission activity which node $v_k$ is engaged will either be blocked or collide with the transmission activity on $(v_i, v_j)$.

**Definition 3. The interference degree of a link $(v_i, v_j)$** is defined as the number of interfering nodes for $(v_i, v_j)$. Let $\hat{V}_I(v_i, v_j)$ denote the set of $v \in V$ containing all interfering nodes of $(v_i, v_j)$, then the interference degree $D_I(v_i, v_j) \equiv |\hat{V}_I(v_i, v_j)|$.

A link with a high interference degree implies multiple nodes can interfere with its transmission activity, causing channel competition and/or collision. This is undesirable because both channel competition and collision degrade the network capacity (i.e., the number of bytes that can be simultaneously transported by the network). Indeed it is the interfering nodes (rather than the communication neighbors) that substantially affect the throughput capacity under the physical model. Hence, the interference degree is a better index than the node degree in quantifying the interference. In Section III, we will show that the interference degree does not necessarily relate to the node degree.

Given the definition of the interference degree, we are in a position to define the link interference graph which is the counterpart of the communication graph under the physical model.

**Definition 4. A link interference graph** represents the interference of a link $(v_i, v_j)$ as $G_I(V_I(v_i, v_j), E_I(v_i, v_j))$, where $V_I(v_i, v_j) = \hat{V}_I(v_i, v_j) \cup v_i \cup v_j$ and $E_I(link_{i,j})$ is the set of edges such that $(w, v_j) \in E_I(v_i, v_j)$, $w \in \hat{V}_I(v_i, v_j) \setminus \{v_j\}$.

### III. INTERFERENCE UNDER THE PHYSICAL MODEL

In this section we show that a small node degree does not directly relate to low interference under the physical model. Hence, the topology rendered by conventional topology control algorithms may not be capacity-efficient. Moreover, we show that the interference can be reduced by adequate power adjustment.

As mentioned in Section II-A, the topology is a graph induced by the transmit power assignment. Most existing topology control algorithms produce topologies by simply assigning the minimum possible power so as to ensure edges exist for network connectivity. Figure 1 gives an example that shows that this type of power assignment does not serve the purpose of mitigating interference under the physical model. Consider a link $(i, j)$ in Figure 1(a) and compare its interference degree against node $j$’s degree. The node degree of $j$ is 2. Let $\beta = 10$, $\alpha = 4$ and $N = 0$, and each node be configured with the minimal power so that it can communicate with its farthest neighbor (i.e., Eq. (1) holds). Under this configuration, the transmission activities of all the other nodes ($A, B, C, D$ or $E$) transmitting lead to $\text{SINR}_{i,j} = 1/0.6^4 = 7.7 < 10$. That is, by Definition 2. all the other nodes are the interfering nodes to link $(i, j)$, rendering the link interference graph of link $(i, j)$ in Figure 1(b). Although the node degree of $j$ is only two, link $(i, j)$ has six interfering nodes, i.e., the transmission activity on link $(i, j)$ may have to compete for channel access with 5 other potential transmissions. As a result, the attainable link capacity is much lower than it is expected to be. Such high interference, induced by graph-model-based topology control (and its associated power assignment), is obviously undesirable.

The above example also demonstrates that the interference degree does not necessarily relate to the node degree. As a matter of fact, the interference degree is affected by several parameters such as $\beta, N, \alpha$ and $p_t$. Among them, $N$ and $\alpha$ are environmentally determined and not controllable. $\beta$ is a controllable parameter, and in the interest of Shannon’s capacity, should be set to a reasonable large value. In this paper we thus focus on adjusting the transmit power $p_t$.

Now we show, by using the same example, that adjusting the transmit power (with the physical SINR model in mind) can indeed mitigate the interference. If the transmit power of node $i$ is raised to 1.5 times of that in Figure 1. Even if any other node transmits concurrently with node i, $\text{SINR}_{i,j}$ now increases to $1.5/0.6^4 = 11.5$. This implies, instead of using the minimum power to maintain network connectivity, an adequate power level can substantially reduce the effect of concurrently transmitting nodes and thus improve the link capacity. Note also that a similar observation is also made by Moscibroda et al. in [10]. Note that the above example considers only peer interference. If the cumulative interference (i.e., interference contributed by multiple, concurrent transmissions) is considered, the interference in the topology induced by graph-model-based topology control will become even more severe.

The inadequacy of graph-model-based topology control is rooted at the fact that the underlying communication topology it induces does not capture the interference appropriately under the physical model. An interesting question is then whether or not there exists a power assignment that enables the communication graph to represent the corresponding interference graph as well. We will address this question in Section IV.

### IV. POWER CONTROL IN KNOWN TOPOLOGIES

In this section, we seek the answer to the following question: given a communication topology, is it possible to find a
power assignment such that the communication graph of the topology is identical to the physical-model-based interference graph? The rationale for enabling the communication graph to represent the interference graph is because the topology rendered by some of topology control algorithms exhibits several desirable properties such as bi-connectivity [9] and low node degree [4], [2]. If we can find a power assignment to enable the communication graph to represent the interference graph, we can invoke the new power assignment procedure after the topology is generated. All the desirable properties are preserved, and yet the adverse effects caused by interference are mitigated. We first formulate the problem as an optimization problem, and then investigate the feasibility of the linear program.

A. Problem Statement

We first define what we mean by the communication graph of a topology representing its interference graph.

Definition 5: Under the physical model, the communication graph of a topology $G(V,E)$ is said to represent its interference graph, if and only if for every edge $\alpha \in E$, both $G_I(v_i, v_j)$ and $G_I(v_j, v_i)$ are the subgraphs of $G$.

Let $G'(V,E')$ be the complement of $G$. By Definition 5, the power assignment $P_l = \{p_l(1), p_l(2), ..., p_l(n)\}$ must satisfy the following constraints: for each pair of neighbors $v_i$ and $v_j$ in $G$,

- An edge $e_{i,j} \in G$ exists.
- Any edge $e'_{k,j} \in G'$ does not exist in $G_I(v_i, v_j)$.

The first constraint implies that the power assignment $p_l(i)$ and $p_l(j)$ guarantees the communication capability between $v_i$ and $v_j$ if $e_{i,j} \in G$, i.e., $p_l(i) \geq d_{i,j}^{\alpha}RX_{\min}/g_{i,j}$ and $p_l(j) \geq d_{i,j}^{\alpha}RX_{\min}/g_{j,i}$. Without loss of generality, we assume that the channel gain is $g_{i,j} = 1 \forall i, j$. The first constraint can then be expressed as

$$p_l(i) \geq d_{i,j}^{\alpha}RX_{\min}, \quad p_l(j) \geq d_{i,j}^{\alpha}RX_{\min}. \quad (4)$$

The second constraint implies that, if edge $e_{k,j}$ does not exist in $G$, the transmit power $p_l(k)$ of node $v_k$ should not be large enough to enable $v_k$ to become an interfering node of link $(v_i, v_j)$ (with node $v_i$ having the transmit power $p_l(i)$), i.e.,

$$\frac{p_l(i)d_{i,j}^{\alpha}}{N + p_l(k)d_{k,j}^{\alpha}} \geq \beta. \quad (5)$$

The above inequality implies that from the perspective of the transmission activity $v_i \rightarrow v_j$, $v_k$’s transmission can simultaneously take place without impairing $v_i$’s transmission. Thus edge $e_{k,j}$ does not exist in $G_I(v_i, v_j)$. Eq. (5) can be re-written as

$$\beta d_{i,j}^{\alpha}p_l(k) - d_{k,j}^{\alpha}p_l(i) \leq -\beta N d_{i,j}^{\alpha}d_{k,j}^{\alpha}. \quad (6)$$

With the two sets of constraints, we can formulate the problem as a linear programming with respect to $p_l(i), i = 1, ..., n$:

$$\text{minimize} \sum_{i} p_l(i)$$

subject to

$$p_l(i) \leq p_{\text{max}}$$
$$p_l(i) \geq \frac{d_{i,j}^{\alpha}RX_{\min}}{g_{i,j}}, \forall e_{i,j} \in G \quad (7)$$
$$\beta d_{i,j}^{\alpha}p_l(k) - d_{k,j}^{\alpha}p_l(i) \leq -\beta N d_{i,j}^{\alpha}d_{k,j}^{\alpha} \quad \text{if edge} e_{i,j} \in G \text{ and edge} e'_{k,j} \in G'$$

If the above linear program has a solution, it gives a feasible power assignment that enables a given communication graph to represent the interference graph.

B. Feasibility of the Problem

To study the feasibility of the linear program formulated, we use the communication graph induced by a representative topology control algorithm – local minimal spanning tree (LMST) [4] and its extensions [6] and [5]. LMST is chosen because as reported in [4], the node degree in its resulting topology is proved to be bounded by six. Moreover, as shown in the simulation study in [4], the average node degree in the resulting topology is comparatively lower than several other algorithms.

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Fig. 1. A low-node-degree topology does not necessarily imply low interference.
A total of 20 topologies are generated by exercising LMST in 20 random networks. Each network has 20 nodes which are uniformly placed in a rectangle area of 400 × 400 m². We first assign to each node the minimal possible power so that Eq. (1) holds for every link in the resulting topology. Based on this assignment and Definition 3, we can compute the interference degree for each link with respect to different values of β. Figure 2 shows the average interference degrees v.s. the average node degree. As anticipated, the minimal power assignment cannot ensure that the interference degree remains small in the interference graph under the physical model (Section III). The gap between the node degree and interference degree is surprisingly large. Moreover, the two average degrees are not linearly related to each other.

Now we investigate whether or not there exists a feasible power assignment to the the linear program given in Section IV-A. By solving the linear program on each topology induced by LMST, we found that no feasible solution exists for most of the cases, suggesting that the domain of p_t defined by the constraints is likely to be infeasible. (Solutions exist for some of the topologies when the number of nodes is no more than 6.) Moreover, most of the infeasibility is caused by the violation of Eq. (6).

To further understand under what condition Eq. (6) is violated, we consider a simple scenario shown in Figure 3. The network has a total of four nodes: 1, 2, 3 and 4. The solid lines mark the links present in the topology (e.g., link (1, 2) and link (3, 4)), while the dotted lines indicate the links not present in the topology (e.g., link (1, 4) and link (3, 2)). Let the distance between nodes 1 and 2, between nodes 3 and 4, between 1 and 4, and between 3 and 4 be respectively denoted as \(a_1, a_2, b_1\) and \(b_2\). Now we consider link (1, 2) first. If node 3 is not an interfering node to this link, then by Eq. (6), we have

\[
\beta a_1^2 p_t(3) \leq b_2^2 p_t(1). \tag{8}
\]

Similarly, by considering link (3, 4), we have

\[
\beta a_2^2 p_t(1) \leq b_1^2 p_t(3). \tag{9}
\]

Eqs. (8) and (9) hold at the same time if and only if the following inequality holds

\[
\frac{\text{SINR}_{\text{min}}^a a_1^2 a_2^2}{b_1^2 b_2^2} \leq 1. \tag{10}
\]

Otherwise, the power assignments \(p_t(1)\) and \(p_t(3)\) contradict with each other. Note that this particular topology can be a subgraph of a larger topology. Hence any power assignment for such subgraph should satisfy the constraint given by Eq. (10); otherwise the power assignment for the whole topology will be infeasible under the physical model. Now we generalize this feasibility constraint.

Definition 5: An alternating cycle \(C_a\) in a topology \(G = (V,C)\) is a cycle that alternates between edges in \(G\) and edges in \(G'\).

For example, \(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1\) is an alternating cycle in Figure 3. Let the length of an edge in \(G\) be denoted as \(a_i\) and that in the complement topology \(G'\) be denoted as \(b_i\). The feasibility constraint can be stated as follows.

Theorem 1: Any power assignment for a topology is infeasible under the physical model if there exists an alternating cycle in \(G\) such that

\[
\text{SINR}_{\text{min}}^a \prod_{i \in C_a} a_i > \prod_{j \in C_a} b_j.
\]

Unfortunately, none of the existing topology control algorithms can ensure that the resulting topology satisfies this constraint. In our experiments, the probability that a power assignment for the resulting topology is feasible diminishes with the increase in the number of nodes (when \(n > 6\), the probability is almost zero). This suggests that it is not likely to find power assignments to a topology induced by graph-model-based topology control to represent the corresponding interference graph. Therefore, as far as mitigating interference (and hence improving network capacity) is concerned, most existing topology control algorithms do not perform well under the physical model. In the next section we will propose a novel algorithm that combine topology control and power control to mitigate interference and improve network capacity.

V. TOPOLOGY CONTROL TO MAXIMIZE SPATIAL REUSE

In this section, we propose a novel algorithm to maximize spatial reuse and improve network capacity. The approach is composed of two component algorithms: (i) T4P that
computes a power assignment that maximizes spatial reuse with a fixed topology, and (ii) P4T that generates a topology that maximizes spatial reuse with a fixed power assignment. By alternately invoking the two component algorithms, both the topology and the power assignment converge to a point that globally maximizes the network capacity.

A. Spatial Reuse Metric

Conceptually, spatial reuse is referred to the capability of a network to accommodate concurrent transmissions. Although a number of studies have been carried out on spatial reuse, there have not been explicit metrics defined to characterize the level spatial reuse. Most topology control algorithms use interference as an implicit metric, based on the intuition that low interference implies high spatial reuse. Although the intuition is correct, we show in Section IV that graph-model-based topology control inadequately captures interference under the physical model. Indeed, the interference degree, rather than the node degree, affects the link capacity. From a link’s point of view, if there are less interfering nodes in its vicinity, it will have more chances to access the channel. From the network’s point of view, if every link has a small number of interfering nodes, then the network will be able to accommodate more concurrent transmissions. Based on the above observation, we use the average interference degree as the metric for spatial reuse. It is obtained by taking all interference degree over all nodes in the network.

B. Topology to Power assignment: T4P

Under the physical model, whether some other concurrent transmission interferes an ongoing transmission of interest depends on several factors. If the transmit power is high, the ongoing transmission may tolerate interference better because of a higher SINR. On the other hand, if every node transmits with high power, the interference is likely high, depending on the relative positions of competing transmitters to the receiver of interest. In Section II, we have defined an interfering node in Eq. (3). Let the left hand side of Eq. (3) be defined as \( \beta_k(i,j) \). Then we define an indicator function to denote whether a node \( k \) is an interfering node to link \( (i,j) \)

\[
I(\beta_k(i,j)) = \begin{cases} 
1, & \beta_k(i,j) < \beta \\
0, & \beta_k(i,j) \geq \beta 
\end{cases}
\]

Locally minimizing the interference degree may cause high interference to others. Hence all the nodes within the interference range must cooperate to achieve some level of global optimality. As such, we formulate the T4P problem as an optimization problem:

\[
\begin{align*}
\text{minimize} & : \sum_{\text{link}(i,j) \in \mathcal{T}} \sum_{k \neq i,j} I(\beta_k(i,j)) \\
\text{subject to} & : P_{\text{min}} \leq P_t \leq P_{\text{max}}.
\end{align*}
\]

The above problem is an integer program because of the existence of indicator functions. Fortunately, as indicated in [11], the hard SINR requirement can be “softened” by the sigmoid function. The sigmoid function is a continuous function expressed as

\[
sig(x) = \frac{1}{1 + e^{-a(x-b)}}. \tag{12}
\]

When \( x \) is greater than the threshold \( b \), \( \sig(x) \) will quickly rise up to 1, and when \( x \) is less than the threshold \( b \), \( \sig(x) \) will quickly drop down to 0. The parameter \( a \) determines how quickly the sigmoid function changes near the threshold. Figure 4 gives two example sigmoid functions. We approximate the integer program by replacing the indicator function with the sigmoid function:

\[
\begin{align*}
\text{minimize} & : \sum_{\text{link}(i,j) \in \mathcal{T}} \sum_{k \neq i,j} \sig(\beta_k(i,j)) \\
\text{subject to} & : P_{\text{min}} \leq P_t \leq P_{\text{max}}. \tag{13}
\end{align*}
\]

where we set the parameter \( b = \beta \). The problem can then be solved by using a sequential quadratic programming (SQP) method [12], [13].

In summary, T4P finds an optimal power assignment given a fixed topology as follows.

**Algorithm 1** Topology to Power: T4P

**Require:** Topology(\( V, E \))

Solve the optimization problem (13) with the SQP method

**Ensure:** Power Assignment \( P_t \)

C. Power assignment to Topology: P4T

The above algorithm T4P determines an optimal power assignment with a given topology. However, the input topology may not be optimal in terms of maximizing network capacity. If different topologies (induced by different topology control algorithms for the same network) are used as input to T4P, different power assignments result. It is obviously undesirable to test out all possible topologies for optimality.
To address this problem, we devise another component algorithm P4T, which generates an optimal connected topology, given a fixed power assignment. The algorithm is similar to the minimum spanning tree algorithm, except that we attempt to find the spanning tree that gives the minimal interference degree. The pseudo code of P4T is given below. Specifically,

**Algorithm 2 Power to Topology: P4T**

**Require:** Power assignment \{p_1(1), p_2(2), ..., p_n(n)\}

for all node pairs \(u, w\) such that distance\((u, w)\) \(\leq\) transmission range do

compute its interference degree by Eq. (3)

end for

sort edges in the non-decreasing order of interference degree, and let \(\tilde{e}_1, \tilde{e}_2, ...\) be the resulting sequence of edges initialize \(n\) clusters, one per node, \(E = \emptyset\) and \(i = 1\)

while the number of cluster \(> 1\) do

for \(\tilde{e}_i(u, w)\) if cluster\((u) \neq\) cluster\((w)\) then

merge cluster\((u)\) and cluster\((w)\)

\(E = E \bigcup \{\tilde{e}_i\}\)

end if

\(i = i + 1\)

end while

**Ensure:** Topology \(T(V, E)\)

Given a power assignment, we compute (by Eq. (3)) the interference degree for every pair of nodes whose distance is less than the maximum transmission range (i.e., the \(d_{i,j}\) value that makes the equality in Eq. (1) hold). The interference degree calculated is considered as the weight of the edge \(\text{edge}_{i,j}\). Initially, each node forms a one-node cluster. Edges are selected in the non-decreasing order of their weights. If the node pair of the selected edge is in different clusters, then the two clusters are merged. The above step is repeated until there is one cluster. Note that P4T not only gives a topology but also implicitly gives \(P_{\text{min}}\) that ensures network connectivity. It can be used as the lower bound for the optimization problem in T4P. In Section V-D, we will prove that the topology induced by P4T is optimal in terms of minimizing the interference degree.

**D. Spatial Reuse Maximizer**

So far we have devised two algorithms: (i) T4P gives a power assignment such that the interference degree given a fixed topology is minimized, and (ii) P4T derives, given a fixed power assignment, a spanning tree that gives the minimal interference degree. To optimize both \(P_t\) and \(T\), we propose an MaxSR. It works by alternatively invoking T4P and P4T until the power assignment converges to a point. Formally we present MaxSR below. Now we prove MaxSR does converge with the following lemma and theorem.

**Lemma 1:** Algorithm P4T gives an connected topology that minimizes the interference degree with a fixed power assignment.

The proof of lemma 1 is similar to Theorem III in [9], which proves that a minimum cost spanning tree algorithm gives an optimum connected graph that minimizes the transmit power. The only difference is that P4T intends to find a spanning tree that gives the minimal interference degree. Hence we can prove Lemma 1 following the same line of argument in [9] except that we replace the edge weight of distance by the edge weight of interference degree.

**Theorem 2:** MaxSR converges to an optimal point.

**Proof:** Let \(D(P_t^{(n)}, T^{(n)})\) be the sum of interference degree after the \(n\)-th iteration. Because T4P intends to minimize the sum of interference degree in a fixed topology, after \((n+1)\)-th running T4P, we must have \(D(P_t^{(n+1)}, T^{(n)}) \leq D(P_t^{(n)}, T^{(n)})\).

Similarly, by Lemma 1, we have

\[
D(P_t^{(n+1)}, T^{(n+1)}) \leq D(P_t^{(n+1)}, T^{(n)}).
\]

Consequently, \(D(P_t^{(n)}, T^{(n)})\) is a monotonic non-increasing function in \(n\). Since \(P_t\) has a lower bound, \(D(P_t^{(n)}, T^{(n)})\) should also be bounded in a connected graph. Thus \(D(P_t^{(n)}, T^{(n)})\) converges, and we conclude that algorithm MaxSR converges.

According to our experiments, Figure 5 illustrates the convergence speed of MaxSR versus the network size, where \(\epsilon = 0.02\). The observation is that the number of iterations is independent of the network size and MaxSR normally converges within 10 iterations. But note that the running time of T4P and P4T should depend on the number of nodes.

**VI. SIMULATION STUDY**

In this section, we carry out a simulation study to evaluate the performance of MaxSR and compare it against three schemes: MaxPow (i.e., all nodes transmit with their maximum transmit power), LMST [4] and CBTC(\(\pi/6\)) [2].

**Metrics That Are of Interest:** In the simulation study, we are primarily interested in the following metrics:

- **Interference Degree:** Given a power assignment, the interference degree can be computed for each link.

**Algorithm 3 SpatialReuseMaximizer**

**Require:** Node set \(V\) and their coordinates \(\{X, Y\}\)

let \(\epsilon\) be a small value

let \(D(T, P_t)\) be the sum of interference degree with given \(T\) and \(P_t\)

initialize \(\Delta = 1\), \(T = T(P_{\text{max}})\) and \(P_t = T4P(T)\)

while \(\Delta > \epsilon\) do

\(D_{\text{old}} = D(T, P_t)\)

\(T_{\text{old}}\) = T4P(T)

\(P_t = T4P(T)\)

\(\Delta = ||D_{\text{old}} - D(T, P_t)||\)

end while

**Ensure:** Power assignment \(P_t\)
• **Network Connectivity**: Connectivity is perhaps the most important criterion for topology control. In our study, we quantify the level of connectivity under the physical model by the number of disconnected flows during the simulation time.

• **Throughput Capacity**: As discussed in Section V-A, interference degree is a good metric for characterizing spatial reuse and hence network capacity improvement. We evaluate the performance of various algorithms with respect to network capacity by keeping track of the saturated throughput in random networks.

  a) **Computation Result**: First we give the computation result of MaxSR against three schemes: MaxPow, LMST and CBTC, with respect to the average interference degree. A total of 10 networks are generated randomly, and for each network a total of 40 nodes are uniformly placed in a rectangle area of 500×500 m². For each network, MaxSR derives both the topology and the power assignment; MaxPow assigns the maximum transmit power to each node and the topology is induced by the power; while LMST and CBTC derive the topology and induce the power assignment by assigning the minimum power so as to maintain the derived topology.

  Based on the topology and the power assignment derived/induced, we then compute the interference degree for each link and take the average over all links. Figure 6 gives the average interference degree under the various algorithms. Not surprisingly MaxPow has the largest average interference degree, confirming the intuition that large power gives rise to high interference. Based on the minimum spanning tree algorithm, LMST gives perhaps the minimum interference among all conventional topology control algorithms. MaxSR, on the other hand, gives the minimum average interference degree among all the algorithms.

  b) **Simulation Setup**: We leverage J-sim [14] to carry out the simulation study for the following reasons: (i) ns-2 does not take into account of the effect of accumulative interference; and (ii) ns-2 computes the interference range, assuming that all nodes use a common transmit power, whereas topology control algorithms assign different levels of transmit power to different nodes.

  In our simulation study, we consider IEEE 802.11-based networks. Table I shows the system parameters used in the simulation. Again a total of 10 networks are generated randomly, and for each network a total of 40 nodes are uniformly placed in a rectangle area of 500×500 m². A total of 20 source-destination pairs are specified. In order to decouple the effect of routing protocols from topology control, we consider the saturated throughput of one-hop flows, i.e., a source and its corresponding destination are so chosen that they are neighbors of each other.

<table>
<thead>
<tr>
<th>TABLE I SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>RXThreshold</td>
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<tr>
<td>Inter-arrival time</td>
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<tr>
<td>CPThreshold</td>
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<tr>
<td>PHY payload</td>
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<tr>
<td>PHY header</td>
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<td>ACK frame</td>
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<td>DATA bit rate</td>
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<td>PHY bit rate</td>
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<td>$\alpha$</td>
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<tr>
<td>Traffic pattern</td>
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<tr>
<td>Trans. protocol</td>
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<td>Routing protocol</td>
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<tr>
<td>MTR time</td>
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<td>CWmin</td>
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<td>CWmax</td>
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<td>Retry limit</td>
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<td>Max txpower</td>
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**Performance Evaluation**: Although we have decoupled the effect of routing protocols from topology control, we have to consider the effect of the carrier sense threshold in IEEE 803.11-based networks. This is because the network capacity depends also on the setting of the carrier sense threshold. On the one hand, if the carrier sense threshold is too small, spatial reuse cannot be fully exploited and the network may encounter the exposed node problem. On the other hand, if the carrier sense threshold is too large, interference becomes severe and the network may encounter hidden node problem. Thus, we will run simulation with different carrier sense thresholds and observe its effect on the network connectivity and capacity.

Figure 7 gives the simulation result of the aggregate throughput v.s. the carrier sense threshold under various algorithms. As anticipated, MaxSR achieves the highest aggregate throughput except when the carrier sense threshold is small.
VI. RELATED WORK

The issue of power control has been studied in the context of topology maintenance, where the objective is to preserve network connectivity, reduce power consumption, and mitigate MAC-level interference [2], [3], [4], [5], [6], [17]. Rodoplu et al. [3] introduced the notion of relay region and enclosure for the purpose of power control. A two-phase distributed protocol was then devised to find the minimum power topology for a static network. In the first phase, each node $i$ executes local search to find the enclosure graph. In the second phase, each node runs the distributed Bellman-Ford shortest path algorithm upon the enclosure graph, using the power consumption as the cost metric.

CBTC($\alpha$) is a two-phase algorithm in which each node finds the minimum power $p$ such that transmitting with $p$ ensures that it can reach some node in every cone of degree $\alpha$. The algorithm has been analytically shown to preserve the network connectivity if $\alpha < 5\pi/6$. It has also ensured that every link between nodes is bi-directional.

Li and Hou [4] devised a Local Minimum Spanning Tree (LMST) algorithm and its variations [5], [6] for topology control and management. In LMST, each node builds its local minimum spanning tree independently with the use of locally collected information, and only keeps on-tree nodes that are one-hop away as its neighbors in the final topology. They have proved analytically that (1) if every node exercises LMST, then the network connectivity is preserved; (2) the node degree of any node in the resulting topology is bounded by 6; and (3) the topology can be transformed into one with bi-directional links (without impairing the network connectivity) after removal of all uni-directional links.

As mentioned in Section I, topologies derived under these graph-model based topology control algorithms may not capture interference adequately under the physical SINR model. As a result, interference may be outrageously high in the topology induced by graph-model based algorithms, rendering sub-optimal network capacity.

Control of transmit power for capacity improvement:

Use of power control for the purpose of spatial reuse and capacity improvement has been treated in the COMPOw protocol [15], the PCMA protocol [16], the PCDC protocol [17], the POWMAC protocol [18], and the PRC protocol [19]. Narayanaswamy et al. [15] developed a power control protocol, called COMPOw. In COMPOW each node runs several routing daemons in parallel, one for each power level. Each routing daemon maintains its own routing table by exchanging control messages at the specified power level. By comparing the entries in different routing tables, each node can determine the smallest common power that ensures the maximal number of nodes are connected.

Monks et al. [16] propose PCMA in which the receiver advertises its interference margin that it can tolerate on an out-of-band channel and the transmitter selects its transmit power
that does not disrupt any ongoing transmissions. Muqattash and Krunz also propose PCDC and POWMAC in [17], [18] respectively. The PCDC protocol constructs the network topology by overhearing RTS and CTS packets, and the computed interference margin is announced on an out-of-band channel. The POWMAC protocol, on the other hand, uses a single channel for exchanging the interference margin information.

Kim et al. [19] studied the relationship between physical carrier sense and Shannon capacity, and showed that the achievable network capacity only depends on the ratio of the transmit power to the carrier sense threshold. They then propose a decentralized power and rate control algorithm, called PRC, to enable each node to adjust, based on its signal interference level, its transmit power and data rate. The transmit power is so determined that the transmitter can sustain a high data rate, while keeping the adverse interference effect on the other neighboring concurrent transmissions minimal.

All the efforts reported in this category focus more on devising practical power control protocols, and have not formally established optimality in the course of algorithm/protocol construction.

Joint topology control and scheduling under the physical SINR model: Moscibroda, Wattenhofer, and Zollinger [8] are the first to consider topology control under the physical model. They focus on reducing the schedule length in topology-controlled networks. They proved that if the signals are transmitted with correctly assigned transmission power levels, the number of time slots required to successfully schedule all links is proportional to the squared logarithm of the network size. They also devised a centralized algorithm for approaching the theoretical upper bound. In a similar problem setting, Brar, Blough, and Santi [20] presented a computationally efficient, centralized heuristic for computing a feasible schedule under the physical SINR model. They did not explicitly consider topology control, although whether or not communication succeeds is determined based on the SINR model. In some sense, MaxSR complements the above two efforts. Recall that MaxSR aims to improve network capacity without assuming any specific scheduling policy. Instead of attempting to reduce the schedule length, we focus on deriving a network topology, along with its power assignment, to maximize the network capacity.

VIII. CONCLUSION

In this paper, we investigate the issue of topology control under the physical SINR model, with the objective of maximizing network capacity. We show that existing graph-model-based topology control captures interference inadequately under the physical model. In order to address the problem, we introduce a new metric for spatial reuse, called the interference degree. It measures the actual interference under the physical model. To mitigate interference and improve spatial reuse, we then propose a centralized approach MaxSR that combines a power control algorithm T4P with a topology control algorithm P4T. We also show via simulation that the topology derived by MaxSR outperforms that induced from existing topology control algorithms by 50-110% in terms of maximizing the network capacity.

We have identified several avenues for future research. We will design, based on the insight shed from the study reported in this paper, a decentralized version of MaxSR that maximizes spatial reuse. We would also like to investigate how to combine MaxSR with a scheduling policy (such as that proposed in [20]) so as to maximize network capacity in both the spatial and temporal domains.

REFERENCES