Fluid Flow and Heat Transfer in Serpentine Channels at Low Reynolds Numbers

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Fluid Flow and Heat Transfer in Serpentine Channels at Low Reynolds Numbers

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Fluid flow and heat transfer in a two-dimensional serpentine channel was investigated numerically for Reynolds numbers between 175 and 725. A fifth-order finite differencing scheme was employed to carry out the computations. Three different geometries were studied. All three geometries consisted of the same wave length and the same shape of top and bottom walls, but different channel heights. In addition, two different inlet/outlet boundary conditions were investigated. It was found that the heat transfer as well as the pressure drop increase with decreasing channel height. For a given channel height, the results show that the time-averaged mean Nusselt number as well as the time-averaged mean friction factor scale linearly with the Reynolds number.

1 Introduction

In the past, a large number of methods have been devised to increase heat and mass transfer in compact heat exchangers. Those methods usually attempt to replace the boundary layer that forms on the heat transfer surface with fluid from the core, thereby increasing the temperature and concentration gradients at the wall. The primary goal is the design of a heat exchanger which yields the highest heat transfer rate with the lowest possible pressure drop. Secondary goals include ease of manufacturability and maintenance.

The particular geometry investigated in the present work is a channel with wavy walls. Most common are wavy wall channels where the top and bottom walls are either offset 0° or 180°. Here we focus on the first case with the walls in phase. This type of channel is commonly referred to as a serpentine or corrugated channel.

Wavy passages have been investigated in a number of earlier studies. The general observation made is that we can expect significant heat transfer enhancement for transitional and unsteady flow, but not for steady flow. When the flow is unsteady, interactions
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>(a)</td>
<td>amplitude-to-length ratio</td>
</tr>
<tr>
<td>(f)</td>
<td>friction factor</td>
</tr>
<tr>
<td>(H)</td>
<td>sum of convective and viscous terms</td>
</tr>
<tr>
<td>(H)</td>
<td>interwall spacing</td>
</tr>
<tr>
<td>(n)</td>
<td>number of cycles</td>
</tr>
<tr>
<td>(p)</td>
<td>pressure</td>
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<tr>
<td>(Nu)</td>
<td>Nusselt number</td>
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<tr>
<td>(Pr)</td>
<td>Prandtl number</td>
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<tr>
<td>(Re)</td>
<td>Reynolds number</td>
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<td>(Str)</td>
<td>Strouhal number</td>
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<tr>
<td>(t)</td>
<td>time</td>
</tr>
<tr>
<td>(T)</td>
<td>temperature</td>
</tr>
<tr>
<td>(u)</td>
<td>2-D cartesian velocity vector</td>
</tr>
<tr>
<td>(\bar{u})</td>
<td>intermediate velocity</td>
</tr>
<tr>
<td>(\bar{u})</td>
<td>average velocity</td>
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### Greek Symbols

<table>
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<tr>
<td>(\lambda)</td>
<td>wavelength</td>
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<tr>
<td>(\rho)</td>
<td>density</td>
</tr>
<tr>
<td>(\nu)</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>dimensionless temperature</td>
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### Superscripts

<table>
<thead>
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<th>Superscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td>time step</td>
</tr>
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### Subscripts

<table>
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<th>Subscript</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>(in)</td>
<td>inlet quantity</td>
</tr>
<tr>
<td>(m)</td>
<td>bulk mean quantity</td>
</tr>
<tr>
<td>(out)</td>
<td>outlet quantity</td>
</tr>
<tr>
<td>(wall)</td>
<td>quantity evaluated at the wall</td>
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</table>

between core flow and the boundary layer provide increased heat and mass transfer. In the present work we have numerically studied the heat transfer and pressure characteristics of the serpentine channel geometry over a range of Reynolds numbers and for three different geometries.

The next section gives an overview of research done prior to our investigation. The third section introduces the governing equations and the numerical procedure used to solve the governing equations. We also present computational details in this section. Following this, we discuss the results of the numerical computations for both steady and unsteady flow. Descriptions of the unsteady flow and temperature fields as well as quantitative data on friction factors and heat transfer rates are given.

## 2 Previous Research

Goldstein and Sparrow [5] were perhaps the first to study fluid flow and heat transfer in a corrugated channel. They investigated local and average heat and mass transfer
characteristics for laminar, transitional and low Reynolds number turbulent flow. It was shown that for laminar flow with a Reynolds number of up to 1200 the heat transfer coefficients were only moderately larger than those for a straight channel. However, a nearly 300% heat transfer enhancement was observed in the higher Reynolds number regime for Reynolds numbers between 6000 and 8000.

O’Brien and Sparrow [12] investigated forced convection heat transfer coefficients and friction factors for flow in a corrugated duct. Experiments were carried out for Reynolds numbers ranging from 1500 to 25000 based on the hydraulic diameter, and Prandtl numbers between 4 and 8. The results showed a heat transfer enhancement of a factor of about 2.5. However, the friction factor for the corrugated channel (0.57) was also found to be significantly higher than the friction factor for a straight duct.

Ektesabi et al. [4] investigated flow in a serpentine channel for Reynolds numbers ranging from 300 - 40000 based on the volumetric equivalent diameter \(D_e = 2H \lambda / L\), where \(L\) is the length along the sinusoidal wall per cycle. It was found that the periodic change in the size of the recirculation zone was accompanied by a sweep and burst cycle of large-scale vortices. In the regime of flow separation the friction factor was strongly affected by the channel geometry and less dependent on the Reynolds number.

Forced oscillatory flow in wavy channels was recently studied by Nishimura [8] for Reynolds numbers between 10 and 500. It was found that a serpentine channel yields better mass transfer and pumping power performance than a converging-diverging channel at low Strouhal numbers. At high Strouhal numbers, however, a converging-diverging channel yielded higher mass transfer rates.

Nishimura and co-workers [9] also investigated mass transfer in a serpentine channel focusing on the transitional flow regime. It was found that at low Reynolds numbers the flow is laminar and two-dimensional, whereas for higher Reynolds numbers the flow becomes turbulent and shows an unsteady three-dimensional vortical structure. In the
latter case, the flow in the troughs showed an intermittent reversed flow pattern. The unsteadiness of the flow resulted in a significant increase in mass transfer for turbulent flow.

Finally, Nishimura et al. [11] investigated the occurrence of longitudinal vortices in channels with narrow spacing. They show that interaction of the Taylor-Görtler kind vortices located at the upper and lower walls of the channel promote transition to turbulence and thereby can lead to an increase in mass transfer.

Gschwind et al. [6] observed a flow instability in the concave part of sinusoidal wavy ducts similar to the Görtler instability at concave walls or the Dean instability in tube flow. Their experimental results demonstrated the possibility of introducing longitudinal vortices in wavy duct flow by centrifugal instability. It was found that the instability exists only for a small range of Görtler numbers and depends strongly on the duct height.

Heat transfer and pressure drop for different inter-wall spacing of the corrugated walls were determined by Sparrow and Comb [13]. They observed that an increase in inter-wall spacing gives rise to a Nusselt number increase of up to 30%. However, the friction factor is also increased by a factor greater than two. The heat transfer coefficient for larger inter-wall spacing was found to be slightly lower than that for smaller inter-wall spacing, but the pressure drop was also lower.

Asako and Faghri [2] employed a finite volume method to study laminar flow in wavy channels with uniform wall temperature. Computations were carried out for Reynolds numbers between 90 and 1000. Asako and Faghri verified some of the earlier experimental studies and concluded that pressure drop and friction factor are higher than the corresponding values for a straight channel.

Asako, Nakamura and Faghri [3] numerically determined heat transfer and pressure drop of a corrugated duct with rounded corners. Reynolds numbers between 100 and 1000 and a Prandtl number of 0.7 were used for the computations. Not surprisingly,
it was observed that rounded corners resulted in a decrease of the friction factor and Nusselt number of up to 80\% when compared to a corrugated channel with sharp corners. However, the heat transfer rate of the round cornered duct was greater than that for the corresponding straight duct under identical pumping power and mean flow conditions.

Another numerical study was carried out by Amano et al. \cite{1}. Using a geometry for their simulation similar to that used by Sparrow and Comb \cite{13} and employing a full Reynolds stress model, Amano and co-workers were able to confirm earlier experimental results.

The authors are not aware of earlier numerical investigations of the serpentine channel geometry in the low Reynolds number regime.

3 Governing Equations and Numerical Procedure

For all computations presented in this paper, the flow is considered to be two-dimensional with no variations in the spanwise direction. The governing equations for conservation of mass, momentum and energy can then be written in the following form:

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

$$\frac{\partial \Theta}{\partial t} + \nabla \cdot (\mathbf{u} \Theta) = \frac{1}{Re \, Pr} \nabla^2 \Theta ,$$

where $\Theta = (T - T_{wall})/(T_{m,in} - T_{wall})$ is the non-dimensionalized temperature. Here, $T_{wall}$ is the wall temperature and $T_{m,in}$ is the mean (mass averaged) temperature at the inlet of the computational domain. The conservation equations are discretized on a curvilinear grid using a finite-volume formulation.

An explicit fractional step scheme is employed to integrate the governing equations numerically. In a fractional step procedure, the governing equations are first solved for
an intermediate velocity field $\mathbf{u}$ given by

$$\frac{\mathbf{u} - \mathbf{u}^n}{\Delta t} = \frac{3}{2} \mathbf{H}^n - \frac{1}{2} \mathbf{H}^{n-1},$$

(4)

where $\mathbf{H} = -\nabla \cdot (\mathbf{uu}) + 1/\text{Re}\nabla^2 \mathbf{u}$ and $\Delta t$ is the time step size. This step is followed by solving a Poisson equation for the pressure $p$:

$$\nabla \cdot (\nabla p^{n+1}) = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}$$

(5)

The new pressure field is then used to update the intermediate velocity field and $\mathbf{u}^{n+1}$ can be obtained from

$$\mathbf{u}^{n+1} = \mathbf{u} - \Delta t \nabla p^{n+1}$$

(6)

The pressure equation is solved using a conjugate gradient method. Spatial discretization uses a collocated scheme with the two Cartesian velocities and pressure situated at the centers of the discrete finite volumes in the computational domain. Mass conservation is satisfied by balancing the mass fluxes at the cell faces.

Computations were performed for three different geometries (Fig. 1). The base geometry consists of a spacing, $H$, of 1.3 units between the top and bottom walls. This measure was chosen to allow comparison of our numerical results with the experimental results of Nishimura [10] and the computational results of Wang and Vanka [14], who both studied a furrowed channel with a phase shift of 180° between the upper and lower walls. Amplitude-to-length ratio, $a$, and wavelength, $\lambda$, were chosen as 0.25 and 2.8, respectively, corresponding to the aforementioned papers by Nishimura and Wang. The remaining two geometries were chosen with inter-wall spacings of 1.0 and 1.6 units, respectively, while amplitude-to-length ratio and wavelength were kept constant.

On the walls, no-slip and constant temperature boundary conditions were enforced. The pressure boundary condition was imposed indirectly on the flow through the zero flux condition at the walls. The pressure directly at the wall was found from a zero derivative condition.
Two different sets of boundary conditions were imposed in the streamwise direction. The initial simulations enforced periodic inlet and outlet conditions for a computational domain that consisted of a single wave, that is

\[
\begin{align*}
  u(0, y) &= u(\lambda, y), \\
  \frac{\Theta(0, y)}{\Theta_{m, in}} &= \frac{\Theta(\lambda, y)}{\Theta_{m, out}},
\end{align*}
\]

where \( \Theta_{m, in} \) and \( \Theta_{m, out} \) are the non-dimensional mass averaged temperatures at inlet and outlet of the computational domain, respectively. These boundary conditions are shown in Figure 1. The imposition of periodicity on the temperature follows along the lines of the procedure presented by Kays and Crawford [7] for the case of a circular pipe. A constant pressure drop is imposed in the streamwise direction, i.e.

\[
p(\lambda, y) - p(0, y) = C \lambda \nu,
\]

where \( C \) is an arbitrary constant. The numerical results presented here were computed for \( C = 20 \) and \( C = 40 \).

A second set of simulations was carried out on a domain that consisted of 14 waves plus straight inlet and outlet sections. In this case, a uniform velocity profile (plug flow) was prescribed at the inlet. At the outlet, a zero velocity gradient was enforced.
In order to obtain accurate numerical results with tolerable computing times, a grid resolution of $128 \times 128$ for the periodic boundary conditions and $64 \times 64$ per wave for the developing flow was chosen for the computations presented here. The choice of these grid resolutions were based on a grid independence study by Wang and Vanka [14] who used the same software as the present authors to study the flow in a furrowed channel.

4 Results

Before we present our numerical results we define the dimensionless numbers by which the performance of the individual channel geometries was evaluated. The Reynolds number was based on the channel height, $H$, and the average velocity across the channel, $\bar{u}$, and is given by

$$Re = \frac{\bar{u}H}{\nu} ,$$

where without loss of generality the density, $\rho$, was assumed to be 1. To evaluate the rate of heat transfer, a Nusselt number was defined as follows:

$$Nu = \frac{[\partial T/\partial y]_{wall} H}{T_{wall} - T_{m,in}} ,$$

where $T_{m,in}$ is the bulk mean temperature at the inlet to the computational domain and $T_{wall}$ is the temperature of the top and bottom walls of the channel. The friction factor was defined for each individual wave as

$$f = \frac{\Delta p}{\frac{1}{2} \rho \bar{u}^2} ,$$

where $\Delta p$ is the pressure drop across a single wave. This particular definition of the friction factor allows us to compare the individual waves with the fully developed case. Finally, to describe the oscillatory behavior of the flow we introduce the Strouhal number

$$Str = \frac{nH}{\Delta t \bar{u}} ,$$

where $n$ is the number of cycles that occur over the time interval $\Delta t$. 
Table 1: Reynolds numbers for the different computations

<table>
<thead>
<tr>
<th></th>
<th>≈ 200</th>
<th>≈ 300</th>
<th>≈ 400</th>
<th>≈ 500</th>
<th>≈ 700</th>
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<tr>
<td>H = 1.0</td>
<td>–</td>
<td>340</td>
<td>–</td>
<td>475</td>
<td>–</td>
</tr>
<tr>
<td>H = 1.3</td>
<td>215</td>
<td>315</td>
<td>390</td>
<td>525</td>
<td>700</td>
</tr>
<tr>
<td>H = 1.6</td>
<td>–</td>
<td>325</td>
<td>–</td>
<td>520</td>
<td>725</td>
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</table>

4.1 Periodic Flow

For each of the three different geometries, solutions were computed for a range of Reynolds numbers. Note that the Reynolds numbers were not constant during a simulation, but varied due to the fact that a constant pressure drop was prescribed rather than a constant mass flux. Consequently, all Reynolds numbers given in this section are time averaged Reynolds numbers. Similarly, all other dimensionless numbers presented are time-averaged numbers. Nusselt numbers and friction factors were also averaged in space and the values provided are therefore global values. To compute the spatial average, Nusselt numbers and friction factors were averaged along the top wall of the channel.

Also, it should be pointed out that it proved rather difficult to match certain average Reynolds numbers exactly by just varying viscosity and pressure drop. Table 1 shows the Reynolds numbers actually reached for the individual geometries, when aiming for Reynolds numbers of 200, 300, 400, 500 and 700.

In addition to the Reynolds numbers given in Table 1, for which the flow has been observed to be unsteady, a steady simulation was carried out for the base geometry for a Reynolds number of 175. For this Reynolds number, the discretized equations were integrated until steady state was reached. The flow was assumed to have converged to steady state as the velocities at representative locations approached time invariant values (Fig. 2a).
Figure 2b shows the streamline plot corresponding to this Reynolds number. We see that a vortex is trapped in both upper and lower cavities. The vortices are blocking part of the channel, thus reducing the effective average height of the channel. The streamlines pass straight through the open space and do not follow the sinusoidal curvature of the channel walls. Compared to a straight channel the heat transfer rate is increased due to energy transfer from the core flow to the vortices. The Nusselt number for this particular Reynolds number was found to be 8.14 compared to 7.54 for a straight channel.

Next the Reynolds number was increased to a value of 215. The velocities at the monitoring locations show that at this stage the flow is unsteady (Fig. 3a). As can be seen from Figure 3a, the flow is subject to a single oscillatory mode. We also note that compared to the streamline plot for Re=175 (Fig. 2b) the vortices have grown and the center of the vortices has shifted downstream (Fig. 3b). Despite the flow now being unsteady, only a marginal increase in the Nusselt number to 8.41 was observed.

At Re=315 we find that in addition to the primary mode of oscillation secondary modes appear in the velocity time signal (Fig. 4a). The instantaneous streamline plot in Figure 4b clearly shows that the flow is asymmetric. This figure also demonstrates the underlying pattern of vortex formation and destruction. A vortex first forms at the
upstream side of the trough. It then grows until it eventually fills the whole cavity, thereby pushing other vortices that are present in the trough into the freestream. We can identify the different stages of vortex development in Figure 4b. A single vortex is filling out the lower cavity, while a vortex is forming in the upper cavity. In the very top right corner of Figure 4b the streamlines indicate a vortex being absorbed by the core flow. The occurrence of secondary modes also results in a significant increase in the rate of heat transfer. The Nusselt number computed for this flow is 13.68.

Next a calculation was carried out for a Reynolds number of 390 (Fig. 5). The comments regarding vortex formation and destruction made earlier apply here as well.
We note also that the secondary modes are stronger and the peaks of the velocity-time signal are more pronounced. The Nusselt number has increased further to a value of 16.44.

The last but one calculation for the base geometry was carried out for a Reynolds number of 525. Again, we notice an increase in "randomness" of the flow. Also, for the first time we see more than two vortices trapped in a cavity at one instant in time. One possible explanation is that the characteristic time of vortex growth, i.e. the time it takes for a vortex to grow and fill out the whole cavity, is now significantly longer than the duration of a vortex formation/destruction cycle. Hence, before a vortex has the time to grow and fill the whole cavity, a new vortex is already forming pushing the old vortex into the core flow. The computation predicts a further increase of the Nusselt number to 20.06.

Finally, Figure 7 shows the velocity time signal and streamline plot for Re=700. As before, we see vortices in various stages of their development in the upper and lower cavities. With a Nusselt number of 28.4, the rate of heat transfer in this case is almost four times as high as the rate of heat transfer in the straight channel.

Summarizing our findings for the base geometry of $H = 1.3$, we can say that the
observed pattern of vortex formation and dissipation existed for all Reynolds numbers at which the flow was unsteady. Furthermore, for the higher Reynolds numbers, vortex generation and dissipation allows for more than one vortex to be trapped in the cavities at one instant in time. We also find that the vortices grow bigger with increasing Reynolds number. Streamlines pass more or less straight through the channel, except for the point where a vortex is either formed from the core flow or absorbed by the core flow. In this case the streamlines follow the shape of the vortex. Computing the corresponding Strouhal numbers based on the mean velocity and the channel height, it is found that the

Figure 6: Base geometry at Re=525; a) velocity time signal, b) streamline plot

Figure 7: Base geometry at Re=700; a) velocity time signal, b) streamline plot
Table 2: Numerical results for base geometry

<table>
<thead>
<tr>
<th>Re</th>
<th>175</th>
<th>215</th>
<th>315</th>
<th>390</th>
<th>525</th>
<th>700</th>
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<tbody>
<tr>
<td>Str</td>
<td>N/A</td>
<td>0.295</td>
<td>0.276</td>
<td>0.261</td>
<td>0.246</td>
<td>0.218</td>
</tr>
<tr>
<td>Nu</td>
<td>8.14</td>
<td>8.41</td>
<td>13.86</td>
<td>16.44</td>
<td>20.06</td>
<td>28.40</td>
</tr>
<tr>
<td>f</td>
<td>0.448</td>
<td>0.413</td>
<td>0.411</td>
<td>0.424</td>
<td>0.460</td>
<td>0.504</td>
</tr>
</tbody>
</table>

Strouhal number decreases slightly with increasing Reynolds number. Table 2 summarizes Strouhal numbers, Nusselt numbers and friction factors for the base geometry.

The second geometry investigated in this study is identical to the base geometry, except that the spacing between the upper and lower walls is increased to 1.6 units. Solutions for Reynolds numbers of 325, 520 and 725 were computed. Figures 8a through 10a show that for all the Reynolds numbers studied the flow is subject to a single mode of oscillation.

The streamline plots 8b through 10b show the same general features as found for the base geometry. The flow is subject to the same process of vortex formation/absorption as seen before. Unlike for the base geometry, however, it can be seen that even at the higher
Reynolds numbers the flow path is significantly less obstructed by vortices ejected from the troughs. Hence, there is only little interaction between the individual vortices, which explains why no additional oscillatory modes arise with increasing Reynolds number.

Furthermore, it was found that the Strouhal numbers are essentially identical for all three Reynolds numbers with values ranging from 0.334 at Re=725 to 0.340 at Re=520. Table 3 gives Strouhal numbers, Nusselt numbers and friction factors obtained for this geometry.
Finally, a third set of computations was carried out for a spacing of 1.0 units between upper and lower walls. Unlike for the previously mentioned case of a channel height of 1.6 units, multiple modes of oscillation are present even for the smallest investigated Reynolds number of 340 (Fig. 11a). The oscillations become more violent with increasing Reynolds number (Fig. 12a).

The streamline plots (Fig. 11b and 12b show that for this particular geometry the vortices can obstruct almost the entire channel leaving only little room for the core fluid to pass through. We observe significant vortex interactions, which cause multifrequency oscillation patterns in the velocity time signals.

Table 4 summarizes the numerical results for the channel of height 1.0 units.

Figure 11: Inter-wall spacing of 1.0 units at Re=340; a) velocity time signal, b) streamline plot
Table 4: Numerical results for inter-wall spacing of 1.0 units

<table>
<thead>
<tr>
<th>Re</th>
<th>340</th>
<th>475</th>
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<tbody>
<tr>
<td>Str</td>
<td>0.778</td>
<td>1.172</td>
</tr>
<tr>
<td>Nu</td>
<td>18.92</td>
<td>28.24</td>
</tr>
<tr>
<td>$f$</td>
<td>0.891</td>
<td>0.891</td>
</tr>
</tbody>
</table>

Figure 13 a and b summarize the numerical results for the periodic inlet and outlet boundary conditions by plotting Nusselt number vs. Reynolds number and friction factor vs. Reynolds number. The plots show that the Nusselt number as well as the friction factor depend linearly on the Reynolds number. However, unlike the friction factor, which only changes moderately with the Reynolds number, we find a significant increase in heat transfer for higher Reynolds numbers. This result would suggest that for a given channel height the throughput should be sufficiently high to take advantage of the predicted heat transfer enhancement at higher Reynolds numbers.

Figure 12: Inter-wall spacing of 1.0 units at Re=475; a) velocity time signal, b) streamline plot
4.2 Developing Flow

In the previous section we presented numerical results for a serpentine channel with periodic inlet and outlet conditions. Physically, this corresponds to an infinitely long channel. For practical applications, however, it is necessary to investigate a finite domain to evaluate the transition distance required to obtain an unsteady flow. It is also important to know how many waves are necessary before the Nusselt number for a given wave approaches that of the periodic regime.

The computational domain used to study the developing flow case consisted of 14 waves plus straight inlet and outlet sections of length $\lambda$. The channel height was chosen to be 1.3 units corresponding to the base geometry investigated in Section 4.1.

Figure 14a shows the velocity-time signal for $Re=300$. The velocity is monitored in waves 12, 13 and 14 at the same relative location within each wave. It can be seen that while the flow is steady in wave 12, the flow is unsteady in waves 13 and 14. Considering the single oscillatory frequency, the flow field at this Reynolds number is similar to a Reynolds number of 215 for the case with periodic inlet/outlet boundary conditions. This similarity also shows in the corresponding streamline plot (Fig. 15) where single vortices occupy the troughs.
Figure 14: Velocity-time signal of developing flow at a) Re=300, b) Re=400, c) Re=500 and d) Re=700, respectively.

The velocity-time signal for Re=400 is shown in Figure 14b which indicates that the point of transition has moved upstream by approximately four waves. Also, in the unsteady regime the flow is still subject to a single oscillatory mode. The difference between steady and unsteady flow can also be seen in the streamline plot (Fig. 16). While the cavities of the leftmost wave, wave 8, contain single vortices indicating stationary flow, we find that further downstream in waves 9 and 10 multiple vortices appear in a single cavity. This indicates that a vortex formation/dissipation cycle similar to that observed in the periodic case exists also for the developing case.

This conclusion is further confirmed in Figures 17 and 18, which depict the streamline patterns at Re=500 and Re=700. Both plots show similar flow fields with single vortices occupying the leftmost wave, indicating steady flow, and multiple vortices in the
unsteady regime. The point of transition, however continues to move further upstream with increasing Reynolds number. The flow transitions in the eighth wave for Re=500 and in the fifth wave for Re=700. The velocity-time signals (Fig. 14c-d) show that even at Re=500 the flow is still subject to a dominant oscillatory mode with a low frequency modulation superimposed. At Re=700 further oscillatory modes appear.

Finally, we compare the rates of heat transfer for the individual Reynolds numbers. In order to allow a meaningful comparison between developing and periodic flows and to have a Nusselt number that is independent of the total length of the channel, a local Nusselt number was computed for each wave of the channel. This local Nusselt number is then plotted against the wave number (Figure 19a). The overall Nusselt number can be found by simply averaging the local Nusselt numbers.

Figure 19a affirms the conclusions drawn earlier regarding the point of transition of the flow. It is clearly seen that the unsteadiness of the flow leads to a significant heat transfer
enhancement. For the cases where transition occurs early in the channel, namely Re=500 and Re=700, the Nusselt number essentially reaches the value for a periodic channel.

5 Conclusions

The present work demonstrates the influence of different inter-wall spacing on heat transfer and friction factor for a serpentine channel. Simulations were performed for two different sets of inlet and outlet boundary conditions. For the case of periodic inlet/outlet conditions, solutions were computed for three different channel heights. For the developing flow only the base geometry was considered. Computations were carried out for several different Reynolds numbers for each individual channel geometry.

In the periodic case, it was observed that the heat transfer rate increases significantly with the Reynolds number, whereas the friction factor changes only moderately. In gen-
For the developing flow the rate of heat transfer becomes a function of the number of waves in the channel. It is seen that the point of transition shifts more and more upstream as the Reynolds number is increased. At higher Reynolds numbers the flow becomes unsteady quite early and the rate of heat transfer reaches the periodic channel value in the last waves.

Future studies should address the effects of variation of channel height in the developing region as well as the effects of spanwise variation on heat transfer and friction factor.

References


