A STUDY OF THE ROUGHNESS
OF CONCEPTUAL RIVER SYSTEMS OR WATERSHEDS

A Contribution to the International Hydrological Decade

by

T. E. HARBAUGH and VEN TE CHOW

Sponsored by
NATIONAL SCIENCE FOUNDATION
RESEARCH GRANT GP-1464

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
SEPTEMBER 1967
A STUDY OF THE ROUGHNESS OF CONCEPTUAL RIVER SYSTEMS OR WATERSHEDS


by

T. E. Harbaugh and Ven Te Chow

Sponsored by
National Science Foundation
Research Grant GP-1464

A Contribution to the International Hydrological Decade,
Approved by the U. S. National Committee for IHD

Department of Civil Engineering
University of Illinois
Urbana, Illinois
September 1967
SYNOPSIS

In this paper, the roughness of a conceptual river system or watershed is investigated by mathematical formulation and laboratory experimentation. By mathematical formulation, the watershed is considered as a quasi-linear distributed dynamic system. The hydraulic behavior of this system representation is described by a set of quasi-linear partial differential equations of continuity and momentum principles which define the spatially varied unsteady flow. The conceptual watershed is composed of the overland flow part and the main channel flow part. The equations are first applied to the overland flow phase due to input rainfall and then to the main channel flow phase using the overland flow as input. Solution of the equations is made in their finite-difference forms on digital computers. The friction slope in the energy equation is expressed by a conceptual measure of the watershed roughness. This conceptual roughness is further evaluated in a laboratory study of the watershed in various shapes and slopes and its sensitivity to depth of flow and raindrop impact on the laboratory watershed is disclosed.
SYNOPSIS

Dans cet article, on étudie la rugosité d'un réseau ou d'un bassin fluvial conceptuel par une représentation mathématique et par des expériences de laboratoire. Mathématiquement, on considère le bassin comme un système dynamique distribué quasi linéairement. Le comportement hydraulique du système ainsi représenté est décrit par un ensemble d'équations aux dérivées partielles quasi-lineaires de continuité et de moments qui définissent l'écoulement varié non permanent. Le bassin conceptuel se compose de la partie en écoulement superficiel et de la partie en écoulement par canal principal. On applique tout d'abord les équations à la phase en écoulement superficiel dû à l'apport pluvial, puis à la phase en écoulement par canal principal alimentée par l'écoulement superficiel. On résout les équations mises sous forme de différences finies à l'aide de machines électroniques. La pente de frottement dans l'équation d'énergie est exprimée par une mesure conceptuelle de la rugosité du bassin. On évalue ultérieurement cette rugosité conceptuelle par une étude en laboratoire du bassin pour des formes et des pentes variées, et on montre sa sensibilité aux variations en profondeur de l'écoulement et aux chocs des gouttes de pluie sur le bassin expérimental.
THE WATERSHED AS A SYSTEM

The direct runoff of a watershed resulting from a given effective rainfall is modified by the internal processes occurring within the watershed boundaries. These internal hydrodynamic processes are highly complex. For purposes of mathematically describing the influence which the watershed has upon the direct runoff, the watershed may be conceptually considered as a system. In order to mathematically define the conceptual watershed system, it is necessary to establish basic principles by which the system shall operate.

Fortunately, the basic concept of conservation of mass and its application through the equation of continuity are available for the study of a watershed system. In addition, the dynamic behavior of the watershed may be idealized through use of the momentum equation for spatially varied unsteady flow. Use of the momentum equation will allow consideration of the temporal and spatial variations of inflows as well as the temporal and spatial variations of resistance to flow encountered within the watershed boundaries. Both the continuity and momentum equations can be expressed in terms of quasi-linear partial differential equations. These equations, such as in the following forms, can thus describe the watershed as a quasi-linear distributed dynamic system: 1,2

\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{\partial y}{\partial t} = \tilde{F} \]  
(1)

\[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + g \frac{\partial y}{\partial x} + \frac{v}{y} \tilde{F} = g(S - S_r) \]  
(2)


where $y$ is the depth of water in feet, $V$ is the velocity of flow in feet per second, $x$ is the distance in feet along the direction of flow, $t$ is the time in seconds, $I$ is the average spatially lateral inflow in cubic feet per second, $g$ is the gravitational acceleration in feet per second per second, $S$ is the slope of the watershed, and $S_f$ is the friction slope.

A watershed system is considered as being composed of two distinct parts: the overland flow part and the channel part. The above set of equations are first applied to the overland flow part for which the rainfall is the spatially varied inflow. The equations are then applied to the channel flow part for which the computed overland flow contributing to the channel flow becomes the spatially varied inflow to be used in the equations. This mathematical representation of the watershed system constitutes a conceptual river system or a conceptual watershed in this paper. The mathematical analysis of this system provides a means of investigating the time response of the watershed system and the effect of roughness upon the time response.

The quasi-linear partial differential equations which are used to represent mathematically the conceptual watershed cannot be solved directly. However, for practical purposes, they can be expressed in finite-difference forms and then solved on digital computers for a given hypothetical watershed boundary and an arrangement of the overland and channel flows.

CONCEPTUAL ROUGHNESS

Evaluation of Manning’s $n$ for given channels or surfaces is a highly subjective task. Use of this measure of roughness was originally established for steady uniform flow, although many investigators have also employed it to unsteady flow problems, particularly in the hydraulic routing of floods. Favorable results have been obtained for many of these investigations; however, for
The use of a constant value of Manning's $n$ appears to produce satisfactory results even in unsteady flow, if the range in depth is not large and if the depth of flow is large with respect to the size of the roughness. However, in studies such as those dealing with spatially varied unsteady flow, the flow depth changes from zero to its maximum, sometimes in a very short period of time; consequently, if the size of the roughness remains constant over the time of flow or over the length of flow, its influence upon the flow field will be of a larger magnitude at small depths of flow than that at relatively large depths of flow.

This is to say that if Manning's $n$ is used to express roughness, its value, for a given surface subjected to spatially varied unsteady flow, will decrease with increasing depths of flow. This behavior is borne out by uniform flow experiments conducted for a range of discharges by Woo and Brater and also earlier by the Corps of Engineers. Rearrangement of their data indicates a variation in Manning's $n$ with flow depth for a 1\% slope as follows:

$$n = 0.00614 y^{-0.166} \quad (\text{masonite}) \quad S = 1\% \quad (3)$$

$$n = 0.0065 y^{-0.166} \quad (\text{concrete}) \quad S = 1\% \quad (4)$$

---


Results of these tests indicate that for small depths of flow, Manning's $n$ is not constant but exhibits a definable function of the depth. However, since the results of these investigators were made for uniform flows, they do not include the force needed to accelerate the lateral inflow or indicate the influence of added turbulence due to raindrop impact forces which were present in many laboratory experiments of spatially varied unsteady flow. It is necessary then to introduce a conceptual measure of roughness for application in spatially varied unsteady flow which still has the ease of use as Manning's $n$. This new conceptual measure of roughness when applied to the proposed conceptual watershed is referred to as conceptual watershed roughness, designated by $N'$ (the prime emphasizes the conceptual nature of this measure). This roughness is further defined according to the formula for the conceptual case of sheet flow as

$$N' = W_c y^x$$  \hspace{1cm} (5)

where $W_c$ is the coefficient of watershed roughness equal to $c_u + c_i + c_{iv}$; and, $c_u$ is the coefficient accounting for the variation of Manning's $n$ with depth, $c_i$ is the coefficient based on added resistance due to raindrop impact produced by a particular rainfall applicator at a point just above the water surface, $c_{iv}$ is the coefficient based on variation of raindrop impact for different depths of flow, $z$ is an exponent, and $y$ is the depth of flow in feet.

The value of $c_u$ is essentially the coefficient noted in Eqs. (3) and (4). The values of $c_i$ and $c_{iv}$ have to be evaluated from experimental data obtained for conditions of spatially varied unsteady flow which is a result of spatially distributed simulated rainfall. The values of the coefficients $c_i$ and $c_{iv}$ depend upon such factors as size of raindrop, velocity of raindrop, intensity of rainfall, type of rainfall applicator, and slope of the plane over which
the spatially varied unsteady flow is occurring. The coefficient $c_i v$ depends upon the additional factor of the depth of flow. The exponent $x$ is assumed to be only a function of the slope. Use of this relationship is readily incorporated into the conceptual watershed computer program since a measure of roughness can be evaluated for each calculated depth in the finite-difference forms of Eqs. (1) and (2).

BEHAVIOR OF THE HYDROGRAPH FROM THE CONCEPTUAL WATERSHED

To properly evaluate the behavior of the hydrograph from the conceptual watershed, computer runs were made for the finite-difference forms of Eqs. (1) and (2), in which the friction slope was determined from the conceptual watershed roughness as a substitute for Manning's $n$. These results are presented in the following sections, with comparisons to laboratory data obtained from the watershed experimentation system developed in the University of Illinois Hydraulic Engineering Laboratory. The testing laboratory watersheds were made of masonite and had the shapes of a square, an inverted triangle and a circle, all having essentially the same size of about 1,040 sq. ft. on the average, a main channel slope of either 1% or 1.75%, and an overland flow slope of 1%. The direction of overland flow was guided by wooden strips laid perpendicular to the main channel flow. Calculated direct runoff hydrographs (DRH) of a conceptual


watershed for various assumed coefficients of watershed roughness are shown in Figure 1. These hydrographs show the influence of increasing the value $W_c$, which is assumed constant over the entire watershed. Many instances can also arise in which the value of the coefficient of conceptual watershed roughness will be different for the overland flow phase as compared to that of the channel flow phase. This situation is demonstrated in Figure 2 where two different coefficients of conceptual watershed roughness are assigned separately to the overland flow and channel flow phases. As can be seen, the relative increase in the coefficient of conceptual watershed roughness for the overland flow phase over the channel flow phase flattens the rising limb of the hydrograph appreciably. Increasing the uniform value of the coefficient of watershed roughness tends to laterally shift the discharge hydrograph along the time axis.

INFLUENCE OF RAINDROP IMPACT ON CONCEPTUAL WATERSHED ROUGHNESS

To determine the coefficient of conceptual watershed roughness for the laboratory testing watershed, laboratory tests for three uniform rainfall intensities were performed on the square watershed with a 1% main channel slope (Figure 3). The trial value of $W_c$ was used in the conceptual watershed computer program until a satisfactory fit between the experimental and theoretical data was achieved. These data are presented graphically in Figure 3. As shown in Figure 3, the coefficient of conceptual watershed roughness is different for the three rainfall intensities. This should be expected since by definition [Eq. (5)] the coefficients $c_i$ and $c_iv$ both vary with intensity. The value of $W_c$ versus intensity plots as a straight line on a log-log paper (Figure 4). The equation of this line is

$$W_c = 0.0092 i^{0.37}$$

(6)
Use of data obtained by Woo and Brater\textsuperscript{5} allows further interpretation to be made regarding the behavior of the coefficient of conceptual watershed roughness in the absence of raindrop impact. The same watershed surface material (masonite) has been used in this experiment as was used by Woo and Brater; reanalysis of their data resulted in a value of $c_u$ equal to $0.00614$. This value of $c_u$ is subtracted from the coefficient of watershed roughness $W_c$ for the various rainfall intensities. Plotting this difference, $c_i + c_{iv}$ versus rainfall intensity results in (Figure 4)

$$W_c - c_u = c_i + c_{iv} = 0.0037 + 0.62$$

Further separation of these two coefficients to produce values attributable to either $c_i$ or $c_{iv}$ will have to await more detailed experimentation into the effect of the impact of individual raindrops on various depths of flow. Some initial work along this line is being undertaken by Palmer\textsuperscript{10} and Harbaugh\textsuperscript{11}, and also at the University of Illinois, but results are not available at this time.

**INFLUENCE OF FLOW DEPTH ON WATERSHED ROUGHNESS**

Results of the conceptual watershed for all four shapes with a conceptual watershed roughness represented by $N^* = 0.018y^{-0.166}$ are presented in Figure 5. It should be noted that this watershed roughness was established for the square laboratory watershed as related in Figure 3. It appears that

\textsuperscript{10} Palmer, R. S.: "Waterdrop Impact Forces," paper presented at 1963 Winter Meeting, American Society of Agricultural Engineers.

the conceptual watershed roughness is independent of the watershed shape under the laboratory testing situation in which sheet flow occurred and all flow conditions were essentially equal except a variation in flow depth and discharge. Therefore, this should not be construed as necessarily being the case of any watershed, laboratory or natural, where sheet flow does not occur and physiographic conditions complicate the flow phenomenon. Figure 5 indicates the behavior of the four shapes if each had a uniform roughness coefficient of 0.018. However, a comparison of these theoretical hydrographs with the experimental hydrographs obtained from the laboratory indicates some variations, particularly in the case of the triangle (Figure 6). This variation can be explained as follows: The depth of flow in the upstream of the channel was considerably higher for the square shape than for the triangle. This higher depth of flow is due to the contributions to the main channel from the perpendicular overland flow strips. However, in the case of the triangle, these contributions in the upstream are relatively small; consequently, the depth of flow is considerably less in both the channel flow and the overland flow areas. As previously noted, the value of $c_{iv}$ has the property of reflecting the relative influence of raindrop impact for different depths of flow. The hydrograph for the triangle is thus influenced by the higher relative value of $c_{iv}$ due to the smaller depths of flow particularly in the overland flow area.

**INFLUENCE OF WATERSHED SLOPE ON THE FLOOD FLOW**

Experimental hydrographs for a main channel slope of 1.75% and overland flow of 1% for all shapes are presented in Figure 7. Inspection of Figure 7 indicates the more rapid time response of the watershed for steeper slopes.
INFLUENCE OF RAINFALL INTENSITY AND DURATION ON THE FLOOD FLOW

The experimental hydrographs of five uniform rainfall intensities on the square watershed with a main channel slope of 1% and an overland flow slope of 1% are presented in Figure 3. The data in this figure indicate that for lower intensities the slope of the rising limb decreases and the time to equilibrium increases.

The experimental hydrographs for three rainfall durations equal to $0.2 \, T_e$, $0.4 \, T_e$, and $0.6 \, T_e$ are presented in Figure 8, where $T_e$ is the time to equilibrium at which the outflow is practically equal to the input rainfall. These results are for the square watershed with a main channel slope of 1% and an overland flow slope of 1%. Data in this figure indicate a definite decrease in both time to peak and discharge due to a decrease in rainfall duration $D$. The particular hydrograph for $D/T_e = 0.4$ was determined from the conceptual watershed using $W_c = 0.016$ for all time during the rainfall. As may be seen in Figure 8, the hydrographs of this calculation are somewhat lower than the experimental hydrographs obtained in the laboratory. This supports the explanation previously made concerning the change of the coefficient of conceptual watershed roughness due to raindrop impact. It is again evident that $W_c$ decreases after cessation of the rainfall as shown by the increase in discharge over the theoretical value sometime after cessation for Curve B. Again, more investigation is necessary for a proper evaluation of $W_c$ for various experimental conditions.

CONCLUSIONS

A river system or watershed is represented conceptually by the quasi-linear distributed dynamic system and thereby described mathematically by
the quasi-linear partial differential equations. This conceptual river system or watershed so represented can be solved by digital computers using the finite-difference forms of the equations. In these equations, a conceptual watershed roughness is introduced as a substitute of Manning's $n$ to evaluate the friction slope. By the analytical solution and the experimentation of the laboratory model of the conceptual watershed, it was disclosed that the coefficient of conceptual watershed roughness is sensitive to the depth of flow and the raindrop impact in addition to the surface roughness.

The influence of raindrop impact was shown to have a significant influence upon the discharge hydrographs resulting from laboratory simulated rainfalls. This is particularly evident on the time to equilibrium of the four different watershed shapes tested. The variation of time to equilibrium for the different shaped watersheds is only 15% in the simulated laboratory watersheds as compared to 64% in the conceptual watersheds; however, the arrangement of the shapes in the order of increasing time to equilibrium is not the same for both approaches. Variation of the time of concentration for different shapes is approximately 16% for both conceptual and simulated laboratory watersheds.

The departure of the time distribution of runoff from a triangular laboratory watershed to that obtained from the conceptual watershed is quite pronounced and is believed due entirely to the influence of raindrop impact, particularly the value of $c_{iv}$. This coefficient, $c_{iv}$, describes the relative influence of raindrop impact for varying depths of flow. These depths of flow are quite small on laboratory watersheds, and particularly so on the upstream of the channel on the triangular-shaped laboratory watershed. It was shown that this particular shape is considerably more influenced by raindrop impact. All laboratory watershed shapes which have small depths of flow are also in-

-13-
fluenced by this raindrop impact; however, it is oftentimes not as noticeable in the hydrographs resulting from these shapes.

The study reported in this paper provides a first step toward the understanding of the new field of watershed hydraulics. Although the conceptual watershed and its laboratory models were applied to a simple system of overland flow contributing to a main channel, it is believed that the approach of investigation could be extended to more complicated artificial or simulated natural river systems.

ACKNOWLEDGEMENTS

This paper represents a portion of the work undertaken by the senior author in his Ph.D. dissertation entitled "Time Distribution of Runoff From Watersheds," University of Illinois, 1966, directed by Dr. V. T. Chow as part of a research project sponsored by the National Science Foundation, Grant NSF-GP-1464.