TURBULENCE STRUCTURE IN COBBLE-BED OPEN-CHANNEL FLOW

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Abstract

The present work reports experimental observations aimed at characterizing the particular turbulence structure of high-Reynolds-number cobble-bed open-channel flow under small relative submergences. Recently available technology (acoustic Doppler velocimetry) allowed for the acquisition of three-dimensional velocity components at different distances from the bed, above and below the average top of the roughness elements, and synchronization of this sensor with a hot-film probe for streamwise velocity allowed for the spatial-temporal resolution of coherent events. Data processing includes the mean flow structure and usual turbulence statistics, evaluation of the prediction capability of low-order cumulant expansions, computation of energy budget terms as well as the characterization of coherent structures using different conditional averaging techniques. Comparisons are also provided to the more extensively investigated structure of ‘canonical’ smooth-wall shear flows. Results clearly show the absence of an equilibrium layer as opposed to the typical constant-turbulent-kinetic-energy flux region characteristic of small-scale roughness flows, hence providing a plausible explanation for the long recognized lack of applicability of the logarithmic law to free-surface flows with large relative roughness. Ejections are shown to dominate the Reynolds stresses above a plane located near the average top of the cobbles, whereas inrushes or sweeps make the biggest contribution to the turbulent momentum transport below this plane. The relative contribution of sweeps and ejections to the Reynolds stress has been found to correlate with the vertical flux of turbulent kinetic energy and the net momentum flux, as can be predicted by cumulant expansions of low order. Conditional averaging of coherent events reveal important resemblances to results under small-scale conditions in terms of Strouhal number, coefficient of variation and probability distribution of bursting period, eulerian length and time scales, etc., thus contributing to the definition of a universal turbulence structure in wall-bounded shear flows.
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The study of the turbulence structure of wall-bounded shear flows constitutes one of the most interesting and active research areas in modern turbulence (Barenblatt, 1993). The challenging feature in these type of flows comes from the fact that the presence of the solid wall naturally imposes some length scales, whereas the thickness of the boundary layer itself still remains as another natural length scale (Tenekees and Lumley, 1990), so that the structure of turbulence is thus determined by the coexistence of at least two length scales. Historically, significant attention has been given to the study of the turbulence structure of flow over smooth surfaces, either zero-pressure-gradient boundary layers or duct flows (Raupach et al., 1991), and increasing research has been conducted lately on flow in smooth-bed open channels (Nezu and Nakagawa, 1993). Experimental as well as numerical results provide to date a vast body of knowledge that allows for the understanding of much of the dynamics involved in smooth-wall conditions. Although most flows in nature take place under environmental, rough conditions, relatively less research has been focused on the turbulence structure of flows over rough walls (Grass et al., 1991), and even less research has dealt with the investigation of turbulence in environmental conditions. Most of the systematic work conducted so far in rough-wall regimes has been restricted to the case of atmospheric flows over plant canopies and wind tunnel studies over simulated vegetation (Raupach et al., 1991). The present work aims at contributing to the understanding of the turbulence structure in mountain rivers by simulating large-scale cobble bed flows in the laboratory, and placing the results in a unified conceptual frame together with the current fluid-mechanic knowledge on wall-bounded shear
layers. The present introductory review is therefore subdivided into three sub-topics: §1.1 Mean flow structure, §1.2 Turbulence structure, and §1.3 Coherent structures in wall-bounded shear flows.

1.1 Mean Flow Structure

It is generally accepted that far enough from the solid boundary (if this region physically exists) the dynamics of shear flows attain a state of development that becomes independent of the length scale(s) imposed by the wall. This region has been traditionally called outer layer and is characterized by weak shear stresses, hence not the site of dominant instability mechanisms related to turbulence production. The only natural length scale in this region is thus the thickness of the boundary layer itself. On the other hand, the region close to the boundary exhibits large values of shear stresses, and is where the turbulence generation achieves its maximum value. This latter region is designated as the inner layer, and its dynamics is characterized by length scales that strongly depend upon the relative magnitude of the viscous and roughness effects. The inner layer has been historically designated sometimes as active, whereas the outer layer has originally been conceived as a passive region (Raupach et al., 1991). For the particular case of open-channel flows the inner region is usually identified with the portion of the flow near the bed up to a distance on the order of 15 or 20% of the mean flow depth (Nezu and Nakagawa, 1993). Accordingly, the term outer layer encompasses two regions, namely the free surface region and the inertial or equilibrium region.

Smooth-wall conditions prevail when the viscosity-related length scale, $v/u_*$ (with $v$ representing the kinematic viscosity of the fluid and $u_*$ the mean shear velocity), is much larger than any representative dimension of the roughness elements (i.e. small roughness Reynolds number, $Re_k$ with $k$ representing the equivalent sandgrain roughness). Under such conditions experimental evidence demonstrates that the turbulent momentum transfer remains a small fraction of the wall shear stress up to some distance identified with the
upper limit of the so-called viscous sublayer (Tennekes and Lumley, 1990). In the other limiting case, viscous length scale much smaller than the representative dimension(s) of the roughness elements (fully rough regime, i.e. high enough Reₖ), a counterpart of the viscous sublayer in smooth-wall conditions is usually defined as "quasi-separated layer" (Nezu and Nakagawa, 1993) or "roughness sublayer" (Raupach et al., 1991). This layer is dynamically influenced by the length scale(s) associated with the elements that determine the roughness at the boundary, and its thickness is as such a roughness-type dependent variable. The direct effect of surface roughness upon the mean flow velocity is commonly expressed by a roughness function defined as (Hama, 1954):

$$\frac{\Delta U}{u_*} = \left( \frac{2}{c_f} \right)_{\text{smooth}}^{1/2} - \left( \frac{2}{c_f} \right)_{\text{rough}}^{1/2}$$

where $c_f$ represents a coefficient of local skin friction and $\Delta U$ is the downward shift in the local mean velocity of the logarithmic part of the velocity profile. Regarding the universality of (1), Perry et al. (1969) have shown that two type of roughness can be distinguished, namely a so-called 'k-type' with its roughness function depending on the value of Reₖ, and a 'd-type' roughness for which the roughness function depends on $du_*/v$ rather than on Reₖ, where d is an outer-layer scale. However, the dynamics of the transition regime between smooth and fully-rough conditions are to-date not completely understood. Lower and upper limits for this regime have been determined experimentally for k-type roughness (Schlichting, 1979; Nikuradse, 1939), where the lower critical transition Reₖ number is generally recognized as the onset of vortex shedding by the roughness elements, but no physical significance has been attributed so far to the upper critical value. Measurements in d-type roughness by Townes and Sabersky (1966) show a linear relationship between Reₖ and the roughness Strouhal number defined as:

$$St = \frac{2\pi k}{\bar{T} u_*}$$

where $\bar{T}$ is the mean period of vortex shedding, for Reₖ < 150, whereas St was found to be
constant and about 10 for $Re_k > 150$. Results by Bandyopadhyay (1987), both in k-type and d-type roughness, confirm the observations of Townes and Sabersky (1966), with the interesting result that in both cases the slope of $St$ versus $Re_k$ for $Re_k < 150$ was similar and almost identical to the one obtained by Townes and Sabersky (1966). Since the physical significance of this slope may be described in terms of the square-root of the stability parameter of Black's (1968) analysis, $u_*(\sqrt{T/v})^{1/2}$ (notice that it is indeed independent of any roughness scale), its universal nature has therefore been understood by Bandyopadhyay (1987) as indicative of the existence of similar structures for the boundary layer both in rough and smooth walls.

The traditional assumption concerning the mean flow is that at large enough local flow Reynolds numbers, $Re_z = z u_*/v$, the overall dynamics of turbulent boundary layers becomes independent of the dominant processes at the wall, i.e. independent of viscosity and/or roughness related scales (herein $x$, $y$ and $z$ represent the streamwise, lateral and bed-normal axis of the Cartesian coordinate system, with $u$, $v$ and $w$ standing for the corresponding components of the instantaneous velocity vector; in what follows capital letters will stand for temporal-mean velocity values and primes will denote temporal fluctuations around the corresponding mean; overbars will denote the temporal averaging operator). The spatial as well as the spectral structure of turbulence have hence been described by asymptotic matching concepts leading to the characterization of an overlap or match region, called inertial sublayer and inertial subrange, respectively. The well-known logarithmic law has been traditionally derived from these hypothesis. More recently however, alternative hypothesis have proved to be identically valid, and the power law has been obtained by assuming incomplete similarity or scaling assumptions instead. This latter process is also referred to as intermediate asymptotic (Barenblatt, 1993).

The structure of open-channel flows presents new challenges compared to duct or wind-tunnel boundary layers due to the particular effects of the free surface both upon the
mean flow and the turbulence structure. Over the years an extensive body of knowledge has provided hydraulic engineers with enough tools to tackle the problem for the more studied case of free-surface flows of large relative submergence \( h/k \gg 1 \), with \( h \) denoting the mean flow depth), where alluvial rivers constitute a typical example (Nezu and Nakagawa, 1993). The hydraulics of mountain and gravel-bed streams (usually of small relative submergence) have historically received less relative attention than the corresponding research on alluvial rivers, probably due to the less relative development of mountain regions. Since the seventies, however, hydraulic engineers have been increasingly concerned with establishing resistance laws for mountain streams, and therefore relate mean velocities to hydraulic characteristics of the river both experimentally and theoretically (Limerinos, 1970; Bathurst, 1978; Bray, 1979; Parker and Peterson, 1980; Jarret, 1983; Thorne and Zevenbergen, 1985; Bathurst, 1985; Aguirre-Pé and Fuentes, 1990; Wiberg and Smith, 1991).

The current universal trend amongst hydraulic engineers and geomorphologist appears to agree in defining three roughness scales based mainly upon mean flow criteria (Bathurst et al., 1981; Bathurst, 1985):

1. **large-scale roughness** conditions are considered to prevail when the mean flow depth is of the same order as the characteristic scale of the roughness elements.

2. on the other limiting condition, the roughness scale is considered small when the ratio between mean flow depth and the characteristic roughness length is large enough for the resistance function "to be described by boundary layer theory" (Bathurst et al. 1981).

3. **intermediate roughness scales** are defined in a range which extends between the previous two extreme limits.

The main difficulty with the above given classification is that it basically lacks a theoretical or more physical sound basis, being more related to the goodness-of-fit of experimental and field observations to the logarithmic resistance law. This explains why the limits of this scale classification are so vaguely defined. For the case of gravel-bed rivers of
small-scale roughness Hey (1979) proposed a variation of the logarithmic law by modifying the values of the coefficients in the classical law of the wall and assuming that a good approximation to the equivalent roughness is \( k = D_{84} \) (with \( D_{84} \) being that size in the granulometric distribution for which 84% is finer). However, in mountain streams characterized by high gradients and boulder beds, studies indicate that the flow resistance is mainly governed by form drag due to the elements, hence "the associated resistance processes are different from those for small-scale roughness" and therefore "it is not possible to use the semilogarithmic resistance equations developed for small-scale roughness" (Bathurst et al. 1981). Instead, it appears that for these conditions a power law would be more suitable (Bathurst, 1978; López, 1992), while a modified version of the logarithmic law would apply for intermediate roughnesses (Thompson and Campbell, 1979).

Laboratory as well as field observations have indeed shown the vertical velocity profile in mountain streams to differ substantially from the logarithmic law, with sometimes a two-zone velocity profile developing, taking and "S" shape (Marchand et al., 1984). As stated by Bathurst (1994), the close relationship between velocity distribution and flow resistance suggest that once the factors that control the existence of the S-shaped profile, or the deviations from the logarithmic law, become completely understood, then the processes which govern the flow resistance will become clear as well. This is probably the reason why, although a lot of different resistance laws have been proposed, and every year "new and improved" relationships appear in the literature, it seems that no further progress will be made until the main dynamics of the turbulence are clarified.

1.2 Turbulence Structure

Since dimensional analysis proved to constitute a reliable tool for determining the main properties of the mean velocity in wall-bounded flows, the same approach has been traditionally followed to establish the properties of turbulence. In this regard, some complementary hypothesis have been developed over the years concerning velocity and length scales, like the wall-similarity, equilibrium layer and attached-eddy hypothesis (Townsend,
1961; Townsend, 1976; Perry et al., 1986; Raupach et al., 1991). Altogether, this body of assumptions leads to a set of predictions for the turbulent length scales, that are comparable with similar assumptions for the mean flow. Accordingly, the structure of turbulence in boundary layers has also been subdivided into an outer and an inner region.

1.2.1 Structure of the outer region

Following dimensional arguments it is thus expected that outer-layer vertical profiles of central moments of velocity components at different orders (either single point or not) should collapse when normalized with the mean shear velocity and the length scale of the outer layer. To the degree of accuracy obtainable with commonly employed measurement techniques this similarity holds for different flow and wall-roughness conditions (Grass, 1971; Raupach et al., 1986; Bandyopadhyay and Watson, 1988; Raupach et al., 1991, Nezu and Nakagawa 1993, etc.). Moreover, Raupach (1981) found a simple proportional relationship between all third-order joint moments of \( u' \) and \( w' \) for wind-tunnel flow both over smooth and over cylinder-roughened surfaces with different densities (herein \( M_{ij} = \frac{\overline{u'w'^j}}{\sigma_u/\sigma_w} \) will denote the \( k \)-order joint moment of \( u' \) and \( w' \), with \( k = i + j \) and \( \sigma_x \) representing the standard deviation of the variable \( x \)). Later Raupach et al. (1986) confirmed the universal character of these proportionalities for wind tunnel flow over bluff elements in a diamond array using a specially designed three-hot-wire anemometer. Concerning other statistics, Bandyopadhyay and Watson (1988) found a constant behavior of the ratio \( -\frac{\overline{u'w'}}{q} \) in the outer layer of a flow with grooved walls, with values very close to results in smooth-wall conditions (Bradshaw 1967), thus providing another confirmation for the wall-similarity hypothesis (\( q \) stands for \( \overline{u'^2 + v'^2 + w'^2} \)).

Recently, however, Krogstad et al (1992) obtained experimental results of a zero-pressure turbulent boundary layer over a mesh-screen that seem to violate some of the consequences derived from wall-similarity assumptions. In particular, they observed larger effects of the roughness on the outer region, and stronger as well as more frequent ejections
and sweep events than for smooth-wall conditions. They suggest that the observed aspects are probably due to the fact that different roughnesses may have different spectral signatures for the vertical velocity fluctuation, and that the departure from wall-similarity assumptions may also indicate a higher degree of interaction between the inner and the outer region than usually accepted.

1.2.2 Structure of the inner region

With respect to flow regions near the solid boundary, the absence of wall-similarity in the roughness sublayer, or in other words the three-dimensionality and the degree of dependence of the turbulence structure near the wall upon the dominant length-scales, becomes evident by comparing for example values of turbulent intensities above plant canopies and three-dimensional roughness as sandgrains (Nezu and Nakagawa, 1993). Some authors suggest this difference to be related to the extent of influence of the spanwise aspect ratio, $\lambda_{AR}$, of the roughness elements upon the eddy-shedding process in the roughness sublayer (Bandyopadhyay and Watson, 1988). The spanwise aspect ratio is defined as the largest spanwise distance obstructing the flow divided by the characteristic height of the roughness element. Unfortunately, as mentioned above, besides some few works on other type of roughness elements (e.g. Andreopoulos and Bradshaw, 1981; Bandyopadhyay and Watson, 1988; etc.), the vast majority of detailed experimental information in the roughness sublayer comes from field observations of atmospheric flows or wind tunnel models of canopy flows (Raupach et al., 1991). This is mainly a consequence of two facts, the first being that common engineering rough-walls consist of sandgrains which do not easily admit measurements within or even close to them, and the second reason being the high turbulence intensities present in this region due to the direct dynamical effects of several roughness length scales, which invalidate the use of traditional anemometry. It is precisely this inherent three-dimensionality of the flow in the roughness sublayer, that prompts the need of the spatial averaging of the conservation equations in order to properly represent the problem in a more tractable one-dimensional frame (Raupach and Shaw, 1982).
Turbulence intensity data for rough beds in open channels are very limited in comparison with those for smooth beds. The direct effect of wall roughness on turbulent intensities was measured for sandgrain roughness by Grass (1971) using the hydrogen-bubble technique and by Nezu (1977) using hot-film anemometry. Their results clearly show roughness effects to exist near the bed, $y/h < 0.30$, in particular for streamwise turbulent intensities, $\sigma_u/\bar{u}_*$, which reach a peak of about 2.0 compared to typical values of 2.8 for smooth walls (Nezu and Nakagawa, 1993). Similar trends are observed for the relative turbulence intensity in the streamwise direction, $\sigma_u/U$, which for example increases from 0.12 for smooth-walls to 0.22 for a rough bed with $k/h = 0.06$ (Nezu, 1977). Bed-normal intensities, on the other hand, show a small increase near the roughness elements compared to smooth-bed conditions, presumably because vertical damping is less severe due to the open nature of the surface (Krogstad et al., 1992) and therefore all three intensities tend to merge suggesting the existence of a redistribution of turbulent energy towards isotropy as large-scale eddies pass energy to smaller ones destroyed due to the elements (Nezu and Nakagawa, 1993).

While in smooth-bed conditions the main generation of turbulence takes place in the buffer region ($10 < z_{rms}/\nu < 30$), this layer is not observable over rough beds, merging with the roughness elements, suggesting thus different mechanisms of turbulence production.

Knowledge of the turbulence structure seems to be even more unclear for flow within roughness elements, where it becomes even more obvious that a variety of length scales may influence the turbulence dynamics. For example there have been reports of "bulges" in the mean velocity profile within vegetation, which if real would imply the existence of counter-gradient momentum transfer (Raupach and Thom, 1981). The mean shear seems to attain a maximum at the top of the roughness and then attenuate within the elements, corresponding with a decrease in mean velocity. This behavior is in line with the decrease in turbulent momentum transfer needed by the flow since roughness elements are absorbing momentum via drag forces. Observations in canopy flows also show normalized standard deviations of velocity decreasing fast within the roughness elements. Regarding higher central
moments of velocity, skewness of streamwise, $M_{30}$, and vertical velocity fluctuations, $M_{03}$, show positive and negative values, respectively, a trend observed in general all within the roughness sublayer, thus inverting signs compared to typical values in inertial and outer layers. Similar trends have been found for the other two third-order joint central moments of $u'$ and $w'$, namely $M_{12}$ and $M_{21}$. This information together with the existence of high values of kurtosis coefficients ($M_{40}$ and $M_{04}$) points out the presence of very high turbulent intermitencies (Maitani, 1979).

Regarding turbulent scales, all evidence seems to indicate that within two-dimensional elements (vegetation with small spanwise aspect ratios) the macro-length scales of turbulence are of the same order as the height of the roughness elements themselves, $h_v$. It seems thus, that appropriate scales for velocity and length within the vegetation are $u_*$ and $h_v$, respectively (Raupach et al., 1991; Gao and Li, 1993), which explains the good degree of collapse obtained in this region for normalized turbulence intensities using these scaling variables, although the range of heights varies by a factor of about 500, whereas the type of roughness ranges from simple rods in a wind tunnel to complex combinations of crown, trunk space, and understory forests (Kaimal and Finnigan, 1994).

1.2.3 Turbulence structure in open channels with intermediate and large-scale roughness

As mentioned before, only few reports exist on the characterization of turbulence in large-scale free-surface flows. Nowel and Church (1979) reported probably one of the first works on the characterization of the turbulence structure in open channels with intermediate roughness scale conditions. They performed turbulence measurements using hot-film anemometry in a recirculating flume with constant flow depth (7.2 cm.) and a bed covered with bricks (i.e. legos) of 0.9 cm. height (i.e. relative roughness of about 1/8) in different density arrays. Their primary objective was to study the effect of roughness spacing on the flow structure. Their results support the spatial subdivision of the flow field in three regions: an inner zone near the bed, a wake zone extending between one and two roughness
heights, and a third region (the outer layer) extending from about 35% of the depth up to the free surface. In the wake region, they found an excess of turbulence production, attributed to the wake shedding above the roughness elements. Spectral results indicated the absence of an inertial subrange, and length scales showed the existence of a marked dependence upon roughness density and distance to the bed.

Nakagawa et al. (1989) reported experimental measurements of turbulence in an open-channel flow over glass beads of small relative submergence, using both hot-film and laser Doppler anemometry above the roughness elements, and covering a relative submergence range between 1.3 and 4.0. They were able to identify the existence of a roughness sublayer, characterized by more uniform mean velocities than predicted by the logarithmic law, by a substantial suppression of the Reynolds stresses and by smaller peaks of turbulent intensities.

More recent laboratory investigations were conducted by Graf and coworkers (1992) at the Laboratoire de Recherches Hydrauliques, Lausanne, for flow over a gravel bed but with relative large submergences. They employed their own developed device, an acoustic Doppler velocity profiler, together with conventional hot-film techniques and Prandtl tubes. Their observations show the existence of an inertial region where mean velocity profiles may be described by the log-law. Of remarkable importance is the fact that their results on the turbulence structure show vertical turbulent intensities to increase towards the bed up to the limit of the wall-region where they start to decrease in disagreement with empirically obtained laws (Nezu and Nakagawa, 1993). Other aspects of turbulence seem to agree with commonly reported results.

1.3 Coherent Structures

Kline et al. (1967) were probably among the first to experimentally identify and investigate the appearance of coherent phenomena in turbulent boundary layers. Observations on smooth-wall boundary layers led them to define and conceive the bursting
process as a randomly occurring phenomena, comprising the lift up of low-speed fluid from the wall, sudden oscillation, bursting and ejection. These bursting events are in turn closely associated with energetic sweeps and inrush events, which transport high-speed fluid towards the bed enhancing near-wall vorticity by lateral spanwise vortex stretching and also generating new vorticity which is subsequently ejected from the wall (Grass et al., 1991). All these cyclical processes are particularly linked to the generation and thus the maintenance of turbulence both in smooth and rough conditions (Grass et al., 1991), involving recurrent instabilities, and are responsible as well for the major contributions to the turbulent momentum transport in wall layers.

The vast majority of coherent structures were observed for the case of pressure-driven boundary layers, with far less observations concerning open channel flows (Nezu and Nakagawa, 1993). In a landmark work, Nezu (1977) conducted a series of hot-film measurements together with hydrogen bubble observations both in smooth and rough-bed open channels covering a wide variety of flow Reynolds and Froude numbers, Re and Fr, respectively. As suggested by Rao et al. (1971), he found that irrespective of this two latter parameters and of distance to the bed, the log-normal probability distribution reasonably fitted the observed distribution of the bursting period, $T_b$, which presented a coefficient of variation, $\sigma_{T_b}/T_b$, of about 1.0 to 1.5 compared to values of 0.5 found using visual observations by Kim et al. (1971) in boundary layers. More recently, synchronization of a high-speed video camera with traditional hot-film anemometry for measuring both streamwise velocity as well as bed-shear stress in smooth-bed conditions allowed for the identification and characterization of five different types of coherent events in open channels (López, 1994; López et al., 1996). In particular, the analysis of low-speed ejections, consisting of downstream convected and up-lifting oscillating shear layers (inclined between 15 and 18° with respect to the bottom), showed that the occurrence of such coherent patterns affects the turbulence structure mainly in the wall region, constituting a plausible mechanism by which energy is extracted from the mean flow by large scale turbulent motions and then further transferred.
towards smaller eddies, while the coherence is lost (García et al., 1995). The intermittent nature of production and dissipation became also noticeable, taking place in about 21% of the total time.

In open channels as well as in boundary layers it has been observed that the structure of the inertial sublayer seems to be dominated by relatively symmetric transport processes, with ejections and sweeps each contributing an almost identical percentage to the total Reynolds stresses (about 75% and 60%, respectively). In the outer layer however strongly asymmetric transport processes prevail, an ejections become more significant with increasing distance to the wall, accounting for roughly 90% of the total stress (Raupach, 1981). The relatively small amount of the total time occupied by these stress-contributing events confirms the strong intermittency characteristic of turbulence in boundary layers. A direct relation has also been observed between the sequence of bursting events and the turbulence energy budget via the turbulent diffusion represented by the third-order cross-correlations of $u'$ and $w'$. Moreover, for the case of open channel flows, Nakagawa and Nezu (1977) found that near the free surface the negative production (from odd-quadrant events) is of the same order of magnitude as the net production, although both are of small absolute value. This latter observation motivated them to suggest that in this region the energy exchange from turbulence to the mean flow (sometimes called backscatter flux) should not be neglected.

Most experimental evidence tends to confirm the existence of similar coherent motions in boundary layers irrespective of wall roughness conditions. Grass (1971) using flow visualizations by the hydrogen-bubble technique demonstrated the existence both of violent ejections and inrush events over transitional and hydraulically-rough open channels. He noted that over a rough wall, ejections were particularly violent, with ejecting fluid "raising almost vertically from the interstices between the roughness elements". Bessem and Stevens (1984) employed crosscorrelation analysis between wall-probe and hot-wire signals and found that large structures, inclined to the wall at approximately $20^\circ$, also exist in rough-wall boundary layers. Results by Grass et al. (1991, 1993) show that similar vortical structures as in
smooth-wall conditions are also present in rough walls, and that they are likewise linked to the bursting events. These investigators also demonstrated that adjacent to a rough boundary a spanwise flow structure exists, which exhibits a well defined cross-flow wavelength proportional to the size of the roughness elements. Osaka and Mochizuki (1987) also observed low-speed streaks at the top of narrow-cavity bar roughness, which tend to disappear for increasing gap space between bars and to reappear at higher separations.

One of the most interesting behaviors of coherent events under smooth and rough conditions becomes evident in the conditional probability distributions of Reynolds stresses close to the wall, i.e. in the region directly influenced by the dominant length scales associated with the solid boundary, either the viscous or the roughness sublayer. Experimental observations both in open channels with sandgrain roughness (Nakagawa and Nezu, 1977) or wind tunnel models of atmospheric canopy flows (Raupach, 1981) show that sweeps are more important than ejections in maintaining Reynolds stresses close to rough walls, as seems also to be the case in the viscous dominated region for smooth conditions (Kim et al., 1987). Moreover, sweeps have been observed to be dominant events not only just above but also within the roughness elements, and even to make strong contributions to the turbulent momentum transfer for very large values of H, while ejections cease to contribute for relatively small values of H, with H representing the size of the hyperbolic sector in the quadrant technique (see Figure 1).

To the writers' knowledge very few experimental research has been conducted to determine the influence upon the turbulence structure of roughness elements with more than one characteristic length scale. Bandyopadhyay and Watson (1988) conducted a series of laboratory measurements in air tunnels in order to study the influence of the spanwise aspect ratio of the roughness elements upon the apparently universal structure of the flux of shear stress as stated above. Apparently, the particular relationship between drag and the spanwise aspect ratio of the roughness elements is due to the effect of \( \lambda_{AR} \) upon the vortex shedding process (Bandyopadhyay, 1987). From this point of view, vegetation or simulated plants have
an aspect ratio $\lambda_{AR} \ll 1$, while transversely grooved elements have $\lambda_{AR} \gg 1$, and sandgrain or gravel constitute an intermediate case with the spanwise aspect ratio of order one, $\lambda_{AR} = O(1)$. Regarding the variation of third-order joint moments of $u'$ and $w'$, Bandyopadhyay and Watson (1988) found that the skewness of $u'$ ($M_{30}$) showed a similar behavior as with plants or sandgrain elements, but an opposite trend was observed for the other third-order joint moments, i.e. they did not change sign in the roughness sublayer ($M_{12}$ remained always negative, while $M_{21}$ and $M_{03}$ where positive everywhere). The fact that in the region directly influenced by the elements, $M_{30}$ is positive regardless of element shape, while $M_{03}$ seems to be negative for smooth walls, and roughness with $\lambda_{AR} \ll 1$ and $\lambda_{AR} = O(1)$ points out the existence of essential differences in the vertical diffusion of turbulence. The authors explained their observations by suggesting the existence of two different types of dominant coherent vortices near the wall. Although they did not perform any flow visualizations nor conditional sampling analysis, they suggested that the wallward transport of momentum observed in three-dimensional roughnesses may be due to the presence of a
necklace vortex, while the outward transport of momentum in their experiments could be modeled by hairpin vortices.

In summary, besides some existing similarities, the mechanism of turbulence production over smooth wall differs from that over a rough boundary, and these differences are also highly dependent on the dominant length scales associated with the roughness elements. Although the production mechanism for a rough surface has not yet been completely clarified (Nezu and Nakagawa, 1993), some investigators suggest that bursting motion over a rough wall may be triggered by the inrush of high-speed fluid parcels coming from the outer region towards the roughness elements, which could explain why sweeps are more important than ejections in maintaining Reynolds stresses close to rough-walls with small or three dimensional aspect ratios \((\lambda_{AR} \ll 1 \text{ or } \lambda_{AR} = O(1))\). The high-speed fluid coming downward would sweep low-fluid parcels trapped within the roughness sublayer, and as a result low-speed fluid would lift up, oscillate and breakdown. Hence the essential difference would consist in that the triggering mechanism is not the instability of the viscous sublayer, but the inrush due to sweep motions, and hence the dominant vortex structure could consist of necklace vortices as proposed by Bandyopadhyay and Watson (1988). On the other \(\lambda_{AR}\) limit \((\lambda_{AR} \gg 1)\), the turbulence structure over rough-walls consisting of roughness elements with high spanwise aspect ratios may be characterized by outward fluxes of momentum which could be modeled by the existence of hairpin vortices. Nezu and Nakagawa (1993) claim that a renewal model, originally proposed by Einstein and Li (1956) and Hanratty (1956) and later extended by Nakagawa and Nezu (1978), may explain this features because it poses the sweep-phase as an initial condition.

Since numerical modeling still invariably deals with smooth surfaces, carefully conducted experiments making use of traditional and new arriving technologies together with deep physical insights will continue to be the only tool for engineering investigations concerning the turbulence structure over rough surfaces and the associated production
mechanisms. Some clues concerning the different mechanisms of turbulent production might be related to the influence of the roughness elements upon the vortex shedding process. The present work aims at helping in the description of the turbulence structure in environmental open-channel flows, as a necessary step towards a better understanding of the different mechanisms that govern momentum as well as scalar transport processes.

Some theoretical considerations concerning the use of cumulant expansions in turbulence research are presented in §2 as well as a brief description of the spatial averaging technique. Experimental facilities and measuring devices are described in §3, while the experimental conditions are presented in §4. Obtained results are shown in §5, and further analyzed and discussed in §6. Lastly, conclusions of the present work are summarized in §7.
CHAPTER 2. THEORETICAL BACKGROUND

In what follows a brief introduction to cumulant expansions is presented in §2.1 and the concepts of spatial averaging are presented in §2.2.

2.1 Low-order cumulant expansions of probability density distributions

The closure problem of turbulence implies that for the specification of any given moment, some knowledge about higher-order moments is required, which usually do not tend to zero with increasing order. Thus, expansions of the probability density function (or its Fourier transform, called characteristic function) of turbulence variables in terms of moments is generally of no practical use. Instead, some functions of the statistical moments exist, which, even for isotropic turbulence do indeed tend to zero as the order increases (Rotta, 1972). These special functions are called cumulants (Monin and Yaglom, 1971; Stuart and Ord, 1987) and are defined as the coefficients of a series expansion of probability density functions in terms of Hermite polynomials (eigenfunctions of singular Sturm-Liouville problems in an infinite domain), or as the coefficients in a power series of the logarithm of the characteristic function. Herein the bivariate cumulant of order $k$ will be designated as $Q_{ij}$ with $k = i+j$. In some cases third-order cumulant expansions (skewness corrections) have proved to be not only necessary but also sufficient for specifying the influence of coherent events upon the structure of turbulence and its main statistical descriptors (Raupach, 1981).

For the sake of identifying the different types of organized motions and their relative contribution to the total turbulent transport of momentum the conditional quadrant
technique (Wallace et al., 1972) constitutes a widely employed method among turbulence researchers. This technique essentially consists in partitioning the $u'-w'$ plane into five subsectors as shown in Figure 1. Therein, events are classified according with the quadrant sectors, and therefore the term ejection is associated with second-quadrant events while sweeps are related to fourth-quadrant events (hereafter called 2QE and 4QE, respectively).

Using low-order cumulant expansions Nakagawa and Nezu (1977) obtained an expression for estimating conditional contributions from each of these regions to the total Reynolds stress, $p$, which proved to described with reasonable accuracy their experimental observations in smooth and (small-scale) rough open channel flows, namely:

$$
\begin{align*}
 p_1(t) &= p_G(t) + \psi^+(t) & p_3(t) &= p_G(t) - \psi^+(t) \\
 p_2(t) &= p_G(t) + \psi^-(t) & p_4(t) &= p_G(t) - \psi^-(t)
\end{align*}
$$

where:

$$
\begin{align*}
 t &= \frac{R}{(1 - R^2)} q \\
 q &= \frac{u'w'}{\sigma_u \sigma_w} \\
 R &= -\frac{u'w'}{\sigma_u \sigma_w} \\
 p_G(t) &= \frac{R}{2\pi} \exp(Rt) \frac{K_0(|t|)}{(1 - R^2)^{1/2}} \\
 \psi^\pm &= \frac{R}{2\pi} \exp(Rt) K_{1/2}(|t|) \frac{|t|^{1/2}}{(1 + R)} \left[ (1 \pm R)(S^\pm + D^\pm) |t| - \left( \frac{2 \pm R}{3} S^\pm + D^\pm \right) \right] \\
 S^\pm &= \frac{1}{2} (M_{03} \pm M_{30}) \\
 D^\pm &= \frac{1}{2} (M_{21} \pm M_{12})
\end{align*}
$$

and $K_n(\cdot)$ represents the $n$th order modified Bessel function of the second kind. The joint moments $M_{12}$ and $M_{21}$ have been called diffusion factors (Nakagawa and Nezu, 1977). A more detailed description of concepts related to cumulant expansions is given in Appendix A.
Moreover, the use of probabilistic averages together with equations (3) and (4) allow for the estimation of the contribution of each sector to the total Reynolds stress as a function of hole size $H$, herein called $RS_{i,H}$:

\[
RS_{i,H} = \int_{-H}^{+H} q \, p_i(q) \, dq \quad (i = 2, 4) \quad \text{and} \quad RS_{i,H} = \int_{-\infty}^{-H} q \, p_i(q) \, dq \quad (i = 1, 3)
\]

\[
RS_{5,H} = 1 - \sum_{i=1}^{4} RS_{i,H} \tag{5}
\]

Likewise the fractional time occupied by each sector as a function of $H$, $T_{i,H}$, is defined as:

\[
T_{i,H} = \int_{H}^{+H} p_i(q) \, dq \quad (i = 2, 4) \quad \text{and} \quad T_{i,H} = \int_{-\infty}^{-H} p_i(q) \, dq \quad (i = 1, 3)
\]

\[
T_{5,H} = 1 - \sum_{i=1}^{4} RS_{i,H} \tag{6}
\]

Nagano and Tagawa (1987) have shown how the use of third-order cumulant expansions yields a simple relationship between the skewness coefficient of a random variable, $s_k$, and the fractional time during which it was positive or negative ($T_+$ and $T_-$, respectively):

\[
s_k = 3 \sqrt{2\pi} \, (T_- - T_+) \tag{7}
\]

In particular, in turbulence investigations, the interest is principally centered on the relative contributions to the total transport of momentum arising from sweeps and ejections as a function of $H$, $\Delta S_{42,H}$, defined as:

\[
\Delta S_{42,H} = RS_{4,H} - RS_{2,H} \tag{8}
\]

The relative contribution from inward and outward interactions may likewise be defined as:

\[
\Delta S_{31,H} = RS_{3,H} - RS_{1,H} \tag{9}
\]

Using equations (3), (4) and (8) the relative contribution from sweeps and ejections can be estimated as (Raupach, 1981):

\[
\Delta S_{42,H} = - \exp[-H'(1 - R)] \left\{ K_1 \left( H'^2 + \frac{2H'}{(1 - R)} + \frac{2}{(1 - R)^2} \right) + K_2 \left( H' + \frac{1}{1 - R} \right) \right\} \tag{10}
\]
where
\[ H' = \frac{R}{1 - R} \frac{H}{R/2\pi} \]
\[ K_1 = \frac{(1 - R)^2}{R/2\pi} \left[ \frac{S^-}{3} + D^- \right] \quad K_2 = -\frac{1 - R}{R/2\pi} \left[ \frac{2 + R}{3} S^- + D^- \right] \]

In particular for \( H = 0 \):
\[ \Delta S_{42,0} = \frac{1}{3 \sqrt[3]{R/2\pi}} \left[ R S^- - 3 D^- \right] \]

According to this approach, then, knowledge of the correlation coefficient \( R \) and the third-order joint moments suffices to specify the relative contribution from sweeps and ejections to the total turbulent transport of momentum. One of the important applications of such expression arises from the fact that in some cases a good correlation has been found among all third-order joint moments (leaving therefore only one unknown to be measured besides \( R \)) and that, in such case, the term appearing in the energy balance equation representing the transport of the vertical flux of turbulent kinetic energy, \( VFTKE \), may be easily related to \( \Delta S_{42,H} \) (Raupach, 1981):
\[ VFTKE = \frac{1}{2} q^2 w' = \frac{1}{2} \left( \overline{w'^2 w'} + \overline{v'^2 w'} + \overline{w'^3} \right) = \phi(\Delta S_{42,H}) \]

2.2 Conditional Sampling Techniques

One common tool for examining phase- and ensemble-average information both from visual observations and velocity signals is the conditional sampling technique. The general definition of a conditional averaging operator, \( CA[.] \), for any arbitrary signal \( q(x, y, z, t) \) over a pre-determined averaging time, \( T \), is (Nezu and Nakagawa, 1993):
\[ \int_{T} q(x_1, y_1, z_1, t + \tau) \cdot I(x_o, y_o, z_o, t) \ dt \]
\[ CS[q(\Delta x, \Delta y, \Delta z, \tau)] = \frac{\int_{T} I(x_o, y_o, z_o, t) \ dt}{\int_{T} I(x_o, y_o, z_o, t) \ dt} \]
where \((x_o, y_o, z_o)\) and \((x_1, y_1, z_1)\) represent the spatial location of the detection and sampling sensors, respectively; and \((\Delta x, \Delta y, \Delta z, \tau)\) denotes the distance and time lag between both probes. The detection function \(I(x_o, y_o, z_o, t)\) is responsible for selecting the coherent motion in question. Several detection functions have been proposed, and in the present work two of them will be employed, namely the variable-interval-time-average technique (VITA), developed by Blackwelder and Kaplan (1976), and the weighted function proposed by Nakagawa and Nezu (1981). In the VITA technique the detection function is:

\[
I(t) = \begin{cases} 
1 & \text{if } \text{Var}(t) > k_v \sigma_q^2 \\
0 & \text{otherwise}
\end{cases}
\]  

(15)

where \(\text{Var}(t) = AV[q'(t, T)]^2 - AV[q'(t, T)]^2\) and \(AV[q'(t, T)] = \frac{1}{T} \int_{t-T/2}^{t+T/2} q'(t) \, dt\).

The weighting function proposed by Nakagawa and Nezu (1981) is a modified version of the traditional quadrant technique. As mentioned before, the subdivision of the \(u'-w'\) plane in quadrants is a widely used methodology for studying the bursting phenomena, where sorting functions for ejections, \(I_e\), and sweeps, \(I_s\), are defined by:

\[
I_e(t) = \begin{cases} 
1 & \text{if } u'(t) < 0 \text{ and } w'(t) > 0 \\
0 & \text{otherwise}
\end{cases}
\]

(16)

\[
I_s(t) = \begin{cases} 
1 & \text{if } u'(t) > 0 \text{ and } w'(t) < 0 \\
0 & \text{otherwise}
\end{cases}
\]

However, since fluid motions defined by (16) also contain interactions (Lu and Willmarth, 1973), a threshold level \(H\) is typically introduced, such that ejections or sweeps occur only if \(|lu'(t)w'(t)| \geq H\sigma_u \sigma_w\). The problem resides in the determination of the value of \(H\), which is more or less arbitrary. Nakagawa and Nezu (1979) defined \(H\) for ejections and sweeps as the value corresponding to 50\% of the contribution to the turbulent momentum transport of each coherent event at \(H=0\). To overcome the difficulty of selecting the value of the hole size \(H\),
Nakagawa and Nezu (1981) proposed a detection function which is weighted by the instantaneous Reynolds stress itself, i.e., $I_{eNN} = u'(t)w'(t)I_c(t)$ for ejections and $I_{sNN} = u'(t)w'(t)I_s(t)$ for sweeps. These detection functions basically assume that a stronger instantaneous Reynolds stress contributes more to the average structure of the bursting phenomenon. The relative advantage of this method is that no arbitrary parameter needs to be introduced.
CHAPTER 3. EXPERIMENTAL APPARATUS AND FACILITIES

The experiments were carried out in a metallic tilting flume 12.20 m long, 0.90 m wide, and 0.60 m high, with the test section located about 6 m downstream from the entrance. Discharge measurements were performed by means of a manometer calibrated using two parallel weighting tanks of 20,000 pounds capacity each. Two types of anemometers were employed, measuring velocities either separately or synchronized: a TSI model 1239-W ruggedized hemisphere hot-film sensor (HF) and a Sontek acoustic Doppler velocimeter (ADV), both illustrated in Figure 2 and 3. The ADV consists of a new available technology that uses remote sensing techniques to measure the three Cartesian velocity components in a single volume at a maximum sampling rate of 25 Hz. The sampling volume is defined by the interception of four acoustic beams and the width of the transmit pulse, and is therefore shaped as a 3 to 9 mm long cylinder, 6 mm in diameter and centered at about 50 mm from the transducer. For the discussion below, it is worth to note that this sensor is able to measure backflow velocities. Furthermore, the capability of the ADV to acquire data at an externally fixed sampling rate as well as to send a synchronization output, was used to synchronize both anemometers, and thus to allow for the study of the temporal-spatial structure of coherent structures. For that purpose the HF sensor was located at a fixed position atop one of the roughness elements, with its sensing part aligned in the z-axis with the measuring volume of the ADV. As will be shown below, relative turbulent intensities in the region near the average top of the cobbles were of the order of 25%, hence a little higher than the maximum values
recommended for hot-film measurements (Goldstein, 1986), but believed to be accurate enough for the purpose of conducting cross-correlation analysis.

Before performing the measurements on the cobble-bed flow, the degree of reliability of the sensing instrument was checked by comparing results obtained using the acoustic sensor in hydraulically smooth-bed conditions with hot-film data, with well accepted semi-empirical expressions and with numerical results of a k-ε turbulence model. The comparison included vertical distributions of normalized mean flow velocities, streamwise, vertical and spanwise turbulent intensities, Reynolds stresses, turbulent kinetic energy, kinematic eddy viscosity, mixing length, power spectra, and estimation of the energy balance equation. Reasonably good agreement was obtained, except for flow regions close to the free surface, where the dimensions of the instrument introduced some local disturbances, and in a small region located approximately 5 cm from the solid boundary, where echo effects are believed to distort the turbulence measurements. As an example Figure 4 illustrates the power spectra for the streamwise velocity fluctuations in smooth-bed conditions as estimated both from ADV and HF data, with a sampling frequency of 25 Hz and 500 Hz, respectively. It can be observed how side-lobe leakage generates a departure of the ADV-data at approximately

![Figure 2 Hot-Film and Acoustic Doppler Sensors](image)
3 Hz, thus a little below the corresponding Nyquist frequency of 12.5 Hz and at about the frequency where the inertial spectral range starts. It must however be said that no windowing of the data was performed in either case to avoid the well-known Gibbs-phenomenon-related problem. In view of that we can assert that the spectral performance of the ADV is similar to the one by the HF, moreover it will be shown below how in certain circumstances even the inertial spectral range can be adequately captured by the acoustic sensor.

Figure 3 Hot-Film and Acoustic Doppler Boards.
Figure 4  Comparison of Estimated Power Spectra for Streamwise Velocity Fluctuations, computed from data obtained with Traditional Hot-Film Anemometry (HF) and with Acoustic Doppler Sensor (ADV)
CHAPTER 4. EXPERIMENTAL CONDITIONS

The roughness elements used were natural cobbles with a mean diameter, $D_m$, of 0.12 m and a standard deviation of 0.02 m, hence constituting a k-type roughness (Perry et al., 1969), with $\lambda_{AR} = O(1)$ (Bandyopadhyay and Watson, 1988), and were placed in a closed-packed arrangement on the channel bed. Six runs of experiments were conducted under uniform flow conditions corresponding to the data show in Table I, where Re is the bulk Reynolds number defined as $R_h \frac{U_m}{\nu}$, with $R_h$ representing the hydraulic radius and $\nu$ the kinematic viscosity of water, and $Fr$ is the Froude number defined as $\frac{U_m}{\sqrt{gh}}$, where $g$ is the gravitational acceleration.

**TABLE I Experimental Conditions for Cobble Bed Flow**

<table>
<thead>
<tr>
<th>Run</th>
<th>Slope (%)</th>
<th>$h_b$ (m)</th>
<th>$U_m$ (m/s)</th>
<th>$u^*$ (m/s)</th>
<th>Re</th>
<th>$Fr$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grav2</td>
<td>0.90</td>
<td>0.25</td>
<td>0.46</td>
<td>0.085</td>
<td>74000</td>
<td>0.29</td>
</tr>
<tr>
<td>Grav3</td>
<td>0.50</td>
<td>0.24</td>
<td>0.28</td>
<td>0.064</td>
<td>44000</td>
<td>0.18</td>
</tr>
<tr>
<td>Grav4</td>
<td>0.25</td>
<td>0.27</td>
<td>0.29</td>
<td>0.056</td>
<td>50000</td>
<td>0.18</td>
</tr>
<tr>
<td>Grav5</td>
<td>0.25</td>
<td>0.27</td>
<td>0.29</td>
<td>0.056</td>
<td>50000</td>
<td>0.18</td>
</tr>
<tr>
<td>Grav6</td>
<td>0.25</td>
<td>0.27</td>
<td>0.29</td>
<td>0.054</td>
<td>50000</td>
<td>0.18</td>
</tr>
</tbody>
</table>
TABLE II Experimental Conditions for Smooth Bed Flow

<table>
<thead>
<tr>
<th>Run</th>
<th>Slope (%)</th>
<th>h (m)</th>
<th>$U_m$ (m/s)</th>
<th>$u^*$ (m/s)</th>
<th>Re</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smol</td>
<td>0.90</td>
<td>0.17</td>
<td>0.28</td>
<td>0.013</td>
<td>35000</td>
<td>0.22</td>
</tr>
<tr>
<td>Smo2</td>
<td>0.90</td>
<td>0.20</td>
<td>0.36</td>
<td>0.019</td>
<td>50000</td>
<td>0.26</td>
</tr>
</tbody>
</table>

The difficulty in determining values of the local bottom shear stress (or shear velocity) in boundary layers is a long recognized problem, commonly associated for rough flows with the determination of the zero-displacement plane for the origin of the wall-normal axis as well as the estimation of the equivalent sandgrain roughness (Raupach et al., 1991). Herein, the spatially averaged value of $u_*$ was estimated from vertical profiles of Reynolds stress assuming the virtual origin of the $z$-axis at an arbitrary distance from the channel bed equal to $D_m/2$. In open channels this presents the disadvantage that it is usually very difficult to determine the influence of both secondary currents and transverse Reynolds stress upon the vertical turbulent transfer of momentum, in particular for flumes with low values of width-to-depth ratios where positive values of $u'w'$ are expected near the free surface (Nezu and Nakagawa, 1993; Dunn, 1996). This disadvantage becomes even more noticeable in highly three-dimensional flows of small relative submergences. In order to make our results independent of the choice for the zero-plane location, most observations presented herein will be plotted as a function of distance to the bed of the flume, $z_b$, using $h_b$ as scaling length (see Figure 5). Although not systematically conducted for that purpose, our observations show the existence of secondary currents affecting the vertical distribution of Reynolds stress, i.e. decreasing the vertical transfer of momentum, so that values of $u_*$ were estimated taking these effects into account (see Figure 6). The exact value of the momentum transfer towards the bed (viza viz the shear velocity) was however practically irrelevant for most of the results shown in the present report.
As can be observed runs Grav4, Grav5 and Grav6 correspond to the same experimental conditions, but measurements were taken at different points in order to account for any variability due to spatial inhomogeneity. When appropriate, results were compared against observations corresponding to two runs with uniform flow in hydraulically smooth-bed conditions (Table II). All measurements were performed on verticals between cobbles, avoiding measurements in separation zones, and checking permanently for the existence of negative streamwise velocities indicating backflow conditions. A maximum of 100 "instantaneous" negative values of local streamwise velocity were taken as acceptable over a total record of 5000 samples (2.0%), which was observed to be fulfilled approximately for the region $z_b/h_b > 0.35$. As will be shown below, all local time-averaged velocity profiles have positive values.

In order to compute higher-order moments with acceptable accuracy, records of at least 200 seconds in length were acquired at 25 Hz, resulting in a minimum of 5000
samples per measuring point. As can be observed from results presented below, such record length is about 1000 times larger than both the estimated macro-time scale of the flow in the streamwise direction, $T_u$, and any time-scale of the detected coherent events (either 2QE or 4QE), and hence allows to obtain reasonable converging values of high-order central moments up to fifth order with averaging time, $\tau$. This is illustrated in Figures 7 a) and b) for flow regions below and above the average top of the roughness elements, respectively (where $\sqrt{\bar{u'}^2}$ represents the standard deviation of the streamwise velocity fluctuations for $\tau = 200$ sec). In Appendix A a more detailed description is given concerning errors in the estimates of higher-order moments.
Figure 7 a) Variation of Turbulence Statistics with Averaging Time
- Grav5 - $z_b/h_b = 0.34$
Figure 7 b) Variation of Turbulence Statistics with Record Length
- Grav5 - $z_b/h_b = 0.69$
For clarity of presentation, results will be subdivided into four subsections, showing the observed mean flow and turbulence intensities in §5.1, the obtained higher-order moments and the prediction capability of low-order cumulant expansions in §5.2, the energy budget in §5.3, and finally results regarding observations of turbulent coherent structures are presented in §5.4.

5.1 Mean Flow Structure and Turbulence Intensities

Figure 8 illustrates a typical vertical distribution of normalized mean velocities, showing a characteristic "S-shape", and therefore a highly sheared profile near the averaged top of the cobbles, as observed by others (Bathurst, 1994). The solid line in this graph indicates a slope equal to the one corresponding to the log-law and the vertical dashed line corresponds to the average top of the cobbles. To further investigate the diffusive properties, Figure 9 depicts normalized values of turbulent eddy viscosity, $v_T$, compared to results for the smooth-bed case and estimations from the semi-empirical expression by Nezu and Rodi (1986). Dimensionless vertical profiles of streamwise turbulent intensities are shown in Figure 10 and 11 using the shear velocity and the local mean flow velocity as scaling variables, respectively. Profiles of the correlation coefficient of Reynolds stress, $-R$, are depicted in Figure 12.

5.2 Higher-order Moments and Prediction Capability of Low-Order Cumulant Expansions

Vertical variations of third-order joint moments are shown in Figures 13 a) and 13 b) for smooth and cobble bed flows, respectively. Good correlations in the form of simple
Figure 8  Vertical Profiles of Normalized Mean Velocity

Figure 9  Vertical Profiles of Normalized Turbulent Eddy Viscosity compared to Values for Large Relative Submergence
proportional relationships could be established among all third joint moments as shown in Figure 14 for cobble bed flow, where \( q \) stands for the linear correlation coefficient of the regression. Thus we have:

\[
M_{21} = a M_{30}, \quad M_{12} = b M_{30}, \quad M_{03} = c M_{30}
\]

with \( a = -0.73, \ b = 0.66 \) and \( c = -0.78 \). In order to check the applicability of the cumulant discard technique to flow over cobble beds, Figure 15 and 16 show comparisons between measured fourth and fifth moments and their values assuming zero fourth and fifth cumulants, respectively. (Note that for the fourth-order cumulants \( Q_{40} \) and \( Q_{04} \) to vanish, the fourth-order moments \( M_{40} \) and \( M_{04} \) (kurtosis) have to be equal to three, and for the fifth-order cumulants \( Q_{50} \) and \( Q_{05} \) to be zero, the moments \( M_{50} \) and \( M_{05} \) have to be equal to ten times the corresponding third-order moments \( M_{30} \) and \( M_{03} \), respectively).
Figure 11 Vertical Profiles of Normalized Streamwise Intensities with Local Mean Velocity as Scaling Variable

Figure 12 Vertical Variation of the Correlation Coefficient of Reynolds Stress
Figure 13 a) Vertical Profiles of Third-order Joint Moments
Smooth Wall
Figure 13 b) Vertical Profiles of Third-order Joint Moments for Cobble Bed – Grav4
Vertical Dashed Line represents location of Average Top of Roughness Elements
Figure 14 Observed Relationship between Third-order Joint Moments
As noted previously, the expressions for third-order cumulant expansions of $p_i(w)$ may be highly simplified using Equation (17) or, similarly, observed relationships between $D-$ and $S-$ (Figure 17) or $D^+$ and $S^+$ (Figure 18), namely:

$$\frac{D^-}{S^-} = 0.77 \quad \frac{D^+}{S^+} = -0.29$$

By defining $D^-/S^- = m^-, S^- = n^- M_{30}$, $D^+/S^+ = m^+, S^+ = n^+ M_{30}$ we may then rewrite $\psi^\pm$ in (4) as:

$$\psi^\pm = M_{30} \frac{n^\pm R}{6\pi} \exp(Rt) K_{1/2}(1) \frac{1}{(1 \pm R)^2} \frac{1}{(1 \pm R)^2} \left[ (1 \pm R)(1 + 3m^\pm) \right]$$

And thus by knowing only $R$ and $M_{30}$ we may estimate relative contributions of coherent events to the turbulent transport of momentum as well as fractional times in each quadrant.
Figure 16 Observed Relationship between Fifth- and Third-order Moments. Line Represents the Relationship for zero Fifth-order Cumulants
Figure 17 Observed Correlation between $D^-$ and $S^-$

$D^- = 0.77 \ S^-$
$q = 0.95$

Figure 18 Observed Correlation between $D^+$ and $S^+$

$D^+ = -0.29 \ S^+$
$q = 0.69$
All the expressions presented in §2.1 for predicting contributions to the Reynolds stress coming from different quadrants, rely on the common assumption that third-order cumulant expansions provide reasonable approximations to the joint probability density functions of $u'$ and $w'$. Therefore the first natural step was to check the degree of approximation of these low-order expansions against observed distributions. Figure 19 compares observed joint probability distributions to estimations from Gaussian, third-, fourth- and fifth-order cumulant expansions. As expected from the results shown in the Figures 15 and 16, the observed degree of approximation is indeed very good, and it may be also observed that although higher-order expansions improve the predictive capability, third-order cumulant expansions provide good enough approximations for most practical purposes. Another indirect check for the degree of approximation of third-order cumulant expansions is depicted in Figure 20, which shows observed values of $M_{30}$ and $M_{03}$ against estimates from (7). Figure 21 shows results of the observed probability density distribution of Reynolds stresses compared to the estimations obtained by cumulant expansions of third order (Nakagawa and Nezu, 1977). Conditional probability distributions of Reynolds stresses in each quadrant for $H=0$ are illustrated in Figures 22 and 23 for flow above ($z_b/h_b = 0.58$) and below ($z_b/h_b = 0.35$) the average top of the cobbles, respectively, for run Grav4. Lines represent predictions of cumulants expansions of third order using Equation (19), hence with only $R$ and $M_{30}$ as input variables.

Percentage contributions from each quadrant-event to the total Reynolds stress, and the fraction of the total time occupied by each of these events, both as a function of the hole size $H$, were computed and results for cobble-bed flow are shown in Figures 24 and 25, for points above ($z_b/h_b = 0.53$) and below ($z_b/h_b = 0.23$) the average top of the roughness elements, respectively, for run Grav3. Again, lines represent predictions of cumulant expansions of third-order using Equation (19), hence with only $R$ and $M_{30}$ as input variables. Figure 26 illustrates the variation of $\Delta S_{42,H}$ as a function of $H$, where solid lines represent the estimates obtained by using (10) and (11) combined with (18) and (19). Likewise, vertical
a) Gaussian

\[ R = 0.45 \]

b) 3rd-order Cumulant Expansion

\[ Q10 = 0.460 \]
\[ Q21 = -0.259 \]
\[ Q12 = 0.274 \]
\[ Q03 = -0.212 \]

Figure 19 a) and b) Observed Joint Probability Density Distribution compared to Estimations from Gaussian and Third-Order Cumulant Expansions
Figure 19 c) and d) Observed Joint Probability Density Distribution compared to Estimations from Cumulant Expansions of Different Orders
distributions of observed fractional times corresponding to 2QE and 4QE are plotted in Figure 27a) and b) for runs Grav4 and Grav6, respectively, where lines represent estimates obtained by (4) and (6) combined with (18) and (19).

As predicted by the cumulant discard technique, good correlations were observed between $\Delta S_{42,H}$ and the third-order joint moments of streamwise and vertical velocity fluctuations. As an example figure 28 illustrates the measured correlation between $\Delta S_{42,H}$ and $M_{30}$, together with the expression that results from substituting Equation (18) and (17) in (12), namely:

$$\Delta S_{42,0} = \frac{n^-}{3} \frac{[R - 3 \ m^-]}{\sqrt{2\pi}} M_{30} = 0.56 \ M_{30}$$

Data processing also included computations of vertical distributions of normalized mean vertical fluxes of turbulent kinetic energy as shown in Figure 29. Since $V\text{TKE}_+\text{ and } \Delta S_{42,H}$ both show similar behavior (both change sign near the average top of
Figure 21 Observed and Estimated Probability Distribution of Reynolds Stress (a) above and (b) below Cobbles
Figure 22 Observed and Estimated Conditional Probability Distribution of Reynolds Stress
Grav4 - $z_b/h_b = 0.58$

Figure 23 Observed and Estimated Conditional Probability Distribution of Reynolds Stress
Grav4 - $z_b/h_b = 0.35$
Figure 24 Observed and Predicted Relative Contribution to Reynolds Stress and Fractional Time – Grav 3 – $z_p/h_p = 0.53$
Figure 25 Observed and Predicted Relative Contribution to Reynolds Stress and Fractional Time - Grav3 - $z_b/h_b = 0.23$
the cobbles) a possible relationship was investigated, and the observed correlation is shown in Figure 30. The line in this graph represents the expression obtained by replacing Equation (17) and constant values of normalized turbulent intensities in (13) assuming

$$\bar{v'}^2 \bar{w'} = \frac{1}{2}(\bar{u'}^2 \bar{w'} + \bar{w'}^3)$$

(Antonia and Luxton, 1971), namely:

$$VFTKE_+ = \frac{VFTKE}{u'_s^3} = \frac{3}{4}(M_{21}\sigma_{u+}^2 + \sigma_{w+} + M_{03}\sigma_{w+}^3)
= \frac{3}{4}(c_1 \sigma_{u+}^2 + \sigma_{w+} + c_2 \sigma_{w+}^3) \Delta S_{42,0} = -0.88 \Delta S_{42,0}$$

where $c_1 = -1.30$ and $c_2 = -1.39$. The strong dependency of the normalized vertical flux of turbulent kinetic energy with $\Delta S_{42,0}$ is emphasized by results depicted in Figure 31, which essentially show the absence of any correlation between this flux and the relative contribution to the Reynolds stresses from the third and first quadrant, $\Delta S_{31,0}$.

![Figure 26 Observed and Predicted Relative Contribution to Reynolds Stress as function of $H$ for different Distances to the Bed - Grav4](image_url)
Figure 27 a) Vertical Distributions of Observed and Estimated Fractional Time for 2QE and 4QE at H=0 – Grav4
Figure 27 b) Vertical Distributions of Observed and Estimated Fractional Time for 2QE and 4QE at H=0 – Grav6
Figure 28 Observed and Estimated Correlation Between $\Delta S_{42,0}$ and $M_{30}$

Figure 29 Observed Profile of Vertical Flux of Turbulent Kinetic Energy
The correlation between $\Delta S_{42,0}$ and the general turbulence dynamics was further investigated by computing net values of turbulent vertical net-momentum fluxes, VNMF, according to similar definitions used by Wei and Willmarth (1991):

\[
VNMF = \frac{< w'_{+} >^2}{T_+} - \frac{< w'_{-} >^2}{T_-}
\]

where the subscript $i$ represents the $i$th value, and $+$ ($-$) stands for values of the variable while $w'$ was upward (downward). $w'$ stands for $w'/\sigma_w$. Figure 32 shows the observed correlation between the net momentum flux and $\Delta S_{42,0}$, whereas Figure 33 illustrates the uncorrelated relationship between the net momentum flux and $\Delta S_{31,0}$.

![Figure 30 Observed Correlation Between $\Delta S_{42,0}$ and the Vertical Flux of Turbulent Kinetic Energy](image)
Figure 31 Observed Correlation Between $\Delta S_{31.0}$ and the Vertical Flux of Turbulent Kinetic Energy

Figure 32 Observed Correlation Between $\Delta S_{42.0}$ and Net Vertical, Local Momentum Flux
5.3 Energy Budget

Spectral analysis by Fast Fourier Transforms of streamwise velocity fluctuations are shown in Figure 34 for flow above \( z_b/h_b = 0.65 \), just atop \( z_b/h_b = 0.48 \) and below \( z_b/h_b = 0.35 \) the average top of the roughness elements for run Grav4. Power spectra for wall-normal velocity fluctuations are depicted in Figure 35, while cospectra are illustrated in Figure 36 for the same run. Figure 37 illustrates vertical distributions of normalized values of production and transport terms in the turbulent kinetic energy balance equation, as well as turbulent dissipation rates evaluated both from energy balance, \( \epsilon_{eb} \), and from the subinertial range in the power spectra of the streamwise velocity fluctuations, \( \epsilon_{ps} \).

5.4 Turbulent Coherent Structures

The period between consecutive ejections and sweeps was computed both using the VITA technique with slope condition (Blackwelder and Kaplan, 1976) and the conditional
Figure 34 Power Spectra of Streamwise Velocity Fluctuations — Grav4 —

Figure 35 Power Spectra of Bed-Normal Velocity Fluctuations — Grav4 —
quadrant analysis (Wallace et al., 1972). The latter allows for the estimation of the period corresponding to each quadrant $T_{bh,i}$ as a function of the hole size $H$. Results of both methodologies were observed to agree very well for $k_v = 1$ and $H=2$, with $k_v$ representing the threshold level in the VITA technique in equation (15). Figure 39 illustrates the variation of the period for each quadrant event as a function of $H$ for two different distances from the wall. For a given $H$, mean values of the period for ejections and sweeps were observed to be almost constant with distance to the bed, whereas its coefficient of variation was observed to be in the average close to 0.90, both for $H=2$ and $H=4$. These two values of $H$ approximately constitute the lower and upper limits of all the observed values corresponding to 50% of the contribution to the Reynolds stress coming from each coherent motion at $H=0$. Probability density distributions of the period between consecutive events were also computed and are compared in Figures 40 a) and b) to the exponential and log-normal distributions.

The fact that the amount of observed ejections and sweeps is very similar (and hence also their periods), has prompted some researchers to affirm that in the average each
Figure 37 Measured Profiles of Energy Budget Terms – Grav4 –
P: Production Term; Td: Turbulent Diffusion Term; $E_{eb}$: Turbulent Dissipation Rate estimated from Energy Balance; $E_{ps}$: Turbulent Dissipation Rate estimated from Power Spectra.
ejection is followed by one sweep event (Nezu and Nakagawa, 1993). Although true as a mean behavior in the long term, this description does not necessarily reflect the individual structure of the bursting phenomenon. In order to explore further on this aspect, Figure 41 shows histograms of the probability of finding one sweep following one ejection, as well as the probability of having one or more second-quadrant events following one ejection all as a function of hole size H, for two different distances from the bed. Figure 42 depicts similar results for the case of sweep-like events.

Since it is strongly believed that the coherent structures dominating the turbulence in the present flow conditions are highly related to the vortex shedding phenomena at the cobble level, the periodicity of the shedding process was estimated following the methodology proposed by Bandyopadhyay (1987). As can be observed in Figure 38, no clear rise in the long-term autocorrelation function, $R_{xx}(\tau)$ (with $\tau$ representing the time lag), is present, hence supporting the fact that the vortex shedding is fairly periodic over short times, and that this time period varied randomly over a wide range as can be clearly observed in the probability density functions of $T_{bi}$, Figures 40 a) and b). Power spectra of velocity fluctuations also show the absence of any distinctive peak corresponding to a dominant frequency. Computations of short-term autocorrelations for the streamwise velocity component, $\tilde{R}_{uu}(\tau)$, confirm this assertion, showing a very periodic phenomenon over a widely varying time period. Figure 43 shows results of short-time autocorrelations, where the existence of fairly periodic vortex shedding in a low-noise background can be observed. Therein solid and dashed vertical lines represent mean values and one standard deviation as computed using the quadrant technique, respectively. Values of mean periods as well as coefficients of variation were found to be very close to the ones determined by the quadrant technique.

Once the mean period was estimated, values of the Strouhal number, $St$, of vortex shedding as defined by (2) could be determined. Figure 44 shows a plot of $St$ versus
Figure 38 Autocorrelation Function Estimates. (a) Streamwise Velocity. (b) Bed-Normal Velocity. (c) Reynolds Stress. — Grav4 —
the roughness Reynolds number $\text{Re}_k$ together with experimental observations by Townes and Sabersky (1966) and Bandyopadhyay (1987) as well as Black's (1968) theory.

The conditional sampling technique proposed by Nakagawa and Nezu (1981) was employed in order to study the time structure of ejections and sweeps, its advantage being that random fine turbulence cancels out. Figure 45 a) and b) show the conditionally averaged patterns of $< u >_e (\tau), < w >_e (\tau)$ and $< uw >_e (\tau)$, as well as $< u >_s (\tau), < w >_s (\tau)$ and $< uw >_s (\tau)$, where the subindex $e$ and $s$ stand for ejections and sweeps, respectively. As can be observed from the previous figures, the conditional averages were found to be asymmetric around $\tau = 0$, as also observed by others for small-scale roughness conditions (Nakagawa and Nezu, 1981) and therefore the computations of the associated time scales are computed separately as proposed by Nakagawa and Nezu (1981), hence:

$$\mathcal{T} < \dot{u} >_\tau = \int_0^\tau \frac{< \dot{u} >_e (\tau)}{< \dot{u} >_e (0)} d\tau$$  \hspace{1cm} (23)$$

$$\mathcal{T} < \ddot{u} >_\tau = \int_0^{\tau^+} \frac{< \dot{u} >_e (\tau)}{< \dot{u} >_e (0)} d\tau$$  \hspace{1cm} (24)$$

where $\tau^-$ and $\tau^+$ represent the time-lag corresponding to the first zero-crossing, respectively. Accordingly the mean value is defined as $\mathcal{T} < \dot{u} >_e = \frac{1}{2} (\mathcal{T} < \dot{u} >_\tau^- + \mathcal{T} < \dot{u} >_\tau^+)$. The time scales for $< w >_e (\tau), < uw >_e (\tau), < u >_s (\tau), < w >_s (\tau)$ and $< uw >_s (\tau)$ are defined in the same manner as (23) and (24). Assuming Taylor's frozen turbulence hypothesis Eulerian time scales, $\mathcal{T}$, were transformed to spatial scales, $\ell$, using the maximum velocity as a surrogate for the convective velocity in order to compare our results with observations by Nakagawa and Nezu (1981). Figure 46 illustrates the observed variation of the length-scales associated with the different events as a function of distance to the bed. Figure 47 depicts the difference between $\mathcal{T} < \dot{\ell} >_\tau^-$ and $\mathcal{T} < \dot{\ell} >_\tau^+$ during the ejection and sweep phase, both for $u'$ and $w'$, with the subindex $'i'$ standing either for $e$: ejections or $s$: sweeps.
In order to further investigate the spatial/temporal structure of coherent events, the streamwise velocity signal from the hot-film sensor was conditionally averaged using the moving ADV as the detecting probe. Results are shown in Figure 48 and 49 for Grav 4, where the subindex HF stands for data corresponding to the hot-film anemometer.

Two different techniques were applied for investigating the macro-scales of turbulent motion, namely the one based on the spectral representation of velocity fluctuations (Nezu and Nakagawa, 1993) and a more recent one based on the wavelet transform of the velocity series (Gao and Li, 1992). The first technique is based on the estimation of the value of length scale that produces the best fit of the von Karman’s formula to the observed spectrum, and has proven to give reliable results in open channels (Nezu and Nakagawa, 1993; López, 1994; Niño, 1995). The dimensional form of von Karman’s spectrum for the streamwise velocity component, \( S_{uu}(k) \), takes the form:

\[
S_{uu}(k) = \frac{2}{\pi} \frac{\sigma_u^2}{\bar{u}} \left[ 1 + \left( \frac{k}{k_o} \right)^2 \right]^{-5/6}
\]  

(25)

where \( k \) represents the one-dimensional wavenumber in the streamwise direction, \( \bar{u} \) stands for the macro length-scale corresponding to \( u \) (hereafter \( L_w \) and \( L_{uw} \) will represent similar scales for the bed-normal velocity component and for the Reynolds stress, respectively), and \( k_o \) is the wavenumber corresponding to the macro scale. Using Taylor’s frozen turbulence approach:

\[
S_{uu}(k) = \frac{U}{2\pi} G_{uu}(f) \\
\]

(26)

together with equation (25) it is easy to show that the frequency spectrum, \( G_{uu}(f) \), becomes:

\[
G_{uu}(f) = \frac{4}{U} \frac{\sigma_u^2}{\bar{u}} \left[ 1 + \left( \frac{2\pi}{U} f \frac{\bar{u}}{\bar{a}} \right)^2 \right]^{-5/6}
\]

(27)

where \( \alpha = \bar{u}k_o \) is a constant of the order of unity. Equation (27) was used to find the value
of $L_u$ that produced the best fit to the observed spectrum, and a similar procedure was followed for estimating $L_w$ and $L_{uw}$. Figure 50 illustrates results of applying this methodology, where the von Karman expression is compared to measured power spectra. Vertical distributions of dimensionless macro-scales are shown in Figure 51. Results for the streamwise velocity component are compared to the semi-empirical expression of Nezu and Nakagawa (1993) for open channels of large relative submergence:

$$\frac{L_u}{h} = \sqrt{\frac{z}{h}} \quad \frac{z}{h} \leq 0.60$$

$$\frac{L_u}{h} = 0.77 \quad \frac{z}{h} > 0.60$$

Comparison between length-scales obtained from former analysis and from the conditional averaging technique are illustrated in Figure 52 for Grav4.

Wavelet transforms were performed by using two types of mother wavelets, namely the Mexican hat (Figure 53) and Morlet's wavelets (Figure 54). Results for run Grav4 for $z_b/h_b = 0.32$ are depicted in Figure 55, where $TS$ represents the time scale of the process. Results of applying the complex Morlet’s wavelet transform ($\omega_0=8$) give a real as well as an imaginary part, from which spectrograms and phase diagrams can be computed and as an example are illustrated in Figures 56, 57 and 58 also for Grav4 $z_b/h_b = 0.32$. Gao and Li (1992) proposed a methodology for estimating principal time scales of coherent structures using wavelet variances, $\Psi_{WL}^2$, which are computed by integrating wavelet-function fluctuations over time for every given timescale (along horizontal lines in Figure 55). These authors identified the scale representative of the majority of the structures as the one corresponding to the first maxima of the wavelet variance, i.e. $d(\Psi_{WL}^2)/dTS = 0$. Results corresponding to the data of Figure 55 are illustrated in Figure 59. The vertical line in this graph indicates the corresponding prediction of the macro-time scale using Fourier transforms.
The Taylor, $\lambda$, and Kolmogorov, $\eta$, turbulence micro-scales were estimated as follows:

$$
\lambda = \sqrt{\frac{15 \nu \sigma_u^2}{\varepsilon}}
$$

$$
\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}
$$

where the value of the turbulent dissipation was estimated from the inertial range in the power spectra of the streamwise velocity fluctuations. Figure 60 illustrates the dimensionless results as function of $z/h$, in order to compare our observations with the following empirical expressions proposed by Nezu and Nakagawa (1993) for free-surface flows of large relative submergences:

$$
\frac{\lambda}{h} = \sqrt{\frac{15 B_1}{2.3 K R_*}} \left(\frac{z}{h}\right)^{1/4} \exp\left(\frac{z}{2h}\right)
$$

$$
\frac{\eta}{h} = K^{-1/4} \left(2.3 R_* \sqrt{\frac{z}{h}} \exp\left(-\frac{z}{h}\right)\right)^{-3/4} \left(\frac{L_x}{h}\right)
$$

where $R_* = h u_*/\nu$, and for high Reynolds numbers $K$ and $B_1$ are close to 0.69 and 1.00, respectively (Nezu and Nakagawa, 1993).

As mentioned before, synchronized measurements from the hot-film and acoustic sensors allowed for the computation of space-time correlation functions, and, using the time lag to the peak together with Taylor's frozen turbulence approach, to estimate the shape of the coherent motions. Figure 61 depicts the observed shape in dimensionless form, compared to other inclined structures typical of different smooth and rough wall-bounded shear layers.
Figure 39 Variation of Period for different quadrant events as a function of Hole size H
Grav4 – a) $z_b/h_b = 0.65; b) z_b/h_b = 0.48$
Figure 40 b) Observed Probability Density Functions for Period between sweeps (4QE) for $H = 2$, compared to Exponential (E) and Log-normal (LN) Distributions - Grav4 -
Figure 41 Relative Distribution in Time of Ejections as a function of Hole size H.
- Grav 4 - (a) $z_b/h_b = 0.65$; (b) $z_b/h_b = 0.48$
Figure 42 Relative Distribution in Time of Sweeps as a function of Hole size $H$.

- Grav4 - (a) $z_b/h_b = 0.65$; (b) $z_b/h_b = 0.48$
Figure 43 Periodicity in the Streamwise Velocity Fluctuation from Short-Time Autocorrelation Analysis - Grav4 - $z_b/h_b = 0.48$
Figure 44 Universal Variation of Strouhal Number, St, with Roughness Reynolds Number, Re_k. ● x Present Data; Bandyopadhyay (1987): ■ k-type groove; □ d-type groove; ▲ Sandgrain grit 36. Townes and Sabersky (1966) d-type grooves; □ 1/8 in. x 1/8 in.; ○ 1/4 in. x 1/4 in.; + 1/2 in. x 1/2 in.; ▲ 1 in. x 1 in.;
Figure 45 a) Conditionally Averaged Time Correlations for Ejection Events
- Grav4 $z_b/h_b = 0.48$. 
Figure 45 b) Conditionally Averaged Time Correlations for Sweep Events
- Grav4 - $z_b/h_b = 0.48$. 
Figure 46 Vertical Profile of Eulerian Length-Scales of Ejection and Sweep Motions using Nakagawa and Nezu’s (1981) Conditional Analysis - Grav4 -
Figure 47 Vertical Profile of Asymmetries in the Conditionally Averaged Patterns of Streamwise and Bed-Normal Velocities during Ejection and Sweep Motions using Nakagawa and Nezu's (1981) Conditional Analysis — Grav4 —
Figure 48 Conditionally Averaged Patterns of (Hot-Film) Streamwise Velocity at the Top of the Cobbles with Detecting Probe (ADV) at $(\Delta x, \Delta y, \Delta z) = (0, 0, 4.4 \text{ cm})$. – Grav4 –
Figure 49 Conditionally Averaged Patterns of (Hot-Film) Streamwise Velocity at the Top of the Cobbles with Detecting Probe (ADV) at $(Δx, Δy, Δz) = (0., 0., 10.0\ cm)$. – Grav4 –
Figure 50 Observed Power Spectra for Streamwise and Bed-Normal Velocity Fluctuations and Co-spectra. Solid lines represent Von Karman Expression used for estimating Length Scales.
Figure 51 Vertical Distribution of Macro-length Scales obtained from Fourier Analysis
Figure 52 Comparison Between Length-scales obtained from Fourier Analysis (solid line) and Conditional Averaging Technique (Nezu and Nakagawa, 1981) – Grav4.
\[ \psi_{a,b}(t) = \frac{1}{\sqrt{a}} \left[ 1 - \left( \frac{t - b}{a} \right)^2 \right] \exp\left[ -\frac{1}{2} \left( \frac{t - b}{a} \right)^2 \right] \]

Figure 53 Representation of Mexican Hat Wavelet for different values of the parameters.
\[ \psi_{a,b}(t) = \frac{1}{\sqrt{a}} \exp[-i\omega_0 \frac{t-b}{a}] \exp[-\frac{1}{2}(\frac{t-b}{a})^2] \]

\[ \omega_0 = 5. \]

\[ \omega_0 = 8. \]

Figure 54 Representation of Real and Imaginary Part of Morlet's Wavelet
Figure 55  Wavelet Transform using Mexican hat.
Grav4: $z_b/h_b = 0.32$. 

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Figure 56 Spectrogram using Morlet's Wavelet. Grav4: $z_b/h_b = 0.32$. 
Figure 57 Phase Diagram using Morlet’s Wavelet.
Grav4: $z_b/h_b = 0.32$. 
Figure 5.8 Spectrogram and Phase Diagram using Morlet's Wavelet. 
Grav4: $z_b/h_b = 0.32$. 
Figure 59 Wavelet Variance as a function of Time-Scale.
Grav4: $z_b/h_b = 0.32$.

Figure 60 Observed Vertical Variation of Dimensionless Taylor and Kolmogorov Micro-scales
compared to Semi-Empirical Expressions (Nezu and Nakagawa, 1993) – Grav4
Figure 61 Dimensionless Shape of Coherent Structures in Different Wall-Bounded Shear Flows: \( \delta \) represents thickness of the boundary layer, and \( h_p \) plant height in open channels with simulated vegetation.
Several results have been presented showing the absence of a turbulence equilibrium layer in the region immediately above the roughness elements. Normalized profiles of mean velocity thus do not follow a logarithmic distribution, at least for $z/h \leq 0.60$, which coincides with the region where the turbulent diffusion of kinetic energy was observed to be of similar order as the production and dissipation terms in the energy equation. Furthermore, this region is characterized by strong coherent patterns in the conditionally averaged $u'$ and $w'$ signals when the moving ADV is used as the detecting probe. At $z/h = 0.60$, $T_d$ was observed to vanish (i.e. VFTKE is almost constant close to this location) and the velocity profile to follow a logarithmic distribution up from this location. The observed absence of wall similarity is obviously caused by the strong influence that length scales associated with the roughness elements have upon the turbulence dynamics, which explains why normalized values of turbulent intensities using the shear velocity as scaling variable do not collapse onto a single curve, but rather depend on the location of the measuring point. These profiles show almost constant values in each sampling vertical, only slightly higher near the average top of the cobbles, which agree with typical observations in the roughness sublayer of atmospheric flows over plant canopies. It is however interesting to see how a normalization using local mean flow velocities leads to a better collapse of the data. Moreover, much higher values of turbulent intensities relative to the local mean velocity are observed than in usual smooth boundary layers, which clearly indicates that the turbulent energy decays at a smaller rate than the local time-averaged velocity. As will be shown later, both the vertical transport
of turbulent energy fluxes and the production term are believed to play an essential role in this observed behavior.

The correlation coefficient of the Reynolds stress shows almost constant values in the vertical above the elements, with an average of \(-0.40\), slightly increasing towards the average top of the cobbles. Since this coefficient represents some measure of the efficiency of turbulence in transferring momentum relative to the absolute amount of turbulence present, one may conclude that this efficiency is almost constant over the entire flow depth, slightly increasing towards the top of the cobbles. In flow regions below the average top of the elements, where the Reynolds stress decreases towards the bottom of the channel due to the less momentum transfer needed as the roughness contribute with form drag to balance the action of gravity, turbulent intensities both in the streamwise and bed-normal directions decrease at a similar rate, hence maintaining \(R\) almost constant.

Third-order joint moments of streamwise and bed-normal velocity fluctuations were observed to change sign near the average top of the roughness elements, and reasonably good correlations were observed to exist among all of them, resembling similar results found by Raupach (1981) for wind tunnel flow over several cylinder-roughened surfaces and by Raupach et al. (1986) for wind tunnel flow over bluff elements. They found values of \(a = -0.50, b = 0.50\) and \(c = 0.75\). In this later study, however, Raupach et al. (1986) observed that these ratios were different for flow within simulated canopy elements, with relative small values of the diffusion coefficients. Results shown in Figure 14 are probably not enough to extract further conclusions other than to affirm that the degree of correlation tends to decrease for flow below the average top of the cobbles, and that good estimates of turbulence descriptors were obtained using a constant ratio for the entire flow depth. A good degree of correlation has been also observed between the relative diffusion, \(D^-\), and relative skewness, \(S^-\), coefficients, with a ratio \(D^-/S^- = 0.77\) very close to the value of 0.70 found by Nakagawa and Nezu (1977) for open channel flows over smooth and small-scale
roughnesses, and to the value of 0.63 that can be inferred from the aforementioned
observations by Raupach (1981). However very poor correlations have been found between
$D^+$ and $S^+$.

Above the cobbles, kurtoses of $u'$ and $w'$ show values very close to three, i.e.
fourth-order cumulants close to zero, with larger deviations below the average top of the
elements. All other fourth-order as well as fifth-order bivariate joint cumulants may be
assumed to vanish for flow regions above the cobbles, hence confirming that low-order
cumulant expansions constitute a reasonable approximation to the joint probability density
functions (see Appendix A).

Estimates of probability density distributions of Reynolds stresses as well as
conditional probabilities in each quadrant by using cumulant expansions of third-order and
observed proportionalities yield reasonably good agreements, although below the averaged
top of the cobbles deviations are generally larger, hence suggesting that higher order
cumulants may be needed in the expansion, as observed in the previous paragraph. However,
it is interesting to note that trends are correctly predicted by the approximation, namely that
2QE make the biggest contributions to the Reynolds stresses above the cobbles, whereas 4QE
contribute the most to the turbulent transport of momentum for flow below the average top
of the elements. Large tails in the probability distribution of Reynolds stress indicate the
existence of high intermittency levels. Conditional probability distributions, on the other
hand, show that both 2QE and 4QE have not only larger values, but also larger tails than the
two interaction events, which points out that the previously mentioned higher intermittency
is essentially caused by even-quadrant events only (2QE and 4QE). Above the cobbles it is
observed that although 2QE show a smaller peak, they show larger values than 4QE for $|q| \
\geq 5$, which coincides with similar observations by Nakagawa and Nezu (1977) for smooth flow
conditions and confirms the dominance of 2QE over 4QE in the turbulent transport of
momentum. Below the average top of the cobbles, however, the opposite is true, with smaller
peaks of 4QE than 2QE, but with larger values of 4QE for $|q| \geq 5$. The fact that $|q| = 5$
consistently marks the border between regions of different behavior is not a coincidence, but rather a consequence of the constancy of $R$ and the ratio $D^-/S^-$. Using expressions (3) and (4) it can be easily shown that the probability distributions of 2QE and 4QE have equal value when:

$$q = q_{24} = \frac{1 + R}{R} \frac{2 + R + 3 \frac{D^-}{S^-}}{1 + 3 \frac{D^-}{S^-}}$$  \hspace{1cm} (33)

Substituting then $D^-/S^- = 0.77$ and $R = 0.40$ in Equation (33) gives $q_{24} = 4.98$.

Concerning odd-quadrant events (1QE and 3QE) it is interesting to note that their relative behavior seems to be unaffected by the presence of the cobbles, with 3QE having a largest peak, but with higher values of 1QE everywhere in the flow for large enough values of $|q|$. The opposite was observed for smooth flow conditions, i.e. with outward interactions having a larger peak and inward interactions showing higher values for $|q| \geq 2$ (see also Nakagawa and Nezu, 1977). According to these results, outward interactions make therefore larger contributions to the backscatter flow of energy, that is energy transferred from the fluctuations towards the mean flow, irrespective of distance from the bed, whereas the opposite seems to be true for equilibrium layers irrespective of wall roughness condition. The limiting value of $|q|$ for which the probability densities of 1QE and 3QE have equal value varies a lot from one measuring point to another, mainly due to the poor correlation found between $D^+$ and $S^+$. Indeed, using third-order expansions it can be shown that $p_1(w)$ becomes equal to $p_3(w)$ for:

$$q = |q_{13}| = \frac{1 - R}{R} \frac{2 - R + 3 \frac{D^+}{S^+}}{1 + 3 \frac{D^+}{S^+}}$$  \hspace{1cm} (34)

and hence substituting $D^+/S^+ = -0.29$ and $R = 0.40$ we obtain an average value of $|q_{13}| = 8.4$. Note that this value differs from the limiting $|q_{13}| = 2$, observed in the results presented by Nakagawa and Nezu (1977).
Figures 24 and 25 are another confirmation of the high intermittency of the turbulent transport of momentum throughout the whole flow field. It can be observed that all four quadrant events inside a hole sector of size $H=1$ occupy between 40 and 50% of the total time, but contribute only a very few percent to the total turbulent transport of momentum, only about 5% of the total Reynolds stress independent of sensor location. Above the average top of the cobbles, the turbulent structure resembles the typical one of equilibrium layers, with 2QE making the biggest contributions to Reynolds stresses. It should be remarked however, that this fact does not mean that an equilibrium layer as such exists above the cobbles. Below the average top of the roughness elements, 4QE make the largest contributions to the turbulent transport of momentum. For example, Figure 25 shows that events inside an hyperbolic sector of size $H=6$ amounted for approximately 95% of the total time, but contributed only to about 50% of the total turbulent momentum transport. So that nearly 50% of the Reynolds stresses were produced in 5% of the total time. Moreover, only 4QE contributed to that amount, since the contribution of all other events is negligible for values of $H>6$. The same proportions were observed, but with 2QE making the biggest contributions, in our experimental results for flow above the cobbles and for the smooth bed conditions, as well as in Nakawaga and Nezu’s observations both in smooth and rough conditions for small-scale roughness flows.

Previous results clearly suggest that a relationship may exist between the third-order joint moments of $u'$ and $w'$ and the value of $H$ above which the contribution of 2QE to the total turbulent transport of momentum can be assumed negligible. Using third-order expansions of equations (3) and (4) for $p_1(q)$ and equation (19) the value $H_2$ was computed as a function of $M_{30}$, where $H_2$ represents the value of $H$ for which $|q|p_2(q)$ is equal to 0.01, or in other words, the value of the hole size above which contributions of 2QE to the total Reynolds stress may be assumed negligible. The same was done for the case of 4QE, i.e. $H_4$ was computed, and results are plotted in Figure 63. It can be observed, that when 2QE dominate the turbulent transport of momentum (i.e. $M_{30}<0$) variations in the value of the
third-order joint moments ($M_{30}$ in the graph) generate only small alterations upon the limit above which $2Q$E are negligible, but highly influence the value of $H$ above which $4Q$E cease to contribute to the Reynolds stress. The opposite is observed for flow regions where $4Q$E make the biggest contributions to the turbulent transport of momentum. This, of course, agrees with the well-known fact that the higher the values of the skewness coefficient the more intermittent becomes the production of Reynolds stresses (and of turbulent kinetic energy), which may be visualized in the graph by noting that the larger the absolute value of $M_{30}$ the larger also becomes the difference between $H_2$ and $H_4$.

The contributions to the total turbulent transport of momentum from both $1Q$E and $3Q$E become negligible even for small values of the hole size, and a tendency is observed for these contributions to vanish for smaller values of $H$ as the average top of the cobbles is approached. For example, for a distance from the wall equal to half the mean diameter of the cobbles their contributions become negligible for $H \approx 9$, whereas near the average top of the cobbles their contribution vanishes for $H \approx 2.5$. Consequently the negative contribution to the Reynolds stresses disappear for $H \approx 2.5$. For example, for a distance from the wall equal to half the mean diameter of the cobbles their contributions become negligible for $H \approx 9$, whereas near the average top of the cobbles their contribution vanishes for $H \approx 2.5$. Consequently the negative contribution to the Reynolds stresses disappear for $H \approx 2.5$. For example, for a distance from the wall equal to half the mean diameter of the cobbles their contributions become negligible for $H \approx 9$, whereas near the average top of the cobbles their contribution vanishes for $H \approx 2.5$. Consequently the negative contribution to the Reynolds stresses disappear for $H \approx 2.5$.

Furthermore, since the odd-quadrant events are associated with the averaged negative production of turbulent kinetic energy, it results worth to note that both their total contribution and the value of $H$ for which their contributions vanish decreases as the zone is approached, where the peak of the production of turbulence is located. Thus, the fractional backscatter flux of energy from turbulent fluctuations towards the mean flow seems to reach a minimum where the production of turbulence attains its peak. Figure 62 schematically summarizes the vertical variation of the joint probability density function of $u'$ and $w'$.

Third-order cumulant expansions provide good estimates of the variation of $\Delta S_{42,H}$ with $H$, only for low values of the hole size ($H<4$). Large deviations were detected for $H>4$ as can also be observed in the results presented by Raupach (1981).
Figure 62 Schematic Vertical Distribution of Joint Probability Distributions
predicted by third-order cumulant expansions between $\Delta S_{42,0}$ and $M_{30}$ is very similar to the one observed by Raupach (1981), where he obtained a proportionality constant of 0.37 ($\Delta S_{42,0} = 0.37 M_{30}$) compared to the value of 0.56 found in the present study.

Results show that although the relative contribution of 4QE and 2QE to the total Reynolds stress varies with distance from the bed, their total contribution to the turbulent momentum transport seems to remain almost constant for $H=0$ irrespective of position and roughly equal to 1.35. Thus, we may write:

$$\frac{RS_4}{RS_2} = \frac{(RS_4 + RS_2) + (RS_4 - RS_2)}{(RS_4 + RS_2) - (RS_4 - RS_2)}$$

or

$$\frac{RS_4}{RS_2} = \frac{(RS_4 + RS_2) + \Delta S_{42,0}}{(RS_4 + RS_2) - \Delta S_{42,0}}$$

Figure 64 shows observations of the ratio $RS_4/RS_2$ as well as the predictions of (36) assuming $RS_4+RS_2=1.35$ and with $\Delta S_{42,0}$ computed from (20). Results of numerical integration of (5) are also plotted.

Computations of vertical fluxes of turbulent kinetic energy reveal the existence of an upward flux above the average top of the cobbles and a downward flux below that imaginary plane. For this to exist a source of turbulent kinetic energy must be located near the top of the cobbles, which is confirmed by results shown in Figure 37. Since the turbulent diffusion term is negative over an important part of the flow depth, it represents thus a sink term in the energy balance, and thus locally generated turbulent kinetic energy as well as energy coming from other regions (for example by advection) is being transported away from the top of the roughness elements. Moreover, results of turbulence production and diffusion indicate both terms to have similar orders of magnitude (with local production being in the average only twice as large as diffusion) for $z/h \leq 0.60$, or in other words, that the turbulence can not be assumed to be in a state of local equilibrium. Thus no equilibrium layer exists, given a plausible explanation for the lack of applicability of the logarithmic law. Hence unlike the
case of smooth or small-scale roughness conditions, where very close to the bed a flow region exists in which energy is being transferred from the inner to the outer region in a state of local equilibrium, it seems that in the case of small relative submergences this energy transfer process is being conducted under non-equilibrium conditions. Note that the vertical diffusion term tends to be positive (thus a source) both at some distance below and above the turbulence generation zone, i.e. indicating an energy supply to these flow regions. Although it was impossible to measure 3D velocities close to the free surface, the obtained information suggests that the region where turbulence is in state of local equilibrium (for large submergences \(0.2 \leq z/h \leq 0.60\)) end up being squeezed between the free-surface and the inner region.

The existence of such different dynamics in the two flow regions delimited by the top of the cobbles has also been correlated with the net momentum flux in these regions. As demonstrated by López and García (1996), the use of third-order cumulant expansions yields:

\[
VNMF = \frac{2M_{03}}{3\sqrt{2\pi}}
\] (37)

and since \(M_{03}\) may be in turn related to \(\Delta S_{42,0}\) as shown before, we get:

\[
VNMF = \frac{2}{0.56} \times \frac{S_{42,0}}{3\sqrt{2\pi}} = -0.37S_{42,0}
\] (38)

which is the line shown in Figure 32. Hence, dominance of \(2QE\) correlate with upward net momentum flux, whereas dominance of \(4QE\) correlate with downward fluxes. Figure 65 shows the excellent agreement between equation (37) and our own experimental observations. As long as a third-order cumulant expansions constitute a good approximation to \(p(w)\), equation (37) clearly shows that a change in sign of the skewness of the vertical velocity fluctuations directly implies a change in sign of the local net momentum flux. In other words, that the dominance of \(2QE\) (\(4QE\)) upon the turbulent transport of momentum also implies the existence of a local, net upward (downward) momentum flux. The fact that the net momentum
flux does correlate with \( \Delta S_{42,0} \) (or \( RS^4/RS^2 \)) gives also a plausible explanation for experimental observations by Wei and Willmarth (1991) (see Figure 66), who noted that the net momentum flux changes sign for \( z_+ < 30 \), hence near the region where Wallace et al. (1972) and Brodkey et al. (1974) found the ratio \( RS^4/RS^2 \) to go up above unity near a smooth wall.

Defining the dimensionless horizontal local, net momentum flux, \( HNMF \), as:

\[
NMF = \int_{-\infty}^{\infty} u^2 p(\hat{u}) \, d\hat{u} - \int_{0}^{\infty} u^2 p(\hat{u}) \, d\hat{u}
\]

and following similar arguments as before yields:

\[
HNMF = \frac{2 M_{30}}{3 \sqrt{2\pi}}
\]

Moreover, noting that:

\[
\Delta S_{42,0} = \frac{(R - 3 K)}{3 R \sqrt{2\pi}} S^-
\]

where \( K = D^-/S^- \), we may write:

\[
\Delta S_{42,0} = \frac{(R - 3 K)}{4 R} (VNMF - \dot{HNMF})
\]

Equation (42) explicitly indicates the existing relationship between local, net momentum fluxes and total stress fractions due to sweeps and ejections for an hyperbolic sector of size \( H=0 \). Similar expressions are easily obtained for any given size \( H \), yielding:

\[
\Delta S_{42,H} = -\frac{1 - R}{4 R} \exp\left[-H'(1 - R)\right] (VNMF - \dot{HNMF})
\]

\[
\left\{(1 - R) (1 + 3 K) \left(H'^2 + \frac{2H'}{1 - R} + \frac{2}{(1 - R)^2}\right) - (2 + R + 3 K) (H' + \frac{1}{1 - R})\right\}
\]

Figure 67 shows the agreement between our experimental observations for cobble-bed open-channel flow and Equation (42), whereas Figure 68 compares the observed and
estimated values of $\Delta S_{H}$ for different sizes $H$ of the hyperbolic sector using Equation (43) at four dimensionless distances from the bed in run Grav4.

Third-order cumulant expansions together with observed proportionalities provide reasonable approximations to the vertical variation of the fractional times occupied by 2QE and 4QE. Concerning the relative time occupied by each event, results clearly show that above the average top of the cobbles ($RS_2 > RS_4$) $T_{b,2} < T_{b,4}$, and below that plane ($RS_4 > RS_2$) $T_{b,4} < T_{b,2}$, for small values of $H$. All these observations confirm that above the elements relative short and intensive 2QE dominate the turbulent transport of momentum, whereas for flow within the elements relative short, intensive and less frequent 4QE make the biggest contribution to the Reynolds stresses.

Mean period of ejections and sweeps were observed to coincide irrespective of hole size $H$, and having also values very similar to the period of vortex shedding estimated from short-time autocorrelation analysis. Moreover, the shape of the short-time autocorrelation function resembles similar results found by Bandyopadhyay (1987) for flow regions close to small-scale sandgrain roughness with $\lambda_{AR} = O(1)$. As observed by others, the probability density function of the bursting period was found to follow a log-normal distribution, with a coefficient of variation ranging from 0.85 to 1.00. These values lie between coefficients of variation observed by visual inspection (Kim et al., 1971) and by hot-film data analysis (Nezu, 1977). Analysis of the particular structure of each bursting event shows several events of the same quadrant occurring together, with an exponential distribution describing the probability of multiple events of the same quadrant following a similar one. The probability of having one sweep followed by one ejection (and vice versa) was estimated to be very close to 20% for $H = 2$, and slightly increasing with $H$.

Values of the Strouhal number for our high roughness Reynolds number confirm the universal behavior of the shedding process irrespective of roughness type and relative submergence, with a value of $St$ very close to 10.0 as determined by Townes and
Sabersky (1966). As pointed out by Bandyopadhyay (1987) this may constitute another proof for the similar structure of wall-bounded shear flows both under smooth and rough conditions.

Results of applying the conditional averaging technique proposed by Nakagawa and Nezu (1981) show similar patterns for $u'$, $w'$ and $u'w'$ as found by others in smooth and small-scale conditions, both for motions during the ejection and sweep phase. Likewise, it was also found that the time-scale of these conditional averages yields the largest values for the streamwise component and the smallest ones for the Reynolds stress. The latter is yet another confirmation of the high intermittent nature of the vertical turbulent momentum transfer, with a pulse-like behavior. In general average time-scales of ejections and sweeps were very similar over the entire flow depth, with the former becoming slightly larger in a small region above the top of the roughness elements. The most striking behavior was found in the asymmetry of the conditionally averaged patterns, were above the roughness elements the difference $\mathcal{T}_{\langle\cdot\rangle_\tau} - \mathcal{T}_{\langle\cdot\rangle_{\tau'}}$ remained always positive during the ejection or sweep phase both for $u'$ and $w'$, thus indicating that both events occur rapidly and then decay slowly, contrary to common observations for ejections in boundary layers, which occur slowly and then decay rapidly. Conditionally averaged patterns of the hot-film velocity signal using the ADV as the detecting probe clearly show that the phase of the bursting motion above the cobbles goes ahead of that at the top of the elements, which agrees with well-known results in boundary layers. Moreover, as in the case of large relative submergences, the perturbations originating near the average top of the cobbles seem to extend up to the free-surface region, although the coherence gets lost as $Az$ increases (Nakagawa and Nezu, 1981). Moreover, as can also be observed in the conditionally averaged patterns of Grass (1971) and Nakagawa and Nezu (1981), when the detecting probe is in the region near the free surface the states where $\langle \hat{u}_{HF} >_e < 0$ (ejection phase) and $\langle \hat{u}_{HF} >_s > 0$ (sweep phase) are more stable and last much longer than the bursting period.
Wavelet transforms using the Mexican hat show a somewhat disorganized turbulent field for small time scales, whereas as the scale increases the distribution in time tends to be organized into discrete events, whose time-scale agrees well with macro-time scales inferred from spectral and conditional analysis. Likewise, spectrograms using Morlet's wavelet transform show the presence of dominant scales in a somewhat organized pattern, in agreement with previous analyses. Phase diagrams indicate some tendency for lines of constant phase to converge to singularities at scales smaller than the dominant coherent scales, which may be an indication of the (multi-)fractal nature of the increasingly sparse small-scale activity.

Macro length-scales estimated from power spectrum using von Karman's expression agree with corresponding scales of coherent structures obtained from conditional averaging analysis, hence indicating that the detected structures are indeed representative of the large-scale, energy-containing scales responsible for the conversion of mean flow energy into turbulence fluctuations. This latter assertion is further confirmed by the fact that, as mentioned before, the observed coherent events (with $H \geq 2$) are the ones that contribute the most to the production of turbulent kinetic energy and the turbulent momentum transfer. It is interesting to note that this macro length-scales are smaller than the values predicted using expressions obtained for large relative submergences (Nezu and Nakagawa, 1993), which once again clearly indicates that the roughness element's scales (and not the outer-layer scales) are the ones dominating the turbulence dynamics in flow with small relative flow depth. Vertical distributions of turbulence micro length-scales (Taylor and Kolomogorov) do however agree with semi-empirical expressions derived for small-scale roughness conditions, which in turn shows that the smallest scales of motion tend to be independent of the particular conditions affecting the largest scales.

Above the cobbles, macro length-scales as well as spatial/temporal cross-correlation analysis reveal the existence of longitudinally elongated structures of the
order of the mean cobble size, correlated in the vertical over regions close to one third of the
element height. It is interesting to note that present results agree with observations of
structures in the roughness sublayer of atmospheric flows over plant canopies (Raupach et al.,
1991). Accordingly, we may assume that the eddies generated or shed by the roughness
elements are of a size similar to the mean cobble height. Remembering then that the existence
of an equilibrium layer means that the turbulence has to be independent of the scales
associated with the wall (or similarly that the influence of the wall condition remains confined
to the inner layer), and considering that the upper limit of the inner layer is about 20% of the
flow depth (Nezu and Nakagawa, 1993), then we may specify the following condition for a
region with turbulence in local equilibrium to exist:

\[ D_m < 0.20 \, h \]  

(44)

The former expression states that the distance between the bed and the upper limit of the inner
layer has to be larger than the size of the roughness-generated eddies. Then we obtain a lower
limit of the relative submergence for a logarithmic distribution to describe the vertical profile
of mean streamwise velocities in steady, uniform open channel flows, namely:

\[ \frac{h}{D_m} \geq 5 \]  

(45)

which is in agreement with empirically determined values (Bathurst, 1985), and provides a
more physically sound explanation for the empirically observed limits.
Figure 63 Estimated Dependence of $H_2$ and $H_4$ upon $M_{30}$. Points correspond to H2-Experimental Observations

Figure 64 Observed and Predicted Relationship between $RS_4/RS_2$ and $M_{30}$
Figure 65 Observed and Predicted Relationship between NMF and $M_{03}$

$$VNMF = \frac{2M_{03}}{3\sqrt{2\pi}}$$

Figure 66 Wei and Willmarth Observations against Estimates from third-order Cumulant Expansion
Figure 67 Observed Correlation Between $\Delta S_0$ and Differential Local, Net Momentum Flux

$$R = \frac{2.31}{4 \, R}$$

Figure 68 Observed and Estimated Relative Contributions to the total Reynolds Stress using Equation (43)
CHAPTER 7. CONCLUSIONS

Results presented herein clearly demonstrate the strong influence that the cobble scales have upon the turbulence dynamics over the entire flow depth, which determines the lack of wall similarity of the turbulence statistics, and as a consequence no local equilibrium region as such exists in the region close to the elements. Large variations of the turbulent transport term in the vertical, and the fact that both local turbulent production and transport terms are of similar order of magnitude constitute other justifications for the absence of a logarithmic velocity distribution close to the elements. However, at a certain distance from the bed \( z/h = 0.60 \) the net vertical transport of turbulent kinetic energy vanishes, with local production and dissipation becoming of similar order of magnitude, and at the same location a tendency is observed for the mean velocity to be logarithmically distributed. It can be therefore hypothesized that by analogy with the structure of boundary layers with small-scale roughness, the 'inner layer' herein extends up to a distance of approximately \( z/D \approx 1.0 \) or \( z/h \approx 0.60 \), where the scale(s) of the turbulence-generating vortices (rather than distance to the wall) dominate the flow dynamics. This latter limit coincides with the lower limit of the so-called free-surface region, and hence the equilibrium layer (or intermediate region, Nezu and Nakagawa, 1993) becomes squeezed between the two former ones.

Scales associated with the roughness elements also exert important influences upon higher-order statistics, but in a local level 'relative similarity' (as opposed to the 'absolute similarity' observed far enough from solid walls) assumptions have been found to exist, with
simple proportionalities relating bivariate third-order joint moments of streamwise and bed-normal velocity fluctuations. Roughness scales in the case of flows with small submergences seem therefore to exert a strong influence upon the spatial variation of turbulence statistics but to maintain their relative functional dependence quite constant (at least above the elements), which is also somewhat confirmed by the collapse obtained when turbulent intensity profiles are normalized with local values of mean velocities.

All these allow for the accurate estimation of joint probability density distributions using rather simple expressions, which ultimately provide a tool for predicting several characteristic turbulence transport processes such as the relative contribution of coherent events to the turbulent transport of momentum, the local net momentum flux and the vertical flux of turbulent kinetic energy. It is strongly believed that this may also constitute a starting point for a better understanding of other related transport processes (sediment, pollutants, heat, etc.) under environmental conditions.

Moreover, the observed relative similarities show the existence of a strong universal turbulence structure for wall-bounded shear flows as can be observed in the excellent agreement obtained when herein observed values of the constants of proportionality among bivariate third-order central moments as well as values of the ratio $D^-/S^-$ are compared with similar results in smooth and very different roughness conditions. Values of the roughness Strouhal number (both from conditional and short-term autocorrelation analysis) and estimations of probability density functions of the bursting period also contribute to the definition of a similar structure of boundary layers both under smooth and very different types of roughness conditions.

An imaginary plane located near the average top of the cobbles seems to divide flow regions with different dynamics. Above this plane relatively short-duration but strong-intensity, high intermittent 2QE make the largest contributions to the turbulent transport of momentum, whereas below that plane 4QE have been observed to contribute the
most to the Reynolds stress. Contrary to typical results under large relative flow submergences, the Eulerian time-scales estimated from the conditionally averaged patterns of ejections and sweeps above this plane indicate that both events occur rapidly and then decay slowly. The location of this dividing plane is furthermore characterized by absolute maxima of the turbulent production, dissipation and transport terms in the energy balance equation, with the latter behaving as a sink term in this region, thus transporting energy away both upwards and downwards, but under local non-equilibrium conditions. As correctly predicted by low-order cumulant expansions, the existence of this particular structure correlates in turn with net upward kinetic energy and momentum fluxes above the elements, whereas net downward fluxes seem to dominate below the average top of the cobbles. As mentioned above, it is strongly believed that this particular structure should exert a strong influence upon other associated transport processes (e.g. entrainment of sediment into suspension), which needs to be addressed by future investigations.

Furthermore, drawing an analogy to the more-known turbulence structure of smooth-wall boundary layers, we may say that a double structure of turbulence seems feasible, with an outer region and an inner region barely interacting most of the time, but with relatively brief intervals with very strong, pulse-like interactions (bursting). Instead of the inherent instability of the viscous sublayer in smooth walls, it seems that the trigger mechanism is herein related to the process that generates the shedding of vortices from the roughness elements, which causes ejections of low-momentum fluids towards outer regions. Although the relative duration of these events is rather short, the vast majority of the Reynolds stress occurs during the appearance of these structures, which are also believed to be the source of new vorticity. This analogy is really striking if one considers not only the similar shape of the mean velocity profile in the viscous sublayer of smooth walls and the one in the inner-layer of cobble-bed flow (where the high shear is the origin of vorticity), but also the similar behavior of third-order joint moments as well as the ratio $RS_4/RS_2$ in these regions.
Above the cobbles, streamwise as well as bed-normal macro-length scales (computed both using spectral and wavelet transforms) have been found to be of similar order as the roughness elements themselves, and spatial/temporal cross-correlation analysis show the existence of structures inclined towards the bed at approximately 20°. Moreover, the period, and also its probability distribution, of detected vortices shed from the cobbles (computed using short-term autocorrelations) coincides with the period of those structures (2QE and 4QE with $2 \leq H \leq 4$) responsible for almost all the production of turbulence as well as the vertical transport of turbulent momentum. All this evidence clearly defines the shape and type of the dominant turbulent structures, identifying them with elongated, inclined (horseshoe?) vortices, quasi-periodically shed from the elements with longitudinal scales of about the size of the cobbles and vertical as well as spanwise scales of the order of $1/3$ the mean roughness diameter. It is suggested that one of the main differences between these roughness-shed vortices and the structures observed under smooth-wall conditions and rough-beds with large relative submergences, may reside in the fact that ejection and sweep events in the former both grow up rapidly and have a relatively slow decay, whereas that is not the case for ejections in the latter.

In flow regions below the average top of the elements the period of coherent events is similar as for structures above this plane, with the major difference being that time-scales for the streamwise velocity during the ejection phase indicate a slow rise of the signal with a relatively rapid decay (for the bed-normal component this in not quite the case although the signal tends to rise as fast as it decays). Taking this into account, together with the change in sign of all bivariate third-order joint moments, we can not rule out the possibility that the structures dominating the turbulence in this region correspond to the necklace-type vortices proposed by Bandyopadhyay and Watson (1988), although obviously more experimental evidence is needed to support this assertion. An alternative speculation can be drawn using the typical evolution of an asymmetric hairpin-like vortex in a turbulent shear flow: the existence of different scales near the elements clearly explains the possibility of
asymmetric vortices near the roughness, and in the presence of turbulent shear this initial distortion has been observed to evolve into head and legs, where further stretching narrows the leg spacing and expands the head (Figure 69), thus generating local regions with wallward flow.

By imposing the condition that the structures associated with the roughness elements be confined into the inner region in open channels, a limit for the relative submergence has been obtained (Equation 45) in order for a logarithmic mean velocity distribution to exist, which agrees with empirical observations reported in the literature.

Figure 69 Schematic Evolution of an Asymmetric Vortex in a Rough Wall with Turbulent Shear.
CHAPTER 8. REFERENCES


CHAPTER 9. NOMENCLATURE

\(\overline{()}:\) Time average operator over turbulence.

\(<.>_{e} \text{ and } <.>_{s}:\) Conditionally averaged patterns of ejections and sweeps

\((.)':\) Fluctuation over time-averaged value.

\((.)_{+}:\) Variable made dimensionless using wall units or inner variables for smooth walls \((u_{*} \text{ and } v)\)

\(a\) and \(b:\) Parameters of wavelet transforms

\(D_{m}:\) Mean sediment diameter.

\(D^{\pm}: 1/2(M_{21} - M_{12})\)

\(\Delta S_{42,0}\) and \(\Delta S_{31,0}:\) Relative contribution of ejections and sweeps, and third and first quadrant to the turbulent transport of momentum

\(\delta_{ij}:\) Kronecker delta

\(f:\) frequency \((\text{Hz})\)

\(Fr: \) Froude number

\(g: \) gravitational acceleration.

\(G_{uw}, G_{ww}\) and \(G_{uw}:\) Frequency spectrum of streamwise, bed-normal and Reynolds stress fluctuations, respectively.

\(h_{b}:\) Mean flow depth measured from the bed of the flume.

\(h: \) Mean flow depth measured from a distance to the bed equal to \(D_{m}/2.\)

\(H: \) size of hyperbolic sector in the classical quadrant technique

\(HNMF: \) Horizontal net momentum flux
$\eta$: Kolmogorov microscale

$k$: Equivalent roughness size.

$\kappa$: Von-Karman's constant (0.40)

$\varepsilon$: Dissipation rate of turbulent kinetic energy ($\varepsilon_{eb}$ and $\varepsilon_{ps}$, dissipation rates estimated from energy balance and power spectrum, respectively).

$k$: Turbulent kinetic energy ($=1/2[u'^2+v'^2+w'^2]$)

$L_u$, $L_w$ and $L_{uw}$: Length-scale of streamwise, bed-normal and Reynolds stress fluctuations.

$L_{i,e}$, $L_{i,s}$: Length scales of ejections and sweeps determined from conditionally averaged signals.

$M_{ij}$: Bivariate joint moment of streamwise and bed-normal velocity fluctuations of order $k = i + j$.

$\mu$: Fluid dynamic viscosity

$\nu$: Fluid kinematic viscosity.

$P$: Turbulence production term.

$\psi^{a,b}(x)$: Wavelet function.

$\Psi_{WL}(TS)$: Wavelet variance as a function of time scale.

$q$: $u'w'/\sigma_u/\sigma_w$

$Q_{ij}$: Bivariate joint cumulant of streamwise and bed-normal velocity fluctuations of order $k = i + j$.

$R$: Correlation coefficient of Reynolds stress

$R_h$: Hydraulic radius.

$R_k$: Roughness Reynolds number

$R_e$: Flow Reynolds number

$RS4$, $RS2$: Contributions of sweeps and ejections to the total Reynolds stress for $H=0$.

$R_{uu}$: Autocorrelation function for streamwise velocity fluctuations

$\bar{R}_{uu}$: Short-term autocorrelation function for streamwise velocity fluctuations

$q$: Fluid density.

$S^\pm$: $1/2(M_{03} - M_{30})$
$S_{0}$: Bed slope.

$S_{u''}$: Wavenumber spectrum of streamwise velocity fluctuations.

$\sigma_u, \sigma_v, \sigma_w$: Standard deviation of the streamwise, spanwise and wall-normal velocity, respectively.

$\mathcal{I}_{i,e}, \mathcal{I}_{i,s}$: Time scales of ejections and sweeps determined from conditionally averaged signals.

$T_{b,e}$ and $T_{b,s}$: Bursting period determined from ejections and sweeps, respectively.

$T_{i,H}$: Fractional time in quadrant 'i' for hole size H.

$TS$: Time scale of velocity signal in wavelet-transformed space.

$T_{t}$: Turbulent transport of turbulent kinetic energy.

$u_1(u), u_2(v), u_3(w)$: Instantaneous streamwise, spanwise and wall-normal velocities, respectively.

$U, V, W$: Mean streamwise, spanwise and wall-normal velocities, respectively.

$u', v', w'$: Streamwise, spanwise and wall-normal velocity fluctuations, respectively.

$u_*$: Mean bed shear velocity.

$VFTKE$: Vertical flux of turbulent kinetic energy.

$VNMF$: Vertical net momentum flux.

$x, y, z$: Right-handed coordinate system representing streamwise, spanwise and wall-normal axis, respectively, measured from flume bed.

$z_b$: Bed-normal axis measured from $D_m/2$
A.1 Errors in Higher-Order Moment Estimates

Taking into account only errors due to statistical considerations, the accuracy of a parameter estimates $\psi$ based on sample values can be specified by the mean square error (MSE) defined as (Bendat and Piersol, 1986):

$$MSE = E[(\hat{\psi} - \psi)^2]$$  \hspace{1cm} (A.1)

where $E[.]$ represents the expected value. This mean square error may be shown to consist of two parts, a variance term that describes the random portion of the error plus a bias term that specifies the systematic part of the total error. For example, for variance estimates of bandwith limited Gaussian white noise the bias error is proportional to the inverse of the bandwith $B$ times the record length $T$. Thus, for long enough records the bias error may be assumed negligible compared to the random portion of the total error. Furthermore, in engineering practice it is usually desired to define a relative error in terms of the quantity being estimated, and therefore a normalized random error, $\varepsilon_r$, is defined as (Bendat and Piersol, 1986):

$$\varepsilon_r = \sqrt{\frac{E[\psi^2] - E^2[\hat{\psi}]}{\psi}}$$  \hspace{1cm} (A.2)

Likewise, the required total record length, $T$, and the required number of averages, $N$, can therefore be specified for bandwidth limited Gaussian white noise as a function of the desired random error in the estimation of the second central moment yielding
(Bendat and Piersol, 1986):

\[
T = \frac{1}{4} B \epsilon_r^2 \quad N = \frac{1}{2} \epsilon_r^2 \quad \text{(A.3)}
\]

On the other hand, it can be shown that for the case of a Gaussian random process (and neglecting variations in the integral time scale of different moments) it requires roughly 16/3 and 113/5 times as long to get similar relative accuracy in the fourth and sixth central moments as in the second central one, respectively.

By means of the preceding expressions it is therefore possible to specify orders of magnitude of the relative random errors for higher-order moments estimates. For example, assuming a bandwidth of 20 Hz and a record length of 180 seconds the normalized random error in the estimation of the second central moment is approximately 1.7%. For the fourth and sixth central moments the error is close to 3.8% and 4.0%, respectively. Although the assumptions of Gaussian bandlimited white noise are very crude, the rough estimations of the normalized random errors made above allow us to affirm that data records 3 minutes long sampled at 25 Hz provide good estimations of higher-order moments, as shown above in Figures 7 a) and b).

**A.2 Cumulant Expansions of Probability Density Functions**

In statistical theory it is useful to define the Fourier transform of a probability density function, which is called a characteristic function. The theory supporting these definitions may be regarded as a special case of the general theory of Fourier transforms applied to distribution functions. Necessary and sufficient conditions for a function to be a characteristic function are described in some advanced statistics references (see Stuart and Ord, 1987) and therefore will be omitted here. Given a probabilistic density function of a variable \( u' \), pdf\( (u') \), its associated characteristic function, \( \Phi(k) \), is thus defined as:
Where it may be observed that \( \Phi(k) \) is nothing but the (ensemble) average of \( \exp(iu\hat{k}) \), or:

\[
\Phi(k) = \langle \exp(iu\hat{k}) \rangle
\] (A.5)

By expanding now the exponential in a power series, yields:

\[
\Phi(k) = 1 + ik \langle \hat{u} \rangle + \frac{(ik)^2}{2!} \langle \hat{u}^2 \rangle + \cdots + \frac{(ik)^n}{n!} \langle \hat{u}^n \rangle
\] (A.6)

The \( n \)-moments \( M_i = \langle (\hat{u})^i \rangle \) of a probability distribution may hence be regarded as a set of descriptive constants that are useful for measuring its properties, as well as specifying it. However, they are not the only possible set of constants for that purpose, and moreover they are sometimes not even the best available set. From a theoretical point of view a more useful set of constants \( Q_i \) exists, which are formally defined by the following mathematical identity (Stuart and Ord, 1987):

\[
\exp \left[ \sum_{i=1}^{\infty} \frac{Q_i}{i!} x^i \right] = \sum_{j=0}^{\infty} \frac{M_j}{j!} x^j
\] (A.7)

(Note that there is no \( Q_0 \)). If in Equation (A.7) we let \( x = ik \), it is readily seen that the right hand side becomes equal to \( \Phi(k) \) (see Equation (A.6)), thus by taking the natural logarithm on both sides we may write (Chatwin, 1970):

\[
\ln[\Phi(k)] = \frac{(ik)^2}{2!} + \frac{(ik)^3}{3!} Q_3 + \frac{(ik)^4}{4!} Q_4 + \cdots + \frac{(ik)^n}{n!} Q_n
\] (A.8)

(Note that \( Q_1 = 0 \) and that in our dimensionless form \( Q_2 = 1 \) as will become clear later). \( \ln[\Phi(k)] \) is called a cumulant generating function as well (Stuart and Ord, 1987). Hence, the probability density function may be recovered by an inverse Fourier transform so that:

\[
\text{pdf}(\hat{u}') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(k) \exp(-iu\hat{k}) \, dk
\] (A.9)
be found by taking moments of varying order (Papoulis, 1987). The function in terms of Hermite polynomials. The expression for the different cumulants may be shown in terms of probability density a Gaussian distribution. They are the coefficients in the expansion of the probability density.

Equation (9.15) clearly shows why cumulants are considered a measure of the deviation from

\[
(9.16) \quad \langle e^{\frac{\xi x^2}{2}} \rangle = \langle x \rangle^2 + \langle x \rangle^4 
\]

we finally get the leading order expansion of probability density of the cumulants as:

\[
(9.17) \quad \langle x^2 \rangle \langle x^4 \rangle = \langle x \rangle^4
\]

with

\[
(9.18) \quad \langle x \rangle^4 = \langle x \rangle^2 \langle x^2 \rangle
\]

Considering now that by definition:

\[
(9.19) \quad \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x^2 \rangle^n}{n!} \right\} = \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x \rangle^{2n}}{n!} \right\}
\]

we now expand the second exponential inside the integral, yields:

\[
(9.20) \quad \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x \rangle^{2n}}{n!} \right\} = \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x \rangle^{2n}}{n!} \right\}
\]

which may be rewritten as:

\[
(9.21) \quad \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x \rangle^{2n}}{n!} \right\} = \exp \left\{ \sum_{n=0}^{\infty} \frac{\langle x \rangle^{2n}}{n!} \right\}
\]
where \( E(u^2) \) explicitly accounts for the deviations from the Gaussian distribution. Hence:

\[
\int_{-\infty}^{\infty} u^m \ E(u^2) \ du = \int_{-\infty}^{\infty} u^m \ pdf(u^2) \ du - \int_{-\infty}^{\infty} u^m \ G(u^2) \ du \\
\int_{-\infty}^{\infty} u^m \ G(u^2) \ \sum_{j=3}^{\infty} \frac{Q_j}{j!} H_j(u^2) \ du = \int_{-\infty}^{\infty} u^m \ pdf(u^2) \ du - \int_{-\infty}^{\infty} u^m \ G(u^2) \ du \\
m! \frac{Q_m}{m!} + a_{m(m-2)} \frac{Q_{(m-2)}}{(m-2)!} = M_m - \frac{2m!}{2^m m!}
\]

where \( a \) represents the moments of the function \( G(x)H(x) \) (Papoulis, 1987). So that the first three cumulants become:

\[
3! \frac{Q_3}{3!} = M_3 \quad Q_3 = M_3 \tag{A.19}
\]

\[
4! \frac{Q_4}{4!} = M_4 - 3 \quad Q_4 = M_4 - 3 \tag{A.20}
\]

\[
5! \frac{Q_5}{5!} + 60 \frac{Q_3}{3!} = M_5 \quad Q_5 = M_5 - 10 M_3 \tag{A.21}
\]

Likewise, the joint probability density distribution of two variables may be expanded in cumulants yielding (Nakagawa and Nezu, 1977):

\[
\text{pdf}(u', v') = G(u', v') \left[ 1 + \sum_{j+k=3}^{\infty} \frac{Q_{jk}}{j!k!} H_{jk}(u', v') \right]
\]  

where:

\[
G(u', v') = \frac{1}{2\pi \sqrt{1 - R^2}} \exp\left\{ - \frac{u'^2 + 2Ru'v' + v'^2}{2(1 - R^2)} \right\}
\]

\[
R = -\frac{E[u' v']}{\sigma_u \sigma_v}
\]

Qjk are the cumulants of two variables, and \( H_{jk} \) are the bivariate Hermite polynomials of order \( m=j+k \). Expressions for bivariate Hermite polynomials up to order six are given by Barndorff-Nielsen and Pedersen (1979) and are defined as:
\[ H_{jk}(X;\Delta) \exp(-\frac{1}{2} X^T \Delta X) = (-1)^{j+k} \frac{\partial^{j+k}}{\partial x^j \partial y^k} \exp(-\frac{1}{2} X^T \Delta X) \]  

where \( X \) is a vector and \( \Delta \) a matrix so that \( X = (x, y) \) and \( \Delta = [\delta_{ij}] \).

In the present work we will be dealing only with bivariate Hermite polynomials up to order five, and it is worth to note that a small error appears in the reference given for \( H_{40} \) and therefore the correct form is reproduced below:

\[ H_{40}(X;\Delta) = \delta_{11}^2 x^4 + 4\delta_{11}^3 \delta_{12} x^3 y + 6\delta_{11}^2 \delta_{12}^2 y^2 + 4\delta_{11} \delta_{12}^3 x y^3 + \delta_{12}^4 y^4 - 6\delta_{11}^2 x^2 - 12\delta_{11}^3 \delta_{12} x y - 6\delta_{11} \delta_{12}^2 y^2 + 3\delta_{11}^2 \]  

(A.26)

### A.3 Sign of Higher-Order Cumulants

The estimated higher-order moments were employed to calculate the corresponding bivariate joint cumulants as (Stuart and Ord, 1987):

**Third-Order Joint Cumulants**

\[ Q_{30} = M_{30} \]
\[ Q_{21} = M_{21} \]
\[ Q_{12} = M_{12} \]
\[ Q_{03} = M_{03} \]

**Fourth-Order Joint Cumulants:**

\[ Q_{40} = M_{40} - 3 \]
\[ Q_{31} = M_{31} + 3 R \]
\[ Q_{22} = M_{22} - 2 R^2 - 1 \]
\[ Q_{13} = M_{13} + 3 R \]
\[ Q_{04} = M_{04} - 3 \]

**Fifth-Order Joint Cumulants:**

\[ Q_{50} = M_{50} - 10 M_{30} \]
\[ Q_{41} = M_{41} + 4 M_{30} R - 6 M_{21} \]

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The sign of all cumulants higher than second order was observed to be highly dependent upon the distance to the wall. In fact, regions close to the imaginary plane delimited by the average top of the roughness elements seemed to constitute zones of change of sign for all joint cumulants $Q_{ij}$, with $i+j > 2$. The observed behavior is illustrated in Figure A.1. As far as cumulant expansions is concerned, it can be observed that the estimated cumulants do not tend to zero as the order is increased. Instead, the fact that higher order expansions provided better estimates to the observed joint probability density distribution is more related to the relative contribution of the different terms, as will be discussed below.

### A.4 Relative Contribution of Different Order-Terms

Preliminary observations showed higher order expansions to yield better estimates to the observed joint probability density function. As mentioned above this fact could not be necessarily attributed to a decrease in the values of the cumulants with increasing order. Therefore, in order to further elucidate the problem, the relative contribution of the different terms in the cumulant expansions were computed for the conditional probability distributions of $\hat{u}$ for different values of $\hat{w}$, herein denoted as $p_c(\hat{u} | \hat{w}) = p_c(\hat{u} | \hat{w}^{(o)})$. Expanding this function in cumulants yields:

$$p_c(\hat{u} | \hat{w}^{(o)}) = pg(\hat{u} | \hat{w}^{(o)}) \{1. + CF_3 + CF_4 + CF_5 + ...\}$$

(A.28)

where $pg(\hat{u} | \hat{w}^{(o)})$ represents the gaussian distribution, and $CF_i$ denotes the correction factor of "ith" order in the cumulant expansion. Thus, the contribution of the fourth and fifth terms

\[
\begin{align*}
Q_{32} &= M_{32} - M_{30} + 6 M_{21} R - M_{12} \\
Q_{23} &= M_{23} - M_{03} + 6 M_{12} R - M_{21} \\
Q_{14} &= M_{14} + 4 M_{03} R - 6 M_{12} \\
Q_{05} &= M_{05} - 10 M_{03}
\end{align*}
\]

where:

$$M_{ij} = \frac{E[\hat{u}^i \hat{w}^j]}{\sigma_u \sigma_w}$$
Figure A.1 Vertical Distribution of Bivariate Cumulants of 4th and 5th Order
relative to the third-order correction was estimated as CF3/CF4 and CF3/CF5, respectively. Results are depicted in Figures A.2 and A.3 for flow above and below the average top of the roughness elements, respectively. By numbering the graphs in each Figure from bottom to top and from left to right with the variable "i" (so that the lower left graph corresponds to j = 1 and the upper right graph to j = 20) each plot represents the relative contributions of either CF3/CF4 or CF3/CF5 to \( p_c(u^* | 0.20 \frac{(j - 10)}{\sigma_u}) \).

As can be observed in the preceding graphs, the ratio of the third-order correction to corrections of higher order is larger than unity for the majority of the computational field. Although a third-order cumulant expansion seems to be a leading-order approximation to the observed joint probability density of \( \hat{u}^* \) and \( \hat{w}^* \), further criteria will be explored in the next section that justify the selection of a third-order, Gramm-Charlier type distribution.

A.5 Goodness-of-Fit of Cumulant Expansions

In engineering practice the chi-square goodness-of-fit test is a commonly used hypothesis to test the equivalence of a probability density function of sampled data to some theoretical density function of interest (Bendat and Piersol 1986). Therein a statistic with an approximate chi-square distribution is employed as a measure of the discrepancy between the observed and the testing density function. Thus, the hypothesis of equivalence is tested by studying the sample distribution of this statistic, which is defined as:

\[
\chi^2 = \sum_{i=1}^{K} \frac{(F_{mi} - F_{ti})}{F_{ti}}
\]  

(A.29)

with \( F_{mi} \) and \( F_{ti} \) representing the measured (or observed) and theoretical frequency at interval i, respectively, and K being the total number of class intervals in which the observations has been grouped. The region of acceptance of the hypothesis is:

\[
\chi^2 \leq \chi^2_{n, \alpha}
\]  

(A.30)
Figure A.2 b) Relative Contribution of Third- and Fifth-Order Cumulant Expansion Terms
File: 1121–05.VEL
Figure A.3 a) Relative Contribution of Third- and Fourth-Order Cumulant Expansion Terms
File: 1121-11.VEL
Figure A.3 b) Relative Contribution of Third- and Fifth-Order Cumulant Expansion Terms
File: 1121-II.VEL
where $\chi^2_{n;\alpha}$ represents the variable with the chi-square distribution. If $X^2$ is less than or equal to $\chi^2_{n;\alpha}$ the hypothesis is accepted at the $\alpha$ level.

One of the main drawbacks of the procedure stems from the subjectivity in selecting the number of class intervals, $K$, in which the total sample will be partitioned. For the analysis of the observed samples class intervals of 40% the standard deviation in each axis were selected, and the value of the expected frequency $F_i$ was checked so that $F_i > 3$ in all intervals (Bendat and Piersol, 1986).

The number of degrees of freedom for each function to be tested is equal to the number of intervals, $K$, minus the number of different independent linear restrictions imposed on the observations. It is worth to note that one restriction is inherent to the procedure, because the frequency in the last class interval becomes unique once all other frequencies are known. Thus, by assuming that each cumulant is linearly independent from one another the degree of freedom of the expansion decreases with increasing order. For example, for the gaussian joint probability density distribution with zero mean $n = K - 1 - 1$, whereas for the third-, fourth- and fifth-order cumulant expansions the degrees of freedom become $n = K - 1 - 4$, $n = K - 1 - 5$ and $n = K - 1 - 6$, respectively.

For all the cases, however, $n$ remains large enough for the following approximation to be adopted (Bendat and Piersol, 1986):

$$\chi^2_{n;\alpha} = n \left[ 1 - \frac{2}{9} \frac{1}{n} + z_{\alpha} \sqrt{\frac{2}{9} \frac{1}{n}} \right]$$  \hspace{1cm} (A.31)

where $\alpha$ is the level of significance of the hypothesis test, and $z_{\alpha}$ represents the desired percentage point for a standard normal distribution.

In the vast majority of the cases studied the sample value $X^2$ decreased with increasing expansion order. However, since the value of $\chi^2_{n;\alpha}$ also decreases for smaller degrees of freedom, it became clear that third-order approximations provided a substantial improvement over the Gaussian distributions, but that the use of higher-order terms did not
improve the estimation compared to the increase of the number of parameters required. As an example the next Table shows some results corresponding to the same sample location as Figures A.3 a) and b):

<table>
<thead>
<tr>
<th>Expansion Order</th>
<th>n</th>
<th>$X^2$</th>
<th>$\chi^2_{n,\alpha}$ $(\alpha=0.05)$</th>
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<tr>
<td>Gaussian</td>
<td>48</td>
<td>128.05</td>
<td>65.16</td>
</tr>
<tr>
<td>Third</td>
<td>46</td>
<td>58.51</td>
<td>62.83</td>
</tr>
<tr>
<td>Fourth</td>
<td>45</td>
<td>56.15</td>
<td>61.65</td>
</tr>
<tr>
<td>Fifth</td>
<td>44</td>
<td>50.88</td>
<td>60.48</td>
</tr>
</tbody>
</table>

The different approximations to the sampled joint distribution corresponding to the example of the previous Table are the ones illustrated in Figure 19.

A.6 Alternative View on Cumulant Expansions

An alternative view to the expansion of probability density functions in terms of cumulants is to consider the Hermite polynomials as a complete function basis in a Hilbert space with particular inner product, and then express any function (for example the probability density function) as the linear combination of these polynomials. Mathematically the Hermite polynomials are the eigenfunctions of the general singular Sturm-Liouville problem in an infinite domain (real axis):

$$-(p f')' + q f = \lambda w f$$  \hspace{1cm} (A.32)

where in particular $p = e^{-x^2}$, $q = 0$ and $w = e^{-x^2}$, with the eigenvalue of $H_k$ being $\lambda_k = 2k$. If that is the case, then it can be demonstrated (Canuto et al, 1988) that the coefficients of the Hermite expansion of a smooth function defined over $(-\infty, \infty)$ decay faster than algebraically.
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