OVERTOPPING RISK FOR AN EXISTING DAM

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A probability-based methodology to evaluate quantitatively and systematically the overtopping risk of dams is formulated. The risk model consists of fault tree analysis and random process modeling of the flood, wind, and other geophysical forces. This study considers, as an example, mainly overtopping induced by occurrences of flood and wind. A load combination model is established to account for the combined effects resulting from concurrence of flood and wind. A complete procedure for evaluating the risk of overtopping induced by flood and wind including a detailed uncertainty analysis of relevant parameters is presented. The methodology can be generalized to consider quantitatively other conditions affecting the safety of a dam, not merely restricted to overtopping.

Four risk computation techniques are studied and compared; namely direct integration method, Monte Carlo simulation method, mean-value first-order second-moment method, and advanced first-order second-moment method. The advanced first-order second-moment method, which linearizes the Taylor series expansion of the performance function at the failure point, and utilizes the first and second moments of the component variables, is shown to be the preferred one at present for risk evaluation of dams.

The overtopping risk of a medium size earth dam located in northern Illinois is evaluated as an example to demonstrate the use of the proposed risk model and procedure. It has been found that the U.S. Dam Safety Inspection Program, using the normal reservoir pool level together with an inflow flood generated by a 24-hour rainfall to assess overtopping risk, is over conservative.
ACKNOWLEDGMENTS

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NOTATION

\( a^o \) = ratio of time to peak to duration of rainfall

\( A \) = event; also watershed area

\( C_d \) = discharge coefficient

\( C \) = an event; also \( \frac{\partial g}{\partial x_i} \) evaluated at means or failure points of \( x_i \).

\( CN \) = runoff curve number

\( d_F \) = duration of \( h_H(t) \)

\( d_W \) = duration of \( h_Z(t) \)

\( D \) = average depth of reservoir in feet along fetch

\( E[Z] \) = first moment of random variable \( Z \)

\( f_L(z) \) = probability density function of load \( L \)

\( f_R(r) \) = probability density function of resistance \( R \)

\( f_X(x) \) = probability density function of variable \( X \)

\( F \) = fetch or length of water surface in miles over which the wind blows

\( F_e \) = effective fetch length in miles

\( F_R(r) \) = cumulative distribution function of variable \( R \)

\( g(\cdot) \) = performance function

\( h_F \) = maximum height of reservoir level raising above initial reservoir level \( H_o \) due to flood

\( h_H(t) \) = height of reservoir level above \( H_o \) at time \( t \) built up by flood

\( h_r \) = wave run-up height

\( h_T \) = wind tide height

\( h_w \) = \( h_T + h_r \)

\( h_Z(t) \) = height of reservoir level above \( H_o \) at time \( t \) built up by wave

\( H_o \) = initial reservoir level

\( H_c \) = height of the crest of dam
\[ H_F = h_F + H_o \]
\[ H_s = \text{height of spillway crest} \]
\[ i_{ep} = \text{peak intensity of effective rainfall hyetograph} \]
\[ i_p = \text{peak intensity of synthetic hyetograph} \]
\[ i(t) = \text{effective rainfall intensity at time } t \]
\[ L = \text{load; also wave length} \]
\[ L_c = \text{maximum value of combined load} \]
\[ n = \text{number of years} \]
\[ N(\mu_R, \sigma_R) = \text{normal distribution function of variable } R \text{ with mean } \mu_R \text{ and standard deviation } \sigma_R \]
\[ p_f = \text{probability of failure, i.e., } P(L > R) \]
\[ p_F = \text{probability of overtopping induced by occurrence of flood} \]
\[ p_W = \text{probability of overtopping induced by occurrence of wind} \]
\[ p_{FW} = \text{probability of overtopping induced by concurrency of flood and wind} \]
\[ P(A) = \text{probability of failure event } A \]
\[ P_F(T) = \text{probability of overtopping due to flood in time period } T \]
\[ P_{FW}(T) = \text{probability of overtopping due to concurrence of flood and wind in time } T \]
\[ P_W(T) = \text{probability of overtopping due to wind in time period } T \]
\[ Q_i = \text{inflow into reservoir} \]
\[ Q_{ip} = \text{peak discharge of inflow hydrograph} \]
\[ Q_{op} = \text{peak discharge of outflow hydrograph} \]
\[ R = \text{resistance} \]
\[ s = \text{sample standard deviation} \]
\[ S_{\text{max}} \] = maximum reservoir storage
\[ t_a \] = time to peak of rainfall hyetograph
\[ t_c \] = time of concentration
\[ t_d \] = duration of inflow hydrograph
\[ t_r \] = rainfall duration
\[ t_s \] = the time when outflow hydrograph begins
\[ T_r \] = return period
\[ u(t) \] = ordinate of instantaneous unit hydrograph at time \( t \)
\[ V \] = overtopping of the dam
\[ V_W \] = wind velocity
\[ X_i \] = variable, component of load or resistance
\[ Z \] = performance variable
\[ \beta \] = reliability index; also scale factor
\[ \delta \] = coefficient of variation
\[ \Phi(\beta) \] = cumulative standardized normal distribution evaluated at \( \beta \)
\[ \lambda \] = model error correction factor
\[ \gamma \] = Euler constant
\[ \sigma \] = standard deviation
\[ \tau \] = relative occurrence time of \( h_F \) and \( h_W \)
\[ \mu_X \] = mean of \( X \)
\[ \mu_F \] = mean occurrence duration of reservoir level \( h_F(t) \)
\[ \mu_W \] = mean occurrence duration of reservoir level \( h_W \)
\[ \nu_X \] = mean occurrence rate of variable \( X \)
\[ \nu_F \] = mean occurrence rate of flood
\[ \nu_W \] = mean occurrence rate of wind
\[ \nu_{FW} \] = mean concurrence rate of flood and wind
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<td>antecedent soil moisture condition</td>
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<td>CDF</td>
<td>cumulative distribution function</td>
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<td>COV</td>
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<td>MFOSM</td>
<td>mean-value first-order second-moment</td>
</tr>
<tr>
<td>PDF</td>
<td>probability density function</td>
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<tr>
<td>PMF</td>
<td>probable maximum flood</td>
</tr>
<tr>
<td>PMP</td>
<td>probable maximum precipitation</td>
</tr>
<tr>
<td>SCS</td>
<td>Soil conservation service</td>
</tr>
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</table>
CHAPTER 1. INTRODUCTION

1.1. General Remarks

A dam, as defined by Stamm (1973), is the primary control structure placed across a river or channel to facilitate a storage of water. Since early days, dams have been constructed for the purposes of flood control, water supply, irrigation, water power utilization, navigation, recreation and other beneficial uses. With increasing demand on water resulting from world population growth and industrialization, the rate of dam construction has substantially increased in recent years. In the United States, the number of dams has been increased from about 1600 before the beginning of this century to approximately 50,000 in the mid-1970's (Miles; 1976). Figure 1.1 shows the growth of the number of non-federal dams in the United States during this period (Department of Army, 1976).

Dams provide not only beneficial but also potentially adverse effects on the living environment. Dams may fail because of improper design, unexpected natural hazards, operational error, act of war or sabotage, and other causes. Failure of a dam usually causes severe loss of properties and lives, and damage to the environment, thus creating a considerable amount of suffering and hardship. For example, on August 11, 1979 an earthfill dam, Machhu II Dam five miles upstream of the industrial town of Morvi in western India, was overtopped by a flood and burst because of the heavy monsoon rain which occurred conjunctively with an inoperative emergency sluice gate rusted from years of disuse. The town, together with 68 villages along the Machhu River, was hit by the
Two thousand or more persons were killed, $15 million of crops were damaged or destroyed, 12,700 housing units were completely destroyed, and 6,700 other homes were partially damaged (Water Power & Dams Construction, 1979).

Many dams also failed in the United States. Recent disasters include: (1) the collapse of Buffalo Creek Dam in West Virginia on February 26, 1972, killing 125 persons and leaving 4,000 homeless; (2) the breach of Teton Dam in Idaho on June 5, 1976, causing the loss of at least 11 lives and about $400 million of property damage; and (3) the collapse of Kelly Barnes Dam in Georgia on November 6, 1977, killing 39 persons and destroying a number of buildings and houses (Rinehart, 1979; Jansen, 1980).

As to the statistics of dam failure, the United States Committee on Large Dams (1975) reported that before the year 1973, out of the 4914 dams exceeding 45 feet in height, 74 failures occurred – a rate of 1.5%. A survey by the International Commissions on Large Dams (1965) of 64 countries indicated that out of 8925 dams reported there were 202 failures (2.3%). Figure 1.2 shows the percentage of dams that have failed in the years since 1900. The percentage of failure has decreased due to improvement of engineering knowledge and design experience. However, due to increases in population and land development in the area downstream of a dam, the damages and losses from dam failures did not decrease.

Prompted by the failure of the Buffalo Creek Dam, the United States Congress passed the National Dam Inspection Act, Public Law 92-367, in August 1972. This law calls for the U. S. Army Corps of Engineers to
Fig. 1.1 Cumulative number of U. S. non-federal dams.

Fig. 1.2 Percentage of dams failed in decades.
inventory and inspect every dam 25 feet or higher or with a capacity to
impond at least 50 acre-feet of water. As of October 1, 1978,
inspection reports on 1793 dams had been completed by the Corps (1978).
A startling number, namely 353 or 20%, of those dams inspected had been
declared unsafe. The procedures specified in the dam safety inspection
program include collection of pertinent data, visual inspection, and
comparison of spillway capacity to peak discharge of specific floods.
Nevertheless, it lacks the systematic and scientific basis for
quantitative assessment of the safety of dams. Moreover, a search of the
literature reveals that there was little past research accomplished that
can be used for a quantitative evaluation of dam safety.

Soon after the failure of Teton Dam, President Carter issued a
memorandum dated April 23, 1977 to more than twenty federal agencies
responsible for dams urging them to undertake a thorough review of dam
safety practices, utilizing new technology, and specifically suggesting
probabilistic or risk-based analysis. In addition to this memorandum,
the reports by the Inter-Agency ad hoc Committee on Federal Dam Safety of
the Federal Coordinating Council for Science, Engineering, and Technology
(1977), and by the Committee of the Safety of Dams of the Assembly of
Engineering, National Research Council (1977), both recommend
implementation of risk and reliability analysis. Although both
committees acknowledged its potential usefulness, risk and reliability
analysis is currently not used in dam safety evaluation, due mostly to
the lack of trained personnel and clearly defined methodology.

Dams fail because of uncertainties in natural environment and human
actions. Clearly, dam failure is not a deterministic event, and hence
quantitative evaluation of dam safety requires probability theory. The present study is an attempt to assess the risk of a dam quantitatively and systematically. Risk models and procedures for evaluating the risk of a dam due to overtopping will be presented.

1.2. Causes of Dam Failure

Earth dams may fail because of overtopping, seepage, piping, instability, destruction and other causes. The statistics of past dam failures due to different causes from various sources were summarized by Baecher et al. (1980) and listed in Table 1.1. It shows that overtopping and seepage or piping are the major causes of failure.

Table 1.1 Causes of dam failure in percentage

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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Overtopping</td>
<td>23</td>
<td>30</td>
<td>28</td>
<td>38</td>
<td>36</td>
</tr>
<tr>
<td>Piping or seepage</td>
<td>40</td>
<td>38</td>
<td>44</td>
<td>44</td>
<td>30</td>
</tr>
<tr>
<td>Sliding</td>
<td>2</td>
<td>15</td>
<td>10</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>35</td>
<td>17</td>
<td>18</td>
<td>9</td>
<td>19</td>
</tr>
</tbody>
</table>

There are many possible factors which are related to causes of dam failure and can be grouped as follows (Yen and Tang, 1979; Sherard et al., 1963):

Hydrological factors - including flood frequency, volume, peak and time distribution of the flood, rainfall-runoff relationship, initial
water stage in the reservoir prior to a flood, sediment in the reservoir and ice and debris.

Hydraulic factors - including spillway capacity, flood routing, waves, intakes, sluice, erosion and scour protection, and faulty gates or valves.

Geotechnical factors - including unfavorable soil conditions such as weak layers, fissured material, adversely oriented joints, seepage, piping, excessive pore pressure, uneven settlement, slope instability during rapid drawdown of reservoir level and landslide.

Seismic factors - including seismic stability of dam, liquefaction, earthquake-induced cracks, tsunamies and waves.

Structural and construction factors - including inadequate structural design, inferior material used, construction errors and poor quality control.

Operational factors - including improper maintenance, incorrect operational procedures, human errors and negligence.

Other factors - such as act of war, sabotage and impact of boats or other materials.

All these factors are subject to uncertainty and may be different from dam to dam and vary with time and space. All the contributing factors and possible causes of failure should be considered systematically in the evaluation of the total risk of a dam. It would require a tremendous amount of information, manpower and time to accomplish this task. The present study concentrates on the evaluation of the part of overtopping risk of a dam due to hydrological and hydraulic factors. Nevertheless, the methodology presented in this study
can be extended to consider other contributing factors in the evaluation of total risk of a dam.

1.3. Objectives and Scope of Study

As the statistics (Table 1.1) show, overtopping is a major cause of dam failure. The overall objective of the present study is to formulate a practical methodology to evaluate systematically and quantitatively the risk of failure in an existing dam due to overtopping. Specifically, the objective can be divided as follows:

1. To review existing procedures for risk calculation and evaluation.
2. To formulate a risk model for evaluating overtopping failure probability induced by different geophysical forces.
3. To assess the effects of various uncertainties on the risk of overtopping of a dam.
4. To apply the developed risk model to evaluate the overtopping risk of an existing dam.

Overtopping of a dam occurs when reservoir water overtopped the dam. Overtopping can result from failure to make timely release of flood water through the spillway and outlets, or from wave actions induced by the occurrence of strong wind, landslide or earthquake. In this study, overtopping risk due to the occurrence of flood and wind will be investigated. To accomplish the objectives, the present study was conducted in the following sequence. In Chapter 2, the relationship between the risk and uncertainties is investigated. A review of
different risk calculation methods is presented in Chapter 3. Comparison of accuracy, consistency, and efficiency among the different risk calculation methods is made. An efficient technique of risk calculation is also proposed. The risk models for overtopping induced by different geophysical forces are developed in Chapter 4. The risk evaluation procedures of overtopping due to floods and wind are also studied. In Chapter 5, a case study of an existing earth dam is used as an example to illustrate the proposed risk models and evaluation procedures. The risk of overtopping induced by floods, and wind are evaluated. The risk evaluation following the guidelines proposed by the U. S. Army Corps of Engineers is also investigated and discussed. The summary, conclusion and suggestions for future research are described in the last chapter.
CHAPTER 2. DEFINITION AND ANALYSIS
OF RISK AND UNCERTAINTY

2.1. Definitions of Risk and Uncertainty

2.1.1. Definition of Risk

Different definitions have been given explicitly or implicitly to the term "risk". For example, risk has been related to (1) the probability of failure (Yen and Tang, 1976), (2) the reciprocal of expected length of time before failure (return period) (Borgman, 1963), (3) the expected cost of failure (Young et al., 1970), and (4) actual cost associated with failure (Bras, 1979). In this study, risk is defined as the probability of failure.

Failure of a system is defined as an event in which the system fails to function with respect to its original design objectives. Dam failure may involve physical damage of the dam structure such as failure of the outlet works. It may also involve injury or loss of lives, property loss or damage, and adverse changes to the environment downstream of the dam. Satisfactory performance of a dam system is governed by generalized load variables and resistance variables. Failure events are usually the joint occurrences of excessive loads and weak resistance of the dam structure and/or possible human errors.

Loads on a dam are those external forces that act on the dam. These forces may be geophysical or from human action. For example, loads may be the flood, earthquake, landslide, wind, ice, act of war, wave, seiche, and hydraulic pressure.
Resistance of a dam is the capacity or the internal strength of the structure to withstand damage, overtopping, breaching, erosion, collapse, and similar disruptions. It depends on the design of the dam which involves its type, shape, dimensions, function, construction process and material, and other factors (e.g., height of the dam crest, type and discharge capacity of the outlet works, and the strength and size of the core).

The risk of a dam is the total risk which is the combination of the probabilities of all the possible occurrences of various failure events. Determination of the acceptable level of risk should be based on economic, social, political, historical and other factors (Yen and Ang, 1971; ASCE Task Committee, 1973; Bertle, 1973; CIRIA, 1977). From the record of dam failures given by the United States Committee on Large Dams (1975), the number of dams failed is plotted in Fig. 2.1 as a function of the number of years of service before failure. The mean service life for the 74 failed dams is 17.4 years. From the data obtained by this 1975 study (74 failures as of 1973 of the 4914 dams over 45 feet high), the probability of dam failure is estimated as $8.7 \times 10^{-4}$ per dam-year which may be regarded as a reference for determining the acceptable risk.

2.1.2. Definition of Uncertainty

Uncertainty is defined as the variabilities of the outcomes in repeated occurrences, observations, or estimation. In dealing with uncertainties in the design of hydraulic structures, Yen and Ang (1971) classified them into two types, namely, objective and subjective uncertainties. The objective uncertainties are measurable or
average service life = 17.4 year

probability of failure = \( \frac{1}{17.4} \times \frac{74}{4914} = 8.7 \times 10^{-4} \) per year

Fig. 2.1 Number of dam failed vs. service life in years.
quantifiable such as sample statistics of observed data, and corresponding deductive probabilistic information. The subjective uncertainties are those that lack a data base for their evaluation and must be described and handled subjectively on the basis of judgment and intuition.

According to the classification of variables discussed in Section 1.2, uncertainty of the variable can also be grouped as hydrologic, hydraulic, geotechnic, structural, construction, seismic and other uncertainties. Usually, uncertainties are expressed in terms of either a probability density function (Davis, 1972), or in terms of coefficient of variation (Ang and Cornell, 1974). Through a combined consideration of all the random variables relating to the failure of the system and their uncertainties, the total risk of a system may be determined accordingly.

2.2. Analysis of Risk and Uncertainty

2.2.1. General Expression for Uncertainty

The uncertainties of the load and resistance and of their component variables can be described mathematically by the probability density functions, $f_L(l)$ and $f_R(r)$, respectively, as shown in Fig. 2.2. The probability density function (pdf) gives the relative likelihood of obtaining the various values of the random variable. The pdf is usually obtained based on a histogram of the observed data. Sometimes it may be established theoretically or prescribed subjectively.

Instead of pdf, either the variance or the coefficient of variation of the variable can also be adopted to denote the degree of uncertainty. The variance of a variable $X$, $\text{Var}(X)$, is defined as
Fig. 2.2 Probability density functions for load and resistance.

Fig. 2.3 Probability density function for performance variable $Z$. 
\[ \text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu_X)^2 f_X(x) \, dx \] (2-1)

in which \( f_X(x) \) is the pdf of \( X \), and \( \mu_X \) is the mean of \( X \) which is defined as

\[ \mu_X = \int_{-\infty}^{\infty} x f_X(x) \, dx \] (2-2)

The mean value estimated from a set of \( n \) samples of \( X \), \( \bar{x} \), is given by

\[ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \] (2-3).

The square root of \( \text{Var}(X) \) is known as the standard deviation, \( \sigma \). Its value may be estimated from samples of \( X \) and denoted as \( S \) by

\[ s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}} \] (2-4).

The coefficient of variation (COV or \( \delta \)) is the ratio between the standard deviation and the mean value, namely

\[ \delta = \frac{\sigma}{\mu_X} = \frac{s}{\bar{x}} \] (2-5).

2.2.2. General Expression for Risk

The failure of a system can be regarded as the event that the load, \( L \), exceeds the resistance, \( R \), of the system. The risk, defined as the probability of failure, can be expressed mathematically as

\[ \text{Risk} = p_f = P \{ L > R \} \] (2-6).

Reliability is defined as the complement of the risk, then

\[ \text{Reliability} = 1 - p_f = P \{ L \leq R \} \] (2-7).
Let the variables $X_i$, $i=1,2,...,n$, represent the component variables of the load, and $X_j$, $j=n+1,...,m$, represent the component variables of the resistance. Some of the $X_i$'s may be identical with some of the $X_j$'s. Then, both $L$ and $R$ can be expressed as functions of these component variables

$$L = g_L(X_i) \quad i = 1, 2, ..., n$$  \hspace{1cm} (2-8)

and

$$R = g_R(X_j) \quad j = n+1, ..., m$$  \hspace{1cm} (2-9)

Since $X_i$ and $X_j$ may be random variables subject to uncertainties, $R$ and $L$ are random variables as well. Let the density function for $L$ be denoted as $f_L(l)$, and that for $R$ as $f(r)$, as shown in Fig. 2.2. The risk can be expressed as

$$p_f = \int_0^\infty \int_0^l f_{R,L}(r,l) \, dr \, dl$$  \hspace{1cm} (2-10)

in which $f_{R,L}(r,l)$ is the joint probability density function of $R$ and $L$. If the resistance $R$ is statistically independent of load $L$, then Eq. (2-10) can be simplified as:

$$p_f = \int_0^\infty \int_0^l f_R(r) f_L(l) \, dr \, dl$$  \hspace{1cm} (2-11)

Rearrange Eq. (2-11) as

$$p_f = \int_0^\infty f_L(l) \left[ \int_0^l f_R(r) \, dr \right] \, dl$$  \hspace{1cm} (2-12)

and, therefore

$$p_f = \int_0^\infty f_L(l) F_R(l) \, dl$$  \hspace{1cm} (2-13)

in which $F_R(l)$ is the cumulative distribution function of $R$.

Alternatively, Eq. (2-6) may be rewritten as

$$p_f = P\{ R - L < 0 \}$$  \hspace{1cm} (2-14)
This means that failure can also be defined as either the event $R - L < 0$, $(R / L) - 1 < 0$, or $\ln (R / L) < 0$. The above three equations can be expressed in a general form as

$$P_f = P \left[ Z < 0 \right]$$

(2-17)

where $Z$ is called performance variable and is equal to $R - L$, $(R / L) - 1$ or $\ln (R / L)$. Failure therefore corresponds to the event of having a negative value of the performance variable, $Z$, as shown in Fig. 2.3.

Since $L$ is a function of $X_i$ and $R$ is a function of $X_j$ as shown in Eqs. (2-8) and (2-9), $Z$ is also a function of $X_i$ and $X_j$, and can be expressed as

$$Z = g \left( X_1, X_2, \ldots, X_n, X_{n+1}, \ldots, X_m \right)$$

(2-18)

The function $g(.)$ describes the functional relationship of the failure event and is called the performance function. When $Z = 0$, the function $g(.)$ represents the failure surface in an $m$-dimensional space.

2.2.3. Sensitivity of Risk to Component Uncertainties

The risk of a system arises from the uncertainties of the load and resistance of the system. Relationships between risk and uncertainties will be studied as follows. Let the load, $L$, and the resistance, $R$, be normally distributed, represented by $N(\mu_L, \sigma_L)$ and $N(\mu_R, \sigma_R)$, respectively. Consequently, the performance variable, $Z$, in Eq. (2-14) is also normally distributed with $N(\mu_R - \mu_L, \sqrt{\sigma_R^2 + \sigma_L^2})$, and the risk
expressed by Eq. (2-11) can be rewritten as (e.g., see Kapur, 1977)

\[ p_f = 1 - \Phi \left[ \frac{\mu_R - \mu_L}{\sigma_R^2 + \sigma_L^2} \right] \]  

(2-19)

or

\[ p_f = 1 - \Phi \left[ \frac{\mu_R}{\mu_L} - 1 \right] \sqrt{\left( \frac{\mu_R}{\mu_L} \right)^2 \delta_R^2 + \delta_L^2} \]  

(2-20)

in which \( \Phi(\beta) \) denotes the cumulative standard normal distribution evaluated at the reliability index, \( \beta \). The value of \( \Phi \) increases as \( \beta \) increases.

From Eq. (2-20), the relationships among \( p_f, \mu_R / \mu_L, \delta_L \) and \( \delta_R \) are shown in Figs. 2.4 and 2.5. Figure 2.4 shows that for constant \( \delta_R \) and \( \delta_L \), \( p_f \) decreases with increasing values of \( \mu_R / \mu_L \). This means that the risk of an existing system can be reduced by increasing the strength or capacity of the system or by reducing the load to the system. Figure 2.5 depicts the effects of \( \delta_R \) and \( \delta_L \) on \( p_f \) for different values of \( \mu_R / \mu_L \). It shows clearly that by increasing \( \delta_R \) and \( \delta_L \), the value of \( p_f \) is higher for \( \mu_R / \mu_L > 1 \), but is lower for \( \mu_R / \mu_L < 1 \).

If the uncertainty in the resistance is not considered, i.e. \( \delta_R = 0 \), the value of \( p_f \) is found to be higher when \( \mu_R / \mu_L < 1 \), and lower when \( \mu_R / \mu_L > 1 \) (Figs. 2.4 and 2.5). For a hydraulic structure design, \( \delta_R \) and \( \delta_L \) may denote hydraulic and hydrological uncertainties, respectively, (Yen, 1975; Tung and Mays, 1980). Therefore if the hydraulic uncertainty is neglected, the risk value will be overestimated when \( \mu_R / \mu_L < 1 \) (with \( p_f > 0.5 \)) and be underestimated when \( \mu_R / \mu_L > 1 \) (with \( p_f < 0.5 \). The
Fig. 2.4 Risk vs. $\mu_R/\mu_L$ for constant $\delta_R$ and $\delta_L$. 
Fig. 2.5 Risk vs. $\delta_R$ and $\delta_L$ for constant value of $\mu_R/\mu_L$. 
concept that the risk is underestimated by ignoring the hydraulic uncertainty as stated by Tung and Mays (1980) is thus not universal.

In addition to the $\mu_R/\mu_L$ ratio and the magnitude of coefficient of variation, (COV), the risk is affected also by the type of probability distribution assigned to the load and resistance. Figures 2.6 and 2.7 show the results of risks evaluated for different distributions of $R$ and $L$ which include normal, log-normal, and uniform distributions. For a given value of $\mu_R/\mu_L$, the risk is found to be less sensitive to the distributions with increasing $\delta_L$ and $\delta_R$. However for given values of $\delta_L$ and $\delta_R$, the risk is more sensitive with increasing values of $\mu_R/\mu_L$. Observe that the risk value evaluated by ignoring the resistance uncertainty is more sensitive to the distributions than the case in which both uncertainties of the load and resistance are considered (Figs. 2.6 and 2.7).

The sensitivity of risk, $p_f$, to the distributions of $R$ and $L$ therefore depends on the $\mu_R/\mu_L$ ratio and the magnitude of $\delta$. Suppose the risk is regarded as sensitive to the distribution whenever the ratio between any two risk values calculated with different distributions exceeds a specific value, e.g., one order in magnitude. Thus the combination of values of $\mu_R/\mu_L$, $\delta_R$ and $\delta_L$ such that risks are sensitive to distributions may be portrayed in Fig. 2.8. The dash lines in Fig. 2.8 denote the risk contours which are constructed from the mean of the risk values calculated from normal, lognormal, and uniform distributions for different values of $\mu_R/\mu_L$, and $\delta_R$ and $\delta_L$. The conclusion that $p_f$ is not sensitive to the type of distribution when $p_f > 10^{-3}$ or $\delta > 0.3$ as pointed out by Ang (1970) and Yen and Ang (1971) is only a gross statement.
Fig. 2.6 Risk for three different distributions of $R$ and $L$ ($\delta_L = \delta_R \neq 0$).
Fig. 2.7 Risk for three different distributions of $L$, $\delta_R = 0$. 

- N: Normal distribution
- LN: Log-normal distribution
- U: Uniform distribution
Fig. 2.8 Sensitivity of risk to distributions of R and L.
CHAPTER 3. CALCULATION OF RISK

3.1. Methods of Risk Calculation - Literature Review

The probability of failure, i.e., the risk, of a structure can be calculated if the statistics and distribution, i.e., uncertainties, of each variable \( X_i \) in Eq. (2-18) and the performance variable \( Z \) in Eq. (2-17) are known. Depending on the availability of the statistical information on each of \( X \)'s and on the complexity of the function \( g(.) \) in Eq. (2-18), various methods can be used to evaluate the risk. The important methodologies proposed by previous investigators relevant to and potentially useful for dam safety evaluation are described briefly as follows.

3.1.1. Method of Return Period

Hydraulic structures such as dams, levees, culverts, storm sewers and wave breakers are traditionally designed to withstand some specific level of hydrologic loading. The design magnitude, \( Q \), for the structure is generally expressed as the magnitude of a hydrologic event, such as precipitation or flood, having a design return period, \( T_R \). This design return period is usually specified by some regulations, codes or guidelines. For example, for safety inspection of dams, the U.S. Army Corps of Engineers (1976) recommended that a small dam with low hazard should safely pass the spillway design flood having a return period equal to 50 to 100 years. For storm sewers in a commercial district, the design rainfall of approximately a 10-year return period is suggested in ASCE's Manuals and Reports on Engineering Practice No. 37 (1969).
The return period, $T_r$, is defined as the average length of time until a specified magnitude of the resistance, $Q$ will be equaled or exceeded, (Chow, 1953, 1964; Ang and Tang, 1975). Thus, if $T_r$ is in years, the probability of an event, $Y$, equal to or greater than the design $Q$, in each year is

$$P(Y \geq Q) = \frac{1}{T_r} \quad (3-1)$$

In natural hydrologic phenomena $Y$ is usually a continuous variable, and hence $P(Y = Q) = 0$. If the simple risk of failure is defined as the probability of the occurrence of $Y$ greater than $Q$ in each year, then the non-failure probability for each year is

$$P(Y < Q) = 1 - \frac{1}{T_r} \quad (3-2)$$

Hence, the risk for an $n$-year period under consideration is

$$P(Y > Q) = 1 - (1 - \frac{1}{T_r})^n \quad (3-3)$$

In deriving Eqs. (3-1), (3-2), and (3-3), two major assumptions are made (Yen, 1970):

(1) Occurrences of the random variable $Y$ are independent between years.

(2) The hydrologic system is time invariant.

However, in a natural system such as in a watershed, there is actually a seasonal variation, as well as geophysical long-term variations. The probability associated with the return period as expressed in Eqs. (3-1), (3-2), and (3-3) nevertheless has been widely used as a measure of the risk for hydrologic structures (e.g., Borgman, 1963, Young et al., 1970, ASCE Task Committee; 1973, James and Lee; 1971). Although the risk
evaluated from Eq. (3-3) considers at best only part of the hydrologic risks, other uncertainties associated with the load and resistance are entirely ignored in this method. Hence the total risk of a complex natural system cannot be evaluated by this method.

3.1.2. Method of Direct Integration

In this method the risk is evaluated through a direct, analytical or numerical integration of the probability density function of the load and resistance as expressed in Eq. (2-11) or (2-13). The exact distribution functions of the load, $f_L(\xi)$, and resistance, $f_R(r)$, should be analyzable and definable. If the distribution functions describe the load and resistance correctly, then the risk evaluated by this method is exact.

Tang (1980) presented a procedure for incorporating the probability model uncertainty into risk evaluation. A direct integration was used to evaluate the hydrologic risk in his illustration. Wood (1977) assessed the overtopping and structural risks analytically with assumed probability density functions of the flood and structural failure modes. No considerations were given to the uncertainties of hydrologic, hydraulic, and model error parameters. Tung and Mays (1980) evaluated the risks for the culvert and levee by estimating first the statistical parameters of load and resistance from uncertainties of their parameters through first-order approximate formula, and then assigning distribution functions to the load and resistance. Risk, as evaluated by using Eq. (2-13) is found to be extremely sensitive to the distribution functions assigned. Therefore, an improper assumption of the
distribution functions of the load and resistance may negate the merit of accuracy of direct integration. Duckstein and Borgardi (1981) also studied the risk of a levee system due to various failure modes such as overtopping, boiling, slope sliding and wind wave erosion. The risk is estimated by direct integration of a joint density function of the load and resistance. However, the selection of the resistance is ambiguous and the determination of the uncertainties of the load and resistance are not presented clearly.

The biggest disadvantage of the direct integration method is the difficulty in the analytical derivation of the appropriate probability density functions of the load and resistance from their component random variables especially when the system is complex, such as for a dam structure and its environment. Nevertheless, once \( f_L(z) \) and \( f_R(r) \) are defined, with the assistance of a computer, most of the integration can be performed numerically, although possibly not economically in terms of time and cost. The direct integration method therefore, is good only for simple systems or when highly accurate risk value is required.

3.1.3. Monte Carlo Simulation Method

Monte Carlo simulation is a process using in each simulation a particular set of values of the random variables artificially generated in accordance with the corresponding probability distribution. The simulation is generally performed by a computer. The expected risk value can be estimated by examining the results of a large number of repetitive simulation runs. For example, first, a set of sample values, \( x_i \)'s, are generated according to the distribution function or statistical property
of each $X_i$ by means of a computer random generator. Next, a corresponding value of the performance variable, $Z$ in Eq. (2-17), can be calculated using Eq. (2-18). With sufficient repetitive samplings and calculations, a set of $Z$ values are obtained. Finally, the risk of the failure event can be evaluated by the ratio of the number of negative $Z$ to the total number of $Z$ being generated.

In the measure of structural safety, Warner and Kaballa (1968) used the Monte Carlo method to obtain the distributions of the resistances, loads, and performance variables of structures. In the design and operation of a multipurpose reservoir system, Askew et al. (1971) used Monte Carlo technique in deriving the optimum contract levels for the system, subject to the constraints imposed by stated maximum permissible risk to meet firm contract deliveries. Bohun and Vischer (1978) applied Monte Carlo simulation to risk and sensitivity analysis with different distribution functions of input data. Duckstein et al. (1981) estimated the probabilities of failure events for mine flooding by the Monte Carlo simulation method. Haan (1972) evaluated error probabilities of stochastic models as a function of the number of observations used in determining the parameters of the stochastic models of hydrologic problems with the Monte Carlo simulation method. Later, Matalas et al. (1975) and Wallis et al. (1977) applied this method to estimate the mean and standard deviation of skewness with several assumed distributions of flood sequences. Chow (1978) used the Monte Carlo method to generate sequences of hydrologic data for the study of the performance of a hydrologic system. Wen (1977) used Monte Carlo simulation results to verify his derivation of the statistics of
combination of extreme loads.

In fact Monte Carlo simulation is perhaps the only solution technique to problems which cannot be solved analytically because of their nonlinear behavior or complex system relationship. Despite its usefulness, the Monte Carlo simulation method has the following disadvantages:

(1) The risk estimated by using this method is not unique, depending on the size of the samples and the number of trials. It is never certain that the resultant statistical descriptor indeed reflects the true moments of the joint probability distribution that is being simulated. The true risk is unknown and can only be approached by infinite samples or trials.

(2) The computer expenses of Monte Carlo simulation increase substantially as the level of accuracy and the number of variables increase. Hiller and Lieberman (1974) recommended that in general, if an equivalent analytical model is available, it should be used instead of Monte Carlo simulation.

3.1.4. Mean-Value First-Order Second-Moment (MFOSM) Method

The evaluation of risk using multiple integration, Eq. (2-10), is a formidable task, even though it can be performed numerically. In actual engineering practice, the distributions of the constituent variables, e.g., $f_{x_1}(x_1), f_{x_2}(x_2), ..., f_{x_{n+1}}(x_{n+1}), f_{x_{n+2}}(x_{n+2}), ..., f_{x_m}(x_m)$, are usually not well defined and very often, information for these variables is limited to their respective means and variances (or COV's). For most engineering practices therefore, approximations that
are consistent with the state and quality of available information may be adequate, and indeed are more sensible. Thus, the first-order second-moment method, abbreviated as FOSM from now on, may be appropriate.

The first order analysis is an approximate probability analysis by truncating the second and higher order terms of the Taylor series expansion of random variables. A second moment analysis utilizes only the first two statistical moments, i.e., the expected value and variance of the random variables, evaluated at the point of Taylor's expansion, in the probabilistic analysis (Yen, 1976). This method was initially published by Mayer in the late 1920's, but it was not until the end of the 1960's with the work of Cornell (1969) that this method began to be developed seriously for engineering applications. When the Taylor series is expanded at the mean of the variables, the method is called mean-value first-order method (MFOSM) and the method can be illustrated as follows (Ang and Tang, 1975; Benjamin and Cornell, 1970).

As discussed in Section 2.2.2., failure of an existing system can be expressed by a performance variable Z which is a function of the resistance and loading variables, X_i:

\[ Z = g(X_1, X_2, \ldots, X_n, X_{n+1}, \ldots, X_m) \quad (3-4) \]

\[ Z = g(X_i) \quad i = 1, 2, \ldots, m \quad (3-5) \]

Expanding the function g(.) in a Taylor series about the mean values \( \bar{x}_i \)'s of the variables \( X_i \)'s yields

\[ Z = g(\bar{x}_i) + \sum_{i=1}^{m} (x_i - \bar{x}_i) \frac{\partial g}{\partial x_i} + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} (x_i - \bar{x}_i)(x_j - \bar{x}_j) \frac{\partial^2 g}{\partial x_i \partial x_j} + \ldots \quad (3-6) \]
where the derivatives are evaluated at $\bar{X}_i = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_m)$. The first-order expansion of $Z$ is

$$Z = g(\bar{X}_i) + \sum_{i=1}^{m} (X_i - \bar{X}_i) \frac{\partial g}{\partial X_i}$$

(3-7)

By taking the first and second moment of $Z$ in Eq. (3-7) and neglecting the terms higher than the second order, one has

$$E[Z] = \bar{Z} = g(\bar{X}_i)$$

(3-8)

$$\text{Var}(Z) = \sum_{i=1}^{m} C_i^2 \text{Var}(X_i) + \sum_{i=1}^{m} \sum_{j=1}^{m} C_i C_j \text{COV}(X_i, X_j)$$

(3-9)

where $C_i$ and $C_j$ are the values of partial derivatives $\partial g/\partial X_i$ and $\partial g/\partial X_j$, respectively, evaluated at $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_m$. If variables, $X_i$'s, are statistically independent, the covariance terms will vanish and Eq. (3-9) becomes

$$\text{Var}(Z) = \sum_{i=1}^{m} C_i^2 \text{Var}(X_i)$$

(3-10)

Therefore,

$$\sigma_Z = \left[ \sum_{i=1}^{m} (C_i \sigma_i)^2 \right]^{1/2}$$

(3-11)

where $\sigma_Z$ and $\sigma_i$ are the standard deviations of $Z$ and $X_i$, respectively.

The reliability index, $\beta$, which is a measure of risk, is defined as

$$\beta = \frac{E[Z]}{\sigma_Z}$$

(3-12)

which is the reciprocal of the coefficient of variation of the variable $Z$. In the MFOSM method, the reliability index may conveniently be considered as the distance from the origin ($Z=0$) to the mean $E[Z]$ measured in standard deviation units. As such, $\beta$ is a measure of the probability that $Z$ will be less than zero as shown in Fig. 3.1. As $\beta$
Fig. 3.1 Definition of reliability index $\beta$.

Fig. 3.2 Comparison of $\beta$ value by MFOSM and AFOSM methods.
increases, the probability $P(Z < 0)$ decreases accordingly.

Several authors, e.g., Hasofer and Lind (1974), consider $\beta$ as a convenient reliability measure since further information on the statistical characteristics of $X_i$'s is dispensable or may be unavailable. However, Rackwitz and Fiessler (1977) pointed out that, for practice, the risk, $p_f$, can be approximately estimated by

$$p_f = 1 - \Phi(\beta) = 1 - \Phi\left(\frac{E[Z]}{\sigma_Z}\right) \tag{3-13}$$

where the value of $\Phi(\beta)$ can be found directly from cumulative standard normal distribution tables in statistics reference books (e.g., Ang and Tang, 1975; Benjamin and Cornell, 1970). The risk evaluated from Eq. (3-13) may be exact when the basic variables, $X_i$'s, are all normally distributed and when the functions $g(.)$ can be expressed as a linear combination of the basic variables, $X_i$'s.

The MFSMO method as suggested in Eqs. (3-8), (3-11) to (3-13) has been widely used as a means of obtaining approximate probability of failure for complex engineering systems. Tang and Yen (1972), Yen and Tang (1975), and Yen (1978) used this method to determine the relationship between the safety factor and the risk of a storm sewer failure by accounting for component uncertainties. Yen (1976) used this method to demonstrate how uncertainties due to pipe roughness could be systematically accounted for and how the risk associated with this design due to those uncertainties could be assessed. Later, Yen et al. (1980) applied this method to the hydraulic design of culverts with hydrologic and hydraulic uncertainties being considered. Predergrast (1979) adopted this method as the basis for evaluating the safety of a concrete gravity
dam. The probabilities of sliding failure and overturning failure of the gravity dam were evaluated by considering various load effects which include reservoir stage, hydrostatic pressure, weight of the dam structure, uplift at the base of the dam and earthquake forces. The hydrologic and hydraulic aspects of dam safety, however, were not considered. Tung and Mays (1980) used Eqs. (3-8) and (3-10) to calculate the mean and variance of the load and resistance from their component variables $X_i$'s. The risk was evaluated by direct integration of Eq. (2-13) with assumed probability distributions of load and resistance. It was shown that with different assumed probability distributions of load and resistance, values of risk may result in significant differences.

The MFOSM method has been shown to be relatively simple, able to account for the various sources of uncertainties, adaptable for reevaluation with additional data and suitable for practical application. However, this method has certain disadvantages and weaknesses, including the following (Cornell, 1972; Ditlevsen, 1973; Lind, 1977):

1. In civil engineering projects, the events of failure often happen at extreme values rather than near the mean of the load and resistance, e.g., high floods or large earthquakes. Such variables are mostly associated with large variance and skewed probability distributions. Furthermore, civil engineering systems usually exhibit nonlinear behavior. The risk value estimated using the MFOSM method which linearizes the performance function $g(.)$ and evaluates at the mean values of variables $X_i$'s, may differ considerably from the actual risk.

2. The risk value depends on how the performance function, $g(.)$,
is formulated. That is, different risk values, $p_f$, or reliability index, $\beta$, are obtained for different formulations of $g(.)$. For example, as shown in Eqs. (2-14) to (2-16), defining $Z = R - L$, $(R / L) - 1$, or $\ln(R / L)$ could yield different values of $p_f$ or $\beta$.

The aforementioned shortcomings could be overcome by considering the higher order terms in the Taylor series expansion of $g(.)$ (Ang and Tang, 1975; Fiessler et al., 1979). However, this improvement requires additional statistical information, such as the skewness, which is usually inaccurate or unavailable in most situations. In addition, the calculation procedure is cumbersome. A technique to improve the accuracy of the first-order second-moment method was proposed by Rackwitz (1976) and recommended in a CIRIA report (1977), which evaluates the performance function $g(.)$ at the "failure point" (i.e. a point on the failure surface) rather than at the mean values of the basic load and resistance variables, $X_i$'s. The methodology is presented in the following section.

3.1.5. Advanced First-Order Second-Moment (AFOSM) Method

The essence of this method is to linearize the performance function $g(X_1, X_2, \ldots, X_m)$ at a likely failure point $(X_1^*, X_2^*, \ldots, X_m^*)$ on the failure surface, such that $g(X_1^*, X_2^*, \ldots, X_m^*) = 0$. The determination of the failure point, $(X_1^*, X_2^*, \ldots, X_m^*)$, however, is generally not a simple task; it is not known a priori, and therefore needs to be determined iteratively. The algorithm for finding this point has been shown by Paloheimo and Hunnus (1974) and in a CIRIA report (1977) as follows, with the assumption that the variables $X_i$'s are uncorrelated.
The first-order expansion of $Z$ at a point $x_i^*$ is expressed in the following form

$$Z = g(x_i^*) + \sum_{i=1}^{m} (x_i - x_i^*) \frac{\partial g}{\partial x_i}$$  \hspace{1cm} (3-14)$$

where $\frac{\partial g}{\partial X_i}$ are evaluated at the point $x_i^* = (x_1^*, x_2^*, \ldots, x_m^*)$ and are denoted as $C_i$. Thus, it can be shown that (3-15)

$$E[Z] = g(x_i^*) + \sum_{i=1}^{m} C_i (\bar{X}_i - x_i^*)$$  \hspace{1cm} (3-15)$$

$$\text{Var}(Z) = \sum_{i=1}^{m} C_i^2 \text{Var}(X_i)$$  \hspace{1cm} (3-16)$$

$$\sigma_Z = \left[ \sum_{i=1}^{m} \left( C_i \sigma_i \right)^2 \right]^{1/2}$$  \hspace{1cm} (3-17)$$

Following Lind (1971), the expression of $\sigma_Z$ is rewritten in a linearized form

$$\sigma_Z = \sum_{i=1}^{m} \alpha_i C_i \sigma_i$$  \hspace{1cm} (3-18)$$

in which $\alpha_i$'s are the sensitivity factors and are evaluated from

$$\alpha_i = \frac{C_i \sigma_i}{\left[ \sum_{j=1}^{m} \left( C_j \sigma_j \right)^2 \right]^{1/2}}$$  \hspace{1cm} (3-19)$$

Substituting Eqs. (3-15) and (3-18) into Eq. (3-12) gives

$$\beta = \frac{g(x_i^*) + \sum_{i=1}^{m} C_i (\bar{X}_i - x_i^*)}{\sum_{i=1}^{m} \alpha_i C_i \sigma_i}$$  \hspace{1cm} (3-20)$$

If $x_i^*$ is on the failure surface, then

$$g(x_i^*) = 0$$  \hspace{1cm} (3-21)$$
Rearranging Eq. (3-20) gives

\[ \sum_{i=1}^{m} c_i (x_i - x_i^*) - a_i \beta \sigma_i = 0 \]  \hspace{1cm} (3-22)

Solving this equation gives

\[ x_i^* = \bar{x}_i - a_i \beta \sigma_i \]  \hspace{1cm} (3-23)

which defines the failure point \( x_i^* \).

For given values of \( \bar{x}_i \) and \( \sigma_i \) of variables \( X_i \), the reliability index \( \beta \) and the failure points \( x_i^* \) may be found by solving Eqs. (3-21) and (3-23) simultaneously. The reliability index, herein, may be used as a measure of failure probability because it represents the (minimum) distance from the mean-values (\( \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m \)) to the true failure surface \( g \left( x_1, x_2, \ldots, x_m \right) = 0 \) (Fig. 3.2). It has been shown that \( \beta \) is invariant no matter how the performance function \( g(X_i) \) is defined for the failure event; e.g. \( \frac{R}{L} - 1 \), \( R - L \), or \( \ln \left( \frac{R}{L} \right) \), (Hasofer and Lind, 1974; and Veneziano, 1974).

The probability of failure is given by Eq. (3-13) as

\[ p_f = 1 - \Phi (\beta) \]  \hspace{1cm} (3-13)

The value of \( p_f \) from this equation is exact within the first order context only when the performance is a linear combination of normally distributed variables \( X_i \). However, the basic variables are usually skewed, non-normal, and nonlinearly related. Accordingly, Eq. (3-13) gives only very approximate probability value unless other actions are taken. Methods for improving the probability value estimated by Eq. (3-13) for non-normal variables have been proposed by Paloheimo (1974), Lind (1977) and Rackwitz and Fiessler (1977), which basically approximated the non-normal distribution with an equivalent normal
distribution for a variable $X_i$. The method proposed by Rackwitz and Fiessler is adopted in this study.

The essence of the approach proposed by Rackwitz and Fiessler (1977) is to transform the non-normal distributions into equivalent normal distributions so that the values of the cumulative distribution function, CDF, and the probability density function, pdf, of the non-normal distributions are the same as those of the equivalent normal distributions at the failure point $x_i^*$; i.e.

$$F_{X_i}(x_i^*) = \Phi \left( \frac{x_i^* - \bar{X}_N}{\sigma_i^N} \right)$$

(3-24)

$$f_{X_i}(x_i^*) = f_N \left( \frac{x_i^* - \bar{X}_N}{\sigma_i^N} \right) / \sigma_i^N$$

(3-25)

In which $F_{X_i}(x_i^*)$ and $f_{X_i}(x_i^*)$ are the CDF and pdf of $X_i$ at $x_i^*$, and $\Phi(\cdot)$ and $f_N(\cdot)$ are the CDF and pdf of the standard normal distribution, respectively. From Eqs. (3-24) and (3-25), the mean, $\bar{x}_i^N$, and standard deviation, $\sigma_i^N$, of the equivalent normal distributions for the basic variables $X_i$'s become,

$$\bar{x}_i^N = x_i^* - \Phi^{-1} \left( F_{X_i}(x_i^*) \right) \sigma_i^N$$

(3-26)

$$\sigma_i^N = \frac{f_N \left( \Phi^{-1} \left( F_{X_i}(x_i^*) \right) \right)}{f_{X_i}(x_i^*)}$$

(3-27)

Replacing $x_i$ with $x_i^N$ and $\sigma_i$ with $\sigma_i^N$, Eqs. (3-19) and (3-23) become

$$\sigma_i = \left( \frac{\sqrt{\sum_{j=1}^{m} \left( c_{ij} \sigma_j^N \right)^2}}{C_{i1} \sigma_i^N} \right)^{1/2}$$

(3-28)
As before, the reliability index, $\beta$, and the failure points $x_i^*$ can be solved from Eqs. (3-21) and (3-29) in conjunction with Eqs. (3-26), (3-27) and (3-28). Since $\beta$ is determined from the normalization of non-normal distributed variables, the probability of failure, $p_f$, therefore, can be well approximated from Eq. (3-13). An iterative algorithm for finding $\beta$ and $x_i^*$ with a fast rate of convergence are shown as follows (CIRIA; 1977):

1. Define the performance function $g(X_i)$ for the failure event.
2. Input the statistics of $X_i$.
3. Choose initial values of $\beta$ and $x_i^*$.
4. Evaluate the partial derivatives $C_i = \partial g / \partial x_i$ for all $i$, at the points $x_i^*$.
5. Calculate $\bar{x}_i^N$ and $\sigma_i^N$ for all $i$, according to Eqs. (3-26) and (3-27).
6. Compute the sensitivity factors $\alpha_i$ for all $i$, according to Eq. (3-28).
7. Re-evaluate values of $x_i^*$ for all $i$, according to Eq. (3-29).
8. Repeat steps (4) to (7) until the values of $x_i^*$ converge within specified limits.
9. Evaluate $Z = g(x_i^*)$.
10. Modify $\beta$ and repeat steps (4) to (9) to achieve $Z = 0$, within specified limits.
11. Compute the probability of failure $p_f$ from Eq. (3-13).
For the calculation of the second moment (Eq. (3-9)) in the FOSM method, the gradient or the sensitivity factor of the performance function for each variable, \( \frac{\partial g}{\partial X_i} \), is required. The analytical function of \( \frac{\partial g}{\partial X_i} \) can be derived easily if the performance function \( g \) is simply related to each variable, \( X_i \). However, the complicated relationship between \( g \) and \( X_i \) can make it difficult to calculate the gradient \( \frac{\partial g}{\partial X_i} \) analytically. In this case, the gradient \( \frac{\partial g}{\partial X_i} \) can be approximated numerically by a forward difference criterion as

\[
\frac{\partial g}{\partial x_i} \approx \lim_{\Delta x_i \to 0} \frac{\Delta g}{\Delta x_i} = \frac{g(x_i + \Delta x_i) - g(x_i)}{\Delta x_i}
\]  (3-30)

in which \( g(x_i + \Delta x_i) \) and \( g(x_i) \) are the values of performance function \( g \) evaluated at points \( x_i + \Delta x_i \) and \( x_i \), respectively.

The AFOSM method requires the basic variables \( X \) be uncorrelated. If basic variables, \( Y \), are found to be correlated, then, an orthogonal transformation of variables \( Y \) to a new set of uncorrelated variables \( X \) is necessary. If \( V_Y \) represents the covariance vector of variables \( Y \), then the new uncorrelated variables \( X \) are shown to be (Hansfer and Lind; 1974)

\[
X = [C] Y
\]  (3-31)

in which \( [C] \) is the eigen vector of the covariance vector \( V_Y \). The standard deviations of the variables \( X \) are the eigenvalues of \( V_Y \) and are the diagonal elements of the following orthogonal vector, \( V_X \),

\[
V_X = [C] V_Y [C]^T
\]  (3-32)

To achieve convergence, the above iterative algorithm requires local differentiability of the performance function as well as local
continuity and monotony of the original density function. If the
performance function is discontinuous, it must be treated as a series of
continuous functions. The search for $E_3$ may become numerically more
complex if the performance function has several local minimums or if the
original density function is discontinuous and bounded. It is the
author's experience that some problems may happen occasionally with this
algorithm. They are:

1. The iteration may diverge or give different $E$ values because of
the local minimums of the performance function.

2. The iterations may converge very slowly when the probability of
failure is very small, for example, $p_f < 10^{-4}$.

3. In the case of bounded random variables, the iteration may yield
some of $X_i^*$ outside of the bounded range of the original density
function. However, if the bounds are strictly enforced, the
iteration may diverge.

In order to improve these numerical problems, a nonlinear optimization
 technique, Generalized Reduced Gradient method, is proposed.

3.2. Proposed Solution Technique for Advanced First-Order Second-Moment
Method

The Generalized Reduced Gradient (GRG) algorithm was first
developed by Abadie and Carpentier (1969) and was shown to be a very
efficient technique for solving nonlinear constraint problems. The
general problem solved by GRG method is in the following form:

$$\text{Min } w(X)$$

subject to

$$g(X) = 0$$

$$X_L \leq X \leq X_U$$
where $X$ is a vector of $n$ variables, $w(X)$ is objective function, $G(X)$ is a vector of $m$ equality constraints which can be linear or nonlinear, $X_L$ is a vector of lower bounds on $X$. $X_U$ is a vector of upper bounds on $X$.

In the AFOSM method, the reliability index $\beta$ and the failure points $x_i^*$ are found by solving Eqs. (3-28) and (3-29) in conjunction with Eqs. (3-26) and (3-27). For applying the GRG method to get $\beta$ and $x_i^*$, these equations are rearranged. Equation (3-29) may be rewritten as

$$x_i^* - \frac{X_i}{N} + a_i \beta c_i^N = 0 \quad \text{for all } i \quad (3-36)$$

This equation can be regarded as equality constraints. Obviously, Eq. (3-36) is a nonlinear constraint in $x_i^*$ because $a_i$, $\frac{X_i}{N}$, and $c_i^N$ are nonlinear functions of $x_i^*$.

Recall that (see Fig. 3.1) safety is defined as the condition

$$g(X_i) > 0 \quad (3-37)$$

and failure as

$$g(X_i) < 0 \quad (3-38)$$

Depending on the values of $X_i$, $g(X_i)$ can either be positive or negative. Therefore, the minimum of the absolute value of $g(X_i)$ should be equal to zero which is equivalent to the criterion of failure surface. Hence, the reliability index $\beta$ and the failure point $x_i^*$ can be found by

$$\min g|\{(x_i^*)\}| \quad (3-39)$$

subject to

$$x_i^* - \frac{X_i}{N} + a_i \beta c_i^N = 0 \quad (3-40)$$

$$\varepsilon_i < x_i^* < u_i \quad i = 1, 2, \ldots m \quad (3-41)$$
where $x_i$ is the lower bound of the variable $X_i$, and $u_i$ is the upper bound of variable $X_i$. The probability of failure is determined by Eq. (3-13).

There are many possible GRG algorithms. The solution technique and software package developed by Lasdon et al. (1975a, 1975b) are used in this study. The solution procedure of this GRG technique involves two sets of variables; namely, $X_b$ which is a set of basic or dependent variables, and $X_n$ which is a set of nonbasic or independent variables. The basic variables are selected such that $\frac{\partial E}{\partial X_b}$ is nonsingular at a feasible location $\bar{X}$. Accordingly, a constraint set

$$E(X_b, X_n) = 0 \quad (3-42)$$

is solved for $X_b$ in terms of $X_n$ to give the basic variables as a function of the nonbasic variables, i.e., $X_b(X_n)$. The objective function is then reduced to

$$w(X_b(X_n), X_n) = W(X_n) \quad (3-43)$$

which is a function of only $X_n$.

The GRG problem is now

$$\text{Min } W(X_n) \quad (3-44)$$

and is subject to the bounds on $X_n$, where $W(X_n)$ is the reduced objective function. The reduced gradient $\nabla W$ is computed in terms of the nonbasic variables as

$$\nabla W = \frac{\partial W}{\partial X_n} = \frac{\partial w}{\partial X_n} - M^T \left[ \frac{\partial E}{\partial X_n} \right] \quad (3-45)$$

where $M^T$, the transpose of the simplex multiplier vector, is determined from

$$\left[ \frac{\partial E}{\partial X_b} \right]^T M = \frac{\partial w}{\partial X_b} \quad (3-46)$$
All partial derivatives are evaluated at the current feasible location $\bar{x}$. The GRG algorithm solves the original problem, Eqs. (3-33) to (3-35), by a sequence of reduced problems, Eq. (3-43), using a reduced gradient method. A search direction $\bar{d}$ is formed from $W(X)$ and a one-dimensional search is initiated to solve the problem

$$\text{Min } W(\bar{x} + a\bar{d}), \quad a \geq 0$$

(3-47)

This minimization is done only approximately and is accomplished by choosing a sequence of positive values ($a_1, a_2, \ldots$) for $a_i$. In order to evaluate each value of $W(\bar{x} + a_i\bar{d})$, the basic variables $X_b(X + a_i\bar{d})$ must be determined. These satisfy the system of equations

$$E(X_b, \bar{x} + a_i\bar{d}) = 0$$

(3-48)

where $\bar{x}, a_i, \bar{d}$ are known and $X_b$ is found by the pseudo-Newton method.

Like other nonlinear programming algorithms, this technique theoretically cannot guarantee the global optimum. If local optima appear to be a problem or the results appear to be suspect, a popular procedure is to try a variety of starting points. It is found that if the minimum of $|g(X_i)|$ in Eq. (3-39) approaches zero, the solutions of $\beta$ and $x_i^*$ usually are the global ones.

3.3. Comparison of Risk Calculation Methods

A comparison is made on the aforementioned four risk calculation methods, namely, the direct integration method, the Monte Carlo simulation method, the MFOSM method and the AFOSM method. The return-period method is not considered because of its incapability of handling the uncertainties of every component variable. The risk value calculated by direct integration is exact and is used as a reference for the comparison of the other three methods. For the AFOSM method the
numerically iterative process and the GRG nonlinear optimization technique are also compared. The comparison of the methods can be made from four aspects which are:

1. The consistency of the evaluated risk values with respect to different forms of the performance function for a failure event.

2. The accuracy of the methods.

3. The sensitivity of the evaluated risk value to the assumed probability distributions for the variables of load and resistance.

4. The computer time and cost required for each of the methods.

In order to have the exact risk value evaluated by direct integration method and to check the accuracy of the other methods, the load and resistance are chosen to be

\[ L = X_1 + X_2 \]  \hspace{1cm} (3-49)

and

\[ R = X_3 \cdot X_4 \]  \hspace{1cm} (3-50)

where \( X_1 \) and \( X_2 \) are the uncorrelated variables of the load, \( L \), and \( X_3 \) and \( X_4 \) are the uncorrelated variables of the resistance, \( R \). The uncertainties of these variables are listed in Table 3.1.

Table 3.1 Uncertainties of variables of load and resistance

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>COV</th>
<th>Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 )</td>
<td>0.5</td>
<td>0.2</td>
<td>Normal or Uniform</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>1.5</td>
<td>0.4</td>
<td>Normal or Uniform</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>1.0</td>
<td>0.005</td>
<td>Log-Normal</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>1.5</td>
<td>0.1</td>
<td>Log-Normal</td>
</tr>
</tbody>
</table>
Different mean values of $X_4$ have been assigned in order to establish a risk curve of the risk value, $p_f$, versus the ratio $\mu_R/\mu_L$. The load variables $X_1$ and $X_2$ are assumed to be normally distributed; and the resistance variables $X_3$ and $X_4$ are assumed log-normally distributed. Since $L$ is a linear combination of independent normally distributed variables, and $R$ is a product of independent log-normally distributed variables, it can be shown (Ang and Tang, 1975) that $L$ is normally distributed and $R$ is log-normally distributed. The probability density functions, pdf, of $L$ and $R$ are

$$f_L(\lambda) = \frac{1}{\sqrt{2\pi} \sigma_L} \exp \left( -\frac{1}{2} \left( \frac{\lambda - \mu_L}{\sigma_L} \right)^2 \right) \quad -\infty < \lambda < \infty \quad (3-51)$$

$$f_R(r) = \frac{1}{\sqrt{2\pi} \xi_R} \exp \left( -\frac{1}{2} \left( \frac{\ln r - \lambda_R}{\xi_R} \right)^2 \right) \quad 0 \leq r < \infty \quad (3-52)$$

in which

$$\mu_L = \mu_1 + \mu_2 \quad (3-53)$$

$$\sigma_L = \sigma_1^2 + \sigma_2^2 \quad (3-54)$$

and

$$\lambda_R = \lambda_3 + \lambda_4 \quad (3-55)$$

$$\xi_R^2 = \xi_3^2 + \xi_4^2 \quad (3-56)$$

in which $\mu_i$ and $\sigma_i$ are the mean and standard deviation of $X_i$, respectively; and $X_i$ and $\xi_i$ are the mean and standard deviation of the natural logarithm values of $X_i$, respectively. With the statistical
parameters of $L$ and $R$ given in Eqs. (3-53) to (3-56), the risk can be evaluated numerically by direct integration of Eq. (2-11) or Eq. (2-13) associated with Eqs. (3-51) and (3-52). Risk values for various values of the mean of $X_4$ and of the ratio $\mu_R/\mu_L$ are listed in Table 3.2.

In order to investigate the sensitivity of the risk calculation methods to the different distributions assigned to the variables, both $X_1$ and $X_2$ are also assumed to be uniformly distributed having the following pdf:

$$f_{X_i}(x_i) = \begin{cases} \frac{1}{b_i - a_i}, & a_i \leq x_i \leq b_i, \ i = 1, 2 \\ 0, & \text{elsewhere} \end{cases}$$

(3-57)

in which $a_i$ and $b_i$ are respectively the lower and upper bounds of variables $X_i$ where $a_1 < a_2 < b_1 < b_2$. By means of Laplace transformation (Appendix A), the pdf of the load, $f_L(\ell)$, becomes

$$f_L(\ell) = \begin{cases} \frac{\ell - a_1 - a_2}{(b_1 - a_1)(b_2 - a_2)}, & a_1 + a_2 \leq \ell < b_1 + a_2 \\ \frac{1}{b_2 - a_2}, & b_1 + a_2 \leq \ell < a_1 + b_2 \\ \frac{b_1 + b_2 - \ell}{(b_1 - a_1)(b_2 - a_2)}, & a_1 + b_2 \leq \ell \leq b_1 + b_2 \\ 0, & \text{elsewhere} \end{cases}$$

(3-58)
Table 3.2 Risk values by different methods with normally distributed $X_1$ and $X_2$

<table>
<thead>
<tr>
<th>$\bar{X}_4$</th>
<th>$\frac{\bar{R}}{L}$</th>
<th>Direct Integration</th>
<th>Monte Carlo</th>
<th>MFSOM</th>
<th>AFOSN</th>
<th>GRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.500</td>
<td>0.750</td>
<td>0.786</td>
<td>0.788 (0.3)**</td>
<td>0.786 (0.0)</td>
<td>0.790 (0.5)</td>
<td>0.790 (0.5)</td>
</tr>
<tr>
<td>1.875</td>
<td>0.938</td>
<td>0.578</td>
<td>0.579 (0.2)</td>
<td>0.577 (-0.2)</td>
<td>0.579 (0.2)</td>
<td>0.584 (1.0)</td>
</tr>
<tr>
<td>2.250</td>
<td>1.125</td>
<td>0.353</td>
<td>0.353 (0.0)</td>
<td>0.352 (-0.3)</td>
<td>0.358 (1.4)</td>
<td>0.366 (3.7)</td>
</tr>
<tr>
<td>2.625</td>
<td>1.313</td>
<td>0.177</td>
<td>0.178 (0.6)</td>
<td>0.177 (0.0)</td>
<td>0.201 (13.6)</td>
<td>0.231 (30.5)</td>
</tr>
<tr>
<td>3.000</td>
<td>1.500</td>
<td>0.740x10^{-1}</td>
<td>0.746x10^{-1} (0.8)</td>
<td>0.750x10^{-1} (1.4)</td>
<td>0.105 (41.9)</td>
<td>0.152 (105.4)</td>
</tr>
<tr>
<td>3.375</td>
<td>1.688</td>
<td>0.260x10^{-1}</td>
<td>0.253x10^{-1} (-2.7)</td>
<td>0.274x10^{-1} (5.4)</td>
<td>0.352x10^{-1} (35.4)</td>
<td>0.104 (300.0)</td>
</tr>
<tr>
<td>3.750</td>
<td>1.875</td>
<td>0.790x10^{-2}</td>
<td>0.799x10^{-2} (11.1)</td>
<td>0.892x10^{-2} (12.9)</td>
<td>0.262x10^{-1} (231.6)</td>
<td>0.749x10^{-1} (-5.2)</td>
</tr>
<tr>
<td>4.125</td>
<td>2.063</td>
<td>0.211x10^{-2}</td>
<td>0.220x10^{-2} (4.3)</td>
<td>0.269x10^{-2} (27.5)</td>
<td>0.127x10^{-1} (251.9)</td>
<td>0.559x10^{-1} (501.9)</td>
</tr>
<tr>
<td>4.500</td>
<td>2.250</td>
<td>0.506x10^{-3}</td>
<td>0.390x10^{-3} (-22.9)</td>
<td>0.770x10^{-3} (52.2)</td>
<td>0.617x10^{-2} (1119.4)</td>
<td>0.432x10^{-1} (8437.5)</td>
</tr>
<tr>
<td>4.750</td>
<td>2.375</td>
<td>0.226x10^{-4}</td>
<td>---</td>
<td>0.216x10^{-3} (855.8)</td>
<td>0.298x10^{-2} (13085.8)</td>
<td>0.344x10^{-1} (152112.4)</td>
</tr>
</tbody>
</table>

* Risk value is estimated from the average of two generated samples of 32,000 each.

** Numbers in parentheses represent the percentage error = \( \frac{\text{Risk} - \text{Risk}_{d.i.}}{\text{Risk}_{d.i.}} \times 100\% \)
Similarly, the risk is evaluated numerically using direct integration, Eq. (2-11) or Eq. (2-13) associated with Eqs. (3-52) and (3-58). The result is listed in Table 3.3.

By using the calculation procedures and equations described in the previous sections, the risk values computed by using the Monte Carlo simulation, MFOSM and AFOSM methods are also listed in Tables 3.2 and 3.3. Three different representations of the performance variable, Z, for the failure event as expressed in Eqs. (2-14), (2-15), and (2-16) are studied for the MFOSM and AFOSM methods. The risk values given in Tables 3.2 and 3.3 by the Monte Carlo method are the average of two simulations, with 32,000 randomly generated samples for each simulation. Based on the risk values given in Tables 3.2 and 3.3, plots of the risk, $p_f$, versus the ratio $\frac{R}{L}$ for the different methods investigated are given in Fig. 3.3. The risk curves evaluated by direct integration are also plotted and serve as a reference for the comparison.

The inconsistency of the MFOSM method to the different representations of $Z$ for the failure event is clearly shown. The risk values calculated from the three different representations of $Z$, namely, $R - L$, $\ln \left( \frac{R}{L} \right)$, and $\left( \frac{R}{L} \right) - 1$, are different, and the differences become more significant as $p_f$ becomes smaller. The risk value calculated by MFOSM method does not vary with the distributions of the variables provided the values of the first two moments remain unchanged. As can be seen from Tables 3.2 and 3.3, and Fig. 3.3, it remains the same no matter which distribution functions are assigned to the variables. This method gives a reasonable approximation when the risk is high, e.g., $p_f > 0.1$ in this example. However, when $p_f$ becomes small, the results
Table 3.3 Risk values by different methods with uniformly distributed $X_1$ and $X_2$

<table>
<thead>
<tr>
<th>$\bar{X}_4$</th>
<th>$\frac{R}{L}$</th>
<th>Direct Integration</th>
<th>*Monte Carlo</th>
<th>MFOSM $P[(R-L)&lt;0]$</th>
<th>MFOSM $P[(\ln \frac{R}{L})&lt;0]$</th>
<th>MFOSM $P[(\frac{R}{L}-1)&lt;0]$</th>
<th>AFOSM Iteration</th>
<th>AFOSM CRG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.500</td>
<td>0.750</td>
<td>0.741</td>
<td>0.742</td>
<td>0.786</td>
<td>0.813</td>
<td>0.848</td>
<td>0.735</td>
<td>0.735</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.1)</td>
<td>(6.1)</td>
<td>(9.7)</td>
<td>(14.4)</td>
<td>(-0.8)</td>
<td>(-0.8)</td>
</tr>
<tr>
<td>1.875</td>
<td>0.938</td>
<td>0.560</td>
<td>0.562</td>
<td>0.577</td>
<td>0.579</td>
<td>0.582</td>
<td>0.563</td>
<td>0.563</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.4)</td>
<td>(3.0)</td>
<td>(3.4)</td>
<td>(3.9)</td>
<td>(0.5)</td>
<td>(0.5)</td>
</tr>
<tr>
<td>2.250</td>
<td>1.125</td>
<td>0.380</td>
<td>0.379</td>
<td>0.352</td>
<td>0.358</td>
<td>0.366</td>
<td>0.393</td>
<td>0.393</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-0.3)</td>
<td>(-7.4)</td>
<td>(-5.8)</td>
<td>(-3.7)</td>
<td>(3.4)</td>
<td>(3.4)</td>
</tr>
<tr>
<td>2.625</td>
<td>1.313</td>
<td>0.207</td>
<td>0.206</td>
<td>0.177</td>
<td>0.201</td>
<td>0.231</td>
<td>0.233</td>
<td>0.233</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.5)</td>
<td>(-14.5)</td>
<td>(-2.9)</td>
<td>(11.6)</td>
<td>(12.6)</td>
<td>(12.6)</td>
</tr>
<tr>
<td>3.000</td>
<td>1.500</td>
<td>$0.770 \times 10^{-1}$</td>
<td>$0.760 \times 10^{-1}$</td>
<td>$0.750 \times 10^{-1}$</td>
<td>0.105</td>
<td>0.152</td>
<td>0.102</td>
<td>0.103</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1.3)</td>
<td>(-2.6)</td>
<td>(36.4)</td>
<td>(97.4)</td>
<td>(32.5)</td>
<td>(33.8)</td>
</tr>
<tr>
<td>3.375</td>
<td>1.688</td>
<td>$0.179 \times 10^{-1}$</td>
<td>$0.183 \times 10^{-1}$</td>
<td>$0.274 \times 10^{-1}$</td>
<td>$0.532 \times 10^{-1}$</td>
<td>0.104</td>
<td>$0.288 \times 10^{-1}$</td>
<td>$0.288 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.2)</td>
<td>(53.1)</td>
<td>(197.2)</td>
<td>(481.0)</td>
<td>(60.9)</td>
<td>(60.9)</td>
</tr>
<tr>
<td>3.750</td>
<td>1.875</td>
<td>$0.263 \times 10^{-2}$</td>
<td>$0.262 \times 10^{-2}$</td>
<td>$0.892 \times 10^{-2}$</td>
<td>$0.262 \times 10^{-1}$</td>
<td>$0.749 \times 10^{-1}$</td>
<td>$0.500 \times 10^{-2}$</td>
<td>$0.498 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-0.4)</td>
<td>(239.2)</td>
<td>(896.2)</td>
<td>(2747.9)</td>
<td>(90.1)</td>
<td>(89.4)</td>
</tr>
<tr>
<td>4.125</td>
<td>2.063</td>
<td>$0.261 \times 10^{-3}$</td>
<td>$0.297 \times 10^{-3}$</td>
<td>$0.269 \times 10^{-2}$</td>
<td>$0.127 \times 10^{-1}$</td>
<td>$0.559 \times 10^{-1}$</td>
<td>$0.286 \times 10^{-3}$</td>
<td>$0.464 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(13.8)</td>
<td>(4417.6)</td>
<td>(32894.7)</td>
<td>(23001.6)</td>
<td>(116.1)</td>
<td>(118.4)</td>
</tr>
<tr>
<td>4.500</td>
<td>2.250</td>
<td>$0.187 \times 10^{-4}$</td>
<td>---</td>
<td>$0.770 \times 10^{-3}$</td>
<td>$0.17 \times 10^{-2}$</td>
<td>$0.432 \times 10^{-1}$</td>
<td>$0.463 \times 10^{-4}$</td>
<td>$0.442 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4417.6)</td>
<td>(32894.7)</td>
<td>(23001.6)</td>
<td>(147.6)</td>
<td>(136.4)</td>
<td></td>
</tr>
</tbody>
</table>

* risk values are estimated from the average of two generated samples of 32,000 each

** numbers in parentheses represent the percentage error = \( \frac{\text{Risk} - \text{Risk}_{d.i.}}{\text{Risk}_{d.i.}} \times 100\% \)
Fig. 3.3 Comparison of risk values calculated by different methods.
by this method may be significantly different from the exact ones obtained by direct integration method. Although the risk values evaluated with the representation $R - L$ do not differ much from the true values when $X_1$ and $X_2$ are normally distributed, it does not guarantee that this representation is a better approximation for any case. Regarding the accuracy, consistency, and sensitivity, the MFOSM method is not recommended for the dam safety evaluation due to the low risk value normally associated with dams, which usually is at a level smaller than $10^{-3}$ as indicated in Chapter 2.

It is found that the risk value calculated by the AFOSM method is invariant with respect to different representations of $Z$ and responsive to the different distributions assigned to the same variables. The results show that the AFOSM method gives very good approximations to the true risk values especially when $X_1$ and $X_2$ are normally distributed. Both numerical iteration method and generalized reduced gradient technique are used to calculate the risk. The computation time of GRG is found to be three times that of the iteration method in most of the runs. However, for the case with the bounded uniform distribution assigned to variables $X_1$ and $X_2$, the computation time of the iteration method increases when the risk value becomes small, and it is greater than that of the GRG method. Figure 3.4 shows a rough comparison of the computation time of these two methods.

The iteration method also diverges frequently when the risk value becomes small. Several runs are often required to achieve a converged risk value. For example, four runs were executed to obtain the risk value for $\mu_R/\mu_L = 2.25$. The GRG technique however, significantly reduced
Fig. 3.4 Relative computation time for Monte Carlo simulation method and AFOSM iteration and GRG techniques.
the number of runs and usually one run is enough. Therefore, for bounded 
probability distribution functions and/or small risk values, the GRG 
technique is may be more efficient than the iteration method based on the 
limited results here.

The result in Fig. 3.3 evaluated by using the Monte Carlo 
simulation method is the average of two randomly generated samples of 
32,000 of each risk value. The data show that with a large enough sample 
the risk value does approach the actual one. Considering all these 
factors, namely, accuracy, invariance and sensitivity, this method is 
superior to the MFOSM and AFOSM methods. However, the accuracy depends 
largely on the number of samples generated. The smaller the $p_f$ value, 
the more samples are needed, and therefore the more computation time is 
required. As mentioned earlier, one major disadvantage of the Monte 
Carlo simulation method is that the appropriate sample size is unknown in 
advance. Table 3.4 illustrates the accuracy of the risk value with 
respect to sample size when $X_1$ and $X_2$ are uniformly distributed. The 
amount of computer time used in the Monte Carlo simulation method 
relative to that of the AFOSM method is also compared in Fig. 3.4.

The statistics of dam failures shows that the average risk is less 
than the level of $10^{-3}$. Clearly, the MFOSM is not suitable for dam 
safety evaluation. Based on the above example, the Monte Carlo 
simulation method gives the best result, but its dependence on the sample 
size and its high computational cost limit its use in practice, 
especially when the number of random variables becomes large. As to the 
accuracy, consistency, and computational cost, the AFOSM method with GRG 
opimization technique is highly recommended for dam safety evaluation.
Table 3.4  Risk values estimated by Monte Carlo simulation method with different generated samples

<table>
<thead>
<tr>
<th>Sample Size ( \mu_R/\mu_L )</th>
<th>100</th>
<th>1000</th>
<th>8000</th>
<th>32,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.500</td>
<td>0.77</td>
<td>0.69</td>
<td>0.742</td>
<td>0.748</td>
</tr>
<tr>
<td>*</td>
<td>0.65</td>
<td>*</td>
<td>0.559</td>
<td>0.570</td>
</tr>
<tr>
<td>2.250</td>
<td>0.45</td>
<td>*</td>
<td>0.389</td>
<td>0.378</td>
</tr>
<tr>
<td>2.625</td>
<td>0.20</td>
<td>0.13</td>
<td>0.223</td>
<td>0.210</td>
</tr>
<tr>
<td>3.000</td>
<td>0.06</td>
<td>*</td>
<td>0.920x10^{-1}</td>
<td>0.770x10^{-1}</td>
</tr>
<tr>
<td>3.375</td>
<td>*</td>
<td>*</td>
<td>0.230x10^{-1}</td>
<td>0.220x10^{-1}</td>
</tr>
<tr>
<td>3.750</td>
<td>*</td>
<td></td>
<td>0.500x10^{-2}</td>
<td>0.600x10^{-2}</td>
</tr>
<tr>
<td>4.125</td>
<td>*</td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>4.500</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

Computation Time (sec)  
| 0.034  | 0.036  | 0.157  | 0.156  | 1.098  | 1.109  | 4.327  | 4.320  |

* sample size becomes inadequate for these risk values
The Monte Carlo simulation method is suggested to be used as a checking tool whenever it is necessary.

3.4. Fault Tree Analysis for Calculation of Total Risk

The risk of dam is the total risk combining the probabilities of all possible failure events. The methods described in the previous sections are used to calculate the risk of a failure event. The fault tree analysis (FTA) described here is an attempt to consider the relationships among the failure events of the dam, so that the individual risks of the dam can be integrated systematically to yield the overall risk of the dam. The FTA also provides a means to identify the potential sources (factors) of failure or critical failure modes therefore proper remedial work or modification can be made to reduce the chance of failure.

Fault tree analysis was developed by Bell Telephone Laboratories in 1962 at the request of the Air Force for use with the Minuteman ICBM system. Later, the FTA technique was applied to evaluate the risk of a variety of systems including nuclear power plants, chemical processing plants, and electrical systems (Vesely, 1975; Cumming, 1975; Powers et al., 1975; Fussell, 1975). Yen (1977) applied FTA to obtain quantitative results for probability based decision making in hydropower operation. Recently, Lambe et al. (1981) used FTA to relate the possible causes of loss contaminated fluid stored in a waste storage dam. The total probability of loss of containment is assessed by combining risk values assigned to each failure events. Duckstein et al. (1981) estimated the reliability of an underground hydraulic system of a mine by a combination of the Monte Carlo simulation method and fault tree analysis.
A fault tree provides a graphical representation of the various component possible failure events which would cause failure of a system. Preparation of a fault tree begins with defining the undesirable event (top event) associated with the performance of a system under consideration. Next, all the potential causes of failure and possible failure events which contribute to the top event are identified. Construction of the fault tree starts with the events that could directly cause the top event, and branches down in proper order to all logical combinations of basic failure events and causes that could bring about the failure event. Thus, the fault tree resembles an inverted converging tree system with no loops and having the branches joined by nodes. The nodes are also called gates in other engineering fields such as electrical, chemical, and nuclear engineering.

Once the fault tree is constructed, the probability of failure of each listed failure event can be analyzed along each of the branches using proper mathematical formulations of failure events and risk calculation methods. The total risk of the system can be assessed through a combination of the nodes and branches of the fault tree.

Figure 3.5 gives the structure of a simple fault tree. The expressions commonly used in preparation of fault trees are illustrated in Fig. 3.6. As shown in Fig. 3.5, there are two kinds of fundamental logic nodes for combining the related failure events in the fault tree construction, namely, the OR and AND nodes. The OR node describes the situation in which the output event (e.g., A in Fig. 3.6) from this node towards the root of the fault tree will exist if one or more of the input events of branches (e.g., $B_1, B_2, \ldots, B_n$ in Fig. 3.6) converging to
Fig. 3.5 Example fault tree.
Symbol Description

**AND node, Intersection, Output A exists if and only if all of** $B_1$, $B_2$, ..., $B_n$ **exist simultaneously.**

**OR node, Union, Output A exists if any of** $B_1$, $B_2$, ..., $B_n$, **or any combination thereof, exists.**

**Identification of a particular event, When contained in the sequence, usually describes the output or input of an AND or OR node.**

**Basic event or condition, usually a malfunction, describable in terms of a specific component or cause.**

**An event purposely not developed further because of lack of information or of insufficient consequence, Could also be used to indicate further investigation when additional information becomes available.**

Fig. 3.6 Fault tree symbols.
this node exist. In probability theory terms, the output event is the union of the input events of the same OR node. The AND node describes the logical operation that requires the coexistence of all input events of this node to produce the output event from this node. In other words, the output event is the intersection of the input events of the same AND node. Hence, the approaches to evaluate probabilities for the occurrence of failure events are different between these two types of nodes. For example, as shown in Fig. 3.5, failure event $B_1$ is contributed by the concurrence of the events $C_1$ and $C_2$. Therefore, the probability of $B_1$ occurring can be evaluated by

$$P(B_1) = P(C_1 \cap C_2)$$  \hspace{1cm} (3-59)

If the events $C_1$ and $C_2$ are independent of each other, then Eq. (3-59) becomes

$$P(B_1) = P(C_1) \cdot P(C_2)$$  \hspace{1cm} (3-60)

The undesirable event $A$ results from the occurrence of either the failure event $B_1$ or the event $B_2$, or both. Therefore, the probability of event $A$ occurring can be evaluated by

$$P(A) = P(B_1 \cup B_2)$$  \hspace{1cm} (3-61)

or

$$P(A) = P(B_1) + P(B_2) - P(B_1 \cap B_2)$$  \hspace{1cm} (3-62)

If $B_1$ and $B_2$ are statistically independent, Eq. (3-62) becomes

$$P(A) = P(B_1) + P(B_2) - P(B_1) \cdot P(B_2)$$  \hspace{1cm} (3-63)

If $B_1$ and $B_2$ are mutually exclusive events, Eq. (3-62) is simplified as

$$P(A) = P(B_1) + P(B_2)$$  \hspace{1cm} (3-64)
A basic assumption of fault tree analysis is that the system and its subsystems or components can have only two conditional modes; namely, they can either operate successfully or fail. No operation is partially successful. In actual systems, partial failures may occur so that the system may operate to a limited degree at other than a successful or failed condition. However, the stipulation of total success or failure generally constitutes the use of worst-case conditions.
4.1. Introduction

The fault tree given in Fig. 4.1 shows briefly the failure events which contribute to the failure of an existing earth dam. Failure of a dam occurs when any one or more of the failure events, namely, overtopping, seepage, piping, instability and others, occurs. Hence, an OR node is used to connect the aforementioned failure events. If $F$ represents the event of dam failure, then

$$F = F_v U F_s U F_p U F_I U F_c$$  \hspace{1cm} (4-1)$$

in which $F_v$, $F_s$, $F_p$, $F_I$, and $F_c$, represent the failure events of overtopping, seepage, piping, instability and others, respectively. The total risk of the dam therefore can be evaluated by

$$P[F] = P[F_v U F_s U F_p U F_I U F_c]$$  \hspace{1cm} (4-2)$$

Actually, for a dam system there are two types of failure; namely, structural failure and non-structural failure. The structural failure may involve physical damage of the dam structure, e.g., burst of dam, so that a proper function of the dam system is no longer possible. The non-structural failure may not cause physical damage of dam but involves the injury or loss of lives, property loss or damage, and adverse changes to the environment downstream of dam. It is most unlikely that a structural failure of a dam would happen before the occurrence of the non-structural failure. The fault tree of dam failure as shown in
Fig. 4.1 Simple fault tree for an existing earth dam.
Fig. 4.1 essentially follows the concept of the non-structural failure. If the structural failure is the concern of the risk evaluation, the mechanism between the non-structural failure and structural failure should be further studied.

To evaluate $P[F]$, all the factors and causes related to the occurrence of each failure event in Eqs. (4-1) and (4-2) need to be considered. This requires a tremendous variation of information, knowledge, work and time, and is beyond the scope of this study due to limitations of time and available computer resources. Therefore, this study is limited to the risk evaluation of overtopping. Failure due to other causes can be evaluated similarly and the total risk can be calculated according to the fault tree (FT) and Eq. (4-2).

Overtopping occurs when the water level of the reservoir behind the dam rises above the crest of the dam. Overtopping may result from: (1) failure to make timely and adequate release of flood through the spillway and flood release outlets; (2) wave action induced by wind, landslide, earthquake and other geophysical forces; and (3) the combination effect of (1) and (2). The failure events of overtopping induced by various combinations of geophysical forces are related to the OR node as shown in Fig. 4.1. The risk of overtopping, $P[F_V]$, thus can be evaluated by

$$P[F_V] = P[E_1 \cup E_2 \cup \ldots \cup E_i \cup \ldots \cup E_n]$$

(4-3)

in which $E_i$ represents the failure event of overtoppings due to the $i$-th geophysical force.

In this chapter risk models to evaluate the risk of failure events $F_V$ and $E_i$ are derived and presented. Because the geophysical forces
acting on a dam vary during the service life of the dam, their stochastic characteristics are considered in the risk models. The performance function which relate the load and resistance to the overtopping is also described. In each occurrence of the adverse geophysical forces, random variables in the performance function are assumed to be stationary; i.e., their probabilistic characteristics remain the same. This study considers the overtopping failure events induced by flood and wind as an example. However, the methodology of risk analysis can be applied to other overtopping failure events as well.

4.2. Risk Model of Overtopping

A dam is subjected to the occurrences of loading induced by various geophysical forces during its service life. Overtopping occurs when the water in the reservoir behind the dam flows over the dam proper. Let $h_G$ denote the height of the reservoir water raised by an occurrence of a geophysical force, $G$, such as a flood. If $H_c$ is the height of the dam embankment and $H_o$ is the stage of the reservoir water level before the occurrence of $G$, both measured from the same datum, then overtopping occurs when $(h_G + H_o > H_c)$. Accordingly, the risk of overtopping, $P_G$, given an occurrence of $G$ can be expressed as

\[
P_G = P[h_G + H_o > H_c]
\]

\[
= P[h_G > H_c - H_o]
\]  \hspace{1cm} (4-4)

The geophysical forces which may cause overtopping usually are the
extreme ones which can be regarded as extraordinary forces of rare occurrence. Most of the time, the magnitudes of these forces such as those of ordinary flood or light breeze are negligible when considering the probability of overtopping. Analysis of historical records shows that the occurrences of these forces can be regarded as random with respect to time, their numbers as well as their duration are subject to statistical variations. To evaluate the risk of overtopping over a given period of time, the random process of the geophysical forces should first be determined.

The occurrence of an extraordinary force is usually assumed to follow a Poisson process (Wen, 1977a; Tang, 1981) which is based on the following assumptions:

1. A force can occur at any time.
2. The occurrence of the force in a given small time interval is independent of that in any other non-overlapping intervals.
3. The probability of occurrence of the force is approximately proportional to the length of the small time interval. The probability of two or more occurrences of the force in the same small time interval is negligible.

On the basis of these assumptions, the probability for a force to occur $n$ times in a given time period $T$ can be described by the Poisson process as

$$P[G \text{ occurs } n \text{ times}] = \frac{(\nu T)^n e^{-\nu T}}{n!} \quad n = 0, 1, 2, \ldots, N$$

(4-5)

in which $\nu$ is the mean rate of occurrence of $G$.

By assuming that the probability distributions, which describe the
uncertainties of \( h_G \), \( H_c \) and \( H_o \) respectively, remain the same in each occurrence of \( G \), then from the multiplicative law of probability theory, the risk of overtopping in \( n \) occurrences of \( G \) can be shown as

\[
P(E_G \mid G \text{ occurs } n \text{ times}) = 1 - (1 - p_G)^n
\]

in which \( E_G \) represents the failure event \( h_G > H_c - H_o \). From the total probability theorem, the risk of overtopping induced by occurrences of a geophysical force in a given service time \( T \), is

\[
P_E(T) = \sum_{n=0}^{\infty} P(E_G \mid G \text{ occurs } n \text{ times}) \cdot P(G \text{ occurs } n \text{ times})
\]

\[
= \sum_{n=0}^{\infty} [1 - (1 - p_G)^n] \cdot (\nu T)^n \frac{e^{-\nu T}}{n!}
\]

\[
= 1 - \exp (-\nu T p_G)
\]

(4-7)

To obtain the total risk of overtopping, the relationships among the overtopping event \( F_V \), and the component overtopping events induced by the various geophysical forces, \( E_i \), as shown in Eq. (4-3) are utilized. Let \( E_1 \cup E_2 \cup E_3 \cdots \cdots \) be the complement of \( E_1 \cup E_2 \cup E_3 \cdots \cdots \) i.e., nonoccurrence of overtopping, Eq. (4-3) becomes

\[
P[F_V] = 1 - P[E_1 \cup E_2 \cup E_3 \cdots \cdots]
\]

\[
= 1 - P[E_1 \cap E_2 \cap E_3 \cap \cdots \cdots]
\]

\[
= 1 - P[E_1] \cdot P[E_2] \cdot P[E_3] \cdots \cdots
\]

\[
= 1 - \prod_{i=1}^{n} P[E_i]
\]

(4-8)
in which \( E_1, E_2, \ldots \) and \( E_n \) are assumed to be statistically independent, and \( P[E_i] \) is the probability of the \( i \)-th nonovertopping event. Therefore, from Eq. (4-7) the probability for event \( E_i \) not to occur in a given time \( T \) is

\[
P_{E_i}(T) = 1 - P_{E_i}(T) = \exp (-v_i T p_i)
\]  

(4-9)

in which \( v_i \) is the mean occurrence rate of the \( i \)-th geophysical force, and \( p_i \) is the risk of overtopping induced in each occurrence of the \( i \)-th geophysical force. By substituting Eq. (4-9) into Eq. (4-8), the total risk of overtopping in a time period \( T \), \( P_F(T) \) is

\[
P_F(T) = 1 - \prod_{i=1}^{n} \exp (-v_i T p_i)
\]

(4-10)

To account for the various possible combinations of forces which would occur during the service life of a dam, tree-type connections with YES and NO paths for different combinations of forces as shown in Fig. 4.2 is helpful. Each path describes one possible combination of forces. For the four possible geophysical forces shown, there are \( n=15 \) possible combinations of forces which could induce overtopping failure. With Eq. (4-10), the total risk of overtopping induced by these four geophysical forces can be represented by the following equation

\[
P_V(T) = 1 - \exp \left\{-T \left[ \sum_{i=1}^{4} v_i p_i + \sum_{ij}^{4} v_{ij} p_{ij} + \sum_{ijk}^{4} v_{ijk} p_{ijk} \right] \right\}
\]

(4-11)
Fig. 4.2 Possible combinations of occurrences of four geophysical forces.
in which \( v_i \) is the mean occurrence rate of one geophysical force; \( v_{ij} \), \( v_{ijk} \) and \( v_{ijkl} \) are the mean rates of coincident occurrences of any two, three, and four forces, respectively; and \( p_i \), \( p_{ij} \), \( p_{ijk} \) and \( p_{ijkl} \) are the risks of overtopping given the coincident occurrence of one, two, three, and four geophysical forces, respectively.

Equation (4-11) shows essentially the same as that given by Wen (1980) for the probability of extreme load combination. The four terms within the bracket of Eq. (4-11) represent the relative contributions to the total risk of overtopping from respective component overtopping events induced by the various combinations of the four geophysical forces. In practical situations not all the terms within the brackets need to be considered. Usually, the fourth term is negligible due to the highly unlikelyhood of simultaneous occurrences of all four forces; the third term is significant only when the product value of the mean occurrence rate, \( v_i \), and the mean duration, \( u_i \), of each component load induced by the respective force is greater than \( 10^{-1} \); otherwise it can be ignored when \( v_i u_i \) smaller than \( 10^{-2} \) (Wen, 1977b).

If two or more independent single forces, each described by a Poisson process, occur simultaneously, then the induced combined load also follows the Poisson process. By assuming the magnitude of the load independent to the duration of the load, Wen (1977a) showed that the mean rate of combined load can be derived from the single load process such

\[
v_{ij} = v_i v_j (u_i + u_j)
\]  

(4-12)

for two concurrent single forces; and

\[
v_{ijk} = v_i v_j v_k (u_i u_j + u_j u_k + u_i u_k u_j)
\]  

(4-13)
for three concurrent single forces. The symbol \( u_i \) stands for the mean duration of the \( i \)-th load induced by a single occurrence of geophysical force in a period of time \( T \).

The assumption of independence for the occurrence, the magnitude and duration of the loads may not be satisfactory for some geophysical forces. Wen and Pearce (1981) developed models for dependent load process. However, because of the complicated calculations involved in the dependent-load models and the lack of the needed information, no attempt was made in the present study to deal with the dependency of the loads. As an initial study of the risk evaluation of dams, and for the purpose of obtaining useful preliminary results, errors due to the independency assumption are tolerated. However, it would be desirable to assess these factors in future studies.

4.3. Risk Modeling for Flood-induced Overtopping

4.3.1. Risk Criterion

Irrespective of their main objectives of construction, dams will store the flood water and attenuate their peak discharges. When the spillway and outlet fail to make timely and adequate release of water behind the dam, the reservoir level, which is built up by the flood, will overtop the crest of the dam. As demonstrated in Eq. (4-4), the probability of overtopping induced by the occurrence of a flood can be defined as

\[
p_F = P[h_F > H_c - H_o]
\]

in which \( h_F \) is the maximum rise of reservoir level built up by the flood. Assuming that the occurrence of floods follows the Poisson process with a
mean occurrence rate $\nu_F$, and the probability functions for $h_F$, $H_o$, and $H_c$ remain the same for each occurrence of flood, then according to Eq. (4-7) the probability of overtopping due to flood in a period of time $T$, $P_F(T)$, can be evaluated from

$$P_F(T) = 1 - \exp\left(-\nu_F T p_F\right)$$  \hspace{1cm} (4-15)

The variables involved in Eqs. (4-14) and (4-15) are discussed in the following section.

4.3.2. Formulation of Performance Function

The risk calculation by means of the advanced first-order second-moment (AFOSM) method requires the formulation of the performance function which, from Eq. (4-14), can be represented as

$$g_F = H_c - H_o - h_F$$  \hspace{1cm} (4-16)

The value of $H_c$ can be determined from direct measurement at the dam site or from the specification of the original design. The initial reservoir level, $H_o$, is a function of reservoir operation and is the result of the water budget among the inflow, outflow, evaporation and other losses. The initial reservoir level $H_o$ can be obtained or approximated on the basis of past observations of the reservoir level, reservoir operation and subjective judgment. The maximum of $H_o$ should not be greater than the top of the dam, $H_c$.

The parameter $h_F$ represents the maximum rise of water level induced by the load of a flood, and its value is a function of the following: (1) the inflow hydrograph of the flood into the reservoir; (2) the initial reservoir stage, $H_o$; (3) the storage-stage relationship of the reservoir;
Fig. 4.3 Schematic sketch of dam and reservoir system.
and (4) the discharge-stage relationship of the dam. The inflow and outflow hydrographs of a flood into and out of an uncontrolled reservoir as well as the change of reservoir stage and storage volume due to flood are shown in Fig. 4.4. Before the occurrence of \( h_F \), the reservoir is being filled whereas after it the reservoir is being depleted. Therefore, the flood water stored in the reservoir reaches its maximum, \( S_{\text{max}} \), at the height \( h_F \). The outflow discharge increases as the uncontrolled reservoir level becomes higher and reaches its maximum at the height \( h_F \). The magnitude of \( h_F \) can be determined from \( S_{\text{max}} \) together with the storage-stage relationship, or from \( Q_{\text{op}} \) with the discharge-stage rating curve as shown in Fig. 4.5. The storage-stage relationship curve usually can be constructed from topographic surveys. The discharge-stage rating curve depends on the type of outflow structure of the reservoir. For example, the relationship of discharge and stage of uncontrolled overflow spillways can be expressed by

\[
Q = C_d L h^{3/2}
\]

(4-17)

where \( Q \) is flow discharge through the spillway, \( C_d \) is the discharge coefficient, \( L \) is effective spillway length, and \( h \) is the height of water above the spillway crest. If the velocity head of the approaching flow in the reservoir is negligible, \( h_F \) can be computed as

\[
h_F = (Q_{\text{op}} / C_d L)^{2/3} + H_s - H_o
\]

(4-18)

in which \( H_s \) is the height of the spillway crest and \( Q_{\text{op}} \) is the peak spillway discharge of the flood.

To obtain the peak discharge \( Q_{\text{op}} \) or the maximum storage volume
Fig. 4.4 Inflow and outflow hydrographs, storage, and stage of an uncontrolled reservoir.
Fig. 4.5 Storage, discharge, and stage curve relation for a reservoir.

Fig. 4.6 Convolution of ERH and IUH (Chow, 1964).
for the determination of h_F, information on the flood hydrograph into the reservoir should be provided. The reservoir routing technique together with the storage-stage curve and discharge-stage curve are used to determine \( Q_{op} \) and \( S_{max} \) from the flood hydrograph. The determination of the flood hydrograph for an ungaged watershed and the reservoir routing technique used in this study are presented in following sections.

### 4.3.2.1. Determination of Flood Hydrograph

Flood water entering a reservoir comes from (1) flood from the streams entering the reservoir; (2) overland flow entering directly from surrounding areas of the reservoir; and (3) precipitation falling directly onto the reservoir surface. The hydrological data most directly useful in determining the flood hydrographs are the actual streamflow records over a considerable period of time at the dam site. Such records are rarely available for small dams. The flood hydrographs, however, may be derived or converted from the precipitation data and streamflow records at nearby locations in the watershed or in watersheds having similar characteristics. Methods and procedures for estimating the flood hydrographs from various records and related information have been described elsewhere (Chow, 1962, 1964; Bureau of Reclamation, 1977).

In view of the fact that most watersheds draining into small dams are ungaged and that information on rainfall is easier to obtain than that of runoff, often the flood hydrograph into a reservoir is derived from using the available rainfall information. Many methods can be used to estimate runoff from rainfall. In this study, as an example of risk evaluation, the flood hydrograph is determined from the convolution of
the effective rainfall and the instantaneous unit hydrograph. Other rainfall-runoff techniques can also be adopted with corresponding formulations for risk evaluation.

The term "effective rainfall" (ER) defined here as the part of total rainfall that contributes entirely to the direct runoff, is the difference between the total rainfall and the abstractions of interception, evaporation, transpiration, depression storage, deep percolation and delayed subsurface runoff. The instantaneous unit hydrograph (IUH) is a conceptual hydrograph of direct runoff resulting from one inch of effective rainfall applied instantaneously and uniformly over the entire watershed. As shown in Fig. 4.6, let \( i(t) \) denote the effective rainfall intensity at time \( t \), \( u(t-\tau) \) the ordinate of IUH at time \( t-\tau \). By the principle of linear superposition, the ordinate of the direct runoff hydrograph (DRH) at time \( t \), \( Q(t) \), is

\[
Q(t) = \int_{0}^{t} u(t-\tau) i(\tau) d\tau
\]

(4-19)

By convoluting the effective rainfall hyetograph (ERH) and IUH through the rainfall duration \( t_r \), the DRH of the watershed can be obtained. For simplicity, the base flow during the occurrence of flood is assumed to be equal to that before the occurrence of flood. Since the base flow is included in the consideration of initial reservoir level \( H_0 \), the DRH is used as the flood hydrograph which contributes to the rise of reservoir level.

To compute the flood hydrograph using the convolution process, the ERH and the IUH of the watershed should be provided. Methods to derive the IUH from rainfall and runoff records of a watershed can be found
elsewhere (Chow, 1964). For an ungaged watershed for which rainfall and runoff records are not available, the IUH can be derived from geomorphologic parameters of the watershed (Cheng, 1982) or from synthetic unit hydrograph techniques, such as those suggested by Snyder (1955) or by the U.S. Soil Conservation Service (1972). In this study, for simplicity, the synthetic triangular unit hydrograph proposed by SCS (1972) and also used by the Bureau of Reclamation (1977), as shown in Fig. 4.7, is adopted as an example to produce the IUH.

Three basic components describe the triangular unit hydrograph. They are: (1) the time to the peak discharge, $t_{up}$, (2) the duration of the hydrograph, $t_{ud}$, and (3) the peak discharge, $q_u$. They can be computed by using the following empirical equations suggested by SCS:

$$t_{up} = \frac{t_{ed}}{2} + t_L \quad (4-20)$$

$$t_{ud} = 2.67 \ t_{up} \quad (4-21)$$

$$q_u = \frac{484 \ A}{t_{up}} \quad (4-22)$$

in which $t_{ed}$ is the duration of the effective rainfall in hr; $t_L$ is the time lag from the center of the effective rainfall to the time of peak in hr., and A is the area of the watershed in square miles. The unit of the peak discharge, $q_u$, in Eq. (4-22) is in cfs.

As defined previously, the IUH is a hypothetical unit hydrograph whose duration of effective rainfall approaches zero as a limit, i.e.,
Fig. 4.7 Triangular unit hydrograph.

Fig. 4.8 Determination of watercourse length and its elevation difference.
\( t_{ed} = 0 \). Therefore from Eq. (4-20), the time to peak discharge, \( t_{up} \), of IUH is given by

\[
t_{up} = t_L
\]  \hfill (4-23)

As pointed out by Chow (1962), the lag time of the IUH for small drainage basins ( < 50 \text{ mi}^2), is very close to, and therefore equivalent to, the time of concentration, \( t_c \), i.e.,

\[
t_{up} = t_c
\]  \hfill (4-24)

The concentration time, \( t_c \), is defined as the time required for the surface runoff from the remotest part of the watershed to reach the point being considered. Methods for estimating \( t_c \) have been discussed by Chow (1962) and others. Among these methods, the empirical formula proposed by Kirpich (1953) is commonly used. Kirpich's formula is

\[
t_c = (11.9 \frac{L_w^3}{H_w})^{0.385}
\]  \hfill (4-25)

in which \( t_c \) is in hours, \( L_w \) is the length of the longest flow path in miles, and \( H_w \) is the elevation difference of \( L_w \) in feet. Figure 4.8 illustrates the measurement of \( L_w \) and \( H_w \) for the watershed upstream of a reservoir. The IUH can be determined by using Eq. (4-24) to compute \( t_{up} \) and using Eqs. (4-21) and (4-22) to compute \( t_{ud} \) and \( q_u \), respectively.

It is well known that the peak discharge and total volume of the flood runoff depend on the depth and time distribution of the contributing rainfall. For determining the temporal pattern of the rainfall, approaches using the mass curve, such as those proposed by Huff (1967), the U.S. Soil Conservation Service (1972) and the British National Environmental Council (1975), are frequently used. The time distribution of rainfall can be obtained from the curve of cumulative
rain depth versus time. However, in nature, no two rainstorms have the same temporal pattern. The use of the mass curve means that all the rainstorms follow the same temporal pattern. Chow and Yen (1976) studied statistically the time distribution of rainfall using the method of moments. Use of a synthetic hyetograph based on the statistics of rainstorm parameters may give a reasonable estimate of rainstorms in the evaluation of risk. Because of its simplicity and feasibility to estimate reasonably the runoff hydrograph, the nondimensional triangular hyetograph (Yen and Chow, 1980) as shown in Fig. 4.9 is used in this study.

The symbol $a^0$ in Fig. 4.9 is the ratio of the time to peak intensity, $t_a$, to the duration of rainfall. With a given $a^0$, rainfall duration, $t_r$, and depth, $D_R$, the time to peak intensity, $t_a$, and the peak intensity of the synthetic hyetograph, $i_p$, can be determined from

$$t_a = a^0 t_r$$

$$i_p = \frac{2D_R}{t_r}$$

The effective rainfall hyetograph is determined by drawing a horizontal line on the hyetograph in such a way that the area of the hyetograph above the horizontal line is equal to the depth of the direct runoff (Fig. 4.10). If $D_Q$ stands for the depth of the direct runoff, then from this geometric relationship the duration, $t_{ed}$, time to peak, $t_{ea}$, and maximum intensity, $i_{ep}$ of the ERH can be obtained by the following equations

$$t_{ed} = t_r \sqrt{\frac{D_Q}{D_R}}$$
Fig. 4.9 Dimensionless triangular hyetograph.

Fig. 4.10 Derivation of ERH from triangular rainfall hyetograph.
For estimating the direct runoff depth from the total rainfall depth for the ungaged watershed, the empirical formula developed by SCS is used

\[ D_Q = \frac{(R - 200 \frac{CN}{CN} + 2)^2}{R + 800 \frac{CN}{CN} - 8} \]  

(4-31)

in which \( CN \) is the runoff curve number and is determined by the type of soil, soil cover, antecedent soil moisture condition and land use condition of the watershed. According to their hydrologic properties, the watershed soils are primarily classified into Groups A, B, C, and D in order from high to low infiltration potential. For each soil group, different \( CN \) values are given according to the conditions of soil cover and land treatment. Antecedent soil moisture conditions are classified into three groups. They are (1) dry moisture (AMC I), (2) average moisture (AMC II), and (3) wet moisture (AMC III). The \( CN \) values can be further adjusted according to the moisture condition of the watershed soil. The classification of the soil groups, the tables giving \( CN \) values for different soil conditions, and detailed procedures for determining \( CN \) values can be seen elsewhere (SCS, 1972; Chow, 1962).

Once the IUH and the ERH are determined, the flood hydrograph, i.e., DRH, can be determined from the convolution integral, Eq. (4-18). However, the numerically tedious convolution integral may cause calculation problem in the risk evaluation. To overcome this disadvantage, Cheng (1982) used Laplace transformation technique to solve the convolution integral. With the assumption that the ERH and IUH
are triangular in shape, the solution of discharge \( Q(t) \) in Eq. (4-19), solved by using the Laplace transformation technique, is

\[
Q(t) = \frac{\int_{0}^{t} q_u(t) \, dt}{6} \left\{ \frac{t^3}{t_e a \, t_e d \, t_u d} - \frac{U_{t_e a}(t)(t - t_{ea})^3}{t_e a \, t_e d \, t_u d} \right. \\
+ \frac{U_{t_e d}(t)(t - t_{ed})^3}{(t_{ed} - t_{ea}) \, t_e a \, t_u d} - \frac{U_{t_u p}(t)(t - t_{up})^3}{t_e a \, t_e d \, (t_{ud} - t_{up})} + \frac{U_{t_u d}(t)(t - t_{ud})^3}{t_e a \, t_u d \, (t_{ud} - t_{up})} \\
+ \frac{U_{t_e a + t_{up}}(t)(t - t_{ea} - t_{up})^3}{t_e a \, t_u d \, (t_{ed} - t_{ea})(t_{ud} - t_{up})} + \frac{U_{t_{ed} + t_{ud}}(t)(t - t_{ed} - t_{ud})^3}{t_{ed} \, t_u d \, (t_{ed} - t_{ea})(t_{ud} - t_{up})} \\
- \frac{U_{t_e a + t_{ud}}(t)(t - t_{ea} - t_{ud})^3}{t_e a \, t_u d \, (t_{ed} - t_{ea})(t_{ud} - t_{up})} - \frac{U_{t_{ed} + t_{up}}(t)(t - t_{ed} - t_{up})^3}{t_{ed} \, t_u d \, (t_{ed} - t_{ea})(t_{ud} - t_{up})} \right. \\
\left. \right\} 
\]

(4-32)

Eq. (4-32) not only eliminates the necessity of the integration but also provides an easier calculation procedure for obtaining the flood hydrograph. Derivation of the above equation is shown in Appendix B. Figure 4.11 shows the flood hydrograph obtained by using Eq. (4-32). The IUH, ERH, and DRH determined using the foregoing procedures are all approximations. Refined analyses from improved methods, such as using infiltration indices and curves rather than constructing a horizontal line to determine the duration of the ERH, are desired and further research is needed for such refinements.

4.3.2.2. Flood Routing in Reservoir

The maximum reservoir height \( h_F \), is determined by routing the
Fig. 4.11 Flood hydrograph convoluted from triangular ERH and IUH.
inflow hydrograph through the reservoir. The basic equation used in the reservoir routing procedure is the continuity relationship

\[ Q_1 - Q_0 = \frac{dS}{dt} \]  \hspace{1cm} (4-33)

in which \( Q_1 \) is the inflow into the reservoir, \( Q_0 \) is the outflow from the reservoir, and \( dS/dt \) is the time rate of change of the volume of water, \( S \), stored in the reservoir. Since the peak discharge \( Q_{op} \) occurs at the maximum stage \( H_F \), which is the sum of \( h_F \) and \( H_0 \), \( dS/dt = 0 \). Therefore, from Eq. (4-33), for an uncontrolled reservoir the peak discharge \( Q_{op} \) occurs at the intersection of the inflow and outflow hydrograph where \( Q_1 = Q_0 \) (Fig. 4.4).

Many flood routing methods have been developed to determine the maximum reservoir height \( h_F \) during the occurrence of a flood. Examples include the Puls method, the Muskingum method, and the graphical method. From a practical viewpoint, it is desirable to simplify the shape of the inflow flood hydrograph and the reservoir routing calculation procedure as illustrated in the next paragraph. Further refinement can be made for better accuracy, if required, by using more complicated routing methods.

For simplicity in computation without losing generality, the inflow hydrograph of the flood into the reservoir is approximated by a simple triangle instead of an actual curvilinear shape as shown in Fig. 4.12. In this triangular approximation the peak discharge and its time, and the total volume of the curvilinear flood inflow hydrograph are retained. The triangular shape may result in shorter runoff time than that of the curvilinear hydrograph. However, this difference is not significant in the determination of the outflow peak discharge \( Q_{op} \) and load \( h_F \).
The simplified reservoir routing scheme using triangular inflow and outflow hydrographs is illustrated in Fig. 4.13. The outflow hydrograph begins at time \( t_s \) when the reservoir is filled up to the crest of the spillway. The rising limb of outflow hydrograph is assumed to be linear with its maximum at the point intersecting the recessing limb of inflow hydrograph, i.e., point P in Fig. 4.13 where \( Q_i = Q_o \).

Given the initial reservoir stage, \( H_o \), the inflow hydrograph, the storage-stage relationship curve, and discharge rating curve, the value of \( h_F \) can be determined. As shown in Fig. 4.13, the shaded area \((ABDP)\) between the inflow and outflow hydrograph is the maximum amount of flood water that is stored in the reservoir. From the geometric relationship, it can be expressed as

\[
S = \frac{1}{2} Q_{ip} t_d - \frac{1}{2} Q_{op} (t_d - t_s)
\]

in which \( S \) is the maximum amount of water stored in the reservoir, \( Q_{ip} \) is the peak discharge of the inflow hydrograph, \( t_d \) is the duration of the inflow hydrograph, and \( t_s \) is the time when outflow hydrograph starts. The volume \( S \) is equal to the net storage volume of the reservoir between the initial reservoir stage, \( H_o \), and the maximum reservoir level \( H_F \). Hence,

\[
S = S_F - S_o
\]

where \( S_F \) and \( S_o \) are the reservoir storage capacities for the stages \( H_F \) and \( H_o \), respectively, and can be obtained from the storage-stage relationship of reservoir as

\[
S_F = f (h_F + H_o)
\]

\[
S_o = f (H_o)
\]
Fig. 4.12 Triangular representation of inflow hydrograph.

Fig. 4.13 Simplified reservoir routing.
where \( f(.) \) denotes the function of the storage-stage relationship. Substituting Eq. (4-34), (4-36) and (4-37) into Eq. (4-35), one obtains

\[
f(H_F) - f(H_0) = \frac{1}{2} Q_{ip} t_d - \frac{1}{2} Q_{op} (t_d - t_s)
\]  

(4-38)

For an uncontrolled spillway, the peak of the outflow hydrograph, \( Q_{op} \), can be determined from Eq. (4-17), i.e.,

\[
Q_{op} = C_d L (H_F + H_o - H_c)^{3/2}
\]  

(4-39)

The substitution of Eq. (4-39) into Eq. (4-38) yields the following implicit function for the maximum reservoir level \( h_F \),

\[
F(h_F) = f(h_F + H_o) - f(H_o) - \frac{1}{2} Q_{ip} t_d - \frac{1}{2} C_d L (h_F + H_o - H_c)^{3/2} (t_d - t_s) = 0
\]  

(4-40)

Since \( F(h_F) \) usually is a nonlinear implicit function of \( h_F \), the numerical calculation technique such as Newton iteration method can be used to solve for \( h_F \).

Values of the peak inflow discharge \( Q_{ip} \) obtained from Eq. (4-32) and the maximum reservoir height \( h_F \) determined from Eq. (4-40) are only approximations. Model error correction factors are used to account for the possible errors which may result from inadequate information, approximation procedures, and the empirical formula used to determine \( Q_{ip} \) and \( h_F \). If \( Q_{ip}^* \) represents the flood peak discharge estimated from Eq. (4-32), then the adjusted peak discharge \( Q_{ip} \) is

\[
Q_{ip} = \lambda_q Q_{ip}^*
\]  

(4-41)

Similarly, if \( h_F^* \) represents the maximum reservoir height calculated from Eq. (4-40), then the adjusted maximum reservoir height \( h_F \) is

\[
h_F = \lambda_F h_F^*
\]  

(4-42)
4.3.3. Variables of Performance Function

All the variables relevant to overtopping failure as described in the preceding section are subject to uncertainties. The statistical information of each variable should be known or predetermined from the past record, direct measurement, related sources and subjective judgement.

The variables of the performance function are divided into the following four groups:

1. **Hydrologic variables** -- which include the rainfall depth $D_R$, duration of hyetograph, $t_r$, ratio of time to peak, $a^0$, runoff curve number, $CN$, and initial reservoir level, $H_o$.

2. **Watershed variables** -- which include the watershed area, $A$, length of watercourse, $L_w$, and elevation difference between the ends of the main water course, $H_w$.

3. **Hydraulic variables** -- which include the spillway discharge coefficient, $C_d$, and the dimensions of outflow structure such as spillway length, $L_s$, and crest height, $h_s$.

4. **Correction variables** -- which account for model error factors such as $\lambda_Q$ and $\lambda_F$.

The uncertainties of these variables are discussed in the following section.

4.3.3.1. Hydrologic Variables

Statistics of the rainfall parameters such as depth and duration can be obtained from the rainfall record in the region of interest. However, rainfall information often is either unavailable or inadequate
for most of the watersheds upstream of dams of small to medium size. In such cases the rain statistics can be estimated from the records at nearby precipitation stations or from other sources. Alternatively, the U. S. Weather Bureau Technical Paper No. 40 (TP-40, Hershfield, 1961) gives rainfall depth contour maps for specified rainfall durations ranging from 0.5 to 24 hours, and is traditionally a standard source of rainfall information for engineering projects in the United States. Hence, it is used in this study to provide rainfall information for the risk evaluation of dams. The probability density function (pdf) of the rain depth for each duration of rainfall can be derived by plotting the raindepths for the given return period on a probability paper of log-normal distribution or Gumbel distribution (Hershfield, 1961). The pdf of Gumbel distribution (Extreme Type I distribution) is

\[ f(x) = \frac{1}{c} \exp \left\{ -\left( \frac{a + x}{c} - \exp \left[ -\frac{a + x}{c} \right] \right) \right\}, \quad -\infty < x < \infty \]  \hspace{1cm} (4-43)

and the cumulative probability function (cdf) of this distribution is

\[ F(x) = \exp \left\{ -\exp \left[ -\frac{a + x}{c} \right] \right\} \]  \hspace{1cm} (4-44)

where \( x \) is the variate representing the rainfall depth, and \( a \) and \( c \) are parameters calculated by

\[ a = \gamma c - \mu \]  \hspace{1cm} (4-45)

and

\[ c = \frac{\sqrt{6}}{\pi} \sigma \]  \hspace{1cm} (4-46)

in which \( \gamma \) is the Euler constant equal to 0.57721, \( \mu \) is the mean, and \( \sigma \) is the standard deviation of the rain depth. A procedure to derive the probability functions for rainfall depth will be discussed in the case study given in Chapter 5.
The rainfall depths obtained from TP-40 are the annual series events. Therefore, the mean occurrence rate of the rainfall is once per year. Because the flood is caused by rainfall, the mean occurrence rate of flood which is derived from the rainfall is also once per year, i.e., $v_F = 1$/year.

The statistics of the rainfall duration, $t_r$, was studied by Chow and Yen (1976) and Alexander (1981) for certain locations in the United States. Their results showed that the probability distribution of $t_r$ can be approximated by an exponential distribution. The pdf and cdf of the exponential distribution are respectively,

$$f(x) = \frac{1}{c} \exp(-x/c)$$

and

$$F(x) = 1 - \exp(-x/c)$$

in which $c$ is equal to the sample mean of $t_r$. By analyzing 455 rainstorms of hourly rainfall record at Urbana, Illinois, Chow and Yen (1976) found that $c = 2.45$ hr.

In nature, rainfall may occur of any duration. Since the occurrence frequencies of rainfall duration are different, the risk value should be weighted. Let $p[V|t_r]$ represent the risk of overtopping based on rainfall information with a duration $t_r$, and $f(t_r)$ denotes the pdf of $t_r$, then the risk of overtopping, $p_F$, can be calculated by

$$p_F = \int_0^\infty p[V|t_r] f(t_r) \, dt_r$$

Equation (4-49) can be approximated in discrete form as

$$p_F = \sum_{n=1}^{\infty} p[V|t_r] [P(t_r + \frac{\Delta t}{2}) - P(t_r - \frac{\Delta t}{2})]$$

where $t_r = n\Delta t$. 

\[ (4-50) \]
in which \( P(t_r - \Delta t/2) \) and \( P(t_r + \Delta t/2) \) are the cumulative probability at the duration \( t_r - \Delta t/2 \) and \( t_r + \Delta t/2 \), respectively.

The ratio \( a_0 \) of the triangular hyetograph for different durations, seasons and locations in the United States is given by Yen and Chow (1977) and Yen et al. (1982). It is found that statistical values of \( a_0 \) are rather consistent for different durations but varies with the season and location. As shown by Yen and Chow (1977), the values of the mean and COV of \( a_0 \) could range from 0.26 to 0.56 and 0.20 to 0.35, respectively.

The type of soil, soil cover, land use condition, and antecedent moisture condition are the factors in determining the value of CN. Table 4.1 lists the values of CN for different hydrologic soil-cover conditions under AMC II. Since AMC I and AMC III represent the dry and wet moisture conditions of the soil, the CN values for these AMC's are different from that for AMC II. Table 4.2 lists the SCS recommended values of CN for AMC I and III in terms of AMC II.

The initial reservoir stage, \( H_0 \), is a result of the water budget balance among the inflow, outflow, evaporation and other losses before the occurrence of the flood, and most likely also depends heavily on the season. The uncertainty of \( H_0 \) can be estimated from the past record of reservoir level and future operation of the reservoir.

4.3.3.2. Watershed Variables

Usually, the watershed area \( A \), main watercourse length, \( L_w \), and elevation difference, \( H_w \), of the watershed are determined from a topographic map or field survey. The errors in determining \( A \), \( L_w \) and \( H_w \),
Table 4.1 SCS runoff curve number for different hydrologic soil-cover conditions for antecedent moisture condition II

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* Close-drilled or broadcast
§ Including right-of-way
Source of Reference SCS (1972)
Table 4.2 SCS runoff curve number for different antecedent moisture conditions

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come mainly from three sources: (1) the uncertainty in delineating the boundary of the watershed and the longest watercourse, (2) the error of the measurement, and (3) the error of the map, if used.

In determining the area of a three-square mile drainage basin from a U. S. Geological Survey 7.5-minute topographic map, Yen (1975) found that the average error of area measurement for 34 persons expressed in terms of the coefficient of variation is $\delta_A = 0.045$. The error may be larger if the measurement is made by fewer persons.

4.3.3.3. Hydraulic Variables

According to Eq. (4-17), the spillway length, $L_s$, spillway crest height, $h_s$, and the discharge coefficient, $C_d$, together with Eq. (4-17) determine the discharge capacity of the spillway. Uncertainties of $L_s$ and $h_s$ come from the measurement errors and possible plugging due to debris, vegetation, tree branches or ice at the spillway crest. The discharge coefficient, $C_d$, usually is a function of side contraction, roughness of the spillway surface and the water level above the spillway crest, and its value should be determined experimentally. Values of $C_d$ for different types of spillway can be found elsewhere (Chow, 1959; King and Brater, 1963; Bureau of Reclamation, 1977).

The height of the dam crest, $H_c$, can be determined from field measurement. Uncertainty of $H_c$ comes from settlement of the foundation at the dam site and measurement errors. If $H_c$ is determined from the specifications of the design, then the uncertainty of $H_c$ should include the possible construction error.
4.3.3.4. Modeling Correction Variables

Modeling correction variables, $\lambda_Q$ in Eq. (4-41) and $\lambda_F$ in Eq. (4-42), are used to account for the model errors because of the use of empirical formulation or approximated mathematical representation of the physical phenomena. The correction variable, $\lambda$, is expressed as

$$\lambda = \frac{\text{correct value}}{\text{approximated value}} \quad (4-51)$$

Uncertainties of the correction variables, therefore, are determined by comparing the results determined from an accurate model, or obtained from actual measurements, to the results obtained from the approximation.

4.4. Risk Modeling for Wind-induced Overtopping

4.4.1. Risk Criterion

Two effects which may contribute to overtopping induced by wind are (1) height of wind tide, $h_T$, above the reservoir water level and (2) height of wave run-up, $h_r$, on the upstream slope of the dam. Overtopping induced by wind without flood occurs when the sum of $h_T$, $h_r$, and the undisturbed reservoir water level, $H_o$, exceeds the crest of dam, $H_c$. The probability of overtopping induced by the occurrence of wind only, without flood and other geophysical forces, therefore, can be defined as

$$P_W = P [h_T + h_r > H_c - H_o] \quad (4-52)$$

Assume that the occurrence of wind follows the Poisson process with mean occurrence rate, $\lambda_w$, and the probability functions which describe $h_T$, $h_r$, and $H_o$ each remain unchanged for each occurrence of wind. The probability of overtopping due to wind alone within a time period $T$, $P_W(T)$, can be evaluated from
A number of formulas have been developed for the computation of the heights of wind tide and wave run-up. The criteria and procedures proposed by the U. S. Army Corps of Engineers and summarized by Saville et al. (1963) have been generally accepted for use in estimating $h_T$ and $h_R$ for inland reservoirs with deep water. These criteria are used in this study.

4.4.2. Determination of Performance Function

Wind tide, or "setup", is the piling up of water at the leeward end of an enclosed body of water, as a result of the horizontal stress on water exerted by the wind. The magnitude of wind tide can be estimated from the following simplified version of Dutch's formula

$$h_T = \frac{V^2 W F}{1400D}$$

where $h_T$ is the setup in feet above the undisturbed water, $V_W$ is the wind velocity in miles per hour, $F$ is the fetch or length of water surface in miles over which the wind blows, and $D$ is the average depth of the reservoir in feet along the fetch.

The height of wave run-up, $h_R$, is the vertical height above the undisturbed water surface that a wave will run up the slope of an embankment. It is a function of the wave characteristics as measured by the ratio between the wave height and wave length, and the slope, roughness, and permeability of the embankment. According to Saville et al. (1963), the wave height in a reservoir is given by the following empirical equation

$$P_W(T) = 1 - \exp\left(-\frac{V_W T}{p_W}\right)$$
where $h_s$ is the average height in feet of the highest one-third of the waves occurring in a particular series called the significant wave, and $F_e$ is the effective fetch length in miles. The wave length, $L$, can be computed from

$$L = 5.12 \, t_w^2$$

(4-56)

where the wave period $t_w$ is given by

$$t_w = 0.49 \, V_w^{0.44} \, F_e^{0.28}$$

(4-57)

Substituting Eq. (4-57) into Eq. (4-56) yields

$$L = 1.23 \, V_w^{0.88} \, F_e^{0.56}$$

(4-58)

From Eqs. (4-55) and (4-58), the ratio between wave height and wave length can be estimated from

$$\frac{h_s}{L} = 0.028 \, V_w^{0.18} \, F_e^{-0.09}$$

(4-59)

With the values of $h_s / L$ and the embankment slope known, the height of the wave run-up, $h_r$, can be estimated from Fig. 4.14. The solid lines in Fig. 4.14 represent the wave run-up on smoothly graded, grassed, and paved slopes. The dashed lines represent the wave run-up on rubble mounds as in breakwaters. The height of wave run-up on a coarse riprap slope is approximately 50% of that for a correspondingly smooth slope.

For the risk calculation using the advanced first-order
Fig. 4.14 Wave run-up ratio versus wave steepness and embankment slope (SaVille et al., 1963).
second-moment method, it is desirable to express the wave run-up height, \( h_r \), as a function of the wave height \( h_s \), the ratio \( h_s / L \) and slope of embankment. Figure 4.15 gives the relationship curves of \( h_r / h_s \) and \( h_s / L \) for different slopes of the embankment. These curves are fitted approximately from the values given in Fig. 4.14 and they can be represented by

\[
h_r = a h_s \exp \left[ -b \left( \frac{h_s}{L} \right) \right]
\]

(4-60)

where \( a \) and \( b \) are coefficients and their values are listed in Table 4.3 for different embankment slopes.

Equations (4-54) and (4-60) used for the determination of the heights of wind tide and wave run-up are derived on the basis of approximation and empirical observations. A model error parameter \( \lambda_w \) is introduced to the results estimated using these equations. Accordingly, the performance function, \( g \), of Eq. (4-52) can be described by

\[
g = H_c - H_o - (h_T + h_r)
\]

\[
= H_c - H_o - \lambda_w \left[ \frac{V^2 F}{1400} + a h_s \exp \left( -b \frac{h_s}{L} \right) \right]
\]

(4-61)

where \( h_s \) and \( h_s / L \) are calculated from Eqs. (4-55) and (4-59), respectively.

4.4.3. Variables in Performance Function

The component variables of load and resistance considered in the performance function, Eq. (4-61), include the wind speed, fetch length, average reservoir depth along the fetch, undisturbed reservoir water
Fig. 4.15 Variation of $h_r/h_s$ vs. $h_s/L$ for different embankment slopes.
Table 4.3 Values of a and b in formula $h_r = a h_s \exp(-b h_s / L)$

<table>
<thead>
<tr>
<th>Slope (Vertical/Horizontal)</th>
<th>Smooth Slope</th>
<th>Rough Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>1:2</td>
<td>2.67</td>
<td>3.73</td>
</tr>
<tr>
<td>1:2.25</td>
<td>2.76</td>
<td>5.45</td>
</tr>
<tr>
<td>1:2.5</td>
<td>2.80</td>
<td>6.88</td>
</tr>
<tr>
<td>1:3</td>
<td>2.75</td>
<td>8.94</td>
</tr>
<tr>
<td>1:4</td>
<td>2.28</td>
<td>10.71</td>
</tr>
</tbody>
</table>

Table 4.4 Relationship between wind over land to that over water (After Saville et al., 1963)

<table>
<thead>
<tr>
<th>Fetch (mi)</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{V_{\text{water}}}{V_{\text{land}}}$</td>
<td>1.08</td>
<td>1.13</td>
<td>1.21</td>
<td>1.28</td>
<td>1.31</td>
<td>1.31</td>
</tr>
</tbody>
</table>
level, crest height of the dam, and modeling error. All these variables are random, with their probability functions assumed to remain unchanged in each occurrence of wind.

The wind velocity, $V_w$, used in estimating the height of wind tide and wave run-up is measured over the water surface of the reservoir and is greater than the corresponding wind velocity measured at land stations. For reservoirs with wind data known from a land station, the wind velocity should be adjusted with ratios given in Table 4.4. Because of the wind direction and other factors, not every occurrence of wind is adverse to the dam. Hence, only the occurrences that may cause significant wave run-up are considered in the risk evaluation. The mean occurrence rate of wind, $\nu_w$, can be determined as the number of wind storms being considered divided by the time period within which the wind storms occurred.

Three probability functions have been widely used to describe the probability distribution of the annual extreme wind speed. They are extreme Type I (Simiu et al., 1979), extreme Type II (Thom, 1968), and Rayleigh distributions (Simiu and Filliben, 1980). The Extreme Type I distribution is also called the Gumbel distribution and its probability functions have been shown in Eqs. (4-43) and (4-44). The probability density functions $f(x)$ and the distribution functions $F(x)$ of the Type II and Rayleigh distributions are as follows.

**Type II (Fréchet) distribution**

\[
f(x) = \frac{b}{a} \left(\frac{x}{a}\right)^{-b-1} \exp \left\{ -\left(\frac{x}{a}\right)^{-b} \right\}
\]

\[
F(x) = \exp\left\{ -\left(\frac{x}{a}\right)^{-b} \right\}
\]
in which $\beta$ and $\alpha$ are the scale and shape parameters, respectively. The Rayleigh distribution is

$$f(x) = \frac{2(x - \gamma)}{\beta^2} \exp\left[-\frac{(x - \gamma)^2}{\beta}\right]$$  \hspace{1cm} (4-64)

$$F(x) = 1 - \exp\left[-\left(\frac{x - \gamma}{\beta}\right)^2\right]$$  \hspace{1cm} (4-65)

in which $\beta$ and $\gamma$ are the scale and location parameters, respectively.

The effective fetch is a weighted fetch by taking into account the effect of difference in reservoir shapes and for obtaining a better correlation between the fetch and wave height. The steps for estimating $F_e$ as given by Saville et al. (1963) are summarized as follows:

1. Locate the maximum fetch line in the reservoir along the direction of the wind.
2. Seven secondary fetch lines radiating from the dam and on each side of the maximum fetch are drawn at $6^\circ$ interval.
3. Multiply the length of each fetch line by the cosine of the angle between the line and the maximum fetch line.
4. Divide the sum of the products in step 3 by the sum of the cosines to obtain the effective fetch distance.

The fetch length may vary with the reservoir level. Higher reservoir levels always result in higher fetch lengths than lower reservoir levels. The fetch lengths $F$ and $F_e$ can be related to, and are functions of the reservoir level $H_o$.

The total number of variables in the performance function may be reduced. The average depth $D$ in Eq. (4-54) can also be expressed as a function of the reservoir level $H_o$. The modeling error, $\lambda_w$, introduced
in Eq. (4-61) is given by comparing the estimated values from the aforementioned equations with the results obtained from actual observations or from improved reliable models.

The fetch, $F$, used in the wind tide computation is frequently taken as substantially longer than the effective fetch $F_e$ used in computing wave characteristics. Wind-tide effects may be transferred, to some extent, around substantial bends in a reservoir, thus warranting the assumption of the existence of a longer fetch than that indicated by a clear straight fetch length. Usually the fetch, $F$, is taken as twice the effective fetch $F_e$.

Since the true probability distribution governing the random mechanism of an event can never be known perfectly, the choice of one particular distribution may give a significantly different answer from that of the other. This has been clearly illustrated in the examples given by Wood and Rodríguez-Iturbe (1975), Tang (1980) and Tung and Mays (1980). Instead of assuming a specific probability distribution for wind speed, it may be conceptually more appealing to assume that each distribution could be potentially correct, and to evaluate the risk composited from the result of each distribution. Let $p_{w,i}$ represent the evaluated risk based on wind speed fitted by a probability distribution $i$, and $P[i]$ denotes the probability that the distribution $i$ is the "true" one among all the $N$ distributions considered, then the risk, $p_w$ of overtopping in an occurrence of wind considering $N$ distributions can be weighted by

$$p_w = \sum_{i=1}^{N} P[i] p_{w,i} \quad (4-66)$$
The magnitude of the weighting factor \( P[i] \) is determined based on collected data, prior knowledge and subjective judgement. By extending the weighting method proposed by Tang (1980) and Tung and Mays (1980), \( P[i] \) may be evaluated by

\[
P[i] = \frac{\text{Var}_i(V_w)}{\sum_{j=1}^{N} \text{Var}_j(V_w)}
\]

(4-67)

in which \( \text{Var}_i(V_w) \) is the variance of \( V_w \) fitted by a probability distribution \( i \).

4.5. Overtopping Risk Induced by Concurrence of Flood and Wind

4.5.1. Risk Criterion for Failure Event due to Concurrence of Component Loads

The combined load of simultaneous occurrences of different natural forces may be more significant than the load induced by a single natural force. However, the maximum value, \( L_c \), of the combined load usually is no greater than the sum of the maximum values, \( x_{im} \), of the individual loads. The magnitude of \( L_c \) depends on the duration, magnitude, shape and occurrence time of each component load (Wen, 1977b; Larrabee and Cornell, 1979). If all the component loads are constant during the coincident occurrence, then \( L_c \) is simply equal to

\[
L_c = \sum_{i=1}^{n} x_{im}
\]

(4-68)

If the intensity of each component load varies with time during the coincident occurrence, then \( L_c \) is given by
Fig. 4.16 Combined load from superposition of concurrent loads.

(a) Constant $X_1(t)$

(b) Variable $X_1(t)$
\[ L_c = \text{MAX} \sum_{i=1}^{n} X_i(t) \]  \hspace{1cm} (4-69)

in which \( X_i(t) \) is the \( i \)-th component load at any time \( t \) in a coincident occurrence. Equations (4-68) and (4-69) are given based on the principle of linear superposition. Figure 4.16 illustrates the superposition of the loads which occur simultaneously.

For the case of simultaneous occurrences of flood and wind, the combined load \( L_c \) can be calculated by

\[ L_c = \text{MAX} \{ h_H(t) + h_Z(t) \} \]  \hspace{1cm} (4-70)

in which \( h_H(t) \) denotes the height of reservoir level at time \( t \) built up by flood (Fig. 4.4), and \( h_Z(t) \) is the sum of wind tide height and wave run-up height at time \( t \). In this study the load \( h_Z(t) \) in the case of concurrence of flood and wind is assumed to vary with time, whereas \( h_w \) in the case of overtopping induced by wind alone is assumed constant. Changes of the load \( h_Z(t) \) come from changes in fetch length and reservoir level due to the occurrence of flood. The probability of overtopping due to a concurrence of flood and wind, \( p_{FW} \), can be defined as

\[ p_{FW} = P \{ L_c > H_c - H_0 \} \]  \hspace{1cm} (4-71)

where \( H_c \) is the height of the crest of the dam, and \( H_0 \) is the initial reservoir level before the simultaneous occurrence of flood and wind.

With the assumption that individual occurrences of flood and wind follow the Poisson process, the probability of overtopping due to concurrence of flood and wind in a period of time \( T \), \( P_{FW}(T) \), can be evaluated from

\[ P_{FW}(T) = 1 - \exp \left(-\nu_{FW} T p_{FW} \right) \]  \hspace{1cm} (4-72)
where $\nu_{FW}$ is the mean concurrence rate of flood and wind and can be expressed by Eq. (4-12)

$$\nu_{FW} = \nu_F \cdot \nu_W \cdot (\nu_F + \nu_W)$$  \hspace{1cm} (4-73)

where $\nu_F$, $\nu_W$ are the mean occurrence rates of flood and wind, respectively; $\nu_F$ and $\nu_W$ are the mean durations of the reservoir level $h_H(t)$ and wave height $h_Z(t)$, respectively.

4.5.2. Formulation of Combined Load and Risk Model

The combined load, $L_c$, in Eqs. (4-69) and (4-70) is a function of the duration, magnitude, shape and occurrence time of each single load. Figure 4.17 shows the combinations of the reservoir height $h_H(t)$ raised by the flood and the wave height, $h_Z(t)$, induced by wind. The bell-shaped curve denotes the changing $h_H(t)$ during the occurrence of the flood, while the shape of $h_Z(t)$ varies with the changing reservoir level during the occurrence of wind. The wave height, $h_Z(t)$, increases with the increase of reservoir level and vice versa (Fig. 4.17-b,c,d). If the wind ends just before the beginning of flood or the wind starts just after the end of flood, then the wave height is assumed to be constant (Fig. 4.17-a,e). From Fig. 4.17, it can be shown that

$$L_c = \begin{cases} 
\text{MAX} (h_F, h_W), & \text{for case (a) & (e)} \\
\text{MAX} [h_F, h_H(t) + h_Z(t)], & \text{for case (b) & (d)} \\
h_F + h_{mz}, & \text{for case (c)} 
\end{cases}$$  \hspace{1cm} (4-74)

where $h_F$ is the maximum of $h_H(t)$, and $h_{mz}$ is the sum of $h_T$ and $h_Z$ calculated at $h_F$, ie. $h(t)$ is a function of $h_F$. Equation (4-74) can be rewritten as
Fig. 4.17 Combined load from concurrency of flood and wind.
\[ L_c = \begin{cases} 
    h_F + h_{mz} & \text{for } |\tau| \leq \frac{d_w}{2} \\
    \text{MAX} \left[ h_F, h_H(t) + h_z(t) \right] & \text{for } |\tau| > \frac{d_w}{2}
\end{cases} \]  

(4-75)

in which \( d_w \) is the duration of \( h_z(t) \), and \( \tau = \tau_1 - \tau_2 \) where \( \tau_2 \) and \( \tau_1 \) are the occurrence time of \( h_F \) and the middle point of \( h_z(t) \), respectively. Figure 4.18 shows a plot of the combined load vs. relative occurrence time for the cases (a) \( h_F \geq h_w \) and (b) \( h_F < h_w \).

Formulation of \( L_c \) when \( \tau > \frac{d_w}{2} \), however, is complicated because the function \( h_z(t) \) is difficult to define, and \( h_z(t) \) varies with reservoir level. The dashed line in Fig. 4.18 indicates the combined load \( L_c \) for \( |\tau| > \frac{d_w}{2} \) which varies in the following range

\[ h_F + h_{mz} \geq L_c \geq \text{MAX} \left( h_F, h_w \right) \]  

(4-76)

Since \( \tau \) is uniformly distributed between \( -(0.5 d_w + t_p) \) and \( (0.5 d_w + d_F - t_p) \), the probability of failure, \( p_{FW} \), due to the concurrence of flood and wind can be shown as

\[
p_{FW} = P \left[ L_c > H_c - H_o \right] \mid \text{coincidence, } d_F, d_w \] 

\[
= \frac{d_w}{d_F + d_w} \cdot P \left[ h_F + h_{mz} > H_c - H_o \right] + \frac{d_F}{d_F + d_w} \cdot P \left[ L_c > H_c - H_o \right] \mid |\tau| > \frac{d_w}{2}
\]  

(4-77)

where \( d_F \) is the duration of \( h_H(t) \). Based on Wen's concept (1977b) for approximating the combined load from a rectangular shaped load and a triangular shaped load, the probability in the last term of Eq. (4-77) can be approximated by the probability evaluated at the average of upper bound and lower bound of \( L_c \) in Eq. (4-76). Thus,

\[
p_{FW} = \frac{d_w}{d_F + d_w} \cdot P[h_F + h_{mz} > H_c - H_o] + \frac{d_F}{d_F + d_w} \cdot P \left[ \frac{h_F + h_{mz} + \text{MAX} \left( h_F, h_w \right)}{2} > H_c - H_o \right]
\]  

(4-78)
Fig. 4.18 Combined load for different relative load occurrence time for
(a) $h_F \geq h_w$, and (b) $h_F < h_w$. 
Since $d_w$ and $d_F$ are also random variables, Eq. (4-78) is further approximated by the mean values of $d_w$ and $d_F$, i.e., $\mu_w$ and $\mu_F$, as

$$P_{FW} = \frac{\mu_w}{\mu_F + \mu_w} P[h_F + h_{mz} > H_c - H_o] + \frac{\mu_F}{\mu_F + \mu_w} P \left[ \frac{h_F + h_{mz} + \text{MAX} (h_F, h_w)}{2} > H_c - H_o \right]$$

$$= \frac{\mu_w}{\mu_F + \mu_w} P[h_F + h_{mz} > H_c - H_o]$$

$$+ \frac{\mu_F}{\mu_F + \mu_w} \text{MAX} \{P[h_F + \frac{h_{mz}}{2} > H_c - H_o], P[\frac{h_F + h_w + h_{mz}}{2} > H_c - H_o]\}$$

(4-79)

The maximum reservoir height, $h_F$, and the total height, $h_w$, of wind tide and wave run-up in Eq. (4-79) can be obtained as described in Sections 4.3 and 4.4.

The duration of wind wave action is assumed equal to the duration of the wind. Therefore, the mean duration, $\mu_w$, can be estimated from the average duration of the strong wind being considered in the risk evaluation. Due to the detention effect of the reservoir, the duration of the reservoir level usually is longer than that of the flood into the reservoir (Fig. 4.4). Like flood hydrographs, often, $h_H(t)$ approaches a triangular shape, and therefore the mean duration $\mu_F$ can be approximated from the mean values of variables as

$$\mu_F = \bar{t}_s + \frac{\bar{Q}_{op} t_d - 251}{Q_{op}}$$

(4-80)
where \( Q_{ip}, Q_{op}, t_s, S_1 \), respectively, are the averages of \( Q_{ip}, Q_{op}, t_d \), and the storage volume, \( S_1 \), of the reservoir between \( H_o \) and \( H_s \) as shown in Fig. 4.13. Equation (4-79) can be modified for risk evaluation of overtopping failure due to coincident occurrences of floods and waves induced by earthquakes or landslides in which the shape of the wave is similar to that of \( h_z(t) \). Once \( p_{FW} \) is determined from Eq. (4-79), the probability of overtopping failure in a period of time \( T \) can be obtained from Eq. (4-72).
CHAPTER 5. RISK EVALUATION OF OVERTOPPING -- CASE STUDY

5.1. Introduction

This chapter demonstrates the evaluation of the risk of overtopping induced by the occurrences of flood and wind. The risk criteria, models, and formulations presented in Chapter 4 are applied to estimate the risk of overtopping. The example system is an earth dam, the Lake in the Hills Dam located near Crystal Lake in McHenry County, Illinois. This dam has been declared unsafe in the National Dam Safety Inspection Program for being unable to pass 0.5 PMF (probable maximum flood) on normal reservoir pool level, i.e., top of the spillway crest (U. S. Army Corps of Engineers, 1978).

The basic procedure of risk evaluation for an overtopping event can be summarized in the following steps:

1. Define the risk criterion and performance function for the overtopping event.
2. Determine the uncertainties of all the variables in the performance function.
3. Perform the risk analysis with suitable risk calculation methods.

The advanced first-order second moment method (AFOSM) with the general reduced gradient technique (GRG) is used as the primary method to calculate the risk. Risk values evaluated by considering various uncertainties, as well as risk calculation by using other methods will be studied and compared.
Detailed procedure for risk evaluation of overtopping induced by either flood or wind alone, and by simultaneous occurrences of both flood and wind will be illustrated accordingly in the following sections. Since the main purpose of this study is to illustrate the application of the proposed methodology, certain formulations and uncertainty values of the variables are hypothetically assumed. Refinement can be made when additional information or measurements become available.

5.2. General Description of Example Dam

The Lake in the Hills Dam is an earth filled structure built in 1926 for recreational purposes. The dam is located on Woods Creek, a tributary to the Fox River in the Illinois River Basin, two miles south of Crystal Lake Airport, in McHenry County, Illinois. Figure 5.1 shows the location of the dam, and Fig. 5.2 shows the watershed upstream of the dam. The dam is approximately 40 feet high and 780 feet long from abutment to abutment. An uncontrolled concrete chute spillway is located at the right-hand side of the abutment. Figure 5.3 gives the general plan of the dam. With an embankment height of 40 feet, the maximum storage capacity is approximately 969 acre-feet. Therefore, the dam is in the intermediate size category. The dam is classified in the high hazard potential category in the U. S. Dam Safety Program because there is a residential subdivision located immediately below the dam which would be seriously affected by the failure of the dam. Pertinent data of the dam are abstracted from inspection report, National Dam Safety Program, (1978) and listed in Table 5.1.
Fig. 5.1 Location map of example dam.
Fig. 5.2 Watershed of example dam.
General Plan

edge of reservoir at spillway crest elevation

scale in feet

spillway

20 0 40

780 ft abutment to abutment

spillway crest length

top of dam, EL. 827.0 ft

crest elevation 822.0

40.0

ground level before construction

Embarkment Profile At Axis

Fig. 5.3 General plan of example dam.
Table 5.1 Pertinent data of Lake in the Hills Dam

<table>
<thead>
<tr>
<th>Watershed area</th>
<th>8.55 (mi²)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dam</strong></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Earth Embankment</td>
</tr>
<tr>
<td>Elevation at top of Dam</td>
<td>827.0 (feet-MSL)</td>
</tr>
<tr>
<td>Height above streambed</td>
<td>40.0 (feet)</td>
</tr>
<tr>
<td>Upstream face slope</td>
<td>1:2 (Verti.:Horiz.)</td>
</tr>
<tr>
<td>Downstream face slope</td>
<td>1:1.75 (Verti.:Horiz.)</td>
</tr>
<tr>
<td>Length</td>
<td>780 (feet)</td>
</tr>
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<td>Top width</td>
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<tr>
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</tr>
<tr>
<td><strong>Reservoir</strong></td>
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</tr>
<tr>
<td>Elevation at normal pool</td>
<td>822.0 (feet-MSL)</td>
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<tr>
<td>Elevation at maximum pool</td>
<td>827.0 (feet-MSL)</td>
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<td>Capacity at maximum pool</td>
<td>969 (Acre-feet)</td>
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<td>Length at maximum pool</td>
<td>1.5 (miles)</td>
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<td></td>
</tr>
<tr>
<td><strong>Spillway</strong></td>
<td></td>
</tr>
<tr>
<td>Elevation at crest</td>
<td>822.0 (feet-MSL)</td>
</tr>
<tr>
<td>Length at</td>
<td>29.8 (feet)</td>
</tr>
<tr>
<td>Type</td>
<td>concrete broad-crest weir</td>
</tr>
</tbody>
</table>
5.3. Evaluation of Overtopping Risk Due to Flood

Based on the analysis given in Chapter 4, the primary procedure and variables involved in the calculation of overtopping risk due to flood are illustrated by the block diagram shown in Fig. 5.4. Depending on the available data and structure of the dam, the variables and procedure shown in Fig. 5.4 can be altered. For example, if the information on the hydrograph of the flood which enters the reservoir is available, then the procedure to determine the flood hydrograph can be replaced by considering the parameters of the flood hydrograph such as the peak discharge and duration of the flood instead of the parameters of rainfall and watershed. Determination of the discharge rating curve, storage-stage relationship and the uncertainties of the variables are presented in the following sections.

5.3.1. Determination of Discharge Rating Curve and Storage-stage Relationship

Hydraulically, the spillway of the Lake in the Hills dam is classified as a type of broad-crest weir. The discharge rating curve of the spillway can be described by Eq. (4-17), i.e.

\[ Q = C_d L_e^{3/2} h \]  

The effective length \( L_e \) is determined from the net crest length, \( L_s \), by considering contraction effect, and is estimated as (Bureau of Reclamation, 1977)

\[ L_e = L_s - 0.2 h \]

The last term in Eq. (5-2) represents the abutment contraction effect. The pier contraction effect is not considered because no bridge pier is located on the spillway.
Fig. 5.4 Procedure for evaluation of overtopping risk induced by occurrence of flood.
The storage-stage relationship of the reservoir above the normal pool of the level for the Lake in the Hills dam is given in Table 5.2. The storage for each stage is obtained from the area measured from the USGS 7.5-minute topographic map for the Crystal Lake quadrangle using the average of planimetered areas at 10-feet contour intervals multiplied by the elevation difference. A polynomial equation is fitted to represent the storage-stage relationship above the normal pool level, and it is

\[
S = a + bH + cH^2 + dH^3 \quad \text{for } H \geq 35 \text{ ft}
\]  

(5-3)

in which

\[
\begin{align*}
S &= \text{storage in cubic feet} \\
H &= \text{stage in feet} \\
a &= -31090206.7564 \\
b &= 28440927.88693 \\
c &= -868026.7852873 \\
d &= 9442.302793737
\end{align*}
\]

The digits of the above numbers are merely for computer calculation and no implication of the numerical significance is proposed. The storage-stage relationship below the normal pool level is assumed to be

\[
S = 21264.392H^2 \quad \text{for } H < 35 \text{ ft}
\]  

(5-4)

No map measurement for storage-stage relationship below the normal pool level was attempted.

5.3.2. Determination of Uncertainties of Variables in Performance Function

The AFOSM method determines the risk by considering the probability
Table 5.2 Storage and stage for Lake in the Hills Dam

<table>
<thead>
<tr>
<th>Elevation (ft above MSL)</th>
<th>Stage (ft)</th>
<th>Storage (in $10^7$ ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 822.0</td>
<td>35.0</td>
<td>2.605</td>
</tr>
<tr>
<td>822.8</td>
<td>35.8</td>
<td>2.801</td>
</tr>
<tr>
<td>824.5</td>
<td>37.5</td>
<td>3.289</td>
</tr>
<tr>
<td>826.0</td>
<td>39.0</td>
<td>3.816</td>
</tr>
<tr>
<td>827.0</td>
<td>40.0</td>
<td>4.221</td>
</tr>
<tr>
<td><strong>827.5</strong></td>
<td>40.5</td>
<td>4.443</td>
</tr>
<tr>
<td>*828.0</td>
<td>41.0</td>
<td>4.678</td>
</tr>
<tr>
<td>829.0</td>
<td>42.0</td>
<td>5.197</td>
</tr>
<tr>
<td>830.0</td>
<td>43.0</td>
<td>5.780</td>
</tr>
<tr>
<td>831.0</td>
<td>44.0</td>
<td>6.434</td>
</tr>
</tbody>
</table>

* Top of spillway crest
** Top of dam
distribution function and statistical parameters such as the mean and standard deviation of the variables. The uncertainties of the hydrologic variables, watershed variables, hydraulic variables, and correction variables are determined in following subsections.

5.3.2.1. Uncertainties of Hydrologic Variables

There is no precipitation station located within the watershed of the Lake in the Hills dam. Therefore, TP-40 is used in this example as the source of data for rainfall. From TP-40, the rainfall depths for durations from 0.5 to 24 hours and return periods from 2 to 100 years are listed in Table 5.3. By plotting the rainfall depths with the corresponding return periods on Gumbel probability distribution paper (Fig. 5.5), the parameters of Gumbel distribution for each rainfall duration can be obtained from the intercepts and slopes of the straight lines fitted by the least squares method (Ang and Tang, 1975). Values of \( \mu \) and \( \sigma \) as shown in Eqs. (4-45), (4-46) duration are given in Table 5.4.

An exponential probability distribution (Eq. (4-43), (4-44)) is assumed for the rainfall duration \( t_r \). The sample mean \( c = 2.45 \) hrs. obtained by Chow and Yen (1976) from an analysis of 455 rainstorms of hourly rainfall record at Urbana, Illinois is assumed to be the mean of \( t_r \) at the example dam site.

The probability density function of \( a^0 \) is assumed to be normally distributed. Because most of the extremely heavy rainstorms occur from March to September in Illinois, the \( a^0 \) derived from rainstorms in this period is more desirable. Based on the results given by Yen and Chow
Table 5.3 Rainfall depths in inches for different durations

<table>
<thead>
<tr>
<th>Return period (Yr)</th>
<th>Rainfall duration (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>1.04</td>
</tr>
<tr>
<td>5</td>
<td>1.27</td>
</tr>
<tr>
<td>10</td>
<td>1.42</td>
</tr>
<tr>
<td>25</td>
<td>1.63</td>
</tr>
<tr>
<td>50</td>
<td>1.82</td>
</tr>
<tr>
<td>100</td>
<td>2.03</td>
</tr>
</tbody>
</table>

Table 5.4 Mean and standard deviation of Gumbel distribution of rainfall of different durations

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>Rainfall duration (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Mean, μ (in.)</td>
<td>1.06</td>
</tr>
<tr>
<td>Std. dev. σ (in.)</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Fig. 5.5 Gumbel distribution fitting of rainfall depths for different rainfall durations for example dam site.
(1977) for Urbana, the mean and standard deviation are assumed to be 0.40 and 0.32, respectively, for Lake in the Hills.

The type of soil, soil cover, land use condition, and antecedent moisture condition are the factors for determining the magnitude of CN value. As shown in the soil map of McHenry County (Ray and Wascher, 1965), the watershed of the Lake in the Hills dam primarily consists of hydrological soil groups B and C. From Table 4.1, a CN = 75 is used for antecedent moisture condition (AMC) II. From Table 4.2 the corresponding CN values for AMC I and AMC III are 57 and 88, respectively. Since, AMC I and AMC III represent the dry and wet moisture of the soil, CN = 57 and CN = 88 are regarded as the lower and upper bounds of the CN value, respectively. The pdf of CN is assumed to be an asymmetric triangular bounded distribution with the maximum likelihood (mode) at CN = 75, i.e., AMC II condition. The mean and coefficient of variation calculated from the equations shown in Fig. 5.6 are 73.3 and 0.087, respectively.

The data for initial reservoir stage, $H_o$, were not available for this example. The value of $H_o$ is assumed to vary between 20 ft and 35 ft (the latter being the height of spillway crest) following a triangular pdf with the mode equal to 30 ft.

5.3.2.2. Uncertainties of Watershed Variables

In this example, a USGS 7.5-minute topographic map is also used to determine the area of the watershed $A_w$, the length of the main water course, $L_w$, and elevation difference between its two ends, $H_w$ (Fig. 5.2). The mean values of the measurements are $A_w$=8.55 square miles, $L_w$ = 4.38 mi and $H_w$ =98 ft. Concerning the error in area measurement, as
Fig. 5.6 Statistics of five simple distributions.
suggested by Yen (1975) as discussed in section 4.3.3.1 serves as a reference. $\delta_A = 0.050$ is adopted for the measurement error of the area of the watershed used in this example. A uniform pdf with a mean of 8.55 square miles is assumed for the variation of the watershed area $A$.

As to the errors in measuring $L_w$ and $H_w$, 36 engineering students were asked to measure $L_w$ and $H_w$ of a watershed from a U.S. Geological Survey 7.5-minute map. The average errors, denoted by the coefficients of variation of $L_w$ and $H_w$ were found to be $\delta_L = 0.77$ and $\delta_H = 0.89$, respectively. In this study the higher values of $\delta_L = 0.8$ and $\delta_H = 0.9$ are adopted. Uniform distribution functions are assumed for both $L_w$ and $H_w$.

5.3.2.3. Uncertainties of Hydraulic Variables

The spillway length, $L_s$, and spillway crest height, $h_s$, given in Table 5.1 were obtained from actual measurement of the dam (U.S. Army Corps of Engineers, 1978) which would likely be the maximum values of $L_s$ and $h_s$. Plugging at the spillway crest may reduce the size of $L_s$, increase $h_s$, and change $C_d$, and consequently, reduce the spillway discharge capacity. In this example, it is assumed that because of plugging, $L_s$ may vary between 29.8 ft and 26.8 ft, and $h_s$ between 5 ft and 4 ft, respectively. Right-angled triangular probability density functions are assumed for $L_s$ and $h_s$ with the modes at $L_s = 29.8$ ft and $h_s = 5$ ft, respectively. The uncertainty due to measurement error is small compared to that of plugging and hence it is ignored.

The spillway of the dam in this example is a type of broad-crest weir. King and Brater (1963) gave the experimental values of $C_d$ for
broad-crest weir with different heights of water head and breadths of the weir. They show that the value of the discharge coefficient, $C_d$, is rather constant with respect to the height of the head when the breadth of the weir is greater than 15 ft. Since the breadth of this spillway is greater than 15 ft (Fig. 5.3), the value of $C_d$ is assumed to be independent of the head. By considering the possible difference of $C_d$ between the model and prototype weirs, and the effects due to plugging on the spillway crest, the value of $C_d$ is assumed to be $2.64 \pm 0.1$. A symmetric triangular pdf is assumed such that $\mu_c = 2.64$ and $\delta_c = 0.015$.

As for the determination of the height of the dam crest, a deterministic value $H_c = 40$ ft is used in this example. The measurement error is ignored. Because the Lake in the Hills dam has been in operation since 1926, the settlement of foundation in the future will be very small and need not be considered.

5.3.2.4. Uncertainties of Correction Variables

As expressed by Eq. (4-41), the uncertainties of $\lambda_Q$ can be obtained by comparing the reliably observed runoff peak discharge to the calculated peak discharge obtained from the method described in Chapter 4. In this example no attempt was made to perform the actual analysis of $\lambda_Q$. A symmetric triangular pdf is subjectively assumed for $\lambda_Q$ with $\delta = 0.2$ and $\mu = 1.0$.

The uncertainty of $\lambda_F$ comes from the error in determining the storage-stage relationship, discharge rating curve and the approximate flood routing method. By using the same flood hydrograph, storage-stage relationship and discharge rating curve, the value of $\lambda_F$ was determined
by comparing the maximum reservoir height $h_F$ calculated by the Puls method to that approximated by the simple routing method presented in Chapter 4. The error which comes from the use of the simple routing method is estimated to be $\delta_1 = 0.050$ with $\mu = 0.916$ for $\lambda_F$ (Appendix C). The error which comes from the storage-stage relationship, discharge rating curve and the use of Puls method is subjectively assumed to be $\delta_2 = 0.04$. The overall error therefore, is $\delta_F = (0.05^2 + 0.04^2)^{1/2} = 0.065$, with $\mu_F = 0.916$. A symmetric triangular pdf is subjectively assumed to describe the distribution of $\lambda_F$. It should be mentioned that the Puls method is not the most accurate one and it is used for illustration only. Other sophisticated methods can be used if more accurate information on $\lambda_F$ is desired.

5.3.3. Calculation and Result of Overtopping Risk Due to Flood

Once the statistics of every variable in the performance function are determined and the necessary formulations for calculation of the maximum reservoir height $h_F$ are provided, the overtopping risk due to flood can be calculated using the advanced first-order second-moment method (AFOSM) with general reduced gradient (GRG) technique.

The use of the GRG computer package developed by Lasdon et al. (1975a, 1975b) requires user input of the objective function and the constraint equations, Eq. (3-34) and (3-35). The logic and relative formulations for preparing the objective functions and the constraint equations are illustrated in Fig. 5.7.
Fig. 5.7 Block diagram of input procedure to GRG package for evaluation of overtopping risk due to flood.
5.3.3.1. Effects of Different Considerations of Uncertainties

The statistics of the variables considered in the evaluation of overtopping risk due to flood are summarized in Table 5.5. Different risk values are obtained depending on the level of uncertainties considered. Six cases representing different levels of uncertainties were investigated as follows.

1. Only the uncertainties of hydrological variables are considered.
2. Uncertainties of both hydrological and watershed variables are considered.
3. Uncertainties of both hydrological and hydraulic variables are considered.
4. Uncertainties of hydrological, watershed and hydraulic variables are considered.
5. Uncertainties of hydrological, hydraulic and correction variables are considered.
6. The uncertainties of all the four groups of variables are considered.

In the above cases, if the uncertainties of certain variables were not considered, the mode or mean values of the variables were assumed in calculating the risk. These values are also listed in column 8 of Table 5.5.

The calculated risk values, $p(V|t)$, for an occurrence of flood based on rainfall information with durations from 0.5 hrs. to 24 hrs., are listed in Table 5.6 and also plotted in Fig. 5.8. The following conclusions were drawn:

1. The risk increases when the possibility of plugging on spillway crest, i.e., uncertainties of hydraulic variables, is considered.
Table 5.5 Uncertainties of variables in overtopping risk due to flood

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Mean (1)</th>
<th>Standard Deviation (2)</th>
<th>Coefficient of Variation (3)</th>
<th>Lower Bound (4)</th>
<th>Upper Bound (5)</th>
<th>Mode (6)</th>
<th>Distribution (7)</th>
<th>Deterministic Value (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>0.5 hr</td>
<td>1.058</td>
<td>0.296</td>
<td>0.280</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>1 hr</td>
<td>1.407</td>
<td>0.381</td>
<td>0.271</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>2 hr</td>
<td>1.678</td>
<td>0.471</td>
<td>0.280</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>3 hr</td>
<td>1.817</td>
<td>0.539</td>
<td>0.297</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>6 hr</td>
<td>2.139</td>
<td>0.639</td>
<td>0.299</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>12 hr</td>
<td>2.508</td>
<td>0.749</td>
<td>0.299</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td>in.</td>
<td>24 hr</td>
<td>2.844</td>
<td>0.889</td>
<td>0.313</td>
<td>0</td>
<td>0</td>
<td>Gumbel</td>
<td></td>
</tr>
<tr>
<td>$t_r$</td>
<td>hr</td>
<td></td>
<td>2.45</td>
<td>2.45</td>
<td>1.0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Exponential</td>
</tr>
<tr>
<td>$a^0$</td>
<td></td>
<td></td>
<td>0.40</td>
<td>0.32</td>
<td>0.800</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>Normal</td>
</tr>
<tr>
<td>CN</td>
<td></td>
<td></td>
<td>73.33</td>
<td>6.36</td>
<td>0.087</td>
<td>57.</td>
<td>88.</td>
<td>75.</td>
<td>Triangular</td>
</tr>
<tr>
<td>$H_o$</td>
<td>ft</td>
<td></td>
<td>28.33</td>
<td>3.12</td>
<td>0.110</td>
<td>20.</td>
<td>35.</td>
<td>30.</td>
<td>Triangular</td>
</tr>
<tr>
<td>$A$</td>
<td>mi$^2$</td>
<td></td>
<td>8.55</td>
<td>0.43</td>
<td>0.050</td>
<td>7.81</td>
<td>9.29</td>
<td>---</td>
<td>Uniform</td>
</tr>
<tr>
<td>$L_w$</td>
<td>mi</td>
<td></td>
<td>4.38</td>
<td>0.35</td>
<td>0.08</td>
<td>3.78</td>
<td>4.98</td>
<td>---</td>
<td>Uniform</td>
</tr>
<tr>
<td>$H_w$</td>
<td>ft</td>
<td></td>
<td>98.</td>
<td>8.82</td>
<td>0.09</td>
<td>83.</td>
<td>113.</td>
<td>---</td>
<td>Uniform</td>
</tr>
<tr>
<td>$C_d$</td>
<td></td>
<td></td>
<td>2.64</td>
<td>0.04</td>
<td>0.015</td>
<td>2.54</td>
<td>2.74</td>
<td>2.64</td>
<td>Triangular</td>
</tr>
<tr>
<td>$L_s$</td>
<td>ft</td>
<td></td>
<td>28.8</td>
<td>0.71</td>
<td>0.025</td>
<td>26.8</td>
<td>29.8</td>
<td>29.8</td>
<td>Triangular</td>
</tr>
<tr>
<td>$h_s$</td>
<td>ft</td>
<td></td>
<td>4.67</td>
<td>0.24</td>
<td>0.051</td>
<td>4.</td>
<td>5.</td>
<td>5.</td>
<td>Triangular</td>
</tr>
<tr>
<td>$\lambda_Q$</td>
<td></td>
<td></td>
<td>1.00</td>
<td>0.20</td>
<td>0.204</td>
<td>0.5</td>
<td>1.5</td>
<td>1.0</td>
<td>Triangular</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td></td>
<td></td>
<td>0.916</td>
<td>0.042</td>
<td>0.046</td>
<td>0.813</td>
<td>1.019</td>
<td>0.916</td>
<td>Triangular</td>
</tr>
</tbody>
</table>

* Value assumed for calculation of the overtopping risk when the uncertainty of a variable is not considered
Table 5.6 Risks of overtopping in occurrence of flood by considering various uncertainties for different rainfall durations

| Variables with Uncertainties | $p[v | t_r]$ | $P_F$ |
|-----------------------------|-------------|-------|
|                            | $t_r = 0.5$ hr | $t_r = 1$ hr | $t_r = 2$ hr | $t_r = 3$ hr | $t_r = 6$ hr | $t_r = 12$ hr | $t_r = 24$ hr |       |
| $D_R$, $a^0$, $CN$, $H_o$  | $0.105 \times 10^{-4}$ | $0.258 \times 10^{-3}$ | $0.175 \times 10^{-2}$ | $0.428 \times 10^{-2}$ | $0.132 \times 10^{-1}$ | $0.257 \times 10^{-1}$ | $0.231 \times 10^{-1}$ | $0.478 \times 10^{-2}$ |
| $D_R$, $a^0$, $CN$, $H_o$, $A$, $L_w$, $H_w$ | $0.120 \times 10^{-4}$ | $0.281 \times 10^{-3}$ | $0.186 \times 10^{-2}$ | $0.447 \times 10^{-2}$ | $0.136 \times 10^{-1}$ | $0.263 \times 10^{-1}$ | $0.238 \times 10^{-1}$ | $0.495 \times 10^{-2}$ |
| $D_R$, $a^0$, $CN$, $H_o$, $C_d$, $L_S$, $h_S$ | $0.153 \times 10^{-4}$ | $0.345 \times 10^{-3}$ | $0.223 \times 10^{-2}$ | $0.530 \times 10^{-2}$ | $0.161 \times 10^{-1}$ | $0.326 \times 10^{-1}$ | $0.333 \times 10^{-1}$ | $0.592 \times 10^{-2}$ |
| $D_R$, $a^0$, $CN$, $H_o$, $A$, $L_w$, $H_w$, $C_d$, $L_S$, $h_S$ | $0.172 \times 10^{-4}$ | $0.371 \times 10^{-3}$ | $0.234 \times 10^{-2}$ | $0.550 \times 10^{-2}$ | $0.165 \times 10^{-1}$ | $0.333 \times 10^{-1}$ | $0.340 \times 10^{-1}$ | $0.609 \times 10^{-2}$ |
| $D_R$, $a^0$, $CN$, $H_o$, $C_d$, $L_S$, $h_S$, $\lambda_Q$, $\lambda_F$ | $0.425 \times 10^{-4}$ | $0.682 \times 10^{-3}$ | $0.359 \times 10^{-2}$ | $0.773 \times 10^{-2}$ | $0.214 \times 10^{-1}$ | $0.417 \times 10^{-1}$ | $0.440 \times 10^{-1}$ | $0.816 \times 10^{-2}$ |
| $D_R$, $a^0$, $CN$, $H_o$, $A$, $L_w$, $H_w$, $C_d$, $L_S$, $h_S$, $\lambda_Q$, $\lambda_F$ | $0.476 \times 10^{-4}$ | $0.719 \times 10^{-3}$ | $0.372 \times 10^{-2}$ | $0.777 \times 10^{-2}$ | $0.218 \times 10^{-1}$ | $0.423 \times 10^{-1}$ | $0.448 \times 10^{-1}$ | $0.830 \times 10^{-2}$ |
Fig. 5.8 Risk of overtopping in an occurrence of flood by considering various uncertainties for different rainfall durations.
2. The risk value becomes larger when more uncertainties are considered.

3. The effect of watershed variable uncertainties is insignificant. Therefore, the uncertainties of watershed variables can be ignored in the risk evaluation to reduce the computer cost.

4. At least up to rainfall duration of 15 hrs., the risk increases with durations.

The risk value listed in Table 5.6 and shown in Fig 5.8 are evaluated based on a rainfall of a given duration, i.e., \( p[V|t_r] \) in Eq. (4-49). Because rainfall may occur with any duration, the risk value should be weighted by Eq. (4-50) considering different frequencies of occurrence of the rainfall duration, i.e.,

\[
p_F = \sum_{n=1}^{\infty} p[V|t_r] \left[ P(t_r + \frac{\Delta t}{2}) - P(t_r - \frac{\Delta t}{2}) \right]
\]  

Table 5.7 shows the computation of risk \( p_F \) by considering the uncertainties of all the variables. The exponential probability function of Eq. (4-47) with the mean equal to 2.45 hrs. is used to calculate \( P(t_r - \Delta t/2) \) and \( P(t_r + \Delta t/2) \) in Table 5.7. Column 6 in Table 5.7 represents the weighted risk contributed by each segment of \( \Delta t \) at different duration \( t_r \). It shows that rainfall with durations from 2 hrs. to 8 hrs. contributed more than 2/3 of the total risk value, and those with durations of greater than 24 hrs. can be ignored. The risk value of overtopping induced by an occurrence of flood \( p_F \), therefore, is 0.00830 by considering uncertainties of all variables. Values of \( p_F \) for other considerations of uncertainties are also calculated and listed in the last column of Table 5.6. Once \( p_F \) is determined, the probability of
Table 5.7 Computation of overtopping risk, $p_r$, in an occurrence of flood

| Rainfall duration $t_r$ (hr) | $p[v|t_r]$ (2) | $P(t_r - 0.5)$ (3) | $P(t_r + 0.5)$ (4) | $(5) = (3) - (4)$ (5) | $(6) = (2) \times (5)$ (6) |
|-----------------------------|----------------|-------------------|-------------------|-------------------|----------------------|
| 0.5                         | $0.476 \times 10^{-4}$ | 0.185             | $0.185 \times 10^{-4}$ | $0.879 \times 10^{-5}$ |
| 1.5                         | $0.203 \times 10^{-2}$ | 0.458             | $0.182 \times 10^{-2}$ | $0.555 \times 10^{-3}$ |
| 2.5                         | $0.566 \times 10^{-2}$ | 0.640             | $0.103 \times 10^{-2}$ | $0.121 \times 10^{-2}$ |
| 3.5                         | $0.100 \times 10^{-1}$ | 0.760             | $0.121 \times 10^{-1}$ | $0.121 \times 10^{-2}$ |
| 4.5                         | $0.147 \times 10^{-1}$ | 0.841             | $0.118 \times 10^{-1}$ |                           |
| 5.5                         | $0.195 \times 10^{-1}$ | 0.894             | $0.104 \times 10^{-1}$ | $0.373 \times 10^{-3}$ |
| 6.5                         | $0.241 \times 10^{-1}$ | 0.930             | $0.854 \times 10^{-3}$ |                           |
| 7.5                         | $0.284 \times 10^{-1}$ | 0.953             | $0.669 \times 10^{-3}$ |                           |
| 8.5                         | $0.323 \times 10^{-1}$ | 0.969             | $0.506 \times 10^{-3}$ |                           |
| 9.5                         | $0.357 \times 10^{-1}$ | 0.979             | $0.104 \times 10^{-2}$ |                           |
| 10.5                        | $0.387 \times 10^{-1}$ | 0.986             | $0.268 \times 10^{-3}$ |                           |
| 11.5                        | $0.412 \times 10^{-1}$ | 0.991             | $0.190 \times 10^{-3}$ |                           |
| 12.5                        | $0.433 \times 10^{-1}$ | 0.994             | $0.133 \times 10^{-3}$ |                           |
| 13.5                        | $0.449 \times 10^{-1}$ | 0.996             | $0.916 \times 10^{-4}$ |                           |
| 14.5                        | $0.462 \times 10^{-1}$ | 0.997             | $0.626 \times 10^{-4}$ | $0.424 \times 10^{-4}$ |
| 15.5                        | $0.470 \times 10^{-1}$ | 0.998             | $0.136 \times 10^{-2}$ |                           |
| 16.5                        | $0.476 \times 10^{-1}$ | 0.999             | $0.599 \times 10^{-3}$ | $0.285 \times 10^{-4}$ |
| 17.5                        | $0.479 \times 10^{-1}$ | 0.999             | $0.398 \times 10^{-3}$ | $0.191 \times 10^{-4}$ |
| 18.5                        | $0.479 \times 10^{-1}$ | 0.999             | $0.265 \times 10^{-3}$ | $0.127 \times 10^{-4}$ |
| 19.5                        | $0.477 \times 10^{-1}$ | 1.000             | $0.176 \times 10^{-3}$ | $0.840 \times 10^{-5}$ |
| 20.5                        | $0.473 \times 10^{-1}$ | 1.000             | $0.117 \times 10^{-3}$ | $0.554 \times 10^{-5}$ |
| 21.5                        | $0.467 \times 10^{-1}$ | 1.000             | $0.779 \times 10^{-4}$ | $0.364 \times 10^{-5}$ |
| 22.5                        | $0.460 \times 10^{-1}$ | 1.000             | $0.518 \times 10^{-4}$ | $0.238 \times 10^{-5}$ |
| 23.5                        | $0.452 \times 10^{-1}$ | 1.000             | $0.344 \times 10^{-4}$ | $0.156 \times 10^{-5}$ |

Summation = $0.830 \times 10^{-2}$
overtopping due to flood in a period of time $T$, $P_F(T)$, can be evaluated from Eq. (4-15). Values of $P_F(T)$ for $T$ from 1 year to 1000 years with various uncertainties are calculated and plotted in Fig. 5.9.

5.3.3.2. Effect of Initial Reservoir Level

In the national dam safety program carried out by the U. S. Army Corps of Engineers, a dam is declared unsafe when the spillway is unable to pass a specific amount of flood, such as the 100-year flood, or 1/2 PMF, or 1 PMF, depending on the category of the dam. To determine the adequacy of the spillway discharge capacity at the maximum pool level (top of dam), the inflow hydrograph is routed through the reservoir surcharge storage, assuming a starting water surface at the bottom of surcharge storage, e.g., the top of the spillway crest (U. S. Army Corps of Engineers, 1975). The flood hydrograph, if necessary, is usually generated from the 100-year rainfall, or 1/2 PMP (Probable Maximum Precipitation), or 1 PMP with duration equal to 24 hrs. With such a deterministic consideration, it often gives a conservative and unjustifiable evaluation.

For illustration, the risk of overtopping due to flood is studied with an initial reservoir level equal to the top of the spillway crest; i.e., $H_o = 35$ ft. Table 5.8 gives the risk values evaluated by assuming $H_o = 35$ ft, and by including the uncertainty of $H_o$ for two different levels of uncertainties of the other variables. From Fig. 5.10, it is obvious that the risk values assumed for $H_o = 35$ ft with no uncertainty is higher than those evaluated by including the uncertainty of $H_o$. The risk $p_F$ weighted from the risks of different rainfall durations is also
Fig. 5.9 Risk of overtopping due to flood in given time period considering various uncertainties.
Table 5.8 Risks of overtopping due to flood assuming $H_o = 35$ ft and considering uncertainty of $H_o$

| Variables with Uncertainties | $p[y | t_r]$ | $p_F$ |
|------------------------------|-------------|-------|
|                             | $t_r = 0.5$ hr | $t_r = 1$ hr | $t_r = 2$ hr | $t_r = 3$ hr | $t_r = 6$ hr | $t_r = 12$ hr | $t_r = 24$ hr |       |
| $D_R$                       | $0.485 \times 10^{-5}$ | $0.199 \times 10^{-3}$ | $0.175 \times 10^{-2}$ | $0.474 \times 10^{-2}$ | $0.152 \times 10^{-1}$ | $0.275 \times 10^{-1}$ | $0.205 \times 10^{-1}$ | $0.531 \times 10^{-2}$ |
| $D_R$, $H_o$                | $0.157 \times 10^{-5}$ | $0.793 \times 10^{-4}$ | $0.830 \times 10^{-3}$ | $0.250 \times 10^{-2}$ | $0.944 \times 10^{-2}$ | $0.213 \times 10^{-1}$ | $0.193 \times 10^{-1}$ | $0.326 \times 10^{-2}$ |
| $D_R$, $a^o$, $CN$          | $0.144 \times 10^{-3}$ | $0.185 \times 10^{-2}$ | $0.818 \times 10^{-2}$ | $0.159 \times 10^{-1}$ | $0.383 \times 10^{-1}$ | $0.637 \times 10^{-1}$ | $0.542 \times 10^{-1}$ | $0.153 \times 10^{-1}$ |
| $C_d$, $L_s$, $h_s$         | $0.425 \times 10^{-4}$ | $0.682 \times 10^{-3}$ | $0.359 \times 10^{-2}$ | $0.773 \times 10^{-2}$ | $0.214 \times 10^{-1}$ | $0.417 \times 10^{-1}$ | $0.440 \times 10^{-1}$ | $0.816 \times 10^{-2}$ |
Rainfall duration, \( t_r \), (hr) - \( t_r \).

Fig. 5.10 Risk of overtopping due to flood calculated for \( H_o = 35 \) ft without uncertainty and uncertainty of \( H_o \) for different rainfall durations.
calculated and listed in Table 5.8. It shows that by assuming $H_o = 35 \text{ ft}$ with no uncertainty, the risk $p_F$ could be overestimated by 60% to 90% as compared to that evaluated by including the uncertainty of $H_o$. The risk value for a rainfall of 24 hrs. duration with $H_o = 35 \text{ ft}$ could be overestimated by 530% to 565% as compared to that evaluated by including the uncertainty of $H_o$ and the distribution of different rainfall durations. The risks in different periods of time $T$, $P_F(T)$ for various assumptions of $H_o$ and $t_r$ are shown in Fig. 5.11.

5.3.3.3. Comparison Of Risk Evaluated by Different Methods

The risk of overtopping due to flood is also evaluated by two other methods: the Monte Carlo simulation method and the mean value first-order second-moment method (MFOSM). Values of the risk which consider the uncertainties of all variables with various rainfall durations are listed in Table 5.9 and shown in Fig. 5.12 together with the results obtained from AFOSM. By using the results obtained from the Monte Carlo simulation method as the reference, it is found that AFOSM does give closer agreement to Monte Carlo results despite the highly nonlinear performance function in this example. However, the results given by MFOSM method considering three different representations of performance function are all overestimated and fail to give correct evaluations. Therefore, the use of MFOSM in risk evaluation of overtopping should not be encouraged.

5.4. Overtopping Risk Due to Wind

The performance function which describes the state of overtopping induced by the occurrence of wind has been shown in Section 4.4.2. to be
Fig. 5.11 Risk of overtopping due to flood in a period of time for several assumptions of $H_o$ and $t_r$. 
Table 5.9 Risks of overtopping due to flood evaluated by Monte Carlo simulation method and by Mean-Value First-Order Second-Moment method*

<table>
<thead>
<tr>
<th>Method</th>
<th>$p(v \mid t_r)$</th>
<th>$t_r = 0.5$ hr</th>
<th>$t_r = 1$ hr</th>
<th>$t_r = 2$ hr</th>
<th>$t_r = 3$ hr</th>
<th>$t_r = 6$ hr</th>
<th>$t_r = 12$ hr</th>
<th>$t_r = 24$ hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>---</td>
<td>0.520 x 10^{-2}</td>
<td>0.348 x 10^{-2}</td>
<td>0.665 x 10^{-2}</td>
<td>0.172 x 10^{-1}</td>
<td>0.343 x 10^{-1}</td>
<td>0.397 x 10^{-1}</td>
<td></td>
</tr>
<tr>
<td>Performance function</td>
<td>$40 - H_o - h_F$</td>
<td>0.281 x 10^{-3}</td>
<td>0.395 x 10^{-2}</td>
<td>0.253 x 10^{-1}</td>
<td>0.545 x 10^{-1}</td>
<td>0.159 x 10^{0}</td>
<td>0.876 x 10^{-1}</td>
<td>0.759 x 10^{-1}</td>
</tr>
<tr>
<td>Performance function</td>
<td>$\ln\left(\frac{40 - H_o}{h_F}\right)$</td>
<td>0.868 x 10^{-1}</td>
<td>0.799 x 10^{-1}</td>
<td>0.115 x 10^{0}</td>
<td>0.147 x 10^{0}</td>
<td>0.223 x 10^{0}</td>
<td>0.123 x 10^{0}</td>
<td>0.105 x 10^{0}</td>
</tr>
<tr>
<td>Performance function</td>
<td>$\frac{40 - h_o}{h_F} - 1$</td>
<td>0.344 x 10^{0}</td>
<td>0.266 x 10^{0}</td>
<td>0.253 x 10^{0}</td>
<td>0.262 x 10^{0}</td>
<td>0.290 x 10^{0}</td>
<td>0.173 x 10^{0}</td>
<td>0.144 x 10^{0}</td>
</tr>
</tbody>
</table>

* Risk is evaluated by considering uncertainties of the following variables, $D_R$, $a^0$, $a$, $CN$, $H_o$, $A$, $L_w$, $H_w$, $C_d$, $L_s$, $h_s$, $\lambda_Q$, $\lambda_F$.

** Risk is not calculated because of large computer expense.
Fig. 5.12 Risk of overtopping due to flood calculated by different methods for different rainfall durations.
in which $H_c$, $H_o$, $V_w$, $F$, $F_e$, and $D$ are the height of the dam, the initial reservoir level, wind velocity, fetch length, effective fetch length, and the average depth of the reservoir along the fetch, respectively. The constants $a$ and $b$ are determined from Table 4.3 for a given upstream slope of the dam. In this example, the upstream slope of the Lake in the Hills dam is 1:2. Therefore, from Table 4.3, $a$ and $b$ are 2.67 and 3.73, respectively.

In estimating the risk using the advanced first-order second-moment method, the variables in the performance function should be statistically independent. For a given direction of wind, the fetches $F$, $F_e$, and the average depth $D$ are proportional to the depth of the reservoir level $H_o$ and intercorrelated with each other. A transformation as described in section 3.1.5. is necessary. However, the complex transformation can be avoided by relating $F$ and $D$ to the reservoir level, $H_o$, so that not only the number of the variables in Eq. (5-6) is reduced but also the intercorrelated problem can be eliminated.

Figure 5.13 gives the procedure and variables for the evaluation of the risk of overtopping due to wind. The reduction and determination of the variables are further discussed in the following paragraphs.

5.4.1. Reduction and Determination of Variables in Performance Function

5.4.1.1. Reduction of Variables

The magnitudes of the fetch, $F$, effective fetch, $F_e$, and average depth, $D$, usually are determined from the map of the reservoir. For the same wind direction $F_e$ is less than $F$ because of the effects of relative
Wind velocity $V_w$

Effective fetch length $F_e$

Average reservoir depth $D$

Upstream slope $S$

Initial reservoir level $H_0$

Fetch length $F$


Determine wind tide $h_T$

Determine wave height $h_s$

Determine wave run-up $h_r$

Performance function $g = 40 - (h_r + h_T + H_o)$

Calculate risk by AFSOM method

Risk value

*: deterministic value 1:2 for lake in the Hills dam

Fig. 5.13 Procedures for evaluation of overtopping risk induced by occurrence of wind.
narrow fetch width of the reservoir. Following the procedures described in Section 4.4.3. Fig. 5.14 gives the computation of \( F_e \) at normal reservoir level \((H_o = 35 \text{ ft})\) of the Lake in the Hills dam with the wind blowing towards the southeast. The magnitude of \( F_e \) is found to be 0.215 mile. Magnitudes of \( F_e \) for other initial reservoir levels \( H_o \) can be obtained using the same calculation procedure. Shorter \( F_e \) is expected for a lower \( H_o \). Because of the lack of information on the reservoir area below the normal pool level, it is assumed that \( F_e \) is a function of \( H_o \) described by

\[
F_e = 0.1181 - 0.004397 \cdot H_o + 0.0002079 \cdot H_o^2 \quad \text{for} \quad 20' < H_o < 35'
\]

(5-7)

when the wind blows to the southeast as shown in Fig. 5.14.

The fetch \( F \) is the length along the direction of wind blow and is usually taken as twice the effective fetch. In this example \( F \) is measured along the maximum fetch line and is also assumed to be a function of \( H_o \) in the form of

\[
F = 0.3384 - 0.01726 \cdot H_o + 0.0005771 \cdot H_o^2 \quad \text{for} \quad 20' < H_o < 35'
\]

(5-8)

The depth \( D \) is the average reservoir level along the direction of wind blow. In this example, \( D \) is assumed to be half of the initial \( H_o \), i.e.,

\[
D = 0.5 \cdot H_o \quad \text{for} \quad 20' < H_o < 35'
\]

(5-9)

By relating \( F_e, F, \) and \( D \) to the initial reservoir level \( H_o \) using Eqs. (5-7), (5-8) and (5-9), the number of variables in Eq. (5-6) is reduced from five to three, namely, \( V_w, H_o \) and \( \lambda_w \).

Since wind may blow in any direction from upstream of the dam, different representations of \( F, F_e \) and \( D \) in terms of \( H_o \) for different
<table>
<thead>
<tr>
<th>$\theta(\degree)$</th>
<th>$X \times 10^3$ (mi)</th>
<th>$\cos \theta$</th>
<th>$X \cos \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>0.11</td>
<td>0.743</td>
<td>0.08</td>
</tr>
<tr>
<td>36</td>
<td>0.12</td>
<td>0.809</td>
<td>0.10</td>
</tr>
<tr>
<td>30</td>
<td>0.14</td>
<td>0.866</td>
<td>0.12</td>
</tr>
<tr>
<td>24</td>
<td>0.32</td>
<td>0.914</td>
<td>0.29</td>
</tr>
<tr>
<td>18</td>
<td>0.35</td>
<td>0.951</td>
<td>0.33</td>
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<td>12</td>
<td>0.38</td>
<td>0.978</td>
<td>0.37</td>
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<tr>
<td>6</td>
<td>0.42</td>
<td>0.995</td>
<td>0.42</td>
</tr>
<tr>
<td>0</td>
<td>0.45</td>
<td>1.000</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>0.22</td>
<td>0.995</td>
<td>0.22</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>0.978</td>
<td>0.15</td>
</tr>
<tr>
<td>18</td>
<td>0.12</td>
<td>0.951</td>
<td>0.11</td>
</tr>
<tr>
<td>24</td>
<td>0.09</td>
<td>0.914</td>
<td>0.08</td>
</tr>
<tr>
<td>30</td>
<td>0.08</td>
<td>0.866</td>
<td>0.07</td>
</tr>
<tr>
<td>36</td>
<td>0.08</td>
<td>0.809</td>
<td>0.06</td>
</tr>
<tr>
<td>42</td>
<td>0.07</td>
<td>0.743</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$E = 13.511 \quad \Sigma = 2.9$

Effective fetch $= \frac{2.9}{13.511} = 0.215$ mile

Fig. 5.14 Computation of effective fetch for Lake in the Hills dam.
wind directions are needed. These relationships can be obtained by the same procedures as used for those of the southeast wind direction. However, in the present study, no attempt was made to establish the relationship for each direction. Only the wave effects caused by the wind blowing southeastward are considered because the maximum effective fetch length is in this direction. For simplicity, it is also assumed that any extreme wind velocity occurring in other directions can also happen in the southeast direction, although the direction of maximum potential wind velocity does not always coincide with that of the maximum fetch. The risk of overtopping due to wind evaluated by the above considerations and assumptions, therefore, is conservative.

5.4.1.2. Uncertainties of Variables in Performance Function

No information on wind is available at the Lake in the Hills dam. The wind record at Chicago Midway Airport, was used to estimate the uncertainties of wind speed, $V_w$, at Lake in the Hills by assuming the same wind condition at both locations. Table 5.10 lists 35 annual extreme fastest-mile wind speeds published by Simiu et al. (1979) for the Midway Airport station. These wind speeds were adjusted and equivalent to those at a level 30 ft above ground. Because the fetch lengths $F$ and $F_e$ are smaller than 0.5 mile the adjustment of the wind speed from ground to water (Table 4.2) is not needed.

In this example risks evaluated with wind speeds described by extreme type I (Eqs. (4-43), and (4-44)), extreme type II (Eqs. (4-62), and (4-63)) and Rayleigh distributions (Eqs. (4-64), and (4-65)) were studied and compared. To fit the wind speed data with probability
Table 5.10 Annual extreme fastest-mile wind speeds for Midway airport, Chicago

<table>
<thead>
<tr>
<th>Year</th>
<th>Wind Speed (mi/hr)</th>
<th>Year</th>
<th>Wind Speed (mi/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1943</td>
<td>44</td>
<td>1961</td>
<td>44</td>
</tr>
<tr>
<td>1944</td>
<td>46</td>
<td>1962</td>
<td>42</td>
</tr>
<tr>
<td>1945</td>
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<td>1947</td>
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<td>1958</td>
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<td>1976</td>
<td>44</td>
</tr>
<tr>
<td>1959</td>
<td>48</td>
<td>1977</td>
<td>51</td>
</tr>
<tr>
<td>1960</td>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
distributions, the probability paper and the least squares method were used. Figures 5.15, 5.16, and 5.17 give the probability plots of the annual maximum series of 35 years of fastest-mile wind speeds. The cumulative probability for each point, $X$, plotted in the probability papers is calculated by the Weibull formula $m/(N+1)$, where $N$ is the number of the observations and $m$ is the rank of $X$ (arranged in increasing order). Values of the statistical parameters fitted from the probability papers for each distribution are given in Table 5.11. To verify the validity of the assumed distributions, the goodness-of-fit test by the Kolmogorov-Smirnov method is performed, and it shows that all these distributions are acceptable models at the 20% significance level (Fig. 5.18).

For the evaluation of the risk of overtopping due to wind alone, it is assumed that no flood occurs concurrently with the wind. The maximum of the initial reservoir level, $H_0$, before the occurrence of the wind, therefore, should not exceed the top of the uncontrolled spillway crest whose height is 35 ft. For the present study the uncertainty of $H_0$ is assumed to be the same as that of the initial reservoir level before the occurrence of flood. The probability distribution of $H_0$ is assumed to be triangular with its lower bound, upper bound and mode equal to 20 ft, 35 ft, and 30 ft, respectively.

The model (correction) variable $\lambda_w$ is the ratio between the observed wave height, $h_w$, and computed $h_w$. In this example, the uncertainty of $\lambda_w$ is subjectively assumed to be described by a symmetric triangular probability distribution with its lower and upper bounds at 0.5 and 1.5, respectively. The probability function and statistical
Fig. 5.15 Extreme type I distribution fitting of wind velocity.
Fig. 5.16 Extreme type II distribution fitting of wind velocity.
Table 5.11 Uncertainties of wind speed, initial reservoir level and correction variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Variance</th>
<th>C.O.V.</th>
<th>Statistic Parameters</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_W$ (mi/hr)</td>
<td>Extreme type I</td>
<td>47.18</td>
<td>5.36</td>
<td>28.72</td>
<td>0.114</td>
<td>$a = -44.77$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_W$ (mi/hr)</td>
<td>Extreme type II</td>
<td>43.12</td>
<td>3.12</td>
<td>9.73</td>
<td>0.719</td>
<td>$\alpha = 44.63$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$V_W$ (mi/hr)</td>
<td>Rayleigh</td>
<td>47.13</td>
<td>5.14</td>
<td>26.39</td>
<td>0.109</td>
<td>$\alpha = 37.30$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$H_0$ (ft)</td>
<td>Triangular</td>
<td>28.33</td>
<td>3.12</td>
<td>9.72</td>
<td>0.110</td>
<td>-</td>
<td>20.0</td>
<td>35.0</td>
</tr>
<tr>
<td>$\lambda_{hw}$</td>
<td>Triangular</td>
<td>1.0</td>
<td>0.289</td>
<td>0.083</td>
<td>0.289</td>
<td>-</td>
<td>0.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Fig. 5.18  Kolmogorov-Smirnov test of wind velocity fitted by extreme Type I, extreme Type II and Rayleigh distributions.
parameters which describe the uncertainties of $H_0$ and $\lambda_w$ are also listed in Table 5.11.

5.4.2. Evaluation of Overtopping Risk Due to Wind

To calculate the risk of overtopping due to wind, the AFOSM method with the GRG technique is used. Figure 5.19 gives the flow chart for preparing the objective function and the constraint equations. Risks are evaluated based on considerations of the following uncertainties.

1. Only the uncertainty of wind speed, $V_w$, is considered.
2. Uncertainties of wind speed and initial reservoir level, $H_0$, are considered.
3. Uncertainties of wind speed, initial reservoir level, and correction variable, $\lambda_w$, are considered.

When the uncertainties of $H_0$ and $\lambda_w$ are not considered in cases 1 and 2, the mode of $H_0$ and mean of $\lambda_w$ are used in calculating the risk. Risks are also evaluated with the wind speed described by different probability distributions, namely, extreme Type I, extreme Type II, and Rayleigh distributions.

Table 5.12 lists the calculated $p_{w,i}$ based on the three levels of uncertainties assumed above, with wind fitted to extreme Type I, Type II, and Rayleigh distributions. Risks evaluated for normal reservoir pool level ($H = 35$ ft) are also given in Table 5.12. Like the risk of overtopping induced by flood, the risk due to wind only, $p_{w,i}$, evaluated for $H_0 = 35$ ft is $6000\%$ to $12000\%$ higher than that evaluated by including the uncertainty of $H_0$.

Depending upon the probability distribution used for the wind
Fig. 5.19 Block diagram of input procedure to GRG package for evaluating overtopping risk due to wind.
Table 5.12  Risk values of overtopping in an occurrence of wind based on different considerations of uncertainties and different probability density functions of wind

<table>
<thead>
<tr>
<th>Variables with Uncertainties</th>
<th>( p_{w,i} )</th>
<th>( \frac{p_w}{p_{w,i}} )</th>
<th>Composite Risk ( p_w )</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_w )</td>
<td>0.*</td>
<td>0.990 x 10^{-8}</td>
<td>0.*</td>
<td>0.149 x 10^{-9}</td>
</tr>
<tr>
<td>( V_w, H_o )</td>
<td>0.535 x 10^{-9}</td>
<td>0.373 x 10^{-6}</td>
<td>0.*</td>
<td>0.562 x 10^{-7}</td>
</tr>
<tr>
<td>( V_w, H_o, \lambda_w )</td>
<td>0.118 x 10^{-6}</td>
<td>0.217 x 10^{-5}</td>
<td>0.*</td>
<td>0.378 x 10^{-6}</td>
</tr>
<tr>
<td>( V_w )</td>
<td>0.304 x 10^{-6}</td>
<td>0.432 x 10^{-4}</td>
<td>0.394 x 10^{-17}</td>
<td>0.661 x 10^{-5}</td>
</tr>
<tr>
<td>( V_w, \lambda_w )</td>
<td>0.124 x 10^{-4}</td>
<td>0.110 x 10^{-3}</td>
<td>0.523 x 10^{-6}</td>
<td>0.222 x 10^{-4}</td>
</tr>
</tbody>
</table>

*Risk value approaches to 0 and lies outside the computer's range

** \( P[i] \) is calculated by \( P[i] = \frac{\text{Var}_i(V_w)}{\sum_{j=1}^{N} \text{Var}_i(V_w)} \) (Eq. 4-67)
speed, the risk values vary significantly. Considerably lower risk values are obtained when Rayleigh distribution is used, while higher risk values are obtained when Type II distribution is used. Table 5.12 also gives the \( P[i] \) value for each distribution by means of Eq. (4-67) and the variance of each distribution listed in Table 5.11. Based on Eq. (4-66) the composite risk \( p_w \) are also given in Table 5.12. The risks of overtopping due to wind in different periods of time \( T \), \( P_w(T) \), for considering various uncertainties based on the composited risk \( p_w \) and the occurrence rate \( v_w = 1 \) are shown in Fig 5.20.

5.5. Overtopping Risk Due to Concurrence of Flood and Wind

Applying the results derived in Section 4.5.2, for treatment of combined loads, the risk of overtopping due to a coincident occurrence of flood and wind, \( p_{FW} \), can be calculated by

\[
p_{FW} = \frac{\mu_w}{\mu_F + \mu_w} P\left[ h_F + h_{mz} > h_c - h_o \right] + \frac{\mu_F}{\mu_F + \mu_w} \text{MAX} \left( P\left[ h_F + \frac{h_{mz}}{2} > h_c - h_o \right] \right),
\]

\[
P\left[ \frac{h_F + h_w + h_{mz}}{2} > h_c - h_o \right]
\]

To obtain the value of \( p_{FW} \), the probability values of three different failure states described by the following performance functions should first be determined.

\[
g_1 = h_c - h_o - (h_F + h_{mz}) \quad (5-11)
\]

\[
g_2 = h_c - h_o - (h_F + 0.5 h_{mz}) \quad (5-12)
\]

\[
g_3 = h_c - h_o - 0.5 \left( h_F + h_w + h_{mz} \right) \quad (5-13)
\]
Fig. 5.20 Risk of overtopping due to wind in a time period by considering various uncertainties.
The logic and procedure for estimating $p_{FW}$ can be illustrated by the block diagram shown in Fig. 5.21.

The uncertainties of the variables and the related formulations for determining the maximum reservoir level, $h_r$, and total height of wind tide and wave run-up, $h_w$, are the same as those described in sections 5.3 and 5.4. To evaluate the risk $p_{FW}$ based on rainfall of different durations and wind speeds described by different probability functions, the weighting methods presented in section 4.3.3.1 and 4.4.3 are also used. If $C_j$, $j=1, 2, 3$, denotes the failure events described by the three performance functions $g_j$, $j=1, 2, 3$ shown in Eqs. (5-11), (5-12) and (5-13), respectively, then $p_{FW}$ can be computed as follows:

1. Calculate the probability of each event $C_j$, $p[C_j | t_r, i]$, for each rainfall duration $t_r$ and each wind probability function $i$.
2. Compute the probability of each event $C_j$, $p[C_j | t_r]$ for each rainfall duration $t_r$, by

$$p[C_j | t_r] = \sum_{i=1}^{N} p[C_j | t_r, i] P[i]$$  \hspace{1cm} (5-14)

in which $P[i]$ is the probability that the probability function $i$ is the "true" one among all the $N$ probability functions considered.

3. Calculate the probability of overtopping due to a concurrence of flood and wind, $p_{FW} | t_r$, for each rainfall duration $t_r$ by using Eq. (5-10), i.e.,

$$p_{FW} | t_r = \frac{\mu_w}{\mu_r + \mu_w} p[C_1 | t_r] + \frac{\mu_F | t_r}{\mu_r + \mu_w} \text{MAX}(p[C_2 | t_r], p[C_3 | t_r])$$  \hspace{1cm} (5-15)

in which $\mu_{F | t_r}$ is the average duration of the reservoir level raised by the flood, i.e., $h_H(t)$, for each $t_r$.

4. Compute the risk of overtopping $p_{FW}$ by

$$p_{FW} = \sum_{n=1}^{\infty} p_{FW} | t_r \cdot [P(t_r + \frac{\Delta t}{2}) - P(t_r - \frac{\Delta t}{2})]$$  \hspace{1cm} (5-16)
Fig. 5.21 Procedure for risk evaluation of overtopping due to occurrences of flood and wind.
in which \( P(t_r - \Delta t/2) \) and \( P(t_r + \Delta t/2) \) are the cumulative probability of rainstorm with a duration less than or equal to \( t_r - \Delta t/2 \) and \( t_r + \Delta t/2 \), respectively.

Values of \( p[C_j | t_r, i] \) calculated with \( t_r \) equal to 0.5, 1, 2, 3, 6, 12 and 24 hr, and wind speed described by extreme Type I, extreme Type II and Rayleigh distributions are listed in Table 5.13. The uncertainties of hydrological, hydraulic, and modeling correction variables, except watershed variable, are considered. Unlike those of risk of overtopping induced by wind only, the differences of \( p[C_j | t_r, i] \) from different probability distributions of wind are found to be insignificant because of the relative small amount of \( h_w \) as compared to that of \( h_F \). Values of \( p[C_j | t_r, i] \) for each duration \( t_r \) are computed by Eq. (5-14) and also listed in Table 5.13. In order to evaluate \( p_{FW|t_r} \) by means of Eq. (5-15), the average durations of \( h_H(t) \) and wave height, \( u_w \), are needed. Magnitudes of \( u_F \) for each \( t_r \) can be determined from Eq. (4-80) using mean values of the variables. Because of the lack of information, magnitude of \( u_w \) is subjectively assumed to be 2 hrs. The \( p_{FW|t_r} \) computed from Eq. (5-5) for \( u_w \) equal to 2 hrs. and \( t_r \) equal to 0.5, 1, 2, 3, 6, 12, and 24 hrs. are listed in Table 5.14. Value of \( p_{FW} \) weighted from Eq. (5-16) are also given in Table 5.14. The risk \( P_{FW}(T) \) in a period of time \( T \) therefore can be evaluated by using Eq. (4-72)

\[
P_{FW}(T) = 1 - \exp(-v_{FW} T \cdot p_{FW})
\]

in which \( v_{FW} = v_{FW}(u_F + u_w) \). The average duration of \( h_H(t) \), \( u_F \), can be obtained by weighting \( u_F|t_r \) in the same way as determining \( p_{FW} \), i.e., by

\[
u_F = \sum_{n=1}^{N} \mu_F|t_r \cdot [P(t_r + \frac{\Delta t}{2}) - P(t_r - \frac{\Delta t}{2})]
\]
Table 5.13 Probability values of $p(C_l | t_r, i)$ and $p(C_r | t_r)$ for given rainfall duration $t_r$ and probability function of wind

<table>
<thead>
<tr>
<th>Extreme Type</th>
<th>Rayleigh</th>
<th>Extreme Type I</th>
<th>Extreme Type II</th>
<th>Extreme Type II + Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk value is calculated by using Eq. (5-14), $p(C_l</td>
<td>t_r, i) = \sum_{i=1}^{3} p(C_l</td>
<td>t_r, i) P[type I]$, in which $P[type I] = 0.443$, $P[type II] = 0.150$, and $P[Rayleigh] = 0.407$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t_r$ (hr)</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(C_l</td>
<td>t_r, i)$</td>
<td>0.150 x 10^{-2}</td>
<td>0.110 x 10^{-1}</td>
<td>0.573 x 10^{-1}</td>
<td>0.120 x 10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p(C_r</td>
<td>t_r)$</td>
<td>0.211 x 10^{-1}</td>
<td>0.221 x 10^{-1}</td>
<td>0.271 x 10^{-1}</td>
<td>0.277 x 10^{-1}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Extreme Type</th>
<th>Rayleigh</th>
<th>Extreme Type I</th>
<th>Extreme Type II</th>
<th>Extreme Type II + Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk value is calculated by using Eq. (5-14), $p(C_l</td>
<td>t_r, i) = \sum_{i=1}^{3} p(C_l</td>
<td>t_r, i) P[type I]$, in which $P[type I] = 0.443$, $P[type II] = 0.150$, and $P[Rayleigh] = 0.407$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t_r$ (hr)</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(C_l</td>
<td>t_r, i)$</td>
<td>0.150 x 10^{-2}</td>
<td>0.110 x 10^{-1}</td>
<td>0.573 x 10^{-1}</td>
<td>0.120 x 10^{-1}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p(C_r</td>
<td>t_r)$</td>
<td>0.211 x 10^{-1}</td>
<td>0.221 x 10^{-1}</td>
<td>0.271 x 10^{-1}</td>
<td>0.277 x 10^{-1}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.14 Risks of overtopping due to concurrences of flood and wind

<table>
<thead>
<tr>
<th>$\mu_w$ (hr)</th>
<th>$t_r = 0.5$ hr</th>
<th>$t_r = 1$ hr</th>
<th>$t_r = 2$ hr</th>
<th>$t_r = 3$ hr</th>
<th>$t_r = 6$ hr</th>
<th>$t_r = 12$ hr</th>
<th>$t_r = 24$ hr</th>
<th>$P_{FW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_F = 6.55$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 6.62$</td>
<td>$0.627 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 6.77$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 6.93$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 7.56$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 31.00$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\mu_F = 35.11$</td>
<td>$0.553 \times 10^{-3}$</td>
<td>$0.479 \times 10^{-2}$</td>
<td>$0.170 \times 10^{-1}$</td>
<td>$0.302 \times 10^{-1}$</td>
<td>$0.679 \times 10^{-1}$</td>
<td>$0.115 \times 10^{0}$</td>
<td>$0.129 \times 10^{0}$</td>
<td>$0.285 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Table 5.15 Risks of overtopping due to occurrence of flood only, of wind only, and concurrence of flood and wind in a period of time $T$

<table>
<thead>
<tr>
<th>Loading</th>
<th>Risk in an Occurrence of Loading</th>
<th>$T = 1$</th>
<th>$T = 5$</th>
<th>$T = 10$</th>
<th>$T = 50$</th>
<th>$T = 100$</th>
<th>$T = 500$</th>
<th>$T = 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flood</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$0.827 \times 10^{-2}$</td>
<td>$0.407 \times 10^{-1}$</td>
<td>$0.796 \times 10^{-1}$</td>
<td>$0.340 \times 10^{0}$</td>
<td>$0.564 \times 10^{0}$</td>
<td>$0.984 \times 10^{0}$</td>
<td>$0.100 \times 10^{1}$</td>
</tr>
<tr>
<td>Wind</td>
<td>$1.0 \times 10^{-6}$</td>
<td>$0.378 \times 10^{-6}$</td>
<td>$0.189 \times 10^{-5}$</td>
<td>$0.378 \times 10^{-5}$</td>
<td>$0.189 \times 10^{-4}$</td>
<td>$0.378 \times 10^{-4}$</td>
<td>$0.189 \times 10^{-3}$</td>
<td>$0.378 \times 10^{-3}$</td>
</tr>
<tr>
<td>Flood &amp; Wind</td>
<td>$0.11 \times 10^{-2}$</td>
<td>$0.285 \times 10^{-1}$</td>
<td>$0.157 \times 10^{-3}$</td>
<td>$0.314 \times 10^{-2}$</td>
<td>$0.157 \times 10^{-2}$</td>
<td>$0.313 \times 10^{-2}$</td>
<td>$0.156 \times 10^{-1}$</td>
<td>$0.309 \times 10^{-1}$</td>
</tr>
<tr>
<td>Total risk of overtopping due to flood and/or wind</td>
<td>$0.830 \times 10^{-2}$</td>
<td>$0.408 \times 10^{-1}$</td>
<td>$0.799 \times 10^{-1}$</td>
<td>$0.341 \times 10^{0}$</td>
<td>$0.565 \times 10^{0}$</td>
<td>$0.984 \times 10^{0}$</td>
<td>$0.100 \times 10^{1}$</td>
<td></td>
</tr>
</tbody>
</table>

*Risks are calculated by considering uncertainties of following variables: $D_R$, $a^0$, $CN$, $H_0$, $A$, $L_w$, $H_w$, $C_d$, $L_s$, $h_s$, $\lambda_Q$, $\lambda_F$*
With \( \mu_{Fit} \) given in Table 5.14 and by using the same procedure as illustrated in Table 5.7, \( \mu_F \) is found to be 7.64 hrs. The \( P_{FW}(T) \) for different time periods \( T \) are shown in Fig. 5.22.

5.6. Total Risk of Overtopping Due to Flood and Wind

The risks of overtopping due to occurrences of flood only, wind only, and the simultaneous occurrences of flood and wind have been evaluated in the previous sections and are summarized in Table 5.15. From Table 5.15, the risk due to a simultaneous occurrence of flood and wind is higher than those due to the sole occurrence of flood or wind. Because of the short fetch of the Lake in the Hills dam-reservoir, the risk induced by wind is relatively small and can be ignored. Table 5.15 also gives risk values of overtopping for each cause for time periods of 1 year to 1000 years. It appears that the risk induced by concurrence of flood and wind in a time period \( T \) becomes very small as compared to that of flood alone because of the relatively small annual occurrence rate, i.e., \( \nu_{FW} = 0.11 \times 10^{-2} \) per year with respect to \( \nu_F = 1 \) per year for the flood and \( \nu_W = 1 \) per year for wind. From Eq. (4-11), the total risk of overtopping due to flood and/or wind in a time period \( T \), \( P_V(T) \), can be calculated by

\[
P_V(T) = 1 - \exp \left[ -T \left( \nu_F P_F + \nu_W P_W + \nu_{FW} P_{FW} \right) \right]
\]

Values of \( P_V(T) \) for \( T \) from 1 year to 1000 years are given in Fig. 5.22. Figure 5.22 also shows the comparison among the total risk and risks due to different causes in a time period \( T \). This comparison shows that flood is the primary contributing factor to overtopping, and that the
Fig. 5.22 Comparison of total risk and risks induced by different geophysical forces.
occurrence of wind alone or the concurrence of flood and wind have relatively negligible effects.
6.1. Summary

The usefulness of risk analysis in dam safety evaluation has been recognized by various federal committees on the safety of dams. Risk analysis provides a quantitative measurement of dam safety so that the safety of different dams can be ranked and compared, and priorities in inspection, remedial work, allocation of funds, and emergency preparedness among different dams can be determined. However, in current practice of the national dam safety program, risk-based analysis has not been implemented. The present study is an initial attempt to develop methodology and procedures which can be used to evaluate systematically and quantitatively the risk of dam failure due to overtopping.

Risk in this study is defined as the probability of unsatisfactory performance which occurs when the load applied to a system exceeds the resistance (or capacity) of the system. Both the load and resistance are subject to uncertainties. Uncertainties can be described in terms of a probability function or statistical parameters such as standard deviation, variance, and coefficient of variation. The effects of different levels of uncertainties of various variables on risk are analyzed. Moreover, the effects of different probability functions and coefficients of variation of variables and ratios of their mean values are also studied.

Various methods have been proposed for risk evaluation. In the design of water resources systems, the return period has often been used
as an index or measure of the risk. However, it considers only the uncertainty of future hydrologic events, e.g., floods or rainfall, and it ignores other sources of uncertainties, such as spillway discharge capacity, and reservoir water level. Hence, at best, the method of return period accounts for only part of the total risk of a complex system such as a dam. Other methods such as direct integration, Monte Carlo simulation, mean-value first-order second-moment (MFOSM), and advanced first-order second-moment (AFOSM) may be capable of accounting for the various uncertainties of different factors or variables contributing to the failure of the system. In order to select a suitable method for risk evaluation of a dam, a comparison of the aforementioned methods based on accuracy, consistency, and efficiency was performed. The AFOSM method is found to be superior to other methods. The AFOSM method is an extension of the MFOSM method which considers the first-order terms of the Taylor series expansion of the performance function, and utilizes the first two statistical moments of the random variables in the risk analysis. The AFOSM method evaluates the risk at the failure point of each variable on the failure surface, instead of evaluating the risk at the mean value of each variable as in the MFOSM method, so that the risk value can be better approximated. The solution scheme for the AFOSM method is further improved by the use of the generalized reduced gradient optimization technique which not only decreases the computation time for system with low risk values, but also provides an efficient algorithm for treating variables with bounded probability distributions.

The statistics given in Chapter 1 indicate that overtopping is a
major failure mode of earth and rockfill dams. Overtopping occurs when the reservoir level raises due to occurrence of flood, wind, earthquake, landslide and other geophysical forces, causing water to flow over the top of the dam. To evaluate the risk of overtopping, the stochastic process of the geophysical forces should be considered. By assuming Poisson process for the occurrences of the geophysical forces and the uncertainties of the other hydrological and hydraulic variables to be invariant in each occurrence of the geophysical force, a risk model for overtopping induced by various geophysical forces was developed. Through the use of fault tree analysis, the overall risk of overtopping was systematically combined from the component risks of overtoppings due to various occurrences of geophysical forces.

In this study, only overtopping due to occurrences of flood and wind are investigated. To derive the performance function for overtopping induced by a flood, the hydrograph of the flood into the reservoir is first obtained by the convolution of effective rainfall hyetograph with instantaneous unit hydrograph. The maximum reservoir water level is subsequently determined through a simplified reservoir routing method using the discharge rating and storage-stage curves. The wind-tide and wave run-up are the additional water heights induced by wind. The empirical equations established by the U. S. Army Corps of Engineers are used to calculate the heights of wind-tide and wave run-up for a given initial reservoir elevation. A load combination model is established to account for the maximum combined effects resulting from concurrence of flood and wind. In this case, the total height of wind-tide and wave run-up is not constant. Instead, it changes
proportionally with the reservoir level which raises with the flood flowing into the reservoir. The magnitude of the maximum combined load depends also on the relative occurrence time between the loads induced by flood and wind.

To illustrate the proposed risk model and methodology, an example is given in Chapter 5 to demonstrate the evaluation of overtopping risk for an earth dam having an uncontrolled spillway. Procedures for analyzing the uncertainties of the variables and flow charts for performing the risk calculation of overtopping due to occurrences of flood and wind are presented. The AFOSSM method with ORG technique is used to calculate risk values. Differences among the risk values obtained by considering different levels of uncertainties and using different risk calculation methods are investigated. Evaluation of the risk from overtopping induced by the occurrence of flood, following the guidelines recommended by the U. S. Army Corps of Engineers for their current dam safety inspection program is discussed in this study.

The methodology presented in this study gives a general procedure of risk assessment. It can be extended to include other conditions affecting dam safety. Formulations of the performance functions considered in Chapters 4 and 5 for overtopping risk evaluation can be varied for different dams.

6.2. Conclusions

The following conclusions are drawn from this study of overtopping risk of a dam.

1. The probabilistic approach of risk analysis considers systematically and quantitatively various uncertainties contributing to
the failure of a dam. It gives a more reasonable risk evaluation than that of the current national dam safety inspection program which is primarily deterministic approach and considers only one special flood condition at normal reservoir pool level among many possible combinations of random nature.

2. Uncertainties of relevant variables can be described by the coefficient of variation (COV) or the probability function of the variables. Risk is found to be a function of the magnitudes of COV, type of probability function, and relative mean values of the load and resistance. If the mean value of the load, \( \mu_L \), is smaller than that of the resistance, \( \mu_R \), then for the same value of \( \frac{\mu_R}{\mu_L} \), the risk value is higher when larger uncertainties or uncertainties of more variables are considered. If the mean value of the load is larger than that of the resistance, however, the reverse is true.

3. Various risk calculation methods such as the direct integration, Monte Carlo simulation, MFOSM, and AFOSM methods were studied and compared. By considering the accuracy, consistency, and efficiency, AFOSM is shown to be the most promising risk calculation method for dam safety evaluation.

4. The comparison of risk calculation methods given in Chapter 2 and the case study of overtopping risk evaluation discussed in Chapter 5 show that different risk values can be obtained by using different risk calculation methods and by considering different uncertainties of the load and resistance. This indicates that in practice it is difficult, if not impossible, to obtain the "absolute" true risk value of a system because it is unlikely to have all influential variables, whether large
or small, to be accounted for completely and thoroughly.

5. In the hydrologic and hydraulic aspects of the current U. S. dam safety program, a dam is declared unsafe for being unable to pass a specific amount of flood water on normal reservoir pool level. Risk of overtopping due to flood evaluated on normal reservoir pool level is found to be higher than that evaluated by considering the possible variation in the reservoir level before the occurrence of flood. It can be further overestimated if the overtopping risk is evaluated based on the flood hydrograph generated by the 24 hours rainfall. This may partly explain why an astonishing number of 20% of the inspected dams were declared unsafe by the U. S. Army Corps of Engineers.

6. Risk values of overtopping due to occurrences of flood and wind are compared. This comparison shows that flood is the primary contributing factor to overtopping, and that the occurrence of wind alone or the concurrence of flood and wind have relatively negligible effects.

6.3. Future Research

Experience obtained from the the present study leads to the following suggestions for possible future research:

1. Risk evaluation of overtopping presented in this study considers only a portion of the total risk of a dam. Other failure events and factors contribute to the failure of a dam such as seepage, piping, instability and human errors should be considered in the overall risk evaluation of the dam. The risk model and procedure developed in this study can be extended and refined to calculate the risk of the aforementioned failure events.
2. More accurate risk values can be obtained if uncertainties of the variables in the performance function are more precisely quantified and described. Consideration and effort should be given to collect and analyze the data and information so that the uncertainties can be more precisely estimated and the prediction of the risk of a dam can be improved.

3. Deterioration of the material or structure of the dam, generally occurs with time of operation. Furthermore, the various geophysical forces could be dependent. These effects should be considered in further study of risk evaluation.

4. Simplified approaches are used in this study to account for rainfall and runoff relationship, reservoir routing and wind-wave relationship. Adoption of more sophisticated and reliable relationships would reduce the uncertainty of model errors, but would require more extensive numerical techniques in the risk evaluation.

5. The ultimate objective of risk evaluation for a dam is to facilitate a rational optimal decision for selecting proper design, remedial measures, or inspection programs among different alternatives. Risk methodologies developed in this study will be major inputs in decision analysis or optimization techniques for achieving these objectives.
APPENDIX A. DERIVATION OF PROBABILITY DENSITY FUNCTION OF SUM OF TWO RANDOM VARIATES

Consider $X_1$ and $X_2$ uniformly distributed as shown below

Let $U_c(t)$ be a step function as

\[
U_c(t) = \begin{cases} 
0 & ; t < c \\
1 & ; t \geq c 
\end{cases}
\]

that $X_1$ and $X_2$ can be represented by the following equations

\[
f_{X_1}(x_1) = \frac{1}{b_1-a_1} \left[ U_{a_1}(x_1) - U_{b_1}(x_1) \right]
\]

\[
f_{X_2}(x_2) = \frac{1}{b_2-a_2} \left[ U_{a_2}(x_2) - U_{b_2}(x_2) \right]
\]

If the load $L = X_1 + X_2$, then the probability density function of $L$ is equal to the convolution of probability density functions of $X_1$ and $X_2$ as

\[
f_L(t) = f_{X_1}(x_1) * f_{X_2}(x_2)
\]

Since the Laplace transform of the convolution of two functions is equal to the product of the separate Laplace transforms. Hence,

\[
\mathcal{L} [ f_L ] = \mathcal{L} [ f_{X_1} ] \cdot \mathcal{L} [ f_{X_2} ]
\]

where $\mathcal{L} [ f ]$ denotes the Laplace transform of function $f$.

Use the definition that

\[
\mathcal{L} [ U_c(t) ] = \int_0^\infty e^{-st} U_c(t) \, dt = \frac{e^{-cs}}{s}
\]
that
\[ \mathcal{L} [f_L] = \frac{1}{(b_1-a_1)(b_2-a_2)} \left[ \frac{-a_1 s - b_1 s}{s} \right] \left[ \frac{-a_2 s - b_2 s}{s} \right] \]

Use the relationship that
\[ \mathcal{L} [t] = \frac{1}{s^2} \]
\[ \mathcal{L} [u_c(t) f(t-c)] = e^{-cs} F(s) ; \quad \mathcal{L} [f(t)] = F(s) \]

the function \( f_L(t) \) is obtained as follows

\[ f_L(t) = \frac{1}{(b_1-a_1)(b_2-a_2)} [t-(a_1+a_2)U_{(a_1+a_2)}(t)-t-(b_1+a_2)]. \]

\[ U_{(b_1+a_2)}(t)-[t-(a_1+b_2)]U_{(a_1+b_2)}(t)+[t-(b_1+b_2)]U_{(b_1+b_2)}(t)] \]

If \( a_1 < a_2 < b_1 < b_2 \) and \( b_1+a_2 < a_1+b_2 \), then

\[ f_L(t) = \begin{cases} \frac{t - a_1 - a_2}{(b_1-a_1)(b_2-a_2)}, & a_1 + a_2 \leq t < b_1 + a_2 \\ \frac{1}{b_2 - a_2}, & b_1 + a_2 \leq t < a_1 + b_2 \\ \frac{b_1 + b_2 - t}{(b_1-a_1)(b_2-a_2)}, & a_1 + b_2 \leq t \leq b_1 + b_2 \\ 0, & \text{elsewhere} \end{cases} \]
APPENDIX B. DERIVATION OF DRH BY USING LAPLACE TRANSFORM TECHNIQUE

For a triangular distribution shown below,

\[
f(t) = \frac{C \left( T_2 - t \right)}{T_2 - T_1}
\]

let \( \mathcal{L} \{ f(t) \} \) denote the Laplace transform of the function \( f(t) \), i.e.,

\[
\mathcal{L} \{ f(t) \} = F(s) = \int_0^\infty e^{-st} f(t) \, dt
\]

\[
= \int_0^{T_1} e^{-st} \frac{Ct}{T_1} \, dt + \int_{T_1}^{T_2} e^{-st} \frac{C \left( T_2 - t \right)}{T_2 - T_1} \, dt
\]

\[
= \frac{CT_2}{s^2} \left[ \frac{1}{T_1 T_2} - \frac{e^{-sT_1}}{T_1 (T_2 - T_1)} + \frac{e^{-sT_2}}{T_2 (T_2 - T_1)} \right]
\]

Since ERH and IUH are assumed to be triangular in shape as shown below,

Then

\[
\mathcal{L} \{ \text{ERH} \} = \frac{i_{ep} t_{ed}}{s^2} \left[ \frac{1}{t_{ea} t_{ed}} - \frac{e^{-st_{ea}}}{t_{ea} (t_{ed} - t_{ea})} + \frac{e^{-st_{ed}}}{t_{ed} (t_{ed} - t_{ea})} \right]
\]

\[
\mathcal{L} \{ \text{IUH} \} = \frac{q_u t_{ud}}{s^2} \left[ \frac{1}{t_{up} t_{ud}} - \frac{e^{-st_{up}}}{t_{up} (t_{ud} - t_{up})} + \frac{e^{-st_{ud}}}{t_{ud} (t_{ud} - t_{up})} \right]
\]

Since the Laplace transform of the convolution of two functions is equal to the product of the separate Laplace transforms,
hence,

\[ L[DRH] = L[ERH] \cdot L[IUH] \]

\[ = \frac{i_{ep} t_{ed}}{s^2} \frac{q_{ut}}{s^2} \left[ \frac{1}{t_{ea} t_{ed}} - \frac{e^{-st_{ea}}}{t_{ea} (t_{ed} - t_{ea})} + \frac{e^{-st_{ed}}}{t_{ed} (t_{ed} - t_{ea})} \right] \]

By taking the inverse of the Laplace transform

\[ t^n = \frac{n!}{s^{n+1}} \quad \text{and} \]

\[ L[U_c(t) f(t-c)] = e^{-cs} F(s) \]

in which the step function is

\[ U_c(t) = \begin{cases} 0; & t < c \\ 1; & t \geq c \end{cases} \]

By taking the inverse of the Laplace transform

\[ DRH = L^{-1} \{ L[DRH] \} \]

\[ Q(t) = \frac{i_{ep} q_{ut} t_{ed} t_{ud}}{6} \left\{ \begin{array}{c} t^3 \\ \frac{t_{ea} t_{ed} t_{up} t_{ud}}{t_{ua} t_{ed} t_{up} t_{ud}} \\ \frac{t_{ua} t_{ed} t_{up} t_{ud}}{t_{ua} t_{ed} t_{up} t_{ud}} + \frac{U_{t_{ed}} (t)(t - t_{ed})^3}{t_{ed} - t_{ea}} \frac{t_{up} t_{ud}}{t_{up} t_{ud}} + \frac{U_{t_{up}} (t)(t - t_{up})^3}{t_{ed} t_{up} (t_{ud} - t_{up})} + \frac{U_{t_{ud}} (t)(t - t_{ud})^3}{t_{ed} t_{up} (t_{ud} - t_{up})} \end{array} \right\} \]
The modeling correction variable, $\lambda_F$, is expressed as

$$\lambda_F = \frac{h_F}{h'_F}$$

where $h_F$ and $h'_F$ are the maximum reservoir height obtained by the Puls method and the proposed routing method (Eqs. (4-39) and (4-40)), respectively.

The Puls method assumes invariable discharge-stage relationships and neglects the variable slope occurring during the passage of flood wave. By expressing Eq. (4-33) in finite time intervals, it becomes

$$\frac{1}{2} \left( Q_{i1} + Q_{i2} \right) \Delta t - \frac{1}{2} \left( Q_{o1} + Q_{o2} \right) \Delta t = S_2 - S_1$$

where the numerical subscripts indicate the routing periods, and $Q_i$, $Q_o$, and $S$ are instantaneous values of inflow, outflow, and storage, respectively, at the beginning of the routing period indicated. Arranging the above equation so that

$$\frac{1}{2} \left( Q_{i1} + Q_{i2} \right) \Delta t + S_1 - \frac{1}{2} Q_{o1} \Delta t = S_2 + \frac{1}{2} Q_{o2} \Delta t$$

Routing is accomplished by substituting the known values in the above equation to obtain $S_2 + 0.5Q_{o2} \Delta t$. Then, $Q_{o2}$ is obtained from the relationship between $Q_{o2}$ and $S_2 + 0.5Q_{o2} \Delta t$. The maximum $Q_{o2}$ obtained by the above procedure is the peak discharge $Q_{op}$. The $h_F$ can be calculated by using Eq. (4-39).

The uncertainty of $\lambda_F$ is determined by comparing $h_F$ and $h'_F$ from same flood hydrographs, storage-stage relationship and discharge rating curve.

Seven different triangular shaped inflow hydrographs were routed by the Puls method and the proposed routing method, respectively, through the reservoir of the Lake in the Hills dam with 36 different initial reservoir levels. For each inflow hydrograph 36 values of $\lambda_F$, i.e., ratio of $h_F/h'_F$ were obtained.

The mean, $\mu_{\lambda_F}$, and coefficient of variation, $\delta_{\lambda_F}$, of $\lambda_F$ calculated from
these 36 values for each inflow hydrograph are listed in the following table.

<table>
<thead>
<tr>
<th>No.</th>
<th>( Q_{ip} ) (cfs)</th>
<th>( t_d ) (hr)</th>
<th>( a^\circ )</th>
<th>( \mu_{\lambda F} )</th>
<th>( \delta_{\lambda F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000</td>
<td>24</td>
<td>0.3</td>
<td>0.908</td>
<td>0.035</td>
</tr>
<tr>
<td>2</td>
<td>2,000</td>
<td>12</td>
<td>0.4</td>
<td>0.921</td>
<td>0.037</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
<td>24</td>
<td>0.4</td>
<td>0.928</td>
<td>0.055</td>
</tr>
<tr>
<td>4</td>
<td>1,000</td>
<td>24</td>
<td>0.3</td>
<td>0.922</td>
<td>0.049</td>
</tr>
<tr>
<td>5</td>
<td>1,500</td>
<td>18</td>
<td>0.35</td>
<td>0.908</td>
<td>0.042</td>
</tr>
<tr>
<td>6</td>
<td>3,000</td>
<td>12</td>
<td>0.3</td>
<td>0.911</td>
<td>0.027</td>
</tr>
<tr>
<td>7</td>
<td>2,000</td>
<td>16.8</td>
<td>0.4</td>
<td>0.913</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Averages of \( \mu_{\lambda F} \)'s and \( \delta_{\lambda F} \)'s are used as the uncertainty of \( \lambda_F \). The Puls method is not the most accurate one for reservoir routing, hence, higher uncertainty value, \( \delta_{\lambda F} = 0.050 \) is used instead of 0.040.
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