Departure-Site Spacing for Liquid Droplets and Jets Falling in Thin-Film Heat Exchangers

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ABSTRACT

When a liquid film falls from one tube to another below it, the flow can take the form of discrete droplets, individual jets or a continuous sheet. Experiments exploring the effects of thermophysical properties and geometrical parameters on the droplet and jet flow patterns are described. Measurements of droplet and jet departure-site spacing are reported for several fluids over a wide range of liquid flow rates, tube sizes and tube spacing. For the conditions of this study, departure-site spacing increased with decreasing Re for high-Ga fluids and was nearly independent of Re for low-Ga fluids. Departure-site spacing increased slightly with tube diameter for small tubes and was nearly independent of tube size for large tubes. Departure-site spacing was nearly independent of tube spacing for the entire range of experiments; however, a relation between jet shape, jet spacing and tube spacing was observed under some conditions. A qualitative study of liquid-jet shapes shows that this flow feature depends strongly on tube spacing, and comparisons to existing models suggest that further work in this area is needed.
INTRODUCTION

Horizontal-tube, falling-film heat exchangers enjoy wide application because they provide good thermal performance with small shell-side liquid inventories[1]. The flow patterns observed in falling-film heat exchangers have been idealized and described by Mitrovic[2] as shown in Figure 1. Mitrovic and co-workers[3] have provided criteria for predicting transitions between the flow modes, and they have presented correlations for jet spacing. Hu and Jacobi[4] extended this work by providing flow observations for a wide range of flow rates and fluid properties; their description of the flow regimes is summarized in Figure 2. Along with an extensive set of transition criteria, Hu and Jacobi described the transitions between falling film modes in terms of competing flow mechanisms: inertia-dominated flows take the sheet mode, gravity or surface-tension dominated flows take the droplet mode, and the jet mode results when these mechanisms compete. The flow maps of Hu and Jacobi capture this competition in $Re-Ga^{1/4}$ space. In a related study, Hu and Jacobi[5] conducted local heat transfer measurements to explore the effect of flow mode on heat transfer behavior. The droplet and jet spacing are especially important in this respect, and this paper is aimed at characterizing these flow features.

In the droplet and jet modes, the liquid falls from the tube at sites that are a fixed distance apart, $\lambda$ (See Figures 1 & 2.). This behavior appears to be related to the Taylor instability. For inviscid, incompressible fluids, Bellman and Pennington[6] found the so-called critical and most dangerous Taylor wavelengths to be given by

$$\lambda_c = 2\pi \sqrt{\frac{\sigma}{g(\rho - \rho_v)}} , \text{ and } \lambda_d = \lambda_c \sqrt{3} . \quad (1)$$

$\lambda_c$ represents the wavelength of the shortest unstable disturbance; whereas, $\lambda_d$ is the disturbance length that grows most rapidly and is expected to appear in application.

A description of departure-site spacing based on the Taylor instability is partially motivated by the work of Zuber[7], who used the idea to predict the maximum heat flux in film boiling. Later, Lienhard and Wong[8] studied the spacing of bubble detachment sites for film boiling from a cylinder, and they predicted the bubble-detachment site spacing to follow
Lienhard and Wong conducted boiling experiments using benzene and isopropanol on cylindrical heaters with diameters from 0.0254 mm to 0.645 mm, and they found the dominant site spacing to be about 25% higher than the most dangerous wavelength.

The departures-site spacing for a liquid falling from a tube is analogous to the vapor detachment situation; however, experimental results show that the simple Taylor-instability approach may not accurately model the physics. Maron-Moalem et al.[9] studied dripping between horizontal tubes as a function of liquid mass flow rate, surface tension, tube diameter and tube spacing. They measured the droplet departure frequency and the distance between droplet producing sites. Maron-Moalem et al. found λ to decrease with increasing flow rate and to increase with droplet departure frequency, tube spacing and tube diameter. For the jet mode, Ganic and Roppo[10] observed λ to be lower than predicted by the Taylor instability and unaffected by flow rate and tube spacing.

Yung et al.[11] concluded that for low-viscosity liquids, like water, ethyl alcohol and ammonia, the instability wavelength most likely to appear at the interface is

\[ \lambda = 2\pi \frac{\sqrt{n\sigma}}{\sqrt[3]{\rho g}} . \]  

For liquids on a horizontal tube, they found that \( n = 2 \) best fit the experimental data, but for thick liquid layers they recommend \( n = 3 \). For high-viscosity liquids, Taghavi and Dhir[12] found that \( \lambda \) is larger than for low-viscosity liquids.

According to Dhir and Taghavi[13], during the dripping of a liquid from the underside of a horizontal tube, \( \lambda \) increases with \( r^* \) for \( r^* < 2 \), and is insensitive to \( r^* \) at higher values of \( r^* \). Tang and co-workers[14] studied the Taylor instability wavelength and accounted for viscous effects in addition to surface curvature. Their model predicted that \( \lambda_d \) increased with \( r^* \) but became insensitive to tube radius for \( r^* > 4 \).

Li and Harris[15] stated that earlier work by Lienhard and Wong[8] was oversimplified. Using a velocity potential and assuming \( \rho >> \rho_v \), they obtained the following expression for the dimensionless most-dangerous wavelength, \( \lambda_d^* = \frac{\lambda_d}{\xi} \):

\[ \lambda_c = \frac{2\pi}{\sqrt{g(\rho - \rho_v)/\sigma + 2/d^2}} , \text{ and } \lambda_d = \lambda_c \sqrt[3]{3} . \]
\[ \lambda_d^* = \frac{2.16 + \sqrt{3 \cdot 0.467r^{1.491}}}{1 + 0.467r^{1.491}} \lambda_c^* \quad \text{where} \quad \lambda_c^* = \frac{2\pi}{\sqrt{1 + 1/2r^2}}. \]  (4)

For \( r^* > 2 \), the \( \lambda_d^* \) predicted by Li and Harris agreed to within 10% of that predicted by Lienhard and Wong; however, for \( r^* \rightarrow 0 \), Li and Harris predict a value about 25% greater than that predicted by Lienhard and Wong.

Very recently, Armbruster and Mitrovic[16] provided a correlation for jet spacing. Their correlation is based on data from two fluids, water and isopropyl alcohol, and is reported to correlate their data to within ±7.5%. Using the current nomenclature, the correlation can be written as:

\[ \lambda = 2\pi \sqrt{2} \sqrt{\frac{g(\rho - \rho_v)}{\sigma} \left( 1 + \left( \frac{Re/14}{Ga^{0.8}} \right) \right)} + \frac{2}{d^2}. \]  (5)

This expression is somewhat unique, in that it explicitly accounts for a flow rate effect on the jet spacing. The size and uncertainty of the data set used to develop Eqn. (5) are somewhat unclear, and no data were provided for droplet spacing.

In addition to the spacing between departure sites, the shape of the falling jet can be important in understanding the falling-film flow. Motivated by this importance, Mitrovic and Ricoeur[17] recently analyzed the heating of free-falling liquid jets. They provided a good summary of the related literature and references to other reviews. In their careful analytical treatment of free-falling capillary jets, Mitrovic and Ricoeur provided predictions of the jet shapes and temperatures. However, these results are limited because they do not consider tube spacing effects; only initial conditions at the jet departure site are considered. In another approach to modeling jet behavior, Bejan[18] considered the buckling of free liquid columns or sheets flowing onto solid surfaces. According to Bejan[18,19], the most striking geometric feature of buckling flows is the deformation of straight fluid streams into sinusoidal shapes with characteristic wavelengths. Bejan considered a flowing liquid jet to be a column compressed by the impulses of the flow. For a round jet, he obtained the characteristic
buckling length to be \( \pi \cdot r_j \). In this approach, only inertia forces are considered and, at low mass flow rates, this view becomes untenable. For static fluids, an alternate method has been adopted to model capillary phenomena. In particular, a simple balance between gravity and surface tension as described by Boucher[20] can be used to predict the free-surface shape for pendant or sessile drops. It is unclear which method—that used by Mitrovic, Bejan, or Boucher—is best for predicting liquid-jet shapes for the falling film.

In this paper, we report experimental observations of the spacing between jet and droplet departure sites for liquids falling between horizontal circular tubes. While the effects of surface tension and density on departure-site spacing are clear through prior work, there is ambiguity as to the roles of tube diameter, tube spacing, and liquid flow rate. One goal of the current work is to resolve this ambiguity. Furthermore, only limited results are available in the literature for the shapes of liquid jets falling between horizontal tubes. Another goal of the current work is to report observations of jet shape and to qualitatively compare these shapes to existing models. It is hoped the results will contribute to a clearer understanding of the intertube falling-film flow behavior.

**METHOD**

**Apparatus & Instrumentation.** The test facility shown in Figure 3 was described earlier[4]; however, a brief description will be summarized in this section for completeness. An open-loop wind tunnel provided for a downward air flow external to the falling liquid film. The wind tunnel inlet was equipped with a 9:1 area contraction, pull-out grids and honeycomb flow straighteners. The transparent test section was 203.2 mm by 304.8 mm, and the test specimens were mounted to the sides of the wind tunnel using fasteners that allowed easy adjustment to tube spacing. The transition section between the fan and expansion was made of a flexible rubber belt to mitigate vibrations caused by the blower. For the same purpose, gaskets were used at each interface between wind-tunnel sections. The wind tunnel provided air velocities from less than 1 m/s up to 15 m/s at the test section inlet, and the approach velocity profiles were flat to within 2.7% at the maximum blower speed.
Liquid was circulated through a closed loop that contained a head tank, a thermally controlled reservoir, a liquid pump and filter, a needle valve and flow meters. The test liquid was delivered from the reservoir to the adjustable constant-head tank by the pump. It was gravity fed from the head tank, through the first flow meter and needle valve, to the upper feed tube in the test section. In the test section, as shown in Figure 4, liquid issued from the upper feed tube, flowed around the lower tubes, and eventually fell into the catching tube. Downstream from the test section, the flow returned to the thermally controlled reservoir.

The liquid feed arrangement followed the method described by Mitrovic[2]*. The top feed tube had inside and outside diameters of 15.86 and 22.22 mm, respectively. Along a 229 mm length on the bottom of this tube, 1.0 mm diameter holes were drilled 1.5 mm apart. A 1.4 mm gap was used between the bottom of the first feed tube and the top of the second tube. The lower feed tube and all dummy tubes always had diameters equal to the test tube. Therefore, the liquid film always fell between tubes of identical diameter in the test section.

The liquid flow rate was measured using the oscillating-piston flow meter upstream of the test section; the flow meter on the discharge line provided a redundant check. These flow meters had an operating range up to 252 ml/s and an uncertainty of ±0.5% of the reading. At flow rates below 36 ml/s, the liquid issuing from the catching tube was collected and weighed. This method had a maximum uncertainty of 2%. The surface tension of the fluids was measured using a Du Nouy ring tensiometer (±0.5%). Air velocity at the inlet of the test section was measured using a Pitot tube and manometer (±0.12 Pa). Air density was inferred from ideal gas behavior and laboratory temperature (±0.1C), humidity (±2%), and pressure (±0.5 mm Hg). A digital camera and a 35-mm camera were used to record flow images; using a reference image, the uncertainty in lengths determined from the images was ±0.5 mm. Thermophysical properties were determined using the measurements outlined above and property correlations from the literature (See [4] & [5] for details).

**Experimental Scope & Procedure.** Assuming shear at the liquid-vapor interface to be negligible, a long tube (>>l), and a liquid density much larger than the vapor density, the falling film behavior can depend on the flow rate per unit length of tube, \( \Gamma \); the dynamic

*Armbruster and Mitrovic[16] recently reported an improved method for distributing the falling film.
viscosity, \( \mu \); the liquid density, \( \rho \); the surface tension, \( \sigma \); the tube diameter, \( d \); the intertube spacing, \( s \); and the gravitational acceleration, \( g \). The data now presented were recorded with a quiescent atmosphere, and the underlying assumptions stated above are tenable. The nominal ranges of the important physical variables in our experiments are given in Table 1.

The dependence of the droplet or jet departure site spacing, \( \lambda \), on the physical variables given in Table 1 can be represented using a Reynolds number \( Re \), modified Galileo number \( Ga \), the dimensionless diameter, \( d^* \) and the dimensionless tube spacing \( s^* \). This set of dimensionless variables implies a particular physical interpretation. If the capillary length scale, \( \xi \), is adopted, then \( Ga^{1/4} \) is interpreted as a ratio of gravity to viscous forces. Likewise, if a viscous scale is adopted, \( Ga^{1/4} \) can be interpreted as the square root of surface tension to viscous forces (see [4]). The interpretation and set of dimensionless parameters is not unique, and alternatives can be found[3,4,21]. The current choice is internally consistent and does not affect the results. The experimental range, physical interpretation and experimental uncertainties for these dimensionless groups are reported in Table 2.

A wide range in thermophysical properties was achieved by conducting experiments with water, ethylene glycol, a water-glycol mixture, and hydraulic oil. The correlations and measurements used to determine the properties of these fluids are described in detail by Hu and Jacobi[4]. The nominal values for \( Ga^{1/4} \) were 530 (water), 165 (water-glycol), 35.7 (ethylene glycol), and 5.2 (hydraulic oil); and the capillary constant, \( \xi \), was 2.64 (water), 2.28 (water-glycol), 2.07 (ethylene glycol), and 1.92 mm (oil). Thus, uncertainties in \( \lambda^* \) ranged from about 2% to 4%. Although the experimental Re range given in Table 2 was possible, the droplet and jet modes only occurred under certain conditions[4]; these conditions are approximately satisfied by the following relation:

\[
Re \leq 1.414Ga^{0.233} .
\]

Before an experiment, tube inclination and alignment were carefully checked, and the feeding tubes were positioned so that the falling liquid impinged at top-dead-center on all tubes in the test section. The test liquid was circulated for several hours to ensure the tubes were fully wetted and the piping system was free of air. A total of 234 observations of departure-site spacing were recorded in this study.
RESULTS AND DISCUSSION

Spacing of Droplet and Jet Departure Sites. In Figure 5, the dimensionless departure-site spacing, $\lambda^*$, is plotted against Re for the four test liquids. Predictions for jet spacing based on Eqn. (5) are shown in the figure over a range of Re for each fluid. Because Eqn. (5) is restricted to the jet mode, it has not been extended into the droplet regime of the plots. However, for purposes of comparison, we extrapolated beyond the $Ga^{1/4}$ range of Eqn. (5) and plotted those results using a dashed line. Therefore, disagreement between the experiments and Eqn. (5) for the low-$Ga^{1/4}$ fluids is not too surprising—Armbruster and Mitrovic did not consider such fluids. The results are in general agreement with Armbruster and Mitrovic[16] and with the results of Maron-Moalem et al.[9]. The data clearly show that $\lambda^*$ depends on Re, especially at low Re and high $Ga^{1/4}$. A decrease in the Re is accompanied by an increase in $\lambda^*$. At low-$Ga^{1/4}$, $\lambda^*$ appears to be insensitive to Re over a wide range. Although the change in $\lambda^*$ with Re is limited to about 30% in the current study, Maron-Moalem et al. reported $\lambda$ to change by more than 100%. Ganic and Roppo[10] reported $\lambda^*$ to be independent of the liquid flow rate. It is somewhat surprising that the spacing between droplets is more dependent on flow rate than is the spacing between jets. As Re $\rightarrow$ 0, the droplet spacing appears to approach the most dangerous Taylor wavelength as given by Equation (1); i.e., $\lambda^*_d \rightarrow 2\pi \sqrt{3} \approx 10.9$.†

The effects of the tube diameter and spacing on $\lambda^*$ are shown in Figures 6 & 7. The data indicate that tube diameter effects are more pronounced at small $d^*$, and $\lambda^*$ increases with $d^*$. This behavior is congruent with the predictions by Li and Harris[15], Lienhard and Wong[8], and Armbruster and Mitrovic[16]. The tube spacing effects shown in Figure 7 indicate that tube spacing does not have a profound influence on $\lambda^*$. However, the data suggest that $\lambda^*$ first decreases with $s^*$, reaches a minimum near $s^* = 5$, then increases slightly. Interesting support to this suggested behavior is provided when the jet-shape data are considered—this relationship will be discussed when the jet shapes are presented.

† It should be noted that this result could also be obtained from Equation (3) with n=3 instead of using the recommended n=2.
Limited experiments were conducted with a concurrent air flow, and it was found that with an air flow \( \lambda^* \) decreases. While this behavior was repeatable, it was difficult to quantify with the current apparatus because of unsteadiness in the air flow associated with the tube wakes. The change in \( \lambda^* \) was small—roughly a 10% decrease in \( \lambda^* \) was observed at the highest air velocity (\( We=200 \)). Due to difficulties in obtaining reliable data, no effort beyond these preliminary observations was pursued.

Because these data span a larger parameter space than prior reports, a regression analysis was conducted to develop a correlation with wider applicability. The results shown in Figure 5 suggest that the \( Re \) dependence is only important at low flow rates and that the regime of \( Re \) dependence is more extensive at high \( Ga^{1/4} \). Therefore, we elected to begin by correlating our results in two regimes. For the 60 observation with \( Re < 50 \), the following relation correlates the data with an RMS error of \( \pm6.3\% \):

\[
\lambda^* = 0.836\Lambda - 0.863\frac{Re}{Ga^{1/4}} ,
\]

where

\[
\Lambda = 2\pi\sqrt{3}/\sqrt{1 + 2/d^2} .
\]

For the 155 observation with \( Re > 100 \), the following correlation represents the data with an RMS error less than \( \pm7.9\% \):

\[
\lambda^* = 0.75\Lambda - 85/Ga^{1/4} .
\]

Finally, motivated by the method of Churchill and Usagi[22] for correlating data bound by asymptotes, the following correlation was developed. Equation (9) correlates all 234 observations with an RMS error less than \( \pm8.3\% \) over the parameter range given in Table 2 :

\[
\lambda^* = \left\{ \frac{0.836\Lambda - 0.863\frac{Re}{Ga^{1/4}}}{1 + \left( \frac{0.836\Lambda - 0.863\frac{Re}{Ga^{1/4}}}{0.75\Lambda - 85/Ga^{1/4}} \right)^2} \right\}^{1/12} .
\]
The success of Eqn. (9) supports the application of dynamic similarity over a wide range of flow conditions. In this respect, it is interesting to note the relation between the departure-site spacing and the physics reflected through the dimensionless parameters. The Reynolds number can be interpreted in the conventional sense as an inertia-to-viscous force ratio. The modified Galileo number can be interpreted as gravity-to-viscous or surface-tension-to-viscous forces, depending on the length scale (see [4]). The departure-site spacing increases as \( Re \) decreases and appears to be independent of \( Re \) at high flow rates. Furthermore, for small \( Ga \), changes in \( \lambda^* \) appear to be confined to a small range of \( Re \). Therefore, these results suggest that viscous effects generally act to increase \( \lambda^* \); inertia, gravity and surface tension act to decrease \( \lambda^* \). When these effects compete, \( \lambda^* \) changes. These trends are in general agreement with the findings of Taghavi and Dhir[12].

It is commonly assumed in modeling free-falling jet shapes, even when surface tension is included, that only curvature around the jet axis is important. This assumption is probably invalid when the jet falls onto a solid surface, especially when the tube spacing is small. As shown in Figures 8 through 11, for small \( s^* \), the jets appear to take the shape of a pendant drop. With an increase in \( s^* \), the jets become elongated, taking the shape of an inverted bell. During the experiments, the bell-shaped jets corresponded to the minimum in \( \lambda^* \) with \( s^* \) discussed in connection with Figure 7. For some fluids, a further increase in \( s^* \) yields jets shaped like the upper frustum of a cone, with a spheroid attached at the apex (see Figure 8d and 10b). For these shapes, curvature around the jet axis is of the same order as off-axis curvature. For a still larger tube spacing, the included angle of the cone appears to decrease, and the spherical base becomes more ring-like. For some conditions, multiple rings or toroids were formed at the jet base. For a larger tube spacing, the ring or rings at the jet base became difficult to discern, and the jet was smooth and linear over its length with small wrinkles at its base. For these shapes, curvature around the jet axis is much more important than off-axis curvature. For water and water-glycol mixtures—high \( Ga^{1/4} \) fluids—these shapes were easy to discern. However, for ethylene glycol the liquid rings and wrinkles at the base of the jet were not observed. The free surfaces of the ethylene glycol jets were generally
smooth and wrinkle free. For hydraulic oil—with the lowest $Ga^{1/4}$—the jets simply took a converging shape, with no obvious presence of the above described shapes (see Figure 11).

For pendant and bell-shaped jets described above, a liquid bridge formed at the bottom of the jet. The size of this bridge, which connected the film on the bottom tube to the impinging jet, did not vary much with flow rate or tube spacing. This bridge is evident in Figures 8a, 9a, and 10a (as well as in other Figures).

The shapes described above are not predicted by the analysis of Mitrovic and Ricoeur[17] in which conditions at the base of the jet are not considered. The work of Bejan[18,19] suggests that asymmetric buckling of the liquid column should occur; however, such behavior was not observed in these experiments (care was taken to observe the jets from several vantage points). While the work of Boucher[20], predicted the general jet shape observed for a small tube spacing, the liquid bridge at the jet base is not predicted, and the behavior at large tube spacing does not resemble Boucher’s predictions. None of the existing models appear to predict the shapes observed through the range of these experiments; a modification of Boucher's approach to account for inertial effects (as presented by Hu[20]) may be useful. In predicting the shape of liquid jets, the effects of $Re$, $Ga$, and $s^*$ are apparently important; outside the range of the current experiments $r^*$ may also be important.

**PRACTICAL SIGNIFICANCE**

In designing falling-film heat exchangers, the wetting characteristics of the flow are very important to the heat transfer performance. Furthermore, local dry-out can be a significant problem in operation, and information on departure-site spacing may be of value to designers in avoiding this problem. A large spacing between droplets or jets might lead to local film dry-out, and the correlations provided by Eqs. (7)-(9) are the most comprehensive to date for assessing droplet and jet spacing. There is clear evidence that heat and mass transfer occurring in the intertube space can be important to the overall performance of falling-film heat exchangers. Jet shape directly affects the interfacial area, and even a qualitative understanding of its behavior may have future practical significance.
CONCLUSIONS

In this paper, we have reported experimental measurements of the droplet- and jet-departure-site spacing using four working fluids and a range of tube diameters, tube spacing, and liquid flow rates. Departure-site spacing depends on flow rate and decreases for an increase in $Re$; this dependence is higher for fluids with a large $Ga^{1/4}$. In the parameter space of these data, a weak dependence on tube diameter was observed, with $\lambda^*$ increasing with tube diameter. The dependence on tube spacing was observed to be weakest; however, the data suggest that, at a particular tube spacing, the jets take a characteristic shape and move slightly together. Departure-site data have been successfully correlated to $Re, Ga^{1/4}, d^*$, and a new more widely applicable correlation has been provided. A discussion of the physical implications of these results was presented. Liquid jet shapes were presented for a range of conditions, and comparisons to existing models suggest that further work in this area is needed. Because falling-film heat exchangers use a range of fluids with concurrent and counter-current vapor flows, future work in this area should include vapor-shear effects with a wide range of fluid properties.
NOMENCLATURE

$d$ diameter, m

$d^*$ normalized tube diameter, $d/\xi$, dimensionless

$Ga$ modified Galileo number (or Kapitza number), $\rho \sigma^3/\mu^4 g$, dimensionless

$g$ gravitational acceleration, m·s⁻²

$n$ constant appearing in Eq. (3), dimensionless

$Re$ Reynolds number, $2\Gamma/\mu$, dimensionless

$r$ radius, m

$r^*$ normalized radius, $r/\xi$, dimensionless

$s$ tube spacing, m

$s^*$ normalized tube spacing, $s/\xi$, dimensionless

$V$ velocity, m·s⁻¹

$We$ Weber number, $\rho_v V^2d/\sigma$, dimensionless

Greek Symbols

$\Gamma$ total liquid mass flow rate per unit length of tube, kg·m⁻¹·s⁻¹

$\Lambda$ length scale defined in Eq. (7), dimensionless

$\lambda$ instability wavelength, spacing between neighboring jets or droplets, m

$\lambda^*$ normalized departure-site spacing, $\lambda/\xi$, dimensionless

$\mu$ dynamic viscosity, kg·m⁻¹·s⁻¹

$\rho$ mass density, kg·m⁻³

$\sigma$ surface tension at gas/liquid interface, kg·s⁻²

$\xi$ capillary constant, $\sqrt{\sigma/\rho g}$, m

Subscripts and Superscripts†

$c$ critical, i.e., above which are unstable

$d$ most dangerous, i.e., most rapidly growing

$j$ of the liquid jet

$v$ of the surrounding vapor or air

† Unsubscripted properties are taken as those of the liquid phase; unsubscripted lengths are taken as those of the tube.
REFERENCES


Table 1 - The experimental range of the relevant physical variables

<table>
<thead>
<tr>
<th>Physical Parameter, Symbol</th>
<th>Experimental Range</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow rate per unit length, $\Gamma$</td>
<td>up to 0.22</td>
<td>kg/m-s</td>
</tr>
<tr>
<td>Liquid dynamic viscosity, $\mu$</td>
<td>$7.9 \times 10^{-4}$ to $4.4 \times 10^{-2}$</td>
<td>N-s/m²</td>
</tr>
<tr>
<td>Mass density of the liquid, $\rho$</td>
<td>780 to 1140</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Gas/liquid surface tension, $\sigma$</td>
<td>$2.2 \times 10^{-2}$ to $6.8 \times 10^{-2}$</td>
<td>N/m</td>
</tr>
<tr>
<td>Tube diameter, $d$</td>
<td>9.5, 12.7, 15.9, 19.0, 22.2 mm</td>
<td></td>
</tr>
<tr>
<td>Tube spacing, $s$</td>
<td>about 5 to 50 mm</td>
<td></td>
</tr>
<tr>
<td>Gravitational acceleration, $g$</td>
<td>fixed at 9.8</td>
<td>m/s²</td>
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Table 2 - Relevant dimensionless parameters, the experimental range, and typical uncertainties associated with propagated measurement uncertainty

<table>
<thead>
<tr>
<th>Dimensionless Number</th>
<th>Physical Interpretation</th>
<th>Experimental Range</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re = 2\Gamma / \mu$</td>
<td>(inertial force/viscous force) — based on film thickness scale</td>
<td>up to ~400</td>
<td>±2%</td>
</tr>
<tr>
<td>$Ga = \rho \sigma^3 / g \mu^4$</td>
<td>(gravitational force/viscous force)⁴ — based on capillary length scale</td>
<td>$7(10^2)$ to $8(10^{10})$</td>
<td>±4%</td>
</tr>
<tr>
<td>$d^* = d / \xi$</td>
<td>reveals tube diameter effects</td>
<td>3 to 10</td>
<td>±4%</td>
</tr>
<tr>
<td>$s^* = s / \xi$</td>
<td>reveals tube spacing effects</td>
<td>2 to 22</td>
<td>±4%</td>
</tr>
<tr>
<td>$\lambda^* = \lambda / \xi$</td>
<td>dimensionless dependent variable; departure-site spacing</td>
<td>--</td>
<td>±4%</td>
</tr>
</tbody>
</table>
Figure 1 — The idealized intertube falling-film modes\cite{2}: (a) the droplet mode, with liquid leaving the tube intermittently; (b) the jet mode, where discrete jets continuously flow from the upper to the lower tubes; and (c) the sheet mode, in which the liquid film forms an unbroken sheet between the tubes.
Figure 2 — The observed falling-film modes reported by Hu and Jacobi\textsuperscript{[4]}. For the fluid shown, \(Ga^{1/4} = 36\), and the modes are classified as (a) droplet, \(Re = 1.3\); (b) droplet-jet, \(Re = 8\); (c) inline jet, \(Re = 21\); (d) staggered jet, \(Re = 41\); (e) jet-sheet, \(Re = 57\); and (f) sheet mode, \(Re = 64\).
Figure 3 — A schematic of the experimental apparatus used to study the intertube falling film modes. An air flow is provided by the open-loop wind tunnel, and the test liquid is circulated in the thermally controlled closed-loop system (from Ref. [4]).
Figure 4 — A schematic of the tubes within the test section. Liquid is issues from the upper feeding tube, flows around a lower feeding tube that helps ensure flow uniformity, and then falls through the test section to the catching tube (from Ref. [4]).
Figure 5 — The effect of liquid Reynolds number, $Re$, on the dimensionless droplet and jet departure-site spacing, $\lambda^*$ for (a) water with $Ga^{1/4} = 530$, $d^* = 6.01$, and $s^* = 5.68$; (b) ethylene glycol, $Ga^{1/4} = 35.7$, $d^* = 7.67$, and $s^* = 7.25$; (c) water-glycol, $Ga^{1/4} = 165$, $d^* = 6.96$, and $s^* = 6.58$; and (d) oil, $Ga^{1/4} = 5.2$, $d^* = 8.27$, and $s^* = 7.81$. 
Figure 6 — The effect of tube diameter, $d^*$, on droplet and jet departure-site spacing, $\lambda^*$. Example data are given as a function of Re for (a) ethylene glycol, $Ga^{1/4}=35.7$ at $s^*=7.25$ for $d^*=4.60$ and 7.67; and for (b) water-glycol, $Ga^{1/4}=165$ at $s^*=6.56$ for $d^*=6.96$ and 9.74.
Figure 7 — The effect of tube spacing, $s^*$, on droplet and jet departure-site spacing, $\lambda^*$. Example data are given for two fluids: water and the water-glycol mixture.
Figure 8 — Liquid jet shapes for water: $Ga^{1/4} \approx 530$, $Re \approx 240$, $d^* \approx 6$ and $s^* = 2.27, 2.65, 3.45, 3.86, 4.81, 7.54$, and 12.0, for (a) through (g), respectively.
Figure 9 — Liquid jet shapes for ethylene glycol, \( \text{Ga}^{1/4} \approx 35.7 \), \( \text{Re} \approx 11 \), \( d^* \approx 7.67 \) and \( s^* = 2.95, 4.11, 4.40, 5.65, \) and 8.74 for (a) through (e), respectively.
Figure 10 — Liquid jet shapes for the water-glycol mixture, $Ga^{1/4} \sim 165$, $Re \sim 108$, $d^* \sim 6.96$, and $s^* = 3.29$, 4.17, 4.78, 5.35, and 6.36 for (a) through (e), respectively.
Figure 11 — Liquid jet shapes for oil, $G_a^{1/4} \sim 5.2$, $Re \sim 3$, $d^* \sim 8.27$ and $s^* = 3.91, 4.74, \text{and } 6.19$ for (a) through (c), respectively.