Modeling and Analysis of Gas Coolers

X. Fang

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For additional information:

Air Conditioning and Refrigeration Center
University of Illinois
Mechanical & Industrial Engineering Dept.
1206 West Green Street
Urbana, IL  61801

(217) 333-3115
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217 333 3115
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MODELING AND ANALYSIS OF GAS COOLERS

Xiande Fang

ABSTRACT

The transcritical cycle of carbon dioxide in air conditioning and heat pump systems needs the research on heat transfer and pressure loss of heat exchangers operating at supercritical pressures. This paper offers a comprehensive survey of single-phase in-tube heat transfer correlations, CO2 supercritical heat transfer, friction factor correlations, the calculation of pressure loss in the supercritical conditions, and the heat transfer and friction factor correlations on the gas cooler air-side. The equations for calculating heat transfer and pressure loss at supercritical pressures both in the fully developed turbulent regime and in the transitional regime are obtained. Based on the mathematical model, a computer simulation program in EES for gas coolers is developed. The verification with experimental data is made thereafter. The prediction agrees with the experimental data very well. Some analyses on gas cooler thermal performance are carried out with the program.

1 INTRODUCTION

Carbon dioxide is considered as a potential alternative refrigerant for car air conditioning and heat pump systems. The capacity and COP of CO2 systems depend on the pressure in the high side, because they operate in a transcritical cycle (Figure 1) under most conditions.

The process path of a CO2 transcritical cycle, as shown in Figure 1, consists of compression (1'-2), supercritical heat rejection (2-3), adiabatic expansion (3'-4), two-phase heat absorption (4-1), and (1-1') and (3-3') if a suction line heat exchanger is used. The heat exchanger in which supercritical heat rejection occurs is called a gas cooler instead of a condenser.

Many of the recent investigations of CO2 as an alternative refrigerant have been performed in the cycle performance (Lorentzen and Pettersen, 1993, Rieberer and Halozan, 1997, and McEnaney et al, 1998) and thermophysical properties (Span and Wagner, 1996, and Vesovic et al, 1990). However, very few studies have been carried out
to quantify local heat transfer coefficients and pressure loss during the heat rejection.

Although no phase change takes place at supercritical pressures, the thermophysical properties of CO$_2$ change drastically during the process (Figure 2). In these circumstances, the heat transfer coefficient and pressure loss are greatly dependent on both the local mean temperature and the heat flux, where the conventional models could not apply (Pitla et al, 1998). Only four papers (Krasnoshchekov et al, 1969, Baskov et al, 1977, Petrov and Popov, 1985, and and Petrov and Popov, 1988) address specifically the heat transfer and pressure loss of CO$_2$ cooled at supercritical pressures. All of them are limited to fully developed turbulent regime.

Several papers related to gas cooler simulation have been published (Schonfeld and Kraus, 1997, Rieberer and Halozan, 1997, and Robinson and Groll, 1998). The heat transfer correlations the authors used are conventional, but pressure loss was not considered. Using Gnielinsky equation (1976), Schonfeld and Kraus (1997) and Rieberer and Halozan (1997) developed their computer programs for gas coolers cooled with water. The heat transfer correlation Robinson and Groll (1988) used is Petuhov-Kirillov (1958) equation. Only Schonfeld and Kraus compared their model predictions with experimental data. They concluded that the heat transfer from supercritical fluids could not be calculated exactly with classical methods of convective heat transfer because their predictions were remarkably higher than the experimental data.

It is not realistic to scrutinize thermal performances of gas coolers experimentally because they are dependent on a number of factors such as gas cooler geometry and materials, air inlet parameters, as well as refrigerant inlet parameters. The modeling of gas coolers is a powerful means to analyze the thermal performances throughout. Also, the mathematical model of gas coolers is the cornerstone of gas cooler design and the system modeling of the CO$_2$ transcritical cycle. This paper includes the following major topics:

- Pressure loss calculation. Based on hydraulic drag factors at constant thermophysical properties and of CO$_2$ specific model in the fully developed turbulent regime, propose an equation that spans both the transitional and the fully developed turbulent regime.
Figure 1 Process Path and Pressure - Enthalpy diagram of the Transcritical Cycle

Figure 2a The Variation of Specific Heat Capacity with Temperature

Figure 2b Variation of Prandtl Number with Temperature

Figure 2c Variation of Density with Temperature

Figure 2d Variation of Heat Conductivity with Temperature
• In-tube heat transfer of gas coolers. Based on the heat transfer model of constant thermophysical properties and a CO₂-specific model in the fully developed turbulent regime, propose equations which are applicable to both the fully developed turbulent regime and the transitional regime.

• Review and compare the air-side heat transfer and friction factor models.

• Develop a simulation program based on the mathematical models. Verify the program with experimental data to check the applicability of the models proposed.

• Carry out the thermal performance analyses with the program.

2 PRESSURE LOSS IN TUBES

2.1 Pressure Loss Equation

The total pressure loss in a section can be calculated by

$$\Delta p = \frac{G^2}{2\rho} \left( f_h \frac{L}{D} + \xi \right)$$  \hspace{1cm} (1)

where hydraulic drag factor $f_h$ is (Petrov and Popov, 1985)

$$f_h = f + f_i$$  \hspace{1cm} (2)

where the inertia factor $f_i$, as in one-dimensional approximation, is expressed as

$$f_i = \frac{8q_w}{G c_p} \left[ - \frac{1}{\rho} \left( \frac{\partial \rho}{\partial t} \right)_p \right]_m$$  \hspace{1cm} (3)

For incompressible fluid flow, $f_i = 0$, Equation (1) is reduced to the commonly used Darcy-Weisbach equation

$$\Delta p = \frac{G^2}{2\rho} \left( f \frac{L}{D} + \xi \right)$$  \hspace{1cm} (4)

Generally, the diameter of ports of louvered fin gas cooler tubes is small. In this circumstance, the some deformation caused by cutting may exist at the port entrance and exit so that the local pressure loss of tubes is commensurable with or even much larger than their distributed pressure loss. Ide’lchik (1966) introduced the calculation methods for various types of the local friction coefficients.

2.2 Darcy-Weisbach Friction Factor

Many equations for the Darcy-Weisbach friction factor have been developed. The
Blasius’ equation (5) and Filonenko’s equation (6) are widely used for the turbulent flow in smooth tubes (Zukauskas and Kami, 1989).

\[ f = \frac{0.316}{Re^{1/4}} \quad (Re \leq 10^5) \]  
\[ f' = (1.82 \log Re - 1.64)^{-2} \quad (1 \times 10^4 \leq Re \leq 5 \times 10^6) \]  

The “smooth” here means that the wall roughness elements are so small that their influence does not extend beyond the laminar sublayer.

There are other opinions about the applicable Reynolds number range of Blasius’ equation (5) and Filonenko’s equation (6). For example, Incropera and DeWitt (1996) introduced \( Re \approx 2 \times 10^4 \) for Blasius’ equation (5) and \( 3000 \leq Re \leq 5 \times 10^6 \) for Filonenko’s equation (6).

Moody and Princeton (1944) introduced Colebrook’s equation. Colebrook, in collaboration with C. M. White, developed an equation which agrees with two extremes of roughness in transition zone.

\[ \frac{1}{f^{0.5}} = -2 \log \left( \frac{R_{st}}{3.7} + \frac{2.51}{f^{0.5} Re} \right) \]  

Since Colebrook’s equation cannot be solved explicitly for \( f \), Althul developed an explicit formula which was modified by Tsal (ASHRAE Handbook of Fundamentals, 1993)

\[ f' = 0.11 \left( \frac{R_{st}}{Re} + \frac{68}{R_{st}} \right)^{0.25} \]

\[ f = \begin{cases} 
  f' & \text{if } f' \geq 0.018 \\
  f = 0.0028 + 0.85f' & \text{if } f' < 0.018 
\end{cases} \]  

Friction factors obtained from Althul’s modified equation are within 1.6% of those obtained by Colebrook’s equation.

Churchill (1977) proposed a more complicated equation for all flow regimes and all relative roughness, which agrees with the Moody diagram (Moody and Princeton, 1944)

\[ f = 8 \left[ \left( \frac{8}{Re} \right)^{12} + \left( \frac{2.457 \ln \left( \frac{1}{(7/Re)^{0.9} + 0.27R_{st}} \right)}{16} + \left( \frac{37,530}{Re} \right)^{16} \right)^{-3/2} \right]^{1/12} \]
The comparison of Blasius' equation (5), Filonenko's equation (6), Althul's modified equation (8), and Churchill's equation (9) is shown in Figure 3. It is seen that Blasius' equation (5) can be valid for Re<1.5×10^5, and Filonenko's equation (6) can be used within Re>8×10^3.

Althul's modified equation (8) has apparently lower prediction than Churchill's equation (9) for large relative roughness conditions. When relative roughness R_r < 0.001, the predictions by Althul's modified equation (8) and Churchill's equation (9) differ only slightly. It is better to limit the application of Equation (8) to R_r ≤ 0.001.

2.3 Friction Factor of Supercritical Cooling

Thermophysical property variations in the cooling conditions at supercritical pressures significantly affect the pressure loss characteristics. The absolute value of the inertia drag which is negative in cooling conditions is commensurable with the friction drag. At some circumstances, this can decrease the total hydraulic drag to negative values, thereby resulting in the appearance of zones with pressure increasing along tubes.

Petrov and Popov (1985) calculated the friction factor of CO_2 cooled in the supercritical conditions in the range of Re_w = 1.4×10^4 - 7.9×10^5 and Re_m = 3.1×10^4 - 8×10^5. They obtained an interpolation equation of the friction factor

\[ f = f_{0w} \frac{\rho_w}{\rho_m} \left( \frac{\mu_w}{\mu_m} \right)^s \]  

(10a)

where \( f_{0w} \), the friction factor at constant thermophysical properties, is calculated by Equation (6) at tube wall temperature T_w, and

\[ s = 0.023 \left( \frac{\dot{Q}_w}{G} \right)^{0.42} \]  

(10b)

Later in 1988, they calculated the friction factor for cooling of supercritical water in the range of Re_w = 2×10^4 - 1.88×10^5 and Re_m = 2.3×10^4 - 2.03×10^5, and derived a friction factor equation as follows:

\[ \frac{f}{f_{0m}} = \left( \frac{\mu_w}{\mu_m} \right)^{1/4} + 0.17 \left( \frac{\rho_w}{\rho_m} \right)^{1/3} \left| \frac{f_{10}}{f_{0m}} \right| \]  

(11)
where \( f_{om} \) is calculated by Equation (6) at mean fluid temperature \( T_m \), and the inertia factor \( f_i \) is given by Equation (3).

They claimed that Equation (11) described their calculated data for water, helium, and carbon dioxide at supercritical pressures with the deviation of no more than ±8% in the boundary conditions of \( T_w = \text{constant} \) and \( q_w = \text{constant} \).

No \( \text{CO}_2 \)-specific experimental correlations are found for the hydraulic drag factor in the cooling conditions at supercritical pressures.

Figure 4 compares hydraulic drag factor calculations, where "\( \text{CO}_2 \)-specific" means the hydraulic drag factor calculated with Equations 2, 3, and 10, "water-specific" denotes that calculated with Equations 2, 3, and 11, and "constant thermophysical property" stands for that calculated with Churchill's equation (9). The predictions of the \( \text{CO}_2 \)-specific equation is over 10% more than those of the water-specific equation and Churchill's equation (9) with maximum 27.5%. The conditions of Figure 4 are: \( D = 0.79 \) mm, \( p_{rin} = 100 \) bar, \( T_{rin} = 120 \) °C, \( T_{ain} = 35 \) °C, air face velocity = 2.5 m/s, \(-130 < q_w/G < -28 \) J/kg, \( 6.7 \times 10^3 < Re_w < 1.81 \times 10^4 \), and \( 7.8 \times 10^3 < Re_m < 1.85 \times 10^4 \).

Figure 5 illustrates the predictions of inertia friction equation and the temperature distribution along the tube. The conditions of Figure 5 is the same as those of Figure 4. In these conditions, the absolute value of the inertia friction is over 10% of that of friction factor when \( L < 0.4 \) m.

Note that \( f_{ow} \) and \( f_{om} \) in Petrov-Popov's equations (10) and (11) are calculated by Filonenko's equation (6) at \( T_w \) and \( T_m \) respectively, and that Equation (6) is only used for the fully developed turbulent flow in smooth tubes. In order to extend the use of Equations (10) and (11) to the transitional regime and rough tubes, we suggest that \( f_{ow} \) and \( f_{om} \) are calculated by Churchill's equation (9) instead of Filonenko's equation (6).

3 HEAT TRANSFER MODELS
3.1 The Efficiency and Overall Heat Transfer Coefficient

The gas cooler studied in this paper is an air-cooled louvered fin heat exchanger, which has the flat tubes with the cross section of several independent ports (Figure 6). The hot and cold fluids move in cross flow, and both are unmixed. The heat transfer in the gas cooler is assumed to be quasisteady.
Figure 3 The Comparison of Friction Factor Equations

Figure 4 Comparison of Hydraulic Drag Factor Calculations
Figure 5 Inertia Factor and Temperature Distribution

Figure 6 Louvered Fin Gas Cooler
Suppose that the heat conduction along tubes can be neglected, and that in the given tube cross section, the refrigerant in all ports has the same thermal state, which can be described with mean parameters. The effectiveness of gas coolers can be calculated by

\[ \varepsilon = 1 - \exp\left\{ \frac{1}{C_r} NTU^{0.22} \left[ \exp\left( -C_r \cdot NTU^{0.78} \right) - 1 \right] \right\} \]  

(12)

where the heat capacity ratio \( C_r \) is

\[ C_r = \frac{C_{\text{min}}}{C_{\text{max}}} \]  

(13)

and the number of transfer units \( NTU \) is

\[ NTU = \frac{UA}{C_{\text{min}}} \]  

(14)

**3.2 Heat Transfer in Tubes**

The heat transfer in gas cooler tubes occurs at supercritical pressure where the thermophysical properties of the fluid change drastically. The great variation in the thermophysical properties causes the heat transfer coefficient to be greatly dependent on both the local mean temperature and the heat flux.
Many studies of forced convection heat transfer in turbulent regime have been done. Dittus–Boelter's equation and Sieder–Tate's equation (Incropera and DeWitt, 1996) are widely quoted by heat transfer textbooks.

Hausen (1959) proposed the following equation:

$$Nu = 0.037(Re^{3/4} - 180)Pr^{0.42}[1 + (\frac{D}{L})^{2/3}](\frac{\mu}{\mu_w})^{0.14}$$  \hspace{1cm} (19)

where fluid dynamic viscosity $\mu_w$ is evaluated at $T_w$, and all other properties are evaluated at $T_m$. The applicable range was suggested to be in $0.6 < Pr < 10^3$ and $2300 < Re < 10^6$. However, some researchers (Gnielinski, 1976) indicated it should be used only in transitional region.

Based on theoretical analyses, Petukhov et al (Table 1) proposed following equation for determining the local heat transfer coefficient of fully developed turbulent flow in long tubes:

$$Nu = \frac{(f/8)RePr}{A_1 + A_2(f/8)^{1/2}(Pr^{2/3} - 1)}$$  \hspace{1cm} (20)

where the friction factor $f$ is calculated by Blasius' equation (5) or Filonenko's equation (6) according to the Reynolds number, all properties are evaluated at $T_m$, and $A_1$ and $A_2$ are shown in Table 1.

Table 1 Coefficients $A_1$ and $A_2$ in Petukhov's Equation (20)

<table>
<thead>
<tr>
<th>Authors/Year</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>Suggested applicable range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petukhov-Kirillov/1958</td>
<td>1.07</td>
<td>12.7</td>
<td>$10^4 - 5 \times 10^6$ $0.5 - 200$</td>
</tr>
<tr>
<td>Petukhov-Popov/1963</td>
<td>$1 + 3.4f$</td>
<td>$11.7 + \frac{1.8}{Pr^{1/3}}$</td>
<td>$10^4 - 5 \times 10^6$ $0.5 - 200$</td>
</tr>
<tr>
<td>Petukhov-Kurganov- Gladuntsov/1973</td>
<td>$1.07 + \frac{900}{Re} - \frac{0.63}{1 + 10Pr}$</td>
<td>12.7</td>
<td>turbulent flow* $0.7 - 5 \times 10^5$</td>
</tr>
</tbody>
</table>

*The applicable range of the Petukhov-Kurganov-Gladuntsov equation is not clear in their paper. They said it was “for fully developed turbulent flow” at first, and mentioned it “has been checked out experimentally over the ranges of $0.7 \leq Pr(Sc) \leq 5 \times 10^5$ and $4 \times 10^3 \leq Re \leq 6 \times 10^5$” later.

Gnielinski (1976) studied Hausen's and Petukhov-Kurganov-Gladuntsov's equations to obtain an equation valid for both the transitional and the fully developed turbulent regimes. He proposed a modified equation with all properties evaluated at $T_m$: 

- 11 -
\[
 Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} 
\]

(21)

The author compared the equation predictions with approximately 800 experimental data in the range of \(2300 < Re < 10^6\) and \(0.6 < Pr < 10^5\), and concluded that the equation described nearly 90 percent of the experimental data to be within ±20%.

The term \((\mu/\mu_w)^{0.14}\) in Hausen's equation (19) is used for considering the influence of large property variations on the heat transfer. Studying the heat transfer of high heat flux densities, Hufschmidt and Burck (1968) proposed a factor \((Pr/Pr_w)^{0.11}\) to modify Petukhov-Kirillov's equation (20).

Gnielinski (1976) adopted \((Pr/Pr_w)^{0.11}\) to modify his equation (21) for liquids, and \((T_m/T_w)^{0.45}\) for gases. For gases, he compared the experimental data of Nusselt number with the results calculated by his equation. They were agreement with each other well when Nusselt number was greater than 400. However, the average deviation of the calculated results is about +20% of the experimental data when Nusselt number was less than 300.

Petukhov, Kurganov and Gladuntsov (1973) suggested the following equation to modify their equation (20):

\[
 Nu_m = Nu_{om} \left( \frac{k_w}{k_m} \right)^{1/3} \left( \frac{c_p_w}{c_p_m} \right)^{1/4} \left( \frac{T_w}{T_m} \right)^{-(0.53+1.5\log\left(\frac{\rho w}{\rho m}\right)/\mu_m)} 
\]

(22)

where \(Nu_{om}\) is calculated with Petukhov-Kurganov-Gladuntsov's equation (20) at \(T_m\). They claimed most of the experimental data were within ±10% of the model prediction.

The modifications to property variations made above are not applicable to the heat transfer at supercritical pressures. The specific characteristics of heat transfer at supercritical pressures have attracted many researchers (Polyakov, 1991). However, most of published papers are related to the heating conditions because it is much more difficult to obtain experimental data on local heat transfer in the cooling conditions.

Krasnoscheko et al (1969) conducted an experiment at supercritical pressures with \(CO_2\) cooled in a long horizontal tube of an inner diameter = 2.22 mm, and derived the following equation from the experimental data:
where $\text{Nu}_{\text{ow}}$ is calculated with Petukhov-Kirillov's equation (20) at $T_w$, and with $m$ given by

$$m = B \left( \frac{c_p}{c_{p,w}} \right)^k$$

and $c_p$ is defined as

$$c_p = \frac{i_m - i_w}{T_m - T_w}$$

$n$, $B$, and $k$ in Equation (23) are given in Table 2.

<table>
<thead>
<tr>
<th>p, bar</th>
<th>80</th>
<th>100</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>0.38</td>
<td>0.68</td>
<td>0.80</td>
</tr>
<tr>
<td>$B$</td>
<td>0.75</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>$k$</td>
<td>0.18</td>
<td>0.04</td>
<td>0</td>
</tr>
</tbody>
</table>

The experiment range of Krasnoshcheko-Kuraeva-Protopopov's equation is $9 \times 10^4 \leq \text{Re}_m \leq 3.2 \times 10^5$ and $6.3 \times 10^4 \leq \text{Re}_w \leq 2.9 \times 10^5$.

The authors compared their calculation with the experimental data of Tanaka et al (1971) (CO$_2$ ascending flow cooled in a vertical tube of an inner diameter = 6 mm), and found large deviations. They thought this was partly due to extrapolation of the value $n$, $B$, and $k$.

Baskov et al (1977) conducted an experiment at supercritical pressures with CO$_2$ ascending flow cooled in a long vertical tube of an inner diameter = 4.12 mm, and found their experimental data were lower than those calculated with Equation (23). They obtained the following equation from their experimental data:

$$\text{Nu}_{\text{w}} = \text{Nu}_{\text{ow}} \left( \frac{c_p}{c_{p,w}} \right)^m \left( \frac{\rho_m}{\rho_w} \right)^n$$

where $\text{Nu}_{\text{ow}}$ is calculated with Petukhov-Kurganov-Gladuntsov's equation (20) at $T_w$, $c_p$. 

- 13 -
is the same definition as Equation (24). When \( T_m/T_{pc} \leq 1 \), \( m = 1.4 \) and \( n = 0.15 \). When 
\( T_m/T_{pc} > 1 \), \( m \) and \( n \) are listed in Table 3.

Table 3 \( m \) and \( n \) in Baskov-Kuraeva-Protopopov’s Equation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( c_p/c_{pw} &gt; 1 )</th>
<th>( c_p/c_{pw} &lt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p, \text{ bar} )</td>
<td>80 100 120</td>
<td>80 100 120</td>
</tr>
<tr>
<td>( m )</td>
<td>1.2 1.6 1.6</td>
<td>0.45 0.45 0.45</td>
</tr>
<tr>
<td>( n )</td>
<td>0.15 0.10 0</td>
<td>0.15 0.10 0</td>
</tr>
</tbody>
</table>

Comparing with the experimental data of Krasnoshcheko et al (1969) and Tanaka et al (1971), Baskov et al (1977) found that the values calculated with Equation (25) were 25% lower than the former on average and most within ± 25% of the latter. They speculated that the divergence might be connected with the difference in the orientation of the tubes and the schemes of the experimental units. However, the authors conducted experiments to compare ascending flow with descending flow, and concluded that in their experimental range \((0.95 \times 10^5 \leq \text{Re}_m \leq 6.44 \times 10^5)\), there was no effect of free convection on the heat transfer.

\( T_{pc} \) is the temperature at which the fluid \( c_p \) has maximum value at the given pressure. It varies with pressure. From the data given by Vargaftik (1975) and Span and Wagner (1996), we derive the following equation for \( \text{CO}_2 \) with maximum deviation less than 0.18 \(^\circ \text{C} \) in the range of 75 bar \( \leq p \leq 150 \) bar:

\[
T_{pc} = 253.0936 + 8.168142p - 0.1683366p^2
\]

(26)

where the unit of pressure \( p \) is bar.

Petrov and Popov (1985) proposed the following equation based on their theoretical calculation for \( \text{CO}_2 \) cooled in the supercritical region with \( 3.1 \times 10^4 \leq \text{Re}_m \leq 8 \times 10^5 \), \( 1.4 \times 10^4 \leq \text{Re}_w \leq 7.9 \times 10^5 \), and \( -350 \leq \dot{q}_w/G \leq -29 \) J/kg:

\[
N_{uw} = N_{uw\text{ref}} \left(1 - 0.001 \frac{\dot{q}_w}{G} \right) \left( \frac{c_p}{c_{pw}} \right)^n
\]

(27a)

where

\[
n = \begin{cases} 
0.66 - 4 \times 10^{-4} (\dot{q}_w/G) & \text{when } \frac{c_p}{c_{pw}} \leq 1, \\
0.9 - 4 \times 10^{-4} (\dot{q}_w/G) & \text{when } \frac{c_p}{c_{pw}} > 1;
\end{cases}
\]

(27b)
and $\text{Nu}_{\text{ow}}$ is calculated with Petukhov-Popov's equation (20) at $T_w$. The authors compared their calculation results by Equation (27) with those by Krasnoshcheko-Kuraeva-Protopopov's equation (23) and Baskov-Kuraeva-Protopopov's equation (25), and found that their results were lower than those by Equation (23), and were not larger than $\pm 15\%$ of those by Equation (25) in the region of $-240 \leq q_w/G \leq -50 \text{ J/kg}$.

Summarizing their calculations for carbon dioxide, water, and helium, Petrov and Popov (1988) obtained a generalized equation for the heat transfer of supercritical cooling

$$\text{Nu}_m = \frac{\frac{f}{8} \text{Re}_m \text{Pr}}{1.07 + 12.7 \sqrt{\frac{f}{8} \text{Pr}^{0.23} \left( \frac{\rho_w}{\rho_m} \right) \left( 1 - A_1 \sqrt{\frac{f}{f_i}} \right)^{-2} \left( 1 - A_2 \sqrt{\frac{f}{f_i}} \right)^{2}}}$$

(28)

where the inertia factor $f_i$ is calculated with Equation (3), the friction factor $f$ is calculated with Equation (11), and

$$\overline{\text{Pr}} = \frac{c_p \mu_m}{k_m}$$

(29)

For CO$_2$, $A_1 = 0.9$, $A_2 = 1.0$, for water, $A_1 = 1.1$, $A_2 = 1.0$, and for helium, $A_1 = 0.8$, $A_2 = 0.5$.

The above literature study shows that in-tube heat transfer under the supercritical pressures has its specific characteristics. The literature on cooling conditions is limited and the results differ from each other considerably.

Figure 7 compares Equations (19) - (21). Petukhov's equation (20) is recommended for use in the range of $10^4 \leq \text{Re} \leq 5 \times 10^6$, where the differences in prediction among its three forms are no more than $5.1\%$ when $1 < \text{Pr} < 150$. Hausen's equation (19) has lowest prediction. Gnielinski's equation (21) is close to Hausen's equation (19) in the transitional regime ($2300 \leq \text{Re} \leq 10^4$) and to Petukhov's equation (20) in the range of $10^4 \leq \text{Re} \leq 5 \times 10^6$. In the range of $10^6 \leq \text{Re} \leq 5 \times 10^6$, the differences in prediction between Gnielinski's equation (21) and Petukhov's equation (20) are no more than $5\%$ when $1 < \text{Pr}$
Therefore Gnielinski's equation (21) is recommended for use in the range of 2300 ≤ Re ≤ 5×10^6. The heat transfer of gas coolers sometimes in the transitional regime so that Equation (21) should be chosen to calculate Nu_{ow}.

Figure 8 shows the comparison of Petukhov's equation (20), Krasnoshchekov-Kuraeva-Protopopov's equation (23), Baskov-Kuraeva-Protopopov's equation (25), and Petrov-Popov's equations (27) and (28). Generally, CO₂-specific equations (Equations 23, 25, 27 and 28) have higher heat transfer prediction than Petukhov's equation (20) which is used for constant thermophysical property fluids. This is because for CO₂ in the supercritical region the great variations in the thermophysical properties exist. The more close to the pseudocritical point the temperature, the greater the variations. The conditions of Figure 8 are the same as those of Figures 4 and 5.

Petukhov's equation (20) has been proved to be suitable for constant thermophysical property fluids. The reason why it is not applicable to CO₂ supercritical cooling is that generally there are great thermophysical property variations in the circumstance. The CO₂-specific equations are all based on Petukhov's equation (20) with the modification to thermophysical property variations. Therefore, if in the range to which both the CO₂-specific equations and Petukhov's equation (20) are applicable, the best of the CO₂-specific equations should have the least deviation from Petukhov's equation (20). Those whose prediction is extremely high or low are excluded. Based on these considerations, Petrov-Popov's equation (27) is preferred.

Based on Gnielinski's equation (21), Petrov-Popov's equation (27), we obtain the following in-tube heat transfer model of gas coolers:

\[
Nu_w = \frac{(f_w/8)(Re_w - 1000)Pr_w}{A + 12.7(f_w/8)^{1/2}(Pr_w^{2/3} - 1)} \left(1 - 0.001 \frac{q_w}{G} \frac{c_p}{c_{p,w}} \right)^n
\]

(30a)

where

\[
A = \begin{cases} 
1 + 7 \times 10^{-8} Re_w & \text{Re}_w < 10^{-6} \\
1.07 & \text{Re}_w \geq 10^{-6}
\end{cases}
\]

(30b)
Figure 7 The Comparison of In-Tube Heat Transfer Equations

Figure 8 The Comparison of Heat Transfer Equations
In Equation (30), \( f_w \) is the friction factor evaluated at \( T_w \) by Blasius' equation (5) or Filonenko's equation (6) according to the Reynolds number \( \text{Re}_w \), and \( \bar{c}_p \), \( n \) are calculated by Equations (24) and (27b), respectively.

Figure 9 compares Equation (30) with Petrov-Popov's equation (27) in the range of \( 10^4 < \text{Re}_w < 10^6 \) and \(-350 < \dot{q}_w/G < -20 \) J/kg. The deviation of Equation (30) from Equation (27) is less than 5.5 %. Equation (30) is used as the in-tube heat transfer model to simulate the gas cooler Hydro MAC 2000 (Figure 10, discussed in detail in the following COMPUTER PROGRAM section). The experimental data (McEnaney et al., 1998, and Yin et al., 1998) is in the range of \( 3500 < \text{Re}_w < 2.5 \times 10^4 \) and \(-115 < \dot{q}_w/G < -3 \) J/kg. The predictions agree with the experimental data very well. Therefore, the recommended applicable range of Equation (30) is \( 3000 \leq \text{Re}_w \leq 10^6 \) and \(-350 \leq \dot{q}_w/G \leq 0 \) J/kg.

### 3.3 Air-Side Heat Transfer

The air side heat transfer coefficient of gas coolers can be determined by

\[
h_a = (\rho \overline{c_p})_a St
\]  

(31)

where

\[
St = j / \text{Pr}^{2/3}
\]  

(32)

From experimental data with louvered fin heat exchanger with triangular channels, Davenport (1983) correlated a dimensional model for the Colburn \( j \) factor which is dimensionless.

\[
j = 0.249 \text{Re}_L^{0.42} L_h^{0.33} \left( \frac{L_1}{F_1} \right)^{1.1} F_1^{0.26} \text{Pr}^{0.26} \quad 300 < \text{Re}_{Dh} < 4000
\]  

(33)

where, \( \text{Re}_{Lp} \) is the Reynolds number based on louver pitch, and the units of characteristic length are in mm. It was reported that 95% of the experimental data were within ±6% of the model prediction.

Based on the experimental data with the heat exchanger with louvered plate-and-tube fin geometry, Achaichia and Cowell (1988) suggested a correlation for the Stanton number \( St \)

\[
St = 1.554 \text{Re}_L^{0.59} \left( \frac{\beta}{\theta} \right)^{0.09} \left( \frac{T_p}{L_p} \right)^{0.09} \left( \frac{F_p}{L_p} \right)^{-0.04} \quad \text{Re}_{Lp} > 75
\]  

(34a)
Figure 9 Comparison of Equation (30) with the Petrov-Popov Equation (27)

Figure 10 Gas Cooler Hydro MAC 2000

- 19 -
\[ \beta = 0.936 - \frac{243}{Re_{L_p}} - 1.76 \frac{F_p}{L_p} + 0.995 \theta \]  

(34b)

They claimed their equation described all the Stanton number data for \( Re_{L_p} > 75 \) to be within 10%.

Chang et al (1994) conducted the experiments with brazed aluminum heat exchangers with rectangular-channel louvered fins. Based on their own experimental data and those of Davenport (triangular channel, 1983), Achaichia and Cowell (plate-and-tube, 1988), and Webb and Jung (rectangular channel, 1992), they obtained

\[ j = 0.291 R e_{L_p}^{-0.589} \varepsilon_f^{0.438} \]  

(35)

where the finning factor \( \varepsilon_f \) is

\[ \varepsilon_f = \frac{A_f + A_{io}}{A_{io}} \]  

(36)

They found that in the range of \( 100 < Re_{L_p} < 700 \) and \( 7 < \varepsilon_f < 12 \) their model described 92% of the Colburn \( j \) factor data to be within \( \pm 10\%\).

Sunden and Svantesson (1992) proposed the Colburn \( j \) factor correlation for louvered fin heat exchangers with rectangular channels as follows:

\[ j = 3.67 R e_{L_p}^{-0.591} \left( \frac{\theta}{90} \right)^{0.239} \left( \frac{F_p}{L_p} \right)^{0.0206} \left( \frac{F_i}{L_p} \right)^{-0.283} \left( \frac{L_h}{L_p} \right)^{0.671} \left( \frac{T_p}{L_p} \right)^{-0.243} \]  

(37)

Chang and Wang (1996) also presented a Colburn \( j \) factor correlation for the heat exchangers with rectangular channels:

\[ j = 0.4361 R e_{L_p}^{0.559} \varepsilon_f^{0.192} \varepsilon_i^{0.0956} \quad 100 < Re_{L_p} < 1000 \]  

(38)

\[ \varepsilon_i = \frac{A_i}{A_f + A_{io}} \]  

(39)

Based on all work above, Chang and Wang (1996) obtained a generalized heat transfer correlation for louvered fin geometry:

\[ j = R e_{L_p}^{-0.49} \left( \frac{\theta}{90} \right)^{0.27} \left( \frac{F_p}{L_p} \right)^{-0.14} \left( \frac{F_i}{L_p} \right)^{-0.29} \left( \frac{T_d}{L_p} \right)^{-0.23} \left( \frac{L_h}{L_p} \right)^{-0.14} \left( \frac{T_p}{L_p} \right)^{0.68} \left( \frac{\delta_f}{L_p} \right)^{-0.28} \left( \frac{\delta_f}{L_p} \right)^{-0.05} \]  

(40)
They claimed that in the range of $100 < \text{Re}_{lp} < 3000$, 89.3% of all the published louvered fin data are correlated within ±15% with Equation (40).

Figure 9 compares Davenport's equation (33), Achaichia-Cowell's equation (34), Chang-Wang-Chang's equation (35), Sunden and Svantesson's equation (37), and Chang-Wang's equations (38) and (40). Davenport's equation (33) which is based on the experimental data of the triangular-channel fin geometry has the lowest prediction when $\text{Re}_{lp} < 380$. Achaichia-Cowell's equation (34) which is correlated from the experimental data of the plate-and-tube fin geometry has the highest prediction when $\text{Re}_{lp} > 80$. Sunden-Svantesson's equation (37) and Chang-Wang's equation (38) come from the experimental data of the rectangular-channel fin geometry, and have good agreement with each other. Chang-Wang-Chang's equation (35) is more close to the rectangular-channel fin equations though the authors claimed it is fitted with the experimental data of rectangular-channel, triangular-channel, and plate-and-tube fin geometries. Chang-Wang's equation (40) is generalized one which is the approximation of the other equations. It is seen that for the given fin geometry, it is better to choose the correlation from the experimental data of the same fin geometry.

### 3.4 Air-Side Fanning Friction Factor $f_F$

The air-side pressure loss of the louvered fin heat exchanger can be calculated with Equation (4). Davenport (1983) correlated an air-side friction factor for the triangular-channel fin geometry.

$$f_F = 5.47 \text{Re}_{lp}^{-0.72} L_h^{0.37} \left( \frac{L_i}{F_i} \right)^{0.89} L_p^{0.2} F_i^{0.23} \quad 70 < \text{Re}_{Dh} < 900 \quad (41a)$$

$$f_F = 0.494 \text{Re}_{lp}^{-0.39} \left( \frac{L_p}{L_p} \right)^{0.33} \left( \frac{L_i}{F_i} \right)^{1.1} F_i^{0.46} \quad 1000 < \text{Re}_{Dh} < 4000 \quad (41b)$$

where, $\text{Re}_{Dh}$ is the Reynolds number based on the hydraulic diameter, and all the units of characteristic length are in mm.

\[ f_F = 10.4 \, \text{Re}^{1.17}_{p_{lp}} \, F_p^{0.05} \, L_p^{1.24} \, L_h^{0.25} \, T_p^{0.83} \quad \text{Re}_{Dh} \leq 150 \] (42a)

\[ f_F = 0.895 \, f_A^{1.07} \, F_p^{-0.22} \, L_p^{0.25} \, L_h^{0.33} \, T_p^{0.26} \quad 150 < \text{Re}_{Dh} < 3000 \] (42b)

where

\[ f_A = 596 \, \text{Re}^{(0.318 \log \text{Re} L_p - 2.25)}_{lp} \] (42c)

and all the units of characteristic length are in mm.

It was reported that Equation (42b) predicted friction factors to be within ±10% of the experimental data.

Chang et al (1994) correlated an air-side friction factor for the rectangular-channel fin geometry

\[ f_F = 0.805 \, \text{Re}^{-0.514}_{lp} \left( \frac{F_p}{L_p} \right)^{-0.72} \left( \frac{F_l}{L_p} \right)^{-1.22} \left( \frac{L_l}{L_p} \right)^{1.97} \quad 100 < \text{Re}_{lp} < 700 \] (43)

The authors compared the predictions of Equation (43) with their own experimental data, and found the equation described most of the experimental data to be within ±15%. However, compared with Webb and Jung’s experimental data (1992), the predictions were slightly higher.

Sahnoun and Webb (1992), and Dillen and Webb (1994) developed more complicated friction factor correlations.

Figure 10 compares Davenport’s equation (41), Achaichia and Cowell’s equation (42), and Chang-Wang-Chang’s equation (43). In the range of 70 < Re_{Dh} < 700, Davenport’s equation (41) remains the lowest prediction, Achaichia and Cowell’s equation (42) holds the highest prediction. The prediction of the latter is over 60% higher than that of the former. When Re_{Dh} < 70, the predictions of Achaichia and Cowell’s equation (42) are about twice as much as those of Chang-Wang-Chang’s equation (43) and Davenport’s equation (41). When Re_{Dh} > 400, the Predictions of Achaichia and Cowell’s equation (42) and Chang-Wang-Chang’s equation (43) agree with each other quite well.
Figure 11 The Comparison of Air-Side Heat Transfer Equations

Figure 12 Comparison of Air-Side Friction Factor Equations
4 COMPUTER PROGRAM

Each pass of the gas cooler is separated into several equal-length segments along the refrigerant flow direction. Each segment is treated as a cross flow heat exchanger whose outlet fluid parameters are calculated based on the mathematical models above. This way, the whole gas cooler is equivalent to a number of heat exchangers in series connection. The length of the segments should be small enough to enable the air sides of all segments to be in the minimum heat capacity.

The program can be used to all louvered fin geometries as mentioned above. The verification is carried out with the experimental data (McEnaney et al., 1998 and Yin et al., 1998) of the gas cooler Hydro MAC 2000 (Figure 8), which is a rectangular-channel louvered fin heat exchanger. The experimental data cover the range of $107 < Re_{lp} < 180$, $220 < Re_{Dh} < 360$, $3500 < Re_w < 2.5 \times 10^4$, $3700 < Re_m < 2.7 \times 10^4$, and $-115 < q_jG < -3 \text{ J/kg}$. Each pass is separated into five equal-length segments.

Because the fin channel of the gas cooler Hydro MAC 2000 is rectangular, Sunden-Svantesson’s equation (37) and Chang-Wang-Chang’s equation (43), whose applicable ranges cover the experimental range of the gas cooler Hydro MAC 2000, are chosen as the air-side heat transfer and friction factor models for the verification, respectively. Table 4 shows the air-side geometric parameters of the gas cooler Hydro MAC 2000 and the heat exchangers for correlating Equations 37 and 43.

Table 4 Air-Side Geometric Parameters of the Louvered Fin Heat Exchangers

<table>
<thead>
<tr>
<th>Variety</th>
<th>$L_p$ (mm)</th>
<th>$L_f$ (mm)</th>
<th>$\theta$ (deg)</th>
<th>$F_p$ (mm)</th>
<th>$F_d$ (mm)</th>
<th>$F_l$ (mm)</th>
<th>$T_d$ (mm)</th>
<th>$T_p$ (mm)</th>
<th>$D_h$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-W-C</td>
<td>1.318-1.693</td>
<td>12.15-17.18</td>
<td>28</td>
<td>1.8-2.2</td>
<td>22-44</td>
<td>16-19</td>
<td>22-44</td>
<td>21-24</td>
<td>2.936-3.642</td>
</tr>
<tr>
<td>S-S</td>
<td>0.5-1.4</td>
<td>5-10.2</td>
<td>18.5-28.5</td>
<td>1.5-2.0</td>
<td>37-57.4</td>
<td>8-12.5</td>
<td>37-57.4</td>
<td>9.5-14</td>
<td>2.609-3.426</td>
</tr>
<tr>
<td>Hydro*</td>
<td>0.9906</td>
<td>7.493</td>
<td>23</td>
<td>1.1213</td>
<td>16.51</td>
<td>8.89</td>
<td>16.51</td>
<td>10.47</td>
<td>1.991</td>
</tr>
</tbody>
</table>

* Some other parameters of the gas cooler Hydro MAC 2000 are: port number =11, port diameter = 0.79 mm, fined tube length = 532 mm, and gas cooler height = 366.4 mm.

Equation (30) is used for the refrigerant-side heat transfer. Equation (1) is used for refrigerant-side pressure loss, where the inertial factor is calculated by Equation (3), the friction factor is calculated by Petrov-Popov’s equation (10) with $f_{ow}$ evaluated by Churchill’s equation (9) at $T_w$. For the gas cooler Hydro MAC 2000, $Re < 2.7 \times 10^4$, and
the pressure drop due to friction is only about 30 % of the total pressure drop (Usually, pressure drops due to inertia, sudden entrance contraction, and sudden exit expansion are about 5%, 35%, and 30% of the total pressure drop of the gas cooler Hydro MAC 2000, respectively). In this case, neglecting roughness has very small effect on the value of the total pressure drop if relative roughness is less than 0.002. Therefore, the port is assumed to be smooth.

A variety of local resistance coefficients of the refrigerant side are given by Ide’lchik (1966). Suppose some deformation caused by cutting exists at port entrances and exits. Considering the entrance to the port as a sudden contraction, the entrance resistance coefficient is of the form

\[ \xi = \xi_w + \bar{\varepsilon}(1.707 - A_{\text{ratio}})^2 \]  \hspace{1cm} (44)

where \( \xi_w \) and \( \bar{\varepsilon} \) are the functions of the entrance Reynolds number. From curve fitting, we obtained

\[ \xi_w = \begin{cases} -0.085 + 78.5/(\ln Re)^{2.62} & 2 \times 10^3 < Re < 7 \times 10^5 \\ 0 & Re \geq 7 \times 10^5 \end{cases} \]  \hspace{1cm} (45)

and

\[ \bar{\varepsilon} = \begin{cases} -0.21 + 0.1028\ln Re & 2 \times 10^3 < Re \leq 5 \times 10^4 \\ 0.64 + 0.025\ln Re & 5 \times 10^4 < Re < 10^6 \end{cases} \]  \hspace{1cm} (46)

\( A_{\text{ratio}} \) is the ratio of the port entrance or exit area to the port cross section area. It can be determined by measuring port diameter and port entrance or exit diameter. If there is no measuring data available, an approximate approach is adjusting \( A_{\text{ratio}} \) value until pressure predictions are reasonable.

Considering the exit from the port as a sudden expansion, the exit resistance coefficient is of the form

\[ \xi = 1 + \xi' (1 - A_{\text{ratio}}) + 2 \sqrt{\frac{\xi'}{(1 - A_{\text{ratio}})}} \]  \hspace{1cm} (47)

where \( \xi' \) is between 0.13 and 0.5.
Figure 13 compares the capacity of the gas cooler Hydro MAC 2000. The program describes the experimental data very well. Ninety-seven percent of 301 experimental data are within ±5% of the program calculation. Figure 14 compares the refrigerant temperatures at the outlet of the gas cooler. The predicted results agree with the measured ones very well. Ninety-seven percent of the calculations do not deviate ±1 °C of the experimental data.

Figure 15 shows the pressure drop comparison. The prediction is correspondent with the experimental data well. The program describes ninety-one percent of the experimental data to deviate less than ±25 kPa.

Figure 16 shows the variation of the pressure drop with the average Reynolds number. It suggests that the pressure drop inside gas cooler tubes can be described with the Reynolds number.

5 ANALYSES

5.1 Error Analyses of Model Predictions

Figure 17 illustrates the relationship between the heat transfer error of the model prediction and the Reynolds number. Generally, the model slightly over-predicts the heat transfer performance when \( Re_w < 10^4 \), and the error decreases with the Reynolds number increasing.

Figure 18 demonstrates the relationship between the pressure drop error of the model prediction and the Reynolds number. Generally, the absolute error slightly increases with the Reynolds number increasing. On the contrary, the relative error decreases with the Reynolds number increasing.

5.2 Influence of Segmentation on Calculation Accuracy

Theoretically, the smaller the segment length, the more accurate the calculation. However, the computing time will increase with the increase of the segment number. It is necessary to determine the suitable number of the segments to make the computing results accurate enough while computing time is acceptable.

Figure 19 shows the influence of the segmentation on the calculation accuracy of the gas cooler Hydro MAC 2000, where the segment ratio is the ratio of the number of the segments in which the air-side heat capacity is greater than that of the refrigerant side to the total segment number. The segment number equals one when each pass is treated as
one segment. When this is done, all the air sides usually have the maximum heat capacity, but actually it is the refrigerant side that has the maximum heat capacity. The segment ratio decreases when the segment number increases. When the segment number is big enough, all the air sides will have the minimum heat capacity so that the segment ratio equals zero.

The in-tube local heat transfer coefficient is the heat transfer coefficient at some point inside the tube, which can be approximated by the average heat transfer coefficient when the segment is short enough. In this case, the air side should be the minimum heat capacity. If the segment number is not chosen reasonably, the refrigerant sides of some segments will be the minimum heat capacity. Then, the heat capacity ratio and the number of transfer units will change unreasonably to cause considerable uncertainty in calculation results.

In our calculation, it is found that when the segment ratio is near zero, the uncertainty in the calculation become negligible. Therefore, the suitable number of the segments should be that which makes the segment ratio be equal to or near zero.

5.3 Influence of Pressure Drop Uncertainty on Capacity Calculation

Figure 20 shows the affect of the pressure drop uncertainty on the model's calculation of the heat rejection of the gas cooler Hydro MAC 2000. The capacity deviation is defined as

\[
\text{Capacity deviation} = \frac{\text{exact value of the capacity} - \text{calculated capacity}}{\text{exact value of the capacity}}
\]

The pressure drop deviation is defined as

\[
\text{Pressure drop deviation} = \frac{\text{exact value of the pressure drop} - \text{calculated pressure drop}}{\text{exact value of the pressure drop}}
\]

It can be seen that the greater the refrigerant mass velocity, the greater the capacity uncertainty due to the pressure drop uncertainty. The results show why in-tube pressure drop should not be neglected in the capacity prediction. At a refrigerant mass velocity of 856 kg/m²-s for example, the calculated capacity will be overestimated by 5% if the pressure drop is neglected.

5.4 Sensitivity Analysis of Gas Cooler Geometry

The following sensitivity analysis takes the Hydro MAC 2000 as a baseline gas cooler. Suppose \( P_{\text{in}} = 100 \) bar, refrigerant mass flow rate = 0.05 kg/s, \( T_{\text{an}} = 45 \) °C, air mass flow rate = 0.6 kg/s.
Figure 13 Capacity: Measured vs. Predicted

Figure 14 Outlet Refrigerant Temperature: Measured vs. Predicted
Figure 15 Pressure Drop: Measured vs. Predicted

Figure 16 Variation of Pressure Drop with Reynolds Number
Figure 17 Heat Transfer Error of The Prediction

![Graph](attachment:graph17.png)

\[ T_{\text{out}} = \text{Refrigerant outlet temperature [C]} \]

\[ \text{Err}_T = \frac{\text{Measured} - \text{Predicted}}{\text{Measured}} \]

![Graph](attachment:graph18.png)

\[ D_p = \text{Pressure Drop [kPa]} \]

\[ \text{Err}_D = \frac{\text{Measured} - \text{Predicted}}{\text{Measured}} \]

![Graph](attachment:graph19.png)

![Graph](attachment:graph20.png)

\[ \text{Err}_D = \frac{\text{Measured} - \text{Predicted}}{\text{Measured}} \]

Figure 18 Pressure Drop Error of The Prediction

- 30 -
5.4.1 The Influence of Tube Depth on Capacity

The tube depth increases 1.49 mm when the port number increases 1. Figure 21 shows the influence of tube depth on capacity, where the port number from 6 to 18 is correspondent to the tube depth from 9.06 mm to 26.94 mm. In the given conditions, the capacity increases by 6 – 10 % when the tube depth increases from 16.51 mm to 26.94 mm, and decreases by 17 – 22 % when the tube depth decreases from 16.51 mm to 9.06.

The friction factor of a rectangular-channel gas cooler is independent on the tube depth according to Chang-Wang-Chang’s equation (43). However, the pressure drop in the channel is in direct proportion to the tube depth.

5.4.2 The Influence of Fin Length on Capacity

As shown in Figure 22, the capacity varies little with the fin length if air mass flow rate remains constant. The gas cooler height will decrease as the fin length decreases, while the air-side friction will increase basically in direct proportion to the fin length decrease.

6 CONCLUSIONS

This paper offers a comprehensive survey of one-phase in-tube heat transfer correlations, CO₂ supercritical heat transfer, friction factor correlations, the calculation of pressure drop in the supercritical conditions, and the heat transfer and friction factor correlations on the gas cooler air-side.

When CO₂ cooled at supercritical pressures, the local heat transfer coefficient and the hydraulic drag factor are greatly dependent on both the local mean temperature and the heat flux. This is because the thermophysical properties of CO₂ change drastically during the process. In these circumstances, the conventional models of the local heat transfer coefficient and the hydraulic drag factor do not apply.

In the case of thermophysical property variations, the inertia drag could not be neglected. When CO₂ cooled at supercritical pressures, the absolute value of the inertia drag, which is negative in cooling conditions, may be commensurable with the friction drag. At some circumstances, this can lead to the decrease of the total hydraulic drag to negative values, thereby resulting in the appearance of zones with pressure increasing along the tube.
Figure 21 The Influence of Tube Depth on Capacity

Figure 22 The Influence of Fin Length on Capacity
Petrov-Popov’s friction factor equations (10) and (11) are valid only in smooth tubes for fully developed turbulent flow conditions. This paper suggests that \( f_{0w} \) and \( f_{0m} \) in Equations (10) and (11) are calculated by Churchill’s equation (9) to extend their use to rough tubes and the transitional regime. The pressure drop predictions of the program with the friction factor calculated by Petrov-Popov’s equation (11) in which \( f_{0m} \) is evaluated by Churchill’s equation (9) at \( T_m \) agree with the experimental data. However, in small refrigerant mass velocity, the prediction usually a little larger than the experimental data. More careful experiments are needed to explain the phenomena.

The following in-tube local heat transfer model for \( \text{CO}_2 \) cooled at supercritical pressures is proposed:

\[
Nu_w = \frac{(f_w/8)(Re_w - 1000)Pr_w}{A + 12.7(f_w/8)^{1/2}(Pr_w^{2/3} - 1)} \left( 1 - 0.001 \frac{q_w}{G} \right) \left( \frac{c_p}{c_{p_w}} \right)^n
\]

where

\[
A = \begin{cases} 
1 + 7 \times 10^{-8} Re_w & \text{Re}_w < 10^{-6} \\
1.07 & \text{Re}_w \geq 10^{-6}
\end{cases}
\]

\( f_w \) is the friction factor evaluated at \( T_w \) by Blasius’ equation (5) or Filonenko’s equation (6) according to the Reynolds number \( Re_w \), and \( c_p \) and \( n \) are calculated by Equations (24) and (27b), respectively.

The recommended applicable range of Equation (30) is \(-350 \leq q_w/G < 0 \) J/kg and \( 3000 \leq Re_w \leq 10^6 \). The predictions of the program based on it agree with the experimental data very well.

The literature on the heat transfer of the louvered fin geometry was reviewed. Chang-Wang’s equation (40) is generalized one which is the approximation of the special-geometry equations, such as Davenport’s equation (33) (triangular-channel), Achaichia-Cowell’s equation (34) (plate-and-tube), and Sunden-Svantesson’s equation (37) and Chang-Wang’s equation (38) (rectangular channel). For the given fin geometry, the correlation from the experimental data of the same fin geometry should be more accurate than the generalized equation.
A computer simulation program has been developed and verified with the experimental data. It describes ninety-seven percent of the experimental data to be within ±5% for capacity, ±1 °C for the outlet refrigerant temperatures, respectively, and ninety-one percent of the experimental data to be within ±25 kPa for pressure drop.

The pass segmentation and pressure drop calculation uncertainty influence the capacity calculating accuracy. Theoretically, the smaller the segment length, the more accurate the calculation. However, the computing time will increase with the increase of the segment number. The segment ratio can be used as a rule to judge the optimum segment length. The suitable segment length should be that which makes the segment ratio be equal to or near zero. The pressure drop should not be neglected.

The capacity increases considerably when the tube depth increases. In the some conditions of the Hydro MAC 2000 for instance, the capacity increases by 6 – 10 % when the tube depth increases from 16.51 mm to 26.94 mm. The pressure drop in the fin channel is in direct proportion to the tube depth.

Lowering fin height can reduce a gas cooler size with only slight affect on the capacity. However, air-side friction will increase basically in direct proportion to the decrease of fin height.

NOMENCLATURE

A  = area
A_{\text{ratio}}  = ratio of the port entrance or exit area to the port cross section area
A_{ta}  = tube area of air side (outside)
C_{\text{max}}  = maximum heat capacity
C_{\text{min}}  = minimum heat capacity
c_{p}  = specific heat
C_{r}  = heat capacity ratio, C_r = C_{\text{min}} /C_{\text{max}}
D  = inner diameter of tubes
f = Darcy-Weisbach friction factor
f_F = Fanning friction factor
f_h = hydraulic drag factor
f_i = inertia factor
F_d = fin depth
F_l = fin length
F_p = fin pitch
G = \rho V, cross-section mass velocity
h = heat transfer coefficient
i = enthalpy
j = Colburn j factor
k = thermal conductivity
L = tube length
l = F_l/2
L_l = louver length
L_p = louver pitch
Nu = Nusselt number
NTU = number of transfer units
p = pressure
Pr = Prandtl number
q_w = heat flux density through tube wall to fluid
Re = Reynolds number
R_{fa} = air side fouling factor
R_{fr} = refrigerant side fouling factor
R_{rt} = relative roughness of tubes
R_{tw} = heat resistance of tube wall
St = Stanton number
T = temperature
T_{cm} = temperature at which the fluid c_p has maximum value in the give pressure
$T_d = \text{tube depth}$

$T_p = \text{tube pitch}$

$UA = \text{overall heat transfer coefficient}$

$V = \text{velocity}$

$\Delta p = \text{pressure loss}$

Greek Symbols

$\theta = \text{louver angle [deg]}$

$\delta = \text{thickness}$

$\mu = \text{fluid dynamic viscosity}$

$\varepsilon = \text{effectiveness of gas coolers}$

$\varepsilon_f = \text{finning factor}$

$\xi = \text{local pressure loss coefficient}$

$\eta = \text{fin efficiency}$

$\rho = \text{density}$

Subscripts

$a = \text{air side}$

$D_h = \text{hydraulic diameter}$

$f = \text{fin}$

$in = \text{inlet}$

$L_p = \text{louver pitch}$

$m = \text{at fluid mean temperature}$

$out = \text{outlet}$

$pc = \text{at pseudocritical point, where the specific heat reaches maximum at the given pressure.}$

$r = \text{refrigerant side}$

$w = \text{at inner tube wall temperature}$

REFERENCES


