STRENGTH IN FLEXURE OF BONDED AND UNBONDED
PRESTRESSED CONCRETE BEAMS

A Thesis
by
J. Warwaruk

Issued as a Part
of the
SIXTH PROGRESS REPORT
of the
INVESTIGATION OF PRESTRESSED CONCRETE
FOR HIGHWAY BRIDGES

August, 1957
UNIVERSITY OF ILLINOIS
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Conducted by THE ENGINEERING EXPERIMENT STATION UNIVERSITY OF ILLINOIS

In Cooperation With THE DIVISION OF HIGHWAYS STATE OF ILLINOIS and U. S. DEPARTMENT OF COMMERCE BUREAU OF PUBLIC ROADS

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I. INTRODUCTION

1. Introduction

The study of the behavior of prestressed concrete beams subject to flexure was initiated at the University of Illinois in 1951. So far, three phases of this investigation have been reported. Billet (1)* has presented a comprehensive study of the behavior of post-tensioned bonded beams. Both Feldman (2) and Allen (3) have studied post-tensioned unbonded beams. The tests reported herein, together with the related analyses, are intended to add further to the knowledge of the behavior of prestressed concrete beams.

2. Object

The object of this report was to study further the effects of several variables on the flexural strength of prestressed concrete beams. The study by Allen (3) included unbonded beams having mild steel at the bottom in addition to the prestressed reinforcement. One series of tests reported herein was undertaken to study the behavior of unbonded beams having mild steel bars added at both the top and bottom, in equal amounts. A second series of tests was undertaken to study the behavior of pretensioned beams as compared to the behavior of post-tensioned bonded beams. And, the third series included beams which had varicus degrees of unbonding in the flexure span and one beam which was unbonded in the shear span only.

3. Scope

Eleven beams were tested. Three beams were designated as Series W and eight as Series J. Series W, which involved the study of unbonded beams with mild

* Numbers in parentheses refer to entries in the bibliography.
steel bars added at the top and bottom, had the percentage of steel as a major variable. Series J, which included all the pretensioned beams, had for its variables the percentage of steel and the unbonded length of the reinforcement. In both series, the concrete strength, although not an intended variable, was different for most beams.

The percentage of steel varied from about 0.2 to about 0.8 and the concrete strength varied from 4000 psi to 5500 psi. All beams were loaded at the third points of a nine-foot span.

4. Acknowledgments

This project was carried out as a part of the Investigation of Prestressed Reinforced Concrete for Highway Bridges in the Structural Research Laboratory of the Department of Civil Engineering of the University of Illinois. This investigation is conducted as part of the Illinois Cooperative Highway Research Program under the sponsorship of the Division of Highways, State of Illinois, and the United States Department of Commerce, Bureau of Public Roads.

The work on this project has been guided by an advisory committee having the following membership: E. F. Kelley, E. L. Erickson, and Harold Allen, representing the Bureau of Public Roads; W. E. Chastain, Sr., W. J. Mackay and C. E. Thunman, Jr., representing the Illinois Division of Highways; and N. M. Newmark, C. P. Siess, I. M. Viest, and N. Khachaturian representing the University of Illinois.

General direction of the investigation was given by Dr. N. M. Newmark and supervision of the program was provided by Dr. Siess. The work was carried out under the immediate supervision of M. A. Sozen, Research Associate in Civil Engineering.
This report was written as a thesis under the direction of Dr. Siess and his assistance in planning the tests and his helpful comments are gratefully acknowledged. Credit for assistance in the development of equipment and analysis, and in conducting the tests and critical study of the manuscript of this report is due Mr. Sozen. Appreciation is expressed to J. G. MacGregor, Research Assistant in Civil Engineering, for his aid in conducting the tests, reducing and interpreting the data, and preparing the figures. Special thanks are given to Wyck McKenzie, Junior Laboratory Mechanic in Civil Engineering, for his aid in fabricating specimens and equipment.

The wire reinforcement was furnished by the American Steel and Wire Division of the United States Steel Corporation.

5. Notation

The symbol \( f \) is used for stress and \( \varepsilon \) for strain. The subscripts \( s \) and \( c \) refer to the steel and concrete, respectively. Other subscripts include: \( e \), referring to conditions at effective prestress after losses; \( cr \), referring to conditions at cracking; \( u \), referring to conditions at ultimate load; and \( y \), referring to yield. The prime (') and double prime ("') superscripts refer to quantities based on the bonded top and bottom non-prestressed mild steel bars, respectively.

**Beam Constants**

- \( A_s \) = total area of prestressed wire reinforcement
- \( A'_s \) = total area of bonded top mild steel reinforcement
- \( A''_s \) = total area of bonded bottom mild steel reinforcement
- \( b \) = width of beam
- \( h \) = depth of beam
- \( d \) = effective depth to wire reinforcement at ultimate
d' = effective depth to top mild steel reinforcement

d'' = effective depth to bottom mild steel reinforcement

\[ p = \frac{A_s}{bd} \]

\[ p' = \frac{A'_s}{bd''} \]

\[ p'' = \frac{A''_s}{bd''} \]

Moments

\[ M_{cr} = \text{bending moment at cracking} \]

\[ M_u = \text{total measured resisting moment at ultimate} \]

Stresses

Concrete

\[ f'_c = \text{compressive cylinder strength determined from 6 by 12-in. control cylinders} \]

\[ f_r = \text{modulus of rupture determined from 6 by 6 by 20-in. control beams} \]

\[ f_{cu} = \text{average concrete stress in the compression zone of a beam at ultimate} \]

Steel

\[ f_{se} = \text{effective prestress after losses} \]

\[ f_{su} = \text{stress in wire reinforcement} \]

\[ f'_s = \text{stress in top mild steel reinforcement at ultimate} \]

\[ f''_s = \text{stress in bottom mild steel reinforcement at ultimate} \]

\[ f_y = \text{yield stress of mild steel reinforcement} \]

Strains

Concrete

\[ \varepsilon_c = \text{concrete strain at any stage} \]
$\varepsilon_{ce} =$ strain at the level of the wire reinforcement due to effective prestress

$\varepsilon_u =$ maximum concrete strain at first crushing

**Steel**

$\varepsilon_{se} =$ strain due to effective prestress, $f_{se}$

$\varepsilon_{su} =$ strain in wire reinforcement at ultimate

$\varepsilon'_{sa} =$ additional strain in wire reinforcement between prestress and ultimate

$\varepsilon_{sa} = \varepsilon'_{sa} + \varepsilon_{ce}$

$\varepsilon'_{su} =$ strain in top mild steel reinforcement at ultimate

$\varepsilon''_{su} =$ strain in bottom mild steel reinforcement at ultimate

$\varepsilon_y =$ yield strain of mild steel reinforcement

**Parameters**

$k =$ ratio of the neutral axis depth to the effective wire reinforcement depth

$k_u =$ ratio of the neutral axis depth to the effective wire reinforcement depth at ultimate

$k_2 =$ ratio of distance between the top of the beam and the center of compression to the neutral axis depth

$F =$ strain compatibility factor relating steel strain to maximum compression concrete strain

$E_s =$ modulus of elasticity of steel

$Q' = E_s p / f_{cu}$
II. DESCRIPTION OF MATERIALS, FABRICATION, AND TEST SPECIMENS

6. Materials

(a) Cement. Marquette Type III Portland Cement was used for all the beams presented in this report with the exception of beam J-1; Incor Type III Portland Cement was used for beam J-1. All the cement was purchased in paper bags from local dealers and stored under proper conditions.

(b) Aggregates. Wabash river sand and gravel were used in casting all the beams. Both aggregates have been previously used in the laboratory and have passed the usual specification tests. The absorption of both fine and coarse aggregate was about one per cent by weight of the surface dry aggregate. Aggregate sieve analysis and fineness modulus of the sand are given in Table 1.

(c) Concrete Mixes. Concrete mixes were designed by the trial batch method. Table 2 lists the proportions of the concrete batches used in each beam. Proportions are in terms of oven dry weights. Location of the batches is discussed in Section 7(a). The following properties for each batch are also listed in Table 2: slump, compressive strength, modulus of rupture, age of test, and aggregate lot. The compressive strength, $f'_c$, is the average strength of a minimum of four 6 by 12-in. control specimens tested at the same time as the beam. Values reported for the modulus of rupture, $f_r$, are results of tests of 6 by 6 by 20-in. control beams loaded at the third points of an 18-in. span.

(d) Reinforcing Wire. Two lots of steel designated as Type IX and Type X were used as prestressing reinforcement for the beams. Both types were manufactured by the American Steel and Wire Division of the United States Steel Corporation and are designated by the manufacturer as "Hard Drawn Stress Relieved Super-Tens Wire". The following steps were involved in its manufacture: hot rolling,
lead patenting, cold drawing and stress relieving. Both types were delivered in coils about six feet in diameter and weighing approximately 300 lb. each. Type IX wire was used in Series W and Type X in Series J. The following heat analyses have been furnished by the manufacturer.

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<tr>
<th>Type</th>
<th>C Percent</th>
<th>Mn Percent</th>
<th>P Percent</th>
<th>S Percent</th>
<th>Si Percent</th>
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<tr>
<td>IX</td>
<td>0.82</td>
<td>0.83</td>
<td>0.010</td>
<td>0.027</td>
<td>0.27</td>
</tr>
<tr>
<td>X</td>
<td>0.81</td>
<td>0.76</td>
<td>0.010</td>
<td>0.027</td>
<td>0.23</td>
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Samples of each of the two types of wire were cut from various portions of the roll and tested in a 120,000-lb. capacity Baldwin hydraulic testing machine. Strains were measured with an 8-in. extensometer employing a Baldwin "microformer" coil and recorded with an automatic recording device. The extensometer had a range of about 4-per cent strain. The average stress-strain relationships for the two types of wires are shown in Figs. 1 and 2.

(e) Reinforcing Bars. Beams of Series W had two No. 3 bars at top and bottom in addition to the prestressed wires. These intermediate grade bars were not prestressed and were fully bonded. Two 10-ft. lengths of bar and a 2-ft. test specimen were cut from each 22-ft. length of laboratory stock. The test specimen was used to obtain a stress-strain curve by the procedure outlined in Section 6(d). Typical stress-strain relationship is shown in Fig. 3.

7. Casting and Curing

(a) Unbonded Beams With Additional Bonded Steel

Beams included in this category are Series W which had, in addition to the unbonded prestressed reinforcement, non-prestressed bonded mild steel bars added at top and bottom of the beam. To hold the intermediate grade bars in place during casting a cage assembly was constructed using plain No. 2 ties spaced at six inches as shown on Fig. 4.
For these specimens, a hole was formed in the lower part of each beam to provide a channel for the wire reinforcement. This hole was approximately 2 by 3 in. with its center about 8 in. below the top of the beam. It was formed by means of a core composed of eight 1/2-in. steel rods, ten 3/8-in. rubber tubes, four 1 by 1 by 1/8-in. angles and a cover of sheet rubber. The core was assembled in the following manner: rods were inserted into a steel template at both ends of the form, rubber tubes were placed as needed between and outside the rods. The angles formed the corners of the hole, and a strip of rubber 4 in. wide was wrapped continuously around the angles. This core was then inserted into the cage assembly and the entire unit placed in the beam form.

All concrete was mixed from three to six minutes in a non-tilting drum type mixer of six cubic feet capacity, and was placed in the form with the aid of a high frequency internal vibrator. In order that the concrete in the region of failure would be from the same batch, the first batch of each mix was placed at the end quarters of the beam and the second in the central half. Twelve 6 by 12-in. control cylinders were cast, four from the first batch and eight from the second batch. One 6 by 6 by 20-in. control beam was cast from each batch.

Several hours after casting, the top surface of the beam was trowelled smooth and the cylinders were capped with neat cement paste. The core was removed from the beam after the concrete had hardened.

The beam and control specimens were removed from the forms after the concrete had gained sufficient strength. All the beams were stored in the air of the laboratory with no special curing.

(b) Bonded and Partially Bonded Beams

This classification includes the beams of Series J which were all pre-tensioned. The tensioning procedure was accomplished by means of a prestressing
frame which provided a reaction for the tensioning force. The frame consisted of
two 12-ft. lengths of extra heavy 3-in. diameter pipe and two bearing plates, 2 by
8 by 20 in. It was built to fit around the form for the beam. The bearing plates
had three rows of 0.206-in. diameter holes to accommodate the various positions of
the wires. Partial unbonding was accomplished by encasing the prestressing wires
in rectangular boxes 3 by 4 in. in cross-section at the desired locations. These
boxes were made from plywood and masonite as shown in Fig. 5. After the box was
assembled, it was then wrapped entirely with a thin sheet of polyethylene plastic
to waterproof the unit and facilitate removal. Fig. 5 also shows a view of the
prestressing frame and a completed box. The length and position of the box varied;
Fig. 6 shows the different arrangements of unbonding used. Upon completion of
tensioning and fabrication of the box, the prestressing frame was lowered into the
beam form and the beam was cast. Procedure for concrete mixing, placing and pre-
paration of control specimens was similar to that for unbonded beams described in
Section 7(a). The sides of the form were released about six hours after casting.
The beam and prestressing frame were moved to the testing area on the third day
following casting. Prestress was not released until one or two days before the
time of testing.

c) Dimensions of Test Specimens

All beams tested were rectangular in cross-section, nominally 6 by 12 in.
Although the beams were cast in metal forms, the dimensions of the individual
beams varied slightly. Actual dimensions are given in Table 3. Beams of Series
W were ten feet long, those of Series J were eleven feet long. The span length
for all tests was nine feet.
8. Prestressing Equipment and Procedure

(a) Equipment

In Series W, since the wires were unbonded, the anchorages had to resist both the prestressing force and the increase in wire tension due to load increases. Two types of anchorage were used. Wedge grip anchors were used for beam W-1 since it had only six wires. Beams W-2 and W-3 had threaded anchorages. All the beams of Series J, because they were pretensioned, had threaded anchorages.

The wedge grip anchorages used in beam W-1 were fabricated in the laboratory machine shop. At the tensioning end of the beam the wedge grips consisted of a three-jaw gripping chuck housed in an internally tapered, externally threaded grip housing outside of which was fitted an internally threaded sleeve. The sleeve was run up against the bearing block or shims to hold the stress after tension was applied. The sleeves and grip housing were fabricated from mild steel. Chuck jaws were commercial "Strandvise" fittings made by the Reliable Electric Company of Chicago. At the opposite end of the beam the wedge grips consisted of a three jaw gripping chuck housed in an internally tapered grip housing.

The threaded anchorage consisted of a special heat-treated nut screwed on the threaded end of a prestressing wire. Wires were threaded 24 threads to the inch for about three inches on each end in an automatic threading machine with specially heat-treated chasers. The threads on the wires were cut to provide a medium fit with the threads in the nuts.

The nuts were specially made in the laboratory machine shop. They were subdrilled with a No. 16 tap drill and tapped with a standard No. 12, 24 threads to the inch, tap. The thread cut on the wires to fit a No. 12 thread in the nut was sufficient to develop at least 190 ksi in the wire.
The nuts were made from 1/2-in. diameter "Buster" alloy punch and chisel steel having the following composition limits: Carbon 0.56 to 0.60 per cent, silicon 0.60 to 0.80 per cent, chromium 1.10 to 1.30 per cent, tungsten 2.00 to 2.30 per cent, vanadium 0.20 to 0.30 per cent. The nuts were 5/8-in. long and hexagonal in cross-section. They were hardened by the following procedure: (1) Pack in charcoal in a closed steel box. (2) Heat for 20 minutes at 1200 deg. F. (3) Heat for 45-60 minutes at 1650 deg. F. (4) Oil quench to slightly above room temperature. (5) Temper 30 minutes at 1000 deg. F. (6) Remove from furnace and air cool.

(b) Post-tensioning

A thirty-ton capacity Simplex centerhole hydraulic ram operated by a Blackhawk pump was used to tension the wires. A U-shaped frame supported by the bearing plate provided a reaction for the jack. To tension the wires, the ram reacted against the frame and a 5/8-in. diameter rod. The thrust was transferred from the ram to the rod through washers and a nut. The rod was directly connected to the wires with a threaded union-connection when threaded wires were used. When wedge grips were used, the threaded union connection screwed into the grip housing. After the wire was tensioned to the desired stress, a nut was turned up against a shim in the case of a threaded anchorage and the sleeve was turned up against the bearing plate in the case of the wedge grips.

The bearing plates used for the post-tensioned beams were 6 by 6 by 1 1/2 in. and were stiff enough to give a fairly uniform distribution of stress at the ends of the beams. The use of stiff bearing plates eliminated the need of special reinforcement near the ends of the beam.

(c) Pretensioning

The same tensioning equipment was used for pretensioning as for post-tensioning. However, since the reinforcement was tensioned before the beam was
cast, a reaction had to be provided for the tensioning force. This consisted of the frame described previously in Section 7(b). To tension the wires, the ram reacted against the frame and a 5/8-in. rod as in the post-tensioned beams. However, instead of the thrust being absorbed by the beam through the jacking frame, it was transferred from the jacking frame to the prestressing frame. The wires were tensioned and secured against the prestressing frame in the same manner as the threaded post-tensioned wires were secured to the bearing plates against the beam.

(d) **Measurement of the Tensioning Force**

The tensioning force in each wire was determined by measuring the compressive strain in cylindrical aluminum dynamometers placed on the wire between the nut or wedge grip and the bearing plate at the end opposite to which the tension was applied. The dynamometer consisted of a 2-in. length of 1/2-in. aluminum rod with a 0.2-in. diameter hole drilled through its center. Strains were measured by means of two Type A7 SR-4 electric strain gages mounted on opposite sides of each dynamometer and wired in series. This arrangement gave a strain reading which was the average of the strains in the two gages thereby compensating small eccentricities of load that might occur. The dynamometers were calibrated using the 6000-lb. range of a 120,000-lb. capacity Baldwin hydraulic testing machine. The calibration constants of the dynamometers were very nearly the same; the strain increment necessary to measure a tensioning stress of 120 ksi in the wire was about 2000 millionths. This large increment of strain allowed a precise measurement of stress in the wires, since the strain indicator used had a sensitivity of two or three millionths.

(e) **Tensioning Procedure**

Before inserting the wires into the channel one end of each wire was threaded through the bearing plates and secured with a nut. Then all wires were
pulled through the hole in the beam at the same time. The wires were then threaded through the other bearing plate and both plates were secured to the ends of the beam with a thin layer of "Hydrocal" gypsum plaster. The dynamometers were then slipped onto the wires at one end of the beam and finally the anchoring nuts were put on each end of each wire. After readings were taken on all the dynamometers the wires were tensioned individually. The jacking frame was attached to the bearing plate and the pull-rod was connected to the wire. The center-hole ram was placed over the pull-rod and each wire was, in turn, tensioned to the desired value of stress, the anchor nut was turned up snug against the shim, and the pressure on the ram was released. Since the beam shortened elastically with the tensioning of the wires it was necessary to overstress each wire in proportion to the number of wires to be subsequently tensioned so that retensioning would not be necessary.

The reinforcement for the pretensioned beams was tensioned in the prestressing frame prior to casting the beam. The ends of the wires were slipped through the end plates of the form and through the bearing plates of the prestressing frame. The dynamometers were then slipped onto one end of the wires and the anchoring nuts were put on each end of the wire. After readings in the unstressed dynamometers were taken, the wires were tensioned individually. The procedure was similar to that of the post-tensioned beams except that the prestressing frame underwent greater elastic shortening than the post-tensioned beam and more adjustment was required to give the wires the desired amount of initial tension.

9. Instrumentation, Loading Apparatus, and Testing Procedure

(a) Instrumentation of Unbonded Beams With Additional Bonded Steel

(1) Electric Strain Gages. Strains in the concrete on the top of the beam were measured with Type Al SR-4 electric strain gages. These gages had a nominal gage length of 3/4 in. and a width of 3/16 in. The method of application
was as follows: the surface of the concrete at the desired location was first ground smooth with a portable grinder. A thin layer of Duco cement was applied to the smooth surface and allowed to dry for 15 to 20 minutes. A layer of Duco cement was applied to the gage which was then mounted on the beam. One-pound weights were used to keep the gages flat while the cement was drying. A half-inch layer of sponge rubber was placed between the gage and the weight. Heat was not used to hasten the drying period since it can be detrimental to the concrete. The gages were applied only a few days prior to testing and were not waterproofed. Location of gages used for all the beams of Series W is shown in Fig. 7.

Strains in the wires were measured with Type A7 SR-4 electric strain gages, which had a nominal gage length of 1/4 in. and a minimum trim width of 3/16 in. They were chosen for their narrow width, short length and flexibility. The surface of the wire was prepared for the gage by polishing with emery cloth and cleaning thoroughly with acetone. Duco cement was used as the bonding agent. Only enough pressure was applied to squeeze out the excess cement and hold the gage to the contour of the wire. Heat lamps were used to hasten the drying of the cement. Following heat drying, while the wire was still warm, a layer of wax was applied to the gage to protect it from moisture. The lead wires were then soldered on and layers of electrical and cloth tape were wrapped entirely around the gage. As the wires were unbonded there was no necessity for positive waterproofing of the gages. The leads from the gages were carried down the reinforcement channel to the dynamometer end of the beam where they were brought out from behind the bearing plate through a small groove formed in the concrete. The gages were placed on at least four wires at midspan of the beam and symmetrical about the center of gravity of the wires.
Strains were read to the nearest 10 millionths with a Baldwin Portable Strain Indicator. Dummy gages for temperature compensation were mounted on unstressed steel blocks.

(2) Mechanical Strain Gages. The distribution of strains in the concrete was measured with a 10-in. Whittemore strain gage. Strains measured with the Whittemore gage were estimated to the nearest millionth. Measurements on all gage lines were taken twice or until the readings agreed within 10 millionths.

The location of the gage lines are shown in Fig. 7. The depth to the lowest line was chosen to correspond to the original depth of the steel. Steel plugs 3/8-in. in diameter and 1/4-in. long, with gage holes drilled to a depth of about 1/8 in. were cemented to the sides of the beam to establish the gage lines.

(3) Deflection Dial Indicators. Deflections at midspan and at each third point were measured with 0.001-in. dial indicators. The indicators were mounted on posts attached to a deflection frame which spanned between piers that supported the beam.

(b) Instrumentation of Bonded and Partially Bonded Beams

(1) Electric Strain Gages. Strains in the concrete on top of the beam were measured with Type A3 SR-4 electric strain gages. These gages were similar in size to the Type Al SR-4 gages but were chosen because of their thin backing which greatly facilitated the mounting procedure. The sequence and method of application of the gages was as described in Section 9(a). Location of top concrete gages for all the beams of the J-series was similar and details are shown in Fig. 8.

Strains in the wires were measured with Type A7 SR-4 electric strain gages. The gage details and method of application are described in Section 9(a). Because all the beams were pretensioned it was necessary to waterproof the gages very carefully since they would be in contact with wet concrete. Waterproofing
was accomplished by pouring melted petrolastic into a preformed container 1 1/2 by 1/2 by 1/2 in. which was attached to the reinforcing wire. A view of a waterproofed gage is shown in Fig. 9. Petrolastic was applied to the gage after the wire was prestressed thus minimizing the possibility of cracking due to stretching the wire. All gages waterproofed in this manner proved satisfactory in that a test for leakage resistance of the gage, just prior to testing of the beam, gave resistance values of 10,000 megohms or greater. The lead wires from the gages were located below the reinforcement wire and were brought out to one end of the beam. Location of gages varied with individual beams but each beam had at least one or more gages at midspan and one gage at the third points; all gages were placed symmetrical about the center of gravity of the wires.

Equipment for measuring strains is described in Section 9(a).

(2) Mechanical Strain Gages. A 10-in. Whittemore strain gage was used to measure the distribution of strain throughout the depth of the beam. The procedure of obtaining strains is described in Section 9(a). The location of gage lines is shown in Fig. 8 and was similar for all the beams of this series.

(3) Deflection Dial Indicators. Deflection measurements and equipment used for bonded and partially bonded beams is as described in Section 9(a).

(c) Loading Apparatus

All beams were tested in a specially constructed frame employing a 30-ton capacity Simplex hydraulic ram operated by a Blackhawk pump. Details of the frame are shown in Fig. 10. The hydraulic jack was used to apply deformation and a 50,000-lb capacity elastic ring dynamometer was used to measure the corresponding load. The dynamometer was equipped with a dial indicator that was calibrated at 111-lb. per division; it was sensitive to about one-tenth of one division.
(d) **Testing Procedure**

After prestressing beams of Series W, no appreciable tensile stresses occurred at the top extreme fiber because the center of gravity of the wire reinforcement was located at the lower kern point of the cross-section. For Series J the center of gravity of the wire reinforcement was approximately one inch outside the kern point and tensile stresses occurred in the top extreme fiber. For all the beams except beam J-7 the magnitude of these stresses was less than that which would cause cracking. Since beam J-7 had a large amount of reinforcement, precautions were taken to insure that no cracking would occur at the top surface. The top of the beam was first prestressed externally by means of two wires. The prestress was then released in the bonded reinforcement. When sufficient load was applied to induce net compressive stresses on the top of the beam, the external wires were removed. Because the behavior of the beam was linear at this stage there was no difficulty in correcting the deflection and strain readings to account for the external wires.

The failure load was reached in about five hours and eight increments of load. The load and deflection readings were taken during, and before and after each increment. All strain readings were recorded and the cracks were marked with ink after each increment. Usually there were about two increments of load up to the cracking load. Thereafter, increments were based on strain and deflection measurements rather than load. A drop-off in load and increase in deflection occurred while strain and deflection readings were being recorded. The beams were loaded until they ruptured completely or failed to develop increased resistance to large deformations.
III. PRESENTATION OF TEST RESULTS

10. Measured and Derived Quantities

All the beams were tested to failure by applying usually seven or eight increments of load. The following quantities were measured immediately following each increment: (1) applied load, from the elastic ring dynamometer; (2) deflections at midspan and at the third points, from the dial gages; (3) concrete strains on the top of the beam in the flexure span, from SR-4 gages; (4) average distribution of concrete strains throughout the depth of the beam, from measurements with a Whittemore gage; (5) steel strains from SR-4 gages mounted on the wires and also indirectly from dynamometers, in the case of unbonded beams; and (6) the distance from the top of the beam to the top of the highest crack.

After the cracking load was exceeded, the height and location of each crack was marked with ink. Usually, photographs were taken in the later stages of the test prior to failure and also following failure. After failure, the depth to the reinforcement, and, in the case of unbonded beams, the depth to the top of the reinforcement channel at the region of failure were determined. The location of the mild steel, when present, was also established by measurements at the region of failure. The width and total depth of the beam were measured at several locations prior to testing.

Since crushing of the concrete usually occurred during the application of a load increment, the corresponding strain readings could not be obtained directly. Consequently, concrete strains corresponding to crushing were extrapolated from plots of strain versus deflection.

Using the measurements mentioned above, the following quantities were derived: (1) the flexural moment in the beam; (2) the depth to the reinforcement; (3) the assumed depth to the neutral axis; (4) the stress in the reinforcement;
and (5) the strains in the concrete. For the unbonded beams, which had relatively large reinforcement channels, the depth to the reinforcement was based on a computation involving the ultimate deflection of the beam and the depth to the top of the channel. The reinforcement in these beams remained horizontal until it came in contact with the top of the channel. The ultimate reinforcement stress was computed using the depth to the reinforcement at failure, measured depth of the compression zone, \( k_2 = 0.42 \), and the physical properties of the beam. This was compared with the ultimate stress obtained from strain measurements. Also computed were the effective strength of the concrete in the beams, \( f_{cu} \), and the parameter \( F_{cu} \). Various measured and derived quantities are given in Table 4.

11. Load-Deflection Characteristics

Load-deflection curves for the unbonded, bonded, and partially bonded specimens are plotted in Figs. 11, 12, 13 and 14. Each curve is the result of about a hundred separate load-deflection readings, most of which were taken on the run. At the end of each increment of load, while strain readings were being recorded, an increase in deflection and corresponding drop-off in load occurred. These decreases were not plotted since they should have had no effect on the envelope load-deflection curve. Discussion of load-deflection characteristics and comparisons with other similar beams are given in Section 18.

12. Strain Measurements

(a) Distribution of Concrete Strains on the Top of the Beam

Strains in the concrete on the top of the beam in the flexure span were measured with Type A1 or A3 SR-4 electric strain gages. These gages and their spacing were chosen because it was desired to obtain not only a measurement of the maximum strains occurring but also their distribution.
The strain distributions measured from unbonded beams with additional bonded reinforcement are shown on Fig. 15. Those measured from pretensioned beams are shown on Figs. 16, 17 and 18. The results of strain distribution measurements for the partially bonded beams are plotted on Figs. 19, 20 and 21. All the plotted distributions are based on strains extrapolated to crushing of concrete except in Fig. 17 where the development of the strain distribution and changes in the crack pattern have also been shown. On this figure, the loads corresponding to stages in the development of the strain distribution are recorded. The maximum values of the measured concrete strains ranged from 0.0035 to 0.0050. These results are discussed in Section 20 of this report in relation to measurements by other investigators.

The distribution of concrete strains in the flexure span is not uniform in any one case. An examination of the different beams shows that the peaks in the distribution of strain occur above crack locations and that the highest peak occurs in the plane of the highest crack. Comparison of beams with low and high steel percentages shows that the distribution throughout the flexure span varies with the crack pattern. Beams having low amounts of steel have fewer cracks resulting in strain distributions which are not as uniform as those in beams with greater amounts of steel and consequently with a larger number of cracks.

The minimum strain occurring in the flexure zone for all the beams was greater than 0.002 with the exception of beam W-3 which had a minimum strain of 0.0019.

(b) Distribution of Strains Over the Depth of the Beam

The distribution of concrete strains over the depth of the beam were obtained by measuring the deformation between gage plugs with a 10-in. Whittemore strain gage.
For Series J beams these readings were taken at five different levels at three locations on each side of the beam. The top three levels were located so as to fall within the uncracked zone of the concrete at failure in most of the specimens. The location of the gage lines are shown in Fig. 8. For Series W beams, mechanical strain measurements were made at four levels as shown in Fig. 7. In the following section, only the results from Series J will be discussed since strains taken at five levels gave results that were more significant.

The strain distribution indicated by the average of the individual gage lines on opposite sides of the beam are shown as solid lines on Figs. 16 to 21. These individual distributions are plotted with respect to datum lines located at the center of their respective gage lines. The distributions obtained by averaging the results of six gage lines at the same level are shown as solid lines at the extreme right on Figs. 16 to 21.

In each beam, the variation of strain before cracking was linear over the depth and fairly consistent between the different gage lines. However, after cracking, the strains obtained on individual gage lines varied, depending on the extent, spacing and type of cracking. Once cracking had progressed part way up the beam, the strains obtained from the gages located in the cracked zone depended primarily on the number of cracks between gage plugs, since the greatest proportion of the total deformation occurred at the crack.

In the fully-bonded beams, which should not have had any stress raisers to predetermine the crack location, cracking occurred at random along the flexure span. In each beam, crack spacing throughout the flexure span was fairly uniform. The total number of cracks varied with the amount of reinforcement in the beam. Beam J-3 (Q' = 14.5) had only three cracks within the flexure span, one within each gage line. These cracks extended vertically to about mid-depth of the beam and
began forking out as shown on Fig. 22. Near failure, a complicated crack pattern had evolved. Beams J-1 ($Q' = 43.3$) and J-7 ($Q' = 55.1$) had cracks which were more closely spaced. Moreover, these cracks did not fork out to the extent that those in beam J-3 did. Fig. 22 shows the crack pattern for beam J-1 and Fig. 31 shows the crack pattern for beam J-7.

The strains measured on all the fully-bonded beams varied from one gage line to another. The greatest amount of distortion in the distribution of strains within a gage line and between different gage lines was noted in beam J-3.

Strains measured in the compressive zone of all the beams were linear and the magnitude of strain at the same level was similar for the various gage lines. The average distribution of strain throughout the flexure span was taken to be the average of the results obtained on the individual gage lines.

For the partially unbonded beams, particularly J-6 and J-8, cracks occurred first at the edges of the unbonded regions. Both these beams had fewer cracks in the flexure span than the comparable fully-bonded beam J-1. A comparison of the strains obtained on the central gage line of these two beams indicates how cracking affects the strains. In beam J-6 the cracking was such that there was no crack within the central gage line until the later stages of loading. A crack branching across from an adjacent crack crossed the gage line above the mid-height of the beam during load increment 4. The resulting distribution of strain for the central gage line at the crushing load is greatly distorted because the measurements indicate very little strain at the lowest level. The strains in the outer gage lines are also distorted; however, in this case the measurements at the lowest level show more strain than would be necessary for a linear distribution. An opposite effect is seen in beam J-8. Because of stress raisers, two cracks occurred within the central gage line and consequently measurements at the lowest level
indicate that more strain occurred there than at the same level in the outer gage lines. The magnitude of the measured strains are shown in Fig. 21. From these plots it is seen that the absence or presence of cracks affects greatly the strain distribution obtained on different gage lines.

Despite the lack of linearity in the plotted readings of the individual gage lines, the strain distributions based on averaging the six readings at each level were linear or nearly linear. On the basis of the observations made throughout the loading history of the beams, this result is to be expected. However, it should not be taken as an indication of linearity of strains over the depth of the beam.

![Sketch A](image)

**SKETCH A**

Sketch A shows the crack pattern at failure for a representative bonded simply-supported prestressed concrete specimen. It is assumed that no major inclined cracks have developed. The gage lines over which the strain measurements were taken are also shown on this sketch. A good portion of the shear span indicated on the figure has no flexural cracks, while the flexure span has several fully-developed cracks.
According to the readings taken during the tests, the strains are linearly distributed over the depth of the beam prior to cracking. Therefore, strains must be linear in the beam shown over the length from each reaction almost up to the extreme flexural cracks. This condition demands that whatever deformation takes place between the two uncracked portions of the beam must be such that it results in a linear distribution of strain when averaged over this distance.

The reported average strain distributions come very close to being linear based on measurements of the deformation over the total cracked portion of the beam. Therefore, it is reasonable that these should indicate linear or nearly linear distributions of strains.

Figs. 16 to 21 also show that the average strain on the top of the beam obtained by extrapolating the strains indicated by the Whittemore strain gage compares well with the average strains obtained from the electric strain gages. The largest variation is for beam J-8. In this beam the average strain indicated by the mechanical strain measurements was about six per cent greater than that indicated by the electrical strain measurements.

The depth of the compressive zone as obtained from measurements of the highest crack was smaller than the average depth indicated by measurements with the Whittemore strain gages. In the computations, the depth of the compression zone was taken as that given by measurements to the highest crack since this represented the actual conditions at the section of failure.

(c) **Steel Strains**

Steel strains for Series W were obtained from dynamometer strains and from electric strain gages. For Series J, only electric strain gages were used. Gage spacing is described in Section 9(b). Steel stresses were obtained from calibration curves for the dynamometers and from a stress-strain curve for the
steel. For the fully-bonded beams, because cracking occurred at random and possibly at some distance from a gage, the gage nearest the crack was chosen to represent the steel strain. The strain thus obtained, after converting to stress, was used for comparison with stress derived from load measurements. The strain gages on the reinforcement in beam J-6, which was unbonded in the flexure span, all gave similar readings. Beam J-8 had two unbonded portions. Fig. 23 shows the location of the gages and the strain readings of different gages during loading until crushing of this beam. Cracking occurred as shown in Fig. 21. For every load, the gages in the unbonded regions gave strain readings which were higher than the bonded gages. This is attributed to the fact that cracks occurred at the edges of the unbonded sections. Steel stresses for all the beams as obtained from strain measurements are compared with stresses from load measurements in Table 4.
IV. GENERAL ANALYSIS OF FLEXURAL STRENGTH OF PRESTRESSED CONCRETE BEAMS

13. Assumptions

The analysis presented in this report is a modification of Stüssi's theory for flexural failure (4). Primarily, it involves conditions of equilibrium and strain compatibility. The analysis is semi-empirical in that the magnitudes of several quantities necessary to determine the ultimate strength of prestressed beams are derived from experimental data.

A general expression for the ultimate flexural moment is derived in Section 14. The derivation and application of this expression depends on several assumptions. These assumptions are:

1. Conditions of statics are valid.
2. At ultimate, the strain compatibility factor, $F$, and the parameter, $k_u$, relate the steel strain to the maximum concrete strain which occurs at the extreme fiber in compression.
3. Concrete crushes at a limiting strain, $\varepsilon_u$.
4. The average stress in the concrete in the compression zone at failure, $f_{cu}^*$, is known.
5. The ratio, depth to the compressive force to depth to the neutral axis, $k_2 = 0.42$.
6. The stress-strain curve for the reinforcement is known.
7. No tension is resisted by the concrete.

Failure in a prestressed concrete beam is a local phenomenon, occurring nearly in a single plane. At the section of failure, Bernoulli's assumption of linear strain distribution with depth is generally not valid. As discussed previously in Section 12, concrete strains appear to be linearly distributed in the
compression zone of the beam. In the tension zone, cracking distorts the concrete strains. For bonded beams, this cracking together with the possible loss of bond adjacent to the crack affects also the distribution of steel strains. The factor F is introduced in the distribution of steel strains. The factor F is introduced in the analysis and, together with the parameter \( k_u \), relates the steel strain with the concrete strain at the extreme fiber in compression. This compatibility factor, F, does not appear by itself in the analysis. It is combined as a product with \( \varepsilon_u \) and appears in the analysis as a parameter \( F\varepsilon_u \).

Measurements of concrete strains at crushing in this investigation and as reported by previous investigators indicate that assumption 3 is valid. A discussion of these results is presented in Section 20.

Assumptions 4 and 5 relate to the conditions within the compression zone of the concrete. The stress distribution is non-linear and probably assumes some shape similar to the stress-strain curve for the concrete obtained from tests of 6 by 12 in. cylinders. Although the concrete stress varies throughout the depth of the compression zone, it is convenient to assume that the effective stress is an average value, \( f_{cu} \). Section 19 presents an empirical relationship for \( f_{cu} \) in terms of 6 by 12-in. cylinder strength. The total compressive force in the concrete is located a distance \( k_2 k_u d \) from the extreme fiber in compression. For a triangular stress distribution, \( k_2 = 0.33 \), and for a rectangular stress distribution, \( k_2 = 0.50 \). Because the actual stress distribution usually lies between these two limiting distributions, an average value of \( k_2 = 0.42 \) has been chosen. This value for \( k_2 \) is reasonable because the range of \( k_2 \) is small and it has been shown by Billet (1) to have a small effect on the ultimate moment.

The assumption that no tension is resisted by the concrete is not entirely true. Tensile stresses exist in the concrete between cracks and in the
region immediately below the neutral axis. Since the magnitude of the tensile stresses is small compared to the compressive stresses, and decreases with increasing steel percentage, no appreciable error should be introduced if tensile stresses are neglected.

14. Derivations

Considering beams reinforced in tension only, the conditions of stress and strain at the failure plane are shown in Fig. 24. They are based on the assumptions stated in the previous section.

If moments about the compression force in Fig. 24 are considered, the following simple expression may be written for the ultimate resisting moment:

$$ M_u = A_s f_{su} d (1 - k_u k_u) $$

The quantity $k_u$ can be evaluated in terms of beam properties from conditions of equilibrium provided that the beam is of constant width in the compression zone:

$$ C = T $$

$$ pbd f_{su} = bk_d f_{cu} $$

$$ k_u = \frac{p f_{su}}{f_{cu}} $$

(2)

The resulting expression is slightly more complicated if the width varies. With the help of Eq. 2, Eq. 1 may be rewritten in a dimensionless form:

$$ \frac{M_u}{f_{cu} bd^2} = k_u (1 - k_u k_u) $$

(3)

If the value of the ultimate steel stress, $f_{su}$, is determined, the ultimate resisting moment can be found from Eq. 1 with the help of Eq. 2. The steel
stress can be evaluated using the strain conditions at ultimate in conjunction with
the stress condition.

In accordance with assumption 2, Section 13, the relation between the
maximum concrete strain, $\varepsilon_u$, and the steel strain, $\varepsilon_{sa}'$, (Fig. 24), can be stated
as:

$$\frac{\varepsilon_{sa}'}{1 - k_u} = \frac{\varepsilon_u}{k_u}$$  \hspace{1cm} (4)

or

$$\varepsilon_{sa}' = F\varepsilon_u \frac{(1 - k_u)}{k_u}$$  \hspace{1cm} (5)

For a prestressed concrete beam, the ultimate steel strain, $\varepsilon_{su}'$, becomes:

$$\varepsilon_{su}' = \varepsilon_{sa}' + \varepsilon_{ce} + \varepsilon_{se}$$  \hspace{1cm} (6)

For practical ranges of the variables, $\varepsilon_{ce}$ is a very small quantity.
Therefore, for the sake of simplicity it can be neglected in Eq. 6, and added to
$\varepsilon_{sa}'$ in Eq. 4. Thus:

$$\frac{\varepsilon_{sa}}{1 - k_u} = \frac{\varepsilon_u}{k_u} F$$  \hspace{1cm} (7)

where:

$$\varepsilon_{sa} = \varepsilon_{sa}' + \varepsilon_{ce}$$  \hspace{1cm} (8)

Equations 6, 7 and 8 combine to give:

$$\varepsilon_{su} = F\varepsilon_u \frac{(1 - k_u)}{k_u} + \varepsilon_{se}$$  \hspace{1cm} (9)

For given values of $p$, $f_{cu}'$, and the stress-strain curve for the steel,
Eqs. 9 and 2 can be solved to give the value for the ultimate steel stress if $F\varepsilon_u$
is known or assumed.
15. The Effect of Variations in the Compatibility Factor on the Flexural Strength

The usefulness of the analysis in predicting the ultimate flexural strength is dependent on the derivation of a satisfactory empirical relationship for the parameter $F_{e_u}$. A study was made to determine how $F_{e_u}$ affects the flexural strength. This was done by using the analysis as presented in Section 14 and assuming a range of $F_{e_u}$ from 0.004 to 0.0001. A practical range of $Q'$ was chosen; $Q'$ ranged from 15 to 45 for effective prestress levels of 120,000 psi, 60,000 psi and 0. The stress-strain curve for Type X wire was used. Figs. 25, 26 and 27 show the values of the computed moments as ordinates and the assumed values of the parameter $F_{e_u}$ as abscissas. On these curves, points corresponding to the ultimate steel stresses of 200 ksi, 220 ksi and 240 ksi are indicated by curves as marked. The 0.2 per cent offset stress for Type X wire is 220 ksi (Fig. 2).

Figs. 25, 26 and 27 indicate that the reduction of the ultimate flexural moment is non-linear as $F_{e_u}$ is decreased. Beams having low values of $Q'$ have a small reduction in the ultimate moment for large decreases in $F_{e_u}$. Considering an effective prestress of 120,000 psi, a 50 per cent decrease in $F_{e_u}$ (from 0.004 to 0.002), causes a ten per cent reduction at the most, in the ultimate moment capacity for the range of $Q'$ studied. For beams having properties such that the stress in the reinforcement at the ultimate load is at least equal to the 0.2 per cent offset stress, the effect of $F_{e_u}$ on the magnitude of the ultimate moment will be small. For beams having properties such that the ultimate steel stress is less than the 0.2 per cent offset stress, the effect of $F_{e_u}$ is more important.

The effects of different levels of prestress can be seen when Figs. 25, 26, and 27 are compared. For the various values of $Q'$, only small variations in the ultimate moment are noted as the level of prestress is varied, whenever the ultimate steel stress is equal to or larger than the 0.2 per cent offset stress.
When the ultimate steel stress is less than the 0.2 per cent offset stress, a given decrease in $F_{e_u}$ causes larger decreases in the ultimate moment as the level of prestress is decreased.

In general it appears from this study that if the ultimate steel stress is in the inelastic range the resisting moment is insensitive to variations in the parameter $F_{e_u}$. When the ultimate steel stress is in the elastic range, the resisting moment is quite sensitive to $F_{e_u}$.

16. Factors Affecting the Compatibility Factor

(a) Strain Concentrations

From tests it has been observed that strain concentration is one of the factors affecting the variation in the compatibility factor, $F$. The effect of strain concentration can be shown qualitatively by considering a section of a bonded beam which has several well developed cracks. Fig. 28 shows such a section. Also shown are representative variations in concrete strain at the top surface of the beam, and in depth of the neutral axis. In the flexure span of a bonded beam, regardless of the bond conditions, there is a location between cracks where there is no relative movement between the steel and the concrete. Points A and B represent such locations. The effects of strain concentrations may be illustrated with the aid of deformation diagrams. Such a diagram is shown in Fig. 28. The horizontal axis of this diagram represents the position of any point between A and B to the left or to the right of the crack. The vertical axis represents the actual deformation of any point relative to a datum plane taken at the crack. The slope of any one of the deformation diagrams at any position, is the corresponding strain at that position.

With the given strain distribution and location of neutral axis, it is possible to obtain a deformation diagram to represent the total demanded deformation
at the level of the reinforcement between points A and B. For purposes of comparison, if it is assumed that the strain compatibility factor $F$ is one, then the deformation diagram is a graphical representation of

$$\sum_{A}^{B} \epsilon_c \frac{(1-k)}{k} \Delta L$$

and is shown as a heavy line in Fig. 28.

The concrete and steel deformations have also been shown in Fig. 28 to compare them with the "demanded" deformations. The concrete deformation between cracks will be very small since the maximum tensile strain that can occur in the concrete before cracking is about 0.0002. If the tensile strains exceed this amount then intermediate cracks will form. The concrete deformation diagram is shown as a dashed line in Fig. 28.

The deformation diagram for the steel varies depending on the conditions of bond. The expected limits of variation are:

1. Complete loss of bond between A and B giving a deformation diagram which is a straight line between A and B.

2. No loss of bond, in which case the steel deformation diagram will correspond to the concrete deformation diagram.

In any beam the bond properties will vary depending on the level of stress in the reinforcement, size and surface quality of the reinforcement, and the quality of the concrete. For beams with poor bond characteristics the actual deformation diagram of the steel will approach Case 1 and will have a maximum slope that is less than the maximum slope of the "demanded" deformation diagram. Since the slopes of the deformation diagrams represent strain, this results in values of $F$ which are less than one. When bond characteristics are excellent, the maximum slope of the steel deformation diagram is greater than the maximum slope of the
"demanded" deformation diagram and as a result $F$ is greater than one. Both of these conditions are shown in Fig. 28.

In Fig. 28, representative variations of strain at the top surface of the beam and depth of neutral axis have been shown. The effect of strain concentration on $F$ can be seen if the strain distribution on top is varied. For a uniform strain distribution and constant depth to the neutral axis, the "demanded" deformation diagram for $F$ equal to one will be a straight line joining A and B. For this case, any amount of bond will yield an actual deformation diagram which will have a maximum slope greater than the maximum slope of the "demanded" deformation diagram, and consequently $F$ will be greater than 1. Now, if the bond conditions remain the same, and the strain concentration is progressively made more acute, the slope of the "demanded" deformation diagram will increase and the value of $F$ will be decreased first to one and then to values below one. Thus, strain concentration together with probable loss of bond will generally give values for $F$ which are less than one.

(b) Loss of Bond in the Shear Span

No bond in the shear span is a major reason for the lower observed values of the compatibility factor $F$ in unbonded beams. For such beams the increase in steel strain with increases in load is uniformly distributed throughout the length of the steel. The effect of no bond in the shear span can also be shown qualitatively by deformation diagrams. However, now it is necessary to consider deformations over the entire length of the beam. For simplicity, it may be assumed that the concrete strains at the top surface are uniformly distributed throughout the flexure span and are negligible in the shear span. This is shown in Fig. 29. Using this strain distribution in conjunction with the assumed location of the neutral axis, a "demanded" deformation diagram is obtained for $F$ equal
to one. This is shown as a heavy line in Fig. 29. Because the largest amount of concrete strain at the top surface occurs within the flexure span, the major portion of the "demanded" deformation is located in this region. In the shear spans, the "demanded" deformations are small compared to the total deformations. Since the steel is unbonded, increases in steel strain are uniformly distributed throughout the length of the steel and the actual deformation diagram for the steel will be a straight line joining points X and Y. The maximum slope of the deformation diagram for the steel corresponding to the actual strain in the steel, is smaller than the maximum slope of the "demanded" deformation diagram. For fully unbonded beams these conditions will always be true and the value of F will be smaller than one. The effect of strain concentration, when superimposed on the effects of unbonding in the shear span, will cause a further decrease in the value of F. On Fig. 28 the deformation of the steel between cracks in the flexure span of an unbonded beam may be represented by a straight line joining points A' and B'. Because cracks are spaced farther apart in unbonded beams as compared to bonded beams, the distance to the crack from point A' or B' will be longer than that from point A or B. The total steel deformation between points A' and B' can be less than the "demanded" deformation because it is unbonded throughout the length of the beam. Thus, on Fig. 28 when strain concentration is neglected, F is the ratio of the slope of A' B' to the slope of line AB. When the effects of strain concentration are included, F is the ratio of the slope of line A'B' to the maximum slope of the "demanded" deformation diagram shown by the heavy line.

17. Unbonded Beams with Additional Bonded Steel

The analysis for unbonded beams with additional bonded steel is similar to the analysis for beams reinforced in tension only which is presented in Section 14. The assumed conditions for stress and strain at the ultimate load are shown
in Fig. 30. The following expression for the ultimate resisting moment is obtained by taking moments of all the forces about the compressive force in the concrete:

\[ M_u = A_s f_{su} d(l - k_2 k_u) + A_s' f_s' (k_2 k_d - d') + A_s'' f_s'' (d'' - k_2 k_d) \]  

This expression can be simplified if it is assumed that:

\[ A_s' = A_s'' \quad \text{and} \quad f_s' = f_s'' = f_y \]

For Series W these assumptions are reasonable since the areas of the top and bottom additional reinforcement were equal and since the measured steel strains indicated yield stresses. Consequently the expression for the ultimate moment becomes:

\[ M_u = A_s f_{su} d(l - k_2 k_u) + A_s' f_s' (d'' - d') \]  

Thus, Eq. 11 indicates that the total resisting moment is the sum of the resisting moment of the prestressed reinforcement and the resisting moment of the mild steel reinforcement.

The quantity \( k_u \) is obtained from the conditions of equilibrium:

\[ C' + C = T + T'' \]

\[ p' \frac{d''}{d} f_s' + k_u f_{cu} = p f_{su} + p'' \frac{d''}{d} f_s'' \]

and assuming that:

\[ A_s' = A_s'' \quad \text{and} \quad f_s' = f_s'' = f_y \]

this reduces to:

\[ k_u = p \frac{f_{su}}{f_{cu}} \]  

This expression for \( k_u \) is the same as that derived previously in Section 14 for beams reinforced in tension only. The ultimate resisting moment can be obtained from Eq. 11 with the aid of Eq. 2 if the yield stress for the mild steel and the ultimate stress for the prestressing steel are known. The yield stress for the
mild steel is obtained by tests as described in Section 6(e). The ultimate stress for the prestressing steel is obtained from strain conditions at the plane of failure in conjunction with the stress-strain curve for the steel.

On the basis of assumption 2 stated in Section 13 the relation between the maximum concrete strain and the steel strain (Fig. 30) can be stated:

\[
\frac{\epsilon_{sa}}{1 - k_u} = \frac{\epsilon_u}{k_u} F \quad (4)
\]

This relation is the same as derived previously for beams reinforced in tension only. In accordance with the derivation in Section 14 the relationship for determining the ultimate steel strain is:

\[
\epsilon_{su} = F\epsilon_{u} \left(\frac{1 - k_u}{k_u}\right) + \epsilon_{se} \quad (9)
\]

For given values of \( p, f_{cu} \) and stress strain curve, Eqs. 9 and 2 can be solved to give the ultimate steel stress if \( F\epsilon_{u} \) is known or assumed.
18. Behavior of Prestressed Concrete Beams Failing in Flexure

(a) Fully Bonded Beams

Fully bonded beams have previously been tested by Billet. All of these beams were post-tensioned and grouted. The beams included in the present series were all pretensioned but their behavior was similar to that of post-tensioned beams. Primarily, two modes of behavior were observed depending on the magnitude of the parameter $Q'$. Beams having small values of $Q'$ are under-reinforced while those with large values are over-reinforced. In both types of behavior the ultimate load was taken as the load observed at first crushing of the extreme compression fiber. In an under-reinforced beam, the steel strains at crushing extended well into the inelastic portion of the stress-strain curve whereas in the case of an over-reinforced beam the steel strain was elastic or extended only slightly into the inelastic portion. Failure of an under-reinforced beam is ductile whereas the failure of an over-reinforced beam is sudden and brittle. Later paragraphs describe both failures in detail.

Upon release of prestress, the beams deflected upward from 0.016 to 0.065 in. depending on the amount of steel. All beams behaved "elastically" up to first cracking. The load-deflection characteristics were straight lines and were similar for all beams since the concrete strengths and the beam cross-sections were similar, and the effect of different amounts of steel was small so far as the apparent stiffnesses of the beams were concerned.

The behavior of the beams from first cracking up to the ultimate load depended on whether the beams were under-reinforced or over-reinforced.

(1) Behavior of Under-Reinforced Beam J-3. Beam J-3 is a typical under-reinforced beam. As the load was increased beyond first cracking, more cracks
developed within the flexure span and in that portion of the shear span over which the applied moment was larger than the cracking moment. For beam J-3 there were three major cracks spaced from 10 to 12 in. within the flexure span and one crack in each shear span. The cracks within the flexure span developed very rapidly at first. Fig. 22 shows that at load increment number 2 (P = 7.5 kips) the cracks had risen to a height greater than mid-height of the beam. During the application of this load increment the cracks started to "fork" out. With further increases in load these "forked" cracks developed further and rose slightly. During the later stages of loading, small vertical cracks formed as branches of the "forked" cracks. Fig. 22 shows the crack pattern immediately after crushing.

The start of cracking produced a very abrupt change in slope of the load-deflection curve for under-reinforced beams as illustrated by the curve for beam J-3 in Fig. 12. Before cracking, the concrete carried tensile stress; after cracking, most of this stress is transferred to the steel. Because the percentage of steel is small in an under-reinforced beam this transfer increases the steel stress and strain appreciably, the neutral axis, which is approximately defined by the crack height, rises rapidly when cracking first occurs and then stabilizes as the stresses in the compressive zone of the concrete approach those of a fully developed stress block. The beam offered further resistance to load as long as the steel strains remained in the elastic region of the stress-strain diagram; however, once inelastic steel strains were produced, large deflections occurred with little increase in load. This continued up to crushing of the concrete. With increased deformation beyond first crushing, the beam continued to deflect at a nearly constant load. During this stage, the zone of crushing grew larger and extended downward into the region which was previously cracked. For beam J-3, the load began to drop off at a deflection greater than 2.4 in. (Fig. 12), and the
crushing progressed rapidly downward. The beam finally came to rest on the intermediate supports at a deflection of 2.7 in. The load-deflection diagram on Fig. 12 indicates that this type of behavior is very ductile and gives sufficient warning before collapse.

(2) Behavior of Over-Reinforced Beam J-7. Beam J-7 is an example of an over-reinforced beam. With increased load after first cracking, cracks developed throughout the flexure span and extended into the shear span. Nine cracks spaced from 3 to 6 in. were noted in the flexure span. Their development was rapid at first, but in the later stages the height of the crack remained nearly constant. These cracks did not rise as high as those in beam J-3 nor did they "fork" out as extensively. The nearly constant height of the cracks was an indication that conditions of a fully developed stress block were being realized.

The load-deflection characteristics beyond cracking did not change abruptly. The deflection increased at a greater rate than the load because the stiffness of the section was now reduced due to cracking. However, because the steel strains were still in the elastic portion of the stress-strain diagram, the beam resisted additional load up to first crushing. Moreover, additional load was resisted beyond first crushing, but in this stage, the zone of crushing grew larger and began moving downward rapidly. Collapse occurred violently with complete destruction of the compressive zone. Fig. 31 shows beam J-7 after first crushing and at complete collapse. A brittle failure of this type is not desirable since there is little or no warning of collapse.

(3) Behavior of Beam J-1. The behavior of beam J-1 was similar in most respects to the behavior of beam J-7. Beam J-1 might be classed as a beam with balanced percentage of reinforcement because at first crushing the steel strains were very near the 0.2 per cent offset stress on the stress-strain diagram.
Increased loading beyond that producing first cracking produced six major cracks, spaced 6 to 8 in. apart within the flexure span. These cracks progressed rapidly to nearly mid-height of the beam and then began to fork out. In the later stages, prior to crushing, the rate of crack development decreased and the height of the cracks appeared to be nearly stable.

The load-deflection characteristics, shown in Fig. 12, did not change abruptly beyond first cracking. Additional load was resisted at all stages up to first crushing. With increased deformation beyond first crushing, the beam offered a small resistance to load; however, during this stage, the zone of crushing increased and progressed downward. Although failure was not as sudden as the failure of beam J-7, it was nevertheless of a brittle nature.

(b) Unbonded Beams With Additional Bonded Mild Steel

The behavior of all beams in this series was similar and was like that commonly associated with over-reinforced beams. The beams deflected upwards from 0.032 to 0.043 in. immediately after prestressing. Their behavior before cracking was "elastic" with load deflection characteristics being linear and comparable for all beams. In all of these beams, first flexural cracking did not produce an abrupt change in the load-deflection curve. Instead, there was a gradual change in the stiffness of the beam beyond this stage. Cracks developed throughout the flexure span and in the shear span wherever the applied moment was larger than the cracking moment. The values of the parameter Q' for these beams ranged from 40 to 85 and the number of cracks observed in the flexure span ranged from 8 to 10. Comparable unbonded beams would have had three cracks or less as shown previously by Allen (3). The large number of cracks in these beams is attributed to the presence of bonded deformed mild steel bars. All cracks extended nearly vertically upward, and in no case was there any extensive forking of cracks. All of these beams failed violently. Crushing was observed first at the top fiber.
and developed downward at a very fast rate with little increase in load. This was followed by a gradual bowing out of the mild steel in the top of the beam. As the deformation was increased, crushing of the concrete was observed through the entire depth of the section. The load started dropping off rapidly, and final collapse took place by a sudden buckling out of the bottom mild steel. Fig. 32 shows the condition of beams W-2 and W-3 after failure.

(c) Partially Unbonded Beams

This series included five beams in which the reinforcement was unbonded in different portions of the span. Fig. 6 shows the locations of the unbonded portions of the beams. Unbonding was limited to the flexure span for all beams except J-5, which was unbonded in the shear spans only. Because unbonding was the only variable considered, the properties of these beams were chosen similar to those of beam J-1 so that direct comparisons could be made. These beams behaved, in general, like over-reinforced beams. Their behavior is described in the following paragraphs, first for the four beams with unbonded portions in the flexure span, and then for beam J-5 which was unbonded in the shear spans.

With release of prestress the beams which were partially bonded in the flexure span deflected upwards from 0.052 to 0.065 in. This deflection is larger than for fully-bonded beams because of the reduced section due to unbonding. The behavior up to cracking was "elastic"; the load deflection diagrams were straight lines and were similar for all beams, however, because of the reduced stiffness, the slopes were less than the slope for beam J-1. First cracking was observed at lower loads than the cracking load for beam J-1. With increased load, cracking extended throughout the flexure span and into the shear span. For these beams, only three or four cracks were noted in the flexure span. These cracks rose rapidly to mid-height of the beam and then began to "fork" out. For beams J-4 and J-8, these cracks began at the edges of the unbonded regions. In all the
beams, increased loading produced extensive forking of cracks, and as the ultimate load was approached, vertical cracks extended up from these "forked" cracks. Beyond first crushing, the beams offered small resistance to load. Crushing extended downward rapidly, and final collapse was quite violent.

Beam J-2 was initially cracked in the flexure span. Beam J-4 failed in shear by crushing of the concrete at the top of an inclined crack and near the loading point.

Fig. 13 shows the load-deflection curves for these beams. In all beams there was no abrupt change in the load-deflection characteristics at first cracking. The beams continued to resist load during all stages up to crushing. Generally, the behavior of the beams was similar to the behavior of beam J-1.

Beam J-5 had unbonded lengths of 30 in. in each shear span. With release of prestress, the beam deflected upward 0.057 in. Most of this deflection occurred within the shear span since the stiffness of the shear span was less than the stiffness of the flexure span. Up to cracking, the load-deflection relationship was linear. Cracks were noted first in the shear span at the edge of the unbonded regions. These cracks developed rapidly and at a load of 9.5 kips were within two inches of the top of the beam. Additional load produced cracking within the flexure span. At a load of 11.5 kips, four cracks spaced 8 to 10 in. apart had risen to mid-height of the beam. As more load was applied the crack development was confined mainly to the flexure span. In this region, the cracks forked out and rose at a slow rate. The widths of the two major cracks in the shear span increased to about 0.25 in. At a load of 18.5 kips slight crushing was noted above these cracks. Additional load did not produce further crushing at this location because of the restraining action of the external stirrups and the loading block. At a load of 21 kips, crushing was noted in the flexure span. With a small
additional load the zone of crushing moved down rapidly and final failure was rapid and violent. Fig. 33 shows beam J-5 in the later stages of loading.

The load-deflection curve of beam J-5 is compared with that of beam J-1 in Fig. 14. The cracking load was lower for beam J-5 because of the reduced stiffness of the section. Beyond cracking, the rate of increase of load appears to be similar for both beams. For similar load levels the center line deflection of beam J-5 is larger than the corresponding deflection for J-1. This larger deflection is attributed to the opening of the cracks in the shear span which made it possible for the steel in the unbonded region to reach a level of stress which was comparable to the level of stress in the flexure span.

19. Average Concrete Stress in the Beam at Ultimate

The average concrete stress in the compression zone of a beam at ultimate is denoted by $f_{cu}$. For beams of Series J, the values of $f_{cu}$ have been evaluated by the following relationship which is a transformation of Eq. 3:

$$f_{cu} = \frac{M_u}{bd^2 k_u (1 - k_2 k_u)}$$  \hspace{1cm} (12)

For Series W a similar expression for $f_{cu}$ was derived by combining Eqs. 11 and 12:

$$f_{cu} = \frac{M_u - A' f_y (d'' - d')}{bd^2 k_u (1 - k_2 k_u)}$$  \hspace{1cm} (13)

In Eqs. 12 and 13 all the quantities used to derive $f_{cu}$ are measured except $k_2$ which was assumed to be 0.42. In Table 4 the derived values of $f_{cu}$ for Series W and J are listed.

The measured values of $f_{cu}$ obtained from these tests, together with those reported by several other investigators for both prestressed and ordinary reinforced
Concrete beams have been plotted against $f'_c$, the strength of 6 by 12-in. cylinders, in Fig. 34. All the plotted points on Fig. 34 fall in a band which indicates a gradual lowering of $f'_c$ as $f'_t$ is increased. The following expression relating $f'_c$ to $f'_t$ was fitted by the least squares method to the plotted data in Fig. 34:

$$f'_c = \frac{f'_t}{0.8 + 0.0001 f'_c}$$

(14)

where $f'_c$ and $f'_t$ are in psi. This expression is plotted on Fig. 34 together with the expressions used previously by Billet and by Feldman and Allen, for prestressed beams.

The observed values of $f'_c$ for Series J lie slightly below the previously reported values for similar bonded and unbonded beams. A possible reason for these lower observed values of $f'_c$ is that 3/8-in. maximum size aggregate was used whereas the previous beams all had 3/4-in. maximum size aggregate.

In Series W the observed $f'_c$ values are considerably lower than previous reported values for $f'_c$. In these beams, the presence of mild steel bars may have affected the conditions of stress within the compression zone of the beam, however, it was observed that these bars did not exhibit tendencies of bowing out until after extensive crushing of the concrete.

20. Concrete Strains on the Top of the Beam

The maximum strain at first crushing of the concrete on the top surface of the beam is designated as $\varepsilon_u$. This quantity does not enter the analysis in Section 14 independently. However, once the parameter $F\varepsilon_u$ is established for any beam, then if $\varepsilon_u$ is known the magnitude of the strain compatibility factor, $F$, can be estimated.

For Series W the maximum concrete strain observed ranged from 0.0032 to 0.0040. In Series J, $\varepsilon_u$ was greater than 0.0040 for all beams. These values of
\( \epsilon_u \) have been plotted against the strength of 6 by 12-in. cylinders in Fig. 35 together with the values of \( \epsilon_u \) reported in references (1), (2), (3) and (5).

There appears to be no significant trend in \( \epsilon_u \) as \( f'_c \) is varied. For the range of \( f'_c \) from 2000 to 7000 psi, most of the observed values of \( \epsilon_u \) lie in the range from 0.0028 to 0.0050. The values of \( \epsilon_u \) reported are results obtained from strain measurements employing SR-4 electric strain gages which were either one or six inches long. It is significant to note that the 1-in. strain gages measured maximum concrete strains which were larger than the strains observed using 6-in. gages. In any beam, cracking affects the distribution of top concrete strains. Thus, whenever a 6-in. gage is used to measure the maximum strain in a region of strain concentration, an average strain over the gage length is obtained. Closely spaced 1-in. gages will give readings which will be higher than those of 6-in. gages, and will describe better the variations in concrete strains. In Fig. 35 an average value of 0.004 has been chosen for the maximum concrete strain \( \epsilon_u \). A large percentage of the reported values of \( \epsilon_u \) are smaller than 0.004, however, as most of these are values obtained from measurements using 6-in. gages, the maximum strains are probably higher than these "average" values.

A check on the reliability of the electric strain gages has been mentioned previously in Section 12(b). Comparisons of average strains on the top of the beam throughout the flexure span obtained by electrical measurements compared favorably with the results of mechanical measurements.

21. Derived Values of the Parameter \( F \epsilon_u \)

The values of the parameter \( F \epsilon_u \) have been derived for Series W and J using a transformation of Eq. 9:

\[
F \epsilon_u = \frac{k_u}{(1 - k_u)} \epsilon_{sa}
\]  

(15)
In computing the value of $F_{E_u}$ the steel strains have been derived from the measured moment at first crushing, and $k_u$ has been obtained from measurements of the crack height. Using the same procedure, values of $F_{E_u}$ have also been evaluated for previously tested bonded and unbonded beams from data presented by Billet (1), Feldman (2) and Allen (3). All of the values thus obtained are plotted in Fig. 36 as a function of $k_u$.

The values of $F_{E_u}$ obtained for Series W lie in the range of values previously observed for unbonded beams. The expression derived by Allen to relate $F_{E_u}$ to $k_u$ for beams loaded at the third points should, therefore, be satisfactory also for unbonded beams with added top and bottom mild steel.

For fully bonded and partially bonded beams Fig. 36 shows that the derived values of $F_{E_u}$ are scattered, particularly for the lower values of $k_u$. This scatter is due primarily to strain concentrations which have been previously discussed in Section 16. To compare the data shown in Fig. 36 a value of 0.004 for the crushing strain, $e_u$, as discussed in Section 20 has been chosen. When there are no strain concentrations in fully bonded beams the strain compatibility factor, $F$, should be at least one. If $e_u$ is chosen to be 0.004, then, with no strain concentrations, $F_{E_u}$ will be 0.004 regardless of $k_u$. This represents the "ideal" bonded beam. The scatter of $F_{E_u}$ observed in Fig. 36 is, then, primarily a consequence of strain concentrations, which vary in bonded beams. Those beams having low values of $Q'$ (and resulting low $k_u$ values), usually have a small number of cracks so that the strain concentrations are probably appreciable in magnitude. Beams having large values of $Q'$ will have cracks which are spaced more closely together than those of low $Q'$ beams and the resulting strain concentrations will be less. It is seen from Fig. 36 that as $k_u$ increases, the values of $F_{E_u}$ approach closer to $F_{E_u} = 0.004$. For $e_u$ equal to 0.004 the derived values of $F$ range from about 0.4 to 1.2.
For the unbonded beams, much lower values of $F_{e_u}$ have been derived. The primary reasons for these low values of $F_{e_u}$ are loss of bond in the shear span and strain concentrations, which have been discussed in Section 16. If loss of bond in the shear span were the only factor affecting the value of $F_{e_u}$ for unbonded beams, it would be possible to relate $F_{e_u}$ derived from the unbonded beams to the value for bonded beams. For practical purposes this could be done by taking the value of $F_{e_u}$ for the unbonded beam and modifying it by the ratio of the total length of the beam to the length of the flexure span. In Fig. 36 the broken line was obtained by multiplying the ordinates of the solid line by three which represents the ratio mentioned for beams loaded at the third points. Since the solid line included both the effects of unbonding in the shear span and strain concentration for unbonded beams, the broken line should be representative of bonded beams having strain concentration characteristics of unbonded beams. Because of the wider spacing of cracks and different deformation characteristics, the strain concentrations in unbonded beams are usually more acute than in bonded beams. For this reason the broken line would be expected to represent a lower limit to the values of $F_{e_u}$ observed in bonded beams, and it does.

The foregoing statements can be summarized in the following sketch:

![Sketch B](image-url)
If it were possible to evaluate quantitatively the effects of strain concentration, a general expression relating $F_\varepsilon_u$ with $k_u$ for both bonded and unbonded beams could be derived. However, because of the scatter of the data from the bonded beams, such a relationship has not been attempted in this report.
VI. SUMMARY AND CONCLUSIONS

The objective of the tests described in this report was to study further the effects of several variables on the flexural strength and behavior of prestressed concrete beams. Two series of beams were tested. In Series W the effect of added top and bottom mild steel in post-tensioned unbonded beams was studied. The behavior of fully bonded and partially bonded beams was studied in Series J.

There were three beams in Series W and eight in Series J. All the beams were rectangular and had a nominal cross-section of 6 by 12-in. Beams of Series W were 10-ft. long and those of Series J were 11-ft. long. All beams were reinforced with cold drawn high strength steel wire reinforcement. The level of prestress was nominally 120,000 psi for all the beams. All beams were loaded at the third points of a 9-ft. span. The major variable for Series W was the percentage of steel. For Series J the variables were the percentage of steel and partial unbonding. The percentage of steel for all the beams ranged from about 0.2 to 0.8. The concrete mixes were designed to yield cylinder strength of 4500 psi, however, the strengths varied from 4000 to 5500 psi.

The analysis presented in this report is a modification of Stussi's theory for flexural failure. A strain compatibility factor has been introduced which, together with $k_u$, relates the steel strain to the maximum compressive concrete strain. An expression for the ultimate steel strain in terms of the parameter $F_{eu}$ has been derived. This analysis is general and applies to bonded or unbonded beams; however, for each case a different value for the parameter $F_{eu}$ must be used. It has been shown that strain concentrations and loss of bond in the shear spans are the primary factors affecting the variations in $F_{eu}$.
The behavior of beams of Series W was similar to beams previously tested which had only additional bottom mild steel (3). For these beams, the moment capacity was larger than for similar unbonded beams. This was due to the resisting moment of the added mild steel reinforcement. The wire reinforcement stresses at ultimate, however, were not increased significantly from those observed in unbonded beams without bonded mild steel. The primary effect of the bonded mild steel was to distribute the cracks and consequently reduce the strain concentrations at the top surface of the beam. The derived values of $F_u$ for these beams, however, were in the range previously observed for unbonded beams. Values of the effective stress in the compression zone at ultimate, $f_{cu}$, were lower than those previously observed for unbonded beams. It is possible that the presence of the mild steel reinforcement affected the stress conditions within the compression zone at ultimate; however, the reinforcement did not bow out until extensive crushing of the concrete occurred.

The behavior of the pretensioned beams was similar to that of the post-tensioned beams tested previously (1). However, larger values of the maximum concrete strain at crushing, $\varepsilon_u$, were measured although the distribution of concrete strains throughout the flexure span was similar. Values of $f_{cu}$ derived for the pretensioned beams were slightly lower than those reported for post-tensioned grouted beams. These values of $f_{cu}$ have been compared with the results of other investigations and a simple relation for $f_{cu}$ in terms of $f'_c$ has been derived. In general, no major differences were noted in the behavior of pretensioned beams as compared to post-tensioned grouted beams.

The behavior of the partially bonded beams was similar to the behavior of a comparable fully bonded beam. The load-deflection curves for the beams with varying degrees of bond in the flexure span were all similar. The level of stress
in the reinforcement at ultimate was comparable to that observed in fully bonded beams. Derived values of $f_{cu}$ were also in the range of values observed for the other bonded beams. For these beams, however, the crack pattern was different. All beams had only three or four cracks within the flexure span. These cracks forked extensively in the later stages of loading and the resulting crack patterns were similar to fully unbonded beams. The behavior of the beam which was unbonded in the shear spans only was like that of a fully bonded beam because failure was prevented in the shear span. The values of $P_{cu}$ derived from the tests of partially bonded beams were comparable to those derived from the tests of fully bonded beams.
VII. BIBLIOGRAPHY


TABLE 1
SIEVE ANALYSIS OF AGGREGATES
Percentages Retained

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<td>19.2</td>
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<td>No. 16</td>
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<td>24.9</td>
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<td>No. 30</td>
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<td>42.5</td>
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<td>No. 50</td>
<td>93.2</td>
<td>93.2</td>
<td>95.0</td>
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<tr>
<td>No. 100</td>
<td>98.9</td>
<td>98.2</td>
<td>98.5</td>
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<tr>
<td>Fineness Modulus</td>
<td>3.10</td>
<td>2.70</td>
<td>3.30</td>
</tr>
</tbody>
</table>
### TABLE 2

**PROPERTIES OF CONCRETE MIXES**

<table>
<thead>
<tr>
<th>Beam</th>
<th>Batch</th>
<th>Cement:Sand:Gravel by weight</th>
<th>Water/Cement by weight</th>
<th>Slump in.</th>
<th>Compressive Strength, $f'_c$ psi</th>
<th>Modulus of Rupture, $f_r$ psi</th>
<th>Age at Test days</th>
<th>Aggregate Lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-1</td>
<td>1</td>
<td>1:2.73:4.20</td>
<td>0.76</td>
<td>3 2 1/2</td>
<td>5490 5430</td>
<td>454 410</td>
<td>8</td>
<td>A</td>
</tr>
<tr>
<td>W-2</td>
<td>1</td>
<td>1:2.80:4.19</td>
<td>0.70</td>
<td>3 4</td>
<td>4750 4760</td>
<td>452 460</td>
<td>6</td>
<td>A</td>
</tr>
<tr>
<td>W-3</td>
<td>1</td>
<td>1:2.77:4.19</td>
<td>0.71</td>
<td>3 1/2 3 1/2</td>
<td>4960 5120</td>
<td>426 435</td>
<td>6</td>
<td>A</td>
</tr>
<tr>
<td>J-1</td>
<td>2</td>
<td>1:3.20:3.50</td>
<td>0.74</td>
<td>2 2 1/2</td>
<td>4270 3970</td>
<td>428 388</td>
<td>8</td>
<td>B</td>
</tr>
<tr>
<td>J-2</td>
<td>2</td>
<td>1:3.19:3.44</td>
<td>0.79</td>
<td>4 4 1/2</td>
<td>4990 5090</td>
<td>492 525</td>
<td>7</td>
<td>B</td>
</tr>
<tr>
<td>J-3</td>
<td>2</td>
<td>1:3.18:3.49</td>
<td>0.77</td>
<td>1 3</td>
<td>5250 5280</td>
<td>477 451</td>
<td>7</td>
<td>B</td>
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<tr>
<td>J-4</td>
<td>2</td>
<td>1:3.33:3.43</td>
<td>0.77</td>
<td>4 4</td>
<td>5430 4970</td>
<td>535 456</td>
<td>7</td>
<td>B</td>
</tr>
<tr>
<td>J-5</td>
<td>2</td>
<td>1:3.20:3.45</td>
<td>0.77</td>
<td>2 3</td>
<td>5060 4900</td>
<td>547 425</td>
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<td>B</td>
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<tr>
<td>J-6</td>
<td>2</td>
<td>1:3.24:3.45</td>
<td>0.70</td>
<td>3 3 1/2</td>
<td>5460 5100</td>
<td>467 475</td>
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<td>C</td>
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<tr>
<td>J-7</td>
<td>2</td>
<td>1:3.20:3.46</td>
<td>0.71</td>
<td>4 1/2 4 1/2</td>
<td>6190 5230</td>
<td>397 418</td>
<td>8</td>
<td>C</td>
</tr>
<tr>
<td>J-8</td>
<td>2</td>
<td>1:3.18:3.42</td>
<td>0.76</td>
<td>3 2 1/2</td>
<td>4410 4110</td>
<td>460 412</td>
<td>7</td>
<td>C</td>
</tr>
<tr>
<td>Beam</td>
<td>b</td>
<td>d</td>
<td>d'</td>
<td>d''</td>
<td>No. of Wires and Area, $A_s$ sq. in.</td>
<td>Wire Type</td>
<td>$A'_s = A''_s$</td>
<td>$f_y$ ksi</td>
</tr>
<tr>
<td>------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
<td>-------------------------------------</td>
<td>-----------</td>
<td>-----------------</td>
<td>-----------</td>
</tr>
<tr>
<td>W-1</td>
<td>6.01</td>
<td>7.30</td>
<td>1.17</td>
<td>10.47</td>
<td>6-0.179</td>
<td>IX</td>
<td>0.22</td>
<td>47.7</td>
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<tr>
<td>W-2</td>
<td>5.97</td>
<td>7.48</td>
<td>1.17</td>
<td>10.53</td>
<td>10-0.299</td>
<td>IX</td>
<td>0.22</td>
<td>47.9</td>
</tr>
<tr>
<td>W-3</td>
<td>6.01</td>
<td>7.51</td>
<td>1.22</td>
<td>10.49</td>
<td>12-0.358</td>
<td>IX</td>
<td>0.22</td>
<td>48.0</td>
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<tr>
<td>J-3</td>
<td>6.01</td>
<td>9.10</td>
<td>----</td>
<td>----</td>
<td>3-0.091</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-1</td>
<td>6.30</td>
<td>9.06</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-7</td>
<td>6.06</td>
<td>9.08</td>
<td>----</td>
<td>----</td>
<td>12-0.362</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-2</td>
<td>6.09</td>
<td>9.00</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-6</td>
<td>6.08</td>
<td>8.90</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-4</td>
<td>6.04</td>
<td>8.95</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>J-8</td>
<td>6.09</td>
<td>8.98</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
<td>----</td>
<td>----</td>
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<tr>
<td>J-5</td>
<td>6.06</td>
<td>9.00</td>
<td>----</td>
<td>----</td>
<td>7-0.211</td>
<td>X</td>
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</table>
TABLE 4

MEASURED AND DERIVED QUANTITIES

<table>
<thead>
<tr>
<th>Beam</th>
<th>M&lt;sub&gt;cr&lt;/sub&gt;</th>
<th>M&lt;sub&gt;u&lt;/sub&gt;</th>
<th>A&lt;sub&gt;fs&lt;/sub&gt;&lt;sup&gt;y&lt;/sup&gt;(d&quot;-d')</th>
<th>k&lt;sub&gt;d&lt;/sub&gt;</th>
<th>k&lt;sub&gt;u&lt;/sub&gt;</th>
<th>f&lt;sub&gt;su&lt;/sub&gt;</th>
<th>f&lt;sub&gt;se&lt;/sub&gt;</th>
<th>F&lt;br&gt;&lt;sub&gt;0&lt;/sub&gt;</th>
<th>f&lt;sub&gt;cu&lt;/sub&gt;</th>
<th>Q'</th>
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<tbody>
<tr>
<td></td>
<td>in. kips</td>
<td>in. kips</td>
<td>in. kips</td>
<td>in.</td>
<td>ksi</td>
<td>ksi</td>
<td>ksi</td>
<td>ksi</td>
<td>psi</td>
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<tr>
<td>W-1</td>
<td>149</td>
<td>166</td>
<td>314</td>
<td>98</td>
<td>1.90</td>
<td>0.260</td>
<td>158</td>
<td>159</td>
<td>186</td>
<td>121.4</td>
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<tr>
<td>W-2</td>
<td>201</td>
<td>194</td>
<td>402</td>
<td>99</td>
<td>2.90</td>
<td>0.388</td>
<td>148</td>
<td>148</td>
<td>162</td>
<td>124.4</td>
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<tr>
<td>W-3</td>
<td>203</td>
<td>204</td>
<td>423</td>
<td>97</td>
<td>3.20</td>
<td>0.426</td>
<td>141</td>
<td>134</td>
<td>147</td>
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<td>J-3</td>
<td>123</td>
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<td>1.50</td>
<td>0.165</td>
<td>240</td>
<td>---</td>
<td>252</td>
<td>118.2</td>
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<tr>
<td>J-1</td>
<td>171</td>
<td>180</td>
<td>361</td>
<td>--</td>
<td>3.35</td>
<td>0.370</td>
<td>215</td>
<td>---</td>
<td>218</td>
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<td>J-7</td>
<td>270</td>
<td>273</td>
<td>569</td>
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<td>0.418</td>
<td>208</td>
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<td>205</td>
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<td>J-2*</td>
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<td>362</td>
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<td>2.45</td>
<td>0.272</td>
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<td>---</td>
<td>219</td>
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<td>J-6</td>
<td>160</td>
<td>173</td>
<td>365</td>
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<td>2.28</td>
<td>0.256</td>
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<td>---</td>
<td>218</td>
<td>112.3</td>
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<td>J-4**</td>
<td>153</td>
<td>171</td>
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<td>--</td>
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<tr>
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<td>167</td>
<td>370</td>
<td>--</td>
<td>2.36</td>
<td>0.263</td>
<td>220</td>
<td>---</td>
<td>220</td>
<td>112.1</td>
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<tr>
<td>J-5</td>
<td>160</td>
<td>171</td>
<td>404</td>
<td>--</td>
<td>2.30</td>
<td>0.207</td>
<td>233</td>
<td>---</td>
<td>222</td>
<td>115.0</td>
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Column

(1) Measured in test
(2) Based from f<sub>u</sub> measured from control specimens
(3) From M<sub>cr</sub>-A<sub>fs</sub><sup>y</sup>(d"-d') and measured k<sub>u</sub> for Series W, M and measured k<sub>u</sub> for Series J
(4) From SR-4 gages
(5) From Eq. 15 using f<sub>su</sub> from column 9 and measured k<sub>u</sub>
(6) From Eqs. 12 and 13
* Beam J-2 was initially cracked in flexure span
** Beam J-4 failed in shear
FIG. 1 STRESS-STRAIN RELATIONSHIP FOR TYPE IX WIRE

Ultimate Stress = 251 ksi

0.2 percent Offset Stress = 199 ksi

E_s = 30,000 ksi

0.195-in. Diameter
8-in. Gage Length
Ultimate Stress = 267 ksi
0.2 percent Offset Stress = 220 ksi

$E = 50,000 \text{ ksi}$

0.196-in. Diameter 8-in. Gage Length

FIG. 2 STRESS-STRAIN RELATIONSHIP FOR TYPE X WIRE
FIG. 3  TYPICAL STRESS-STRAIN RELATIONSHIP FOR NO. 3 BARS

- $f_y = 48$ ksi
- $E_s = 30,000$ ksi
- 8-in. Gage Length
- Ultimate Stress = 75 ksi
Reinforcing Cage for Series W Beams.

No. 3 Mild Steel Deformed Bars
No. 2 Plain Ties

Cross-section of Beam Showing Location of Top and Bottom Mild Steel Bars.

FIG. 4 DETAILS OF ADDED STEEL FOR SERIES W BEAMS
Prestressing Frame and Completed Boxes.

Cross-section of Beam Showing Box for Partial Unbonding of Beams.

FIG. 5 DETAILS OF BOX FOR SERIES J BEAMS
Beams W-1, W-2, and W-3.

Beams J-2 and J-6.
(Note - Beams J-1, J-3 and J-7 similar except fully bonded)

Beam J-4

Beam J-8

Beam J-5

FIG. 6 LOCATION OF UNBONDED SECTIONS
Electric Strain Gages on Top of Beam

Whittemore Strain Gage Lines on Sides of Beam

FIG. 7 LOCATION OF GAGES ON CONCRETE - SERIES W
Electric Strain Gages on Top of Beam

Whittemore Strain Gage Lines on Sides of Beam

FIG. 8 LOCATION OF GAGES ON CONCRETE - SERIES J
FIG. 9 WATERPROOFED GAGE
FIG. 10  LOADING FRAME TEST SET-UP
FIG. 11  LOAD-DEFLECTION CURVES FOR BEAMS WITH ADDED MILD STEEL TOP AND BOTTOM

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f'_{c}$</th>
<th>p percent</th>
</tr>
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<tbody>
<tr>
<td>W-1</td>
<td>5430</td>
<td>0.408</td>
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<td>W-2</td>
<td>4760</td>
<td>0.670</td>
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<tr>
<td>W-3</td>
<td>5120</td>
<td>0.792</td>
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X First Crushing
FIG. 12 LOAD-DEFLECTION CURVES FOR FULLY BONDED BEAMS
FIG. 13 LOAD-DEFLECTION CURVES FOR BEAMS PARTIALLY UNBONDED IN FLEXURE SPAN
FIG. 14 LOAD-DEFLECTION CURVE FOR BEAMS J-1 AND J-5
FIG. 15 CONCRETE STRAIN DISTRIBUTION ON THE TOP SURFACE OF SERIES W BEAMS AT ULTIMATE
FIG. 16  STRAIN DISTRIBUTIONS IN FLEXURE SPAN FOR BEAM J-3
Fig. 17  Strain Distributions in Flexure Span for Beam J-1
Distribution of Top Concrete Strain at Ultimate

Distribution of Concrete Strains Over the Depth of the Beam at Ultimate

FIG. 18 STRAIN DISTRIBUTIONS IN FLEXURE SPAN FOR BEAM J-7
Measured Strain (SR-4 gages)

Average Strain (SR-4 gages)

Distribution of Top Concrete Strain at Ultimate

Distribution of Concrete Strains Over the Depth of the Beam At Ultimate

FIG. 19 STRAIN DISTRIBUTIONS IN FLEXURE SPAN FOR BEAM J-5

<table>
<thead>
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<th>Load Increment</th>
<th>Total Applied Load - kips</th>
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<tbody>
<tr>
<td>1</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>9.7</td>
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<tr>
<td>3</td>
<td>11.3</td>
</tr>
<tr>
<td>4</td>
<td>12.8</td>
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<td>5</td>
<td>15.3</td>
</tr>
<tr>
<td>6</td>
<td>17.4</td>
</tr>
<tr>
<td>7</td>
<td>18.8</td>
</tr>
<tr>
<td>8</td>
<td>20.9</td>
</tr>
<tr>
<td>9 (Crushing)</td>
<td>21.2</td>
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</table>
FIG. 20 STRAIN DISTRIBUTION IN FLEXURE SPAN FOR BEAM J-6
FIG. 21  STRAIN DISTRIBUTION IN FLEXURE SPAN FOR BEAM J-8
FIG. 22 COMPARISON OF CRACKING FOR BEAMS WITH LOW AND HIGH VALUES OF Q'
FIG. 23  MEASURED DISTRIBUTIONS OF STEEL STRAINS FOR BEAM J-8

See Fig. 21 For Applied Load Corresponding To Load Number
FIG. 24  ASSUMED CONDITIONS OF STRESS AND STRAIN FOR BEAMS REINFORCED IN TENSION ONLY
FIG. 25  EFFECT OF $F_{e u}$ ON ULTIMATE MOMENT CAPACITY
FIG. 26  EFFECT OF $F_{e u}$ ON THE ULTIMATE MOMENT CAPACITY
FIG. 27  EFFECT OF $F_{\epsilon_u}$ ON ULTIMATE MOMENT CAPACITY
FIG. 29 DEFORMATION DIAGRAM FOR AN UNBONDED BEAM
FIG. 30 ASSUMED CONDITIONS OF STRESS AND STRAIN FOR BEAMS WITH ADDED TOP AND BOTTOM MILD STEEL
FIG. 31 OVER-REINFORCED BEAM AT FAILURE
FIG. 32  UNBONDED BEAMS WITH ADDITIONAL BONDED MILD STEEL AT FAILURE

Beam W-2
Q' = 70

Beam W-3
Q' = 85
FIG. 33 CRACK PATTERN FOR BEAM UNBONDED IN SHEAR SPANS ONLY
FIG. 34  RELATIONSHIP FOR AVERAGE CONCRETE STRESS IN THE BEAM AT ULTIMATE

\[ f_{cu} = \frac{f'_c}{0.8 + 0.0001f'_c} \]
FIG. 35  MAXIMUM CONCRETE STRAIN AT FIRST CRUSHING VS. CONCRETE STRENGTH
FIG. 56 RELATION BETWEEN $F_{e_u}$ AND $k_u$