THE EFFECTS OF STRESS, TIME, AND TEMPERATURE ON THE UPPER YIELD POINT OF STRUCTURAL STEEL

By
R. N. WRIGHT
and
W. J. HALL

A Technical Report
on a Research Program
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THE NATIONAL SCIENCE FOUNDATION
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Department of Civil Engineering
University of Illinois
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Richard Newport Wright III, Ph.D.
Department of Civil Engineering
University of Illinois, 1962

The upper yield point of structural steel defines the beginning of gross inelastic behavior and therefore is a significant factor in evaluating the response of steel structures. This dissertation is concerned with investigation of a theoretical mechanism for the initiation of yielding which permits prediction of the upper yield point in structural steel as a function of stress, temperature, and time.

The principal objective of this study is to determine whether the upper yield point can be predicted with the degree of exactness desirable for use in engineering research and design. The mechanism for the initiation of yielding is based upon the dislocation theory of plastic flow. Yield criteria are developed for two different conceptions of the critical event in yield initiation; the critical number theory yield criteria consider yielding to be triggered by a critical number of dislocations, while critical stress theory criteria consider the attainment of a critical stress on an obstacle to initiate yielding. Yield criteria which are intended to account for the effects of aging and back stress are presented in addition to the simpler criteria which neglect these effects. Techniques for the empirical evaluation of the material parameters appearing in the yield criteria are developed, and the expressions required for the application of the criteria are derived.

A test program designed to provide data for the evaluation of the criteria and to explore the range of validity was carried out on a 0.17 percent carbon steel in the as-rolled condition. No significant preferability for either the
critical stress or critical number theory yield criteria can be shown for the range of yield point studied - from 32 to 112 ksi axial stress. The simplest form of yield criterion considered, the critical number yield criterion of Eq. (2.8), is demonstrated to be valid for stress levels in excess of the upper yield stress level attained at room temperature with an axial stressing rate of 30 ksi/second.

It appears to be necessary to consider the effects of aging and back stress to obtain yield criteria which will be valid for lower levels of yield stress. An introductory study of these effects is carried out which indicates that both phenomena are likely to be effective at low yield stress levels, and further that the pre-yield aging phenomenon is probably stress sensitive.

Evaluations of the yield criterion of Eq. (2.8) with the test data from this and other investigations indicate that no one form of the activation energy function is applicable to all low carbon steels. However, it does appear that the following form for the activation energy stress derivative

\[
\frac{dU}{d\tau} = -\frac{K}{\tau^a}
\]

(where K and "a" are constants) provides a satisfactory fit to the data in the applicable region and is convenient for use. At the present time, the material parameters of the criteria cannot be accurately inferred from direct measurements of physical and chemical properties but must be obtained from an analysis of limited yield point data for the particular steel of interest.
ACKNOWLEDGMENT

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The author wishes to express his gratitude to his adviser Professor W. J. Hall, and to Professor G. M. Sinclair for their guidance in the planning of the investigation. Mr. H. S. Hamada, who worked closely with the author in the conduct of the program, was extremely helpful in every phase of the experimental work.

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CHAPTER 1. INTRODUCTION

1.1 Significance of the Upper Yield Point

Although the upper yield point in structural steel is not commonly used as a structural design parameter, its variation as a function of stress, time, and temperature is of considerable engineering significance. Several examples of areas in which an ability to predict the upper yield point may play an important part in current research, and later be reflected in engineering design practice, follow.

In the area of brittle fracture mechanics the upper yield stress level, which must be reached in order to initiate inelastic behavior, has great significance. If the yield level is greater than the stress level required to propagate a crack, the desired ductile behavior of the material may not be realized. Studies of the inelastic behavior of steel are concerned, at least indirectly, with the requirements for initiating plastic flow. The initiation of plastic flow is defined by the upper yield point - as a function of stress, temperature, and time. Investigators of the behavior of composite structural elements, such as reinforced concrete members, often are faced with situations in which they attempt to analyze the behavior of a composite element without sufficient knowledge of the behavior of the constituent elements. Efforts to predict the behavior of steel in low-cycle high-stress fatigue should be aided by an ability to predict the effects of the stress-time history on the initiation of plastic flow. In designing to resist rapid or impact loadings it is important to be able to predict the yield stress even if inelastic behavior is tolerable. These are a few of the examples which can be cited of areas in
which there is reason to believe that a better understanding of the upper yield point phenomenon may contribute significantly to engineering knowledge.

1.2 Background

Research in solid state physics and theoretical metallurgy in recent years has increased significantly the knowledge of the physical mechanisms governing the inelastic behavior of metals. Theories based on dislocation concepts of plastic flow have been advanced which have made possible the development of criteria for predicting the stress level at which steel will yield as a function of the stress-temperature history of loading.

The discussion of dislocation concepts of plastic flow which follows will not be preceded by an exposition of the dislocation theory. Texts on dislocation theory have been prepared by leaders in the development of the theory: A. H. Cottrell (9) and W. T. Read (26). Articles surveying the theory and its applications by Parker (24), Cuff and Shetky (11), and Dash and Tweet (12) are recommended for obtaining a rapid grasp of the physical significance of the theory.

The earliest explanation advanced for the phenomenon of plastic flow was that the atoms of a crystal slid as a whole over the atoms in the other side of the slip plane. However, Frenkel's calculation of the theoretical shearing stress, based upon the interatomic forces, resulted in a value of critical shearing stress for slip several orders of magnitude greater than experimentally observed values. The concept that slip results from a movement of imperfections, called dislocations, was advanced by Taylor, Orowan, and Polanyi in 1934. Theoretical and experimental studies have advanced the understanding of the effects of dislocations on the behavior of crystals to such a

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1 The numbers in parentheses following a name refer to bibliographical entries.
level that most of the present studies of the mechanisms of fatigue, brittle fracture, creep, and plastic flow are based upon dislocation models.

The original studies of the effects of loading rate on yield stress, flow stresses, and ultimate strength of metals preceded the advent of dislocation theories of plastic flow. Nadai (23) describes the results of exploratory studies of the effects of straining rate on flow stresses, flow stress is defined as the stress required at a given plastic strain and temperature to maintain a given rate of straining. Prominently mentioned by Nadai are the early work of Ludwik, and the more recent studies of Manjoine and Nadai. With the work of Cottrell and his associates in the late 1940's the dislocation theories of plastic flow reached a level of development useful to the investigators of rate effects on the yield level of steel. The abruptness of the transition is reflected, for example, in the work of the Clark and Wood group - from the phenomenological study appearing in Clark and Wood's 1949 paper (7) to the extensive discussion and use of dislocation concepts in Vreeland, Wood, and Clark's 1953 paper (29).

1.3 Objectives and Scope

The major objective of this study was to evaluate experimentally the range of validity of several criteria which have been proposed for predicting the upper yield point in low carbon steel. While investigators developing the criteria have supported them with results from tests of simple stress-time histories prior to yielding, no thorough investigation of the performance of the criteria with general pre-yield stress-time histories has been performed. As used herein, "general pre-yield stress-time history" means that the stress level may vary irregularly before yielding.
In addition, techniques of testing and data study for evaluation of the material parameters appearing in the criteria are presented and discussed in detail. An effort is made to determine whether these material parameters are functions of directly measurable physical properties.

The influence of stress-time history upon the upper yield point is described in Chapter 2, beginning with the development of yield criteria based upon dislocation concepts of plastic flow initiation. The formulation of the yield criteria follows the development used by Campbell (5) fairly closely, although attention also is given to the concept preferred by Hendrickson (17), namely, that a critical stress is responsible for the initiation of yielding. Yield criteria considering aging are developed using a mechanism suggested by Campbell and Duby (6). Also, yield criteria approximating the influence of back stresses are presented.

The program of study required to evaluate and validate the yield criteria is developed in Chapter 3. The approach to the problem is essentially empirical - largely because theoretically developed expressions for the activation energy function, such as those of Cottrell and Bilby (8), are rather complex and therefore poorly adapted to evaluation by experimental means.

In the light of the requirements for experimental work developed in Chapter 3, a test program was conducted as described in Chapter 4. Evaluations of the yield criteria from the test data are presented in Chapter 5. The simplest yield criterion is applied to tests of general stress-time history to determine how closely it will predict the yield points observed in the tests.

In Chapter 6 the form of the activation energy function which was determined to be appropriate for the steel used in this investigation is applied, with the simplest yield criterion, to data from other investigations. An effort
is made to determine the range of applicability of a particular activation energy function, and to determine whether correlations can be made between simple physical and chemical properties and the material parameters appearing in the yield criterion.

The conclusions which can be drawn from the results of the investigation are presented in Chapter 7. In addition, recommendations concerning work required to clarify major points of uncertainty in the yield initiation mechanism are presented.

1.4 Notation

- a: power of stress in the activation energy derivative.
- b: measure of the rate of aging (seconds\(^{-1}\)).
- \(b_y\): \(b\) evaluated at the yield stress level.
- \(b_{ym}\): \(b\) evaluated at the minimum yield stress level.
- e: base of natural logarithms.
- t: time (seconds).
- \(t_c\): time interval at constant stress.
- \(t_d\): delay time interval.
- \(t_y\): time of yield.
- \(t_{ym}\): time at which the minimum yield level is attained.
- A: constant in the activation energy function (cal/mol).
- B: constant in the activation energy function for the stress dependent portion.
- C: constant in the single temperature yield criterion (seconds).
- D: diffusion coefficient (seconds\(^{-1}\)), constant in the single temperature yield criterion for the stress dependent portion.
- E: Young's modulus (ksi).
F  Fahrenheit degrees of temperature.
K  Kelvin degrees of temperature, constant in the activation energy derivative.
N  number of free dislocations.
N_c critical number of free dislocations.
Q  damage increment (seconds).
R  gas constant, 1.986 cal/mol-°K.
T  absolute temperature (°K).
U  activation energy for dislocation release (cal/mol).
U_e first order approximation for U.
U_i U formulated to consider back stress.
U_{iy} U_i evaluated at the yield stress level.
U_Y U evaluated at the yield stress level.
U_{ym} U evaluated at the minimum yield stress level.
U_a activation energy for aging (cal/mol).
v  frequency factor (seconds⁻¹).
σ  axial stress (ksi), the significance of subscripts is the same as given for τ below.
τ  octahedral shearing stress (ksi).
τ_c critical stress on obstacle.
τ_i effective stress on source considering back stress.
τ_o constant with units of stress, stress on obstacle.
τ_{sy} static upper yield stress.
τ_Y upper yield stress.
τ_{ym} minimum upper yield stress.
τ_{ym} stress rate (ksi/second).
τ* theoretical stress rate required to attain a given τ_Y neglecting aging and back stress.
cal calories.
exp exponential.
ksi $10^3$ pounds per square inch.
ln natural logarithm.
log logarithm to the base 10.
mol gram molecular weight.
CHAPTER 2. DEVELOPMENT OF YIELD CRITERIA

2.1 Factors in the Dislocation Mechanism for Yielding

The concept that slip in metal crystals is produced by the movement of dislocations through the crystal lattice provides one explanation for the low value of shearing stress required to produce slip. However, the dislocation concept alone is inadequate to explain many of the phenomena associated with plastic flow. Why is an upper yield point observed in many body centered cubic lattice metals, that is, why is a larger stress required to initiate slip than to maintain steady plastic flow after slip begins? If the passage of a single dislocation through the crystal lattice produces one lattice spacing (about $3 \times 10^{-8} \text{ cm}$) of displacement in the slip plane, how are enough dislocations provided to produce large plastic strains? Why is the theoretical shearing stress required to move a dislocation through an otherwise perfect crystal lattice lower than the observed resistance of the metal to plastic deformation? These are some of the phenomena which must be explained before a dislocation theory can be considered to give an adequate representation of the processes involved in plastic flow.

Observed flow stresses are generally higher than the level required to move a dislocation through an otherwise perfect lattice since this level of crystal perfection is rare. Intersecting dislocations in other crystallographic planes, precipitate particles, and solute atoms form obstacles to the movement of a dislocation in the lattice of an individual crystal. The misalignment of crystals at grain boundaries impedes the transmittal of slip from grain to grain. Actually, the transmission of slip to an adjacent grain
is not considered to occur by passage of dislocations into the next grain, but by a concentration of stress at the grain boundary which causes dislocations in the next grain to move. In this study the grain boundaries, precipitate particles, etc., which impede dislocation movement, will be termed "obstacles". Detailed discussion of the interaction of dislocations with obstacles may be found in Cottrell (9). Here, it will be considered that the stress on an obstacle is proportional to the product of the applied stress and the number of dislocations restrained by the obstacle.

The question of the source for the large number of dislocations required to produce significant plastic strain was answered by Frank and Read (14). They postulated a mechanism by which a single dislocation line, locked at one or both ends by obstacles, can produce an unlimited number of dislocations. The details of the mechanism of the F-R, or Frank-Read, source are covered in most texts describing dislocation theory, i.e. Read (26). Examples of the experimental work which has resulted in pictures of F-R sources in operation are shown by Dash (12).

The explanation of the sharp yield point in body centered cubic lattice metal was proposed by Cottrell and Bilby (8). The lattice immediately below the extra half plane of atoms of an edge dislocation is expanded and offers favorable (low energy) sites for small impurity atoms such as carbon and nitrogen. Given time, such impurity atoms will diffuse to this position. This arrangement of impurity atoms is termed an "atmosphere" of the dislocation. As a result of the presence of the atmosphere the dislocation has a lowered misfit energy. A movement of the dislocation away from its atmosphere requires that its misfit energy be increased, and therefore requires more stress than is required to move the dislocation in the lattice. The atmosphere thus "locks" the dislocation. Because of the distortion of the body centered cubic lattice
by interstitial impurity atoms, an interaction of an atmosphere with screw dislocations also is possible. These atmosphere locking phenomena are considered responsible for the appearance of an upper yield point. The concept is bolstered by the disappearance of the sharp yield phenomenon at temperatures high enough to preclude the existence of an atmosphere along a dislocation line. A quantitative description of the interaction between solute atoms and dislocations is given by Cottrell (9).

Dislocations issued from a F-R source but held up by an obstacle exert a stress on the dislocation source. This "back stress" is of opposite sign to the applied stress and lowers the effective stress on the source. If the obstacle is sufficiently strong the back stress can become great enough to prevent issuance of further dislocations by the source. Vreeland (28) gives a theoretical analysis of the build up of a dislocation array at an obstacle in which he develops an expression for the number of dislocations in the array. A theoretical expression for back stress probably could be attained from an extension of his analysis.

2.2 Mechanism of Yield Initiation

Yield criteria are formulated herein for yielding initiated by the following process. The critical F-R source in the most favorably oriented crystal has a dislocation line locked by an atmosphere. As the stress level rises the source becomes active and releases dislocations. It is assumed that each dislocation issued must be removed from the locking atmosphere, that the dislocations move freely until held up at the grain boundary, and that yielding begins when the number of dislocations at the grain boundary becomes sufficient to activate F-R sources in the adjacent grain. The picture is highly simplified, many F-R sources should begin to release dislocations at
approximately the same stress level in many grains of a polycrystalline specimen, and the chain of slip must spread from grain to grain until a slip band forms across the specimen. However, this simple yield mechanism does contain the essential steps of yield initiation:

1. Dislocation sources become active and release dislocations.
2. Released dislocations are halted by obstacles which must be overcome before general yielding begins.

The process of yield initiation is confirmed by experimental evidence, data showing very small anelastic strains - on the order of $30 \times 10^{-6}$ in/in - occurring prior to yielding. This is termed "microstrain" and apparently is produced by the movement of dislocations from source to obstacle. The fact that micro-strain is measurable indicates that many sources release appreciable numbers of dislocations prior to yielding. The fact that at constant stress microstraining will occur and end without general yielding shows that back stress can cause sources to cease to release dislocations. Discussion of, and data on, the microstraining phenomenon are given by Vreeland (28), Hendrickson (16), and Smith (27). The onset of microstraining marks the activation of dislocation sources, this is apparently the mechanism responsible for the appearance of a proportional limit.

Certain secondary effects complicate the formulation of the yield mechanism. As mentioned above, back stress causes the stress driving the F-R source to become progressively smaller than the applied stress. In addition, if the loading time is fairly long, diffusion of small impurity atoms to sites locking the released dislocations can occur. This phenomenon, which will lower the stress acting on the obstacle, will be termed "aging" or "relocking" in further discussion. Also, it is possible that the atmosphere locking the dislocation of the F-R source will become disordered as dislocations are rapidly
released, making each successive dislocation easier to release. Efforts will be made to evaluate approximately the influence of the first two above-mentioned effects; the latter effect is neglected in the formulation of the yield criteria.

The stress driving the dislocations is logically the resolved shearing stress in the plane of slip. The octahedral shearing stress is here assumed to be representative of this stress level for polycrystalline specimens. Since only uniaxial stress tests were performed in this study, no direct proof of the validity of this assumption can be put forward. However, studies of the influence of stress state on the resistance to yielding of metals quoted by Nadai (23) show good correlation with octahedral shear stress for biaxial stress states.

2.3 Number of Dislocations Issued by a Dislocation Source

The energy required to remove a dislocation from its locking atmosphere and source is supplied from two sources, the applied stress and thermal energy. The dislocation vibrates thermally about its equilibrium position and an occasional thermal oscillation has sufficient energy to break the dislocation away. Let the thermal energy required to free the dislocation be denoted by the term "activation energy" and represented by U. As the applied stress level increases the activation energy decreases since more of the energy requirement is provided by stress. It is not essential to this mechanism that the dislocation of the F-R source be locked by an "atmosphere." Energy must still be applied to bow a free dislocation to its critical radius, the energy can come from applied stress and thermal oscillations.

The probability that a single thermal oscillation will attain the energy level U is given by the Maxwell-Boltzmann probability distribution: \( \exp(-U/RT) \);
for U in cal/mol, R the universal gas constant - 1.986 cal/°K-mol, and T the absolute temperature in degrees Kelvin.

If the frequency of vibration of the dislocation line is \( \nu \) seconds\(^{-1}\), the statistical rate of dislocation release becomes

\[
\frac{dN}{dt} = \nu \exp \left( - \frac{U}{RT} \right)
\]  

(2.1)

The frequency factor \( \nu \) is a function of the interatomic forces or lattice spacing and is thus relatively insensitive to stress and temperature. It is considered to be constant here.

The number \( N \) of dislocations issued by a dislocation source at a time \( t_1 \) can be determined by integration of Eq. (2.1). If no dislocations had been issued up to time \( t = 0 \)

\[
N = \nu \int_0^{t_1} \exp \left( - \frac{U}{RT} \right) dt
\]

(2.2)

The reduction in the number of free dislocations caused by relocking can be estimated by a formulation suggested by the work of Campbell and Duby (6). The rate of aging can be considered to be proportional to the rate of diffusion of the solute atoms forming the atmosphere

\[
b = D \exp \left( - \frac{U_a}{RT} \right)
\]

(2.3)

where \( b \) represents the rate of relocking in seconds\(^{-1}\), \( D \) is a diffusion coefficient in seconds\(^{-1}\), and \( U_a \) is an activation energy for diffusion in cal/mol.

Narrowing the definition of \( N \) to the number of dislocations which have been issued by the source and remain free, and considering the rate of relocking of the free dislocations to be proportional to the number present

\[
\frac{dN}{dt} = \nu \exp \left( - \frac{U}{RT} \right) - bN
\]

(2.4)
If it is assumed that $b$ is a function of temperature alone, that neither $D$ nor $U_a$ are stress dependent, and that temperature is constant; Eq. (2.4) may be directly integrated. For $N = 0$ at $t = 0$

$$N = \nu \exp (-bt_{1}) \int_{0}^{t_{1}} \exp (-\frac{E}{R T} + bt) \, dt$$

(2.5)

An approximate integration of Eq. (2.4) for the special case of constant stress rate, for a stress sensitive rate of relocking $b$, is presented in Chapter 3.

The effects of back stresses on the number of dislocations released by a source are difficult to formulate satisfactorily. However, the concept is simple; the activation energy should be a function of the effective stress on the source. The effective stress may be expressed as the applied stress minus the back stress, or

$$\tau_e = \tau - \tau_b$$

$U$ is then $U(\tau_e)$ and requires formulation of the back stress to be defined. The back stress should be a function of the grain size - a measure of the distance from the obstacle of the grain boundary - and the number of dislocations, $N$, in the array. Formidable difficulties are involved in the integration of expressions such as Eq. (2.2) if $U$ is an unknown function of both the applied stress and $N$. Because of the complexity of the situation only a gross approximation to the influence of back stress is attempted.

The experimental data shown in Chapters 5 and 6, Fig. 5.1, 6.1, etc., indicate that for constant stress rate testing as the rate of loading approaches zero, or, for constant stress testing, as the duration of loading approaches infinity, the yield level trends toward a limit independent of temperature, $\tau_{ym}$, the minimum yield level. If this minimum yield level is
taken to be a measure of the back stress level which the obstacle can cause to be exerted on the source, the activation energy can be taken as a function of an effective stress on the source

$$\tau_i = \tau - \tau_{ym}$$  \hspace{1cm} (2.6)

Denoting $U(\tau_i)$ as $U_i$, an expression similar to Eq. (2.2) is obtained

$$N = \nu \int_{t_{ym}}^{t_i} \exp \left( - \frac{U_i}{RT} \right) dt$$  \hspace{1cm} (2.7)

Here $N$ represents the number of dislocations issued in the time interval $t_{ym} < t < t_i$, where $t_{ym}$ is the time at which $\tau$ reaches $\tau_{ym}$. In Eq. (2.7) the number of dislocations released in the interval $0 < t < t_{ym}$ is neglected.

In the following sections yield criteria are established based upon:

Eq. (2.2), which yields the simplest forms; Eq. (2.5), which considers aging; and Eq. (2.7), which estimates back stress effects.

2.4 Yield Criteria for a Critical Number of Dislocations

The value of $N$ obtained from Eq. (2.2), (2.5), or (2.7) is considered representative of the number of dislocations restrained by the obstacle which resists the onset of general yielding. Yielding may be considered to begin when $N$ reaches a critical value $N_c$. The concept that yielding is initiated by a critical number of dislocations will be called hereafter the "critical number theory".

If the time at which yielding begins is denoted $t_y$, criteria may be developed from Eq. (2.2), (2.5), and (2.7).

The criterion neglecting aging or back stress effects from Eq. (2.2) becomes
The criterion considering aging effects from Eq. (2.5) is

\[ \frac{N_c}{v} = \int_{0}^{y} \exp \left( -\frac{U}{RT} \right) dt \]  

(2.8)

The criterion considering back stress effects from Eq. (2.7) is

\[ \frac{N_c}{v} = \exp (-bt_y) \int_{0}^{y} \exp \left( -\frac{U}{RT} + bt \right) dt \]  

(2.9)

The criterion considering back stress effects from Eq. (2.7) is

\[ \frac{N_c}{v} = \int_{t_y}^{t_{ym}} \exp \left( -\frac{U}{RT} \right) dt \]  

(2.10)

Since the type of experimental work conducted in this study will not permit separation of the quantities \( N_c \) and \( v \), the factor \( N_c/v \) will be considered to be a single constant.

2.5 Yield Criteria for Critical Stress on an Obstacle

As previously mentioned, the stress, \( \tau_o \), exerted on an obstacle by the dislocation array it holds back is expressed by

\[ \tau_o = N\tau \]

where \( \tau \) is the applied stress. If yielding is considered to be initiated when the stress on the obstacle reaches a critical level \( \tau_c \), the basic critical stress theory yield criterion takes the form

\[ \tau_c = N_c \tau_y \]  

(2.11)

Equations (2.8), (2.9), and (2.10) can be used to define \( N_c \). Yield criteria are obtained by substituting the above equations in Eq. (2.11).

The yield criterion neglecting aging and back stress comes from Eq. (2.8)
\[ \frac{\tau_C}{\nu} = \tau_y \int_0^{t_y} \exp \left( -\frac{U}{RT} \right) dt \] (2.12)

The yield criterion considering aging comes from Eq. (2.9)

\[ \frac{\tau_C}{\nu} = \tau_y \exp (-bt_y) \int_0^{t_y} \exp \left( -\frac{U}{RT} + bt \right) dt \] (2.13)

The yield criterion considering back stress comes from Eq. (2.10)

\[ \frac{\tau_C}{\nu} = \tau_y \int_{t_{ym}}^{t_y} \exp \left( -\frac{U}{RT} \right) dt \] (2.14)

The concept of a critical stress on the obstacle, called the "critical stress theory," seems physically preferable to the critical number theory. If sources must be activated in adjacent grains to initiate yielding, the stress concentration on the grain boundary would appear to be a better measure of the quantity increasing the effective stress on the F-R sources in the adjacent grain than the number of dislocations held back at the grain boundary. However, the yield criteria based upon the critical number theory are somewhat simpler in form, and therefore easier to apply. In the work which follows, the criteria for both theories are developed for evaluation with test data in order to determine which one leads to the more satisfactory representation of the behavior.
CHAPTER 3. TECHNIQUES FOR VALIDATING YIELD CRITERIA

3.1 Procedure

The process of validation of the yield criteria is made up of two parts. In the first part, the unknown factors of the yield criteria are evaluated, i.e., for the criterion of Eq. (2.8) the constant $N_c/v$ and the function $U(\tau)$ are established. It is shown that tests of a simple stress-time history are useful in this phase to permit integration of the criteria for unknown $U$. The second part of the process of validation is concerned with showing whether the criteria, as evaluated, are indeed valid. That is, whether the criteria can successfully predict the upper yield point for tests of general pre-yield stress-time history.

In this chapter the equations and techniques required for the processes of evaluation and validation of the yield criteria are developed. The actual validation and comparison of the criteria appears in Chapter 5.

3.2 Evaluating the Yield Criteria Using Constant Stress Data

The constant stress test is ideally a test in which the stress rises instantaneously to a level that is maintained constant until yielding occurs. The constant stress condition only can be approximated because a finite rise time, an overshoot at first maximum, and oscillations in the stress level usually are encountered. Damping can eliminate the overshoot and oscillations but will increase the rise time. A stress-time curve for a typical, damped, constant stress test is shown in Fig. 3.1. The stress value $\tau_y$ is representative of the average stress level after the rise. The choice of the time interval representative of the duration for which the stress is maintained
Figure 3.1  Stress-Time Curve for a Constant Stress Test.
prior to yield varies among investigators. Some use the time \( t_c \), time at constant stress, or the time interval from the time when the stress first reaches the level selected as \( \tau_y \) to the time when yielding occurs. Other investigators use the time interval \( t_d \), delay time, or the time interval from the time at which the stress reaches a "static" yield level for the material to the time of yielding. The difference between \( t_c \) and \( t_d \) is rarely significant, and reports quoting one rarely present enough data to obtain the other, so the two quantities will be considered as one and reported here as \( t_d \).

A good reason for being unconcerned about which time interval, \( t_c \) or \( t_d \), is reported is the difficulty in establishing the end of the interval, i.e., the time of yielding, \( t_y \). Because of microstraining, bending under imperfectly axial loading, initiation of yielding at a point in the specimen away from strain gages, and, perhaps, the basic nature of the process, the transition from zero strain rate for the elastic specimen to a high plastic strain rate in the lower yield region is gradual. Some investigators have used the drop of the stress, itself often gradual. Massard (22) used the time at which the secant modulus, \( \frac{\sigma}{\epsilon} \), dropped to \( 20 \times 10^3 \) ksi in steel. Other investigators have used the intersection of the projected linear portion of the plastic strain-time curve with the projected line of constant elastic strain. Definition becomes difficult in lightly damped machines where the oscillations in stress may be appreciable. The amount of unrecovered strain may increase with each oscillation of stress, but can general yielding be said to have occurred when the strain does decrease somewhat for a small decrease in stress? The figures in Keenan (20) show how difficult it can be to select the time of yielding.

**Critical Number Theory, Aging and Back Stress Negligible** - The applicable criterion, given by Eq. (2.8), is

\[
\frac{N_c}{v} = \int_0^{\tau_y} \exp \left( - \frac{U}{RT} \right) dt
\]
where \( N_c/v \) is a constant, \( U \) is a function only of stress, and temperature is considered to be constant during the test. Idealizing the stress-time function prior to yield as having zero rise time to the level \( \tau_y \), and a duration of \( t_d \) leads to

\[
\frac{N_c}{v} = \exp \left( - \frac{U_y}{RT} \right) \int_0^{t_d} dt
\]

where \( U_y \) is the value of \( U \) for \( \tau = \tau_y \). Integration yields

\[
\frac{N_c}{v} = \exp \left( - \frac{U_y}{RT} \right) t_d \tag{3.1}
\]

Data from a program of constant stress tests at several different temperatures have a form similar to Fig. 3.2, curves of \( \tau_y \) vs. \( \log t_d \) for constant temperature. If the logarithm of Eq. (3.1) is taken

\[
\log t_d = U_y \left( \log e \right) \frac{1}{RT} + \log \frac{N_c}{v}
\]

Therefore, a plot of the test data, for a given \( \tau_y \) but different \( T \), with \( \log t_d \) as the ordinate, and \( 1/RT \) as the abscissa, should be a straight line of slope \( U_y \log e \) since \( U_y \) is a constant for a given \( \tau_y \), and the intercept on the ordinate is \( \log N_c/v \). Each stress level should plot with the same intercept if \( \log N_c/v \) is a constant. A typical plot of this kind, called a "plot for \( U \)" , is shown in Fig. 3.3. The slopes of the lines can be measured and the corresponding values of \( U_y \) plotted against the corresponding \( \tau_y \) to determine the function \( U(\tau) \).

Critical Stress Theory, Aging and Back Stress Negligible - Equation (2.12) is integrated for stress constant at \( \tau_y \) for an interval \( t_d \) in the same manner described above.

\[
\frac{\tau_c}{v} = \tau_y \exp \left( - \frac{U_y}{RT} \right) t_d \tag{3.2}
\]
If the logarithm of Eq. (3.2) is taken

\[
\log \tau_y + \log t_d = U_y (\log e) \frac{1}{RT} + \log \frac{\tau_c}{v}
\]

it is obvious that the same technique of study by a plot for \( U \) similar to Fig. 3.3 is appropriate. The only differences are that the ordinate of the plot becomes \( \log \tau_y + \log t_d \), and the common intercept becomes \( \log \frac{\tau_c}{v} \).

Some indication as to the preferability of the critical stress or critical number theory can be seen on a plot for \( U \). If the critical number theory plot is tried, but the intercept appears to become successively smaller as \( \tau_y \) increases, the critical stress theory will show a better common intercept since a term \( \log \tau_y \) appears in the ordinate. However, the data scatter is likely to be great enough to obscure this indication.

**Critical Number Theory, Aging Considered - Integrating Eq. (2.9)**

\[
\frac{N_c}{v} = \exp (-bt_y) \int_0^{\tau_y} \exp \left( - \frac{U}{RT} + bt \right) dt
\]

Considering the stress to be constant at \( \tau_y \) for a time interval \( t_d \)

\[
\frac{N_c}{v} = \exp (-bt_d - \frac{U}{RT}) \int_0^{t_d} \exp (bt) dt
\]

which leads to

\[
\frac{N_c}{v} = \exp \left( - \frac{U}{RT} \right) \frac{1}{b} \left[ 1 - \exp (-bt_d) \right]
\]  

(3.3)

Note that as \( t_d \) becomes small Eq. (3.3) converges to Eq. (3.1). Therefore, \( U \) and \( N_c/v \) can be evaluated by the technique described for Eq. (3.1). Deviations from the theoretical picture of Fig. 3.3 can be expected to occur for greater \( t_d \). With \( U \) and \( N_c/v \) known Eq. (3.3) can be solved for the rate of
relocking b. Recall that b is expected to be a function of temperature as defined by Eq. (2.3).

**Critical Stress Theory, Aging Considered** - Equation (2.13) is integrated in the manner described above to yield

\[
\frac{\tau_c}{\nu} = \tau_y \exp \left( - \frac{U}{\nu R T} \right) \frac{1}{b} \left[ 1 - \exp \left( -bt_d \right) \right]
\]  

(3.4)

The values of \( \frac{\tau_c}{\nu}, U, \) and \( b \) may be obtained by the method of study described above.

**Critical Number Theory, Back Stress Considered** - The criterion to be integrated is given by Eq. (2.10)

\[
\frac{N_c}{\nu} = \int_{\tau_{ym}}^{\tau} \exp \left( - \frac{U_i}{\nu R T} \right) dt
\]

Recall that \( U_i \) represents \( U(\tau - \tau_{ym}) \), where \( \tau_{ym} \) is an experimentally observed minimum yield level. For the constant stress condition \( U_i \) will be constant at \( U_{iy} \), \( U(\tau_y - \tau_{ym}) \), and the integration limits will be \( 0 < t < t_d \).

\[
\frac{N_c}{\nu} = \exp \left( - \frac{U_{iy}}{\nu R T} \right) t_d
\]  

(3.5)

The technique for evaluation of \( U_i \) and \( N_c/\nu \) is identical to that described for Eq. (3.1).

**Critical Stress Theory, Back Stress Considered** - Equation (2.14) is integrated in the manner described above to give

\[
\frac{\tau_c}{\nu} = \tau_y \exp \left( - \frac{U_{iy}}{\nu R T} \right) t_d
\]  

(3.6)

The technique for evaluation of \( U_i \) and \( \tau_c/\nu \) is that described for Eq. (3.2).
3.3 Evaluating the Yield Criteria Using Constant Stress Rate Data

As just noted the use of constant stress data is desirable because the evaluation of the criteria is simple and exact. But, it is difficult to perform constant stress tests which will produce stress-time curves reasonably close to the ideal form. For experimental simplicity it is often convenient to use constant stress rate testing to evaluate the yield criteria.

A typical shape for the stress-time curve, when an attempt is made to obtain constant stress rate, is shown in Fig. 3.4. The peak stress value occurring as the strain rate increases abruptly (for a soft, relatively low spring stiffness, testing machine such as used in this study) is considered the upper yield stress. It is necessary to determine the constant stress rate \( \dot{\tau} \) which represents the stress-time function. Irregularities in the stress rate when the stress is less than 90 percent of \( \tau_y \) are rarely significant since the quantity \( \exp(-U/RT) \) decreases rapidly as the stress decreases. The tendency for the stress rate to decrease just prior to the peak stress presents a more serious problem. The leveling off of the stress rate can be considered to be due to increased flexibility of the specimen caused by microstraining, yielding in the specimen away from the strain gages and partial yielding from non-axiality of the load. In this work the leveling off of the stress rate is neglected and the rate otherwise representative of the last 10 percent of the stressing to yield is used. This procedure may slightly overestimate the stress rate associated with a given \( \tau_y \).

Critical Number Theory, Aging and Back Stress Negligible - Equation (2.8) is integrated

\[
\frac{N_c}{\nu} = \int_{0}^{\tau_y} \exp \left( -\frac{U}{RT} \right) \, dt
\]
Figure 3.4  Stress-Time Curve for a Constant Stress Rate Test
Since the experimental stress-time history is represented by $\tau_y$ and $t$ the time variable is replaced by stress with the substitution $t = \tau / \dot{\tau}$, and Eq. (2.8) becomes

$$\frac{N_c}{\nu} = \frac{1}{\dot{\tau}} \int_0^{\tau_y} \exp \left( - \frac{U}{RT} \right) dt$$

With $U$ a function of $\tau$ alone the integrand is in an appropriate form, but an approximation is required in order to integrate the function of an unknown $U(\tau)$. The quantity $\exp \left( - \frac{U}{RT} \right)$ decreases very rapidly as stress decreases. Therefore, an expansion of $U$ about the value at the maximum stress level will give a good value for the integral. Since the maximum stress level for the constant stress rate test is $\tau_y$, $U$ will be expanded about $U_y$, the value of $U$ for $\tau = \tau_y$. The approximate expression for $U$, retaining only first order terms, is

$$U \sim U_e = U_y + \left. \frac{dU}{d\tau} \right|_y (\tau - \tau_y) \quad (3.7)$$

The substitution of $U_e$ for $U$ in Eq. (2.8) gives an integrable expression since both $U_y$ and $\left. \frac{dU}{d\tau} \right|_y$ are constants. The expression to be integrated is then

$$\frac{N_c}{\nu} = \frac{RT \exp \left( - \frac{U_y}{RT} \right)}{\dot{\tau}} \int_0^{\tau_y} \exp \left[ - \left. \frac{dU}{d\tau} \right|_y \frac{1}{RT} (\tau - \tau_y) \right] dt$$

and the integration yields

$$\frac{N_c}{\nu} = - \frac{RT \exp \left( - \frac{U_y}{RT} \right)}{\dot{\tau} \left. \frac{dU}{d\tau} \right|_y} \left[ 1 - \exp \left( \frac{\tau_y}{RT} \left. \frac{dU}{d\tau} \right|_y \right) \right] \quad (3.8)$$

The term $\exp \left( \frac{\tau_y}{RT} \left. \frac{dU}{d\tau} \right|_y \right)$ is small compared to 1, so Eq. (3.8) is well approximated by...
\[
\frac{N_c}{v} = \frac{RT \exp \left( - \frac{U_y}{RT} \right)}{\frac{dU}{d\tau}}
\] 

(3.9)

In order to evaluate \( U \) and \( \frac{N_c}{v} \) the logarithm of Eq. (3.9) is taken

\[
\log RT - \log \tau = U_y \left( \log e \right) \frac{1}{RT} + \log \frac{N_c}{v} + \log \left| \frac{dU}{d\tau} \right|
\] 

(3.10)

A plot for \( U \) similar to Fig. 3.3 can be prepared from constant stress rate data. The abscissa is again \( 1/RT \), the ordinate becomes \( \log RT - \log \tau \).

Again, data for a given \( \tau_y \) but different \( T \) should plot on a straight line, the slope of which is the \( U \log e \) appropriate to that stress level. The intercepts at the ordinate do not fall at a common point, unless \( U \) is linear in \( \tau \), since the quantity \( \log \left| \frac{dU}{d\tau} \right| \) is included in the intercept. The intercept normally becomes lower as \( \tau \) increases because \( \frac{dU}{d\tau} \) commonly decreases as \( \tau \) increases.

For the data studied in this investigation a form for \( U \) such that

\[
\frac{dU}{d\tau} = - \frac{K}{\tau^a}
\]

could be found which would give good fit to the data. A plot for \( U \) with a common intercept can be constructed when \( \frac{dU}{d\tau} \) has this form since Eq. (3.10) becomes

\[
a \log \tau_y + \log RT - \log \tau = U_y \left( \log e \right) \frac{1}{RT} + \log \frac{N_c}{v} + \log K
\] 

(3.11)

It is necessary to use a process of trial and correction to establish the proper value for "a". An "a" of 1, which leads to the Yokobori (31) form for activation energy

\[
U = A - B \ln \tau = B \ln \frac{\tau_0}{\tau}
\]

appears reasonable for the first trial.

Critical Stress Theory, Aging and Back Stress Negligible - Equation (2.12) is integrated by a similar process of approximate integration
\[ \frac{\tau_c}{\nu} = - \frac{\tau_y \exp \left( - \frac{U}{RT} \right)}{\frac{dU}{dt} \mid_y} \]  \hspace{1cm} (3.12)

The evaluation of \( U \) and \( \tau_c / \nu \) is carried out by the same process described for Eq. (3.9). The only difference is in the additional log \( \tau_y \) term in the ordinate of the plot for \( U \).

Critical Number Theory, Aging Considered - The criterion to be integrated is given by Eq. (2.9)

\[ \frac{N_c}{\nu} = \exp \left( -bt_y \right) \int_0^{\tau_y} \exp \left( - \frac{U}{RT} + bt \right) dt \]

If \( b \), the measure of the rate of aging, is considered to be a function of temperature alone, and temperature is assumed to remain constant during the stressing, Eq. (2.9) may be integrated for constant stress rate by changing the variable of integration to \( \tau \) and using the \( U \) of Eq. (3.7) in place of \( U \).

\[ \frac{N_c}{\nu} = \exp \left( - \frac{bt_y \cdot U}{\tau \cdot RT} \right) \int_0^{\tau_y} \exp \left[ - \frac{dU}{d\tau} \mid_y \left( \frac{\tau - \tau_y}{RT} + \frac{bt}{\tau} \right) \right] d\tau \]

\[ \frac{N_c}{\nu} = \exp \left( - \frac{U}{RT} \right) \left[ 1 - \exp \left( \frac{\tau_y}{RT} \frac{dU}{d\tau} \mid_y - \frac{bt}{\tau} \right) \right] \]

and since \( \exp \left( \frac{\tau_y}{RT} \frac{dU}{d\tau} \mid_y - \frac{bt}{\tau} \right) \) is small compared to 1.

\[ \frac{N_c}{\nu} = \exp \left( - \frac{U}{RT} \right) \left[ b - \frac{\tau}{RT} \frac{dU}{d\tau} \mid_y \right] \]  \hspace{1cm} (3.13)
In an investigation of the strain aging process for plastically strained steel, Brittain (3) shows that the activation energy for strain aging, comparable to $U_a$ of Eq. (2.3), is stress dependent. It is also possible that the diffusion coefficient $D$ is stress dependent. In order to study the stress dependence of the pre-yield aging process it is necessary to consider such dependence in the development of the yield criteria.

Returning to Eq. (2.4) and specializing it for constant stress rate by setting

$$\frac{d\tau}{dt} = \frac{b}{t}$$

then

$$\frac{dN}{d\tau} + \frac{b(\tau)}{t} N = \frac{\nu}{t} \exp \left( - \frac{U}{RT} \right)$$

Consider then Eq. (2.3) for $b$

$$b = D \exp \left( - \frac{U}{RT} \right)$$

If $\frac{dU}{d\tau} < 0$, and the coefficient $D$ is such that a function $G = b/\dot{\tau}$ is regular, i.e. integrable by infinite series, a procedure of integration may be carried out based on similar approximations to those used in the derivation of Eq. (3.9).

Let

$$I (\tau) = \int G \, d\tau = \int \frac{b}{\dot{\tau}} \, d\tau$$

$$H (\tau) = - \frac{U}{RT}$$

Equation (2.4), becomes

$$\frac{dN}{d\tau} + GN = \frac{\nu}{t} \exp \left( H \right)$$

Integrating

$$\frac{N_0}{\nu} = \frac{\exp \left( -I \right)}{t} \int_0^{\tau} \exp \left( I + H \right) \, d\tau$$

Approximating functions $I$ and $H$ by expansions about $\tau = \tau_y$ which retain only first order terms
\[ I \sim I_y + G_y (\tau - \tau_y) \quad \text{where:} \quad I_y = I(\tau_y) \]
\[ H \sim H_y + \dot{H}_y (\tau - \tau_y) \quad G_y = \frac{b(\tau_y)}{\dot{\tau}} = \frac{b_y}{\dot{\tau}} \]
\[ H_y = -U_y/RT \]
\[ \dot{H}_y = -\frac{1}{RT} \frac{dU}{d\tau} \bigg|_y \]

\[ \frac{N_c}{v} = \exp \left( \frac{H_y}{\tau} \right) \left[ 1 - \exp \left( -G_y (\tau_y - \dot{H}_y \tau_y) \right) \right] \]

Resubstituting
\[ \frac{N_c}{v} = \exp \left( -\frac{U_y}{RT} \right) \left[ 1 - \exp \left( \frac{b_y \tau_y}{\dot{\tau}} + \frac{\tau_y}{\dot{\tau}} \frac{dU}{d\tau} \bigg|_y \right) \right] \]

Again \[ \exp \left( \frac{b_y \tau_y}{\dot{\tau}} + \frac{\tau_y}{\dot{\tau}} \frac{dU}{d\tau} \bigg|_y \right) \] is small compared to 1.

\[ \frac{N_c}{v} = \exp \left( -\frac{U_y}{RT} \right) \left[ 1 - \exp \left( \frac{b_y \tau_y}{\dot{\tau}} + \frac{\tau_y}{\dot{\tau}} \frac{dU}{d\tau} \bigg|_y \right) \right] \]
\[ \text{where} \quad b_y = D(\tau_y) \exp \left( -\frac{U_a(\tau_y)}{RT} \right) \]

It is interesting to note the similarity of Eq. (3.13) and (3.14). Note too, that any function \( b(\tau) \) established by study of data using Eq. (3.14) must be reviewed to insure that approximations made in the derivation of Eq. (3.14) are justifiable.

Equations (3.13) and (3.14) may be evaluated by a process of trial and correction. At higher stress rates and lower temperatures the effects of aging should become small, so that fair values for \( N_c/v \) and \( U \) should result from Eq. (3.9). Let the values of \( \dot{\tau} \) determined by the use of Eq. (3.9) be denoted as \( \dot{\tau}^* \), the theoretical stress rate required to reach a given yield stress...
level for negligible aging

\( t^* = \frac{RT \exp \left( -\frac{U_y}{RT} \right) \nu \frac{d\nu}{dt} y}{N_c \frac{du}{v}} \)  \hspace{1cm} (3.15)

Equation (3.14) solved for \( b \) in terms of \( t^* \) gives

\[ b_y = -\frac{\frac{dU}{dt} y}{RT} \left[ t^* - t \right] \]  \hspace{1cm} (3.16)

Thus, for trial values of \( N_c/v \) and \( U \) from a study of Eq. (3.9), values for \( b \) can be obtained by using the test data values for \( t \) in Eq. (3.16). A trial function for \( b \) can then be substituted in Eq. (3.16) to obtain \( t^* \) values which can be used in a plot for \( U \), in place of the data \( t \), to improve the evaluation of \( N_c/v \) and \( U \). The process is continued until reasonable fit to the data is attained.

Equation (3.16) implies the existence of a minimum yield point, the yield level for which \( t \) is zero. Designating the minimum yield level as \( \tau_{ym} \) (a function of \( T \)), \( U_{ym} \) as the corresponding \( U(\tau_{ym}) \), and \( b_{ym} \) as \( b(\tau_{ym}) \); a solution of Eq. (3.16) for \( \dot{t} = 0 \) yields

\[ \frac{b_{ym}}{b} \exp \left( \frac{U_{ym}}{RT} \right) = 1 \]  \hspace{1cm} (3.17)

Note that Eq. (3.17) can be obtained by letting \( t_d \) approach infinity in Eq. (3.3). Equation (3.17) is not explicit in \( \tau_{ym} \) but by taking the logarithm of the equation a direct solution for the temperature at which a given \( \tau_y \) is \( \tau_{ym} \) can be made.

\[ \frac{RT}{N_c} \log \frac{U_{ym} - U_{am}}{\log D \nu - \log D_m} \]  \hspace{1cm} (3.18)

where \( D \) and \( U_{am} \) are the factors of Eq. (2.3) appropriate to \( \tau_{ym} \).
Critical Stress Theory, Aging Considered - The required expressions are obtained by operations upon Eq. (2.13) which are essentially identical to the preceding operations. The resulting relations are:

The yield criterion from integration of Eq. (2.13) is

\[
\frac{\tau_c}{\nu} = \frac{\gamma_y \exp \left( - \frac{U_y}{RT} \right)}{b_y - \frac{i}{RT} \left. \frac{dU}{d\tau} \right|_y}
\]  \hspace{1cm} (3.19)

The theoretical stress rate for negligible aging is

\[
\dot{\tau}^* = \frac{\gamma_y \exp \left( - \frac{U_y}{RT} \right)}{\frac{\tau_c}{\nu} \left. \frac{dU}{d\tau} \right|_y}
\]  \hspace{1cm} (3.20)

Equation (3.16) for the study of \( b \) is unchanged. The equation for minimum yield point, again not explicit in \( \gamma_y \) since \( U_y \) is \( U(\gamma_y) \) is

\[
\gamma_y = b \frac{\gamma_c}{\nu} \exp \left( - \frac{U_y}{RT} \right)
\]  \hspace{1cm} (3.21)

The explicit equation for the temperature at which a given \( \gamma_y \) is \( \gamma_y \) is

\[
RT = \frac{(U_y - U_{am}) \log e}{\log \gamma_y - \log \frac{\tau_c}{\nu} - \log D_m}
\]  \hspace{1cm} (3.22)

Critical Number Theory, Considering Back Stress - Equation (2.10) is integrated by essentially identical operations to those used in the derivation of Eq. (3.8). The differences are that \( U_i \) is \( U(\gamma_i) \), and that the lower limit of integration is \( \gamma_y \) rather than zero. The criterion resulting is

\[
\frac{N_i}{\nu} = \frac{RT \exp \left( - \frac{U_i}{RT} \right)}{\left. \frac{i}{RT} \frac{dU}{d\tau} \right|_y} \left\{ 1 - \exp \left[ \frac{(\gamma_i - \gamma_y) \left. \frac{dU}{d\tau} \right|_y}{RT} \right] \right\}
\]  \hspace{1cm} (3.23)
The term \( \exp \left[ \frac{(\tau - \tau_{ym})}{RT} \frac{dU_i}{d\tau} \right] \) is not negligible when \( \tau \) is near \( \tau_{ym} \); but does lose significance rapidly as \( \tau \) becomes larger. Taking the logarithm of Eq. (3.23) in order to evaluate \( U_i \) and \( N_c/\nu \) with a plot for \( U_i \)

\[
-\log \hat{\tau} + \log RT + \left\{ \log \left[ 1 - \exp \left( \frac{(\tau - \tau_{ym})}{RT} \frac{dU_i}{d\tau} \right) \right] \right\} = \frac{U_i \log e}{RT} + \log \frac{N_c}{\nu} + \log \frac{dU_i}{d\tau} \tag{3.24}
\]

Equation (3.24) may be studied in the manner discussed for Eq. (3.10) by neglecting the bracketed term in the first trial for values of \( U_i \) and \( N_c/\nu \). The plot for \( U \) can then be improved by including the bracketed term in the ordinate of the plotted points, and \( U_i \) and \( N_c/\nu \) improved as required.

**Critical Stress Theory, Considering Back Stress** - A similar technique of approximate integration is used with Eq. (2.14). The resulting criterion

\[
\frac{\tau_c}{\nu} = \frac{\tau_y RT \exp \left( - \frac{U_i \nu}{RT} \right)}{\hat{\tau} \frac{dU_i}{d\tau} \frac{dU_i}{d\tau}} \left\{ 1 - \exp \left[ \frac{(\tau - \tau_{ym})}{RT} \frac{dU_i}{d\tau} \right] \right\} \tag{3.25}
\]

is evaluated in the same manner discussed for Eq. (3.23).

3.4 Studying the Validity of the Criteria for a General Stress-Time History of Loading

The technique for studying the validity of the criteria neglecting aging and back stress effects is presented in this section. Similar techniques are applicable to the criteria considering aging and back stress.

**Critical Number Theory** - The critical number theory criterion of Eq. (2.8)

\[
\frac{N_c}{\nu} = \int_0^{\tau_y} \exp \left( - \frac{U}{RT} \right) dt
\]
is rearranged in the form

\[ \frac{N}{\nu} = \sum_{t=0}^{t=t_t} Q_{rs} \tag{3.26} \]

where \( Q_{rs} \) represents the "damage" occurring during the time interval between \( t_r \) and \( t_s \) of the loading

\[ Q_{rs} = \int_{t_r}^{t_s} \exp \left( -\frac{U}{RT} \right) dt \tag{3.27} \]

Yield is predicted to occur when the damage increments \( Q_{rs} \) sum to \( N_c/\nu \).

If the stress is constant during the time interval at \( \tau_r \)

\[ Q_{rs} = \exp \left( -\frac{U_r}{RT} \right) \left[ \tau_s - \tau_r \right] \tag{3.28} \]

Equation (3.28) results from an integration procedure identical to that used to obtain Eq. (3.1).

If the stress level varies linearly in the time interval, i.e. \( \dot{\tau} \) is constant, with \( \tau_r \) the maximum stress level in the time interval and \( \tau_s \) the minimum

\[ Q_{rs} = -\frac{1}{|\dot{\tau}|} \exp \left( -\frac{U_r}{RT} \right) \frac{RT}{\dot{\tau}} \left( 1 - \exp \left[ (\tau_r - \tau_s) \dot{\tau} \right] \right) \tag{3.29} \]

Equation (3.29) results from an integration identical to that shown in the derivation of Eq. (3.8). Note that the bracketed term is not similar to 1 when \( \tau_r - \tau_s \) is small.

The actual stress-time history should be approximated by linear segments, \( \dot{\tau} = 0 \) or constant, since integration of the yield criterion is difficult for higher order stress-time variations.
Critical Stress Theory - The yield criterion of Eq. (2.12) is arranged in the form

\[
\frac{\tau}{v} = \tau_y \sum_{t=0}^{t=t_y} Q_{rs}
\]  

(3.30)

yielding is predicted to occur when the damage increments \(Q_{rs}\) sum to \(\frac{1}{\tau_y v} \frac{\tau}{v}\).

The damage increments \(Q_{rs}\) for constant stress and constant stress rate are those given above in Eq. (3.28) and (3.29).

3.5 Study of Data at a Single Temperature Level

Yield criteria permitting predictions of the influence of stress-time history on the yield level at a specific temperature are developed in this section. It is necessary to use data taken at the same temperature for the evaluation. The discussion is based upon the criterion of Eq. (2.8), the critical number theory criterion neglecting aging and back stress, and upon an assumed form for activation energy. A similar approach may be followed with other criteria and other forms for \(U\).

The form of the criterion to be considered is

\[
\frac{N}{\tau} = \int_0^{\tau_y} \exp \left( - \frac{U}{RT} \right) dt
\]  

(2.8)

If it is assumed that \(\frac{dU}{dt} = - \frac{K}{\tau^a}\) where "\(a\)" is a constant, then

\[
U = A - \frac{K}{1 - a} \tau^{1-a} \quad \text{for} \quad a \neq 1
\]  

(3.31a)

\[
U = A - K \ln \tau \quad \text{for} \quad a = 1
\]  

(3.31b)

Substituting for \(U\) in Eq. (2.8) and forming new constants...
Evaluation of the criterion of Eq. (3.32) to obtain values for the constants a, C, and D requires a minimum of three data points.

**Evaluation with Constant Stress Data** - For constant stress conditions the criterion takes the form

\[ C = \int_{0}^{t_y} \exp \left[ D \frac{1}{(1-a)} \right] \, dt \quad \text{for } a \neq 1 \]  

\[ C = \int_{0}^{t_y} \frac{1}{D} \, dt \quad \text{for } a = 1 \]  

Taking the logarithm of Eq. (3.33a)

\[ \log t_d = \log C - D (\log e) \frac{1}{(1-a)} \]  

The proper value of "a" is that for which \( \log t_d \) will plot against \( \frac{1}{(1-a)} \) as a straight line. The slope of the line is \(-D \log e\) and the intercept on the \( \log t_d \) axis for \( \frac{1}{(1-a)} = 0 \) gives \( \log C \).

The value for "a" must come from a trial and correction procedure. It is convenient to begin with a trial of \( a = 1 \).

\[ C = \frac{1}{D} \frac{1}{t_y} \quad \text{for } a = 1 \]  

Taking the logarithm of Eq. (3.33b)

\[ \log t_d = \log C - D \log \frac{1}{(1-a)} \]  

If "a" is indeed 1 a plot of \( \log t_d \) versus \( \log \frac{1}{(1-a)} \) will be linear, with a slope of \(-D\) and an intercept for \( \frac{1}{(1-a)} = 1 \) of \( \log C \). If instead, the curve is concave upward "a" should be greater than 1, if concave downward "a" should be less than 1.
Evaluation with Constant Stress Rate Data - For constant stress rate the
criterion of Eq. (3.32a) is integrated by the same approximate procedure used
in the derivation of Eq. (3.9)

\[ C = \frac{\tau_a}{\tau} \exp \left[ D \frac{\tau^{(1-a)}}{\tau} \right] \quad \text{for} \quad a \neq 1 \]  

(3.35a)

Taking the logarithm of Eq. (3.35a)

\[ a \log \tau_y - \log \tau - \log |1-a| = -D \log e \tau^{(1-a)}_y + \log C + \log |D| \]

(3.36a)

The proper value for "a" is that for which \( (a \log \tau_y - \log \tau - \log |1-a|) \)
will plot versus \( \tau^{(1-a)}_y \) as a straight line. The slope of the line is
\((-D \log e)\) and the intercept at \( \tau_y = 0 \) gives \( (\log C + \log |D|) \).

The value for "a" must come from a trial and correction process. It is
appropriate to begin by trying \( a=1 \). For \( a=1 \) the criterion of Eq. (3.32b)
may be directly integrated.

\[ C = \frac{1}{\tau} \int_0^{\tau_y} \tau^D \, dt \]

\[ C = \frac{\tau_y^{D+1}}{(D+1)} \quad \text{for} \quad a = 1 \]

(3.35b)

Taking the logarithm of Eq. (3.35b)

\[ \log \tau = (D + 1) \log \tau_y - [\log C + \log (D + 1)] \]

(3.36b)

If "a" is actually 1, \( \log \tau \) will plot against \( \log \tau_y \) as a straight line. The
slope of the line is \((D + 1)\) and the intercept for \( \tau_y = 1 \) is \([- \log C - \log (D + 1)]\). If the curve is not straight but concave upward "a" should be
less than 1, if concave downward "a" should be greater than 1.
For General Stress-Time History Prior to Yield - Equation (3.32) is rarely directly integrable so the general stress-time curve is most easily handled by approximation with linear segments. Applying the technique of Section 3.4

\[ t = t_Y \]
\[ C = \sum_{t=0} Q_{rs} \]  

(3.37)

The equation for the damage increment \( Q_{rs} \) for a time interval \( \Delta t \) in which the stress is constant at \( \tau_r \) is

\[ Q_{rs} = \exp \left[ D \tau_r^{(l-a)} \right] \Delta t \quad \text{for} \quad a \neq 1 \]  

(3.38a)

\[ Q_{rs} = \tau_r^D \Delta t \quad \text{for} \quad a = 1 \]  

(3.38b)

The equation for the damage increment \( Q_{rs} \) for a time interval \( \Delta t \) in which \( \tau \) is constant, the greatest stress is \( \tau_r \) and the smallest stress is \( \tau_s \) is

\[ Q_{rs} = \frac{\tau_s^E \exp \left[ D \tau_r^{(l-a)} \right]}{\tau_r^{D(l-a)} D(l-a)} \left\{ 1 - \exp \left[ -D(l-a) \left( \frac{\tau_r - \tau_s}{\tau_r} \right) \right] \right\} \]

for \( a \neq 1 \)  

(3.39a)

\[ Q_{rs} = \frac{\tau_r^{(D-l)} - \tau_s^{(D+1)}}{\tau_r^{(D+1)}} \quad \text{for} \quad a = 1 \]  

(3.39b)

The above expressions may be used to predict the upper yield point when a single temperature level is of interest.

3.6 Conversion of Expressions from Octahedral Shear to Particular Stress States

For data taken from torsional or axial stress testing, it is convenient to evaluate the criteria using the measured stress values. For any defined
stress terms: \( \tau_{ym}, \tau_c \), etc.; the other stress state level giving the same octahedral shearing stress should be substituted. The \( N_c/v \) factor requires no alteration. Constants in expressions for \( U \) should be so altered that the same \( U \) value will result for equivalent stress levels.

As an example consider Eq. (3.12)

\[
\frac{\tau_c}{v} = -\frac{\tau_y}{v} \frac{RT \exp \left( - \frac{U_y}{RT} \right)}{\frac{dU}{d\tau}}
\]

where \( \tau \) is defined as octahedral shearing stress. If the material parameters are

\[
\frac{\tau_c}{v} = 6.87 \times 10^{-7} \text{ ksi-seconds}
\]

\[
U = \frac{183,200}{\tau} \text{ ksi-cal/mol}
\]

in terms of uniaxial stress for which

\[
\sigma = \frac{3}{\sqrt{2}} \tau
\]

Equation (3.12) is now written

\[
\frac{\sigma_c}{v} = -\frac{\sigma_y}{v} \frac{RT \exp \left( - \frac{U_y}{RT} \right)}{\frac{dU}{d\sigma}}
\]

and the material parameters become

\[
\frac{\sigma_c}{v} = 1.46 \times 10^{-6} \text{ ksi-seconds}
\]

\[
U = \frac{388,600}{\sigma} \text{ ksi-cal/mol}
\]
3.7 Selection of the Stress-Time Function for Criteria Evaluation

It is clear that constant stress testing greatly simplifies criteria evaluation. However, it is difficult to perform constant stress tests which reasonably approximate the ideal form.

Since the testing machine used for this study would have required considerable modification to damp it enough to get constant stress tests without large overshoot, and since the damping would increase the rise time of the load to a degree severely limiting the attainable range of $t_d$, it was decided to use constant stress rate tests for evaluating the yield criteria.
4.1 Specimens and Material

All tests carried out in this study were performed by applying axial load to specimens of either 0.2 or 0.1 sq. in. area. Plans for the specimens are shown in Fig. 4.1.

The specimens were fabricated from a 3/4-in. hot-rolled fully-killed A-7 steel plate, Bethlehem Heat 57T240, C. E. Department code number A-2. In the machining, cutting rates were held to such a level that the specimen did not become too hot to handle. The specimens were finished by hand polishing to an average 15 micro-inch rms surface roughness as measured by a Type PAC Profilometer.

The specimens were tested in an as-rolled condition. The only heat treatment of any type occurred during the curing of strain gages - a period of 2 to 24 hours at 200°F. The shorter time should be adequate to lock any dislocations freed in the machining, judging by the data presented by Brock (4) and Smith (27).

The results of a check analysis of the chemical composition are given in Table 4.1.

| Table 4.1. Plate A-2. Chemical Composition in Percent |
| C  | Mn | Si | P  | S  | Ni | Cr | Cu | Al |
| .17 | .89 | .18 | .020 | .034 | .025 | .05 | .08 | .06 |

The grain size was ASTM 6. Photomicrographs of the steel are shown in Fig. 4.2.