BEHAVIOR OF MULTISTORY REINFORCED CONCRETE FRAMES DURING EARTHQUAKES

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A Report on a Research Project Sponsored by
THE NATIONAL SCIENCE FOUNDATION
Research Grant GI 30760X

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
NOVEMBER, 1972
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This report is based on a thesis written by S. Otani under the supervision of M. A. Sozen.
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CHAPTER 1

INTRODUCTION

1.1 Object and Scope

The work described in this report was undertaken to understand the inelastic response of reinforced concrete multistory frames to earthquake motions through experimental and analytical studies.

The experimental work included tests of small-scale one-bay three-story frames subjected to base motions simulating one horizontal component of representative earthquake-motion records.

An analytical model was developed to calculate the dynamic response of a multistory frame for a given base motion, material properties and geometry of the structure. The model recognizes stiffness changes along the length of the frame members caused by cracking of the concrete, yielding of the reinforcement, and stress reversals.

The analytical model was tested by comparing its output with the experimental results. An important feature of the investigation was the availability of response measurements on structures, which had deformed well into the yield range, with definite knowledge of the base motion and structural properties.

In addition, the test results were used to study the applicability of analyses based on linear response in the design of reinforced concrete frames.
1.2 Review of Previous Research

A considerable amount of work on the investigation of the behavior of buildings during strong motion earthquakes has preceded this effort. Previous researches on the dynamic tests of reinforced concrete frame buildings and nonlinear dynamic analyses of frame structures will be briefly discussed in this section.

Vibration measurements in a ten-story reinforced concrete building were reported by Fukutomi (1931). In this study, the fundamental frequencies in two directions and crude mode shapes were studied through oscillations of the building induced by microtremors and natural earthquakes.

With a development of sophisticated vibration exciters, steady-state forced vibration tests were conducted on the actual full-scale structure at very low amplitudes (Kawasumi et al 1956, Hisada et al 1956, Hudson 1962 b, Nielsen 1964, Bouwkamp et al 1966, Matthiesen et al 1966), in which frequency and damping characteristics of a variety of structures and their foundations were studied.

Shiga et al (1966 and 1970) studied the behavior of one-story one-bay reinforced concrete models to periodic large base excitations. Inelastic response of the test structures was evaluated from the viewpoint of linearly elastic response theory by determining equivalent linear stiffnesses and viscous damping factors at different levels of vibration.

Takeda et al (1970) studied the inelastic response of reinforced concrete to strong motion earthquakes. Two reinforced concrete cantilever columns were subjected to a base disturbance in one horizontal direction.

* References are arranged in alphabetical order in the List of References. The number in parentheses refers to the year of publication.
Measured response signals were compared with those of an analytical model based on the geometry and material properties of the test specimen and a hysteresis rule. It was concluded that an analytical model for a reinforced concrete system should be based on a static force-deflection relationship which included the changes in stiffness as a function of the previous loading history.

Gulkan and Sozen (1971) reported the test results of one-story one-bay reinforced concrete frame models subjected to simulated earthquake motions. Each frame model was idealized in the analysis as a single-degree-of-freedom system with stiffness characteristics determined by the primary load-deflection curve and Takeda's hysteresis rule (1970). It was pointed out that Takeda's hysteresis rule sometimes provided higher stiffness and larger hysteresis area than those measured in a static test when the load was alternated in a small amplitude range. However, the agreement obtained between experiment and analysis was reported to be, in general, satisfactory particularly in the simulated earthquake tests.

A multistory frame structure was initially idealized as a shear-beam system. The nonlinear behavior of the system was studied by many investigators with the development of high-speed electronic digital computers. Penzien (1960) studied the behavior of six-degree-of-freedom systems with elasto-plastic stiffness characteristics subjected to an earthquake motion.

Berg and Dadeppo (1960) studied numerical methods to analyze a rectangular plane framework with elasto-plastic members. Berg (1961) also
analyzed a four-story one-bay plane frame with the members having ideal elasto-plastic moment-rotation characteristics.

Goel (1967) analyzed multistory one-bay frames to strong motion earthquakes. Ramberg-Osgood function was adopted to represent the moment-curvature relationship of beam members, while linearly elastic moment-curvature relationship was assigned to column members. Although curvature distribution was nonlinear over a beam member, the point of contraflexure always remained at the center of the beam because of symmetry of the frame. Therefore, moment-rotation relationship of beam member was uniquely determined as a function of moments at ends of the beam. The equations of motion were solved by Gill's version of the fourth-order Runge-Kutta numerical procedure using a finite increment of time.

Clough, Benuska and Wilson (1965) analyzed twenty-story three-bay frame structures to an earthquake motion. Each member was assumed to have bilinear moment resistance. Instead of using bilinear rotational springs at the ends of a member, two parallel elements were introduced in a member: an elasto-plastic component to represent yielding characteristic, and a fully elastic element to represent strain hardening.

Aoyama and Sugano (1968) adapted this method for the analysis of reinforced concrete frames by using a three-component model to represent three characteristic stages: elastic, post-cracking, and post-yielding. Stiffness properties of each component element were proposed to be based on experimental results of the members.

Giberson (1967) discussed and compared two nonlinear models: a two-component model developed by Clough et al (1965) and a one-component
model with equivalent nonlinear springs at the ends of a member. The one-
component model was favored because of its versatility in utilizing various
hysteresis models. Moment-rotation characteristics of the equivalent
nonlinear springs were to be calculated from the curvature distribution and
a fixed point of contraflexure at the center of the member, unless stiff-
ness characteristics of the springs were provided.

Suko and Adams (1971) also used the concept of the equivalent
nonlinear springs in analyzing multistory multibay steel frames. All
inelastic deformation of a member was lumped in rotations of two equivalent
springs at the ends of the member. The location of the contraflexure
point was determined for each member in the structure at the elastic stage,
and was assumed to remain constant during the analysis of the frame.
CHAPTER 2

OUTLINE OF EXPERIMENTAL WORK

Three-story one-bay reinforced concrete empty frame models were tested under a series of simulated earthquake motions on the University of Illinois Earthquake Simulator (referred to as the earthquake simulator). The test specimens represent approximately one-eighth of full-scale three-story reinforced concrete structures.

A test frame (Fig. 2.1) consisted of three beams of identical section properties and length at each story level, and of two continuous columns from the base to the top of the frame. The distance between the two column center lines measured 36.0 in. The dimensions of a column were 2.5 by 2.5 in. with gross reinforcement ratio of 3.2 percent (four No. 2 deformed bars). The dimensions of girders were 2.5 by 3.0 in. with gross reinforcement ratio of 2.67 percent (four No. 2 deformed bars).

Small aggregate concrete was used in all the specimens. The details of the specimens and the material properties are described in Appendix A.

Two identically designed frames were fastened onto the earthquake simulator platform parallel to each other and to the direction of motion. In order to increase stiffness and to prevent failure in the transverse direction the two frames were connected at each floor level outside of the beam-column joints by rigid steel racks. Approximately 1885-lb steel weight including weight of a steel rack was attached at each floor level to simulate the dead and live loads and also to develop horizontal inertia forces under earthquake conditions. The steel weights were concentrated
outside of the beam-column joints so that the static gravity load did not disturb the specimen except in the columns. Including the weight of the specimen, the effective story weight was 1960 lb.

The frames were subjected to a base motion in one-horizontal direction parallel to their planes. The table motion was intended to simulate one horizontal component of recorded earthquakes in California. The earthquake records chosen were the NS component of the 1940 El Centro record (Imperial Valley earthquake) and the N21E component of the 1952 Taft record (Tehachapi shock). In order to simulate the relative relation between the natural frequencies of the frames and the frequency content of the earthquake, the time axis of the earthquake record was compressed by the factor of 2.5. The amplitude of the earthquake records was chosen arbitrarily, and was increased from one earthquake test run to another until the capacity of the earthquake simulator was reached. The performance of the earthquake simulator is described in Appendix B. The method of scaling test models is discussed briefly in Appendix C with a special emphasis on the relation between dimensional scale and time scale.

Three independent dynamic tests D1, D2 and D3 were carried out. The test variables were the intensity of the "first earthquake" and the strength of the frames. Frames D11, D12, D21 and D22 in Tests D1 and D2 had the same design strength. The strength of Frames D31 and D32 in Test D3 was smaller than that of the first group. The maximum base accelerations measured on the earthquake simulator platform in the test runs are listed in Table 2.1.
Displacements relative to the base of the specimen and absolute accelerations were recorded at the three beam levels parallel to the motion as the fundamental responses of the frames. Vertical accelerations on the steel racks, transverse accelerations at the third-level beam-column joints and strains in the reinforcement of the first-story columns and beam were recorded to provide supplementary information.

Observed test results are described in Chapter 5 with a special emphasis on the maximum response values, waveforms and crack patterns in the test frames.
CHAPTER 3

LOAD-DEFORMATION CHARACTERISTICS OF FRAME ELEMENTS

3.1 Material Properties

Deformational properties of the test frames are based on the measured stress-strain relationship of the reinforcement and the concrete. The stress-strain relationship was idealized by mathematical functions based on average properties of the materials in order to simplify the computations for the moment-curvature relationship of a section. Two functions were assumed for the behavior of the reinforcement and the concrete under constantly increasing loading.

(a) Stress-Strain Relationship of the Concrete

A parabola combined with a straight line as was proposed by Hognestad (1951) was adopted in this analysis with some modification. Accordingly,

\[
\begin{align*}
     f_c &= 0 & \varepsilon_c \leq \varepsilon_t \\
     f_c &= f'_c \left[2 \frac{\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right] & \varepsilon_t \leq \varepsilon_c \leq \varepsilon_o \\
     f_c &= f'_c \left[1 - Z (\varepsilon_c - \varepsilon_o)\right] & \varepsilon_o \leq \varepsilon_c \\
\end{align*}
\]

and

\[
\begin{align*}
     \varepsilon_t &= \varepsilon_o \left(1 - \sqrt{1 - \frac{f_t}{f'_c}}\right) & (3.2) \\
     f_t &= -6.0 \sqrt{f'_c} & (3.3)
\end{align*}
\]
where

\[ f_c = \text{stress of the concrete} \]
\[ f' = \text{compressive strength of the concrete, measured from 4 by 8-in. concrete cylinder tests} \]
\[ f_t = \text{tensile strength of the concrete, given by Eq. 3.3, based on splitting tests of 4 by 8-in. concrete cylinders} \]
\[ \varepsilon_c = \text{strain of the concrete} \]
\[ \varepsilon = \text{strain at which } f' \text{ is attained} \]
\[ \varepsilon_t = \text{strain given by Eq. 3.2, which is consistent with Eq. 3.1} \]
\[ Z = \text{constant which defines the descending slope of the stress-strain curve. The numerical value of 100.0 was used in this analysis.} \]

The proposed curve is shown in Fig. 3.1, in comparison with a measured stress-strain curve. If the strain \( \varepsilon_o \) at the maximum stress is taken directly from a test, the proposed curve shows some discrepancy from the measured response.

From a technical viewpoint it was difficult to read an \( \varepsilon_o \) at the right moment of the maximum stress during a compression test of a concrete cylinder. Therefore, the \( \varepsilon_o \) was chosen so that Eq. 3.1 gave the same strain \( \varepsilon_c \) as a strain measured at 70 percent of compressive strength. The proposed curve with a revised \( \varepsilon_o \) is shown in Fig. 3.1. It compares more favorably with the measured curve.
The modified strains $\varepsilon_o$ are plotted in Fig. 3.2 with respect to compressive strength of the concrete. The general trend is that the strain $\varepsilon_o$ at compressive strength increases with the compressive strength but the points in the figure scatter too widely to formulate a precise relation.

The descending slope was not measured during a compression test because of the limitations of the testing machine. Therefore, the constant $Z$, which defines the slope of a descending branch, was arbitrarily chosen as 100.0. According to Hognestad (1951), the constant $Z$ will be approximately 43 for $\varepsilon_o = 0.003$. The effect of the constant on the moment-curvature relation will be discussed in a later section.

(b) **Stress-Strain Relationship of the Steel**

A piecewise linear stress-strain relationship was assumed for the reinforcing steel. Accordingly,

\[
\begin{align*}
\sigma_s &= E_s \varepsilon_s & &\varepsilon_s \leq \varepsilon_y \\
\sigma_s &= f_y & &\varepsilon_y \leq \varepsilon_s \leq \varepsilon_{sh} \\
\sigma_s &= f_y + E_{sh} (\varepsilon_s - \varepsilon_{sh}) & &\varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_{su} \\
\sigma_s &= f_{su} & &\varepsilon_{su} \leq \varepsilon_s
\end{align*}
\]  

(3.4)

in which

$\sigma_s =$ stress of the steel

$f_y =$ yield stress of the steel
\( f_{su} \) = ultimate stress of the steel
\( \varepsilon_s \) = strain of the steel
\( \varepsilon_y \) = strain at which \( f_y \) is attained
\( \varepsilon_{sh} \) = strain at which strain hardening commences
\( \varepsilon_{su} \) = strain at which \( f_{su} \) is attained
\( E_s \) = elastic Young's modulus of the steel
\( E_{sh} \) = modulus to defined stiffness in strain hardening range.

Young's modulus \( E_s \) of the steel was assumed to be 29,000,000 psi in this analysis. The slope for the strain hardening was found from the yield stress \( f_y \), the ultimate stress \( f_{su} \), the strain \( \varepsilon_{sh} \) at the strain hardening, and the arbitrarily chosen ultimate strain \( \varepsilon_{su} \). A typical example of idealized stress-strain relationship of the reinforcing steel is shown in Fig. 3.3. The stress-strain relationship of the reinforcing steel was assumed to be symmetric with respect to the origin of the relationship.

3.2 Moment-Curvature Relationship of a Section

The primary moment-curvature curve, which is defined as the moment-curvature curve for a constantly increasing load, provides a good index to understand the behavior of the section under consideration even in the case of reversed loadings.

The numerical values of bending moments and curvatures corresponding to flexural cracking and yielding can be calculated from the geometry of the section, the amount of existing axial load, the properties of concrete and reinforcing steel, and with Bernoulli's Hypothesis, which
assumes a linear strain distribution across the depth of the section.

(a) **Flexural Cracking**

Flexural cracking of a reinforced concrete section was assumed to occur when the stress at the tensile extreme fiber of the section exceeded the tensile strength of the concrete.

In addition to the material properties given in Section 3.1, stress-strain relationship of the concrete was approximated as being linearly elastic. Young's modulus $E_c$ of the concrete was calculated as the secant modulus at a stress equal to 40 percent of the compressive strength. Furthermore, the tensile strength of the concrete was assumed to be equal to the tensile strength from splitting tests of 4 by 8-in. concrete cylinders. The use of modulus of rupture resulted in values which were unrealistically high.

The cracking moment of the section can be written as

$$M_c = Z_e (f_t + \sigma_a)$$

$$Z_e = \frac{(I_c + n_1s)}{x} \quad \text{(3.5)}$$

in which

- $M_c$ = cracking moment of a section
- $f_t$ = tensile strength of the concrete found from the splitting tests of 4 by 8-in. concrete cylinders
\[ \sigma_a = \text{axial stress existing in the section} \]
\[ I_c = \text{moment inertia of a concrete section along the neutral axis} \]
\[ I_s = \text{moment inertia of the steel along the neutral axis} \]
\[ x = \text{distance from the neutral axis to the extreme tensile fiber of the section} \]
\[ E_s = \text{Young's modulus of reinforcing steel (} = 29.0 \times 10^6 \text{ psi)} \]
\[ E_c = \text{secant modulus of the concrete at } 0.4 f_c^' \]

The material properties used are listed in Table 3.1. The values were taken from a specimen which had average concrete properties of the test frames in a series.

(b) Flexural Yielding

The flexural yielding was defined as a stage when the tensile reinforcement yielded in tension, which generally causes a drastic change in the slope of a moment-curvature diagram. If the tensile reinforcement is arranged in multi-layers, the stiffness change takes place gradually from the commencement of a yielding of the furthest layer reinforcement steel to that of the closest layer to the neutral axis of the section.

If strain and stress distributions were assumed as shown in Fig. 3.4, strains and curvature are related by the assumption of linear strain distribution:

\[ \phi = \frac{\varepsilon_c}{c} \]

\[ = \frac{\varepsilon_s^' / (c - d')}{(c - d')} \]  \hspace{1cm} (3.6)

\[ = \frac{\varepsilon_s}{(d - c)} \]
in which

\[ \phi = \text{curvature} \]

\[ \varepsilon_c = \text{concrete strain at the extreme compressive fiber} \]

\[ \varepsilon_s' = \text{strain in the compressive reinforcement} \]

\[ \varepsilon_s = \text{strain in the tensile reinforcement} \]

\[ d' = \text{distance from the extreme compressive fiber to the center of compressive reinforcement} \]

\[ d = \text{distance from the extreme compressive fiber to the center of tensile reinforcement} \]

\[ c = \text{depth of the neutral axis}. \]

From the equilibrium condition of the resultant force

\[ \int_{-c'}^{c} f_c b \, dx + A_s' f_s' - A_s f_s = N \]  \hspace{1cm} (3.7)

in which

\[ f_c = \text{stress in the concrete} \]

\[ f_s' = \text{stress in the compressive reinforcement} \]

\[ f_s = \text{stress in the tensile reinforcement} \]

\[ b = \text{width of the cross section} \]

\[ A_s' = \text{area of the compressive reinforcement} \]

\[ A_s = \text{area of the tensile reinforcement} \]

\[ N = \text{axial load acting on the section} \]

\[ c' = \text{distance from the neutral axis to the point of the maximum tensile stress}. \]
The stresses \( f'_{c}, f'_{s}, \) and \( f_{s} \) can be calculated by Eq. 3.1 and 3.4 for given strains \( \varepsilon'_{c}, \varepsilon'_{s}, \) and \( \varepsilon_{s} \), respectively.

The integration in Eq. 3.7 can be evaluated if a strain \( \varepsilon_{c} \) at the compressive fiber and depth \( c \) to the neutral axis are given:

\[
X = \int_{c'}^{c} f_{c} b \, dx
\]

Strain at distance \( x \) from the neutral axis can be expressed

\[
\varepsilon = \frac{x}{c} \varepsilon_{c}
\]

and rewritten in a differential form

\[
dx = \frac{c}{\varepsilon_{c}} \, d\varepsilon
\]

Let strain at \( c' \) be \( \varepsilon_{t} (<0) \), then \( X \) can be rewritten as

\[
X = \frac{bc}{\varepsilon_{c}} \int_{\varepsilon_{t}}^{\varepsilon_{c}} f_{c} \, d\varepsilon
\]

If the integration \( X \) is evaluated by using Eq. 3.1
(1) $\varepsilon_t < \varepsilon_c < \varepsilon_o$

$$X = \frac{bc}{\varepsilon_c} \int f_c \left[ \left( \frac{\varepsilon_c}{\varepsilon_o} \right) \varepsilon_c - \frac{1}{3} \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 \varepsilon_c - \left( \frac{\varepsilon_t}{\varepsilon_o} \right) \varepsilon_t \right.$$  
$$\left. + \frac{1}{3} \left( \frac{\varepsilon_t}{\varepsilon_o} \right)^2 \varepsilon_t \right]$$  

$$= \frac{bc}{\varepsilon_c} \int f_c \left[ \frac{2}{3} \varepsilon_o + \frac{1}{3} \left( \frac{\varepsilon_t}{\varepsilon_o} \right)^2 \varepsilon_o - \left( \frac{\varepsilon_t}{\varepsilon_o} \right) \varepsilon_o \right.$$  
$$\left. + \left[ 1 - \frac{2}{3} \left( \varepsilon_c - \varepsilon_o \right) \right] \left( \varepsilon_c - \varepsilon_o \right) \right]$$  

(3.8)

(2) $\varepsilon_o < \varepsilon_c$

$$X = \frac{bc}{\varepsilon_c} \int f_c \left[ \frac{2}{3} \varepsilon_c + \frac{1}{3} \left( \frac{\varepsilon_t}{\varepsilon_c} \right)^2 \varepsilon_c - \left( \frac{\varepsilon_t}{\varepsilon_c} \right) \varepsilon_c \right.$$  
$$\left. + \left[ 1 - \frac{2}{3} \left( \varepsilon_c - \varepsilon_o \right) \right] \left( \varepsilon_c - \varepsilon_o \right) \right]$$

If $\varepsilon_s = \varepsilon_y$, then the unknowns $\phi, \varepsilon_s', \varepsilon_c$ and $c$ can be uniquely solved by Eq. 3.6 and 3.7, although Eq. 3.7 is a nonlinear equation.

Bending moment $M$ at the depth $x$ can be calculated:

$$M = \int_{-c'}^{c} f_c \eta \, dn + X (x - c) + A_s' f_s' (x - d')$$  
$$+ A_s f_s (d - x) + N (x - \frac{D}{2})$$  

(3.9)

in which

$D =$ total depth of the section.

The integration $Y$ in Eq. 3.9 can be evaluated if Eq. 3.1 is substituted:
\[ Y = \int_{-c'}^{c} f_c \, b \, \eta \, d\eta \]

(1) \( \varepsilon_t \leq \varepsilon_c \leq \varepsilon_o \)

\[ Y = b \, f'_c \left( \frac{c}{\varepsilon_c} \right)^2 \left[ \left( \frac{\varepsilon_c}{\varepsilon_o} \right) \varepsilon_c^2 \left( \frac{2}{3} - \frac{1}{4} \frac{\varepsilon_c}{\varepsilon_o} \right) - \left( \frac{\varepsilon_t}{\varepsilon_o} \right) \varepsilon_t^2 \left( \frac{2}{3} - \frac{1}{4} \frac{\varepsilon_t}{\varepsilon_o} \right) \right] \]

\[ (3.10) \]

(2) \( \varepsilon_o \leq \varepsilon_c \)

\[ Y = b \, f'_c \left( \frac{c}{\varepsilon_o} \right)^2 \left[ \frac{5}{12} \frac{\varepsilon_o^2}{\varepsilon_o^2} - \frac{\varepsilon_t^3}{\varepsilon_o^2} \left( \frac{2}{3} - \frac{1}{4} \frac{\varepsilon_t}{\varepsilon_o} \right) \right. \]

\[ + \frac{1}{2} \left( 1 + 2\varepsilon_o \right) \left( \varepsilon_c^2 - \varepsilon_o^2 \right) - \frac{7}{3} \left( \varepsilon_c^3 - \varepsilon_o^3 \right) \left. \right] \]

In the current analysis a bending moment was evaluated along the plastic centroid of the section.

The above mentioned procedure can be easily applied in the case where a strain at the extreme compressive fiber is given. It may not be practical to solve Eq. 3.6 and 3.7 directly because the solution may not be available in a closed form. Therefore, a recommended procedure is first to draw interaction diagrams of the section at a strain either in the reinforcement or at the compressive fiber and then to read the bending moment at the given axial load. The material properties as well as section
properties used are listed in Table 3.2. The axial load-bending moment and axial load-curvature interaction diagrams of columns are shown in Fig. 3.5 for the idealized material properties. The calculated values are listed in Table 3.3 for two different series of tests.

3.3 Factors that Affect Moment-Curvature Relationship

Some factors that affect the moment-curvature relationship of the reinforced concrete sections used in this study are briefly discussed.

(a) Properties of the Concrete

Properties of the concrete could be summarized by the three indices which described the stress-strain relationship of the concrete. These indices are compressive strength $f'_c$, strain $\varepsilon_o$ at which the maximum stress is attained, and descending slope $Z$ after the maximum stress is attained.

Moment-curvature relationship is compared in Fig. 3.6 to see the effect of the descending slope $Z$ in a concrete stress-strain relationship. All the other material properties and the section properties were kept the same. The value $Z$ does not have any influence on the moment-curvature diagram up to the yielding of the tensile reinforcement, because the compressive fiber strain has not reached $\varepsilon_o$, hence has not experienced the descending strain at the yielding of the tensile reinforcement. The difference between two curves is negligible in a practical sense even after yielding. However, if the descending slope is steeper in a concrete stress-strain curve, the straight descending branch crosses the zero stress axis at a smaller limiting strain. Therefore, the compressive concrete starts
to be ineffective at a smaller limiting strain, which results in the failure of the section at a smaller curvature.

Moment-curvature relationships are compared in Fig. 3.7 to see the effect of the compressive strength $f'_c$ of the concrete combined with the strain $\varepsilon_o$. The upper two curves were from the sections in the first two series of the tests, and the lower two curves from the sections in the last series of the tests. The difference between the two groups was the location of tensile and compressive reinforcement and the properties of the concrete. The comparison should be made between the two curves of a group. The cracking point is affected by the compressive strength of the concrete because the cracking moment is proportional to the tensile strength of the concrete (Eq. 3.5), which is closely related to the compressive strength of the concrete. At and beyond yielding, the section with a stronger concrete gave a larger moment and a larger curvature for the same compressive fiber strain $\varepsilon_c$. If the difference of 13.4 percent and 21.7 percent in the compressive strength of the concrete is recognized for the upper and the lower groups, respectively, the influence of the compressive strength can be considered to be small.

(b) Properties of the Reinforcement

Moment-curvature relationships are compared in Fig. 3.8 to note the effect of the properties of the reinforcement. Three different stress-strain curves were considered, but it was not intended to study the effects of individual index values such as yield and ultimate stresses, yield, strain-hardening and ultimate strains. Elastic Young's modulus of steel
was assumed to be 29,000,000 psi. The properties of the concrete were not exactly same, but the compressive strength of 5,140 psi and 5,340 psi should not make appreciable difference from the observation in (a) of this section.

The curvatures were almost the same for the same compressive fiber strain. The yield moments were affected by the yield stresses of the reinforcement. With the commencement of strain-hardening in the tensile reinforcement, the section starts to carry additional moment. The second slope seems to be closely related to the strain-hardening slope of the reinforcement. This can be observed in the same figure, where the third curve in Fig. 3.8 was calculated from a section with the steepest strain-hardening slope.

(c) Arrangement of Reinforcing Steel

It may be obvious that the location of reinforcing steel affects the moment-curvature relationship of a section. This can be observed in Fig. 3.7, where the upper two curves are calculated from section with the less concrete cover depth.

If the distance from the compressive fiber to the tensile reinforcement is longer, the moment and curvature for the same compressive fiber strain are larger, hence, the start of strain hardening is earlier. The cracking point is also affected due to the contribution of steel to the moment inertia of the section.

(d) Existence of Axial Load

Moment-curvature relationships are compared in Fig. 3.9 to see the effect of axial load. Curves 2 through 4 were calculated for the axial
stresses in the first-, second- and third-story columns during a test. If an axial stress is larger, the bending moment is larger and the curvature is smaller for a given compressive fiber strain. The cracking point is affected in the same way.

3.4 Idealization of Moment-Curvature Relationship

The primary moment-curvature curve can be expressed, without too much distortion, by two points and one slope, i.e., (1) a flexural cracking point, (2) a yielding point and (3) a slope after yielding of the tensile reinforcement.

Although the stress-strain relationship of the reinforcement has a strong influence on the shape of the moment-curvature curve of a section, the tensile tests of 10 coupons from a single No. 2 deformed bar gave scattering yield stresses and ultimate stresses (Appendix A.1(b)). Therefore, it was decided to use the average values to define the stress-strain relationship of the reinforcement.

It was not necessary to take into consideration the difference in concrete properties in the series of test frames since the properties of concrete had much less influence on the shape of the moment-curvature curve than that of the reinforcement. Therefore, the average values were used to define the stress-strain relation of the concrete.

Fig. 3.9 (a) and (b) show the calculated representative moment-curvature relationship for the first two test series and the last test series, respectively. In simplifying the moment-curvature relationship, the calculated cracking point was adopted without modification. After the
section cracks, the moment-curvature relationship was assumed to follow a line which connected the origin and the calculated yield point until the moment reached the yielding moment. After the yielding moment is reached, the moment-curvature relationship was assumed to follow a line which connected the yield point and a point calculated at the compressive fiber strain $\varepsilon_c = 0.008$.

The idealized moment-curvature relationship is shown in Fig. 3.10 (a) and (b) for a beam and the third-story column in comparison with the calculated curves.

Idealized moment-curvature relationships were developed for one beam and each column at the three stories and for two different arrangements of the reinforcement.

3.5 Moment-Deformation Relationship of a Cantilever Beam

In analyzing the overall response of the frame, the portion of the beam or column between the varying point of contraflexure and the joint was considered as a basic unit. Thus the moment-deformation relationship of a cantilever is one of the fundamental steps of the analytical method used.

To construct the moment-deformation curve of a cantilever, the bending moment was assumed to be distributed linearly over the length of the beam with zero moment at the free end and the maximum moment at the fixed end. The deformation was calculated at the free end as displacement and rotation.
Shear deformation as well as axial deformation was neglected in calculating deformation. Therefore, rotation and displacement can be computed from the distribution of curvature over the beam and the boundary conditions.

An idealized moment-curvature relationship which is described in Section 3.4 is used in order to compute free end rotation and displacement. An idealized moment-curvature relationship can be expressed as three straight lines:

\[
\phi = \frac{M}{EI} \quad M \leq M_c
\]

\[
\phi = \frac{\phi_y M}{M_y} \quad M_c \leq M \leq M_y \tag{3.11}
\]

\[
\phi = \phi_y \left[1 + \frac{1}{EI_y} \left(\frac{M}{M_y} - 1\right)\right] \quad M < M_y
\]

in which

- \(EI\) = initial flexural rigidity for transformed section
- \(M\) = bending moment
- \(M_c\) = cracking moment
- \(M_y\) = yielding moment
- \(\phi\) = curvature
- \(\phi_c\) = cracking curvature
- \(\phi_y\) = yielding curvature
- \(EI_y\) = slope in the moment-curvature curve after yielding.
If a moment is given, then the curvature is calculated by Eq. 3.11. As bending moment varied linearly in the cantilever, the curvature distribution could be defined if the fixed end moment was given.

Rotation at the free end of a cantilever beam can be calculated from a curvature distribution by simply computing area under the curvature diagram along the length of a beam. Displacement at the free end of a cantilever beam can be calculated from a curvature distribution by computing the first moment of the curvature diagram along the free end. For a uniform cantilever beam with a triangular distribution of bending moment, the curvature diagram along the beam is defined by the fixed end moment. Hence, the free end rotation and the displacement are also defined by the fixed end moment.

If the free end rotation, the free end displacement, and the fixed end moment are denoted as \( R(M) \), \( D(M) \) and \( M \), respectively, then the first two variables are expressed as the function of the last variable as follows:

\[
(1) \quad M < M_c
\]

\[
R(M) = \frac{L}{2} \cdot \frac{M}{EI}
\]

\[
D(M) = \frac{L^2}{3} \cdot \frac{M}{EI}
\]

(3.12)
(2) \( M_c < M \leq M_y \)

\[
R(M) = \frac{L}{2} \left[ (1 - \lambda_c^2) \frac{\phi_y}{M_y} M + \lambda_c \phi_c \right]
\]

\[
D(M) = \frac{L^2}{3} \left[ (1 - \lambda_c^3) \frac{\phi_y}{M_y} M + \lambda_c^2 \phi_c \right]
\]

\[
\lambda_c = \frac{M_c}{M}
\]

(3) \( M_y < M \)

\[
R(M) = \frac{L}{2} \left[ (1 - \lambda_y) \left\{ \lambda_y + \frac{1}{E I_y} (1 - \lambda_y) \right\} + \lambda_y - \lambda_c^2 \frac{\phi_y}{\lambda_y} \right] + \frac{L}{2} \lambda_c \phi_c
\]

\[
D(M) = \frac{L^2}{6} \left[ (2 + \lambda_y) (1 - \lambda_y) \left\{ \lambda_y + \frac{1}{E I_y} (1 - \lambda_y) \right\} + \lambda_y (1 + \lambda_y) - 2 \lambda_c^3 \frac{\phi_y}{\lambda_y} + \frac{L^2}{3} \lambda_c^2 \phi_c \right]
\]

\[
\lambda_c = \frac{M_c}{M}
\]

\[
\lambda_y = \frac{M_y}{M}
\]

in which \( L \) = length of the cantilever beam.
It should be noted that the free end rotation is always proportional to the length of the cantilever beam and that the free end displacement is always proportional to the square of the length of the cantilever. Therefore, if the moment-rotation and the moment-displacement relationships are prepared for a cantilever of unit length, then the relationships can be modified for cantilever beam of any length by simply being multiplied by the length and the square of the length, respectively.

Moment-rotation relationship for a cantilever of unit length was calculated (Fig. 3.11) for the two idealized moment-curvature relationships. The corresponding moment-displacement relationships are idealized by the three straight lines connecting the origin, cracking, yielding, and ultimate points successively. The ultimate point was defined as a point when the moment reached a moment at which the extreme compressive fiber strain reaches 0.008. The ultimate point is not meant to represent the point when the member fails, but is defined merely to represent a point after yielding. The idealized moment-rotation and moment-displacement relationships are also shown in Fig. 3.11 and 3.12 by broken lines. The numerical values are listed in Table 3.4 for the idealized moment-deformation relationships.

3.6 Rotation due to Bond Slippage at the Ends of a Member

In addition to the deformation of members in a frame, rotation due to the slip of the tensile reinforcement along its embedded length was considered at the ends of a member.
Bond stress was assumed to be constant along the development length with a magnitude \( u \) as indicated in Fig. 3.13. From the equilibrium of forces, the development length \( L \) is computed,

\[
L = \frac{A_s f_s}{\pi D u}
\]

(3.15)

in which

- \( A_s \) = area of the reinforcement
- \( f_s \) = stress in the reinforcement
- \( D \) = diameter of a reinforcing bar
- \( u \) = average bond stress

As the bond stress is constant over the development length, the steel stress decreases linearly with the distance and becomes zero at the distance of the development length. Therefore, the elongation \( \Delta L \) of the reinforcement over the development length yields

\[
\Delta L = \frac{L f_s}{2 E_s}
\]

(3.16)

in which

- \( E_s \) = Young's modulus of the reinforcement.

If enough development length is provided at each end of a member, then the elongation can be rewritten as

\[
\Delta L = \frac{1}{8} \cdot \frac{D}{E_s u} f_s^2
\]

where the area of steel is replaced by \( \frac{\pi}{4} D^2 \).
If the compressive reinforcement does not slip and the joint concrete is rigid, the rotation $R$ due to the slip can be evaluated by the expression,

$$ R = \frac{\Delta L}{(d - d')} \quad (3.17) $$

in which

- $d$ = depth of the tensile reinforcement
- $d'$ = depth of the compressive reinforcement.

If the relation between a bending moment and a stress in the tensile reinforcement is assumed as

$$ \frac{f_y}{\sigma_s} = \frac{M}{M_y} $$

in which

- $f_y$ = yield stress of the tensile reinforcement
- $M$ = bending moment at the end of a member
- $M_y$ = yielding moment at the end of a member

Then the rotation is related to the moment by the expression:

$$ R = \frac{1}{8} \frac{D}{E_s u} \left( \frac{f_y}{M_y} \right)^2 M^2 \frac{1}{d - d'} \quad (3.18) $$

This relation is plotted in Fig. 3.14. The relationship was idealized by a bilinear relationship as is shown in Fig. 3.14 by a broken line. The first break point was found at $M = \frac{1}{2} M_y$, and the second point at $M = M_y$ on the original curve.
Bond stress $u$ was assumed to be given by the expression

$$u = 6.5 \sqrt{\frac{f_t}{c}}$$

(3.19)

The calculated rotations at the yielding of the tensile reinforcement and the break-point are listed in Table 3.5.
CHAPTER 4

FRAME ANALYSIS

4.1 Introductory Remarks

This chapter describes the method of analysis for a reinforced concrete frame subjected to static load reversals and dynamic base disturbances. The method was developed to study the behavior of a reinforced concrete frame in an inelastic range.

The major difficulty in analyzing a reinforced concrete frame dynamically as well as statically is that the range of inelastic deformation of a member extends far from the member ends, which makes the concept of equivalent inelastic hinges at the member ends unrealistic unless the location of the hinges can be shifted as a function of the amount of inelastic deformation and load history. Furthermore, the difficulty also lies in that the inelastic deformation of a member is a function of the location of the contraflexure point. In the case of dynamic analysis, the lack of knowledge on the behavior of reinforced concrete members under load reversals makes the problems more difficult to solve.

The following structural analysis method was developed to analyze a reinforced concrete empty frame. The stiffness matrix of an inelastic member was constructed by taking into account the location of the contraflexure point and the distribution of curvature along the member. The effect of the curvature distribution was simplified by considering free-end deformations of a cantilever representing the portion of a member between the end of a member and the contraflexure point. The effect of load
history was taken into account by using Takeda's hysteresis rule (Takeda et al, 1970). The detailed derivation of the flexibility matrix for a single member is discussed in Appendix E.

4.2 Assumptions

An engineering problem is generally idealized and reorganized as a mathematical problem, which can be solved either in a closed form or by a numerical method. In the formulation of a mathematical model, engineering judgment has an important role in determining which characteristic is most significant and will best approximate the solution. This section treats assumptions used in formulating the mathematical model for the analysis of the test frames.

In order to simplify the solution of the problem, eight limiting assumptions were made:

(1) The frame was idealized as a plane frame. The analysis is limited in this plane. Out-of-plane action was recorded during the test, but the magnitudes were small compared with in-plane action, which justifies two-dimensional analysis in the plane of the frame.

(2) Every member in the structure was treated as a massless line member represented by its centroidal axis. The centroidal axes of all the members in the structure lay in the plane in which the response of the structure was considered. If the individual members are slender and their overall behavior along their centroidal axis is of major concern rather than across their sections, the line member idealization is reasonable.
(3) The analysis is limited to the small deformation which retains the initial configuration.

(4) Axial and shear deformations of the frame members were ignored. When forces are applied at the ends of a slender member, the total deformation comes largely from flexural deformation. Unless a member is very slender, or unless the structure itself is very slender, the effect of axial deformation can be neglected. In addition, axial and shear deformations of a member to load reversals in an inelastic range were not clearly understood. Therefore, the analysis was limited to flexural deformation.

(5) The free end rotation and deflection of all the members as a cantilever were assumed to follow the Takeda's hysteresis rule (Takeda et al, 1970). This assumption was adopted because very few research results are available on the hysteretic behavior of reinforced concrete members to load reversals.

(6) Yield and cracking moments of a member were assumed to be independent of the change in axial load, although they were calculated for the initial axial load.

(7) The idealized frame was assumed to be fixed at the base of the first-story columns on an infinitely rigid foundation.

(8) All the mass in the structure was assumed to be concentrated at beam-column connections of the structure.

4.3 Structural System

Three independent displacement components should be considered
in a general plane frame analysis: two mutually perpendicular translations in a plane and one rotation along an axis normal to the plane.

In the course of the following analysis, the right-hand screw rule was adopted to describe the positive directions of x, y and z axes, as shown in Fig. 4.1.

The above coordinate system was used to describe all the displacement and force components of the structure, the members and the joints. In other words, the global coordinate system was used to describe the structure, the response of the structure, and the responses of the members and the joints. The usage of the global coordinate system eliminated the transformation from a local coordinate system to the global coordinate system in assembling a structural stiffness matrix, and from the global coordinate system to a local coordinate system in finding member forces.

The structure considered in this analysis was a regular rectangular empty frame with arbitrary numbers of stories and bays. In other words, for each column line from the first to the top stories, the centroid axes of all the columns must lie on the same line. Similarly, for each story level from the first bay to the last, centroid axes of all the beams at the level were on the same line. Dead loads were attached to the column-beam joints of each floor so that the beams did not carry their weight as in the test. First-story columns were fixed at their bases. The end rotation caused by slip of the reinforcement was included in the member properties.
The assumptions made for the analysis reduced the number of degrees of freedom of the structure as follows. The assumption of infinite axial rigidity resulted in the same lateral and vertical displacements at all joints of a given level. The assumption of small deformation and the assumption that all the deformation results from bending moment alone eliminated the vertical displacement component from the analysis. Therefore, the number of degrees of freedom was reduced to the number of stories in the structure in order to describe lateral story displacements and the number of joints in order to describe joint rotations. The corresponding forces were lateral loads at each floor level and bending moments at each joint.

4.4 Stiffness Matrix of a Member

A member was defined in the analysis as a structural element that connected two adjacent structural joints. The member (horizontal or vertical) consisted of a flexible line element with two rotational spring elements at its ends, and of two rigid zones outside of rotational springs (Fig. 4.2).

A stiffness matrix was developed for a simply supported member with external bending moments applied at the supports. Flexibility characteristics were first studied for constituent elements of a member under an incremental load by assuming piecewise linear force-deformation relationships. The flexibility relations were then assembled for a member and transformed into a stiffness relation.
(a) Flexibility Relation of a Rotational Spring Element

A rotational spring element was used to simulate a rotation due to the slip of the tensile reinforcement along its embedded length. An idealized moment-rotation relationship is described in Section 3.6.

A small increment of bending moment in a rotational spring element results in an incremental rotation proportional to the incremental moment if a piecewise linear moment-rotation relationship is assumed. The proportionality constant of a linearly elastic system is called a flexibility constant. For a piecewise linear moment-rotation relationship, the flexibility constant $f$, in general, is a function of the existing moment or rotation and of a load history of the spring.

The relation between an incremental moment $\Delta M$ and the resultant rotation $\Delta R$ was expressed in the form

$$\Delta R = f \cdot \Delta M \quad (4.1)$$

in which the flexibility constant $f$ of the spring element was assumed to be expressed by the simplified Takeda's system (Takeda et al, 1970) with a bilinear primary force-deflection relation.

(b) Flexibility of an Elastic Prismatic Line Element

The formation of a flexibility matrix, which transforms end external moments to end rotations of a simply supported member, was considered in the case of a linearly elastic line element.
From the assumptions made in Section 4.2, shear and axial deformations of a member were neglected. Incremental elastic rotations $\Delta \theta^e_{A'}$ and $\Delta \theta^e_{B'}$ at the ends of an element $A'B'$ (Fig. 4.3) can be expressed as a function of incremental external moments $\Delta M_{A'}$ and $\Delta M_{B'}$, in a matrix form

\[
\begin{bmatrix}
\Delta \theta^e_{A'} \\
\Delta \theta^e_{B'}
\end{bmatrix} =
\begin{bmatrix}
\frac{L}{3EI} & -\frac{L}{6EI} \\
-\frac{L}{6EI} & \frac{L}{3EI}
\end{bmatrix}
\begin{bmatrix}
\Delta M_{A'} \\
\Delta M_{B'}
\end{bmatrix}
\] (4.2)

in which

$EI = \text{flexural rigidity of the elastic line element}$

$L = \text{length of the element } A'B'$

The sign convention was adopted so that clockwise external moment and end rotation gave positive values.

(c) **Flexibility Relation of an Inelastic Line Element**

Deformation of an inelastic line element can be divided into elastic and inelastic deformations, which can be calculated separately. The elastic end rotations of a simply supported line element can be expressed by Eq. 4.2. The formulation of a flexibility matrix of a simply supported line element $A'B'$ due to inelastic deformation was considered with incremental external bending moments $\Delta M_{A'}$ and $\Delta M_{B'}$.

An attempt was made to express end rotations of a simply supported member by deformation of two cantilevers. Detailed derivation is described in Appendix E.
End rotations $\Delta \theta_A^P$ and $\Delta \theta_B^P$ due to inelastic deformation was rewritten in the form

$$\begin{bmatrix} \Delta \theta_A^P \\ \Delta \theta_B^P \end{bmatrix} = \begin{bmatrix} f_{11}^{\prime\prime} & f_{12}^{\prime\prime} \\ f_{21}^{\prime\prime} & f_{22}^{\prime\prime} \end{bmatrix} \begin{bmatrix} \Delta M_A^P \\ \Delta M_B^P \end{bmatrix}$$

(4.3)

in which $f_{11}^{\prime\prime}$, $f_{12}^{\prime\prime}$, $f_{21}^{\prime\prime}$, and $f_{22}^{\prime\prime}$ were calculated from any of Eq. E.14, E.15, and E.16 (Appendix E). It should be noted that the flexibility matrix calculated here is not symmetric ($f_{12}^{\prime\prime} \neq f_{21}^{\prime\prime}$).

(d) Flexibility and Stiffness Relations of a Line with Two Rotational Spring Elements

Incremental end rotations $\Delta \theta_A^e$ and $\Delta \theta_B^e$ of a flexible line element with two rotational springs at its ends were calculated by simply adding contributions from the constituent elements,

$$\Delta \theta_A^e = \Delta \theta_A^{e_i} + \Delta \theta_A^p + \Delta \theta_A^r$$

$$\Delta \theta_B^e = \Delta \theta_B^{e_i} + \Delta \theta_B^p + \Delta \theta_B^r$$

(4.4)

Hence, incremental end rotations were calculated from the incremental external moments $\Delta M_A^e$ and $\Delta M_B^e$ in a matrix form as shown in Eq. 4.5 by combining Eq. 4.1, 4.2 and 4.3
The stiffness matrix of the same member was obtained by simply inverting the flexibility matrix in Eq. 4.5, which resulted in the form

\[
\begin{bmatrix}
\Delta \theta_{A'} \\
\Delta \theta_{B'}
\end{bmatrix} =
\begin{bmatrix}
 f'_{11} & f'_{12} \\
 f'_{21} & f'_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta M_{A'} \\
\Delta M_{B'}
\end{bmatrix}
\]

The stiffness matrix given here is not symmetric because the flexibility matrix for an inelastic member is not symmetric \((f'_{12} \neq f'_{12})\).

(e) Treatment of Rigid Zones at the Ends of a Member

The flexural rigidity of a beam-column joint core*, in general, is much greater than that of a member. Therefore, the core can be treated as a rigid zone as far as flexural deformation is concerned. Two rigid zones \(AA'\) and \(BB'\) of length \(\lambda_A L\) and \(\lambda_B L\) were attached to a flexible part \(A'B'\) of length \(L\) outside of rotational springs (Fig. 4.2).

Moments \(\Delta M_A\) and \(\Delta M_B\) at the rigid ends of member were related to moments \(\Delta M_{A'}\) and \(\Delta M_{B'}\) at the ends of a flexible part (Fig. 4.4) through a

\* Joint core is the part common to both the beam and the column.
coordinate transformation in the form

\[
\begin{bmatrix}
\Delta M_A \\
\Delta M_B
\end{bmatrix} = \begin{bmatrix}
1 + \lambda_A & \lambda_A \\
\lambda_B & 1 + \lambda_B
\end{bmatrix} \begin{bmatrix}
\Delta M_A' \\
\Delta M_B'
\end{bmatrix}
\]

(4.7)

Similarly, rotations \(\Delta \theta_A\) and \(\Delta \theta_B\), at the ends of a flexible part including rotations from rotational springs were related to rotation \(\Delta \theta_A\) and \(\Delta \theta_B\) at the rigid ends as shown by Eq. 4.8.

\[
\begin{bmatrix}
\Delta \theta_A' \\
\Delta \theta_B'
\end{bmatrix} = \begin{bmatrix}
1 + \lambda_A & \lambda_B \\
\lambda_A & 1 + \lambda_B
\end{bmatrix} \begin{bmatrix}
\Delta \theta_A \\
\Delta \theta_B
\end{bmatrix}
\]

(4.8)

By combining Eq. 4.6, 4.7 and 4.8, the moment-rotation relationship of a simply supported member can be written in the form

\[
\begin{bmatrix}
\Delta M_A \\
\Delta M_B
\end{bmatrix} = \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix} \begin{bmatrix}
\Delta \theta_A \\
\Delta \theta_B
\end{bmatrix}
\]

(4.9)

Each member was treated as a unit in the analysis of the frame. The member could be either a beam or a column depending on the orientation of the principal axis of the member.
4.5 Formation of the Structural Stiffness Matrix

A member stiffness matrix was constructed based on a behavior of a simply supported member with external bending moments applied at the supports. The force-displacement relations of a column and a beam were studied separately in terms of the generalized forces and displacements of a frame. The generalized displacements of a frame were lateral story displacements \( \{U\} \) and joint rotations \( \{\Theta\} \). The corresponding forces were lateral load \( \{P\} \) at each floor level and bending moments \( \{M\} \) at each joint.

As all the joints in a story level experienced the same translational displacements, the principal axis of a beam was always parallel to the initial principal axis of the beam. Therefore, incremental member end rotations are the same as the incremental joint rotations of the structure. The force-displacement relation of a beam in terms of generalized forces and displacement took the same form as is shown in Eq. 4.9:

\[
\begin{pmatrix}
\Delta M_A \\
\Delta M_B
\end{pmatrix} =
\begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\begin{pmatrix}
\Delta \Theta_A \\
\Delta \Theta_B
\end{pmatrix}
\]

where

\( \Delta M_A, \Delta M_B = \text{contribution to joint bending moments at A and B of a structure from member AB} \)

\( \Delta \Theta_A, \Delta \Theta_B = \text{incremental joint rotation at A and B of a structure.} \)

A relative story displacement causes the principal axis of a column to rotate from the initial vertical position (Fig. 4.5). In order
to take into account the rigid body rotation of the principal axis of a
column, incremental end rotations $\Delta \theta_A$ and $\Delta \theta_B$ of a simply supported member
should be calculated from incremental joint rotations $\Delta \theta_A$ and $\Delta \theta_B$, and
incremental lateral story displacements $\Delta U_A$ and $\Delta U_B$ of the structure as

$$
\begin{bmatrix}
\Delta \theta_A \\
\Delta \theta_B
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\frac{1}{L(1+\lambda_A + \lambda_B)} & \frac{-1}{L(1+\lambda_A + \lambda_B)} \\
\frac{1}{L(1+\lambda_A + \lambda_B)} & \frac{-1}{L(1+\lambda_A + \lambda_B)}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_A \\
\Delta \theta_B \\
\Delta U_A \\
\Delta U_B
\end{bmatrix}
$$

The incremental lateral reactions at the supports of a simply
supported column were related to the incremental story forces $\Delta P_A$ and $\Delta P_B$
contributed from column AB. Therefore, the contributions to generalized
forces of a frame from a column were expressed by Eq. 4.12.

$$
\begin{bmatrix}
\Delta M_A \\
\Delta M_B \\
\Delta P_A \\
\Delta P_B
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\frac{1}{L(1+\lambda_A + \lambda_B)} & \frac{-1}{L(1+\lambda_A + \lambda_B)} \\
\frac{-1}{L(1+\lambda_A + \lambda_B)} & \frac{-1}{L(1+\lambda_A + \lambda_B)}
\end{bmatrix}
\begin{bmatrix}
\Delta M_A \\
\Delta M_B \\
\Delta P_A \\
\Delta P_B
\end{bmatrix}
$$

The force-displacement relation of a column was expressed by simply com-
bining Eq. 4.9, 4.11 and 4.12.
The formulation of the structural stiffness matrix was accomplished by adding force contributions from all the members in a frame at each story and joint. The force-displacement relation was arranged in the form

\[
\begin{pmatrix}
\Delta P \\
\Delta M
\end{pmatrix} =
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\begin{pmatrix}
\Delta U \\
\Delta \Theta
\end{pmatrix}
\]

(4.13)

in which

- \(K_{11}\) = submatrix of size, number of stories by number of stories
- \(K_{12}\) = submatrix of size, number of stories by number of joints
- \(K_{21}\) = submatrix of size, number of joints by number of stories
- \(K_{22}\) = submatrix of size, number of joints by number of joints
- \(\Delta P\) = incremental story force vector
- \(\Delta M\) = incremental joint moment vector
- \(\Delta U\) = incremental story displacement vector
- \(\Delta \Theta\) = incremental joint rotation vector

### 4.6 Static Analysis of Frames to Constantly Increasing Lateral Loads

An application of the proposed frame analysis method is discussed for a static case where a known set of lateral loads were applied to a reinforced concrete frame at a very small load increment up to the failure. The behavior of the structure depends greatly on the load distribution chosen in the problem. A set of external lateral loads were
increased monotonously in one direction. The structure was treated as a
linearly elastic system during a load increment. Member forces were cal-
culated at the end of each load increment. If a member force exceeded a
limiting stress, the member stiffness was modified for the next load
increment. This procedure is similar to the limit analysis, although
the method developed here recognizes more realistic moment-curvature
relationships for reinforced concrete members.

Only the lateral external loads were considered. The external
moments at the joints in a structure were assumed to be zero. Then Eq.
4.13 can be rewritten in two matrix equations:

\[
\{\Delta P\} = [K_{11}] \{\Delta U\} + [K_{12}] \{\Delta \Theta\}
\]

(4.14)

\[
\{\Theta\} = [K_{21}] \{\Delta U\} + [K_{22}] \{\Delta \Theta\}
\]

From the second equation of Eq. 4.14, \{\Delta \Theta\} can be solved as

\[
\{\Delta \Theta\} = -[K_{22}]^{-1} [K_{21}] \{\Delta U\}
\]

(4.15)

Therefore, the first equation of Eq. 4.14 can be reduced to

\[
\{\Delta P\} = [K] \{\Delta U\}
\]

(4.16)

in which

\[
[K] = [K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}]
\]

(4.17)
The matrix \([K]\) can be called a "reduced" structural stiffness matrix, which related lateral displacements to lateral forces. Eq. 4.16 is solved for lateral displacements from a given set of lateral load and a known structural stiffness. Joint rotations are calculated by Eq. 4.15 from known lateral displacements. Member forces are calculated by a stiffness relation of the member and joint displacements.

### 4.7 Dynamic Analysis of Frames to Base Motions

(a) **Equation of Motion of an Undamped System**

The equation of motion of a structure was developed from the equilibrium conditions of story forces and joint moments. Forces are induced by structural resistance and mass inertia. By the assumption that the rotational and mass inertia of members and joints were ignored, mass inertia was considered only at each story level.

It was assumed that the structural properties do not change during a small time increment. The equation of motion without damping was written for a small time increment as

\[
[M] \{\Delta \ddot{U}\} + [K_{11}] \{\Delta U\} + [K_{12}] \{\Delta \Theta\} = -[M] \{\Delta \ddot{x}_0\}
\]

\[
[K_{21}] \{\Delta U\} + [K_{22}] \{\Delta \Theta\} = 0
\]  

in which

\([M] = \text{diagonal story mass matrix}\)
\([K_{11}], [K_{12}], [K_{21}], [K_{22}] = \) submatrices of a structural stiffness matrix in Eq. 4.13

\(\{\Delta U\}\) = incremental story displacement vector

\(\{\Delta \Theta\}\) = incremental joint rotation vector

\(\{\Delta \ddot{U}\}\) = incremental story acceleration vector

\(\{\Delta \ddot{X}_0\}\) = incremental base acceleration vector

The structural characteristics were revised at each time step after the incremental responses were calculated.

From the second part of Eq. 4.18, joint rotations were solved as

\(\{\Delta \Theta\} = -[K_{22}]^{-1} [K_{21}] \{\Delta U\}\)  (4.19)

If Eq. 4.19 is substituted into the first part of Eq. 4.18, the equation of motion yields to

\([M] \{\Delta \ddot{U}\} + [K] \{\Delta U\} = -[M] \{\Delta \ddot{X}_0\}\)  (4.20)

in which

\([K] = [K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}]\)  (4.21)

(b) Treatment of Damping

The word "damping" has been used for a long time in structural dynamics with damping recognized as a resisting force as well as a source of energy dissipation. Damping was probably introduced in mechanical vibration due to the fact that the motion of any mechanical system under
oscillation tends to diminish its amplitude with time if the energy is not supplied to the system, a phenomenon which could not be explained by elastic mechanical vibrations without introducing some kind of energy absorbing mechanism either in or out of the system.

Rayleigh (1877) considered two types of damping in his book, "The Theory of Sound", in which he said, "If we suppose that each particle of the system is retarded by forces proportional ... to the absolute velocities; ... it is equally important to consider such (retarding forces) as depend on the relative velocities of the parts of the system, ..." Since then the damping has been generally treated as a viscous type which is proportional to absolute or relative velocities.

It can be claimed that damping has been introduced to reconcile the existence of some unknown sources of energy absorption, and that the viscous type damping was adopted because of mathematical simplicity.

If the mechanism and source of damping are clearly known quantitatively, then damping should be considered at the member level in a structural analysis. However, a precise quantitative value of damping is seldom available. Therefore, damping was considered at the structural level as some unknown source of energy dissipation in this analysis.

The expression should be simple and easy to solve. Consequently, viscous type damping was adopted: forces which are proportional to velocities of each floor relative to floors and proportional to velocities of each floor relative to the base of the structure.
Mathematically, this is essentially the same as introduced by Rayleigh, except that velocities are measured from the base of the structure rather than from the absolute reference. The reason for this difference is that the supports of mechanical system do not move in absolute coordinates in Rayleigh's study.

The form of the damping matrix $[C]$ in the equation of motion was assumed to have the form

$$[C] = C_1 [M] + C_2 [K]$$

where $C_1$ and $C_2$ are constants. The equation of motion is rewritten


in which

$\{\Delta U\}$ = incremental story velocity vector

Equation 4.23 can be found in the most textbooks on structural dynamics.

(c) Modal Analysis of Test Frames

It has been known that a linearly elastic system with small damping effect has its own frequencies and shapes of vibration. The formulation of an eigen value problem can be found in many textbooks on structural dynamics.

Since the mass matrix $[M]$ is a diagonal matrix with positive diagonal elements, there exist a matrix $[M]^{-1/2}$ such that
Consider matrices $[\tilde{C}]$ and $[\tilde{K}]$ expressed in Eq. 4.24,

$$[\tilde{C}] = [M]^{-1/2}[C][M]^{-1/2}$$

$$[\tilde{K}] = [M]^{-1/2}[K][M]^{-1/2}$$

then they are symmetric and nonnegative since $[C]$ and $[K]$ are symmetric and nonnegative. Furthermore, since the damping matrix used is a linear combination of the mass and stiffness matrices, the system possesses classical normal modes.

Let

$$\{u\} = [M]^{-1/2}\{\phi\}\{z\}$$

(4.25)

in which

$$[\phi] = \text{real orthogonal matrix such that } [\phi]^T[\phi] = [I],$$

which diagonalizes matrices $[\tilde{C}]$ and $[\tilde{K}]$

$\{z\} = \text{vector with modal response components}$

Then the equation of motion can be written in the form

$$\ddot{z} + [\phi]^T[\tilde{C}][\phi]\{z\} + [\phi]^T[\tilde{K}][\phi]\{z\} = -[\phi]^T[M]^{1/2}\{\ddot{x}_o\}$$

(4.26)

Matrices $[\phi]^T[\tilde{C}][\phi]$ and $[\phi]^T[\tilde{K}][\phi]$ are diagonal matrices with diagonal
elements $2_s \beta_s \omega$ and $s \omega^2$, respectively: $\omega$ is a circular frequency of the $s$th mode, and $\beta_s$ is a damping factor of the $s$th mode.

Damping factors and circular natural frequencies are related to constants $C_1$ and $C_2$ in Eq. 4.22 in the following fashion:


$$= C_1 [i] + C_2 [\phi]^T [\tilde{K}][\phi]$$

Or for each row of the matrix equation

$$2_s \beta_s \omega = C_1 + C_2 s \omega^2 \quad (4.27)$$

Therefore, $C_1$ and $C_2$ are determined for given damping factors from two different modes.

Let

$$[\psi] = [M]^{-1/2} [\phi] \quad (4.28)$$

where $[\psi]$ = transformation matrix which consists of mode shape vectors.

Equation 4.26 can be rewritten in the form

$$\ddot{Z} + D[2_s \beta_s \omega] \{\dot{Z}\} + D[s \omega^2] \{Z\} = -[\psi]^T [M] \{\ddot{X}_o\} \quad (4.29)$$
in which

\[ D[2^s \beta_s \omega] = \text{diagonal matrix with diagonal elements } 2^s \beta_s \omega \text{ for } s = 1, \ldots, N \]

\[ D[\omega^2] = \text{diagonal matrix with diagonal elements } \omega^2 \text{ for } s = 1, \ldots, N \]

\[ N = \text{number of degrees of freedom of the system.} \]

or

\[ \ddot{\mathbf{z}} + D[2^s \beta_s \omega] \{\dot{z}\} + D[\omega^2] \{z\} = -\{C_s\} \ddot{\chi}_o \]  

(4.30)

in which

\[ C_s = \sum_{i=1}^{N} m_i \psi_i \text{ for } s = 1, 2, \ldots, N \]

\[ C_s \text{ is called a modal participation factor.} \]

From Eq. 4.30, the linearly elastic response of the system can be calculated in the following way. The response at a particular time can be expressed as the sum of all modal contributions. Each modal contribution can be calculated as product of the mode shape and the response of a single-degree-of-freedom system at that particular time and the modal participation factor which is the sum of story masses multiplied by mode shape vector at corresponding story levels. As linear operation alone was used in the above derivation, stress and strain responses are also calculated as the sum of modal contributions in stress and strain responses.
(d) **Numerical Solution of the Equation of Motion**

The equation of motion can be solved numerically assuming the properties of the structure does not change between two time steps. Linear variation of acceleration over the time interval was assumed in solving the equation of motion by a numerical method. With this assumption, the incremental velocity $\{\Delta \dot{U}\}$ and acceleration $\{\Delta \ddot{U}\}$ can be expressed in terms of incremental displacement $\{\Delta U\}$ and the previous step acceleration $\{\ddot{U}\}$, velocity $\{\dot{U}\}$ and displacement $\{U\}$.

\[
\{\Delta \dot{U}\} = 3\{\Delta U\} / \Delta T - 3\{\dot{U}\} - \frac{1}{2}\{\ddot{U}\} \Delta T
\]

\[
\{\Delta \ddot{U}\} = 6\{\Delta U\} / \Delta T^2 - 6\{\ddot{U}\} / \Delta T - 3\{\dot{U}\}
\] (4.31)

Equation 4.23 combined with Eq. 4.31 can be solved for the incremental displacement $\{\Delta U\}$ in the form

\[
\{\Delta U\} = [A]^{-1}\{B\}
\] (4.32)

in which

\[
\]

\[
\{B\} = [M] \left[6 \{\dot{U}\} / \Delta T + 3 \{\ddot{U}\} - \{\Delta \ddot{X}_o\}\right]
\] (4.33)

\[+ [C] \left[3 \{\dot{U}\} + \frac{1}{2} \{\ddot{U}\} \Delta T\right] \]
The incremental velocity and accelerations were calculated from Eq. 4.31. The incremental joint rotations were calculated from Eq. 4.19.

(e) Correction of Unbalanced Forces

When the responses of a frame were calculated in the incremental form, the total forces may violate the equilibrium condition at the joints and at the story level. Two major sources of the error were considered.

The equation of motion was solved by a numerical procedure between two time steps for the incremental responses. The responses at a time step was calculated as the sum of all the increments to that time step. The equilibrium condition for forces was satisfied incrementally, but the accumulation of numerical errors may cause the unbalance of resultant forces. The accumulation of numerical error was not corrected in the analysis.

The force-deformation relationships of structural members were idealized to be piecewise linear, while the responses from the equation of motion may result in the deviation of member forces from the assumed force-deformation relationships. In the dynamic analysis used, the deviation of forces was corrected so as to satisfy the force-deformation relationships at each time step. Due to the correction, the equilibrium condition was violated in the equation of motion. The unbalanced moments at each joint were detected by simply adding incremental member end moments at the joint, and were included in the equation of motion as correction terms.
The unbalanced story forces were hard to detect because the total story forces should be examined for the equilibrium. It was thought that the assumption of the damping mechanism might cause a larger error in the story response than that from unbalanced story force. Therefore, the story forces were corrected only for the terms that were resulted from the unbalanced moments at the joints.
CHAPTER 5

OBSERVED RESPONSES DURING SIMULATED EARTHQUAKE TESTS

5.1 Introductory Remarks

This chapter describes the observed behavior of the test specimens subjected to simulated earthquake motions. Three specimens D1, D2, and D3, each consisting of two identically designed frames, were tested on the earthquake simulator platform under a series of simulated earthquake motions. The outline of the experimental work was described in Chapter 2.

The spectrum intensity, which is a measure of the intensity of base motion, was used as a basis for comparing the behavior of the test specimens under different base motions. The values of the spectrum intensity should not be directly compared with those calculated from a real earthquake because of the difference in linear and time scales between full-scale structures and the test specimens: the test specimens represented approximately one-eighth of a prototype structure, and the original time axis was compressed by a factor of 2.5 in the test.

Observations were made on the recorded signals during a test run and on the crack patterns after a test run. Response signals were studied for their maximum values, waveforms and frequency components.

5.2 Index to Define the Intensity of Base Motion

It is necessary to define an index for the intensity of base motion in addition to maximum acceleration in order to have a basis for comparing structural response to different earthquake motions.
When an earthquake waveform is fed into the command center of the earthquake simulator, the amplitudes of the earthquake signal are linearly proportioned by a potentiometer, designated "Span". The value at which the "Span" dial is set represents the best index for the intensity of base motion. This value, however, does not have an engineering meaning by itself, and does not work as a reference if the input earthquake signal is changed.

The maximum acceleration on the test platform is a good index for representing the intensity of a given base motion. Measured maximum base accelerations are compared in Fig. 5.1 with the dial setting of "Span" in Test D1, where the "Span" setting was increased from one test run to another without any change in the input earthquake motion up to the fifth run. The maximum accelerations increased with "Span" settings, but they were not proportional to the "Span" settings. This may be because the base acceleration contains high frequency components and the maximum value is very sensitive to the existence of high frequency noise. Consequently, the maximum base acceleration is not considered to be a good index.

Housner (1952) proposed spectrum intensities to develop a measure of the intensity of ground motion. The spectrum intensity is defined to be the area under a velocity response spectrum curve between periods of 0.1 and 2.5 sec

\[
S_{I_{\beta}} = \int_{0.1}^{2.5} S_{V}(\beta, T) \, dT \quad (5.1)
\]
in which

\[ S_{I_B} = \text{spectrum intensity at damping } \beta \]
\[ S_v(\beta, T) = \text{velocity response curve} \]
\[ \beta = \text{damping factor in percent} \]
\[ T = \text{period of a linearly elastic system} \]

Spectrum intensities of base acceleration signals measured on the earthquake simulator platform were calculated between 0.4 and 1.0 sec and periods in order to be consistent with the time scale of 1/2.5.

Spectrum intensities are compared with the "Span" dial settings in Fig. 5.2 for damping factors 0, 5 and 20 percent of critical. Solid circles were calculated from acceleration records measured at the base of the north frame in Test D1. Open circles were calculated from acceleration records measured at the base of the south frame. The discrepancy between solid and empty circles should be largely due to the error in calibrating the two signals. Spectrum intensities for the three damping factors can be observed to be proportional to the "Span" dial settings. Therefore, the spectrum intensity was chosen to define the intensity of base motion. A damping factor of 0.20 is used to calculate the spectrum intensity in this report because the shape of the response spectrum curve is less sensitive to the input waveform for larger damping factors.

Spectrum intensities for damping factors of 0.0 and 0.05 are compared with that for a damping factor of 0.20 in Fig. 5.3 for sixteen test runs in Tests D1 through D3. Spectrum intensities at the damping factor of 0.0 were affected by the shape of base acceleration record, while those at a damping factor of 0.05 were not.
Spectrum intensities are compared with maximum base accelerations in Fig. 5.4 for all sixteen test runs. Up to a spectrum intensity of approximately 120.0 in., a linear relation exists between the maximum acceleration and the spectrum intensity, but after that the maximum acceleration increases more rapidly than the spectrum intensity. Evidently, there was distortion of the waveform beyond accelerations of approximately 1.2 g.

5.3 General Observations

(a) Instrumentation

The detailed description of instrumentation can be found in Appendix A.4. A brief description is presented in this section. Behavior of the specimen was measured in terms of displacements at the three beam levels relative to the base of the frame, and absolute accelerations at the same three levels.

Deformation of the test frames was measured at the three beam levels and the base girder relative to the steel A-frames (Fig. A.19 in Appendix A). The displacement measurement at the base girder indicated that the base of the test frames did not move relative to the A-frames more than 0.01 in., well within the accuracy of the instruments. The natural frequency of the steel A-frames was 70 Hz. Inspection of the displacement records revealed no motion at that frequency range at any level. Therefore, the measured displacement signals at the three levels were assumed to represent displacements relative to the base of the frame.
The displacement waveforms reported in this chapter were measured on the south frame in Test D1 and on the north frame in Tests D2 and D3. The choice of the frame was arbitrary: signals reported were those which were recorded on the same tape as the acceleration signals.

Accelerations were measured at two locations on a steel rack at each of the three levels in the fundamental direction of motion. The signals measured at east side of the steel racks are reported here. Base accelerations were measured at the top of the base girders of the test frames. The signals measured on the south frame are presented with their response spectra. The choice of gages was arbitrary, but the waveforms measured at two locations of the same level were almost identical (Appendix A.4).

Base shears and overturning moments for a single frame were calculated from the measured acceleration signals at the three levels combined with the story weight (980 lb/story) and the story heights. Base shear was defined as a lateral force acting in the first story and was calculated at each time step as an algebraic sum of the products of the story masses and the acceleration amplitudes at the corresponding levels. Base overturning moment was defined as the base moment on the structure as a whole due to story lateral forces and was calculated at each time step as an algebraic sum of the products of the level height from the base and lateral forces at the corresponding levels. The overturning effect of gravity loads acting through the sidesway displacements (the P-Δ effect) was ignored in calculating base moments.
(b) **Base Motion**

The acceleration signals of the El Centro (1940) NS component and the Taft (1952) N21E component were used as input to the earthquake simulator. The original time axes were compressed by a factor of 2.5 and the amplitudes of acceleration was increased after each run. Properties of the base motions are listed in Table 2.1.

The El Centro record was used in the first two to five runs with increasing intensities of the motion until the capacity of the earthquake simulator was reached. Then the Taft record was used.

Response spectra of a base motion were calculated for each run of the tests, and plotted on a tripartite logarithmic graph by calculating maximum displacement response of a series of linearly elastic systems to the base motion. The frequency of the systems was varied from 0.5 Hz to 50 Hz, which covered the frequencies of most of practical structures in the model time scale. Damping factors of 0, 5, and 20 percent of critical were used.

Response spectra (a) through (e) in Fig. 5.27 were made for the base motions, which intensity was increased after each run without changing their input earthquake waveform (the El Centro 1940 NS component) to the earthquake simulator. The shapes of the response spectra were quite similar (in a logarithmic plot) for different runs. The difference among them was in magnitude.

Response spectra (f) in Fig. 5.27 was made for the base motion which simulated the Taft (1952) N21E component. If response spectra (e) and (f) in Fig. 5.27 are compared, the difference can be noticed.
(1962) pointed that a Fourier spectrum of ground acceleration was similar to the zero-damped velocity response spectrum. In other words, the zero-damped velocity response spectrum indicates approximately the frequency content of the ground motion. When the two response spectra are compared in this sense, the simulated El Centro acceleration waveform is seen to contain large-amplitude frequencies over a wider range than the simulated Taft acceleration waveform.

(c) Measured Waveforms

Measured response waveforms are shown in Fig. 5.20 through 5.25, Fig. 5.33 through 5.36, and Fig. 5.44 through 5.47. Detailed observations are presented separately in Section 5.4, 5.5, and 5.6 for each test.

General observations about the measured waveforms are:

(1) Base acceleration waveform simulated closely the original acceleration time history without noticeable noise (Fig. A.26 in Appendix A).

(2) Base shear waveform was governed by the "first mode" component for the first one or two runs in a test, but was gradually influenced by the "second mode" component as the runs were continued. The "third mode" component did not show up noticeably in the waveform.

(3) Base overturning moment waveform was almost exclusively governed by the "first mode" component. The waveform was quite similar to the third-level displacement waveform in the early runs of each test.
(4) Acceleration waveforms contained more of "higher mode" components, especially at the lower two levels. As the base acceleration was increased, the "higher mode" components became more perceptible even in the third-level waveform, in which the "first mode" component had prevailed.

(5) The displacement waveforms were very smooth at the three levels. As the frame became damaged, the center of oscillation shifted after a large oscillation due to inelastic deformation. The three signals oscillated in the same phase.

(6) Damping factors of a specimen measured in a free vibration part of a run ranged from 4 to 7 percent of critical for the first mode even after the specimen was severely damaged.

The terms "first mode", "second mode", and "third mode" were used to describe the phase relationship of the three signals. "First mode" implies that all three level signals oscillated in the same phase. "Second mode" implies that only two adjacent level signals were in the same phase. "Third mode" implies that the first- and the third-level signals were in phase.

Existence of certain frequencies associated with these three modes was observed during the test runs, although the frequencies changed evidently related to the amount of structural damage. The existence of such frequencies can be best demonstrated by examining a smooth base shear waveform which is obtained by summing three acceleration signals with higher frequency components (for example, see Fig. 5.20 (b), (d), (e), and (f)). For the sum to be smooth, those higher frequency components should
be contained in at least two level waveforms and completely out of phase. The usage of the terms is consistent with the vibration modes associated with a linearly elastic system.

(d) **Change in Dominant Frequencies**

Frequencies associated with each mode of vibration were measured. The first-mode frequency was found on displacement signal traces. The second- and the third-mode frequencies were more easily identified on the first- or the second-level acceleration signal traces. A period of three to ten cycles of clearly identified oscillation was measured, and the average frequency was determined. As the test progressed, the period of oscillation changed noticeably even in two adjacent cycles.

The ratio of the measured lowest frequency in a run to calculated elastic natural frequency was plotted for each mode as a function of the average of positive and negative extreme displacements at the third level (Fig. 5.5, 5.6, and 5.7). The calculated natural frequencies are listed below.

**Calculated Frequencies**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Specimens D1 and D2 (Hz)</th>
<th>Specimen D2 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Mode</td>
<td>7.23</td>
<td>6.89</td>
</tr>
<tr>
<td>Second Mode</td>
<td>23.8</td>
<td>22.8</td>
</tr>
<tr>
<td>Third Mode</td>
<td>41.5</td>
<td>39.8</td>
</tr>
</tbody>
</table>
The calculation was based on uncracked transformed sections without rigid zones in joints nor rotational springs at the ends of members. The ratio decreased with increasing amplitudes of oscillation. The decay rate was slower for higher modes. A free vibration test before the earthquake Test D1 indicated that the frequencies were approximately 80 percent of the calculated frequencies. The first-mode frequency at the end of the last run was reduced to about a quarter of the calculated frequency.

(e) Crack Patterns

Shrinkage cracks were observed at the corners of each test frame along transverse reinforcement in the columns and along column longitudinal reinforcement in the base girder. Furthermore, in spite of the careful handling, unavoidable cracks were added in the beam-column connections when the steel racks were secured to the test frames. Vertical cracks in the base girder along steel pipes formed when the frames were fastened to the earthquake simulator platform.

Crack patterns were marked after each run of each test on both frames. The crack patterns presented here refer to the south frame.

Cracks were generally observed first at the base of the first-story columns, in the first- and the second-level joint cores, and at the ends of the first- and second-level beams. Diagonal cracks in the joint cores were observed even in the first run. Cracks gradually spread along the first- and the second-story columns, and the first- and the second-level beams.
Diagonal cracks in the first story columns appeared when the maximum base acceleration exceeded approximately 0.7 g. As the test proceeded, cracks spread in the frame. Wide cracks were observed at the base of the first-story columns, at the ends of the first-level beam, and at the top of the second-story columns. Sometimes wide cracks were observed at the ends of the second-level beam.

Diagonal cracks in the joint might also have been influenced by the constraint from the steel racks which were very stiff and tended to stay parallel to the test platform, whereas the joint cores tended to rotate.

(f) Maximum Response

Maxima were picked automatically during the electronic data reduction process from the measured response waveforms for each run. As the three specimens have comparable strengths, their responses are compared directly. The intensity of base motion was defined by the spectrum intensity ($S_{120}$) at 20 percent of critical damping as discussed in Section 5.2.

Maximum base shears and overturning moments are shown in Fig. 5.8 and 5.9. In both figures, the maximum values increase linearly with spectrum intensity up to a spectrum intensity of approximately 10.0 in., and then appear to reach a plateau at a base shear of 2.8 kip and a base overturning moment of 110 kip-in. Those values are larger than the ones predicted by an elasto-plastic mechanism analysis to be discussed in Chapter 6. When the intensity of base motion was increased slightly after "yielding", the maximum values tended to decrease. The fact that Specimen D3 was slightly weaker than the others did not show up in the figures.
Maximum base overturning moments are plotted with respect to maximum base shears in Fig. 5.10. Base shear is the total force that the structure carries at a particular instance, and is distributed at each floor level. The distributed lateral forces cause a base overturning moment. Therefore, a comparison of base shear and overturning moment indicates an "equivalent height" at which the base shear can be concentrated to produce the same overturning effect at the base. From the figure, the "equivalent" height can be located above the second level, but below the mid-height of the third-story column. The "equivalent height" is at the second level for a uniform lateral-force distribution, and at 40.7 in. from the base for a triangular distribution.

Maximum accelerations at each level are compared with spectrum intensities in Fig. 5.11. The maximum accelerations increased with spectrum intensity up to a spectrum intensity of approximately 10 in. After that the maximum acceleration increased at a very slow rate. Maximum acceleration in each run increased with height of the level up to the "yielding", and after that the maximum accelerations at the first level increased at a higher rate than the other two. The amount of increase in the maximum acceleration after the "yielding" seemed to depend on the activity of the second-mode component, which was most active in the first-level acceleration signal. The difference in strength of the specimens did not show up in the comparisons.

The total displacement range is compared with spectrum intensity in Fig. 5.12. The total displacement was used as an index for the maximum
displacement in this report. The reasons for this are as follows:

(1) The absolute maximum displacement is affected by the location of the zero axis, which could have been shifted between test runs due to the influence of temperature change on the electronic recording devices, or due to an accidental shock given to a specimen while crack patterns of a specimen were studied.

(2) It is easy to find the extreme displacements in the two directions during a data reduction process.

(3) The total displacement range was independent of the location of the zero axis.

(4) As a specimen oscillated almost evenly in the two directions during a test run, the total displacement range was not too far from twice the maximum displacement.

The average extreme displacement increased almost linearly with spectrum intensity. Specimens D1 and D2 behaved similarly, whereas Specimen D3, which is weaker than the other two specimens (see Chapter 2), gave larger displacements at the three levels.

Maximum base shears are compared with average extreme displacements at the first level in Fig. 5.13, which shows a trend similar to that in Fig. 5.8. The maximum base shear seemed to have reached a limit at approximately 2.8 kip.

The maximum base shear and overturning moment are generally smaller than the ones calculated by assuming all the maximum lateral forces to occur at the same moment. As noted in (c) of this section, the
first-level acceleration waveform contained higher frequency components, and its maximum occurred at a time different from the time for the maxima in upper level acceleration waveforms. The ratios of measured maximum base shear and overturning moment to those calculated from the maximum lateral forces are plotted in Fig. 5.14. The ratio tends to decrease after a spectrum intensity of 20 in. This is consistent with the observation of response waveforms that higher mode components became dominant as the damage progressed in a specimen. The decay rate is faster for the base shear, which is consistent with the observation of response waveforms that the base shear waveform contained more higher frequency components than the base overturning moment.

5.4 Test D1

The intensity of base motion was increased in this test from Run D1-1 to Run D1-5 using the El Centro (1940) NS component as input for the earthquake simulator. The intensity of base motion was intended to increase by 50 percent after each run. When the capacity of the earthquake simulator was reached, the Taft (1952) N21E component was used at full capacity.

Observed behavior of the specimen is summarized in Table 5.1. Immediately before the test a small shock was applied to the earthquake simulator platform. The natural frequencies of the test specimen were determined to be 5.7 Hz for the first mode, and 18.7 Hz for the second mode. The third mode was not discernible in the acceleration trace.
(a) Run D1-1

The maximum base acceleration was measured to be 0.24 g. The spectrum intensity ($S_{120}$) at a 20 percent of critical damping was found to be 4.55 in. The measured response waveforms are shown in Fig. 5.20, and the response spectra in Fig. 5.27 (a). Crack patterns were not recorded after this run.

Large displacements occurred at 1 and 5 sec from the beginning of the base motion. The large oscillations lasted a few seconds. Response waveforms were generally smooth and governed by the first-mode component except the first-level acceleration waveform, in which the second-mode component was also observed. The displacement waveforms at the three levels were very similar.

The frequency associated with the first mode decreased with time: 4.2 Hz in the first 4.5 sec period, 4.0 Hz in the middle 4.5 sec period, and 3.6 Hz in the last 4.5 sec period.

(b) Run D1-2

The maximum base acceleration was measured to be 0.40 g. The spectrum intensity was 7.06 in. Measured response waveforms are shown in Fig. 5.21, the crack pattern in Fig. 5.26 (b), and the response spectra in Fig. 5.27 (b). The waveforms in this run were similar to those in Run D1-1. The free vibration part decayed at a faster rate than before.

The frequency associated with the first mode decreased to 3.4 Hz at the end. The second- and the third-mode frequencies were measured to be 12 Hz and 23 Hz respectively, in the latter half of the run.
Cracks were observed at the ends of the three beams, at the top and the bottom of the first-story columns. Diagonal cracks were observed in some joint cores in the first and second levels. Cracks along beam members were not examined closely.

(c) Run Dl-3

The maximum base acceleration was measured to be 0.52 g. The spectrum intensity was 10.7 in. Measured waveforms are shown in Fig. 5.22, crack pattern in Fig. 5.26 (c), and response spectra in Fig. 5.27 (c).

Large responses were observed at 1, 2, and 5 sec from the beginning of the base motion. The second-mode component was clearly seen in base-shear, overturning-moment and third-level acceleration waveforms overlapping the first-mode component. Higher mode components became more active in the second-level acceleration waveform. Peaks of the acceleration signals did not occur simultaneously due to the existence of higher mode components. Large oscillations in displacement waveforms diminished at a faster rate than before. The latter half of the displacement waveforms had an almost uniform amplitude of approximately half the maximum amplitude.

The first-mode frequency was 3.3 Hz for the first 6-sec period and 3.0 Hz to 2.8 Hz for the rest of the run. The second- and the third-mode frequencies were 13 Hz and 22 Hz, respectively, in the first 4-sec period, but the second-mode frequency dropped to 12 Hz after a large oscillation at 5 sec.

Many cracks were found along the first- and the second-level beams, but no cracks in the third-level beam. More cracks were observed
in the beam-column connection at the first and the second levels. New cracks were observed at the top of the third-story columns.

(d) Run D1-4

Maximum base acceleration was measured to be 0.84 g, the spectrum intensity 15.7 in. Measured waveforms were shown in Fig. 5.23, and crack pattern in Fig. 5.26 (d), and response spectra in Fig. 5.27 (d).

The base overturning moment waveform was very smooth and dominated by the first-mode component. The second-mode component was not dominant in the base-shear waveform, as it was in Run D1-3. On the other hand, higher mode components were more perceptible in the acceleration waveforms than in Run D1-3. Frequency in the displacement waveforms changed from one cycle to another in the first 4.0 sec. The first-mode frequency was approximately 2.9 Hz at 8 sec and 2.4 Hz in free vibration. The second-mode frequency was 21 Hz at 3 and 8 sec.

Diagonal cracks were observed in the first-story columns, and at the top of the second-story columns. New cracks were found in the third-level beam. The region of flexural cracks spread from the joint cores in the first- and the second-story columns. The damage in first-level joint cores increased. The bottom of the first-story columns were damaged badly in flexure.

(e) Run D1-5

Maximum base acceleration was 1.42 g, and the spectrum intensity was 21.3 in. The El Centro (1940) NS component was used as input. Measured waveforms are shown in Fig. 5.24, crack pattern in Fig. 5.26 (e), and response spectra in Fig. 5.27 (e).
Large displacements were observed in the first 3 sec from the beginning of the base motion. The base-shear waveform was governed by the first- and the second-mode components, while the base-overturning moment waveform was dominated by the first-mode component. The second-mode component became perceptible in the third-level acceleration waveform. It should be noted that the smooth base-moment waveform was calculated from the "jagged" acceleration waveforms at the three levels.

The displacement signals oscillated in the same phase, but not necessarily in a well defined frequency. The center of oscillation shifted in both directions with time.

The first-mode frequency for free vibration was 2.0 Hz, and the damping factor was 7 percent of critical. The third-mode frequency was 18 Hz in free vibration.

New cracks were observed in the joint cores at the first and the second levels, and also at the bottom of first-story columns. Diagonal cracks were found at the top of the second-story columns. Existing cracks around the joint cores opened wider.

(f) Run D1-6

The maximum base acceleration was 3.16 g, and the spectrum intensity was 30.4 in. The Taft (1952) N21E component was used as input. The measured waveforms are shown in Fig. 5.25, crack pattern in Fig. 5.26 (f), and response spectra in Fig. 5.27 (f).

The specimen responded in a different fashion to the base motion: the large oscillations occurred over the duration of the base motion. Higher mode components were more perceptible in the acceleration waveforms than in
the previous runs. Displacement signals oscillated nearly in the same phase at the three levels, but it was difficult to find a constant periodicity in the signals.

The frequency and the damping factor associated with the first mode were measured in a free vibration part to be 1.7 Hz and 6 percent of critical.

Some concrete spalled off from a first-level joint core. Crushing of concrete was seen at the bottom of the first-story columns. Crushing occurred only at the outer faces of the columns and was visible for a vertical distance of 0.5 in. Further damage was concentrated in the joint cores at the first and second levels.

(g) Maximum Response

Maximum accelerations in east and west directions are compared for each level in Fig. 5.15. Maxima in both directions indicate a similar trend for each level. The difference in magnitude was within 20 percent except for the third-level maximum acceleration in Run D1-6, in which the difference reached 50 percent due to a sharp spike in the waveform (Fig. 5.25 (f)).

Absolute maximum accelerations are compared with spectrum intensity ($S_{120}$) in Fig. 5.16. Up to a spectrum intensity of 21 in., a maximum acceleration in a run increased with a story height.

Total displacement ranges at three levels are compared in Fig. 5.17. These ranges increased almost linearly with spectrum intensity with a ratio of 1.0 : 1.9 : 2.3 (first level : second level : third level),
which indicates that large displacements took place in the first and the second stories.

Measured maximum base shears are compared in Fig. 5.18 with calculated base shears from measured maximum accelerations at the three levels. The calculated base shear was always larger than the measured base shear. Their difference in magnitude became wide after 'yielding' at a spectrum intensity of 7 in. The discrepancy resulted from the fact that the maximum accelerations at the three levels did not occur simultaneously.

Measured maximum base-overturning moments are compared in Fig. 5.19 with calculated base-overturning moments from measured maximum accelerations at the three levels. A trend is observed similar to the one for the base shear.

5.5 Test D2

El Centro (1940) NS component was used as input in the first two runs, and Taft (1952) N21E component in the last two runs. The specimen had the same design values as the one in Test D1. The only variable was the intensity of the base motion in the first run. Observed behavior of the specimen is summarized in Table 5.2.

(a) Run D2-1

The maximum base acceleration was measured to be 0.86 g. The spectrum intensity ($S_{120}$) was 15.8 in. The base motion was similar in intensity and waveform to that for Run D1-4. The measured waveforms are shown in Fig. 5.33, crack pattern in Fig. 5.37 (b), and response spectra of the base motion in Fig. 5.38 (a).
The response waveforms in this run were comparable to those in Run D1-4. However, the higher mode components were more discernible in the waveforms of Run D1-4. The specimen in Test D1 had been subjected to three previous runs of lower intensity before Run D1-4. The strong correlation between the acceleration and displacement responses in Runs D1-4 and D2-1, except in the first one sec period when Specimen D2 was being "broken", suggests that the overall response of a partially damaged specimen to a base motion more intense than the ones it has experienced earlier would be reasonably similar to the response of a new specimen to the same base motion. Except for the initial period, each new test run with a higher intensity can be treated as an independent test.

During Run D2-1, large oscillations occurred at 1, 2, and 5 sec from the beginning of the base motion. The base-shear waveform was governed by the first-mode component modified by a fraction of the second-mode component. The base-overturning moment signal oscillated primarily in the first mode. The acceleration waveforms at lower levels contained more of higher mode components. The second-mode component prevailed in the first-level acceleration signal. The displacement waveforms were smooth and virtually in phase. The first-level displacement waveform contained a fraction of the second-mode component. The center of oscillation in the displacement waveforms at the three levels shifted after one sec. The displacement waveforms after two sec from the beginning of the base motion were nearly identical to those in Run D1-4.

The measured frequencies, obtained by measuring a period of three to ten cycles of the response signal, are given in the following table.
<table>
<thead>
<tr>
<th>Time from the Beginning of Base Motion (Sec)</th>
<th>First Mode (Hz)</th>
<th>Second Mode (Hz)</th>
<th>Third Mode (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>-</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>2.0</td>
<td>3.1</td>
<td>-</td>
<td>24</td>
</tr>
<tr>
<td>3.0</td>
<td>3.3</td>
<td>-</td>
<td>21</td>
</tr>
<tr>
<td>4.0</td>
<td>-</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>5.0</td>
<td>3.3</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>7.5</td>
<td>3.0</td>
<td>-</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>12</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>11</td>
<td>-</td>
</tr>
</tbody>
</table>

The frequency or stiffness reduction was less in this run than in Run D1-4.

Cracks formed in the first- and the second-level beams and columns. Wide cracks were observed at the ends of the first- and the second-level beams, and at the base of the first-story columns. Fine cracks were observed in the joint cores at the three levels. Diagonal cracks were found in the first-story columns, and the first- and the second-level joint cores. Some cracks in the columns developed from the existing shrinkage cracks.

(b) Run D2-2

The maximum base acceleration was measured to be 1.10 g, and the spectrum intensity (S_{1,20}) was 19.3 in., which represented approximately an increase of 25 percent with respect to Run D2-1. Measured waveforms are
shown in Fig. 5.34, crack pattern in Fig. 5.37 (c), and response spectra in Fig. 5.38 (b).

The response waveforms in this run were also very similar to the ones in Run D1-4. Large oscillations occurred within the first 3 sec. Oscillations at 5 sec were somewhat smaller than the ones in Run D2-1. The acceleration waveforms at the three levels contained more of the second-mode components than they did in Run D2-1. The third-mode component was seen in the second-level acceleration waveform. The center of oscillation in the displacement waveforms shifted in both directions in the first 3 sec and was stabilized after that.

The measured frequencies are listed below.

| Time from the Beginning of Base Motion Sec | First Mode Measured Frequency Second Mode Measured Frequency Third Mode Measured Frequency |
|------------------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1                                        | 3.0                             | 12                              | -                               |
| 2                                        | 3.0                             | 13                              | -                               |
| 3                                        | -                               | 11                              | 20                              |
| 4                                        | -                               | 10                              | 20                              |
| 5                                        | 3.3                             | -                               | 19                              |
| 6                                        | -                               | 10                              | 19                              |
| 8                                        | 2.7                             | -                               | 20                              |
| 9                                        | 2.8                             | 10                              | -                               |
| 12                                       | 2.3                             | 11                              | 20                              |

Wide cracks were observed in the first level-joint cores. Fine diagonal cracks were seen near the base of the first-story columns. Wide
cracks were also observed at the base of the first-story columns and at the ends of the first- and the second-level beams.

(c) Run D2-3

Taft (1952) N21E component was used in this and the following run. The maximum base acceleration was 1.21 g. The spectrum intensity ($S_{20}$) was 20.1 in. The intensity of the base motion was nearly the same as in Run D2-2. Measured response waveforms are shown in Fig. 5.35, crack pattern in Fig. 5.37 (d), and response spectra in Fig. 5.38 (c).

The response waveforms were quite different from the ones recorded in Run D2-2. Large amplitude oscillations did not extend to more than one cycle, and were separated by medium amplitude oscillations.

The second-mode oscillation was perceptible in acceleration waveforms at all three levels, especially at the first and the second levels. The displacement waveforms drifted east after oscillations at 2, 3, and 4 sec.

The measured frequencies are listed below.

<table>
<thead>
<tr>
<th>Time from the Beginning of Base Motion</th>
<th>First Mode Hz</th>
<th>Measured Frequency</th>
<th>Third Mode Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sec</td>
<td></td>
<td>Second Mode Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Third Mode Hz</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.7</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>4.5</td>
<td>2.6</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>2.7</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>2.3</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>End</td>
<td>2.1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
The specimen was damaged so badly that it was difficult to determine the oscillating frequency.

Cracks were found in all three level beams. Not many new cracks were found in the first-story columns, but the existing cracks opened wider. Wide flexural cracks were observed at the base and approximately one inch above the base of the first-story columns. Crossing diagonal cracks (X-patterns) were seen at the lower and upper parts of the first-story columns. Wide cracks were also seen at the ends of the first-level beam.

(d) Run D2-4

The maximum base acceleration was measured to be 3.43 g. The spectrum intensity ($S_{120}$) was 28.4 in. The intensity of base motion was increased by approximately 40 percent with respect to Run D2-3. The measured response waveforms are shown in Fig. 5.36, crack pattern in Fig. 5.37 (e), and response spectra in Fig. 5.38 (d). The damping factor measured in a free-vibration test before this run was 7 percent of critical.

The general waveforms were similar to the ones in Run D1-6. The second-mode component was more prevalent in the acceleration waveforms. The displacement waveforms indicate that the specimen responded as if it had very little lateral resistance.

The existing cracks, especially at the ends of members, opened wider. Some concrete spalled off from a first-level joint core. Wide diagonal cracks were seen in the first-level joint cores. The third-level joint cores looked sound, although flexural cracks at the top of the third-story columns were wide. Anchorage cracks along the main reinforcement of
columns were seen in the base girder. Crushing of concrete was observed at the outer faces of the base of the first-story columns. New cracks were found in the second- and the third-level beams.

(e) **Maximum Response**

Maximum acceleration in east and west directions are compared for each level in Fig. 5.28. The maxima are of a similar amplitude for the four test runs, which indicates that the specimen was in the "yield stage" from Run D2-1. Similar values are observed in both directions. At the second and the third levels, the maximum values occurred in the west for the simulated El Centro base motions and in the east for the simulated Taft base motions. Absolute maximum accelerations at the three levels are compared in Fig. 5.29. The third-level acceleration was always the largest of the three, however, the second-level acceleration was mostly smaller than the first-level acceleration.

Total displacement ranges at three levels are compared in Fig. 5.30. From the observation of displacement waveforms in this test (see (g), (h), and (i) of Fig. 5.33 through 5.36), the maxima in the east and the west occurred simultaneously at all three levels. Displacement ranges increased more or less linearly with spectrum intensity with a ratio of 1.0 : 1.9 : 2.7 (first level : second level : third level).

Measured maximum base shears are compared in Fig. 5.31 with calculated base shears from measured maximum accelerations at the three levels. The measured maximum base shear did not increase with spectrum intensity, while the calculated value increased and became almost twice as large as
the measured value at a spectrum intensity of 28.4 in. The measured extreme values in the east and the west are comparable for each run.

Measured maximum base overturning moments are compared in Fig. 5.32 with calculated base overturning moments from measured maximum acceleration at the three levels. The calculated value increased at a faster rate than the measured value, indicating that the maximum accelerations at the three levels did not occur at the same time.

5.6 Test D3

The frames of this specimen were designed to have weaker beam and column sections than the ones in the previous two tests, by 5 percent in a column yield moment and by 8.5 percent in a beam yield moment. Weight at each story and overall dimensions were kept the same.

Similar base motions were used in this test as in Test D2: El Centro (1940) NS component for the first two runs and Taft (1952) N21E component for the last two runs.

Maximum measured responses are summarized in Table 5.3. Free vibration tests were carried out before the first earthquake test run by applying a small shock to the earthquake simulator platform. The frequency associated with the first mode oscillation was measured to be 5.3 Hz, 5.2 Hz, and 5.1 Hz for the three successive free vibration tests. The frequency was observed to become smaller after each test.

Shrinkage cracks were seen in the base girder along the longitudinal reinforcement of columns (Fig. 5.48 (a)). Cracks were seen in the columns and at the end of the beams.
(a) Run D3-1

The maximum base acceleration was measured to be 0.61 g. The spectrum intensity ($S_{120}$) at a 20 percent of critical damping was calculated to be 12.3 in. The measured response waveforms are shown in Fig. 5.44, crack pattern in Fig. 5.48 (b), and response spectra in Fig. 5.49 (a).

The first- and the second-level displacement signals were not recorded in this run due to failure in the amplifiers. Overall response of the specimen was very similar to the one in Run D2-1. Large oscillations were observed at 1, 2, and 5 sec from the beginning of the base motion. The large oscillation tended to diminish quickly. The base-shear waveform was dominated by the first-mode component with a fraction of the second-mode component. The base-overturning moment waveform was smooth and governed by the first-mode component.

Peaks of the acceleration waveforms did not occur simultaneously at the three levels because of the existence of the second- and the third-mode components. The second-mode component was observed prevalent in the first-level acceleration waveform. A fraction of the third mode component could be seen in the second-level acceleration waveform. The third-level displacement waveform was smooth and dominated by the first-mode component.

The frequency associated with the first-mode oscillation was measured to be 3.1 Hz at the beginning and 2.6 Hz at the end of the run. The second-mode frequency was 13 Hz at the beginning and 10 Hz at the end. The third-mode frequency was 20 Hz at 4.0 sec.

After this test run, a free vibration test was carried out under a small base shock. The frequencies for the first and the second modes
were 2.7 and 10.4 Hz, respectively, which were fairly consistent with the ones measured during the dynamic test.

Cracks were well spread along the columns from the first to the third story. Crossed diagonal cracks were seen in the joint cores at all levels. Diagonal cracks were found in the first- and the second-story columns. Damage in the beams was less than that in the columns.

(b) Run D3-2

The maximum base acceleration was measured to be 1.10 g. The spectrum intensity (S1,20) was 18.2 in. The measured response waveforms are shown in Fig. 5.45, crack pattern in Fig. 5.48 (c), and response spectra in Fig. 5.49 (b).

Large oscillations were observed within the first 3 sec. The overall waveforms were similar to the ones recorded in Run D2-2. The base-shear and overturning-moment waveforms contained appreciable amounts of higher mode components. Higher mode components were seen more often in the acceleration waveforms at all the levels than in Run D3-1. The second-mode component was prevalent in the first-level acceleration waveform.

The center of oscillation in the displacement waveforms drifted after large oscillations within the first 3 sec from the beginning of the base motion. The displacement signals were generally in phase.

The first-mode frequency was 2.7 Hz at the beginning and 2.2 Hz at the end of the run. The second-mode frequency was 12 Hz at the beginning and 10 Hz at the end. The third-mode frequency was 20 Hz. Free vibration tests after this run indicated frequencies of 2.2 Hz for the first mode and 9.8 Hz for the second mode.
New cracks were found in the beams at all levels. Cracks in the columns were observed at smaller spacing. More cracks formed in the joint cores at all levels. Tensile cracks perpendicular to the main reinforcement of the columns and anchorage cracks along the main reinforcement of the columns were observed in the base girder. Diagonal cracks were seen in the first- and the second-level joint cores. Wide cracks were seen at the ends of beams at the first and the second levels. The bottoms of the first- and the second-story columns were severely damaged.

(c) Run D3-3

Taft (1952) N21E component was used in this and the following run. The maximum base acceleration was 0.93 g. The spectrum intensity was 19.0 in. The intensity of base motion was approximately the same as in the preceding run. Measured response waveforms are shown in Fig. 5.46, crack pattern in Fig. 5.48 (d), and response spectra in Fig. 5.49 (c).

Large oscillations were observed over the duration of the base motion. Each large oscillation was followed by a few cycles of relatively small oscillations. Overall waveforms were similar to the ones recorded in Run D2-3.

The second-mode component was more perceptible in the acceleration waveforms at all three levels. The center of oscillation shifted to east after a large oscillation at 2 sec.

It was difficult to determine a frequency of oscillation during the dynamic test. Free vibration tests indicated frequencies of 1.7 Hz for the first mode, 8.0 Hz for the second mode, and 16 Hz for the third
mode. A damping factor measured in free vibration after the imposed base motion had ceased was 4 percent of critical.

More cracks formed in the joint cores at all levels and at the lower part of the first-story columns. Few new cracks were found in the rest of the specimen. Shell concrete spalled off from a joint core at the first level. A diagonal crack was seen at the lower part of second-story column.

(d) Run D3-4

The maximum base acceleration was 2.14 g, and the spectrum intensity was 25.3 in. The maximum base acceleration was double of that in the last run, but the spectrum intensity indicated that the effectiveness of the base motion increased only by 33 percent. The measured response waveforms are shown in Fig. 5.47, crack pattern in Fig. 5.48 (e), and response spectra in Fig. 5.49 (d).

The specimen seemed to be damaged so badly that the periodicity of oscillation changed rapidly. Overall waveforms were similar to the ones recorded in Run D1-6 and Run D2-4. The center of oscillation in the displacement waveforms seemed to shift with the base displacement.

(e) Maximum Response

Measured maximum accelerations in both directions are compared for each level in Fig. 5.39. At the second and the third levels the maxima occurred in the west for the simulated El Centro base motions, while they occurred in the east for the simulated Taft base motions. The same was observed in Test D2. The maxima did not occur simultaneously at the three levels.
The measured maximum accelerations at the three levels are compared in Fig. 5.40. The third-level acceleration was always the largest as was observed in Tests D1 and D2. The first-level acceleration was larger than the second-level acceleration in Runs D3-3 and D3-4.

Total displacement ranges at the three levels are compared in Fig. 5.41. These ranges increased almost linearly with the spectrum intensity with a ratio of 1.0 : 1.7 : 2.1 (first level : second level : third level). This ratio is similar to the one found in Test D1 (1.0 : 1.9 : 2.3), indicating that a large displacement took place in the first and the second stories. From the observation of displacement waveforms, these maxima occurred almost at the same moment at all three levels in both east and west directions.

Measured maximum base shears are compared in Fig. 5.42 with calculated base shears from the measured maximum accelerations at the three levels. The measured maximum values stayed in a range from 2.5 to 3.0 kip, while the calculated values increased with the spectrum intensity.

Measured maximum base-overturning moments are compared in Fig. 5.43 with calculated base-overturning moments from the measured maximum accelerations at the three levels. The measured extreme values differed by approximately 20 percent in the two directions as was observed in Test D2. The difference between the measured and the calculated overturning moments was not so wide as in the comparison of base shear.
CHAPTER 6

DISCUSSION OF THE TEST RESULTS IN RELATION TO LINEAR-RESPONSE ANALYSIS

6.1 Introductory Remarks

The studies in this chapter were made to evaluate the test results from the viewpoint of methods routinely available to design engineers.

The structural effects of the linearly elastic response in the three modes were summed using four different techniques. Three of these were variations of spectral modal analysis: (a) maxima from first mode only, (b) absolute sum of maxima, and (c) square root of the sum of the squares of maxima. The fourth was maxima obtained from a step-by-step response history analysis of the entire frame to the ground motion.

All analyses were made for two different assumptions about member stiffness based on uncracked transformed and fully cracked sections. The test frames were idealized as a plane frame as described in Chapter 4. Modal characteristics of the test frames were determined by solving an eigen value problem on a digital-computer. The natural frequencies and mode shape vectors of oscillation are compared for idealized frames in Table 6.1. Effects of rigid zones in beam-column connections (referred to as rigid joints)* and rotational springs at the ends of members

* A member without a "rigid joint" has the same stiffness from center to center of joints. A member with a rigid joint has infinite flexural stiffness from center to face of joint.
(referred to as rotational springs)* were studied for the test frames based on uncracked transformed sections. As it would be anticipated, the highest frequencies for this stiffness assumption were calculated for a frame with rigid joints and without rotational springs. The introduction of springs reduced the natural frequencies for all three modes by approximately 20 percent. Without rigid joints and rotational springs as commonly assumed in design, the natural frequencies deviated from the ratio $1 : 3 : 5$ (first mode : second mode : third mode), which is found for a uniform shear-beam structure, due to the existence of flexible beams.

The mode shape vectors were not so sensitive to the structural stiffness as the natural frequencies. The mode shape vectors were already multiplied by the participation factors. Therefore, the values represent displacements at each level if the spectral displacement response is taken as unity, or acceleration at each level if the spectral acceleration response is taken as unity (Fig. 6.1).

Lateral shears and overturning moments were calculated for an arbitrary 1.0 g spectral acceleration response from the mode shape vectors, story weight, and story height. Calculated lateral shears and overturning moments are compared for each mode in Fig. 6.2.

It is observed in Fig. 6.1 that the first mode component will dominate at the third level in acceleration and displacement responses if the spectral responses for the three modes are the same. The effect of

* See Section 3.6.
higher mode components is larger at the first- and the second-level responses than that at the third-level response.

It is observed in Fig. 6.2 that the first-mode component has a dominant effect on story-shear and overturning-moment distributions if acceleration spectral values are comparable for all three modes. The second-mode component had some influence on the first story shear, and the third-level overturning moment. The third-mode component can be ignored in calculating story shear and overturning moment unless its acceleration spectral response is much greater than those of the other two.

The chapter also contains a static limit analysis of the test frames to provide a basis for evaluating the dynamic base shears and overturning moments.

6.2 Analysis of Test Frames Based on Uncracked Transformed Sections

(a) Spectral Modal Analysis

Displacements and accelerations were obtained from the elastic response spectra for each test run for the calculated frequencies of a test frame. The values were interpolated linearly from two adjacent calculated points on linear coordinates. A critical damping factor of 5 percent was used for all three modes. Displacement and acceleration responses are listed in Table 6.2 for all three modes.

The spectral displacement for the first mode is about ten times greater than that for the third mode. On the other hand, the spectral accelerations are comparable in most cases for all three modes.
If the combined effect of the spectral displacements and the mode shape is considered, the calculated displacement waveforms at all three levels are governed by the first-mode component. On the other hand, the calculated first-level acceleration signal would contain approximately equal contributions from the three mode components, but the first-mode component would become dominant for the upper levels. The lateral shear and overturning moment waveforms would be governed by the first-mode component.

The maximum response of a linearly elastic system subjected to a ground motion can be estimated by combining the modal characteristics of the system with the use of the spectral response of the ground motion. The "upper bound" of the response can be obtained by adding the absolute maximum modal components based on the assumption that the maxima for the modal components occur at the same moment. It was proposed (Rosenblueth 1952) that the "probable" maximum response is given approximately by the square root of the sum of the squares (root mean square) of the maximum modal components. From the preceding discussion the "first mode" component is expected to give good approximations of the displacement and base moment. Those three values are listed for acceleration response in Table 6.4, for displacement response in Table 6.5, for base shear in Table 6.6, and for overturning moment response in Table 6.7.

A large difference among these three values is seen in acceleration response, which indicates that the effect of higher mode components is large in the acceleration response, especially at the lower levels.
The absolute sum of first-level accelerations is about three times as large as the first-mode acceleration. The difference in magnitudes of values obtained from these three methods is small for the third-level acceleration, which indicates that the first-mode component has a dominant influence on the third-level acceleration waveform.

The difference among the response values from the three spectral analysis methods was practically nothing for displacements at the three levels and in the base-overturning moment, which indicates that the first-mode component dominates in the displacement and the base-overturning-moment response.

In the case of base shear response, the "first-mode" and the "probable" values were almost the same, while the "upper bound" value was a little larger than the two, which indicates that there exist higher mode components in the response, but the response is dominated by the first-mode component.

"Probable" maximum accelerations are plotted against the spectrum intensity of base motion in Fig. 6.3. The maximum accelerations at all three levels increased almost linearly with the spectrum intensity for the simulated El Centro base motions, which is consistent with the similarity in the shape of response spectra at different intensities of an earthquake motion. For the simulated Taft base motion, the first-level "probable" maximum acceleration was not proportional to the spectrum intensity of the base motion, while the maximum accelerations at the other two levels were nearly proportional to the spectrum intensity. For the same
spectrum intensity of base motion, simulated Taft base motions gave larger accelerations at all three levels for this particular set of modal frequencies.

"Probable" maximum displacements are plotted against the spectrum intensity of base motion in Fig. 6.4. The maxima increased almost linearly with the spectrum intensity for both simulated earthquake motions at all three levels. Simulated Taft base motions gave larger accelerations at all three levels for the same spectrum intensity than simulated El Centro base motions.

(b) Response-History Analysis

Linearly elastic response of the test frames to a base motion was calculated based on uncracked transformed sections of the members by a step-by-step numerical method on a digital computer. Damping factors for the first and the second modes were assumed to be 5 percent of critical in order to be consistent with the modal analysis. The damping factor for the third mode was determined by Eq. 4.27, in which constants $C_1$ and $C_2$ were determined by the first- and the second-mode damping factors and the corresponding circular frequencies.

In calculating the stiffness of the structure, rigid zones in a joint core and rotational springs at the ends of a member were not included. Base acceleration records used were those measured on the base girder of the south frame. The interval of the records was 0.002 sec. The response calculation was made at the same interval. Calculated maximum values are listed in Table 6.4 through 6.7. The maximum acceleration
and displacement are compared with the spectrum intensity \( S_{20} \) of base motions in Fig. 6.5 and 6.6. Maximum accelerations to simulated El Centro base motions (Fig. 6.5) increased nearly proportionally with the spectrum intensity at all three levels, but those to simulated Taft base motions did not at the first and the second levels. Maximum displacements at each level increased almost proportionally with the spectrum intensity with different proportionality constants for the two types of base motion.

The calculated maxima from the direct solution are compared with those from the modal analysis methods in Table 6.4 through 6.7. Maximum displacements calculated from the spectral modal analysis methods agreed favorably with those from the direct solution at all three levels for simulated El Centro base motions. However, maximum displacement from the direct solution sometimes exceeded the "upper bound" values from the spectral analysis by as much as 20 percent. This resulted from the fact that the spectral value at a given frequency was taken by linear interpolation from two adjacent calculated points. If the shape of a response spectrum is sensitive to a frequency component, the linear interpolation sometimes gives an inaccurate result.

The "probable" maximum accelerations (Table 6.4) from the modal analysis gave a good estimate of within 20 percent to those from the direct solutions for simulated El Centro base motions. However, in a case of a simulated Taft base motion, the difference reached to 40 percent (Run D1-6), which again indicates that the linear interpolation of spectral values will sometimes lead to very crude results.
The maximum base shears and overturning moments (Table 6.6 and 6.7) from the direct solution were predicted very well by the first-mode response in the case of simulated El Centro base motions. For simulated Taft base motions, the direct solution gave larger values than the absolute sum of modal maxima in Tests D1 and D2.

6.3 Analysis of Test Frames Based on Fully Cracked Sections

(a) Spectral Modal Analysis

Stiffness characteristics of the test frames were calculated on the basis of fully cracked sections and the stress-strain curve for concrete described in Chapter 3. Flexural stiffness of a member was taken as the slope of a line connecting the origin and the yield point in a moment-curvature diagram. Rigid zones in beam-column connections and rotational springs at the ends of members were not considered in order to limit the analyses to the "routine" domain.

The calculated natural frequencies and mode shape vectors are listed in Table 6.1. As would be expected, the mode shape vectors for the three modes are almost identical to those calculated based on uncracked transformed sections. The natural frequencies are approximately 30 percent smaller than the corresponding frequencies based on uncracked transformed sections without rigid zones at beam-column connections and rotational springs at the ends of members.

Although the mode shape vectors are almost identical to the ones based on uncracked transformed sections, the frame is expected to
respond to a base motion in a different manner from the one based on uncracked sections because of the difference in modal frequencies.

Displacements and accelerations were obtained from the elastic response spectra of the measured base motion at the calculated frequencies of the system. The spectral values were found by linear interpolation from two adjacent calculated points on linear coordinates. A critical damping factor of 5 percent was used for all three modes. The response values are listed in Table 6.3. The first-mode displacement was in most cases ten times as large as the second-mode displacement; and approximately thirty to fifty times as large as the third-mode displacement. The first-mode acceleration was the largest for the simulated El Centro base motions. The accelerations were of similar magnitudes for the three modes.

Maxima calculated from the first-mode component alone, from sum of the absolute maxima of the three modal components, and from root mean square of the maxima of the three modal components are listed in Table 6.4 through 6.7.

The "probable" maximum acceleration at the third level (Table 6.4) was almost the same as the first-mode maximum acceleration. The "probable" maximum accelerations deviated from the first-mode maximum acceleration at the lower levels, indicating that the higher-mode components were also active. The "upper bound" accelerations were more than 20 percent larger than the first-mode maximum accelerations at the third level, and they became two to three times as large as the first-mode maxima at the first level.
The maximum displacements of the first mode component, the "upper bound," and the "probable" are practically the same, which indicates that displacement response is controlled by the first-mode component, and that the contributions to the displacement response from higher mode components are very small for the system considered.

The "probable" maximum base shears were almost the same as the maximum first-mode base shears, while the "upper bound" base shears were 15 to 30 percent larger than the maximum first-mode accelerations.

The maximum base-overturning moments of the three spectral analysis methods were practically the same, indicating that the first-mode component dominates in the base-overturning moment waveforms.

Calculated maximum accelerations are compared with the spectral intensity \( S_{1,20} \) for the three levels in Fig. 6.7. Those based on simulated El Centro base motions increased almost linearly with the spectral intensity at all three levels, while those based on simulated Taft base motions showed the same tendency but with a larger scatter.

Calculated maximum displacements are compared with the spectral intensity for the three levels in Fig. 6.8. The displacements increased almost linearly with spectral intensity except for some deviating data from Test D3. Specimen D3 was more flexible than the other two. Its natural frequency was approximately 90 percent of that of Test Frames D1 and D2.

(b) Response-History Analysis

Linearly elastic response of the idealized test frames was
calculated based on fully cracked sections of members by a step-by-step numerical method. The idealized frames were the same as described in (a) of this section. Damping factors of 5 percent of critical were used for the first and the second modes. The third mode damping factor was determined by Eq. 4.27.

Calculated maximum values are listed in Table 6.4 through 6.7. Calculated maximum accelerations at each level are compared with the spectral intensity (\(S_{1,20}\)) at 20 percent of critical damping in Fig. 6.9. Calculated maximum accelerations at all three levels increased almost linearly with the spectral intensity for simulated El Centro base motions. For the simulated Taft base motions, maximum accelerations showed a similar trend but with more scatter.

Maximum displacements at all three levels (Fig. 6.10) increased almost linearly with the spectrum intensity for the two types of base motion. Data points which deviated from a straight relationship were calculated for Specimen D3.

Maxima from the direct solution can be compared with those from modal analysis methods in Table 6.4 through 6.7. The "probable" maximum acceleration (Table 6.4) based on the modal analysis agreed favorably with maximum accelerations from the direct solution. The "upper bound" acceleration overestimated maximum accelerations at the first and the second levels, while the first-mode accelerations underestimated them at the same levels.
Maximum displacements (Table 6.5) at all three levels from the three modal analysis methods agreed favorably with those from the response-history analysis. Maximum base shears (Table 6.6) and base moments (Table 6.7) from the first-mode component agreed favorably with those from the response-history analysis as would be expected from the mode shape (Fig. 6.2).

Maximum response based on fully cracked sections can be compared with that based on uncracked sections in Table 6.4 through 6.7. Maximum accelerations (Table 6.4) based on fully cracked sections were slightly larger than those based on uncracked sections at all three levels for simulated El Centro base motions, while their relation is opposite for simulated Taft base motions. Maximum displacements (Table 6.5) based on cracked sections were larger than those based on uncracked sections at all three levels for the two types of base motion. Maximum base shears (Table 6.6) and overturning moments (Table 6.7) based on cracked sections were larger than those based on uncracked sections for simulated El Centro base motions. The relation was reversed for simulated Taft base motions.

6.4 Elasto-Plastic Collapse-Mechanism Analysis of Test Frames

(a) Elasto-Plastic Collapse Mechanism

The strength of a single test frame was estimated by assuming elasto-plastic moment-curvature relationship for the structural members. Flexural rigidity of a prismatic member was determined as a slope of a line which connects the origin and the yield point in a moment-curvature
diagram. This stiffness estimate may be reasonable for the idealized moment-curvature relationship in Section 3.4, since the cracking moment was less than 20 percent of the yielding moment (Fig. 3.10). The uncracked part of a member becomes shorter as the member is stressed severely. The contribution of the uncracked part to the member deformation is small because bending moment and the corresponding curvature are small in the uncracked region.

When the bending moment reached the yielding moment of a section, the section was assumed to carry no more moment, forming a plastic hinge. Yield moments were assumed to be constant and independent of the axial load due to the overturning effect. In general, a reinforced concrete section is capable of carrying a moment more than the one associated with yielding of the tensile reinforcement due to the strain-hardening of steel. However, the strength over the yield level is not considered in a structural design.

The process of a collapse-mechanism formation depends on the force distribution over the height of the structure. Two different lateral force distributions at the three beam levels were considered: uniform and reversed triangular distributions. If the first mode component is dominant in the acceleration waveforms at the three levels, the (reversed) triangular distribution would be a good approximation of dynamic lateral loads in a static analysis. On the other hand, if the effect of higher mode components is considered to be dominant, lateral forces at lower levels become comparable to the third-level lateral force, hence a uniform
distribution of lateral loads would be a good approximation.

Three basic admissible mechanism modes are compared in Fig. 6.11 for a single test frame. Joint cores were assumed to have the same flexural rigidity as connecting members in calculating mechanism loads. Base shear at the formation of the collapse mechanism due to a uniform distribution was higher than that due to a triangular distribution. The collapse mechanism consists of plastic hinges at the base of the first-story columns, at the top of the second-story columns, and at the ends of the first-level beam. The location of plastic hinges in the mechanism mode is consistent with the observed location of wide cracks in the test frames. The test frames had base shear coefficients of 0.65 for Specimens D1 and D2, and 0.60 for Specimen D3 under a uniform lateral load distribution.

(b) Formation of Collapse Mechanism

The formation of the collapse mechanism of a single test frame was followed by applying at the three levels a set of lateral forces of either the same magnitude at the three levels or the magnitudes proportional to the height of each level from the base. The amplitude of lateral forces was constantly increased. Whenever a bending moment exceeded the yield moment at a member end, a plastic hinge was assigned to the point. The effect of gravity loads was not included in this analysis.

The first plastic hinges generally formed at the ends of the first-level beam, and the second set of hinges at the base of the first-story columns. After the formation of the third set of plastic hinges
at the ends of the second-level beam, the collapse mechanism was completed by the formation of plastic hinges at the top of the second-story columns. In the case of specimens subjected to uniform lateral loads, the order of the first and the second plastic hinge formations was reversed. The plastic hinges at the ends of the second-level beam are not required to form the collapse mechanism.

Behavior of a frame with rigid zones in a beam-column connection (referred to as rigid joints) is compared with the one ignoring the rigid zones in a beam-column connection in Fig. 6.12 through Fig. 6.15 for the two lateral force distributions and for the two test specimens. The frame with rigid joints was initially stiffer than the other, but it gave almost the same displacements as the other frame at the formation of the collapse mechanism. The base shear at the mechanism formation was higher for the frame with rigid joints. Overall force-deformation curves for the two frames were similar for a given lateral load distribution.

The effect of lateral force distributions is compared in Fig. 6.16 for a frame without rigid joints. Smaller displacements were observed in the frame subjected to a uniform distribution at the same base shear. The base shear at the mechanism formation was higher for the uniform distribution, while the displacements at the mechanism was larger for the triangular distribution.

Collapse mechanism formation in Specimens D1 and D3 is shown in Fig. 6.17. The weaker Specimen D3 gave the larger displacements at all three levels for a given base shear, and the less base shear at the
formation of the collapse mechanism. The two frames behaved in a similar manner largely because the yield moments of the columns and the beams in Specimen D3 were reduced from those in Specimen D1 by similar ratios.

Displacements at the three beam levels are plotted in Fig. 6.18 and 6.19 for each step when new plastic hinges were formed in test frames without rigid joints. The second story had initially the largest story deflection per story height for the two lateral force distributions. The first story gave the largest story deflection per story height at the formation of the collapse mechanism. The first-level displacements were comparable for the two force distributions at the mechanism formation, while the third-level displacement of a frame subjected to the triangular force distribution was larger than that to the uniform force distribution, because more forces were distributed at upper levels in the triangular distribution than in the uniform distribution. The ratio of the three displacements was approximately 1.0 : 1.9 : 2.3 (first level : second level : third level) for the uniform force distribution, and 1.0 : 1.2 : 2.6 for the triangular force distribution at the formation of the collapse mechanism.

6.5 Discussion of the Test Results

The behavior of the test frames to simulated earthquake motions cannot be explained precisely by an elastic response analysis or by an elasto-plastic analysis. However, the following discussion was attempted in order to show the relationship of predictions based on routinely available methods of analysis to the actual results of the three-story frames tested.
The initial first-mode frequencies of the test specimens were 5.7 Hz for Test D1 and 5.3 Hz for Test D3, which were approximately 22 percent smaller than those calculated based on uncracked transformed sections, but approximately 14 percent larger than those calculated based on yield sections. The frequencies associated with three fundamental modes dropped immediately after one large oscillation in the test, the waveforms cannot be predicted by the elastic response analysis based on the stiffness assumptions used in this chapter.

(a) Acceleration Response

Measured acceleration waveforms were observed to have higher mode components especially at lower two levels. Initially the frequencies associated with the three fundamental modes were in the constant acceleration range in a simplified response spectrum, hence, the spectral accelerations were comparable for the three modes. Although the mode shape vectors might constantly have changed during a test run as structural damage progressed, the shape of the first-mode oscillation could be approximated as reversed triangular because the deflected shape required the least energy for the structure. Therefore, amplitudes of the first-mode component were small at the first level, and higher mode components were likely to appear in the lower levels.

As the structural damage progressed, the first-mode frequency fell into the constant velocity range, while the second- and third-mode frequencies stayed in the constant acceleration range, hence, the share
of the first-mode component in acceleration waveforms decreased and higher-mode components became visible in the third-level acceleration waveform.

Measured maximum accelerations are compared with calculated maximum accelerations based on the two stiffness assumptions by a modal analysis method (root mean square) in Fig. 6.20 through 6.22, and by a response-history analysis method in Fig. 6.23 through 6.25. Maximum accelerations at the three levels were predicted reasonably well for the first three runs in Test D1 by the elastic response analysis methods based on either of the two stiffness assumptions. Calculated maximum accelerations tended to increase almost linearly with the intensity of base motion, while the measured maximum accelerations tended to reach 'yield' level at a spectral intensity of approximately 10 in., and after which rate of increase was much less. The discrepancy between the measured and the calculated values was observed after a spectrum intensity of approximately 10 in. especially at the upper levels. Calculated maximum accelerations at the second and third levels were in most cases greater than the measured maximum accelerations.

Calculated maximum accelerations based on the modal analysis and the response-history analysis showed similar trends with respect to the measured maximum accelerations. Although it is not practical to use an elastic response analysis to predict acceleration response of a system to an intense base motion, if the elastic response analysis were to be used, the root-mean-square method of spectral modal analysis would be sufficient to obtain maximum values.
(b) **Displacement Response**

From modal analyses based on uncracked transformed sections as well as on fully cracked sections, it was expected that the first-mode component would be dominant in displacement waveforms at all levels as demonstrated in (g) through (i) of Fig. 5.20 through 5.25. Although the observed frequencies were smaller than the calculated frequencies, the ratio of the observed frequencies associated with the three modes was more or less constant. Furthermore, because the smoothed spectral displacement increases as frequency decreases, the spectral displacement for the first mode is much larger than those for higher modes. Hence the first-mode component was dominant in displacement signals even in an inelastic system.

As the damage increased in the test specimens, higher mode components were also observed in the displacement signals. This may be explained as follows. A typical response spectrum at a critical damping factor of 5 percent (for example, see Fig. 5.27(a) for the simulated El Centro base motion) for the tests can be idealized to have constant acceleration response at frequencies above 5.0 Hz, constant displacement response at frequencies below 1.0 Hz, and constant velocity response at frequencies between 1.0 and 5.0 Hz. In the constant-acceleration response range, displacement increases rapidly as frequency decreases. The rate of increase in displacement response becomes slower in the constant-velocity range. Initially, the frequencies associated with the three modes were in the constant acceleration range in the response spectra.
As the test progressed, the first-mode frequency fell into the constant velocity range. Hence, the rate of increase in the spectral displacement associated with the first mode became low. While the second- and the third-mode frequencies were still in the constant acceleration range to give increasing displacement response at a faster rate as the frequencies decreased with the amount of structural damage. In this manner, the higher frequency component became appreciable in the displacement waveforms. So far the phenomena were discussed only from the spectral viewpoint. The mode shape of vibration associated with the three modes should be considered as well. However, the effect of mode shape vectors was small compared with the effect of spectral response on the displacement response.

Measured maximum displacements are compared with calculated maximum elastic displacements based on uncracked transformed and fully cracked sections. Measured displacements represent one-half of the total displacement range during a test run. Calculated displacements represent maximum displacements computed by modal analysis (RMS) shown in Fig. 6.26 through 6.28 and response-history analysis shown in Fig. 6.29 through 6.31. It should be mentioned that these two methods to calculate maximum displacements gave values of a similar magnitude for each test.

Calculated maximum displacements (Fig. 6.26(c), 6.27(c), and 6.28(c)) based on fully cracked sections agreed favorably with the measured displacements at the third level for simulated El Centro base motions. The comparison of the maximum displacements at the first and second levels was quite poor for the two types of base motion and for two different
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As the test progressed, the first-mode frequency fell into the constant velocity range. Hence, the rate of increase in the spectral displacement associated with the first mode became low. While the second- and the third-mode frequencies were still in the constant acceleration range to give increasing displacement response at a faster rate as the frequencies decreased with the amount of structural damage. In this manner, the higher frequency component became appreciable in the displacement waveforms. So far the phenomena were discussed only from the spectral viewpoint. The mode shape of vibration associated with the three modes should be considered as well. However, the effect of mode shape vectors was small compared with the effect of spectral response on the displacement response.

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Calculated maximum displacements (Fig. 6.26(c), 6.27(c), and 6.28(c)) based on fully cracked sections agreed favorably with the measured displacements at the third level for simulated El Centro base motions. The comparison of the maximum displacements at the first and second levels was quite poor for the two types of base motion and for two different
stiffness assumptions. In most cases, the calculated displacements were smaller than the measured displacements. The comparison is also found to be poor for the simulated Taft base motions at the three levels and for the two stiffness assumptions. The calculated displacements based on uncracked transformed sections were smaller than those based on fully cracked sections. Calculated maximum displacements based on uncracked sections always gave less than one-half of the measured maximum displacements for simulated El Centro base motions, and mostly less than two-thirds of them for simulated Taft base motions.

The ratio of the measured maximum displacements at the three levels, was approximately \(1.0 : 1.9 : 2.3\) (first level : second level : third level) for Test D1, \(1.0 : 1.9 : 2.7\) for Test D2, and \(1.0 : 1.7 : 2.1\) for Test D3, whereas, the corresponding ratio for the first mode was \(1.0 : 2.6 : 3.7\). Therefore, it is difficult to have a good agreement at the three levels simultaneously based on the elastic response analysis, which is governed by the first-mode component. The ratio of the three level displacements at the formation of the collapse mechanism was \(1.0 : 1.9 : 2.3\) for the uniform lateral load distribution, and \(1.0 : 1.2 : 2.6\) for the triangular lateral load distribution. The uniform distribution gave the more favorable ratio.

The maximum base shears are compared with the maximum first-level displacements for the measured and calculated values in Fig. 6.32. Calculations were based on elasto-plastic mechanism analysis without rigid zones in a beam-column connection. The measured displacements represent
one-half of the total displacement range at the first level during a test run. It should be noted that the analysis method underestimated the displacement at a base shear below the formation of the first set of plastic hinges.

(c) Base Shear Response

Base shear was calculated as the algebraic sum of all lateral forces on the structure at each time step. As the intensity of the base motion was increased in successive test runs, a tendency was observed in the base-shear waveform to vary from a relatively smooth response reflecting the first-mode (Fig. 5.20(b)) to a response with measurable higher mode components (Fig. 5.25(b)).

If the mode shape of oscillation is considered, all three lateral forces act in the same direction for the first mode. Two lateral forces act in the same direction for the second and third modes, while the third force acts in the opposite direction to reduce the contribution to base shear of the two forces. Therefore, the first-mode component is expected to govern the base shear if the spectral accelerations are comparable for the three modes.

As the structural damage progressed, spectral acceleration for the first-mode component decreased because of the decreasing frequency, which fell into the constant-velocity range. On the other hand, spectral accelerations for the second and third modes did not change much as they remained in the constant-acceleration range. Therefore, the higher mode components became visible in base shear waveforms as structural damage progressed.
Measured maximum base shears are compared with those calculated in the elasto-plastic mechanism analysis in Fig. 6.32. The horizontal axis represents the calculated first-level maximum displacement, and one-half of the total first-level displacement range for the test results. Up to the displacement at which the collapse mechanism was calculated, the measured maximum base shears from Test D1 fell reasonably close to the calculated curve. After that, however, the measured maximum base shears were nearly 50 percent greater than the calculated base shear at the formation of the mechanism.

If yield moments were assumed at the top and base of the first-story columns inside-to-inside of the first-level beam and the base girder, then base shears become 2.40 kip for a single frame in Tests D1 and D2, and 2.28 kip for the single frame in Test D3. The measured maximum base shears at a displacement greater than 0.4 in. were larger than these values.

Figure 3.5 shows a partial interaction diagram corresponding to the development of the tensile strength of the steel in the tension reinforcement. With an axial load of 1.47 kip, the first-story column could carry an ultimate bending moment of 12.3 kip-in. for Specimens D1 and D2, and 11.4 kip-in. for Specimen D3. Therefore, the upper bound base shear that a single frame could take was 3.28 kip for Test D1 and D2, and 3.04 kip for Test D3. The measured maximum base shears were less than these upper bound values.
The moment-carrying capacity of a column tends to decrease as the axial load decreases if the section is designed so that the tension reinforcement yields prior to compression failure in the concrete. When the effect of overturning moment is considered, one column will be unburdened of its axial load and lose its moment-carrying capacity, while the other column increases its moment-carrying capacity due to the increasing axial load. Therefore, a larger part of the existing base shear should be carried by the compression side column, which may cause shear failure in the column although the shear strength also increases with an increasing axial load.

Measured maximum base shears are compared with maximum base shears calculated from the root-mean-square method of the spectral modal analysis in Fig. 6.33, and from the response-history analysis method in Fig. 6.34. The calculated base shears based on either of the two methods agreed favorably with the measured maximum base shears for the first two runs in Test D1. In the rest of the test runs, the calculated values were larger than the measured. The difference in the calculated maximum base shears between the two stiffness assumptions was small for the simulated El Centro base motions, and was smaller for the modal analysis than for the response-history analysis. The difference was large for the simulated Taft base motions.

Figure 6.35 was plotted in an attempt to interpret the test results from the viewpoint of current design practice. The ordinates represent the ratios of the calculated to the measured base shear for the
three-story test frames. The calculated values were taken from the linear response-history analysis based on fully cracked sections at a damping factor of 5 percent of critical for the first and the second modes. In essence, the ordinates are the "actual" reduction in base shear which can be ascribed to inelastic action. The abscissas represent the ratios of the measured maximum first-level displacement to the calculated "yield" deflection at the first level. The yield deflection was found to be 0.082 in. from Fig. 6.32(a) for Specimens D1 and D2 under the triangular force distribution without rigid zones in the joints, and 0.092 in. from Fig. 6.32(b) for Specimen D3. In essence, the abscissas are the "ductility" reached at the first beam level.

The straight line A in the figure shows equal values of the reduction and the ductility. The curve B shows the following relation

\[ \gamma = \sqrt{2\mu - 1} \]

in which

- \( \gamma \) = force reduction factor
- \( \mu \) = ductility factor

It is seen that data points fall below both Curves A and B. Data points indicate that the structure reached a ductility factor of 6.0 in order to reduce the base shear from the calculated elastic response base shear by 50 percent.

The reduction factor is strongly influenced by the way the structural stiffness and damping are evaluated. Therefore, this figure should
not be treated as the absolute measure, but it would be wrong to assume in the design of this structure that the elastic base shear calculated with a damping factor of 5 percent of critical could be reduced by the ductility factor $\mu$ or by the factor $\sqrt{2\mu - 1}$.

(d) **Base-Overturning Moment Response**

Overturning moment at the base of a structure was calculated as the algebraic sum of the products of lateral forces and corresponding heights from the base. The moment should be resisted by the bending moment at the base of the first-story columns and axial forces in the first-story columns.

Base-overturning moment waveforms were observed to be almost exclusively governed by the first-mode component. This may be explained as follows. The three lateral loads act in the same direction for the first mode, hence, each lateral force contributes effectively to the base moment. On the other hand, one lateral force acts in the direction opposite to the other two forces for the second and the third modes, hence, base moments for the second and third modes become very small compared with the one for the first mode (Fig. 6.2). Base moments of the higher two modes were less than 2 percent of that of the first mode for the same spectral acceleration response in the case shown in Sections 6.2 and 6.3. Therefore, the first-mode component would dominate the base-moment waveforms.

Measured base-overturning moments are compared with those calculated in the elasto-plastic mechanism analysis in Fig. 6.36. The horizontal axis represents the first-level displacement for the mechanism analysis,
and one-half of the total first-level displacement range for the test results. Up to a displacement at which the collapse mechanism was calculated, the maximum base-overturning moments from Test D1 were reasonably close to the calculated curve. After that, however, the measured maximum overturning moments were more than 50 percent greater than the calculated base moment corresponding to the formation of the mechanism in all three tests. This is a discrepancy due to the assumption of elasto-plastic response based on the yield stress of the steel. The measured base moments are compatible with the actual strength of the frames as demonstrated below.

From the equilibrium conditions, the following relation should hold for a single frame;

\[ OTM = M_{B1} + M_{B2} + P_c L \]  \hspace{1cm} (6.1)

where

- \(OTM\) = base overturning moment of a single frame
- \(M_{B1}, M_{B2}\) = bending moment at the base of the first-story columns
- \(P_c\) = axial force in the first-story column due to overturning effect
- \(L\) = center-to-center distance of the first-story columns
  
  \(= 36.0\) in.

The moment-carrying capacity increases almost linearly with existing axial load in a column (Fig. 3.5) if the axial load is less than the balance load.
The change in the axial load due to the overturning effect in the two columns is of the same magnitude, but of opposite signs. Therefore, the sum of the bending moments at the bases of the first-story columns can be assumed to be constant, and given by twice the moment-carrying capacity at the initial axial load. The ultimate moments at $\varepsilon_{cu} = 0.004$ with an axial load of 1.47 kips were read from Fig. 3.5 to be 9.6 kip-in. for Specimens D1 and D2, and 9.1 kip-in. for Specimen D3. The maximum base moments were measured to be 132 kip-in. in Run D1-6 for Tests D1 and D2, and 117 kip-in. in Run D3-4 for Test D3. The change in axial force was calculated from Eq. 6.1 as to be 3.1 kip for Run D1-6, and 2.8 kip for Run D3-4. These calculated values indicate that a first-story column experienced tensile force of 1.6 kip in Run D1-6, and 1.3 kip in Run D3-4. However, the columns were capable of carrying such tensile forces as demonstrated in the axial load-bending moment interaction diagrams (Fig. 3.5).

The measured maximum base moments are compared with those calculated based on the root-mean-square method of the spectral modal analysis in Fig. 6.37, and based on the step-by-step response analysis method in Fig. 6.38. These two methods gave similar values. The calculated maxima agreed favorably for the two runs of Test D1. In the rest of test runs, the calculated maximum base moments were larger than the measured. The calculated values based on the uncracked transformed sections of members were more than three times as large as the measured for the simulated Taft base motions.
Chapter 7

Discussion of the Test Results in Relation to Nonlinear Analysis

7.1 Introductory Remarks

The nonlinear response of the test frames was studied analytically utilizing the structural analysis method developed in Chapter 4, and the material properties evaluated in Chapter 3. The test frame was idealized as a regular plane frame standing on an infinitely rigid foundation. In forming structural stiffness, the portion of a member between an end and the contraflexure point was taken as a basic unit. Nonlinear response was determined for each basic unit by a primary response of a cantilever and Takeda's hysteresis (Takeda et al, 1970). Rotation due to bond slip of the tensile reinforcement at the ends of a member was assumed to follow a bilinear primary curve with a simplified hysteresis rule based on Takeda (Appendix F). The hysteresis rules used in this analysis were adopted arbitrarily. However, Takeda's hysteresis was shown to be efficient in predicting the dynamic behavior of reinforced concrete cantilever columns (Takeda et al, 1970) and reinforced concrete one-story one-bay frames (Gulkan and Sozen, 1971). A joint core was treated as infinitely rigid despite the fact that joint cores developed X-shaped cracks in the experiments. The effect of the gravity load was not included in the analysis. Change in axial load in a column due to the overturning effect was not considered in evaluating the stiffness of the column. The analysis contained no limit to the flexural strength of individual members. None was needed because of the very low slope of the force-displacement curve after yielding. Maximum moments and displacements reached in individual members were logged.
The analytical model mentioned above was treated as the "standard" frame in both static and dynamic analyses.

7.2 Static Analysis of Test Frames

The strength of a single test frame was evaluated for a constantly increasing static load. Two different load distributions were used in the static analysis: (a) equal loads at the three levels and (b) loads varying proportionally with the height of the level.

The analysis method developed in Chapter 4 allows as a member flexibility matrix any of Eq. E.14, E.15, and E.16 in Appendix E. Equation E.14 was used for the static analysis because the expected amount of computational operations was not prohibitive.

The structural response was calculated for incremental lateral loads as follows:

1. The structural stiffness matrix was formulated corresponding to location of the contraflexure points and stiffness characteristics of members in the preceding step.

2. Incremental structural displacements were calculated from the structural stiffness and the incremental loads.

3. Incremental member forces were calculated from joint displacements.

4. If the location of the contraflexure point moved more than a limiting value (0.0002 of member length in this analysis) from the assumed position, a new structural stiffness matrix was formulated, and steps (2) through (4) were repeated until the location of the contraflexure point converged using the same incremental load.
If member forces exceed a limiting value, the member stiffness was modified in accordance with the hysteresis rules.

The incremental loads were chosen so that any member would have an incremental member force of less than 20 percent of its cracking moment.

Hysteresis rules were taken into account in step (5), because some members were unloaded due to the stiffness changes in the structure. This phenomenon was also observed in the elasto-plastic limit analysis described in Section 6.4.

General response of the idealized frame can be summarized as follows in relation to Fig. 7.1. The frame was linearly elastic up to a base shear at which the cracking moment was first reached at the bottom of the first-story columns, or at the ends of the first level beam, or both at the same load step. After first cracking took place, the frame gradually lost its stiffness with additional cracks in the members. The stiffness of the frame was drastically reduced by the yielding at the base of the first-story columns, or at the ends of the first-level beam, or both simultaneously. The order of cracking and yielding of the members was very similar to that calculated in the elasto-plastic limit analysis. After the first or second set of yield hinges, the stiffness of the frame did not change appreciably.

The effects of rigid zones in joint cores and rotational springs that simulate bond slip of the tensile reinforcement at the ends of the members are studies for the uniform loads and the triangular loads in Fig. 7.1 and 7.2. The ordinates of the figures represent total forces applied to the frame, and the abscissas are the first-level displacements.
As would be expected, a frame without rigid zones gave the largest displacement for a base shear, and the one without rotational springs gave the smallest displacement. At first yielding in a frame, the frame without rigid zones deflected 1.7 to 2.0 times the frame without rotational springs.

Effect of modeling the inelastic-deformation properties of the frame members is studied for uniform loads and triangular loads in Fig. 7.3 and 7.4. Inelastic behavior of a member was considered over an entire length in Eq. E.14 (circular symbols in the figures), while inelastic behavior was concentrated to two inelastic springs at the ends of the member in Eq. E.16 (triangular symbol). A frame model with equivalent inelastic springs was slightly more flexible than the one with inelastic members, but the difference was small. The effect of rigid zones or rotational springs was by far greater than that of modeling inelastic stiffness of the member.

The calculated load-deflection responses of the frames used in Tests D1 and D3 are compared in Fig. 7.5. Although the frame in Test D3 was slightly weaker than the one in Test D1, the force-displacement relations were almost identical. However, cracking and yielding were calculated at different base shears and displacements.

The effect of load distributions is studied in Fig. 7.6 and 7.7. As would be expected, uniform load distribution resulted in a larger base shear for the same first level displacement. Measured maximum base shears are compared with one-half of the first-level total displacement range in Fig. 7.6. Measured points fell reasonably close to the calculated curves.

Calculated base moments are plotted against first-level displacements for the two load distributions in Fig. 7.7. The difference between
the two curves is small. Measured maximum base moments are also shown in
the same figure. In general, the measured values gave larger base moments
at the same first-level displacement than the calculated curves.

The difference between the data and the calculated curves for base shear and base moment may be partially attributed to the force
distribution.

Displacements at the three levels are plotted for uniform and triangular loads in Fig. 7.8 and 7.9. Large relative displacements were
observed in the first and second stories for both load distributions. This was consistent with the locations of yielding at the first and second stories.

7.3 Nonlinear Response-History Analysis

(a) The Method of Analysis

Nonlinear response history of the idealized test frames was calculated for the measured base motions in the tests. In order to evaluate the
member stiffness, Eq. E.15 in Appendix E was used, in which the member
flexibility was based on stress history of the member and location of the contraflexure point at the last step. When bending moments at the ends of
a member are small, the location of the contraflexure point shifts very rapidly even for a small change in the moments, which causes nonconvergence
in the numerical solution. Therefore, the location of the contraflexure point was frozen to the previous point if a bending moment at either end of
a member was less than a specified value, which was determined for each
response-history analysis. The value varied from 0.8 to two times the cracking moment for the member depending on the intensity of base motion.
Every test run was simulated analytically starting, in each case, with an undamaged specimen. Two analytical solutions were obtained for each test run: with and without viscous damping. The damping matrix was proportional to the stiffness matrix with a damping factor for the first mode of 2 percent of critical at the initial elastic stage.

The time interval in the response calculation was 0.0005 sec.

(b) General Observations about Calculated Response Waveforms

Calculated response histories of the idealized test frames to the measured base motions are shown in Fig. 7.10 through 7.21 for Test D1, in Fig. 7.26 through 7.40 for Test D2, and in Fig. 7.46 through 7.53 for Test D3. The overall characteristics of the calculated waveforms are similar to the measured response waveforms described in Section 5.3:

(1) acceleration waveforms contained higher frequency components, especially at the first level, and

(2) displacement, base shear, and base-moment waveforms were smooth, and governed by the first-mode component.

These observations are consistent with the modal characteristics of the structure and response spectra of base motions as described in Section 6.5. Briefly, base shear and base moment were likely to be dominated by the first-mode because of the characteristics of the mode shapes. Displacements at the three levels were governed by the first mode because of a much larger spectral displacement at the first-mode frequency than those at the second- and third-mode frequencies. Spectral accelerations at the three mode frequencies were comparable. Therefore, the higher mode components were noticeable in the waveforms, especially at the lower levels.
(c) Effects of Structural Characteristics

The effects of various idealization of structural components and damping are studies in Fig. 7.26 through 7.34. Calculated maximum response values were listed in Table 7.1 and 7.2. In order to give the basis for comparison, an idealized test frame with rigid zones in joint cores, with rotational springs at the ends of the members having a simplified Takeda's hysteresis rule, and without viscous-type damping was taken to be 'standard.' The base motion used was that measured in Test Run D2-1. Member stiffness was based on Eq. E.15 in Appendix E for the 'standard' frame.

Calculated response waveforms of the standard frame without viscous damping are shown in Fig. 7.26. The effect of viscous damping was studied in Fig. 7.27 through 7.29, in which a critical damping factor of 2 percent for the first mode at the initial elastic stage was used. The damping matrix was assumed to be proportional to the structural stiffness matrix \( C_1 = 0 \) and \( C_2 \neq 0 \) in Eq. 4.22) in Fig. 7.27, to be proportional to the mass matrix \( C_1 \neq 0 \) and \( C_2 = 0 \) in Fig. 7.28, and to be combination of the two \( C_1 \neq 0 \) and \( C_2 \neq 0 \) in Fig. 7.29. For the uncracked structure, damping factors for each mode are tabulated below.

<table>
<thead>
<tr>
<th>Type of Damping</th>
<th>Initial Damping Factors for Three Modes</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_1 )</td>
<td>( \beta_2 )</td>
</tr>
<tr>
<td>(1) No damping</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(2) Damping proportional to stiffness</td>
<td>0.02</td>
<td>0.066</td>
</tr>
<tr>
<td>(3) Damping proportional to mass</td>
<td>0.02</td>
<td>0.006</td>
</tr>
<tr>
<td>(4) Combination of (2) and (3)</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>
If the damping matrix is proportional to the stiffness matrix in a linearly elastic structure, larger damping factors are automatically assigned to the higher modes. If a damping matrix is proportional to the mass matrix, smaller damping factors are assigned to the higher modes.

Acceleration waveforms in Fig. 7.27 (damping matrix proportional to stiffness matrix) were observed to have less discernible higher mode components than those in Fig. 7.26 (no viscous damping). Damping proportional to structural stiffness was effective in reducing higher mode components in inelastic response. Displacement waveforms in these two figures were almost identical. If base shear waveforms are compared, the one with no damping will be seen to contain more of higher mode components.

The amount of higher mode components in acceleration waveforms in Fig. 7.28 (damping matrix proportional to mass matrix) is similar to that in Fig. 7.26 (no damping). Overall displacement waveforms in Fig. 7.28 are similar to those in Fig. 7.26. However, the maximum displacements at the three levels in Fig. 7.28 were approximately 85 percent of those in Fig. 7.26.

Some of the higher mode components were reduced in acceleration waveforms of Fig. 7.29 (combination of damping proportional to mass and stiffness) in relation to those of Fig. 7.26 (no damping). Base shear, base moment and displacement waveforms in Fig. 7.29 were almost identical to those in Fig. 7.28.

From the comparison of acceleration waveforms for the three different cases, damping proportional to structural stiffness is most effective in reducing the amount of higher mode components without distorting base shear, base moment and displacement waveforms from those in the
"standard" structure. The effect of viscous-type damping on the first-mode response was observed to be measurable if the damping matrix was proportional to the mass matrix. If the damping matrix was proportional to the structural stiffness matrix, the effect of viscous-type damping on the first-mode response in inelastic range was observed to be small due to the reduction in stiffness. Hysteresis energy dissipation would be of more importance in a large inelastic response than that due to viscous-type damping.

Effect of modeling the inelastic-deformation properties of the frame members was studied by using Eq. E.16 in Appendix E. Equation E.16 assumes that all inelastic action of a member is concentrated at the ends of the member as equivalent inelastic springs. The member stiffness is determined considering the location of the contraflexure point. The response waveforms are shown in Fig. 7.30. The overall response waveforms were quite similar to those of the "standard" frame (Fig. 7.26). The maximum base shear, base moment, accelerations at all three levels, and the third-level displacement agreed with those of the "standard" frame. The first- and second-level maximum displacements were approximately 1.2 times those of the "standard" frame.

If the location of the contraflexure point was assumed to be fixed at the midspan of a frame member in evaluating the stiffness of "equivalent inelastic springs" at the ends of the member, the overall response waveforms (Fig. 7.31) were almost identical to those of the "standard" frames. The maximum response values agreed very well with those of the "standard" frame.
The effect of rigid zones in joint cores of a frame was studied by comparing Fig. 7.26 and 7.32. The rigid zones were ignored in the response analysis in Fig. 7.32. As would be expected, the frame without rigid zones in joint cores responded in a longer period than the standard frame; however, the difference was small. The maximum displacements of the frame without rigid zones were approximately 1.2 times larger than those of the "standard" frame, while the maximum base shear, base moment, and accelerations at three levels were of similar magnitudes for the two frames. The overall response waveforms were similar for the two frames.

The rotational springs were not included in the response analysis shown in Fig. 7.33. As would be expected, the frame without the springs is stiffer than the "standard" frame. More of higher mode components can be observed in the acceleration waveforms in Fig. 7.33 than in those in Fig. 7.26. Maximum base shear and base moment of the frame without rotational springs were almost the same as those of the standard frame. Maximum displacements of the frame without rotational springs were approximately 85 percent of those of the standard frame.

The effect of hysteresis rules for rotational springs on the response waveforms was studied by comparing Fig. 7.26 and 7.34. In Fig. 7.34, a regular bilinear hysteresis rule was assigned to the rotational springs with the same primary response curve as those in the "standard" frame. The overall response of the two different frames was similar with large-amplitude oscillations at 1.0, 2.0, and 5.0 sec. If base moment, base shear, and displacement waveforms in Fig. 7.26 and 7.34 are compared, the waveforms for the two frames are seen to be almost identical due to the
fact that both frames have exactly the same properties except for the hysteresis rules of the rotational springs. However, when acceleration waveforms at the three levels were compared, higher mode components are found to be more perceptible for the frame with bilinear rotational springs than for the "standard" frame. The difference in higher mode response may be attributed partially to the difference in the amount of hysteretic energy dissipation, and partially to the difference in the stiffness at low-amplitude oscillation related to the two hysteretic rules as illustrated in Fig. 7.45. For a small oscillation in which stress remains within the upper and lower stress boundaries, the bilinear system does not dissipate hysteretic energy, while the degrading system does. As for the stiffness, it would be clear that the degrading system is more flexible than the bilinear system.

The calculated maximum stresses in each member of the frames with different structural characteristics and dampings were seen to be comparable in Table 7.2. Some difference was observed only in member stresses below the yielding level, as would be expected from the fact that a stress does not increase much after the yielding.

(d) General Comparison of Calculated Response with Measured Response

Comparison of calculated and measured response waveforms is summarized in this section. The distribution of large-amplitude oscillations along the time axis, the maximum values, and frequency components were used as indices for the comparison.

The response-history analysis referred to the "standard" frames which had the following special features discussed in the preceding section:
(a) rigid joint cores, (b) rotational springs at ends of frame members to represent slip of reinforcing bars, and (c) with or without damping proportional to stiffness with a damping factor of 2 percent of critical for the first mode of the uncracked frame.

Calculated response waveforms are shown in Fig. 7.10 through 7.21 for Test D1, in Fig. 7.26 through 7.40 for Test D2, and in Fig. 7.46 through 7.53 for Test D3. In each figure, (a) the measured base motion, (b) calculated base shear, and (c) base moment are shown. The calculated acceleration waveforms at the three levels are shown in (d) through (f). The calculated displacement waveforms at the three levels are shown in (g) through (i). The waveforms of each signal start at the same moment.

Calculated maximum structural response values are listed in Table 7.3 through 7.5 for each test, and are compared with those measured in Fig. 7.22 through 7.25 for Test D1, in Fig. 7.41 through 7.44 for Test D2, and in Fig. 7.54 through 7.57 for Test D3.

The following general observations emerge from a comparison of the measured and calculated quantities.

1. The analytical model was stiffer than the test specimen. The large discrepancies in oscillating periods of the model and the specimen were observed especially at a low-amplitude of oscillation.

2. The simulation was poor for a low intensity base motion in which yielding was calculated in only a few members.

3. The comparison was poor for any test run in which the test specimen had been damaged extensively in the preceding test runs.

4. The comparison was favorable for a test run in which the El Centro (NS) 1940 base motion was simulated at a high intensity.
(5) Viscous damping proportional to stiffness was effective in reducing higher mode components in calculated acceleration waveforms.

(6) Comparison of the acceleration waveforms was found to be more favorable for the model with damping in terms of the amount of higher mode components.

(7) In general, calculated maximum accelerations and displacements at the third level agreed favorably with those measured.

(8) Calculated maximum accelerations and displacements at the lower beam levels sometimes showed discrepancies up to approximately 40 percent of those measured.

(9) Calculated maximum base shears and base moments agreed favorably with those measured (within 15 percent).

Specific observations will be recorded separately for the three tests in Sections 7.4, 7.5, and 7.6.

7.4 Calculated Response for Test D1

The "standard" frame with and without viscous damping was analytically subjected to the base motions measured in Test D1. Six test runs were analyzed independently.

Calculated response waveforms are shown in Fig. 7.10 through 7.21, and are to be compared with the measured waveforms shown in Fig. 5.20 through 5.25. Calculated maximum structural response values are compared with those measured in Fig. 7.22 through 7.25. Calculated maximum member response values are listed in Table 7.6. In this table, the term deflection coefficient refers to the ratio of the attained deflection to the yield deflection of a cantilever beam. In the analysis, each member is made up of two cantilever beams.
(a) Test Run D1-1

Measured response waveforms are shown in Fig. 5.20. Calculated response waveforms are shown in Fig. 7.10 for the model without damping, and in Fig. 7.11 for the model with damping.

Large-amplitude oscillations were measured at 1.0 to 2.0 sec, and 5.0 to 7.0 sec, while large-amplitude oscillations were calculated at 1.0 to 2.5 sec, and 10 to 11 sec.

Maximum base shear and base moment calculated with no damping were almost the same as those measured. However, the times for the calculated and measured maxima were not identical. Base shear and base moment waveforms calculated with damping were almost free of higher mode components, and their maxima were approximately 80 percent of those calculated without damping.

The calculated maximum accelerations at three levels agreed favorably with the measured maxima (within 20 percent), although they were not always calculated at the same moments as the measured maxima.

The comparison of the measured and the calculated displacement waveforms was quite poor in amplitudes, frequency components and distribution of large-amplitude oscillations. The amplitudes of displacement calculated with damping were less than those calculated without damping.

One strain gage on the longitudinal reinforcement at the bottom of a first-story column indicated yielding in the steel. However, the maximum moment calculated without damping at the same location reached only 75 percent of the yielding moment.

(b) Test Run D1-2

Measured response waveforms are shown in Fig. 5.21. Calculated
response waveforms are shown in Fig. 7.12 with no damping and in Fig. 7.13 with damping.

Large amplitude oscillations were observed at 1.0 sec, 2 sec, and 5.0 to 6.0 sec. The analytical models could simulate the large-amplitude oscillations at 1.0 and 2.0 sec, but they failed to do so between 5.0 to 6.0 sec. Instead, the analytical models exaggerated the oscillations between 10.0 and 11.0 sec.

The maximum base moment and base shear calculated with or without damping were in favorable agreement with those measured (within 15 percent).

Although the amount of higher mode components in acceleration waveforms was more satisfactorily simulated by the model with damping, the maximum accelerations were predicted better by the model without damping.

Simulation of displacement waveforms was poor in terms of amplitudes, distribution of large oscillations and frequency components. Displacement waveforms calculated with and without viscous damping were very similar.

Moments at the ends of the first-level beam and at the bottom of the first-story columns were calculated (without damping) to be barely over the yield level.

(c) Test Run D1-3

Measured response waveforms are shown in Fig. 5.22. Calculated response waveforms are shown in Fig. 7.14 for the frame with no damping, and in Fig. 7.15 for the frame with damping.

Large-amplitude oscillations were measured at 1.0 sec, 2.0 sec, and 5.0 sec. Large oscillations at the same locations were calculated, but
at a higher fundamental frequency. Distribution of large oscillations in the calculated waveforms was closer to that in the measured waveforms of the last test run.

In the base-shear and base-moment waveforms, large-amplitude oscillations were well simulated by the analytical models with and without damping. The model failed to reproduce higher mode components in the measured waveforms.

Maximum accelerations at the second and third levels were predicted better by the model without damping, but the acceleration at the first level was predicted better by the model with damping.

The maximum displacement waveforms calculated with and without damping were similar. The calculated maxima at the three levels were approximately 75 percent of those measured.

Yielding was calculated (without damping) at the ends of the first- and second-level beams and at the bottom of the first-story columns. The locations of yielded joints were not sufficient to form a collapse mechanism for the frame.

(d) Test Run D1-4

Measured response waveforms are shown in Fig. 5.23. Calculated waveforms are shown in Fig. 7.16 for the frame without viscous damping, and in Fig. 7.17 for the frame with viscous damping.

Large-amplitude oscillations were observed at 1.0 to 3.0 sec, and medium-amplitude oscillations at 5.0 sec and 7.0 to 9.0 sec. The analytical models with or without damping reproduced these medium- and large-amplitude oscillations successfully.
The base-moment and base-shear waveforms and maxima were very well simulated by the analytical models with or without damping.

The times for the large-amplitude oscillations in the calculated displacement waveforms compared well with the measured times; however, medium-amplitude oscillations at around 8.0 to 10.0 sec were not successfully simulated. The maximum displacements were predicted successfully by the models both with and without damping.

Yielding was calculated for the model without damping at the ends of the first- and the second-level beams, at the top of the second- and the third-story columns, and at the bottom of the first-story columns.

(e) **Test Run D1-5**

Measured response waveforms are shown in Fig. 5.24. Calculated waveforms are shown in Fig. 7.18 for the model without damping, and in Fig. 7.19 for the model with damping.

Large-amplitude oscillations were measured at 1.0 to 2.5 sec, which were followed by medium-amplitude oscillations with relatively large peaks at 5.0, 7.0, and 11.0 sec. The analytical models with and without damping could successfully simulate those large and medium oscillations.

The base shear, base moment and displacement waveforms calculated with or without damping were quite similar. The maximum response values calculated with or without damping were comparable to those measured.

In the analytical model without damping, yielding was calculated at the ends of all the members except at the top of the first-story column, and at the bottom of the second- and third-story columns. Deflection coefficients were calculated to be greater than 8.0 in the first-level beam and at the bottom of the first-story columns.
(f) **Test Run D1-6**

Measured response waveforms are shown in Fig. 5.20. Calculated waveforms are shown in Fig. 7.20 for the idealized frame without viscous damping, and in Fig. 7.21 for the frame with viscous damping. The base motion in this run simulated the N21E component of Taft (1952) record. The test frame had been already damaged badly in the preceding five test runs. The analysis did not take this damage into account.

The calculated base shear and base moment waveforms were quite poor compared with those measured, in terms of frequency components and distribution of large-amplitude oscillations. However, the analytical models with and without damping simulated the large-amplitude oscillations reasonably well.

The maximum base shears and base moments calculated with and without damping agreed favorably with those measured.

The measured acceleration waveforms were relatively well simulated by the analytical models with or without damping. Higher mode components were more perceptible in the measured and the calculated waveforms.

The displacement waveforms calculated with and without damping were similar. Large residual displacements at the three levels were calculated without damping. The measured displacement waveforms changed their center of oscillation with time. A similar tendency was observed in the calculated waveforms only to a limited extent.

7.5 **Calculated Response for Test D2**

The results of nonlinear dynamic response analysis for the Test D2 are compared with the measured values. Comparisons were made on the
basis of waveforms (the measured waveforms in Fig. 5.33 through 5.36, and the calculated waveforms in Fig. 7.26 through 7.40), and maximum response values (Fig. 7.41 through 7.44). The maximum response of the members is listed in Table 7.7 for the analytical model without viscous damping.

The analysis referred to the idealized "standard" frames with and without viscous damping. Four test runs were analyzed independently.

(a) Test Run D2-1

Measured response waveforms are shown in Fig. 5.33. Calculated response waveforms are shown in Fig. 7.26 for the model without damping, and in Fig. 7.27 for the model with damping.

Large-amplitude oscillations were observed at 1.0, 2.0, and 5.0 sec, and medium-amplitude oscillations at 7.0 to 10.0 sec in the test. The analytical models with and without damping simulated the large-amplitude oscillations successfully, but failed to simulate the medium-amplitude oscillations.

The calculated waveforms (base shear, base moment, accelerations and displacements) with and without damping were almost identical with the measured waveforms up to approximately 3.0 sec from the beginning of the test.

The base shear and base moment waveforms calculated with and without damping were almost identical except for a small amount of higher mode components. The maximum base shear and base moment calculated with and without damping agreed favorably with the measurements.

The maximum displacements calculated with and without damping agreed very well with those measured.
Yielding was calculated (no viscous damping) at the ends of the first- and second-level beams, at the base of the first-story columns, and at the top of the second- and third-story columns. The locations of yielded joints were sufficient to form the collapse mechanism of a frame with elasto-plastic members. Deflection coefficients of 5.7 and 7.3 were calculated at the ends of the first-level beam and at the base of the first-story columns, respectively.

(b) Test Run D2-2

Measured response waveforms are shown in Fig. 5.34. Calculated response waveforms are shown in Fig. 7.35 for the analytical model without damping, and in Fig. 7.36 for the model with damping.

Large-amplitude oscillations were observed at 1.0 to 2.5 sec, and medium-amplitude oscillations at 5.0 sec, 7.0 to 9.0 sec, and at 11.0 sec. The analytical models with and without damping were successful in simulating these relatively large oscillations.

The analytical models with and without damping simulated well the measured base-shear and base-moment waveforms. The calculated maximum base shear and base moment agreed favorably with those measured.

Distribution of acceleration amplitude along the time axis was calculated satisfactorily for the models with and without damping. The maximum accelerations at the three levels were also calculated satisfactorily.

The residual displacements at the end of the base motion were calculated in the same direction as that measured. The effect of viscous damping in the calculated displacement waveforms was small. The maximum displacements at the three levels were calculated satisfactorily.
Yielding of the members was calculated at the same locations as in the preceding run.

(c) Test Run D2-3

Measured response waveforms are shown in Fig. 5.35. Calculated waveforms are shown in Fig. 7.37 for the idealized frame without damping, and in Fig. 7.38 for the frame with damping.

In the measured base shear and base moment waveforms, large-amplitude oscillations were separated by relatively small oscillations, whereas, large oscillations were more or less uniformly spread in the calculated waveforms.

In the calculated displacement waveforms, the fundamental mode frequency could be observed to be different at the beginning and at the end of the record. The residual displacement was calculated in the opposite direction to that measured.

(d) Test Run D2-4

Measured response waveforms are shown in Fig. 5.36. Calculated waveforms are shown in Fig. 7.39 for the model without damping, and in Fig. 7.40 for the model with damping.

Large peaks were observed in the measured base-shear and base-moment waveforms at 2.0, 3.0, 4.0, 5.5, and 9.5 sec. The calculated waveforms also peaked at the same locations. However, large oscillations in the calculated waveforms were not as clearly separated as those in the measured waveforms. The maximum base shear and base moment were calculated with and without damping closely to those measured, although the times for
the maximum values did not coincide with the times at which maxima were measured.

Distribution of amplitudes along the time axis in the calculated acceleration waveforms was somewhat similar to that in the measured waveforms.

Comparison of the measured and the calculated displacement waveforms was poor.

7.6 Calculated Response for Test D3

The measured response in Test D3 is compared with the calculated response from the nonlinear dynamic analysis of the "standard" frame with and without damping using the measured base motions. Four test runs were analyzed independently. The measured waveforms are shown in Fig. 5.44 through 5.47. The calculated waveforms are shown in Fig. 7.46 through 7.53. The measured and the calculated maximum response values are compared in Fig. 7.54 through 7.57. The maximum response of the members is listed in Table 7.8 for the analytical model without viscous damping.

(a) Test Run D3-1

Measured response waveforms are shown in Fig. 5.44. Calculated response waveforms are shown in Fig. 7.46 for the model without damping, and in Fig. 7.47 for the model with damping.

Large-amplitude oscillations were observed in the measured waveforms at 1.0 and 2.0 sec, and medium-amplitude oscillations at 5.0 sec and 7.0 to 10.5 sec. The analytical models with or without damping could simulate the measured waveform closely up to 2.5 sec from the beginning of
the test, during which time the largest response was measured in the test. The medium-amplitude oscillations at 5.0 sec were successfully simulated by the analytical models, but those between 7.0 and 10.5 sec were not.

Minor differences were observed in the base-shear and base-moment waveforms calculated with and without viscous damping. The calculated maximum base shear and base moment were approximately 80 percent of those measured.

The maximum accelerations calculated without damping were closer to the measurements than those calculated with damping.

The displacements at the first- and second-beam levels were not measured in this test run due to the failure in the amplifiers. The displacements calculated with damping were generally smaller than those calculated without damping. The maximum displacement at the third level was calculated closely without damping. The large displacements calculated with damping were approximately 80 percent of the measured values.

Yielding was calculated at the ends of the first- and second-level beams, at the top of the second- and third-story columns, and at the bottom of the first-story columns.

(b) Test D3-2

Measured response waveforms are shown in Fig. 5.45. Calculated response waveforms are shown in Fig. 7.48 for the model without damping, and in Fig. 7.49 for the model with damping.

Large-amplitude oscillations were observed at 1.0 to 3.0 sec, and medium-amplitude oscillations at 5.0 sec, 7.0 to 8.0 sec, and 11.0 sec. The analytical models with and without viscous damping simulated those large
oscillations, although some discrepancies were found in frequency components and amplitudes.

The calculated base-shear and base-moment waveforms were close to those measured except for low-amplitude oscillations. The calculated waveforms with and without damping were almost identical except for the higher mode components. The maximum base shear and base moment were calculated in good agreement with those measured.

The calculated acceleration waveforms at the three levels were comparable to those measured. The calculated maximum accelerations (with or without damping) at the three levels were in agreement (within 10 percent) with those measured except the maximum acceleration at the first level calculated with damping (30 percent off).

The displacement waveforms calculated with and without damping were almost identical for this test run. Large oscillations in the calculated displacement waveforms were separated by small oscillations. However, the large oscillations in the measured waveforms continued having a similar amplitude. The maximum displacements at the second and third levels were in good agreement with those measured. The maximum displacement at the first level was calculated to be approximately two-thirds of that measured.

(c) Test Run D3-3

Measured response waveforms are shown in Fig. 5.46. Calculated response waveforms are shown in Fig. 7.50 for the model without damping, and in Fig. 7.51 for the model with damping.

The test specimen had been damaged in the preceding two test runs. The intensity of base motion in this run was almost the same as that in the
last run. Therefore, it may not be fair to compare the measured and the calculated responses, for the calculation was based on the idealized frame without initial damage.

Large-amplitude oscillations in the measured base-shear and base-moment waveforms were generally separated by medium- or small-amplitude oscillations. However, those in the calculated waveforms continued with comparable amplitudes. The maximum base shear and base moment were calculated closely to those measured, although the calculated maxima were not found at the same time.

Comparison of the measured and calculated acceleration waveforms was poor in terms of frequency components and amplitudes.

The measured displacement waveforms shifted the center of oscillation after a large oscillation at 1.5 sec, at which the analytical model had not suffered a degree of damage similar to the test specimen. The comparison of the displacement waveforms was poor.

(d) Test Run D3-4

Measured response waveforms are shown in Fig. 5.47. Calculated response waveforms are shown in Fig. 7.52 for the model without damping, and in Fig. 7.53 for the model with damping.

Comparison of the measured and the calculated waveforms in this run was poor. However, the maximum base shear, base moment, and accelerations at the three levels were calculated satisfactorily.

The calculated maximum displacements were 40 to 90 percent of those measured. The larger discrepancy was observed at the lower beam levels.
Yielding was calculated at the ends of all the beams, at the top of the second- and third-story columns, and at the bottom of the second- and third-story columns. A large change in the fundamental frequency can be seen in the calculated base moment waveforms by comparing oscillations during the first and last 2.0 sec of the test.
CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Object and Scope

The main objective of this investigation was to study experimentally and analytically the response of multistory reinforced concrete frames subjected to a strong base motion.

Small three-story one-bay reinforced concrete frames were tested using the University of Illinois Earthquake Simulator (dimensions of the test specimen in Fig. A.12, material properties in Table 3.2). Each specimen consisting of two parallel frames, carried approximately 2,000 lb at each story (Fig. 2.1). Each test involved a series of simulated earthquake motions of increasing intensity in one horizontal direction parallel to the planes of the frames. The earthquake records were the NS component of 1940 El Centro and the N21E component of 1952 Taft. Three tests were conducted with four to six runs in each test. The maximum base acceleration varied from 0.24 to 3.4 g (response spectra in Fig. 5.27). Limiting base shear coefficient of the specimens (base shear corresponding to collapse mechanism for a "triangular" lateral load distribution divided by total weight of test structure) was approximately 0.6. Measurements in the tests included accelerations and displacements at all levels.

8.2 Observed Response to Simulated Earthquake Motions

(1) All three specimens withstood, without collapse, the base motions of maximum accelerations of 2.1 to 3.4 g. The relative intensity index (maximum base acceleration coefficient divided by the limiting base
shear coefficient of the specimen) reached a maximum of approximately 6.0.
(For an SDF system with a period of 0.1 to 0.3 sec and a damping factor of
5 to 10 percent of critical, the relative intensity index should not exceed
approximately 0.5 for the system to remain elastic).

(2) The measured maximum first-level displacement was approximately
one-twentieth of the story height.

(3) The measured maximum base shears were approximately 1.7 times
the base shear calculated for the collapse mechanism (load at each level
proportional to height, no strain hardening). The measured maximum base
shear was approximately 1.3 times the calculated base shear assuming yield
moments at the top and the bottom of the first-story columns (clear height).

(4) The measured maximum base (overturning) moments were approxi-
mately 1.8 times the base moment calculated for the collapse mechanism. Due
to this large base moment, the first-story column developed net tensile
stress, which was not calculated for the collapse mechanism.

(5) The fundamental frequency of a specimen varied from approxi-
mately 80 percent of the calculated natural frequency (based on uncracked
transformed sections, without rigid zones in joint cores, and without rota-
tional springs at ends of the frame members) before the first test run to
25 percent at the end of the last test run.

(6) Spectrum intensity with a damping factor of 20 percent of
critical was found to be a better index to define an intensity of base
motion than maximum base acceleration. Total displacement range at each
level increased almost linearly with spectrum intensity.
(7) Damping factors measured after the base motion had subsided ranged from 4 to 7 percent of critical for the first mode even after the specimen was severely damaged.

(8) The three beam levels oscillated generally in the same phase.

(9) Base shear and base moment were generally in phase with the third-level displacement.

(10) Acceleration waveforms contained higher frequency components, especially at the lower two beam levels.

8.3 Test Results in Relation to Calculated Linearly Elastic Response

Linear response analysis of an idealized frame was made for each base motion measured in the tests. In predicting the response of a reinforced concrete system, linear analysis is handicapped not only by the possible yielding of the structure but also by the drastic changes in stiffness which occur due to progressive cracking. The analyses were made for the frame members with uncracked transformed and fully cracked sections using a critical damping factor of 5 percent for the first two modes in response-history analysis, and 5 percent for all three modes in spectral modal analysis.

(1) Calculated maximum accelerations based on either uncracked transformed or fully cracked sections at the three levels agreed favorably with those measured for low intensity base motion, as would be expected from the fact that the initial fundamental frequencies of the specimen and the idealized elastic frames were well in the "constant acceleration" range of the response spectrum. However, when the intensity of base motion increased, the calculated maximum accelerations deviated from those measured because of cracking and yielding in the test specimen.
(2) Calculated maximum displacements at the third level based on fully cracked sections agreed favorably with those measured for simulated El Centro base motions. The comparison of measured and calculated (fully cracked sections) maximum displacements at the first and the second levels was observed to be poor. The calculated maximum displacements at all three levels based on uncracked transformed sections were always smaller than two-thirds of the corresponding measured maximum displacements.

(3) The "actual" reduction in base shear was much less than the "ductility" $\mu$ reached at the first beam level, or less than $\sqrt{2\mu - 1}$. The "actual" reduction was defined as a ratio of the calculated (linearly elastic, fully cracked sections, and a critical damping factor of 5 percent for the first two modes) to the measured maximum base shear. The "ductility" was defined as a ratio of the measured maximum first-level displacement to the calculated "yield" deflection of the frame at the first level for triangular load distribution in elasto-plastic limit analysis.

(4) The maximum accelerations at the three levels calculated from the square root of the sum of the squares of the maximum modal components agreed favorably with those calculated from response-history analysis.

(5) The maximum displacements at the three levels calculated from the first-mode component alone agreed favorably with those calculated from response-history analysis.

(6) The maximum base shears and base moments calculated from the first mode component alone agreed favorably with those calculated from response-history analysis.
8.4 Test Results in Relation to Calculated Nonlinear Response

Nonlinear response history was calculated by a method developed in this report. In analyzing the overall response of the frame, the portion of the beam or column between the varying point of contraflexure and the joint was considered as a basic unit. The stiffness characteristics of the basic unit were determined by the primary force-deflection and rotation relations of a cantilever and Takeda's hysteresis rule (Takeda et al, 1970). Rotation due to bond slip of the tensile reinforcement was simulated by placing, at ends of the frame members, bilinear rotational springs with a simplified hysteresis based on Takeda. The joint core, the part common to both the beam and the column, was assumed to be infinitely rigid. The idealized frame without viscous damping was taken to be the "standard".

The effects of various idealizations of structural components and damping observed in the course of the analysis are summarized as follows:

1. Damping proportional to stiffness was very effective in reducing amplitudes of higher frequency components without changing fundamental waveform.

2. Damping proportional to mass reduced amplitude of fundamental wave without affecting the amount of higher frequency components.

3. Simplified Takeda's hysteresis used in rotational springs had capability of dissipating more energy and providing lower stiffness at low-amplitude oscillations than bilinear hysteresis.

Comparisons of calculated response waveforms with those measured are summarized as follows:

1. The analytical model was stiffer than the test specimen, especially at low-amplitude oscillations.
(2) Large oscillations were favorably simulated by the analytical model, if the intensity of base motion was relatively high, and if the damage in the test specimen was small prior to the test run.

(3) The analytical model with viscous damping proportional to stiffness (a first-mode damping factor of 2 percent of critical at the initial uncracked stage) was preferable to the one without viscous damping in the amount of higher frequency components in the calculated acceleration waveforms.

(4) The analytical model predicted closely the maximum accelerations and displacements at the third level, and the maximum base shears and base moments.

8.5 Concluding Discussion

The three-story reinforced concrete frames resisted very strong base motions, in the plane of the frame, without collapse. The severity of the ground motion in relation to the strength of the structure is reflected by the relative intensity index (defined in Section 8.2) which reached 6.0 in the tests. The measured maximum first-level deflection exceeded approximately 10 times the yield deflection calculated for the collapse mechanism (load at each level proportional to height, and no strain hardening). The observed heavy damage was limited to the bases of the first-story columns. All damage was attributable to axial load and/or flexure without complications due to shear and bond stresses. It should be noted that all frame members and joint cores were provided with web reinforcement to carry the entire expected shear.
Because of strain hardening in the reinforcement, the measured base shears and base moments were considerably larger than quantities calculated from idealized elasto-plastic models. For the same reason, columns developed net axial tensile forces during the more severe test runs, a conclusion which could not be obtained by routine elasto-plastic analysis.

Within the domain of linearly elastic analysis, a relatively elaborate response-history analysis provided no better insight into the behavior of the test structures than the simple spectral-response analysis based on a spectrum calculated for small frequency intervals.

The total base shear obtained from linear-response analysis (fully cracked sections, damping factor of 5 percent of critical) could not be reconciled with the measured base shear in relation to the attained ductility for the first level (by dividing the calculated base shear by the ductility factor \( \mu \) or by \( \sqrt{2\mu - 1} \)).

The inelastic model reproduced the response of the test frames satisfactorily for large-amplitude response. The model was able to account for behavioral characteristics related to yielding and strain hardening of the reinforcement, cracking of the concrete, major shifts in the point of contraflexure in the frame members, and stiffness changes due to load reversals.
LIST OF REFERENCES


Table 2.1 Summary of Base Motions

<table>
<thead>
<tr>
<th>Earthquake Designation*</th>
<th>Measured Max. Acc.,g</th>
<th>Spectrum Intensity** (S_{120}) in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Test D1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run 1 El Centro (NS) 1940</td>
<td>0.24</td>
<td>4.55</td>
</tr>
<tr>
<td>Run 2 El Centro (NS) 1940</td>
<td>0.40</td>
<td>7.06</td>
</tr>
<tr>
<td>Run 3 El Centro (NS) 1940</td>
<td>0.53</td>
<td>10.7</td>
</tr>
<tr>
<td>Run 4 El Centro (NS) 1940</td>
<td>0.84</td>
<td>15.7</td>
</tr>
<tr>
<td>Run 5 El Centro (NS) 1940</td>
<td>1.42</td>
<td>21.3</td>
</tr>
<tr>
<td>Run 6 Taft (N21E) 1952</td>
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<td>30.4</td>
</tr>
<tr>
<td>(2) Test D2</td>
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<td></td>
</tr>
<tr>
<td>Run 1 El Centro (NS) 1940</td>
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</tr>
<tr>
<td>Run 2 El Centro (NS) 1940</td>
<td>1.10</td>
<td>19.3</td>
</tr>
<tr>
<td>Run 3 Taft (N21E) 1952</td>
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<td>20.1</td>
</tr>
<tr>
<td>Run 4 Taft (N21E) 1952</td>
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<td>28.4</td>
</tr>
<tr>
<td>(3) Test D3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run 1 El Centro (NS) 1940</td>
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<td>12.3</td>
</tr>
<tr>
<td>Run 2 El Centro (NS) 1940</td>
<td>1.10</td>
<td>18.2</td>
</tr>
<tr>
<td>Run 3 Taft (N21E) 1952</td>
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<td>19.0</td>
</tr>
<tr>
<td>Run 4 Taft (N21E) 1952</td>
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</tr>
</tbody>
</table>

*The time axis of the original earthquake acceleration record was compressed by 2.5. The amplitudes were varied.

**Spectrum intensity was calculated for a damping factor of 20 percent of critical and a period range from 0.04 to 1.0 sec.
Table 3.1  Material Properties Used in Calculating the Cracking Point

<table>
<thead>
<tr>
<th></th>
<th>Specimens D1 and D2</th>
<th>Specimen D3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus of the Steel, psi</td>
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<td>29,000,000</td>
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<tr>
<td>Secant Modulus* of the Concrete, psi</td>
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<td>3,010,000</td>
</tr>
<tr>
<td>Tensile Strength of the Concrete, psi</td>
<td>470</td>
<td>368</td>
</tr>
<tr>
<td>Effective Moment Inertia** of Columns, in.</td>
<td>4.51</td>
<td>4.37</td>
</tr>
<tr>
<td>Effective Moment Inertia** of Beams, in.</td>
<td>7.63</td>
<td>7.18</td>
</tr>
</tbody>
</table>

* Secant modulus at 40 percent of compressive strength of the concrete

**Based on uncracked transformed sections
Table 3.2 Material Properties Used in Calculating the Yield Moment and Curvature

<table>
<thead>
<tr>
<th>Specimens D1 and D2</th>
<th>Specimen D3</th>
</tr>
</thead>
</table>

Properties of the Concrete
- Compressive Strength $f'_c$, psi
- Strain at $f'_c$
- Slope Constant $Z$ for Descending Branch

Properties of the Reinforcement
- Yield Stress $f_y$, psi
- Ultimate Stress $f'_{su}$, psi
- Yield Strain $\varepsilon_y$
- Strain Hardening Strain $\varepsilon_{sh}$
- Ultimate Strain $\varepsilon_{su}$

Dimensions of the Cross Sections
- Width $b$, in.
- Total Depth $D$, in.
  - Column
  - Beam
- Depth of the Tensile Reinforcement $d$, in.
  - Column
  - Beam
- Depth of the Compressive Reinforcement $d'$, in.
  - Column
  - Beam
- Area of the Tensile Reinforcement $A_s$, in.$^2$
  - Column and Beam
- Area of the Compressive Reinforcement $A'_{s'}$, in.$^2$
  - Column and Beam

<table>
<thead>
<tr>
<th></th>
<th>Specimens D1 and D2</th>
<th>Specimen D3</th>
</tr>
</thead>
<tbody>
<tr>
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<td>5,050</td>
</tr>
<tr>
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<td>0.00280</td>
<td>0.00280</td>
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<tr>
<td>Slope Constant $Z$ for Descending Branch</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Yield Stress $f_y$, psi</td>
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<td>42,600</td>
</tr>
<tr>
<td>Ultimate Stress $f'_{su}$, psi</td>
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<td>66,500</td>
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<tr>
<td>Yield Strain $\varepsilon_y$</td>
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<td>0.00147</td>
</tr>
<tr>
<td>Strain Hardening Strain $\varepsilon_{sh}$</td>
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<td>0.016</td>
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<tr>
<td>Ultimate Strain $\varepsilon_{su}$</td>
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<td>0.065</td>
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<tr>
<td>Width $b$, in.</td>
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<td>2.5</td>
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<tr>
<td>Total Depth $D$, in.</td>
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</tr>
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<td>2.5</td>
</tr>
<tr>
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<tr>
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<td>Column</td>
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</tr>
<tr>
<td>Beam</td>
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<td>0.10</td>
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<td>Area of the Tensile Reinforcement $A_s$, in.$^2$</td>
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<td>0.10</td>
</tr>
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Table 3.3 Calculated Moment-Curvature Relationship

<table>
<thead>
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<th>Stage</th>
<th>First-Story Column</th>
<th>Second-Story Column</th>
<th>Third-Story Column</th>
<th>All Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M*</td>
<td>φ**</td>
<td>M</td>
<td>φ</td>
</tr>
<tr>
<td>(a) Specimens D1 and D2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cracking</td>
<td>2.51</td>
<td>0.176</td>
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<td>0.157</td>
</tr>
<tr>
<td>Yielding</td>
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<td>1.215</td>
<td>8.59</td>
<td>1.185</td>
</tr>
<tr>
<td>ε_c**** = 0.002</td>
<td>9.40</td>
<td>3.53</td>
<td>8.97</td>
<td>3.76</td>
</tr>
<tr>
<td>ε_c = 0.004</td>
<td>9.56</td>
<td>9.20</td>
<td>9.15</td>
<td>9.50</td>
</tr>
<tr>
<td>ε_c = 0.006</td>
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<td>9.82</td>
<td>14.50</td>
</tr>
<tr>
<td>ε_c = 0.008</td>
<td>10.76</td>
<td>18.90</td>
<td>10.40</td>
<td>19.27</td>
</tr>
<tr>
<td>ε_c = 0.010</td>
<td>11.32</td>
<td>23.60</td>
<td>10.96</td>
<td>23.95</td>
</tr>
<tr>
<td>(b) Specimen D3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cracking</td>
<td>2.11</td>
<td>0.160</td>
<td>1.84</td>
<td>0.140</td>
</tr>
<tr>
<td>Yielding</td>
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<td>1.289</td>
<td>8.16</td>
<td>1.254</td>
</tr>
<tr>
<td>ε_c**** = 0.002</td>
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<td>3.20</td>
<td>8.56</td>
<td>3.41</td>
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<td>ε_c = 0.004</td>
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<td>8.72</td>
<td>8.26</td>
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<td>8.93</td>
<td>12.31</td>
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<tr>
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<td>16.34</td>
<td>9.28</td>
<td>16.65</td>
</tr>
<tr>
<td>ε_c = 0.010</td>
<td>9.92</td>
<td>20.28</td>
<td>9.59</td>
<td>20.59</td>
</tr>
</tbody>
</table>

* M = Moment, kip-in.  ** φ = Curvature, x 10^-3 1/in.  *** ε_c = Compressive fiber strain
Table 3.4  Idealized Moment-Rotation and Moment-Displacement Relationships of a Unit Length Cantilever

<table>
<thead>
<tr>
<th>Member</th>
<th>M*</th>
<th>Cracking R**</th>
<th>Cracking D***</th>
<th>Yielding M</th>
<th>Yielding R</th>
<th>Yielding D</th>
<th>Ultimate M</th>
<th>Ultimate R</th>
<th>Ultimate D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Specimens D1 and D2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam</td>
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<td>0.0494</td>
<td>0.0329</td>
<td>9.67</td>
<td>0.423</td>
<td>0.289</td>
<td>12.75</td>
<td>2.68</td>
<td>2.33</td>
</tr>
<tr>
<td>First-Story Column</td>
<td>2.51</td>
<td>0.0881</td>
<td>0.0461</td>
<td>9.01</td>
<td>0.585</td>
<td>0.401</td>
<td>10.76</td>
<td>2.40</td>
<td>2.07</td>
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<tr>
<td>Second-Story Column</td>
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<td>0.0786</td>
<td>0.0524</td>
<td>8.59</td>
<td>0.573</td>
<td>0.392</td>
<td>10.40</td>
<td>2.25</td>
<td>1.94</td>
</tr>
<tr>
<td>Third-Story Column</td>
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<td>0.0691</td>
<td>0.0587</td>
<td>8.16</td>
<td>0.559</td>
<td>0.381</td>
<td>10.04</td>
<td>2.13</td>
<td>1.82</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Beam</td>
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<td>0.0407</td>
<td>0.0271</td>
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<td>0.473</td>
<td>0.321</td>
<td>10.52</td>
<td>1.62</td>
<td>1.39</td>
</tr>
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<td>First-Story Column</td>
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<td>0.0535</td>
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<td>9.62</td>
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<td>0.611</td>
<td>0.416</td>
<td>9.28</td>
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<td>1.35</td>
</tr>
<tr>
<td>Third-Story Column</td>
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<td>0.0593</td>
<td>0.0395</td>
<td>7.75</td>
<td>0.599</td>
<td>0.406</td>
<td>8.93</td>
<td>1.78</td>
<td>1.45</td>
</tr>
</tbody>
</table>

* M = Moment, kip-in.
** R = Free end rotation, x 10^-3 rad
*** D = Free end displacement, x 10^-3 in.
<table>
<thead>
<tr>
<th>Member</th>
<th>Break-Point Moment, kip-in.</th>
<th>Break-Point Rotation, rad.</th>
<th>Yield-Point Moment, kip-in.</th>
<th>Yield-Point Rotation, rad.</th>
</tr>
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<tbody>
<tr>
<td>(a) Specimens D1 and D2</td>
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<td></td>
<td></td>
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<td>9.01</td>
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<td>0.000605</td>
<td>8.59</td>
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<tr>
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<td>0.000605</td>
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<td>0.00242</td>
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<td>0.00265</td>
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<td>7.75</td>
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Table 5.1 Summary of Observed Response in Test D1

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<th>Response Designation</th>
<th>D1-1</th>
<th>D1-2</th>
<th>D1-3</th>
<th>Test</th>
<th>Run</th>
<th>D1-5</th>
<th>D1-6</th>
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<tbody>
<tr>
<td>(a) Spectrum Intensity, in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$S_{10}$</td>
<td>13.2</td>
<td>20.1</td>
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<td>45.2</td>
<td>60.5</td>
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<td>16.1</td>
<td>23.9</td>
<td>32.2</td>
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<tr>
<td>$S_{120}$</td>
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<td>10.7</td>
<td>15.7</td>
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<tr>
<td>(b) Frequencies, Hz</td>
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<td>3.6--3.4</td>
<td>3.3--2.8</td>
<td>3.0--2.4</td>
<td>2.8--2.0</td>
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<td>13--12</td>
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<td>13--10</td>
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<tr>
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<td>22</td>
<td>21</td>
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<tr>
<td>(c) Base Acceleration, g</td>
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<td>0.53</td>
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<td>(f) Acceleration, g</td>
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<td>(g) Displacement*, in.</td>
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<td>0.56</td>
<td>--</td>
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<td>1.92</td>
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<tr>
<td>(h) $\frac{3}{2}P_{\max_i}^{**}$</td>
<td>0.90</td>
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<td>0.69</td>
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<td>(i) $\sum_{i=1}^{3}P_{\max_i}^{<strong>h_i}^{</strong>*}$</td>
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<td>0.80</td>
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*One half of total displacement range in a test run.  **Maximum lateral load of a level in a test run.  ***Height of a level from the base.
Table 5.2 Summary of Observed Response in Test D2

<table>
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<tr>
<th>Response Designation</th>
<th>Test D2-1</th>
<th>Test D2-2</th>
<th>Test D2-3</th>
<th>Test D2-4</th>
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<tbody>
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<tr>
<td>$S_{10}$</td>
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<td>89.9</td>
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<td>30.8</td>
<td>43.0</td>
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<td>(b) Frequencies, Hz</td>
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<td></td>
<td></td>
<td></td>
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<td>3.0--2.3</td>
<td>2.7--2.1</td>
<td>3.2--1.8</td>
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<tr>
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<td>12--10</td>
<td>10--9.1</td>
<td>10--8.0</td>
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<td>20</td>
<td>20--16</td>
<td>17--16</td>
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<td>(c) Base Acceleration, g</td>
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<td>109</td>
<td>103</td>
<td>125</td>
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<td>Third Level</td>
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<td>1.50</td>
<td>1.60</td>
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<td>(g) Displacement*, in.</td>
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<td>0.39</td>
<td>0.53</td>
<td>0.59</td>
<td>0.79</td>
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<td>Second Level</td>
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<td>0.98</td>
<td>1.07</td>
<td>1.47</td>
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<td>Third Level</td>
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<td>1.44</td>
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<td>2.17</td>
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<tr>
<td>(h) Base Shear/ $\sum_{i=1}^{3}</td>
<td>P_{max}</td>
<td>_{i}$**</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>(i) Base Moment/ $\sum_{i=1}^{3}</td>
<td>P_{max}</td>
<td><em>{i}h</em>{i}$***</td>
<td>0.83</td>
<td>0.74</td>
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</table>

*One half of total displacement range in a test run. **Maximum lateral load of a level in a test run. ***Height of a level from the base.
<table>
<thead>
<tr>
<th>Response Designation</th>
<th>D3-1</th>
<th>D3-2</th>
<th>D3-3</th>
<th>D3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Spectrum Intensity, in.</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>33.4</td>
<td>51.0</td>
<td>60.8</td>
<td>83.1</td>
</tr>
<tr>
<td>$S_{15}$</td>
<td>18.2</td>
<td>27.4</td>
<td>29.4</td>
<td>39.4</td>
</tr>
<tr>
<td>$S_{120}$</td>
<td>12.3</td>
<td>18.2</td>
<td>19.0</td>
<td>25.3</td>
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<tr>
<td>(b) Frequencies, Hz</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>First Mode</td>
<td>3.1-2.6</td>
<td>2.7-2.2</td>
<td>2.7-1.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Second Mode</td>
<td>13-10</td>
<td>12-9.4</td>
<td>10-9.3</td>
<td>11-10</td>
</tr>
<tr>
<td>Third Mode</td>
<td>22-20</td>
<td>21-20</td>
<td>19-18</td>
<td>19-17</td>
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<tr>
<td>(c) Base Acceleration, g</td>
<td>0.61</td>
<td>1.10</td>
<td>0.93</td>
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<td>(d) Base Shear, kip</td>
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<td>2.92</td>
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<tr>
<td>(e) Base Moment, kip-in.</td>
<td>108</td>
<td>112</td>
<td>101</td>
<td>117</td>
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<tr>
<td>(f) Acceleration, g</td>
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<td></td>
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<tr>
<td>First Level</td>
<td>1.16</td>
<td>1.13</td>
<td>1.50</td>
<td>1.47</td>
</tr>
<tr>
<td>Second Level</td>
<td>1.17</td>
<td>1.32</td>
<td>1.04</td>
<td>1.44</td>
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<td>Third Level</td>
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<td>1.68</td>
<td>1.54</td>
<td>1.82</td>
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<tr>
<td>(g) Displacement*, in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First Level</td>
<td>0.67</td>
<td>0.73</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>Second Level</td>
<td>1.14</td>
<td>1.26</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>Third Level</td>
<td>0.95</td>
<td>1.40</td>
<td>1.54</td>
<td>2.10</td>
</tr>
<tr>
<td>(h) Base Shear/ $\sum_{i=1}^{3}</td>
<td>P_{max}</td>
<td>_i$**</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>(i) Base Moment/ $\sum_{i=1}^{3}</td>
<td>P_{max}</td>
<td>_i<strong>h_i$</strong>*</td>
<td>0.79</td>
<td>0.75</td>
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</table>

*One half of total displacement range in a test run. **Maximum lateral load of a level in a test run. ***Height of a level from the base.
<table>
<thead>
<tr>
<th></th>
<th>Members with Uncracked Sections</th>
<th>Members with Fully Cracked Sections</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1st Mode</td>
<td>2nd Mode</td>
</tr>
<tr>
<td>(a) Specimens D1 and D2</td>
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<td></td>
</tr>
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<td>23.8</td>
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<td>0.042</td>
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<tr>
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<tr>
<td>Third Level</td>
<td>1.256</td>
<td>-0.341</td>
</tr>
<tr>
<td>Second Level</td>
<td>0.893</td>
<td>0.332</td>
</tr>
<tr>
<td>First Level</td>
<td>0.345</td>
<td>0.382</td>
</tr>
<tr>
<td>(b) Specimen D3</td>
<td></td>
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</tr>
<tr>
<td>Frequencies, Hz</td>
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<td>22.8</td>
</tr>
<tr>
<td>Periods, sec</td>
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<td>0.044</td>
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<tr>
<td>First Level</td>
<td>0.343</td>
<td>0.382</td>
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<sup>a</sup>Mode shape vectors are normalized and multiplied by the participation factors.
Table 6.2 Spectral Response of Test Frames
(Uncracked Section, $\beta=0.05$)

<table>
<thead>
<tr>
<th>Test Run</th>
<th>Displacement, in.</th>
<th>Acceleration, g</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Mode</td>
<td>2nd Mode</td>
</tr>
<tr>
<td>D1-1 (El Centro)</td>
<td>0.086</td>
<td>0.007</td>
</tr>
<tr>
<td>D1-2 (El Centro)</td>
<td>0.145</td>
<td>0.018</td>
</tr>
<tr>
<td>D1-3 (El Centro)</td>
<td>0.232</td>
<td>0.021</td>
</tr>
<tr>
<td>D1-4 (El Centro)</td>
<td>0.357</td>
<td>0.035</td>
</tr>
<tr>
<td>D1-5 (El Centro)</td>
<td>0.490</td>
<td>0.043</td>
</tr>
<tr>
<td>D1-6 (Taft)</td>
<td>0.956</td>
<td>0.090</td>
</tr>
<tr>
<td>D2-1 (El Centro)</td>
<td>0.370</td>
<td>0.030</td>
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<tr>
<td>D2-2 (El Centro)</td>
<td>0.460</td>
<td>0.035</td>
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<tr>
<td>D2-3 (Taft)</td>
<td>0.676</td>
<td>0.035</td>
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<td>D2-4 (Taft)</td>
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<tr>
<td>D3-1 (El Centro)</td>
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</tr>
<tr>
<td>D3-2 (El Centro)</td>
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<tr>
<td>D3-3 (Taft)</td>
<td>0.675</td>
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<td>D3-4 (Taft)</td>
<td>0.912</td>
<td>0.058</td>
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Table 6.3 Spectral Response of Test Frames (Fully Cracked Section, $\beta=0.05$)

<table>
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<tr>
<th>Test Run</th>
<th>Displacement, in.</th>
<th>Acceleration, g</th>
</tr>
</thead>
<tbody>
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<td>2nd Mode</td>
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<td>D1-1 (El Centro)</td>
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<td>D1-2 (El Centro)</td>
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<td>0.029</td>
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<tr>
<td>D1-3 (El Centro)</td>
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<td>D1-4 (El Centro)</td>
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<tr>
<td>D1-5 (El Centro)</td>
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<td>0.091</td>
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<tr>
<td>D1-6 (Taft)</td>
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<td>0.16</td>
</tr>
<tr>
<td>D2-1 (El Centro)</td>
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<td>0.07</td>
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<tr>
<td>D2-2 (El Centro)</td>
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<td>D2-3 (Taft)</td>
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<td>D3-2 (El Centro)</td>
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<td>0.08</td>
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<tr>
<td>D3-3 (Taft)</td>
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<td>0.08</td>
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<td>D3-4 (Taft)</td>
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<td>Earthquake Type</td>
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Table 6.4  Comparison of the Measured Maximum Accelerations with the Calculated Maxima from Linear-Response Analysis

(b) Test D2

<table>
<thead>
<tr>
<th>Earthquake Test Run Type</th>
<th>Measured Maximum Acceleration, g</th>
<th>Response-History</th>
<th>Spectral Analysis</th>
<th>Fully Cracked Section</th>
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<td>1.61</td>
<td>2.97</td>
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<td>3.18</td>
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<tr>
<td>D2-1 El Centro</td>
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<td>1.77</td>
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*One-half of the total displacement range in a test run.*
Table 6.5  Comparison of the Measured Maximum Displacements with the Calculated Maxima from Linear-Response Analysis

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*One-half of the total displacement range in a test run.
Table 6.5  Comparison of the Measured Maximum Displacements with the Calculated Maxima from Linear-Response Analysis

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Note: One-half of the total displacement range in a test run.
Table 6.6  Comparison of the Measured Maximum Base Shears with the Calculated Maxima from Linear-Response Analysis

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<td>El Centro</td>
<td>81.4</td>
<td></td>
<td>78.4</td>
<td>77.5</td>
<td>78.6</td>
<td>77.5</td>
</tr>
<tr>
<td>D1-3</td>
<td>El Centro</td>
<td>74.9</td>
<td></td>
<td>127</td>
<td>125</td>
<td>126</td>
<td>125</td>
</tr>
<tr>
<td>D1-4</td>
<td>El Centro</td>
<td>105</td>
<td></td>
<td>198</td>
<td>193</td>
<td>195</td>
<td>193</td>
</tr>
<tr>
<td>D1-5</td>
<td>El Centro</td>
<td>106</td>
<td></td>
<td>266</td>
<td>266</td>
<td>269</td>
<td>266</td>
</tr>
<tr>
<td>D1-6</td>
<td>Taft</td>
<td>132</td>
<td></td>
<td>692</td>
<td>508</td>
<td>513</td>
<td>508</td>
</tr>
<tr>
<td>D2-1</td>
<td>El Centro</td>
<td>109</td>
<td></td>
<td>206</td>
<td>199</td>
<td>201</td>
<td>199</td>
</tr>
<tr>
<td>D2-2</td>
<td>El Centro</td>
<td>109</td>
<td></td>
<td>259</td>
<td>240</td>
<td>242</td>
<td>240</td>
</tr>
<tr>
<td>D2-3</td>
<td>Taft</td>
<td>103</td>
<td></td>
<td>431</td>
<td>356</td>
<td>358</td>
<td>356</td>
</tr>
<tr>
<td>D2-4</td>
<td>Taft</td>
<td>125</td>
<td></td>
<td>608</td>
<td>501</td>
<td>507</td>
<td>501</td>
</tr>
<tr>
<td>D3-1</td>
<td>El Centro</td>
<td>108</td>
<td></td>
<td>135</td>
<td>136</td>
<td>137</td>
<td>136</td>
</tr>
<tr>
<td>D3-2</td>
<td>El Centro</td>
<td>112</td>
<td></td>
<td>206</td>
<td>206</td>
<td>208</td>
<td>206</td>
</tr>
<tr>
<td>D3-3</td>
<td>Taft</td>
<td>101</td>
<td></td>
<td>352</td>
<td>330</td>
<td>332</td>
<td>330</td>
</tr>
<tr>
<td>D3-4</td>
<td>Taft</td>
<td>117</td>
<td></td>
<td>478</td>
<td>448</td>
<td>451</td>
<td>448</td>
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</tbody>
</table>
Table 6.8  Comparison of Response at the Formation of the Mechanism

<table>
<thead>
<tr>
<th>Force Distribution</th>
<th>Specimens D1 and D2</th>
<th>Specimen D3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Displacement, in.</td>
<td>Displacement, in.</td>
</tr>
<tr>
<td></td>
<td>1st Level 2nd Level 3rd Level</td>
<td>1st Level 2nd Level 3rd Level</td>
</tr>
<tr>
<td>(a) Uniform Distribution</td>
<td>Base Shear,kip</td>
<td>Base Shear,kip</td>
</tr>
<tr>
<td>Without Rigid Joints</td>
<td>0.260 0.494 0.601</td>
<td>1.92 0.297 0.568 0.699</td>
</tr>
<tr>
<td>With Rigid Joints</td>
<td>0.257 0.477 0.567</td>
<td>2.03 0.290 0.543 0.658</td>
</tr>
<tr>
<td>(b) Triangular Distribution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Rigid Joints</td>
<td>0.279 0.568 0.729</td>
<td>1.73 0.314 0.650 0.836</td>
</tr>
<tr>
<td>With Rigid Joints</td>
<td>0.279 0.559 0.706</td>
<td>1.85 0.315 0.631 0.802</td>
</tr>
</tbody>
</table>
Table 7.1 Effect of Idealized Structural Characteristics on Calculated Maximum Structural Response (Nonlinear Analysis)

<table>
<thead>
<tr>
<th>Structural Response</th>
<th>Standard Frame$^a$</th>
<th>Equivalent Inelastic Springs$^{***}$</th>
<th>No Rigid Zones in Joint Cores</th>
<th>No Rotational Springs</th>
<th>Bilinear Rotational Springs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_1 = 0.0$</td>
<td>$C_2 \neq 0.0$</td>
<td>$C_2 = 0.0$</td>
<td>Inflection Point Variable Fixed</td>
<td></td>
</tr>
<tr>
<td>Maximum Acceleration, g</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Level</td>
<td>1.43</td>
<td>1.30</td>
<td>1.34</td>
<td>1.31</td>
<td>1.44</td>
</tr>
<tr>
<td>Second Level</td>
<td>1.19</td>
<td>1.10</td>
<td>1.07</td>
<td>1.05</td>
<td>1.49</td>
</tr>
<tr>
<td>First Level</td>
<td>1.03</td>
<td>1.10</td>
<td>0.92</td>
<td>1.05</td>
<td>1.28</td>
</tr>
<tr>
<td>Maximum Displacement, in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third Level</td>
<td>1.18</td>
<td>1.02</td>
<td>1.12</td>
<td>1.05</td>
<td>1.30</td>
</tr>
<tr>
<td>Second Level</td>
<td>0.96</td>
<td>0.82</td>
<td>0.93</td>
<td>0.84</td>
<td>1.08</td>
</tr>
<tr>
<td>First Level</td>
<td>0.47</td>
<td>0.38</td>
<td>0.46</td>
<td>0.39</td>
<td>0.48</td>
</tr>
<tr>
<td>Maximum Base Shear, kip</td>
<td>2.66</td>
<td>2.48</td>
<td>2.61</td>
<td>2.47</td>
<td>2.66</td>
</tr>
<tr>
<td>Maximum Base Moment, kip-in.</td>
<td>95.0</td>
<td>91.8</td>
<td>95.4</td>
<td>92.6</td>
<td>95.8</td>
</tr>
</tbody>
</table>

$^a$ The idealized frame with rigid zones in joint cores and rotational springs (simplified Takeda's hysteresis) at ends of the frame members.


$^{***}$ Inelastic deformations are lumped in rotations of the inelastic springs at ends of the frame members.
Table 7.2 Effect of Idealized Structural Characteristics on Calculated Maximum Member Response (Nonlinear Analysis)

<table>
<thead>
<tr>
<th>Member</th>
<th>Response</th>
<th>( C_1^{***=0.0} )</th>
<th>( C_2^{***=0.0} )</th>
<th>( C_1 \neq 0.0 )</th>
<th>( C_2 \neq 0.0 )</th>
<th>Equivalent Inelastic Springs</th>
<th>No Rigid Zones in Joint Cores</th>
<th>No Rotational Springs</th>
<th>Bilinear Rotational Springs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Third-Level Beam</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>8.69</td>
<td>8.49</td>
<td>8.57</td>
<td>8.50</td>
<td>8.64</td>
<td>8.82</td>
<td>8.32</td>
<td>8.58</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>0.88</td>
<td>0.86</td>
<td>0.87</td>
<td>0.86</td>
<td>0.88</td>
<td>0.86</td>
<td>0.90</td>
<td>0.84</td>
</tr>
<tr>
<td>Second-Level Beam</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>10.52</td>
<td>10.37</td>
<td>10.47</td>
<td>10.40</td>
<td>10.60</td>
<td>10.74</td>
<td>10.55</td>
<td>10.58</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>2.94</td>
<td>2.60</td>
<td>2.83</td>
<td>2.66</td>
<td>2.62</td>
<td>2.86</td>
<td>3.00</td>
<td>3.07</td>
</tr>
<tr>
<td>First-Level Beam</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>11.73</td>
<td>11.34</td>
<td>11.65</td>
<td>11.40</td>
<td>11.84</td>
<td>11.78</td>
<td>11.68</td>
<td>11.65</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>5.71</td>
<td>4.81</td>
<td>5.52</td>
<td>4.94</td>
<td>4.95</td>
<td>4.65</td>
<td>5.59</td>
<td>5.51</td>
</tr>
<tr>
<td>Top</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>8.38</td>
<td>8.19</td>
<td>8.29</td>
<td>8.21</td>
<td>8.29</td>
<td>8.47</td>
<td>8.32</td>
<td>8.41</td>
</tr>
<tr>
<td>Third-Level Column</td>
<td>Deflection Coefficient***</td>
<td>5.76</td>
<td>4.05</td>
<td>4.96</td>
<td>4.29</td>
<td>6.89</td>
<td>8.62</td>
<td>5.28</td>
<td>6.09</td>
</tr>
<tr>
<td>Bottom</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>3.07</td>
<td>3.36</td>
<td>3.37</td>
<td>3.26</td>
<td>4.52</td>
<td>3.38</td>
<td>4.35</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>1.05</td>
<td>1.22</td>
<td>1.12</td>
<td>1.16</td>
<td>2.71</td>
<td>1.81</td>
<td>1.75</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
<td>0.32</td>
<td>0.32</td>
<td>0.30</td>
<td>0.49</td>
<td>0.32</td>
<td>0.46</td>
<td>0.23</td>
</tr>
<tr>
<td>Top</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>9.50</td>
<td>9.19</td>
<td>9.31</td>
<td>9.22</td>
<td>10.31</td>
<td>9.44</td>
<td>9.88</td>
<td>9.69</td>
</tr>
<tr>
<td>Second-Level Column</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>6.67</td>
<td>6.05</td>
<td>5.78</td>
<td>6.08</td>
<td>6.22</td>
<td>5.93</td>
<td>7.68</td>
<td>6.82</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>2.98</td>
<td>2.30</td>
<td>2.57</td>
<td>2.37</td>
<td>3.79</td>
<td>2.37</td>
<td>3.83</td>
<td>3.39</td>
</tr>
<tr>
<td>Bottom</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>6.79</td>
<td>6.09</td>
<td>6.39</td>
<td>6.11</td>
<td>7.15</td>
<td>6.38</td>
<td>7.70</td>
<td>7.23</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>0.71</td>
<td>0.62</td>
<td>0.66</td>
<td>0.62</td>
<td>0.75</td>
<td>0.66</td>
<td>0.83</td>
<td>0.77</td>
</tr>
<tr>
<td>Top</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>12.13</td>
<td>11.22</td>
<td>11.71</td>
<td>11.40</td>
<td>12.18</td>
<td>12.18</td>
<td>11.98</td>
<td>11.97</td>
</tr>
<tr>
<td>First-Level Column</td>
<td>Maximum Moment, kip-in. ( D_{\text{max}} ) ( \times 10^{-4} ) in./in.</td>
<td>29.34</td>
<td>21.92</td>
<td>25.97</td>
<td>23.46</td>
<td>33.75</td>
<td>33.75</td>
<td>28.16</td>
<td>28.08</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient***</td>
<td>7.30</td>
<td>5.46</td>
<td>6.48</td>
<td>5.85</td>
<td>5.76</td>
<td>5.76</td>
<td>7.02</td>
<td>7.00</td>
</tr>
</tbody>
</table>

* Calculated maximum free-end displacement of a unit length cantilever

** Calculated maximum deflection divided by the yield deflection of a cantilever.

*** Constants to define the damping matrix \( [C] = C_1 [M] + C_2 [K] \).

**** The idealized frame with rigid zones in joint cores and rotational springs (simplified Takeda's hysteresis) at ends of the frame members.

**** Inelastic deformations are lumped in rotations of the inelastic springs at ends of the frame members.
## Table 7.3 Test D1. Measured and Calculated (Nonlinear Analysis) Maximum Response

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Level</td>
<td>Second Level</td>
<td>Third Level</td>
<td>First Level</td>
<td>Second Level</td>
</tr>
<tr>
<td>D1-1</td>
<td>Measured</td>
<td>0.37</td>
<td>0.59</td>
<td>0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calculated (B**=0.0)</td>
<td>0.41</td>
<td>0.57</td>
<td>0.61</td>
<td>0.074</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>0.33</td>
<td>0.45</td>
<td>0.53</td>
<td>0.051</td>
<td>0.12</td>
</tr>
<tr>
<td>D1-2</td>
<td>Measured</td>
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<td>0.88</td>
<td>1.10</td>
<td>0.17</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>0.58</td>
<td>0.79</td>
<td>1.01</td>
<td>0.104</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>0.49</td>
<td>0.68</td>
<td>0.90</td>
<td>0.100</td>
<td>0.23</td>
</tr>
<tr>
<td>D1-3</td>
<td>Measured</td>
<td>0.88</td>
<td>1.10</td>
<td>1.40</td>
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<td></td>
<td>Calculated (B=0.0)</td>
<td>1.19</td>
<td>1.03</td>
<td>1.31</td>
<td>0.22</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>0.73</td>
<td>0.81</td>
<td>1.09</td>
<td>0.22</td>
<td>0.43</td>
</tr>
<tr>
<td>D1-4</td>
<td>Measured</td>
<td>0.88</td>
<td>1.20</td>
<td>1.50</td>
<td>0.44</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.27</td>
<td>1.28</td>
<td>1.46</td>
<td>0.42</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>0.97</td>
<td>1.06</td>
<td>1.32</td>
<td>0.40</td>
<td>0.81</td>
</tr>
<tr>
<td>D1-5</td>
<td>Measured</td>
<td>1.10</td>
<td>1.40</td>
<td>1.45</td>
<td>0.61</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.20</td>
<td>1.33</td>
<td>1.71</td>
<td>0.60</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>1.03</td>
<td>1.20</td>
<td>1.65</td>
<td>0.56</td>
<td>1.17</td>
</tr>
<tr>
<td>D1-6</td>
<td>Measured</td>
<td>1.69</td>
<td>1.50</td>
<td>2.60</td>
<td>0.84</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>2.12</td>
<td>1.78</td>
<td>1.93</td>
<td>0.74</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>2.20</td>
<td>1.55</td>
<td>1.94</td>
<td>0.72</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

** One-half of total displacement range.

** Damping factor for the first mode at the initial elastic stage.
Table 7.4 Test D2. Measured and Calculated (Nonlinear Analysis) Maximum Response

<table>
<thead>
<tr>
<th>Test Run</th>
<th>Response</th>
<th>Maximum Acceleration, g</th>
<th>Maximum Displacement, # in.</th>
<th>Max. Base Shear, kip</th>
<th>Max. Base Moment, kip-in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Level</td>
<td>Second Level</td>
<td>Third Level</td>
<td>First Level</td>
</tr>
<tr>
<td>D2-1</td>
<td>Measured</td>
<td>1.29</td>
<td>1.15</td>
<td>1.61</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.03</td>
<td>1.19</td>
<td>1.43</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>0.92</td>
<td>1.07</td>
<td>1.34</td>
<td>0.39</td>
</tr>
<tr>
<td>D2-2</td>
<td>Measured</td>
<td>1.27</td>
<td>1.32</td>
<td>1.50</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.48</td>
<td>1.36</td>
<td>1.87</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>1.16</td>
<td>1.14</td>
<td>1.58</td>
<td>0.51</td>
</tr>
<tr>
<td>D2-3</td>
<td>Measured</td>
<td>1.49</td>
<td>1.24</td>
<td>1.60</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.22</td>
<td>1.44</td>
<td>1.79</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>1.08</td>
<td>1.44</td>
<td>1.73</td>
<td>0.50</td>
</tr>
<tr>
<td>D2-4</td>
<td>Measured</td>
<td>2.02</td>
<td>1.74</td>
<td>2.04</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.97</td>
<td>1.77</td>
<td>2.00</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.02)</td>
<td>2.03</td>
<td>1.59</td>
<td>1.79</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

* One-half of total displacement range.

** Damping factor for the first mode at the initial elastic stage.
Table 7.5 Test D3. Measured and Calculated (Nonlinear Analysis) Maximum Response

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Level</td>
<td>Second Level</td>
<td>Third Level</td>
<td>First Level</td>
</tr>
<tr>
<td>D3-1</td>
<td>Measured</td>
<td>1.16</td>
<td>1.17</td>
<td>1.53</td>
<td>-</td>
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<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>0.98</td>
<td>1.00</td>
<td>1.34</td>
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<td>0.24</td>
</tr>
<tr>
<td>D3-2</td>
<td>Measured</td>
<td>1.13</td>
<td>1.32</td>
<td>1.68</td>
<td>0.67</td>
</tr>
<tr>
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<td>Calculated (B=0.0)</td>
<td>1.19</td>
<td>1.37</td>
<td>1.64</td>
<td>0.50</td>
</tr>
<tr>
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<td>Calculated (B=0.02)</td>
<td>0.87</td>
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<td>1.59</td>
<td>0.47</td>
</tr>
<tr>
<td>D3-3</td>
<td>Measured</td>
<td>1.50</td>
<td>1.04</td>
<td>1.54</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>Calculated (B=0.0)</td>
<td>1.17</td>
<td>1.40</td>
<td>1.65</td>
<td>0.46</td>
</tr>
<tr>
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<td>Calculated (B=0.02)</td>
<td>0.91</td>
<td>1.29</td>
<td>1.58</td>
<td>0.47</td>
</tr>
<tr>
<td>D3-4</td>
<td>Measured</td>
<td>1.47</td>
<td>1.44</td>
<td>1.82</td>
<td>0.98</td>
</tr>
<tr>
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<td>Calculated (B=0.0)</td>
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<td>1.62</td>
<td>1.77</td>
<td>0.60</td>
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<tr>
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<td>Calculated (B=0.02)</td>
<td>1.66</td>
<td>1.53</td>
<td>1.73</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

* One-half of total displacement range.

** Damping factor for the first mode at the initial elastic stage.
## Table 7.6 Test D1. Calculated Maximum Member Response (No Damping)

<table>
<thead>
<tr>
<th>Test Run</th>
<th>Response</th>
<th>Beam</th>
<th>First Level</th>
<th>Second Level</th>
<th>Third Level</th>
<th>First-Story Column</th>
<th>Second-Story Column</th>
<th>Third-Story Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First Level</td>
<td>Second Level</td>
<td>Third Level</td>
<td>Top Bottom Top Bottom</td>
<td>Top Bottom</td>
<td>Top Bottom</td>
</tr>
<tr>
<td>D1-1</td>
<td>Maximum Moment, kip-in.</td>
<td>7.99</td>
<td>6.08</td>
<td>3.35</td>
<td>3.55</td>
<td>7.11</td>
<td>4.55</td>
<td>3.38</td>
</tr>
<tr>
<td></td>
<td>$D_{\text{max}}$, $10^{-4}$ in./in.</td>
<td>2.30</td>
<td>1.63</td>
<td>0.67</td>
<td>1.13</td>
<td>3.01</td>
<td>1.76</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient**</td>
<td>0.80</td>
<td>0.56</td>
<td>0.23</td>
<td>0.28</td>
<td>0.75</td>
<td>0.45</td>
<td>0.29</td>
</tr>
<tr>
<td>D1-2</td>
<td>Maximum Moment, kip-in.</td>
<td>9.74</td>
<td>9.11</td>
<td>5.05</td>
<td>4.15</td>
<td>9.01</td>
<td>5.89</td>
<td>5.37</td>
</tr>
<tr>
<td></td>
<td>$D_{\text{max}}$, $10^{-4}$ in./in.</td>
<td>3.34</td>
<td>2.70</td>
<td>1.27</td>
<td>1.45</td>
<td>4.03</td>
<td>2.47</td>
<td>2.19</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient</td>
<td>1.16</td>
<td>0.93</td>
<td>0.44</td>
<td>0.36</td>
<td>1.01</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>D1-3</td>
<td>Maximum Moment, kip-in.</td>
<td>10.34</td>
<td>9.90</td>
<td>6.74</td>
<td>5.71</td>
<td>9.84</td>
<td>8.17</td>
<td>5.40</td>
</tr>
<tr>
<td></td>
<td>$D_{\text{max}}$, $10^{-4}$ in./in.</td>
<td>7.29</td>
<td>4.40</td>
<td>1.86</td>
<td>2.27</td>
<td>10.74</td>
<td>3.69</td>
<td>2.21</td>
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<tr>
<td></td>
<td>Deflection Coefficient</td>
<td>2.52</td>
<td>1.52</td>
<td>0.64</td>
<td>0.57</td>
<td>2.68</td>
<td>0.94</td>
<td>0.56</td>
</tr>
<tr>
<td>D1-4</td>
<td>Maximum Moment, kip-in.</td>
<td>11.55</td>
<td>10.55</td>
<td>8.70</td>
<td>6.90</td>
<td>12.51</td>
<td>9.30</td>
<td>6.71</td>
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<tr>
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<td>$D_{\text{max}}$, $10^{-4}$ in./in.</td>
<td>15.31</td>
<td>8.68</td>
<td>2.55</td>
<td>2.90</td>
<td>32.41</td>
<td>10.00</td>
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<td>Deflection Coefficient</td>
<td>5.30</td>
<td>3.00</td>
<td>0.88</td>
<td>0.72</td>
<td>8.09</td>
<td>2.55</td>
<td>0.74</td>
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<tr>
<td>D1-5</td>
<td>Maximum Moment, kip-in.</td>
<td>13.05</td>
<td>11.59</td>
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<td>9.89</td>
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<td>25.18</td>
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<td>Deflection Coefficient</td>
<td>8.70</td>
<td>5.39</td>
<td>1.11</td>
<td>0.70</td>
<td>8.42</td>
<td>3.86</td>
<td>0.35</td>
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<tr>
<td>D1-6</td>
<td>Maximum Moment, kip-in.</td>
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<td>13.54</td>
<td>10.91</td>
<td>8.37</td>
<td>14.46</td>
<td>11.40</td>
<td>10.07</td>
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<tr>
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<td>$D_{\text{max}}$, $10^{-4}$ in./in.</td>
<td>31.86</td>
<td>28.44</td>
<td>11.09</td>
<td>3.67</td>
<td>31.69</td>
<td>27.92</td>
<td>16.54</td>
</tr>
<tr>
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<td>Deflection Coefficient</td>
<td>11.03</td>
<td>10.20</td>
<td>3.84</td>
<td>0.92</td>
<td>7.90</td>
<td>7.12</td>
<td>4.23</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

* Calculated maximum free-end displacement of a unit length cantilever.

** Calculated maximum deflection divided by the yield deflection of a cantilever.
<table>
<thead>
<tr>
<th>Test Run</th>
<th>Response</th>
<th>Beam</th>
<th></th>
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<th></th>
<th>First-Story Column</th>
<th>Second-Story Column</th>
<th>Third-Story Column</th>
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<tbody>
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<td></td>
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<td>Bottom</td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td>D2-1</td>
<td>Maximum Moment, kip-in.</td>
<td>11.73</td>
<td>10.52</td>
<td>8.70</td>
<td>6.79</td>
<td>12.13</td>
<td>9.50</td>
<td>6.67</td>
</tr>
<tr>
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<td>$D_{\text{max}}^{\text{a}}$, $\times 10^{-4}$ in./in.</td>
<td>16.50</td>
<td>6.49</td>
<td>2.55</td>
<td>2.84</td>
<td>29.34</td>
<td>11.68</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient$^{\text{a}}$</td>
<td>5.71</td>
<td>2.94</td>
<td>0.88</td>
<td>0.71</td>
<td>7.31</td>
<td>2.98</td>
<td>0.74</td>
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<tr>
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<td>$D_{\text{max}}^{\text{a}}$, $\times 10^{-4}$ in./in.</td>
<td>24.75</td>
<td>15.52</td>
<td>2.82</td>
<td>3.06</td>
<td>30.86</td>
<td>13.64</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient</td>
<td>8.56</td>
<td>5.38</td>
<td>0.98</td>
<td>0.76</td>
<td>7.65</td>
<td>3.48</td>
<td>0.78</td>
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<td>D2-3</td>
<td>Maximum Moment, kip-in.</td>
<td>12.71</td>
<td>10.91</td>
<td>9.10</td>
<td>6.90</td>
<td>12.44</td>
<td>10.06</td>
<td>8.61</td>
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<td>22.95</td>
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<td>2.69</td>
<td>2.90</td>
<td>31.91</td>
<td>16.43</td>
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<td>Deflection Coefficient</td>
<td>7.95</td>
<td>3.84</td>
<td>0.93</td>
<td>0.72</td>
<td>7.95</td>
<td>4.20</td>
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<tr>
<td>D2-4</td>
<td>Maximum Moment, kip-in.</td>
<td>13.40</td>
<td>12.08</td>
<td>9.90</td>
<td>7.75</td>
<td>13.12</td>
<td>11.01</td>
<td>8.67</td>
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<td>$D_{\text{max}}^{\text{a}}$, $\times 10^{-4}$ in./in.</td>
<td>27.49</td>
<td>18.80</td>
<td>4.38</td>
<td>3.35</td>
<td>37.40</td>
<td>24.55</td>
<td>4.65</td>
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<tr>
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<td>Deflection Coefficient</td>
<td>9.51</td>
<td>6.50</td>
<td>1.52</td>
<td>0.84</td>
<td>9.32</td>
<td>6.26</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

$^{\text{a}}$ Calculated maximum free-end displacement of a unit length cantilever.

$^{\text{a}}$ Calculated maximum deflection divided by the yield deflection of a cantilever.
Table 7.8 Test D3. Calculated Maximum Member Response (No Damping)

<table>
<thead>
<tr>
<th>Test Run</th>
<th>Response</th>
<th>Beam</th>
<th>First Level</th>
<th>Second Level</th>
<th>Third Level</th>
<th>First-Story Column</th>
<th>Second-Story Column</th>
<th>Third-Story Column</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3-1</td>
<td>Maximum Moment, kip-in.</td>
<td>10.24</td>
<td>9.65</td>
<td>8.16</td>
<td>5.24</td>
<td>10.16</td>
<td>8.48</td>
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<tr>
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<td>$D_{\text{max}}$, $x 10^{-4}$ in./in.</td>
<td>12.06</td>
<td>8.30</td>
<td>2.92</td>
<td>2.32</td>
<td>17.04</td>
<td>6.86</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>Deflection Coefficient**</td>
<td>3.76</td>
<td>2.58</td>
<td>0.91</td>
<td>0.54</td>
<td>4.00</td>
<td>1.66</td>
<td>0.68</td>
</tr>
<tr>
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<td>Maximum Moment, kip-in.</td>
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<td>10.70</td>
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<td>5.52</td>
<td>11.85</td>
<td>9.15</td>
<td>8.17</td>
</tr>
<tr>
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<td>$D_{\text{max}}$, $x 10^{-4}$ in./in.</td>
<td>22.63</td>
<td>15.01</td>
<td>4.73</td>
<td>2.49</td>
<td>30.53</td>
<td>12.46</td>
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<tr>
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<td>Deflection Coefficient**</td>
<td>7.06</td>
<td>4.68</td>
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<td>0.58</td>
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<td>3.00</td>
<td>1.02</td>
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<td>Maximum Moment, kip-in.</td>
<td>11.74</td>
<td>10.33</td>
<td>8.95</td>
<td>5.10</td>
<td>11.62</td>
<td>9.90</td>
<td>7.27</td>
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<td>Deflection Coefficient**</td>
<td>6.75</td>
<td>3.94</td>
<td>1.19</td>
<td>0.53</td>
<td>6.74</td>
<td>4.51</td>
<td>0.87</td>
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<td>$D_{\text{max}}$, $x 10^{-4}$ in./in.</td>
<td>24.87</td>
<td>19.13</td>
<td>8.17</td>
<td>2.93</td>
<td>35.55</td>
<td>18.18</td>
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<td>Deflection Coefficient**</td>
<td>7.75</td>
<td>5.96</td>
<td>2.54</td>
<td>0.69</td>
<td>8.34</td>
<td>4.38</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Note: The idealized frame with rigid zones in joint cores and rotational springs to represent bond slip of the reinforcement at ends of the frame members.

* Calculated maximum free-end displacement of a unit length cantilever.

** Calculated maximum deflection divided by the yield deflection of a cantilever.
Fig. 2.1 Tests in Progress
Fig. 3.1 Comparison of Assumed and Measured Stress-Strain Relationships for Concrete

\[ f_c = f'_c \left[ 2 \frac{\varepsilon_c}{\varepsilon_o} - \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right] \quad \varepsilon_c \leq \varepsilon_o \]

\[ f_c = f'_c \left[ 1 - 100 \left( \varepsilon_c - \varepsilon_o \right) \right] \quad \varepsilon_o \leq \varepsilon_c \]

\[ f'_c = 5150 \text{ psi} \]
Fig. 3.2 Comparison of Modified Strain $\varepsilon_o$ with Compressive Strength of the Concrete

$\varepsilon_o$: strain at the maximum stress of the concrete
Fig. 3.3 Idealized Stress-Strain Relationship of the No. 2 Deformed Bar

No. 2 Deformed Bars

- $f_y = 42.5$ ksi
- $f_{su} = 66.5$ ksi
- $\varepsilon_{sh} = 0.016$
- $\varepsilon_{su} = 0.065$
Fig. 3.4 Distribution of Stress and Strain over a Cross Section
Fig. 3.5 Axial Load-Bending Moment and Axial Load-Curvature Interaction Diagrams of the Columns
Frame D11

$N = 1.47 \text{ kip}$

- $Z = 50.0$
- $Z = 100.0$

$\varepsilon_c = 0.002$

$\varepsilon_c = 0.004$

$\varepsilon_c = 0.006$

$\varepsilon_c = 0.008$

- Yielding
- Cracking

$Z$: Constant to define the descending slope of concrete stress-strain curve

$\text{Curvature, } x 10^{-2} \text{ 1/in.}$

Fig. 3.6 Effect of the Descending Slope of Concrete Stress-Strain Curve on a Moment-Curvature Relationship
Fig. 3.7 Effect of the Properties of Concrete on a Moment-Curvature Relationship
Fig. 3.8 Effect of the Properties of Reinforcement on a Moment-Curvature Relationship

- $f_y$ (psi) $f_{su}$ (psi) $\varepsilon_{sh}$ $\varepsilon_{su}$ $f'_c$ (psi) $\varepsilon_o$

<table>
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<tr>
<th></th>
<th>44,300</th>
<th>67,000</th>
<th>0.0125</th>
<th>0.060</th>
<th>5340</th>
<th>0.00270</th>
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<td>0.016</td>
<td>0.065</td>
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<tr>
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<td>65,500</td>
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<td>0.0575</td>
<td>5140</td>
<td>0.00296</td>
</tr>
</tbody>
</table>

- $E_s = 29,000,000$ psi
- $Z = 100.0$
- $N = 1.47$ kip
Fig. 3.9 Calculated Moment-Curvature Relationships
Fig. 3.10 Comparison of Idealized and Calculated Moment-Curvature Relationships
Fig. 3.11 Fixed End Moment-Free End Rotation Relationship of a Cantilever
Fig. 3.12 Fixed End Moment-Free End Displacement Relationship of a Cantilever
Fig. 3.13 Rotation Mechanism Due to Slip of Tensile Reinforcement

Fig. 3.14 Moment Rotation Relationship at a Member End Due to Slip of Tensile Reinforcement
Fig. 4.1 A Frame Structure and its Coordinate System

Fig. 4.2 A Typical Member in a Structure
Fig. 4.3 Deformation of Elements of a Member

(a) Rotational Spring

(b) Elastic Simply Supported Beam

(c) Inelastic Simply Supported Beam
Fig. 4.4 Moment Distribution and Deformation of a Beam Member
Fig. 4.5 Deformation of a Column Member

(a) Initial Configuration

(b) Deformed Configuration
Fig. 5.1 Comparison of the Maximum Base Acceleration with the "Span" Dial Setting
Fig. 5.2 Comparison of Spectrum Intensity with "Span" Dial Setting
Fig. 5.3 Comparison of Spectrum Intensities of Different Damping Factors

- ○ $S_{1,0}$ (El Centro)
- △ $S_{1,0}$ (Taft)
- ● $S_{1,5}$ (El Centro)
- ▲ $S_{1,5}$ (Taft)

$\beta = $ Damping Factor
Fig. 5.4 Comparison of Maximum Base Acceleration with Spectrum Intensity
Fig. 5.5 Change in First-Mode Frequency with Total Displacement Range
Fig. 5.6 Change in Second-Mode Frequency with Total Displacement Range

One-Half of the Total Third-Level Displacement Range, in.
Fig. 5.7 Change in Third-Mode Frequency with Total Displacement Range
Fig. 5.8 Comparison of Measured Maximum Base Shear with Spectrum Intensity

Fig. 5.9 Comparison of Measured Maximum Base Moment with Spectrum Intensity
Fig. 5.10 Effective Height for Base Overturning Moment
Fig. 5.11 Comparison of Measured Maximum Acceleration at each Level with Spectrum Intensity
Fig. 5.12 Comparison of Total Displacement Range with Spectrum Intensity
Fig. 5.13 Comparison of Measured Maximum Base Shear with Total Displacement Range at the First Level
Fig. 5.14  Comparison of Measured Maximum Base Shear and Moment with Those Calculated from Measure Maximum Acceleration at each Level.
Fig. 5.15 Measured Maximum Acceleration at each Level in Test D1
Fig. 5.16 Measured Maximum Acceleration in Test D1
Fig. 5.17 Comparison of Total Displacement Range with Spectrum Intensity in Test D1

- ○ Third Beam Level
- △ Second Beam Level
- □ First Beam Level

Test D1
Runs 1 Thru 6
Fig. 5.18 Measured Maximum Base Shears and Those Calculated from Maximum Accelerations (Test D1)

Fig. 5.19 Measured Maximum Base Moments and Those Calculated from Maximum Accelerations (Test D1)
Fig. 5.20 Observed Response, Test Run DI-1
(d) First-Level Acceleration, g

(e) Second-Level Acceleration, g

(f) Third-Level Acceleration, g

Fig. 5.20 (Contd) Observed Response, Test Run D1-1
Fig. 5.20 (Contd) Observed Response, Test Run D1-1
Fig. 5.21 Observed Response, Test Run D1-2
Fig. 5.21 (Contd) Observed Response, Test Run DI-2
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 5.21 (Contd) Observed Response, Test Run D1-2
Fig. 5.22 Observed Response, Test Run D1-3
Fig. 5.22 (Contd) Observed Response, Test Run D1-3
(g) First Level Displacement, in

(h) Second Level Displacement, in

(i) Third Level Displacement, in

Fig. 5.22 (Contd) Observed Response, Test Run DI-3
Fig. 5.23 Observed Response, Test Run D1-4
Fig. 5.23 (Contd) Observed Response, Test Run D1-4
Fig. 5.23 (Contd) Observed Response, Test Run D1-4
Fig. 5.24 Observed Response, Test Run DI-5
Fig. 5.24 (Contd) Observed Response, Test Run D1-5
Fig. 5.24 (Contd) Observed Response, Test Run D1-5
Fig. 5.25 Observed Response, Test Run D1-6
Fig. 5.25 (Contd) Observed Response, Test Run DI-6
Fig. 5.25 (Contd) Observed Response, Test Run D1-6
Fig. 5.26 Crack Patterns Observed in Test D1
(b) After Test Run D1-2

Fig. 5.26 (Contd) Crack Patterns Observed in Test D1
Fig. 5.26 (Contd) Crack Patterns Observed in Test D1

(c) After Test Run D1-3
(d) After Test Run D1-4

Fig. 5.26 (Contd) Crack Patterns Observed in Test D1
(e) After Test Run D1-5

Fig. 5.26 (Contd) Crack Patterns Observed in Test D1
(f) After Test Run D1-6

Fig. 5.26 (Contd) Crack Patterns Observed in Test D1
Fig. 5.27 Test D1. Linear Response Spectra ($\beta=0.0, 0.05, 0.20$)
Fig. 5.27 (Contd) Test D1. Linear Response Spectra ($\beta=0.0, 0.05, 0.20$)
Fig. 5.27 (Contd) Test D1. Linear Response Spectra ($\beta=0.0, 0.05, 0.20$)
Fig. 5.28 Measured Maximum Acceleration at each Level in Test D2
Fig. 5.29 Measured Maximum Acceleration in Test D2

Fig. 5.30 Comparison of Total Displacement Range with Spectrum Intensity in Test D2
Fig. 5.31 Measured Maximum Base Shears and Those Calculated from Maximum Accelerations (Test D2)

Fig. 5.32 Measured Maximum Base Moments and Those Calculated from Maximum Accelerations (Test D2)
Fig. 5.33 Observed Response, Test Run D2-1
Fig. 5.33 (Contd) Observed Response, Test Run D2-1
Fig. 5.33 (Contd) Observed Response, Test Run D2-1
Fig. 5.34  Observed Response, Test Run D2-2
Fig. 5.34 (Contd) Observed Response, Test Run D2-2
Fig. 5.34 (Contd) Observed Response, Test Run D2-2
Fig. 5.35 Observed Response, Test Run D2-3
Fig. 5.35 (Contd) Observed Response, Test Run D2-3
Fig. 5.35 (Contd) Observed Response, Test Run D2-3
Fig. 5.36 Observed Response, Test Run D2-4
Fig. 5.36 (Contd) Observed Response, Test Run D2-4
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 5.36 (Contd) Observed Response, Test Run D2-4
Fig. 5.37 Crack Patterns Observed in Test D2
(b) After Test Run D2-1

Fig. 5.37 (Contd) Crack Patterns Observed in Test D2
Fig. 5.37 (Contd) Crack Patterns Observed in Test D2

(c) After Test Run D2-2
(d) After Test Run D2-3

Fig. 5.37 (Contd) Crack Patterns Observed in Test D2
(e) After Test Run D2-4

Fig. 5.37 (Contd) Crack Patterns Observed in Test D2
Fig. 5.38 Test D2. Linear Response Spectra
(B=0.0, 0.05, 0.20)
Fig. 5.38 (Contd) Test D2. Linear Response Spectra ($\phi=0.0, 0.05, 0.20$)
Fig. 5.39 Measured Maximum Acceleration at each Level in Test D3
Fig. 5.40 Measured Maximum Acceleration in Test D3

Fig. 5.41 Comparison of Total Displacement Range with Spectrum Intensity in Test D3
Fig. 5.42 Measured Maximum Base Shears and Those Calculated from Maximum Accelerations (Test D3)

Fig. 5.43 Measured Maximum Base Moments and Those Calculated from Maximum Accelerations (Test D3)
Fig. 5.44 Observed Response, Test Run D3-1
Fig. 5.44 (Contd) Observed Response, Test Run D3-1
Fig. 5.44 (Contd) Observed Response, Test Run D3-1
Fig. 5.45 Observed Response, Test Run D3-2
Fig. 5.45 (Contd) Observed Response, Test Run D3-2
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 5.45 (Contd) Observed Response, Test Run D3-2
Fig. 5.46 Observed Response, Test Run D3-3
Fig. 5.46 (Contd) Observed Response, Test Run D3-3
Fig. 5.46 (Contd) Observed Response, Test Run D3-3
Fig. 5.47 Observed Response, Test Run D3-4
Fig. 5.47 (Contd) Observed Response, Test Run D3-4
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 5.47 (Contd) Observed Response, Test Run D3-4
(a) Before the Test

Fig. 5.48 Crack Patterns Observed in Test D3
Fig. 5.48 (Contd) Crack Patterns Observed in Test D3

(b) After Test Run D3-1
(c) After Test Run D3-2

Fig. 5.48 (Contd) Crack Patterns Observed in Test D3
(d) After Test Run D3-3

Fig. 5.48 (Contd) Crack Patterns Observed in Test D3
(e) After Test Run D3-4

Fig. 5.48 (Contd) Crack Patterns Observed in Test D3
(a) D3-1 (Based on El Centro 1940, NS)

(b) D3-2 (Based on El Centro 1940, NS)

Fig. 5.49 Test D3. Linear Response Spectra (β=0.0, 0.05, 0.20)
Fig. 5.49 (Contd) Test D3. Linear Response Spectra (R=0.0,0.05,0.20)
Mode shape vectors are already multiplied by the modal participation factors.

Fig. 6.1 Comparison of Mode Shape
Fig. 6.2 Comparison of Story-Shear and Overturning Moment for Three Modes of a Single Frame Due to 1.0 g Spectral Acceleration
Fig. 6.3 Maximum Accelerations Calculated by Spectral Modal Analysis (RMS) Based on Uncracked Sections
Fig. 6.4 Maximum Displacements Calculated by Spectral Modal Analysis (RMS) Based on Uncracked Sections
Fig. 6.5 Maximum Accelerations Calculated by Linear Response-History Analysis (Uncracked Sections)
Fig. 6.6 Maximum Displacements Calculated by Linear Response-History Analysis (Uncracked Sections)
Fig. 6.7 Maximum Accelerations Calculated by Spectral Modal Analysis (RMS) Based on Fully Cracked Sections
Fig. 6.8 Maximum Displacements Calculated by Spectral Modal Analysis (RMS) Based on Fully Cracked Sections
Fig. 6.9 Maximum Accelerations Calculated by Linear Response-History Analysis (Fully Cracked Sections)
Fig. 6.10  Maximum Displacements Calculated by Linear Response-History Analysis (Fully Cracked Sections)
Fig. 6.11 Comparison of Base Shears at the Formation of Mechanism for a Single Test Frame
Fig. 6.12 Formation of Collapse Mechanism in Frames (D1) with and without Rigid Joints (Uniform Load)
Fig. 6.13 Formation of Collapse Mechanism in Frames (D1) with and without Rigid Joints (Triangular Load)
Fig. 6.14 Formation of Collapse Mechanism in Frames (D3) with and without Rigid Joints (Uniform Load)
Fig. 6.15 Formation of Collapse Mechanism in Frames (D3) with and without Rigid Joints (Triangular Load)
Fig. 6.16 The Effect of Lateral Force Distribution on the Formation of the Collapse Mechanism
Fig. 6.17 Collapse Mechanism Formation in Frames D1 and D3
Specimens D1 and D2 without Rigid Joints

(a) Uniform Lateral-Force Distribution

(b) Triangular Lateral-Force Distribution

Fig. 6.18 Calculated Displacements at Formation of Plastic Hinges in Frame D1
Specimen D3 without rigid zones

(a) Uniform Lateral-Force Distribution

Specimen D3 without rigid zones

(b) Triangular Lateral-Force Distribution

Fig. 6.19 Calculated Displacements at Formation of Plastic Hinges in Frame D3
Fig. 6.20 Test D1. Measured and Calculated (RMS) Maximum Accelerations
Fig. 6.21 Test D2. Measured and Calculated (RMS) Maximum Accelerations
Fig. 6.22 Test D3. Measured and Calculated (RMS) Maximum Accelerations
Fig. 6.23 Test D1. Measured and Calculated (Linear Response-History Analysis) Maximum Accelerations
Fig. 6.24 Test D2. Measured and Calculated (Linear Response-History Analysis) Maximum Accelerations
Fig. 6.25 Test D3. Measured and Calculated (Linear Response-History Analysis) Maximum Accelerations
Fig. 6.26 Test DI. Measured and Calculated (RMS) Maximum Displacements
Fig. 6.27 Test D2. Measured and Calculated (RMS) Maximum Displacements
Fig. 6.28 Test D3. Measured and Calculated (RMS) Maximum Displacements
Fig. 6.29 Test DI. Measured and Calculated (Linear Response-History Analysis) Maximum Displacements
Fig. 6.30 Test D2. Measured and Calculated (Linear Response-History Analysis) Maximum Displacements
Fig. 6.31 Test D3. Measured and Calculated (Linear Response-History Analysis) Maximum Displacements
Fig. 6.32 Comparison of Measured Maximum Base Shears with Those Calculated by Limit Analysis
Fig. 6.33 Measured and Calculated (RMS) Maximum Base Shears
Fig. 6.35 Reduction in Calculated Base Shear vs. "Ductility"
(Calculated Base Shear from Linear Response-History Analysis, Fully Cracked Section)
Fig. 6.34 Measured and Calculated (Linear Response-History Analysis) Maximum Base Shears
Fig. 6.36 Comparison of Measured and Calculated (Limit Analysis) Maximum Base Moments

(a) Tests D1 and D2

(b) Test D3
Fig. 6.37 Measured and Calculated (RMS) Maximum Base Moments
Fig. 6.38 Measured and Calculated (Linear Response-History Analysis) Maximum Base Moments
With rigid joint cores and rotational springs
△ Without rigid joint cores
■ Without rotational springs
Open symbols before yielding in any member

(a) Frame Type for Tests D1 and D2

(b) Frame Type for Test D3

Fig. 7.1 Effect on Static Response of Assumed Structural Characteristics (Equal Loads at All Levels)
Fig. 7.2 Effect on Static Response of Assumed Structural Characteristics (Triangular Load Distribution)
Frame with rigid joint cores and rotational springs
△ Frame with inelastic deformations lumped at joints
Open symbols before yielding in any member

Fig. 7.3 Effect of Modeling Inelastic-Deformation Characteristics of Members (Equal Loads at All Levels)
Frame with rigid joint cores and rotational springs

Frame with inelastic deformations lumped at joints

Open symbols before yielding in any member

Fig. 7.4 Effect of Modeling Inelastic-Deformation Characteristics of Members (Triangular Load Distribution)
Fig. 7.5 Calculated Base Shear vs. First-Level Displacement Curves

(a) Uniform Force Distribution

- Frame Type for Tests D1 and D2
- Frame Type for Test D3

Open symbols before yielding in any member

(b) Triangular Force Distribution

Frames with rigid joint cores and rotational springs
Fig. 7.6 Comparison of Calculated (Static Analysis) and Measured Base Shear

Frames with rigid joint cores and rotational springs

(a) Tests D1 and D2

(b) Test D3
Fig. 7.7 Comparison of Calculated (Static Analysis) and Measured Base Moment

(a) Tests D1 and D2

(b) Test D3

Frames with rigid joint cores and rotational springs
Frame with rigid joint cores and rotational springs

- Third Level Displacement
- Second Level Displacement
- First Level Displacement

Open symbols before yielding in any members

Tests D1 and D2

Fig. 7.8 Comparison of Calculated Displacements at the Three Levels (Equal Loads at All Levels)
Frame with rigid joint cores and rotational springs

- Third Level Displacement
- Second Level Displacement
- First Level Displacement

Open symbols before yielding in any members

Tests D1 and D2

Fig. 7.9 Comparison of Calculated Displacements at the Three Levels (Triangular Load Distribution)
Fig. 7.10  Test Run D1-1. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.10 (Contd) Test Run D1-1. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.10 (Contd) Test Run D1-1. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.11 Test Run DI-1. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.11 (Contd) Test Run DI-1. Calculated Response Standard Frame, $\beta = 0.02$
Fig. 7.11 (Contd) Test Run DI-1. Calculated Response
Standard Frame, $\beta = 0.02$
Fig. 7.12 Test Run D1-2. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.12 (Contd) Test Run DL-2. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.12 (Contd) Test Run D1-2. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.13 Test Run D1-2. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.13 (Contd) Test Run D1-2. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.13 (Contd) Test Run D1-2. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.14 Test Run D1-3. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.14 (Contd) Test Run DI-3. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.14 (Contd) Test Run D1-3. Calculated Response
(Standard Frame, \( \beta = 0.0 \))
Fig. 7.15 Test Run D1-3. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.15 (Contd) Test Run D1-3. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.15 (Contd) Test Run D1-3. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.16 Test Run D1-4. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.16 (Contd) Test Run D1-4. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.16 (Contd) Test Run D1-4. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.17 Test Run D1-4. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.17 (Contd) Test Run D1-4. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.17 (Contd) Test Run D1-4. Calculated Response (Standard Frame, $\beta = 0.02$)

(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Time, sec
Fig. 7.18 Test Run 01-5. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.18 (Contd) Test Run D1-5. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.18 (Contd) Test Run D1-5. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.19 Test Run DI-5. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.19 (Contd) Test Run D1-5. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.19 (Contd) Test Run D1-5. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.20 Test Run D1-6. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.20 (Contd) Test Run D1-6. Calculated Response
(Standard Frame, $\beta = 0.0$)
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 7.20 (Contd) Test Run D1-6. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.21 Test Run D1-6. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.21 (Contd) Test Run D1-6. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.21 (Contd) Test Run DI-6. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.22 Test D1. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Accelerations.
Fig. 7.23 Test D1. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Displacements
Test D1

Fig. 7.24 Test D1. Measured and Calculated (Nonlinear Analysis) Maximum Base Shears
Fig. 7.25 Test D1. Measured and Calculated (Nonlinear Analysis) Maximum Base Moments
Fig. 7.26 Test Run D2-1. Calculated Response
(Standard Frame, \( C_1 = C_2 = 0.0 \))
Fig. 7.26 (Contd) Test Run D2-1. Calculated Response
(Standard Frame, $C_1 = C_2 = 0.0$)
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 7.26 (Contd) Test Run D2-1. Calculated Response
(Standard Frame, $C_1 = C_2 = 0.0$)
Fig. 7.27 Test Run D2-1. Calculated Response
(Standard Frame, $C_1 = 0.0$, $C_2 \neq 0.0$)
Fig. 7.27 (Contd) Test Run D2-1. Calculated Response
(Standard Frame, $C_1 = 0.0$, $C_2 \neq 0.0$)
Fig. 7.27 (Contd) Test Run D2-1. Calculated Response
(Standard Frame, $c_1 = 0.0$, $c_2 \neq 0.0$)
(g) First-Level Displacement, in.
(h) Second-Level Displacement, in.
(i) Third-Level Displacement, in.
Fig. 7.28: Test Run D2-1. Calculated Response (Standard Frame, $C_1 \neq 0.0$, $C_2 = 0.0$)
Fig. 7.28 (Contd) Test Run D2-1. Calculated Response (Standard Frame, $C_1 \neq 0.0$, $C_2 = 0.0$)
Fig. 7.28 (Contd) Test Run D2-1. Calculated Response (Standard Frame, $C_1 \neq 0.0, C_2 = 0.0$)
Fig. 7.29 Test Run D2-1. Calculated Response
(Standard Frame, C₁ ≠ 0.0, C₂ ≠ 0.0)
Fig. 7.29 (Contd) Test Run D2-1. Calculated Response (Standard Frame, $C_1 \neq 0.0$, $C_2 \neq 0.0$)
Fig. 7.29 (Contd) Test Run D2-1. Calculated Response (Standard Frame, $C_1 \neq 0.0$, $C_2 \neq 0.0$)
Fig. 7.30 Test Run D2-1. Calculated Response (Standard Frame, $C_1 = C_2 = 0.0$, Inelastic Deformation Lumped at Member Ends, Variable Inflection Point)
Fig. 7.30 Test Run D2-1. Calculated Response (Standard Frame, 
(Contd) $c_1 = c_2 = 0.0$, Inelastic Deformation Lumped at 
Member Ends, Variable Inflection Point)
Fig. 7.30 Test Run D2-1. Calculated Response (Standard Frame, (Contd) \( C_1 = C_2 = 0.0 \), Inelastic Deformation Lumped at Member Ends, Variable Inflection Point)
Fig. 7.31 Test Run D2-1. Calculated Response (Standard Frame, $C_1 = C_2 = 0.0$, Inelastic Deformation Lumped at Member Ends, Fixed Inflection Point)
Fig. 7.31 Test Run D2-1. Calculated Response (Standard Frame, (Contd) $C_1 = C_2 = 0.0$, Inelastic Deformation Lumped at Member Ends, Fixed Inflection Point).
Fig. 7.31  Test Run D2-1. Calculated Response (Standard Frame, (Contd)  \( C_1 = C_2 = 0.0 \), Inelastic Deformation Lumped at Member Ends, Fixed Inflection Point)
Fig. 7.32 Test Run D2-1. Calculated Response (Standard Frame, \(C_1 = C_2 = 0.0\), No Rigid Zone in Joint Core)
Fig. 7.32 Test Run D2-1. Calculated Response (Standard Frame, (Contd) \(C_1 = C_2 = 0.0\), No Rigid Zone in Joint Core)
Fig. 7.32 Test Run D2-1. Calculated Response (Standard Frame, (Contd) \( C_1 = C_2 = 0.0 \), No Rigid Zone in Joint Core)
Fig. 7.33 Test Run D2-1. Calculated Response (Standard Frame, $C_1 = C_2 = 0.0$, No Rotational Springs)
Fig. 7.33  Test Run D2-1.  Calculated Response (Standard Frame, (Contd)  $C_1 = C_2 = 0.0$, No Rotational Springs)
Test Run D2-1. Calculated Response (Standard Frame, (Contd) $C_1 = C_2 = 0.0$, No Rotational Springs)
Fig. 7.34 Test Run D2-1. Calculated Response (Standard Frame, $C_1 = C_2 = 0.0$, Bilinear Rotational Springs)
Fig. 7.34 (Contd) Test Run D2-1. Calculated Response (Standard Frame, $C_1 = C_2 = 0.0$, Bilinear Rotational Springs)
Fig. 7.34 Test Run D2-1. Calculated Response (Standard Frame, (Contd) \(c_1 = c_2 = 0.0\), Bilinear Rotational Springs)
Fig. 7.35 Test Run D2-2. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.35 (Contd) Test Run D2-2. Calculated Response
(Standard Frame, $\beta = 0.0$)

- (d) First-Level Acceleration, g
- (e) Second-Level Acceleration, g
- (f) Third-Level Acceleration, g
Fig. 7.35 (Contd) Test Run D2-2. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.36 Test Run D2-2. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.36 (Contd) Test Run D2-2. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.36 (Contd) Test Run D2-2. Calculated Response
(Standard Frame, β = 0.02)

(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.
Fig. 7.37 Test Run D2-3. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.37 (Contd) Test Run D2-3. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.37 (Contd) Test Run D2-3. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.38 Test Run D2-3. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.38 (Contd) Test Run D2-3. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.38 (Contd) Test Run D2-3. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.39 Test Run D2-4. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.39 (Contd) Test Run D2-4. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.39 (Contd) Test Run D2-4. Calculated Response (Standard Frame, $\beta = 0.0$)
(a) Measured Base Acceleration, g

(b) Base Shear, kip

(c) Base Moment, kip-in.

Fig. 7.40 Test Run D2-4. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.40 (Contd) Test Run D2-4. Calculated Response (Standard Frame, $\beta = 0.02$)
(g) First-Level Displacement, in.

(h) Second-Level Displacement, in.

(i) Third-Level Displacement, in.

Fig. 7.40 (Contd) Test Run D2-4. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.41 Test D2. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Accelerations
Fig. 7.42 Test D2. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Displacements
Test D2

Fig. 7.43 Test D2. Measured and Calculated (Nonlinear Analysis) Maximum Base Shears
Fig. 7.44 Test D2. Measured and Calculated (Nonlinear Analysis) Maximum Base Moments.
(a) Bilinear Hysteresis System

(b) Takeda Hysteresis System

Fig. 7.45 Comparison of Bilinear and Takeda Hysteresis Systems
Fig. 7.46 Test Run D3-1. Calculated Response
(Standard Frame, β = 0.0)
Fig. 7.46 (Contd) Test Run D3-1. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.46 (Contd) Test Run D3-1. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.47 Test Run D3-1. Calculated Response (Standard Frame, \( \beta = 0.02 \))
Fig. 7.47 (Contd) Test Run D3-1. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.47 (Contd) Test Run D3-1. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.48 Test Run D3-2. Calculated Response
(Standard Frame, $\beta = 0.0$)
(d) First-Level Acceleration, $g$

(e) Second-Level Acceleration, $g$

(f) Third-Level Acceleration, $g$

Time, sec

Fig. 7.48 (Contd) Test Run D3-2. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.48 (Contd) Test Run D3-2. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.49 Test Run D3-2. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.49 (Contd) Test Run D3-2. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.49 (Contd) Test Run D3-2. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.50 Test Run D3-3. Calculated Response  
(Standard Frame, $\beta = 0.0$)
(d) First-Level Acceleration, g

(e) Second-Level Acceleration, g

(f) Third-Level Acceleration, g

Fig. 7.50 (Contd) Test Run D3-3. Calculated Response
(Standard Frame, \( \beta = 0.0 \))
Fig. 7.50 (Contd) Test Run D3-3. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.51 Test Run D3-3. Calculated Response
(Standard Frame, $\beta = 0.02$)
(d) First-Level Acceleration, g

(e) Second-Level Acceleration, g

(f) Third-Level Acceleration, g

Fig. 7.51 (Contd) Test Run D3-3. Calculated Response
(Standard Frame, \( \beta = 0.02 \))
Fig. 7.51 (Contd) Test Run D3-3. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.52 Test Run D3-4. Calculated Response
(Standard Frame, \( \beta = 0.0 \))
Fig. 7.52 (Contd) Test Run D3-4. Calculated Response
(Standard Frame, $\beta = 0.0$)
Fig. 7.52 (Contd) Test Run D3-4. Calculated Response (Standard Frame, $\beta = 0.0$)
Fig. 7.53 Test Run D3-4. Calculated Response
(Standard Frame, $\beta = 0.02$)
Fig. 7.53 (Contd) Test Run D3-4. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.53 (Contd) Test Run D3-4. Calculated Response (Standard Frame, $\beta = 0.02$)
Fig. 7.54 Test D3. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Accelerations
Fig. 7.55 Test D3. Measured and Calculated (Nonlinear Response-History Analysis) Maximum Displacements
Fig. 7.56 Test D3. Measured and Calculated (Nonlinear Analysis) Maximum Base Shears
Fig. 7.57 Test D3. Measured and Calculated (Nonlinear Analysis) Maximum Base Moments
APPENDIX A.

DESCRIPTION OF EXPERIMENTAL WORK

A.1 Materials

Properties of the materials used in the tests are described in this section. Test results of control specimens are listed separately for concrete and steel.

(a) Concrete

High-early-strength cement (Type III) was used in casting all the specimens. Fine lake sand and Wabash River sand were used as fine and coarse aggregate. The mix proportion was 1:1:4 (cement : fine aggregate : coarse aggregate) by dry weight. The water-cement ratio was nominally 0.7 by weight. However, this ratio was increased at the time of mixing to insure sufficient workability. All the aggregate, cement and mix proportion have been used previously in small scale frame specimens (Fiorato, 1970 and Gulkan, 1971) in the Structural Research Laboratory of the Civil Engineering Department at the University of Illinois, Urbana.

A frame specimen and control specimens were cast from a single batch. The minimum number of control specimens were

6 of 2 by 2 by 8-in. prisms for the modulus of rupture test,
2 of 4 by 8-in. cylinders for splitting tests,
4 of 4 by 8-in. cylinders for compressive tests,
4 of 2 by 2 by 2-in. cubes for compressive tests,
and 2 of 6 by 12-in. cylinders for compressive tests.

A typical stress-strain curve for the concrete is shown in Fig. A.1. The stress-strain relationship was determined from compression tests of
4 by 8-in. cylinders with a mechanical extensometer of a 5-in. gage length. Due to the limitations of the testing machine, the descending branch was not measured after the maximum stress was reached.

The compressive strength of the concrete was determined from compressive tests of 4 by 8-in. cylinders. The frequency distribution of compressive strength is plotted in Fig. A.2. The average compressive strength from 43 cylinders tests was 5,050 psi with a range from 3,280 to 6,050 psi.

Compressive strength was also measured using compressive tests of 2 by 2 by 2-in. cubes and 6 by 12-in. cylinders. The strength found from these tests are compared with one from 4 by 8-in. cylinders in Fig. A.3. A compressive strength from cubes gave the highest and the one from 4 by 8-in. cylinders gave the lowest value from the same batch of the concrete.

The average secant modulus determined from a stress-strain curve at 40 percent of the compressive strength of a 4 by 8-in. cylinder is compared with the compressive strength in Fig. A.4.

A tensile strength was determined using splitting tests of 4 by 8-in. cylinders. The modulus of rupture was determined using 2 by 2 by 8-in. prisms loaded at the mid-span and simply supported at 3 in. from the loading point. The tensile strength and the modulus of rupture of the concrete are compared with the compressive strength in Fig. A.5. The relation between tensile strength \( f_t \) and compressive strength \( f_c \) was found to be approximately

\[
    f_t = 6.0 \sqrt{f_c} \tag{A.1}
\]
and the relation between the modulus of rupture \( f_r \) and the compressive strength \( f'_c \)

\[
f_r \approx 13.7 \sqrt{f'_c}
\]  

(A.2)

The frequency distribution is plotted for the tensile strength in Fig. A.6. The average from 26 splitting tests was 430 psi with a scatter range from 247 to 577 psi. The modulus of rupture is compared with tensile strength in Fig. A.7. The modulus of rupture was generally found to be more than twice as large as the tensile strength. The test results are summarized in Table A.1.

(b) Reinforcement

Number 2 deformed bars were used as longitudinal reinforcement in columns and beams. Number 3 deformed bars were used as longitudinal reinforcement in the base girders. Number 14 gage plain wires were used exclusively as stirrup reinforcement in columns and beams.

Number 2 deformed bars (Fiorato, 1970) were purchased from the Triangle Steel and Supply Company in California and annealed at 1200°F for two hours by Fred A. Snow and Co. of Chicago. When the No. 2 deformed bars were prepared for use in the specimens, the surface was covered with rust and appeared reddish-brown. No special treatment was carried out to clean the surface.

Surfaces of the No. 3 deformed bars and the No. 14 gage wires were clean.
Stress-strain curves of the No. 2 and 3 deformed bars were obtained during a tensile test using a mechanical extensometer of a 2.0-in. gage length and an engineering scale. The extensometer was used for elongation up to approximately 1.5 percent strain, and then was replaced by the engineering scale until the maximum stress was reached. Typical stress-strain curves are shown in Fig. A.8 and A.9.

Most of stress-strain curves of the No. 2 deformed bars had distinct yield plateaus up to strains of 0.8 to 2.0 percent, and reached the ultimate stresses at 16 to 28 percent strain. A single No. 2 deformed bar was cut into ten pieces of 14 in. long from one end to the center of the bar. Each coupon was subjected to tensile stresses up to failure. Measured yield and ultimate stresses are plotted in Fig. A.10. The plot shows an approaching 10 percent of the average yield stress scatter in a single bar. Due to this fact, an average yield stress of 42,600 psi obtained from sixty-four coupons was used in the analysis rather than individual yield stresses determined from tests of coupons which were taken from the same bars used in the frame specimens. The frequency distribution of yield stresses is shown in Fig. A.11. By the same token, an average ultimate stress of 66,500 psi from fifty-eight coupons was adopted in the analysis. An average strain at the commencement of strain hardening was 1.6 percent from strain readings of sixteen coupons. The ultimate strain ranged from 15 to 27.5 percent for a 2-in. gage length.

Number 3 deformed bars had an average yield stress of 47,500 psi obtained from tests of fifteen coupon specimens with a scatter range of
44,500 to 51,000 psi. An average ultimate stress from the same set of specimens was 71,600 psi with a scatter range of 67,700 to 74,500 psi.

Number 14 gage plain wire has a nominal diameter of 0.0800 in. and a nominal area of 0.00503 in.² Sixteen coupon specimens were taken at random from the same lot as was used in the frame specimens, and tested in tension. The average yield stress was 39,600 psi with a scatter range of 37,400 to 42,100 psi. The average ultimate stress was 54,400 psi with a scatter range of 50,700 to 57,700 psi. Strain at the ultimate stress ranged from 15.0 to 30.5 percent for a 2.0-in. gage length.

A.2 Description of the Specimens

A total of nine test frames were built. Three identically designed frames are treated as a set, in which two frames were subjected to a series of dynamic base motions and the remaining one frame was subjected to static load reversals. The static test results are not reported here.

The first set and the second set contained identical specimens. The third set was different from the other two only in the arrangement of longitudinal reinforcement.

(a) Dimensions

The overall nominal dimensions of the one-bay three-story test frame are given in Fig. A.12. Three beams at the first, second, and third stories had identical section properties. The columns were continuous from the base to the top with the same cross-sectional properties except for the amount of stirrup reinforcement.
The clear story height was 15.0 in. inside-to-inside of the beams. The ends of all three beams protruded 6.0 in. from the center of the columns. The top end of the columns protruded 7.0 in. from the center of the third-story beam.

A column had dimensions of 2.5 by 2.5 in. and was 67.5 in. high measured from the bottom face of the base girder. Beams were 2.5 in. wide, 3.0 in. deep and 48.0 in. long. The base girder had dimension of 2.5 by 8.0 in. and was 60.0 in. long. All the specimens were built within a fabrication error of 0.06 in. Two vertical holes were made at the protruding parts of a beam, 2.0 in. from the end in order to support a steel rack. The hole was reinforced with a steel pipe of 9/16 in. inside diameter.

Five vertical holes were made in the base girders on 12.0-in. centers in order to fasten the frame to the earthquake simulator platform. The holes were reinforced with steel pipes (9/16-in. inside diameter).

(b) Longitudinal Reinforcement

Arrangement of the longitudinal reinforcement is shown in Fig. A.12. Four No. 2 deformed bars were used as longitudinal reinforcement in beams and columns. Eight No. 3 deformed bars were used in the base girder.

Gross reinforcement ratio was 2.67 percent for a beam, 3.20 percent for a column, and 4.4 percent for the base girder.

Arrangement of reinforcement of the first six frames was different from the last three frames. Detailed cross sections of a beam, a column and the base girder are shown in Fig. A.13 and A.14.

(c) Lateral Reinforcement

Number 14 gage plain wires were used exclusively as transverse
reinforcement in the test frames. The arrangement of the transverse reinforcement is shown in Fig. A.15. Stirrup reinforcement was provided at every 6.0 in. in a beam in order to hold longitudinal reinforcement in place during the casting of concrete.

In the base girder (Fig. A.16), six stirrups at every 0.5 in. were provided on both sides of the column longitudinal reinforcement. One stirrup was placed on both sides of each steel pipe sleeve.

Eight hoops were provided at every 3/4 in. from each beam-column connection on the second- and third-story columns. Eight hoops at every 3/4 in. from bottom and top faces of the first-story columns were provided in frames D11 and D12. Twelve hoops at every 1/2 in. were provided in the same location in the rest of the frames.

The core of each beam-column connection was reinforced with seven hoops as shown in Fig. A.15. Three of these enclosed only the column reinforcement. The other four also extended to the cantilevered portion of the beam and enclosed the hooked ends of the beam reinforcement as well as the column reinforcement.

A total of ten hoops were provided along the column longitudinal reinforcement in the column-base girder connection.

(c) Casting and Curing

The reinforcement cage (Fig. A.17), with steel pipes for the holes, was tied into position in a casting form, which consisted of a 3/4-in. plywood board at the bottom and 1/4-in. steel plates at the sides of the frame. The steel pipe sleeves were also securely tied to the steel
sides by 1/2-in. diameter steel bars. The reinforcement cage was held in position by means of 1/4-in. nuts as spacers and thin steel wires. The concrete was placed in the form and vibrated externally not to disturb the reinforcement cage. Control specimens were cast at the same time from the same batch of concrete.

After the concrete surface was struck off and trowelled smooth, frames D11 and D12 were left exposed to the laboratory air for twenty-four hours until the casting form was removed. The rest of the frames were covered with wet burlap and plastic sheet immediately after the concrete surface was struck off and trowelled smooth.

The casting form was struck down twenty-four hours after the concrete was cast. The frame was then covered with wet burlap and plastic sheets and kept moist. The wet burlap and plastic sheets were removed a week after casting. The specimen was then kept in the laboratory until the time of testing.

A.3 Test Procedure

The frame tests were carried out with due cares in order to acquire reliable test results. The procedure is described in this section.

(a) Performance Check of the Earthquake Simulator

The performance of the earthquake simulator is quite sensitive to the "tuning" of its various components. It may not always produce the same fidelity with respect to a given input waveform although all the controls are in their nominally correct positions. Consequently, it is preferable, though not always necessary, to tune the entire system before starting a series of tests.
Before installing the test frames on the platform, the earthquake simulator was run several times to calibrate the actuator travel against maximum platform acceleration for the particular waveform to be used in the tests. The plots of table accelerations versus time in their runs were also examined for fidelity with respect to the input waveform.

Besides checking the performance of the earthquake simulator, these runs provided information for controlling the intensity of the platform motion during the tests of the reinforced concrete frames.

(b) Installation of the Test Frames

After the performance of the earthquake simulator was decided to be acceptable, the two test frames were placed on the test platform (Fig. A.18). Initially, the bolts which tied the frame to the platform were left loose so that an accidental shock during installation of the frames and the steel weights should not damage the frames. Steel racks carrying the steel weights were put in place one level at a time starting from the first level.

All the steel plate weights were fastened to two parallel steel box sections on the floor, and then lifted by the crane to a position approximately a half inch below the specified location. The steel angles were then attached to the box sections. After tightening all the bolts, the steel rack was lifted and secured in the specified location by tightening long bolts hung from the frames. The bolts connecting the frames and a steel rack were again left loose so that an accidental shock should not damage the frames.
Immediately before the test, the bolts were tightened fast taking care not to crack the frames.

(c) **Calibration of Instruments**

Test data were all recorded on magnetic tape recorders in terms of electric voltage. In order to be able to translate the voltage into a physical measurement, the relation between the physical measurement and the voltage should be known. Therefore, mechanical calibration signals were recorded on the tape recorder before the test started. This calibration process was accomplished by subjecting each displacement gage to a known mechanical displacement, or by subjecting each acceleration gage to the earth gravity in the measuring axis. The calibration was made before and after the test to check the influence of temperature change on electronic devices and to confirm the initial results.

(d) **Test Run**

The selected earthquake waveform was fed into the earthquake simulator by a tape recorder. The starting of the input tape recorder may cause an electric spike. If the spike is transmitted to the earthquake simulator and reproduced on the test platform, the shock may be much stronger than that caused by a programmed earthquake motion. In order to avoid this accidental shock, the pressure in the actuator was cut when the input tape recorder was started. When the input and the recording tape recorders reached working speed, the actuator was pressurized first at "low" and then "high" before starting the run.

After free vibration of the frames subsided, the hydraulic pressure was cut from the actuator, and the tape recorders were stopped.
(e) **Examination of Crack Patterns**

After each run, Detection Ink\(^\ast\), which contains small fluorescent particles, was sprayed on the surface of the frames. The small fluorescent particles in the ink penetrated into a concrete crack and reflected "black light" showing the crack patterns clearly. Cracks on the frames were marked with pencil and indentified.

(f) **Adjustment of Instrumentation**

After each run, every channel of the tape recorder was checked to see if it was recording properly. From the rough observation of the recorded signals, the gain settings for the next run were determined. From the viewpoint of data reduction, it is preferable to maintain the same gain settings throughout the test, rather than modifying them between runs.

A.4 **Instrumentation**

Three different types of sensors (Fig. A.19) were used during each run of the tests: accelerometers, displacements transducers and electric strain gages. A typical wiring diagram for instrument output is shown in Fig. A.20.

Eight accelerometers were installed to measure horizontal accelerations parallel to the imposed direction of motion: an accelerometer at the top of the base girder of each frame, and two accelerometers on the rigid steel rack at each story level (Fig. A.19). Six accelerometers were installed with their axes vertical direction, two each steel rack at each story level, in order to measure vertical vibrations.

\(^\ast\) Flaw Detection Ink (CermoreFluorescent), Burmah Oil Trading Ltd., Lobitos Division, Manchester, England.
All of the accelerometers (Fig. A.21) mentioned above measured the absolute acceleration of the point of installation in the direction of the principal axis of the accelerometer. Four Kistler Model 305T/515T Servo Accelerometer/Amplifier Systems, and twelve ENDEVCO A-116-15 Accelerometers and accompanying ENDEVCO Amplifiers were used. In order to avoid high frequency noise in an acceleration signal, which was considered to be of little engineering significance, either a built-in or an additional low-pass filter (DC to 100 Hz) was used with the amplifier. A 1.0-g calibration signal was generated by changing the principal axis of a gage from the horizontal position to the vertical position.

Relative displacements were measured between two points by differential transformers (Fig. A.22). Two A-shaped rigid steel frames were fastened on the earthquake simulator platform to provide a reference point (Fig. A.23). Displacements of the test frames were measured with respect to the reference frames at four levels: top of the base girder and mid-heights of the three beams.

Two DC-type differential transformers (Hewlett-Packard Co.) with ± 0.25 in. travel limit were used with DC amplifiers on the base girder of the two frames.

AC-type differential transformers (Schaevitz Engineering Inc.) with ± 3.0 in. travel limit were used at the three beam levels of each test frame.

Calibration signals were generated by using a 0.25-in. metal block gage for a DC-type differential transformer, and by using a 1.0-in. metal block gage for an AC-type differential transformer. Linear response between distance and the gage output was checked within the operating range.
High elongation strain gages (HE-121-B, Micro-Measurements Co.) were used on the No. 2 deformed bars in the two diagonal corners of the first-story columns (at the face of the beam or the girder) and in the first-story beam (at the face of a column). Strain gages were installed in the Frames D12, D22, and D32. Gages (Fig. A.24) were all coated with waterproofing flexible materials.

A.5 Data Reduction

The dynamic response measurements were recorded by three analog magnetic tape recorders, each with a capability of recording thirteen voltage signals and one audio signal. The audio channel of each tape contained a short description of the test runs. A common signal (the original earthquake acceleration record) was recorded in Channel 1 of all three units as a time reference so that data on the three tapes could be synchronized for analysis of the test results. The input earthquake acceleration waveform was used as a common signal to the three tape recorders.

Each test segment of tape containing the response signals of the specimens was converted to digital data; this included portions before the earthquake motion began and after the response of a specimen subsided. The conversion system was that available in the Department of Civil Engineering, University of Illinois, Urbana (Appendix D). Average values of the portion before the base motion had started (approximately 1800 point per channel) were assumed to represent zero values of the signals. These zero values were assumed to be correct throughout the test run because the duration was less than 20 sec, and the recording system was stable during the test run.

Although low-pass filters were used in the instrumentation, digitized records contained high-frequency noise. In order to avoid the noise
in the analysis and presentation of test results, the following formulas were used to smooth the digitized signals. For all cases,

\[ Y_i = \frac{1}{4} (X_{i-1} + 2X_i + X_{i+1}) \]

and for presentation of recorded signals,

\[ Y_i = \frac{1}{1024} (X_{i-5} + 10X_{i-4} + 45X_{i-3} + 120X_{i-2} + 210X_{i-1} \]

\[ + 252X_i + 210X_{i+1} + 120X_{i+2} + 45i+3 + 10X_{i+4} + X_{i+5}) \]

in which

\[ X = \text{recorded value} \]
\[ Y = \text{smoothed value} \]

Subscripts indicate locations of points relative to the current Step \( i \). The effect of smoothing recorded signals can be observed in Fig. A.25.

The base acceleration waveforms measured in Test Run D2 are compared with the input acceleration waveforms in Fig. A.26. The comparison of the waveforms are seen to be generally favorable.

The first-level acceleration waveforms measured at the two different locations on the steel rack in Test Run D1-1 are compared in Fig. A.27. Those two signals are almost identical. The first-level displacement waveforms measured at the two frames in Test Run D1-1 are compared in Fig. A.28.
Table A.1  Measured Average Response of Concrete Control Specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Water-Cement Ratio</th>
<th>Age, Days</th>
<th>Secant Modulus*, $10^6$ psi</th>
<th>Compressive Strength, psi</th>
<th>Tensile Strength, psi</th>
<th>Splitting Modulus of Rupture</th>
</tr>
</thead>
<tbody>
<tr>
<td>D11</td>
<td>0.80</td>
<td>76</td>
<td>3.12</td>
<td>4,790</td>
<td>5,940</td>
<td>420</td>
</tr>
<tr>
<td>D12</td>
<td>0.77</td>
<td>49</td>
<td>3.16</td>
<td>5,140</td>
<td>5,920</td>
<td>470</td>
</tr>
<tr>
<td>D21</td>
<td>0.73</td>
<td>203</td>
<td>3.32</td>
<td>5,210</td>
<td>6,100</td>
<td>440</td>
</tr>
<tr>
<td>D22</td>
<td>0.73</td>
<td>168</td>
<td>3.30</td>
<td>5,430</td>
<td>6,570</td>
<td>505</td>
</tr>
<tr>
<td>D31</td>
<td>0.83</td>
<td>175</td>
<td>3.00</td>
<td>4,340</td>
<td>4,820</td>
<td>270</td>
</tr>
<tr>
<td>D32</td>
<td>0.75</td>
<td>53</td>
<td>3.33</td>
<td>5,280</td>
<td>7,230</td>
<td>425</td>
</tr>
</tbody>
</table>

* Measured at 40 percent of compressive strength in compression test of 4x8-in. cylinders.
Table A.2  Measured Average Response of Number 2 Deformed Bars

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Stress, psi</th>
<th>Strain Hardening</th>
<th>Ultimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yield</td>
<td>Ultimate</td>
<td></td>
</tr>
<tr>
<td>D11</td>
<td>42,100</td>
<td>67,100</td>
<td>0.011</td>
</tr>
<tr>
<td>D12</td>
<td>41,700</td>
<td>66,800</td>
<td>0.014</td>
</tr>
<tr>
<td>D21</td>
<td>42,300</td>
<td>65,600</td>
<td>0.016</td>
</tr>
<tr>
<td>D22</td>
<td>41,900</td>
<td>68,300</td>
<td>0.018</td>
</tr>
<tr>
<td>D31</td>
<td>42,300</td>
<td>66,100</td>
<td>0.018</td>
</tr>
<tr>
<td>D32</td>
<td>42,900</td>
<td>67,400</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Fig. A.1 Representative Stress-Strain Relationship of the Concrete

\( f' = 5050 \text{ psi} \)
Fig. A.2 Distribution of the Measured Compressive Strength of 4 by 8 in. Cylinders
Fig. A.3 Comparison of Compressive Strength of Concrete from Three Types of Control Specimens
Fig. A.4 Variation of Secant Modulus with Square Root of Compressive Strength of Concrete
Fig. A.5 Variation of Tensile Strength and Modulus of Rupture with Compressive Strength of Concrete

Square Root of Compressive Strength of Concrete in psi

- Modulus of Rupture $f_r$
- Tensile Strength $f_t$

$f_t = 6.0 \sqrt{f_c}$

$f_t = 13.7 \sqrt{f_c}$
Fig. A.6 Distribution of Tensile Strength of the Concrete

Fig. A.7 Comparison of Modulus of Rupture and Tensile Strength of Concrete
Fig. A.8 Representative Stress-Strain Relationship of Number 2 Deformed Bars

Number 2 Deformed Bar

\[
\begin{align*}
E_s &= 29,000,000 \text{ psi} \\
\sigma_y &= 42,600 \text{ psi} \\
\sigma_{su} &= 66,500 \text{ psi}
\end{align*}
\]
Fig. A.9 Representative Stress-Strain Relationship of Number 3 Deformed Bar

Number 3 Deformed Bar

- $E_s = 29,000,000$ psi
- $f_y = 47,500$ psi
- $f_{su} = 71,600$ psi
Fig. A.10  Distribution of Yield and Ultimate Stress of Number 2 Deformed Bar along its Length

Fig. A.11  Distribution of the Measured Yield Stress of Number 2 Deformed Bars
Fig. A.12 Overall Dimensions and Arrangement of Main Reinforcement of a Test Frame
Fig. A.13 Details of Specimens D1 and D2
Fig. A.14 Details of Specimen D3
Fig. A.15 Arrangement of Stirrup Reinforcement in a Test Frame

unit in inch
Fig. A.16 Detail of Reinforcement in the Base Girder

Fig. A.17 A Typical Reinforcement Cage in the Casting Form
Fig. A.18 Test Specimen with Steel Weights
Fig. A.19 Location of Gages during a Simulated Earthquake Test
Acceleration:  
Displacement:  
Strain:  

Channel | Tape Recorder No. 1  
--- | ---  
1. Common Signal*  
2. Hor. Acc. at the base of Frame A*  
3. Hor. Acc. at the base of Frame B  
4. Hor. Acc. at the first beam level* (East)  
5. Hor. Acc. at the first beam level (West)  
6. Hor. Acc. at the second beam level* (East)  
7. Hor. Acc. at the second beam level (West)  
8. Hor. Acc. at the third beam level* (East)  
9. Hor. Acc. at the third beam level (West)  
10. Hor. Displ. at the base of Frame A  
11. Hor. Displ. at the first level of Frame A*  
12. Hor. Displ. at the second level of Frame A*  
13. Hor. Displ. at the third level of Frame A*  

Channel | Tape Recorder No. 2  
--- | ---  
1. Common Signal  
2. Vertical Acc. on the test platform  
3. Vertical Acc. at the first beam level  
4. Vertical Acc. at the second beam level  
5. Vertical Acc. at the third beam level  
6. Transverse Acc. at the third beam level  
7. Transverse Acc. at the third beam level  
8. Transverse Acc. at the first beam level  
9. Transverse Acc. at the first beam level  
10. Hor. Displ. at the base of Frame B  
11. Hor. Displ. at the first level of Frame B  
12. Hor. Displ. at the second level of Frame B  
13. Hor. Displ. at the third level of Frame B  

Channel | Tape Recorder No. 3  
--- | ---  
1. Common Signal  
2. Strain** at the base of the first-story column  
3. Strain at the base of the first-story column  
4. Strain at the base of the first-story column  
5. Strain at the base of the first-story column  
6. Strain at the top of the first-story column  
7. Strain at the top of the first-story column  
8. Strain at the top of the first-story column  
9. Strain at the top of the first-story column  
10. Strain at the end of the first level beam  
11. Strain at the end of the first level beam  
12. Strain at the end of the first level beam  
13. Strain at the end of the first level beam  

* The signals were recorded on an oscillograph.  
** Strains were measured on the longitudinal reinforcement.

Fig. A.20 Typical Wiring Diagram of Gages
Fig. A.21 Kistler and Endevco Accelerometers

Fig. A.22 AC-Type and DC-Type Differential Transformers
Fig. A.23 The A-Frame and Test Specimen

Fig. A.24 Detail of Strain Gage Coating
Fig. A.25 Effect of Smoothing Measured Signals
Fig. A.26 Comparison of Input and Measured Base Acceleration Signals
(d) Original Taft (N21E) 1952 Acceleration Record, g

(e) Measured Base Motion Signal in Test D2-3, g

(f) Measured Base Motion Signal in Test D2-4, g

Fig. A.26 (Contd) Comparison of Input and Measured Base Acceleration Signals
Fig. A.27 Comparison of Output Waveforms of Two Accelerometers
Fig. A.28 Comparison of Output Waveforms of Two Displacement Gages
B.1 Description of the Facility

The University of Illinois Earthquake Simulator (referred to as the earthquake simulator) is currently housed in the Structural Research Laboratory of the Civil Engineering Department in Urbana campus.

Because detailed description of the system hardware have been published elsewhere (Sozen et al, 1969 and Gulkan, 1971), only brief description of the facility is given here. The system consists of (1) a hydraulic ram equipped with a servo-valve, (2) a power supply, (3) a command center, and (4) a test platform.

The test platform is 12 by 12 ft in plan. It is supported by four flexure plates as shown in Fig. B.1. The flexure plates have flexural joints at each end, which can rotate with very small resistance allowing free motion of the platform in one horizontal direction. The test bed plate is drilled and tapped for 1/2-in. bolts on 12-in. centers in two directions.

Free vibration of the platform on its supports has a natural period of 2.5 sec. The resistance provided by the flexure plates was measured to be 90.0 lb/in. with the force applied at the level of the platform. From these two items of information, the effective weight of the platform was evaluated as approximately 5,500 lb.

The test platform is actuated by a 75,000 lb capacity hydraulic ram to a maximum velocity of 20.0 in./sec and to a maximum displacement of ± 2.0 in. under dynamic conditions.
The command center can accept almost any signal in terms of electric voltage as long as the signal does not demand more than the capacity of the hydraulic ram. Input form can be displacement, velocity and acceleration time histories, although the motion of the hydraulic ram is controlled by a displacement command, which can be acquired by electronic integration from a velocity or an acceleration time history. The manufacturer of the system (MTS Co.) recommended the displacement input for the following reasons:

1. The ram is controlled by a displacement sensor attached to the ram.
2. By inputting displacement command, it is possible to avoid problems introduced by the integrating circuit within the system, and
3. The integrating circuit within the system includes an electronic safety device to limit the maximum displacement of the ram. This safety device may distort the integrated waveform especially at the low frequency range.

B.2 Performance Test under Earthquake Motions

Despite the advice from the system manufacturer to use displacement records as input, it was decided to experiment with an acceleration input to see whether it could lead to a better earthquake simulation.

The digitized earthquake time histories were converted into an analog signal using the IBM 1800 directly connected to an IBM 360-75 computer. The procedure is described in Appendix D. A Sangamo 3500 14-channel tape recorder was used in recording the analog signals. Preliminary
trials indicated that less noise signal was generated at higher tape play speeds and that this was independent of the recording speed. Accordingly, the tape play speed was set at 60.0 in./sec and the record speed at 15.0 in./sec. Four sets of record were made at different time scales. Each set of earthquake time-histories were compressed with respect to time by 2.5, 5.0, 7.5 and 10.0.

During the performance tests, a concrete weight of 4,000 lb was securely fastened to the test platform to represent the weight of a test specimen. Two servo-accelerometers were used to measure the platform motion. The servo-accelerometers with accompanying amplifier had flat response from DC to 200 Hz.

Four earthquake acceleration histories digitized by Amin (1966), two components of El Centro 1940 and two components of Taft 1952 were used as input. The input signals were recorded and played at time compression ratio of 2.5, 5.0, 7.5, and 10.0.

The measured acceleration records on the platform were digitized by using the IBM 1800.

(a) Waveform of Simulated Earthquake

The prototype and simulated acceleration histories of the NS component of El Centro (1940) record are shown in Fig. B.2. The vertical scale of simulated earthquake record represents the measured acceleration divided by earth gravity.

No record is shown for the displacement input at the time compression ratio of 2.5. In this run, the fundamental waveform was successfully reproduced, but the high-frequency noise which accompanied it made
it impossible to show the waveforms in the figure to a reasonable scale, the trend of which can be observed if the waveforms for the displacement input at the time compression ratio of 5.0 and 10.0 are compared.

The waveform for the displacement input $\gamma = 5$ was well reproduced although the relative magnitudes of some of the acceleration spikes were distorted in the initial period of the earthquake. At $\gamma = 10$, the reproduction of the earthquake was satisfactory except for the reduction in the number of zero crossings. The prototype acceleration history contained approximately 230 zero crossings compared with 120 zero crossings in the simulated record at $\gamma = 10$.

The waveform was excellent for the record obtained by inputting the acceleration signal at $\gamma = 2.5$. The noise which was observed for the displacement input was virtually nonexistent in this case. At $\gamma = 5$, distortion of the waveform at intenser range was further exaggerated, with the total number of zero crossings of 130.

The total number of zero crossings in the simulated acceleration records are plotted in Fig. B.3 as a ratio of the total number of zero crossings in the prototype acceleration record. The horizontal axis represents the time compression ratio. There is a definite trend, for both types of input, for the number of zero crossings to decrease as the time compression ratio is increased. Although the zero crossings were not regularly spaced with respect to time during the course of the acceleration history, this trend does indicate that there is an approximate limit to the number of "cycles" that the system can handle. The trend implies that the system cannot handle more than approximately 40 zero crossings per
second. Thus, an earthquake acceleration record which contains approximately eight zero crossings per second could be reproduced closely up to approximately a time compression ratio of 5. At higher time compression ratios, the number of zero crossings in the simulated record would be less than that for the original record.

(b) Response Spectra of Simulated Earthquake

Calculated response spectra for prototype and simulated earthquake motions of the NS component of the El Centro (1940) record are given in Fig. B.4 for the damping factors of 0.0, 0.05 and 0.10 of critical. In order to provide a direct comparison between the response to the prototype motion and to the simulated earthquake motions, the simulated acceleration time histories were "normalized" before the calculations for the response spectra were made. The maximum acceleration measured for the simulated record was set equal to the maximum acceleration in the prototype record. The actual time of the simulated motions were stretched by the time compression ratio.

Response spectra are shown for the prototype El Centro acceleration record and for the simulated earthquake accelerations of \( \gamma = 5 \) of displacement input and \( \gamma = 2.5 \) and 10 of acceleration input. In general, the comparison of the response spectra to the prototype and simulated earthquake was more favorable than the comparison of the waveforms for the prototype and simulated acceleration histories.

(c) Noise in the Displacement Input

If the waveforms of time compression ratio of 5 for displacement input and acceleration input are compared, the existence of high frequency
noise can be easily recognized in the waveform of the displacement input. This high frequency noise is less conspicuous in the waveforms for the higher time compression ratios of displacement input.

The observed noise may be related to the existence of a small noise signal either on an input magnetic tape or input electronic circuit. This may be explained roughly as follows. Suppose the displacement time history $d(t)$ is expanded by Fourier series as

$$d(t) = \sum_{i=1}^{\infty} d_i \sin \omega_i t \quad (B.1)$$

in which

- $d_i =$ amplitude of $i^{th}$ component
- $\omega_i =$ circular frequency of $i^{th}$ component
- $t =$ time.

If the displacement waveform is fed into the earthquake simulator and is exactly reproduced on the earthquake simulator, then the simulated acceleration time history $a(t)$ will be

$$a(t) = \sum_{i=1}^{\infty} -\omega_i^2 d_i \sin \omega_i t \quad (B.2)$$

Equation B.2 indicates that amplitude of a displacement component is amplified proportional to square of its circular frequency, and that if there exists a high frequency component noise with even a small amplitude in a displacement waveform, the high frequency noise is exaggerated in an
acceleration waveform. This may be the main reason for the displacement input to cause such large high frequency noise.

The noise frequency is independent of a time compression ratio. Therefore, the noise amplitude stays constant for any time compression ratio, while the acceleration amplitudes of simulated earthquake increase with the square of a time compression ratio. Hence, the noise level is reduced to only a fraction in the waveform.

In the case of acceleration input, the noise may not be amplified because of the following reason. Suppose the acceleration time history is expanded by Fourier series as

$$ a(t) = \sum_{i=1}^{\infty} a_i \sin \omega_i t $$

(B.3)

in which

$$ a_i = \text{amplitude of } i^{th} \text{ component.} $$

If the acceleration signal is exactly integrated in the electronic circuit, then the displacement signal should have the following forms:

$$ d(t) = \sum_{i=1}^{\infty} -\frac{a_i}{\omega_i^2} \sin \omega_i t $$

(B.4)

If the displacement waveform is precisely reproduced on the platform, the acceleration should be measured as

$$ a(t) = \sum_{i=1}^{\infty} a_i \sin \omega_i t $$

(B.5)
This waveform is exactly the same as the original input acceleration, which indicates that there should not be any amplification of noise from the input signal.
Fig. B.1 Arrangement of Ram and Test Platform
Fig. B.2 Comparison of Measured Platform Acceleration Waveforms with the Original Earthquake Waveform
Fig. 8.2 (Contd) Comparison of Measured Platform Acceleration
Waveforms with the Original Earthquake Waveform
Fig. B.3 Variation of Number of Zero Crossings with the Time-Compression Ratio
Fig. B.4 Comparison of Response Spectra Calculated from the Original and the Measured Records \((\beta = 0.0, 0.05, 0.10)\)
Fig. B.4 (Contd) Comparison of Response Spectra Calculated from the Original and the Measured Records ($\gamma = 0.0, 0.05, 0.10$)

(c) Measured Platform Motion ($\gamma = 2.5$, Acc. Input)

(d) Measured Platform Motion ($\gamma = 10.0$, Acc. Input)
APPENDIX C.

SCALING OF A MODEL

When a small scale model is used in an experiment, the scale of the model should stay within a certain limit so that the test results from a model could be projected to describe the behavior of a prototype structure.

In order to describe engineering quantities, three basic independent units are necessary: units of length, time, and force. Also, three independent dimensions can be chosen to express dimensions of any engineering quantity: length (L), time (T), and force (F). The word "independent" is used because a dimension of any of the three cannot be expressed as a combination of the other two. The choice of the three fundamental dimensions are arbitrary as long as they are mutually independent.

Subscripts m and p were used to denote model and prototype, respectively. Three constants $\alpha$, $\beta$, $\gamma$ were used to show how the length, time and force were linearly scaled from a prototype to a model;

\[ \alpha(L)_p = (L)_m \]
\[ \beta(T)_p = (T)_m \]
\[ \gamma(F)_p = (F)_m \]

(C.1)

If a model is linearly scaled from a prototype, then an engineering quantity in a model can be expressed as a product of a constant scale
factor and the corresponding engineering quantity in the prototype. For example, model acceleration $A_m$ can be expressed as a product of prototype acceleration $A_p$, length scale factor $\alpha$ and time scale factor $\beta$, in the form

$$A_m = \frac{\alpha}{\beta^2} A_p$$

In other words, two accelerations $A_m$ and $A_p$ are linearly related by the factor $\alpha/\beta^2$, which is uniquely defined by the choice of $\alpha$ and $\beta$.

In the current tests $\alpha$ and $\beta$ were arbitrarily chosen as 1/8 and 1/2.5, respectively, so that $A_m$ did not depart too far from $A_p$ because the gravity acceleration is the same in a model and a prototype. Originally $A_m$ was intended to be equal to $A_p$, but an earthquake record of time scale $1/\sqrt{8}$ was not available. A scale factor $\gamma$ is determined by the condition that the pertinent properties of the materials were comparable in model and prototype, specifically Young's modulus $E$.

$$E_m = \frac{\gamma}{\alpha^2} E_p$$

hence

$$\gamma = \alpha^2 = \frac{1}{64}$$

The total amount of weight per story is determined by the scale factor assuming approximately 200 lb/ft$^2$ average weight over an 8 by 8 ft floor area.
By using this scaling system, the following relations exist between the model and the "arbitrary" prototype:

\[ \alpha = \frac{1}{8} \]

\[ \beta = \frac{1}{2.5} \]

\[ \gamma = \frac{1}{64} \]

\[ D_m = \alpha \cdot D_p \]

\[ V_m = \frac{\alpha}{\beta} \cdot V_p \]

\[ A_m = \frac{\alpha}{\beta^2} \cdot A_p \]

\[ \dot{\varepsilon}_m = \frac{1}{\beta} \cdot \dot{\varepsilon}_p \]

\[ \sigma_m = \sigma_p \]

\[ S1_m = \alpha \cdot S1_p \]

in which

- \( D \) = displacement
- \( V \) = velocity
- \( \dot{\varepsilon} \) = strain rate
- \( \sigma \) = stress
- \( S1 \) = spectrum intensity
APPENDIX D

DIGITAL-TO-ANALOG AND ANALOG-TO-DIGITAL CONVERSION

Development of data processing facilities has been of great importance in the test of a structure to a simulated earthquake motion. The facilities should be capable of (1) generating an analog signal from digital data, and (2) converting analog data into digital form at a reasonably high speed and with sufficient accuracy.

In simulated earthquake tests of a structure on the earthquake simulator, the motion of the test platform is controlled by an electric analog signal. Although original strong motion records of an earthquake were usually in graphical analog form, the records have been converted and are available in digital form. Therefore, the digital records have to be converted to an electric analog of the motion to be used in the earthquake simulator system.

On the other hand, all the measured response signals of the test structure are recorded continuously on magnetic tapes in terms of electric voltage. These electric analog signals have to be converted to digital form at some stage of the analysis in order for a high-speed digital computer to have access to the test data due to the fact that more sophisticated analytical methods have been developed and are available for use on a digital computer than on an analog computer.

The following paragraphs describe briefly the digital-to-analog and analog-to-digital conversion procedures which were carried out along with the simulated earthquake tests. The procedures were developed in three stages.
When the earthquake simulator was first put into operation, the conversion procedure (analog-to-digital and digital-to-analog) involved manual operation. In generating an electric analog of an earthquake motion, the digital record was converted manually to a graphical analog by being plotted on a sheet of paper with dark ink. The graphical analog was converted to an electric analog using a photo-electric curve follower. The position of the curve follower was sensed as electric voltage and was recorded on an analog magnetic tape. This process, however, was found to be inadequate in reproducing an original earthquake acceleration on the earthquake simulator.

At this stage, the analog-to-digital process was carried out by converting a graphical analog trace to digital data using a system called "Oscar" (Benson-Lehner Co.). The Oscar consists of (1) an illuminating screen, (2) two straight rulers, (3) a digital voltmeter, and (4) a digital output device. The graphical trace was placed on the screen, and was followed manually with the intersection of the two rulers. The coordinates of the intersection were sensed by the digital voltmeter, and were punched either on a paper tape or a series of cards. This process was found impractical because of its slow conversion rate and the large amount of labor required.

Later, the IBM 1800 Data Acquisition and Control System, which was housed in the Department of Computer Science of the University of Illinois at Urbana-Champaign, was introduced and used with the aid of the IBM 360/50 digital computer for analog-to-digital and digital-to-analog conversion. The IBM 1800 system could convert a series of digital data
into an electric analog at a conversion rate of 10,000 : 20,000 : 30,000 or 40,000 points per second. The analog signal was recorded on an FM magnetic tape. The IBM 1800 system could convert up to 16 channels of electric analog signals into digital data, and write the values on a digital magnetic tape (9 tracks, 1600 BPI). An electric analog signal was sampled every millisecond or at slower rates.

Currently, an analog-to-digital and digital-to-analog converter system was assembled in the Civil Engineering Department of the University of Illinois at Urbana-Champaign. The system consists basically of (1) a 16-bit word mini-computer of 8-K memory, (2) a 12-bit analog-to-digital and digital-to-analog converter, and (3) a 9 track 800-BPI (byte per inch) 75-IPS (inch per second) digital tape drive. The mini-computer controls the operation of the entire system and stores digital data in two buffers. Up to 14 electric analog signals may be processed simultaneously in the system. Continuous analog signals are converted at a constant rate to digital data, which are stored in a buffer of the mini-computer. When the buffer is filled up, the contents of the buffer are written on a digital magnetic tape, while the other buffer stores successive digitized data. In addition to the basic elements, some peripheral components are also available, such as a teletype to feed data for a governing computer program, an oscillograph and oscilloscope to monitor electric signals, television sets for miscellaneous visual monitoring of the experiment. After the analog-to-digital conversion is finished, the digital data are read from the digital tape and converted back to an electric analog signal to check the validity of the conversion process on some analog display devices.
Digital-to-analog conversion is carried out in the reverse way. A series of digital data is read from a digital magnetic tape and stored in the two buffers, alternately. The contents of a buffer are converted into analog signals, which are constantly dumped out. If the buffer is emptied, then the contents of the other buffer are used.
APPENDIX E.

FORMULATION OF THE FLEXIBILITY RELATION FOR AN INELASTIC MEMBER

The formulation of a flexibility matrix of a simply supported member AB due to inelastic deformation is described in this section. External moments $M_A$ and $M_B$ are applied at supports A and B, respectively (Fig. E.1). No additional loads are applied to the member. Shear and axial deformations are ignored.

The main objective of the method is to analyze a member in which inelastic deformation zones extend so far from the ends that the equivalent spring idealization is not realistic.

The assumed deformed configuration of a simply supported member is shown in Fig. E.1. The point of contraflexure on the member is indicated by C, corresponding to C' on the moment diagram.

The length of the member and segments $AC'$ and $C'B$ are denoted by $L$, $\lambda_A L$ and $\lambda_B L$, respectively. The coefficients $\lambda_A$ and $\lambda_B$ are related to the external moments $M_A$ and $M_B$ as follows;

$$\lambda_A = \frac{M_A}{M_A + M_B}$$

$$\lambda_B = \frac{M_B}{M_A + M_B}$$

(E.1)

Two lines AD and BE tangent to the deformed curve were drawn at beam ends A and B, where D and E fall on a line perpendicular to straight
line AB at point C. Two straight lines CF and CG were drawn at point C parallel to AD and EB, respectively. Points F and G lie on a line perpendicular to AB at B. Lines AD and BG intersect at H.

The following relation holds from the geometry in Fig. E.1:

\[ \overline{BH} = \overline{CD} + \overline{EC} - \overline{FG} \]  \hspace{1cm} (E.2)

If beam segments \( \overline{AC} \) and \( \overline{CB} \) are considered as two cantilevers fixed at A and B, and free at C, \( \overline{EC} \) and \( \overline{CD} \) are recognized as free end deflections \( D(M_A, \lambda_A L) \) and \( D(M_B, \lambda_B L) \). The angles at point C between the deformed beam and straight lines CF and CG are the free-end rotations \( R(M_A, \lambda_A L) \) and \( R(M_B, \lambda_B L) \).

Therefore, length FG can be written as

\[ \overline{FG} = \lambda_B L \{ R(M_B, \lambda_B L) - R(M_A, \lambda_A L) \} \]  \hspace{1cm} (E.3)

The slopes of tangents at A and B are \( \theta_A \) and \( \theta_B \).

Equation E.2 can be rewritten with the definitions made above in the form

\[ \overline{BH} = D(M_A, \lambda_A L) + D(M_B, \lambda_B L) + \lambda_B L \{ R(M_A, \lambda_A L) - R(M_B, \lambda_B L) \} \]  \hspace{1cm} (E.4)

Because \( \overline{BH} \) is also the product of the slope \( \theta_A \) at A times the length of the member, Eq. E.4 is rearranged in the form
Similarly

\[ \theta_a = \frac{1}{L} [D(M_A, \lambda_AL) + D(M_B, \lambda_BL)] \]
\[ + \lambda_BL \{ R(M_A, \lambda_AL) - R(M_B, \lambda_BL) \} \] (E.5.a)

\[ \theta_B = \frac{1}{L} [D(M_A, \lambda_AL) + D(M_B, \lambda_BL)] \]
\[ + \lambda_AL \{ R(M_B, \lambda_BL) - R(M_A, \lambda_AL) \} \] (E.5.b)

A sign convention in calculating free-end deformations of a cantilever was adopted so that a positive external fixed-end moment \( M \) gave rise to positive free-end displacement and rotation for a positive cantilever length \( \lambda L \). As noted in Section 3.5, the free-end displacement is proportional to the square of the cantilever length, while free-end rotation is proportional to the cantilever length:

\[ D(M, \lambda L) = (\lambda L)^2 D(M) \]
\[ R(M, \lambda L) = \lambda L R(M) \] (E.6)

in which \( D(M) \) and \( R(M) \) were free-end displacement and rotation of the unit length cantilever. The use of Eq. E.6 commits the analysis to logging moment history on the basis of the moments at the ends of the members only. This may lead to errors in calculating deformations as described below.

Consider the beam AB (Fig. E.3) subjected first to positive and then negative end moments \( M_A \) and \( M_B \). It is assumed that \( M_A \) is larger than
the yield moment and \( M_B \) is smaller than the cracking moment. After the existence of these conditions, the beam is yielded and cracked over the indicated lengths for both directions of loading.

If an arbitrarily chosen moment distribution shown in Fig. E.3.b is reached after the events shown in Fig. E.3.a, the deformation calculation will be based on the moment at end B which has always remained below the cracking moment, therefore, the beam will be assumed in the calculation to be completely uncracked which will result in an underestimate of the deformations.

To eliminate this drawback would require keeping track of the moment history at several stations along the length of the beam leading to an unwieldy computer program. In view of the fact that errors are introduced only when the point of inflection shifts substantially and only in the presence of small moments, it was decided to ignore this effect in the analysis. It should also be noted that the error introduced is smaller for deflection than it is for rotation.

Equation E.5 was rewritten in the form

\[
\theta_A = L\left[\lambda_A^2 D(M_A) + \lambda_B^2 D(M_B) + \lambda_A \{\lambda_A (R(M_A) - \lambda_A R(M_A))\}\right]
\]

\[
\theta_B = L\left[\lambda_A^2 D(M_A) + \lambda_B^2 D(M_B) + \lambda_B \{\lambda_B (R(M_B) - \lambda_B R(M_B))\}\right]
\]

If external moments \( M_A \) and \( M_B \) are increased by \( \Delta M_A \) and \( \Delta M_B \), respectively, then the resultant end rotations and the location of the
The point of contraflexure are related in the form

\[ \theta_A + \Delta \theta_A = \lambda \left[ (\lambda_A + \Delta \lambda_A)^2 D(M_A + \Delta M_A) + (\lambda_B + \Delta \lambda_B)^2 D(M_B + \Delta M_B) \right. \]

\[ + (\lambda_B + \Delta \lambda_B) \left( (\lambda_A + \Delta \lambda_A) R(M_A + \Delta M_A) \right) \]

\[ - (\lambda_B + \Delta \lambda_B) R(M_B + \Delta M_B) \right] \tag{E.8.a} \]

\[ \theta_B + \Delta \theta_B = \lambda \left[ (\lambda_A + \Delta \lambda_A)^2 D(M_A + \Delta M_A) + (\lambda_B + \Delta \lambda_B)^2 D(M_B + \Delta M_B) \right. \]

\[ + (\lambda_A + \Delta \lambda_A) \left( (\lambda_B + \Delta \lambda_B) R(M_B + \Delta M_B) \right) \]

\[ - (\lambda_A + \Delta \lambda_A) R(M_A + \Delta M_A) \right] \tag{E.8.b} \]

\[ \lambda_A + \Delta \lambda_A = \frac{M_A + \Delta M_A}{M_A + M_B + \Delta M_A + \Delta M_B} \tag{E.9} \]

\[ \lambda_B + \Delta \lambda_B = \frac{M_B + \Delta M_B}{M_A + M_B + \Delta M_A + \Delta M_B} \]

Incremental end rotations \( \Delta \theta_A \) and \( \Delta \theta_B \) are calculated by subtracting Eq. E.7 from Eq. E.8.
Equation E.9 was rearranged to calculate change $\Delta \lambda_A$ in the location of the point of contraflexure in the form

$$
\Delta \theta_A = L \left[ (\lambda_A + \Delta \lambda_A)^2 D(M_A + \Delta M_A) - \lambda_A^2 D(M_A) \right. \\
+ (\lambda_B + \Delta \lambda_B)^2 D(M_B + \Delta M_B) - \lambda_B^2 D(M_B) \\
+ (\lambda_A + \Delta \lambda_A) (\lambda_A + \Delta \lambda_A) R(M_A + \Delta M_A) - (\lambda_B + \Delta \lambda_B) R(M_B + \Delta M_B) \\
- \lambda_B \left\{ \lambda_A R(M_A) - \lambda_B R(M_B) \right\} \right] 
$$

(E.10.a)

$$
\Delta \theta_B = L \left[ (\lambda_A + \Delta \lambda_A)^2 D(M_A + \Delta M_A) - \lambda_A^2 D(M_A) \right. \\
+ (\lambda_B + \Delta \lambda_B)^2 D(M_B + \Delta M_B) - \lambda_B^2 D(M_B) \\
+ (\lambda_A + \Delta \lambda_A) (\lambda_B + \Delta \lambda_B) R(M_B + \Delta M_B) - (\lambda_A + \Delta \lambda_A) R(M_A + \Delta M_A) \\
- \lambda_A \left\{ \lambda_B R(M_B) - \lambda_A R(M_A) \right\} \right] 
$$

(E.10.b)

Equation E.9 was rearranged to calculate change $\Delta \lambda_A$ in the location of the point of contraflexure in the form

$$
\Delta \lambda_A = \frac{\lambda_B \Delta M_A - \lambda_A \Delta M_B}{M_A + M_B + \Delta M_A + \Delta M_B} 
$$

(E.11)

If stiffnesses $SD(M)$ and $SR(M)$ can be defined between the previous and the present steps for free-end displacement and rotation with a
knowledge of the load history (Fig. E.2),

\[
D(M_A + \Delta M_A) = D(M_A) + \frac{\Delta M_A}{SD(M_A)}
\]

\[
D(M_B + \Delta M_B) = D(M_B) + \frac{\Delta M_B}{SD(M_B)}
\]  \(\text{(E.12)}\)

\[
R(M_A + \Delta M_A) = R(M_A) + \frac{\Delta M_A}{SR(M_A)}
\]

\[
R(M_B + \Delta M_B) = R(M_B) + \frac{\Delta M_B}{SR(M_B)}
\]

then Eq. E.10 can be rearranged into the form

\[
\begin{bmatrix}
\Delta \theta_A \\
\Delta \theta_B
\end{bmatrix} =
\begin{bmatrix}
f_{11} & f_{12} \\
f_{21} & f_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta M_A \\
\Delta M_B
\end{bmatrix}
\]  \(\text{(E.13)}\)
in which

\[ f_{11} = L \left\{ \frac{(\lambda_A + \Delta \lambda_A)^2}{SD(M_A)} + \frac{(\lambda_A + \Delta \lambda_A) (\lambda_B + \Delta \lambda_B)}{SR(M_A)} + \lambda_B X \right\} \]

\[ f_{12} = L \left\{ \frac{(\lambda_B + \Delta \lambda_B)^2}{SD(M_B)} - \frac{(\lambda_B + \Delta \lambda_B)^2}{SR(M_B)} - \lambda_X \right\} \]

\[ f_{21} = L \left\{ \frac{(\lambda_A + \Delta \lambda_A)^2}{SD(M_A)} - \frac{(\lambda_A + \Delta \lambda_A)^2}{SR(M_A)} + \lambda_B Y \right\} \]

\[ f_{22} = L \left\{ \frac{(\lambda_B + \Delta \lambda_B)^2}{SD(M_B)} + \frac{(\lambda_A + \Delta \lambda_A) (\lambda_B + \Delta \lambda_B)}{SR(M_B)} - \lambda_Y \right\} \]

\[ X = \{(2\lambda_A + \Delta \lambda_A) D(M_A) - (2\lambda_B + \Delta \lambda_B) D(M_B) + (\lambda_B - \lambda_A - \Delta \lambda_A) R(M_A) \]

\[ + (2\lambda_B + \Delta \lambda_B) R(M_B) \} / (M_A + M_B + \Delta M_A + \Delta M_B) \]

\[ Y = \{(2\lambda_A + \Delta \lambda_A) D(M_A) - (2\lambda_B + \Delta \lambda_B) D(M_B) - (2\lambda_A + \Delta \lambda_A) R(M_A) \]

\[ + (\lambda_B - \lambda_A - \Delta \lambda_A) R(M_B) \} / (M_A + M_B + \Delta M_A + \Delta M_B) \]  \hspace{1cm} (E.14)

It should be noted that the flexibility matrix is not symmetric, and that the flexibility matrix contains \( \Delta \lambda_A, \Delta \lambda_B, \Delta M_A \) and \( \Delta M_B \), which are
unknown at the time the flexibility matrix is constructed. An iteration procedure is to be used to determine the unknowns, which involves the analysis of the total frame at each iteration.

One alternative method to avoid the iteration procedure is to assume that the point of contraflexure does not move during a load increment, then, elements of the flexibility matrix can be simplified to

\[ f_{11} = \lambda_A \left\{ \frac{\lambda_A}{SD(M_A)} + \frac{\lambda_B}{SR(M_A)} \right\} \]

\[ f_{12} = \lambda_B \left\{ \frac{1}{SD(M_B)} - \frac{1}{SR(M_A)} \right\} \]

\[ f_{21} = \lambda_A \left\{ \frac{1}{SD(M_A)} - \frac{1}{SR(M_B)} \right\} \]

\[ f_{22} = \lambda_B \left\{ \frac{\lambda_B}{SD(M_B)} + \frac{\lambda_A}{SR(M_B)} \right\} \]  \hspace{1cm} (E.15)

Again the flexibility matrix is not symmetric.

If the analysis assumes the equivalent springs at the ends of a member, in which all the plastic deformation of a member is concentrated in two equivalent springs at its ends, then the following relation holds
for inelastic deformation of a unit length cantilever beam;

\[ SD(M) = SR(M) \]

therefore, off diagonal elements of the flexibility matrix become zero

\[ f_{12} = f_{21} = 0 \]

(E.16.a)

and the diagonal elements are reduced to

\[ f_{11} = \frac{\lambda_A L}{SD(M_A)} \]

\[ f_{22} = \frac{\lambda_B L}{SD(M_B)} \]

(E.16.b)

Giberson (1967) proposed the equivalent spring idealization for \( \lambda_A = \lambda_B = 0.5 \). Suko (1971) used the equivalent springs for an arbitrary initial location of contraflexure.

The flexibility matrix of a simply supported inelastic member was formulated as a function of cantilever deformation of the length between a support and the point of contraflexure. In order to calculate exact values of the elements of the flexibility matrix, the relation shown in Eq. E.14 was used for the analysis. The iteration procedure tends to cause difficulty when the incremental external moments become comparable to the existing external moments.
Fig. E.1 Geometry of Deflected Member
Fig. E.2 Incremental Force-Deformation Relationships
Fig. E.3 Damaged Region in a Frame Member
HYSTERESIS RULES USED IN NONLINEAR FRAME ANALYSIS

Assumed stiffness characteristics of the analytical model have a very important role in determining the dynamic response. Two hysteretic systems for reinforced concrete were adopted from Takeda et al (1970): (a) the original Takeda hysteresis system (Fig. F.1) with a tri-linear primary curve was used for the frame members, and (b) a simplified version of the Takeda hysteresis system (Fig. F.2) with a bilinear primary curve was used for the rotational springs which represented bond slip of the tensile reinforcement at the ends of the frame members.

The following definitions and notations are used to simplify the description of the hysteretic rules.

- loading = an increase, without change in sign, of the absolute value of the force
- unloading = a decrease, without change in sign, of the absolute value of the force
- load reversal = change in sign of the force with respect to the one in the last load step
- $K$ = stiffness to be used in the next load increment
- $O$ = origin of the primary curve
- $C$ = cracking point on the primary curve
- $Y$ = yielding point on the primary curve
- $U$ = ultimate point on the primary curve
- $P$ = point at which current calculation begins
- $U_1$ = point at which unloading begins
\( X_i \) = load reversal point or intersection point of the force-deflection curve with the displacement axis.

\( S_i \) = slope

\( F(A) \) = force at point A on the force-deflection curve

\( D(A) \) = deflection at point A on the force-deflection curve

\( S(AB) \) = slope of line segment \( AB \)

At each load step, the force active is assumed to be positive in describing the hysteresis rules. An apostrophe is used to indicate the corresponding point in negative region of the force-deflection diagram.

In the following section, the rules of the hysteresis system are described in terms of the definition given above. An example is provided below to help in the interpretation of the rules.

Example

Rule 3: loading on the primary curve after yielding

3.1 loading: \( K = S(YU) \), go to rule 3.

3.2 unloading: unloading point = \( U_m \),

\[
S_1 = S(C') \times \left[ \max \left\{ D(U_m), D(U'_m) \right\} / D(Y) \right]^{0.5}
\]

\( K = S_1 \), go to rule 4.

Interpretation

Rule 3 governs if loading starts on the primary curve after yielding has occurred.

If the load continues in the same direction (3.1 loading), the path of the force-deflection curve follows the primary curve for that increment with the slope \( K \) defined by the primary curve (slope of segment \( YU \)). For the next increment, rule 3 continues to govern.
If the load is decreased (3.2 unloading) from the point $U_m$, the slope is defined by the product of the slope $(C'Y)$ and the ratio of the maximum deflection reached in either direction to the yield deflection. Calculation for that increment uses the slope $S_1$. The next increment is governed by rule 4.

F.1 Takeda's Hysteresis System

Takeda's hysteresis system has a trilinear primary curve with two break-points at cracking $C$ and yielding $Y$. The primary curve is assumed to be symmetric about its origin.

The following set of rules were used to determine the stiffness of a frame member at each load level (Fig. F.1).

Rule 1: elastic stage

1.1 loading

1.1.1 $F(P) \leq F(C)$ : $K = S(OC)$, go to rule 1.
1.1.2 $F(P) > F(C)$ : $K = S(CY)$, go to rule 2.

1.2 unloading and load reversal : $K = S(OC)$, go to rule 1.

Rule 2: loading on the primary curve up to yielding

2.1 loading

2.1.1 $F(P) \leq F(Y)$ : $K = S(CY)$, go to rule 2.
2.1.2 $F(P) > F(Y)$ : $K = S(YU)$, go to rule 3.

2.2 unloading : unloading point = $U_m$, $S_1 = S(OC')$, $K = S_1$, go to rule 5.

Rule 3: loading on the primary curve after yielding

3.1 loading : $K = S(YU)$, go to rule 3.
3.2 unloading: unloading point = $U_m$,

$$S_1 = S(\overline{CY}) \times \left[ \frac{\max \{ D(U_m), D(U'_m) \}}{D(Y)} \right]^{0.5}$$

$K = S_1$, go to rule 4.

Rule 4: unloading from point $U_m$ on the primary curve after yielding

4.1 loading
4.1.1 $F(P) \leq F(U_m)$ : $K = S_1$, go to rule 4.
4.1.2 $F(P) > F(U_m)$ : $K = S(YU)$, go to rule 3.

4.2 unloading: $K = S_1$, go to rule 4.

4.3 load reversal
4.3.1 uncracked in the negative range: $K = S_1$, go to rule 15.
4.3.2 otherwise: load reversal point = $X_o$, $S_2 = S(\overline{X_oU_m})$,

$K = S_2$, go to rule 6.

Exception in 4.3.2

If $F(Y) > F(U_m)$ and $S(\overline{X_oY}) > S(\overline{X_oU_m})$

then $S_2 = S(\overline{X_oY})$, $U_m = Y$, $K = S_2$, go to rule 6.

Rule 5: unloading from point $U_m$ on the primary curve before yielding

5.1 loading
5.1.1 $F(P) \leq F(U_m)$ : $K = S_1$, go to rule 5.
5.1.2 $F(P) > F(U_m)$ : $K = S(CY)$, go to rule 2.

5.2 unloading: $K = S_1$, go to rule 5.

5.3 load reversal
5.3.1 uncracked in the positive range: $K = S_1$, go to rule 14.
5.3.2 otherwise: the same as 4.3.2.
Rule 6: loading toward point $U_m$ on the primary curve

6.1 loading

6.1.1 $F(P) \leq F(U_m) : K = S_{O_m}$, go to rule 6.

6.1.2 $F(P) > F(U_m) :$ the same as 2.1.

6.2 unloading: unloading point = $U_o$, $K = S_1$, go to rule 7.

Rule 7: unloading from point $U_o$ after rule 6.

7.1 loading

7.1.1 $F(P) \leq F(U_o) : K = S_1$, go to rule 7.

7.1.2 $F(P) > F(U_o) : K = S_{O_m}$, go to rule 6.

7.2 unloading: $K = S_1$, go to rule 7.

7.3 load reversal: load reversal point = $X_1$, $K = S_{X_1 U_m}$, go to rule 8.

Rule 8: loading toward point $U_m$ on the primary curve

8.1 loading

8.1.1 $F(P) \leq F(U_m) : K = S_{X_1 U_m}$, go to rule 8.

8.1.2 $F(P) > F(U_m) :$ the same as 2.1.

8.2 unloading: unloading point = $U_1$, $K = S_1$, go to rule 9.

Rule 9: unloading from point $U_1$ after rule 8

9.1 loading

9.1.1 $F(P) \leq F(U_1) : K = S_1$, go to rule 9.

9.1.2 $F(P) > F(U_1) :$ the same as 8.1

9.2 unloading: $K = S_1$, go to rule 9.

9.3 load reversal: load reversal point = $X_2$, $K = S_{X_2 U_o}$, go to rule 10.

Rule 10: loading toward point $U_o$

10.1 loading

10.1.1 $F(P) \leq F(U_o) : K = S_{X_2 U_o}$, go to rule 10.
10.1.2 $F(P) > F(U_o)$: the same as 6.1.

10.2 unloading: unloading point = $U_2$, $K = S_1$, go to rule 11.

**Rule 11:** unloading from point $U_2$ after rule 10

11.1 loading

11.1.1 $F(P) \leq F(U_2)$: $K = S_1$, go to rule 11.

11.1.2 $F(P) > F(U_2)$: the same as 10.1.

11.2 unloading: $K = S_1$, go to rule 11.

11.3 load reversal: load reversal point = $X_3$, $K = S(X_3U_1)$, go to rule 12.

**Rule 12:** loading toward point $U_1$

12.1 loading

12.1.1 $F(P) \leq F(U_1)$: $K = S(X_3U_1)$, go to rule 12.

12.1.2 $F(P) > F(U_1)$: the same as 8.1.

12.2 unloading: unloading point = $U_3$, $K = S_1$, go to rule 13.

**Rule 13:** unloading from point $U_3$ after rule 12

13.1 loading

13.1.1 $F(P) \leq F(U_3)$: $K = S_1$, go to rule 13.

13.1.2 $F(P) > F(U_3)$: the same as 12.1.

13.2 unloading: $K = S_1$, go to rule 13.

13.3 load reversal: the same as 9.3.

**Rule 14:** loading in the uncracked direction after cracking in the other direction

14.1 loading

14.1.1 $F(P) \leq F(C)$: $K = S_1$, go to rule 14.

14.1.2 $F(P) > F(C)$: $K = S(CY)$, go to rule 2.

14.2 unloading: $K = S_1$, go to rule 14.

14.3 load reversal: $K = S_1$, go to rule 5.
Rule 15: loading in the uncracked direction after yielding in the other direction

15.1 loading

15.1.1 $F(P) \leq F(C) : K = S_1$, go to rule 15.

15.1.2 $F(P) > F(C) : \text{let } Q \text{ be point on the curve for rule 15 and } F(Q) = F(C), \text{ then } K = S(QY), \text{ go to rule 16.}$

15.2 unloading: $K = S_1$, go to rule 15.

15.3 load reversal

15.3.1 $F(P) \leq F(U_m) : K = S_1$, go to rule 4.

15.3.2 $F(P) > F(U_m) : K = S(YU), \text{ go to rule 3.}$

Rule 16: loading toward the yield point after rule 15

16.1 loading

16.1.1 $F(P) \leq F(Y) : K = S(QY), \text{ go to rule 16.}$

16.1.2 $F(P) > F(Y) : K = S(YU), \text{ go to rule 3.}$

16.2 unloading: $U_m = Y, U_o = P, S_2 = S(QY), K = S_1$, intersection of $YQ$ and deflection axis $= X_o$.

F.2 Simplified Hysteresis System

Takeda's hysteresis system was simplified and modified for a system with a bilinear primary curve. This can be accomplished by setting the cracking point to be the same as the origin of the primary curve in Takeda's hysteresis system. The simplified Takeda's hysteresis rules are similar to the one proposed by Clough and Johnston (1966). The simplified model includes more rules for small-amplitude load reversals.

The following set of rules were used to determine the stiffness of a rotational spring at each load step (Fig. F.2).
Rule 1: elastic stage

1.1 loading

1.1.1 $F(P) \leq F(Y) : K = S(OY)$, go to rule 1.

1.1.2 $F(P) > F(Y) : K = S(\overline{YU})$, go to rule 2.

1.2 unloading and load reversal : $K = S(\overline{OY})$, go to rule 1.

Rule 2: loading on the primary curve after yielding

2.1 loading : $K = S(\overline{YU})$, go to rule 2.

2.2 unloading : unloading point = $U_m$, $K = S(\overline{OY})$, go to rule 3.

Rule 3: unloading from point $U_m$ on the primary curve

3.1 loading

3.1.1 $F(P) \leq F(Y) : K = S(\overline{OY})$, go to rule 3.

3.1.2 $F(P) > F(Y) : K = S(\overline{YU})$, go to rule 2.

3.2 unloading : $K = S(\overline{OY})$, go to rule 3.

3.3 load reversal : load reversal point = $X_0$, $K = S(\overline{X_0U_m^*})$, go to rule 4.

Rule 4: loading toward point $U_m$ on the primary curve

4.1 loading

4.1.1 $F(P) \leq F(U_m) : K = S(\overline{X_0U_m})$, go to rule 4.

4.1.2 $F(P) > F(U_m) : K = S(\overline{YU})$, go to rule 2.

4.2 unloading : unloading point = $U_o$, $K = S(\overline{OY})$, go to rule 5.

Rule 5: unloading from point $U_o$ after rule 4

5.1 loading

5.1.1 $F(P) \leq F(U_o) : K = S(\overline{OY})$, go to rule 5.

5.1.2 $F(P) > F(U_o)$ : the same as 4.1.

5.2 unloading : $K = S(\overline{OY})$, go to rule 5.

* If $U_m$ in the new direction is not defined yet, $U_m$ is taken as yielding point $Y$. 
5.3 Load reversal: load reversal point = $X_1$, $K = S(X_1 U_m)$, go to rule 6.

**Rule 6:** loading toward point $U^*_m$ on the primary curve

6.1 Loading

6.1.1 $F(P) \leq F(U^*_m) : K = S(X_1 U_m)$, go to rule 6.

6.1.2 $F(P) > F(U^*_m) : K = S(YU)$, go to rule 2.

6.2 Unloading: unloading point = $U_1$, $K = S(OY)$, go to rule 7.

**Rule 7:** unloading from point $U_1$ after rule 6

7.1 Loading

7.1.1 $F(P) \leq F(U_1) : K = S(OY)$, go to rule 7.

7.1.2 $F(P) > F(U_1) : $ the same as 6.1.

7.2 Unloading: $K = S(OY)$, go to rule 7.

7.3 Load reversal: load reversal point = $X_2$, $K = S(X_2 U_o)$, go to rule 8.

**Rule 8:** loading toward point $U_o$

8.1 Loading

8.1.1 $F(P) \leq F(U_o) : K = S(X_2 U_o)$, go to rule 8.

8.1.2 $F(P) > F(U_o) : $ the same as 4.1.

8.2 Unloading: unloading point = $U_2$, $K = S(OY)$, go to rule 9.

**Rule 9:** unloading from point $U_2$ after rule 8

9.1 Loading

9.1.1 $F(P) \leq F(U_2) : K = S(OY)$, go to rule 9.

9.1.2 $F(P) > F(U_2) : $ the same as 8.1.

*U in load level 6 is of the sign opposite to the sign of U in load level 4.*
9.2 unloading: $K = S(OY)$, go to rule 9.

9.3 load reversal: load reversal point = $X_3$, $K = S(X_3U_1)$, go to rule 10.

Rule 10: loading toward point $U_1$

10.1 loading

10.1.1 $F(P) \leq F(U_1)$: $K = S(X_3U_1)$, go to rule 10.

10.1.2 $F(P) > F(U_1)$: the same as 6.1

10.2 unloading: unloading point = $U_3$, $K = S(OY)$, go to rule 11.

Rule 11: unloading from point $U_3$ after rule 10

11.1 loading

11.1.1 $F(P) \leq F(U_3)$: $K = S(OY)$, go to rule 11.

11.1.2 $F(P) > F(U_3)$: the same as 10.1

11.2 unloading: $K = S(OY)$, go to rule 11.

11.3 load reversal: load reversal point = $X_2$, $K = S(X_2U_0)$, go to rule 8.
Fig. F.1 Takeda's Hysteresis Rules
Fig. F.1 (Contd) Takeda's Hysteresis Rules
Fig. F.2 Simplified Takeda's Hysteresis Rules
APPENDIX G

COMPUTER PROGRAM FOR NONLINEAR RESPONSE-HISTORY ANALYSIS

The flow diagram of the computer program for nonlinear response-history analysis is shown in Fig. G.1. The method of analysis is described in Chapter 4. The program was prepared to analyze a reinforced concrete frame with less than 10 members. The program was written in Fortran IV. The total core space required to run the program was approximately 110 kilobites in addition to temporary disk space in which calculated response values were stored. It took approximately 11 minutes of computing time on the IBM 360/75 computer for the program to complete a response analysis of a three-story one-bay frame subjected to a 13.0 seconds of base motion at a 0.0005 sec time interval. Calculated response values were plotted at the end of analysis on a CALCOMP plotter.
Fig. G.1 Flow Diagram of Computer Program for Nonlinear Response-History Analysis
Iteration due to shift of inflection point

Compute:
1. new structural matrix

Compute:
1. incremental structural response from equation of motion
2. incremental member response

Compute:
1. total structural response
2. total member response
3. member stiffness based on hysteresis rules

Modify:
1. member response be compatible with hysteresis rules and compute unbalanced force due to the modification

Record:
1. maximum responses

End of earthquake record

Fig. G.1 (Contd) Flow Diagram of Computer Program for Nonlinear Response-History Analysis
Plot: 1. base acceleration signal
2. base shear signal
3. base moment signal
4. acceleration response signals
5. displacement response signals

Write: 1. maximum structural response values
2. maximum member response values

End

Fig. G.1 (Contd) Flow Diagram of Computer Program for Nonlinear Response-History Analysis
APPENDIX H
NOTATION

H.1 Terminology

Certain terms were introduced and used in this report. Although such terms are defined when they are first introduced in the text, they are listed below for convenient reference.

attained ductility factor = ratio of measured maximum deformation response to calculated yield deformation response

deflection coefficient = ratio of the attained deflection to the yield deflection of a cantilever

first mode = phase relationship in which three level signals oscillate in the same phase

force reduction factor = ratio of calculated elastic force response to corresponding measured response

fully cracked section = section of which flexural rigidity is defined as the slope of a line connecting the origin and the yield point in a moment-curvature diagram

joint core = part of a structure common to both beam and column

limiting base shear coefficient = index value for the strength of a frame, defined as the ratio of base shear corresponding to collapse mechanism for a triangular lateral load distribution (elasto-plastic members) to total weight of the frame
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>relative intensity index</td>
<td>index value for the intensity of the base motion relative to the strength of the structure, defined as the ratio of the maximum base acceleration in terms of &quot;g&quot; to the limiting base shear coefficient of the structure</td>
</tr>
<tr>
<td>response-history analysis</td>
<td>response analysis based on direct numerical integration of the equation of motion by a step-by-step procedure</td>
</tr>
<tr>
<td>rigid joint</td>
<td>joint core with infinite flexural stiffness</td>
</tr>
<tr>
<td>RMS</td>
<td>response value calculated by the square root of the sum of the squares of the maximum modal components (root mean square)</td>
</tr>
<tr>
<td>rotational spring</td>
<td>a spring at the ends of frame members which simulates bond slip of the tensile reinforcement</td>
</tr>
<tr>
<td>second mode</td>
<td>phase relationship in which only two adjacent level signals oscillate in the same phase</td>
</tr>
<tr>
<td>spectrum intensity</td>
<td>index to define the intensity of base motion, based on the area under a velocity response spectrum curve in a specified frequency range (Housner, 1952)</td>
</tr>
</tbody>
</table>
spectral modal analysis = response analysis based on modal characteristics and linearly elastic response spectra

standard frame = analytical model in Chapter 7 with (1) rigid zone in joint cores, (2) rotational springs at the ends of flexible elements, having simplified Takeda hysteresis rule, and (3) stiffness of member defined by Eq. E.15 in Appendix E

third mode = phase relationship for which the first- and the third-level signals are in phase

triangular distribution = distribution of lateral loads proportional to the height of each level

uniform distribution = distribution of lateral loads equal at each level

H.2 Symbols

All symbols used in this report are defined where they are first introduced in the text. The symbols which appear more than once in the text are listed below for convenient reference.

$A_s$ = area of (tensile) reinforcement

$A'_s$ = area of compressive reinforcement

$[C]$ = damping matrix

$C_1, C_2$ = constants to define a damping matrix as shown in Eq. 4.22

$[\tilde{C}]$ = matrix defined by Eq. 4.24

$D$ = total depth of a section in Eq. 3.9

$D$ = diameter of a reinforcing bar in Eq. 3.15
\( D_{\text{max}} \) = calculated maximum free end displacement of the frame members as a unit-length cantilever
\( D(M) \) = free end displacement of a unit length cantilever due to fixed end moment \( M \)
\( D(M_A, \lambda_A L) \) = free end displacement of a cantilever of length \( \lambda_A L \) due to fixed end moment \( M_A \)
\( \tilde{D}[^*] \) = diagonal matrix with diagonal element \([^*]\)
\( E_c \) = secant modulus of concrete at a stress equal to 40 percent of the compressive strength
\( E_s \) = Young's modulus for steel
\( E_{sh} \) = modulus of the steel to define slope in strain hardening range
\( E_I \) = initial flexural rigidity for a transformed section
\( E_{Iy} \) = slope of the moment-curvature relationship after yielding
\( I_c \) = moment inertia of a concrete section along the neutral axis
\( I_s \) = moment inertia of the steel along the neutral axis
\( [K] \) = "reduced" structural stiffness matrix
\( \tilde{K} \) = matrix defined by Eq. 4.24
\( K_{ij} \) = submatrix of a structural matrix shown in Eq. 4.13
\( L \) = length of a cantilever in Eq. 3.12 through 3.14
\( L \) = length of a flexible member \( A'B' \) in Eq. 4.2 and in Appendix E
\( L \) = center-to-center distance of the first-story columns in Eq. 6.1
\( [M] \) = structural mass matrix
\( M \) = bending moment
\( M_A \) = moment at end \( A \) of member \( AB \)
\( M'_A \) = moment at end \( A' \) of flexible element \( A'B' \)
\( M_c \) = cracking moment of a section

\( M_y \) = yielding moment of a section

\( N \) = axial load acting on a section (Chapter 3)

\( N \) = number of degrees of freedom of a system (Chapter 4)

\( P_A \) = lateral force at level A in a frame

\( R \) = rotation due to bond slip of tensile reinforcement

\( R(M) \) = free end rotation of a unit length cantilever due to fixed end moment \( M \)

\( R(M_A, \lambda_A L) \) = free end rotation of a cantilever of length \( \lambda_A L \) due to fixed end moment \( M_A \)

\( S_D(M) \) = slope of free end displacement-fixed end moment curve of a cantilever of unit length

\( S_R(M) \) = slope of free end rotation-fixed end moment curve of a cantilever of unit length

\( U_A \) = story displacement at level A relative to the base

\( \dot{U} \) = story velocity relative to the base

\( \ddot{U} \) = story acceleration relative to the base

\( Z \) = constant which defines the descending slope of stress-strain curve of concrete in Eq. 3.1

\( \{Z\} \) = modal spectral response vector

\( b \) = width of a cross section

\( c \) = depth of the neutral axis from the extreme compressive fiber

\( c' \) = distance from the neutral axis to the point at which \( E_t \) is attained

\( d \) = distance from the extreme compressive fiber to the center of tensile reinforcement
\( d' \) = distance from the extreme compressive fiber to the center of compressive reinforcement

\( e \) = superscript to indicate "elastic"

\( f \) = flexibility coefficient of a rotational spring in Eq. 4.1

\( f'_c \) = stress of concrete

\( f'_{ij} \) = flexibility coefficient of flexible element \( A'B' \) (i, j = 1 or 2)

\( f''_{ij} \) = inelastic flexibility coefficient of flexible element \( A'B' \) (i, j = 1 or 2)

\( f_r \) = modulus of rupture of concrete

\( f_s \) = stress in tensile reinforcement

\( f'_s \) = stress in compressive reinforcement

\( f_{su} \) = ultimate stress of steel

\( f_t \) = tensile strength of concrete, given by Eq. 3.3

\( f_y \) = yield stress of steel

\( k_{ij} \) = stiffness coefficient of member \( AB \) (i, j = 1 or 2)

\( k'_{ij} \) = stiffness coefficient of flexible element \( A'B' \) (i, j = 1 or 2)

\( p \) = superscript to indicate "inelastic"

\( u \) = average bond stress between a reinforcing bar and concrete, given by Eq. 3.19

\( x \) = distance from the neutral axis to the extreme tensile fiber of a section in Eq. 3.5

\( \Delta \) = incremental value

\( \Delta L \) = elongation of reinforcement over a development length

\( \Delta T \) = time increment in numerical solution of the equation of motion
\(\Delta \theta_A\) = incremental rotation at end A of member AB

\(\Delta \theta_{A'}\) = incremental rotation at end A' of flexible element A'B'

\(\theta_A\) = joint rotation at A of a frame

\([\psi]\) = matrix defined by Eq. 4.28

\(\beta\) = damping factor at the initial elastic stage (for the first mode)

\(\beta_s\) = s \(^{th}\) mode damping factor

\(\gamma\) = force reduction factor

\(\varepsilon\) = strain

\(\varepsilon_c\) = strain of concrete

\(\varepsilon_o\) = strain at which the compressive strength of concrete is attained

\(\varepsilon_s\) = strain in (tensile) steel

\(\varepsilon_s'\) = strain in compressive steel

\(\varepsilon_{sh}\) = strain at which strain hardening of steel commences

\(\varepsilon_{su}\) = strain at which ultimate stress is reached in an idealized stress-strain curve of steel

\(\varepsilon_t\) = strain at which tensile strength of concrete is attained, given by Eq. 3.2

\(\varepsilon_y\) = strain at which the yield stress of steel is attained

\(\lambda_A\) = ratio of the length of rigid zone AA' to the length of a flexible element A'B'

\(\mu\) = attained ductility factor

\(\sigma_a\) = axial stress existing in a section

\(\phi\) = curvature

\([\phi]\) = real orthogonal matrix which diagonalizes matrices [C] and [K]
\( \phi_c \) = curvature of a section at cracking
\( \phi_y \) = curvature of a section at yielding
\( \omega \) = circular frequency
\( \omega_s \) = \( s \)th mode circular frequency