TIME-DEPENDENT DEFORMATIONS AND LOSSES IN CONCRETE BRIDGES BUILT BY THE CANTILEVER METHOD

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A method is presented for the analysis of time-dependent deformations of post-tensioned concrete bridge superstructures which are erected by cantilever methods. The method takes into account any arbitrary creep and shrinkage properties of the concrete, friction between strand and ducts, relaxation of the steel stress, and construction loads. A step-by-step procedure is presented which takes into account the creep of concrete under variable stress, variations in Young's modulus of concrete, and all elastic changes in stress accompanying the construction of additional segments.

The analysis method was used to study the effects of variations of the parameters on the long-term behavior of two bridges which were built and designed following quite different criteria. Excellent agreement between measured and computed curvatures and strains were found in the two cases for which experimental data were available.

Post-tensioned Concrete, Bridges, Cantilever Erection, Creep, Shrinkage, Relaxation of Steel Stresses, Long-term Behavior, Prestress Losses, Computer Analysis, Precast Segments, Cast-in-Place Segments
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The contents of this report reflect the views of the authors who are responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Illinois Department of Transportation. This report does not constitute a standard, specification, or regulation.
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1. INTRODUCTION

1.1 General Remarks

In highway bridge construction there is an increasing trend toward the use of longer spans. This trend is the result of a number of different requirements relating to safety, economy, function, and aesthetics (2). As a consequence, it became apparent that conventional precast prestressed concrete girders would soon be limited by their maximum transportable weights and/or lengths. A solution to this problem was the development of precast prestressed segmental construction in which the benefits of both precasting and post-tensioning could be combined advantageously.

The segmental cantilevered bridge construction, in which the sections may be precast or cast-in-place, has as a main object the elimination of falsework and temporary supports which are always a nuisance for navigation (for bridges over river) or for road traffic. This method is basically a progressive construction of a cantilever in segments and tying them through the medium of prestressing to the segments already completed. Generally, the cantilevers are constructed from the pier simultaneously on both sides, so that the unbalanced moment during construction is kept to the minimum. Sometimes the shore cantilever may have to be supported on the abutment or may have to be shortened as compared to the river side cantilever. The latter case may require provision of extra dead weight with non-structural concrete. Cantilever construction has also been possible with the cantilever on one side only, with the shore side being a short cantilever which is anchored to the ground by means of tendons or effectively counterbalanced by a counter-weight block. At midspan, the cantilevers can be made continuous or connected by hinges.
Prestressed concrete members undergo time-dependent deformations as a result of creep and shrinkage of the concrete and relaxation of the prestressing reinforcement. The rates of creep and shrinkage of the concrete and relaxation of the prestressing steel are greatest during the early ages and decrease continuously with time (under constant environmental conditions). The rate of long-time deformations of concrete members also diminishes with time, but measurable deformations occur for many years. Specifications about prestress losses are based largely on consideration of research results. Precise determination of prestress losses in a given situation is very complex and requires detailed information on the materials to be used, methods of curing, environmental conditions, and other detailed construction information that usually is not available to designers. For large and/or special structures, such as segmental cantilever bridges, it may be necessary to obtain this information in order to maintain control of the geometry of the bridge during construction so that different cantilevers can meet at correct grade and alignment. This high performance skill is the privilege of only a few firms, who can ensure systematic construction procedures, accurate pre-planning, and careful follow-up week by week, and keep secret in their files all the data obtained in the field.

1.2 Previous Studies

Not many reports have been published on the time-dependent behavior of segmental cantilever bridges, and all of the known studies are at least briefly discussed in this section.
A study of the long-term deflections of segmental cantilevered concrete bridges connected by hinges at midspan was done by U. Keijer (14, 15). That work includes a parameter variation study where the influence of different creep and shrinkage coefficients, modulus of elasticity of concrete, modulus of elasticity and relaxation of the prestressing steel, density of concrete, weight of the form work, and the initial prestressing force on deflections were considered. The conclusions drawn were that the creep coefficient and the shape of the concrete creep-time curve have the dominating influences on the magnitude and development of the deflections, and that the other parameters affect mainly the deformations during the period of construction. It was also found that when a creep function containing a logarithm function is assumed, good agreement between the observed and calculated values of deflections can be obtained.

In Keijer's investigation, creep and shrinkage strains are obtained from equations as a function of time, which simplifies computations, especially if several creep curves are to be used. The program has a handicap of not being able to handle arbitrary creep and shrinkage strains from field data tests. The creep function was modified so the calculated time-dependent deflections fitted the measured. No parameter variation of the wobble and curvature coefficient, or of the number of creep curves was done, and the losses were not studied. An attempt to obtain all the field data of the bridges used in Keijer's work did not succeed, as he no longer had easy access to the data.

According to Reference 16, many measurements of creep and shrinkage strains over long periods of time, and deflection measurements of several segmental cantilevered concrete bridges, have been taken in Japan.
Unfortunately, the purpose of those measurements have been to estimate exact values of creep and shrinkage, that is, final values of shrinkage, and values of creep coefficient. The structures considered are inadequately described, and the measured values consequently cannot be compared with computed values.

In Reference 9, a method of computing displacements and losses in multistage prestressed members is presented and can be applied to the calculation of losses and deflections of segmental cantilever concrete bridges. A computer program based on that method of analysis has been developed (8). Since this computer program was not developed with the single purpose of being used for segmental cantilever bridges, its use demands too many hand computations before the input data can be given, that is, it makes necessary the computation of the prestress force and dead load moment at each section of the bridge. It can not handle field data of creep and shrinkage, neither computes deflections. Furthermore, only handles bridges with prismatic sections which make the program obsolete for any but the shortest spans.

The University of Texas at Austin has published various reports about problems related with segmental cantilever bridges. The Corpus Christi Bridge (12) has been studied there in detail, and thanks to Dr. John E. Breen, a copy of the construction plans was obtained and that data was used in this investigation. S. Kashima (13), also from the University of Texas, built and load tested of a model of the Corpus Christi bridge, but time-dependent deformations were not taken into account.

In France, M. Belmain and Y. Le Bourdelles (6), did a study of a cantilever concrete bridge over the Oise River. This work was directed
to study shrinkage and creep strains in this type of bridges without getting involved with prestress losses and deflections. Therefore, their conclusions were about how good C.E.B. (7) and the French Code are in predicting creep and shrinkage. They reported some creep and shrinkage data obtained from field tests that have been used in this investigation.

Finally, there is a book written by Y. Guyon (11) that has a very interesting chapter about this type of bridges, and where some data used in this investigation was obtained.

1.3 Object and Scope of Investigation

The main object of this investigation is to give an exposition of the nature of the time-dependent deformations and prestress losses in segmental cantilever bridges connected by hinges at midspan, using precast and cast-in-place segments and subjected to either fluctuating or constant environmental conditions.

The scope of the investigation may be divided in two parts:
1. The development of a computer program and
2. Analytical study of the time-dependent deformations and prestress losses.

The computer program is based on:
1. A rational method of analysis for predicting the time-dependent stresses, strains, loss of prestress and deflections of a segmental cantilever bridge under any environmental conditions. Both rate of creep and superposition methods for predicting creep strains under variable stress conditions are included.
The analytical study includes:

1. A variation of the different parameters to determine the sensitivity of the creep, shrinkage and prestress losses, and deformations of the structure. The major parameters investigated in this study were: curvature and wobble coefficients, age of the segments, type of prestressing strands (stress-relieved strands and low-relaxation strands), environmental conditions, modulus of elasticity of concrete, and loads applied during construction.
2. METHOD OF ANALYSIS

2.1 Introduction

The prestress force applied to prestressed concrete members decreases continuously during the lifetime of the structure as a result of the combined effects of creep and shrinkage strains of the concrete and relaxation of the steel. Of the numerous factors affecting the magnitude of creep, only the effect of stress will be considered.

There are two distinct methods of analysis suitable for calculating creep under variable stress in concrete structures:

1. The rate of creep method, and
2. The superposition method.

In the following, both the rate of creep and the superposition methods will be briefly described in terms of time-dependent stress-strain relationships in concrete members.

An iterative solution of the differential equation of stress and strain is introduced, and a suitable model for calculating stresses, strains, and the time-dependent deformations of the structure, in a form useful for computer programming, is presented. The details of the computer program are presented in Appendix B.

2.2 The Rate of Creep Method

If the variation of a unit creep curve (creep caused by a unit stress) with time is defined as \( \frac{d \varepsilon_{\text{cre}}(t)}{dt} \), the change in creep strain of a concrete element subjected to a stress \( f(t) \), for a time interval \( dt \), within the working stress range, is assumed to be \( f(t) \frac{d \varepsilon_{\text{cre}}(t)}{dt} \). Therefore, creep under variable stress can be expressed as:
\[ \varepsilon_{cr}(t) = \int_{0}^{t} f(t) \frac{d \varepsilon_{cru}(t)}{dt} \, dt \]  

\( f(t) \) and \( \frac{d \varepsilon_{cru}(t)}{dt} \) are independent variables

where:

\( \varepsilon_{cr}(t) = \) total creep strain at time \( t \),
\( \varepsilon_{cru}(t) = \) unit or specific creep strain as a function of time, \( i.e., \) creep strain at time \( t \) caused by a constant unit stress, and
\( f(t) = \) concrete stress as a function of time.

This method (22) integrates all elementary increments of creep, each of which is computed from the unit creep curve and the particular stress acting during the applicable element of time. This method disregards the entire earlier history of the stress because it is assumed that even when the magnitude of stress changes with time, the concrete will creep at the rate \( f(t) \frac{d \varepsilon_{cru}}{dt} \). In the case when the stress is removed, \( f(t) \frac{d \varepsilon_{cru}}{dt} \) is equal to zero which means that concrete does not undergo any creep recovery with time after removal of the stress; concrete, however, experiments some creep recovery. This method is expected to underestimate creep under increasing stress and vice versa. For small changes in stress, this method will still give reliable answers, but for large variation of stresses, such as stresses induced by large loads applied at later times, the errors involved may be appreciable, unless corresponding creep versus time curves are used.
2.3 Superposition Method

This method, based on McHenry's hypothesis (18), states that the creep strains produced in concrete at any time \( t \) by an increment of stress applied at any time \( t_0 \) are independent of the effects of any other stress increment applied either earlier or later than \( t_0 \). This method of analysis predicts creep recovery and generates stress-strain curves of a shape similar to experimental results. It is assumed that the concrete creeps in tension at the same rate as it creeps in compression. Creep under variable stress can be obtained by superimposing appropriate creep curves introduced for corresponding changes in stress at different time intervals. This is true if creep is proportional to applied stress. By making use of this method, the time-dependent linear relation between stress and creep strain can be written in the form of

\[
\varepsilon_{cr}(t) = f_0 \varepsilon_{cr}(t, t_0) + \sum_{t_0}^t \Delta F(t_i) \varepsilon_{cr}(t, t_i) \quad (2.2)
\]

where:

\( f_0 \) = initial stress in concrete at the time of first loading, \( t_0 \),

\( \varepsilon_{cr}(t, t_0) \) = unit creep strain at time \( t \), for concrete loaded at the age \( t_0 \),

\( \Delta F(t_i) \) = additional stress increments or decrements applied at time \( t_0 < t_i < t \),

\( \varepsilon_{cr}(t, t_i) \) = unit creep strain at time \( t \) for concrete loaded at age \( t_i > t_0 \).
This method does take into account the entire history of the loading, and it is expected to overestimate creep under increasing stress and vice-versa. There are two practical disadvantages to the use of this method. First, it requires the time-dependent unit creep curves for the concrete loaded at different ages, which implies a large number of tests. Second, the amount of numerical calculation involved limits its application unless a computer is available.

2.4 Prediction Methods for Creep and Shrinkage

No matter what method of analysis is used to compute the time-dependent losses and deformations in concrete structures, creep and shrinkage strains and relaxation of the steel versus time relationship are assumed to be known and can be considered as step functions or can be expressed as equations. When no creep and shrinkage test data for a specific structure are available, a preliminary prediction of creep and shrinkage values at different times is required, and therefore the reliability of the method of analysis will greatly depend upon the method of predicting creep and shrinkage strains, and relaxation of the steel.

Several methods have been suggested for predicting the creep and shrinkage values of concrete, all of them being empirical. Since the discussion of these methods is not the purpose of this investigation, just two of them will be mentioned here, being the second method the one used throughout this study. They are described below.
(I) ACI-Committee 209 Recommendations.

The ACI-Committee 209, Subcommittee 2 (3) has presented a method for predicting creep, shrinkage and temperature effects in concrete members. The method is generally consistent with the ACI (4), and represents a nominal approach for design purposes, and does not give exact results by any means. According to the Committee the procedure of this method is as follows:

A) Strength and Elastic Properties

The concrete strength versus time relationship is based on the cement type, age, and curing conditions. The following equations approximate average values:

Moist cured concrete, type I cement

\[ f'_{c}(t) = \frac{t}{4.0 + 0.85 t} \quad f'_{c}(28) \quad (2.3) \]

Moist cured concrete, type III cement

\[ f'_{c}(t) = \frac{t}{2.3 + 0.92 t} \quad f'_{c}(28) \quad (2.4) \]

Steam cured concrete, type I cement

\[ f'_{c}(t) = \frac{t}{1.0 + 0.95 t} \quad f'_{c}(28) \quad (2.5) \]

Steam cured concrete, type III cement

\[ f'_{c}(t) = \frac{t}{0.7 + 0.98 t} \quad f'_{c}(28) \quad (2.6) \]
The modulus of elasticity is computed from the equation

$$E_c(t) = 33.0 W^{1.5} \sqrt{f'_c(t)} \text{ psi} \quad (2.7)$$

where

- $f'_c(t) = $ concrete strength at time $t$, in lbs/in.$^2$,
- $t = $ age of the concrete in days,
- $E_c(t) = $ modulus of elasticity at time $t$, in lbs/in.$^2$, and
- $W = $ unit weight of the concrete in lbs/cu ft.

B) Creep

The following equation gives creep strain versus time relationship

$$\varepsilon_{cr}(t) = \frac{f_0}{E_{co}} \frac{t^{0.6}}{10 + t^{0.6}} C_{cu} \quad (2.8)$$

where

- $E_{co} = $ modulus of elasticity of concrete at the time of initial loading, and
- $C_{cu} = $ ultimate creep coefficient, having an average value of 2.35.

This is the standard creep equation for a concrete satisfying the following conditions; A slump of 4 in. or less, under a relative humidity of 40 percent, a minimum thickness of member of 6 in. or less, and loading age of 7 days for moist cured, and 1 to 3 days for steam cured.
C) Shrinkage

The following equation gives shrinkage strains versus time relationship

\[ \varepsilon_{sh}(t) = \frac{t}{a + t} \varepsilon_{shu} \]  \hspace{1cm} (2.9)

where \( \varepsilon_{sh}(t) \) = shrinkage strain at time \( t \),

\[ a = \text{a constant having a value of 35.0 any time after 7 days of age for moist cured concrete, and 55.0 after 1-3 days of age for steam cured concrete.} \]

\[ \varepsilon_{shu} = \text{ultimate shrinkage strain, having a value of 800 x } 10^{-6} \text{ in./in. for moist cured concrete, and 730 x } 10^{-6} \text{ in./in. for steam cured concrete.} \]

This is the standard shrinkage equation for concrete under the same conditions as the ones mentioned for the standard creep equation. When these conditions are not satisfied, the ultimate creep coefficient and the ultimate shrinkage strain values are affected by correction factors that depend on the following variables:

a. Time of initial loading and time initial shrinkage is considered.

b. Relative humidity of environment.

c. Minimum thickness of member.

d. Slump of the mix.

e. Cement content.

f. Percentage of fines in mix.

g. Percentage of entrained air in concrete.
(II) European Concrete Committee (C.E.B.) Recommendations

As mentioned earlier, this investigation is based on the 1970 C.E.B. recommendations (7) to predict creep and shrinkage strains versus time relationship, so that, time-dependent deformations of concrete members can be computed. The details of this method are as follows:

A. Strength and Elastic Properties

The compressive concrete strength is obtained from tests at required ages on cylindrical specimens. When experimental evidence of compressive strength is not available at different times, C.E.B. gives a table, for normal concrete and temperatures (15-20°C), that contains the values of the ratios between the compressive strength at an age of t days and the compressive strength at an age of 28 days, these values have been plotted in a graph form (see Fig. 2.1).

Once the compressive strength at a time t is known, the tangential modulus of elasticity is computed from the equation

\[ E_c(t) = 66,000 \sqrt{f_c'(t)} \]  \hspace{1cm} (2.10)

where

\[ E_c(t) \text{ and } f_c'(t) \text{ are in N/cm}^2 \]

or

\[ E_c(t) = 79,400 \sqrt{f_c'(t)} \]  \hspace{1cm} (2.11)

where

\[ E_c(t) \text{ and } f_c'(t) \text{ are in lbs/in.}^2. \]
For deformations under prolonged loads and for normal aggregate concretes and lightweight aggregate concretes, the secant modulus must be used instead, being this equal to 90 percent of the tangent modulus.

B) Creep

According to the C.E.B. recommendations, the magnitude of deformations due to creep under working conditions can be evaluated making use of the theory of linear creep, which for a constant stress leads to the estimation of the actual creep strain using the equation

$$\varepsilon_{cr}(t) = \frac{f_0}{E_{cs}(28)} \phi(t)$$

(2.12)

where

$$E_{cs}(28) = \text{secant modulus of concrete at 28 days, and}$$

$$\phi(t) = \text{creep coefficient, is expressed as the product of five partial creep factors}$$

$$\phi(t) = K_c K_d K_b K_e K_t$$

(2.13)

where

$$K_c = \text{creep factor, depends on the relative humidity of air (see Fig. 2.2),}$$

$$K_d = \text{creep factor, depends on the age of the concrete at the time of loading and the type of cement at a temperature of 20°C (see Fig. 2.3),}$$

$$K_b = \text{creep factor, depends on the composition of the concrete in terms of cement content and water-cement ratio (see Fig. 2.4),}$$
$K_e = \text{creep factor, depends on the theoretical thickness of the cross section of the member, } d_m \text{ (see Fig. 2.5), and}$

$K_t = \text{creep factor, gives the rate at which the creep strain will develop with time, depending on the theoretical thickness of the cross section of the member (see Fig. 2.6).}$

If the concrete hardens at temperatures, $T$, other than 20°C, the age of concrete at time of loading should be modified by the corresponding degree of hardening using the following equation:

$$D = \sum \Delta t (T + 10^\circ) \quad (2.14)$$

where:

- $D = \text{represents the degree of hardening at the moment of loading, an equivalent age in days (see Fig. 2.3), and}$
- $\Delta t = \text{represents the number of days during which hardening has taken place at } T^\circ \text{C.}$

The theoretical thickness, $d_m$, takes the effects of size and shape of the member on creep and shrinkage into account, and is defined as follows:

$$d_m = \frac{\text{Area of cross section}}{\frac{1}{2} \text{(Perimeter exposed to the atmosphere)}} \quad (2.15)$$

C) Shrinkage

The C.E.B. recommendations evaluates the shrinkage strains in concrete, under constant environmental conditions and at any instant, as the product of five partial shrinkage factors:
\[ \varepsilon_{sh}(t) = \varepsilon_c K_b K_e K_p K_t \] (2.16)

where:

\( \varepsilon_c \) = shrinkage factor, depends on the relative humidity of the air (see Fig. 2.7),

\( K_b \) = shrinkage factor, depends on the composition of the concrete in terms of cement content and water-cement ratio. Same as \( K_b \) creep factor (see Fig. 2.3),

\( K_e \) = shrinkage factor, depends on the theoretical thickness of the cross section of the member, \( d_m \) (see Fig. 2.8),

\( K_p \) = shrinkage factor, depends on the geometric percentage (p) of longitudinal reinforcement area \( (A_{st}) \) with respect to the cross-sectional area of the member \( (A_c) \)

\[ p = 100 \frac{A_{st}}{A_c} \], and

\[ K_p = \frac{100}{100 + np} \]

\( n = 20 \) with regard to the effects of creep, and

\( K_t \) = shrinkage factor, gives the rate at which the shrinkage strain will develop with time, depending on the theoretical thickness of the cross section of the member. Same as \( K_t \) creep factor (see Fig. 2.6).

These two methods of predicting creep and shrinkage in concrete members have been compared and some cases of disagreement were found (19). There is also a very significant difference in their procedures that lies in the prediction of the rate at which creep and shrinkage occur.
2.5 Relaxation of Prestressing Reinforcement

Relaxation is defined as the loss in stress in prestressing reinforcement that occurs at constant strain. Relaxation characteristics of prestressing reinforcement are of interest in prestressed concrete design, even though pure relaxation does not exist under practical conditions since prestressing reinforcement is not subjected to constant strain, but it is generally agreed that conditions in a beam closely approximate this condition. At present, a knowledge of the losses resulting from relaxation is required primarily in relation to the serviceability of a prestressed member. The amount of relaxation loss to be expected in a given steel depends on several factors, of which the following can be mentioned: type of prestressing reinforcement, ratio of initial stress to yield stress, time, and temperature. Despite the large variations that may take place in the amount of relaxation loss, it is possible to estimate this loss with sufficient accuracy for design purposes.

In this investigation, the loss of stress due to relaxation for stress-relieved strands are estimated using the equation developed by Magura, Sozen and Siess (17). This equation gives relaxation losses versus time relationship including both initial stress and yield stress as variables too, and assuming a relatively constant temperature of about 70°F. The equation has the following form:

\[ f_s(t) = f_{si} \left[ 1 - \frac{\log t}{c} \left( \frac{f_{si}}{f_y} - 0.55 \right) \right] \] (2.17)

where:
\[ f_s(t) = \text{stress on the prestressing reinforcement at time } t, \]
\[ f_{si} = \text{the initial steel stress,} \]
\[ f_y = \text{steel yield stress, measured at an offset strain of } 0.001, \]
\[ t_s = \text{time after stressing, in hours,} \]
\[ \log t_s = \text{logarithm of time } t_s \text{ base 10, and} \]
\[ c = \text{relaxation constant equal to 10.0 (for stress-relieved strand).} \]

In this equation the critical parameter appears to be the ratio of initial stress to the steel yield stress. When this ratio is less than 0.55, the relaxation loss is very small and can be neglected for practical purposes. Relaxation continues for many years although the rate is reduced greatly soon after stressing. Temperature variations can have a critical effect on relaxation if the range is abnormally high, however, under ordinary working conditions this variable can be ignored.

The steel industry is trying to produce prestressing reinforcement exhibiting a negligible relaxation loss; so far, the steel manufacturers (21) have come out with a new kind of strand that shows lower relaxation losses than those obtained using Eq. 2.17 with a relaxation constant equal to 10.0. They claim that the constant in Eq. 2.17 should be 45.0. In addition, this steel has a higher yield stress which also lowers relaxation losses.

In prestressing concrete members, changes of the prestressing force continuously take place because of the creep and shrinkage of the concrete and relaxation of the steel itself. In cantilever construction, the prestressing reinforcement suffers big instantaneous changes of stresses caused by the prestressing steel that is going to hold subsequent segments.
All these changes must be taken into account for a better estimation of the long-time relaxation losses. The way that this is done is explained in Appendix B.

2.6 Assumptions and Definitions

The assumptions of this analysis are concerned mainly with the parameters related to the material properties and the environment, and are as follows:

1. Concrete has a linear stress-strain relationship under short-time loading at the stress level of interest.
2. Steel has a linear stress-strain relationship under short-time loading.
3. The modulus of elasticity of the concrete versus time relationship is known and can be considered as a step function.
4. The modulus of elasticity of the steel is known.
5. Strains are linearly distributed over the depth of the cross section of the member.
6. Creep strains are proportional to the stresses up to 40 percent of the concrete strength.
7. Shrinkage strains are distributed uniformly over the depth of the cross section of the member.
8. Elastic strains and creep and shrinkage strains in concrete are additive phenomena within the elastic range, that is, the principle of superposition can be applied to obtain the total strains and stresses at different time intervals.
9. The unit creep strain versus time relationship for constant stress in the concrete is known at every time that a new
segment is built (up to 9 segments), and can be considered as a step function.

10. The shrinkage strain versus time relationship, for the concrete, is known and is the same for each segment, and can be considered as a step function.

11. The number of different unit creep strain versus time relationships for constant stress are the same for each segment, and in the same order.

12. Stress in concrete is constant during each time interval.

13. Stress in steel is constant during each time interval.

14. The relaxation of the steel versus time relationship is known and can be considered as step function.

15. The effects of nontensioned steel on creep and shrinkage strains of the concrete are neglected.

16. Sequence of casting, curing and stressing of each segment are known.

17. Adequate bond between concrete and post-tensioned steel exists so that changes in strain in steel are equal to the changes in strain in the concrete at the same level.

2.7 Numerical Integration Procedure

The determination of prestress losses, in prestressed concrete members, requires detailed information on the materials to be used, methods of curing, environmental conditions, and construction methods. The difficulty in estimating prestress losses lies in that some of the losses are function
of both time and level of stress in the concrete and steel. This relationship cannot be represented in an algebraic formula, since the stress is constantly changing as a result of the losses in prestress that are occurring, which makes it impossible to obtain a closed form solution. Consequently, the most accurate method of computing stress loss is to employ a numerical integration procedure that takes into account the different variables.

In this study, the integration procedure is based on the rate of creep method. This method is treated as a step by step numerical procedure which converts the creep, shrinkage and relaxation versus time relationship into step functions having independent effects, and satisfying the assumptions outlined in Section 2.6.

The computer program developed in this investigation is described in Appendix B. It uses an iterative procedure similar to the one mentioned by the ACI Comm. 435 (5) and is based on the rate of creep method. The procedure is also able to handle different numbers of creep curves, so the revised rate of creep method, which is essentially a small modification of the rate of creep method made by Mossiossian and Gamble (20), and the superposition method can be applied. The program analyzes segmental concrete bridges constructed using the balanced cantilever method.

The following steps of the numerical integration for the time interval being considered are carried out for each section of the constructed segments. The term "steel" means each steel cable that is present at each section of the constructed segments during that time interval.

The numerical integration, starting at the time of transfer of pre-stress can be summarized in the following steps:
For the first time interval:
1. First segment is built.
2. Compute steel stresses taking the losses caused by wobble and curvature coefficients into account.
3. Compute moments caused by the steel stresses found in step 2, dead load, and construction loads.
4. Compute total concrete stresses at the end of the first time interval by using beam theory.
5. Compute total concrete strains at the end of the first time interval by dividing the total stresses found in step 4 by the corresponding modulus of elasticity of concrete.

For each later time interval:
6. Compute corresponding incremental creep and shrinkage strains at the level of steel due to stresses caused by prestressing force, and dead and construction loads.
7. Compute prestress losses due to creep and shrinkage of the concrete as the product of change of concrete strain at the level of the steel found in step 6 times the modulus of elasticity of the steel.
8. Compute prestress losses due to relaxation of the steel.
9. Compute the total prestress losses by adding the prestress losses due to creep and shrinkage of the concrete found in step 7 to the prestress losses due to relaxation of the steel in step 8.
10. Compute the elastic change of concrete stress by considering the total prestress losses found in step 9 as a load equal to the stress found in step 9 multiplied by the steel area and applied to the cross section at the center of gravity of the steel.

11. Update the total stresses by adding algebraically the elastic change of stress found in step 10 to the total stresses existing at the beginning of time interval.

12. Compute the elastic change in strain by dividing the elastic change of stress found in step 10 by the corresponding modulus of elasticity of the concrete.

13. Compute the net change in strain by adding algebraically the elastic change in strain found in step 12 and the incremental creep and shrinkage strain found in step 6.

14. Update the total strain by adding algebraically the net change in strain found in step 13 to the total strain existing at the beginning of the interval.

If a new segment is not built during this time interval, steps 11 and 14 are the total stresses and strains respectively at the end of this interval, so consider next time interval and proceed as before, beginning from step 6, otherwise, go to next step:

15. Compute prestress force, and the incremental moments due to the prestress force, dead and construction loads caused by erecting the new segment.

16. Compute the elastic change of concrete stress caused by the forces found in step 15.
17. Compute the elastic change in strain by dividing the change of stress found in step 16 by the corresponding modulus of elasticity of the concrete. This change is called the elastic shortening.

18. Compute total stress losses caused by the elastic shortening by multiplying the elastic change in strain at the level of the steel found in step 17 times the modulus of elasticity of the steel.

19. Compute the elastic change of concrete stress by considering the total prestress losses found in step 18 as a load equal to the stress found in step 18 multiplied by the steel area applied at the center of gravity of the steel.

20. Compute the elastic change in strain by dividing the elastic change of stress found in step 19 by the corresponding modulus of elasticity of the concrete.

21. Compute the net elastic change of concrete stress by adding algebraically the changes of stresses found in steps 16 and 19.

22. Compute the net elastic change in strain by adding algebraically the change of strains found in steps 17 and 20.

23. Compute the total stresses at the end of time interval by adding algebraically the net elastic change of stress found in step 21 to the total stresses existing in step 11.

24. Compute the total strains at the end of time interval by adding algebraically the net elastic change of strain found in step 22 to the total strains existing in step 14.
25. Consider next time interval and proceed as before, beginning from step 6.

Once total strains are found, curvatures and deflections are easily computed.
3. PRESENTATION AND DISCUSSION OF FACTORS AFFECTING TIME-DEPENDENT BEHAVIOR

3.1 General

The time-dependent behavior of precast prestressed concrete members is affected by many factors in different ways. Creep and shrinkage of the concrete and relaxation of the prestressing steel have long been known to be the main contributors of this time-dependent behavior and prestress losses.

In cantilever segmental concrete bridges, the time-dependent deformations and prestress losses begin as soon as the prestressing strands are anchored in the concrete. These deformations develop due to the influence of two effects: the sustained transverse loading which tends to deflect the cantilever downward, and the effect of the prestressing force which tends to deflect it upward. From then on, deformations and prestress losses are affected by the creep and shrinkage of concrete and relaxation of the steel, with these phenomena occurring simultaneously and affecting each other continuously throughout the life of the structure. However, for purpose of analysis, it will be assumed that the principle of superposition is valid and the single effects can be treated individually, as long as all the other time-dependent variables are included and taken as reasonable values.

Although the main parameters influencing the time-dependent deformations and prestress losses of prestressed concrete structures are creep and shrinkage of the concrete and relaxation of the prestressing steel, there are some important environmental and construction factors which affect the magnitudes and rates of creep and shrinkage of the concrete and relaxation of the steel, which in turn will influence the time-dependent behavior of these structures. Environmental factors such as humidity and temperature.
have a great effect in the rates and magnitudes of creep and shrinkage of the concrete. Construction factors greatly influencing the magnitude of the creep and shrinkage of concrete are the age of concrete at the time of construction, loads used for construction purposes, and duration and type of curing; those having a significant influence on the relaxation losses are the wobble and curvature coefficients of friction between the tendons and the ducts.

In order to obtain a better understanding of the time-dependent behavior of segmental cantilever concrete bridges, it is important to know how each of these factors affects the magnitudes of the time-dependent deformations and prestress losses.

The effects studied and discussed in this investigation are the environmental conditions, type of prestressing strands, age of the segments at the time of construction, wobble and curvature coefficients, loads used for construction purposes, and because in this type of bridge construction there are substantial changes in stresses each time that a new segment is erected, the variation of modulus of elasticity of concrete and the number of creep curves used were included. Although in this type of construction the elastic losses might have a great influence on the relaxation losses, they were not studied particularly, but they obviously were taken into account in the analysis.

The Corpus Christi and Oise Bridges (see Appendix A) have been analyzed to illustrate the effects of those parameters mentioned above.

In this study, the standard conditions for the analysis of the Corpus Christi bridge are based on a wobble coefficient of 0.0002/ft, a curvature coefficient of 0.25, zero for the construction load, stress-relieved strand type for relaxation, 70 percent relative humidity for creep
and shrinkage computed according to the 1970 C.E.B. recommendations, and the segments being 30 days old at the time of post-tensioning.

The standard conditions for the Oise bridge are based on a wobble coefficient of 0.0015/ft, a curvature coefficient of 0.25, zero for construction load, 70 percent relative humidity for creep and shrinkage computed according to the 1970 C.E.B. recommendations, and the segments being 7 days old at the time of post-tensioning. No relaxation losses occurred in this bridge because of the low initial steel stresses.

The modulus of elasticity of concrete used for the standard conditions in both bridges was computed according to the C.E.B. recommendations (see Eq. 2.11).

In connection with most of the figures a theoretical axial force is mentioned and can be different for each section. Its value is equal to the sum of the forces obtained by multiplying the final area of steel going through the section in question by its original anchor stress, without friction losses. Also, when it is said that a certain loss increases it means an increase in the loss relative to the theoretical axial force.

Since the way that stresses in the concrete are built up plays a very important role in this chapter, Figs. 3.1 and 3.2 show then at two different sections for each bridge, using the standard conditions mentioned earlier. The stresses at the top and bottom of each section are plotted versus time, and illustrate the major changes in stress during construction followed by the relatively minor changes in stress in the several years following completion of the cantilevers.

Throughout this study, compressive stresses and strains are positive. Curvatures, where the top shortens relative to the bottom, are also positive, as well as the upward deflections.
3.2 Effects of Creep

Creep is a time-dependent deformation that increases strains in concrete with time, even at constant stress. These strains lead to decreases in strain in the prestressed steel and to reductions in the prestressing steel force as a consequence. These two effects cause changes in curvature, which in turn change the deflections. The influence on creep of the relative humidity of the air in which the concrete is stored has been known for some time, and it is one of the main factors affecting its magnitude.

According to the C.E.B. recommendations, the relative humidity affects the creep strains obtained from Eq. 2.13 through the coefficient $K_C$ (see also Fig. 2.2). These strains are for constant environmental conditions.

The effects of changing the creep values (by changing the relative humidity) on creep losses at the sections studied are shown in Figs. 3.3 and 3.4. It can be seen that the lower the relative humidity, the greater the creep losses. The creep losses decrease as the section studied moves away from the pier. There are some differences in the initial shapes of these curves for the two bridges, because of the rates at which the stresses are built up.

In the Corpus Christi Bridge, the decrease of the relative humidity from 80 to 50 percent causes a decrease in the axial prestress force of 36 kips, which represents a drop from 80.3 to 79.8 percent of the theoretical axial force. The effect on the relaxation losses consists in a change of
8 kips, which represents a decrease in the relaxation losses from 7.8 to 7.7 percent of the theoretical axial force. These changes in the creep and relaxation losses decrease the concrete stress at the top from 538 psi to 524 psi, and increase the one at the bottom from 1552 psi to 1555 psi. In the Oise Bridge, this decrease in the relative humidity causes a diminishing of 292.0 kips in the axial prestress force, which represents a drop from 75.7 to 74.3 percent of the theoretical axial force. As mentioned before, there is no relaxation loss in this bridge. These changes, decrease the stress at the top from 734 psi to 688 psi, and increase those at the bottom from 1040 psi to 1048 psi. All of these data are at the section over the pier and 2000 days after the construction started; at any other section the changes are smaller.

Analyzing all these data, it can be said that the effect of creep on the axial prestress force and relaxation losses in the precast Corpus Christi Bridge can be neglected in terms of losses for practical purposes. The same can not be said about the cast-in-place Oise Bridge, where the change in axial prestress force represents a higher percentage of loss of the theoretical axial force, and the change in stress at the top of the section is also high.

The effect of creep on deflections in two of the sections analyzed can be observed in Fig. 3.5, and can be said that the higher the relative humidity the smaller the change in deflections with time, and that the change in relative humidity is more critical for the Oise Bridge. Figures 3.6 and 3.7 also show the effect on creep on deflections of the bridges as construction progresses and at 2000 days after construction started. It can be seen that during construction the higher the relative humidity the
higher the deflection, but at 2000 days after construction started, the
higher the relative humidity the smaller the deflection. This is caused
by the way at which stresses are built up. The differences during con­
struction are never large, but they become more important after long
periods of time.

3.3 Effects of Shrinkage

Shrinkage strains are usually assumed to be uniformly distributed
over the depth of the cross section and along the length of the member,
and to be independent of stress and creep for design purposes. Consequently,
shrinkage will primarily affect the loss of prestress, which in turn will
affect the time-dependent behavior of the bridge in a degree that will de­
pend on the eccentricity of the strands. While there are many factors
affecting shrinkage characteristics of concrete in different degrees, one
of the most influential factors is the relative humidity of the air, which
is varied in this study to obtain shrinkage variations.

According to the C.E.B. recommendations, the relative humidity,
under constant climatic conditions, affects shrinkage strains obtained
from Eq. 2.16, through the basic shrinkage coefficient \( \varepsilon_c \) (see also Fig.
2.7).

The effects of changing the relative humidity for shrinkage,
while maintaining a constant relative humidity of 70 percent for creep, on
shrinkage losses can be observed in Figs. 3.8 and 3.9. It can be said that
the higher the relative humidity the lower the shrinkage losses. This
variation of relative humidity affects both bridges in the same proportion,
because of the fact that their theoretical thicknesses are very close to the same value. Also, for a constant relative humidity, the further the section is from the pier the higher the shrinkage losses because of the higher capacity to shrink that the section has after having all the pre-stressing steel in. Therefore, the last section built will have the highest percentage of shrinkage losses, where the loss is expressed as a percentage of the theoretical axial force for that section.

In the Corpus Christi Bridge, at the section over the pier, the effect of reducing the relative humidity for shrinkage from 80 to 50 percent causes a drop in the creep losses of 2 kips, which represents a decrease in creep losses from 1.65 to 1.63 percent of the theoretical axial force. The relaxation losses are affected more than the creep losses with this variation in shrinkage strains, having a drop of 8 kips, which represents a decrease in relaxation losses from 7.92 to 7.82 percent of the theoretical axial force. The axial prestress force is the most affected, as it was pointed out before, decreasing its value by 47 kips, which represents a drop in axial prestress force from 80.7 to 80.0 percent of the theoretical axial force. These changes decrease the stresses at the top from 549 to 531 psi, and increase the ones at the bottom from 1550 to 1554 psi. The difference in percentages of all these changes, except creep losses and stresses, are not much higher as the section analyzed moves away from the pier. The higher change of percentage in relaxation losses can be explained by the fact that steel stresses are higher as the section moves away from the pier since friction losses are smaller. The higher percentage of change in shrinkage losses is caused, as mentioned before, by the higher...
capacity of the section to shrink. The higher change of percentage in axial prestress force is mainly caused by that higher shrinkage capacity of the section. In Fig. 3.1 the great variation of stresses as the section moves away from the pier can be observed. Since creep losses are proportional to stresses, this variation explains the smaller change of percentage in creep losses as the section moves away from the pier. The inertia and area of the sections along the bridge do not vary that much, while the prestress axial force decreases drastically as the section moves away from the pier so changes in prestress axial force represents smaller variation of stresses in the concrete.

In the Oise Bridge, at the section over the pier, this effect of reducing the relative humidity for shrinkage causes a reduction in creep losses of 24 kips, which represents a decrease in creep losses from 4.5 to 4.3 percent of the theoretical axial force. The axial prestress force is affected by a drop of 302 kips, which represents a decrease from 75.8 to 74.2 percent of the theoretical axial force. These changes decrease the stresses at the top from 736 to 689 psi, and increase the ones at the bottom from 1040 to 1048 psi. The difference in percentages of all these changes, except creep losses and stresses, are not much higher as the section analyzed moves away from the pier. The explanation for this can be the same as the one given for the Corpus Christi Bridge.

Analyzing all these data, it can be said that the effects of varying the relative humidity for shrinkage on the relaxation losses in the Corpus Christi Bridge is not much but can not be neglected. The effect on creep losses can be neglected in the Corpus Christi Bridge but not in the Oise Bridge. The effect on axial force is high in both bridges, as expected,
being higher in the Oise Bridge. All of these data are obtained 2000 days after construction of the bridges started.

The effect of shrinkage on deflections in two of the analyzed sections can be observed in Fig. 3.10, and it can be seen that the higher the relative humidity the smaller the change in deflections, and that the effect is higher in the Oise Bridge. All sections of both bridges were studied, and it can be said that shrinkage did not affect deflections during construction.

3.4 Effects of Relaxation

In addition to creep and shrinkage of concrete, relaxation of the prestressing steel is also an important parameter having an influence on the time-dependent behavior of prestress concrete structures.

In this investigation only two types of prestressing strands have been considered: Stress-relieved strands and low-relaxation strands. Both types are Grade 270.

The relaxation losses of the prestressing steel are predicted using Eq. 2.17, which is based on the results of constant-strain relaxation tests, and where the relaxation constant $C$ has a different value for each type of prestressing strand, as discussed in Art. 2.5.

In the Corpus Christi Bridge, the effect of each type of prestressing strands on the relaxation losses can be seen in Fig. 3.11. The advantages of the low-relaxation strands are quite obvious, with a reduction in relaxation losses of more than 6 percent of the theoretical axial force at the section over the pier. The relaxation losses, for each type of prestressing strands, are higher as the section analyzed moves away from
the pier. Therefore, the last section built will have the highest percentage in relaxation losses, when the loss is expressed as a percentage of the theoretical axial force for that section.

Higher prestress level implies larger creep losses; therefore, creep losses are expected to be higher when low-relaxation strands are used. This is corroborated by Fig. 3.12, and it can be said that the higher the relaxation losses the smaller the creep losses. The creep losses, when low-relaxation strands are used instead of stress-relieved strands, increase by 0.24 percent of the theoretical axial force at the section over the pier. This change decreases as the section analyzed moves away from the pier, and it is a consequence of smaller stresses in the concrete, but it can not be neglected. Therefore, the highest creep losses are obtained at the section over the pier when low-relaxation strands are used. In this figure, the difference in creep losses, between using low-relaxation strands and strands with zero relaxation, is very small, and can be practically neglected.

The effect of each type of prestressing strands on the prestress force can be seen in Fig. 3.13, and it can be said that the higher the relaxation losses the smaller the prestress force. In this figure, when low-relaxation strands are used instead of stress-relieved strands, the prestress force is increased by 5.5 percent of the theoretical axial force at the section over the pier, as can be observed. This increase is higher as the section considered moves away from the pier, even though this effect is reduced by the fact that shrinkage losses, as mentioned before, also increase in that direction and tend to decrease the prestress force. It can also be observed that the difference in prestress force, between using low-relaxation strands and strands with zero relaxation, is about 1.5
percent of the theoretical axial force at the section over the pier. This
difference increases as the section studied moves away from the pier. This
effect is mainly caused by the fact that relaxation losses also increases in
that direction; the fact that creep losses decrease in that direction also
increases this difference but in a smaller degree.

If instead of using stress-relieved strands, low-relaxation strands
are used, the stresses in the concrete at the section over the pier in­
crease from 532 to 680 psi at the top, and those at the bottom decrease
from 1554 to 1522 psi. This change is smaller as the section analyzed moves
away from the pier, but can not be neglected.

The effect of the relaxation losses on deflections in one of
the sections can be observed in Fig. 3.14, and can be said that the higher
the relaxation losses the higher the change in deflections with time.
Figure 3.15 also shows the effect of relaxation losses on deflections of
this bridge as construction progresses and at 2000 days after construction
started. It can be seen that the higher the relaxation losses, the higher
the deflections both during construction and at 2000 days after construction
started.

3.5 Effects of Variation of Modulus of Elasticity of Concrete

It has been known that the modulus of elasticity of concrete varies
with time, and can be calculated using the methods explained in Art. 2.4.
The computer program developed in this investigation allows one to change
the modulus of elasticity every time that a new segment is built, up to four
times (see Appendix B). \( E_C \) was taken as a function of \( \sqrt{f_C'} \), following the
C.E.B. recommendations, and \( f_C' \) increased with time following the C.E.B.
values.
In the Corpus Christi Bridge, the effects of varying the concrete modulus with time on time-dependent deformations were compared with results of identical analysis, assuming the concrete modulus constant and it can be said that the effects of variations of the modulus of elasticity of concrete on time-dependent deformations and losses were small, and stresses were not affected at all. The major differences between results for relaxation, creep losses, and prestress force occurred at the section over the pier. At other sections, the changes, if any, were smaller. The changes in deflection was small, reduced 2 percent at the last section built. This change was mainly because of the change of curvature at the section over the pier which was caused by the change in elastic strains.

In the Oise Bridge, the same type of comparison was done and it can be said that the effects of variation of the modulus of elasticity of concrete on time-dependent deformations and losses were small even though they were more pronounced here than in the Corpus Christi Bridge. Stresses were affected in this case. The change in deflection was small, with a 10 percent reduction at the last section built. It was also mainly because of the change in curvature at the section over the pier.

In general, it can be stated that a reasonable increase in the modulus of elasticity of concrete during construction did not have a major effect on losses of the above mentioned bridges. Its effect on deflection, caused mainly by the change in curvature at the section over the pier which in turn was caused by the instantaneous elastic deformations due to construction of subsequence segments can not be neglected.
3.6 Effects of Age of the Segments at the Time of Construction

In the segmental cantilever construction, the concrete in the segments at the time of the construction might be very young as in the case of the cast-in-place Oise Bridge, or relatively old as in the precast Corpus Christi Bridge. Therefore, it is of interest to know the effects of age of the segments at the time of construction on time-dependent deformations and losses of these structures.

The parameters involved in this case are creep, shrinkage, and modulus of elasticity of concrete. It is well known that concrete members loaded at early age yield larger creep strains for a given time under sustained load compared to the same concrete at later age. The earlier the segment is erected the greater the capacity to shrink, and smaller the modulus of elasticity.

In order to study the effects of age of the segments on time-dependent deformations and losses of segmental cantilever concrete bridges, the Corpus Christi and Oise Bridges have been analyzed assuming segments with different ages at the time of construction.

The Corpus Christi Bridge has been analyzed assuming the segments to 7, 14, 30, 60, and 90 days old at the time of construction. The effects of age of the segments on creep losses are shown in Fig. 3.16, and it can be seen, as was expected, that the younger the concrete the higher the creep losses. Also, for a constant age the creep losses are smaller as the section studied moves away from the pier because of smaller concrete stresses as the section moves in that direction. In Fig. 3.17, the effects of the age of the segments on shrinkage losses are shown, and it can be said that the younger the segments the higher the shrinkage losses. It can be
observed that the change in shrinkage losses is not much when the age of the segments varies from 7 to 14 days, but this change increases as the age increases, up to 60 days old. Beyond 60 days, the differences become smaller. This behavior is a direct consequence of the variation with time, for this theoretical thickness, of the coefficient $k_t$ of Eq. 2.16, and can be seen in Fig. 2.6. From the above observation, it can be said that the time interval between erection of segments would also play an important role on the shrinkage losses in bridges with large theoretical thickness, when they are erected at early ages. About the effects of the age of the segments on relaxation losses, it can be said that the older the segments the higher the relaxation losses. This can be explained by the fact that steel stresses and strains are higher because of the decrease in creep and shrinkage losses. The relaxation losses, at the section over the pier and with increasing age of the segments, are as follows: 7.71, 7.78, 7.88, 7.96, and 7.99 percent of the theoretical axial force. The variation in relaxation losses, caused by the aging of the segments, is smaller as the section analyzed moves away from the pier. The prestress force is the most affected by the age of the segments. This effect on the prestress force is a product of the combination of creep, shrinkage and relaxation losses. The relaxation losses have an effect contrary to the creep and shrinkage losses on the prestress axial force, since as the segments get older when erected, the creep and shrinkage losses decrease which increases the prestress force, but the relaxation losses increase which decreases the prestress force. It can be said that the younger the segments the smaller the final prestress force. The value of the prestress force, at the section over the pier and with increasing age of the segments, is: 79.70, 80.02, 80.43, 80.76, and 90.93 percent of the theoretical axial force. The variation of the prestress force is smaller as the
section studied moves away from the pier. When the age of the segments varies from 7 to 90 days at the time of construction, the stresses, at the section over the pier, increase at the top from 521 to 556 psi, and decrease at the bottom from 1556 to 1549 psi. These changes in stresses are smaller as the section analyzed moves away from the pier. All these data are for 2000 days after construction started.

The Oise Bridge was also analyzed assuming segments to be 7, 14, 28, 60, and 90 days old at the time of construction. The effects of age of the segments on creep and shrinkage losses are shown in Figs. 3.18 and 3.19, respectively. What was said for the Corpus Christi Bridge about these losses, can also be said for the Oise Bridge. In this bridge as in the Corpus Christi, the younger the segments the smaller the prestress axial force. Here, this effect is a product of the combination of just creep and shrinkage losses. At the section over the pier and in an increasing age of the segments, the value of the prestress axial force is: 75.22, 76.06, 76.77, 77.50 and 77.89 percent of the theoretical axial force. The variation of the prestress axial force is smaller as the section analyzed moves away from the pier. When the age of the segments varies from 7 to 90 days at the time of construction, the stresses, at the section over the pier, increase at the top from 714 to 800 psi, and decrease at the bottom from 1043 to 1029 psi. These changes in stresses are smaller as the section moves away from the pier. These data are for 2000 days after construction started.

Analyzing all these results, it can be said that the effects of age of the segments at the time of construction on creep and shrinkage losses are very important, as the creep losses are reduced by more than 50 percent as the age of segment increases from 7 to 90 days. Almost the same
can be said about the shrinkage losses, but in a smaller percentage. It can be observed that the Oise Bridge was affected more than the Corpus Christi Bridge by these two losses, mainly as a result of the smaller initial stresses in its prestressing steel. The effect of age of the segments on relaxation losses in the Corpus Christi Bridge is not much but cannot be neglected. The effect on prestress force is important in both bridges, being higher in the Oise Bridge as a direct consequence of the creep and shrinkage losses. The effect on concrete stresses was also higher in the Oise Bridge.

The effect of age of the segments at the time of construction on deflections in two of the sections studied are shown in Fig. 3.20, it can be seen that the older the segments the smaller the change in deflections with time, and that this change is higher in the Corpus Christi Bridge. In these two bridges, the differences in deflections during construction, when the segments are 7 and 90 days old when erected, are not large but they become large after long periods of time. For example, in the Oise Bridge, the deflection at 2000 days at the end of the cantilever, section 13, for segments 7 days old at the time of construction is reduced by 42 percent if the segments are 90 days old when erected. However, the difference at the time when the construction of the bridge ended, 77 days, was almost zero.

3.7 Effects of the Number of Creep Curves

The computer program developed in this investigation can handle up to nine different creep curves, which allows the use of different methods of analysis suitable for computing creep under variable stress in concrete members. These methods were discussed in Chapter 2.
To illustrate the effects of the number of creep curves on time-dependent deformations and losses, the Corpus Christi and Oise Bridges have been analyzed using different numbers of creep curves, increasing them one at a time, up to nine.

The effects of the number of creep curves on the time-dependent behavior in two of the sections analyzed in the Corpus Christi Bridge can be studied in Table 3.1, which is self-explanatory. First of all, it should be said that since shrinkage strains are independent of stresses and creep strains, the shrinkage losses are independent of the number of creep curves used and have a constant value, 1.28 and 1.091 percent of their respectively theoretical axial forces at section 1 and 5 respectively, 2000 days after construction started. In Table 3.1, it can be observed that when the creep losses increase, the relaxation losses and the prestress force decrease, and vice versa. The variation of the relaxation losses is only the consequence of the variation of creep losses because, as mentioned before, an increase in creep losses means smaller stresses in the prestressing steel which in turn means smaller relaxation, and vice versa. The change in prestress force is a consequence of both relaxation and creep losses. The effect of number of creep curves on creep losses does not have a definite direction, that is, the creep losses do not increase or decrease with the increase of the number of creep curves, because of the way that concrete stresses were built up. In the results for section 5, in this table, it can be noticed that there are no changes except in deflections, when more than six creep curves are used. This happens because only five more segments are erected after section five, and the computer program is set up (see Appendix B) to use a new creep curve every time that changes in stresses,
at any section, are caused by the erection of a new segment. The change in
deflection at section 5, when using more than six creep curves, is caused
by the effects of those creep curves on the sections erected earlier. In
Fig. 3.21 the effect of the number of creep curves on deflections with time
at one of the sections studied in the Corpus Christi Bridge is shown. It can
be seen that there is not a definite direction in the change in deflection
with the number of creep curves. This is also caused by the way that con-
crete stresses, at the sections, were built up.

The effects of the number of creep curves on the time-dependent
behavior of two of the sections studied in the Oise Bridge, 2000 days after
construction started, are shown in Table 3.2. It can be seen that the big
change in creep losses occurred when two creep curves were used. This
happened mainly because of a big decrease in creep strains as a result of
the variation of creep with the age of the concrete at the time of loading,
as given by the coefficient $K_d$ of Eq. 2.13, and can be seen in Fig. 2.3.
In this case, the variation of creep losses does not have a definite direction
either, that is, the creep losses do not consistently increase or decrease
with the increase of the number of creep curves because of the way that
concrete stresses are built up. It can be noticed that the variation of
creep losses, when more than three creep curves are used, is smaller, caused
by the fact that the change in the creep strains of the creep curves are
smaller, as a consequence of the variation, as mentioned above, of the
coefficient $K_d$, and the time intervals between erection of segments. The
variation in the prestress force, as explained earlier, is a consequence of
only the creep losses, and decreases when the creep losses increase and
vice versa. In the part b of this table, it can be noticed that the results
do not change when more than seven creep curves are used. The explanation for this is similar to the one given above for section 5 of the Corpus Christi Bridge, but in this case, six segments are erected after section 7. The shrinkage losses are constant, and equal to 2.85 and 3.0 percent of their theoretical axial force at sections 1 and 7, respectively, at 2000 days after construction started. In Fig. 3.22, the effect of the number of creep curves on deflections with time of one of the sections in the Oise Bridge is shown. It can be seen that the change in deflection decreases as the number of creep curves increases. In this bridge, the concrete stresses are built up in a more uniform way than in the Corpus Christi Bridge. In addition, the effects of aging during construction on the creep characteristics of the concrete were more important than in the Corpus Christi Bridge.

In general, it can be said that the variation of relaxation and creep losses, prestress force, and concrete stresses in the Corpus Christi Bridge caused by varying the number of creep curves can be neglected for practical purposes. However, the variation of the deflection cannot be neglected. The same cannot be said about the Oise Bridge, where the effects of the number of creep curves on the variation of creep losses are important and the computed change in deflection which can be reduced by 50 percent.

3.8 Effects of Curvature Coefficient

In post-tensioned members, the profile of the prestressing steel can be varied at the designer's will. When the cable profile is curved, normal stresses are created between the cable and the sheathing or the
surrounding concrete, inducing friction as a consequence. Loss of prestress is caused by this induced friction, and is dependent on the curvature coefficient, also called the coefficient of friction, and the stress exerted by the cable on the surrounding material.

To study the effects of the curvature coefficient on the time-dependent behavior of segmental cantilever bridges, the Corpus Christi and Oise Bridges have been studied assuming different values of the curvature coefficient in the range recommended by AASHO (1) and ACI (4).

The Corpus Christi Bridge has been analyzed assuming the curvature coefficient equal to 0.15, 0.25, and 0.3. The value of 0.25 was recommended by the designers. The effect of the curvature coefficient on the prestress force is shown in Fig. 3.23. It can be seen that the smaller the curvature coefficient the higher the prestress force. It is also observed that the variation of prestress force caused by the variation of the curvature coefficient is higher as the section in question is closer to the pier. This can be explained by the fact that the closer the section is to the pier, the greater the number of cables that goes through, and the higher the prestress losses caused by the curvature coefficient. In Fig. 3.24, the effect of the curvature coefficient on creep losses is shown. It can be seen that the higher the curvature coefficient the smaller the creep losses. Also, the variation of creep losses caused by the variation of the curvature coefficient is smaller as the section studied moves away from the pier. This is mainly due to the higher decrease in concrete stresses as the section is closer to the pier because of higher losses in prestress force. Figure 3.25 shows the effect of the curvature coefficient on the relaxation losses with time. It is observed that the higher
the curvature coefficient the smaller the relaxation losses. This is because of the reduction in initial steel stresses caused by the prestress losses. For a constant curvature coefficient, it can be noticed that the relaxation losses are higher as the section analyzed moves away from the pier, as a result of higher steel stresses. Also, the variation of relaxation losses caused by the variation of the curvature coefficient is smaller as the section studied moves away from the pier. This is mainly due to the higher effect of the curvature coefficient on the prestress losses as the section gets closer to the pier. When the curvature coefficient decreases from 0.3 to 0.15, the stresses at the section over the pier increase from 519 to 587 psi at the top, and those at the bottom decrease from 1556 to 1543 psi. These changes in concrete stresses are smaller as the section analyzed moves away from the pier. Shrinkage losses are independent of the curvature coefficient, and have values of 1.280, and 1.304 percent of the theoretical axial force at sections 1 and 5, respectively. All these data are for 2000 days after construction started.

The same values of curvature coefficient used above were used in the Oise Bridge. The effect of the variation of the curvature coefficient on the prestress force is shown in Fig. 3.26. It can be seen that the higher the curvature coefficient the smaller the prestress force. The explanation for this can be the same one given above for the Corpus Christi. It can also be observed that as the section studied moves away from the pier the higher their percentage of prestress force. This is because losses due to curvature and wobble coefficient are smaller, and so is the time elapsed for other losses to occur. In Fig. 3.27, the effect of the curvature coefficient on creep losses is shown. These effects are similar to the ones observed in
the Corpus Christi Bridge, and the same explanation can be given. In this bridge, Oise, the variation of the curvature coefficient from 0.3 to 0.15 increases the concrete stresses at the section over the pier from 682 to 788 psi at the top, and decreases those at the bottom from 1048 to 1033 psi. These changes in concrete stresses are also smaller as the section in question moves away from the pier. Shrinkage losses are 2.848 and 2.996 percent of the theoretical axial force at section 1 and 7, respectively. These data are for 2000 days after construction started.

Based on these observations, it can be stated that the effect of the variation of the curvature coefficient on the prestress force is important in both bridges, being higher in the Oise, and can not be neglected. The effect on the creep losses is not much, but can not be neglected in the Oise Bridge. Relaxation losses are the second most affected, after prestress losses, by the variation of this coefficient, increasing the relaxation losses at the section over the pier by 33 percent when the coefficient is reduced from 0.3 to 0.15 in the Corpus Christi bridge. Obviously, this effect can not be neglected. The effect on concrete stresses is also important in both bridges, being higher in the Oise.

The effect of the variation of the curvature coefficient on deflections in two of the sections analyzed are shown in Fig. 3.28. It can be observed that the higher the curvature coefficient the higher the change in deflections with time, and that this change is higher in the Oise Bridge. In these two bridges, the deflection at the end of the cantilever, when the curvature coefficient varies from 0.15 to 0.3, increases from 0.35 to 0.44 inches in the Corpus Christi, and from 0.05 to 0.23 inches in the
Oise Bridge, when construction ends. At 2000 days after construction started, the increase is from 0.91 to 1.03 inches in the Corpus Christi, and from 0.77 to 1.13 inches in the Oise. Therefore, it can be concluded that the effect of the curvature coefficient on deflections is very important, being more critical in the Oise Bridge, and it must be taken into consideration in the design.

3.9 Effects of Wobble Coefficient

In addition to the primary curvature of the cable caused by the variation of eccentricity, secondary curvature caused by small vertical and horizontal deviation from the theoretical path may occur in post-tensioned members because of unavoidable undulations in the cable during its installation and setting, or errors in construction. This effect creates additional normal stresses which causes additional losses, and is described as the wobbling effect. These additional losses are dependent on the length of the cable, the friction between the contact materials, called the wobble coefficient, and the quality of construction.

In order to illustrate the effects of the wobble coefficient on the time-dependent deformations and losses in segmental cantilever bridges, the Corpus Christi and Oise Bridges were analyzed assuming different values of the wobble coefficient in the range recommended by AASHO (1).

The Corpus Christi Bridge was analyzed assuming values of the wobble coefficient equal to 0.002/ft, and 0.0002/ft. The value of 0.0002/ft was recommended by the designers. The effect of the wobble coefficient on the prestress force is shown in Fig. 3.29. It can be seen that the higher the
wobble coefficient the smaller the prestress force. For a constant wobble coefficient, it can be observed that the percentage in prestress force is higher as the section studied moves away from the pier. This is because both friction losses and time-dependent losses are smaller in that direction. It is also noticed that the variation in prestress force caused by the variation of the wobble coefficient is higher as the section analyzed is closer to the pier. This is because of the larger distance from the jacking end of the cable which causes larger losses. Figure 3.30 shows the effect of the wobble coefficient on creep losses. It can be seen that the higher the wobble coefficient, the smaller the creep losses. Also, the variation of creep losses due to the variation of the wobble coefficient is smaller as the section studied moves away from the pier. This is mainly caused by the higher decrease in concrete stresses as the section studied is closer to the pier because of the higher losses in prestress force. In Fig. 3.31, the effect of the wobble coefficient on the relaxation losses with time is shown. It can be observed that the higher the wobble coefficient the smaller the relaxation losses. This is caused by a decrease in initial steel stresses as a result of greater losses due to wobble. For a constant wobble coefficient, it can be noticed that the relaxation losses are higher as the section in question moves away from the pier, as a result of higher steel stresses. Thus, the highest relaxation losses are obtained at the last section built. Also, the variation of relaxation losses due to the variation of the wobble coefficient is smaller as the section studied moves away from the pier. This is mainly caused by the higher effect of the wobble coefficient on the steel stresses as the section is closer to the pier. When the wobble coefficient decreases from 0.002/ft to 0.0002/ft, the
stresses at the section over the pier increase from 431 to 542 psi at the top, and those at the bottom decrease from 1572 to 1552 psi. These changes in stresses are smaller as the section analyzed moves away from the pier. Shrinkage losses are independent from the wobble coefficient and have values of 1.280 and 1.304 percent of the theoretical axial force at section 1 and 5, respectively. All these data are for 2000 days after construction started.

The Oise Bridge was analyzed assuming values for the wobble coefficient of 0.0002/ft, 0.0015/ft, and 0.002/ft. Figure 3.32 shows the effects of the wobble coefficient on prestress force. It can be seen that the higher the wobble coefficient, the smaller the prestress force. For a constant wobble coefficient, it can be noticed that the percentage in prestress force is higher as the section moves away from the pier. It is also observed that the variation of prestress force caused by the variation of the wobble coefficient is higher as the section analyzed is closer to the pier. The explanations for all of this are similar to the ones given above for Corpus Christi. The effect of the wobble coefficient on creep losses is shown in Fig. 3.33. These effects are similar to those observed in the Corpus Christi bridge, and the same explanation can be given. When the wobble coefficient varies from 0.002/ft to 0.0002/ft, the stresses at the section over the pier increase from 637 to 926 psi, and those at the bottom decrease from 1055 to 1012 psi. These changes in stresses are smaller as the section in question moves away from the pier. Shrinkage losses have a value of 2.848 and 2.996 percent of the theoretical axial force at sections 1 and 7, respectively. All these data are for 2000 days after construction started.
In general, it can be said that the prestress force is the most affected, in both bridges, by the variation of the wobble coefficient. Therefore, its effect can not be neglected. The effect of the wobble coefficient on creep losses is not much in Corpus Christi, but still cannot be neglected. Relaxation losses are the second most affected, increasing by 61 percent at the section over the pier when the wobble coefficient decreases from 0.002/ft to 0.0002/ft, and can not be neglected. The effect on concrete losses is also very important in both bridges, being higher in the Oise bridge.

Figure 3.34 shows the effect of the wobble coefficient on deflections in two of the sections analyzed. It can be seen that the higher the wobble coefficient the higher the change in deflections with time, and that this change is higher in the Oise bridge. In these two bridges, the deflection at the end of the cantilever, when the wobble coefficient varies from 0.0002/ft to 0.002/ft, increases from 0.41 to 0.51 inches downwards in Corpus Christi, and from 0.09 upward to 0.27 inches downward in Oise, by the time of end of construction. At 2000 days after construction started, the increase is from 0.99 to 1.13 inches in Corpus Christi, and from 0.49 to 1.21 inches downward in the Oise bridge. From these data it can be concluded that the effect of the wobble coefficient on deflections is very important in both bridges, being more critical in the Oise Bridge where an increase of 147 percent occurred.

3.10 Effects of Construction Loads

In the cantilever construction the segments can be precast or cast-in-place. Precast segments are erected using a variety of methods.
One of them is when the segments are lifted into place by a truck or crawler crane supported on the bridge itself. A frame containing hoisting winches may be supported on the last segment placed and stressed. For cast-in-place segments, traveling forms can be used which hold the falsework and construction materials cantilevered from the previously constructed segments.

To study the effect of construction loads on the time-dependent behavior of segmental cantilever bridges, the Corpus Christi and Oise Bridges have been analyzed assuming some construction load acting permanently at the end of the progressing cantilever.

The precast Corpus Christi Bridge has been analyzed assuming a permanent construction load of 25 kips. The bridge was designed for this construction load plus the weight of a segment. The results obtained from this analysis and those obtained with zero construction load, 2000 days after construction started, have been compared. When the construction load was used, the relaxation and creep losses, and prestress force were smaller, though not very much. The prestress force was the most affected, and decreased from 80.433 to 80.367 percent of the theoretical axial force at the section over the pier. The concrete stresses did not change at the top but increased at the bottom in 1 psi. All these changes are smaller as the section studied moves away from the pier. About the effect of construction load on deflections, it can be said that Section 5 had an increase in deflection, when construction load was used, from 0.32 to 0.33 inches. Section 10, the end of the cantilever, had an increase from 0.41 to 0.44 inches at the end of construction, and from 0.99 to 1.02 inches at 2000 days after construction started.
The cast-in-place Oise Bridge has been analyzed assuming a permanent construction load equivalent to the average weight of a segment, 136.54 kips. The results from this analysis have been compared with those from a similar analysis but without construction load, at 2000 days after construction started. In the case when construction load was considered, the creep losses and prestress force were smaller, decreasing from 4.396 to 4.270, and from 75.215 to 74.960 respectively at the section over the pier, these values in percentages of their theoretical axial force. The concrete stresses decreased from 717 to 714 psi at the top, and those at the bottom increased from 1043 to 1049 psi. All these changes are smaller as the section analyzed moves away from the pier. The effect of construction load on deflection, caused an increase from 0.19 to 0.24 inches in section 7. In section 13, end of the cantilever, the increase was from 0.17 to 0.33 inches at the end of construction, and from 1.01 to 1.18 inches 2000 days after construction started.

Based on all of that data above, it can be said that the long-time effects of the construction load on the Corpus Christi Bridge is very small and can be neglected for practical purposes, but the effect on the concrete stresses during time of construction does have to be studied because it is in the order of the hundreds of lb/in.\(^2\). Although the long-time effects of construction load on the Oise Bridge are not large, they should not be neglected, especially the change in deflections. It is not useless to again remind the designer to study the changes in concrete stresses caused by the construction loads during time of construction. Also, it can be noticed that the difference in deflection just after the end of construction and 2000 days after construction started is still the same, for both bridges.
3.11 Comparison of Computed and Measured Results

In order to study the correlations between the results of analysis and the actual behavior of a segmental cantilever bridge under field conditions, analyses were carried out for the Oise Bridge and a small scale model of the Corpus Christi Bridge (see Appendix A). In such analyses the initial steel stresses, material and section properties, and time interval of construction were assumed to be the same as in the actual structure.

Oise Bridge, a more detailed description of this bridge, its construction, materials and observations of the long-term deformations are presented in References 6, and 11. For the analysis of this bridge, the creep and shrinkage data obtained under laboratory storage conditions, 20°C and 50 percent relative humidity, were used, taking into account the effects of size and shape of the specimens, and the relative humidity of the surrounding environment according to the C.E.B. recommendations. The modulus of elasticity of concrete was obtained from test of specimens (5.81 x 10^6 psi), and its variation with time was very small. The 1970 C.E.B. recommendations were also used to predict the creep and shrinkage strains of the bridge for a relative humidity of 70 percent (which was the relative humidity under field conditions), and to compute the modulus of elasticity of concrete. These predicted values of creep and shrinkage were about one third of those measured. No reasonable explanation for these differences has been found.

In these analyses, a construction load equal to the average weight of a segment plus 20 kips of forms was used for creep purposes, although neither the actual construction load nor its distribution is known. Figure 3.35 shows the measured and calculated curvatures during the construction of
the cantilever, at two different sections. All the data for the analyses was obtained from test specimens. This figure also shows the computed curvatures obtained using all the data based on the C.E.B. recommendations.

From Fig. 3.35, it can be seen that the agreement between the computed and measured values of curvatures is very good. Also, the values of curvature obtained using the data based on the C.E.B. recommendations are about one half of those measured. This is a consequence of the difference between the measured values of creep and shrinkage and the ones predicted.

The distributions of total strains in the outside and inside faces, and over the depth of the web of Section 1A, at different stages of construction, were measured. Figure 3.36 shows the measured and computed values of total strains at the centroid of section 1A, and also shows the computed total strains at the centroid using the data based on the C.E.B. recommendations. From this figure, it can be seen that the computed total strains are in good agreement with the measured total strains in the field. The total strains calculated using the data based on the C.E.B. recommendations are about one half of those measured. Figure 3.37 shows the computed deflection of the cantilever at all stages of erection using the data obtained from tests, and the data based on the C.E.B. recommendations. It can be noticed that the direction of the variation of deflection at each stage of erection is the same in both cases, with the variation being higher when the test data was used. Measured values are not available.

A detailed description of the small scale model of the Corpus Christi Bridge, its analysis, construction, materials, and observations of the time-dependent deformations are presented in Reference 13. During the erection work the temperature and relative humidity varied between 75-90°F and 50-70 percent respectively. The segments were between five to six months old
when erected. For the analysis of this model, all the data used was obtained from tests, except the creep and shrinkage data which was obtained following the C.E.B. recommendations, using 50 percent relative humidity for creep, and 60 percent for shrinkage. Figure 3.38 shows the measured and calculated total strains at the top slab of segment M1 (see Appendix A) during construction of the cantilever. In this figure, SIMPLA2 is a University of Texas computer program which provides an analysis at each stage of erection using the finite segment technique without taking into account time-dependent deformations caused by creep and shrinkage of concrete and relaxation of the steel. Figure 3.39 shows the measured and computed total strains at the bottom slab of segment M1 during construction of the cantilever.

From Figs. 3.38 and 3.39, it can be seen that the computed total strains, using beam theory taking into account time-dependent deformations caused by creep and shrinkage as a method of analysis, are in very good agreement with the measured total strains. Figure 3.40 shows the computed deflections of the cantilever during construction. The trend of these deflections agreed very well with the experimental ones, but the magnitudes are smaller. During the construction of the model there were difficulties in measuring deflections because they were so small in comparison to the span. There were also some small errors in alignment of end faces of some precast segments which showed up in the initial stages of construction and strongly influenced the deflected shape. Because of these problems, no direct comparisons of deflections are shown here.
4. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

Based on the results of the parametric study pursued in this investigation of the time-dependent deformations and prestress losses in segmental cantilever concrete bridges, where the effects of varying environmental conditions, member size, age of the segments at the time of construction, type of prestressing strands, wobble and curvature coefficients, and construction loads were taken into account, some conclusions and recommendations can be made. They can be summarized as follows:

1. The effects of varying environmental conditions, especially the relative humidity and temperature of the air, have a significant influence on creep and shrinkage strains and as a consequence on time-dependent behavior of bridge structures, with creep and shrinkage losses increasing, and relaxation losses and prestress force decreasing when the relative humidity decreases. Shrinkage and relaxation losses are higher, and creep losses smaller as the section studied moves away from the pier. As a consequence of these losses, there is a large decrease in concrete stresses with time at the top of the sections. The effects on deflections during construction are not large, but they become more important after long periods of time.

2. The type of prestressing strands has a great influence on the time-dependent behavior of cantilever concrete bridges. When low-relaxation strands are used instead of stress-relieved
strands, relaxation losses decrease greatly, even though they are partially offset by an increase in creep losses. When this change in type of strands occurs, creep losses and prestress force increase, and there is a large increase of concrete stresses at the top of the sections. As the section studied moves away from the pier, relaxation losses (as a percentage of the theoretical axial force) are higher, the change in creep losses and concrete stresses is smaller, and the change in prestress force is higher. The effect on deflection is important after long periods of time.

3. The estimated values of modulus of elasticity of concrete at time of major stress changes are important because of direct effects on deflections, which are caused only by the change in elastic strains. Losses are barely affected by this variation of modulus.

4. Age of the segments at the time of construction obviously makes some difference in the time-dependent behavior of these bridges because the concrete might be very young for the cast-in-place segments and have all the potential for creep and shrinkage, or relatively old for precast segments where their effect can be reduced in half or more. As a consequence, change in deflection can also be reduced with this aging of the segments, though relaxation losses increase.

5. The number of creep curves used for the analysis has a definite influence on the time-dependent deformations and
losses of these type of bridges though not a definite
direction. That is, the losses and deformations do not
consistently increase or decrease with the increase of
the number of creep curves, but change as a result of
the way that stresses in the concrete were built up.

6. In segmentally cantilevered bridges where the prestressing
steel has an inclined profile in webs, the effect of the
curvature coefficient has a significant influence on its
behavior. A higher curvature coefficient means smaller
creep and relaxation losses, prestress force, and
stresses, but higher change in deflections.

7. Due to the nature of this type of construction, it was
obviously expected the wobble coefficient to have a
significant effect on the behavior of these bridges,
an effect that is similar to that of the curvature
coefficient but more pronounced.

8. Construction loads tend to have a higher effect on the
behavior of cast-in-place than in precast segmental bridges,
and deflection is the most affected part.

9. During the course of the investigation, it was observed
that bridges with cast-in-place segments were more sensitive
to the different parameters studied than those with precast
segments.

10. Generalizations in these type of bridges are very difficult,
because each bridge has a different path of building up the
stresses in the concrete, and that path plays a very important
role in the behavior of the bridge.
11. Deflection is more affected by the following parameters in their order of importance: wobble and curvature coefficients, and creep.

12. Most of these type of bridges are made continuous at mid-span, and as a result there is a need to make the computer program able to do that analysis. This modified program should be able to account for time-dependent pier deformations if the super structure is fixed to the piers.

13. Once these bridges are made continuous, a redistribution of moments caused by time-dependent deformations occurs, and studies of these effects are recommended. This can also be done using the computer program in conjunction with a method proposed by M. Thenoz (23).

14. Computed results were compared with data from field tests but there was not enough data to make any conclusive type of correlation. Therefore more field data is obviously needed.

15. Further study is needed to determine a set of factors for the estimation of prestress losses in these bridges.
5. SUMMARY

The main objective of this investigation was to give an exposition of the nature of the time-dependent deformations and prestress losses of segmental cantilever concrete bridges, using precast and cast-in-place segments subjected to either field exposure conditions or to a constant environment. As a result, a better understanding of the main parameters influencing the time-dependent deformations and prestress losses of these types of bridges was gained.

The scope of this investigation was divided in two parts:
1. The development of a computer program required to compute the time-dependent deformations, stresses and prestress losses.
2. The analytical study of the main parameters influencing the time-dependent deformations and prestress losses.

The computer program is based on a rational method of analysis for predicting the time-dependent stresses, strains, loss of prestress, and deflections of segmental cantilever concrete bridges. This method of analysis is presented in Chapter 2. The computer program has been written in Fortran IV for the IBM-360/75 and is described in Appendix B.

The analytical study included the application of the rate of creep (22), revised rate of creep (20), and the superposition methods (18) to predict the time-dependent strains, stresses, curvatures, losses, and deflections of segmental cantilever concrete bridges.

The main parameters affecting the time-dependent behavior of bridges considered in this study were: Environmental conditions (relative
humidity of the air), type of prestressing strands (stress-relieved and low-relaxation strands), modulus of elasticity of concrete, age of the segments at the time of construction, number of creep curves, curvature and wobble coefficients and loads applied during construction. Also, some computed results were compared with measured data. The results of these analyses were discussed in Chapter 3. A number of important conclusions concerning these analyses were presented in Chapter 4.

For purposes of analysis, it was assumed that the principle of superposition was applicable and that the single effects of each of the main parameters could be treated individually while the other parameters were maintained at realistic values.
LIST OF REFERENCES


2. American Association of State Highway and Transportation Officials (AASHTO), "Interim Specifications for Bridges 1975," Interim 5, Section 6 - Prestressed Concrete.


### Table 3.1
Effects of the Number of Creep Curves on Losses, Stresses, and Deflections in the Corpus Christi Bridge, 2000 Days after Construction Started

<table>
<thead>
<tr>
<th>No. of Creep Curves</th>
<th>Relaxation Losses</th>
<th>Creep Stresses</th>
<th>Axial Force</th>
<th>Concrete Stresses</th>
<th>Deflection</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Section 1</td>
<td></td>
<td></td>
<td>Top (psi)</td>
<td>Bottom (psi)</td>
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<tr>
<td>1</td>
<td>7.882</td>
<td>1.644</td>
<td>80.443</td>
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<tr>
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<td>1552</td>
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<td>409</td>
<td>671</td>
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**Convention:**
Compression in concrete positive (psi)
Downward deflection negative
Relaxation and Creep Losses, and Axial Force in percentage of the Theoretical Axial Force
Table 3.2
Effects of the Number of Creep Curves on Losses, Stresses, and Deflections in the Oise Bridge, 2000 Days After Construction Started

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<tr>
<th>No. of Creep Curves</th>
<th>Creep Losses</th>
<th>Axial Force</th>
<th>Concrete Stresses</th>
<th>Deflection</th>
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<td>737</td>
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Convention:
Compression in concrete positive (psi)
Downward deflection negative Creep Losses, and Axial Force in percentage of the Theoretical Axial Force
Fig. 2.1 Ratio of Compressive Strength at Age $t$ to that at 28 Days According to European Concrete Committee

Fig. 2.2 European Concrete Committee Creep Prediction Factor Coefficient $K_c$ vs. Relative Humidity
Fig. 2.3 European Concrete Committee Creep Prediction Factor Coefficient $K_d$ vs. Age at Loading

Fig. 2.4 European Concrete Committee Creep Prediction Factor Coefficient $K_b$ vs. Mix Properties

$c = 500 \text{ kg/m}^3$

$100 \text{ lbs/yd}^3 = 59.49 \text{ kg/m}^3$
Fig. 2.5 European Concrete Committee Creep Prediction Factor Coefficient $K_e$ vs. Theoretical Thickness

Fig. 2.6 European Concrete Committee Creep Prediction Factor Coefficient $K_t$ vs. Time
Fig. 2.7 European Concrete Committee Shrinkage Prediction Factor Coefficient $\varepsilon_c$ vs. Relative Humidity

Fig. 2.8 European Concrete Committee Shrinkage Prediction Factor Coefficient $K_e$ vs. Theoretical Thickness
Fig. 3.1 Time-Dependent Variation of Stresses in Corpus Christi Bridge
Fig. 3.1 (Continued)
Days Since Construction Started

Section 5

Stresses (psi)

Fig. 3.1 (Continued)
Fig. 3.1 (Continued)
Days Since Construction Started

Fig. 3.2 Time-Dependent Variation of Stresses in Oise Bridge
Fig. 3.2 (Continued)
Days Since Construction Started

Section 7

Top

Bottom

Stresses (psi)

Fig. 3.2 (Continued)
Days Since Construction Started

Section 7

Fig. 3.2 (Continued)
Fig. 3.3 Effect of Variation in Relative Humidity for Creep on Creep Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.3 (Continued)
Fig. 3.4 Effect of Variation in Relative Humidity for Creep on Creep Losses (as percentage of the Theoretical Axial Force) in Oise Bridge
Fig. 3.5 Effect of Variation in Relative Humidity for Creep on Deflection
Days Since Construction Started

Oise Bridge
Section 7

Fig. 3.5 (Continued)
Fig. 3.6 Effect of Variation in Relative Humidity for Creep on Deflected Shape during Construction and at 2000 Days, Corpus Christi Bridge
Fig. 3.6 (Continued)

Sections

80% R.H.

Deflection (in.)

2000 Days
Fig. 3.7 Effect of Variation in Relative Humidity for Creep on Deflected Shape during Construction and at 2000 Days, Oise Bridge
Fig. 3.7 (Continued)

80% R.H.

Deflection (in.)

Sections

80% R.H.

2000 Days

Ms Reference Room
Civil Engineering Department
E106 E. H. Building
University of Illinois
Urbana, Illinois 61801
Fig. 3.8 Effect of Variation in Relative Humidity for Shrinkage on Shrinkage Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Days Since Construction Started

Section 5

Shrinkage Losses (Percent)

Fig. 3.8 (Continued)
Fig. 3.9 Effect of Variation in Relative Humidity for Shrinkage on Shrinkage Losses (as percentage of the Theoretical Axial Force) in Oise Bridge
Days Since Construction Started

Corpus Christi Bridge
Section 5

Fig. 3.10 Effect of Variation in Relative Humidity for Shrinkage on Deflection
Days Since Construction Started

Oise Bridge
Section 7

Deflection (in)

Fig. 3.10 (Continued)
Fig. 3.11 Effect of Type of Prestressing Strands on Relaxation Losses (as percentage of the Theoretical Axial Force)
Fig. 3.11 (Continued)
Corpus Christi Bridge
Section I

Fig. 3.12 Effect of the Type of Prestressing Strands on Creep Losses (as percentage of the Theoretical Axial Force)
Fig. 3.12 (Continued)
Fig. 3.13 Effect of Type of Prestressing Strands on the Prestress Force (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Days Since Construction Started

Section I

Prestress Force (Percent)

Fig. 3.13 (Continued)
Days Since Construction Started

Section 5

Prestress Force (Percent)

Low-Relaxation Strands
Stress-Relieved Strands
No Relaxation

Fig. 3.13 (Continued)
Fig. 3.13 (Continued)
Fig. 3.14 Effect of Type of Prestressing Strands on Deflection
Fig. 3.15 Effect of Type of Prestressing Strands on Deflected Shape during Construction and at 2000 Days, Corpus Christi Bridge
Fig. 3.16 Effect of the Age of the Segments (days) at the Time of Construction on Creep Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge.
Days Since Construction Started

Fig. 3.16 (Continued)
Fig. 3.17 Effect of the Age of the Segments (days) at the Time of Construction on Shrinkage Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.17 (Continued)
Fig. 3.18 Effect of the Age of the Segments (days) at the Time of Construction on Creep Losses (as percentage of the Theoretical Axial Force) in Oise Bridge
Days Since Construction Started

Creep Losses (Percent)

Fig. 3.18 (Continued)
Fig. 3.19 Effect of the Age of the Segments (days) at the Time of Construction on Shrinkage Losses (as percentage of the Theoretical Axial Force) in Oise Bridge.
Fig. 3.19 (Continued)

Days Since Construction Started

Section 7

Shrinkage Losses (Percent)

Fig. 3.19 (Continued)

Days Since Construction Started
Fig. 3.20 Effect of the Age of the Segments (days) at the Time of Construction on Deflection
Fig. 3.20 (Continued)
Fig. 3.21 Effect of the Number of Creep Curves Used in Analysis on Deflections in Corpus Christi Bridge
Days Since Construction Started

Section 7

Fig. 3.22 Effect of the Number of Creep Curves Used in Analysis on Deflections in Oise Bridge
Fig. 3.23 Effect of the Curvature Coefficient on the Prestress Force (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.23 (Continued)
Fig. 3.24 Effect of the Curvature Coefficient on Creep Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.25 Effect of the Curvature Coefficient on Relaxation Losses (as percentage of the Theoretical Axial Force)
Days Since Construction Started

Corpus Christi Bridge
Section 5

Relaxation Losses (Percent)

Fig. 3.25 (Continued)
Fig. 3.26 Effect of the Curvature Coefficient on the Prestress Force (as percentage of the Theoretical Axial Force) in Oise Bridge
Fig. 3.26 (Continued)
Days Since Construction Started

Fig. 3.26 (Continued)
Days Since Construction Started

Section 7

Fig. 3.26 (Continued)
Fig. 3.27 Effect of the Curvature Coefficient on Creep Losses (as percentage of the Theoretical Axial Force) in Oise Bridge
Days Since Construction Started

Fig. 3.27 (Continued)
Days Since Construction Started

Corpus Christi Bridge
Section 5

Deflection (in.)

Fig. 3.28 Effect of the Curvature Coefficient on Deflection
Days Since Construction Started

Oise Bridge
Section 7

Fig. 3.28 (Continued)
Fig. 3.29 Effect of the Wobble Coefficient on the Prestress Force (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.29 (Continued)
Days Since Construction Started

Section 5

Fig. 3.29 (Continued)
Days Since Construction Started

Section 5

Fig. 3.29 (Continued)
Fig. 3.30 Effect of the Wobble Coefficient on Creep Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.31 Effect of the Wobble Coefficient on Relaxation Losses (as percentage of the Theoretical Axial Force) in Corpus Christi Bridge
Fig. 3.31 (Continued)

Relaxation Losses (Percent)

Days Since Construction Started

Section 5

0 400 800 1200 1600 2000

0.002

0.0002
Fig. 3.32 Effect of the Wobble Coefficient on the Prestress Force (as percentage of the Theoretical Axial Force) in Oise Bridge
Days Since Construction Started

Fig. 3.33 Effect of the Wobble Coefficient on Creep Losses (as percentage of the Theoretical Axial Force) in Oise Bridge
Fig. 3.34 Effect of the Wobble Coefficient on Deflection

Days Since Construction Started

Corpus Christi Bridge
Section 5
Days Since Construction Started

Oise Bridge
Section 7

Deflection (in.)

Fig. 3.34 (Continued)
Fig. 3.35 Measured and Calculated Curvatures vs. Construction Time in Oise Bridge
Fig. 3.35 (Continued)
Fig. 3.36 Measured and Calculated Total Strains at the Centroid vs. Time, in Oise Bridge

Days Since Construction Started

Compressive Strain, $10^{-6}$

Measured
Outside Face

Calculated

Measured
Interior Face

Calculated According To C.E.B.
Fig. 3.37 Calculated Deflected Shape during Construction, Oise Bridge

All Data Based On Field Cyls.
All Data According to C.E.B.

Fig. 3.37 (Continued)
Fig. 3.38 Measured and Calculated Total Strains at the Top Slab of Segment M1 during Construction of the Cantilever
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<th>M1, S1</th>
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<th>M3, S3</th>
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<th>M7, S7</th>
<th>M8, S8</th>
<th>M9, S9</th>
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Compressive Strain, $10^{-6}$

Corpus Christi Bridge Scale Model

Fig. 3.39 Measured and Calculated Total Strains at the Bottom Slab of Segment M1 During Construction of the Cantilever
Fig. 3.40 Calculated Deflected Shape during Construction of the Scale Model of the Corpus Christi Bridge
In this investigation three different bridges were selected for the study of the time-dependent deformations and prestress losses of segmental cantilever concrete bridges. The bridges considered were:

1. The bridge over the Oise River in France. This is a three-span cast-in-place cantilever bridge that has been effectively counter-balanced on the short side by extra dead weight, and made continuous at midspan, as it is shown in Fig. A.1. This figure also shows the profile of the prestressing strands and some properties of the sections. In this bridge the segments have equal length, except the first segment that is larger. The depth varies, with the lower face of the bridge following a parabolic profile. A more detailed information about this bridge is given in Reference 6, and 11.

2. The prototype and a scale model of the Corpus Christi bridge. Figure A.2 shows the properties of these bridges and the profile of the prestressing steel. These are pre-cast cantilever concrete bridges which are also made continuous at midspan. More detailed information about these bridges is given in Reference 13.
Fig. A.1 Bridge over the Oise River
Fig. A.2 Corpus Christi Bridge
APPENDIX B

COMPUTER PROGRAM

B.1 Introduction

The estimation of prestress losses in prestressed members is essential for the prediction of actual stresses and displacements. Precise determination of prestress losses in a given situation is very complex and requires detailed information on the materials to be used, methods of curing, environmental conditions, and other detailed information that usually is not available to designers. For structures such as segmental cantilever bridges, it may be necessary to obtain this information in order to maintain control of the geometry of the bridge during construction so that different cantilevers can meet at correct grade and alignment.

The computer program developed in this study has been written in Fortran IV for the IBM-360/75, and is designed to compute losses caused by creep, shrinkage, and elastic shortening of concrete, and relaxation of steel, as well as deformations in segmental cantilever concrete bridges connected by hinges at midspan. The assumptions for this numerical step by step method of analysis have been outlined in Article 2.6. In addition, beam theory analysis was used for computing stresses, that is, shear lag was neglected. A general flow chart for the computer program is shown in Fig. B1.

B.2 Input and Output Data

The input data consists of the following geometrical, physical, and time-dependent properties of the bridge structure:
1. Number of time intervals since construction started.
2. Number of segments.
3. Number of sections, their areas, moments of inertia, and distances from the pier.
4. Number of prestressing strands, their areas, profiles, stresses and section of anchorage, and time interval of post-tensioning.
5. Modulus of elasticity of steel, and its relaxation characteristics.
6. Wobble and curvature coefficients of friction.
7. Unit weight of concrete and variation of its modulus with time.
8. Unit creep and shrinkage strain data.

A computer manual for the input data has been written and is available upon request.

The output of the analysis contains, at each section and time interval, the following:

1. All input data.
2. Relaxation, creep, and shrinkage losses.
3. Moment caused by dead and construction loads.
5. Concrete stresses at the top, bottom and centroid.
6. Elastic, creep, creep plus shrinkage, and total strains at the top, bottom and centroid.
7. Curvatures.
8. Deflections.
B.3 Creep and Creep Losses

The computer program is able to handle unit creep strain data obtained from laboratory or field conditions directly, or generate it using the coefficients derived from the graphs recommended by C.E.B. (7) that are also described in Chapter 2.

The Rate of Creep Method (22), the Revised Rate of Creep Method (20), or the Superposition Method (18), can be used to compute the creep strains since each increment of stress in the concrete, at each section, is carried independently and several unit creep strain curves, for different ages of loading, are used. The creep strain at each section, at any time interval, is computed as the sum of the product of each increment of stress in the concrete times the unit creep increment of the proper unit creep curve corresponding to the time interval. The creep strains at the level of each tendon are calculated, so the creep losses, as the sum of the creep strains at the level of each tendon times the modulus of elasticity of the steel times the tendon area, can be calculated.

B.4 Shrinkage and Shrinkage Losses

The computer program is able to handle shrinkage strain data obtained from laboratory or field conditions directly, or generate it using the coefficients derived from the graphs recommended by C.E.B. (7) that are also described in Chapter 2.

In the analysis only one shrinkage strain curve, common to all segments, is considered. Thus, the shrinkage losses at each section, at any time interval, is computed as the product of the modulus of elasticity
of steel times the increment of shrinkage strain corresponding to the time interval times the number of tendons times the tendon area.

B.5 Relaxation and Relaxation Losses

In this analysis Eq. 2.17, developed by Magura, Sozen and Siess (17), is used to compute pure relaxation. This equation was derived from tests conducted under constant strain conditions and expresses relaxation losses in a mathematical form including both time and initial stress.

During the lifetime of the structure, changes in prestress force are continuously taking place. Sometimes such stress changes occur instantaneously, as is the case when construction of a new segment takes place, or they may occur gradually as a result of creep and shrinkage strains. These stress changes must be taken into account for a better estimation of the long-time relaxation losses. The relaxation losses at each section, at any time interval, is computed in the following way: (1) the stress in each tendon is known at the beginning of the time interval I, (2) a hypothetical initial stress at time \( t = 0 \) is computed based on this stress, by solving Eq. 2.17 to find the stress when \( t = 0 \), and (3) the relaxation losses during the time interval I is found, using as the initial stress at \( t = 0 \) the hypothetical initial stress just found. This is done with all the tendons of each section.

B.6 Elastic Recovery

The analysis takes the changes in axial force and moment caused by the instantaneous deformations (whenever it occurs, especially when a new new segment is built), creep, shrinkage and relaxation losses and adds
them up at the end of each time interval. The change in force is used to
compute elastic changes of concrete and steel stresses and strains correspon-
ding to these losses. The stress changes are added to the stresses
existing at the beginning of the time interval to obtain the final stresses.
The strain changes due to this force, the direct strains due to creep and
shrinkage, and the initial strains are summed to give the final strains.
If these stress and strain recoveries are ignored, the loss of prestress
is overestimated, and equilibrium between tension in the steel and compression
in the concrete and also strain compatibility between changes in steel strain
and changes in concrete strain at the steel level, during any given time in-
terval, cannot be satisfied, which violates the assumptions made in develop-
ing this numerical procedure.

B.7 Strain

In this analysis up to four different values of modulus of
elasticity of concrete are used, one every time that an increment of stress
caused by the construction of a new segment occurs at the section in question,
up to the fourth increment. The fourth value is used from then on.

The analysis keeps track of the total strain, at any time interval,
at the top, bottom and centroid of each section by dividing the increment of
stresses (including elastic recovery) during the interval for the corresponding
value of modulus of elasticity and adding it to the total strains that
existed at the beginning of the interval.

B.8 Curvatures and Deflections

The analysis uses the total strains at the top and bottom, and
the depth of each section to compute the curvature at each time interval
by subtracting the bottom from the top strain and dividing the result by the depth.

Once the values of curvature are computed, they are considered as a distributed load and a numerical integration procedure (10) is used to compute deflections.
Fig. B.1 Flow Chart for the Computer Program
APPENDIX C

NOTATION

The notation used throughout this study are defined where they first appear. For convenience they are summarized below:

a  a constant having a value of 35 any time after 7 days of age for moist cured concrete, and 55 after 1-3 days of age for steam cured concrete.

c  relaxation constant, equal to 10 for stress-relieved strand, and 45 for low-relaxation strand.

$C_{cu}$  ultimate creep coefficient, having an average of 2.35.

D  degree of hardening of concrete at the time of loading.

dm  theoretical thickness of the member.

$E_c(t)$  modulus of elasticity of concrete at time t, in psi.

$E_{co}$  modulus of elasticity of concrete at the time of initial loading.

$E_{cs}(28)$  secant modulus of elasticity of concrete at 28 days.

f(t)  concrete stress as a function of time.

$f_o$  initial stress in concrete at the time of first loading, $t_o$.

$f_s(t)$  stress of the prestressing reinforcement at time t.

$f_{si}$  initial steel stress.

$f_y$  steel stress, measured at an offset strain of 0.001.

$f_c'(t)$  concrete strength at time t, in psi.

$K_b$  influence of composition of the concrete mix on creep and shrinkage.
$K_c$ creep factor, depends on the relative humidity of air.

$K_d$ creep factor, depends on the age of the concrete at the time of loading and the type of cement.

$K_e$ influence of specimen shape and size on creep and shrinkage.

$K_p$ influence of longitudinal reinforcement on shrinkage.

$K_t$ time-strain relationship for creep and shrinkage.

log logarithm to base 10.

t age of the concrete, in days.

$T^\circ C$ temperature in centigrades.

$T^\circ F$ temperature in fahrenheit.

$t_o$ age of concrete at the time of first loading.

$t_s$ time after steel stressing, in hours.

$W$ unit weight of the concrete in pcf.

$\Delta F(t_i)$ additional stress increments or decrements applied at time $t_o < t_i < t$.

$\Delta t$ number of days during which hardening has taken place at $T^\circ C$.

$\varepsilon_c$ shrinkage factor, depends on the relative humidity of the air.

$\varepsilon_{cr}(t)$ total creep strain at time t.

$\varepsilon_{cr}(t)$ creep strain at time $t_i$ caused by a constant unit stress.

$\varepsilon_{cr}(t, t_i)$ unit creep strain at time $t_i$ for concrete loaded at age $t_i > t_o$.

$\varepsilon_{cr}(t, t_o)$ unit creep strain at time $t$, for concrete loaded at the age $t_o$.

$\varepsilon_{sh}(t)$ shrinkage strain at time t.

$\varepsilon_{shu}$ ultimate shrinkage strain, having a value of $800 \times 10^{-6}$ in./in.

for moist cured concrete, and $730 \times 10^{-6}$ in./in. for steam cured concrete.

$\phi(t)$ creep coefficient, is expressed as a product of five partial factors.