ANALYTICAL STUDIES OF LARGE SCALE COLUMN TESTS

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by

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Approved by
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I. INTRODUCTION

1. Object and Scope

In 1949, as a result of questions arising out of the design of certain compression members of the Calcasieu River Bridge at Lake Charles, Louisiana, the column research project now in progress at the University of Illinois was first initiated. The first phase of this program was concerned with the testing of four large scale structural steel columns and had as its primary objective the determination of the manner of failure and the load carrying capacity of each of these members. Two of the columns were fabricated from structural elements comprising plates and angles while the remaining two columns were fabricated from elements comprising channels and wide flange beams. The specimens simulated certain compression members of the Calcasieu River Bridge. The results of these tests have been presented in two progress reports (1,2)* prepared for the Louisiana Department of Highways and the Bureau of Public Roads, Department of Commerce.

In addition to the investigation described above, a related program of column tests was simultaneously undertaken by the National Bureau of Standards. Two large perforated cover plate columns fabricated from plates and angles were tested in order to study the ability of perforated cover plates to properly transmit shear and distribute stress. Reports of these tests have been prepared by the National Bureau of Standards (3,4).

To effectively evaluate and utilize the data obtained from the Illinois and Bureau tests an analytical study of the experimental results has been undertaken. The results of this investigation are presented herein and serve as a supplement to the information included in the original reports.

* Numbers in parenthesis refer to the list of references at the end of this report.
The ultimate objective of this phase of the overall column research project is to explain, on a quantitative and qualitative basis, the behavior of the large scale columns and to correlate the test results with the theoretical predictions wherever possible. In pursuing this objective a detailed study of the following significant topics has been made and are presented in the sections which follow.

i. Effect of residual stresses on the behavior of the columns.
ii. Theory of torsional-flexural buckling for open section columns.
iii. Effective lengths and critical loads for test columns.
iv. Mechanism of failure of test columns.
v. Buckling of web plate developed in one test.
vi. Perforated cover plate column tests.

Items i. through v. are concerned with the Illinois tests. Item vi. is a study of the Bureau column data.

In the Appendix to this report is presented information concerning the dimensions of the specimens, test measurements, and results of the original large scale column tests conducted at the University of Illinois. This section is included for the convenience of the reader and also for the benefit of those persons who have only recently been included on this research program distribution list.

2. Acknowledgements

The studies described herein were performed as part of a research program in Tests of Large Steel Columns sponsored by the Bureau of Public Roads, Department of Commerce and constitute a portion of the Structural Research Program of the Department of Civil Engineering of the University of Illinois. The entire project is under the general direction of Dr. N. M. Newmark, Research Professor of Civil Engineering. The authors wish to thank Mr. Philip Chow, formerly Research Graduate
Assistant in Civil Engineering, who assisted greatly in various technical aspects of this work, such as computing data, preparing drawings, etc. We are also grateful for the assistance of many others who helped in numerous ways but are not mentioned specifically.
II. A STUDY OF RESIDUAL STRESSES

3. Theoretical Considerations

This chapter contains a study of the following question: did residual stresses exist in the columns tested at the University of Illinois, and if so did they influence the ultimate load-carrying capacity of these members? Osgood (5) has demonstrated theoretically that residual stresses are an important factor affecting the ultimate strength of columns. Other research work presently being conducted seems to verify this conclusion.

The most direct method of determining whether initial residual stresses were present in the test specimens would have been to section the columns and measure the relaxation strains. This could not be done since extra lengths of the component elements were not available. However, from the measurements of average strain and overall shortening which were obtained during the tests, it is possible to detect yielding in the columns and in an approximate sense the size of the area involved and the corresponding seriousness of this phenomenon. With this information the existence or absence of initial residual stresses may be inferred, but not their distribution either across the section or lengthwise between sections.

Stated briefly, Osgood's conception is that at any load P large enough to cause yielding in some of the fibers, the area of each fiber which has yielded should be transformed by multiplying by the ratio of the tangent modulus to the original modulus. Then the properties of this transformed area (such as centroid, location of the principal axes, and principal moments of inertia) can be determined. Euler's buckling formula can then be used with the properties of this transformed area to give the length of column which will buckle under the load P.
This analysis may be extended somewhat to consider the significance of the slope of the curve of load vs. strain at the centroid of the transformed section. Consider a column of arbitrary cross section sustaining an axial load $P$, which causes at any section in the column a uniform strain and a bending strain. The increment of axial load equivalent to a small increment of uniform strain, may be evaluated as follows:

Denote $\Delta \varepsilon$ = a small uniform strain increment, hence the strain at the centroid of the transformed section, or, in fact, anywhere along the transformed area bending axis.

then the increase in load $\Delta P$ is

$$\Delta P = \int_{A} \Delta \sigma \, dA = \int_{A} \Delta \varepsilon \, E_{xy} \, dA$$

where $E_{xy}$ is the tangent modulus, a function of $x$ and $y$, $\Delta \sigma$ is the stress increment corresponding to $\Delta \varepsilon$, and $dA$ is the small area over which $\Delta \sigma$ acts. Then

$$\lim_{\Delta \varepsilon \to 0} \frac{\Delta P}{\Delta \varepsilon} = \frac{dP}{d\varepsilon} = \int_{A} E_{xy} \, dA$$

or

$$\frac{dP}{d\varepsilon} = E_i \int_{A} \frac{E_{xy}}{E_i} \, dA$$

where $E_i$ is the initial modulus of elasticity or any desired reference value.

The cross section is now transformed by multiplying each element of area $dA$ by the ratio $E_{xy}/E_i$. If we denote

$$dB = \frac{E_{xy}}{E_i} \, dA$$

then

$$\frac{dP}{d\varepsilon} = E_i \int_{A} dB = E_i B$$
The area B is a reduced area which has an elastic stiffness equal to the actual stiffness of the original section. The ratio B/A is the proportion of area acting at original effectiveness in direct stress.

According to this development the slope of the average stress-average strain diagram, is as follows:

\[
\frac{d\sigma}{d\epsilon} = \frac{1}{A} \frac{dP}{d\epsilon} = E_i \frac{B}{A} = E_r
\]

where \(E_r\) is the average or effective modulus for the column.

If mild steel is idealized as a perfect elasto-plastic material (no strain hardening) then

\[
E_{xy} = E_i \quad \text{for stresses below the yield point.}
\]

\[
E_{xy} = 0 \quad \text{for stresses above the yield point.}
\]

In this case B is the area which is still acting elastically and the ratio B/A is that proportion of the original area which is still elastic. This ratio may be found from the measured slopes of the average stress-average strain diagram by the use of Equation (1). Where these relations are extended to the entire column, the assumption must be made that the residual stresses are the same at every cross section.

4. Application of Theory to Test Data

In accordance with the above development the stress-strain diagrams presented in Figs. 1 to 4 may be interpreted to disclose the relationship of the load to the internal yielding of the columns. The stress is computed by dividing the applied load by the gross area of the cross section. The strain is computed in two ways. The average strains determined from electric strain gages are used to plot the closed circles in these figures. These values are given in Column 8, Tables 2 and 3 of Progress Report No. 1 and in Column 9, Table 2, and Column 8, Table 3 of Progress Report No. 2. These strains are an average
of a large number of gage readings taken at various stations along the length of the column and are believed to be good average values for the entire column. The curves of Figs. 1, 2, 3 and 4 are drawn through the closed circles. The strains used to plot the open circles are computed by dividing the overall shortening readings by the length of the column. Since the lateral bending of the column is never large enough to influence the overall shortening these strains should closely represent average values. In Figs. 1, 2 and 3 the average strains computed from overall shortening measurements agree well with the average strains computed from strain gage readings, but are somewhat erratic, due possibly to the less accurate method of measurement. Where only one value is plotted for any stress reading the two points overlap. In Fig. 4 the two sets of strains are displaced horizontally from each other, but the slopes of the two stress-strain curves agree reasonably well except at high values of stress.

The slopes of the stress-strain curves have been determined at a number of different stresses and are plotted in Fig. 5 for all four columns. As is well known, moduli obtained in this fashion must be considered to be only approximate because of the personal factor involved in sketching the stress-strain curve and in determining the tangents to this curve (by eye). In this respect the slopes measured at the tips of the stress-strain curves are particularly subject to error. Despite these inaccuracies, the trends are still reliable.

Since the strain gages were symmetrically placed about both axes of symmetry, the average of these gage readings represents the strain at the centroid of the section. However, in the previous development $\Delta \varepsilon$ is the shortening of the fiber at the centroid of the transformed section. In the latter stages of the test the bending moments due to lateral deflection of the column cause an unsymmetrical yielding which varies along the length of the column. In this case the centroid of the transformed section shifts away from its original
position forward the side with tensile bending stress. Now when additional load \( \Delta P \) is applied, additional bending about the transformed axis occurs simultaneously with shortening, and this bending increment causes compressive strain in the fiber at the centroid of the complete section. This bending registers as compressive strain on the average of the strain gages. Therefore near ultimate load when the lateral bending stresses are large, the slopes, and hence the tangent moduli, measured from the stress-strain curve are probably too small.

5. Interpretation of Test Results

It should be observed that the columns composed of angles and plates (52-C series) had similar stress-strain curves and the columns composed of a wide flange beam and channels (57-C series) also behaved alike. The stress-strain curves for the 52-C series are fairly straight up to a sharp break near ultimate load. On the other hand, the stress-strain curves for the 57-C series depart from linearity at a lower stress and then curve gradually and uniformly to failure. These trends may also be observed in Fig. 5 which clearly demonstrates the similarity of behavior for the columns of each series. In connection with the early departure from linearity of the stress-strain curves of the 57-C series columns, it is interesting to recall that Leuders' lines were visually detected in the web of column 57-C-1 at a load of only 600 kips; similar yield lines were noted in the web of 57-C-2 but not before a load of 1200 kips, which is almost the maximum load.

Some important measurements related to the residual stress studies are summarized in the following table. The average compressive yield points given in line 1 are explained in the Appendix. Line 2 gives the initial moduli of elasticity found from Figs. 1, 2, 3, and 4. These values are less than the overall
### SUMMARY OF RESIDUAL STRESS MEASUREMENTS

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<td>1</td>
<td>Avg. Compressive Yield Point, psi.</td>
<td>41600</td>
<td>40300</td>
<td>47200</td>
<td>42700</td>
</tr>
<tr>
<td>2</td>
<td>Initial Modulus of Elasticity, ksi.</td>
<td>27700</td>
<td>27000</td>
<td>29600</td>
<td>28200</td>
</tr>
<tr>
<td>3</td>
<td>Average Stress at $E = 25000$ ksi. psi.</td>
<td>23800</td>
<td>23000</td>
<td>18000</td>
<td>17200</td>
</tr>
<tr>
<td>4</td>
<td>Estimated Bending Stress at Load Defined in Line 3, psi.</td>
<td>3800</td>
<td>4000</td>
<td>500</td>
<td>300</td>
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Averages obtained from the coupon tests. The moduli obtained from tensile coupon tests varied from 31200 ksi to 25000 ksi with an average of about 29500 ksi. There were too few coupon tests to attempt to find an average value for each column. Therefore, it is not possible to draw very definite conclusions from the measured initial moduli. However, it is interesting that the initial moduli for the 52-C series columns were less than the values for the 57-C series. This condition was possibly due to the larger number of rivet holes in the 52-C series columns.

The average stress at which initial yielding occurred was difficult to determine accurately; instead the average stress at which the effective modulus of elasticity dropped to 25000 ksi was measured from Fig. 5 and tabulated in Line 3 of the above table. Line 4 contains the bending stress at midsection which occurred simultaneously with the average stress of Line 3. The bending stress was computed* from the strains measured at the four corners of the

---

*Bending stress = average stress on gross area x ratio of the difference to the sum of the strains at corresponding points on opposite sides of the bending axis.
mid-section of each column. For all columns the sum of the average stress and the bending stress was nowhere near the yield point of the material, despite the fact that approximately 15 percent of the cross section had apparently yielded. It is believed that initial residual stresses caused this behavior. Note that this amount of yielding occurred at a maximum stress of 17500-18500 psi in the 57-C series whereas for the 52-C series the corresponding maximum stress was 27000-27600 psi. This difference is compatible with the theory of initial residual stresses because these stresses, having been caused by unequal cooling after rolling, are much larger in heavy rolled sections than in plates or angles. Residual stresses as large as the material yield point have been found in some rolled sections. For the columns under investigation the maximum residual stresses appear to have been approximately 26000 psi for the 57-C series and 14000 psi for the 52-C series.

Under load the history of a steel column containing large residual stresses is probably somewhat as follows. First yielding occurs early in the test due to high initial residual stresses. The amount of yielding increases as the load increases. This premature yielding decreases the bending stiffness of the column and so the lateral deflections grow at an accelerated rate. The bending stresses cause increased yielding in the compressive side, and the column is weakened still further. This process continues until finally the effective area and the corresponding moment of inertia are so reduced that the Euler load for this transformed section is reached and failure occurs.

Recently investigators at Lehigh University have found that the tangent modulus theory seems applicable to steel columns when the tangent modulus is found from a stress-strain curve which has been determined from a test of a short stub of the entire section. Figures 1, 2, 3, and 4 are approximations to this required stress-strain curve, particularly at the lower stresses where only a small amount of bending has yet occurred. Therefore, the tangent moduli which
we desire for a similar analysis are presented in Fig. 5. The tangent moduli obtained from these plots were used to find, in the usual manner, the curves of critical stress vs. slenderness ratio presented in Figs. 6 and 7. These are shown in solid line while the customary Euler curves based on the initial modulus are shown in dotted line. The ultimate stress determined by the test is also plotted for each of the columns. In the plots of the test points the slenderness ratios are computed from the "effective lengths" which are presented in Section 8 of this report.

It will be noted that in columns 52-C-1, 52-C-2, and 57-C-2 the test points are quite a bit above the tangent modulus curve. This is easily explained for the first two columns. The tangent modulus curves are probably too low for columns 52-C-1 and 52-C-2 because the tangent moduli used to determine these curve are too small for the high average stresses; the tangent moduli are too small because of the large, early bending action in these columns. This is especially pronounced in Fig. 6 (b) where the tangent modulus curve breaks downward very sharply and unnaturally. Column 57-C-1 gives an excellent experimental check of the theory, probably because very little bending occurred until just before failure. No explanation can be offered for column 57-C-2 except to note that none of this data is very precise, and also the extension of the effective length concept to the prediction of ultimate load is uncertain.
III. ANALYSIS OF BUCKLING BEHAVIOR

6. General Solution

The purpose of this chapter is to present an analysis of the buckling behavior of the four open-section columns tested at the University of Illinois. In these tests the lateral deflections were accompanied by an appreciable twisting of both flanges; this twisting deformed the original cross-section as is shown in Fig. 6. The twisting occurred along the entire length of the column. The twisting began at low loads in the elastic range and, in general, increased in magnitude with increasing load. The variation of the angle of rotation of the flanges with respect to the length of the column was rather erratic, particularly at low loads. In general, the direction of twist seemed to depend upon the direction of curvature; the direction of twist was always such as to toe in the flanges on the concave side of the buckle and toe out the flanges on the convex side. The maximum rotation of the flanges normally occurred near midheight during the test and always occurred at midheight after large deformations had been produced subsequent to the ultimate load.

The above observations suggest that the rotations of the flanges are a basic component of the deformation of the column and are not just a local phenomenon. This interpretation is borne out by the analysis.

Primary failure (as opposed to local instability) of columns of the open-section type considered in this report is characterized by lateral buckling in a direction either normal to the web (the y direction) or parallel to the web (the x direction. See Fig. 8 (a) for directions). If the columns buckle in the x direction no twisting of the flanges should occur because the flanges are symmetrical about the centroidal x axis of the section. In this case the critical load is simply the Euler load; for the columns tested under this
contract and, in general, this load exceeds the critical load for buckling in the y direction and hence is of little significance. If the columns buckle in the y direction, the deflection is accompanied by twisting of the flanges. The critical load for this case is always less than the Euler load for lateral buckling without distortion of the cross-section, but usually the difference is negligible. The following analysis is concerned exclusively with the latter case.

The column is considered to be separated into three components, two flanges and a web. The flanges are assumed to maintain the shape of their cross-section during bending and twisting. On the other hand the web is considered to be flexible; when the flanges rotate the web is bent as shown in Fig. 6 (b). The problem then is to determine the critical load for two flanges connected by a flexible web which resists relative rotation and relative deflection. It seems reasonable to assume that both flanges will deflect equally, and in this case from considerations of symmetry the rotations must be equal. With these assumptions the problem is simplified to that of determining the critical load of a single flange which may buckle freely in the y direction, cannot deflect at all in the x direction, and is elastically restrained by the web against rotation.

The columns were tested on flat ends. The following analysis is made for a column with hinged ends, loaded axially at the ends. This analysis therefore applies to the test columns when an effective length equal to the distance between points of inflection is used in the solution.

Consider the representation of the column cross-section given in Fig. 8 (a). The principal axes passing through the centroid of the flange section are chosen as the x and y axes. The z axis is perpendicular to the x and y axes; therefore the z axis passes through the centroid of the flange at every section, before deflection occurs. The following notation is used in this study.

\[ x_0 \quad (y_0 = 0) \] coordinates of the shear center of the flange.
v  deflections of the flange shear center axis in the y direction.

β  angle of rotation of cross-section, positive as shown.

h  distance of point of attachment of flange and web to the shear center of the flange.

t  thickness of web plate.

b  width of web plate, i.e. distance between flanges.

E  modulus of elasticity.

μ  Poisson's ratio, taken as 0.27 in this analysis.

I_x', I_y'  moments of inertia of the flange section about the x and y axes, respectively.

A, A', A''  area of column section, flange section, and web, respectively.

C_1  non-uniform torsion constant for flange.

C  uniform torsion constant for flange.

l  effective length of column.

σ  uniform compressive stress.

Other symbols used are defined in the text.

Let us consider the forces which act on a very small length of the column, dz, when the column is subjected to a compressive load and a small disturbance from the initially straight equilibrium position occurs. This deflection and twisting of the column produces both internal resisting and self-excited disturbing forces, as described in the following paragraphs.

As a result of the deflection and twist the ends of the small length of column will be slightly rotated with respect to each other. The initial uniform compressive stresses act normal to the cross section at each end and
therefore a component of force normal to the axis of the column is produced by the bending and twisting. The lateral force acting on the column is divided into two parts. That acting on a small segment of flange is equal to the product of the total force on the flange, \( \sigma A' \), multiplied by the angle of rotation between the ends of the centroidal axis of the flange segment. Thus, if the lateral force on a flange is denoted as \( q' \), one obtains

\[
q' = -\sigma A' \frac{d^2(v + x_0 \beta)}{d\xi^2} \, d\xi
\]

This force is directed away from the center of curvature and acts through the centroid of the flange. Similarly the lateral force on the web \( q'' \) is equal to the product of the total force carried by the web multiplied by the angle of rotation between the ends of the centroidal axis of the deformed web. For convenience this angle is taken as equal to the angle for the flange segment. Then one obtains the following expression for the lateral force on the web.

\[
q'' = -\sigma A'' \frac{d^2(v + x_0 \beta)}{d\xi^2} \, d\xi
\]

This force also is directed away from the center of curvature. The third (and last) disturbing force is a torque acting on the flange which is produced by the interaction of the compressive forces and the twisting of the flange. This torque has the following value:

\[
\text{disturbing torque} = T_d = \sigma \left( I_x' + I_y' \right) \frac{d^2B}{d\xi^2} \, d\xi
\]

Three resisting forces oppose the deflection of the column. First, the flanges possess a resistance against lateral deflection. The force of this resistance acting on a segment of length \( d\xi \) is denoted as \( V \), and is given by the following expression:

\[
V = -\frac{E I_x'}{d^2} \frac{d^2v}{d\xi^2} \, d\xi
\]

This force acts through the shear center of the flange. Secondly, the flanges possess torsional resistance. The torque of this resistance acting on a segment
of length $dz$ is given by the following expression:

$$\text{resisting torque } T_r = C_l \frac{d^4 \beta}{dz^4} dz - C \frac{d^2 \beta}{dz^2} dz$$

This expression is based upon the common theory of non-uniform torsion and is not derived herein. Similarly the expression for disturbing torque above is not derived herein. Finally, the web restrains the rotation of the flanges. As the following solution will reveal, the deflection and twisting configurations are described by a half sine wave. This wave is so long in comparison with the width of the web that at any point along the length of the column the web may be considered to be in pure bending in the lateral direction. When a flat plate of infinite length but finite width, $b$, and thickness, $t$, has hinged edges along both parallel edges and is loaded with equal uniform moments along each edge, the relation between the edge slope $\beta$ and the edge moments $m''$ is as follows:

$$m'' = \frac{Et^3}{6(1-\mu^2)b} \beta \ dz = \frac{Et^3}{5.6b} \beta \ dz$$

This expression is considered to satisfactorily state the moment furnished by the web in resisting twisting of the flanges. The rotation of the edges of the web caused by the lateral forces $q''$ uniformly spread over the width is therefore considered to be negligible in comparison with the rotation of the flanges.

The equations of equilibrium for the flange section may now be written. In the equations below the infinitesimal $dz$ has been eliminated by factoring.

For equilibrium for forces in the $y$ direction, we have

$$V + q'' + \frac{q''}{2} = 0$$

or

$$EI_x' \frac{d^4 \nu}{dz^4} + \sigma A' \frac{d^2 \nu}{dz^2} (\nu + \chi \beta) = 0 \quad (2)$$

Equation (2) is now integrated twice between the limits $z = 0$ and $z = \lambda$ (with boundary conditions $\frac{d^2 \nu}{dz^2} = \nu = \beta = 0$ at both ends of the interval) to
yield the following familiar equation.

\[ EI_x \frac{d^2 v}{dz^2} + \sigma \frac{A}{2} (v + x_o \beta) = 0 \]  

From a consideration of the equilibrium of the torques on the flange section one obtains the following equation.

\[ -q' x_o - \frac{q''}{2} h + m'' + T_r + T_d = 0 \]

or

\[ \sigma A' x_o \left( \frac{d^2 v}{dz^2} + x_o \frac{d^2 \beta}{dz^2} \right) + \sigma \frac{A''}{2} h \left( \frac{d^2 v}{dz^2} + x_o \frac{d^2 \beta}{dz^2} \right) + \frac{E t^3}{5.6 b} \beta + C_i \frac{d^4 \beta}{dz^4} - C \frac{d^2 p}{dz^2} + \sigma (I_x' + I_y') \frac{d^2 \beta}{dz^2} = 0 \]

These terms may now be rearranged as follows.

\[ C_i \frac{d^4 \beta}{dz^4} + \frac{d^2 p}{dz^2} \left[ \sigma (I_x' + I_y') + \sigma A' x_o + \sigma \frac{A''}{2} h x_o - C \right] + \frac{d^2 v}{dz^2} \left[ \sigma A' x_o + \frac{\sigma A''}{2} h \right] + \frac{E t^3}{5.6 b} \beta = 0 \]  

With the following notation,

\[ \bar{x}_o = x_o + \frac{A''}{2A'} h \quad \text{and} \quad \bar{I}_o^2 = \frac{I_x' + I_y'}{A'} + x_o^2 + \frac{h x_o A''}{2A'} \]

Eq. (4) may be simplified to Eq. (5).

\[ C_i \frac{d^4 \beta}{dz^4} + \left( \sigma A' x_o^2 - C \right) \frac{d^2 \beta}{dz^2} + \sigma A' \bar{x}_o \frac{d^2 v}{dz^2} + \frac{E t^3}{5.6 b} \beta = 0 \]

The ends of the column are assumed to be free to warp and rotate with respect to the x axis but the ends cannot rotate with respect to the z axis.

These end conditions may be expressed as follows: at \( z = 0 \) and \( z = l \),

\[ v = \beta = 0 \quad \text{and} \quad \frac{d^2 v}{dz^2} = \frac{d^2 \beta}{dz^2} = 0 \]

A solution satisfying all these conditions can be obtained in the form

\[ v = B_1 \sin \frac{\pi z}{l} \quad \text{and} \quad \beta = B_2 \sin \frac{\pi z}{l} \]

On substitution of these expressions into Eqs. (3) and (5) we obtain Eqs. (6) and (7).

\[ \left( \frac{\sigma A}{2} - \frac{E t^3}{l^2} \right) B_1 + \frac{1}{2} \sigma A x_o B_2 = 0 \]  

\[ \sigma A' \bar{x}_o B_1 + \left( \sigma A' \bar{I}_o^2 - C - \frac{\pi^2}{l^2} C_i - \frac{E t^3}{5.6 b} \frac{l^2}{\pi^2} \right) B_2 = 0 \]
To simplify the expressions, introduce the following notation

\[ \sigma_1 = \frac{2 \pi^2}{A} \frac{EI_x}{l^2}, \quad \sigma_2 = \frac{1}{A^2 \rho_0^2} \left( C + \frac{\pi^2}{l^2} C_1 + \frac{Et^3}{5.6 b} \frac{l^2}{\pi^2} \right) \]

It is interesting to note that \( \sigma_1 \) is the critical stress for lateral buckling of the column in the y direction when no twisting is permitted, and \( \sigma_2 \) is the critical stress for the flange when only rotations but no deflections of the flanges are allowed. Equations (6) and (7) may now be rewritten as follows.

\[
\begin{align*}
(\sigma - \sigma_1) B_1 &+ \sigma x_0 B_2 = 0 \\
\sigma x_0 B_1 + \rho_0^2 (\sigma - \sigma_2) B_2 &= 0 
\end{align*}
\]

Equating to zero the determinant of Eqs. (8), we obtain

\[
\begin{vmatrix}
(\sigma - \sigma_1) & \sigma x_0 \\
\sigma x_0 & \rho_0^2 (\sigma - \sigma_2)
\end{vmatrix} = 0
\]

which gives the following quadratic equation for calculating the critical stress.

\[
\sigma^2 \left( 1 - \frac{x_0 x_0}{\rho_0^2} \right) - \sigma \left( \sigma_1 + \sigma_2 \right) + \sigma_1 \sigma_2 = 0
\] (9)

Equation (9) has two roots, one lower than either \( \sigma_1 \) or \( \sigma_2 \) and the other higher than either \( \sigma_1 \) or \( \sigma_2 \). The lower value is the significant value for engineering purposes.

Since Eqs. (8) are homogeneous, the rotation \( \beta \) and the deflection \( v \) may have any values so long as these values bear a certain relation to each other.

This relation is as follows:

\[
\frac{\beta}{v} = \frac{B_2}{B_1} = \frac{\sigma_1 - \sigma_0}{x_0 \sigma_0}
\] (10)

where \( \sigma_0 \) is the critical stress for which Eqs. (8) become identical.
7. Analyses of Test Sections

In order to apply the solution just obtained it is necessary to divide the column sections into two flanges and a web. This is done on the basis of judgment. For the columns of this report the components were chosen as shown in Fig. 9. The pertinent section properties are also given in this figure. The uniform torsion constant $C$ is computed from values determined from torsion tests of the full section of columns 52-C-1 and 57-C-1; these tests were conducted at Lehigh University (10).

The critical stresses were computed from Eq. 9. Graphs showing the variation of critical stress with the effective length are presented in Figs. 10, 11, and 12 for column sections corresponding to test columns 52-C-1, 52-C-2, and the 57-C series. In each figure three curves are shown. The curve in solid line which is designated as the "true buckling curve" represents the solution of Eq. (9). In addition to this solution, upper and lower limits of the elastic buckling stress are given by the two dotted curves. The upper dotted line represents the Euler relation, to wit,

$$\sigma_0 = \frac{2\pi^2EI_x'}{A\ell^2}$$

This value of the critical stress is based upon the assumption that no distortion of the cross section occurs; this is, therefore, equivalent to an assumption that the web possesses infinite flexural stiffness in the transverse direction. Since this is the largest possible web stiffness, the Euler solution gives the upper limit to the critical stress. The dotted curve representing the lower limit is obtained by assuming that the web has no flexural stiffness; in this case the expression for $\sigma_2$ degenerates to the following formula,

$$\sigma_2 = \frac{1}{A'F_0^2} \left( C + \frac{\pi^2}{\ell^2} C_1 \right)$$
The lower limit is very conservative; in fact it is more conservative than assuming the thickness of the web plate to vanish, because in such a case $x_o$ and $f_o^2$ are reduced and $x_o x_{o_1}$ and $\sigma_1$ are increased, all of these changes tending to increase the critical stress of the flange.

Some very important conclusions may be drawn from the graphs of Figs. 10, 11, and 12. To begin with, the ultimate load for all four of the columns is not lowered by the twisting of the flanges. The true curve and the Euler curve are indistinguishable except in the region of short columns which have very high values for the critical stresses, well above the yield point of the material. Therefore, the ultimate load is not lowered for these particular sections for any length of column.

If the elastic restraint offered by the web is completely neglected, theory would indicate that the ultimate load should be lowered appreciably for column sections 52-C-1 and 52-C-2 for effective lengths less than about 450 in. Even when both the true critical stress and the lower limit are above the yield point of the material, the magnification of the initial eccentricities would be sufficiently different in the two cases to cause a difference in the computed ultimate load. However, for columns with effective lengths greater than 450 in., the ultimate load would not be lowered by very much. For column sections 57-C-1 and 57-C-2 the theoretical ultimate load is not lowered appreciably for any length even when the elastic restraint of the web is completely neglected.

From the above comments it is quite clear that the critical stress and the ultimate load are not sensitive to small changes in the restraint offered by the web. The ultimate load is far more sensitive to other effects. As we shall see in a later section, the critical stresses could not be reliably determined from the data of these tests. For these reasons the theory proposed in the previous section cannot be experimentally verified on the basis of these important
quantities. However, the rotation of the flanges per unit of deflection, \( \beta/v \), is much more sensitive to the rotational restraint of the web. For this reason, the variation of \( \beta/v \) with the effective length is plotted in Fig. 13 for all four column sections. The relative rotations \( \beta/v \) are not plotted for the case where the web restraint is neglected because these values of \( \beta/v \) are so large that they are completely out of the range of the graph for lengths less than about 350 in., and a smaller vertical scale was not desired. Unfortunately the relative rotation \( \beta/v \) is also fairly sensitive to small changes in effective length in the range of the lengths actually obtained in the column tests. Before we can attempt to verify experimentally the buckling theory it is therefore necessary to determine the effective lengths of the test columns. This is done in the next section.

8. Effective Length

The effective length of a column is that length which when substituted into the Euler formula for hinged-ended columns yields a critical load equal to the critical load of the given column. When a column is restrained at the ends, the effective length is less than the true length of the column; in this case the effective length is equal to the distance between points of inflection of the elastic axis when deformed into the buckling configuration which corresponds to the lowest critical load.

To determine the critical load of the test columns from Eq. (9) of Section 6 or from Figs. 10, 11, or 12, the effective length must be known. Furthermore, to make comparisons between the ultimate loads obtained from the tests and the ultimate loads predicted by various theories, the effective length must be known because these theories are for hinged-ended columns. Therefore, an attempt is made in this section to evaluate the effective lengths for the four columns tested at the University of Illinois.
For the columns of interest, the effective lengths cannot be determined accurately or reliably from the test data, for the following reasons. The effective length has to be determined from a study of either the deflected shape under load or the corresponding curvatures since the restraint conditions at the ends are not known. Both of these values depend to a large extent on the initial crookedness of the column until the applied load approaches the critical load, at which time the deflection component of the lowest mode becomes magnified so much more than those of the higher modes that the deflection shape becomes predominantly the shape of the lowest mode. At this time the distance between inflection points is the effective length. However, these columns failed at a load far less than the elastic critical load, so that this relative magnification of the lowest deflection mode was never attained in the elastic range. We may perhaps conclude that the measured distance between points of contraflexure under the largest load before localized yielding commenced should be the most accurate value of the effective length which we can obtain. In this connection it may be noted that uniform yielding along the length of the column, such as that assumed to be produced by residual stresses, will not cause a change in the effective length. However, localized yielding which results in the development of a semi-plastic hinge at some section, such as at midheight, causes a sharp curvature at the section and renders inaccurate a determination of the effective length from the deflected shape. Of course a semi-plastic hinge develops gradually and the load at which its effect on the deflections becomes predominant cannot be determined accurately but must be estimated.

Finally one might question whether the effective length concept is valid for comparisons of the ultimate load with the theoretical ultimate loads. During the tests, after ultimate load had been passed and large deflections had been produced, the columns were observed to have developed plastic hinges at
both ends and at midsection. Therefore the final distance between points of inflection was one-half of the length. This final shape is believed to have little significance. The assumption is made that the first plastic hinge develops at or about the instant at which the ultimate load is obtained and therefore the usual analyses are valid up to this load. We do not have much interest in the action of the column beyond this load.

We may summarize the above discussion as follows: the effective length is assumed to have the same significance with regard to the ultimate load as it does for the critical load, but unfortunately since these columns were not slender the ultimate load was not a large enough proportion of the critical load to eliminate the influence of the initial crookedness in the determination of the effective length. Let us now consider the experimental evidence.

The effective length of a column may be determined from the experimental results by at least two different methods. One method consists in estimating the points of inflection from lateral deflection curves such as are shown in the Appendix, Figs. A6 to A9. However, this procedure has the obvious objection that these points cannot be selected with any appreciable degree of accuracy. Accordingly, an alternative method described by Schuette and Roy (6) was used. Briefly the method consists in "measuring curvature rather than deflection and establishing from such measurements the points of zero curvature, which define the inflection points."

The curvatures along the length of the column may be computed from the readings of the strain gages located at sections A to E (see Figs. A3 and A4). Assuming that the strains vary linearly through the depth, the curvature is equal to the quotient of the differences between the strains at two points opposite each other in a particular cross section \((\varepsilon_1 - \varepsilon_2)\) divided by the distance between the points, measured perpendicular to the bending axis. For the columns under
discussion, the distance between the strain gages was constant so that it was sufficient to plot the strain difference \( (\varepsilon_1 - \varepsilon_2) \) versus the gage position along the column length in order to determine the points of zero curvature. In relation to the total length of the test columns the distance between gage points along the length was greater than might be desired. At this point it is well to note that non-uniform torsion causes normal stresses and strains, and these strains will register in the above computations as curvatures. However, if the bending and twisting are in phase with each other, as assumed in this theory, then the strains due to both effects will be zero at the same inflection point, and hence the twisting stresses will not affect the determination of the points of inflection; otherwise they will. Nevertheless, the results offer some interesting studies.

Figures 14 through 17 show the average strain differences for gages at each of the cross sections of the various columns plotted against the column length, for several different loads. It should be realized that these strain differences are probably not very accurate, percentagewise, because they are a difference between two large and nearly equal strains. Since we desire to stay as much as possible in the elastic range, the deflections are small and the bending moments are small; therefore, the greater part of the strain is due to the average stress. The curves presented in Figs. 14 to 17 are reasonable because they are averages of data from several sets of gages; the plots of individual pairs of opposing gages were erratic at times.

Another point of interest is the fact that the strain gages at a section often indicate that the distribution of strain through the depth is not linear as is assumed. This may be seen in Fig. 12 of the First Progress Report or Figs. 22, 23, and 36 of the Second Progress Report. Both of these observations are possibly due to local bending of some of the plates to which the strain gages were attached. Usually gages were not attached on opposite faces of a plate at a point so that it
was not possible to detect local bending in the plate nor to correct for it. The subsequent interpretation of the data is believed to be reasonable, but it is well to bear in mind some of the limitations of the data when judging the significance of the observed trends.

Series C-l

Fig. 14 shows the increase in the curvature of column 52-C-l caused by various loads in excess of the initial 8000-lb load. The strain differences plotted are the average of the two pairs of gages situated at the flange corners and the two pairs of gages on the flange toes. On the basis of these plots it appears that the procedure gives satisfactory results for this particular column.

The distances between points of inflection, measured from Fig. 14, decrease with increasing load as shown in the following table.

<table>
<thead>
<tr>
<th>Load Kips</th>
<th>Distance Between Points of Inflection</th>
<th>Ratio: Distance Between Points of Inflection Divided by Total Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>303 inches</td>
<td>0.73</td>
</tr>
<tr>
<td>800</td>
<td>298</td>
<td>0.72</td>
</tr>
<tr>
<td>900</td>
<td>284</td>
<td>0.68</td>
</tr>
<tr>
<td>1000</td>
<td>276</td>
<td>0.67</td>
</tr>
</tbody>
</table>

In the residual stress study it was concluded that appreciable yielding had taken place under a load of 880 kips, which produced an average stress of 23800 psi. It was also shown that a considerable bending stress existed at that time. Therefore, probably the most reasonable value of the effective length is the distance between points of inflection for a load of 800 or 900 kips. On this basis the effective length may be approximated as an average of the two, or about 291 in., which is 70 percent of the total length of the column. It is reassuring to note that the distance between points of inflection changed very little as the
load increased from 700 kips to 1000 kips; the latter load is 84 percent of the ultimate load.

The curvature plots for column 57-C-1 are shown in Fig. 15. The strain differences plotted are the averages of the two pairs of gages situated at the flange corners and are based on an initial load of 8 kips. The curvatures shown in Fig. 15 for a load of 1000 kips are quite small, indicating that little bending was experienced at this load. Although a large amount of yielding due to initial residual stresses has occurred up to this load, apparently no semi-plastic hinge has yet developed. As the load is increased from 1000 to 1200 kips, the deflections and curvatures increase appreciably. The curvature diagram, however, still does not appear to give a reliable indication of effective length; indeed if one were to use the curvature diagrams at 1000 and 1200 kips for this purpose, one would have to conclude that the effective length is less than one-half of the true length, which is not possible. As the load increased from 1200 to 1280 kips (which is 96 percent of the maximum load) the deflections and curvatures grew very rapidly indicating that a semi-plastic hinge developed during this load increment. The curvature diagram of Fig. 15 shows that at a load of 1280 kips the distance between points of inflection was approximately 204 in., or 49 percent of the total column length. The position of the inflection points at this load very nearly occurred at the quarter points of the column. These observations indicate that the column acted as if both ends were fixed.

The degree of end-fixity may also be found by studying the angular movement of the bearing blocks through which load was applied to the columns. Fig. 18 shows the rotation of these blocks over the complete range of load for the test of column 57-C-1. In this figure and also in corresponding Figs. 19 and 20 for columns 52-C-2 and 57-C-2, only the rotations in the E-W direction (shown in solid line) are important since the columns failed by lateral bending in this
plane with very little bending in the perpendicular N-S direction. Column 57-C-1 was allowed to sit on the spherical bearing blocks while the load was increased to 100,000 lb. Then wedges were inserted to prevent further rotation of each spherical head. The effect of this procedure is illustrated in the figure. At the top bearing plate there was a noticeable tipping or adjustment of the spherical block during this first increment of load, at which time the column became "seated". For subsequent loads the graph fluctuates about a vertical line not far removed from the position first assumed by the block at 100,000 lb. A considerable amount of this fluctuation may be attributed to the difficulty in repeating a level reading because of the sensitivity of the leveling bubble to rotation about a plane perpendicular to the face of the spherical head. Hence, to a close approximation, the vertical dotted line shown in the figure represents the variation of the upper head movement in the E-W direction for loads greater than 100,000 lb. It may be concluded, therefore, that there was no appreciable movement of the top bearing block with increase of load. This same statement applies, and even to a greater degree, to the lower head. Unfortunately, no rotation measurements were taken for column 52-C-1, so that a direct comparison of this aspect of the test with the difference in effective lengths noted between the two specimens of the C-1 series is not possible.

To summarize, the measurements of the distance between points of inflection from the curvature diagrams establish the fact that a very large end restraint was obtained in this test. The measurements of the rotation of the bearing blocks also indicate a large end restraint. Column 57-C-1 is believed to have been tested with essentially fixed ends; hence, the effective length is estimated to be 207 in., or one-half of the total column length.
Series C-2

The curvature plots for the columns of this series, based on an initial load of 5,000 lb, are shown in Figs. 16 and 17. The strain differences plotted are the averages for the two pairs of gages situated at the flange corners. The head rotation-load relations for the columns are shown in Figs. 19 and 20. By comparing these figures with the corresponding plots of the C-1 series the effect of the degree of end fixity on the column curvature is markedly indicated.

The curvature diagrams for column 52-C-2, which are presented in Fig. 16, may be somewhat unexpected since curvatures approximately symmetrical about the midheight are usually obtained. In these diagrams for all three loads only one point of inflection exists and it is located somewhat below midheight.

All three curvature diagrams indicate that sizeable restraining moments exist at the base. The point of inflection is about 1/4 in. or 35 percent of the total length from the base, which is a greater distance than is reasonable. The point of inflection should be a maximum of 30 percent of the column length from the base when the base is fixed and the top is hinged (negative restraint at the top is excluded). Fig. 19 shows that very little E-W rotation of the bottom bearing block occurred, thereby implying that this end was highly restrained. Finally, the general shape of the deflection configuration presented in Fig. A7 also indicates great restraint at the base as the end slope is approximately vertical. It seems safe to conclude that the base is direction fixed as well as position fixed.

The top of column 52-C-2, on the other hand, does not appear to be highly restrained. Fig. 19 shows that the top bearing head rotated in an E-W direction continuously as the deflections increased with increasing load. The importance of this bearing block rotation on the behavior of the column may be judged from Fig. A7. The dotted line extending from the top of the column to
section A-2 represents the computed position of the column center line arising from the head rotation of $33.5 \times 10^{-4}$ radians at the 950,000 lb load (assuming continuous contact between the head and the column base plate). It may be seen that the projection of the bearing block slope at this load is somewhat less than would be expected from the rest of the deflection curve. This indicates that some end restraint probably existed at the top.

The curvature diagrams of Fig. 16, for loads of 800 and 900 kips each, indicate the existence of an end moment at the top tending to increase rather than resist the deflections. This is impossible because a negative end restraint is impossible. This indication is probably caused by a combination of small curvatures and large torsional strains. At the top of the column the massive spherical head prevents warping of the cross-section due to torsion and thereby large normal stresses and strains are induced. At higher loads the increased bending action must become more prominent as the curvature diagram for a load of 950 kips, when extended to the top, indicates the existence of very little curvature at the top.

To summarize, the end conditions developed in the test of column 52-C-2 appear to be a fixed support at the base and a small restraint (we do not know how much) at the top. Therefore, the effective length must be less than 70 percent of the total length or 290 in.; the value which applies to a fixed-hinged column. Even a small end restraint at the top will shorten appreciably the effective length. In Fig. 16 the distance from the top to the point of inflection is about 270 in. or 65.5 percent of the total length. This value checks the Lundquist plot of the next section and is in the right range; therefore, it is adopted as a reasonable value for the effective length. However, we cannot be definite on this value.

The end conditions developed in the test of column 57-C-2 appear to be exactly the same as those of column 52-C-2. The curvature diagrams of Fig. 17
for column 57-C-2 are similar though generally less reasonable than those of Fig. 16 for column 52-C-2. Note that these curvature diagrams are all for very high loads, near the ultimate; it was necessary to use the data from such large loads because very little bending occurred at lower loads. Possibly a semi-plastic hinge has developed, but nevertheless the data is still useful. In Fig. 17 at the top of the column are shown curvatures which decrease to a small value at the largest load shown; and these curvatures are of a sign which corresponds to a disturbing end moment rather than a restraining end moment. As before, a "negative" restraint is impossible. As shown in Fig. 20, the E-W rotations of the top bearing block are appreciable, and they increase with increasing deflections and loads. Finally, the importance of this bearing block rotation on the behavior of the column may be judged from Fig. A9. The dotted line extending from the top of the column to section \( \Delta -2 \) represents the computed position of the column center line arising from the head rotation of \( 28 \times 10^{-4} \) radians at the 1250 kips load. It may be seen that the projection of the base plate slope at this load is less than would be expected for the rest of the deflection configuration; a small end restraint probably causes a reverse curvature near the top end.

With regard to the restraint at the base, Fig. 17 shows the existence of large restraint moments, Fig. 20 shows the complete absence of rotation of the bottom bearing block during loading, and Fig. A9 shows that the lateral deflection configuration at 1250 kips appears to have a vertical tangent at the bottom and a general shape to be expected of a column fixed at the bottom and only lightly restrained at the top. It seems safe to conclude that the base was essentially fixed.

The effective length of column 57-C-2, as tested, cannot be determined accurately. It is certainly less than 290 in., which applies to a fixed-hinged column, and more than 207 in., which applies to a fixed-fixed column.
9. Critical and Ultimate Load of Columns

(a). Southwell and Lundquist Methods

Owing to various kinds of imperfections columns begin to deflect with the beginning of loading. Such deflections will in turn immediately cause an increase in the moments and deflections. As the deflections continue to increase with load, the stress at the fibres on the concave side of the column may exceed the yield point of material. The stiffness of the column is then reduced. Finally the column usually fails by inelastic buckling before the Euler load is reached.

A number of methods exist for deducing the buckling loads for such columns from the test data. They are all due essentially to Southwell (7) who, in 1932, presented a generalized method of analyzing experimental observations in problems of elastic stability. Briefly, Southwell's method is concerned with the interpretation of simultaneous readings of load and deflection. As originally proposed it requires that the initial deflection reading be taken at zero loading. Such readings are somewhat questionable. In a more general treatment Lundquist (8) proved that the following equation holds:

\[
\frac{\delta - \delta_1}{P - P_1} = \frac{\delta - \delta_1}{P_0 - P_1} + \frac{f_1(z)}{P_0 - P_1}
\]

where

\( P \) and \( \delta \) are any load (below the critical value) and the corresponding deflection, respectively;

\( P_1 \) and \( \delta_1 \) are any arbitrarily selected initial values of \( P \) and \( \delta \), respectively, below \( P_0 \);

\( P_0 \) is the critical value of \( P \);

\( f_1(z) \) is the deflection configuration of the first characteristic mode present in the total deflection shape at load \( P_1 \), a function of the length, \( z \).

The term \( \delta - \delta_1 \) is the amount by which the lateral deflections are increased when the axial load in the column is increased from \( P_1 \) to \( P \). For any assumed \( P_1 \), the difference \( P_0 - P_1 \) is constant. Also for any particular cross section at which
\( \delta - \delta_1 \) is measured the term \( f_1(z) \) is a constant. Hence, if \( \frac{\delta - \delta_1}{P - P_1} \) is plotted against \( \delta - \delta_1 \), a straight line is obtained for this equation. This line cuts the horizontal axis \( (\frac{\delta - \delta_1}{P - P_1} = 0) \) at the distance \( f_1(z) \) from the origin, and the inverse slope of the line is \( P_0 - P_1 \).

Thus it is seen that we can deduce \( P_0 \) without letting \( P \) actually attain that value in the experiment. On this basis, from the test results of an imperfect column the buckling load of the corresponding perfect column can be estimated. Even if the bar reaches the yield point before buckling, the Euler load can often be determined from the test data within the elastic range. Southwell indicated that the method had certain limitations, namely that the deflections must not be so large as to impair elasticity of the material nor the load and deflection so small that their ratio will not be determinable with accuracy. His method assumes that the support conditions remain constant. This method relies upon the fact that the first characteristic mode is predominant in the total deflection configuration as the load approaches the critical. It is possible to improve the results by choosing the position for the deflection measurements at that point where the deflections of the first mode are largest and the deflections of the second mode are smallest (preferably zero); in this way the most troublesome of all modes, the second, may be eliminated. In summarizing Southwell states "trial alone will reveal whether the method will be successful in any particular instance". Lundquist's procedure is also subject to these same limitations. However, it has the advantage that the deflection corresponding to some relatively high load can be selected as the initial value. Hence the uncertainties which may occur in the test data while the specimen becomes thoroughly seated at the low loads will not affect the subsequent data at the higher loads. For this reason Lundquist's procedure has been used in estimating the critical loads of the columns in this investigation.
In a recent paper (9) C. T. Wang shows that Southwell's method (and hence also Lundquist's) though originally proposed for the case of elastic buckling may also be valid for inelastic buckling provided the assumption that the reduced modulus of the material is approximately a constant throughout the length of the column can be justified. In this case an estimate of the ultimate load is obtained rather than the Euler load. He suggests that the assumption may be considered valid if the experimental results check the theoretical predictions when the method is applied. In our case Wang's extension is of no value since the ultimate is obtained by test and need not be estimated.

(b) Application to Test Data

Figures 21 and 22 show the load and lateral deflection data measured in the tests plotted according to Lundquist's extension of Southwell's construction. In selecting the initial load \( P_1 \), consideration was given to the shape of the diagrams of the lateral deflection-load curves.

Figure 21(a) shows the results of this method when applied to column 52-C-1 based on an initial load \( P_1 = 500 \) kips; the corresponding deflection \( \delta_1 \) at the center of the specimen was approximately 0.52 in. It was necessary to use a high initial load in order to obtain a satisfactory plot. The curve represents the best fitting line for all the plotted points and it leads to an estimated critical buckling load of 1590 kips. The six points on this curve correspond to loads of 600, 700, 750, 850, 900 and 950 kips which is a stress range of 16-26 ksi. As may be seen from Fig. 5, over this range the column developed partial inelastic action. Hence, the actual value of the modulus of elasticity, \( E \), which should be used in Euler's theoretical expression should be less than the measured elastic value of 27,700 ksi. Using an average value for the range of plotted points of 25,000 ksi for the modulus and an effective length of 291 in., the critical load from Euler's formula is 1750 kips, which is 10 percent greater
than the value from the Lundquist plot. However, in this computation the critical load is quite sensitive to the effective length, which could not be determined with great accuracy. Thus, if the effective length in the above computation is increased by only 12 in. to 303 in., the critical load is reduced to 1620 kips.

The Lundquist curves for column 52-C-2 were based on an initial load of 400 kips and are shown in Fig. 21 (b). The solid line represents the plot for the inelastic deformation while the broken line represents the plot for the elastic deflection data. The six points on the dotted line correspond to loads of 500, 600, 700, 750, 800, and 850 kips, a stress range of from 14.5 to 25 ksi; for almost this entire range the column behaved elastically, as may be seen in Fig. 5. The five points on the solid line are for loads of 850, 900, 950, 975, and 1000 kips, a stress range of from 25 to 29 ksi; this is a stress range in which occurred rapid yielding and a constantly decreasing effective modulus. The elastic buckling load estimated from the figure is 2320 kips which is only 5 percent in excess of the theoretical Euler load of 2210 kips which is based on the measured modulus of elasticity of $E = 27,000$ ksi and the effective length (as obtained from the curvature studies) of 270 in. This excellent agreement suggests that the elastic modulus and the effective length adopted for this column are approximately correct.

The solid line plotted through the points of Fig. 21 (b) which represent the inelastic region leads to an estimated ultimate load of 1100 kips which is only 2.5 percent greater than the test value of 1075 kips. This result is in agreement with Wang's extension into the inelastic region of Southwell's plot.

The results for the Lundquist plots for the 57-C series of columns, based on an initial load of 400,000 lb are shown in Figs. 22 (a) and (b). The curves give an estimation of the ultimate strength for the columns since they were fitted to points which were in the inelastic range. In Fig. 22 (a) the five points through which the line is plotted correspond to loads of 1000, 1050, 1100,
1150, and 1200 kips, a stress range of from 25 to 30 ksi. In Fig. 22 (b) the eight top points through which the line is plotted correspond to loads of 900, 950, 1000, 1050, 1100, 1150, 1175, and 1200 kips, a stress range of from 22 to 30 ksi. As can be seen in Fig. 5, these points are well within the inelastic range. For column 57-C-1 the estimated value was 1378 kips or 3.6 percent in excess of the actual ultimate load, whereas the excess for column 57-C-2 was only 0.8 percent. For these columns it was not possible to obtain any estimate of the elastic buckling load. This is explained by the fact that these specimens were initially so straight that almost no bending occurred at the low (elastic) loads. Also, the effect of the high residual stresses which were noted for these members tended to produce a reduced modulus at lower loads than in the 52-C series. This is evident by inspection of Fig. 5.

10. Interpretation of Analyses

As pointed out in Section 7, the general solution developed in Section 6 can best be verified by a study of the ratio of the flange rotation to the deflection for each column. The following table contains the information for such a study.

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Quantity</th>
<th>52-C-1</th>
<th>52-C-2</th>
<th>57-C-1</th>
<th>57-C-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Load for Measurements, kips</td>
<td>800</td>
<td>850</td>
<td>1200</td>
<td>1200</td>
</tr>
<tr>
<td>2</td>
<td>Effective Length, in.</td>
<td>291</td>
<td>270</td>
<td>207</td>
<td>270?</td>
</tr>
<tr>
<td>3</td>
<td>Max. Rotation of Flanges, radians</td>
<td>0.0014</td>
<td>0.0040</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>4</td>
<td>Maximum Deflection, in.</td>
<td>0.23</td>
<td>0.18</td>
<td>0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>Experimental Ratio, ( \beta/v, \frac{\text{rad}}{\text{in.}} )</td>
<td>0.006</td>
<td>0.022</td>
<td>0.018</td>
<td>0.009</td>
</tr>
<tr>
<td>6</td>
<td>Theoretical Ratio, ( \beta/v, \frac{\text{rad.}}{\text{in.}} ) (From Fig. 13)</td>
<td>0.007</td>
<td>0.026</td>
<td>0.029</td>
<td>0.010</td>
</tr>
</tbody>
</table>
The experimental ratio, $\beta/v$, must be determined at some definite load or loads. In the first line of the above table are given the loads chosen for this determination. For columns 52-C-1 and 52-C-2 these loads are essentially in the elastic range. But for columns 57-C-1 and 57-C-2 it was necessary to use large loads in the inelastic range because the deflections and rotations were just too small and erratic at lower loads. The effective lengths presented in line 2 are those adopted in Section 8, except for column 57-C-2 which had not been given a value. However, in Section 8 it was noted that the end restraint conditions for 57-C-2 appeared to be similar to those for 52-C-2, and therefore the effective length of that column was adopted here for column 57-C-2. The uncertainty of the value is indicated by the question mark.

In line 3 is given the maximum rotation of the flange (for the load in line 1), computed by dividing the average opening and closing measurement (taken from curves such as given in Figs. A10 and A11) by the depth of the section (15 in.). The maximum deflections tabulated in line 4 were measured from Figs. A6, A7, A8, and A9. These deflections were measured from the thrust line drawn through the inflection points; they are not the deflections from the original position. The maximum rotations and deflections should occur at the same section, but in some cases they do not, perhaps because of the limited number of measurement sections, experimental inaccuracies, and a possible phase shift of the bending and twisting waves caused by the end conditions of the test.

The experimental values of the $\beta/v$ ratio tabulated in line 5 are equal to the quotient of line 3 divided by line 4. Finally in line 6 are tabulated the theoretical values of $\beta/v$ which correspond to the effective lengths tabulated in line 2; these theoretical values are read from Fig. 13.

The theoretical and experimental ratios in lines 5 and 6 agree remarkably well for all four columns, especially so since the theoretical values are
very sensitive to the web thickness (as discussed in Section 7) and to some extent to the effective length. One may conclude that the assumed role of the web in the buckling theory is verified for these four experiments. This is encouraging but further experimental proof using a wider range of variables is required before the entire development can be totally accepted.

With the aid of the concepts developed in the preceding sections it is now possible to understand the behavior of these four columns. It seems quite remarkable that the four columns should develop almost equal strengths when the mechanism of failure for the 52-C series differs radically from that for the 57-C series.

In the 52-C series the columns had considerable initial crookedness (see references 1 and 2 for initial shape), no doubt a result of the make-up of the section which was fabricated from flexible angle and plate elements. Initial residual stresses existed but were of moderate size and extent for the same reason. The immediate cause of failure in all columns was widespread yielding. However, it seems probable that this extensive yielding occurred prematurely in the 52-C series because of the initial crookedness and residual stresses. Of the two factors the initial crookedness is probably more significant than the residual stresses, possibly lowering the ultimate load by as much as 10 percent.

The columns of the 57-C series were initially very straight. However, they contained large residual stresses. As load was increased on these columns, progressive yielding began at a low load, but very little lateral bending occurred at this time. Finally the section was so weakened by this yielding that suddenly rapid lateral bending developed and failure occurred. The ultimate load in this case was probably lowered by 10 or 15 percent because of the large initial residual stresses.

It is interesting to note that the difference in the effective length
between columns 57-C-1 and 57-C-2 apparently did not affect the ultimate stress which was fairly close for the two. Possibly the effective length was not an important factor in these two tests because of the very small initial crookedness and the great reduction in flexural stiffness caused by the extensive yielding at high loads, which, in effect, transformed both columns into long, slender columns where the Euler curve flattens out. Certainly if one of the 52-C series columns had been restrained at the ends to the extent that column 57-C-1 was restrained, the strength would have been greatly increased. Insofar as the difference between 52-C-1 and 52-C-2 is concerned, although a considerable difference exists in the effective length, the \( l/r \) values are quite close, being 72 for 52-C-1, 69 for 52-C-2, and incidentally 50 for 57-C-1 and 65 (based upon \( l = 270 \) in.) for 57-C-2.

Since yielding is the immediate cause of failure in these columns (as in most columns), the normal stresses caused by the twisting of the flanges are of interest. The significant twisting stress is a compression which occurs at the corner (or heel) of the channel on the concave (or compressive) side of the deflected column. The compressive stress due to twisting is additive to the compressive stress due to flexure. The twisting stress is related to the flexural stress by the following equation:

\[
\frac{\text{Compressive Stress at Heel of Channel Due to Torsion}}{\text{Compressive Stress at Outer Fiber Due to Flexure}} = \frac{e \beta}{v}
\]

where \( e \) is the distance from the shear center to the centerline of the channel web or flange plate. The value of \( e \) is 0.67 in. for the 52-C series and 0.88 in. for the 57-C series. The values of \( \beta/v \) for both series are quite small (see Fig. 13) so that the normal stresses due to twisting never exceed three percent of the flexural stress, and are usually much less than this. Consequently, these stresses may be neglected.
IV. COMMENTS ON WEB PLATE BUCKLING IN COLUMN 52-C-2

11. General Description

This chapter is devoted to a discussion of the buckling behavior of the web plate of column 52-C-2. The other three columns in the C-1 and C-2 series of tests were of customary proportions and did not exhibit any local web buckling. In column 52-C-2 the web was deliberately made very thin (the ratio of the unsupported width of web between the nearest lines of rivets to the web thickness was approximately 54) in order to investigate the effect of the web thickness on the ultimate strength of the column. As expected, buckling waves were observed over the entire length of the web before the column as a whole failed.

The character of the buckling which took place in the web of column 52-C-2 can be fairly well determined from a study of the longitudinal web strains presented in Fig. A14 of the Appendix and the web deflection curves along the center line of the web presented in Fig. A13. The strain gages (marked by crosses in Fig. A4) were located on both faces along the center line of the web from section C to section D. The spacing of these gages as measured from section C was nominally 4 in. c-c over the first 4 ft., one space at 7 1/2 in. and five spaces at 8 in. c-c. The deflection readings were taken at spacings of between 4 and 5 in. between sections B and C.

The first significant fact disclosed by these curves is the extreme irregularity of the lengths of the web buckles. The half wave lengths varied from 6 to 12 in. in a haphazard manner. The average length of the buckles as determined by both the strain and deflection readings lay between 9 and 10 in.

The second significant fact which is disclosed by Fig. A14 is that at or before a load of 950,000 lb. the strains on the concave side of the buckles exceeded the yield point strain of the material in at least four buckles between sections C and D. The gages usually were not located at the exact peaks of the
buckles since there was no way of telling beforehand just where these would occur. Hence the maximum strain was not determined for many of the buckles.

Both facts stated above lead to the conclusion that the buckling observed was plastic in nature. The buckles probably were essentially in the shape of the initial irregularities of the plate. These initial irregularities grew as the axial load was increased; some alteration of the buckling pattern occurred during this time. But before a regular, well-developed pattern of waves was evolved yielding occurred in the outer fibers on the concave side in some of the buckles and these buckles then increased rapidly in size producing the buckling phenomenon which was observed.

Because of the complexity of the cross section, an elastic buckling analysis of the web plate is very difficult; considering the plastic nature of the buckling such an analysis does not seem justified. However, a rough approximation of the buckling load can be made easily. The principal uncertainty lies in the boundary conditions along the unloaded edges of the web plate. If the flanges of the column do not restrain the rotations of the edges of the web plate, we may consider the plate as simply supported at both edges, in which case the plate has a transverse width of 20.5 in. between hinges. On the other hand—if the flanges are fixed against rotation the web plate will be practically fixed at the line of rivets because of the angles which reinforce the web plate at both edges. And in this case the width of plate between fixed edges is only 17 in. Since the web plate is very long compared to the width, the plate will buckle into a configuration which gives the absolute minimum critical load. The lengths of the buckles for the two extreme cases cited above are as follows:

(a) hinged edges 20.5 in. apart, length = 20.5 in.
(b) fixed edges 17 in. apart, length = 11 in.

Note that the length of buckle for fixity at the line of rivets, case
(b), is slightly larger than the average length of the buckles developed in the test. Perhaps the influence of the angles to each side of the web plate extends somewhat beyond the line of rivets.

Another experimental verification of the relative fixity of the flanges against rotation due to web buckling was the lack of any observation of twisting in-and-out of the flanges with a half wave length of 9 to 10 in., as was detected in the web plate. If the flanges were rigidly connected to the web plate, as believed, it would be impossible for the edge of the web plate to have a sinusoidally varying edge rotation unless the flanges also had such a twisting configuration. Of course the movement would not have to be large to effect a considerable reduction in restraint, and it could have been overlooked.

A convenient analysis can be made if the actual section is simplified into an equivalent I section, by the use of tables and charts such as given by Bleich. Using reasonable dimensions such analyses lead to the conclusion that the flanges of the actual column provide sufficient restraint to raise the critical load to at least 95 percent of the load for fixed edges. Therefore, the crude analyses lead to the same conclusions as the experimental observations.

The critical stress, assuming that the web plate is fixed at the line of rivets is as follows: (take $E = 30 \times 10^6$ psi, $\mu = 0.28$)

$$\sigma_{cr} = \frac{69.1 \ E \ t^2}{12(1-\mu^2) \ b^2} = 63,000 \text{ psi}$$

This value is over twice as great as the ultimate stress developed. The true critical stress for the web plate of column 52-C-2 is probably somewhat less than this value.

Possibly the most significant question to be asked about the web buckling is whether it lowered the ultimate strength of the column as a whole. It could do this in two ways. In the first place if the web should buckle sufficiently to avoid carrying any stress above the buckling stress then additional load above
this level would have to be taken completely by the flanges. This action would probably cause yielding in the flanges at a lower load than if the web would take its proportionate share of the load up to failure. Secondly, buckling of the web might lower the restraint against torsional failure of the flanges. However, the wave length of the web buckling is so small compared to the wave length of the flange twist that from theoretical grounds it does not seem possible that the web buckling would lessen the restraint to the flange rotation. Therefore, we may neglect this second point and concentrate on the first action.

The effect of the local buckling of the web on the ability of the web to resist additional applied load may be determined by comparing the load - average strain curve for the web with a similar curve for the flanges. The average load-average strain curves for the forty-five SR-4, A-11 gages located on both faces of the web and the forty-nine SR-4, A-11 gages located on the flanges are presented in Fig. 23. The two curves have identical shapes and are in close agreement with one another for the entire range of data. This fact means that the web continued to take a proportionate share of the total load applied to the column even after a pattern of wave buckles existed; the web did not buckle out of the way of further load increases.

Another indication of this action is provided by the measurements of deflections of the buckles. In Fig. A13 the maximum difference in elevation between adjacent buckles at a load of 950,000 lb (ult. load = 1,075,000 lb) is shown to be approximately 0.035 in. In order to buckle effectively out of the way this value would have to be at least equal to the thickness of the plate. The very small deflections actually developed are insufficient for complete buckling.

These two observations appear to indicate that the web was fully effective in resisting the applied load up to failure of the column as a whole, and therefore the local web buckling which occurred before the collapse of the column probably did not lower the ultimate strength of the column.
12. Discussion

The purpose of this chapter is to comment on the results of two tests of large perforated cover plate columns conducted at the National Bureau of Standards, Washington, D. C. These columns represent certain structural details of compression members for the Calcasieu River Bridge at Lake Charles, Louisiana. The objectives of the tests were to determine the effectiveness of the perforated cover plates in heavy full sized sections and to compare their behavior in this usage with their action in the series of tests on column components performed also by the National Bureau of Standards by Stang, Greenspan, and associates.

The two perforated cover plate columns are denoted as TC-1 and TC-2. Reports on the tests have been issued by the National Bureau of Standards (3,4).

The columns were fabricated (riveted construction) from the following structural elements, arranged as shown.

- 4 Angles 4" x 4" x 1/2"
- 2 Web Plates 28" x 3/4"
- 2 Web Plates 28" x 5/8"
- 2 Perforated Cover Plates
  - 18" x 3/8" for TC-1
  - 18" x 3/4" for TC-2
The perforations in the cover plates were of ovaloid shape, 10 in. x 20 in., and spaced at 3 ft. centers. Both specimens had an overall length of 22.7 ft. Both columns were tested with flat ends.

The important section properties for the two columns are presented in the following table, along with values of the ultimate load, maximum stress, and average yield point stress of the coupons.

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Description</th>
<th>TC-1</th>
<th>TC-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Area Through Net Section, in.²</td>
<td>98.0</td>
<td>104.0</td>
</tr>
<tr>
<td>2.</td>
<td>Area Through Gross Section, in.²</td>
<td>105.5</td>
<td>119.0</td>
</tr>
<tr>
<td>3.</td>
<td>Moment of Inertia about Y-Y axis, net section, in.⁴</td>
<td>8699</td>
<td>9001</td>
</tr>
<tr>
<td>4.</td>
<td>Moment of Inertia about Y-Y axis, gross section, in.⁴</td>
<td>8761</td>
<td>9126</td>
</tr>
<tr>
<td>5.</td>
<td>Moment of Inertia about X-X axis, net section, in.⁴</td>
<td>8830</td>
<td>10092</td>
</tr>
<tr>
<td>6.</td>
<td>Moment of Inertia of chord about centroidal, Y'-Y' axis, in.⁴</td>
<td>59.0</td>
<td>76.6</td>
</tr>
<tr>
<td>7.</td>
<td>Maximum Slenderness Ratio (for gross section)</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>8.</td>
<td>Maximum Slenderness Ratio for Chord</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>9.</td>
<td>Ultimate Load, kips</td>
<td>3300</td>
<td>3490</td>
</tr>
<tr>
<td>10.</td>
<td>Avg. Maximum Stress in Net Section, ksi.</td>
<td>33.7</td>
<td>33.6</td>
</tr>
<tr>
<td>11.</td>
<td>Avg. Maximum Stress on Gross Section, ksi.</td>
<td>31.3</td>
<td>29.4</td>
</tr>
<tr>
<td>12.</td>
<td>Avg. Yield Point of Coupons, ksi. (Tension)</td>
<td>41.3</td>
<td>35.3 (Compression)</td>
</tr>
</tbody>
</table>

Lines 1 to 5 give pertinent values of the area and moment of inertia; the term "net section" refers to a section through the center of one of the perforations and the term "gross section" refers to a section which does not cut through the perforation. As usual the areas of the rivet holes are not deducted. In line 6 is given the
moment of inertia of one of the chords about its centroidal axis; the chords are
the two elements of the column to each side of the perforations. The maximum
slenderness ratio, \( \ell/r \), for the column as a whole, given in line 7, is computed
with the value of the radius of gyration of the gross section because this is less
than that for the net section; the length used in this computation is the full
length of the column of 272.4 in. The maximum slenderness ratio for the chord is
based upon the full length of the perforation of 20 in.

The average compressive yield points of the coupons, listed in line 12, are convenient figures to use, but they do not give much information about these columns. Both values were computed by weighting the individual coupon results in proportion to the areas of the elements from which the coupons were taken. The average yield point for TC-1 was obtained from tensile tests of samples furnished by the fabricator of the column. No indication is given in the report of the number of tests made nor the exact element of the column which the coupon represented although the type of element usually can be inferred since the average value for each thickness of plate was given. It would be interesting to know the individual values because the low values have some significance. No compressive coupon tests were made for TC-1. The average tensile values for each type of element are given in the table below. In view of the uncertainties concerning the control tests, little significance can be placed on the ratio of the average stress to the yield point stress based on these tabulated values.

In column TC-2 the same tensile tests were made as for TC-1 but in addition coupons for tensile and compressive specimens were cut from the actual column after it had been tested to failure. One coupon was cut from each element in the column for each type of test. Thus there were four tension and four compression tests for the four angles and two tension and two compression tests for each pair of plates. The compressive yield points were probably raised by the
strains of the column test if they were affected at all. The results of these final control tests were reported by the Bureau. They are summarized in the table below along with the previously averaged data for the tensile tests on the samples of virgin material.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Column TC-1 Avg Tensile Yield Point ksi</th>
<th>Virgin Matl. Avg Tensile Yield Point ksi</th>
<th>Column TC-2 After Column Test Avg Tensile Yield Strength (0.2% offset) ksi</th>
<th>Avg Compressive Yield Strength (0.2% offset) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles</td>
<td>40.1</td>
<td>36.8</td>
<td>37.2</td>
<td>36.5</td>
</tr>
<tr>
<td>Perforated Plates</td>
<td>40.5</td>
<td>35.4</td>
<td>32.5</td>
<td>32.2</td>
</tr>
<tr>
<td>Outer Cover Plates</td>
<td>39.2</td>
<td>39.6</td>
<td>28.3</td>
<td>32.2</td>
</tr>
<tr>
<td>Inner Cover Plates</td>
<td>43.7</td>
<td>39.9</td>
<td>32.4</td>
<td>38.3</td>
</tr>
<tr>
<td>Average, Weighted</td>
<td>41.3</td>
<td>31.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>According to Areas</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the table above, the tensile yield points of the virgin material for TC-1 are somewhat higher than the corresponding values for TC-2. More striking, however, are the differences between the three coupon tests for column TC-2, and particularly the fact that the yield strengths from the compression tests are less than the yield points determined from tensile tests of the virgin material. Compressive tests normally indicate higher values than tensile tests. For column TC-2 the compressive yield strengths are believed to be the most significant values and only these values are discussed further. The average compressive strengths for the perforated plates and the outer cover plates are quite small; the two tests had compressive yield strengths of 30.6 and 33.8 ksi for the perforated plates and 33.7 and 30.8 ksi for the outer cover plates. These values are quite low, but such low values occasionally occur in thick plates. Also, very likely the yield strength would have varied appreciably for identical
coupons taken from different locations of the same plate. For this reason a large number of coupon tests are really needed to establish an average value. Nevertheless, the average compressive yield point of 35.3 ksi. appears to be reasonable and is adopted herein.

To return to the first table of this chapter, the average stress on the net section of TC-2 at ultimate load was 33.6 ksi. This value exceeds the compressive yield point of two of the coupons and is very nearly equal to the yield strength of two more coupons. In other words, at the ultimate load both perforated plates and both outer cover plates should be yielding plastically, or on the verge of yielding plastically. This maximum stress on the net section of 33.6 ksi is 95 percent of the average compressive strength of all coupons. Certainly the strength of column TC-2 is satisfactory.

Column TC-1 sustained a maximum load of 3300 kips which corresponds to a stress on the net section of 33.7 ksi., a value slightly higher than the maximum stress for TC-2. However, since the average tensile yield point is higher, this maximum stress is only 82 percent of the yield point. In view of the uncertainty of the average coupon value for the yield point, it is suggested that this evaluation be disregarded. Column TC-1 sustained a stress on the net section as high as did column TC-2, and should be considered to have satisfactory strength also.

In line 7 of the first table of this chapter the slenderness ratios for the two columns are given as 30 and 31. These values are conservative since the effective length of the columns was probably 50 to 75 percent of the total length used in the computations. The Bureau recommends that the columns be considered to be fixed at the ends, which reduces the slenderness ratio to around 15 to 16. These columns are so stocky that flexural action and elastic buckling are out of the question. The slenderness ratios of the chords are also very low so that the chords do not constitute a source of weakness.
The load versus center line compressive strain curves for both columns are straight up to a high load and then curve off sharply. One may infer from this that initial residual stresses were small for these columns. Calculations of the effective moduli of these columns from the elastic portions of the load-center line compression curves give the following results:

Column TC-1, $E = 25100$ ksi based on net section stress
and $E = 23100$ ksi based on gross section stress.

Column TC-2, $E = 24200$ ksi based on net section stress.
and $E = 21200$ ksi based on gross section stress.

These values are far below the values of modulus of elasticity determined from the coupon tests. This behavior was also noted in the Illinois tests and was attributed to the presence of rivet holes.

The deflection-load curves show that the deflections were quite small until the load was near the ultimate; then suddenly the deflections increased rapidly. Strains read on many strain gages on the column indicated strains large enough to cause yielding at the load at which the center line compression readings and deflections began to increase rapidly.

Both column TC-1 and TC-2 apparently behaved alike. The failure probably was initiated by general yielding over the section. Very small deflections and flexural stresses existed at this time. As the load was increased, the yielding progressed and deflections began. From pictures taken after failure, the deflections appear to be a combination of flexure and shearing translation through one of the center perforations. But the greatest shearing force normally occurs near the ends. Possibly this translation is a manifestation of lateral buckling of the chords and is not caused by a weakness against shearing force. At any rate, the translation was accompanied by buckling of the cover plates and of the perforated plate around the hole. Probably, however, all these effects are
only a result of the widespread yielding which was the basic cause of failure.

The ratio of the maximum measured strain on the edge of the perforation to the average strain on the net section (in the elastic range) was about 1.72 for column TC-1 and 1.84 for column TC-2. In the previous tests on column components by the National Bureau of Standards, this concentration factor was about 1.9, based on net area. This agreement is excellent.

It may be observed in these tests and in the previous tests that the total load carried by the perforated plate does not change appreciably with respect to the length. The average strain in the perforated plate across a section not containing the perforation is less than the average strain in the contiguous angles and cover plates, and the average strain in the perforated plate across a section containing the hole is greater than the average strain in the contiguous elements. Apparently the rivets cannot distribute the load (so as to equalize the stresses) as rapidly as the cross-sectional area changes. The average strain across the net section in the perforated plate appears to be approximately 18 percent greater than the average strain across the cover plates for the same section; this figure is based upon scant information furnished by these two tests and is rather uncertain, but interesting nevertheless. This behavior is disturbing since it means that the perforated plates probably will yield to each side of the perforations considerably before the rest of the column will yield, assuming all components to have equal yield strengths. It might be advisable in design to neglect the area of the perforated cover plate when computing the stability of the chord but to consider the full area across a net section as effective in other computations.

In the previous study by the Bureau the effective area factors for the perforated plates were found. In these tests such a determination is not feasible because the perforated plates constitute only a small part of the total area.
Small variations in the properties of the cover plates and angles would destroy the value of the computation.

We may summarize the previous discussion by the following four conclusions:

(a) The initial failures were probably caused by widespread yielding. The maximum stress (across the net section) resisted by column TC-2 was 95 percent of the compressive yield strength. Column TC-1 probably did as well but this percentage value is uncertain.

(b) The perforated plates performed adequately. For these columns the full area of the perforated plate across a perforation could justifiably be counted as effective.

(c) The perforated plates were not required to resist shear, at least in the elastic range. No information has been obtained relative to their effectiveness in a column subjected to a large shearing force. More experimental work is needed on this question.

(d) The general behavior of the perforated plates in the heavy, full size section was similar to their behavior in the previous tests of column components conducted by the National Bureau of Standards.
VI. SUMMARY

13. Conclusions

1. A theory of flexural buckling accompanied by rotation of the flanges has been presented for columns of open section similar to those tested in this program. The predictions of this theory have been verified by the tests, where comparisons were possible. However, the tests are not sufficient to provide a complete verification of the proposed theoretical solution. The particular solutions for the four test columns, presented in the form of graphs, show that the ultimate load is not lowered by the twisting of the flanges. The critical stresses for this theory are indistinguishable from the critical stresses found by the Euler theory, except for short columns where the critical stresses far exceed the yield point of the material. Furthermore, the significant longitudinal compressive stresses induced by the twisting of the flanges are less than 3 percent of the outer fiber flexural stresses in the flanges and, hence, may be neglected.

2. The effective lengths of the columns, as tested, were found to be 291 in. for 52-C-1, 270 in. for 52-C-2, 207 in. for 57-C-1, and the effective length of column 57-C-2 is estimated to be 270 in., although this value is less certain than the others. None of the values of the effective length presented above can be accepted without reservations.

3. The immediate cause of failure for all columns was plastic yielding. However, the factors which caused the yielding prematurely and lowered the strength for the two series of columns are quite different. In the 52-C series the columns had considerable initial crookedness. To aggravate this condition, the effective lengths are greatest for this series. Initial residual stresses existed but were of moderate size. The initial crookedness is believed to have
been mainly responsible for the lowering of the ultimate load. On the other hand, the columns of the 57-C series were exceptionally straight but contained large initial residual stresses. Very little bending occurred until a large amount of yielding had taken place. The initial residual stresses are believed to be mainly responsible for the lowering of the ultimate load for the 57-C series.

4. The web plate buckling observed in the test of column 52-C-2 did not appear to lower the strength of the column. The shape of the deflected surface indicates that the web plate behaved as if fixed along the line of rivets.

5. The perforated cover plate columns tested by the National Bureau of Standards were short heavy sections. As usual for stocky columns, the initial failures were probably caused by widespread yielding. The maximum stress across the net section resisted by column TC-2 was 95 percent of the average compressive yield strength; the maximum stress for column TC-1 was slightly greater than for column TC-2. The perforated cover plates performed adequately in these columns. The full area of the perforated plate across a perforation could justifiably be counted as effective. The general behavior of the perforated plates in the full size sections was similar to their behavior in the previous tests of column components conducted by the National Bureau of Standards.
APPENDIX

SUMMARY OF PROGRESS REPORTS NOS. 1 AND 2 ON LARGE SCALE COLUMN TESTS

1. General

The information presented in this Appendix summarizes the pertinent data concerning dimensions of the specimens, test measurements, and results of the original large scale column tests conducted at the University of Illinois. This section is included for the convenience of the reader and also for the benefit of those persons who have only recently been included on this research program distribution list. A complete report of these tests may be found in references 1 and 2. All figures of this Appendix have been taken with only minor changes from the original two reports; they have been distinguished from the other figures by the prefix A and are placed behind the regular figures.

II. Test Specimens

Four large scale steel columns were tested at the University of Illinois. Two of the columns (designated as 52-C-1 and 52-C-2) were fabricated from plates and angles and two (designated as 57-C-1 and 57-C-2) were fabricated from channels and wide flange beams arranged in the manner shown in Appendix Figs. A1 and A2. Columns 57-C-2 and 57-C-1 were identical specimens. Columns 52-C-1 and 52-C-2 were also identical with the exception that the web plate of the latter was 1/8 in. thinner than the web plate of the former. All specimens had an overall length of 34.5 ft. and were tested on flat ends between spherical bearing blocks. The members were unsupported laterally. At an early stage of each test wedges were forced into place between the bearing blocks of each spherical head in order to provide a fixed bearing block.
iii. Test Measurements and Procedures

The locations at which the test measurements were taken are illustrated in Figs. A3 and A4. The measurements were as follows:

(a) Strain Measurements: In general, SR-4 electric strain gages, Type A-II were used to measure longitudinal strains at selected points on five different sections of the column. These sections (designated as A, B, C, D, and E) were located at the center, the quarter points, and 2 ft - 4 1/2 in. from each end of the specimen. For the C-1 column tests check measurements of the strain were also made with a mechanical dial gage using a 24 in. gage length. A-II electric strain gages were also mounted along the center line of both faces of the web extending over a quarter of the length of column 52-C-2.

(b) Flange-Twist and Deflection Measurements: Flange twist and deflection measurements in the weak and strong directions were taken 12 in. above and below the strain measuring sections noted above for the C-1 and C-2 test series and also at sections midway between these locations for the latter series. These measuring sections are designated as AB, AT, BB, BT, CB, CT, DB, DT, EB, and ET for the C-1 series (see Fig. A3), and the measuring sections are designated by numbers \( \Delta -1 \) through \( \Delta -14 \), inclusive, for the C-2 series (see Fig. A4). Deflection measurements in the strong direction were made relative to fine music wires (held vertical by weights) by means of a conical pointed scale. The movement of a mirror scale (attached to the column) with respect to the music wires was used to detect deflection in the weak direction. Overall column shortening was measured by the vertical movement of the weights which held the deflection wire taut. Opening and closing of the flanges (flange-twist) was recorded by means of a mechanical gage mounted on a deflectometer bracket and set between the outstanding flange toes. Measurements of the rotation of the supporting spherical bearing blocks were made with a 10 in. sensitive level bubble for the tests of the
C-2 series specimens and also for the test of 57-C-1. An estimate of the deflected shape of the web relative to the web-flange angles over one quarter of the length of column 52-C-2 was obtained with the aid of three mechanical dials mounted on a movable bracket; the results are presented in Fig. A13.

(c) Testing Procedure: The tests were conducted in a Southwark Emery 3,000,000 lb Universal hydraulic testing machine. A general view of a specimen in the machine is shown in Appendix Fig. A5. In general, the load was applied continuously to failure.

Tensile coupon tests were made from samples furnished by the fabricator. However, the amount of material furnished for control specimens was insufficient (material for nine coupon specimens for the four columns), and therefore additional tension and compression tests were made from specimens cut from the column section after the testing was completed. The extra tests are described in the Appendix of the Second Progress Report (2). The extra coupons were cut from around the quarter sections of the columns, where by visual observation the least amount of residual bending strain appeared to remain. Even with the additional tests the number of control specimens is insufficient to certify average values computed from them. Nevertheless averages are convenient figures to use and so are presented in the following table. The averages are weighted in proportion to the areas of the elements involved.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Column Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>52-C-1</td>
</tr>
<tr>
<td>Avg. Tensile Yield Point, Virgin Matl., ksi.</td>
<td>41.2</td>
</tr>
<tr>
<td>Avg. Tensile Yield Point, Pretested Matl., ksi.</td>
<td>39.9</td>
</tr>
<tr>
<td>Avg. Compressive Yield Point, Pretested Matl.</td>
<td>41.6</td>
</tr>
</tbody>
</table>

*Not an average but test of one coupon from web of wide flange beam.
iv. **Additional Information**

Several figures from the previous reports have been included herein for general information and because they are involved in the previous discussions.

The lateral deflection configurations for various loads are presented in Figs. A6, A7, A8, and A9 for the four columns. The deflections shown are relative to the deflections which existed at the low load used for the initial readings. The initial deflection configurations are also given in the original reports but are not included herein. The differences between the bending behavior of the 52-C series and the 57-C series is clearly shown in these figures.

Typical curves showing the change in distance between flange toes at various sections of columns 52-C-2 and 57-C-2 are presented in Figs. A10 and A11. The measurements were made on both sides of the column and these measurements are plotted in the figure, each to the side it represents. At any one section and at any particular load the opening of the flange (+ value) on one side should be equal to the closing (- value) on the other side unless the shape of the cross section of the flange has changed or errors in measurement are made. Usually the readings are about equal. Note the great difference in the amount of opening and closing of the flanges for the two columns; part of this is due to the difference in lateral deflections and part to the difference in torsional strength of the flanges of the two columns. Figure A12 shows two views of column 52-C-1 at midsection after failure. Although exaggerated by the large deflections developed after failure, this figure provides a graphic illustration of flange twist.

Figures A13 and A14 present the measurements pertaining to the plate buckling of column 52-C-2. The shape of the web deflections along the center of the web plate (Line B) are presented in Fig. A13 for various loads. The deflected shape is found between Sections B and C and is measured relative to the angles connecting the flange and web. Note that the deflection waves do not oscillate
about the original zero position but are displaced bodily eastward. This translation is caused by the rotation of the flanges and the corresponding transverse flexure of the web, as shown in Fig. 8. The longitudinal strains along the center of the web plate are presented in Fig. A14 for various loads. The strains are given on both sides of the plate; therefore the curvatures can be computed and the general pattern of buckling waves can be visualized. The strain readings are for the web plate between Sections C and D. The significance of the information in these two figures is discussed in Chapter IV.

Summary From First Two Reports.

1. The ultimate load resisted by columns 52-C-1, 52-C-2, 57-C-1, and 57-C-2 was 1190, 1075, 1330, and 1265 kips, respectively; these loads correspond to stresses on the gross section of 32.3, 31.3, 33.0, and 31.3 ksi, respectively. The maximum stresses range from 70 to 82 percent of the average coupon yield points, depending on what values for the average yield points are used.

2. Lateral deflections of the column were always accompanied by flange rotations.

3. Local buckling of the web occurred in column 52-C-2, as anticipated. Inter-rivet buckling of the flange plate occurred on both columns 52-C-1 and 52-C-2, but was considered to be a secondary effect arising from the primary failure of the column.

4. Shear bands appeared on the webs of columns 57-C-1 and 57-C-2 at approximately 45 percent and 95 percent of the ultimate load, respectively. Yielding was clearly indicated by spalling of the mill scale in horizontal and diagonal bands.
LIST OF REFERENCES


FIG. 2
AVERAGE STRESS-STRAIN CURVE FOR COLUMN 52-G-2

- Average of SR-4 gages
- Average overall shortening
FIG. 3
AVERAGE STRESS-STRAIN CURVE FOR COLUMN 57-C-1
FIG. 4
AVERAGE STRESS-STRAIN CURVES FOR COLUMN 57-C-2
FIG. 6
COLUMN CURVES BASED ON INITIAL AND TANGENT MODULI FOR COLUMNS 52-C-1 AND 52-C-2
FIG. 7
COLUMN CURVES BASED ON INITIAL AND TANGENT MODULI FOR COLUMNS 57-C-1 AND 57-C-2
FIG. 8

DISPLACEMENTS OF CROSS SECTION DURING DEFLECTION
### Properties of Sections

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units</th>
<th>52-C-1</th>
<th>52-C-2</th>
<th>57-C-1</th>
<th>57-C-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>in$^2$</td>
<td>36.90</td>
<td>34.38</td>
<td>40.36</td>
<td></td>
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<tr>
<td>$A'$</td>
<td>in$^2$</td>
<td>13.79</td>
<td>13.60</td>
<td>16.47</td>
<td></td>
</tr>
<tr>
<td>$A''$</td>
<td>in$^2$</td>
<td>9.32</td>
<td>7.19</td>
<td>7.42</td>
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<tr>
<td>$I_x'$</td>
<td>in$^4$</td>
<td>302</td>
<td>302</td>
<td>352</td>
<td></td>
</tr>
<tr>
<td>$I_y'$</td>
<td>in$^4$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>lb-in$^2$</td>
<td>$20.3 \times 10^6$</td>
<td>$20.3 \times 10^6$</td>
<td>$49.2 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$C_l$</td>
<td>lb-in$^4$</td>
<td>$81.5 \times 10^8$</td>
<td>$81.5 \times 10^8$</td>
<td>$157 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td>$x_0$</td>
<td>in</td>
<td>1.20</td>
<td>1.20</td>
<td>1.47</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>in</td>
<td>2.60</td>
<td>2.60</td>
<td>1.83</td>
<td></td>
</tr>
</tbody>
</table>

**FIG. 9** ASSUMED DIVISION OF COLUMN SECTION WITH PROPERTIES OF COMPONENT PARTS
FIG. 10 THEORETICAL RELATION BETWEEN CRITICAL STRESS AND LENGTH FOR COLUMN SECTION 52-C-1
FIG. 11  THEORETICAL RELATION BETWEEN CRITICAL STRESS AND LENGTH FOR COLUMN SECTION 52-C-2
FIG. 12  THEORETICAL RELATION BETWEEN CRITICAL STRESS AND LENGTH FOR COLUMN SECTIONS 57-C-1 AND 57-C-2
FIG. 13

RELATION BETWEEN $\frac{\beta}{v}$ AND EFFECTIVE LENGTH
FIG 14 DIAGRAMS OF AVERAGE STRAIN DIFFERENCE SHOWING LOCATION OF POINTS OF INFLECTION FOR COLUMN 52-C-1
FIG. 15 DIAGRAMS OF AVERAGE STRAIN DIFFERENCE SHOWING LOCATION OF POINTS OF INFLECTION FOR COLUMN 57-C-1
FIG. 16 DIAGRAMS OF AVERAGE STRAIN DIFFERENCE SHOWING LOCATION OF POINTS OF INFLECTION FOR COLUMN 52-C-2
FIG. 17 DIAGRAMS OF AVERAGE STRAIN DIFFERENCE SHOWING LOCATION OF POINTS OF INFLECTION FOR COLUMN 57-C-2
FIG. 18 ROTATION OF LOADING HEADS, COLUMN 57-G-1
FIG. 19 ROTATION OF LOADING HEADS, COLUMN 52-C-2
FIG. 20 ROTATION OF LOADING HEADS, COLUMN 57-C-2
FIG. 21 LUNDQUIST PLOTS FOR COLUMNS 52-C-1 & 52-C-2
SLOPE = 1.022 \times 10^{-6}

P_{cr} - P_i = 9.78 \times 10^5

P_{cr} = (P_{cr} - P_i) + P_i

P_{cr} = (9.78 + 4) \times 10^5

= 13.78 \times 10^5 \text{ LB}.

\[ a \] COLUMN 57-C-1

\[ \delta - \delta_1 \text{ in inches} \]

SLOPE = 1.142 \times 10^{-6}

P_{cr} - P_i = 8.75 \times 10^5

P_{cr} = (P_{cr} - P_i) + P_i

P_{cr} = (8.75 + 4) \times 10^5

= 12.75 \times 10^5 \text{ LB}.

\[ b \] COLUMN 57-C-2

\[ \delta - \delta_1 \text{ in inches} \]

OCUP ST PLOTS FOR COLUMNS 57-C-1 & 57-C-2
Figure 23: Average Load-Strain Curves for Web and Flanges of Column 52-C-2.
FIG. A1  
DETAILS OF TEST COLUMN: 52-C-1 AND 52-C-2

MILL AFTER ASSEMBLY:
1/2 CONTINUOUS FILLET WELD

MATERIAL:
CARBON STEEL: ASTM A7-46
RIVET STEEL: ASTM A41-39

NOTE: WORKMANSHIP IS TO BE IN ACCORDANCE WITH AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICIALS STANDARD SPECIFICATIONS FOR HIGHWAY BRIDGES (1944 EDITION).

1 WEB PL. 20" x 7/16 x 34'-6" FOR 52-C-1
1 WEB PL. 20" x 5/16 x 34'-6" FOR 52-C-2

1 COV. PL. 15" x 3/8" x 34'-6"       4 LBS. 3" x 3" x 3/8" x 34'-6"
1 COV. PL. 15" x 3/8" x 34'-6"       4 LBS. 3" x 3" x 3/8" x 34'-6"
NOTE: WORKMANSHIP IS TO BE IN
ACCORDANCE WITH AMERICAN
ASSOCIATION OF STATE HIGHWAY
OFFICIALS STANDARD SPECIFI-
CATIONS FOR HIGHWAY BRIDGES
(1944 EDITION).

MATERIAL:
CARBON STEEL, A.S.T.M. A7-46
RIVET STEEL, A.S.T.M. A141-39

1 1/2" CONTINUOUS FILLET WELD

MILL AFTER ASSEMBLY

FIG. A2 DETAILS OF TEST COLUMN 57-C-1 AND 57-C-2
FIG. A3  LOCATION OF GAGE POINTS AND SECTIONS

* INDICATES LOCATION OF SR-4 STRAIN GAGES AT SECTIONS A TO E INCLUSIVE.

→→ INDICATES LOCATION OF DEFLECTION MEASUREMENTS.

* INDICATES WIRES USED FOR DEFLECTION MEASUREMENTS.
COLUMNS 52-C-2 AND 57-C-2

* INDICATES LOCATION OF SR-4 STRAIN GAGES, TYPE A-11, AT SECTIONS A TO E INCLUSIVE AND ALONG WEB CF COLUMN 52-C-2.

@ INDICATES LOCATION OF ADDITIONAL SR-4 STRAIN GAGES, TYPE A-9, AT SECTION C.

→ INDICATES LOCATION OF DEFLECTION MEASUREMENTS.

* INDICATES WIRES USED FOR DEFLECTION MEASUREMENTS.

△ INDICATES SECTIONS AT WHICH DEFLECTIONS WERE MEASURED.

FIG. A4 LOCATION OF GAGE POINTS AND SECTIONS
FIG. A5 GENERAL VIEW OF COLUMN 52-C-2 IN 3,000,000-LB. TESTING MACHINE
FIG. A6 LATERAL DEFLECTION CURVES FOR COLUMN 52-C-1
FIG. A7 LATERAL DEFLECTION-LOAD CURVES FOR COLUMN 52-C-2
FIG. A8 LATERAL DEFLECTION CURVES FOR COLUMN 57-C-1
FIG.A9  LATERAL DEFLECTION-LOAD CURVES
FOR COLUMN 57-C-2
FIG. A10 CHANGE IN DISTANCE BETWEEN FLANGE TOES AT VARIOUS SECTIONS OF COLUMN 52-C-2

(+ indicates an opening of flanges
(-) indicates a closing of flanges

Load on Column in 100,000's of Lb.

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<th></th>
<th>8</th>
<th>4</th>
<th>0</th>
<th>0</th>
<th>4</th>
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<th>12</th>
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<td>Δ-2</td>
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<td>(+)</td>
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<td>Δ-4</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
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<td>Δ-6</td>
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<td>Δ-7</td>
<td>(-)</td>
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0.020" Deflection
Load on Column in 100,000's of Lb.

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<tbody>
<tr>
<td>+1</td>
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<tr>
<td>+2</td>
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<tr>
<td>+7</td>
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(+ ) indicates an opening of flanges  
(- ) indicates a closing of flanges

FIG.A11 CHANGE IN DISTANCE BETWEEN FLANGE TOES AT VARIOUS SECTIONS OF COLUMN 57-C-2
FIG. A12 EAST AND WEST FACES OF COLUMN 52-C-1 AFTER FAILURE

(a) East Face of Column 52-C-1
At Section C after Failure

(b) West Face of Column 52-C-1
At Section C after Failure
FIG. A13 WEB DEFLECTION - LOAD CURVES ALONG LINE B FOR COLUMN 52-C-2
FIG. A14 WEB STRAIN-LOAD CURVES FOR COLUMN 52-C-2
# DISTRIBUTION LIST

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