THE SHEAR STRENGTH OF SIMPLE-SPAN REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

By

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and
N. M. NEWMARK

Technical Report
to
OFFICE OF THE CHIEF OF ENGINEERS
Contract DA-49-129-eng-248

UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
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A Technical Report of a Research Project
Sponsored by
THE OFFICE OF THE CHIEF OF ENGINEERS
DEPARTMENT OF THE ARMY

In Cooperation With
THE DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS

Contract DA-49-129-eng-248

Urbana, Illinois
April 1953
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I. INTRODUCTION

1. Object and Scope of Investigation

This report deals exclusively with shear failures in simply-supported reinforced concrete beams without any form of web reinforcement and subjected to one or two equal concentrated loads. The object of this investigation was to correlate the results of all previous research and to determine, by the help of new tests, the transition range between shear and tension failures.

The investigation was conducted in three phases. The first phase was a review of the research carried out previously in the field of diagonal tension and shear. An empirical expression was derived for shearing strength of simple reinforced concrete beams having no web reinforcement. This expression was used as a guide for planning the experimental program.

In the second phase, 13 simple beams were tested. These beams had a column stub at midspan which was cast integrally with the beams. The purpose of the column stub was to simulate the moment and shear conditions adjacent to a column in a framed structure. Load was applied through the column stub. Variables were the steel percentage $p$ and concrete strength $f'_c$. The object of this phase of the investigation was to determine criteria for predicting whether such beams will fail in shear or in flexure.
Based on the test results, a fundamentally new empirical equation was derived for the shearing strength of simple reinforced concrete beams. In the third phase of the investigation all previous and available test data were analyzed in the light of the empirical equation. It is shown herein that the empirical equation can be interpreted by means of the conventional theory of compression failures of reinforced concrete beams.

2. Acknowledgments

The tests and studies reported herein were made as a part of an investigation of the relation between load and deformation for reinforced concrete joints and members conducted in the Structural Research Laboratory of the Engineering Experiment Station of the University of Illinois. The project was originally sponsored by the Office of Naval Research under Contract N6-onr-07134, Task Order 34, Project No. NR-064-372, with funds supplied by the Armed Forces Special Weapons Project. This investigation of shearing strength was undertaken under a supplement to the above contract, with funds furnished principally by the Ohio River Division Laboratories, Corps of Engineers, U. S. Army. After 1 February 1953, the project has been sponsored by the Office of the Chief of Engineers, U. S. Army, under Contract No. DA-49-129-eng-248, with funds supplied by the Armed Forces Special Weapons Project.

The program of the investigation is guided by Dr. N. M. Newmark, Research Professor of Structural Engineering. The program is under the immediate direction of Dr. C. P. Siess, Research Associate Professor of Civil Engineering.
The manuscript of this report was reviewed by Mr. J. H. Appelton, Research Associate, and his helpful comments are appreciated.

3. Notation

The following notation is used in this report:

- \( a \) = distance from end support to a concentrated load
- \( b \) = width of beam
- \( C \) = internal compressive force in concrete; (also various numerical coefficients as defined in text)
- \( d \) = distance from centroid of tension reinforcement to compression face of beam
- \( \varepsilon_u \) = ultimate strain in concrete, taken as 0.004
- \( E_c \) = modulus of elasticity of concrete
- \( E_o \) = slope of stress-strain curve for reinforcing steel in work-hardening region
- \( E_s \) = modulus of elasticity of reinforcing steel
- \( f'_c \) = compressive strength of 6 by 12-in. concrete cylinders
- \( f'_o \) = modulus of elasticity of reinforcing steel
- \( f'_y \) = yield stress of tension reinforcement
- \( f'_y \) = yield stress of compression reinforcement
- \( f_v \) = allowable tensile stress in web reinforcement
- \( \varepsilon_0 \) = steel strain at beginning of work-hardening
- \( (f'_o)_{av} \) = average compressive stress in compression zone of concrete
- \( K \) = \((\sin\alpha + \cos\alpha)\sin\alpha\), where \( \alpha \) = angle between web bars and axis of beam

\[ (f'_o)_{av} = f'_y - E_o \varepsilon_0, \text{ where } \varepsilon_0 = \text{steel strain at beginning of work-hardening} \]
kd = depth of compression zone of concrete as determined from an equivalent section transformed to concrete

kd.s = depth of compression zone of concrete at shear failure

k1 = \frac{C}{k_2 f'c k_b d}, a parameter which determines the magnitude of the compressive force C. It is the ratio of the average compressive stress to the maximum compressive stress in concrete

k2 = fraction of the depth of compression zone which determines the position of the compressive force C in concrete

k3 = ratio of maximum compressive strength of concrete in beam to compressive strength of standard test cylinders

L = span length of test beam

M_y = bending moment at first yielding of tension reinforcement

M = bending moment

n = \frac{E_s}{E_c} = elastic modular ratio, taken as 5 + \frac{10,000}{f'c}

n' = \frac{f_y}{f'c} = plastic modular ratio

p = \frac{A_s}{bd}, where A_s = area of tension reinforcement

p' = \frac{A_s'}{bd}, where A_s' = area of compression reinforcement

r = \frac{A_v}{bs \sin \alpha}, where A_v = area of web reinforcement and s = spacing of web bars along axis of beam

T = force in tension reinforcement

td = distance between centroids of tension and compression reinforcements
\( V = \) shearing force

\( v = \) nominal shearing stress in concrete, \( v = \frac{V}{bjd} \) or
\[
\frac{V}{bk} \frac{d}{s}
\]

as defined in text

\( v_u = \) nominal shearing stress at ultimate load
II. REVIEW OF EARLIER RESEARCH

It is estimated that close to 1000 beams have been tested in
this country in an attempt to determine their strength in shear (1)*. In
this number are included both simple and restrained beams, and beams both
with and without web reinforcement. Not all of these beams, however,
failed in shear; furthermore, a great many of the early beams were of
abnormally low concrete strength, and in some cases the exact concrete
strength is not known. Since concrete strength is one of the major variables
influencing the shearing strength of a beam, a considerably smaller number
of beams is available for a quantitative analysis to evaluate their strength
in shear.

This investigation is concerned only with simple beams having no
web reinforcement. A brief summary of the results of previous investiga-
tions of such beams is given in the following paragraphs.

4. Early Empirical Equations

At the beginning of the century, so-called diagonal tension or
shear failures were considered to be failures in diagonal tension; that is,
failures due to the principal tensile stress reaching the tensile strength
of the concrete. Talbot carried out considerable research both on the
strength of concrete in pure shear and on beams failing in diagonal tension.

* Numbers in parentheses refer to corresponding entries in the Bibliography.
In 1906 Talbot (2) concluded that although it is difficult to devise a form of test specimen and a method of testing which will satisfactorily determine the resistance of concrete to pure shear (due to complications caused by the accompanying compressive, tensile, bulging, and bursting stresses), it appeared that the shearing strength was, in general, at least 50 per cent of the compressive strength, and that it might exceed 75 per cent. Since the test values of the nominal unit shearing stress, \( v = \frac{V}{bkd} \), were considerably below what was considered to be the shearing strength of concrete, it was believed that reinforced concrete beams did not fail in shear and that what had been called shearing failures were really diagonal tension failures.

Since the real value of diagonal tension is generally difficult to determine, Talbot (3) considered the shearing unit stress \( v \) as a measure of diagonal tension. The nominal shearing unit stress \( v \) was derived by considering the amount of horizontal tensile stress transmitted from steel to concrete. This stress was considered as uniformly distributed over a horizontal section just above the plane of the reinforcing bars, giving actually the horizontal unit shearing stress at that level. Since the vertical unit shearing stress must equal the horizontal unit shearing stress, and since no tension was considered acting in the concrete, it was concluded that there was no change in the intensity of the shearing stresses between the level of reinforcement and the neutral axis. Above the neutral axis, the intensity of shearing stress must decrease according to the laws governing the state of stress in homogeneous rectangular beams. Although there was some question about the presence of tension in concrete and the changes in the intensity of the shearing stresses in the compression zone, the nominal
shearing unit stress \( v \), as given by the formula \( v = \frac{V}{bjd} \), was used as a basis of comparison of tests results. The actual diagonal tension stress was understood to be considerably larger, up to more than two times the value of \( v \). From test results, Talbot concluded that the shearing strength of a beam, as measured by \( v \), depended on the richness and the tensile strength of the concrete.

In 1909 Talbot (4) reported the results of additional tests and concluded that the shear strength of a beam as measured by \( v \) increases as the quality of concrete increases, that \( v \) increases with age of the concrete (since \( f'_c \) goes up with age), that \( v \) increases as the steel percentage \( p \) increases, and as the span length \( L \) decreases. It is interesting to note that already in this bulletin Talbot listed the main variables considered by most of the present investigators.

In 1927 Richart (5) reported the results of an extensive series of tests made earlier at the University of Illinois under Talbot’s direction. Since by that time the concept of the truss analogy had been introduced in this country and since the strain gage was available for measuring strains in the web reinforcement, much emphasis was placed in determining the relationship between the nominal shear stress \( v \) and the stress \( f_v \) in the web reinforcement. It was observed, however, that the maximum stresses obtained in the web reinforcement were, in general, much less than would be indicated by the truss analogy equations. Hence, a modified truss analogy formula was introduced: \( v = C + r f_v \), where the factor \( C \) was found to vary between 90 and 200 psi and a statement was made that \( C \) probably depends upon \( f'_c \) and \( p \). Thus for beams without web reinforcement the expression would be \( v = C(f'_c,p) \),
but no further attempt was made to continue Talbot's earlier investigation to establish the variables determining the shearing unit strength $v$.

In 1926 Slater, Lord, and Zipprodt (6) reported two formulas for shearing strength, $v = 60 + 25b^1 + rf_v$ and $v = (0.005 + r)f_v$, which, however, were derived for thin-webbed, I-shaped reinforced concrete beams. For beams without web reinforcement the first formula would yield $v$ as a function of the web thickness $b^1$ whereas the second formula considers the shear resisted by concrete only indirectly. In this investigation again major emphasis was placed on measuring stresses in the web reinforcement and on comparing different systems of web reinforcement. Relatively few beams were tested without some form of web reinforcement, hence the conclusions reached applied primarily for beams with reinforced webs.

By this time, two schools of thought influenced design practice in the United States. In one it was assumed that about one third of the vertical shear was carried by the concrete and two thirds by the web reinforcement. The second method involved the assumption that the concrete carried a constant portion of the working shearing unit stress (such as 40 or 50 psi or a given proportion of $f'_c$) and that the web reinforcement carried the rest. The latter school of thought seems to have been generally accepted by the specification writing bodies in this country. Thus, it was provided that the allowable shearing unit stress $v$, as a measure of diagonal tension, should be a certain percentage of the compressive strength of the concrete. For example, the ACI Building Code (318-51) specifies that the allowable stress is $v = 0.03f'_c$.

The coefficient 0.03 is apparently based on the minimum values of shearing stresses obtained in tests, divided by a factor of safety.
However, a study of simple beams with no web reinforcement which were tested between the years 1905 and 1952 and which failed in shear, revealed that the shearing unit stress at failure varied between the limits $0.02 f'_c$ and $0.16 f'_c$. Hence, the factor of safety varied from less than 1 to more than 5 for beams designed according to the present ACI Code. It must be pointed out, however, that in a large number of the early beams there is some doubt about the compressive strength of the concrete used in the test beams, which might account for some of the very low values of $v$ obtained.

5. Moretto’s and Clark’s Equations

Based on tests of 44 simple beams, Moretto (7) derived in 1945 the following empirical equation: 

$$v = K r f_y + 0.10 f'_c + 5000 \, p$$

for beams with welded vertical and inclined stirrups. This was the first modern attempt to evaluate in quantitative terms the contribution of the various elements of a beam to its strength in shear. Other variables besides the percentage and inclination of web reinforcement were the concrete strength, and to a minor extent, the percentage of tension reinforcement. Since all beams were tested with third-point loading, one of the main variables relating the relative magnitudes of moment and shear at a section where failure occurs was not included in this equation.

For beams without web reinforcement Moretto’s equation would yield 

$$v = 0.10 f'_c + 5000 \, p$$

at failure. However, only four of the test beams had no web reinforcement and the above equation was not intended to apply to such beams. Although in this equation the shearing strength of a beam was expressed in terms of separate contributions by the various components (web reinforcement, concrete, and longitudinal reinforcement), it cannot be
assumed that the effect of removing one of the sources of shearing strength can be represented simply by reducing the corresponding term to zero. Such a procedure is not justified in view of the empirical nature of the equation, since so few tests of beams without web reinforcement were made.

In 1951, Clark (8) reported tests on 62 simple beams involving the following variables: concrete strength, percentage of tension reinforcement, percentage of web reinforcement, and the ratio of depth of beam, \( d \), to shear span, \( a \). He obtained the following equation for shearing strength of simple beams: \( v = 7000 p + 0.12 f'_c \left( \frac{d}{a} \right) + 2500 f_t \). Since 12 of the test beams were without web reinforcement, the previous equation was also intended to be applicable for such beams, yielding \( v = 7000 p + 0.12 f'_c \left( \frac{d}{a} \right) \) at failure.

Clark's equation is the first to account quantitatively for all of the variables listed by Talbot in 1909 as influencing the shearing strength of beams with no web reinforcement. For such beams there is some similarity between Clark's and Moretto's equations; both consider the shearing unit stress \( v \) to be a linear function of \( f'_c \) and \( p \). Clark, however, introduced one additional variable, the ratio of shear span to the effective depth, in his test program whereas this ratio was not a variable in Moretto's tests. A fundamentally greater difference between the two equations lies in the way in which the effect of the web reinforcement is considered to contribute to the shearing strength of beams. This can be explained by the fact that in all beams tested by Clark, vertical stirrups with the same yield strength were used, whereas in Moretto's beams both the inclination and yield strength of the web reinforcement was varied. This also demonstrates how limited is our knowledge regarding the strength of reinforced
concrete beams in shear and shows that much work remains before this factor can satisfactorily be explained.

In the present study, both Moretto's and Clark's equations have been checked against the results obtained from other investigations. In addition, two more empirical equations have been investigated:

\[ v = K r f_y + C_1 f'_c + C_2 p \left( \frac{d}{a} \right) \]
\[ v = K r f_y + (C_1 f'_c + C_2 p) \frac{d}{a} \]

All these attempts to relate the shearing strength of simple reinforced concrete beams to a linear function of \( f'_c \) and \( p \) have failed to give a good correlation with test results.

A study of beams without web reinforcement made by the writer indicated that best agreement with the results of all available investigations of such beams was found with the equation:

\[ \frac{V}{b j d} = v = [0.08 f'_c + 22,000] \frac{d}{a} \]  

Although the agreement was still not good, the above equation was deemed to be the best available and was used as a guide in planning the experimental program.
III. EXPERIMENTAL PROGRAM

6. Planning of Tests

The experimental program was planned primarily to yield criteria for predicting whether the test beams would fail in shear or in flexure. In addition, beams failing in shear were to be analyzed to give quantitative information about the shearing strength of the test beams.

Equation (1), as stated in the previous chapter, was used as a preliminary empirical equation for shear failures:

\[
v = \frac{V}{bd} = 0.08 f'_c + 22,000 \frac{p}{d}
\]  

Since \( M = V a \), Eq. (1) can be written as follows:

\[
\frac{M}{bd^2 f'_c} = 0.08 + 22,000 \frac{p}{f'_c d}
\]  

The ultimate moment for flexural failures is given very closely by the following equation which was derived in a previous technical report:

\[
\frac{M}{bd^2 f'_c} = \frac{p f}{f'_c} \left( 1 - \frac{k^2}{k_1 k_2} \frac{f'}{f'_c} \right)
\]  

Since the percentages of reinforcement to be used for the test beams were such as to give yielding of reinforcement at flexural failures \( f'_y = f'_s \) may be substituted in Eq. (3); and using \( \frac{k^2}{k_1 k_2} = 0.5 \), Eq. (3) can be written as:

\[
\frac{M}{bd^2 f'_c} = \frac{p f}{f'_c} \left( 1 - 0.5 \frac{f'}{f'_c} \right)
\]
Equations (2) and (3a) are plotted in Fig. 1 as functions of the parameter $\frac{P}{f_c}$, for $f_y = 45,000$ psi and $j = 7/8$. The point of intersection between shear and flexural failure curves gives a theoretical transition point between the two types of failures.

Figure 1 was used to determine the range of variables to be used in the test beams. Two design concrete strengths were selected, 2500 and 4000 psi, and the steel percentage $p$ was varied to give both shear and flexure types of failures. The actual physical properties of the test beams are described in the following section.

7. Test Beams

A total of 13 beams were tested. All beams had a rectangular cross section of 6 by 12 in. and were tested on a span of 9 ft. The overall length of the test beams was 10 ft. A column stub 6 in. high was cast integrally with the beams at midspan; the length of the stub was 6 in. for the first beam and 12 in. for the other beams. All column stubs were reinforced with six No. 4 vertical bars, having a length of 14 in. Details of the beams are shown in Fig. 2.

The beams were loaded at midspan through the column stub. The beams were reinforced in tension only and had no web reinforcement. The only variables in these tests were the compressive strength of the concrete, $f_c'$, and the percentage of reinforcement, $p$. Two values of design concrete strength, 2500 and 4000 psi were used. The percentage of tensile reinforcement varied from 0.34 to 4.11. The physical properties of the test beams are given in Table 1.
8. Materials

(a) Cement - Type I Lehigh Portland Cement was used in all beams.

(b) Aggregate - The aggregates were a Wabash River Valley sand having an average fineness modulus of 3.1 and a Wabash River Valley gravel of 1-in. maximum size.

(c) Reinforcing Steel - All reinforcing bars used were intermediate grade, Hi-Bond type bars meeting ASTM Designation A 305-50T. The physical properties of the reinforcing steel are included in Table 1. All bars were straight, 2 in. shorter than the overall length of the beam, and placed in one layer.

9. Fabrication and Curing

Before the reinforcement was assembled, 6-in. gage lines for mechanical strain gages were marked on the two outer bars and the gage holes punched and drilled. Corks of 1 3/8 in. in outside diameter were wired to the bars over the gage holes in order to form core holes in the sides of the beam and provide access to the gage holes.

Concrete was mixed in a 6-cu. ft. capacity non-tilting drum mixer. Two batches of concrete were used for each beam, and both the beams and six 6 by 12-in. control cylinders for each batch were cast in steel forms. Concrete was placed in the forms with the aid of an internal vibrator, the first batch in the ends of the beam and the second batch in the middle. After the initial set the concrete was struck off and trowelled smooth.

The forms were removed one day after casting and the specimens placed in a moist room. Seven days after casting, the specimens were
removed from the moist room and were stored in the air of the laboratory. The beams were tested at an age of about 28 days, at which time 3 control cylinders from each batch were tested. The other 3 cylinders were tested at 7 days.

10. Test Apparatus

The beams were tested on a 9-ft. span in a 300,000-lb. capacity Riehle screw-type testing machine. The load was applied at midspan through the column stub. To provide access to the entire length of beam during testing, the beam was offset in the testing machine. The testing machine was used to apply the load but a 125,000-lb. elastic-ring dynamometer was used to measure the applied load.

For a more detailed description of the test apparatus see page 14 of a previous technical report (9). The same test apparatus was used in both investigations.

11. Testing Procedure and Measurements

Loads were applied in from four to six approximately equal increments up to failure in shear or to first yielding of reinforcement. The number of load increments after yielding depended on the ductility of the beam. The development of cracks was carefully observed and recorded by photographs taken with a 35-mm. camera at every significant change in the crack pattern.

In all tests the following quantities were measured:

(a) Tensile Strain - Strains in the tensile reinforcement were measured with a mechanical type strain gage on six-inch gage length.
A Berry type gage was used until the strains exceeded its range, thereafter a direct-reading type gage was employed. The Berry type gage had a sensitivity of 0.00003 in. per in. and the direct-reading gage 0.0006 in. per in. A total of 17 gage lines were located on each side of the beam as shown in Fig. 2. In the early tests, strains were measured at all 17 gage locations, while in the later tests, readings on some of the gage lines at the ends of the beam were omitted.

(b) Concrete Strain - Strains on the top surface of the beam were measured with Type A-11, SR-4 electrical strain gages. Gage locations are shown in Fig. 2.

(c) Deflections - The deflections of the beam were measured to the nearest 0.01 in. by means of a steel scale at eleven locations on each side of the beam, as shown in Fig. 2. In addition, the midspan deflection was measured to the nearest 0.001 in. with a dial indicator.
IV. TEST RESULTS

12. Modes of Failure

The test beams were designed to give both tension and shear failures. Out of the total of 13 beams, six failed in shear and seven in tension.

The typical failure in shear was a sudden destruction of the compression zone just above the diagonal crack. Simultaneously with shearing off of the compression zone, all beams split along the tensile reinforcement. It was noticed that the amount of tensile reinforcement influenced the manner of shear failure. When the steel percentage \( p \) was relatively small, the formation of diagonal cracks was gradual and could be observed visually. The final collapse occurred after the diagonal cracks were well developed. With an increasing steel percentage the formation of diagonal cracks was more sudden and the final collapse followed shortly after the cracks had formed. For the highest steel percentage, 4.1 per cent, the failure was very sudden with only a slight tendency for diagonal cracking before the final collapse. Fig. 3 shows a typical beam of this group, Beam 8-4, after shear failure.

Moment-deflection curves for beams which failed in shear are given in Fig. 6. It is seen that the maximum deflection at failure was very small.

Beams which failed in tension cracked under a relatively small load. As the load was increased the tension reinforcement yielded. After the reinforcement had yielded, the beams underwent a relatively large
deflection with little further increase in load. When the concrete in the compression zone reached its limiting strain, the maximum load-carrying capacity of the beams was reached and compression failure occurred.

Moment-deflection curves for beams which failed in tension are shown in Fig. 7. It is seen that these beams exhibited much larger deflections at ultimate moment than the beams which failed in shear; note that different scales for deflection are used in Figs. 6 and 7. The relative magnitudes of these deflections depend on the parameter $q$ which is a measure of the depth of the compression zone. When $q$ is very small, there is ample concrete remaining after crushing of concrete on the top of the beam, and the compressive force is transmitted to a lower portion of the beam. The lever arm of the internal moment is thereby reduced but the load remains near maximum, primarily due to the fact that the steel stress, being in the strain-hardening region, increases. Finally the lever arm of the internal moment is considerably reduced and the load on the beam drops off gradually. As the quantity $q$ increases, the magnitude of deflection at ultimate moment decreases and the drop in load is more sudden. This phenomenon is seen in Fig. 7 and is discussed in detail in a previous technical report (9).

In the present series of tests one additional factor influenced the load-deflection characteristics of a beam. Fig. 7 shows that beams S-1, S-9, and S-10, which were in the transition region between tension and shear failures, had little or no load-carrying capacity after the ultimate moment was reached and the concrete crushed. These beams with well-developed diagonal cracks collapsed after first crushing of concrete in the same manner.
as beams which failed in shear and are subsequently called tension-shear failures in this report. Fig. 4 shows a photograph of a typical representative of this group, Beam S-9, after failure. The remaining beams in the tension group failed gradually as described above. Figure 5 shows Beam S-6 in this group after failure.

13. Test Results

Test results for the 13 test beams are given in Table 2.

The quantity $\frac{M}{bd^2f'_c}$, as determined from the test results, is plotted against the parameter $\frac{p}{f'_c}$ in Fig. 8 which is similar to Fig. 1. Beams failing in different modes are marked with different symbols, as noted in the figure. The values of $\frac{M}{bd^2f'_c}$ as predicted by equations (2) and (3a) are plotted as continuous lines. The value of $j$ varied from 0.823 to 0.885 for beams which failed in shear or secondarily in shear; the average value, $j = 0.854$, is used for Eq. (2) in Fig. 8. For beams which failed in tension the average value of yield strength, $f_y = 44,410$ psi, is used in Eq. (3a).

All beams which failed in tension, either with or without a secondary shear failure, fall slightly higher than the theoretical curve; however, as seen in Table 3, the steel stress at failure was above the yield stress of the reinforcement. When reinforcing steel is considered acting with its actual stress at failure, good agreement is obtained between the test results and the predicted values. The average yield stress in Eq. (3a) was used only as a convenience in plotting the theoretical values.
No satisfactory agreement is found, however, between the beams which failed in shear, or secondarily in shear, and the preliminary empirical equation (2) for shear failures. It is seen that the scatter in test results is very considerable. Since Eq. (2) is fundamentally in the form of

\[ v = \frac{V}{bd} = \left[ C_1 f'_c + C_2 P \right] \frac{d}{a}, \]

the wide scatter of points in Fig. 8 shows that no set of numerical constants in the above equation would give good correlation with the test results. Furthermore, since the ratio \( \frac{d}{a} \) was kept constant for the test beams, this plot suggests also that no expression in the form of

\[ v = C_1 f'_c \left( \frac{d}{a} \right) + C_2 P \]

or

\[ v = C_1 f'_c + C_2 P \left( \frac{d}{a} \right) \]

can be found to give good correlation with the test results. The ratio \( \frac{d}{a} \), being a constant, can only modify the numerical coefficients \( C_1 \) and \( C_2 \), depending upon which of the two it is considered to be influencing, while retaining the equation's linear form.

This is the same as trying to draw a straight line through the points which failed in shear in the hope of finding a particular line which would pass through or close to all such points. It is seen in Fig. 8 that this is not possible.

Upon closer examination of Fig. 8, however, it is noticed that the points falling farthest from the curve representing Eq. (2) are either of low concrete strength or, while being of higher concrete strength, have a rather large percentage of reinforcement. For easier identification, all beams of the lower concrete strength are circled in Fig. 8. This suggests that it may be possible to represent the shearing strength \( v \) either by a higher than first degree function of \( f'_c \) and \( P \) or by inserting one additional constant in Eq. (2). From the locations of the different points in Fig. 8 it can be seen that equations in the following form can be used to represent the shear strength of the test beams:
\[ v = \frac{V}{bdj} = \left[ C_1 f'_c - C_2 (f'_c)^2 + C_3 p \right] \frac{d}{a} \]  
(4)

or

\[ v = \frac{V}{bdj} = \left[ C_1 + C_2 f'_c + C_3 p \right] \frac{d}{a} \]  
(2a)

Neglecting slight variations in the quantity \( j \) and including its effect in the values of the numerical coefficients, the coefficients of equations (4) and (2a) were determined from the test results, and the equations were rewritten as follows:

\[ \frac{M}{bd} \frac{f'_c}{f'_c} = 0.27 - \frac{2.8 f'_c}{10^6} + 5700 \frac{P}{f'_c} \]  
(4a)

and

\[ \frac{M}{bd} \frac{f'_c}{f'_c} = 0.091 + \frac{250}{f'_c} + 5700 \frac{P}{f'_c} \]  
(2b)

These two equations give very nearly the same results in the range of \( f'_c \) for which they were derived, from about 2000 to 4500 psi; outside this range, however, the results become widely different. Since the variation of \( f'_c \) in the test beams was not large enough to determine which of the two equations had more general applicability outside the scope of the test beams, both equations were checked against the results of previous investigations. It was found that, in general, an equation of the type of Eq. (4) gave better agreement with the test results than an equation of the type of Eq. (2a), the numerical coefficients, however, were different for different investigations. Furthermore, the coefficients \( C_1 \) and \( C_2 \) in Eq. (4) could be expressed as constants whereas in Eq. (2a) they seemed to be
functions of $f'_c$. This suggests that the shear strength $v$ must be a higher than first degree function of $f'_c$ and $p$ and Eq. (2b) was therefore eliminated from further consideration.

Equation (4a) for shear failures and Eq. (3a) for flexural failures are plotted in Fig. 9 as functions of the parameter $p/f'_c$. For Eq. (4a) the two dashed straight lines refer to the average values of concrete strength, $f'_c = 2200$ psi for the low group and $f'_c = 4200$ psi for the high group; and the two broken continuous lines correspond to the particular values of concrete strength for each beam.

It is seen from Fig. 9 that there is good correlation between the test results and the values predicted by Eq. (4a). Table 2 shows that the maximum deviation from the predicted value is 11.8 per cent for beams which failed in shear; beams which failed in tension with a secondary shear failure tend to be low.

Despite this satisfactory agreement, Eq. (4a) cannot have general applicability since it was based on only a very limited number of tests and, as pointed out before, it did not give satisfactory agreement with previous test results. The range of steel percentage $p$ was rather wide, but essentially only two values of concrete strength, 2200 and 4200 psi, were used. Furthermore, only one beam having the lower concrete strength failed in shear. In addition, one of the main variables influencing the shear strength of beams, the ratio of $\frac{d}{a}$, was not varied in these tests. Thus Eq. (4a), like any other previous empirical equation, is not applicable outside the range of variables for which it was derived. For these reasons another attempt is made in the following chapter to derive a general expression for shearing strength of reinforced concrete beams.
V. ANALYTICAL STUDIES

14. Derivation of Empirical Equation

After the formation of a diagonal tension crack a reinforced concrete beam, when not failing in tension, will fail either in the compression zone of concrete, in bond, or by splitting along the tensile reinforcement.

Failure by destruction of the compression zone just under a concentrated load is most common. Thus the failure takes place at the section of maximum moment and maximum shear. However, the real cause of failure has not been generally understood. It has been suggested that this type of failure is the result of the principal flexural stresses, compressive or tensile, or of the maximum shearing stress.

Previous investigations have indicated that the shearing unit stress $\tau$ is a function in the following form:

$$\tau = \frac{V}{bjd} = F(p, f'_c, \frac{d}{a})$$

But as was seen before, all of the empirical equations suggested by different investigators have failed to give good agreement with all of the available test results.

In this investigation it was first assumed that the total shearing force $V$ is resisted solely by the compression area of concrete. For beams without compressive reinforcement, the area of the compression zone is given by $k_d b$, where the quantity $k_d$ refers to the depth of the compression zone at shear failure. Thus the shearing unit stress is given
by $v = \frac{V}{k_s db}$. It was further assumed that the ultimate shearing unit stress, $v_u$, is a function of $f'_c$. Test results have shown that the shear capacity of the compression zone decreases as the moment-shear ratio, $M/V$, increases. This effect has usually been taken into consideration by the $d/a$-ratio, and there seems to be a linear relationship between this ratio and the shear capacity of the beam. Since both the horizontal compressive stresses and the vertical shearing stresses are assumed to be resisted by the same compressive area, it seems more reasonable to consider the shear-compressive force ratio $V/C$ rather than the $M/V$-ratio as influencing the ultimate load in shear. For the type of beam under consideration it can be written that $\frac{V}{C} = \frac{jd}{a}$. Thus the ultimate shearing stress $v_u$ can be expressed as follows:

$$v_u = \frac{V}{k_s db} = \frac{jd}{a} f'(c')$$

(6)

It is noticed that this expression can be rewritten in a different way:

$$\frac{Va}{bd^2 f'_c} = k_s JF(f'_c) \quad \text{or} \quad \frac{M}{bd^2 f'_c} = k_s JF(f'_c)$$

(7)

These equations are in a form which suggest that the criterion for shear failures is a limiting moment rather than an ultimate shearing stress. There is some supporting evidence for this observation in previous test results. Beams with no web reinforcement tested by Clark (8) had the ratio of $d/a$ as the only variable; all these beams failed at a nearly constant moment, although the total shear force at failure depended upon
the location of the loads on the beams. Turneaure and Maurer (10) reported a series of tests on small mortar beams with the \( \frac{d}{a} \)-ratio as the only variable, and their results again show that the ultimate moment was nearly the same for all positions of loads. Furthermore, the empirical equations (2) and (4a), derived previously, also have the form of a moment equation. Thus the so-called shear failures seem to be failures in compression, the beams failing at a limiting average compressive stress or a limiting total compressive force in the compression zone of the concrete. This type of failure differs from failures in flexural compression only because the compressive area is reduced because of diagonal tension cracking.

The above Eq. (7) can easily be derived by considering the average compressive stress in the concrete. The following relationships can be written for a section where failure occurs:

\[
C = \frac{M}{jd} = \frac{Va}{jd}
\]

\[
C = A_c \left( f_c \right)_{av}
\]

\[
A_c = k_s db
\]

\[
\left( f_c \right)_{av} = F_{l_c} \left( f_c' \right), \text{ at failure.}
\]

Then

\[
C = k_s db F_{l_c} \left( f_c' \right)
\]

\[
\frac{M}{bd^2} = k_s J F_{l_c} \left( f_c' \right) \quad \text{or} \quad \frac{M}{bd f_c'} = k_s J F_{l_c} \left( f_c' \right)
\]
In the new empirical equation (7), there are two main unknowns: the depth of the compression zone $k_s d$ and the limiting average compressive stress, related to $F(f'_c)$. The quantity $j$ can be considered as a constant since it does not vary over a great range.

The depth of the compression zone can be determined accurately for flexural failures, both in tension and in compression, by considering statical equilibrium and the strain relations involved. For shear failure, however, no theoretical relationship relating the extent of diagonal tension cracking and the properties of the beam has been found. From previous investigations, it can be shown qualitatively that $k_s$ is a function of $f'_c$ and $\mu$. Furthermore, this function must be a complex one since different empirical equations considering $v$ as a linear function of $f'_c$ and $\mu$ have failed to agree with test results.

In this analysis, it was considered that $k_s$ is related to $k$ as determined by the "straight line" theory, based on an equivalent section transformed to concrete. It was considered that if $k_s$ is either a constant proportion of the "elastic" $k$, or a proportion which depends on $f'_c$, the empirical equation can still be written as

$$\frac{M}{bd^2f'_c} = k F(f'_c)$$  \hspace{1cm} (8)$$

The unknown function $F(f'_c)$ in the above Eq. (8) must be evaluated empirically. In the following articles, it is evaluated first for beams tested in connection with this investigation and then for other available test data.
(a) Test Data of This Investigation - In order to evaluate the unknown function \( F(f'_c) \) in Eq. (8), the value of \( k \) was determined for each beam by the well-known equation

\[
k = \sqrt{(pn)^2 + 2pn - pn}
\]

which is valid for beams with tensile reinforcement only. The modular ratio \( n \) was determined by Jensen's formula (11)

\[
n = 5 + \frac{10,000}{f'_c}
\]

which has been found to give reliable results.

The quantity \( \frac{M}{bd^2 f'_c k} \) is plotted against \( f'_c \) in Fig. 10. It is seen that the function \( F(f'_c) \) can be represented for the test beams by a linear expression: \( F(f'_c) = 0.73 - \frac{7.3 f'_c}{10^5} \), where \( f'_c \) is expressed in pounds per square inch. For this group of beams, Eq. (8) becomes then:

\[
\frac{M}{bd^2 f'_c} = k(0.73 - \frac{7.3 f'_c}{10^5})
\]

It is seen that Eq. (8a) agrees quite well with the test results. The maximum deviation from the average line is 11 per cent.

(b) Other Test Data - In the analysis of other test data, attention was directed only to simple reinforced concrete beams subjected to concentrated loads and having no web reinforcement.
Out of the available test beams only those were selected for which complete information was given, such as the cylinder strength $f'_c$, steel percentage $p$, cross section and span of the beam, location of loads, and the maximum load obtained.

A total of 30 beams were included in the analysis. These were the beams reported by Clark (8), Moretto (7), Richart (5), Richart and Jensen (12), and Gaston, Siess, and Newmark (9). The range of the test variables for the different groups of beams are listed in Table 4.

The quantity $\frac{M}{bd^2f'_ck}$ for all of the beams in Table 4 is plotted against $f'_c$ in Fig. 11. It is seen that the concrete strength for most of the test specimens was between 3000 and 5000 psi. Within these limits $F(f'_c)$ can be approximated by a linear equation: $F(f'_c) = 0.57 - \frac{4.5f'_c}{10^5}$. Substitution of this expression into Eq. (8) gives the following equation for moment at which a simply supported beam without web reinforcement and under concentrated loads fails in shear:

$$\frac{M}{2bd f'_c} = k(0.57 - \frac{4.5f'_c}{10^5}) \quad (8b)$$

The agreement between Eq. (8b) and test results is satisfactory. Most of the test beams are well within ±15 per cent of the predicted value. One of Clark's beams is low; Clark, however, tested 3 identical beams in a group and the remaining two show very good agreement with the predicted values.
In this connection, it must be pointed out that all compression failures are very sensitive to the compressive strength of the concrete at the section of failure. The compressive strength reported for a test beam is the average strength obtained from control cylinders. Since even control cylinders can have wide differences in their strength, it is not expected that a test specimen is of uniform concrete strength. If the compressive strength at the section of failure happens to be greatly different from the average strength of the control cylinders, the test beam may fail at a load widely different from the predicted load. It is believed that most of the scatter in test results can be attributed to the variation of concrete strength from the average value.

(c) Effect of Column Stub - It is recalled that all beams tested in connection with this investigation were provided with a column stub at midspan which was cast integrally with the beams. The purpose of the column stub was to simulate a beam-column connection in a framed structure. The beams were loaded with one concentrated load through the column stub. All other test beams analyzed in the previous section were simple beams without a column stub and loaded with two concentrated loads. In most cases, loads were applied at the third-points of the span; however, for beams tested by Clark (8) the position of loads on the beam was one of the test variables.

Equations (8a) and (8b) show that beams of the present series failed at a somewhat higher load than could be predicted for beams without column stubs. It is possible that the presence of a column stub had a strengthening effect against shear failures.
Another difference in the test beams, the use of either one or two concentrated loads, is not believed to have influenced the ultimate load. Although no beams without a column stub and loaded with only one load at midspan could be found for comparison, the form of Eq. (8) suggests that the position of the loads is important only as far as it affects the bending moment. Hence the fact that only one load was employed in the present series of tests should not have changed the effect of the a/d-ratio.

It is noticed that for low values of $f'_c$ the increase in strength due to the column stub was larger than for high values of $f'_c$, 21 per cent for $f'_c = 2200$ psi and 11 per cent for $4200$ psi. The limited number of test specimens, however, does not permit any definite conclusion regarding the possible reasons for these differences.

15. Theoretical Interpretation of Empirical Equation

The new empirical Eq. (8b) can be interpreted in the light of the conventional theory of compression failures of reinforced concrete beams. The only modification is in the depth of the compression zone. The following stress block is assumed.

\[
\begin{align*}
T &= pbd f_s \\
C &= k_2 k'_d \\
M &= C d (1 - k_2 k'_s) \\
M &= k_1 k'_c k'_s b d^2 (1 - k_2 k'_s)
\end{align*}
\]

\[
\frac{M}{b d^2 f'_c} = k_1 k'_c k'_s (1 - k_2 k'_s) \quad \text{(11)}
\]
The parameters \( k_1k_3 \) and \( k_2 \) have been determined experimentally by previous investigators. In Fig. 12 the values of \( k_1k_3 \) as obtained by Gaston (9) and in a prestressed concrete investigation at the University of Illinois (unpublished), have been plotted against \( f_c' \). There is considerable scatter in the measured values as would be expected in an investigation of this kind. A reasonable approximation, however, can be obtained by a linear relationship between \( k_1k_3 \) and \( f_c' \). When \( f_c' \) is within the limits of 2000 and 6000 psi, the parameter \( k_1k_3 \) can be approximated as follows:

\[
k_1k_3 = 1.37 - \frac{10.8 f_c'}{10^5} = 2.4 \left( 0.57 - \frac{4.5 f_c'}{10^5} \right) \tag{12}
\]

Substitution of this function into Eq. (11) gives:

\[
\frac{M}{bd^2 f_c'} = 2.4 \left( 0.57 - \frac{4.5 f_c'}{10^5} \right) k_s (1 - k_2 k_s) \tag{13}
\]

It is noticed that this equation is in the same form as the previously found empirical Eq. (8b). Equating the two yields a relationship between \( k_s \) and \( k \):

\[
k_s = \frac{k}{2.4 (1 - k_2 k_s)} \tag{14}
\]

For the beams analyzed in this investigation, \( k \) varied from 0.32 to 0.53. This variation and the use of \( k_2 = 0.45 \) limits the quantity \( (1 - k_2 k_s) \) to the range 0.89 to 0.94 with an average value of 0.92. Hence, \( k_s \) is practically a constant fraction of \( k \), the depth of the compression zone computed by the "straight line" theory.
This finding explains why the previous attempt to use the value of $k$ as a measure of the depth of the compression zone $k_s d$ at shear failures gave fairly good agreement with test results. It does not explain, however, why these two quantities are related. It might be a coincidence that the variables $f'_c$ and $p$ which determine $k_s$ enter into the computations to determine $k$ in a similar manner.

16. Beams Reinforced Both in Tension and Compression

Equations (8b) and (11) were derived for beams without compression reinforcement. For beams reinforced both in tension and compression Eq. (11) can be modified as follows:

$$ M = k_1 k_2 f'_c k_s b d^2 (1 - k_2 k_s) + f'_s p' b d^2 t $$

(15)

where $t_d$ is the distance between the centers of the tension and compression reinforcements, $f'_c$ is the stress in the compression reinforcement, and $p'$ is the ratio of compression reinforcement.

Since the ultimate strain in the concrete is approximately 0.0040 and the yield strain for reinforcing bars around 0.0017, yielding of the compression reinforcement precedes crushing of the concrete in most flexural compression failures. For shear compression failures, however, diagonal cracks extend higher than the vertical tension cracks and it is conceivable that a beam can fail either before or after the compression reinforcement yields. Expressions for ultimate shear moment for both of these two cases are derived in the following paragraphs.
If it is first assumed that compressive reinforcement has reached its yield stress $f'_c$ at shear failures and that $k_s$ is still given by

$$k_s = \frac{k}{2.4(1-k_s 2_s)},$$

Eq. (15) for maximum shear moment can be written as:

$$\frac{M}{bd 2 r'_c} = k \left(0.57 - \frac{4.5 f'_c}{5} \right) + n'p't$$  \hspace{1cm} (16)

Since Eq. (16) assumes that compression reinforcement has yielded while the tension reinforcement is still elastic, the elastic modular ratio $n$ has to be used for the tension reinforcement and the plastic modular ratio, $n'_c = \frac{f'_c}{f'_t}$, for the compression reinforcement in computing the quantity $k$.

If, however, it is assumed that a beam fails before the compression reinforcement yields, an expression for maximum shear moment can be derived by making the same assumptions as in the case of Eq. (8). The presence of compression reinforcement is considered as increasing the compression area by an amount of $np'bd$, the steel area transformed to concrete:

$$A_c = bkd + np'bd = bd (k + np')$$  \hspace{1cm} (17)

In this expression the elastic modular ratio $n$ is used for both compression and tension reinforcement in computing the quantity $k$. The modified compression area leads to the following equation which corresponds to Eq. (8) for beams without compression reinforcement:

$$\frac{M}{bd 2 r'_c} = (k + np') F (r'_c)$$  \hspace{1cm} (18)
The use of $F(f'_0) = 0.57 - \frac{4.5 f'}{10}$ as for all other previous data gives for the ultimate shear moment:

$$\frac{M}{2bd f'_0} = (k + np') (0.57 - \frac{4.5 f'}{10})$$  \hspace{1cm} (18a)

It is seen that equations (16) and (18a), based on different assumptions, are greatly different. Equation (16) gives a much higher ultimate moment that Eq. (18a). Unfortunately, there is no published data available to check the above assumptions and the validity of the equations.

Most earlier investigators, however, have considered that the presence of compressive reinforcement does not affect the shearing strength of a beam. This observation seems to invalidate Eq. (16), and it is believed that Eq. (18a) considers the effect of compression reinforcement more correctly. According to Eq. (18a) the shear strength of a beam with compression reinforcement is but little greater than that of a beam without: $p'$ decreases the value of $k$ while adding the term $np'$, so that the quantity $(k + np')$ is but little greater than the value of $k$ for a beam without compression reinforcement.

17. Transition Region Between Shear and Tension Failures

When the physical properties of a reinforced concrete beam are such that it fails in tension with a secondary crushing of concrete, moment at first yielding and at ultimate load can be computed by theoretical expressions. In order to determine the theoretical transition point between shear and tension failures, the quantities $\frac{M}{2bd f'_0}$ for these two moments and
an expression representing the shear strength of the beam can be plotted against the parameter $p/f'_c$. A hypothetical set of such curves is shown in Fig. 13a. As an aid in discussion, they are drawn out of true proportion, the ultimate moment being actually only from 10 to 20 per cent larger than the moment at first yielding.

These curves suggest three alternatives for the theoretical transition point. First, it can be assumed that the transition point is determined by the ultimate moment and shear curves, point $a$ on Fig. 13b. This alternative means that when $p/f'_c = a$, the beam reaches both its ultimate flexural load and ultimate deflection before failure. When $p/f'_c$ is between $a$ and $b$, the beam yields first but fails secondarily in shear before either the ultimate flexural load or deflection are reached. And at $p/f'_c = b$ the beam fails in shear as soon as the reinforcement starts to yield.

Figure 13c shows the second alternative. It assumes that the shear strength is reduced as soon as first yielding occurs. Thus the theoretical point of transition is at $c$, at some point before the intersection of the ultimate moment and shear curves. From $c$ to $b$ the beam yields but fails in shear before either the ultimate flexural load or the load corresponding to failure in shear is reached.

The third alternative is shown in Fig. 13d. This assumes that the theoretical transition point is at $h$; before $h$ the beam develops its flexural capacity, although from $a$ to $b$ the flexural capacity is greater than its strength in pure shear. This alternative can be defended by assuming that the first yielding takes place at tension cracks, hence all rotation
thereafter occurs at the tension cracks and the progress of diagonal cracks is arrested. For \( p/f'_c > b \) the beam fails in shear.

Very little is known about the actual behavior of a beam in the transition region between shear and tension failures. In the following paragraphs an attempt is made to analyze the results of the present series of tests in the light of the above discussion. Although the number of test beams was limited, it is believed that some qualitative information can be obtained.

Theoretical moments at first yielding and at flexural ultimate were computed by the following equations which were derived in the previous technical report (9):

\[
\frac{M_{pf}}{\frac{b}{f'_c}} = \frac{p f'_c}{f'_c} \left( 1 - \frac{k_2}{3} \right) \tag{19}
\]

\[
\frac{M_{ult}}{\frac{b}{f'_c}} = \frac{p f'_c}{f'_c} \left( 1 - \frac{k_2}{k_1} \frac{p f'_c}{f'_c} \right) \tag{3}
\]

The steel stress \( f_s \) in Eq. (3) was calculated by the following equation, also previously derived (9):

\[
f_s = \sqrt{\frac{E k_3 f'_c}{p}} + \frac{1}{4} (E_0 \varepsilon_u - f'_o)^2 - \frac{1}{2} (E_0 \varepsilon_u - f'_o) \tag{20}
\]

The values of \( \frac{M}{\frac{b}{f'_c}} \) for the test beams as calculated by Eqs. (19) and (3) are given in Table 3 and plotted against \( p/f'_c \) in Fig. 14. It is seen that all beams which failed in tension, except S-10, exhibit
moments which are slightly less than the theoretical moments given by Eq. (3). This difference is primarily due to the fact that the idealized stress-strain curve used for reinforcing steel in Eq. (20) becomes in error on the high side for higher values of steel strain. It is observed, however, that all tension failures, including those which have been called tension-shear failures, fall low by approximately the same proportion. This seems to indicate that all such beams developed their full flexural strength at failure. Moment-deflection curves in Fig. 7 also suggest that the three beams in the tension-shear group developed their full or close to full flexural deflection before failure. These beams differed from the other tension failures only in the manner of final collapse. After the flexural load-carrying capacity was reached, these beams failed by a sudden collapse similar to that of shear failures.

Equation (8a) representing shear failures is also plotted in Fig. 14. This curve and curves corresponding to the measured ultimate moment and to the moment at first yielding as given by Eq. (19) permit us to check the three previously suggested alternatives for the transition point between tension and shear failures.

It is seen that beams S-8, S-6, S-7, and S-12 are well below the loads which correspond to shear failure. These beams reached their ultimate flexural capacity and then the load gradually dropped off. Their behavior indicates a region to the left of point a in Fig. 13b or 13d.

Beam S-1 failed at a load which corresponds to both the ultimate flexural and shear capacity. After the ultimate load-carrying capacity was reached, the beam failed by a sudden collapse. This beam seems to represent point a in Fig. 13b or 13d.
Beam S-9 failed at the ultimate load in flexure but slightly below the ultimate in shear. This refers to a point slightly to the left of \(a\) in Fig. 13a or 13d. The final failure was a sudden collapse.

Beam S-10 failed at the ultimate load in flexure and slightly above the ultimate in shear. Judging from the shear and moment at first yielding curves, this beam seems to correspond approximately to the point \(b\) in Fig. 13d.

The remaining beams failed in shear. It is seen that the moment at failure was considerably below the moment which would cause yielding of the reinforcement. These beams refer to a region to the right of point \(b\) for all alternatives in Fig. 13.

The above findings seem to agree with Alternative 3 of the previous discussion. This suggests, referring to Fig. 13d, that the transition point between shear and tension failures is at \(b\). When \(p/f'\) > \(b\), a beam fails in shear. When \(a < p/f'_0 < b\), a beam reaches its ultimate flexural capacity and develops its full flexural deflection. The final failure after crushing of concrete is, however, very sudden and similar to that of a shear failure. When \(p/f'_c < a\), a beam reaches its ultimate load in flexure and then shows a gradual drop in its load-deflection curve. With an increasing value of \(p/f'_c\) the drop in load becomes more marked and at values of \(p/f'_c\) approaching \(a\) the final failure may be a sudden collapse similar to that in the region from \(a\) to \(b\).

In the above comparison it was tacitly assumed that Eq. (6a) was the true measure of shear strength of the beams under consideration. It is recalled, however, that in deriving Eq. (6a) there was a deviation between the measured and predicted values up to 11 per cent. This difference might invalidate some of the conclusions reached above and shows the need for future research.
VI. SUMMARY AND CONCLUSIONS

18. General Summary and Discussion

Both the test results of this investigation and a review of previous research indicated that the shear strength of a simple reinforced concrete beam without web reinforcement and loaded with one or two concentrated loads is not a linear function of $f'_c$ and $p$, as it has been usually assumed. Results of the present series of tests suggested an empirical equation in the following form:

$$v = \frac{V}{b j d} = \left[ C_1 f'_c - C_2 (f'_c)^2 + C_3 p \right] \frac{d}{a} \quad (4)$$

Taking the quantity $j$ as a constant and including its effect in the numerical coefficients, the coefficients of the above equation were evaluated for the beams with column stubs and the equation was rewritten as follows:

$$\frac{M}{b d f'_c} = 0.27 - \frac{2.8 f'_c}{10^5} + 5700 \frac{p}{f'_c} \quad (4a)$$

Equation (4a) gave satisfactory agreement with the beams for which it was derived. But like any other empirical equation, derived for a certain series of tests, Eq. (4a) was not expected to have general applicability. The number of test beams was rather limited and one of the main variables, the ratio of the shear span to the effective depth of the
beams was kept constant. Furthermore, Eq. (4) is still an expression for the nominal shearing unit stress $v$ at failure. It is recalled, however, that the equation for the nominal shearing unit stress, $v = \frac{V}{bd}$, was derived from considerations of the state of stress in the tension reinforcement. It was an expression for the horizontal shearing unit stress at the level of reinforcing bars which also had to equal the vertical shearing unit stress at the same level. Since concrete was not considered to carry any tension, it was concluded that there could not be any change in the vertical shearing stress from the level of reinforcement to the neutral axis of beam. The formation of a diagonal crack, however, radically changes the state of stress in a reinforced concrete beam. There cannot be any transfer of stress across a crack. Hence this popular conception of the distribution of shearing stresses in a reinforced concrete beam cannot be true and it is believed that any agreement between an empirical expression based on the nominal shearing unit stress $v$ and test results is coincidental.

For these reasons, an attempt was made to derive a general expression for the shear strength of simple reinforced concrete beams without web reinforcement and under concentrated loads. It was first assumed that the total shear force $V$ was resisted solely by the compression area of the concrete above the neutral axis. It was further assumed that the ultimate shearing unit stress, based on the compression area $bd$, was a function of $f'_c$. These assumptions and the observation that the shear strength of a beam decreases as the moment–shear ratio increases were used to derive the following expression for the shear strength of a beam:
The form of the above equation suggested that the criterion for shear failures was a limiting moment rather than an ultimate shearing stress. This observation was supported by test results reported by Clark (8) and Turneaure and Maurer (10). Thus it was concluded that shear failures are failures in compression, the beams failing at a limiting average compressive stress in the compression zone of concrete. This type of failure is different from flexural compression failures only because the compressive area of the concrete is reduced as the result of diagonal tension cracking, since diagonal tension cracks extend higher than the tension cracks.

In the absence of any theoretical means to determine the depth of the compression zone, \( k_s d \), at shear failures, the quantity \( k_s \) as determined by the straight line theory was taken as measure of \( k_s \). It was considered that if \( k_s \) is either a constant proportion of the "elastic" \( k \), or a proportion which depends on \( f'\), the empirical equation can still be written as

\[
\frac{M}{bd^2 f'_c} = k F(f'_c) \tag{8}
\]

The unknown function \( F(f'_c) \) was evaluated from test results. Thus the following two equations were obtained for the moment at which a beam fails in shear:

\[
\frac{M}{bd^2 f'_c} = k (0.73 - \frac{7.3 f'_c}{10^5}) \quad \text{for beams with a column stub} \tag{8a}
\]
and

\[
\frac{M}{2} = k \left( 0.57 - \frac{4.5 f'}{f_c} \right) \quad \text{for beams without a column stub}
\]

Equation (8a) was based on 9 test specimens for which \( f' \) ranged from 2140 to 4690 psi. Maximum deviation from the predicted value was 11 per cent. Equation (8b) was based on 30 tests from five different investigations for which \( f' \) varied from 2230 to 4760 psi. Other test variables for these beams are listed in Table 4. The test values agreed with the predicted values within ±15 per cent. This agreement was much better than that given by other empirical expressions when beams from different investigations were included in the comparison. Furthermore, the agreement was equally good for a rather wide range of test variables.

As is shown by Eq. (8a), beams with a column stub failed at a higher load than beams without stubs. This was apparently due to some strengthening effect of the column stub. The increase in strength varied from 11 to 21 per cent, being lower for higher values of concrete strength.

The empirical equation (8b) was interpreted in the light of the conventional theory of compression failures of reinforced concrete beams. It was found that Eq. (8b) was related to the following theoretical expression:

\[
\frac{M}{2} = k_1 k_2 k_3 (1 - k_2 k_3) \quad \text{(11)}
\]
The quantity $k_1k_3$ had been determined experimentally by previous investigators. For $f'_c$ limited between 2000 and 6000 psi it could be approximated as follows:

$$k_1k_3 = 2.4 \left( 0.57 - \frac{4.5 f'_c}{10^5} \right)$$

Equations (8b), (11), and (12) yielded the following relationship between $k_s$ and $k$:

$$k_s = \frac{k}{2.4 \left( 1 - k_2k_s \right)}$$

Since the quantity $(1 - k_2k_s)$ varies over a very limited range, $k_s$ is practically a constant proportion of the "elastic" $k$. This finding provides an explanation as to why the quantity $k$ is a fairly good measure of diagonal tension cracking and permits the use of the following rather simple expression for the ultimate moment at shear failures:

$$\frac{M}{bd f'_c} = k \left( 0.57 - \frac{4.5 f'_c}{10^5} \right) \quad (8b)$$

The above equation is limited to simple beams without web reinforcement and under one or two concentrated loads. All such beams fail directly under a load, hence in a region of maximum moment and maximum shear. These two are related by $M = va$, where $a$ is the distance from the end support to the load point.

Different variables have the following effect on the above equation (8b):
(a) Ratio of $\frac{a}{d}$ -- Equation (8b) considers shear failures as compression failures. In that sense, the ratio $\frac{a}{d}$ loses its usual meaning; that is, affecting the shearing strength of concrete. The quantity $a$ relates the magnitude of the applied load to the moment at failure, $M = Va$, and the effective depth $d$ affects both the lever arm of internal moment and the area of the compressive zone. For the beams analyzed, the ratio of $a/d$ varied from 1.17 to 3.43. Differences in this ratio do not seem to have any effect on the agreement between the test results and the predicted values. However, in only one series of tests (8) was the $\frac{d}{a}$-ratio used as a test variable, and in that case the beams were near yielding when shear failure occurred.

(b) Tensile Reinforcement -- The amount of tensile reinforcement affects the size of the compressive area. It was found empirically that moment at failure could be related to $k$ which is the depth of the compression zone computed by "straight line" theory. For beams without compressive reinforcement $k$ is given by $k = \sqrt{(pn)^2 + 2pn - pn}$. When the empirical equation was interpreted in the light of flexural compression failures, it was found that the depth of the compression zone at shear failure was practically a constant proportion of $k$, or $k_s = \frac{k}{2.4(1 - k_2k_3)}$. The latter procedure implies that the parameter $k_1k_3$ which is a measure of the total compressive force in concrete remains the same both for flexural and shear compression failures and that the failure criteria is still a limiting compressive strain in the concrete.

(c) Compressive Strength of Concrete -- The shear strength of a beam is directly proportional to the following function of $f'_c$: 
4.5 f' \left(0.57 - \frac{0.57 - c}{10^5}\right) k. It is seen that as f' increases both the quantity

\frac{4.5 f'}{10^5} \left(0.57 - \frac{0.57 - c}{10^5}\right)

which represents the effect of \(k_1 k_3\) and the value of \(k\) decrease. Thus the shear strength is not a linear function of \(f'\) as is usually specified in building codes. As an example, for a beam with 1 percent tension reinforcement an increase of \(f'\) from 2500 psi to 5000 psi increases the shear strength only 36 percent.

(d) Type of Loading -- Equations (8a) and (8b) were derived for simple beams loaded with one or two concentrated loads. This type of loading is a special case among all possible loading conditions. Failure takes place in the region of maximum shear, \(V\), and maximum moment, \(M = V a\).

A number of restrained beams have been tested previously under concentrated loads to evaluate their strength in shear (13). However, all such beams have been loaded in such a way as to make them statically determinate and only a few of them had no web reinforcement. Test results show, however, that whenever all modes of failure except shear have been excluded, the above conception of shear failures as compression failures is valid and Eq. (8b) can be used directly.

It is not expected, however, that the new empirical equation can be applied directly to beams, either simply supported or continuous, under uniform load. The writer has not been able to find any test data on beams under uniform load. However, Bach and Graf (14) in Germany have tested some T-beams, simply supported and without web reinforcement, under eight equal and equally spaced concentrated loads. These beams did not fail in the region of maximum shear and no moment at the support, but at some distance
inside the support. Thus a particular combination of shear and moment seemed to have caused the beams to fail in shear. It is conceivable that a critical section can be found for this type of loading at which the failure load can be related to the equation for limiting moment. The test data available at the present time, however, is not sufficient to determine such a critical section.

(e) Column Stub -- Beams tested in connection with this investigation failed at a somewhat higher load, Eq. (8a), than that predicted by Eq. (8b) for beams without a column stub. The increase of strength varied from 11 to 21 per cent, being higher for lower values of concrete strength.

(f) Compression Reinforcement -- Although no test data were available for beams reinforced both in tension and compression, it was concluded that the effect of compression reinforcement can be included in the analysis by considering $f'_c$ in computing both the "elastic" $k$ and the transformed concrete area. This procedure leads to the following equation:

$$\frac{M}{bd^2f'_c} = (k + np') (0.57 - \frac{4.5 f'_c}{10^5})$$  \hspace{1cm} (18a)

19. Conclusions

For simple beams without web reinforcement and loaded with one or two concentrated loads the load at shear failure can be predicted with a fair degree of accuracy by the following empirical equation:

$$\frac{M}{bd^2f'_c} = k (0.57 - \frac{4.5 f'_c}{10^5})$$  \hspace{1cm} (8b)
All available test results agreed with the predicted values within ±15 per cent.

The present test beams, loaded through a column stub which was cast integrally with the beams, failed at a somewhat higher load given by the equation:

$$\frac{M}{bd f'_c} = k(0.73 - \frac{7.3 f'}{10^5})$$

(8a)

This equation agreed with the beams for which it was derived within ±10 per cent.

These two equations are applicable in the range and combination of test variables considered in this report.

20. **Recommended Future Research**

One of the reasons for our limited knowledge of the shear strength of reinforced concrete beams seems to lie in the conventional approach to the problem. Since the introduction of the concept of truss analogy some 50 years ago, major emphasis has been placed on the evaluation of the contribution of web reinforcement to shear strength. The contribution of the beam itself, without the benefit of any web reinforcement, has remained a relatively unknown quantity. Furthermore, any uncertainties with regard to the contribution of web reinforcement have reflected directly on the contribution of the beam itself, thus rendering both questionable.

Our first problem, therefore, should be the evaluation of shear strength of a beam without web reinforcement. The contribution of the web
reinforcement should be treated separately and it should be determined whether or not these two quantities are additive.

The evaluation of the shear strength of a beam with no web reinforcement involves variations in the compressive strength of concrete, in the amount of longitudinal reinforcement, and in the type of loading. The findings of this report indicate that shear failures of beams under concentrated loads are failures in compression as modified by the effect of diagonal cracking. A comprehensive test program for this type of loading could easily be performed to check the validity of this new conception for the complete range of all variables involved. One variable should be varied at a time, and both shear and tension failures should be obtained. Beams which fail in tension could be used to throw more light on the behavior of a beam in the transition region between the two types of failure.

Beams under distributed load introduce additional unknowns. The use of concentrated loads determines the section of failure, whereas for distributed loads the factors determining this section should be investigated first. If the location of the main diagonal crack in a beam with distributed loading can be predicted by theory, it is expected that the shear capacity of a beam can be determined by the same procedure as that developed herein for beams under concentrated loads.

The second major problem is the evaluation of the contribution of web reinforcement to the shear strength of a beam. Both conventional design procedure and various empirical equations express the contribution of web reinforcement as an independent quantity, not influenced by the
strength of the beam without web reinforcement. This can be expressed as follows:

\[ v = F_1(r) + F_2(p, f'_c, \frac{d}{a}) \]  

(A)

The present ACI code simply assumes that the contribution of the beam itself is a given proportion of the concrete strength, the allowable unit shearing stress being specified as \( v_c = 0.03 f'_c \). The difference between the total shear and that assumed to be carried by the concrete is assigned to the web reinforcement.

However, as long as it has not been shown conclusively that the contribution of the web reinforcement does not depend also on the contribution of the beam itself, the following alternative must be investigated for the total shear strength of a beam:

\[ v = F \left[ p, f'_c, \frac{d}{a}, F_1(r) \right] \]  

(B)

These two alternatives can be expressed in terms of moment-equations as follows:

\[ \frac{M}{bd^2 f'_c} = F_1(r) + F_2(p, f'_c) \]  

(A-1)

\[ \frac{M}{bd^2 f'_c} = F \left[ p, f'_c, F_1(r) \right] \]  

(B-1)

This investigation is currently being extended to beams with web reinforcement. The preliminary results indicate that the effect of web reinforcement is not directly additive to the contribution of the beam
itself, but that it also depends on the shear strength of the beam without web reinforcement. Thus a certain percentage of web reinforcement would not increase the shear strength by a given amount but by an amount which depends on the shear strength of the beam if no web reinforcement had been provided. This suggests that an equation of the type of Eq. (B-1) should be applicable for beams with web reinforcement.

The third main problem to be investigated is the behavior of a beam in the transition region between shear and flexural failures. It is important not only that the possibility of a premature shear failure be eliminated, but also that a beam can, after the first yielding of reinforcement, develop its full flexural load-carrying capacity and especially its full flexural deflection. The increase in the load from the first yielding to the flexural ultimate is, in general, very small and perhaps without any practical significance, but the full flexural deflection adds a major portion to the energy-absorbing capacity of a beam.
VII. BIBLIOGRAPHY


<table>
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<tr>
<th>Beam No.</th>
<th>$f'_c$ (psi)</th>
<th>Reinf. Bar Sizes</th>
<th>$p$</th>
<th>$p/t'_c$ $(10^{-5} \text{ in}^2/\text{lb})$</th>
<th>$d$ (in.)</th>
<th>$a^*$ (in.)</th>
<th>$a/d$</th>
<th>$f_y$ (psi)</th>
<th>$f_o$ (psi)</th>
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* Distance from center of end support to edge of column stub.

** Based on "straight-line theory" with $n = 5 + \frac{10000}{f'_c}$. 
### Test Results
#### U.S. Engineers 248

6 by 12-in. Simple Beams

<table>
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<tr>
<th>Beam</th>
<th>Mode of Failure</th>
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<th>p/f'c (10^-5 in²/lb)</th>
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* S = Shear; T-S = Tension with shear-type final collapse; T = Tension
### TABLE 3

**ANALYSIS OF BEAMS FAILING IN TENSION**

**U. S. ENGINEERS 248**

6 by 12-in. Simple Beams

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<td>S-8</td>
<td>T</td>
<td>0.235</td>
<td>45,000</td>
<td>52,000</td>
<td>57,500</td>
<td>0.096</td>
<td>0.127</td>
</tr>
<tr>
<td>S-12</td>
<td>T</td>
<td>0.137</td>
<td>43,600</td>
<td>58,200</td>
<td>70,900*</td>
<td>0.055</td>
<td>0.120</td>
</tr>
</tbody>
</table>

**Notes:**
- **T-S = Tension with shear-type final collapse**
- **T = Tension**
- *Calculated $f_s$ very excessive due to approximations made in stress-strain curve for steel.*
TABLE 4
RANGE OF VARIABLES OF OTHER TESTS

Beams with no Web Reinforcement

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Entry in Bibliography</th>
<th>Number of Beams</th>
<th>$f'_c$ (psi)</th>
<th>p (in.)</th>
<th>b (in.)</th>
<th>d (in.)</th>
<th>a (in.)</th>
<th>a/d</th>
<th>Loading Positions (2 equal loads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clark</td>
<td>(8)</td>
<td>12</td>
<td>3120-3765</td>
<td>0.0098</td>
<td>8</td>
<td>15.4</td>
<td>36;30;2.34;1.95;24;18</td>
<td>2.34;1.95;1.56;1.17</td>
<td>Various</td>
</tr>
<tr>
<td>Moretto*</td>
<td>(7)</td>
<td>4</td>
<td>3335;3540</td>
<td>0.0186</td>
<td>5.5</td>
<td>19.5</td>
<td>32</td>
<td>1.64</td>
<td>1/3-points</td>
</tr>
<tr>
<td>Richart</td>
<td>(5)</td>
<td>4</td>
<td>3700-4530</td>
<td>0.0233</td>
<td>8</td>
<td>21</td>
<td>36</td>
<td>1.72</td>
<td>1/3-points</td>
</tr>
<tr>
<td>Richart**</td>
<td>(12)</td>
<td>6</td>
<td>2230-4760</td>
<td>0.0280</td>
<td>8</td>
<td>21</td>
<td>32</td>
<td>1.53</td>
<td>1/3-points</td>
</tr>
<tr>
<td>and Jensen</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaston</td>
<td>(9)</td>
<td>4</td>
<td>4020-4750</td>
<td>0.0138;0.0190</td>
<td>6</td>
<td>10.5</td>
<td>36</td>
<td>3.43</td>
<td>1/3-points</td>
</tr>
<tr>
<td>Total Range</td>
<td>30</td>
<td>2230-4760</td>
<td>0.0098</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.43</td>
<td>1/3-points</td>
</tr>
</tbody>
</table>

* Average values reported for each pair of companion specimens.

** Includes only those beams made of concrete with natural sand and gravel aggregates.
FIG. 1  \( \frac{M}{bd^2f_c'} \) VERSUS \( \frac{p}{f_c'} \) FOR DESIGN OF TEST BEAMS
FIG. 2 TEST BEAM. LOCATION OF STRAIN GAGES & DEFLECTION TARGETS
FIG. 3 BEAM S-4 AFTER FAILURE

FIG. 4 BEAM S-9 AFTER FAILURE

FIG. 5 BEAM S-6 AFTER FAILURE
FIG. 6  MOMENT-DEFLECTION CURVES FOR SHEAR FAILURES
FIG. 7  MOMENT-DEFLECTION CURVES FOR TENSION FAILURES

<table>
<thead>
<tr>
<th>Beam</th>
<th>q</th>
<th>p</th>
<th>f'c</th>
</tr>
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<tbody>
<tr>
<td>S-10</td>
<td>0.254</td>
<td>0.0139</td>
<td>2280</td>
</tr>
<tr>
<td>S-9</td>
<td>0.193</td>
<td>0.0093</td>
<td>2140</td>
</tr>
<tr>
<td>S-1</td>
<td>0.165</td>
<td>0.0146</td>
<td>3940</td>
</tr>
<tr>
<td>S-8</td>
<td>0.106</td>
<td>0.0062</td>
<td>2640</td>
</tr>
<tr>
<td>S-6</td>
<td>0.101</td>
<td>0.0093</td>
<td>4150</td>
</tr>
<tr>
<td>S-7</td>
<td>0.069</td>
<td>0.0062</td>
<td>4070</td>
</tr>
<tr>
<td>S-12</td>
<td>0.060</td>
<td>0.0034</td>
<td>2480</td>
</tr>
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\[ \frac{M}{bd^2f_c} \]
FIG. 8 \[ \frac{M}{bd^2 f_c'} \text{ VERSUS } \frac{p}{f_c'} \]
PRELIMINARY EQUATION 2 FOR SHEAR FAILURES

NOTE:
- \( f_c' = 2140-2680 \) For Circled Points
- \( f_c' = 3800-4690 \) For All Other Points
FIG. 9

\[ \frac{M}{bd^2f_c^t} \] Versus \[ \frac{p}{f_c^t} \]

Equation 4a for Shear Failures

NOTE:

- \( f_c = 2140-2680 \) For Circled Points
- \( f_c^t = 3800-4690 \) For All Other Points
Mode of Failure
- Shear
- Tension-Shear

Figure 10: $\frac{M}{bd^2f_c^l k}$ versus Concrete Strength. U.S. Engineers 248
FIG. II  \( \frac{M}{bd^2f'_c} \) VERSUS CONCRETE STRENGTH.
OTHER TEST DATA
FIG. 12  $k_1 k_3$ VERSUS CONCRETE STRENGTH

$2.4\left(0.57 - \frac{4.5f'_c}{10^5}\right)$
FIG. 13 THEORETICAL TRANSITION REGION BETWEEN SHEAR AND TENSION FAILURES

(a) Hypothetical Curves

(b) Alternative 1

(c) Alternative 2

(d) Alternative 3

\[
\frac{M}{bd^2f_c} \quad \text{Shear}
\]

First Yielding

Ultimate Flexural Capacity

\[ \frac{p}{f_c} \]

\[ a \quad b \]

\[ a \quad b \]

\[ c \quad b \]
FIG. 14 TRANSITION BETWEEN SHEAR AND TENSION FAILURES. U.S. ENGINEERS 248
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