THE FLEXURAL AND SHEAR STRENGTH OF REINFORCED CONCRETE BOX CULVERTS

By

LAUPA ARMAS

A. LAUPA, C. P. SIESS

N. M. NEWMARK

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A. Laupa, C. P. Siess
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I. INTRODUCTION

1. Introduction

Reinforced concrete box culverts are customarily designed by computing moments and shears for a given loading with the aid of some type of frame analysis involving the "elastic" properties of the structure. Side walls are designed to resist bending moment combined with axial thrust due to the vertical loads, and shear due to the lateral loads. Top and bottom slabs are designed for bending moment, with small thrusts ignored, and for large shears due to vertical loads.

Under existing specifications, and for proportions and loads typical for culverts under high fills, the depth of the members will usually be controlled by shear in the horizontal members at the face of the column. The amount of reinforcement will depend on the moments and on the axial thrust if it is considered.

The results of tests made at the Ohio River Division Laboratories of the Corps of Engineers suggested that culverts designed by conventional procedures have a factor of safety against failure in shear greater than the factor of safety against failure in flexure. Since recent investigations of the shear strength of reinforced concrete members have yielded new information on the mechanism of shear failure, it is believed that an optimum design procedure for reinforced concrete box culverts can be developed. The optimum condition is one in which the relative strengths in shear and in flexure are such that the structure can develop its full flexural strength and deformation without prior failure in shear and without the structure being over-designed in shear.
The elements that make up a culvert may be considered to be partially restrained beams, subjected to both distributed and axial loads. Thus, the condition of stress is similar to that commonly encountered in reinforced concrete frames under combined flexure, shear, and axial load. Any information developed about the load-carrying capacity of reinforced concrete box culverts can easily be extended to throw more light on the design of elements of a reinforced concrete frame.

2. Object and Scope of Investigation

The object of this research program is to establish by analyses and by studies of the available test data criteria for the structural design of reinforced concrete box culverts.

Stage 1 of this investigation involved the collection, correlation, and analysis of the existing data on the ultimate strength in shear of reinforced concrete beams. A report on this stage of the work was rendered in September, 1953.(

The scope of this report includes Stages 2, 3, and 4 as specified in Contract DA-33-017-eng-222 between the University of Illinois and the Ohio River Division Laboratories, Corps of Engineers, U. S. Army. These stages cover the following phases of the work:

Stage 2 -- Application of the results obtained in Stage 1 to the problem of shear in frames representative in whole or in part of those used in culverts. This will involve the consideration of the effects of axial load in the members and the effects of continuity in indeterminate structures.

* Numbers in parentheses refer to corresponding entries in Bibliography.
Stage 3 -- Collection, correlation, and analysis of the available data on the ultimate strength in flexure of reinforced concrete frames. This will involve consideration of the inelastic behavior of reinforced concrete members in flexure at or near ultimate load, the formation of "plastic hinges", and the consequent redistribution of moments.

Stage 4 -- An attempt to correlate the results of the studies in Stages 1-3 with the results of the culvert tests made by the Ohio River Division Laboratories, considering the behavior and strength of the test specimens with respect to both shear and flexure.

In this report, Stage 2 is covered primarily in Chapter III, Stage 3 in Chapter II, and Stage 4 in Chapter IV.

3. Acknowledgment

The studies reported herein were made as a part of a research program to establish by analysis and by studies of the available test data criteria for the structural design of reinforced concrete box culverts. This research program is conducted by the Structural Research Laboratory in the Engineering Experiment Station of the University of Illinois in cooperation with the Ohio River Division Laboratories, Corps of Engineers, U. S. Army, under Contract DA-22-017-eng-222. This Investigation was initiated in May 1953.

The program of the investigation is guided by Dr. N. M. Newmark, Research Professor of Structural Engineering. The immediate supervision of the program is provided by Dr. C. P. Siess, Research Associate Professor of Civil Engineering. Preliminary studies in connection with Chapter II were made by Dr. Sabri Sami.
4. Notation

The following notation is used in this report:

\( A \) = different symbols as defined in text, given by Eqs. 18 and 36

\( A_s \) = area of tension reinforcement of a section

\( A'_s \) = area of compression reinforcement of a section

\( B \) = given by Eq. 39

\( b \) = width of member

\( C \) = internal compressive force in concrete

\( C' \) = force in compression reinforcement

\( c \) = ratio between lateral and vertical loads on a culvert

\( D \) = total depth of member

\( d \) = distance from centroid of tension reinforcement to compression face of member

\( E_c \) = modulus of elasticity of concrete

\( E_o \) = see Fig. 1

\( E_s \) = modulus of elasticity of reinforcing steel

\( \varepsilon_c \) = strain in extreme compression fiber of concrete

\( \varepsilon_p \) = given by Eq. 14; see Fig. 4

\( \varepsilon_u \) = ultimate compressive strain in concrete, taken as 0.004

\( \varepsilon_o \) = steel strain at beginning of strain hardening, see Fig. 1

\( \varepsilon_s \) = strain in tension reinforcement

\( \varepsilon'_s \) = strain in compression reinforcement

\( \varepsilon_y \) = yield strain in tension reinforcement

\( f_c \) = compressive stress in extreme fiber of concrete, given by "straight-line" theory
Notation (Cont'd)

\( f'_c \) = compressive strength of standard 6 by 12-in. concrete test cylinders

\( f_o \) = see Fig. 1

\( f_s \) = stress in tension reinforcement

\( f'_s \) = stress in compression reinforcement

\( f_y \) = yield stress in tension reinforcement

\( f_{ult} \) = ultimate stress in reinforcement

\( k_2 \) = ratio of the average compressive stress in beam at failure to the strength of standard 6 by 12-in. test cylinders, given by Eq. 1

\( k_2 \) = fraction of the depth of compression zone which determines the position of the compressive force \( C \) in concrete, taken as 0.45

\( k_d \) = depth of compression zone of concrete as determined by "straight-line" theory

\( k_{d_u} \) = depth of compression zone of concrete at ultimate flexural capacity

\( k_{d_y} \) = depth of compression zone of concrete at first yielding of tension reinforcement

\( L \) = span of culvert, see Fig. 11

\( M \) = bending moment at a section

\( M_{na} \) = given by Eq. 21

\( M_s \) = resisting shear moment at beam-column connection subjected to no axial load, given by Eq. 31

\( M'_s \) = resisting shear moment at beam-column connection subjected to axial load, given by Eq. 41

\( M_u \) = resisting moment at a section at ultimate flexural capacity

\( M_y \) = resisting moment at a section at first yielding of tension reinforcement

\( n = \frac{E_s}{E_c} = \gamma \) = elastic modular ratio, taken as \( 5 + \frac{10,000}{f'_c} \)
Notation (Cont'd)

\( P \) = axial load at a section, applied at the mid-depth of the section

\( P_t \) = total vertical load on culvert

\( P_{1y} \) = total vertical load at formation of the first plastic hinge

\( P_{2y} \) = total vertical load at formation of the second plastic hinge

\( P_u \) = total vertical load at failure

\( p = \frac{A_s}{bd} \)

\( p' = \frac{A'_s}{bd} \)

\( t_d \) = distance between centroids of tension and compression reinforcement

\( V \) = shearing force at a section

\( w \) = uniform load on culvert

\( x \) = given by Eq. 45

\( z = \frac{D}{2L} \)
II. ANALYSIS OF FLEXURAL STRENGTH OF CULVERTS

The ultimate flexural strength of a reinforced concrete member depends both on the type of stress and the type of loading to which the member is subjected. With regard to the type of stress, the following combinations can be present: pure flexure; combined flexure and shear; combined flexure and axial load. The type of loading can be either static or dynamic loading. These types can be further subdivided by considering single short-time load application, sustained working loads, sustained high loads, and repeated loads.

In this investigation only the single short-time load application was considered. For this type of loading, the flexural strength of a reinforced concrete member in the region of pure flexure can be determined with a fair degree of accuracy by existing methods (1). In the case of combined flexure and shear, the flexural strength of the member remains unchanged provided that the proper amount of web reinforcement is used to prevent a premature shear failure (2). The addition of axial load, however, changes the flexural strength of a reinforced concrete member. The ultimate resisting moment of a member which would fail in tension under pure flexure first increases as the magnitude of the axial load increases; then, at a certain magnitude of the axial load, the mode of failure changes from tension to a balanced failure. Any further increase in the axial load beyond this point will lower the ultimate flexural moment and produce a compression type of failure. Likewise, the ultimate flexural moment of a member which would fail in compression under pure flexure will immediately decrease as any axial load is applied.
The elements of a culvert are in general stressed under combinations of flexure, shear, and axial load. Since culverts of the type considered in this report are statically indeterminate, their ultimate load-carrying capacity does not necessarily correspond to the load producing first yielding in the reinforcement. The applied load may still be increased beyond this stage and the ultimate load-carrying capacity will not be increased beyond this stage and the ultimate load-carrying capacity will not be reached until the applied moment equals the ultimate flexural resisting moment at some section of the culvert. In order to predict the ultimate load that a culvert can resist, it is necessary to determine the ultimate flexural resisting moments at the critical sections as well as the moments corresponding to first yielding of the reinforcement. The moments produced by the applied loads must of course also be determined.

The calculations for these various resisting and applied moments are described in the following sections of this chapter.

5. Assumptions of the Analysis

The following assumptions have been made by most of the recent investigators in developing expressions for the flexural capacity of reinforced concrete structural members. The validity of these assumptions has been established experimentally and they have led to expressions which have given satisfactory agreement with test results (1, 3, 4, 5):

(a) The strain distribution is linear throughout the depth of the member.

(b) The maximum flexural capacity is reached when concrete crushes at a limiting strain. The values of limiting strain as found by the
investigators have ranged from 0.0034 to 0.004; in this investigation it is assumed that the concrete crushes at a strain of 0.004.

(c) No tension is resisted by the concrete.

(d) The stress-strain relationship of the reinforcing steel is known. In the theory, the actual stress-strain curve of the steel is approximated by an idealized curve consisting of three straight lines as shown in Fig. 1.

(e) Perfect bond exists between the concrete and steel.

(f) In addition to the above assumptions, the properties of the concrete stress-block must be known. These properties have been determined experimentally for two limiting stress conditions, namely vertically cast concentrically loaded columns and horizontally cast beams under pure flexure (Figs. 2a and 2b). In the first case, the limiting concrete stress of 0.85 $f'_c$ is an experimentally determined average value for columns tested with flat ends. This value is assumed to include the effects of shape, size, and vertical casting position of the columns. It is not known whether it would be valid for horizontally cast members tested as columns.

For horizontally cast beams under pure flexure, the type of stress-block shown in Fig. 2b has been found to give reliable results. This stress-block does not require the determination of the actual stress distribution in the concrete. The internal compressive force is defined at the ultimate load by two parameters, $k_1$ and $k_2$, and its location by a third parameter $k_3$. Furthermore, the parameters $k_1$ and $k_2$ can be interpreted as one parameter $k_1 k_2$ since they never appear separately in the analysis. The parameter $k_1 k_2$ is the ratio of the average compressive stress in the beam at failure to the strength of standard 6 x 12-in. test cylinders. The
Numerical value of $k_1k_\bar{z}$ has been found to depend on the value of concrete strength; the following expression has been determined from test results (3, 1):

$$k_1k_\bar{z} = \frac{3000 + 0.5 f'_c}{1500 + f'_c}$$

(1)

This value of $k_1k_\bar{z}$ has given good correlation between measured and computed ultimate moments for beams tested under pure flexure. However, for $f'_c < 3000$ psi, Eq. 1 should be used with caution because of lack of sufficient experimental data for this range of concrete strengths.

For members under combined flexure and axial load, Hognestad has used the stress-block shown in Fig. 2c to develop an ultimate theory (4). This stress-block was determined experimentally for eccentrically-loaded vertically-cast columns. The maximum stress in flexure was chosen the same as that for concentrically-loaded flat-ended column specimens, $0.85f'_c$. Thus, this stress-block for combined flexure and axial load is compatible with one of the two limiting stress conditions, that of pure axial load. For the other limiting stress condition, that of pure flexure, however, Hognestad's stress-block does not yield results in agreement with tests. A comparison between Hognestad's and the $k_1k_\bar{z}$-type stress-blocks shows that the difference between the two increases as the steel percentage $\bar{z}$ increases, or as the concrete strength $f'_c$ decreases.

It is conceivable that the properties of the concrete stress-block are influenced by the type of stress to which the structural member is subjected. Furthermore, the casting position of the member might have some effect. In the case of reinforced concrete box culverts the critical members are the horizontally-cast members which are subjected primarily to flexure
with some axial load. For this reason the $k_1 k_2$-type of stress-block is used in the analysis rather than Hognestad's stress-block which is applicable to vertically-cast columns under primarily axial load with some flexure. Moreover, it is important to note that the properties of both of these stress-blocks have been determined from short-time static tests. In practice, creep under sustained loads might have a larger effect on the load-carrying capacity of a member than the differences resulting from using one of these stress-blocks in preference to the other.

6. **Resisting Moments at Ultimate Flexural Capacity**

An analysis of a reinforced concrete box culvert involves the determination of the bending moments $M$, shears $V$, and axial loads $P$ at different sections of the culvert due to the external loads on the structure. When it is desired to determine the flexural capacity of a section, the shearing force $V$ can be neglected and the effect of the external loads can be considered as a bending moment $M$ and axial load $P$ on the section. It is seen later that the culverts are analyzed with respect to their geometric center-lines. Consequently, the axial load $P$ must be considered as applied at the mid-depth of the section.

For any value of axial load there corresponds a certain moment at which the section fails in flexure. The ultimate flexural resisting moment can be determined by considering the static equilibrium of the section and the strain relations involved. Figure 3 shows the applied moment $M$ together with the corresponding axial load $P$ and the resulting distribution of strain and the magnitude and location of the internal forces at failure. Summation of forces parallel to the axis of the member gives:

$$ P = C + C' - T $$ (2)
Taking moments about the tension reinforcement gives

\[ M + P(d - D/2) = C_d (1 - k_u k_u') + C'td \]  

The relationship between moment and axial load is uniquely determined by the applied loads and the stiffness of the members for an elastic structure. The corresponding values of \( M \) and \( P \) at failure can be calculated from Eqs. 6 and 8 with the aid of the condition of compatibility of strains. In an indeterminate structure loaded into the inelastic range, however, \( M \) is a function of the nature and the degree of inelastic action and is not known in advance. This difficulty can be overcome by constructing an interaction diagram for a certain range of values of \( P \) and \( M \). Such an interaction diagram can easily be obtained by assuming arbitrary values of strain for the tension steel and calculating the corresponding values of moment and axial load.

Since the concrete strain in the extreme fiber was assumed to be 0.004 at the flexural ultimate load, any chosen steel strain determines the corresponding location of the neutral axis of the section:

\[ k_u = \frac{\epsilon_u}{\epsilon_u + \epsilon_s} = \frac{0.004}{0.004 + \epsilon_s} \]
The corresponding strain in the compression steel is derived from Fig. 3 as follows:

\[ \frac{\epsilon_u + \epsilon_s}{d} = \frac{\epsilon'_s + \epsilon_s}{td} \]  

from which

\[ \epsilon'_s = t\epsilon_u - \epsilon_s (1 - t) = 0.004t - \epsilon_s (1 - t) \]  

Steel stresses for given values of strain are determined from the following expressions, derived from the idealized stress-strain diagram of Fig. 1.

\[ \begin{align*}
\epsilon_s &< \epsilon_y & f_s &= E_s \epsilon_s \\
\epsilon_y &< \epsilon_s < \epsilon_0 & f_s &= f_y \\
\epsilon_0 &< \epsilon_s & f_s &= f_o + E_o \epsilon_s
\end{align*} \]

Calculating \( k_u \), \( f'_s \), and \( f^' \) and using \( k_2 \) equal to 0.45, the values of \( M \) and \( P \) which correspond to the assumed value of \( \epsilon_s \) can be determined from Eqs. 6 and 8. Suitable selection of \( \epsilon_s \)-values permits the construction of any desired range of the interaction diagrams.

Figure 6 shows such interaction diagrams for typical rectangular sections reinforced in tension only. On this figure, loads are in kips, moments are in inch-kips, and dimensions are in inches. These curves are drawn for \( f'_c \) equal to 3000 and 5000 psi and \( p \) equal to 0.01, 0.02, and 0.05. The reinforcing steel was assumed to have the following properties: \( f_y = 45,000 \) psi, \( E_s = 30,000,000 \) psi, and no strain-hardening region. The curves are shown up to the point of balanced failure, that is crushing of concrete is simultaneous with yielding of tension reinforcement. For higher axial loads the corresponding flexural capacity decreases rapidly.
It is seen in Fig. 6 that axial load increases the ultimate flexural moment of the under-reinforced sections; that is, those having $p = 0.01$ or $0.02$ in this figure. Furthermore, the more under-reinforced a section is, the higher is the potential increase in its flexural capacity with increasing axial load. The term under-reinforced refers to a section which would fail by yielding of tension reinforcement before crushing of concrete if subjected to pure flexure. It is seen further that the rate of increase in moment is a maximum for low values of axial load and that it levels off at about one half of the axial load corresponding to the point of balanced failure. Further increase in the axial load results in a rapid decrease in the flexural capacity of the section. Likewise, the flexural capacity of an over-reinforced section, such as the one having $p = 0.05$ in Fig. 6, decreases as soon as any axial load is applied. However, sections having these characteristics are seldom encountered in culverts.

7. **Resisting Moments at First Yielding**

It was shown in the previous section that expressions for the resisting moments at flexural ultimate could be written with the aid of a concrete stress-block of which only the magnitude and location of the internal compressive force was known. In order to establish interaction diagrams at first yielding, however, the actual stress-strain relationship for the concrete in the beam must be known. Figure 4 shows the type of stress-strain diagram assumed in this analysis. The diagram consists of a parabolic and a constant stress portion. The parabolic distribution of stress is expressed by the following equation:
The constant maximum stress $k_f f_c'$ was selected so as to make the total area under the stress-strain curve equal to $k_f f_c'$, the average concrete strength in flexure as given by Eq. 1. Thus the following expression was obtained:

$$k_f = A - \sqrt{A^2 - 2\Delta k_1 k_3}$$

(17)

where

$$A = \frac{0.75 E_c \epsilon_u}{f_c'}$$

(18)

$$k_1 k_3 = \frac{3000 + 0.5 f_c'}{1500 + f_c'}$$

(1)

The actual shape of the stress-strain diagram in concrete has now been assumed and the relationship between moment and axial load at first yielding of tension reinforcement can be computed. This can be done by considering a section loaded as shown in Fig. 5. Summation of forces parallel to the axis of member given:

$$P = C + C' - T$$

(2)
The moment of the compressive stress-block about the neutral axis is:

\[ M_{na} = (k_d d)^2 b y f' c \left[ \frac{2}{3} \left( \frac{e_c}{\epsilon_p} \right) - \frac{1}{4} \left( \frac{e_c}{\epsilon_p} \right)^2 \right] \]  

(21)

The magnitude of the resisting moment can be determined by taking moments with respect to the tension reinforcement of the section:

\[ M = M_{na} + C d (1 - k_y) + C'td - P(d - D/2) \]  

(22)

The procedure for the construction of an interaction diagram at first yielding is the same as that at flexural ultimate. Since in the present case the yield strain of reinforcing steel is known, suitable values of concrete strain in the top fiber of the section are assumed. This permits the determination of \( k_y \) and \( \epsilon'_s \):

\[ k_y = \frac{\epsilon_c}{\epsilon_c + \epsilon_y} \]  

(23)

and

\[ \epsilon'_s = t \epsilon_c - \epsilon_y (1 - t) \]  

(24)

The magnitudes of the axial load \( P \) and the moment \( M \) which correspond to the assumed value of \( e_c \) are then calculated from Eqs. 2 and 22.

Whenever the assumed strain \( e_c \) is larger than \( \epsilon_p \), the above equations are not valid. A new set of equations can easily be set up by considering
the conditions of static equilibrium of the section. However, in most cases this is not necessary since concrete strains smaller than $\varepsilon_p$ produce a sufficiently large range of $P$ and $M$ for the interaction diagram.

8. **Moments Produced by Applied Loads**

The analysis of reinforced concrete box culverts with regard to moments and axial loads produced by applied loads is divided into two phases. The first phase considers the structure elastic, before yielding of any section. The second phase considers the inelastic behavior of the structure; that is, the formation of plastic hinges, subsequent redistribution of moments, and ultimate collapse.

(a) **Elastic Analysis**

Elastic analysis of the culverts is performed with respect to the geometric center-lines of the structure. The dimensions of the structure and the type of loading considered are shown in Fig. 7. It is assumed that the members are infinitely stiff from the point of intersection of their center-lines to faces of the columns, a distance $D/2 = z_1$. It is further assumed that for the type of structure under consideration the horizontal and vertical members have the same uniform stiffness along their clear spans.

Fixed end moments are calculated at the face of the columns:

- Hor. Member \[ 1/12 \ v L^2 \ (1 - 2z)^2 \]
- Vert. Member \[ 1/12 \ c w L^2 \ (1 - 2z)^2 \]

Fixed end moments at the intersection of the center-lines are obtained by adding to the above moments at the column face the moments produced by the shear at the column face section and by the applied load $w$ between the two sections. This gives the following moments:
Hor. Member \[ \frac{1}{12} wL^2 (1 + 2z - 2z^2) \]
Vert. Member \[ \frac{1}{12} cwl^2 (1 + 2z - 2z^2) \]

Taking into account also the fixed end moments resulting from the applied loads outside the center-lines of the members, the unbalanced fixed end moment is distributed according to the stiffness and carry-over factors as given by the lengths and stiffnesses of the members. For square box culverts considered in the present analysis, the stiffness factors are equal for both the horizontal and vertical members and the carry-over moments balance themselves because of symmetry of the structure. Consequently, the unbalanced fixed end moment is divided equally between the horizontal and vertical members and the following moments are obtained as final moments at the midspan and column face sections:

Moment at Column Face:
Hor. Member \[ \frac{1}{24} wL^2 \left[ (1 - 10z + 16z^2) + c(1 + 2z - 8z^2) \right] \] (25a)
Vert. Member \[ \frac{1}{24} cwl^2 \left[ c(1 - 10z + 16z^2) + (1 + 2z - 8z^2) \right] \] (25b)

Moment at Midspan:
Hor. Member \[ \frac{1}{24} wL^2 \left[ c(1 - z - 2z^2) - c(1 + 2z - 8z^2) \right] \] (25c)
Vert. Member \[ \frac{1}{24} cwl^2 \left[ 2c(1 - z - 2z^2) - (1 + 2z - 8z^2) \right] \] (25d)

The sum of midspan and column face moments on a member must equal the total static moment for the clear span. This moment is determined solely by the external loading. Equations 25 indicate that the division of the total static moment between midspan and column face sections is a function of the ratio \( c \) of lateral to vertical loading, and the ratio \( z = D/2L \). Furthermore, since the value of \( z \) generally remains rather small, it is seen that the magnitude of the elastic moments depends primarily on the loading ratio \( c \).
If the total vertical load $P_t$ is considered as the basis of comparison, the following substitution can be made in the above equation:

\[ wL (1 + 2z) = P_t \]  
\[ wL = \frac{P_t}{1 + 2z} \]

(26a)  
(26b)

Since both the resisting moments and the theoretical elastic moments due to the applied loads have been determined, the loading history of a structure can be traced. Experience has indicated that the horizontal members are the critical members for a reinforced concrete box culvert. Thus, Fig. 8 shows the elastic moments plotted against the total vertical load $P_t$ both at midspan and at the column face section of a horizontal member.* The resisting moments, both at first yielding and at flexural ultimate are also shown for the two sections. This has been done by relating the computed axial loads from an interaction diagram to the total vertical load $P_t$ as follows:

\[ \text{Hor. Member} \quad P = cP_t/2; \quad P_t = 2P/c \]  
\[ \text{Vert. Member} \quad P = P_t/2; \quad P_t = 2P \]

(27a)  
(27b)

Depending on the resisting moments of the two sections and on the relative magnitudes of the elastic moments, yielding can occur either simultaneously at both sections or first at one of the sections. In Fig. 8 it is assumed that the midspan section yielded first at a load $P_{ly}$.

Since the elastic analysis cannot be used beyond the first yielding at any section, the subsequent loading history, the redistribution of moments,

* This representation is based on that used previously by Glanville and Thomas, Ref. (8).
and the final collapse load are determined by the inelastic properties of the structure. This phase of loading is covered in the following subsection.

(b) Inelastic Analysis

Yielding of the tension reinforcement at some section introduces a so-called plastic hinge at that section. Since the reinforcing steel is yielding, the section can rotate without appreciable increase in its resisting moment. Only when the steel strain reaches the strain hardening region, will the steel stress again increase and thus permit a further increase in the resisting moment of the section.

In Fig. 8 it was assumed that the elastic moment at midspan reached the curve of resisting moment at first yielding at point a. Thus, the first plastic hinge is introduced at the midspan section at load $P_{ly}$ while the column face section remains elastic at point b. It is assumed in this analysis that with further increase in the external loads the resisting moment at the section of the first plastic hinge increases along the first yielding curve. However, the total static moment on the member must remain proportional to the applied vertical load. Since this moment equals the sum of the elastic moments at the two sections, the difference between the elastic moment and the resisting yield moment at midspan must be redistributed to the still elastic column face section. This can be accomplished by a simple graphical construction. As seen in Fig. 8, the distances $x$ must be equal for any vertical line to the right of load $P_{ly}$.

Redistribution of moments produces a marked increase in the applied moment at the column face, and finally, the curve of first yielding is reached at point c. This introduces the second plastic hinge in
the structure at a load $P_{2y}$. Both sections of the member have now yielded and the midspan moment has reached point $d$.

The relationship between moments and load after the formation of the second plastic hinge depends on the distribution of angle changes in the structure and on the moment-angle change characteristics of the sections. If the plastic hinges were true hinges, all angle changes would be concentrated at the locations of the hinges. Allowing for some rotation of the joints, the concentrated angle change at midspan would be more than twice as large as that at the column face. Furthermore, if steel strains were assumed to be in the strain hardening region, any increase in the moment would be proportional to the angle change. Thus, the increase in the total static moment would be divided in the ratio one to two or more between the column face and midspan sections. However, the actual distribution of angle changes at a plastic hinge is different from that at a true hinge. Figure 18 shows the moment diagram produced by uniform load on the horizontal member of a particular culvert. From this moment diagram the actual distribution of angle changes was computed for the member with the aid of moment-angle change relationships as determined for the particular sections. It is seen that at the column face yielding can occur over a rather limited length of the member. At midspan, the change in moment is much more gradual and both yielding and rotation is spread out over a much longer portion of the beam. Although the ratio between the maximum values of angle change at these two sections depends on the section properties, it remains in the neighborhood of one rather than two or more as indicated for true hinges. For this reason, it is
assumed in this analysis that any increase in the total static moment after the formation of two plastic hinges is divided equally between the midspan and column face sections.

Figure 8 shows a graphical method for dividing the increase in the total static moment equally between the midspan and column face sections. An arbitrary vertical line is drawn at distance $h$ from the origin. The distance $2y$ between the elastic moments on this vertical line gives the corresponding total static moment. Another vertical line is traced the same distance $h$ from load $P_2y$ and distances $y$ on this line measured from horizontal lines through points $d$ and $c$ determine the distribution of the applied moment between the two sections.

The ultimate collapse of the structure will occur whenever an applied moment line intersects the corresponding curve of the resisting moment at flexural ultimate. For the structure shown in Fig. 8, this occurs at point $e$ at midspan. The corresponding load $P_u$ is the maximum load the structure can resist although the applied moment at the column face is below its flexural ultimate at point $f$.

The above assumed relationship between the applied moments and the total vertical load is an approximation. It is believed, however, that it is sufficiently close to the true behavior of the structure so that it can be used without significant error in predicting the ultimate flexural capacity of a reinforced concrete box culvert. However, in order that a structure can develop its ultimate flexural capacity, care must be taken that the plastic hinges can develop their ultimate resistance without premature failure either in shear or in bond.
III. ANALYSIS OF SHEAR STRENGTH OF CULVERTS

9. Shear Moment for Combined Flexure and Shear

In Stage (1) of this research program equations were derived for the shear moment of reinforced concrete beams under combined flexure and shear (2). The following equation was obtained for beams without web reinforcement:

\[ M_s = bd^2 f_c' (k + np') F(f_c') \]  

(28)

For laboratory test specimens loaded through steel bearing plates the function \( F(f_c') \) was found to be

\[ F(f_c') = 0.57 - \frac{4.5 f_c'}{10^5} \]  

(29)

Beams loaded through integrally cast column stubs at midspan were found to have somewhat higher shear strength than that indicated by Eq. 29. For such beams the following value of \( F(f_c') \) was evaluated in a previous technical report (6):

\[ F(f_c') = 0.73 - \frac{7.3 f_c'}{10^5} \]  

(30)

Equation 29 was determined for beams with concrete strengths less than 6000 psi. No test data were available for higher values of \( f_c' \). For a given steel percentage \( p \), the quantity \( k f_c' F(f_c') \) reaches its maximum at about \( f_c' = 6000 \) psi. Therefore, this value of concrete strength should be used as a limit in the use of Eq. 29. Beams with higher values of \( f_c' \) should be treated as having \( f_c' = 6000 \) psi until sufficient test
data are available for this range of concrete strength. Likewise, the limit of Eq. 30 is \( f'_c = 5000 \) psi. For higher values of concrete strength, \( f'_c = 5000 \) psi should be used in the calculations.

Since a beam with a column stub simulates a beam-column connection in a framed structure, Eq. 30 is used in the present analysis. Furthermore, the contribution of compression reinforcement to the shear strength of a member is usually rather small. If this effect is neglected, the shear moment equation can be written as follows:

\[
M_s = bd^2 f'_c k (0.73 - \frac{7.3 f'_c}{10^5})
\]  

(31)

10. Shear Moment for Combined Shear, Flexure, and Axial Load

The shear strength of a member under combined shear, flexure, and axial load has received very little attention in the past. Practically no tests on such members have been reported in the literature. As a consequence, the effect of an axial load to the shear strength of a member must be determined analytically. In this section an attempt is made to extend the expressions previously derived for members under combined shear and flexure to include the effect of the axial load. As before, the shear strength of a section is related to an ultimate moment its compression zone can resist before failure. The effect of the axial is considered both in determining the elastic "straight-line" \( k \) and in taking moments about the tension reinforcement to determine the load-carrying capacity of the compression zone.

Figure 9 shows the applied and the internal forces at a section for a straight-line analysis for its shear strength. Summation of forces parallel to the axis of the member gives
\[ T + P = C \]  \hspace{1cm} (32)

where

\[ T = pbdn \left( \frac{1-k}{k} \right) f_c \]  \hspace{1cm} (33)

\[ C = \frac{1}{2} bkdf_c \]  \hspace{1cm} (34)

Moments taken about tension reinforcement:

\[ M + P(d - D/2) = Cd(1 - k/3) \]  \hspace{1cm} (35)

Substituting:

\[ M/P = A \]  \hspace{1cm} (36)

From Eqs. 35 and 36:

\[ P = \frac{Cd(1 - k/3)}{A + d - D/2} \]  \hspace{1cm} (37)

From Eqs. 32 and 37:

\[ T = C \left[ 1 - \frac{d(1 - k/3)}{A + d - D/2} \right] \]  \hspace{1cm} (38)

Using:

\[ 1 - \frac{d(1 - k/3)}{A + d - D/2} = B \]  \hspace{1cm} (39)

and substituting \( C \) and \( T \) from Eqs. 33 and 34, the value of \( k \) is determined from Eq. 38:

\[ k = \sqrt{\left( \frac{pn}{B} \right)^2 + \frac{2pn}{B} - \frac{pn}{B}} \]  \hspace{1cm} (40)

where

\[ B = 1 - \frac{d(1 - k/3)}{A + d - D/2} \]  \hspace{1cm} (39)

\[ A = M/P \]
It is seen that the value of $k$ must be first assumed in Eq. 39 and then calculated from Eq. 40. The correct value of $k$ is obtained when the assumed and calculated values are the same.

After the value of elastic $k$ is known, the shear moment for combined shear, flexure, and axial load can be written by taking moments about the tension reinforcement. The ultimate strength of the section will be reached when the sum of the applied moment and the moment produced by axial load equals the resisting moment of the section, given by Eq. 31. This condition is represented by

$$M' + P(d - D/2) = bd^2f'c_0 (0.73 - \frac{7.3 f'_c}{10^5})$$

where $M'_S$ designates the shear failure moment corresponding to a given value of $P$. It is noticed that for $P = 0$, Eq. 41 for combined shear, flexure, and axial load reduces to Eq. 31 for combined shear and flexure only.

For the purposes of plotting an interaction diagram for a combination of axial load and shear moment, the following procedure can be used:

1. Select an arbitrary value of $B$
2. Compute the corresponding value of $k$ from Eq. 40
3. Compute the corresponding value of $A$ from Eq. 39 which can be rewritten as:

$$A = d\left(\frac{1 - k/3}{1 - B}\right) - d + D/2$$

4. Compute the corresponding values of $M'_S$ and $P$ from Eq. 41 which can be rewritten as:
\[ P(A + d - D/2) = bd^2 f'c'(0.73 - \frac{7.3 f'}{105}) \]  

(41a)

where

\[ M'_s = AP \]  

(36)

When suitable values of \( B \) are selected, an interaction diagram is easily obtained for any desired range of axial load and shear moment, \( M'_s \).

The applied moments on a structure can be compared with the shear moments at critical sections for shear failure. Whenever the applied moment exceeds the resisting shear moment at some critical section, the member fails in shear. Studies in connection with Stage 1 of this investigation showed that the following sections may be critical for shear failure:

(a) Sections where maximum shear and maximum moment coincide, provided that the quantity \( M/Vd \) is in the range of shear-compression failures. See Fig. 23 and Sections 18 and 20 in Ref. (2).

(b) Sections in the region of maximum moment and minimum shear where the quantity \( M/Vd \) reaches a critical value. See Section 21 in Ref. (2).

In the case of culverts under uniform loading there are two possible critical sections for shear failure. Maximum shear and maximum moment occur together at the column face section, case (a). The value of \( M/Vd \) for incipient failure at that section must be investigated with respect to the range of shear-compression. It is possible that for certain combinations of loading the value of \( M/Vd \) is outside the range of shear-compression failures so that the shear strength at that section is
larger than that given by Eq. 41. However, in the report on Stage 1 (Ref. 2) it was not possible to set limits for the range of shear-compression failures in terms of values of M/Vd; these limits must be determined with the aid of experimental data, which at present are not available.

The second critical section, case (b), is in the midspan region at a point in the span where the ratio M/Vd reaches a critical value. This critical M/Vd could not be determined definitely because of lack of sufficient experimental data. It was set tentatively at about 4.5 in Reference (2) and this value is used also in the present analysis. The location of this section can be found as follows:

Moment at distance x from midspan with respect to tension reinforcement:

\[ M = M_{\text{max}} + P(d - D/2) - \frac{1}{2} wx^2 \]  \hspace{1cm} (42)

where \( M_{\text{max}} \) = moment at midspan
\( P \) = axial load in member

Shear at distance x from midspan:

\[ V = wx \]  \hspace{1cm} (43)

Then

\[ \frac{M}{Vd} = \frac{M_{\text{max}} + P(d - D/2) - \frac{1}{2} wx^2}{xwd} = 4.5 \]  \hspace{1cm} (44)

and

\[ x = \sqrt{(4.5d)^2 + \frac{2}{w} \left[ M_{\text{max}} - P(d - D/2) \right]} - 4.5d \]  \hspace{1cm} (45)

The magnitude of the applied moment at section x is given by:

\[ M = M_{\text{max}} - \frac{1}{2} wx^2 \]  \hspace{1cm} (46)
Shear moments and flexural resisting moments together determine the mode of failure and the ultimate load-carrying capacity of a structure. These resisting moments can be calculated for any given culvert and plotted as interaction diagrams for moment and axial load. Figure 10 shows such a graph for a particular case. Resisting moments are plotted against the total vertical load both at midspan and at column face. $M_u$ refers to the ultimate flexural moment, $M_y$ to the flexural moment at first yielding, and $M'_S$ to the shear moment. The shear moment for "midspan" applies over the entire positive moment region; this requires the assumption that the amount of tension reinforcement remains constant in that region. The graph in Fig. 10 is similar to that shown in Fig. 8 except that shear moments are given in addition to the flexural resisting moments.

The applied moments, determined as in the case of Fig. 8 are shown for the column face and the midspan sections. In addition, the elastic moment is given at a section in the midspan region where $M/V_d = 4.5$, determined from Eq. 45. The points of intersection between the elastic moments and the resisting moments determine the behavior of the structure under increasing applied loads. In the present case, the first resisting moment intersected is $M_y$ at the point $a$ for the midspan moment. A plastic hinge is thus produced at that section at load $P_{ly}$. It is seen that the applied moment at the critical section for shear failure in the midspan region is below the corresponding shear moment, at point $e$. Likewise, the applied moment at the column face, point $b$, is smaller than both $M_y$ and $M'_S$ at that section. After the first plastic hinge has formed, the midspan moment increases along $M_y$ and the additional increase in the total static moment is redistributed to the column face section. The applied
moment at that section intersects first the shear moment curve at point \( g \). Consequently, the structure fails in shear at load \( P_u \).

Depending on the relative positions of the resisting moment curves, different modes of failure are possible. The structure may fail in shear before any plastic hinges have developed, or after developing one plastic hinge as shown in Fig. 10. When the flexural resisting moments are critical, the structure will fail in flexure either at the column face or at midspan as shown in Fig. 8.
IV. CORRELATION BETWEEN THEORY AND CULVERT TESTS

11. Description of Culverts and Test Results

The Ohio River Division Laboratories, Corps of Engineers, U. S. Army, have reported tests on four reinforced concrete box culverts (7). All culverts had the dimensions shown in Fig. 11. The main test variable was the ratio between lateral and vertical loading, c. In addition, the effective depth $d$ of Culvert No. 2 was somewhat different from that of the other culverts and the concrete strength $f'_c$ for Culvert No. 7 was higher than that for other culverts.

All culverts had the same amount of reinforcement except that the inner vertical bars were omitted in Culvert No. 2. Hi-bond, 3/8-in. reinforcing steel bars were used in all tests. The arrangement of reinforcement is shown in Fig. 11. The following physical properties of the reinforcement were reported:

\[ f_y = 44 \text{ ksi} \]
\[ f_{ult} = 73.1 \text{ ksi} \]
\[ E_s = 28,500 \text{ ksi} \]

The stress-strain relationship in the strain-hardening region of steel was not reported; the following properties have been assumed on the basis of tests performed on approximately similar bars in the Structural Research Laboratory of the University of Illinois:

\[ f_o = 31.5 \text{ ksi} \]
\[ E_o = 886 \text{ ksi} \]

These quantities are defined in Eq. 1.
Strains were measured with SR-4 electric strain gages both on the reinforcing steel and on the concrete. The locations of the gages on the reinforcement are shown in Fig. 11. Strain gages were placed on the two intermediate bars at each gage location. It should be noted that Gage Locations 1 and 3 were at different distances from the column face in different culverts. To place the gages on the reinforcing bars, 1 1/2 by 1/2-in. slots were formed along the entire 12-in. width of Culverts No. 2 and 5. It is thus seen that Culvert No. 2 had a continuous 1/2-in. deep slot adjacent to the column face in the compression zone of the top horizontal member. This reduction in the compression area of the concrete at that section must be taken into account by using $d = 4.25\text{ in.}$ in the analysis. In Culverts No. 6 and 7, 1 1/4 by 1 1/2 by 1/2-in. depressions were formed at each strain gage, thus exposing only two of the five bars at each gage location.

The culverts were loaded through coil spring and bearing plate assemblies as shown in Fig. 12. The total vertical load was applied by a universal testing machine; the lateral load by compressing the lateral springs. Although this loading arrangement was intended to simulate uniform loading, deflection of culvert members under load resulted in some non-uniform distribution of spring loads at the higher loads.

Dial gages were used to measure deflections of all springs on the top horizontal member and on the side of the culverts. However, the gages were removed prior to reaching the anticipated failure load and deflection readings were not available up to the maximum load.

During application of test loads the lateral load was applied first followed by the corresponding vertical load. Lateral loads were
applied by simultaneously turning the nuts of each pair of horizontal rods, commencing with the top pair. The vertical load was applied at the rate of 15,000 lb per minute and generally in 10,000-lb increments.

The physical properties of the midspan and column face sections of the horizontal members, and the test results are summarized in Table 1. Table 2 shows the loads at which yielding was observed at Gage Locations 1 and 2. The reported loads are the average values for two gages at Gage Location 1 and four gages at Gage Location 2.

12. **Flexural Resisting Moments**

The interaction diagrams for the flexural resisting moments and axial loads were calculated by the method outlined in Chapter II. As a typical example, Table 3 shows the computations for such a diagram at first yielding of the column face section of Culvert No. 6. Since at first yielding the yield strain of the tension reinforcement remains constant, arbitrary values of concrete strain are selected for the extreme compression fiber of the member. Moments and axial loads with respect to the geometric center of the section are then calculated from the corresponding stresses for each assumed strain distribution.

Table 4 shows the same method used to calculate the interaction diagram for the ultimate flexural moment and axial load at the column face section of Culvert No. 6. The ultimate compressive strain of the concrete remains constant and arbitrary values of steel strain are selected. In both cases a certain strain distribution results in no axial load. This corresponds to the loading condition under pure flexure.
The flexural resisting moments with corresponding axial loads are tabulated in Table 5 for all culverts. Since Culvert No. 2 was tested with no lateral load, the resisting moments were calculated for the case of pure flexure only. It was chosen to interpret the tests on the basis of total vertical load, thus the axial loads in Table 5 are converted into equivalent vertical loads with the aid of Eq. 27a.

The resisting flexural moments are shown graphically for all culverts in Figs. 13 through 16. The moments are plotted against the total vertical load $P_t$.

13. 

Shear Moments

The interaction diagrams for the shear moment and axial load were calculated by the method outlined in Chapter III. Table 6 shows the calculations for Culverts No. 5, 6, and 7. These culverts have the same reinforcement and two of them have the same concrete strength, $f'_c = 5000$ psi. The concrete strength for Culvert No. 7 was 6900 psi and as discussed in Section 9, any concrete strength greater than 5000 psi should be treated as $f'_c = 5000$ psi. Consequently, the same interaction diagram applies in all three cases.

Arbitrary values of $B$ between one and zero were selected and the corresponding values of $P$ and $M_s$ were calculated. Finally, the values of axial load $P$ were converted into equivalent values of the total vertical load $P_t$ with the aid of Eq. 27a. As the load ratio $c$ is different for different culverts, the interaction diagrams are not the same for all culverts in terms of the total vertical load although they are the same in terms of the actual axial load $P$. 
Since the effect of compression reinforcement was neglected in the above analysis, the column face and midspan sections have identical properties and the same shear moments.

The shear moments for Culvert No. 2 which was tested with no lateral load, were calculated separately. The following moments were obtained from Eq. 31:

Midspan Section:
\[ d = 4.75 \text{ in.}; \quad M'_s = 151.2 \text{ in.-kips} \]

Column Face Section:
\[ d = 4.25 \text{ in.}; \quad M'_s = 126.6 \text{ in.-kips} \]

Figures 13 through 16 show the shear moments plotted against \( P_t \) together with the flexural resisting moments of the culverts. The shear moments are shown as dashed lines.

14. **Applied Moments**

The elastic applied moments are calculated with the aid of Eqs. 25 for all culverts and are tabulated in Table 7. The moments are expressed in terms of the total vertical load \( P_t \). It is noticed that the sum of midspan and column face moments on either member is a constant and equal to the total static moment between the sections.

The elastic moments are shown as dashed lines in Figs. 13 through 16 for the horizontal members of the culverts. These moments together with flexural and shear resisting moments provide the information necessary to analyze structures. A complete loading history of each individual culvert is described in the following sections. Attention is directed to both the elastic and inelastic behavior, the mode of failure, and the ultimate load of the culverts.
15. **Culvert No. 2**

Figure 13 shows the complete loading history of Culvert No. 2. Since this culvert was tested with no lateral load, all resisting moments remain constant with increasing vertical load. The theoretical elastic moments are plotted against $P_t$ and are shown as dashed lines.

It is seen in Fig. 13 that the first resisting moment intersected by the elastic moments is the midspan moment at first yielding. Consequently, this point should determine the load producing the first plastic hinge, $P_t = 45$ kips. From measured strains as recorded in Table 2, however, it is seen that the midspan section yielded at 60 kips. This discrepancy is caused by differences between the assumed and the actual stiffnesses of the structure. In the analysis, it was assumed that the horizontal and vertical members have the same uniform stiffness along their clear spans. This assumption is valid only for an uncracked, essentially elastic structure. After the structure cracks, the stiffness of the members is changed and the changes are not necessarily the same at all sections. In the present case, all sections have the same amount of tension reinforcement while the theoretical moments are widely different. For the horizontal member the theoretical moment at midspan is more than six times larger than that at column face. This means that different sections crack at different loads and to a different extent. The column face section remains stiffer than the midspan section and attracts more moment than indicated by the analysis. Thus, moments are partially redistributed even before any section has yielded. Furthermore, this redistribution of moments can take place either from section to section of the same member or from one member to another. The whole mechanism
of moment redistribution before the formation of plastic hinges in a structure is extremely complicated and defies theoretical analysis. However, it is recalled that Culvert No. 2 was not reinforced according to the theoretical moments. When the amount of reinforcement at a section is in a more realistic proportion to its theoretical moment, unequal changes in stiffness are not as likely to occur and the magnitude of the actual moment is in better correlation with the theoretical moment.

Since the actual load at the formation of the first plastic hinge could be obtained from the measured strains, this load is taken as the starting point in the inelastic analysis of the culvert. The elastic moment lines are rotated in such a way that the midspan moment intersects the first yielding line at 60 kips and that the sum of midspan and column face moments remains the same as before. This condition is shown with solid lines in Fig. 13.

After the first plastic hinge has formed at the midspan section, all increase in the total static moment is redistributed to the column face section. This was evident from the tests in which the load-strain curves for Culvert No. 2 showed a marked increase of strain at the column face section at about 60 kips. The increased applied moment at the column face is seen to intersect the yield resisting moment first, and the second plastic hinge is formed at a load of 72 kips. Since Gage Location 2 was 2.25 in. from the column face, this load cannot be checked directly from the measured strains. However, an approximate calculation based on the magnitude of yield moments and on the strains measured at Gage Location 2 shows that the section must have yielded at about the load determined from Fig. 13.
The moment redistribution after the formation of the first plastic hinge can be determined also on the basis of the theoretical elastic moments. This is shown as the dashed lines in Fig. 13. Although the load for the first plastic hinge as predicted by theory is considerably smaller than the actual load, the second plastic hinge forms at the same load as that given by the actual behavior of the structure. Thus, the stiffness of a structure affects only its elastic moments and the load at which the first plastic hinge forms. The inelastic behavior of the structure and its ultimate load are determined by the resisting moments and are not influenced by stiffness. This is easily seen in Fig. 13 where the sum of yield moments at midspan and at the column face is a constant. When the second plastic hinge forms, the total static moment between the two sections must equal the sum of their yield moments. This determines a particular value for the load at the second plastic hinge, and it is immaterial how the elastic moments were divided between the sections and at what load the first plastic hinge formed.

After two plastic hinges are formed, the increase in the total static moment is divided equally between the midspan and column face sections as discussed in Section 8-b. Figure 13 shows that the resisting moment at flexural ultimate is reached first at midspan at a load of 98 kips. The measured ultimate load, was 110.73 kips, however. This difference is caused by the fact that the calculated total static moment was based on uniformly distributed load while the actual load distribution becomes exceedingly non-uniform as the structure deflects. It is recalled that only the total applied load was measured. This total load was distributed to the culverts through a spring assembly.
At small deflections of the structure the distribution of spring loads was practically uniform. When the structure yields, however, deflections increase rapidly and more and more of the total load is carried by the springs near the ends of the span. The true distribution of spring loads must be known in order to calculate the actual static moment on the member. This can be determined by calculating the deflected shape of the member at the ultimate load and computing the true forces in each spring with the aid of known spring constants.

The relationship between moment and angle change must be calculated first to determine the deflection of a member. Angle change is related to strains in the extreme fibers of a section in the following manner:

\[ \theta = \frac{e_c + e_s}{d} \]  

(47)

For the ultimate and yield moments, the magnitudes of the strains were obtained in previous calculations for the flexural resisting moments. For intermediate points a cut and try procedure was used. The steel strains at the beginning of strain hardening and at other arbitrary values were taken as known quantities. The corresponding values of concrete strain were determined from the condition of static equilibrium of the internal forces on a section. For Culvert No. 2 with no axial load the sum of the internal forces in the compression zone must equal the tension force in the reinforcement since the section was subjected to pure flexure. As an example, Fig. 17 shows such a moment-angle change relationship for the midspan section of Culvert No. 2. For a member subjected to combined flexure and axial load the condition of static equilibrium is more involved.
For any assumed value of $\epsilon_s$ the corresponding value of $\epsilon_c$ must be so selected that the sum of the internal forces equals a given value of axial load. Thus, the moment-angle change relationship depends on the axial load on the member. Since in the present analysis it is necessary to determine the deflections at the ultimate load of the culvert, the moment-angle change relationship must be determined for the axial load which corresponds to the measured ultimate load.

After the moment-angle change relationship was determined for both the midspan and column face sections, the moment diagram at the ultimate load was plotted for the horizontal member. In order to do this, the uncorrected moments at the measured ultimate load were reduced by such a percentage as to make the midspan moment equal to the ultimate resisting moment at that section. Figure 18 shows such a moment diagram for Culvert No. 2. The corresponding distribution of angle change along the member can now be obtained with the aid of moment-angle change diagrams. This is shown in the same figure.

The deflection diagram of the member can now be calculated by treating the angle change diagram as applied loading on a simple-span beam and computing the corresponding bending moment diagram. In so doing, it was noticed that very nearly the same result was obtained with the greatly simplified angle change diagram shown by dashed lines in Fig. 18. This approximate diagram is determined by the ultimate angle change at midspan and by the location of yield moments on the member. The distance $2a$ between midspan and the location of yield moment is divided into two equal parts. In the center half it is considered that angle change remains constant and equal to $\phi_u$; in the outer halves, that it decreases
linearily to zero. No distributed angle change is considered outside this midspan region. With this approximate diagram it is assumed further that the deflected member remains straight from the intersection of center lines to the inner load point at midspan as shown in Fig. 19.

After the deflection diagram is determined, the actual distribution of spring loads at the ultimate load can be calculated with the aid of known spring constants. The corresponding total static moment can now be computed. Table 8 shows the maximum deflection $\delta$ and the ratio between the actual and the theoretical total static moments for all culverts. This ratio is used as a correction factor to reduce the calculated moments at the maximum load in order to account for non-uniform load distribution due to the deflection of the structure.

The magnitude of applied moments is corrected in Fig. 13 from the point of second plastic hinge to failure with the proper correction factor, 0.87 from Table 8. Since the correction factor is computed for the deflection at the ultimate load, the corrected moments are applicable only in the vicinity of the maximum load. The dashed line in Fig. 13 suggests the actual path of moment for the range of loads from yielding to ultimate.

The corrected moments should determine the maximum load the culvert can resist. Figure 13 shows that the ultimate resisting moment is reached first at the midspan section at a load of about 111 kips. This load is in a very good agreement with the measured ultimate load.

The final failure of Culvert No. 2 was a sudden and definite break at a diagonal crack in the column face region of the bottom horizontal member. This type of failure cannot be classified as a shear failure, however, since the structure had developed plastic hinges both at midspan
Furthermore, the midspan section had reached its ultimate flexural moment. In general, very little is known about the shear strength of a member after the member has yielded. In a previous technical report the authors described a few tests on simple-span beams loaded through a column stub where both shear and flexural failures were obtained (6). Although not enough experimental evidence is available, it is believed that whenever a section has yielded, the predominance of flexural cracks inhibits the appearance of diagonal cracks so that the member should be able to reach its ultimate flexural capacity without a premature shear failure. In the present case it is believed that the final break was the result of bond slip rather than a shear failure. The tension bars were cut off at 8 in. from the column face. At Gage Location 2, 2.5 in. away from the column face, yielding was noted at 100 kips. Thus, only about a 5-in. length of bar was available to transmit the force in steel at yielding to concrete. Assuming uniformly distributed bond stresses, this length of anchorage indicates bond stresses equal to about 830 psi. Any slip of the bars due to the high bond stresses would result in a diagonal crack and immediate collapse of the culvert. The final diagonal crack started at the free ends of the cut-off bars indicating the possibility of bond slip. This type of failure could have been prevented by a better arrangement of reinforcement, either by the use of continuous bars or by bending the bars down.

16. Culvert No. 5

Figure 14 shows the loading diagram of Culvert No. 5. This culvert was loaded with both lateral and vertical loads, the ratio o
between the two being 0.3. The flexural resisting moments are seen to
increase with increasing total load while the shear moment remains
approximately constant.

Observation of the theoretical elastic moments indicates that
the first resisting moment reached is the yield moment at midspan. Thus,
the first plastic hinge should have formed at that section at a load of
about 68 kips. The section actually yielded at 85 kips. This difference
is again caused by the fact that the stiffness of the structure was
reduced unequally because of unequal cracking at the midspan and column
face sections. As an example, the horizontal members had the same amount
of tension reinforcement while the theoretical moment at midspan was more
than two times larger than that at the column face.

After the first plastic hinge formed, the moment at midspan is
considered to increase along the first yielding line and the additional
increase in the total static moment is carried over to the column face
section where the moment is seen to reach the first yielding line before
it reaches the shear moment line. Thus, the second plastic hinge could
form at the column face section at a load of 102 kips. This load cannot
be checked directly from the measured strains because Gage Location 2 was
placed 0.25 in. from column face and strains were measured only up to
100 kips. However, the measured strains were as great as 80 percent of
the yield strain at the last reading.

After the culvert has developed two plastic hinges, the increase
in the total static moment is divided equally between the midspan and
column face sections. The uncorrected moments are seen to reach the flex­
ural ultimate at midspan at 121 kips. However, these moments again must
be reduced by a correction factor to account for deflections and the resulting non-uniformity of loading. Table 8 gives the correction factor to be used for Culvert No. 5 as 0.93. The corrected moments intersect the ultimate resisting moment at midspan at 133 kips. The measured maximum load was 140 kips. It was also reported that this load was the working limit of the spring assembly used in testing. However, in view of Fig. 14 it is likely that the maximum measured load was also the ultimate load this culvert could resist.

In considering moment redistribution after the formation of either one or two plastic hinges, it must be recalled that a shear failure is not possible at a section which has previously yielded. This is associated with the fact that whenever a section yields, the predominance of the flexural cracks inhibits the progress of the diagonal cracks necessary for a shear failure. Thus, it is seen in Fig. 14 that Culvert No. 5 could reach its ultimate flexural capacity even though the applied moment at the column face section intersected the shear moment line before failure. In general, the shear moment at a section loses its significance as soon as the section yields.

17. Culvert No. 6

Culverts No. 5 and 6 were identical in every respect except for the ratio between lateral and vertical loading. This ratio \( c \) was 0.3 for Culvert No. 5 and 0.6 for Culvert No. 6.

The interaction diagrams for resisting moments and axial loads tabulated in Tables 5 and 6 depend only on the physical properties of the sections and not on the load ratio. Thus, these diagrams for flexural and shear moments are identical for both culverts in terms of absolute units.
of moment and axial load. Only when the calculated axial load $P$ is expressed in terms of the total vertical load $P_t$, do the interaction diagrams depend also on the load ratio. In this case the scale of the axial load is shifted to correspond to the given relationship between the axial load in the member, determined by the lateral load, and the total vertical load on the member.

Figure 15 shows the loading diagram of Culvert No. 6. A comparison of Figs. 14 and 15 shows the effect of different load ratios on the resisting moments of the sections. It is seen that with increasing $P_t$, Culvert No. 6 ($c = 0.6$) has a much larger increase in the flexural resisting moments than Culvert No. 5 ($c = 0.3$). The shear moments, however, do not change appreciably and remain approximately constant for both culverts.

The theoretical elastic moments are shown in Fig. 15 with solid lines. The first plastic hinge occurred at midspan at 114 kips. However, it is also noted that for this range of loads the shear moment in the mid-span region is smaller than the yield moment. Consequently, the magnitude of moment at the critical section for shear failure must be investigated for the midspan region. This moment is shown with the dashed line for a section given by $M/Vd = 4.5$. The location of the section was determined by Eq. 45 and the magnitude of the critical moment by Eq. 46. It is seen that the moment at midspan reached its yield moment before the shear moment was reached at the critical section. Thus, the plastic hinge could form without a previous failure in shear.

The load at the first plastic hinge as given by the theoretical elastic moments agrees with the load at which the midspan section yielded
in the tests. This is to be expected since in this culvert the ratio of lateral to vertical load was such that the resulting theoretical moments were proportional to the amount of tension reinforcement used in the sections. As before, all sections had the same amount of tension reinforcement, but as seen in Table 7, the magnitude of the theoretical moments is nearly the same both at the midspan and at the column face sections of the horizontal member. Furthermore, the column face moment for the vertical member has almost the same magnitude. Thus, it is likely that cracking reduced the stiffness of the culvert uniformly so that the relative stiffnesses of the members remained the same as assumed in the analysis.

After the first plastic hinge was formed, the midspan moment was considered to increase along its first yielding line and the additional increase in the total static moment was carried over to the column face section. This moment is seen to intersect the shear moment line first. Consequently, the shear capacity at the column face section was reached at 122 kips. This indicates that the culvert should have failed in shear before developing yielding at the column face. This analysis is verified by the observed behavior of the culvert. The culvert failed in shear at the column face section at a load of 120 kips. Thus, both the mode of failure and the ultimate load are in good agreement with the predicted behavior of the structure.

It is reported that the culvert continued to take load after the shear failure took place (7). The final collapse occurred at 140 kips. This can be explained by the fact that the shear cracks separated each of the horizontal members into two cantilever parts and a free body, held
together by the axial load. Only two of the six springs acted on the middle portion of the member. This combination of circumstances permitted some additional increase in the total applied load, especially since any deflection tended to redistribute the spring loads to the cantilever portions of the member. Finally the middle free body was pushed out from between the cantilever ends and the culvert collapsed.

18. **Culvert No. 7**

Culvert No. 7 was identical to Culvert No. 5 except for a higher concrete strength. The loading history of Culvert No. 7 is recorded in Fig. 16.

It is seen that the first plastic hinge should occur at midspan at 69 kips while this section actually yielded at 80 kips. This difference is again caused by unequal reduction of stiffness of the culvert because of cracking, and is comparable to that observed in Culvert No. 5.

After redistribution of moments, the moment at the column face is seen to reach first the yield moment line. The second plastic hinge is predicted for 102 kips. The measured strains indicate that the column face section actually yielded at about 115 kips. It is likely that at a load close to the second plastic hinge the deflection of the member had already changed the uniform distribution of spring loads and reduced the corresponding total static moment. The reported load-strain curves show a corresponding decrease in the rate of change of strains at the column face at and above 100 kips.
After two plastic hinges have formed, the increase in the total static moment is divided equally between the two sections and the moments are reduced by the correction factor given in Table 8. The predicted ultimate load is seen to be 155 kips. The maximum measured load was 160.7 kips which is in good agreement with the predicted value.
V. SUMMARY AND GENERAL DISCUSSION

19. Summary of Analysis

The foregoing analysis of reinforced concrete box culverts was divided into two separate parts. The first part involved the determination of the resisting moments of the various parts of the structure. Both flexural and shear resisting moments were considered. The second part of the analysis involved the determination of the applied moments, that is the moments produced by the applied loads on the culvert. Starting with elastic moments, the formation of plastic hinges and the subsequent redistribution of moments were considered. The analysis was completed by a comparison of the applied moments with the corresponding resisting moments. A culvert was considered to fail as soon as the applied moment at any section reached the ultimate resisting capacity of that section. Depending on which ultimate resisting moment is reached first, the culvert fails either in flexure or in shear.

The resisting moments were expressed in terms of interaction diagrams for moment and axial load. The flexural resisting moments, both at first yielding and at ultimate capacity, were found to exhibit a very important characteristic. Unlike the resisting moments of structural steel members, the resisting moments of reinforced concrete members which are under-reinforced increase as the magnitude of the axial load on the members increases. This increase in the moment capacity occurs in a range of axial loads from zero to a certain value at which the mode of failure of the member changes from tension to compression. The more under-reinforced is a member, the larger is the potential increase in its
flexural capacity. In some cases, this increase can be more than 100 percent of the flexural capacity of the same member under pure flexure with no axial load. In a sense, an increase in the axial load is equivalent to increasing the amount of tension reinforcement of the member. After the point of balanced failure is reached, any further increase in the axial load results in a rapid decrease in the flexural capacity of the member. Likewise, the flexural capacity of an over-reinforced member decreases as soon as any axial load is applied.

The shear resisting moments, however, were found to remain practically constant for any value of axial load. Consequently, the shear capacity of a member does not change appreciably with an increasing axial load on the member.

The applied moments on the culvert were considered in two stages. The first stage involved the determination of the elastic moments, that is the magnitude of moments before the formation of any plastic hinges. These moments are affected by the stiffness of the culvert. After an elastic moment reaches the flexural resisting moment at first yielding at some section, the section yields and can rotate without any appreciable increase in the applied moment. Further increases in the applied loads must be analyzed by considering the inelastic behavior of the structure. It was assumed in this analysis that after the formation of the first plastic hinge the moment increases along the first yielding curve at that section. Since the culverts analyzed were indeterminate to the first degree, the known moment at the section of the plastic hinge makes them statically determinate. Consequently, any redistribution of moments in the inelastic stage of loading is determined solely by considerations of
statics and is not affected by the stiffness of the culvert. In fact, the sum of the moments at any two sections must remain the same as calculated for the elastic structure although they may be divided differently between the two sections. Thus, moment is redistributed from the section of the first plastic hinge to the adjacent still elastic sections in such a way that the total static moment remains in a constant proportion to the total applied load.

As soon as the applied moment reaches the corresponding yield moment at another section, the second plastic hinge forms in the structure. Any further increase in the total static moment between the sections of two plastic hinges must now be divided between them according to the distribution of angle changes in the structure and the moment-angle change characteristics of the sections concerned. This was approximated in the present analysis by dividing the moment increase equally between the two sections.

The ultimate flexural capacity of the culvert is reached as soon as the applied moment reaches the corresponding ultimate resisting moment at either section with a plastic hinge. However, this type of failure is possible only when premature failures in shear are prevented. In a complete analysis, both flexural and shear resisting moments are compared with the applied moments. Depending on the relationship between these different moments, both shear and flexural failures are possible. The culvert can fail in shear either before any plastic hinges have developed, or after the formation of the first plastic hinge. It is believed, however, that after a section has yielded, the culvert cannot fail in shear at that section. Thus, after the formation of the first plastic hinge a
shear failure can occur only at a still elastic section. Likewise, after two plastic hinges have developed, a culvert should be able to reach its ultimate flexural capacity without a premature shear failure.

20. **Summary of Test Results**

The Ohio River Division Laboratories of the Corps of Engineers tested four reinforced concrete box culverts. The ratio $c$ between the lateral and vertical loading was the main test variable. Figures 11 and 12 show the dimensions of the culverts and the method of loading. Table 1 summarizes the physical properties and ultimate loads of individual culverts.

Figures 13 through 16 show the loading diagrams for each culvert. The midspan and the column face sections of the horizontal members were considered. The flexural resisting moments were calculated by the method outlined in Chapter II. These moments are listed in Table 5. The shear moments were calculated by the method outlined in Chapter III and they are listed in Table 6. The elastic applied moments were obtained with the aid of Eqs. 25. Table 7 lists these moments for different values of the load ratio $c$. All moments were plotted against the total vertical load $P_t$.

The predicted and the observed loads at the formation of the first plastic hinge are compared in Table 9. It is seen that Culvert No. 6 yielded at about the predicted load while the observed loads are higher than the predicted loads for the other culverts. This was explained by differences between the actual stiffness of a culvert and that assumed in the analysis. The calculated moments were based on the assumption that the culverts are infinitely stiff at the intersection of the members and
have the same uniform stiffness along the clear spans of all members. This assumption is valid for an uncracked structure. Since cracking reduces the stiffness of a section, the above assumption can be valid for a cracked structure only when the stiffness is reduced uniformly at all sections. All culverts tested had equal amounts of tension reinforcement. Table 7 shows that the theoretical moments were approximately equal both at midspan and at the column face of the horizontal member and at the column face of the vertical member of Culvert No. 6. Thus, these sections cracked at about the same load at to about the same extent. The relative stiffnesses of the members of Culvert No. 6 remained essentially the same as assumed in the analysis, and the magnitudes of the theoretical moments are in good agreement with the actual moments as indicated by the observed load at first yielding. In other culverts the theoretical moments were widely different at different sections. This is seen in Table 7 and in addition, Table 9 lists the ratios between the theoretical moments at midspan and at the column face of the horizontal member for individual culverts. The sections cracked at different loads and to a different extent. Since maximum moment occurred at midspan, this section cracked first. The column face section remained relatively stiffer than the mid-span section and attracted more moment than indicated by the analysis. Moments were thus partially redistributed even before the midspan section yielded. Consequently, the first plastic hinge formed at a higher load than that predicted. As seen in Table 9, the larger was the difference
between the theoretical moments, the larger was the relative difference between the observed and the predicted loads.

The predicted ultimate loads are compared with the measured loads in Table 10. In order to determine the load-carrying capacity of the culverts, the magnitude of the applied moments was reduced by a correction factor to allow for the non-uniform distribution of spring loads after the formation of the second plastic hinge.

Three culverts failed in flexure and one in shear. Culvert No. 2 had no lateral load. It failed in tension at a load in good agreement with the predicted failure. The final failure was reported as a sudden break at a diagonal crack at the column face section of the horizontal member. Bond stresses were investigated in the cut-off tension reinforcement at that section, and it is believed that the final collapse was the result of bond slip rather than a failure in shear. This type of secondary bond failure can be prevented by proper arrangement of the reinforcement.

Culverts No. 5 and 7 were tested with the load ratio $\alpha$ equal to 0.3. Both culverts failed in tension at a load slightly higher than the predicted load. These culverts were able to resist a much higher load than Culvert No. 2 with no lateral load. This is primarily due to the fact that the lateral loads produced an axial load in the horizontal members. This axial load increased the flexural resisting moments of the members and permitted the culverts to resist a higher load at failure.

Culvert No. 6 failed in shear. This culvert was subjected to the highest lateral load used in the tests, $c = 0.6$. Both the mode of failure and the ultimate load were predicted by the theory. The magnitude of the
The behavior of Culvert No. 6 can best be explained by a comparison of Figs. 14 and 15. These figures show the loading diagrams for Culverts No. 5 and 6 which were identical except for different load ratios. It is seen that the shear moments $M_s$ are practically the same for both culverts while the flexural resisting moments depend on the value of $c$. Culvert No. 5 failed in flexure because the increase in the yield moment with increasing total load was relatively small at the column face section. The applied moment reached the yield resistance of the section before its shear capacity. Culvert No. 6, however, was subjected to larger lateral loads, and the magnitude of the axial loads in the horizontal members was correspondingly larger. The flexural resisting moments increase rapidly with increasing total load while the shear moment remains unchanged. Although the flexural capacity of the culvert was greatly increased, this potential increase could not be utilized because the mode of failure was changed. The applied moment reached the resisting shear moment before the resisting yield moment and the culvert failed in shear.

21. General Discussion

(a) Flexural Resisting Moments

Section 5 lists the assumptions made in developing expressions for the flexural resisting moments. Most of the assumptions are well verified by experiments and can be used with confidence. More uncertainty reigns in the use of a limiting concrete strain and in the stress-strain diagram for the concrete. In the first case, however, previous studies have shown
that small variations in the ultimate concrete strain have relatively little effect on the predicted flexural strength of a member (3).

Moreover, this effect is entirely negligible for small percentages of tension reinforcement and for small magnitudes of axial load on the member. The properties of the concrete stress-block, however, have a more important effect on the predicted values of flexural strength. The $k_1k_2$-type stress-block was used in the present analysis. Since this stress-block has been determined for the condition of pure flexure, its use is limited to members subjected primarily to flexure. Additional experimental research must be carried out in order to determine the properties of a concrete stress-block for a general case, that is stresses varying from pure flexure to pure axial load. The $k_1k_2$-type stress-block was converted into an equivalent stress-strain diagram as shown in Fig. 4 to develop expressions for the resisting moment at first yielding. These expressions could be checked with equations previously determined for pure flexure (1). It was found that the two gave practically identical results. Since, for a culvert, the axial load is small in comparison to the bending moments, it is believed that the use of the above stress-strain diagram did not introduce great error in the analysis.

Since the stress-strain relationship was not known for the strain-hardening region of the reinforcement used in these tests, it had to be assumed. This introduced an additional assumption in the analysis. The assumed values were based on the characteristics of approximately similar bars tested previously in the Structural Laboratory of the University of Illinois (1).

The interaction diagrams for moment and axial load are uniquely determined by the physical properties of a section and do not depend on the
load ratio \( c \). For any value of axial load there corresponds a certain value of moment at which the section fails. However, whenever the resisting moments in the interaction diagrams are plotted as functions of the total vertical load \( P_t \), they do depend on the load ratio \( c \). This is so because the load ratio determines the relationship between the total vertical load and the corresponding axial load in a member as shown by Eqs. 27. It is seen from these equations that the axial load in the horizontal member is a fraction directly proportional to the load ratio \( c \) of the total vertical load. Thus, as the value of \( c \) increases, the magnitude of axial load in the member increases and a correspondingly larger resisting moment is obtained for any value of \( P_t \) from the interaction diagram. For vertical members the magnitude of axial load is always one half the corresponding total vertical load. Consequently, the resisting moments for the vertical members of the culvert do not depend on the load ratio.

(b) Formation of Plastic Hinges

All four culverts under consideration developed their first plastic hinge at the midspan section of the horizontal member. This was determined by the particular combination of resisting and applied moments as given by the physical properties of the sections and by the load ratio \( c \).

Table 7 shows the effect of \( c \) on the elastic moments. It is seen that for small values of \( c \) the maximum moment occurs at midspan of the horizontal members. As \( c \) increases, the midspan moment decreases. After a certain value of \( c \) the maximum moment will occur at the column face section of a vertical member. Finally, with equal vertical and lateral loads,
equal maximum moments are obtained at the column face sections of both horizontal and vertical members.

As was seen before, the resisting flexural moments are affected differently by the load ratio. The resisting moment of the midspan section increases as \( c \) increases while the resisting moment at the column face of the vertical member remains unchanged. However, assuming identical sections, the unchanged resisting moment at the column face is always larger than the resisting moment at midspan. Only when the total lateral load equals the total vertical load do the two resisting moments become equal. Thus, culverts of ordinary proportions develop their first plastic hinge at the midspan section of the horizontal member. Only with high values of \( c \) it is possible that the column face of the vertical member yields first, unless the two sections are reinforced differently.

After the structure has developed its first plastic hinge, a culvert of the type considered here becomes statically determinate. Subsequent redistribution of moments depends only on the requirements of statics and not on the stiffness of the structure. In the present analysis it was assumed that the moment at the location of the first plastic hinge increases along its first yielding curve. In reality, the magnitude of this moment depends on the moment-angle change relationship of the section and on the angle change to which the section is subjected. However, since the magnitude of moment can increase but little before strain hardening of the tension steel takes place, the above assumption is a conservative approximation. If the actual increase in moment at the section were somewhat larger than that assumed, less moment would be redistributed to other sections and the second plastic hinge would form at a somewhat higher load.
Since the moment at the first plastic hinge increases but little, most of the increase in the total static moment is redistributed to the other still elastic sections. In analyzing the test culverts it was considered that the moment redistribution was such that the column face section of the horizontal member reached its yield moment next. The magnitude of the applied moments was also checked at the column face section of the vertical member. It was found that in all culverts the second plastic hinge formed simultaneously at the column face of both the horizontal and vertical members. As an example, Fig. 20 shows the loading diagram for the column face section of the vertical member of Culvert No. 5. The second plastic hinge is seen to develop at 100 kips. This is practically the same load at which the column face section of the horizontal member yielded, 102 kips as seen in Fig. 14.

In a more general case it is possible that the second plastic hinge forms at the column face of the vertical member while the column face moment for the horizontal member is still much below yielding. In this case, it is conceivable that the culvert can fail even before the second plastic hinge has formed in the horizontal member. This case is shown in Fig. 21 where a loading diagram is presented for midspan and for both column face sections of a culvert. Such a loading diagram is easily constructed by use of the condition that the sum of moments at any two sections must remain in a constant proportion to the total vertical load at any stage of the loading.

After two plastic hinges have developed in a culvert, any increase in the total static moment for the two sections must be divided between them in a way that satisfies the deformation conditions of the culvert.
This problem was discussed in more detail in Section 8. It was seen that plastic hinges refer to a region of the member where yielding occurs rather than to a true hinge at a fixed location. Although the midspan region has a larger rotation than the column face region, the intensity of angle change was seen to be approximately the same at both sections. For this reason, it was decided to divide all increase in the total static moment equally between the two sections with plastic hinges. This approximation yielded satisfactory results for the culverts tested. It was also used for the hypothetical culvert analyzed in Fig. 21.

The ultimate flexural capacity of a culvert is reached whenever the applied moment reaches the corresponding ultimate resisting moment at any section. For the test culverts the magnitude of moments had to be reduced to account for the non-uniform distribution of spring loads resulting from large deflections of the structure.

The agreement between the analysis and the tests was considered quite satisfactory in view of the number of assumptions made in developing the theory. However, there is a possibility that some of these approximations might have canceled each other for the particular culverts under consideration. In order to test the general validity of the analysis, a more comprehensive and more rigorously controlled test series should be undertaken.

(c) Shear Moments

Expressions for the resisting shear moments were developed in Section 10. These were based on the previously determined equations for combined flexure and shear only (2). The added effect of the axial load
was considered both in determining the elastic "straight-line" k and in considering the total moment about the tension reinforcement of the section. The validity of the equations developed is limited with regard to the values of concrete strength. Equation 41, which is applicable to beam-column connections, can be used with concrete strengths not exceeding 5000 psi. For higher strengths than this, Eq. 41 may be used but the value of \( f'_c = 5000 \text{ psi} \) should be substituted for the actual concrete strength until more test data are available.

A method was outlined for the calculation of an interaction diagram for the shear moment and axial load at a section. It was found that the magnitude of shear moment does not change appreciably with increasing axial load. Consequently, when the interaction diagram is converted into a resisting moment diagram in terms of the total vertical load, the shear moment is not affected by the load ratio \( \phi \). This is unlike the flexural resisting moments which were seen to depend on the value of \( \phi \).

(d) Critical Sections for Shear Failure

The location of critical sections for shear failure was discussed in Section 10. At the midspan section of a member, the moment is a maximum and the shear is zero; diagonal cracks do not develop and shear failure is not possible. Moving out from this section, moment decreases slowly and shear increases. Studies covering Stage 1 of this investigation suggested that shear failure is possible at a critical value of \( M/V_d \), at a section where the shear is sufficient to produce diagonal cracks and the moment is large enough to produce a shear failure. This critical value of \( M/V_d \) was set tentatively at about 4.5. Since no shear failures took place
in the midspan region, this critical value of $M/Vd$ cannot be checked with the aid of the present tests. More experimental studies should be carried out to investigate the location of the critical section for shear failure in the midspan region.

Both moment and shear have their local maximum values at the column face section. Provided that the quantity $M/Vd$ is within the range of shear compression failure, this section constitutes a critical section for shear failure. Because of lack of experimental data, however, it was not possible to determine the shear compression range of $M/Vd$ in previous studies. Simple-span beams tested under one or two symmetrical concentrated loads were found to fail in shear up to $M/Vd$ equal to 4.8. This was the highest value of $M/Vd$ for most of the test specimens. A few other beams having values of $M/Vd$ considerably higher than 4.8 failed in flexure at a load higher than their strength in shear compression (2).

Culvert No. 6 in the present test series failed in shear. Figure 15 shows the loading diagram for the horizontal member of this culvert. The measured ultimate load is in good agreement with the predicted shear capacity of the culvert. The ratio $M/Vd$ was found to be equal to 1.1 at failure. The shear moment for the column face section of the vertical member was also investigated for this culvert. Figure 22 shows the corresponding loading diagram. It is seen that the applied moment reached the shear moment of this section at a load considerably smaller than the measured ultimate load. Since the culvert did not fail at the predicted load of this section, the corresponding value of $M/Vd$ was investigated. It was found that $M/Vd$ was equal to 2.6 at the predicted load for shear failure. Test data on beams loaded through bearing blocks indicate that
This value of $M/V_d$ is within the shear compression range. In the present case, however, the critical section is at the intersection of two members, a section similar to a beam-column connection in a framed structure. Furthermore, the critical section is subjected to an axial load in addition to the distributed transverse load. The ratio $M/V_d$ was considered at the column face section where both moment and shear have their maximum values. Diagonal cracking, however, must start at some distance away from the column face where the ratio $M/V_d$ is different from that at the column face. It is conceivable that under such conditions the shear compression range of $M/V_d$ is more limited than that for ordinary test beams loaded with concentrated loads through bearing plates and subjected to no axial load. This is an uncertainty which should be investigated experimentally.

Culverts No. 5 and 7 which failed in flexure were also checked with respect to their shear moments at the column face sections of the vertical member. Figure 20 showed the corresponding loading diagram for Culvert No. 5. It is seen that the shear moment at this section was also reached before the culvert failed in flexure. The corresponding value of $M/V_d$ was found to be 5.5. Culvert No. 7 exhibited a similar behavior. Culvert No. 2 had no shear at this section, thus no shear failure was possible.
VI. BIBLIOGRAPHY


TABLE 1

PHYSICAL PROPERTIES AND TEST RESULTS OF CULVERTS

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<thead>
<tr>
<th>Culv. No.</th>
<th>$f'_{c}$</th>
<th>Horizontal Member</th>
<th>Column Face</th>
<th>Ratio Lateral to Vertical Load</th>
<th>Ult. Load $P_u$</th>
<th>Mode of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Midspan $A_s$</td>
<td>Column Face $A'_s$</td>
<td>d</td>
<td>c</td>
<td>kips</td>
</tr>
<tr>
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<td>4.75</td>
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<td>0.55</td>
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<td>0.55</td>
<td>55</td>
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</tr>
</tbody>
</table>

* With a secondary sudden collapse along a diagonal crack

** Working limit of the springs

*** Shear failure at 120k; final collapse at 140k
TABLE 2
LOADS PRODUCING YIELDING AT GAGE LOCATIONS 1 AND 2

<table>
<thead>
<tr>
<th>Culv. No.</th>
<th>(P_u)</th>
<th>(P_{ulast}) at last reading</th>
<th>(P) at yielding of Gage Loc. 1 at midspan</th>
<th>(P) at yielding of Gage Loc. 2 at col. face</th>
<th>Distance at face to column x</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>110.73</td>
<td>100</td>
<td>60</td>
<td>100</td>
<td>2.25</td>
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<tr>
<td>5</td>
<td>140</td>
<td>100</td>
<td>85</td>
<td>---</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>120(140)</td>
<td>130</td>
<td>115</td>
<td>130</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>160.73</td>
<td>160</td>
<td>80</td>
<td>120</td>
<td>0</td>
</tr>
</tbody>
</table>

* Distance from column face to gage; see Fig. 12.
**INTERACTION DIAGRAM FOR MOMENT AT FIRST YIELDING AND AXIAL LOAD**

**COLUMN FACE SECTION OF CULVERT NO. 6**

- $b = 12$ in.; $D = 5.5$ in.; $d = 5$ in.; $t = 0.9$
- $f' = 5$ ksi; $k_y = 1.068$ (Eq. 17); $\epsilon_p = 0.00249$ (Eq. 14)
- $A_s = A' = 0.55$ in$^2$; $f_y = 44$ ksi; $E_s = 28,500$ ksi; $\epsilon_y = 0.00154$

\[
P = C + C' - T (2)
\]

\[
M = M_{na} + C'd(1 - k_y) + C'td - P(d - D/2) (22)
\]

\[
M_{na} = (k_y d)^2 b k y f' \left[ \frac{2}{3} \frac{\epsilon_o}{\epsilon_p} - \frac{1}{4} \left( \frac{\epsilon_o}{\epsilon_p} \right)^2 \right] (21)
\]

\[
C = k_d b k y f' \left[ \frac{\epsilon_o}{\epsilon_p} - \frac{1}{3} \left( \frac{\epsilon_o}{\epsilon_p} \right)^2 \right] = 320.4 \frac{\epsilon_o}{\epsilon_p} - \frac{1}{3} \left( \frac{\epsilon_o}{\epsilon_p} \right)^2 \]

\[
k_y = \frac{\epsilon_o}{\epsilon_o + \epsilon_y} = \frac{\epsilon_o}{\epsilon_o + 0.00154} \quad (23)
\]

\[
\epsilon_y = t \epsilon_o - \epsilon_y (1 - t) = 0.9 \epsilon_o - 0.000154 \quad (24)
\]

<table>
<thead>
<tr>
<th>$\epsilon_o$</th>
<th>$k_y$</th>
<th>$\epsilon_o/\epsilon_p$</th>
<th>$1/3(\epsilon_o/\epsilon_p)^2$</th>
<th>$(1)-(2)$</th>
<th>$C$</th>
<th>$\epsilon_s$</th>
<th>$C'$</th>
<th>$P$</th>
<th>$M_{na}$</th>
<th>$C'd(1-k_y)$</th>
<th>$C'td$</th>
<th>$2.25P$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0014</td>
<td>.476</td>
<td>.592</td>
<td>.095</td>
<td>.457</td>
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<td>.0107.7</td>
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<td>.522</td>
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<td>.431</td>
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<td>171.3</td>
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<td>.405</td>
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<td>.044</td>
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<td>53.1</td>
<td>157.2</td>
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<tr>
<td>.0008</td>
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<td>.034</td>
<td>.287</td>
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<td>16.2</td>
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<td>40.1</td>
<td>56.5</td>
<td>142.6</td>
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</tr>
<tr>
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<td>.026</td>
<td>.255</td>
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<td>8.9</td>
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<td>.213</td>
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<td>73.4</td>
<td>25.6</td>
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<td>109.7</td>
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</tbody>
</table>
### TABLE 4

**INTERACTION DIAGRAM FOR ULTIMATE FLEXURAL MOMENT AND AXIAL LOAD**

**COLUMN FACE SECTION OF CULVERT NO. 6**

\[ b = 12 \text{ in}; D = 5.5 \text{ in}; d = 5.0 \text{ in}; t = 0.9 \]

\[ f' = 5 \text{ ksi}; k = 0.846 \text{ (Eq. 1)}; k_2 = 0.45 \]

\[ A_s = A' = 0.55 \text{ in}^2; f_y = 44 \text{ ksi}; f_o = 31.5 \text{ ksi}; E_s = 28,500 \text{ ksi}; E_o = 886 \text{ ksi}; \epsilon_y = 0.00154; \epsilon_o = 0.0141 \]

\[ P = C + C' - T \]  \hspace{1cm} (2)

\[ C = k_1 k' b d k_u = 253.8 \text{ kips} \]

\[ M = 6d(1 - k_2 k_u) + C' d - P(d - D/2) \]  \hspace{1cm} (3)

\[ C' = A' f' = 0.55 f' \]

\[ M = 50(1 - 0.45k_u) + 4.50' - 2.25P \]  \hspace{1cm} (4)

\[ T = A' f' = 0.55 f' \]

\[ k_u = \frac{\epsilon_u}{\epsilon_u + \epsilon_s} = \frac{0.004}{0.004 + \epsilon_s} \]  \hspace{1cm} (5)

\[ \epsilon_s < \epsilon_y: f' = E_s \epsilon_s = 28,500 \epsilon_s; \epsilon_y < \epsilon_s < \epsilon_o: f' = f_y = 44.0; \epsilon_o < \epsilon: f' = f_o + E_o \epsilon_s = 31.5 + 886 \epsilon_s \text{ (Eqs. 12)} \]

<table>
<thead>
<tr>
<th>( \epsilon_s )</th>
<th>( k_u )</th>
<th>( C )</th>
<th>( \epsilon' )</th>
<th>( f' )</th>
<th>( C' )</th>
<th>886 ( \epsilon_s )</th>
<th>( f_s )</th>
<th>( T )</th>
<th>( P )</th>
<th>0.45 ( k_u )</th>
<th>1-0.45 ( k_u )</th>
<th>5(1-0.45 ( k_u ))</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>.0180</td>
<td>.182</td>
<td>46.2</td>
<td>.0018</td>
<td>44.0</td>
<td>24.2</td>
<td>15.9</td>
<td>47.4</td>
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<td>44.3</td>
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<td>.918</td>
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TABLE 5

INTERACTION DIAGRAMS FOR FLEXURAL RESISTING MOMENTS AND AXIAL LOAD

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<th>Culvert No. 2</th>
<th>Culvert No. 5</th>
<th>Culvert No. 6</th>
<th>Culvert No. 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'_{c} = 5000 ) psi; ( c = 0 )</td>
<td>( f'_{c} = 5000 ) psi; ( c = 0.3 )</td>
<td>( f'_{c} = 5000 ) psi; ( c = 0.6 )</td>
<td>( f'_{c} = 6900 ) psi; ( c = 0.3 )</td>
</tr>
<tr>
<td>( P )</td>
<td>( M )</td>
<td>( 2P/c )</td>
<td>( P )</td>
</tr>
<tr>
<td>kips in-k</td>
<td>kips</td>
<td>kips</td>
<td>kips in-k</td>
</tr>
<tr>
<td>0</td>
<td>103.4</td>
<td>-</td>
<td>0</td>
</tr>
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</table>
TABLE 6
INTERACTION DIAGRAMS FOR SHEAR MOMENTS AND AXIAL LOADS
CULVERTS NO. 5, 6, and 7

b = 12 in; D = 5.5 in; d = 5 in; t = 0.9
f'c = 5 ksi; n = 7

\[ A_s = 0.55 \text{ in}^2; \ p = 0.00917; \ pn = 0.0642 \]

\[ k = \sqrt{(pn/B)^2 + 2pn/B - pn/B} \]  \hspace{1cm} (40)

\[ A = \frac{d(1 - k/3)}{1 - B} - d + D/2 = \frac{5(1 - k/3)}{1 - B} - 2.25 \]  \hspace{1cm} (39a)

\[ P(A + d - D/2) = bd^2f'_c/k (0.73 - \frac{7.3 f'_c}{10^5}) \]  \hspace{1cm} (41a)

\[ P(A + 2.25) = 547.5 \text{ k} \]

\[ M'_s = AP \]  \hspace{1cm} (36)

<table>
<thead>
<tr>
<th>B</th>
<th>pn/B</th>
<th>k</th>
<th>A</th>
<th>547.5k</th>
<th>P</th>
<th>M'_s</th>
<th>P_t = P/0.15</th>
<th>P_t = P/0.30</th>
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<tr>
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TABLE 7
ELASTIC APPLIED MOMENTS FROM EQUATIONS 25

<table>
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<th>Culv. No.</th>
<th>Horizontal Member</th>
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<th>Vertical Member</th>
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<td></td>
<td></td>
<td>Col. Face Midspan</td>
<td>Col. Face Midspan</td>
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<td></td>
</tr>
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<td>0</td>
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<td>-1.418 Pt</td>
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<td>-1.665 Pt</td>
<td>-0.018 Pt</td>
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</table>

Note: \( P_t \) = total vertical load
\( c \) = ratio between vertical and lateral load

TABLE 8
CORRECTION FACTORS FOR TOTAL STATIC MOMENT

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<thead>
<tr>
<th>Culv. No.</th>
<th>( \Phi_u )</th>
<th>2a</th>
<th>( S ) in.</th>
<th>( K ) kip/in.</th>
<th>Corr. Factor</th>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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### TABLE 9

**LOADS AT FORMATION OF FIRST PLASTIC HINGE**

<table>
<thead>
<tr>
<th>Culv. No.</th>
<th>Load Ratio ( c )</th>
<th>Ratio of Theoretical Moments at Midspan and Column Face</th>
<th>( P_{\text{theor.}} ) (kips)</th>
<th>( P_{\text{test}} ) (kips)</th>
<th>( \frac{P_{\text{test}}}{P_{\text{theor.}}} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>5.66</td>
<td>45</td>
<td>60</td>
<td>1.33</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>2.28</td>
<td>68</td>
<td>85</td>
<td>1.25</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>2.28</td>
<td>68</td>
<td>80</td>
<td>1.18</td>
</tr>
<tr>
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<td>1.17</td>
<td>114</td>
<td>115</td>
<td>1.01</td>
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</table>

### TABLE 10

**LOADS AT ULTIMATE STRENGTH OF CULVERTS**

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<tr>
<th>Culv. No.</th>
<th>Load Ratio ( c )</th>
<th>Measured ( P_u ) (kips)</th>
<th>Mode of Failure</th>
<th>Predicted ( P_u ) (kips)</th>
<th>Mode of Failure</th>
<th>Ratio ( \frac{P_u}{P_u\text{-predicted}} )</th>
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</table>
FIG. 1 IDEALIZED STRESS-STRAIN DIAGRAM FOR REINFORCING STEEL

\[ f_s = f_0 + E_c \varepsilon_s \]

\[ f_s = E_s \varepsilon_s \]

\[ \sigma = E \varepsilon \]

\[ f_y \]

\[ f_0 \]

\[ \varepsilon_y \]

\[ \varepsilon_0 \]

Strain

(a) AXIALLY LOADED COLUMN

\[ C = k_1 k_3 f_c' k_u b d \]

\[ \varepsilon_u = 0.004 \]

\[ k_1 k_3 = \frac{3000 + 0.5f_c'}{1500 + f_c'} \]

(b) \( k_1 k_3 \)-TYPE FOR PURE FLEXURE

\[ C = 0.85 k_1 b c f_c' \]

\[ \varepsilon_u = 0.0038 \]

\[ \varepsilon_0 = 2 \times 0.85 f_c' E_c \]

\[ E = 1,800,000 + 460 \times 0.85 f_c' \]

(c) HOGNESTAD'S FOR ECCENTRICALLY LOADED COLUMNS

D

0.85f_c'

FIG. 2 ASSUMED STRESS BLOCKS IN CONCRETE AT ULTIMATE
FIG. 3 STRAINS AND INTERNAL FORCES AT ULTIMATE FLEXURAL CAPACITY

FIG. 4 APPROXIMATE STRESS-STRAIN DIAGRAM IN CONCRETE

FIG. 5 STRAINS AND INTERNAL FORCES AT FIRST YIELDING
FIG. 6  TYPICAL INTERACTION DIAGRAMS FOR ULTIMATE MOMENT AND AXIAL LOAD

- Tension reinforcement only
- $f_y = 45,000$ p.s.i.
- $E_s = 30,000,000$ p.s.i.
- No strainhardening in steel

- $f_c' = 3000$ p.s.i.
- $f_c' = 5000$ p.s.i.
FIG. 7 DIMENSIONS AND TYPE OF LOADING FOR REINFORCED CONCRETE BOX CULVERT USED IN ANALYSIS
FIG. 8 APPLIED AND FLEXURAL RESISTING MOMENTS 
HORIZONTAL MEMBER OF CULVERT
FIG. 9  STRAIGHT-LINE ANALYSIS FOR SHEAR MOMENT

\[ \frac{D}{2} - \frac{d}{3} \]

\[ \frac{d}{2} - \frac{D}{2} \]

\[ \frac{d}{1 - k/3} \]

\[ \text{Moment at section } M/V_d = 4.5 \]

\[ \text{TOTAL VERTICAL LOAD } P_t, \text{ APPLIED AND RESISTING MOMENTS} \]

\[ \text{COLUMN MOMENT} \]

\[ \text{IDSPAN MOMENT} \]

\[ \text{Elastic Moments} \]

\[ P_y, P_u \]

\[ M_x, M_y, M'_x, M'_y, M_{ax}, M_{ay} \]
5 - No. 3 deformed bars in all culverts
5 - No. 3 deformed bars in culverts 5, 6, 7

Note:
- Electric strain gages on steel

<table>
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<th>x (in.)</th>
<th>y (in.)</th>
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<tr>
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</table>

FIG. 11 DIMENSIONS AND REINFORCEMENT OF CULVERTS TESTED

FIG. 12 LOADING ASSEMBLY ON CULVERTS
FIG. 13 LOADING DIAGRAM OF CULVERT NO. 2
FIG. 14 LOADING DIAGRAM OF CULVERT NO. 5
Moment at Section $M_{yud}=4.5$

Measured Ultimate Load, 120 k

Theoretical and From Measured Strains

FIG. 15 LOADING DIAGRAM OF CULVERT NO. 6
FIG. 16 LOADING DIAGRAM OF CULVERT NO. 7
FIG. 17 RELATIONSHIP BETWEEN MOMENT AND ANGLE CHANGE MIDSPAN SECTION OF CULVERT NO. 2
FIG. 18  MOMENT-ANGLE CHANGE DIAGRAMS AT ULTIMATE LOAD
HORIZONTAL MEMBER OF CULVERT NO. 2

FIG. 19  APPROXIMATE DEFLECTION DIAGRAM OF TOP MEMBER
FIG. 20 LOADING DIAGRAM FOR COLUMN FACE SECTION VERTICAL MEMBER OF CULVERT NO. 5

- Measured Ultimate Load, 140 kips
- Yielding at Midspan of Horizontal Member, 85 kips
- Theoretical
- From Measured Strains

MOMENT (in.-k)

TOTAL VERTICAL LOAD, P (kips)
FIG. 21 TYPICAL LOADING DIAGRAM FOR PLASTIC HINGES AT MIDSPAN OF HORIZONTAL MEMBER AND AT COLUMN FACE OF VERTICAL MEMBER.
FIG. 22  LOADING DIAGRAM FOR COLUMN FACE SECTION VERTICAL MEMBER OF CULVERT NO. 6