A STUDY OF THE EFFECTS OF
TIME-DEPENDENT VARIABLES: EFFECTS
IN PRESTRESSED CONCRETE

THesis PRESTRESSED CONCRETE FOR HIGHWAY BRIDGES 03 PROG. REPT. (PART)

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Urbana, Illinois 61801

A THESIS
by
G. McLEAN

Issued as a Part
of the
THIRD PROGRESS REPORT
of the
INVESTIGATION OF PRESTRESSED CONCRETE
FOR HIGHWAY BRIDGES

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JUNE, 1954
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS
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G. McLean

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FOR HIGHWAY BRIDGES

Conducted by
THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

In Cooperation with
THE DIVISION OF HIGHWAYS
STATE OF ILLINOIS

and

THE BUREAU OF PUBLIC ROADS
U. S. DEPARTMENT OF COMMERCE

Urbana, Illinois
June 1954
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I. INTRODUCTION

1. The General Problem

The successful use of prestressed concrete dates from the realization that considerable creep and shrinkage deformations occur in a prestressed concrete member subsequent to the release of the prestressing force onto the concrete. These deformations may, over a period of one or two years, become several times as large as the initial elastic strain in the concrete. There are two primary results. Firstly, a deflection under dead load, similar to, but opposite in direction from, the creep deflection of a concrete arch; and secondly, a decrease in the tension in the prestressing steel, which may be as large as 30,000 psi, or even larger. The decrease in prestress, while having little effect on the ultimate strength of under-reinforced beams, does reduce the load at which tension first appears in the concrete, normally a governing factor in the design of a prestressed concrete flexural member.

Prestressed concrete is a practical method of construction only because the loss in prestress with time approaches a constant value; it can be an economic method only if this loss is a relatively small percentage of the initial prestress. This condition is obtained by using high tensile steel, initially stressed to 60 to 70 percent of its ultimate strength. At such stresses, however, this steel, while under constant strain, suffers a relaxation of stress; this phenomenon of stress relaxation at constant strain is comparable to creep, which is
the increase in strain which occurs in materials subjected to a constant stress. The relaxation loss must also be allowed for in the design of prestressed concrete members. It is convenient, and has thus become common practice, to assume a value for the percentage loss in prestress resulting from all the time-dependent effects.

2. **Object of the Investigation**

    It has been the object of this investigation to determine the individual and collective effects on a prestressed concrete member of the time-dependent properties of the materials constituting that member. That is, this investigation aims to replace an arbitrary assumption of the loss in prestress by a more rational method of estimating the changes that will occur in time, having regard to the actual circumstances and materials used in the structure.

    Experimental emphasis has been placed on determining the relaxation properties of the hard-drawn steel wire used in prestressed concrete, for two reasons: Firstly, the novelty of the high stresses at which it is used, and secondly, the results of laboratory experiments on steel can be applied practically with greater reliability than can experiments on concrete.

3. **Scope of the Investigation**

    (a) **Relaxation Tests on Prestressing Wire**

    These tests were made to investigate the effects on the relaxation of the wire of:
(i) The type of wire.
(ii) The stress level.
(iii) Preliminary overstressing.

All the wire tested was hard-drawn wire manufactured by the American Steel and Wire Division of the United States Steel Corporation, some of which had been further treated as indicated in the following diagram:

```
Cold-Drawn Wire
       /     \
  /       \
/         \ Stress-Relieving
     |       |
     |       |
OR       SR
         |     |
         |     |
S0       S0
```

The processes of straightening and stress-relieving are subsequently designated by the letters S and R, respectively. Relaxation tests on wires of types OR, SR, and S0, were carried out from initial stresses of 50 to 80 percent of the ultimate strength of the wire. The duration of the majority of the tests was about 1000 hours, but some were extended to 7000 hours.

A number of the type OR wires were subjected to a preliminary period of over-stressing at a stress approximately 10 percent above that
from which the relaxation was subsequently measured. This series of tests was initiated because of the recommendations that have been made by various individuals and organizations which imply that temporary overstressing for a short period greatly reduces the loss in stress due to relaxation.

(b) Creep and Shrinkage of Concrete.

A study was made of the existing literature on creep and shrinkage of concrete. The important work of R. E. Davis and of W. H. Glanville and other authors was studied with the aim of estimating the magnitude of those effects and the variables upon which they depend. It was concluded that for a given concrete under given storage and service conditions, the variables to be considered in analytical studies, and in planning of beam tests, were:

(i) The age of the concrete at time of loading.
(ii) The magnitude of the sustained stress.
(iii) The length of period under sustained stress.

Various theoretical expressions incorporating these factors were examined.

(c) Beam Deformations.

The application of the results of simple creep and relaxation tests to the behavior of prestressed concrete beams was considered. The investigation included the formation of theoretical expressions for the effects of the time-dependent variables, and also the initiation of long-time tests of prestressed concrete beams, the object of which is to check the validity of the theoretical expressions.
4. Acknowledgments

The studies reported herein were made as a part of the Investigation of Prestressed Concrete for Highway Bridges conducted in the Structural Research Laboratory of the Engineering Experiment Station of the University of Illinois, in cooperation with the Illinois Division of Highways and the United States Bureau of Public Roads.

General direction of the investigation was given by Dr. N. M. Newmark, and supervision of the program was provided by Dr. C. P. Siess. The work was carried out under the immediate supervision of J. H. Appleton, Research Associate in Civil Engineering.

Appreciation is expressed to Dr. Siess and Mr. Appleton for their helpful suggestions and generous assistance at all times during the investigation, and during the preparation of this report. The help of Asst. Prof. Kiyoshi Okada of Kyoto University, Japan, in the theoretical studies is also gratefully acknowledged.

The steel wire tested was donated by the American Steel and Wire Division of the United States Steel Corporation.
II. PROPERTIES OF HARD-DRAWN STEEL WIRE

5. Introduction

An important prerequisite for the tensioning material in prestressed concrete is that the permissible working stress, and the value of the elastic modulus at that stress, should be such that the loss in prestress due to creep and shrinkage of the concrete is a relatively small proportion of the initial stress. Steels that satisfy this requirement fall into two main groups: (a) cold-drawn wires, up to 0.276 in. in diameter, and (b) high strength bars. The present discussion is limited to the former group.

Hard-drawn wire is produced from high-carbon open-hearth steel. Ingots are first rolled into rods, which are then patented. This latter process consists in heating to a temperature above the transformation range, and then quenching in a hot lead bath. This destroys the two-phase pearlitic structure of the hot-rolled rod, and results in a sorbitic structure, which is capable of withstanding subsequent cold drawing. The rods are then cleaned and drawn through wire-drawing soap to the required size, the number of drafts depending upon the strength required. Drawing is a cold working process; slippage occurs between the grains, moving the crystals from their original random orientation, and producing a fibrous type of structure.
6. Survey of Wire Properties

The most commonly available and most comprehensive piece of information on the properties of cold drawn wire is the stress-strain relationship. It is natural, therefore, that attempts have been made to interpret all aspects of the physical behavior of the wire in terms of this relationship. Of primary interest to the designer in prestressed concrete are the following characteristics:

(a) Modulus of elasticity
(b) Ultimate strength
(c) Ductility
(d) Relaxation loss

The first two are readily obtainable from the stress-strain curve; ductility requires some consideration of yield strength. Relaxation loss is not obviously deducible from this information. Guyon\(^1\)*, however, has used an empirical expression for the relaxation loss as a function of the slope of the curve at the initial tensioning stress employed.

Because hard-drawn wire has no definite yield point, various conventions are employed to define the shape of the stress-strain curve in the region of yield. This region includes initial stress levels which are commonly employed in prestressed concrete. Bannister\(^2\) recommends that the yield be defined by the stress at 0.7 percent elongation; this

\* Superscripts refer to the references on page 44.
method has the disadvantage that a knowledge of the stress at zero strain in the test is presupposed. The stress corresponding to 0.2 percent permanent set is also frequently used to define the yield characteristics, and will be referred to in this report; this definition may be criticized on the grounds of the lack of precise knowledge of the initial tangent modulus of elasticity.

7. Effects of Manufacturing Processes on Wire Properties

A typical stress-strain curve for as-drawn wire is shown in Fig. 1. This curve is noteworthy for the small range of stress in which Hooke's Law is at all applicable, and also for the complete absence of any yield point.

When drawn wire is coiled directly from the wire drawing block it acquires a permanent curvature of about 12 in. radius, and is subsequently shipped in coils of about 2-ft diameter. Because of its high degree of curvature, this wire is inconvenient to use. Therefore, it is commonly mechanically straightened; the effects of straightening can be observed in the stress-strain curve in Fig. 1. It should be noted that although straightening increases the total elongation of the wire, it further decreases its elastic properties. This alteration of the shape of the stress-strain curve makes on-site straightening of as-drawn wire an unsatisfactory process. The shape of the curve after straightening cannot be known accurately, and hence the accuracy of prestressing is reduced.

Controlled low-temperature heat-treatment has been employed by wire manufacturers to increase the elastic range of as-drawn wires.3,4
This treatment is variously known as stress-relieving, or strain-aging. It is a continuous process in which the wire is pulled through a lead bath at temperatures of 750-800 deg F for a few seconds. Its effect on the elastic properties of the wire can be seen in Fig. 1; there is also a notable increase in the total elongation of the wire before fracture.

The stress-relieving treatment is carried out on equipment having large diameter reels, and hence stress-relieved wire has much less curvature than wire coiled directly from the wire-drawing block; the free radius of wire which is shipped in 5-ft diameter coils is about 3 ft. This is an added advantage to the prestressing engineer, as further straightening is unnecessary.

The important changes in the properties of the wire resulting from the heat treatment also include a change in the loss of stress due to relaxation. Changes in relaxation loss resulting from the various manufacturing processes, and their relation to changes in the stress-strain curve, are of primary interest in this study.
III. SUMMARY OF PUBLISHED WORK ON RELAXATION

8. Scope of Summary

A considerable amount of experimental work on creep and relaxation of high-tensile steels at room temperatures has been carried out in recent years. Much of this work, however, has been devoted to creep, probably because extensive experience existed of creep testing of steels at elevated temperatures, and of other materials. Also, creep tests are simpler to perform than relaxation tests. The condition of the steel in prestressed concrete more nearly approximates to relaxation than to creep. Thus, because no relationship between the creep and relaxation characteristics of high-tensile steel has satisfactorily been shown to exist, the value of the creep data is limited. Attention in this report is therefore concentrated on the results of relaxation tests, to the exclusion of the creep test data.

The various reports on relaxation tests of high-tensile steel wire include experimental results presented in a variety of different ways. For ease of reference, abstracts of these results are tabulated here in as uniform a manner as possible.

9. Work of Professor G. Magnel

The published work$^5$ of Prof. Magnel on steel relaxation appears to be limited to tests of two specimens; these were 82-ft lengths of 5 mm cold-drawn wire of 216,000 psi ultimate strength. It was concluded that the relaxation occurred rapidly. In the first test, on a wire initially
stressed to 123,000 psi (57 percent of ultimate), the relaxation was considered to be complete after 12 days, the loss being 12 percent of the initial stress. In the second test, the wire was preliminarily overstressed to 137,000 psi for 2 minutes, and then reduced to the same initial stress, namely, 123,000 psi; the relaxation was considered to be complete after about 2 days, the loss being 3.6 percent of the initial stress.

10. Work of the Swiss Federal Testing Laboratory

The comprehensive 1946 Swiss report on prestressed concrete includes some results of relaxation tests on 3.2 mm hard-drawn Swedish wire, with an ultimate strength of 279,000 psi and 0.2 percent offset stress of 240,000 psi. Relaxation-time relationships were plotted for tests initiated at stresses of 56 to 76 percent of the ultimate strength of the wire; each test was continued until it was considered that the relaxation loss was complete. The results of the tests are summarized in the following table:

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 1 day</th>
<th>Loss at end of test</th>
<th>Duration of test</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
<td>ksi</td>
<td>Percent of initial stress</td>
</tr>
<tr>
<td>156.0</td>
<td>55.9</td>
<td>2.4</td>
<td>1.5</td>
</tr>
<tr>
<td>185.0</td>
<td>66.3</td>
<td>7.1</td>
<td>3.8</td>
</tr>
<tr>
<td>213.0</td>
<td>76.3</td>
<td>12.8</td>
<td>6.0</td>
</tr>
</tbody>
</table>

These results indicate that the percentage loss of stress resulting from relaxation increases considerably with increase in the initial stress level.
11. Work Reported by M. Dawance

This work was carried out at the Building and Public Works Laboratory in Paris at the request of the Société Technique pour L'Utilisation de la Précontrainte. Relaxation tests on wire alone were reported by M. Dawance in 1948⁷; a further paper⁸ in 1952 gives the results for the loss in stress of the wire due to the combined effects of steel relaxation and creep and shrinkage of concrete.

The wires were stretched in a frame 1-metre long, and the stress measured by observing the frequency of the wire in lateral vibration, the stress being proportional to the square of this frequency. Tests were carried out from different initial stresses, those on 5-mm wire extending to 10 days, and those on 2.5-mm wire extending to 300 days. The observations from some of the longer period tests were plotted on a logarithmic time scale, and the remainder on an arithmetic time scale. The following values have been taken from these plots.

(a) Tests on 2.5-mm Wire, Ultimate Strength 284,000 psi

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 1000 hours</th>
<th>Loss at 300 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
<td>Percent of 0.2 o/o proof</td>
</tr>
<tr>
<td>152.0</td>
<td>53.5</td>
<td>67.3</td>
</tr>
<tr>
<td>156.0</td>
<td>54.9</td>
<td>69.0</td>
</tr>
<tr>
<td>156.0</td>
<td>54.9</td>
<td>69.0</td>
</tr>
<tr>
<td>203.0</td>
<td>71.5</td>
<td>89.9</td>
</tr>
<tr>
<td>256.0</td>
<td>90.1</td>
<td>113.1</td>
</tr>
</tbody>
</table>
(b) Tests on 5-mm Wire, Ultimate Strength 224,000 psi.

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
<th>Loss at 240 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
<td>Percent of 0.2 o/c proof</td>
</tr>
<tr>
<td>152.0</td>
<td>67.9</td>
<td>82.9</td>
</tr>
<tr>
<td>182.0</td>
<td>81.2</td>
<td>99.2</td>
</tr>
<tr>
<td>213.0</td>
<td>95.1</td>
<td>116.0</td>
</tr>
</tbody>
</table>

12. Work Reported by G. T. Spare

The American Steel and Wire Division of the U. S. Steel Corporation has conducted both creep and relaxation tests on 0.192-in. diameter wire of 250,000 psi ultimate strength. Initial results have been reported by Spare. Relaxation tests were made on 100-ft long specimens, and the results plotted on a logarithmic time scale. The following approximate values were taken from the graphs.

(a) Tests on As-Drawn Wire

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
<th>Loss at 1000 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
<td>ksi</td>
</tr>
<tr>
<td>141.0</td>
<td>56.4</td>
<td>4.0</td>
</tr>
<tr>
<td>155.0</td>
<td>62.0</td>
<td>7.0</td>
</tr>
<tr>
<td>172.0</td>
<td>68.8</td>
<td>11.0</td>
</tr>
</tbody>
</table>
14. (b) Tests on Stress-Relieved Wire

<table>
<thead>
<tr>
<th>Initial Stress (ksi)</th>
<th>Loss at 100 hours (Percent of initial stress)</th>
<th>Loss at 1000 hours (Percent of initial stress)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 hours</td>
<td>1000 hours</td>
</tr>
<tr>
<td>180.0</td>
<td>72.0</td>
<td>13.0</td>
</tr>
<tr>
<td>195.0</td>
<td>78.0</td>
<td>21.0</td>
</tr>
</tbody>
</table>

13. Work reported by Clarke and Walley

N. W. B. Clarke and F. Walley have described the experimental work carried out at the Field Test Unit of the British Ministry of Works, part of the Building Research Station of the Department of Scientific and Industrial Research. Twenty-three 1000-hour relaxation tests on 30-ft lengths of hard-drawn commercially available wires of various diameters were reported. Observations of stress were again plotted on a logarithmic time scale. The following values for the relaxation losses in 0.20-in. wire are taken from the tables in the paper.

(a) Tests on As-Drawn Wire, 225-251,000 psi Ultimate Strength

<table>
<thead>
<tr>
<th>Initial Stress (ksi)</th>
<th>Loss at 100 hours (Percent of initial stress)</th>
<th>Loss at 1000 hours (Percent of initial stress)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 hours</td>
<td>1000 hours</td>
</tr>
<tr>
<td>70.0</td>
<td>28</td>
<td>2.0</td>
</tr>
<tr>
<td>120.0</td>
<td>48</td>
<td>5.0</td>
</tr>
<tr>
<td>130.0</td>
<td>53</td>
<td>6.0</td>
</tr>
<tr>
<td>142.0</td>
<td>60</td>
<td>9.0</td>
</tr>
<tr>
<td>152.0</td>
<td>64</td>
<td>9.3</td>
</tr>
<tr>
<td>190.0</td>
<td>76</td>
<td>13.0</td>
</tr>
</tbody>
</table>
(b) Tests on Stress-Relieved Wire, 251,000 psi Ultimate Strength

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
<th>Loss at 1000 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
<td>Percent of 0.1 o/o proof</td>
</tr>
<tr>
<td>70.0</td>
<td>28</td>
<td>33</td>
</tr>
<tr>
<td>130.0</td>
<td>52</td>
<td>61</td>
</tr>
<tr>
<td>170.0</td>
<td>68</td>
<td>80</td>
</tr>
</tbody>
</table>

These results indicate two effects:

1. The increase of relaxation loss with increase in initial stress.

2. The reduced relaxation loss of the stress-relieved wire relative to the hard-drawn wire at initial stresses in the range of 28 to 68 percent of the ultimate strength.

14. Work of J. L. Bannister

Bannister\(^2\) measured the relaxation of stress of 0.20-in. hard-drawn wire at the University of Wales, using a lever-type testing machine. His results include those for both as-drawn and heat-treated wire, and also for wire that had been heat-treated under a tension of 166,000 psi. The following data has been abstracted from the figures and tables in Bannister's paper.
(a) Tests on As-Drawn Wire, 226,000 psi Ultimate Strength

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
</tr>
<tr>
<td>134.4</td>
<td>59.5</td>
</tr>
<tr>
<td>157.0</td>
<td>69.5</td>
</tr>
<tr>
<td>179.5</td>
<td>79.4</td>
</tr>
</tbody>
</table>

(b) Tests on Heat-Treated Wire, 234,000 psi Ultimate Strength

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
</tr>
<tr>
<td>134.4</td>
<td>57.4</td>
</tr>
<tr>
<td>157.0</td>
<td>67.1</td>
</tr>
<tr>
<td>179.5</td>
<td>76.7</td>
</tr>
</tbody>
</table>

(c) Test on Wire Heat-Treated Under Tension, 236,000 psi Ultimate Strength

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 100 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of ultimate</td>
</tr>
<tr>
<td>157.0</td>
<td>66.5</td>
</tr>
</tbody>
</table>

The heat-treated wire shows improved relaxation properties relative to the as-drawn wire only at the lowest initial stress level, namely 134,400 psi. At higher stresses, the heat-treated wire exhibits more relaxation loss than the as-drawn wire. The wire that had been heat-treated under tension exhibits much less relaxation loss than wire of either of the other types when tested at the same stress level.
15. Work Reported by Professor R. de Strycker

Considerable work has been carried out in Belgium on creep and relaxation of prestressing wire under the auspices of the "Comité pour l'étude du fluage des métaux aux températures ordinaires" and has been reported by Professor R. de Strycker.11,12,13,14

Emphasis was placed in these tests on the correlation of creep and relaxation data, and the deduction of the latter from the former. The effects of heat-treatment on the relaxation properties of the wire were studied, and it was concluded that optimum heat treatments ranged from 2 minutes at 300 deg C to 10 minutes at 225 deg C.

Some tests were also made in which the wire was subjected to a preliminary period of overstress before the commencement of the relaxation test. It was concluded that overstressing reduced the relaxation in the first few hours under load, but had no effect on the remaining stress after 3 days.

16. Work at Imperial College, London

Dr. F. W. Gifford has recently reported15 the results of a series of 10 relaxation tests on 0.20-in. diameter prestressing wire, in which the variables were initial stress level and preliminary overstressing. The specimens were 210 in. long; the tension in a wire was measured by observing the lateral deflection resulting from a small known load applied at the center. The reported duration of the tests was 420 days; the load remaining in each wire was plotted on an arithmetic time scale.

The average wire properties were:
Ultimate strength
233,000 psi

0.1 percent proof stress
210,000 psi

0.2 percent proof stress
218,000 psi

This is apparently a stress-relieved wire. It was certified to have a minimum 0.1 percent proof stress of 179,000 psi, and it was supplied in 8-ft diameter coils, requiring no straightening.

The following values have been taken from the graphs in the paper.

(a) Wire not Preliminarily Overstressed

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 1000 hrs.</th>
<th>Loss at 400 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of 0.1 o/o proof</td>
<td>Percent of 0.2 o/o proof</td>
</tr>
<tr>
<td>209.0</td>
<td>90</td>
<td>99.5</td>
</tr>
<tr>
<td>184.0</td>
<td>80</td>
<td>87.6</td>
</tr>
<tr>
<td>159.0</td>
<td>70</td>
<td>75.7</td>
</tr>
<tr>
<td>130.0</td>
<td>60</td>
<td>61.8</td>
</tr>
<tr>
<td>104.0</td>
<td>50</td>
<td>49.5</td>
</tr>
</tbody>
</table>

(b) Wires Preliminarily Overstressed for 2 minutes at 5 percent of the Ultimate Strength above the Required Initial Stress

<table>
<thead>
<tr>
<th>Initial Stress</th>
<th>Loss at 1000 hrs.</th>
<th>Loss at 400 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>ksi</td>
<td>Percent of 0.1 o/o proof</td>
<td>Percent of 0.2 o/o proof</td>
</tr>
<tr>
<td>205.0</td>
<td>90</td>
<td>97.6</td>
</tr>
<tr>
<td>184.0</td>
<td>80</td>
<td>87.6</td>
</tr>
<tr>
<td>166.0</td>
<td>70</td>
<td>79.0</td>
</tr>
<tr>
<td>130.0</td>
<td>60</td>
<td>61.8</td>
</tr>
<tr>
<td>104.0</td>
<td>50</td>
<td>49.5</td>
</tr>
</tbody>
</table>
Gifford concluded that for the wire tested,

1. The stress loss at 420 days is 5 percent or less for initial stresses up to 60 percent of the ultimate.

2. The effect of preliminary overstressing is small at initial stresses up to 60 percent of the ultimate, but is significant at higher stresses.

17. General

Because of the variety of origin of the wire tested, and the varying properties, duration of test, and method of presentation, close comparison of the results, as quoted herein, is not warranted. However, these results do indicate the scope of the problem, and the order of magnitude of the relaxation loss to be expected from common initial stresses.

Further tests have been made by Godfrey, Janiche and Thiel\textsuperscript{16}, Gravoin\textsuperscript{17}, Rainieri\textsuperscript{18}, Schrier\textsuperscript{19}, Simon and Zerovain\textsuperscript{20}, and others.
IV. MEASUREMENT OF RELAXATION OF STEEL

18. The Vibration Method

As has been previously noted, a vibration technique was employed by Dawance in his relaxation tests in Paris. This technique appeared to have the important advantages that it (1) required a comparatively small amount of space, and (2) did not require the continuous use of a testing machine. For these reasons it was decided to adopt the method in this laboratory.

The method depends essentially upon the following relationship between the natural frequency of lateral vibration of a stretched string and the tension in the string.

\[ f' = \frac{1}{2L} \sqrt{\frac{T \cdot g}{w}} \]  

(1)

where \( f' \) = the natural frequency of vibration,  
\( L \) = the length of string between nodes,  
\( T \) = the stress, and  
\( w \) = the specific gravity.

\( g \) = the acceleration of gravity.

For a steel wire, which has bending stiffness, a correction must be made for this stiffness according to the equation

\[ f^2 = f_1^2 + f'^2 \]  

(2)

where \( f \) is the natural frequency of the stretched wire, \( f' \) is the natural
frequency neglecting bending stiffness, as in equation 1, and \( f_1 \) is the natural frequency of the wire when resting as a simply supported beam, untensioned, on two supports a distance \( L \) apart,

\[
f_1 = \frac{\pi}{2L^2} \sqrt{\frac{E.I}{\mu}}
\]

(3)

where \( \mu \) is the mass per unit length of the wire. Equation 2 is an approximation, and is applicable only when the stiffness of the wire is small. Hence

\[
f'^2 = \frac{1}{4.L^2} \frac{Tg}{\mu} \quad \text{from 1}
\]

\[
f'^2 = f^2 - \frac{\pi^2}{4.L^2} \frac{E.I}{\mu} \quad \text{from 2 and 3}
\]

Therefore

\[
T = \frac{4wL^2}{g} (f'^2 - K_1)
\]

where \( K_1 \), for a particular diameter wire, depends only on the distance between the nodal points.

In practice, owing to lack of absolute rigidity of the testing frame

\[
T = \frac{4wL^2}{g} (f'^2 - K_1 - K_2)
\]

(4)

where \( K_2 \) is a constant of the test frame and mode of vibration. This relationship enables the stress in a stretched wire to be computed from
an experimentally obtained value of the natural frequency of lateral vibration of the wire.

19. Test Equipment

Test frames consisted of 3-ft lengths of 8 by 8-in. wide-flange beams section with heavy plates welded across the ends, as seen in Figs. 2 and 3. In each frame, the end plates were drilled to accommodate 4 wires stretched between them. Quarter-inch screws, mounted in holes drilled and tapped in the beam flanges, and adjusted so as to bear against the stretched wires, provided definite nodal points when the wires were vibrated. These screws, replaced earlier knife-edges which had proved troublesome to adjust.

Both threaded and split conical wedge anchorages were employed. In the former a hardened steel nut was turned up as far as possible on a thread cut on the wire; this anchorage could be relied upon to develop only about 70 percent of the tensile strength of the wire. For higher stresses an improved anchorage was required. A conical split collet, bearing on an internally tapered stud, was machined and hardened in the laboratory, but failed to give satisfactory results when tested. A grip was subsequently developed that incorporated the three hardened tapered wedges from a commercial 6 BWG-size Strandvise grip, again bearing on an internally tapered stud. The component parts of this anchorage are shown in Fig. 4. This grip was successfully used in the tests that commenced at the higher stress levels. Because wedge-type grips necessarily suffer from pull-in effects, light Last Word dial gages were mounted on the test
wires so that they indicated any movement of the wire in the grip relative to the end plate of the test frame. One of these gages may be seen in Fig. 3.

The equipment for stressing the wires is shown in Figs. 5 and 6; Fig. 5 shows a wire anchored by a threaded nut, but the stressing arrangement was precisely the same for a wire anchored by a wedge grip. The pull rod of a center-pull hydraulic jack was coupled with a dynamometer which in turn was coupled with the anchorage stud by means of a threaded adaptor. When a wire was stressed the extension was taken up by a threaded sleeve, which was turned up to bear against the end plate of the testing frame.

Four SR-4 electric resistance strain gages, connected in series in pairs, were mounted on the dynamometer; one pair mounted longitudinally occupied the active arm, and the other pair mounted laterally occupied the dummy arm of a Wheatstone bridge circuit. This circuit, when completed by a Baldwin Portable Strain Indicator, enabled the strain in the dynamometer to be measured. The dynamometer was calibrated in a testing machine for loads up to 7000 lbs, corresponding to a stress of over 240,000 psi in a 0.192-in. diameter wire. The dynamometer thus gave the stress in the wire directly during the period of stressing.

The electrical apparatus employed to vibrate the wire, to observe resonant vibration of the wire, and to measure the frequency of vibration, is shown diagrammatically in Fig. 8, and also in the photographs, Figs. 6 and 7. The main components were as follows:

(1) An oscillator, with variable frequency output.
(2) A frequency counter which counted the number of cycles in 30 seconds of the oscillator output, and hence gave the oscillator frequency correct to 1/10 of a cycle per second.

(3) An electromagnetic vibrator, fed by the oscillator through a variable-output amplifier. The vibrator was mounted about 1/32 in. away from the wire, at its midpoint.

(4) An ear-phone, also mounted close to the wire, to pick up any forced vibration of the wire.

(5) A cathode-ray oscilloscope; the output of the oscillator was fed directly into the horizontal deflecting plates, and the current generated in the ear-phone by the vibrating wire was fed into the vertical deflecting plates.

When the oscillator frequency coincided with the natural frequency of the wire a figure eight-type picture was obtained on the oscilloscope. The picture was of this form because the wire made one complete oscillation for both the positive and negative half-cycles of the driving current. The third mode of vibration of the wire was employed, for two reasons: (1) It reduced the effects of uncertainties regarding the end conditions of the wires, and (2) it raised the resonant frequency of the wire to a pitch at which it was audible, and hence could be located approximately by ear. Thus the resonant position was indicated by

(a) sound,
(b) appearance of Lissajou figure on the oscilloscope,
(c) maximum vertical dimension of the figure on the oscilloscope. This increased greatly at resonance necessitating reduction in
The amplification of the oscillator output. The maximum vertical size of the oscilloscope figure was the criterion employed in determining the precise position of resonance.

Test Procedure

The procedure for tensioning a wire was as follows: The wire was stretched by the hydraulic jack in about five increments of load, up to the required initial stress. At each step the wire was vibrated, and the oscillator frequency adjusted, until the resonant frequency of the wire was obtained; this frequency, as indicated by the frequency counter, was noted. The strain indicator was balanced, and thus the strain in the dynamometer, and hence the stress in the wire was also obtained.

From these observations made during the stressing of the wire, a calibration graph of stress against the square of the frequency was drawn for the wire; this is the graphical representation of equation 4. When the required stress was reached, the threaded anchoring sleeve, shown in Fig. 4, was turned up to bear hard against the end-plate of the testing frame. The pressure in the jack was then released, and the wire vibrated immediately to obtain the resonant frequency. The corresponding stress read from the calibration graph was the initial stress level of the relaxation test. In the case of wires anchored in wedge grips, the dial gages mounted at the ends of the wire were also adjusted immediately after the release of the jack pressure. The wire was subsequently vibrated after suitable intervals of time, and the resonant frequency and the dial gage readings (if any) were noted. Several readings were taken in the
first hour of the test and later at greater intervals, in accordance with the early rapid, and later less rapid, rate of relaxation.

The procedure described, and the final form of the equipment used, resulted from experience gained in the preliminary tests. These had indicated two major faults in the original experimentation:

1) A second apparent resonance position was obtained, near the true resonant frequency, when the wire was vibrated in its first mode. This effect was much reduced when the wire was vibrated in its third mode, obviating any possibility of confusing a secondary resonance frequency with the true one.

2) Excessive apparent relaxation losses were obtained from early tests due to:

   a) Too much threading on the wires.

   b) Unobserved pull-in of the wedge grips.

Reduction of the amount of threading so that the nut was always fully turned up to the end of the thread, and the use of the small dial gages to measure any movement of the wires in the anchorages, removed these faults. The dial gages indicated relative movements between the ends of the threaded wires and the end-plates of the frame, equivalent to a stress loss in the wire of about 200 psi. This has been neglected. The relative movements in the case of wires anchored by wedge grips were considerably larger, and were therefore compensated for in the computations of stress after relaxation; to the stress obtained from the frequency measurement was added the relative movement multiplied by the modulus of elasticity of the steel.
The accuracy of the tests is limited initially by the accuracy of the calibration of the dynamometer, namely, about 5 lbs. This is equivalent to approximately 170 psi in the 0.192 in. wire. The accuracy of the measurement of stress by the vibration procedure was checked by connecting a vacuum-tube voltmeter across the ear-phone output; the magnitude of the response was observed at frequencies in the neighborhood of resonance, and the observations plotted in Fig. 9. It appears that the accuracy is about 100 psi. The tests were performed in an area where the temperature range was fairly large; it is considered, however, that this should not have had any great effect, as the wires were stressed against a steel frame undergoing the same temperature range. Inaccuracies due to temperature changes could apparently only have resulted from rapid changes that occurred immediately prior to the measurement of stress. However the actual degree of accuracy obtained in the measurements may better judged from the graphs of the results.
V. RELAXATION TEST RESULTS

21. Scope of Results Presented

The results are presented for four series of tests as outlined in the following table:

<table>
<thead>
<tr>
<th>Series No.</th>
<th>No. of Tests</th>
<th>Type of Wire</th>
<th>Properties of Wire</th>
<th>Range of Initial Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ultimate Strength ksl</td>
<td>0.1 percent proof stress ksl</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>SO</td>
<td>244.0</td>
<td>150.0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>OR-1</td>
<td>250.0</td>
<td>206.0</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>OR-2</td>
<td>266.0</td>
<td>218.0</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>SR</td>
<td>240.0</td>
<td>201.0</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>OR-2</td>
<td>266.0</td>
<td>218.0</td>
</tr>
</tbody>
</table>

The initial stress was the main variable in each series. Series 1, 2, and 3 were tested in order to observe the effect of the different manufacturing processes on the relaxation characteristics of the wire. The tests tested in Series 4 were overstressed approximately 10 percent above the intended initial stress for a period of 15 minutes prior to the commencement of the test. This series of tests was made in order that the effect of initial over stressing might be indicated by comparison of these results with those of Series 2.

22. Manufacture of Wire Tested

All the hard-drawn wire tested was made from basic open-hearth steel of the following analysis range:

- Carbon: 0.75-0.86 \( \text{a/o} \)
- Manganese: 0.50-0.90 \( \text{a/o} \)
- Phosphorus, max.: 0.045 \( \text{a/o} \)
- Sulphur, max.: 0.050 \( \text{a/o} \)
The as-drawn wire was subsequently treated as described below. Wire S0 was mechanically straightened. Wire OR was stress-relieved in a continuous process, and supplied in 5-ft diameter coils. Wire SR was straightened, cut into 15-ft lengths, and then stress-relieved.

Stress-relieving was carried out in hot lead at about 800 deg F for a period of between 5 and 15 seconds.

Nominal diameter of the wire was 0.192 in.; actual diameters ranged from 0.192 to 0.194 in. Wire stresses reported were calculated on the basis of actual diameters.

23. Test Results

The results for each series of tests are presented graphically in the following ways:

(a) Stress plotted on an arithmetic time scale.

(b) Stress plotted on a logarithmic time scale.

(c) Initial wire stress, as a percentage of the ultimate strength, plotted against the percentage loss in stress at 1000 hours.

Plots of type (a) indicate the very rapid stress losses that occur in the first 100 hours, and the subsequent much lower rate of loss. The semi-log plots of type (b) serve to demonstrate the apparently continuing nature of relaxation. The plots of type (c) show, that for all the types of wire tested, the percentage relaxation loss increases with increase in the initial stress. It would be desirable to obtain some empirical relationship from which, knowing some wire property, the relaxation loss from any given initial stress could be calculated. Various
methods of plotting the results of all the tests in order to obtain such a general relationship were attempted, but without success.

24. Discussion of Test Results

(a) Effect of stress-relieving on 1000-hour losses. Figs. 13 and 25 show the variation of percentage loss of stress at 1000 hours with the initial stress level, for wires of types 30 and 3R respectively. The two types of wire suffer approximately the same percentage loss for initial stresses in the range of 55-65 percent of the ultimate. However, at higher stresses the stress-relieved wire suffers somewhat greater losses, the loss at 75 percent of the ultimate being 11.8 percent, compared with 8.8 percent for the non-stress-relieved wire.

(b) Curvature of the semi-log graphs. It can be observed that straight lines were fitted successfully to the points plotted on a logarithmic time scale for the straightened hard-drawn wire, type 30, in Fig. 12. In the case of the stress-relieved wires, Figs. 16, 18, and 24, this was not possible; these semi-log graphs have a continuous, though slight, curvature. It is worth noting that de Strycker observed a similar effect; he found that heat-treated wires showed a curvature from the very beginning of the experiment, but that deviation from a straight line could only be perceived after several days for hard-drawn wires. Although this curvature on semi-log graphs does preclude linear extrapolation out to very long periods of relaxation, it does not necessarily imply that the losses will become excessive within any reasonable length of time.

(c) Effect of straightening. No as-drawn wire was tested, because the high degree of curvature of this wire prevented it from being
introduced into the testing frame without some initial straightening. Hence, there is no direct comparison of results between straightened and as-drawn wire. Comparison of the results of the tests in Series 2 and 3 probably does not indicate the effect of straightening for two reasons: (1) The stress-relieving subsequent to straightening of wire SR would destroy the state of internal stress in the wire resulting from the straightening process, and hence would remove all the metallurgical effects of straightening. (2) Wires OR-1 and OR-2 had rather different stress-strain and relaxation properties, and there was no apparent simple correlation between them. Thus no useful result could be obtained from comparing their relaxation properties with those of the SR wire. The very low relaxation losses exhibited by the wire OR-2 was a notable feature of the tests; the ratio of 0.2 percent proof stress to the ultimate strength of this wire was also unusually high. A further interesting point is that wires OR-1 and OR-2 were distributed commercially as essentially the same product; the analyses differed slightly, but both were within the manufacturer’s range of tolerance.

(d) Effect of initial overstressing. The effect of initial overstressing may be observed by comparison of the relaxation loss curves in Figs. 20 and 28 for wire of type OR-2. For convenience, the curve in Fig. 20 is shown as a dotted line in Fig. 28. For initial stresses in the range 50-65 percent of the ultimate strength, overstressing appears to have little effect; certainly the difference in percentage relaxation loss at any given initial stress in this range is no greater than the scatter of the results. At higher stresses temporary overstressing does
produce some reduction in relaxation loss. At an initial stress of 70 percent of the ultimate, the 1000-hour loss without preliminary over-stressing is about 4.3 percent; with preliminary over-stressing, the loss is about 3.6 percent. However, the results presented here are inadequate to make any definite conclusions on the range of initial stresses in which this over-stressing is of value, or of the magnitude of the reduction in loss. In general, the effect appears to be small. It must be noted that these tests were made on the wire OR-2, that exhibited unusually low relaxation losses.
VI. CREEP AND SHRINKAGE OF CONCRETE

25. Introduction

The problems of creep and shrinkage in concrete have been the subject of considerable experimental research in the last thirty years. In particular, extensive programs have been carried out at the University of California and at the Building Research Station in England. The subject is a complex one, and the number of variables involved is large, because of the intrinsic nature of concrete. It is generally accepted that the hydration of cement is accompanied by the formation of a colloidal gel, and that the subsequent behavior of the concrete under continuously varying climatic and loading conditions depends upon the movement of water into and out of this gel.

26. Shrinkage

Hydration of the cement, and evaporation from the surface of a concrete member, set up capillary forces in the concrete which induce shrinkage of the gel; this shrinkage follows an exponential law with respect to time, and thus approaches a limiting value. The amount of shrinkage depends upon the total amount of water in the concrete, and hence in practice, upon the water-cement ratio and the richness of the mix. Other factors which affect the amount of shrinkage are: the moisture conditions in storage and in service of the member, the character and gradation of the mineral aggregate, the chemical composition and fineness of the cement, and the size of cross-section. Accepted values of the
Final shrinkage strain are 0.0002 to 0.0005. 0.0002 to 0.0003 is commonly assumed for post-tensioned prestressed concrete members which have had time to mature before the tensioning of the steel; higher values are used for pretensioned work.

27. Creep

Creep of concrete is taken to mean the deferred strain which occurs in a loaded concrete member in excess of that due to shrinkage. The external loading sets up stresses which affect the movements of the water in the concrete, and hence creep depends upon all the factors which affect shrinkage. It also depends upon the magnitude of the external load, the period of sustained load, and the age at the time of loading.

From theoretical considerations of the transfer of stress from the cementing material to the aggregate, F. G. Thomas \(^{22}\) obtained an exponential expression for the variation of creep strain with time. Both exponential and hyperbolic curves have been successfully fitted to experimental data.

The experimental work of both Davis \(^{23,24,25}\) and Glanville \(^{26}\) leads to the conclusion that creep strains are proportional to the applied stresses, with the qualification that this does not apply at high stresses, at which the creep is relatively greater in proportion to the applied stress. Davis \(^{24}\) also concluded that for a given intensity of sustained stress and a given storage condition, the rate of creep after a year or so under load was practically independent of the age at time of loading. These conclusions have been accepted by numerous engineers and used in the formulation of theories for the effects of creep on plain and
Reinforced concrete structures; such work has been published by Ban and Strada, 27 Dischinger, 28 Lorman, 29 McHenry, 30 Seed, 31 and Whitney. 32

The conclusion that creep strain is proportional to applied stress has led to creep strains being quoted in multiples of the initial elastic strain. Usual values for the final creep strain of a dense vibrated concrete are 1.0-2.0 times the initial elastic strain; higher values of 2.0 or more times the initial elastic strain were obtained by Glanville and others in their tests of hand-tamped concretes, of relatively high water-cement ratio.
Assumptions Regarding Creep of Concrete

Two assumptions are made regarding the creep of concrete:

(1) For different specimens of the same concrete, loaded at the same time but at different stress levels, the creep strains measured at any later time will be proportional to the applied stresses.

(2) For a given concrete, the rate of creep strain at any time under any given intensity of stress is independent of the previous stress and strain history.

These assumptions are based on the experimental work of Iman and Davis, as discussed in Section 27.

Creep of Concrete under Variable Sustained Stress

The assumptions stated in the preceding section are combined in the equation

$$\frac{d\varepsilon_c}{dt} = \frac{f}{E} \frac{d\phi_t}{dt}$$

(1)

where $\varepsilon_c$ = creep strain at time $t$ due to stress $f$,

$\phi_t$ = creep time function at time $t = \frac{\text{creep strain at time } t}{\text{elastic strain}}$

$E =$ modulus of elasticity assumed not to vary with time.

If the function $\phi_t$ and a varying stress $f_t$ are as shown diagrammatically in the following figure,
then the total strain at any time $t = T$ due to the stress $f_t$ is

$$\delta_T = \frac{f_T}{E} + \int_0^T \frac{d\varepsilon_t}{dt} \, dt$$

where $\varepsilon_t$ is the creep strain under the applied stress at time $t$.

If we now substitute for $\frac{d\varepsilon_t}{dt}$ from equation 1, we have,

$$\delta_T = \frac{f_T}{E} + \int_0^T \frac{f_t}{E} \frac{d\phi_t}{dt} \, dt$$

(2)

However we have assumed that $\phi$ is a function of time only. Thus

$$\delta_T = \frac{f_T}{E} + \int_{\phi_0}^{\phi_T} \frac{f_t}{E} \, d\phi$$

(3)

This is a general expression for the total, elastic and creep, strain at any time $t = T$, for concrete subjected to a continuously variable stress $f_t$. 
Deformations in a Bonded Prestressed Concrete Beam

The following notation is employed in the subsequent derivation:

- $A_c =$ area of concrete section.
- $e =$ eccentricity of steel with respect to the center of gravity of the concrete section.
- $k =$ radius of gyration of concrete section with respect to its center of gravity.

\[
\frac{1}{A_c'} = \frac{1}{A_c} \left(1 + \frac{e^2}{k^2}\right)
\]

- $A_s =$ area of prestressing steel
- $E_c =$ modulus of elasticity of the concrete, assumed constant.
- $E_s =$ modulus of elasticity of the steel.
- $f_c =$ stress in the concrete at the level of the steel.
- $f_{ce} =$ initial stress in the concrete at the level of the steel.
- $f_s =$ stress in the steel.
- $f_{se} =$ initial stress in the steel.
- $P =$ initial steel force.
- $P'_t =$ loss in steel force from all effects at time $t$, assumed to be equal to $P_E + P_R$, where
  
  - $P_E =$ elastic decrease in the steel force due to creep and shrinkage of the concrete,
  
  - $P_R =$ loss in steel force due to relaxation under the initial strain, taken as $R\sigma$, $R$ being a constant for any given initial steel stress. This implies that steel relaxation follows the same shape of curve with respect to time as does concrete creep.

- $S_{tt} =$ shrinkage time function of the concrete, equal to a constant times the creep time function.

\[
P'_t = \frac{dP_t}{d\phi}
\]
The changes that occur over a period of time in a prestressed concrete beam result from three factors, namely: relaxation of steel stress, creep of the concrete under stress, and shrinkage of the concrete. These factors result in a decrease in the prestressing force and deformations in the beam. The state of stress and deformation in the beam after these changes have occurred is defined by the remaining prestressing force. It is the purpose of the following analysis to obtain an expression for the total loss in the prestressing force resulting from all effects.

An expression relating all the variables, as governed by the assumptions that have been made, is obtained by equating the changes in strain in the steel and the concrete at the level of the steel, at time t. The change in concrete strain consists of three parts:

(1) Concrete Creep. This part is divided into two terms:
(a) A term for the creep of the concrete, considering the initial prestress force to be acting throughout the time t; (b) A term for the creep strain resulting from the loss in prestress force \( P'_t \), considered as a continuously varying negative force on the concrete section; this term is obtained from Eq. 3.

Hence, concrete creep strain = \( \frac{f_{cs}}{E_c} \sigma_t - \frac{1}{A'_c E_c} \int_{\sigma_0}^{\sigma_t} P'_t \, d \phi \)
(2) Elastic strain resulting from the loss in prestress force $P'_t$, again considered as a continuously variable negative force. This strain is

$$E_c \frac{f_{ce}}{E_c} \dot{\phi}_t - \frac{1}{A'_c} \left[ P'_t + \int_{\phi_0}^{\phi_t} P'_t \, d\phi \right] + S\phi_t = \frac{P_{E}}{A_E S_s}$$

(3) Shrinkage strain = $S\phi_t$.

Thus, equating the strain changes in steel and concrete,

$$f_{ce} \frac{f_{ce}}{E_c} \dot{\phi}_t - \frac{1}{A'_c} \left[ P'_t + \int_{\phi_0}^{\phi_t} P'_t \, d\phi \right] + S\phi_t = \frac{P'_t - P_R}{A_E S_s}$$

However $P_E = P'_t - P_R$

Hence, we obtain

$$f_{ce} \frac{f_{ce}}{E_c} \dot{\phi}_t - \frac{1}{A'_c} \left[ P'_t + \int_{\phi_0}^{\phi_t} P'_t \, d\phi \right] + S\phi_t = \frac{P'_t - P_R}{A_E S_s}$$

(4)

Differentiating with respect to $\phi$,

$$f_{ce} \frac{f_{ce}}{E_c} - \frac{1}{A'_c} \left( \ddot{P}'_t + P'_t \right) + S = \frac{P'_t - P_R}{A_E S_s}$$

Rearranging the terms,

$$P'_t \left[ 1 + \frac{A E S_s}{A'_c E_c} \right] + P'_t \frac{A E S_s}{A'_c E_c} = \frac{A E S_s}{A'_c E_c} f_{ce} + \frac{P_R}{A_E S_s} + S A E S_s$$

Substituting $\frac{A E S_s}{A'_c E_c} = r$, and dividing through by $1 + r$. 

\[ \dot{P}_t + \frac{r}{1+r} P_t = \frac{1}{1+r} \left( \phi A'_{ce} + SA\bar{y}_s + R \right) + \frac{\phi R}{1+r} \]  

Now, since \( P_R = R\phi_t \)

\[ \dot{P}_R = R = \text{Constant} \]

Thus all the terms on the right hand side of Eq. 5 are constants, and the equation is a linear first order differential equation in \( \dot{P}_t \), which may be solved by a standard method. The solution is

\[ P'_t = e^{-\int_0^t \frac{r}{1+r} d\phi} \left[ \int_0^t \frac{r}{1+r} d\phi + C \right] \]

where \( B = \frac{1}{1+r} \left( \phi A'_{ce} + SA\bar{y}_s + R \right) \)

and \( C \) is a constant of integration.

Since we take \( \phi = \phi_0 = 0 \) at \( t = 0 \), we obtain

\[ P'_t = e^{-\int_0^t \frac{r}{1+r} d\phi} \left[ \frac{1}{1+r} \left( \phi A'_{ce} + SA\bar{y}_s + R \right) + \frac{r}{1+r} e^{\frac{r}{1+r} \phi_t} \right] + C \]

\[ = A'_{ce} + \frac{SA\bar{y}_s + R}{1+r} \phi_t + Ce^{\frac{r}{1+r} \phi_t} \]

But \( P'_t = 0 \) at \( t = 0 \). Hence

\[ C = - \left[ A'_{ce} + \frac{R + SA\bar{y}_s}{r} \right] \]

\[ P'_t = \left[ P + \frac{R + SA\bar{y}_s}{r} \right] \left[ 1 - e^{-\frac{r}{1+r} \phi_t} \right] \]

(6)
This equation gives the total loss in prestress force, $P_t$ at time $t$, in terms of the initial prestress force $P$, the creep time function of the concrete at time $t$, the factors $S$ and $R$ which define the concrete shrinkage and the steel relaxation, respectively, the elastic moduli of the steel and concrete, and the properties of the section.

In the above analysis no assumption has been made of any particular form for the creep function $\phi_t$, but it has been assumed that concrete creep and shrinkage and steel relaxation all vary according to the same function of time. This is only approximately true. However, this assumption leads to the result in Eq. 6 that only the final values of the shrinkage factor $S$, and the relaxation factor $R$, at time $t$, are required for calculating the total loss in prestress $P_t$. The value of the creep function $\phi_t$ in Eq. 6 is the creep strain at time $t$, divided by the elastic strain. The values of $\phi_t$, $S$, and $R$ as substituted, will be the correct final values at time $t$ as obtained from separate tests on concrete and steel specimens.

The total stress change in the concrete at the level of the steel is a relatively small proportion of the total stress. In this small stress range, the assumption that the rate of strain is independent of the stress and strain history is probably reasonably good. Similar assumptions, in rather broader terms, have been made previously by McHenry and others.

The validity of Eq. 6, as a relatively simple means of obtaining the resultant effect of the time-dependent variables in a prestressed concrete structure, will be checked by tests of beams which have been designed for this purpose.
An examination was made of the time-dependent factors which reduce the effective prestress force, and which result in deferred deformations in a prestressed concrete member. The factors studied were (a) relaxation of hard-drawn steel wire used in prestressed concrete, and (b) creep and shrinkage of the concrete.

The work on steel relaxation consisted of (1) a survey of the published data on the subject, and (2) an experimental program. The variables considered in this program were (a) the initial stress level, (b) the effect of the processes involved in the manufacture of the wire, and (c) the effect of initial overstressing of the wire. It was found that: (1) The percentage loss in steel stress due to relaxation increased with increase in the initial stress level, (2) Stress-relieving treatments increased the relaxation loss at initial stresses in excess of 65 percent of the ultimate, but had no observable effect on relaxation at lower stresses. (3) Overstressing reduced the relaxation loss only slightly, and only at initial stresses in excess of 65 percent of the ultimate.

An examination was made of the literature on creep and shrinkage of concrete to provide a basis for theoretical work and the planning of future tests. Reasonable assumptions were made, based on this published work. With these assumptions a relatively simple expression for the total loss in prestress due to all the time-dependent effects was derived.
IX. REFERENCES


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23. Davis, R. E., and Davis, H. E., "Flow of Concrete under the Action of Sustained Loads", American Concrete Institute, Vol. 27, 1951.


FIG. 1  STRESS-STRAIN CURVES OF COLD-DRAWN WIRE
2. GENERAL VIEW OF TESTING FRAMES

3. DETAILS AT END OF TESTING FRAME
4. DETAILS OF WEDGE GRIPS

5. DETAILS OF WIRE-STRESSING EQUIPMENT
6. VIEW OF EQUIPMENT USED IN STRESSING OPERATION

7. VIEW OF EQUIPMENT USED IN MEASUREMENT OF WIRE STRESS
FIG. 8 VIBRATION EQUIPMENT FOR MEASURING WIRE STRESS
FIG. 9 FREQUENCY-RESPONSE CURVE
FIG. 10 STRESS–STRAIN CURVE OF TYPE SO WIRE

0.2% proof stress = 190,000 psi
FIG. 11 RELAXATION OF STRAIGHTENED HARD-DRAWN WIRE, TYPE SO
FIG. 12 RELAXATION OF STRAIGHTENED HARD-DRAWN WIRE, TYPE SO
FIG. 13 RELAXATION LOSS OF TYPE SO WIRE VERSUS STRESS LEVEL
FIG. 14 STRESS-STRAIN CURVE OF TYPE OR-1 WIRE

0.2% proof stress = 215,000 psi
FIG. 15 RELAXATION OF STRESS-RELIEVED WIRE, TYPE OR-1
FIG. 17 STRESS–STRAIN CURVE OF TYPE OR–2 WIRE

0.2% proof stress = 237,000 psi
FIG. 18 RELAXATION OF STRESS-RELIEVED WIRE, TYPE OR-2
FIG. 19 RELAXATION OF STRESS-RELIEVED WIRE, TYPE OR-2
FIG. 20 RELAXATION LOSS OF TYPE OR WIRE VERSUS STRESS LEVEL
0.2% proof stress = 208,000 psi

FIG. 21 STRESS–STRAIN CURVE OF TYPE SR WIRE
FIG. 22 RELAXATION OF STRAIGHTENED AND STRESS-RELIEVED WIRE, TYPE SR
FIG. 23 RELAXATION OF STRAIGHTENED AND STRESS-RELIEVED WIRE, TYPE SR
FIG. 24 RELAXATION OF STRAIGHTENED AND STRESS-RELIEVED WIRE, TYPE SR
FIG. 25 RELAXATION LOSS OF TYPE SR WIRE VERSUS STRESS LEVEL
FIG. 26  RELAXATION OF PRELIMINARY OVERSTRESSED WIRE, TYPE OR-2
FIG. 27 RELAXATION OF PRELIMINARILY OVERSTRESSED WIRE, TYPE OR-2
FIG. 28 RELAXATION LOSS OF PRELIMINARILY OVERSTRESSED WIRE, TYPE OR-2, VERSUS STRESS LEVEL