LOAD-DEFORMATION CHARACTERISTICS OF
SIMULATED BEAM COLUMN CONNECTIONS
IN REINFORCED CONCRETE

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and N. M. Newmark

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1. **Object**

This project was undertaken to obtain data which would be useful in predicting the energy-absorbing capacities (a measure of the resistance to dynamic loads) within the plastic range for beam and column structures of reinforced concrete.

The energy absorbing capacity of the individual members of a structure is a function of the area under the load-deformation diagram for that member. Therefore, the principal objective of this investigation was to develop procedures for the computation of load-deformation characteristics of reinforced concrete members subjected to both shear and flexure.

The initial phase of this investigation, conducted by Gaston (1)*, established relationships for the prediction of load-deformation characteristics of members subjected to pure flexure. To simulate these conditions, tests were made on beams simply supported and loaded at the third points.

The primary purpose of the investigation reported herein was to develop relationships for the load-deformation characteristics at typical beam-column connections. To simulate these conditions tests were made on beams simply supported and loaded at midspan through a column stub.

2. **Scope**

Tests were made on 25 beams with properties varying over a wide enough range to cover those expected in practice. The principal variables

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* Numerals in parentheses refer to list of references at end of this report.
were the concrete strength, \( f_c \), and the percentages of tension reinforcement, \( p \), and compression reinforcement, \( p' \). Other variables investigated were:

(a) The effect of the column stub.

(b) The effect of having no bond between the tension reinforcement and the concrete.

(c) The effect of loading in one direction on the properties in the opposite direction.

(d) The effect of having very small percentages of tension reinforcement in beams with no compression reinforcement.

The range of each variable is given in Table 3.

Because it is generally agreed that shear failures are undesirable, properly designed structures should be adequately reinforced in this respect. Consequently, the beams tested by Gaston (1) and those reported herein were provided with either clamp-on or ordinary stirrups as required to insure flexural failures.

There are several critical stages of behavior which define the load-deformation characteristics of reinforced concrete members in flexure. These stages are, in order:

(a) Yielding of the tension reinforcement, marking the beginning of inelastic action with an accompanying increase in deflection and thus in energy-absorbing capacity.

(b) First crushing of the concrete, indicative of the first permanent damage to the member, and representing a stage at which analytical expressions for moments and deformations can be derived.

(c) Second crushing stage, representing crushing throughout the entire compression zone, and indicative of a significant amount of permanent damage to the member.
(d) Maximum load-carrying capacity. For beams without compression reinforcement or with only small amounts, this stage would usually correspond closely to the second crushing stage. However, if reasonable amounts of compression reinforcement are used, this stage corresponds to buckling of the compression bars and may be significantly beyond the stage described in (c).

(e) Post-maximum stage. Beyond maximum load, the moment-carrying capacity decreases with increased deflection.

In this report, the greatest attention has been paid to the first four stages listed above. At yielding and at first crushing, the behavior of the beam can be described analytically. However, at the second crushing stage and at maximum load, the damage to the compression zone was such that analytical expressions requiring the use of stress-strain relations could not be applied. Instead, at maximum load, empirical expressions were resorted to. The post-maximum stage is given little or no attention in this report; because of buckling of the compression reinforcement and advanced deterioration of the concrete, the behavior in this stage was both unreliable and unpredictable except in general terms, and any energy-absorbing capacity developed after maximum load was considered of doubtful value.

3. Outline of Test Program

The test program can be divided into six series of beam tests, as follows:

(a) Series S, which consisted of four beams reinforced in tension only.

(b) Series B, which contained two beams reinforced in both tension and compression and were the only beams which did not possess a column stub.
(c) Series T, which consisted of 15 beams reinforced both in tension and compression.

(d) Six beams of Series T which were inverted and reloaded. These beams have been assigned numbers with the suffix I.

(e) Beam T-16, reinforced in tension with plain bars lubricated in an attempt to prevent bondage with the concrete. The compression reinforcement in this beam consisted of standard deformed bars.

(f) Series N, which consisted of three beams having no compression reinforcement and very small percentages of tension reinforcement.

The properties of the beams in each series can most readily be seen in Table 3 and 4.

4. Acknowledgment

The tests and studies reported herein were made as part of an investigation of the relation between load and deformation for concrete joints and members conducted in the Structural Research Laboratory of the Engineering Experiment Station of the University of Illinois. This project was sponsored by the Office of the Chief of Engineers, U. S. Army, under Contract No. DA-49-129-eng-248, with funds supplied by the Armed Forces Special Weapons Project.

The program of the investigation was guided by Dr. N. M. Newmark, Research Professor of Structural Engineering. The work reported herein was carried out as a thesis for the M. S. degree under the direction of Dr. C. P. Siess, Research Associate Professor of Civil Engineering.

Acknowledgment is given to Mr. J. R. Gaston who conducted a major portion of the tests. Appreciation is expressed to Mr. J. H. Appleton for
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5. Notation

The following notation is used in this report:

- \( A_s \): area of tension reinforcement
- \( A_s' \): area of compression reinforcement
- \( a \): depth of stress block in concrete at first crushing
- \( b \): width of rectangular beam
- \( C_1 \): compressive force in concrete
- \( C_2 \): compressive force in compression reinforcement
- \( d \): depth from compression face of beam to centroid of tension reinforcement
- \( d' \): distance between the centroids of the compression and tension reinforcement
- \( E_c \): modulus of elasticity of concrete; assumed approximately equal to 1,800,000 psi + 530 \( f_c' \)
- \( E_s \): modulus of elasticity of the reinforcing steel assumed to be 30,000,000 psi
- \( f_c \): computed concrete stress on top of beam (straight line theory)
- \( f_{s'} \): stress in tension reinforcement
- \( f_{s'}' \): stress in compression reinforcement
- \( f_{y'} \): yield point of tension reinforcement
- \( f_y' \): yield point of compression reinforcement (obtained from test in tension)
\( f_{ult} \) = ultimate tensile strength of steel reinforcement

\( k_d \) = depth to neutral axis of transformed section for beams reinforced in tension only (straight line theory)

\( k_d' \) = depth to neutral axis of transformed section for beams reinforced in tension and compression (straight line theory)

\( k' \) = \( d'/d \)

\( k_1, k_2 \) = coefficients defining the magnitude and position of the internal compressive force in the concrete (Fig. 38)

\( k_3 \) = ratio of concrete compressive stress in the beam at failure to compressive strength of standard test cylinders, \( f'_c \)

\( L \) = length of beam span

\( M_y \) = bending moment at yield point

\( M_c \) = bending moment at first crushing of concrete

\( M_{max} \) = maximum bending moment developed

\( n \) = \( E_s/E_c \) = modular ratio

\( p \) = \( A_s/bd \)

\( p' \) = \( A'_s/bd \)

\( q \) = \( pf_y/f'_c \)

\( q' \) = \( pf_y - p f'_y \)

\( f'_c \)

\( T \) = total force in tension reinforcement

\( w \) = width of column stub or bearing block

\( \Delta_y \) = midspan deflection at yield point

\( \Delta_c \) = midspan deflection at crushing of concrete
\[ \Delta_{\text{max}} \] = midspan deflection at maximum moment carrying capacity

\[ \varepsilon_s, \varepsilon_c \] = strain in tension and compression steel, respectively

\[ \varepsilon_y, \varepsilon_y \] = strain in tension and compression steel, respectively, at yield point

\[ \varepsilon_{\text{wh}}, \varepsilon_{\text{wh}} \] = strain in tension and compression steel, respectively, at work hardening

\[ \varepsilon_{\text{ult}}, \varepsilon_{\text{ult}} \] = ultimate strain at fracture in tension and compression steel, respectively, from tension test

\[ \varepsilon_u \] = concrete strain at first crushing

\[ \phi_y \] = maximum curvature in beam at yielding

\[ \phi_c \] = maximum curvature in beam at crushing

\[ \phi_{\text{max}} \] = maximum curvature in beam at maximum load-carrying capacity
II. DESCRIPTION OF TEST SPECIMENS, MATERIALS, AND FABRICATION

6. Description of Test Specimens

The test specimens were all of 6 by 12-in. in cross section and 10-ft. long with a span between supports of 9 ft. The beams of series S, T, and N had a column stub of dimensions 6 by 6 by 12-in. cast integrally on the top of the beam at midspan as shown in Fig. 1. Beams T-15, T-16, N-2, and N-3 also had a column stub of the same dimensions cast integrally on the bottom of the beam directly beneath the top stub as indicated by the broken line in Fig. 1.

The load was applied either through the column stub or through an 8-in. long by 2-in. thick steel bearing block for beams B-34 and B-35 and the inverted beams T-8I, T-10I, T-12I, T-13I, and T-14I which had no stub on the top of the beam as tested.

The principal variables and their ranges of values, omitting the inverted beams, were as follows:

- Tension steel percentage, \( p \) 0.17 to 5.10%
- Compression steel percentage, \( p' \) 0 to 4.08%
- Concrete Strength, \( f'_c \) 1905 to 6407 psi
- Ratio, \( p'/p \) 0 to 2.202

\[
q = \frac{pf'_Y}{f'_c}
\]

\[
q' = \frac{pf'_Y - p'f'_Y}{f'_c}
\]

0.0132 to 0.9520

-0.2804 to 0.4488
Stirrups when used consisted of No. 3 deformed bars bent into a rectangular loop with both ends hooked around a longitudinal reinforcing bar on the tension side of the beam. This corresponds to method "n" as used by Gaston (See Ref. 1, Fig. 4). The spacing was either 4 or 6 in. and was chosen so that the stirrups would be capable of carrying all of the predicted maximum shear force at a unit stress not in excess of their yield point.

7. Materials

The cement and aggregates used in the test beams had substantially the same properties as those used by Gaston (1).

The reinforcing steel was of intermediate grade meeting the requirements of ASTM designation A15-39. All the bars used as longitudinal reinforcement were deformed to meet the requirements of ASTM designation A305-50T, except those in beam T-16 which were plain round bars. A photograph of samples of these bars is given in Fig. 6 of Ref. (1). The steel was purchased in 22 ft. lengths.

Properties of each bar determined from tension tests are presented in Table 2. A typical stress–strain diagram for the steel bars is given in Fig. 2. The tension tests were made in a 120,000-lb. capacity Baldwin Southwark Tate-Emery hydraulic testing machine. The strains for the most part were measured with an 8-in. extensometer and recorded with an automatic recording device. However, for the later tests, it was found necessary to use an 8-in. dial extensometer to obtain readings for the large strains after work hardening required for comparison with those developed in the tension reinforcement of the test beams.
8. Fabrication and Curing

(a) Preparation of Steel Reinforcement. The steel reinforcement was prepared in the following manner:

(i) Two pieces, each slightly less than 10-ft. long, were cut from each bar, leaving a smaller length which was used as the tension test specimen. In Beam T-16, 1-in. thick steel plates were welded to the ends of the unbonded tension reinforcement to provide end anchorage.

(ii) Gauge points for the mechanical strain gauges were then marked and punched and the gauge holes drilled.

(iii) Electricians tape was placed over each gauge hole for protection during casting.

(iv) Corks were wired to the bars at each gauge-hole location so that core holes were formed through the concrete providing access to the gauge holes in the tension reinforcement. Although these core holes may have influenced the crack formations the effect was negligible since cracks would have developed in any event before the steel stresses became large.

After the stirrups and ties had been fabricated, the reinforcement was assembled and placed as follows:

(i) The stirrups were placed around the longitudinal reinforcement and securely wired to it to form a unit.

(ii) The unit was then placed in the steel forms and proper spacing assured by spacers and chairs to give approximately one inch of concrete cover at all surfaces.

(iii) In the beams with compression reinforcement, SR-4 electric resistance strain gauges were mounted on the compression steel at
the positions indicated in Fig. 3. These gauges were of Type A-11 with 1-in. gauge lengths. They were carefully waterproofed with Petrolastic asphaltic compound or Cycleweld C-14 cement so that they would function properly after being embedded in the concrete. The mounting of these gauges was facilitated by grinding off the lugs over half the perimeter of the bar for a distance of 1.5 in. That the yield point and the ultimate strength of a bar are little affected by removal of the lugs has been shown by Hognestad (Ref. 2, p. 18).

(b) Casting of Concrete. All concrete was mixed in a non-tilting drum-type mixer of 6-cu. ft. capacity and placed in the forms with the aid of a high frequency laboratory-type internal vibrator. Two batches were required for each beam. The strength of these batches varied even though the mixer was first conditioned with a "butter mix". The first batch was placed in the outer quarters of the beam while the last was placed in the central half so that the critical central section would have uniform concrete strength. The properties of the mixes are given in Table 1.

The consistency of the concrete was difficult to control because the moisture content of the aggregate varied. This is evident in the variation of slump from 1/2 to 6 in.

One day after the beams were cast the steel forms were removed and the beams stored in a moist room for six additional days after which they were stored in air until tested. Beams N-2 and N-3 however were kept in the moist room until testing began. Six 6 by 12-in. control cylinders were cast from each batch of concrete and were vibrated and cured in the same manner as the beams. Three cylinders from each batch were tested at seven days to give an indication of the compressive strength. The other three cylinders from each batch were tested on the day the beam was tested.
SR-4 electric strain gauges were mounted on the top surface of the beam near the column stub. Their number and position varied with the beams and are best illustrated by Fig. 4. Gauge points for mechanical concrete strain measurements on the sides of beams T-5 to T-16 inclusive were made by cementing small steel cylinders to the side of the beams. Each cylinder had a gauge hole drilled into the center of the plane surface facing outward. Their positions are indicated in Fig. 3.
III. DESCRIPTION OF APPARATUS AND TEST PROCEDURE

9. Description of Test Apparatus

(a) Loading Apparatus. The beams were loaded in a 300,000 lb. capacity Riehle screw-type testing machine with the arrangement shown in Fig. 5. The beam was offset in the machine to provide access to its entire length for mechanical strain gauge readings. The pedestal and jack on the west end (left side of Fig. 5) supported the loading beam so that it remained approximately level during testing. Load was measured by means of a 50,000-lb. capacity elastic-ring dynamometer placed immediately above the column stub. (Fig. 5).

(b) Instrumentation. Strains in the tension steel were measured on six-inch gauge lengths through the core holes. A Berry-type mechanical strain gauge was used for strains less than approximately one percent. A direct-reading mechanical gauge equipped with a 0.001-in. dial indicator was used for strains greater than one percent. The precision of the Berry readings were on the order of 0.00003 strain, and those of the direct-reading gauge approximately 0.001. The six-inch gauge lines on the tension reinforcement extended across the full span of the beam as shown in Fig. 1.

The compressive strains on the top surface of the concrete were measured with SR-4 electric resistance strain gauges. The arrangement, number, and type of these gauges differed with the beam series and are shown in Fig. 4.

The strains in the compressive steel were measured with Type A-11 SR-4 electric resistance gauges of 1-in. length at locations indicated in Fig. 3.
Concrete strains along the sides of beams T-5 to T-16 inclusive were measured with the Berry and direct mechanical gauges used for the tension reinforcement.

Concrete strains along the sides of beam N-1 were measured with Type A-9 SR-4 electric resistance strain gauges centered below the faces of the column stub at distances of 2-in. and 10-in. from the bottom of the beam.

The deflections of the beams with respect to the bed of the testing machine were measured with a steel scale divided into hundredths of an inch. The measurements were made at nine locations marked by paper targets cemented to the beams at the locations shown in Fig. 1. The midspan deflection was more accurately measured with a 0.001-in. dial indicator deflectometer. The beams of Series N had a special arrangement for measuring deflections at mid-span and at the quarter-points from the top of the beam so that there was no necessity to have dials beneath the beams if a sudden collapse occurred.

10. Description of Test Procedure

After the beams had cured from 24 to 56 days they were placed in the testing machine and prepared for testing.

The beam was then loaded in increments. Before yielding, the beam was loaded through four to six approximately equal increments of load as measured by the dynamometer dial. After yielding, the loading rate was governed by the increments of deflection, since the deflections increased rapidly with little increase in load. Loading was continued beyond the maximum load until complete collapse occurred or until it was evident that the specimen was unstable. During the application of each increment, the load and the midspan deflection were observed simultaneously.
After the application of each increment of loading the machine was stopped and the following data were recorded:

(a) The maximum dynamometer load obtained during the increment.

(b) Deflections along the beam together with the midspan deflection indicated by the dial gauge.

(c) Strains in the concrete from both the electric resistance and mechanical gauges.

(d) Strains in the compression steel from the electric resistance gauges.

(e) Strains in the tension steel from the mechanical gauges.

(f) Notes as to the general behavior of the beam.

(g) Photographs were taken at each significant change in the appearance of the beam. The cracks and crushing zones were marked with black pencils in order that they be easily seen in the photographs. Positive film strips were obtained from the 35 mm. negatives and projected on a screen for later study.

The three concrete cylinders from each batch were tested on the same day the beam was tested.

The recording of all the data listed above, together with the application of each increment of loading, required approximately 20 minutes. The complete test of one beam required from five to eight hours.
IV. RESULTS OF TESTS

11. Nature of Observations

The nature of the observations were such as to permit a complete study of the actual behavior throughout the stages to failure, especially in the region near the column stub where the plastic hinge formed. They were also selected so that the validity of any assumptions in the analysis could be determined.

Loads and corresponding midspan deflections were observed simultaneously during the application of load and recorded at frequent intervals. Plots of load vs. deflection are most helpful in obtaining an overall picture of the behavior of a beam. Load-deflection curves for the various beams tested are presented in Figs. 6 through 29, and are referred to further in the subsequent portions of this chapter.

The point at which first visible crushing occurred is indicated on each of these curves by a legend and arrows. In addition, the points marked "C" correspond to first crushing as indicated by the strain measurements, as explained subsequently in Section 21; and the points marked "M" correspond to the maximum load.

The measurements taken after each increment of load have been described in Section 10; the purpose of each type of measurement was as follows:

Deflections were measured at several points along the span in order that the deflected shape of the beam could be plotted. Such a plot clearly indicated a concentration of rotation at either side of the column stub at loads beyond yielding, indicative of the formation of a "plastic hinge".
Strains on the top concrete surface near the column stub were measured to detect any localized effects at the intersection of the top beam surface and the face of the column stub and also to determine the strain at which the concrete began to crush. The strain measurements in the concrete with mechanical gauges along the side of the beam were made to assist in the determination of the location of the neutral axis and also in the determination of strains at the top of the beam after the SR-4 gauges had ceased to function. Plots of load-strain curves for Beam T-11 and also a typical distribution of concrete strain near the stub section at yielding, are shown in Figs. 31 and 32, respectively.

Strains in the compression steel were measured to permit a study of the general behavior and also to test the applicability to the center-loaded beams of the assumptions used by Gaston (1) that the compression steel had yielded at maximum load. A plot of load versus strain in the compression steel is shown in Fig. 31 for one beam.

The tension steel strains were also very important, as the major portion of the angle change was caused by these strains, especially after yielding occurred. They were measured over six-inch gauge lengths with no attempt to study the local bond failures in the vicinity of the cracks. However the measurements did indicate that bond failures occurred in the column stub section so that strains across this section were substantially constant and equal to those on either side adjacent the stub.

The general nature of the tension steel-strain distribution at yield loads is shown in Fig. 32, and at maximum load in Figs. 34 and 35.
12. Behavior and Mode of Failure of Test Beams

The behavior of the beams differed appreciably because of the large range of variables. It is therefore necessary to separate the beams into the following groups:

(a) Series S, with no compression reinforcement
(b) Series B and T, with compression reinforcement
(c) Beam T-16, with plain unbonded bars
(d) Series N, with no compression reinforcement and having very small amounts of tension reinforcement.

Tables 1, 2, 3 and 4 should be referred to for assistance in picturing the properties of the beams in the several series.

(a) The first group of beams, together with those in Series N, differed from those of the other series in that they had no compression reinforcement or stirrups. The S beams differed from the N beams in that they had much larger percentages of tension reinforcement as well as lower concrete strengths. Nevertheless the S beams were still considered to be under-reinforced as judged by the criterion that the tension steel in an under-reinforced beam yields before the concrete crushes. The behavior of the beams in Series S was essentially elastic until the tension reinforcement yielded, although there was a change of slope in the load-deflection curve as cracks developed in the tension zone. After yielding, the beams underwent large deflections with a gradual increase in load until the concrete crushed. Because the percentage of reinforcement was small, the tension steel had strained into the work-hardening region. The load-deflection curve then dropped off rapidly, except in the case of beams S-7 and S-12 (Figs. 3 and 9). After yielding, the load level for these beams fluctuated in a manner
which suggested bond failures. Generally speaking, bond failures enable a beam to attain larger deflections.

The behavior of these beams was substantially the same as that of similar beams loaded at the third points, as indicated by the comparison of beams S-8 and T11b in Fig. 27.

(b) Series B and T are the most extensive test series and form the major part of the investigation. The beams in these two series were reinforced both in tension and compression and were adequately reinforced in shear to insure flexural failures.

A general picture of their behavior is shown by a typical load-deflection curve such as that for T-2 or T-6 in Fig. 16. The first part of the curve is typically elastic, the load increasing linearly with little increase in deflection. Yielding of the steel then takes place, and plastic behavior of the beam is thus initiated and accompanied by large deflections with slight increase in load-carrying capacity. First crushing of the concrete takes place usually soon after yielding, but in some cases before yielding. However, this does not indicate that the beam has failed, since the compression reinforcement is still capable of carrying compressive forces. Ultimate failure occurs by crushing of the concrete and buckling of the compression reinforcement or by fracture of the tension reinforcement. Usually there is a leveling off of the load-deflection curve before the actual drop begins at maximum load, probably indicating the start of compression bar buckling or instability of the concrete in the compression zone.

Before yielding occurs, the behavior of these beams was substantially the same as the behavior of those in Series S with no compression reinforcement. However, after first crushing took place, their behavior differed
because the compression reinforcement then began to resist a greater portion of the compression force. After the concrete in the compression zone begins to behave inelastically, it undergoes lateral deformation. This deformation strains the stirrups and the compression reinforcement and to this action there is a corresponding reaction confining the concrete in much the same manner as spiral reinforcement confines the concrete core in a spiral column. The photograph of beam T-3 in Fig. 26a illustrates this behavior by showing a stirrup bent out by this lateral deformation. Because the compression zone under these conditions of restraint is able to resist forces and deformations much better than in a beam not reinforced in compression and supplied with stirrups, the beams with compression reinforcement have a considerable reserve of ductility after first crushing. However, the degree of effectiveness varies, and the following examples considering two extreme cases should be noted. A beam without compression reinforcement which is "over-reinforced" by conventional standards, that is a beam in which the concrete crushes before the tension reinforcement has yielded, has its ductility increased considerably by the addition of compression reinforcement. However, for beams without compression reinforcement that have very small percentages of steel and high concrete strengths, the ductility is not increased to any great extent by the addition of compression reinforcement, since the compression zone is already strong enough to resist a force as great as that which can be developed by the small percentage of tension reinforcement, even if the tension steel enters the work-hardening range, or in some cases, reaches its ultimate strength as in the case of the grossly under-reinforced beams of Series N.
The beams of the Series B and T can be said to have two stages of failure; that is, (a) crushing of the concrete, and (b) reaching of the maximum load-carrying capacity. At each of these stages two possible conditions may exist:

(a) **At first crushing of the concrete.** There are two modes of failure at this stage corresponding to the two modes of ultimate failure in beams with no compression reinforcement:

(i) First crushing of the concrete while the tension steel is still in the elastic range. This case of first crushing was exhibited by beam T-3. (Fig. 14).

(ii) First crushing of the concrete after the steel has yielded. All beams of Series T and B were of this type, with the exception of T-6 and T-3.

(b) **At maximum load carrying capacity.** Here again it was found necessary to divide the failure into two distinct modes:

(i) Crushing of the concrete and buckling of the compression reinforcement. In this type of failure the compression zone undergoes such large deformations that the concrete finally becomes so disintegrated that it is able neither to prevent the compression reinforcement from buckling nor to resist the compression force when the compression bar does buckle. However, even though the beam has lost most of its moment-carrying capacity at this point it is still able to undergo some further deflection. Most of the beams were of this type.
(ii) Fracture of the tension reinforcement. In beams with very
tough compression zones relative to the tension reinforce-
ment, that is beams with high values of $p'$ or $f'_c$ and low
values of $p$, the compression zone is able to resist a
force greater than $A_s f_{ult}$ and therefore fracture of the
tension reinforcement is inevitable. Large deformations
accompany this mode of failure, as indicated by beam
T-11 which failed in this manner. (See Figs. 10 and
13).

Although the general behavior of the beams of Series B and T was
similar, there was a marked difference in the movement of the neutral axis
as the load was applied which had a direct effect on the stress in the com-
pression reinforcement. To illustrate the two types of behavior that were en-
countered, consider beams T-11 and T-6. Beam T-11 had a very high ratio of
$p'/p$ and a high value of $f'_c$. It exhibited normal behavior until the tension
reinforcement yielded. Then, with large strains in the tension steel, the
cracks advanced to a position which was actually above the level of the com-
pression steel, with the result that tension was produced in the compression
steel. With further strains in the tension steel, the concrete at the top of
the beam reached its limiting strain and began to crush above the compression
steel. The neutral axis then gradually moved down to a position below the
compression steel which therefore again acted in compression. The relatively
large quantity of compression reinforcement remained elastic throughout the
test and thus provided a continuing source of resistance against either tension
or compression creating a tendency to draw the neutral axis toward it.
Beam T-6 with a low ratio of $p'/p$ and a low concrete strength exemplifies the second type of behavior which was more typical of the majority of the beams. The neutral axis always remained below the centroid of the compression reinforcement. The strains in the compression steel were always in compression and increasing. As the loading on such a beam progressed, the compression steel strain exceeded the yield point strain and then increased, in some cases, to the beginning of work hardening. However, buckling usually occurred before any strains larger than those at work hardening were observed.

The differences in behavior of the various beams in Series B and T can also be seen by the actual appearance of the beams at the critical stages during the loading. Photographs of beams T-6 and T-11 having extreme properties (that is very high values of $p$ and low values of $f_c'$ or vice versa), together with beam T-4 which has intermediate properties, are shown in Figs. 11, 12 and 13, at three stages of behavior: (a) at first crushing; (b) at the second crushing stage, marked by the convergence of the crushed zones with the tension cracks; and (c) at the maximum load-carrying capacity.

The first stage of crushing is shown in the top photograph of each figure. A comparison of these photographs clearly indicates the greater advancement of the tension cracks in beam T-11 which had a very low value of $q'$ than for beam T-6 which had a very high value of $q''$. In fact the tension crack in beam T-11 advanced above the centroid of the compression reinforcement, as predicted by theory. The second stage of crushing is shown in the center photograph of each figure. The previous comments concerning the differences at first crushing are also applicable to the second stage. However, in all three beams, the zone of crushing had increased in depth and, in beams T-4 and T-6, the tension cracks had advanced further. The lower photographs illustrate the three beams at maximum load-carrying capacity.
They show clearly the greater depth of the crushing zone developed in the beams with higher values of \( q' \), and the greater damage incurred by cracking in the lower sections of the beams with smaller values of \( q' \).

While the characteristics of the beams reinforced in tension only differed little from their counterparts in the third-point loading series, differences were noted in the behavior of beams in Series B and T as compared to the corresponding beams reported by Gaston (1). The most marked of these differences is the greater degree to which the tension steel went into work hardening. As a result, the maximum moments were greater for the beams loaded at midspan, as can be seen from the plot in Fig. 30. In Fig. 30, the quantity \( M_{\text{max}}/bd^2f'_c \) is plotted as a function of \( q = pf'_y/f'_c \). The use of the dimensionless parameter \( q \) in expressions for the ultimate moment-carrying capacity of beams failing in a region of pure flexure has been discussed by Gaston (1) and others. The lower curve in the plot is based on the equation

\[
M_{\text{max}}/bd^2f'_c = q(1-q/2)
\]

which represents Gaston's results for the third-point loaded beams reinforced in tension only (see Ref. 1, Fig. 30). Lying immediately above this curve is a group of open circles representing the measured maximum moments for third-point loaded beams reinforced both in tension and compression (1). These points show the effect of increasing the maximum moment by the addition of compression reinforcement. This effect results from both the increased moment arm and the increased strength of the compression zone. The numbers assigned to each point on Fig. 30 represent the values of a dimensionless parameter \( q' = (pf'_y - pf'_y)/f'_c \) which has for beams with compression reinforcement a significance corresponding to that of the quantity \( q \) for beams reinforced in tension only. This parameter will be discussed further in the following chapter. Lying above the open circles is
a group of solid circles representing the test results for the beams of Series B and T, loaded at midspan through a column stub and having both tension and compression reinforcement. Although there is a tendency for these beams to have higher values of \( \frac{M_{\text{max}}}{bd^2f_c} \) because they have higher values of \( p' \) and thus lower values of \( q' \), a careful study shows that even for beams with the same cross sectional properties, that is, equal values of \( q' \), the solid circles lie at a higher level than the open circles, indicating that the maximum moments developed in the midspan loaded beams were higher than those developed in the third-point loaded beams with comparable properties. The reason for the higher maximum moments in the mid-span loaded beams is that the compression zone is able to resist a greater force, allowing the tension reinforcement to go farther into work hardening and develop higher stresses and strains. In the comparison which follows between beams C4zm and C4xma of the third point loaded beam series and the comparable beams T-2 and T-5 of the midspan series, measured strains indicated that the stresses in the compression steel of all four beams were at the yield point. Therefore the noted increase in the resistance of the compression zone for the midspan loaded beams was due to the greater stresses carried by the concrete. This was possible because the concrete in the compression zone of the midspan loaded beams was required to resist the maximum force over only a short length where the maximum moment was localized, rather than over one-third the length of the beam.

The moment-deflection curves for beams C4zm and T-2 are shown in Fig. 28 while those for C4xma and T-5 are shown in Fig. 29. The yield points of beams T-2 and T-5 are approximately 15 and 10 percent higher, respectively, than those of the corresponding third-point loaded beams; this can be attributed primarily to the 10 and 6 percent increases, respectively, in the yield point of the tension reinforcement. Both of the midspan loaded beams, T-2 and
T-5 had higher ratios of maximum moment to yield moment than the corresponding third-point loaded beams, as has been mentioned previously. It may be noted also that the slopes of the moment-deflection curves for the midspan span loaded beams after yielding were greater than the slopes for the third-point loaded beams. This is not contradictory to theoretical considerations which indicate that the moment-angle change relationships should be the same for beams having similar properties. The deflections of the beams in Series B and T, within the plastic range, resulted primarily from a concentration of angle change in the vicinity of the column stub, whereas the deflections of the third-point loaded beams resulted chiefly from angle changes distributed uniformly over the middle third of the span. Consequently, at the same midspan deflection, the maximum angle change at the point of maximum moment was appreciably higher for the beams loaded at midspan than for those loaded at the third-points. This also explains why the tension steel apparently went into work hardening at smaller deflections for the midspan loaded beams. (See positions marked on Fig. 28 and Fig. 29).

(c) Beam T-16

This special case is discussed in Section 17 under the effect of using plain bars for the tension reinforcement.

(d) Series N

This series consisted of three beams with a very small percentage of tension reinforcement and a relatively high concrete strength. (See Tables 3 and 4). In contrast to normally reinforced beams, cracking was a critical stage in the behavior of these lightly reinforced beams. The major portion of the tension force was resisted by the concrete before cracking occurred, and with the formation of a crack, all of the force was transferred
more or less suddenly to the tension reinforcement. The beams were designed in such a manner that the tension reinforcement would be stressed up to or beyond its yield point when it was called upon to resist the force corresponding to the load causing cracking.

Beams N-2 and N-3 were cured moist for the entire period prior to testing, and were tested in a moist condition, while N-1 was allowed to dry out. This was done in order to investigate the effect of shrinkage on the modulus of rupture and the load causing cracking of the concrete.

Beams N-2 and N-3 had cracking loads appreciably higher than N-1, as shown in Fig. 25. This resulted from the higher modulus of rupture developed in the wet concrete as a consequence of less shrinkage (4). After cracking, the load on beam N-1 dropped to a value corresponding to the yield point of the steel, but at a deflection of about one inch the load had increased again to the value causing cracking. This increase resulted from the added stress due to work-hardening of the reinforcement. The beam finally failed by fracture of a single reinforcing bar at a deflection of 3.3 inches.

The behavior of beam N-2 was similar to that of N-1; however, it had about a 30 percent greater cracking load due to the effect of testing wet. After cracking the load dropped and then increased to a maximum at about 1.5 inches but was nevertheless lower than the cracking load. As in the case of beam N-1, beam N-2 failed by fracture of the single reinforcing bar at a deflection of 2.7 in.

The third beam, designated N-3, was also tested in a wet condition. After cracking occurred in this beam, the load dropped about ten percent, then, with increased deflection, the load fluctuated from values below to values about equal to the initial cracking load and then began to drop off steadily.
At a deflection of about 6 in. There was a definite bond failure in this beam. The beam was badly damaged by cracks when loading was stopped at a deflection of about 9.5 inches. This is shown in Fig. 26b.

13. **Effect of Column Stub**

Beams B-34 and B-35 were the only beams without a column stub tested with midspan loading in this investigation. Their behavior can be compared with that for beams T-1 and T-5, respectively, which had properties most nearly comparable. The comparisons are based on the load-deflection curves in Figs. 6 and 7. The only significant difference is that first crushing seemed to occur sooner in the beams with stubs. This was probably a result of the stress concentrations believed to be present at the intersection of the top surface of the beam and the face of the column stub. The variations in the yield loads for B-34 and T-1 can be accounted for almost entirely by the difference in the yield points of the tension steel. It can be concluded therefore that the addition of the column stub in itself had practically no effect upon the load deformation characteristics. Consequently, any differences between the behavior of beams loaded at midspan and at the third-points probably should be attributed to the nature of the loading rather than to the presence of the column stub.

14. **Effect of Compression Reinforcement**

The general effect of adding compression reinforcement and the accompanying ties or stirrups can be studied with the aid of load-deflection diagrams. There are various ways in which comparisons may be made. One method is to compare beams having equal percentages of tension reinforcement...
and concrete strengths; this is done in Fig. 8 in which beams T-9 and S-8 are compared. It can be seen in this figure that both the maximum moment carrying capacity and the deflection are greater for beam T-9 with compression reinforcement and ties. Needless to say, the steel strains at maximum load were also considerably greater in beam T-9, as is shown in Fig. 34. Because the concrete in the compression zone was confined and toughened by the compression reinforcement and ties, it could undergo greater strains without decrease in resistance and thereby permitted the development of large strains in the tension steel. It is also to be noted that beam T-9 failed prematurely by fracture of a tension bar at a gauge hole while the strain was still well below the ultimate strain obtained in the tension test of a specimen taken from the same bar. Therefore there would have been a greater difference between the two beams if beam T-9 had failed normally. However, it can generally be said that the ductility of a beam is increased by the addition of compression reinforcement.

A second method of comparison is to consider beams with equal values of $q'$, since this parameter seems to be a measure of ductility, as affected by the addition of compression reinforcement. Consider beams S-7 and S-12 as compared to T-1, for which load-deflection curves are plotted in Fig. 9. Although these beams were chosen so as to have approximately equal values of $q'$, their other properties were different and the maximum loads which are primarily a function of $p$ and $f_y$ were quite different. It is intended however to base comparisons on the ductility; that is the ratio of deflection at maximum load to that at yielding. The curve for T-1 was fairly characteristic, but the curves for S-7 and S-12 fluctuated up and down after yielding and attained rather large deflections, probably because of bond failures. For
this reason, no definite conclusion can be reached since the modes of failure were not entirely similar.

The above discussion has dealt with the effect of a column stub cast on the top of the beam. Beams T-15, T-16, N-2 and N-3 also had a stub of the same dimensions cast on the bottom of the beam. Although most of the beams could not be compared directly with similar beams having the stub only on the top, the results of the tests and the observed behavior of these beams suggests that the presence of a column stub on the tension side of the beam had no significant effect.

15. Effect of Varying the Percentage of Compression Reinforcement

In this case, comparisons are made in terms of both the load-deflection curves and the distribution of steel strain at or near maximum load-carrying capacity. The corresponding curves are plotted in Figs. 10 and 35, respectively, for beams T-1, T-7, and T-11, all with tension steel percentage, $p$, equal to 1.39 and concrete strengths all within the range 3900 to 4500 psi, but with compression steel percentage $p'$ varying from 0.63 to 4.0. It can be seen that by increasing $p'$ the maximum load-carrying capacity is increased only slightly if any, but the deflections at maximum moment are increased appreciably. However, two important facts should be pointed out:

(a) Beams T-7 and T-11 which had relatively high values of $p'$ failed by fracture of a tension reinforcing bar rather than by crushing of the concrete and buckling of the compression reinforcement as in T-1. The mode of failure exhibited by T-1, that is, crushing of the concrete and buckling of the bars, would be the more desirable of the two modes as the member...
still remains in one piece and additional deflection is possible after the
maximum load-carrying capacity is reached, even though the load-carrying
capacity decreases.

(b) If it is desired to obtain the maximum energy-absorbing cap-
acity in the most economical manner, adding compression reinforcement in
relatively large quantities, as was done in beam T-11, would seldom be
practical. Returning to the actual cases of beams T-1 and T-11 for an ex-
ample, increasing the compression reinforcement by a factor of six increased
the energy absorbing capacity by only about one third.

16. Effect of Increasing p, q and q' with the Ratio p'/p Remaining
   Approximately Constant

Beams with high concrete strength and with low concrete strengths
are considered separately.

Load-deflection curves for beams T-1, T-2, and T-3, with high con-
crete strengths are shown in Fig. 14, and those for beams T-4, T-5, and
T-6, with low concrete strengths are shown in Fig. 15. The effects in both
cases were found to be the same and can be summarized in four definite parts.
As p, p', and q' increase in magnitude, the following effects can be noted in
Figs. 14 and 15:

(a) The yield and maximum loads increase.

(b) There is a tendency for the deflections at maximum moment to
increase even though it can be seen that T-2 has a greater deflection at maxi-

moment than T-1. However in the subsequent theoretical analysis it is
shown that the deflection of T-2 was larger than would normally be expected.
(c) The ratio of maximum load to the yield point load and the corresponding ratio of deflections at those loads decrease.

(d) The slope of the declining portion of the curve after maximum moment becomes steeper.

17. Effect of Increasing Concrete Strength with Other Properties Remaining Nearly Constant

For this case, beams T-2 and T-6 are compared, since their properties are similar except for concrete strength, which was 3858 psi for T-2 and 1905 psi for T-6.

Comparison of the load-deflection curves in Fig. 16 indicates a large increase in the deflection at maximum moment with an increase of concrete strength. The ratios of maximum load to yield load and the corresponding ratios of deflections at these loads increase as the concrete strengths increase. These effects may be expected since increasing the concrete strength should increase its ability to resist compression forces, thereby permitting the tension steel strains and stresses to reach higher values with consequent increases in load and deflection before failure.

18. Effect of Loading in One Direction on Behavior Under Loading in the Opposite Direction

In this study, comparisons are made for five pairs of beams. One beam of each pair was tested in the inverted position after having been previously tested in the normal position. The load-deflection curves for the inverted test are in each case compared with the virgin load-deflection curve for a beam having properties as similar as possible to those of the
Inverted beams. Load-deflection curves for the following beams are given in the figures indicated:

(a) T-10I vs. S-12 Fig. 17
(b) T-12I vs. T-2 Fig. 18
(c) T15I vs. T-7 and T-14I Fig. 19
(d) T14I vs. T-7 and T-8 Fig. 19
(e) T-8I vs. T-5 Fig. 20

The extent to which each beam was loaded and damaged before being inverted is indicated by the load-deflection curves in Fig. 21. The origin of the load-deflection curves for the inverted series is shown in this figure. The pairs of beams are discussed in the same order as listed above.

(a) Beams T-10I and S-12

Although Beam T-10I differed from Beam S-12 in that it had compression reinforcement, comparison of the behavior of the two beams shows the characteristic differences exhibited by all the inverted beams. The first obvious difference in behavior is the manner in which the load-deflection curve changes its slope gradually from the elastic to the plastic stage rather than having a definite yield point (Fig. 17). Another feature is that beam T-10I has lost a considerable portion of its load-carrying capacity since its maximum load is about equal to that of beam S-12 even though beam S-12 has a lower concrete strength and no compression reinforcement.

The large difference in deflections between those two beams was due to two causes: First, the deflection of beam T-10I was relatively small be-
Because it was limited by the bar fracture. And second, a fairly large deflection was developed by beam S-12 as a result of a probable bond failure. The load-deflection curves in Fig. 21 show the extent to which the various beams were loaded before being inverted. Beam T-10 is shown to have been loaded slightly past the first stage of crushing.

(b) **Beams T-12I and T-2**

Although Beams T-12I and T-2 have different properties in some respects (Table 3), it can still be seen in Fig. 18 that the energy absorbing capacity of T-12I was not substantially reduced even though T-12 was loaded until the second stage of crushing occurred, marked by the convergence of the crushed zones with the tension cracks. As before, the load-deflection curve of T-12I did not exhibit a definite yield point.

(c) and (d) **Beams T-14I and T-15I vs. T-7 and T-8**

These four beams are compared on the basis of load-deflection curves in Fig. 19. Here again the general characteristics of the inverted beams are shown. It can also be noted that the load-deflection curve of the inverted beam T-14I falls below those of beams T-7 and T-8 during the early stage of loading although their maximum load-carrying capacities and corresponding deflections are comparable. Both T-14 and T-15 were loaded beyond the first stage of crushing.

(e) **Beams T-8I and T-5**

A comparison of the load-deflection diagrams for these beams clearly shows that a considerable portion of the energy absorbing capacity for beam T-8 was lost. The reason is directly related to the extent beam T-8 was loaded before being inverted. Beam T-8 was loaded beyond both the second crushing stage until the maximum load-carrying capacity was reached and the compression bars began to buckle. The behavior of T-8I suggests a criterion
for determining the extent to which a beam must be loaded in one direction before losing a major part of its energy-absorbing capacity in the opposite direction. That is, if a beam is loaded in one direction until its maximum moment capacity is reached it will probably lose a considerable portion of its energy absorbing capacity, whereas a significantly smaller amount of damage was observed if the initial loading extended only into the crushing range.

A more quantitative picture of the reduction in moment-carrying capacity can be obtained by comparing ratios of the measured maximum moments of inverted beams with the maximum moments computed by the empirical relationship

\[ M_{\text{max}} = A f_y d' (1.50 - 1.08 q) \]

which is derived subsequently. These ratios, given in Table 7, vary in magnitude from 0.44 to 1.02, the 0.44 being for T-81 which had been loaded up to the maximum load-carrying capacity. The next larger ratio was 0.86 for T-121, which was loaded to the second stage of crushing, and the remaining beams which were loaded to or slightly past first crushing all had ratios above 0.90.

In addition to the general behavior discussed above, there was also faulting observed (Fig. 22) across nearly vertical cracks near the column stubs. These cracks, which extended throughout the depth of the beam, were developed as a result of tension forces having been present in both the upper and lower sections of the beam. Furthermore, the stirrups were not in a position to offer resistance to this faulting since it occurred across cracks which developed between the stirrups.

19. **Effect of Using Unbonded Tension Reinforcement**

A rational analysis indicates that if the tension reinforcement in a midspan loaded beam is unbonded along the entire span its stress will
constant throughout its length at any stage of loading. The behavior of the tension reinforcement is therefore similar to that of the tie rod in a tied arch. The total strain over the entire length of the tension reinforcement should be greater than that for a bonded beam where the strain is concentrated over the section of greatest moment in the vicinity of the column stub. Therefore, in contrast to the bonded beams, the unbonded beams should exhibit larger deflections. However, the maximum moments will not be as large because the uniform strains in the tension reinforcement will not extend as far into the work hardening region before the concentrated angle change near the stub becomes large enough to cause crushing of the concrete.

In order to verify this hypothesis, Beam T-16 was tested with the tension bars coated with oil in an attempt to prevent bond. Comparisons of the load-deflection curve for beam T-16 with those for T-1 and B-34 are shown in Fig. 24. Beams T-1 and B-34 had similar properties except that bonded deformed bars were used for the tension reinforcement in the latter. A careful study of this figure leads to several conclusions concerning the behavior of the unbonded beam and the validity of the hypothesis advanced above.

(a) The yield deflection of beam T-16 was approximately twice as great as that of beam T-1. As predicted by theory, this was caused by the strains in the tensile steel being of constant value over the entire span, as shown by the measured distributions in Fig. 33. The deflection at maximum load was also greater for beam T-16. However, at maximum load-carrying capacity the strains were not uniformly distributed along the span since the strains corresponding to yield stresses in the reinforcement were not necessarily constant. Furthermore, at large deflections, frictional bond
as developed between the reinforcing bars and the adjacent concrete and the stress could therefore vary somewhat from point to point.

(b) The maximum load-carrying capacity of beam T-16 was decreased. In fact, the load actually began to decrease immediately after yielding, and subsequently only occasionally reached a value above the yield load. This also is in agreement with the theory.

(c) After yielding, the load-deflection curve had up and down fluctuations which could be attributed to the building up and releasing of the resisting moments as each of a series of progressive bond slippages or failures occurred.

(d) The downward slope of the deflection curve beyond maximum and for beam T-16 is quite steep as compared to that for beam T-1 or B-34. The appearance of beam T-16 at maximum load is quite different from that of the comparable bonded beams (Fig. 23). Rather than having numerous cracks along the tension zone, there was only one major crack on either side of the column stub. This resulted from the absence of bond required to transfer tensile stresses back into the concrete adjacent to the first crack that formed.

In summary, the effect of using unbonded tension reinforcement tends to be comparable with that predicted by the hypothesis. The energy-absorbing capacity of the unbonded beam up to yielding seemed to be increased. However, although slightly greater deflections were developed by the unbonded beam, the maximum moment was less and the overall energy-absorbing capacity was no greater, if not less, than that for the similar bonded beams.
V. EXPRESSIONS FOR CRITICAL MOMENTS AND DEFORMATIONS

Moments and Deflections at Yielding of the Tension Reinforcement

Yielding of the tension reinforcement is the first critical stage in the behavior of a reinforced concrete member since it marks the boundary between elastic and inelastic behavior, corresponding to an abrupt change in slope of the load-deflection curve. The moment referred to as the yield moment of the member is well defined on the load-deflection curves in Figs. 28 through 29.

Expression for Yield Moment

The prediction of the yield moment can be made by means of expressions developed by Gaston (1) using the same assumptions and method of analysis.

To develop these expressions for yield moment the following factors must be known:

(a) The force in the tension reinforcement.
(b) The distance between the tension force and the center of compression.

The tension force is the product of the yield point of the tension steel, \( f_y \), and the area of the tension steel, \( A_s \). The distance or moment between the tension force and center of compression is \( d \). This distance is determined by assuming that the stresses in the concrete are distributed linearly, as was done by Gaston (1). The yield moment is the product of the tension force and the moment arm. Therefore the expressions for the yield moments are:
(a) For beams without compression reinforcement

\[ M_y = A_s f_y j d \]  
(1)

where

\[ j = (1 - k/3) \]  
(2)

\[ k = \sqrt{2 n p + (n p)^2 - n p} \]  
(3)

\[ n = E_s / E_c \]  
(4)

\[ E_s = 30,000,000 \text{ psi} \]  
(5)

The value of \( E_c \), the initial tangent modulus of elasticity for the concrete may be determined as a function of \( f'_c \) from the following empirical equation.

\[ E_c = 1,800,000 \text{ psi} + 530 f'_c \]  
(6)

(b) For beams with compression reinforcement

\[ M_y = 1/2 k' b d^2 f_c (1 - k'/3) + k'' d A'_s f'_s \]  
(7)

where

\[ k' = \sqrt{2 [n p + (1 - k')(n - 1)p' + (n - 1)p'] + [n - 1] p' + n p} \]  
(8)

\[ f_c = \frac{p f_y}{1/2 k' + p (k + k' - 1) n} \]  
(9)

\[ f'_c = \frac{(k'' + k' - 1)}{k'} f_c n \]  
(10)
Comparison of Measured and Computed Yield Moments

The computed values for the yield moments are compared with the measured values in Table 5. For beams without compression reinforcement, the ratios of the measured to computed yield moments averaged 1.10 and ranged from 1.06 to 1.17. For beams with compression reinforcement, the ratio of measured to computed yield moments averages 1.03 and ranges from 0.95 to 1.08. These values compare well with the corresponding ratios for the third point loaded beams (1). The computations in this section and the subsequent sections always took into account the moment due to the weight of the beam.

There are several factors which may lead to errors in the computation of yield moment:

(a) The yield point of the tension steel may differ from that indicated by the test specimen even though the specimen was obtained from the same bar. The difference between measured and computed yield moment for beam 5-12 must be accounted for in this fashion because the computed moment would not be as high as the measured moment even if the moment arm for the force $F_y$ was taken as large as $d$.

(b) The concrete strength may also differ somewhat from that indicated by the cylinder tests; however, this would change the results very slightly.

\[ n = \frac{E_s}{E_c} \]  
\[ E_s = 30,000,000 \text{ psi} \]  
\[ E_c = 1,800,000 \text{ psi} + 530f_c' \]
(c) In beams with high values of $p$ and low values of $f'_c$ and $p'$, that is, high values of $q$ and $q'$, large concrete stresses are developed at the yield moment. Hence, the use of a secant modulus of elasticity for the concrete would be more appropriate than the initial tangent modulus. This would give higher values of $n$, and would therefore yield higher values of $k$ or $k'$. Higher values of $k$ or $k'$ would in turn give lower values of $j$.

The ultimate effect is that of lowering the computed yield moments. However, since the ratio of the measured to computed moments is usually greater than one, even for the beams with high values of $q$, it is apparent that this effect is probably not present to any marked degree or, if present, is offset by other sources of error.

In conclusion, the use of the conventional "straight line" expressions have been found to be quite satisfactory for the prediction of the yield moments.

**Midspan Deflections at Yield Moment**

The prediction of the yield deflection is based on essentially the same principle as that used by Gaston (1), with an additional assumption as to the distribution of angle change through the column stub or bearing block section.

The midspan deflection of a simple beam supported at two points $A$ and $B$ is equal to the bending moment at midspan for a beam simply supported at $A$ and $B$ and loaded normal to the line connecting $A$ and $B$ with the portion of the curvature or $M/EI$ diagram between $A$ and $B$. (See Ref. 3, section 153). Therefore, if $M/EI$, that is the curvature, is known at each point along the beam, the midspan deflection can readily be computed.
The curvature or angle change may be computed from the expression \( \frac{1}{E} I \), where \( I \) is the moment of inertia of the section of the beam transformed to concrete (1). However, the curvature may be more readily determined by using a geometrical expression since it has fewer terms and the calculations can utilize information obtained previously from the yield moment computations. That is, the curvature is equal to the strain in the tension steel divided by the distance from the centroid of the tension steel to the neutral axis. At the critical section, where the greatest moment at yielding occurs, the curvature \( \phi_y \) is as shown in Fig. 36.

For beams with tension reinforcement only

\[
\phi_y = \frac{\varepsilon_y}{(1-k)d}
\]  

(11)

For beams with compression reinforcement

\[
\phi_y = \frac{\varepsilon_y}{(1-k')d}
\]  

(12)

The values of curvature at yield moment at other sections of the beam are assumed to be as follows: From the critical section at the face of column stub or bearing block to the support, the curvature is assumed to vary linearly from \( \phi_y \) to 0. Through the width of the stub or bearing block, the curvature is assumed to remain constant and equal to \( \phi_y \). Pictorially the assumed distribution of curvature is shown in Fig. 37.

Using the principle stated previously, the expression for computing the deflection at yielding is therefore

\[
\Delta_y = \frac{\phi_y}{24} (2L^2 + 2wL - w^2)
\]  

(13)
Comparison of Measured and Computed Yield Deflections

The ratios of measured to computed values for the yield deflections are shown in Table 5. These ratios average 1.13 and range from 0.98 to 1.29 for the B and T Series, and average 1.00 with a range of 0.90 to 1.10 for the S Series.

The actual curvatures tend to be higher than those computed, especially in beams with high values of q, primarily because of the effect of using the initial tangent modulus rather than the secant modulus. The effect is to increase k or k' as shown previously for the moment computations, and in turn to reduce $1 - k$ or $1 - k'$ with the result that the computed deflections should be lower than the measured values. The comparisons of measured and computed yield deflections support this conclusion as evidenced by the average ratio for the B and T Series of 1.13, and also by the tendency for this ratio to increase with higher values of q. The average ratio of measured to computed deflection for Series S is 1.00, where the average value of q was comparatively lower. As in the case of yield moments, these ratios compare well with the corresponding ratios for the third-point loaded beams.

The assumption that $M/EI$ or curvature varies linearly from the face of the stub or edge of the bearing block to the supports is correct, provided that EI remains constant, since the moment is governed by statics and varies linearly. The assumption of a constant value of $E_c$ along the length of the beam is valid provided the stresses in the concrete are not so great as to introduce an appreciable difference between the secant and tangent moduli. For beams with high values of q and accompanying high concrete stresses at yield, the decrease in concrete stress at sections away from midspan could conceivably
cause some variation in the effective modulus. Similarly, the varying height of the cracks as a function of the varying moment along the span could introduce variations in the moment of inertia. However, neither of these effects can conveniently be taken into account, nor does this seem necessary in view of the relatively good agreement between measured and computed deflections. The assumption that the distribution of curvature remains constant over the width of the stub or bearing block was considered to be justified after the values of measured tension steel strains and measured concrete strains near the stub had been studied.

An example of the measured distribution of tension steel strains is given in Fig. 37. The validity of the assumption regarding the tension steel strains is apparent. It should be noted, however, that this condition would result only if the width of the column stub or bearing block is small enough that propagation of cracking and bond failure may occur across the full width of the stub.

An example of the measured concrete strains is given in Fig. 32. High values of strain near the face of the stub are seen to be present. However, through the stub, the concrete strains were probably smaller since the depth of the cross section was increased.

In view of these considerations and the comparisons of measured and computed values of yield deflections, the assumption that curvature was constant and equal to $\theta_y$ over the width of the stub was used, since it was simple and partially compensated for the high strain concentration near the face of the stub.

In conclusion, the use of expressions similar to those used by Gaston was found to be satisfactory for the computation of yield deflections.
Moments and Deflections at First Crushing

The first critical stage of behavior was yielding of the tension reinforcement. The second critical stage is first crushing of the concrete at the critical section near the column stub. The amount of damage that has occurred at this stage probably does not seriously impair the future usefulness of the member (Section 18).

This stage of behavior for midspan-loaded beams corresponds more or less to the ultimate stage for beams reinforced in tension only and subjected to pure flexure. However, in the midspan-loaded beams the crushing is localized and further increases in both the moment-carrying capacity and deflections take place before the maximum load-carrying capacities and corresponding deflections are reached (Section 12). This effect is rather insignificant for the beams reinforced only in tension as shown by the comparisons of first crushing moments and deflections with the maximum moments and deflections (Figs. 8 and 9). However, the effect is quite considerable for beams with both tension and compression reinforcement (Figs. 6 and 7), since these beams had an appreciable reserve of strength and ductility after first crushing occurred.

Expressions for Crushing Moment

The predictions of the first crushing moments can be made by a trial and error procedure with conditions similar in some respects to those used by Reissner for the ultimate moments.

The crushing moment can be computed if the compression forces and the moment arm between the centroid of these compressive forces and the centroid of the tension force are known. The compressive forces can be computed on the basis of the assumption that the stress block in the concrete is
fully developed when the concrete begins to crush at a limiting strain of 0.004. Stresses in the compression and tension reinforcement must correspond to strains compatible with the assumed limiting concrete strain. These conditions are shown in Fig. 38. From statics, the tension force must equal the compressive force; that is

\[ T = C_1 + C_2 \]  \hspace{1cm} (14)

and

\[ A_s f_s = k_1 k_3 f'_c a b + A'_s f'_s \]  \hspace{1cm} (15)

from which

\[ a = \frac{A_s f_s - A'_s f'_s}{k_1 k_3 f'_c b} \]  \hspace{1cm} (16)

In equation 16 the values of \( f'_c \), \( b \), \( A_s \), and \( A'_s \) are known, and the value of the parameter \( k_1 k_3 \) was computed from the following empirical expression:

\[ k_1 k_3 = 1.37 \frac{10.8f'_c}{10^5} \]

However, three variables are still unknown. Since the compression steel could be in the yield range, in the elastic range, or in some cases in tension, no single assumption could be made as to the magnitude of its stress. Furthermore the tension steel stress could be in the elastic, yielding, or work

This expression differs from that used by Gaston in Reference (1) and given in Eq. 14 of that reference; nevertheless it is an adequate representation of the data plotted in Fig. 19 of Reference (1).
hardening range, and again no definite value could be assigned. The procedure used therefore, was to assume a value for the depth of the compression zone a, and a value of 0.004 for the unit strain in the concrete on the top of the beam at the critical section near the column stub. Stresses compatible with strains, assuming a linear strain distribution, were then calculated for both the tension and compression reinforcement. If equation 15 was satisfied when the values of these stresses were substituted in it, the corrected value of had been chosen. If the equation was not satisfied, new values of a were assumed and the calculations repeated until the equation was solved.

In the beams without compression reinforcement the solution is simplified since

\[ T = C_1 \]  

(17)

and

\[ A_{s_1}^{f_1} = k_1 k_{f_1}^{c_1} \]  

(18)

from which

\[ a = \frac{A_{s_1}^{f_1}}{k_1 k_{f_1}^{c_1}} \]  

(19)

However, from the strain diagram (Fig. 38)

\[ \frac{a}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_s} \]  

(20)

and

\[ a = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_s} \]  

(21)
Solving equations 19 and 21 one obtains

\[
\frac{A_s f_s}{k_1 k_2 f_c' b} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_s} d \quad (22)
\]

\[
f_s = \frac{k_1 k_2 f_c' b d c_u}{A_s (\varepsilon_u + \varepsilon_s)} \quad (23)
\]

All the terms in the equation are known or have been assigned values except \( f_s \) and \( \varepsilon_s \). Equation 23 thus expresses a relationship between \( f_s \) and \( \varepsilon_s \). The stress-strain curve is another such relationship. When these two relationships are satisfied simultaneously, the correct values of \( f_s \) and \( \varepsilon_s \) are known. This solution was obtained graphically using equation 23 and the actual stress-strain curve for the tension reinforcement.

Once \( f_s \) is known, the tensile force \( T = A_s f_s \) can be computed. Also, the depth to the neutral axis \( \bar{d} \) can be obtained from equation 19, and the compressive force \( C_1 \) can be easily computed.

Once the compressive stresses have been determined for beam either with or without compression reinforcement, the moment arms of the compressive forces were then required to compute the crushing moment. The moment arm of the compression steel force about the centroid of the tension reinforcement is simply \( \bar{d}' \). The moment arm for the compressive force in the concrete was assumed to be \( d - k_2 a \), where \( k_2 \), the distance from the top of the concrete to the centroid of the compressive force \( C_1 \), was assigned a value of 0.42.

For beams with compression reinforcement, the expression for the crushing moment is, therefore,

\[
W_c = A_s f_s \bar{d}' + k_1 k_2 f_c' ab(d - k_2 a) \quad (24)
\]
where $f'_s$ and $a$ are computed from the trial and error procedure. The corresponding expression for crushing moment for beams without compression reinforcement is

$$M_c = k_f f'_c ab(d-k_x a)$$  \hspace{1cm} (25)

Comparisons of Measured and Computed Crushing Moments

The stage of behavior of the beams corresponding to the crushing moment computed as described above is not easily defined. At first, an attempt was made to correlate the computed crushing moments with the measured moments corresponding to the stage of first crushing as indicated on the load-deformation curves; these points on the curves were based principally on the written description of the test phenomena. Since the correlation between the computed moments and these measured moments was considered unsatisfactory, a better physical indication of "first crushing" was sought. Further study indicated that a satisfactory measure of first crushing corresponding more closely to the computed moments was obtained from a consideration of the compressive strains in the concrete as measured by the electrical resistance gauges. By this criterion, first crushing was assumed to have occurred when the observed strains reached a maximum value and began to decrease (Fig. 31), probably as a result of detachment of the strain gauge owing to crushing of the concrete. The stage in the test at which this occurred is indicated on the load deflection curves by a vertical line marked "C".

Measured and computed moments at first crushing are compared in Table 6. The ratio of measured to computed moment varied from 0.80 to 1.26. This variation is considerably larger than that for yield moments in Table
but this was to be expected because of the difficulties in obtaining both measured and computed values. However, the rational procedure used for computing the moments at first crushing is considered satisfactory.

**Expressions for Crushing Deflection**

The angle change at the critical section at the face of the column stub or bearing block at first crushing can be determined easily by geometrical considerations in a manner similar to that used for yielding. That is, since the depth of the compression zone is known together with the strains in the tension reinforcement and in the concrete on top of the beam, the expression for the curvature or angle change can be determined by any of the three following expressions.

\[
\psi_c = \frac{\varepsilon_s}{d-a} = \frac{\varepsilon_u}{a} = \frac{\varepsilon_s + \varepsilon_u}{d} \quad (26)
\]

However, the variation of curvature along the length of the beam is difficult to determine, because the above expressions cannot be extended to apply at any section except the one at which the greatest moment occurs. In addition, the strains on the top of the concrete or in the compression steel were quite variable and difficult to estimate.

An empirical approach was therefore adopted, assuming the midspan deflection to result entirely from concentrated angle change at each face of the column stub. This concentrated angle change was assumed to be equal to the computed curvature at the critical section, \((\varepsilon_s + \varepsilon_u)/d\) distributed over a constant length derived empirically by comparison with the measured deflections. Satisfactory results were obtained by assigning the value \(d\) to this required length. The concentrated angle change was therefore equal to
\( \Delta_c = (\varepsilon_s + \varepsilon_u) (L/2 - w/2) \) \hspace{1cm} (27)

Equation 27 applies to beams reinforced in tension only as well as to beams reinforced in both tension and compression.

Comparison of Measured and Computed Crushing Deflection

The measured deflection at first crushing was taken as that corresponding to the measured moment determined as described previously. These deflections are indicated by the line marked "C" on the load-deflection curves.

The ratios of measured to computed crushing deflections shown in Table 6 have an average value of 1.04 and a range of 0.80 - 1.40. The method was therefore considered to be satisfactory.

Although the assumption that the deflection is caused by a concentrated angle change is fairly close to the actual conditions for beams with low values of \( q \), it is far from the true conditions for beams with high values of \( q \), since crushing may occur before yielding, in which case the distribution of curvature is similar to that at yielding. Nevertheless, the results given in Table 6 show no consistent differences for beams with high and low values of \( q \).

Although in previous sections a second stage of crushing marked by the convergence of the crushed zone with the tension cracks was discussed,
no attempt was made to develop expressions for either moments or deflections at this stage. There are two reasons why this was not done: (a) It would have been extremely difficult to define this stage of behavior accurately in terms of measured quantities, and (b), no rational basis could be found for computing moments and deflections at this stage of behavior.

Ductility Factor at First Crushing

In connection with the design of blast-resistant structures, N. M. Newmark (5) has introduced the "ductility factor" which is defined as the ratio of the maximum to the yield deflection. A lower limit to the ductility factor for the beams tested in this investigation is given by the ratio of deflection at first crushing to deflection at yield. Values of this ratio, in terms of measured deflections, are plotted in Fig. 39 as a function of the ratio \( q'/q_{cr} \), and in Fig. 40 as a function of \( q/q_{cr} \). The values plotted in these figures are listed in Table 8.

The quantity \( q_{cr} \) is the value of \( q \), or \( q' \), for which crushing of the concrete will occur simultaneously with yielding of the tension reinforcement. The expression for \( q_{cr} \) is given in Reference 1, equation 23, as

\[
q_{cr} = \frac{k_1 k_3}{1 + \frac{\epsilon_Y}{\epsilon_u}}
\]

The values given in Table 8 were computed for

\[
\epsilon_u = 0.004
\]
\[
\epsilon_Y = \frac{f_Y}{E_s}
\]
\[
E_s = 30 \times 10^6 \text{ psi}
\]
The plot of ductility factor versus $q'/q_{cr}$ in Fig. 39 shows a trend but little positive correlation. This is not entirely unexpected since the yield deflection is not a function of $q'$ and the crushing deflection is related to $q'$ only if the compression reinforcement had yielded at the time the concrete crushed which was true only in certain beams. The curve shown on Fig. 39 is equation A28 from Fig. A9 in the Appendix. This curve represented fairly well the ductility factor at maximum load for the beams reinforced in both known and compression but loaded at the third-points. It is evident from Fig. 39 that the ductility at first crushing of the beams loaded at midspan was appreciably less than that given by the curve for corresponding values of $q'/q_{cr}$. There are at least two reasons for this:

(a) The beams loaded at midspan had relatively large amounts of compression reinforcement, as indicated by the low or negative values of $q'$, with the result that the compression reinforcement was seldom at the yield point when crushing began. And (b), the first crushing stage corresponds usually to a smaller amount of deformation than that at maximum load for the third-point loaded beams; actually, the second stage of crushing for the beams loaded at midspan would be expected to correspond more closely with the maximum load for the beams loaded at the third-points.

A much better correlation was obtained between the ductility factor at first crushing and the ratio $q/q_{cr}$, as may be seen from Fig. 40. The curve shown on this figure is again equation A28 from Fig. A9 of the Appendix. Although this curve appears to fit the plotted points well, the significance of this agreement is obscure since the points on Fig. 40 are plotted as a
function of $q/q_{cr}$ while those on Fig. A9 for beams loaded at the third-points are plotted as a function of $q'/q_{cr}$. It seems, probable, therefore, that the agreement of both of these sets of data with the same equation, using $q$ in one case and $q'$ in the other is only fortuitous and that little significance should be attached to it. Nevertheless, comparison of Figs. 40 and A9 shows clearly that the ductility factors at first crushing for beams loaded at midspan are less than those at maximum load for beams loaded at the third points, since $q$ will always be greater than $q'$ for beams with compression reinforcement.

22. Moments and Deflections at Maximum Moment

After the beams with both tension and compression reinforcement passed through the stages of yielding and first crushing they were still capable of resisting additional moment and undergoing large deflections before the maximum moment was developed. However, by the time that the maximum moment had been reached the beams were very badly damaged, as evidenced by their appearance and by their inability to develop their potential resistance when loaded in the opposite direction (section 18).

In some of the beams of Series S with no compression reinforcement there was an increase in moment-carrying capacity after first crushing, but the amount was negligible. However, in all of these beams the deflection increased considerably after crushing. This behavior could possibly be attributed to bond failures.

The moments and corresponding deflections or rotations at maximum load are undoubtedly of interest as indications of the total energy-absorbing capacity that can be realized before complete collapse under a single loading occurs. However, the importance of this stage should not be over-emphasized.
since the damage to the beams at maximum load was such that any structure composed of similar members would be incapable of resisting further load or withstanding subsequent loads applied in the opposite direction.

Expressions for Maximum Moment

The expression for maximum moment was developed by an empirical procedure since inherent difficulties were encountered when an attempt was made to derive rational relationships. Some of these difficulties were as follows:

(a) The concrete in the upper portion of the beam at the critical section near the column stub had crushed to such an extent that the original cross-section of the beam had been reduced.

(b) The tension steel stresses were usually well above the yield point since the steel had strained into the work hardening range. After work hardening, only the maximum tensile force and final strains were recorded in most of the tension bar tests and therefore the value of the tension steel stress was quite difficult to estimate.

(c) The compression steel stresses were not always at the yield point as assumed by Gaston (1) since there were some beams in the midspan-loaded series with relatively high values of p.

(d) In addition, it was believed that the combination of stirrups and compression reinforcement confined the concrete in the compression zone so that it was able to continue resisting large forces even though considerable deformation took place.

The empirical relationship for maximum moment was derived by arbitrarily assuming that the moment arm between the centroid of the tension reinforcement and the centroid of the compressive forces was equal to the
distance $d'$; that is the resultant of the compressive forces acted at the level of the compression reinforcement. A nominal tension steel stress could then be computed by dividing the measured maximum moment by the product of the area of the tension steel and the distance $d'$. When this nominal stress was divided by the yield stress of the steel a ratio was obtained which could be related to the dimensionless quantity $q' = (p f_y - p' f'_y)/f'_c$. A graph of this ratio versus $q'$ is shown in Fig. 41. A straight line fitted through these points by statistical methods yields the equation

$$\frac{M_{\text{max}}}{A_s d'} = 1.50 - 1.08 q'$$

The empirical expression for maximum moment could then be derived by rearranging the terms in the above equation, thus:

$$M_{\text{max}} = A_s d' f_y (1.50 - 1.08 q')$$

Comparison of Measured and Computed Maximum Moments

The use of the derived empirical equation yielded excellent results as shown by the ratio of measured to computed maximum moments in Table 7. The average ratio was 1.00 with a range of 0.97 to 1.05 for the Series B and C. The equation also yielded quite satisfactory results for Series S by assuming $d' = 9$ in. The average ratio of measured to computed moment for Series S was 1.00 with a range of 0.96 to 1.09. The corresponding ratios for the inverted beams averaged 0.93 with one value as low as 0.44. This clearly indicated the loss in maximum moment-carrying capacity as a result of having been loaded in the opposite direction.
The theoretical basis for using the dimensionless parameter
\[ q' = \left( \frac{p f'_y - p f'_y}{f_c} \right) \]
cannot be shown directly by a rigorous analysis but a general qualitative picture of the influence of \( q' \) can be given by considering the effect that each variable in \( q' \) has on the ratio of the nominal stress \( f_s \) to \( f_y \). First, if the concrete strength is increased, the compression zone will be able to resist a greater compressive force and therefore the tension steel will be forced to strain further into work hardening before the maximum moment is reached, thus increasing the ratio of nominal stress to yield stress. Second, if \( p f'_y \) is increased, the effect will be identical to the effect of increasing the concrete strength. And last, if \( p f'_y \) is decreased, the ratio of \( f_s \) to \( f_y \) will again be increased because the tension steel will have to undergo further strains before its force is increased enough to equal the force that can be resisted in the compressive zone. In the above discussion each variable was changed so as to decrease the value of \( q' \) with a consequent increase in the ratio of \( f_s \) to \( f_y \). That is, each change resulted in an increase in moment.

In view of the relatively good comparisons and the simplicity of the derived expression it can be concluded that this empirical approach to the prediction of maximum moment was satisfactory. However, since the expression derived is empirical, it must be considered limited in scope to beams corresponding to those tested.

**Deflections at Maximum Moment**

An attempt was made to develop relationships for computing deflections at maximum load. However, as in the case of maximum moments, difficulty was encountered in deriving rational expressions, primarily
because the properties of the beam could not be defined precisely at this stage.

Deflection depends primarily on strains whereas moment is a function chiefly of stress. Since the tension steel was in the work-hardening range at failure and the compression steel was usually in the yield range, it is evident that the strains were much more indeterminate than the stresses. For beams failing by crushing of the concrete and buckling of the compression reinforcement, the deflection at failure was very sensitive to small changes in the strength of the compression zone. In such cases, a small increase in the strength of the compression zone required a considerable increase in the tension steel strain in order to develop the additional increase in tensile stress. Three beams failed by fracture of the tension reinforcement. In these cases, the deflection at failure would depend to a large extent on the ultimate strain developed by the steel. The steel strain at failure in the tests of tension coupons varied from 14.2 to 29.4 percent; however, in the tests, fracture occurred at even lower strains owing to the presence of strain gauge holes drilled in the bars. For example, the ultimate steel strain at fracture of the bar in beam T-9 was only 4.7 percent, and the deflection at failure for this beam was consequently relatively small.

The deflection or rotation at failure is primarily a function of the maximum curvature, which can be expressed as \((\varepsilon_s + \varepsilon_c')/d\)', where \(\varepsilon_s\) and \(\varepsilon_c\) are the maximum strains in the tension and compression reinforcement respectively. It has been pointed out in connection with the discussion of maximum moment that the tension steel strain increases as the value of \(q\)' decreases. However, the compressive steel strain, \(\varepsilon_c\)', tends to decrease as \(q\)' decreases,
an effect opposite to that for tension steel strains. Nevertheless, since the tension steel strain is the dominate term, it seems reasonable to assume that the deflections at maximum moment will vary in some manner as a function of $q$.

No success was achieved in any attempt to relate the deflection at maximum load to the parameter $q^1$ although several different plots were tried. A fairly good correlation was obtained, however, between deflection and the quantity $q'' = (p f_s - p' f'_y)/f'_c$, where $f_s$ is the effective stress at maximum load determined from the empirical relation shown on Fig. 41; that is,

$$f_s = (1.50 - 1.08 q^1) f'_y.$$

This correlation is shown in Fig. 41, and affords a purely empirical method of estimating the maximum deflection for beams similar to those tested. There is, however, theoretical justification for using the parameter $q''$. If we equate the forces on a section as was done in the calculation of crushing moment, the following equations are obtained:

\[
T = C_1 + C_2
\]  \hspace{1cm} (28)

\[
p b df_s = p' b df'_s + k_{14} f'_c ab
\]  \hspace{1cm} (29)

If we assume that $f'_s = f'_y$ and $f_s = the nominal stress computed for maximum moment,

\[
p b df'_s = p' b df'_y + k_{14} f'_c ab
\]  \hspace{1cm} (30)
6.0. \[ q = \frac{p_s - p_f^*}{k_1 k_f^*} \] 

The quantity \( k_1 k_3 \) is usually close to one. Furthermore, since the strain in the compression steel is relatively small, and varies partially as a function \( q' \) over a small range, it would be expected that the deflection at maximum load would vary in some manner with \( q' \).

**Ductility Factor at Maximum Moment**

The ratio of deflection at maximum load to deflection at yield may be considered an upper limit on the ductility factor for beams tested in this investigation. This ratio was studied as a function of several different parameters involving the properties of the beams, and plots versus \( q' \), \( q/q_{cr} \), and \( q'/q_{cr} \) are shown in Fig. 43, 44, and 45, respectively.

The plot of ductility factor at maximum load versus \( q' \) in Fig. 43 shows a general trend for the beams in Series T and B; the low value for beam T-9 is a direct result of the fracture of a tension reinforcing bar at unusually low value of strain. The results for the beams of Series S, without compression reinforcement, do not seem to follow the general trend well. Also shown on Fig. 43, are points corresponding to the beams loaded at the third-points (1). In general, these points lie well below those for the beams loaded at midspan as would be expected from the previous discussions of the differences in behavior under the two types of loading.

A plot versus \( q/q_{cr} \) is shown in Fig. 44. No correlation is observed would any be expected since the parameter \( q \) takes no account of the compression reinforcement and its significant effect on the behavior at maximum
load. This figure is included primarily to permit a comparison between the ductility factor at maximum load and the corresponding factor at crushing as shown in Fig. 40. The curve from Fig. 40 is reproduced on Fig. 44. A comparison of this curve with the plotted points on Fig. 44 shows clearly the great increase in deflection between crushing and maximum load. The only exceptions are the beams of Series S, without compression reinforcement, since the maximum load for these beams was reached shortly after crushing began.

In Fig. 45, the ratio of deflection at maximum to deflection at yield is plotted as a function of the ratio \( q/q_{cr} \). Although considerable scatter is in evidence, a general trend can be seen. The curve shown on the figure lies below all points except those for beam T-9, which failed by fracture of a bar at a very low strain, two beams of Series S, without compression reinforcement, one beam of Series B, without a column stub; and beam T-5. This curve may be considered to represent a reasonable minimum value for the ductility factor at maximum load applicable to typical beam-column intersections.
VI. SUMMARY

2. Summary

The object of this investigation was to study the load-deformation characteristics of simulated beam-column connections in reinforced concrete. Tests were performed on 25 simply supported beams loaded at midspan through column stub.

The preparation of the test specimens and the manner of carrying out the tests were described in Chapters II and III, while the results of the tests were presented and discussed in Chapter IV. For the discussions of the behavior of the beams in Chapter IV, the tests were divided into four groups:

(a) Series S, with no compression reinforcement.
(b) Series B and T, with compression reinforcement.
(c) Beam T-16, with plain unbonded tension bars.
(d) Series N, with no compression reinforcement, and having very small amounts of tension reinforcement.

The behavior of the beams in Series S was similar to that expected from former tests and analyses and differed little from the behavior of similar beams in the third-point loaded series. However, in some cases large deflections were developed after maximum moments as a result of probable bond failures.

Series B and T, the most extensive test series, consisted of 17 tests. The behavior of these beams, some of which had relatively large amounts of compression reinforcement, differed from those of Series S in that greater maximum moments and deflections were developed. Furthermore, these maximum deflection moments were also greater than those for similar beams loaded at the third-
points. Two modes of ultimate failure were exhibited by the beams in Series B and T: Crushing of the concrete and buckling of the compression reinforcement, for most of the beams tested; or fracture of the tension reinforcement, which occurred only for beams with very low values of $q^1$.

The beams of Series B, loaded at midspan but without an integrally cast column stub, behaved in much the same manner as the beams of Series T with the column stub. This suggests that the greater strength and ductility of beams loaded at midspan was a result of the type of loading and the resulting moment gradient rather than of the presence of the column stub in itself. This conclusion, however, must be qualified in view of the fact that only two beams without stubs were tested.

Six beams of Series T were tested in the inverted position after having been loaded in the upright position to a point beyond first crushing of the concrete. These inverted beams exhibited no definite yield point but showed little loss in energy-absorbing capacity unless they had been loaded to near their maximum moment capacity in the other direction.

The three beams of Series N were tested in order to study the behavior of beams with very small percentages of tension reinforcement. The tests indicated that their behavior was characterized by a sudden decrease in load-carrying capacity with the formation of tension cracks. This effect resulted from the small area of steel present being unable to resist the tension force which was transferred more or less suddenly from the concrete in cracks developed. However, in general the behavior of these beams was fairly ductile after cracking, and in one beam, a maximum moment in excess of the cracking moment was reached.
Chapter V was devoted to the derivation of expressions for computing the critical points on the load-deflection curves. Satisfactory expressions are developed for computing the moments at yielding, crushing, and at the maximum moment together with the deflections at yielding and at first crushing. Although the development of an analytical expression for the computation of the deflections at the maximum moments was not considered justified, a plot was presented to show the relation between these deflections and a dimensionless parameter involving the known properties of the beams.

**Ductility Factors**

The ductility factors based on deflections measured in the tests shown in Fig. 40 in terms of deflection at first crushing and in Fig. 44 in terms of deflection at maximum load. The values in Fig. 40 are believed to represent a reasonable minimum and corresponds to deflections producing little damage to the concrete on the compression side of a member. The values in Fig. 44 should be considered a maximum for any given beam since the range to a beam at maximum load was usually such that no further reliable predictable structural resistance should be expected of it. The ratio of maximum to the minimum value of the ductility factor, corresponding to maximum load and first crushing, respectively, ranged from about 3.0 for values of \( q \) to as much as 8.0 for the beams with negative values of \( q \).

This, of course, presents a problem as to what value should be used for ductility factor in any particular case; however, no generally applicable rule can be given to this question on the basis of the tests reported here.

The choice necessarily involves judgement leading to a decision regarding the amount of damage that may be permissible in any given case.
The range of ductility factors in terms of $q/q_{cr}$ in Fig. 40 or $q'/q_{cr}$ in Fig. 44 is quite large. Although the $q$ parameter may in certain cases have almost any value in the range considered, the values for typical signs according to current specifications for working stresses vary over a wider range. For beams reinforced in tension only, $q'/q_{cr}$ is identical with $q/q_{cr}$ and ranges from about 0.20 for structural grade reinforcement to about 0.35 for hard or rail grade. If beams are reinforced in both tension and compression, the value of $q'/q_{cr}$ will usually be very similar to that for beams with tension reinforcement only; that is, in the range 0.20 to 0.35.

However, the value of $q/q_{cr}$ may be on the order of 0.40 or 0.50.

If beams are designed for moment in one direction and then are loaded under moment of opposite sign, the tension and compression reinforcement are interchanged. For rectangular beams, the value of $q/q_{cr}$ under reversed loading will usually lie in the range 0.10 to 0.20 but may be as high as 0.30, while the values of $q'/q_{cr}$ will be negative and may range from -0.20 to -0.30.

At a beam-column intersection, T-beams will usually be designed for "negative moment"; that is, with the flange on the tension side of the beam. Under reverse loading, the quantities $q$ and $q'$ must be computed for beam width equal to the width of the flange. Consequently, $q/q_{cr}$ will be relatively small positive values, and $q'/q_{cr}$ will have small negative values.

In summary, typical values of $q/q_{cr}$ for beams will range from 0.20 to 0.40, but under certain circumstances may be as low as 0.10 or as high as 0.50. Typical values of $q'/q_{cr}$ will range between -0.35 and +0.35.
Tied columns, reinforced symmetrically will have values of $q/q_{cr}$ roughly corresponding to those given for beams, and values of $q'/q_{cr}$ equal to zero. Although test data are not available it seems reasonable to believe that the ductility factors for spiral columns would be greatly in excess of those given in this report for beams at maximum load. This would result from the confinement of the concrete and compression reinforcement within the spiral.

25. **Rotations at Beam-Column Intersection**

The analysis of frames subjected to lateral loading requires knowledge of the moment-rotation characteristics at the beam-column intersections; that is, at the joints of the frame. Although the results of the tests have been discussed in the report primarily on the basis of midspan deflections, these may easily be converted into rotations at the column stub.

At the yield moment, the midspan deflection resulted from angle changes varying linearly from zero at the ends of the span to a maximum value adjacent to the column stub. For purposes of analyses, however, dividing the midspan deflection by the distance from the end of the beam to the face of the column stub yields an equivalent rotation which may be considered to be concentrated at the beam-column intersection for purposes of analysis.

Practically all of the midspan deflection occurring after yielding is the result of angle changes which could, for all practical purposes, be considered as concentrated at the column stub. Therefore, the rotation at first crushing or at maximum load can be approximated very closely as the midspan deflection divided by the distance from the end of the beam to the face of the column stub.
Ductility factors in terms of rotation will be the same as the corresponding factors in terms of midspan deflections since the span of the beam does not change. However, it must be pointed out and emphasized that the ductility factors reported herein are a function of the span-depth ratios, d, of the beams and must be modified for beams having proportions different from those of the beams tested. The reasons for this are discussed below.

Since the midspan deflection at yield results from angle changes distributed along the entire length of the beam, it varies as the square of the span length. That is, we may write that

$$\Delta_y = \text{a function of } \phi_y L^2$$

where $\phi_y$ is the maximum angle change adjacent to the column stub and the angle change is distributed along the span in a manner such as that shown in Fig.

The conditions are different at first crushing and at maximum load. At these stages, the greater portion of the midspan deflection, that is, all the deflection subsequent to yielding, is the result of angle changes concentrated in a relatively small region adjacent to the column stub. These angle changes, summed up over some length may be treated as a concentrated angle change, or rotation, acting at the face of the stub. For deflections at first crushing it was found in Section 21 that good agreement with the test results could be obtained by summing the maximum angle change $\phi_c$ over a distance equal to the depth of the beam, d. The expression for deflection at first crushing may then be written as
\[ \Delta_c = \text{a function of } \phi_c \, d \, L \]  

Here \( \phi_c \) is the maximum angle change at first crushing. (Eq. 26).

From equations 32 and 33 we may write

\[ \frac{\Delta_c}{\Delta_y} = \frac{\phi_c}{\phi_y} \frac{d}{L} \]  

(34)

This is evident from equation 34 that the ratio of \( \Delta_c/\Delta_y \) as a function of the properties of the cross-section embodied in the \( \phi \) terms will not be independent of \( L/d \). For beams having ratios \( L/d \) greater than those in these tests, the ductility factor at first crushing could be less than that obtained from Eq. 40, and vice versa. In the tests reported herein, all beams were tested for a span of 9 ft., and the values of \( d \), the effective depth to the steel, averaged about 10.5 in.; the resulting value of \( L/d \) is 108/10.5 = 10.3. For different values of \( L/d \) it is suggested that the ductility factor for first crushing may be obtained from the following expressions:

\[ \frac{\Delta_c'}{\Delta_y'} = \frac{\Delta_c}{\Delta_y} \frac{10.2}{L/d} \]  

(35)

Here the primed deflections refer to beams having any given value of \( L/d \) and unprimed values on the right hand side correspond to those obtained from tests described in this report. It must be emphasized that this method of considering the effect of \( L/d \) is only approximate and that no results are reliable from tests with varying values of \( L/d \).
The ductility factor at maximum load should vary with span length in the same manner as the ductility factor at first crushing (Eqs. 34 and 35). However, there is no direct evidence that the depth of the beam enters at maximum load in exactly the same manner as at first crushing. It seems reasonable to assume, however, that the ductility factor at maximum also varies as function of I/d, and that a correction like that given by equation (35) can be used at maximum load as well as at first crushing.

In summary, ductility factors in terms of rotations are the same as in terms of deflections and may be estimated from Fig. 40 for first crushing and Fig. 45 for maximum load. These figures refer to beams having an I/d ratio of about 10.3; for different values of I/d, the ductility factors obtained from the figures should be modified as indicated by equation (35).
REFERENCES


### Table 1

Properties of Concrete Mixtures

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Weight Ratios
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*Test data were available for bars used in this beam; therefore, values given are average properties of No. 10 bars used in Beams T-11 and T-12.


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TABLE 4

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Average 0.98
Range 0.80 - 1.26

Average 1.04
Range 0.80 - 1.40

TABLE 6
COMPARISON OF MEASURED AND COMPUTED VALUES FOR CRUSHING MOMENT AND DEFLECTION
### TABLE 7
COMPARISON OF MEASURED AND COMPUTED VALUES OF MAXIMUM MOMENT

<table>
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Average 1.00  
Range 0.96 - 1.09

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Average 1.00  
Range 0.97 - 1.05

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<td>25.7</td>
<td>1.02</td>
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Average 1.00  
Range 0.97 - 1.05

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<td>1.02</td>
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Average 0.93  
Range 0.86 - 1.02
### TABLE 8

**DUCTILITY FACTORS AT CRUSHING AND AT MAXIMUM MOMENT**

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<thead>
<tr>
<th>Beam</th>
<th>( q_{cr} )</th>
<th>( q / q_{cr} )</th>
<th>( q' / q_{cr} )</th>
<th>( \Delta c / \Delta y )</th>
<th>( \Delta_{max} / \Delta y )</th>
<th>( \Delta_{max} / \Delta c )</th>
<th>(meas.)</th>
<th>(meas.)</th>
<th>(meas.)</th>
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<td>0.150</td>
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<td>0.074</td>
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FIG. 1 LOCATIONS OF DEFLECTION MEASUREMENTS AND STRAIN GAUGE LINES ON TENSION REINFORCEMENT
FIG. 2 TYPICAL STRESS–STRAIN CURVE FOR REINFORCING BAR IN TENSION
6" Mechanical Strain Gauge Lines (Berry and Direct)

\[ X = 1\frac{1}{2}'' \text{ Beams } T-5 \text{ to } T-15 \text{ Inclusive, } 3'' \text{ Beam } T-16 \]

\[ Y = 2'' \text{ Beams } T-5, 3'' \text{ Beams } T-6 \text{ to } T-16 \text{ Inclusive} \]

\( T \) (T-series)

\( B \) (B-series)

Location of SR-4, A-II Strain Gauges on Compression Steel

FIG. 3 LOCATIONS OF MECHANICAL STRAIN GAUGE LINES ON CONCRETE ON SIDE OF BEAM AND SR-4 GAUGES ON COMPRESSION REINFORCEMENT
FIG. 4 LOCATION OF STRAIN GAUGE LINES ON CONCRETE ON TOP OF BEAM
FIG. 5 ELEVATION VIEW OF TESTING APPARATUS AND BEAM
FIG. 6 EFFECT OF COLUMN STUB ON LOAD—DEFORMATION CHARACTERISTICS (B-34)
FIG. 7 EFFECT OF COLUMN STUB ON LOAD—DEFORMATION CHARACTERISTICS (B-35)
Effect of Adding Compression Reinforcement on the Load-Deformation Characteristics

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<td>0.62</td>
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<td>$p/p'$</td>
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**FIG. 8**

Deflection at Midspan in Inches

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- First Crushing
- T-9 with comp. reinf.
- premature bar fracture
- C
- M
- S-8 no comp. reinf.
FIG. 9 EFFECT OF COMPRESSION REINFORCEMENT ON BEAMS OF APPROXIMATELY EQUAL $q'$
FIG. 10 EFFECT OF INCREASING $p'$ IN BEAMS WITH HIGH $p$ AND $f'_c$
a) At first crushing

b) At second stage of crushing

c) At maximum load

FIG. 11 VIEWS OF BEAM T-6 AT VARIOUS STAGES IN TEST
a) At first crushing

b) At second stage of crushing

c) At maximum load

FIG. 12 VIEWS OF BEAM T-4 AT VARIOUS STAGES IN TEST
a) At first crushing

b) At second stage of crushing

c) At maximum load

FIG. 13 VIEWS OF BEAM T-11 AT VARIOUS STAGES IN TEST
FIG. 14 EFFECT OF INCREASING $p$ AS $p'/p$ REMAINS APPROXIMATELY CONSTANT (HIGH $f'_c$)
FIG. 15 EFFECT OF INCREASING $p$, $q$ AND $q'$ AS $p'/p$ REMAINS APPROXIMATELY CONSTANT (LOW $f_c'$)
FIG. 16 EFFECT OF CONCRETE STRENGTH ON THE LOAD-DEFORMATION CHARACTERISTICS
FIG. 17 EFFECT OF LOADING IN ONE DIRECTION ON PROPERTIES IN THE OPPOSITE DIRECTION - T-10 I
FIG. 18 EFFECT OF LOADING IN ONE DIRECTION ON THE PROPERTIES IN THE OPPOSITE DIRECTION—T-121
FIG. 19 EFFECT OF LOADING IN ONE DIRECTION ON PROPERTIES IN THE OPPOSITE DIRECTION - T-14 I, T-15 I
FIG. 20 EFFECT OF LOADING IN ONE DIRECTION ON PROPERTIES IN THE OPPOSITE DIRECTION - T-8 I
FIG. 21 VIRGIN LOAD-DEFLECTION CURVES FOR BEAMS SUBSEQUENTLY TESTED IN INVERTED POSITION
FIG. 22  FAULTING ADJACENT TO COLUMN STUB (BEAM T-15)

FIG. 23  APPEARANCE OF BEAM T-16 AT END OF TEST
FIG. 24 EFFECT OF USING PLAIN BARS FOR TENSION REINFORCEMENT
Cracking Load N-2
Cracking Load N-3
Cracking Load N-1
N-1 bar fracture
N-2 bar fracture
Unloading Curve N-3

Deflection at Midspan in Inches

Load in Kips

Expanded Scale

FIG. 25 LOAD-DEFLECTION DIAGRAMS FOR BEAM SERIES N

N-1 N-2 N-3
p 0.17 0.17 0.31

f_c' 6138 5649 6407
(a) BEAM T-3 SHOWING STIRRUP BENT OUT BY LATERAL DEFORMATION OF CONCRETE

(b) VIEW OF BEAM N-3 AT END OF TEST

FIG. 26 PHOTOGRAPHS OF BEAMS T-3 AND N-3
FIG. 27 COMPARISON OF BEAMS WITHOUT COMPRESSION REINFORCEMENT
LOADED AT MIDSPAN AND AT THE THIRD-POINTS

- S-8
- T-1 Lb

\[ \begin{array}{c|c|c}
 p & 0.62 & 0.62 \\
 p' & 0 & 0 \\
 f_c' & 2642 & 2520 \\
 f_y & 45,000 & 46,000 \\
\end{array} \]
FIG. 28 COMPARISON OF BEAMS WITH COMPRESSION REINFORCEMENT LOADED AT MIDSPAN AND AT THE THIRD-POINTS—BEAMS T-2 AND C4zn
FIG. 29 COMPARISON OF BEAMS WITH COMPRESSION REINFORCEMENT LOADED AT MIDSPAN AND AT THE THIRD-POINTS. BEAMS T-5 AND C4×na.
FIG. 30 COMPARISONS OF MAXIMUM MOMENTS FOR BEAMS LOADED AT MIDSPAN AND AT THIRD-POINTS
FIG. 31 LOAD-STRAIN CURVES FOR BEAM T-II

Comp. Strain in Concrete at Top of Beam

Two Strain Curves for Comp. Steel
FIG. 32 DISTRIBUTION OF STRAINS OVER LENGTH OF BEAM T-5 AT YIELDING
Fig. 33 Distribution of strain in tension steel over length of beam T16 up to yielding.
FIG. 34 EFFECT OF ADDING COMPRESSION REINFORCEMENT ON STRAIN IN TENSION STEEL AT MAXIMUM LOAD
FIG. 35 EFFECT OF INCREASING $p'$ ON THE TENSION STEEL STRAINS AT MAXIMUM MOMENT
NOTE: In Beams Without Compression Reinforcement $k' = k$

Cross Section

Strains

Stresses

FIG. 36 ASSUMED CONDITIONS AT YIELDING
FIG. 37 ASSUMED DISTRIBUTION OF CURVATURE AT YIELD MOMENT
FIG. 38 ASSUMED CONDITIONS AT FIRST CRUSHING

Cross Section

Strains

Stresses

$C_2 = A_s f_s$

$C_1 = k_1 k_2 f_s' ab$

$d - k_2 a$

$T = A_s f_s$
FIG. 39 DUCTILITY FACTOR AT CRUSHING AS FUNCTION OF $q'/q_{cr}$
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EMPIRICAL EXPRESSIONS FOR MIDS燕AN DEFLECTIONS
OF BEAMS LOADED AT THIRD-POINTS

1. Introduction

This Appendix is concerned exclusively with the derivation and representation of empirical expressions for the deflections at midspan of the beams previously tested under third-point loading (1)*. In the previous report (1), rational procedures were developed for computing angle changes and deflections at both yield and maximum load. These procedures, however, are in some cases rather complex and relatively extensive calculations are involved. An attempt was made, therefore, to derive empirically much simpler expressions to predict the deflections observed in the tests.

Empirical expressions are presented herein for midspan deflection at yield and at maximum load, for beams with tension reinforcement only as well as for beams reinforced in both tension and compression, and tested under third-point loading. In addition, an expression is derived for the rigidity factor, expressed as the ratio of the deflection at maximum load to the deflection at yield.

Acknowledgment

The studies described herein were made by Lt. Comdr. H. L. Murphy, U.S. Engineer Corps, U. S. Navy, and graduate student in Civil Engineering

*Numbers in parentheses refer to list of references following main text of this report.
at the University of Illinois. Appreciation is expressed to J. H. Appleton, research Associate in Civil Engineering for his many valuable suggestions in connection with these studies.

5. Beams Reinforced in Tension Only: Midspan Deflection at Yield

Although the object of these studies was to develop empirical expressions, it was desired that these expressions be rational in form; that is, they should involve the proper variables in the proper manner. It was shown in the previous report (1) that midspan deflection at yield, \( \Delta_y \), could be expressed as a function of the maximum angle change \( \Phi_y \), and the span of the beam \( L \), in the following form:

\[
\Delta_y = \text{Constant} \times \Phi_y \frac{L^2}{2}
\]

(A1)

Here the constant depends on the nature of the loading or, more specifically, on the distribution of moment and angle change along the span.

At yield, the maximum angle change may be derived from the strain relations shown in Fig. 36 of this report. The resulting expression

\[
\Phi_y = \frac{\varepsilon_y}{(1-k)d}
\]

(A2)

where \( \varepsilon_y \) is the yield strain for the reinforcement, \( d \) is the effective depth of the beam, and \( kd \) is the distance from the top of the beam to the neutral axis in a beam without compression reinforcement. From equations A1 and A2 we may write

\[
\Delta_y = \text{Constant} \times \frac{\varepsilon_y}{(1-k)} \frac{L^2}{2d}
\]

(A3)
In order to obtain a dimensionless expression, equation A3 is rewritten as

$$\Delta \frac{y}{L^2} = \text{Constant} \times \frac{c_y}{(1-k)}$$  \hspace{1cm} (A4)

In equation A4, the quantity \((1-k)\) varies inversely with the percentage of reinforcement, \(p\), and directly with the modulus of elasticity of the concrete, \(E_c\). This follows from the fact that, according to the conventional straight line theory, \(k\) is a function of \(pn\), where \(n\) is the ratio \(E_s/E_c\). We may then write

$$\Delta \frac{y}{L^2} = \text{function of } c_y \frac{p}{E_c}$$  \hspace{1cm} (A5)

However, \(E_c\) is in turn a function of the compressive strength of the concrete, \(f'_c\); one relation frequently given is

$$E_c = 1000 f'_c$$  \hspace{1cm} (A6)

A linear relation such as this, however, is valid over only a limited range, and a better expression has been found to be

$$E_c = 1000 k_1k_2 f'_c$$  \hspace{1cm} (A7)

Here \(k_1k_2\) in this case is given by the expression

$$k_1k_2 = \frac{3000 + 0.5f'_c}{1500 + f'_c}$$  \hspace{1cm} (A8)

We may now write

$$\Delta \frac{y}{L^2} = \text{function of } c_y \frac{p}{k_1k_2 f'_c}$$  \hspace{1cm} (A9)

Equation A9 was used as the basis of the first empirical studies.
Two different empirical expressions have been derived for predicting yield point deflection. First, equation A9 was rewritten as

$$\frac{\Delta y d}{L^2} = \text{function of } \frac{f_y}{E_s} k_1 k_f c$$

from which

$$\frac{\Delta y d}{L^2 f_y} = \text{function of } \frac{p}{k_1 k_f c}$$

Since $E_s$, the modulus of elasticity of steel may be considered constant.

On the basis of equation A11, a plot was made with measured values of $p/k_1 k_f c$ as abcissas and corresponding values of $\Delta y d/L^2 f_y$ as ordinates, as shown in Fig. 1. The values plotted are listed in Table A1. It must be noted that in some cases the values of deflection differ from those given in the previous report (1). In these studies, all deflections have been corrected to eliminate a small error in measurement produced as a result of deflection of the testing machine base.

A curve was fitted to the points in Fig. A1 by the least-squares method and its equation determined as

$$\frac{\Delta y d}{L^2 f_y} \times 10^9 = 4.840 + 0.3446 \times 10^6 \frac{p}{k_1 k_f c}$$

The coefficient of correlation, $r$, for this line is 0.86 and the standard error of estimate, $S_y$, for $\Delta y d/L^2 f_y \times 10^9$ is 0.79. From equation A12 we obtain the following expression for midspan deflection, taking

$p = 30,000,000$ psi
\[
\Delta \frac{d}{L^2} = \frac{f}{E_s} \left( 0.1452 + \frac{10.338 P}{k_1 k_2 f' c} \right) \quad (A13)
\]

A second empirical equation in the same form as equation A13 was derived from the plot shown in Fig. A2. The line shown on this figure fits the points with a coefficient of correlation of 0.98 and a standard error of estimate in terms of the ordinates plotted of 0.17. The resulting equation for yield deflection is

\[
\Delta \frac{d}{L^2} = \frac{f}{E_s} \left( 0.1331 + \frac{12975 P}{k_1 k_2 f' c} \right) \quad (A14)
\]

It is not possible to choose between equations A13 and A14 on the basis of coefficients of correlation or standard errors of estimate since these have significance only in terms of the quantities plotted on the graphs and are not necessarily a true measure of the precision with which the equations will predict the observed deflections. However, on the basis of other considerations, it is believed that equation A13 is to be preferred.

**Beams Reinforced in Tension Only: Midspan Deflection at Maximum Load**

At maximum load, the maximum angle change may be derived from the strain relations shown in Fig. 38 of this report, or preferably those shown on page 22 of Reference 1. From strains we obtain:

\[
\phi_m = \frac{\varepsilon_u}{a} \quad (A15)
\]

where \(\varepsilon_u\) is the strain in the concrete at the top of the beam, and \(a\) is the distance from the top of the beam to the neutral axis. If the stress
the tension reinforcement at failure is equal to the yield point, $f_y$, can be shown (equation 8a in Reference 1) that

$$a = \frac{p_f y}{k f'_c d}$$

(A16)

Since it has been found that $e_u$ may be considered as a constant, we may write

$$\phi_m = \frac{\text{Constant}}{p_f y} \frac{d}{k f'_c}$$

(A17)

Hence, in this case also, we may consider the midspan deflection, $\Delta_m$, to be a function of $\phi_m$ and $L^2$, we may write

$$\frac{\Delta_m d}{L^2} = \text{function of} \quad \frac{k f'_c}{p_f y}$$

(A18)

In Fig. A3, the measured midspan deflections at maximum load in the form of $\Delta_m d/L^2$ have been plotted versus the corresponding values of $f'_c/p_f y$ for the beams tested. The values plotted are given in Table A1. The line fitted through the points on Fig. A3 by the least-squares method yielded the equation

$$\frac{\Delta_m d}{L^2} \times 10^3 = -0.1 + \frac{0.356 k f'_c}{p_f y}$$

(A19)

with a coefficient of correlation of 0.96 and a standard error of estimate in terms of the plotted ordinates of 0.33. Since this line did not pass
through the origin, a second line, passing through the origin, was fitted to the plotted points. The equation of this line was

\[
\frac{\Delta_m}{L^2} = \frac{3.415}{10^4} \frac{k_k f'}{p_f y}
\]  
(A20)

Equation A20 is recommended for use.

5. Beams Reinforced in Tension Only: Ductility Factors

The ductility factor is defined as the ratio of deflection at maximum load to deflection at yield; that is, \( \frac{\Delta_m}{\Delta_y} \). However, for convenience in deriving the expressions, the inverse ratio \( \frac{\Delta_y}{\Delta_m} \) is used.

Dividing equation A13 by equation A20, we obtain

\[
\frac{\Delta_y}{\Delta_m} = \frac{10^4}{3.415 \sqrt{E_s}} \frac{p_f y}{k_k f'} \left( 0.1452 f_y + 10338 \frac{p_f y}{k_k f'} \right)  
\]  
(A21)

where

\[
\frac{p_f y}{k_k f'} = \bar{q}
\]  
(A22)

Taking \( E_s = 30 \times 10^6 \) psi, we may write equation A21 as

\[
\frac{\Delta_y}{\Delta_m} = \frac{1}{10245} \bar{q} (0.1452 f_y + 10338 \bar{q})
\]

\[
\frac{\Delta_y}{\Delta_m} = \bar{q} \left( \frac{f_y}{70600} + 1.01 \bar{q} \right)
\]  
(A23)
It is desirable, however, that the ductility factor involve the term \( \tilde{q}_{cr} \), the value of \( \tilde{q} \) for which the beam fails simultaneously in tension and compression; that is, the steel reaches the yield point at the same time the strain in the concrete reaches \( \varepsilon_u \). From equation 23 in Reference 1

\[
\tilde{q}_{cr} = \frac{1}{1 + \frac{\varepsilon_y}{\varepsilon_u}} \tag{A24}
\]

since \( \tilde{q} = q/k \). In equation A24

\[
\varepsilon_y = \frac{f_y}{E_s} = \frac{f_y}{30,000,000}
\]

and

\[
\varepsilon_u = 0.004
\]

Thus

\[
\tilde{q}_{cr} = \frac{1}{\frac{f_y}{120,000} + \frac{f_y}{120,000}} = \frac{120,000}{f_y} \tag{A25}
\]

and

\[
f_y = \frac{1 - \tilde{q}_{cr}}{\tilde{q}_{cr}} 120,000 \tag{A25a}
\]

The term \( \tilde{q}_{cr} \) can now be introduced into equation (A23) by substituting the expression in equation A25a for \( f_y \). This yields, after simplification:

\[
\frac{\Delta_y}{\Delta_m} = 1.7 \frac{\tilde{q}}{\tilde{q}_{cr}} - \tilde{q} (1.7 - 1.01 \tilde{q}) \tag{A26}
\]
Since the ratio $\Delta_y/\Delta_m$ should equal zero for $\bar{q}/\bar{q}_{cr} = 0$, and should equal one for $\bar{q}/\bar{q}_{cr} = 1$, equation A26 was further modified, as follows

$$\frac{\Delta_y}{\Delta_m} = \frac{\bar{q}}{\bar{q}_{cr}} \left[ 1.7 - 1.7 \frac{\bar{q}}{\bar{q}_{cr}} + 1.01 \frac{\bar{q}^2}{\bar{q}_{cr}^2} \frac{\bar{q}}{\bar{q}_{cr}} \right]$$

(A27)

This equation thus yields $\Delta_y/\Delta_m$ as a function of the ratio $\bar{q}/\bar{q}_{cr}$ and the term $\bar{q}_{cr}$ itself, which is a function only of $f_y$. Equation A27 has been plotted in Fig. A4 for values of $f_y$ equal to 40,000 and 55,000 psi, representing the range of values included in the tests. The data are taken from Table A2. Since the two curves are relatively close together, the effect of $f_y$ in itself is seen to be small; the principal variable is $\bar{q}/\bar{q}_{cr}$. It may be noted further that the empirical equation yields a value of $\Delta_y/\Delta_m$ very close to one for $\bar{q}/\bar{q}_{cr} = 1$.

The points plotted on Fig. A4 are ratios of measured deflections to those at maximum load, obtained from the results of the tests (1). The agreement with the empirical equations is believed to be satisfactory. However, for a range of $\bar{q}/\bar{q}_{cr}$ from zero up to 0.5 to 0.6, it is possible to represent the results of the tests by a very simple expression of the form

$$\frac{\Delta_y}{\Delta_m} = 0.75 \frac{\bar{q}}{\bar{q}_{cr}}$$

(A28)

This equation also agrees well with the empirical equation over the range indicated, but lies appreciably below the values from equation A27 for higher values of $\bar{q}/\bar{q}_{cr}$.

In Fig. A5, the ductility factor itself, $\Delta_m/\Delta_y$, is plotted
versus \( q / q_{cr} \). The points and curves on this figure correspond to those on Fig. A4. Furthermore, it can be seen from Fig. A5 that the simple relation given by equation A28 represents reasonably well a lower limit for the ductility factor.

A6. Beams Reinforced in Tension and Compression: Midspan Deflection at Yield

Two expressions have been derived for midspan deflection at the yield point for the beams reinforced in both tension and compression. The first of these is similar in form to equations A13 and A14 for beams without compression reinforcement. In Fig. A6, the term \( \Delta y d / L^2 f_y \) has been plotted as a function of \( p / k_c f'_c \) for each beam tested. The line shown on the figure was fitted by the least-squares method and fits the plotted points with a coefficient of correlation of 0.74 and a standard error of estimate in terms of the plotted ordinates of 0.33. The resulting expression is

\[
\Delta y d = \frac{f_y}{E_s} \left( 0.1926 + 2.75 \frac{p}{k_c f'_c} \right) \quad (A29)
\]

It may be noted from Fig. A6 and equation A29 that the term \( p / k_c f'_c \) has much less effect on yield deflection for the beams with both tension and compression reinforcement than for the beams reinforced in tension only. This is believed to result from the presence of the compression reinforcement which, however, is not included explicitly in Eq. A29. The depth to the neutral axis, and thus the angle change and deflection at yield, is influenced to some extent by the compression reinforcement. In the beams tested, the ratio of compression to tension
reinforcement varied between 0.45 and 0.60 except for one beam for which the ratio was 0.73. As a result, the position of the neutral axis varied relatively little from beam to beam as the quantity \( p/k \times f' \) was varied. Consequently, equation A29 may not be valid for beams having ratios of compression to tension reinforcement significantly different from those used in these tests.

A second equation for yield deflection was derived as shown in Fig. A7. The parameter used in this case was \((p + p')/k \times f'\), where \(p'\) is the percentage of compression reinforcement. The resulting equation is

\[
\frac{\Delta_y}{L^2} = \frac{\Delta_y}{E} \left[ 0.1930 + 1838 \frac{(p + p')}{k \times f'} \right]
\]

(A30)

Although excellent correlation is evident from Fig. A7, an equation involving \((p + p')\) has no rational basis and must therefore be viewed with some suspicion. Since the ratio \(p'/p\) varied only from 0.45 to about 0.60, the term \((p + p')\) in Fig. A7 and equation A30 bears a fairly definite relation to \(p\) alone. In view of this situation, the correlation shown in Fig. A7 may have resulted solely from a fortuitous combination of circumstances. Nevertheless, the correlation was so good that it was considered desirable to include it in this report.

17. Beams Reinforced in Tension and Compression: Midspan Deflection at Maximum Load

The beams with both tension and compression reinforcement were treated in much the same manner as that described in Section A4 of this appendix for beams reinforced in tension only. In this case, however,
the distance \( a \) in equation A15 and Fig. 38 is given by the following expression instead of by equation A16.

\[
a = \frac{p_f - p'_f}{k \frac{k_f'}{c} y} d
\]

(A31)

where \( f'_y \) is the yield point of the compression reinforcement. This equation is valid only if both the tension and the compression reinforcement have reached their respective yield points at maximum load. This will usually be true for the tension reinforcement, but may not be true for the compression reinforcement if large amounts of steel are used or if \( f'_y \) is high. However, in order to obtain a reasonably simple empirical expression for deflection at maximum load, it was necessary to use the relationship given by equation A31 even though it would not always apply strictly.

In Fig. A8, the measured midspan deflections at maximum load, in the form of \( \Delta_m d/L^2 \), are plotted as a function of \( k k_f'/(p_f - p'_f) \) in a manner similar to Fig. A3. As was also the case in Fig. A3, the line fitted by the least-squares method missed the origin by a small amount, and a second line was therefore forced through the origin. The resultant expression is

\[
\frac{\Delta_m d}{L^2} = \frac{4.290}{10^4} \frac{k \frac{k_f'}{c}}{p_f - p'_f} y
\]

(A32)

It is evident from Fig. A8 that the correlation in this case is much poorer than it was for the beams reinforced in tension only (Fig. A3). One reason for this might be that the compression reinforcement was not
always at its yield point. However, it must be recalled from Reference (1) that the type of tie used in these beams varied considerably and that this variable was found to affect the deformations attained at maximum load. Since the type of tie is not considered in Fig. A8 or equation A32, a considerable scatter is to be expected.

A8. Beams Reinforced in Tension and Compression: Ductility Factors

An attempt to derive an expression for the ductility factor as the quotient of equations A29 and A32 led to no correlation whatsoever with the results of the tests. The plot shown in Fig. A9 was therefore prepared. In this figure, the ratio of $\frac{\Delta_m}{\Delta_y}$ is plotted as a function of $\frac{\tilde{q}'}{\tilde{q}_{cr}}$, where

$$\tilde{q}' = \frac{p'y - p'f'}{k1k2c} \quad (A33)$$

Figure A9 is similar to Fig. A5 except for the use of $\tilde{q}'$ in place of $\tilde{q}$. This substitution is rational in view of the analyses presented in Reference 1.

Curves representing equations A27 and A28, with $\tilde{q}'$ substituted for $\tilde{q}$, are shown on Fig. A9 in exactly the same manner as on Fig. A5; it can be seen that the results of the tests are represented fairly well by these equations. Therefore, since $\tilde{q}'$ reduces to $\tilde{q}$ when compression reinforcement is not present, the ductility factor for all of the beams tested under third-point loading can be represented by the expression.

$$\frac{\Delta_m}{\Delta_y} = \frac{\tilde{q}_{cr}}{0.75 \tilde{q}'} \quad (A34)$$
where $q_{cr}$ is given by equation $A25$ and $q'$ by equation $A33$. This equation is valid only if the ratio $q'/q_{cr}$ does not exceed about 0.6.

A9. Limitations of Empirical Expressions

The expressions presented in this Appendix have all been obtained by empirical studies involving only a limited number of test specimens. Therefore, strictly speaking, they should be limited in application to beams having dimensions and properties similar to those tested. Nevertheless, the fact that the form of the equations was in each case based on a rational analysis, together with the dimensionless form of presentation, provides some basis for the belief that these equations may be valid over a somewhat greater range.

The beams tested were reinforced with bars having values of yield point, $f_y$, varying from 40,000 to about 56,000 psi. Although this range is not large numerically, it probably includes most values of static yield strength likely to be encountered for intermediate grade bars; lower values may be expected for structural grade and higher values for hard or rail grade. Nevertheless, since both rational analysis and the results of these studies indicate that yield strength should have a direct effect on the yield deflection, it is believed that the expressions for yield deflection are valid for an appreciably greater range than that found in the tests. A similar argument may be advanced regarding the $f_y$ term in the expressions for deflection at maximum load and ductility factors. It is suggested therefore that the expressions given herein be considered valid for values of $f_y$ outside the range used in the tests.
A more serious limitation on the applicability of the expressions is imposed when beams with compression reinforcement are considered. As was previously pointed out, the ratio of compression to tension reinforcement, $p'/p$, varied from only 0.45 to 0.60 with but one value as high as 0.73. It seems probable, therefore, that equation A29 for yield deflection should not be used for ratios of $p'/p$ less than 0.45. Similarly, since equation A32 for deflection at maximum load does not reduce to equation A20 when $p' = 0$, it should probably not be used outside the range of $p'/p$ mentioned above. These limitations are not in themselves too restrictive since the range of $p'/p$ stated will include many typical members.

Since equation A34 for the ductility factor seems to be applicable to beams without compression reinforcement as well as to beams with values of $p'/p$ ranging up to 0.60, no lower limit on $p'/p$ need be set for this equation. An upper limit of about $p'/p = 0.60$ should be imposed, however, and it is evident that equation has no meaning for values of $p'/p$ such that $\tilde{q}'$ equals zero or becomes negative. Furthermore, equation A34 is not valid for values of $\tilde{q}'/\tilde{q}_{cr}$ greater than about 0.6.

All of the tests considered in this Appendix were made with third-point loading in order to obtain a constant moment over a significant length of the beam. Because the distribution of angle change, and thus the deflection, is related directly to the distribution of moment, the results obtained in these tests cannot be used without modification for other types of loading. It is evident from the results presented in the main body of this report that the magnitude and distribution of angle change and the consequent rotations and deflections are vastly different for beams loaded...
at midspan and at the third points. It is believed, however, that the results presented in this Appendix in the form of empirical expressions for deflections and ductility factors can be applied to predict, at least approximately, the behavior of simple beams subjected to uniformly distributed loads.

At first yielding, the distribution of angle change along the beam should be almost identical with the distribution of moment. This condition was assumed in connection with the analytical procedure developed in Reference 1 for predicting deflection at yield, and the agreement obtained with the results of the tests would seem to confirm this assumption. On this basis, therefore, the deflection at yield may be compared for beams with third-point and uniform loading. At the load producing a maximum moment equal to the yield moment, $M_y$, the midspan deflection of a beam under third-point loading may be expressed as:

$$
\Delta_y = \frac{23}{216} \frac{M_y L^2}{EI} \quad (A35)
$$

For a beam with uniform loading, the deflections corresponding to a maximum moment equal to $M_y$ may be written as

$$
\Delta'_y = \frac{5}{48} \frac{M_y L^2}{EI} \quad (A36)
$$

From equations $A35$ and $A36$, the ratio of midspan deflection for uniform loading to that for third-point loading is

$$
\frac{\Delta'_y}{\Delta_y} = 0.98 \quad (A37)
$$
t midspan and at the third points. It is believed, however, that the results presented in this Appendix in the form of empirical expressions or deflections and ductility factors can be applied to predict, at least approximately, the behavior of simple beams subjected to uniformly distributed loads.

At first yielding, the distribution of angle change along the beam should be almost identical with the distribution of moment. This condition was assumed in connection with the analytical procedure developed in Reference 1 for predicting deflection at yield, and the agreement obtained with the results of the tests would seem to confirm this assumption. On this basis, therefore, the deflection at yield may be compared for beams with third-point and uniform loading. At the load producing a maximum moment equal to the yield moment, $M_y$, the midspan deflection of a beam under third-point loading may be expressed as:

$$\Delta_y = \frac{23}{216} \frac{M_y L^2}{EI}$$  \hspace{1cm} (A35)

or a beam with uniform loading, the deflections corresponding to a maximum moment equal to $M_y$ may be written as

$$\Delta_y' = \frac{5}{48} \frac{M_y L^2}{EI}$$  \hspace{1cm} (A36)

From equations A35 and A36, the ratio of midspan deflection for uniform loading to that for third-point loading is

$$\frac{\Delta_y'}{\Delta_y} = 0.98$$  \hspace{1cm} (A37)
It may be assumed, therefore, that the midspan deflection at yield for a beam under uniform loading will be approximately the same as the corresponding deflection for the same beam under third-point loading. This results, of course, from the fact that the moment diagrams are very similar for the two types of loading and, at yield, the angle-change diagrams are thus also similar.

At maximum load, the relation between deflections for the two types of loading is not so easily established. Although the moment diagrams are still similar in shape, this is no longer true for the angle change diagrams. The angle change at maximum moment may be very much greater than that at yield (depending on the value of $\bar{q}/\bar{q}_{cr}$) because of the inelastic action after yielding. For beams with third-point loading, the maximum angle change is distributed over the entire middle third of the span and is responsible for a major portion of the midspan deflection. With uniform load, however, the maximum angle change is relatively concentrated in the region near midspan and the resulting deflection will be less if the magnitude of the maximum angle change is the same as for the beams loaded at the third-points. This may not be the case, however, since the localization of the maximum moment region may permit the development of greater local angle changes, as was found to be the case for beams loaded at midspan. Since these two effects act in opposite directions, it is difficult to make any precise predictions regarding the relative magnitudes of maximum deflections under third-point and uniform loading.

For beams reinforced in tension and compression, and with well-tied compression reinforcement, it does not seem unreasonable to assume that the deflections at maximum load would be comparable for the two types of loading.
However, any difference that might exist should be assumed in the direction of smaller deflections for the case of uniform load.

For beams reinforced in tension only, it is possible that the maximum angle change developed will be no greater for uniform loading than for third-point loading, and the deflection will thereby be decreased because of the differences in the angle change diagrams. For this case, the midspan deflection at maximum load for beams subjected to uniform load should be taken as about 0.75 to 0.80 the corresponding deflection for beams loaded at the third points.

If the midspan deflection at maximum load is reduced as indicated above, there will be a corresponding reduction in the ductility factor, since the yield deflection is unchanged. It must be noted, however, that the reduction in midspan deflection for uniform loading will be of the order indicated only for beams with moderate ductility. As the value of \( \xi \) approaches \( \xi_{cr} \), the ductility factor approaches unity for any type of loading.

10. Summary

Empirical expressions have been presented for computing midspan deflections at yield and at maximum load for simple-span reinforced concrete beams loaded at their third points. The equations are rational in form, but the coefficients involved were determined empirically from the results of the tests reported in Reference 1. These expressions serve two purposes: (1) They provide relatively simple means for estimating deflections or ratios of deflections. And (2), they serve to show clearly the effects of the several variables on the deflections and ductility factors.
The following expressions have been recommended:

For midspan deflection at yield.

Beams reinforced in tension only,

\[
\frac{\Delta y d}{L^2} = \frac{f_y}{E_s} \left( 0.1452 + \frac{10.338 P}{k k_f' c} \right) \quad (A13)
\]

Beams reinforced in both tension and compression, and having \( p'/p \) not more than about 0.60.

\[
\frac{\Delta y d}{L^2} = \frac{f_y}{E_s} \left( 0.1926 + \frac{2475 P}{k k_f' c} \right) \quad (A29)
\]

For midspan deflection at maximum moment

Beams reinforced in tension only

\[
\frac{\Delta m d}{L^2} = \frac{3.415}{10^4} \frac{k k_f' c}{p_f y} \quad (A20)
\]

Beams reinforced in both tension and compression, and having \( p'/p \) in the range 0.45 - 0.60.

\[
\frac{\Delta m d}{L^2} = \frac{4.290}{10^4} \frac{k k_f' c}{p_f y - p_f y'} \quad (A32)
\]

or ductility factor at maximum load

\[
\frac{\Delta_m}{\Delta_y} = \frac{\bar{\delta}_{cr}}{0.75 \bar{\delta}'} \quad (A34)
\]
where \( q_{cr} \) is given by equation A25 and \( q' \) by equation A33, and \( q'/q_{cr} \) is not more than about 0.60.

Equations A13 and A29 for deflection at yield may be used also for beams subjected to uniformly distributed loads. Similarly, Eq. A32 should be applicable also to the case of uniform load for beams reinforced in both tension and compression. However, for beams reinforced in tension only, the deflections for uniform load may be only about three-quarters those given by equation A20 for small values of \( q'/q_{cr} \), increasing to equal deflections for \( q'/q_{cr} \) equal to one.

For beams reinforced in both tension and compression, equation A32 should yield satisfactory values of the ductility factor for uniform loading. If compression reinforcement is not present, the ductility factors for uniform load may be about one-fourth less than those given by equation A32. However, as \( q'/q_{cr} \) approaches a value of one, the ductility factor should also approach one for either type of loading.
### Deflections for Beams Reinforced in Tension Only

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<th>f'c</th>
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<th>p</th>
<th>f_y</th>
<th>Δy</th>
<th>Δm</th>
<th>p/kkL3c</th>
<th>Δd/L2</th>
<th>ΔdΔ/</th>
<th>ΔmΔ/</th>
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* L = 108 in.
**TABLE A2**

**DUCTILITY FACTORS FOR BEAMS REINFORCED IN TENSION ONLY**

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<tr>
<th>Beam</th>
<th>$q_{cr}$</th>
<th>$\bar{q}$</th>
<th>$\frac{\bar{q}}{\bar{q}_{cr}}$</th>
<th>$\frac{\Delta y}{\Delta m}$</th>
<th>$\frac{\Delta m}{\Delta y}$</th>
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<td>0.118</td>
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## Deflections for Beams Reinforced in Tension and Compression

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<th>f'_y</th>
<th>Δ_y</th>
<th>Δ_m</th>
<th>p</th>
<th>k k'_1 c</th>
<th>k k'_1 c</th>
<th>Δ_y/L^2</th>
<th>Δ_y/(p+p')</th>
<th>k k'_1 c</th>
<th>Δ_m d*</th>
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<td>psi</td>
<td>o/o</td>
<td>in.</td>
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* L = 108 in.
TABLE A4
DUCTILITY FACTORS FOR BEAMS REINFORCED IN TENSION AND COMPRESSION

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<th>Beam</th>
<th>$\tilde{q}_{cr}$</th>
<th>$\tilde{q}$</th>
<th>$\tilde{q}'$</th>
<th>$\frac{\tilde{q}'}{\tilde{q}_{cr}}$</th>
<th>$\Delta_y$</th>
<th>$\Delta_m$</th>
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FIG. A1 BEAMS REINFORCED IN TENSION ONLY: MIDSPAN DEFLECTION AT YIELD — EQUATION A13

\[
\frac{\Delta_y d}{L^2 f_y} \times 10^9 = 4.840 + 0.3446 \cdot \frac{p}{k_1 k_3 f_c'} \times 10^6
\]

\[
r = 0.86
\]

\[
S_y = 0.79 \times 10^{-9}
\]

\[
\Delta y d = \frac{f_y}{E_s} \left[ 0.1452 + 10.338 \cdot \frac{p}{k_1 k_3 f_c'} \right]
\]
\[ \Delta y^d \frac{L^2}{f_y p k_1 k_3 f_c^2} \times 10^3 = 0.4325 + 0.4436 \frac{k_1 k_3 f_c^2}{p} \times 10^{-5} \]

where:
- \( r = 0.98 \)
- \( S = 0.17 \times 10^{-4} \)

FIG. A2 BEAMS REINFORCED IN TENSION ONLY: MIDSPAN DEFLECTION AT YIELD

EQUATION A 14
\[ \frac{\Delta_m d}{L^2} \times 10^3 = -0.1 + 0.35 \frac{k_1 k_3 f'_c}{p f_y} \quad (A\ 19) \]

\[ r = 0.96 \]
\[ S_y = 0.33 \times 10^{-3} \]

\[ \frac{\Delta_m d}{L^2} = \frac{3.415 \times 10^4}{10^4} \frac{k_1 k_3 f'_c}{p f_y} \quad (A\ 20) \]

(Least squares line through origin)

FIG. A3 BEAMS REINFORCED IN TENSION ONLY: MIDSPAN DEFLECTION AT MAXIMUM LOAD
FIG. A4  INVERSE OF DUCTILITY FACTOR FOR BEAMS

REINFORCED IN TENSION ONLY
FIG. A5 DUCTILITY FACTORS FOR BEAMS REINFORCED IN TENSION ONLY

\[
\frac{\Delta_m}{\Delta_y} = \begin{cases} 
\text{Eq. A28} \\
\text{Eq. A27} - f_y = 40,000 \text{ psi} \\
\text{Eq. A27} - f_y = 55,000 \text{ psi}
\end{cases}
\]
\[
\frac{\Delta y_d}{L^2 f_y} \times 10^9 = 6.42 + 0.825 \frac{p}{k_1 k_3 f'_c} \times 10^6
\]

\[r = 0.74\]

\[S_y = 0.33 \times 10^{-9}\]

\[
\frac{\Delta y_d}{L^2 f_y} = \frac{f_y}{E_s} \left[ 0.1926 + 2475 \frac{p}{k_1 k_3 f'_c} \right] (A \ 29)
\]

**FIG. A6** BEAMS REINFORCED IN TENSION AND COMPRESSION

MIDSPAN DEFLECTION AT YIELD—EQUATION A 29
FIG. A7 BEAMS REINFORCED IN TENSION AND COMPRESSION: MIDSPAN DEFLECTION AT YIELD—EQUATION A 30
\[
\frac{\Delta_m d}{L^2} \times 10^3 = -0.1003 + 0.4412 \frac{k_1 k_3 f'_c}{p_{f_y} - p'_{f_y}}
\]

\[r = 0.83\]

\[S_y = 0.83 \times 10^{-3}\]

\[
\frac{\Delta_m d}{L^2} = \frac{4.290}{10^4} \frac{k_1 k_3 f'_c}{p_{f_y} - p'_{f_y}}
\]  \hspace{1cm} (A 32)

FIG. A8 BEAMS REINFORCED IN TENSION AND COMPRESSION: MIDSPAN DEFORMATION AT MAXIMUM LOAD
FIG. A9 DUCTILITY FACTORS FOR BEAMS REINFORCED IN TENSION AND COMPRESSION
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