DEVELOPMENT OF DESIGN CRITERIA FOR SIMPLY SUPPORTED SKEW SLAB-AND-GIRDER BRIDGES

by

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Elastic analyses, using the finite element method, were done on 108 single span skew slab-and-girder bridges. Each structure had 5 girders and stiffnesses were representative of bridges with pretensioned I-girders or steel I-beams. Spans ranged from 40 to 80 ft, girder spacings from 6 to 9 ft, and the skew angle from zero to 60 degrees. The loadings were multiple point loads representing two HS20 AASHTO vehicles, and the loads were positioned to produce maximum bending moments in the girders. Convergence studies to evaluate the precision of the finite element models were also done, and comparisons were made with the results of other studies.

An extensive parametric study was done to determine the most important variables and to gain an understanding of the response of the skew bridge. Expressions for the design moments in interior and exterior girders were then developed. These take into account the span and spacing of girders, the stiffness of the girders relative to the slab stiffness, and the angle of skew. The format is the use of the static moment for a girder, with modifications to this moment based on girder span and spacing, slab to girder stiffness ratio, and skew angle. A similar study was done to obtain factors for the calculation of deflections, starting with the deflection of a simple beam.

Key Words
Highway Bridges, Skew Bridges, Moment Distributions, Slab and Girder Bridges, Finite Element Analysis

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>TITLE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.1 General</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.2 Historical Review</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1.3 Purpose and Scope of Investigation</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1.3.1 Purpose</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1.3.2 Scope</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1.4 Method of Approach and Arrangement of Presentation</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1.5 Notation</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>IDEALIZATION OF THE BRIDGE AND INTRODUCTION OF THE PARAMETERS USED</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.1 General</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.2 Idealization of the Bridge</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>2.3 Introduction of the Parameters Used and their Range of Application</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>2.3.1 General</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>2.3.2 Parameters Defining the Geometry of the Bridge</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2.3.3 Parameters Defining the Elastic Properties of the Materials</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>2.3.4 Parameters Defining the Structural Properties of the Bridge Members</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>2.3.4.1 General</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>2.3.4.2 The Flexural Slab Stiffness $D$</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>2.3.4.3 The Flexural Composite Girder Stiffness $E_g I_c$</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>2.3.4.4 The Torsional Girder Stiffness $G_g J$</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>2.3.4.5 The Dimensionless Stiffness Ratio $H$</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>2.3.5 Parameters Defining the Structural Loading Conditions</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>2.3.5.1 Live Load</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>2.3.5.2 Dead Load</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>2.4 Summary of the Parameters Used in the Parametric Study</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>METHOD OF STRUCTURAL ANALYSIS</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>3.1 General</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>3.2 The Finite Element Method</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>3.3 The Finite Elements Used in this Study</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>3.3.1 Degenerated Thin Shell Isoparametric Element</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>3.3.2 Eccentric Isoparametric Beam Element</td>
<td>41</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>3.4 The Behaviour of the Finite Elements Used</td>
<td>44</td>
<td></td>
</tr>
<tr>
<td>3.4.1 Bending Behaviour</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>3.4.2 Plane Stress Behaviour</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>3.5 Finite Element Mesh Choice: Convergence Study on a Typical Bridge</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>3.6 Comparisons with Previous Bridge Solutions</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>3.6.1 Example Problem: BRIDGE-1</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>3.6.2 Example Problem: BRIDGE-2</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td>3.6.3 Example Problem: BRIDGE-3</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>3.6.4 Example Problem: BRIDGE-4</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>4 DISCUSSION OF RESULTS</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>4.1 General</td>
<td>68</td>
<td></td>
</tr>
<tr>
<td>4.2 Errors in the Bottom Fibre Stresses of the Girders</td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>4.3 Differences in Results for Bridges Which Have the Same b/a and H Ratios</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>4.4 Bridges with more than Five Girders</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>4.5 Influence of Girder Torsional Stiffness</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>4.6 Influence of the End Diaphragms</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>4.7 Locations of the Trucks for Maximum Girder Bending Moments</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>4.8 Results of the Parametric Study</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>4.8.1 General</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>4.8.2 Influence of the Stiffness Parameter H and the Geometric Parameter b/a</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>4.8.2.1 Effect of Varying the Stiffness Parameter H</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>4.8.2.2 Effect of Varying the Parameters b and b/a</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>4.8.3 Effect of Varying the Angle of Skew α</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>4.9 Comparison with the AASHTO Design Recommendations for Right Bridges</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>5 DESIGN CRITERIA FOR RIGHT AND SKEW SLAB-AND-GIRDER BRIDGES</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>5.1 General</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>5.2 Design Criteria Format for Girder Bending Moments</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>5.3 Criteria for Right Bridges</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>5.3.1 Exterior Girders</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>5.3.2 Interior Girders</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>5.4 Criteria for Skew Bridges</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>5.5 Proposed Analysis Procedure for Slab-and-Girder Bridges</td>
<td>102</td>
<td></td>
</tr>
</tbody>
</table>
5.6 Girder Deflections due to Truck Loads ........................................... 104
5.7 Girder Bending Moments due to Dead Load ................................ 106
  5.7.1 Curbs and Parapets ............................................................ 107
  5.7.2 Roadway Resurfacing Load ................................................. 109

6 SUMMARY AND CONCLUSIONS ..................................................... 111

  6.1 Summary ..................................................................................... 111
  6.2 Conclusions .............................................................................. 113
    6.2.1 Conclusions Regarding Design Criteria ......................... 113
    6.2.2 Conclusions Regarding the Behaviour of the Bridge ........ 114
    6.2.3 Conclusions Regarding the Method
          of Structural Analysis ....................................................... 116
    6.2.4 Conclusions Regarding Errors that can be Expected .......... 117
  6.3 Recommendations for Further Research ................................. 118

LIST OF REFERENCES ........................................................................ 119

TABLES ............................................................................................. 131

FIGURES .......................................................................................... 171
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Properties of Supporting Girders Used in the Parameter Study</td>
<td>132</td>
</tr>
<tr>
<td>3.1</td>
<td>Element In-Plane Behaviour: Rectangular Cantilever Beam</td>
<td>133</td>
</tr>
<tr>
<td>3.2</td>
<td>Element In-Plane Behaviour: Skew Cantilever Beam</td>
<td>133</td>
</tr>
<tr>
<td>3.3</td>
<td>Deflection Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)</td>
<td>134</td>
</tr>
<tr>
<td>3.4</td>
<td>Girder Bending Moment Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)</td>
<td>135</td>
</tr>
<tr>
<td>3.5</td>
<td>Girder Axial Force Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)</td>
<td>136</td>
</tr>
<tr>
<td>3.6</td>
<td>Deflection Convergence for $\alpha = 60$ degrees (Mesh 1,2,3)</td>
<td>137</td>
</tr>
<tr>
<td>3.7</td>
<td>Girder Bending Moment Convergence for $\alpha = 60$ degrees (Mesh 1,2,3)</td>
<td>138</td>
</tr>
<tr>
<td>3.8</td>
<td>Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 1,2,3)</td>
<td>139</td>
</tr>
<tr>
<td>3.9</td>
<td>Summary of the Maximum % Change in Results Between Mesh 3 and Mesh 1,2</td>
<td>140</td>
</tr>
<tr>
<td>3.10</td>
<td>Deflection Convergence for $\alpha = 60$ degrees (Mesh 4,2,5)</td>
<td>141</td>
</tr>
<tr>
<td>3.11</td>
<td>Girder Bending Moment Convergence for $\alpha = 60$ degrees (Mesh 4,2,5)</td>
<td>142</td>
</tr>
<tr>
<td>3.12</td>
<td>Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 4,2,5)</td>
<td>143</td>
</tr>
<tr>
<td>3.13</td>
<td>Summary of the Maximum % Change in Results Between Mesh 5 and Mesh 2,4</td>
<td>144</td>
</tr>
<tr>
<td>3.14</td>
<td>Girder Bending Moment Convergence for $\alpha = 60$ degrees (Mesh 2,5)</td>
<td>145</td>
</tr>
<tr>
<td>3.15</td>
<td>Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 2,5)</td>
<td>146</td>
</tr>
<tr>
<td>3.16</td>
<td>Example Problem: BRIDGE-2</td>
<td>147</td>
</tr>
<tr>
<td>3.17</td>
<td>Example Problem: BRIDGE-3</td>
<td>148</td>
</tr>
</tbody>
</table>
Table Page

3.18 Example Problem: BRIDGE-4 ................................................................. 149

4.1 Errors in the Bottom Fibre Stresses in Supporting Girders which Result from the use of the Effective Flange Width Concept ......................................................... 150

4.2 Percentage Girder Bending Moment Differences Obtained from Three Bridges with the same H and b/a Ratios Loading Condition: A Single Point Load ........................................................................ 151

4.3 Percentage Girder Bending Moment Differences Obtained from Two Bridges with the same H and b/a Ratios Loading Condition: Two AASHTO HS20-44 Trucks .......................................................... 151

4.4 Effect of an Increase in the Number of Girders on the Girder Moments ......................................................................................................................... 152

4.5 Effect of Girder Torsional Stiffness on the Girder Bending Moments (1) .............................................................................................................................. 153

4.6 Effect of Girder Torsional Stiffness on the Girder Bending Moments (2) .............................................................................................................................. 154

4.7 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 0$ degrees .................................................................................. 155

4.8 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 30$ degrees .................................................................................. 155

4.9 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 45$ degrees .................................................................................. 156

4.10 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 60$ degrees .................................................................................. 156

4.11 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 0$ degrees .................................................................................. 157

4.12 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 30$ degrees .................................................................................. 157
<table>
<thead>
<tr>
<th>Table</th>
<th>Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha$ = 45 degrees</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.13</td>
<td></td>
<td>158</td>
</tr>
<tr>
<td>4.14</td>
<td></td>
<td>158</td>
</tr>
<tr>
<td>4.15</td>
<td></td>
<td>159</td>
</tr>
<tr>
<td>4.16</td>
<td></td>
<td>159</td>
</tr>
<tr>
<td>4.17</td>
<td></td>
<td>160</td>
</tr>
<tr>
<td>4.18</td>
<td></td>
<td>160</td>
</tr>
<tr>
<td>4.19</td>
<td></td>
<td>161</td>
</tr>
<tr>
<td>4.20</td>
<td></td>
<td>161</td>
</tr>
<tr>
<td>4.21</td>
<td></td>
<td>162</td>
</tr>
<tr>
<td>4.22</td>
<td></td>
<td>162</td>
</tr>
<tr>
<td>4.23</td>
<td></td>
<td>163</td>
</tr>
</tbody>
</table>
Table

<table>
<thead>
<tr>
<th>Table</th>
<th>Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.24</td>
<td>........................................................................................................................................ 163</td>
</tr>
<tr>
<td>4.25</td>
<td>........................................................................................................................................ 164</td>
</tr>
<tr>
<td>4.26</td>
<td>........................................................................................................................................ 164</td>
</tr>
<tr>
<td>4.27</td>
<td>........................................................................................................................................ 165</td>
</tr>
<tr>
<td>4.28</td>
<td>........................................................................................................................................ 165</td>
</tr>
<tr>
<td>4.29</td>
<td>........................................................................................................................................ 166</td>
</tr>
<tr>
<td>4.30</td>
<td>........................................................................................................................................ 166</td>
</tr>
<tr>
<td>4.31</td>
<td>........................................................................................................................................ 167</td>
</tr>
<tr>
<td>4.32</td>
<td>........................................................................................................................................ 167</td>
</tr>
<tr>
<td>4.33</td>
<td>........................................................................................................................................ 168</td>
</tr>
<tr>
<td>4.34</td>
<td>........................................................................................................................................ 168</td>
</tr>
</tbody>
</table>
Table 4.35 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 8.25 ft; Angle of Skew $\alpha = 0$ degrees ................................................................. 168

4.36 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6.75 ft; Angle of Skew $\alpha = 60$ degrees ................................................................. 169

4.37 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 7.5 ft; Angle of Skew $\alpha = 60$ degrees ................................................................. 169

4.38 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 8.25 ft; Angle of Skew $\alpha = 60$ degrees ................................................................. 169

5.1 Maximum Girder Bending Moments $M_{cg}$ for Dead Load: Curbs and Parapets ................................................................. 170

5.2 Maximum Girder Bending Moments $M_{cg}$ for Dead Load: Roadway Resurfacing ................................................................. 170
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Geometry of the Typical Skew Slab-and-Girder Bridge Considered</td>
<td>172</td>
</tr>
<tr>
<td>2.2</td>
<td>Girder Properties</td>
<td>173</td>
</tr>
<tr>
<td>2.3</td>
<td>Relative Truck Locations</td>
<td>174</td>
</tr>
<tr>
<td>3.1</td>
<td>Compatibility Problem Between an Eccentric Beam Element and a Shell Element (1)</td>
<td>175</td>
</tr>
<tr>
<td>3.2</td>
<td>Compatibility Problem Between an Eccentric Beam Element and a Shell Element (2)</td>
<td>176</td>
</tr>
<tr>
<td>3.3</td>
<td>Nodal Degrees of Freedom and Forces Acting on the QLSHELL Element</td>
<td>177</td>
</tr>
<tr>
<td>3.4</td>
<td>Eccentric Assembly of Beam and Shell Elements</td>
<td>178</td>
</tr>
<tr>
<td>3.5a</td>
<td>Plan View of two QLSHELL Elements Showing the Incompatibility due to Differential V-displacements in the Beam Element</td>
<td>179</td>
</tr>
<tr>
<td>3.5b</td>
<td>Incompatibility due to $\theta_x$ Rotations in the Shell Elements</td>
<td>179</td>
</tr>
<tr>
<td>3.6</td>
<td>Rhombic Plate Subjected to a Uniformly Distributed Load: Deflections</td>
<td>180</td>
</tr>
<tr>
<td>3.7</td>
<td>Rhombic Plate Subjected to a Uniformly Distributed Load: Maximum Principal Moments</td>
<td>181</td>
</tr>
<tr>
<td>3.8</td>
<td>Rhombic Plate Subjected to a Uniformly Distributed Load: Minimum Principal Moments</td>
<td>182</td>
</tr>
<tr>
<td>3.9</td>
<td>Skew Cantilever Beam: Geometry and Mesh Layout</td>
<td>183</td>
</tr>
<tr>
<td>3.10</td>
<td>Skew Cantilever Beam: Vertical Deflection at Point A Relative to the Deflection Obtained From Mesh 4</td>
<td>184</td>
</tr>
<tr>
<td>3.11</td>
<td>Geometry and Structural Properties of the Bridge Used in the Convergence Study</td>
<td>185</td>
</tr>
<tr>
<td>3.12</td>
<td>Finite Element Mesh Models Used in the Bridge Convergence Study</td>
<td>186</td>
</tr>
<tr>
<td>3.13</td>
<td>Slab Action in Very Skew Short Bridges</td>
<td>187</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.14</td>
<td>Midspan Axial Force in the Slab in the Longitudinal Direction</td>
<td>188</td>
</tr>
<tr>
<td>3.15</td>
<td>Midspan Bending Moment in the Slab in the Transverse Direction</td>
<td>189</td>
</tr>
<tr>
<td>3.16</td>
<td>Example Problem BRIDGE-1: Geometry, Member Properties and Mesh Layout</td>
<td>190</td>
</tr>
<tr>
<td>3.17</td>
<td>Example Problem BRIDGE-1: Deflection at the Location of the Load</td>
<td>191</td>
</tr>
<tr>
<td>3.18</td>
<td>Example Problem BRIDGE-1: Distribution of the Longitudinal Direction Axial Force in the Deck (Taken from Ref. 63)</td>
<td>192</td>
</tr>
<tr>
<td>3.19</td>
<td>Example Problem BRIDGE-1: Strong-Axis Bending Moments in the Girders (Taken from Ref. 63)</td>
<td>193</td>
</tr>
<tr>
<td>3.20</td>
<td>Example Problem BRIDGE-2: Geometry and Member Properties</td>
<td>194</td>
</tr>
<tr>
<td>3.21</td>
<td>Influence Lines for Girder Bending Moment $M_{cg}$ at Midspan due to a Point Load P Moving Transversely Across the Bridge at Midspan: $b/a = 0.05$ (Taken from Ref. 112)</td>
<td>195</td>
</tr>
<tr>
<td>3.22</td>
<td>Example Problem BRIDGE-3 and -4: Plan View and Cross Section</td>
<td>196</td>
</tr>
<tr>
<td>4.1</td>
<td>Midspan Girder Bending Moment Influence Lines for a Point Load P Moving Along the Skew Centre Line</td>
<td>197</td>
</tr>
<tr>
<td>4.2</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 40$ ft; $b = 6$ ft</td>
<td>198</td>
</tr>
<tr>
<td>4.3</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 60$ ft; $b = 6$ ft</td>
<td>199</td>
</tr>
<tr>
<td>4.4</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 80$ ft; $b = 6$ ft</td>
<td>200</td>
</tr>
<tr>
<td>4.5</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 40$ ft; $b = 9$ ft</td>
<td>201</td>
</tr>
<tr>
<td>4.6</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 60$ ft; $b = 9$ ft</td>
<td>202</td>
</tr>
<tr>
<td>4.7</td>
<td>Maximum Girder Bending Moment Variation with H: $a = 80$ ft; $b = 9$ ft</td>
<td>203</td>
</tr>
</tbody>
</table>
Figure | Page
--- | ---
4.8 Maximum Girder Bending Moment Variation with b/a by Changing b: \(\alpha = 0\) degrees | 204
4.9 Maximum Girder Bending Moment Variation with b/a by Changing b: \(\alpha = 60\) degrees | 205
4.10 Maximum Girder Bending Moment Variation with b/a by Changing b: \(\alpha = 0\) degrees | 206
4.11 Maximum Girder Bending Moment Variation with b/a by Changing b: \(\alpha = 60\) degrees | 207
4.12 Girder Midspan Deflection Variation with b/a by Changing b: \(\alpha = 0\) degrees | 208
4.13 Girder Midspan Deflection Variation with b/a by Changing b: \(\alpha = 60\) degrees | 209
4.14 Girder Midspan Deflection Variation with b/a by Changing b: \(\alpha = 60\) degrees | 210
4.15 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 6\) ft; \(H = 5\) | 211
4.16 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 6\) ft; \(H = 10\) | 212
4.17 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 6\) ft; \(H = 20\) | 213
4.18 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 6\) ft; \(H = 30\) | 214
4.19 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 9\) ft; \(H = 5\) | 215
4.20 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 9\) ft; \(H = 10\) | 216
4.21 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 9\) ft; \(H = 20\) | 217
4.22 Maximum Girder Bending Moment Variation with b/a by Changing a: \(b = 9\) ft; \(H = 30\) | 218
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.23</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 40$ ft; $b = 6$ ft</td>
<td>219</td>
</tr>
<tr>
<td>4.24</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 60$ ft; $b = 6$ ft</td>
<td>220</td>
</tr>
<tr>
<td>4.25</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 80$ ft; $b = 6$ ft</td>
<td>221</td>
</tr>
<tr>
<td>4.26</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 40$ ft; $b = 9$ ft</td>
<td>222</td>
</tr>
<tr>
<td>4.27</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 60$ ft; $b = 9$ ft</td>
<td>223</td>
</tr>
<tr>
<td>4.28</td>
<td>Maximum Girder Bending Moment Variation with $\alpha$: $a = 80$ ft; $b = 9$ ft</td>
<td>224</td>
</tr>
<tr>
<td>5.1</td>
<td>Q-values for Exterior Girder Bending Moments in Right Slab-and-Girder Bridges</td>
<td>225</td>
</tr>
<tr>
<td>5.2</td>
<td>Q-values for Interior Girder Bending Moments in Right Slab-and-Girder Bridges</td>
<td>226</td>
</tr>
<tr>
<td>5.3</td>
<td>Interior Girder Skew Reduction Factor $Z$ for Bending Moments</td>
<td>227</td>
</tr>
<tr>
<td>5.4</td>
<td>Exterior Girder Skew Reduction Factor $Z$ for Bending Moments</td>
<td>228</td>
</tr>
<tr>
<td>5.5</td>
<td>Consistent Interior Girder Skew Reduction Factor $Z$ for Bending Moments</td>
<td>229</td>
</tr>
<tr>
<td>5.6</td>
<td>Consistent Exterior Girder Skew Reduction Factor $Z$ for Bending Moments</td>
<td>230</td>
</tr>
<tr>
<td>5.7</td>
<td>X-values for Interior Girder Midspan Deflections in Right Slab-and-Girder Bridges</td>
<td>231</td>
</tr>
<tr>
<td>5.8</td>
<td>Interior Girder Skew Reduction Factor $Y$ for Midspan Deflections</td>
<td>232</td>
</tr>
<tr>
<td>5.9</td>
<td>X-values for Exterior Girder Midspan Deflections in Right Slab-and-Girder Bridges</td>
<td>233</td>
</tr>
<tr>
<td>5.10</td>
<td>Exterior Girder Skew Reduction Factor $Y$ for Midspan Deflections</td>
<td>234</td>
</tr>
</tbody>
</table>
1.1. General

The slab-and-girder bridge is named so because it consists of two major types of structural members. These are: a) a reinforced concrete slab which serves as the roadway and distributes the concentrated loads imposed by vehicle wheels to b) a number of flexible girders which span in the direction of the traffic and carry all the loads to the abutments.

A skew slab-and-girder bridge is one in which the abutments are not perpendicular to the girders. Many skew highway bridges have already been built in grade separations where the intersecting roads are not perpendicular to one another. They are also necessary where natural or existing man-made obstacles prevent a perpendicular crossing and consequently they are commonly found in mountainous areas. In many cases, the lack of space at complex intersections and in congested built-up areas may also require bridges to be built on skew alignment.

The slab-and-girder bridge system is a favoured structural choice both on economic and aesthetic grounds. The use of some kind of shear mechanism which ensures composite action between the girders and the slab makes it possible to use smaller supporting girders. If steel I-beam or precast prestressed concrete girders are used, expensive shoring can be avoided because these can support the weight of the wet cast-in-place slab concrete. This makes construction relatively rapid and easy and minimizes traffic interruption when it is a problem.

The basic design problem is to determine the distribution of wheel loads among the girders so that the girders can be proportioned to be sufficiently stiff and strong. This has been studied for decades by many researchers who used different approaches to solve the problem. Very little research has been done on skew slab-and-girder
bridges because of the large amount of work involved. Studies on the behaviour of slab-and-girder bridges were limited to right bridges until the advent of the electronic digital computer which made very extensive numerical solutions possible. Research on skew slab-and-girder bridges has had limited impact on bridge design, so much so that even the current (1985) AASHTO Standard Specifications for Highway Bridges (5)* provide the practicing design engineer with absolutely no guidance regarding the effects of skew on the behaviour of a bridge. Therefore, research on skew slab-and-girder bridges with the goal to develop design criteria which include the effects of skew appears desirable. This is the purpose of the present study.

1.2. Historical Review

A program of systematic and coordinated research began in 1936 at the University of Illinois in an attempt to answer some of the questions regarding the design of highway bridges. This research, which was done in cooperation with the Illinois Division of Highways and the Bureau of Public Roads between 1936 and 1954, is summarized by Newmark and Siess (76). All of the bridge problems were solved by a combination of mathematical analyses and laboratory tests. Laboratory tests were done on small scale models of highway bridges and on full scale elements of such bridges. The experimental and analytical results were compared and wherever possible correlated with results of field observations. The design recommendations based on this research have had a significant impact on the existing state of the art of bridge design.

A major contribution to the analysis of right slab-and-girder bridges was made in 1938 by Newmark (72) who developed a method which correctly accounts for the action of a slab continuous over noncomposite supporting girders. The method is derived from the moment distribution method of analysis developed by Hardy Cross.

* The numbers in parentheses refer to the list of references.
The solution is found in the form of an infinite trigonometric series where each term is obtained by numerical calculations involving fixed-end moment, stiffnesses and carry-over factors applied to an analogous continuous beam. This method of analysis is exact in the sense that it leads to formulas in terms of infinite series that satisfy the fundamental differential equation of the theory of flexure of slabs.

Newmark and Siess (75) used this method to analyse a large number of right slab-and-girder bridges which enabled them to determine the structural behaviour. The distribution of load according to their analytical results is in excellent agreement with the distribution determined from measured strains in quarter scale bridge model tests (78). The well-known S/5.5 wheel load fraction which is currently used for the design of the interior girders in slab-and-girder bridges is based on the analytical results of their study. The method of analysis is, however, limited in that neither composite action, girder torsion nor skew bridges can be considered.

Since then, many researchers have further investigated the behaviour of right slab-and-girder bridges using various methods of analysis and including the effects of composite action, girder torsion and transverse diaphragms. Many of these are listed in the references (8, 20, 36, 63, 85, 86, 87, 112,). By contrast, research in the area of skew slab-and-girder bridges which gives some guidance to the practicing bridge engineer is still lacking in the literature. In 1940-1941 Newmark, Siess and Peckham (77) tested five quarter scale simply supported, skew slab-and-girder bridge models. At that time it was not feasible to carry out finite difference analyses on skew slab-and-girder bridges, because the calculations had to be done by hand. The laboratory tests were too limited in scope to lead directly to any design recommendations for skew slab-and-girder bridges. However, Newmark (92) proposed a rational method to take the effects of skew into account approximately.

Chen (14) analysed 18 simply supported, skew slab-and-girder bridges in 1953 using a digital computer which could solve 39 simultaneous equations. The five-girder
bridges analysed had different dimensions and various angles of skew. Chen computed influence surfaces for the midspan bending moments of the girders of the 18 bridges and used these to determine the midspan bending moments in the girders of 72 bridges subjected to AASHTO H-type standard truck loads. He derived from these results a set of empirical relations which can be used to determine wheel load fractions in skew slab-and-girder bridges.

Some of the insufficiencies in Chen's work are as follows:

1. As a result of computer limitations, he was forced to use a rather coarse 8x8 finite difference grid. He compared his results for right bridges with the exact solutions obtained by Newmark (75) which were in good agreement. He then assumed that his finite difference grid was also sufficient to obtain accurate results for skew bridges. There was no independent study of the influence of skew on the solution accuracy. Experience in dealing with the finite difference method has shown, however, that convergence of the solution deteriorates with increase in skew.

2. In the finite difference method an applied concentrated load is converted to a uniformly distributed load which acts on an area equal to the area contained within four adjacent grid lines. This means that a wheel load was distributed over one eighth of the length and width of a bridge. The length over which it is distributed is unrealistically large if the span of the bridge is for instance 80 ft.

3. Chen's work does not include the effects of girder torsion and composite action.

4. Both Chen's and Newmark's wheel load fractions are based on the distribution of only one of the axle loads from each truck on a bridge.

5. A major weakness in Chen's work is the method which he used to express the effects of skew on the wheel load fractions. For large angles of skew where the reduction in girder bending moments as the consequence of skew is significant, very large scatter exists in his wheel load fraction data points. For instance, for a
skew angle of 60 degrees the scatter is as much as 55%. Any beneficial effect of skew is completely lost if a conservative empirical relation is determined from data with such large scatter.

In 1957 Hendry and Jaeger (39) determined the effect of skew on the load distribution by applying their method of grid-frame analysis by the distribution of harmonics to interconnected girders in skew bridges with 3 or 4 longitudinal girders. In the grid-frame analysis method the deck and girders are replaced by an equivalent grillage of interconnected beams with stiffnesses approximately equal to the stiffnesses of the sections of the slab and girders which are replaced. Fujio, Ohmura and Naruoka (30) proposed formulas to determine midspan bending moments in the interior girders of skew grillage bridges. Their formulas were based on a finite difference analysis of orthogonal anisotropic skew plates proposed by Naruoka and Ohmura (69).

In 1966 Gustafson (36) developed a finite element matrix method to analyse skew plates with eccentric integral stiffeners. He used this method to analyse two skew slab-and-girder bridges. The purpose of his study was the development of the method and his computer program.

Mehrain (63) developed finite element computer programs in 1967 to analyse skew composite slab-and-girder bridges. His main objectives were the development of his programs and to study the convergence and accuracy using different finite elements. He removed some kinematic incompatibilities which exist in Gustafson's eccentric girder modelling. His deflection results compared well with those obtained experimentally from a series of tests on plastic bridge models.

In more recent work by Powell and Buckle (85, 86, 87) various computer programs were developed and tested on many types of slab-and-girder bridges. They compared results between the different programs which were based on the following idealizations: ribbed plate, equivalent anisotropic plate, equivalent grid and equivalent isolated girder. They concluded that the isolated girder idealization does not lead to
consistent maximum design values. The ribbed plate, equivalent plate and equivalent grid give about the same results. In skew bridges the grid idealization may underestim­
ate the transverse flexural deck stiffness.

In 1983 Kennedy and Grace (45) analytically determined the effect of diaphragms on the distribution of load in skew slab-and-girder bridges subjected to point loads. They found that the transverse distribution of a point load is enhanced by diaphragms and that the effect of diaphragms is more pronounced in relatively wide bridges with large skew angles.

Many other researchers investigated the behaviour of skew unstiffened slabs, skew slabs with edge beams and other types of skew bridges which are not multiple­
girder bridges.

1.3. Purpose and Scope of Investigation

1.3.1. Purpose

The main purpose of this study is to develop a reliable method of analysis for simply supported, skew slab-and-girder bridges based on linear elastic analysis. Such a method of analysis should be easy to use, should approximate the true behaviour of a bridge with acceptable accuracy and should preferably be in a form familiar to practic­
ing engineers.

In order to develop this simplified analysis procedure it is necessary to find a mathematical model and analytical method of analysis which can accurately predict the behaviour of a skew slab-and-girder bridge. The finite element method is chosen for this purpose. It is dangerous to use computer output blindly, but when the structure being analysed is very complex and no exact solutions exist with which computer results can be compared, one is often forced to rely upon these results. Therefore, it is very important to know if the finite elements which are used are capable of
providing the correct solution. The objectives of this study can be listed as follows:

1. To determine the effect of skew distortion on the behaviour of the finite elements which are used and to determine if the use of these elements ensures convergence to the exact solution.

2. To determine the finite element mesh which provides a solution close to the converged 'correct' solution by doing a convergence study on the bridge. The 'correct' solution is defined as the solution when the finite element results have converged completely.

3. To verify the accuracy of the solutions presented in this study by comparing them with existing solutions for slab-and-girder bridges.

4. To use the selected mesh which provides results close to the 'correct' solution to study the behaviour of skew slab-and-girder bridges by varying the parameters which determine the behaviour of the bridge.

5. To interpret and process the data obtained from the parametric study to develop a simplified, accurate analysis procedure for the maximum girder bending moments in skew slab-and-girder bridges.

1.3.2. Scope

The typical skew bridge considered consists of a reinforced concrete slab of uniform thickness supported by five precast prestressed concrete or steel I-beam girders. The girders are identical, prismatic and equally spaced. The bridge is simply supported at the abutments. Full composite action occurs between the slab and girders. The torsional stiffness of the girders is taken into account. The type of skew considered is such that the abutments are parallel to each other. The span of the bridge varies from 40 to 80 ft, the girder spacing from 6 to 9 ft and the angle of skew $\alpha$, as defined in Fig. 2.1, from 0 to 60 degrees. The slab thicknesses and girder properties used cover
the practical ranges for this type of bridge. The results can also be applied to bridges with steel I-beams if a minor modification is made. A total of 108 two-lane slab-and-girder bridges subjected to two HS20-44 AASHTO standard trucks are analysed. It is assumed that the bridge behaves in a linearly elastic manner. The emphasis is on the maximum bending moments in the girders.

The following limitations are imposed:

1. The bridge has only end diaphragms.
2. The stiffening effect of the curbs is ignored.
3. Only I-shaped girders are considered.
4. The length of the slab overhangs at the edge girders is 19 inches for all bridges considered. This is not of much importance because it is shown in Section 2.2 that a change in the overhang length has little influence on the maximum edge girder bending moment.
5. No truck wheel can get closer than two feet from an edge girder.

The idealization of the bridge model and the bases for certain assumptions and limitations regarding the modelling are discussed in detail in Section 2.2.

1.4. Method of Approach and Arrangement of Presentation

The method of approach follows the same order as the objectives stated in Section 1.3.1.

The idealization of the bridge and the bases for certain assumptions are discussed in Chapter 2. The parameters which determine the behaviour of the bridge are introduced and their ranges of variation are determined. This is followed by a discussion of the loading conditions considered for live and dead load.

Chapter 3 is devoted to the method of analysis. Problems in using the finite element method to model eccentric stiffeners are discussed. The particular elements
used are described and quality tests are done on the shell element, which is used to model the deck, to determine the influence of skew distortion on its behaviour. This is followed by a convergence study which determines the degree of refinement of the finite element mesh needed to give adequate results for the bridge. Four example bridges are analysed using this mesh and the results are compared with existing solutions.

The results of the parametric study are presented in Chapter 4. The effects of varying certain parameters are discussed in detail. This is preceded by a few important topics namely: the effect of increasing the number of girders; the effect of the end diaphragms at the abutments; the influence of girder torsional stiffness; consistency of the parameters; the magnitude of calculated girder bending stress errors and a discussion on the locations of trucks which result in maximum girder bending moments. Chapter 4 is concluded with a comparison between the present analytical results for right slab-and-girder bridges and the AASHTO design provisions.

In Chapter 5 a reliable, practical method of analysis is developed for simply supported, skew slab-and-girder bridges. The expected maximum errors using this analysis procedure are also indicated.

Chapter 6 gives a summary of this report and deals with the conclusions reached. Recommendations for further research are made.

1.5. Notation

The following symbols are defined where they are first introduced in the text. They are listed below for convenient reference.

A the identifier for the edge girder, as shown in Fig. 2.1.
$A_g$ cross sectional area of a prefabricated girder effective in tension.

$A_s$ cross sectional area of the effective flange of an interior composite T-section girder.

$A_{sx}$ cross sectional area of a prefabricated girder effective in shear in the vertical direction.

$A_{sy}$ cross sectional area of a prefabricated girder effective in shear in the horizontal direction.

$a$ span of the bridge in feet.

$B$ the identifier for the first interior girder, as shown in Fig. 2.1.

$b$ the girder spacing in feet.

$b/a$ ratio of the girder spacing to span.

$b_{eff}$ effective flange width of a composite T-section girder.

$b/Q$ wheel load fraction for maximum girder bending moment.

$b/X$ wheel load fraction for girder midspan deflection.

$C$ the identifier for the centre girder, as shown in Fig. 2.1.

$c$ dead load per unit length of a curb and parapet.

$D \frac{E_s t^3}{12(1 - \mu^2)}$ flexural stiffness of the slab per unit width.

$E$ Young’s modulus of elasticity.

$E_g$ modulus of elasticity of the supporting girders.

$E_s$ modulus of elasticity of the slab.

$e$ eccentricity of the centre of gravity of a prefabricated girder with respect to the midsurface of the slab.
G \quad E/[2(1 + \mu)] \quad \text{shear modulus.}

G_s \quad \text{shear modulus of the material in the slab.}

G_g \quad \text{shear modulus of the material in the supporting girders.}

H \quad \frac{E_g I_{cg}}{aD} \quad \text{dimensionless stiffness parameter which is a measure of the bending stiffness of an interior composite girder relative to that of the slab.}

I_{cg} \quad \text{bending moment of inertia of an interior composite T-section girder.}

I_{gx} \quad \text{bending moment of inertia of a prefabricated girder about the strong axis.}

I_{gy} \quad \text{bending moment of inertia of a prefabricated girder about the weak axis.}

J \quad \text{torsional moment of inertia.}

k_1, k_2 \quad \text{constants.}

L \quad \text{span of a beam.}

M_{cg} \quad \text{total bending moment acting on a composite T-section girder.}

M_d \quad \text{design bending moment in a girder obtained from the simplified analysis procedure in Chapter 5.}

M_g \quad \text{bending moment acting on an isolated prefabricated girder.}

M_s \quad \text{integral of the longitudinal bending moments in the flange of a composite T-section girder.}

M_{\text{static}} \quad \text{maximum static bending moment in an isolated beam subjected to half the load of one AASHTO HS20-44 truck.}

N \quad \text{number of subdivisions in a finite element mesh.}

N_g \quad \text{axial force acting on an isolated prefabricated girder.}
P  a point load representing half the load of one heavy axle of an AASHTO HS20-44 truck.

Q  distribution factor for maximum girder bending moment.

R  ratio of the vertical stiffness of an interior composite T-section girder to the vertical stiffness of the section of the slab effective in the transverse direction. It is proportional to \( H(b/a)^3 \).

t  thickness of the slab.

u  displacement in the x-direction.

v  displacement in the y-direction.

w  displacement in the z-direction.

X  distribution factor for girder midspan deflection.

x  cartesian coordinate.

Y  skew reduction factor for girder midspan deflection.

y  cartesian coordinate.

Z  skew reduction factor for maximum girder bending moment.

z  cartesian coordinate.

\( \alpha \)  angle of skew as defined in Fig. 2.1.

\( \Delta \)  girder midspan deflection.

\( \Delta_{\text{static}} \)  midspan deflection of an isolated beam subjected to half the load of one HS20-44 truck located such to produce the maximum static bending moment in the beam.

\( \theta_n \)  rotation about an axis normal to the abutments as shown in Fig. 2.1.
\( \theta_x \)
rotation about the x-axis.

\( \theta_y \)
rotation about the y-axis.

\( \theta_z \)
rotation about the z-axis.

\( \mu \)
Poisson’s ratio, taken as 0.2.

\( \sigma \)
bending stress at a distance \( z \) from the neutral axis of a girder.

\( \omega \)
uniformly distributed dead load per unit area resulting from the resurfacing of the roadway between the faces of the curbs.
CHAPTER 2
IDEALIZATION OF THE BRIDGE AND INTRODUCTION
OF THE PARAMETERS USED

2.1. General

This chapter is divided into four main sections. In Section 2.2 a brief description is given of the idealized highway bridge considered. This is followed by a discussion of the bases of the assumptions and idealizations. In Section 2.3 parameters are introduced which relate to the loading conditions, to the geometry of the bridge and to the material and structural properties of the bridge members. The idealizations concerned with the material properties are discussed in Section 2.3. Section 2.4 consists of a summary of the parameters and their ranges used in this study and briefly describes their general effects on the structural behaviour of the bridge.

2.2. Idealization of the Bridge

The plan view and cross section of the slab-and-girder bridge considered are shown in Fig. 2.1. The cross section of the actual bridge is idealized as shown. The span of the bridge, a, is the length of the bridge in the longitudinal direction, that is, the direction in which the traffic moves. The girder spacing, b, is the shortest distance between two girders and is measured transverse to the direction of traffic movement. The angle of skew $\alpha$ is defined as the angle between the transverse and skew directions as indicated in Fig. 2.1.

The following assumptions and idealizations are made:

1. The bridge deck is idealized as a horizontal slab of uniform thickness. The material in the slab is homogeneous, elastic and isotropic.
2. The slab is supported by five identical equally spaced parallel eccentric I-shaped girders. The girders are elastic and prismatic, that is, the girder cross section remains the same along the length of the girder. The eccentricity, \( e \), indicated in Fig. 2.1 is the distance between the centre of gravity of a supporting girder and the midsurface of the slab.

3. The edge of the slab and the girder ends are simply supported at the two abutments unless specifically indicated otherwise. At any point along the two support edges the vertical deflection and rotation normal to the abutments, \( \theta_n \), are zero. This zero normal rotation \( \theta_n \) is shown as a vector using the right-hand rule in Fig. 2.1. The normal rotational constraint at a support, \( \theta_n = 0 \), has the same effect as an end diaphragm which is rigid in bending in its own plane.

4. Except for the imaginary diaphragms at the support edges, no other diaphragms exist.

5. To simplify the problem it is assumed that full composite action occurs between the supporting girders and slab. This means that there is no shear slip at a girder-slab interface.

6. The girder-slab interaction occurs along a line, that is, the girders have no width.

7. The stiffening effect of the curbs and parapets is ignored.

8. The width of the slab overhang at the two edge girders is 19 inches for all cases studied.

   Note that for the convergence study in Section 3.5 and the example problems discussed in Section 3.6, not all of the above assumptions are true. To enable comparison with previous solutions, the assumptions on which those solutions were based are followed.

   Some of the assumptions listed above need justification and are discussed in the following paragraphs.
The choice to analyse bridges which have only five girders is made mainly to limit the computational cost. It is concluded in Section 3.5 that four rows of finite elements are necessary between adjacent girders to ensure accurate results. This means that if there are more girders, the total number of slab elements required is increased. In Section 4.4 it is shown that the results for a five-girder bridge may conservatively be applied to bridges with more than five girders.

For precast concrete girders, the transfer of shear on the girder-slab interface can be accomplished by means of bond between the two elements and the use of vertical ties to prevent separation. Effective bond is ensured if the top surface of the precast beam has been left rough. Shear studs welded to the top flange are normally used in the case of a concrete slab on steel I-beams. According to Ref. 96, the transfer of horizontal shear between the slab and precast concrete girders is usually no problem at service load levels. Siess (73) showed that for a concrete slab on steel I-beams some shear slip does occur at service load, but the assumption that no slip occurs is still reasonable if the shear transfer mechanism is properly designed.

The maximum span of the simply supported bridges considered in this study is 80 ft. It is unlikely that the designer will use an I-shaped prestressed concrete girder with a top flange width of more than 20 inches for spans in this range. The economical girder spacings used are normally not less than 6 ft. Neglecting the width of the girders is thus not unreasonable and it can be assumed that the girder-slab interaction occurs along a line. I-shaped girders are assumed, thus girders of box-section are not considered. It is shown in Section 4.8.2 that an increase in the slab stiffness results in a better load distribution with smaller bending moments in the girders. Because the width of the girders is, in effect, a stiffening of the slab, the results obtained by ignoring the girder width are conservative.

The effect of the possible stiffening of the edge girder by the curb and parapet was investigated by Newmark (75). He found that an increase of 20% in the exterior
girder bending stiffness resulted in a difference of less than 4% in the maximum influence value for bending moment at midspan of the exterior girder. However, the increase in exterior girder bending stiffness as the result of the stiffness contribution of the curb can be much more than 20% with the consequence that the girder bending moments are effected by more than 4%. This is verified by a series of field tests done by Douglas (26), Guilford (33, 34) and Lin (55) on actual bridges. Their investigations consistently revealed that when the curbs are monolithic with the slab, they do have a moderate influence on the edge girder bending moment.

Despite this fact, designers normally ignore the stiffening effect of the curbs because they are not considered as load-carrying members. The structural designer does not wish to rely on a possible strengthening of the edge girder. There are many different types of curbs and traffic railings which might be used and they all have unknown stiffness contributions. Some curbs are monolithic, others are precast concrete units bolted to the slab and others are concrete parapets with expansion joints at short intervals.

With respect to the interior girders of a bridge, it is safe to ignore the effect of the curbs and parapets because small bending moment reductions occur in the interior girders when the curbs are included in the analysis. If the curb-to-slab connection is such that it does stiffen the edge girder and a larger edge girder design moment occurs, then the bending moment of inertia of the edge girder also has to be larger. With this larger inertia the additional load on the edge girder might be carried without exceeding the allowable bending stresses. This is likely because the contribution of a curb can easily double the edge girder bending moment of inertia, while the fraction of the load that the edge girder carries can never become twice as much as without the curb. It is thus likely that by ignoring the effect of the curbs, a conservative edge girder design will still result. It is this course which is followed throughout the rest of this study.
With regard to the width of the overhang, the designer usually has the freedom to choose the width of the overhang as he wishes. There are normally no limitations except for the aesthetic requirement that the face of the exterior girder should not be flush with the face of the slab, since this would present an unpleasant appearance of large depth.

The choice to use the width of the overhangs as 19 inches is based on the following practical consideration. In view of the uncertainty of the edge girder stiffness associated with the curb and the resulting uncertainty of the magnitude of the maximum edge girder bending moment, it is desirable to prevent the occurrence of the controlling design moment in the edge girder. This can be done by increasing the minimum possible distance between the edge girder and the truck wheel nearest to the edge girder. The influence line for edge girder bending moment in Fig. 3.21 clearly indicates that the truck wheel closest to the edge girder is the most effective load producing moments in the edge girder. It also shows that the moment in the edge girder is very sensitive to the location of the closest wheel, especially when the girder spacing, b, is small. If the minimum possible distance between the edge girder and nearest wheel is increased, a significant moment reduction results. The magnitude of this reduction depends on the geometry and stiffness of the bridge. Work on this subject, that by Sithichaikasem (112) on right bridges, shows that if the truck wheels are always at least two feet away from the edge girder in short span bridges (b/a = 0.1 or larger), the maximum design moment always occurs in one of the interior girders. He also showed that the edge girder bending moment may be the controlling moment if this two feet minimum distance is not kept.

Chen (14) reported that there is a tendency for the edge girder to become the controlling girder when skew is introduced. This is verified in Section 4.8.3. Thus, it is even more important for skew bridges to keep the nearest truck wheels at least two feet away from the edge girder.
Considering the above mentioned points, the decision made is to keep the truck wheels at least two feet away from the edge girders. This can be done by positioning the face of the curb directly above the edge girder. According to AASHTO provision 1.2.5, the outer truck wheels are always two feet away from the face of the curb and, therefore, also two feet away from the edge girder. The length of the overhangs is not the real issue, but it determines the location of the face of the curb relative to the edge girder which has a very important effect on the moments in the edge girder. A 19 inch overhang is required to attach an Illinois standard concrete curb such that the face of the curb is directly above the edge girder. Fortunately, the edge girder bending moment is insensitive to changes in the length of the overhang if the wheel is still kept two feet away from the edge girder. An investigative analysis on a practical skew bridge subjected to truck loads reveals that the maximum edge girder bending moment increases by only 3% if the overhang length is changed from 19 to 39 inches. The same change produces a reduction in the maximum moment in the centre girder of 0.4%.

2.3. Introduction of the Parameters Used and their Range of Application

2.3.1. General

The parameters used to study the behaviour of the bridge are classified as follows:

1. Parameters defining the geometry of the bridge.
2. Parameters defining the elastic properties of the materials.
3. Parameters defining the structural properties of the bridge members.
4. Parameters defining the loading conditions.
These parameters are discussed individually in the sections which follow.

2.3.2. Parameters Defining the Geometry of the Bridge

There are three parameters which determine the geometry of the bridge. They are: the angle of skew $\alpha$, the bridge span, $a$, and the girder spacing, $b$. These three parameters are defined in Section 2.2. Wherever it is convenient a fourth dimensionless parameter, the girder spacing to span ratio $b/a$, is used in conjunction with either $a$ or $b$. The behaviour of the bridge is sensitive to changes in $b$ and the $b/a$ ratio. The influence of the parameters $b$, $b/a$ and $\alpha$ on the behaviour of the bridge is discussed in detail in Chapter 4.

The following values for the angle of skew are used: $\alpha = 0, 30, 45$ and $60$ degrees. A survey by Kennedy (48) in 1969 showed that in the Canadian Province of Ontario about 35% of the total bridge deck area that had been built by that time was on skew alignment. The percentage of total deck area is distributed as shown in the table below:

<table>
<thead>
<tr>
<th>$\alpha$ Degrees</th>
<th>Percentage of Total Deck Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>65</td>
</tr>
<tr>
<td>1 - 30</td>
<td>21</td>
</tr>
<tr>
<td>30 - 45</td>
<td>9</td>
</tr>
<tr>
<td>45 - 60</td>
<td>4</td>
</tr>
<tr>
<td>$&gt;60$</td>
<td>1</td>
</tr>
</tbody>
</table>

Because only 1% of the total deck area is on skews of more than 60 degrees, the angle of skew is limited to 60 degrees in this study.

Since this study is concerned with simply supported bridges, the span is limited to 80 feet. Three values are used for the span: $a = 40, 60$ and $80$ ft. Spans shorter than 40 ft are not considered because it is likely that the engineer will choose an economic
slab-type bridge for spans in this range. It should be realized that a 40 ft gap that could be spanned by a 40 ft right bridge requires a 80 ft span if $\alpha = 60$ degrees.

Girder bending moment results for right bridges reported by Sithichaikasem (112) and others, and results for skew bridges reported by Chen (14) show a very smooth variation with $b/a$. Therefore, it is estimated that by using three spans for each $b$-value, enough data points will be generated to determine the behaviour of the bridge.

The girder spacing, $b$, is one of the most important parameters which determines how the truck load is distributed to the various girders in a bridge. This is reflected in the AASHTO design specifications because the girder bending moments may be calculated using the wheel load fraction—$b/5.5$.

In order to obtain an economic design, the engineer has to change the girder spacing, slab thickness and number of girders according to the span of the bridge. The current (1985) trend is to use larger girder spacings, thereby reducing the number of girders necessary. As reported in the optimization study in Refs. 88 and 89 fewer but stronger girders are more economic. The girder bending moment results for skew bridges reported by Chen (14) and those for right bridges reported by Sithichaikasem (112), indicate a nearly linear variation with $b/a$. The following girder spacings are used for the 108 bridges analysed: Forty-eight of the bridges have a girder spacing of $b = 6$ ft. Another group of 48 bridges has a girder spacing of $b = 9$ ft. To determine if the girder bending moments are also linear in $b$ for skew bridges, two bridges are analysed with $b = 6.75$ ft, eight with $b = 7.5$ and two are analysed with $b = 8.25$ ft. It is found that for all practical purposes a linear relation in $b$ does exist. This is discussed in Section 4.8.2.2. It is, therefore, possible to extrapolate the results linearly when moderate changes in girder spacings is needed.
2.3.3. Parameters Defining the Elastic Properties of the Materials

This study is concerned with the linear elastic behaviour of a bridge under service loads. It is assumed that the bridge slab is made of reinforced concrete, which is idealized as a homogeneous, elastic, isotropic material. The girders may be steel I-beams or prestressed concrete girders. The effect of cracks in the slab concrete on the stiffness of the slab is discussed in Section 2.3.4.2.

The elastic properties of the materials of the bridge members are the Young's modulus of elasticity, Poisson's ratio and the shear modulus. The following notation is used:

\[ E_s = \text{modulus of elasticity of the slab.} \]
\[ E_g = \text{modulus of elasticity of the supporting girders.} \]
\[ \mu = \text{Poisson's ratio.} \]
\[ G_s = \text{shear modulus of the slab; } G = \frac{E}{2(1 + \mu)}. \]
\[ G_g = \text{shear modulus of the supporting girders.} \]

In order to reduce the loss of prestressing force due to creep, the concrete used for prestressed concrete girders is normally of much higher quality than that used for the slab concrete. If prestressed concrete girders are used, the ratio \( E_s/E_g \) is approximately 0.8. Motarjemi (68) showed that the girder bending moments are insensitive to changes in \( E_s/E_g \). The value of 0.8 is used in this study except if otherwise indicated. The ratio of \( E_s/E_g \) is of course much smaller for steel I-beams, but this does not matter because a steel beam may be transformed to an equivalent prestressed concrete girder, by using the appropriate modular ratio.

The value of Poisson's ratio for the slab concrete is taken as 0.2. This value is also used to determine the shear modulus \( G_g \) of the supporting girders in so far as the resistance against shear deformations is concerned. Motarjemi (68) showed that a variation in Poisson's ratio for the slab concrete from 0.05 to 0.25 has no significant
influence on the results.

2.3.4. Parameters Defining the Structural Properties of the Bridge Members

2.3.4.1. General

The behaviour of a bridge slab stiffened by eccentric girders is very complex. The structural action of a slab by itself is complex. The slab behaviour is complicated by the fact that the slab is continuous over supports and that the girders which act as supports are flexible. Because full composite action is considered, the behaviour is further complicated by the girder eccentricity which causes axial forces in the slab with resulting shear lag effects. A very involved analytical model is necessary to take all these effects into account. This model is discussed in Chapter 3.

There is a large number of variables which determine the structural properties of a bridge. The amount of work involved to consider all of these variables in a parametric study is prohibitive. It is, therefore, necessary to eliminate as many variables as possible without simplifying the structure so much that the structural behaviour is thereby altered. This can be done by ignoring the unimportant variables and by combining others to bring about new ones which have the controlling effects on the structural behaviour.

The parameters which have the controlling effects are determined by recognizing the major structural actions in a slab-and-girder bridge. These are as follows:

1. The bridge slab distributes truck loads over the width of the bridge. To do this, it acts in flexure in the transverse direction, similar to a beam continuous over flexible supports. The flexural rotation of the slab at the support girders is resisted by torsional stiffness in the girders.
2. The eccentric girders act together with the slab to form strong, stiff composite T-section girders which carry all the load to the abutments in flexure.

It is, therefore, necessary to combine the variables which determine the flexural slab stiffness and those which bring about the flexural composite girder stiffness and the torsional girder stiffness. This is done in the sections which follow. The girder and slab stiffnesses are then combined to form a single dimensionless parameter.

2.3.4.2. The Flexural Slab Stiffness $D$

The flexural slab stiffness per unit width is

$$D = \frac{E_s t^3}{12(1 - \mu^2)} \quad (2.1)$$

where $t$ is the slab thickness. It is assumed that the slab is made of reinforced concrete. The value of $\mu$ is taken as 0.2. Motarjemi (68) showed that changes in the value of $\mu$ does not have any significant effects. The thickness of the slab, $t$, depends on the girder spacing, $b$, and is normally between six and ten inches. The modulus of elasticity of the slab $E_s$ is taken as: $E_s = 0.8 E_g$, for the reason explained in Section 2.3.3.

It is well known that the flexural stiffness $D$ of a reinforced concrete slab varies with the degree and extent of cracking which is present. Longitudinal cracks caused by bending moments in the transverse direction reduce the transverse flexural stiffness of the slab at the location of the cracks, because the effective slab thickness there is smaller. However, there are usually fairly large sections of the slab which remain intact between cracks. The average flexural stiffness of the slab is thus only slightly reduced by cracking. Newmark (77, 78) did a series of tests on quarter scale concrete bridge models which showed that the distribution of load to the steel I-beam girders, as determined from measured strains, was in excellent agreement with the
distribution predicted by elastic service load analyses. In this analysis Newmark computed the flexural slab stiffness from the gross concrete section ignoring the reinforcement. The full thickness, t, of the slab is used in computing the flexural stiffness in the present study.

2.3.4.3. The Flexural Composite Girder Stiffness \( E_g I_g \)

The second major structural action in the bridge originates from the composite girders which carry all the load to the abutments. In the discussion which follows, the parameters and effects which determine the flexural composite girder stiffness are stated. The number of parameters is then reduced by elimination and combination to give only one convenient parameter.

The girders supporting the slab may be steel I-beams or prestressed concrete girders. For all practical purposes, prestressed concrete girders behave elastically like steel I-beams because design allowable stress requirements prevent the girder from cracking under service load conditions.

A precast prestressed concrete girder is shown in Fig. 2.2a. The following notation is used to define the section properties of the girder:

\( I_{gx} \) = bending moment of inertia about the strong axis, x-x.

\( I_{gy} \) = bending moment of inertia about the weak axis, y-y.

\( J \) = torsional moment of inertia about the shear centre (s.c.).

\( e \) = eccentricity of the centre of gravity (c.g.) of the supporting girder with respect to the midsurface of the slab.

\( A_g \) = gross area effective in tension.
\( A_{sx} = \text{area effective in shear in the vertical direction taken as } A_g/1.2 \text{ for a rectangular girder section.} \)

\( A_{sy} = \text{area effective in shear in the horizontal direction.} \)

To reduce the number of variables, the girder parameters \( I_{gy} \) and \( A_{sy} \) are ignored because they have insignificant influence on the behaviour of the structure. Analyses of two practical skew bridges with prestressed concrete girders show that the inclusion of girder weak-axis bending and weak-axis shear stiffness cause less than a 0.2% change in the distribution of load to the girders. Furthermore, by ignoring \( A_{sy} \), the complicated problem of calculating the location of the shear centre of a thick-walled section is avoided. The location of the shear centre does not enter the solution as far as \( J \) and \( A_{sx} \) are concerned because the cross section is \( y \)-axis symmetric.

The number of variables can further be reduced by combining several to form a new one. A major simplification is possible by using the composite T-section girder stiffness as parameter, thus avoiding the numerous possible variations in \( I_{gx}, A_g \) and \( e \) when they are considered as independent parameters. The composite T-section girder is shown in Fig. 2.2b. The composite T-section stiffness can easily be determined using the effective flange width concept and the transformed area method.

If an isolated T-beam with a wide flange is subjected to bending, the web causes a variation in the compressive bending stress in the top flange. This bending stress varies from a maximum value above the web to a minimum at the ends of the flange. The variation occurs as the result of in-plane shear deformations in the flange, known as shear lag. The effective flange width concept is a tool which permits the simple and rapid calculation of approximate stresses in a composite beam. The shear lag effect in the flange is taken into account approximately by transforming the real T-beam to another T-beam that has an effective flange width in which the bending stress is constant over the width. The work of many researchers who developed this technique is discussed in Ref. 96.
In the present study, the effective flange width recommendations made in the AASHTO Specifications for Highway Bridges (5) are used. If the AASHTO deflection criteria regarding the thickness of the slab is met, it is found that in nearly all cases the effective width equals the girder spacing, \( b \).

The transformed area method is used to transform the slab material to equivalent prestressed concrete material. The effective flange width \( b_{\text{eff}} \) as indicated in Fig. 2.2b, is multiplied by the ratio \( E_s/E_g \) which is always taken as 0.8. If steel I-beams are used, then they are also transformed to equivalent prestressed concrete by using the appropriate modular ratio. With a composite T-section consisting now of only prestressed concrete, the composite moment of inertia \( I_{cg} \) can be calculated from

\[
I_{cg} = I_{gx} + \frac{0.8 b_{\text{eff}} t^3}{12} + \frac{A_g A_s e^2}{A_g + A_s} \quad \text{where} \quad A_s = 0.8 b_{\text{eff}} t. \tag{2.2}
\]

The parameters \( I_{gx}, b_{\text{eff}}, t, A_g \) and \( e \) are thus replaced by only one parameter—\( I_{cg} \).

The required composite girder flexural stiffness is \( E_g I_{cg} \), where \( E_g \) is the modulus of elasticity of the precast prestressed concrete supporting girders. The paragraphs which follow explain the advantage of using the \( I_{cg} \) parameter.

The designer can use the total moment on the composite T-section girder \( M_{cg} \) and the bending moment of inertia of the composite T-section \( I_{cg} \) to calculate the approximate bending stress \( \sigma \) at any point a distance \( z \) from the neutral axis of the composite section using the well known formula:

\[
\sigma = M_{cg} z / I_{cg} \tag{2.3}
\]

The total bending moment on the composite T-section \( M_{cg} \) is made up of the three components shown in Fig. 2.2a.

\[
M_{cg} = M_g + N_g e + M_s \quad \text{where}, \tag{2.4}
\]
\[ M_g = \text{bending moment acting on an isolated supporting girder.} \]

\[ N_g = \text{axial force acting on an isolated supporting girder.} \]

\[ e = \text{eccentricity as defined before.} \]

\[ M_s = \text{integral of the longitudinal bending moments in the flange of the composite T-section girder.} \]

\( N_g e \) is the moment couple resulting from the eccentricity of the supporting girder. \( N_g \) and \( M_g \) are directly available from the finite element program used. To obtain \( M_s \), numerical integration must be used because the slab moments are only available at certain points. The effect of \( M_s \) is very small and is usually less than 3\% of the total moment \( M_{cg} \).

An alternative way to determine the stresses in the supporting girder is to use the supporting girder properties and forces which act on the supporting girder alone. The bending stress \( \sigma \) at any point a distance \( z \) from the neutral axis of the supporting girder can be calculated from:

\[
\sigma = \frac{N_g}{A_g} + \frac{M_g z}{I_{gx}} \tag{2.5}
\]

Here, no approximation is involved and the stresses obtained are more reliable. However, the values of \( N_g \) and \( M_g \) depend on many variables: \( A_g, I_{gx}, b_{eff}, e \) and \( t \), whereas, the value of \( M_{cg} \) depends only on one parameter—\( I_{cg} \).

The advantage of using \( I_{cg} \) as parameter is that for a specific value of \( I_{cg} \), only one analysis is necessary which represents a large number of bridges with different slab thicknesses and different supporting girders. However, using the composite girder stiffness \( I_{cg} \) as parameter, an approximation is introduced because the value of \( I_{cg} \) depends on the effective flange width which approximates the influence of shear lag in the slab. The magnitude of girder stress errors which result from this approximation is discussed in Section 4.2.
The major difference between steel I-beams and prestressed concrete girders in a slab-and-girder bridge is the effect of their torsional stiffness on the load distribution. Badaruddin (8) showed that the torsional stiffness of a steel I-beam has negligible influence on the distribution of truck loads to the girders and can, therefore, be ignored. Prestressed concrete girders have considerably larger torsional stiffness, but Badaruddin showed that the effect of their torsional stiffness on the distribution of truck loads to the girders is only in the order of 5%. On the other hand, the torsional stiffness of girders of box-section, which are not considered in this study, has a large influence.

The torsional stiffness of a girder is made up of two parts: The St. Venant torsional rigidity and warping rigidity. Badaruddin (8), Sithichaikasem (112) and Motarjem (68) showed that the effect of the warping rigidity in bridges which are not curved is insignificant for both steel I-beams and prestressed concrete girders. Warping torsion is thus not considered in this study.

The St. Venant torsional stiffness of the supporting girders is $G_g J$, with $G_g = \frac{E_g}{[2(1 + \mu)]}$ the shear modulus of the girder material and $J$ the torsional moment of inertia. It is necessary to determine if the effect of girder torsion on the distribution of truck loads becomes more important for bridges built on skew alignment. This effect of torsion is discussed in Section 4.5.

McGee (62) showed that linear elastic theory may be used to predict the torsional stiffness of a prestressed concrete beam before cracking. The torsional behaviour of a compound cross section is very complex. In order to calculate the torsional moment of inertia $J$ of a prestressed concrete girder, the cross section is idealized as shown in Fig. 2.2c. Following Bach's (7) approximation, the section is subdivided into rectangles and $J$ is calculated as the sum of the torsional inertia of each rectangle. According to Timoshenko (116), $J$ for a rectangular cross section is: $J = kx^3y$, where $k$ is a
constant depending on the ratio \( y/x \) with \( x \) the smaller and \( y \) the larger dimension of the rectangle.

Bach's approximation is conservative because the shear flow between rectangles is ignored. An analytical test done shows that better accuracy is obtained if the torsional moment of inertia for rectangle No 4 in Fig. 2.2c is subtracted, because it is already included in the twisting stiffness of the slab.

The torsional stiffness \( G_J \) obtained in this way is not very accurate. Fortunately, the influence of torsion on the girder bending moments is small. In Section 4.5 it is shown that a 47% variation in the torsional stiffness results in a maximum change in girder bending moments of only 1.3%. A 100% reduction in torsional stiffness results in girder bending moment variations of less than 7%. Laboratory tests on prestressed concrete members have shown that their torsional rigidity decreases far more than their flexural rigidity as soon as cracking occurs. These uncertainties about the true torsional stiffness and the fact that the girder bending moments are rather insensitive to changes in the girder torsional stiffness make it unnecessary to attempt to determine \( J \)-values using a more refined method. The use of the simplified girder cross section along with Bach's approximation is thus acceptable.

Because the bridge behaviour is so insensitive to girder torsional stiffness, the torsional stiffness is not used as a major parameter in this study. However, it is taken into account. The girders used in the analyses are actual standard precast pretensioned prestressed concrete girders which are used in practice (89). The torsional stiffness calculated as described above is used in each bridge analysis. The girder properties for each analysis are presented in Table 2.1.

Although the properties used in the analysis are expressed for prestressed concrete girders, the results can also be used for steel I-beams by applying the minor modification recommended in Section 4.5.
The main influence of torsion on the behaviour of the bridge is the restraints of the rotation of the slab at the girders. Therefore, the slab is stiffened by this effect. A stiffer slab distributes truck loads better so that smaller design moments occur. The influence of girder torsional stiffness is discussed in detail in Section 4.5.

2.3.4.5. The Dimensionless Stiffness Ratio \( H \)

The behaviour of a slab-and-girder bridge depends on the geometry of the bridge as well as on the structural properties of the bridge members. Newmark (75), on the basis of analyses of bridges using girders having no eccentricity, showed that the flexural stiffness of the slab and the flexural stiffness of the girders need not be considered as two separate parameters. They can be combined to form a new, very convenient dimensionless stiffness parameter. Newmark defined the parameter \( H \) as the ratio of the longitudinal bending stiffness of an isolated girder, to the transverse bending stiffness of a width of slab equal to the span of the bridge:

\[
H = \frac{E_s I_{gx}}{aD}
\]  

(2.6)

Because \( D \) is the flexural stiffness of the slab per unit width, it is necessary to multiply \( D \) by some width in order to make \( H \) dimensionless. The span, \( a \), serves this purpose. It should not be interpreted that \( a \) is the width of the slab effective in the transverse direction. \( H \) is simply a convenient dimensionless stiffness parameter. A large \( H \)-value means that the bridge has very large, stiff girders.

A minor modification is necessary to apply \( H \) in this study. The moment of inertia of the supporting girders \( I_{gx} \) should be replaced by the composite T-section moment of inertia \( I_{cg} \). The modified \( H \) used in the present study is then:

\[
H = \frac{E_s I_{cg}}{aD}
\]  

(2.7)
In making this modification, the accuracy of the H-value depends now on the accuracy of the approximation for the effective flange width. In Section 4.3 the consistency of girder bending moment results is tested for bridges which have the same H-value, but in which the variables determining H are different.

There are further uncertainties regarding the real value of H. What are the real values of $E_s$ and $E_g$? How much does the effect of slab cracking influence the average flexural stiffness of the slab? Fortunately, as shown in Section 4.8.2.1, the girder bending moments are not particularly sensitive to moderate variations in H, especially for large values of H.

A study of bridge design manuals shows that the H-value of practical bridges normally falls between $H = 5$ and $H = 30$. In multi-span bridges, the girder depth is chosen to satisfy requirements for the longest span. For aesthetic reasons, the same girder is normally also used for the shorter spans. The shorter spans thus have large H-values. The values of H used in this study are: $H = 5, 10, 20$ and $30$. A smaller increment in H is used after $H = 5$, because a variation in H in that range has a more pronounced effect on the behaviour of the bridge.

In the analyses, the properties of actual standard prestressed concrete girders which are used in practice (89) are used. The properties of the particular girder used for each value of H are listed in Table 2.1.

It should be noted that the H-value is based on the flexural composite stiffness of an interior girder. The H-value of the exterior girder which has a smaller flange width is different. However, where exterior girder bending moments and deflections reported in this study are related to an H-value, H always corresponds to the value for an interior girder.
2.3.5. Parameters Defining the Structural Loading Conditions

2.3.5.1. Live Load

In this study the emphasis is on the distribution of truck loads among the girders in the bridge. The HS20-44 standard truck considered is a tractor truck with semi-trailer and is in accordance with the AASHTO Standard Specifications for Highway Bridges (5). It represents a large number and variety of actual truck types and loadings to which the bridge might be subjected under actual traffic conditions.

Figure 2.3 shows the locations of the wheel loads and the transverse location of one truck relative to another. The loading produced by one truck consists of three axle loads with the axles spaced 14 feet apart. Each axle transfers two point loads to the bridge deck. The centre to centre distance between the centroids of wheels on each side of an axle is six feet.

Each truck occupies the central portion of a ten-foot wide load lane, one truck per lane. These ten-foot load lanes can be placed anywhere in the entire roadway width of the bridge, which is the clear distance between the faces of the two curbs, to produce maximum moments in whichever girder is considered. This means that no load can act closer than two feet from the face of a curb or edge girder, which is a much desired condition as discussed in Section 2.2. As shown in Fig. 2.3 it also means that the minimum transverse distance between the wheel centroids of two trucks in adjacent loading lanes is four feet.

The AASHTO specifications make provision for the length of the semi-trailer to vary such that the rear axle spacing is between 14 and 30 feet. Because simply supported bridges are dealt with, the 14-foot axle spacing is used to obtain maximum girder bending moments in the bridge.

Girder bending moment influence lines across the width of the bridge, many of which are reported in previous research, clearly indicate that the transverse truck
Spacing should be as small as possible in order to obtain the maximum moment in anyone of the girders. Therefore, only the four-foot minimum distance between adjacent truck wheel centroids is used in the analyses.

Two of the three axles of an HS20-44 standard truck carry the same load. The front axle carries only one quarter of the load carried by each of the other two axles. The truck loading parameter P is defined as half the load acting on one of the heavy axles of a truck. The total weight of a truck is thus 4.5 P. The value of P should be increased according to the AASHTO provision for impact. The trucks in adjacent loading lanes may travel in the same or in opposite directions, whichever case produces the maximum required effect.

If three or four of the traffic lanes on a bridge are occupied simultaneously, it may result in girder bending moments which are larger than the corresponding moments obtained if only two traffic lanes are loaded. However, it is unlikely that three or more lanes will be occupied in such a way that all trucks are producing their maximum contribution to the moment in the particular girder under consideration. It is also very unlikely to have all of these trucks loaded to their maximum capacity. These considerations are recognized in provision 1.2.9 of the AASHTO specifications which allow for a reduction in girder design moments obtained from loading conditions in which three or more traffic lanes are loaded.

Both Badaruddin (8) and Sithichaikasem (112) found that if the girder moments obtained from load cases in which three or more traffic lanes are loaded are multiplied by their appropriate reduction factors, it always results in design moments smaller than those obtained from load cases with two lane loading. Therefore, only two traffic lanes are loaded in this study.

As discussed in Section 4.4, skew bridges behave the same as right bridges with respect to this phenomenon. This means that two-lane loading also controls in skew bridges.
The two other types of vehicle loading specified in the AASHTO specifications are not of importance for the range of spans considered in this study. They are 1) lane loading which represents an approximation of a truck train which normally governs for spans longer than 147 ft and 2) a two-axle military loading with axles spaced at 4 ft which tends to control in bridges with spans shorter than 37 ft.

2.3.5.2. Dead Load

Two types of dead load are also considered. The loading parameter $c$ presents a line load of intensity $c$ caused by the weight of a curb and parapet. It is assumed that these line loads act on top of the edge girders.

The second type of dead load considered is defined by the loading parameter $\omega$, which is a uniformly distributed load of intensity $\omega$ applied to the total deck area between the faces of the curbs. This load simulates the additional dead load which occurs when the roadway is resurfaced.

2.4. Summary of the Parameters Used in the Parametric Study

1. The girder spacing $b$:

The girder spacing values used are $b = 6, 6.75, 7.5, 8.25$ and 9 ft. When $b$ increases, the design bending moments in the girders increase, because the deck area carried by each girder becomes larger and more wheel loads can be applied to the larger area.

2. The geometric ratio $b/a$:

The $b/a$ values used are such that the span, $a$, is 40, 60 and 80 ft for each $b$-value used. An increase in the value of $b/a$ corresponds to a decrease in the ability of the slab to distribute the loads.
3. The bridge stiffness ratio $H$:

The stiffness ratios used are $H = 5, 10, 20$ and $30$. If $H$ is very large, it means that the slab is relatively very flexible. A girder which carries a point load will deflect under the load while the other girders deflect a negligible amount, because the slab is too flexible to transfer large loads to them. This means that the loaded girder has to carry nearly all the load by itself. Thus, the larger the value of $H$, the larger are the girder bending moments that occur. It is explained in Section 4.8.2.1 that this is in most cases not true of the edge girders when the bridge is subjected to truck loads.

The effects of the parameters on the structural behaviour are discussed in detail in Chapter 4.
CHAPTER 3
METHOD OF STRUCTURAL ANALYSIS

3.1. General

It is important to realize that the correctness of the results obtained from a specific mathematical model depends on the underlying assumptions and simplifications which are made to formulate the model. Recent development in the finite element method of analysis makes it possible to model a bridge in a more realistic manner than heretofore possible. The girders can be modelled as eccentric stiffeners so that shear lag in the slab and composite action are properly taken into account. In such a model it is no longer necessary to estimate an effective flange width and to compute the modified flexural stiffness of the composite girders. Of all the methods of analysis available the finite element method is the most powerful, versatile and flexible.

3.2. The Finite Element Method

This method of analysis is so commonly and widely used today that even a brief description of the basic principles on which it is based is superfluous. A description of the method is found in Refs. 19 and 124.

Using the ordinary widely-used finite elements which have Hermitian shape functions for vertical deflections and linear shape functions for axial deflections to model a composite plate structure with eccentric beams causes a compatibility error which affects the accuracy of the solution. This error was pointed out and discussed by Mehrain (63), Gupta (35), Balmer (9) and Miller (65), and Chu and Schnobrich (17).

Linear constraint equations, which might be thought of as representing rigid links, can be used to couple the nodal degrees of freedom of the eccentric beam element to the nodal degrees of freedom of the shell element. The nodal displacements
of the beam are now dictated by those of the shell element to which it is attached. The compatibility error occurs because the axial and bending deformations of the composite assembly are now coupled but have different assumed variations.

Figure 3.1a shows two four-node shell elements and a standard two-node beam element connected to form the eccentric assembly. The displacements u, v, w and rotations \( \theta_x \) and \( \theta_y \) of the beam and shell element are defined in Fig. 3.1a. Figure 3.2 shows a side view of the eccentric assembly. To simplify the discussion it is assumed that the beam has no torsional stiffness or weak-axis bending stiffness.

The shell and beam element both have Hermitian cubic shape functions for vertical deflection w and linear shape functions for axial displacement u. The w-displacements of the beam and shell at any section abcd (Fig. 3.2a) are completely compatible because they have the same w shape functions and because there are no deformations in the z direction. The vertical distance between the midsurface of the shell elements and the centroid of the beam element is the eccentricity e. The Kirchhoff assumption: a line normal to the midsurface (centroid) of a beam or plate remains normal to the deformed midsurface, implies that the u-displacement at the centroid of the beam at any section abcd equals the u-displacement at the midsurface of the plate plus the eccentricity e times the rotation of the plate midsurface at that section. This is indicated in Fig. 3.2b. Because the rotation of the midsurface of the plate varies quadratically between E and F (w is cubic in x; slope = \( \frac{dw}{dx} \)) it implies that the u-displacement at the centroid of the beam also varies quadratically between G and H. But, the axial stiffness of a standard beam element is derived by using a linear shape function for the u-displacement. Therefore, an incompatibility is apparent. This incompatibility manifests itself in the form of shear slip at the beam-slab interface as shown in Fig. 3.2a.

It is not difficult to see this compatibility error in an intuitive way. The moment in the composite T-section girder varies linearly along the member because w is cubic
in x and \( M = E I_{\text{composite}} \frac{d^2w}{dx^2} \). The stress \( \sigma_x \) at the centroid of the stiffener and at the midsurface of the plate can be calculated from: \( \sigma_x = Mz/I_{\text{composite}} \). Thus the axial stress there also varies linearly across the plate and stiffener. But \( \sigma_x = E(du/dx) \) which is constant over the element. Thus the \( u \) shape functions should be quadratic in \( x \) for \( \sigma_x \) to vary linearly along the beam and shell elements.

Miller (65) and Mehrain (63) solved this problem by using exactly the same beam and shell elements, but included an additional \( u \)-displacement degree of freedom at the centre of the shell and beam element to provide quadratic \( u \)-displacement shape functions. Fig. 3.1b shows the additional nodes and degrees of freedom which they used. Chu and Schnobrich (17) used a similar approach.

Using a standard beam element as an eccentric stiffener gives overflexible results as the consequence of the incompatibility in the axial displacement field of the beam. This is shown in Fig. 3.17 and discussed in Section 3.6.1. This compatibility problem does not arise when separate sets of shape functions are used for deflections and rotations. This approach is taken in the present study.

3.3. The Finite Elements Used in this Study

The formulation of the finite elements used in this study is based on the Mindlin theory in which the average shear deformations are automatically taken into account, even if their influence is negligible.

3.3.1. Degenerated Thin Shell Isoparametric Element

Three-dimensional elements are satisfactorily specialized to produce two-dimensional thin shell elements by using explicit integration through the thickness of the slab and by assuming:
1. Lines originally normal to the midsurface of the shell remain straight after deformation.

2. All points on a line originally normal to the midsurface have the same vertical displacement \( w \). Thus a normal line is inextensible during deformation.

Although the normals to the midsurface are assumed to remain straight after deformation, they do not have to stay normal to the deformed midsurface. Thus this model provides the means for taking the average shear deformation into account. This formulation was originally introduced by Ahmad (3).

Figure 3.3 shows the typical thin shell element resulting from the original three-dimensional problem. Each node has six degrees of freedom—three displacements: \( u \), \( v \), \( w \); and three rotations: \( \theta_x \), \( \theta_y \), \( \theta_z \). Displacements and rotations are interpolated from nodal degrees of freedom using separate quadratic Lagrangian shape functions. The element has no stiffness associated with the \( \theta_z \) rotational degrees of freedom. The moments, shear and membrane forces which provide static equilibrium in a shell are also indicated in Fig. 3.3.

The reduced integration technique (Ref. 19, p. 263) is used to compute all terms in the stiffness matrix of the element to avoid excessive stiffness associated with membrane and shear locking which are well known characteristics for this type of element. A disadvantage of using reduced integration is that it can produce some zero energy modes. However, it is possible to prevent all of these modes if two adjacent nodal values of the same type are constrained in at least one element.

In the bridge analyses it is found that some 'soft modes' occur. The correct boundary conditions at the skew supports are not sufficient to prevent these soft modes. In-plane \( v \)-displacement (transverse) and \( \theta_x \) and \( \theta_y \) oscillations occur. The \( v \)-displacement oscillation is prevented by using the following technique. Two rows of additional shell elements with a very small stiffness are overlaid on top of the two rows of shell elements at the two support edges. The stiffnesses of these additional
elements are calculated using full integration. They can thus not deform into the bothersome zero energy modes and, therefore, suppress these soft modes in the whole bridge slab. This technique was used by Milford (64).

It is not possible to get rid of the $\theta_x$ and $\theta_y$ oscillations using this technique. Belytschko (11) shows in a recent paper how this element can be modified to avoid all of these soft modes. Because the present analyses compare well with other exact solutions, it is believed that these oscillations are not important. Thus a Belytschko type modification is not used in this study.

The element used to model the bridge slab is a nine-node Lagrangian-type isoparametric thin shell element called QLSHELL. Details of the development of the element can be found in Ref. 64. Additional information concerning this type of element can also be found in Refs. 1 and 12.

3.3.2. Eccentric Isoparametric Beam Element

Jirousek (42) introduced an eccentric isoparametric beam element which exhibits the required displacement compatibility with Ahmad's (3) isoparametric thin shell element. This eccentric beam element is suitable to stiffen the shell element. The formulation is based on the Timoshenko beam theory which takes into account both an average shear deformation and the shear centre location. The stiffener has no thickness, that is, the interaction between the stiffener and shell occurs along a line. In a real shell-to-beam connection the beam resists transverse curvature in the shell segment which is in contact with the beam. This effect is lost with the above assumption.

The formulation of the stiffness matrix for this eccentric beam element is based on quadratic Lagrangian displacement shape functions. Displacements and rotations in the element are interpolated from the nodal degrees of freedom using separate sets of shape functions for displacements and rotations. Rigid links connect the nodal degrees of freedom of the beam to those of the shell element. Thus the
displacements in the beam element are dictated by those in the shell. These links are always connected to the three nodes on the contact boundary between two shell elements. Figure 3.4 shows an eccentric assembly consisting of two shell elements with one beam element underneath. The nodal degrees of freedom and forces acting on the beam element are also indicated. The beam resists strong- and weak-axis bending, strong- and weak-axis shear deformations, St. Venant torsion and axial force resulting from the composite action. Reduced integration is used to compute all terms in the stiffness matrix of this element.

The effect of restraint of warping on torsion is not taken into account. It has been pointed out by various researchers (8, 68, 112) that in bridges which are not curved this torsion plays an insignificant role.

There is one incompatibility in this eccentric assembly which is unavoidable. Figure 3.5a shows a plan view of two shell elements connected to one eccentric beam element placed under them. To simplify the figure the following is assumed:

1. The u, w, \( \theta_x \) and \( \theta_y \) displacements at all nodes in the two shell elements are zero.
2. All nodal v-displacements in the two shell elements are zero except for the one node at the middle of the contact boundary between the two shell elements as indicated in Fig. 3.5a.
3. To simplify the discussion it is assumed that the centre of gravity and the shear centre of the beam coincide.

Because the nodal displacements of the beam element are dictated by the nodal displacements of the shell (as a result of the rigid links) the deformed centroidal axis of the beam stays directly underneath the deformed boundary between the two shell elements. The indicated v-displacement is the same then in the beam below, because \( \theta_x = 0 \). The v-displacement gives rise to weak-axis bending in the beam which causes \( \theta_z \) rotations in the beam. Note that it is only differential v-displacements between the
three beam nodes which can cause weak-axis bending with resulting $\theta_z$ rotations in the beam.

These $\theta_z$ rotations are transferred back to the shell through the rigid links. However, the shell elements have no stiffness associated with $\theta_z$ rotations meaning that the shell is not aware of these $\theta_z$ rotations. Thus no strains are caused in the shell elements due to $\theta_z$ rotations. The in-plane deformations in the shell which should occur as the consequence of $\theta_z$ rotations are also shown in Fig. 3.5a. Unlike the average bending shear strains in the shell which couple $\theta_x$ and $\theta_y$ with the w degrees of freedom, the shell in-plane shear strains depend only on the u and v degrees of freedom. The $\theta_z$ degrees of freedom are uncoupled.

The $\theta_x$ nodal rotations in the shell which result from bending moments in the shell are shown in Fig. 3.5b. These rotations give rise to torsion in the beam, but also cause differential v displacements at the beam centroid. The magnitude of these v-displacements at the beam nodes is $e$ times $\theta_x$, where $e$ is the eccentricity between the beam centroid and the midsurface of the shell. Again, these displacements cause $\theta_z$ rotations in the beam.

Fortunately, this incompatibility is not important in a slab-and-girder bridge. There are no large in-plane shears and associated planar bending in the transverse direction of the bridge slab which may cause significant differential v displacements. The total torsional resistance which the beam can offer consists of two parts: the torsional stiffness of the beam plus the weak-axis bending stiffness which acts with a lever arm e. It is in this weak-axis bending resulting from torsion and other differential v-displacements where the compatibility error occurs.

Analyses on two practical skew bridges with prestressed concrete girders show that the effect of weak-axis bending stiffness on the distribution of truck loads to the girders is less than 0.2%. This makes the compatibility error even more acceptable.
Note that the major behaviour of the eccentric assembly, which is composite strong-axis bending, is modelled correctly. There is no shear slip at a beam-shell interface or incompatibility in the displacement fields. The eccentric isoparametric beam element is called QLBEAM.

3.4. The Behaviour of the Finite Elements Used

Accidental errors in the results of a structural analysis are often brought to light when these results are interpreted with good engineering judgement. It is dangerous to use computer output blindly, but when the structure being analysed is very complex, the engineer is often forced to rely on these results. Therefore, it is very important to know if the finite elements used to model the structure can provide the correct solution for the types of loads that can be present.

It is difficult to determine the influence of element skew distortion on the behaviour of shell elements which model a skew bridge slab by testing those elements in the bridge structure itself, because no exact solutions exist with which results can be compared.

When the shape of a rectangular element is changed to become a parallelogram element, it has decreased ability to assume deformation patterns contained in its assumed displacement fields. This may cause a degradation in element behaviour, specifically for large angles of skew. The detrimental effect of skew distortion on the evaluation of the Jacobian compounds the effect of large aspect ratios in rectangular elements. This ill conditioning can lead to large numerical errors. It is therefore necessary to perform quality tests on the element by using it in less complicated structures for which other sources of solution are known.

The behaviour of the element itself depends on the element aspect ratio, other shape distortions like the skew mentioned above and the complexity of the true deflected shape which it has to approximate. The complexity of the true deflected
shape depends on the boundary conditions of the structure and on the type of loading applied. However, because the finite element method is based on a piecewise approximation of the true deflected shape, with mesh refinement a single element only has to deform into a rather smooth shape. Therefore, the response of a single element in a fine mesh does not depend greatly on details of the loads or of the structure in which it is used. If a distorted element in a fine mesh has the ability to provide correct results, then it should also provide correct results in another structure with a fine mesh and elements with the same aspect ratio and skew distortion independent of the type of loading.

The response of the eccentric beam stiffeners depends only on the displacements received through the three rigid links attaching it to the slab, thus skew in the slab has no influence on their behaviour.

The QLSHELL finite elements which model the bridge slab resist bending as well as in-plane membrane forces as the consequence of composite action with the eccentric girders. In the two sections which follow, the bending and membrane behaviour of the QLSHELL elements which model the skew bridge slab are determined by using them in other structures for which solutions are known.

3.4.1. Bending Behaviour

The bridge slab is modelled with nine-node Lagrangian shell elements called QLSHELL. In order to study the bending behaviour of this element when skew is introduced, a simply supported rhombic plate subjected to a uniformly distributed normal load is analysed. The angle of skew and element mesh layout are varied. The results are compared with those obtained by Morley (67). He solved the problem by choosing the displacement function as an infinite series of eigenfunctions of the biharmonic equation in polar coordinates.
Figures 3.6 through 3.8 show the deflection, and maximum and minimum principal moments at the centre of the plate as a percentage of Morley's solution. The Linear Damping Function (LDF) finite element solutions as obtained by Mehrain (63) are also indicated. The full plate is subdivided into equal sized rhombic finite elements with $N = 4, 8, 12, \text{ and } 16$ referring to the number of these subdivisions in each direction. The element aspect ratio is one. Poisson's ratio is taken as 0.3.

From Fig. 3.6, the following four important conclusions are drawn:

1. For the angle of skew $\alpha = 0$ degrees the converged deflection, that is, the deflection obtained from $N = 16$, is approximately 1.4% too large. Morley's solution for the case where the angle of skew is zero degrees is correct and is the same as the exact solution given by Timoshenko (116). However, those solutions do not take shear deformations into account. Although shear deformations which are included in the formulation of the QLSHELL element contribute a little to this 1.4% excess in displacement, the 1.4% additional displacement is due mainly to the reduced integration technique used in calculating the stiffness matrix of the element.

2. The accuracy of the solution decreases as the angle of skew increases. A similar observation was made by Morley in the form of slower convergence in his series solution. The accuracy decreases rapidly for angles of skew more than 40 degrees. For example, an examination of Fig. 3.6 reveals that for $N = 8$, at 45 degrees skew, the error relative to the converged result ($N = 16$) is 2.5 times the error relative to the converged result ($N = 16$) at 40 degrees skew. At 60 degrees the error is eight times the error at 40 degrees.

3. Even when the angle of skew is 60 degrees, which means a very large distortion from the original rectangular shape of the element, the behaviour of the element is not changed in such a way that it converges to a wrong result. With mesh refinement, the solution for $\alpha = 60$ degrees will converge to one having
approximately the same error (relative to Morley's) as the solution for the case when $\alpha = 0$.

4. The QLSHELL element used in this study converges considerably faster than Mehrain's LDF element. The 4x4 mesh ($N = 4$) with QLSHELL elements has the same number of equations to be solved as the 8x8 mesh of LDF elements. However, at 60 degrees skew the error is 6.2%, whereas, the error of the $N = 8$ LDF element is as large as 27.8%. Static condensation can be used to eliminate the centre (ninth) node in the QLSHELL element. The condensed element then has the same number of degrees of freedom and the same band width as the LDF element, so that the computational cost will be the same for the two elements. Bearing this in mind, we may compare the two $N = 4$ solutions on a fair computational cost basis. The superiority of the QLSHELL condensed element is then even more pronounced.

The fact that the converged deflection stays consistently about 1.55% too large over the whole range of skew investigated, indicates that the QLSHELL element can be used with confidence to model plates in bending with skew ranging from 0 to 60 degrees. Although the accuracy decreases with increasing skew, sufficient accuracy can be maintained by refining the mesh. How much refinement is necessary must be determined by doing a convergence study on the real bridge structure. This is done in the Section 3.5.

Studies made on the same rhombic plate using a point load showed similar behaviour. As mentioned before, with a fine mesh the ability of an element to provide correct results does not depend on the type of loading.

Figures 3.7 and 3.8 show that for the QLSHELL element the error in the maximum and minimum principal moments at the centre of the plate is greater than the error in the deflections when the angle of skew is more than 40 degrees. Even though the central deflection for $\alpha = 0$ degrees converges to a value 1.4% too large, the
moments converge very closely to the exact value found by Timoshenko (116). The QLSHELL element gives good stress results over a wide range of skew, even for a coarse mesh. This can not be said of the LDF element, therefore, the QLSHELL element is superior to the LDF element. A test done shows that the QLSHELL element behaves much better than a similar eight-node serendipity shell element when skew distortion is introduced.

3.4.2. Plane Stress Behaviour

For the smaller standard precast I-beam sections, up to 70% of the composite bending stiffness of a girder and the contributory width of the slab results from the couple formed by the axial force in the slab and the eccentricity between the slab mid-surface and girder centroid. Therefore, in-plane resistance and shear lag in a bridge slab have an important influence on the stresses in the girders.

In order to study the in-plane membrane behaviour of the nine-node Lagrangian shell element (QLSHELL) the deep cantilever beam in Fig. 3.9 is analysed. It has a length to depth ratio of four and is loaded with a parabolically varying end shear. The u- and v-displacements of all the element nodes at the fixed-end boundary are constrained. The different finite element meshes used are shown in Fig. 3.9.

The elasticity solution coincides with beam theory for this problem, except in the proximity of the built-in end where the full clamping condition constitutes a mixed problem of elasticity for which no closed form solution exists. The theoretical end deflection w, taken from Ref. 28 is:

\[
w = \frac{PL^3}{3EI} + \frac{(4 + 5\mu)}{2} \frac{PL}{Et} = 0.3558 \text{ in.} \tag{3.1}
\]

It is exact if the root is free to warp with the points B, C, and D in Fig. 3.9 fixed. The theoretical vertical deflection w is thus an upper bound for the true deflection.
The results for the vertical end deflection at point A and the horizontal stress at point E are compared in Table 3.1 for the case where \( \alpha = 0 \). The solutions obtained by using the Linear-Linear Strain Parallelogram element (LLSP) of Mehrain (63) are also indicated.

The difference between the exact and the converged displacement and stress results are less than 0.06\%. The QLSHELL element provides the best results. The conclusion is that the QLSHELL element can model in-plane behaviour correctly, even for a coarse mesh. The accuracy of the element must be checked when skew is introduced.

Because of the lack of exact or other theoretical solutions, the influence of skew on the convergence is determined by comparing end deflections with the results obtained from the most refined mesh. Table 3.2 gives the vertical deflection at point A which is shown in Fig. 3.9, for \( \alpha = 0, 30, 40 \) and 60 degrees. In Fig. 3.10 the vertical deflection at point A is expressed in terms of the deflection obtained from the most refined mesh. The abscissa represents a measure of the number of equations to be solved. For fully compatible finite element systems, the deflections increase as the mesh is refined, provided that full integration is used in calculating the element stiffness matrix and the coarser mesh is always contained as part of the finer mesh. In the present case these requirements are not met and hence the results showing smaller deflections for Mesh 4 than for Mesh 3 are not inconsistent.

Behaviour similar to the previous bending behaviour is observed. The accuracy of the solution drops rapidly for \( \alpha > 40 \) degrees, especially when \( \alpha > 50 \) degrees. Table 3.1 shows that when \( \alpha = 0 \) degrees, very good in-plane displacements and stresses resulted. Therefore, bearing in mind that the in-plane behaviour shows similar characteristics as the bending behaviour, it is reasonable to assume that with sufficient mesh refinement the 'correct' result will be approached for cases where \( \alpha \neq 0 \).
The worst aspect ratio (= side ratio) of the elements considered is 1.78. However, a test done on a rectangular cantilever beam with only one element and a parabolically varying shear force at the free end, shows that for an element aspect ratio of 15, the stress errors at the Gauss points are less than 0.001%.

The above element behaviour tests show that the nine-node Lagrangian shell element is capable of yielding highly satisfactory results in bending and in in-plane action under the skew conditions investigated. A major reason for the choice of a nine-node Lagrangian element rather than an eight-node serendipity element is its superior characteristics in distorted shape. The behaviour of the eight-node serendipity element is not presented in this study.

3.5. Finite Element Mesh Choice: Convergence Study on a Typical Bridge

The element behaviour tests in the previous section show that the nine-node Lagrangian shell element (QLSHELL) behaves well when it is distorted into a parallelogram, even when $\alpha = 60$ degrees. The larger the angle of skew, the more the mesh has to be refined to ensure that the 'correct' solution is approached.

In this section, a typical bridge subjected to a point load of 2000 lb is analysed using different finite element models. The purpose is to determine how much the mesh must be refined to yield the converged 'correct' solution. Because of their importance the deflections, moments and axial forces in the girders near midspan are chosen as bases for this comparison. For monotonic convergence, the smaller the differences between the results from a certain mesh and a more refined mesh, the closer the approximation to the final converged 'correct' solution. This concept is used to determine when the results are close enough to the 'correct' solution.

Figure 3.11 shows the plan view, cross section, properties and reference points of the bridge. The bridge is simply supported all along the two support edges (not only pinned at the girder ends). The girder properties are approximately those of an
Illinois Type-36 standard precast I-beam. It is assumed that the girder centroid and the shear centre coincide. The width of the girders is not taken into account. The different finite element models used are shown in Fig. 3.12.

Instead of a skew network, it is also possible to use rectangular elements in the central area of the slab together with triangular elements to fit the skew boundaries at the support edges. However, this kind of mesh is not suitable for a parametric study because the whole mesh has to change each time the geometry of the bridge changes.

The more complicated the true deflected shape of a structure, the more finite elements must be used to approximate this shape correctly. Because the deflected shape in the span direction is always a fairly smooth curve (no inflection-points or sharp changes in slope), 14 elements in the span direction are chosen as a start-off point.

Wheel loads act in nearly all cases somewhere between the structural nodes of the shell elements which model the bridge slab. A point load on a shell element can not be applied directly to the element, but is transformed to work-equivalent forces which act at the nodes of the particular element. This causes a difference between the actual load and the loading applied to the finite element model. A point load is thus distributed over the length (and width) of the element. If the span of the bridge is large and only a few elements are used in the span direction, a point load is distributed over quite a long distance. To minimize this effect, more elements must be used in the span direction. Tests done on a simply supported beam with 14 elements and point loads between the nodes showed that bending moments can be interpolated from the Gauss-points with errors less than 1.2%. Since wheel loads are not really concentrated loads, the true error is less.

This error should be even less in a slab-and-girder bridge for the wheel loads act, in most cases, on the slab and not directly on top of the girders. By the time that the load reaches the girders, it is distributed by the slab over some length of the girder. Whether the applied load is a true point load or a distributed load should not have a
large influence.

The procedure used to study the convergence of results involves keeping the number of elements in the span direction constant at 14 and increasing the number of elements between girders from two until convergence is achieved. The number of elements between girders necessary to give a solution close enough to the converged 'correct' solution is now kept constant and the number of elements in the span direction is varied until the results converge again.

The girder vertical deflection, bending moment and axial force results are presented in Tables 3.3 through 3.5 for the case where $\alpha = 0$. The girder forces tabulated are those at the element Gauss-point just to the left of the reference point. For the bridge in Fig. 3.11 the location is 10.778 inches left of the reference point. Stress smoothing is not necessary since no significant oscillation occurs. The percentage difference between the results from the different meshes are also indicated. The results obtained from the most refined mesh, Mesh 3, are used as bases for the percentage difference comparison. In Tables 3.6 through 3.8 similar results are presented for the case where the angle of skew is 60 degrees.

Note that the bending moments $M_g$ and axial forces $N_g$ tabulated are those acting on the supporting girders and not on the composite T-section girders.

A summary of the maximum percentage change in results is presented in Table 3.9. The percentage change becomes larger for a skew bridge. This trend is similar to the previous slower convergence observed in the tests on element behaviour. An investigation of the tables reveals that the difference between results of Mesh 3 and Mesh 1 is too large for Mesh 1 to be close to the converged 'correct' solution. However, the results of Mesh 2 are very close to those of Mesh 3 which has two times as many elements. This shows that both Mesh 3 and Mesh 2 give solutions close to the converged 'correct' one.
In Table 3.9 the values in brackets are the more realistic maximum percentage change values. For instance, the maximum difference of 3.4% in bending moment comes from the bending moment at F, as a result of the load at A. The bending moment at F is of no great importance since F is close to the support and the design will be governed by the much larger moment near midspan. Note that the percentage change in the influence of the load at F on the bending moment near A, which is of more importance, is less than 0.1%.

When the bridge is loaded with trucks these maximum percentage differences between the most refined mesh and the coarser meshes do not represent the over-all relative error that will occur using one of the coarser meshes. For example, the 1.6% bending moment error in Table 3.9 comes from the bending moment at D, as a result of the load at K. With equal loads at K and D, the influence of the K-load on the total bending moment at D is only about 22.2%. Thus, the global difference in the total moment at D is only \((22.2 \times 1.6) = 0.4\%\), which is very small.

The conclusion drawn from Tables 3.3 through 3.9 is that four elements between girders is enough to ensure sufficient stress accuracy in the girders for the particular structural geometry and properties considered. It remains now to check if the original choice of 14 elements in the span direction is good enough.

To accomplish this, the finite element models: Mesh 2, Mesh 4 and Mesh 5 shown in Fig. 3.12 are used. Because the worst convergence is observed when the bridge is skew, only the case when \(\alpha = 60\) degrees is studied. The number of elements between girders is now kept constant at four—the optimal number previously determined. The results are presented in Tables 3.10 through 3.12. For Mesh 4 and Mesh 5 the tabulated girder forces at a location 10.778 inches left of the reference points are calculated using quadratic interpolation from the values at the Gauss-points. No stress smoothing is necessary because no significant oscillation occurs in the results. A summary of the maximum percentage change in results between the
different meshes is given in Table 3.13. The most refined mesh, Mesh 5, is used as bases for the comparison.

The difference between results of Mesh 4 and Mesh 5 is too large for Mesh 4 to be close to the converged 'correct' solution. However, the results of Mesh 2 are very close to those of Mesh 5 which has two times as many elements. This shows that both Mesh 2 and Mesh 5 are close to the converged 'correct' solution.

In a bridge with a short span and a large angle of skew as shown in Fig. 3.13, there is a tendency for the slab to span in the shortest possible direction. The moments in the shell elements and the influence of skew on the QLSHELL element behaviour is then even more important. It is believed that the bridge geometry used in this convergence study, span/width = 1.4, is such that it takes this detrimental effect into account.

Two questions remain to be answered before a final choice can be made regarding the mesh to use in this study. The first question to answer concerns the bridge properties. What happens to the accuracy of the solution when the ratio of the stiffness of the girders to the stiffness of the slab changes? The properties of the typical bridge in Fig. 3.11 on which the previous work is based corresponds to a small stiffness ratio. The type of girders used is actually too small to carry AASHTO HS20-44 truck loads for the particular span and girder spacing considered.

In order to answer this question, the bridge in Fig. 3.11 is analysed again, but this time using the largest possible practical girders. The following beam properties are used which correspond approximately to those of an Illinois Type-54 standard precast beam:

\[
\begin{align*}
I_{\text{strong}} &= 213715 \text{ in}^4 \\
I_{\text{weak}} &= 15078 \text{ in}^4 \\
J_{\text{torsion}} &= 10647 \text{ in}^4 \\
A_{\text{tension}} &= 599 \text{ in}^2 \\
A_{\text{shear}} &= 300 \text{ in}^2 \\
e &= 33.03 \text{ in}
\end{align*}
\]
The composite girder stiffness of these new girders is 2.9 times the stiffness as used before. Because the forces in the girders increase as their stiffness increases the point load is reduced from 2000 to 20 lb to limit the magnitude of the results which are tabulated. To ensure maximum errors the angle of skew is taken as 60 degrees. The calculated girder bending moments and axial forces for Mesh 2 and Mesh 5 are presented in Tables 3.14 and 3.15. Mesh 1 is not used because the previous work shows that it is incapable of providing reliable results.

The percentage difference comparison between the results for the two meshes indicates that relative large differences occur compared to the case when the bridge has the very flexible girder properties as indicated in Fig. 3.11. This shows that for a fixed bridge geometry, as the girder stiffness increases, the accuracy of the solution deteriorates. However, the very large differences occur in comparatively small girder forces. The important differences are those for the forces in a girder directly below the point where the load is applied because the effect of a load is the largest at the point where it is applied. For instance, if the load is applied to the edge girder at midspan (point A) the edge girder midspan moments obtained from Mesh 2 and Mesh 5 differ only by 3.2%. A similar difference for the centre girder moment at midspan with the load at point C is 0.4%.

A closer look at the percentage differences for both bending moments and axial forces indicates that the differences in results between the two meshes for forces in the edge girders (points A, F and E) are in most cases larger than for any other girder in the bridge. It also shows that the effects of loads located on an edge girder, on the forces in the interior girders, differ much more than the effects of loads which are not located on an edge girder, on the forces in the interior girders. This shows that whatever causes these large differences has a larger effect on the two edge panels (AB and DE) of the bridge than on the interior panels.
As mentioned before, the very stiff girders cause results with large differences in small numbers. To illustrate the insignificance of these large percentage differences, consider the bending moment in the edge girder at midspan (point A). To obtain a maximum moment at point A, a point load should be located at point A. From Mesh 2 this gives a bending moment of 1144 in-lb at A which differs 3.2% from the result obtained from the more refined Mesh 5. If a load is also applied to the other edge girder at midspan its contribution to the moment at point A is only .2030 in-lb. This .2030 value differs by 53.7% from the value obtained from Mesh 5. However, because the total moment at point A is now only 1144 + .2030 in-lb the difference in results between meshes for the total moment at point A is still about 3.2%. The large 53.7% difference is thus nothing to be concerned about.

There are two possible explanations why increasing the girder stiffness in a particular bridge harms the solution accuracy. The first possible explanation is that the slab acts like a continuous beam over flexible supports in the transverse direction of the bridge. If the slab is very stiff relative to the supports, that is, the supports (girders) are very flexible, then the slab curves gently with a smooth deflected shape. However, if the supports are very stiff relative to the slab, the deflected shape of the slab is much more complicated with larger changes in curvature and more inflection points. A more complicated deflected shape in the transverse direction of a bridge requires more shell elements between the girders to approximate this shape correctly. Thus, if the stiffness of the girders in a bridge increases, the shell elements in the slab may not be able to accommodate the more complicated deflected shape of the bridge cross section. This causes a reduction in the reliability of the results.

The second possible explanation is that by increasing the girder stiffness the stiffness of the structure in the span direction may become much larger than the stiffness in the transverse direction. This may produce ill conditioning in the structural global stiffness matrix which give rise to errors during the equation solving
process. A HARRIS-800 computer is used for all calculations in this study.

In order to investigate how many shell elements are necessary between this very stiff girders to approximate the true transverse direction deflected shape correctly, the bridge is analysed with four elements (Mesh 2), eight elements (Mesh 3) and also for a mesh with six elements between girders. The results show no significant difference in girders forces between any of these three meshes. This clearly indicates that four elements between girders are enough to approximate the transverse direction deflected shape correctly and that the errors are probably caused by numerical problems in the equation solving process.

To see if the accuracy is improved by increasing the number of elements in the span direction when the bridge has very stiff girders a new mesh is used. The results for the 59.5 ft span bridge with the very stiff IL-54 girders are already known for Mesh 2 with 14 elements and for Mesh 5 with 28 elements in the span direction. The new mesh also has four elements between girders but has 20 elements in the span direction.

Compared to Mesh 2, the new mesh uses three times as much computer time, but the differences in results compared to Mesh 5 do not reduce accordingly. For instance, the difference of 3.2% in bending moment at midspan of the edge girder with a load there, decreases only slightly to 3.1%. Also, the 0.4% difference in bending moment for the central girder at midspan with a load there is reduced only to 0.3%.

The conclusion that can be drawn from the above analyses is that there is a limit to mesh refinement after which the increase in computational cost is not justified because it is not accompanied by more reliable results. It is important to keep in mind that this numerical problem exists for bridges with girders which are very stiff compared to the stiffness of the slab in the transverse direction. However, from the analyses above it seems that this effect on girder forces is only in the order of 3%. 
The second question which remains to be answered before the final choice of mesh can be made relates to the geometry of the bridge. What happens to the accuracy if Mesh 2 is stretched out to accommodate a large span? The aspect ratio of the shell elements will then increase. A study of bridge design manuals shows that the smallest practical ratio of girder spacing to span is 0.04. For Mesh 2 this means an element aspect ratio of seven.

In order to answer this question, the bridge is analysed again but this time for a span of 150 ft and a girder spacing of 6 ft. This gives a girder spacing to span ratio of 0.04—the smallest ratio used in practice. For Mesh 2 this means an element aspect ratio of seven. The slab properties and overhangs remain the same as in Fig. 3.11. The girders which are used for this span are PCI Type-6 standard beams. To ensure maximum errors the angle of skew is again taken as 60 degrees. The differences in results between Mesh 5 and Mesh 2 are about one third as large as the differences in the previous bridge with the stiff IL-54 girders and a span of 59.5 ft.

The same bridge with a span of 150 ft and PCI Type-6 standard beams is analysed again with everything the same except that the girder spacing is reduced from six to four feet. This means that the element aspect ratio for Mesh 2 is increased further from seven to eleven. The results of this analysis show that the larger of the previous differences for the six feet girder spacing case becomes smaller now. This shows that the element aspect ratio is not the important issue. The decrease in the girder spacing from six feet to four feet increases the element aspect ratio but it also decreases the relative stiffness of the girders. If the girder spacing becomes smaller the stiffness of the slab is increased.

It is believed that the better results obtained for the larger aspect ratio case is caused by this decrease in the girder stiffness relative to the slab stiffness which reduces the ill conditioning in the structural stiffness matrix.
The final choice of the finite element mesh which is used to study the distribution of wheel loads on simply supported, skew slab-and-girder bridges is based on this convergence study. Four shell elements are always used between the girders and 14 elements are always used in the span direction.

The important design forces in a girder result from a load applied close to the point where the girder forces are considered. Because the differences in girder force results obtained from the two different meshes are small for such a point it is believed that the above mentioned mesh and the finite elements used in this study are capable of providing results which can be considered as correct for all practical purposes.

As mentioned before, stresses in the girders are used as bases for this convergence study. For the sake of completeness and to prevent losing track of how the slab itself responds, the slab results for two load cases of the bridge shown in Fig. 3.11 are plotted in Figs. 3.14 and 3.15. The slab axial forces in the span direction and the bending moments in the transverse direction are shown for load cases: load at C and load at H. The section across the bridge is parallel to the abutments and at a location 10.778 inches to the left of midspan. The angle of skew is 60 degrees. Figure 3.14 shows that there is no significant difference between the axial force results obtained from Mesh 2 and Mesh 3. This indicates that four elements between girders are sufficient to model the shear lag in the slab correctly. For clarity, only the values at the Gauss-points are plotted for Mesh 2.

In Fig. 3.15 the only differences between the bending moment results obtained from Mesh 2 and Mesh 3 occur in the regions with high stress gradients. The more refined Mesh 3 can represent these sharp changes better. The difference between results from Mesh 2 and Mesh 3 become smaller as the distance between the load and point considered increases. The bending solution will improve if the load is not applied at only one single node as it is in the majority of cases. The slab bending moments are not discussed in this study.
3.6. Comparisons with Previous Bridge Solutions

In the previous sections the behaviour of the QLSHELL element which is used in this study is tested in a very distorted configuration: skewed and a large aspect ratio. A convergence study is also made on a typical bridge structure to determine the size of the finite element mesh which provides good results. So far no solution for a bridge has been compared with any existing solutions.

Studies on skew slab-and-girder bridges with girders modelled as eccentric stiffeners are very limited in the literature. Gustafson (36), Mehrain (63), Powell (85, 86) and Perret (81) reported results. Sithichaikasem (112) and Wong (123) used eccentric stiffeners for right bridges, but forces acting directly on the stiffeners were not reported in their work.

In this section four example bridges are analysed and the results compared with previous solutions.

3.6.1. Example Problem: BRIDGE-1

The plan view, cross section and member properties of this right bridge are shown in Fig. 3.16. The bridge is simply supported all along the support edges. All figures for this example are taken from Ref. 63. Mehrain (63) chose the loading and geometry of this example problem such that large local stress gradients occur. The loading consists of a single point load and no transverse diaphragms are present. Considering the thickness of the slab the girder spacing is large. He also chose the slab such that the ratio of the total longitudinal flexural stiffness of the slab to the flexural stiffness of the three girders combined is 0.5. Normally this ratio is much less than 0.15 in practical bridges. Except for the fact that the bridge is not skew the above mentioned geometry and structural configuration ensure the unfavourable conditions which may occur in a real bridge.
Figure 3.16 also shows one of the finite element meshes used. The symmetry of the structure and loading condition make it possible to model only one quarter of the structure. The quarter panel is divided into a 6x6, 4x4, 2x2, or 1x1 finite element mesh. (N = 6; N = 4; N = 2; N = 1)

Figure 3.17 shows the deflection under the load. Mehrain used various types of finite elements to model the bridge. He also indicated the solution obtained using a program developed by Scordelis (102). The solution by Scordelis is called MULTPL and is based on the Goldberg-Leve elasticity equations. It is solved by harmonics expansion of the applied load and employs the direct stiffness method. Scordelis treated the girders as vertical plates. The three COMDEK and REFDEK finite element solutions in Fig. 3.17 are those by Mehrain.

The COMDEK(EL) solution is obtained by using a standard beam element as an eccentric stiffener. The overflexibility is caused by the incompatibility in the axial displacement field of the eccentric beam as described in Section 3.2. However, the solution converges if the mesh is refined. As shown by Gupta (35), the reduction in error is proportional to the square of the number of elements in the span direction.

COMDEK(CD) and COMDEK(SM) are solutions with other types of beam elements. Mehrain's REFDEK solution is the most refined and uses additional nodes at the centre of the beam and shell elements to make the eccentric assembly fully compatible under pure strong-axis bending. These additional nodes are discussed in Section 3.2 and shown in Fig. 3.1b.

The superiority of the present solution is clearly visible. Even with only one QLSHELL element an excellent result is obtained. To ensure a better comparison with Mehrain's work a large shear area (1000 in^2) is used for the QLBEAM elements to suppress shear deformation in the beams. The converged deflection in the present solution is within 1.0% of the solution by Scordelis and within 0.6% of Mehrain's REFDEK solution.
Figure 3.18 shows the distribution of the longitudinal direction axial force in the slab. To reduce the congestion of lines, only the values at the Gauss-points of the present solution are plotted. The large decrease in axial force per unit length in the transverse direction shows that large in-plane shear deformations are present. This represents the shear lag. The present solution follows the REFDEK solution very closely. The largest difference in results occurs close to the point where the load is applied. The present solution with $N = 6$ is compared with the REFDEK $N = 4$ solution. Therefore, it is not fair to state that the maximum difference in results is 8.5%. With a REFDEK $N = 6$ solution the difference will be much smaller because a finer mesh will be able to approximate the steep slope near the load point better. The present solution is closer to Mehrain’s solution than it is to the solution by Scordelis. It must be remembered that the solution by Scordelis includes shear deformation which has some influence on the results.

Figure 3.19 shows the strong-axis bending moment in the girders. The present solution is close to both the solutions by Mehrain and by Scordelis. The only significant difference in results occurs in the region with high stress gradient, that is, in the region where the load is applied. This maximum difference is 5.4%. Again, this maximum difference will be smaller if the present $N = 6$ solution is compared with a $N = 6$, instead of $N = 4$, REFDEK solution.

The conclusion is that the present solution agrees well with those by Mehrain and by Scordelis. It can be expected that some differences will occur in areas with high stress gradients. It should be noted that the choice of structure BRIDGE-1 is such that large in-plane shear deformations occur in the slab and that the importance of slab bending is exaggerated.
3.6.2. Example Problem: BRIDGE-2

The cross section, plan view and member properties of this bridge with $\alpha = 40$ degrees is shown in Fig. 3.20. The bridge has seven girders, three diaphragms and is subjected to a single point load of 18 kips. The bridge is pin supported at the girder ends only. Present results are compared with those by Powell (86) who used other types of finite elements. He modelled the bridge with 20 elements in the span direction and two elements between girders. The present solution is obtained by using 14 elements in the span direction and four elements between girders. A large shear area ($10000 \text{ in}^2$) is used for the beams in the present solution to suppress the shear deformations that are not taken into account in Powell’s work. From the convergence study in Section 3.5 it is obvious that two elements between girders are not enough to provide a good solution. However, Powell used a shell element which has a higher order of displacement continuity.

A comparison between the results is made in Table 3.16. Powell compared his GENDEK-5 results with those using the COMDEK(CD) program developed by Mehrain. The GENDEK-5 program by Powell uses the same eccentric stiffener (CD) which Mehrain uses in COMDEK(CD). The plate bending element (Q19) used in GENDEK-5 was developed by Felippa (18, 28) and is called CP in Mehrain’s work.

The maximum difference in vertical deflections between the present solution and those by Powell is 1.4%. The difference in deflections is the largest at the point where the load is applied. This difference dies out as the distance from the point where the load is applied increases.

The diaphragm at midspan has torsional stiffness. Thus a bending moment jump occurs in the girders at midspan. It is not clear from Powell’s (86) report if the girder bottom fiber stress reported is just to the left or just to the right of the diaphragm centroid at midspan. The girder bottom fiber stress of the present solution is given for both locations: just to the left and just to the right of midspan. The girder bottom
fiber stress obtained from the present analysis at the point where the load is applied is up to 7.5% larger than the values obtained by Powell. On the other hand, the girder stress obtained at a point far away from the the load is up to 27% smaller.

The cause of this difference in stress results can reasonably be explained as follows. The explanation consists of two parts:

1. In Mehrain's tests on the rhombic plate (See Fig. 10 in Ref. 63) it is shown that the plate bending element (CDF) used in the COMDEK(CD) program is only slightly stiffer at 40 degrees skew than the plate bending element which Powell used in GENDEK-5. It is also shown that both of these plate bending elements have more excessive stiffness than the LDF element at 40 degrees skew. In Fig. 3.6 a comparison is made between the bending behaviour of the current QLSHELL element and Mehrain's LDF element. It shows that for a 4x4 mesh the deflection obtained with the QLSHELL element at 40 degrees skew is less than 1% in error, whereas the solution using the LDF element is nearly 20% too stiff. This means that the bridge as analysed by using the GENDEK-5 or COMDEK(CD) programs behaves as if it has a stiffer slab than the slab used in the present solution. This brings us to the second point of the explanation.

2. To explain the differences in stress when the load is applied at point 1 or point 2 in Fig. 3.20, reference is made to Fig. 3.21 which is taken from Ref. 112. Although Fig. 3.21 is for a five-girder right bridge, it shows that, if the stiffness of the slab becomes smaller (H becomes larger) and the load is applied at point 1 (C), the moment in the girder under the point load becomes larger. This explains the 3.12% increase in bottom fiber stress at point 1, as tabulated in Table 3.16. The figure shows a reduction in bending moment for girders which are more than one girder spacing away from point 1 (C). This explains the -15.3 or -27.2% decrease in bottom fiber stress at point 2 (A). Figure 3.21 also shows that when the load is applied at point 2 (A) and the slab stiffness becomes 20% smaller (H becomes
larger) that the moment in the girder below the point load increases slightly while the moment at point 1 (C) becomes smaller. This explains the 0.66 or 1.91\% increase at point 2 (A) and the -26.7 or -6.49\% decrease in girder bottom fiber stress at point 1 (C).

To summarize, the differences between the present analysis and the results by Powell and Mehrain is mainly due to the fact that their shell elements do not behave as well as the QLSHELL element under skew conditions. Their bridges behave as if they have thicker slabs than they actually do.

3.6.3. Example Problem: BRIDGE-3

Newmark (72,75) obtained an exact solution for the right bridge shown in Fig. 3.22. It is exact in the sense that the solution to the differential equation is exact and to the extent that the assumptions made are satisfied. The angle of skew \( \alpha = 0 \) and the five girders have no eccentricity with respect to the midsurface of the slab. There are no overhangs at the edge girders and the torsional stiffness of the girders is not taken into account. The ratio of the girder spacing to span \( b/a \) is 0.2 and the stiffness ratio \( H \) is two. Poisson's ratio is taken as zero. The bridge is simply supported all along the two support edges. The bridge is loaded with a unit point load at midspan at different transverse locations.

Chen (14) obtained an approximate solution to this problem using the finite difference approach. He used a rather coarse 8x8 finite difference grid. Their solutions together with the present finite element solution using four elements between girders and 14 in the span direction are presented in Table 3.17. Only the girder bending moment coefficients at midspan are given. The actual girder bending moments are obtained by multiplying the tabulated coefficients by the span and the value of the point load. In the present solution a large shear area is assigned to the girders to suppress the shear deformations which are not taken into account in
Newmark’s or Chen’s formulations.

Both Newmark and Chen reported bending moment coefficients to three decimal places. If the bending moment results of the present solution are rounded off to three decimal places the values obtained agree 100% with the three-decimal exact results reported by Newmark. On the other hand, Chen’s finite difference solution is in error up to a maximum of 5% at certain points. The fact that the present finite elements perform so well strengthens the previous convergence test conclusion that the particular Mesh 2 used is sufficient to obtain good results.

Chen used this example to compare results with Newmark’s exact solution in order to obtain an idea of the accuracy of his finite difference solutions. Based on these results thus erroneously concluded that the finite difference method and network used should also provide reasonable accurate results for skew bridges. There was no independent study of the influence of skew on the solution accuracy. Indeed, in the absence of existing theoretical skew bridge solutions available at that time (1954) comparisons were not possible.

3.6.4. Example Problem: BRIDGE-4

The bridge in Fig. 3.22 is analysed again with the following changes:
1. The angle of skew $\alpha = 60$ degrees.
2. The stiffness ratio $H$ is changed to five. Poisson’s ratio is still zero.

In Table 3.18 a comparison is made between the present results and those obtained by Chen (14). The girder bending moment coefficients at midspan are listed. The loading consists of a unit point load at midspan which moves across the bridge on the skew centre line.

Table 3.18 shows differences between the two solutions which are as large as 42%. Since some of Chen’s bending moment coefficients are reported with only one significant digit, it is not possible to list a realistic percentage difference between the
solutions for all values. For those coefficients which have more than one significant digit an approximate percentage difference is also indicated.

It is very reasonable to believe that the present solution is much closer to the truth. Chen obtained his results from a very coarse 8x8 finite difference grid. He was limited to a coarse grid because the electronic computer he used in 1954 could only handle a maximum of 39 simultaneous equations.
CHAPTER 4
DISCUSSION OF RESULTS

4.1. General

A parametric study is done to determine the behaviour of a simply supported, skew slab-and-girder bridge subjected to two-lane truck traffic. The loading consists of two AASHTO HS20-44 trucks. The parameters introduced in Chapter 2 are varied one by one to determine their effect on the distribution of wheel loads to the girders. The typical skew slab-and-girder bridge considered is shown in Fig. 2.1. The idealizations and assumptions introduced are discussed in detail in Chapter 2.

The study of girder bending moment influence surfaces provides valuable insight into the effects of the parameters on the distribution of a single point load. However, if the maximum girder bending moments due to truck loads are of primary interest these can be determined more efficiently by subjecting the bridge to truck loads rather than to wheel loads one at a time. A truck moving across a bridge can be considered as a unit because the locations of its wheels relative to each other remain constant. The girder bending moments due to the weight of a truck at a certain location on the bridge are thus determined more conveniently directly rather than by the effects of individual wheel loads.

The maximum bending moments in the girders are obtained from the bending moment envelope diagrams which result when the trucks are moved as units in a step-wise fashion along the span. The transverse locations of the trucks which produce maximum girder bending moments are determined by trial and error. Only the maximum girder bending moments are reported.

The following topics are discussed in Chapter 4:
Section 4.2: Errors in the Bottom Fibre Stresses of the Girders

The results of the parametric study consist of values for \( M_{cg} \)—the total bending moment which acts on a composite T-section girder. A designer normally calculates the bending stress at any point a distance \( z \) from the neutral axis of a composite girder by using the formula: \( \sigma = \frac{M_{cg} z}{I_{cg}} \), where \( M_{cg} \) is the total moment acting on the composite section and \( I_{cg} \) is the moment of inertia of the composite section. However, bending stresses obtained in this manner are only approximately correct because the value of \( I_{cg} \) depends on the effective flange width—a concept which approximates the effect of shear lag in the slab. The accuracy of this approximation is of present interest.

It is mentioned in Section 2.3.4.3 that the 'correct' bending stresses in a supporting girder can be obtained by using Eq. 2.5: \( \frac{N_g}{A_g} + \frac{M_g z}{I_{gx}} \), where \( N_g \) and \( M_g \) are the axial force and bending moment which act on the supporting girder alone. \( I_{gx} \) and \( A_g \) are the moment of inertia and cross sectional area of the supporting girder. However, as explained in Section 2.3.4.3, \( N_g \) and \( M_g \) are not reported in this study because their values depend on too many variables. They are replaced by \( M_{cg} \).
In this section a comparison is made between the approximate and 'correct' bending stresses in the supporting girders using the two methods of stress calculation as described above. Furthermore, the influence of the longitudinal slab moment $M_s$ on these stresses is also investigated. The bottom fibre stresses in the supporting girders are used as bases for comparison because of their importance to a designer.

The bottom fibre stresses in the supporting girders of five right bridges are listed in Table 4.1. The results are for spans of 60 and 80 ft. The smallest and largest values of the stiffness parameter $H$ are used. The tabulated bending stresses are those corresponding to the maximum possible girder bending moments due to two HS20-44 trucks on the bridge. For convenience, the rear wheel load $P$ is assumed to be ten kips. This assumption does not affect the comparative magnitudes of the stress errors under consideration. The table shows that the bottom fibre stress errors are between -6.0% and +3.1% which is quite acceptable for design purposes.

The longitudinal slab moment $M_s$ is only 3.3% or less of the total bending moment $M_{cg}$ which acts on a composite T-section. $M_s$ is larger in bridges which have smaller $H$ values. Table 4.1 reveals that the stress errors do not increase much when the contribution of the slab moment $M_s$ to the total moment $M_{cg}$ is ignored. In many cases it even happens that more accurate stresses result when the moment in the slab is ignored. Because the inclusion of the contribution of $M_s$ to $M_{cg}$ does not ensure smaller stress errors, it is ignored. This avoids integrating the slab moments, which would be quite inconvenient in skew bridges.

It is found that the stress errors are significantly larger when a bridge is subjected to a single point load, especially in those girders far away from the point where the load is applied. A single point load presents a much more nonuniform loading condition than the 12 wheels of two trucks. The shear lag effect is more pronounced for nonuniform loading conditions. The effective flange width concept does not approximate the shear lag effect in this case as well as it does for a more uniform loading.
Since the stress errors in Table 4.1 are small, it is reasonable to use the composite T-section moment of inertia $I_{cg}$ as parameter. This simplifies the problem tremendously. All of the $M_{cg}$ bending moments reported in Chapter 4 exclude the longitudinal bending moment $M_s$ in the slab. Although the data in Table 4.1 are obtained from only five bridges, they do cover a large range of bridges. However, it is not implied that the occurrence of larger errors is impossible. Table 4.1 gives an idea of the magnitude of errors which can be expected if designers use the effective flange width concept. This should be borne in mind when the data in this report are used to determine design stresses in the girders.

4.3. Differences in Results for Bridges Which Have the Same b/a and H Ratios

Different bridges which have noncomposite girders and the same $H$ and $b/a$ ratios always give identical results for point loads (75). In the present study the value of $H$ depends on the composite girder moment of inertia $I_{cg}$, which is a function of the effective flange width. The effective flange width concept approximates the effect of shear lag in the slab which depends among other things on the slab thickness, girder spacing, span, eccentricity of the girders and the loading condition. It is thus obvious that some differences in results will occur even when two bridges with composite girders have the same $H$ and $b/a$ ratios.

The purpose of this section is to investigate if the magnitudes of these expected differences are small enough to justify the assumption that the behaviour of a bridge is adequately defined by the two parameters $H$ and $b/a$, whether the girders act compositely or not.

To accomplish this, three skew five-girder bridges with $\alpha = 60$ degrees are analysed for a single point load as well as for the wheel loads of two AASHTO HS20-44 trucks. The three bridges are called Bridge 1, 2 and 3 and have nothing to do with
the example problems in Chapter 3. Both Bridge 1 and Bridge 2 have a span of 56.25 ft and a girder spacing of nine feet. Bridge 3 has the same b/a ratio as the other two bridges, but the span is 37.5 ft and the girder spacing is six feet. The H-value of all three bridges is 15.56. To avoid unrealistic differences between the three bridges, practical girders and slab thicknesses which are suitable for the girder spacings and spans considered are chosen from design manuals.

Bridge 1 consists of a 7.5 inches thick slab supported by Illinois Type-42 precast concrete girders. Bridge 2 consists of a 10.125 inches thick slab and AASHTO-PCI Type-45 girders. Bridge 3 has a 6.891 inches thick slab and Illinois Type-36 girders. These bridges have different slab thicknesses, girder eccentricities, girder properties and girder spacings (in one case). Therefore, differences in the results obtained from bridges with the same H and b/a ratios, but with different girder properties and slab thicknesses can be demonstrated.

The results of this investigation are reported in Tables 4.2 and 4.3. A, B and C refer to the edge, second and centre girder respectively. A single point load is applied on the skew centre line at midspan directly above one of the girders. The maximum percentage differences between the maximum bending moments in girders A, B and C obtained from the three bridges are reported in Table 4.2.

The largest bending moment differences in girders A, B and C are 1.6, 6.6 and 5.2% respectively. The differences in the bending moments in the two girders on the other side of the longitudinal centre line are even larger. The important differences are those for the bending moment in the loaded girder because these are differences in large numbers. Here we have a 1.3% difference for girder A and 2.7% for both girders B and C. The magnitudes of these more important percentage differences are acceptable.

The only differences which are of practical importance are those when the bridges are subjected to truck loads because this loading condition controls the design.
Because two truck loads present a more uniformly distributed loading condition than a single point load, the shear lag effect is less and smaller girder bending moment differences should result. These bending moment differences for truck loads acting on the same bridges are listed in Table 4.3. They are smaller than those for point loads. When the two trucks are located such that large bending moments result in the girder under consideration, the differences in results between Bridge 1 and 2 are 1.0% for girder A, 1.2% for girder B and 1.8% for girder C. Bridge 3 has different values of \( a \) and \( b \). Because the distances between the wheels of a truck are constant, truck load results for Bridge 3 cannot be compared with those for the other two bridges.

The girder bending moment differences for truck loads as discussed above are small. Therefore, it is reasonable to assume that whether the girders and slab act compositely or not, \( H \) and \( b/a \) define the behaviour of a bridge adequately for practical purposes.

4.4. Bridges with more than Five Girders

The present parametric study is done on bridges which have five girders. It is necessary to determine whether these results can be used for the design of bridges with more than five girders because at times six-, seven- and nine-girder bridges are build.

Both Badaruddin (8) and Sithichaikasem (112) who analysed right bridges showed that if more girders are added at the same girder spacing, virtually no change occurs in the distribution of a point load. Badaruddin further showed that the maximum girder bending moments caused by two trucks are reduced slightly if the number of girders is increased. The question that arises is whether this is still true when skew is introduced.

Figure 4.1 shows girder bending moment influence diagrams for a point load moving along the skew centre line at midspan of a five-, six- and seven-girder skew
bridge. The bridge has the following properties: \( H = 5 \), \( b/a = 0.1 \) and \( \alpha = 60 \) degrees. Girders A, B, C, D, and E are the girders of the original five-girder bridge. The girder bending moment influence values are calculated at a section 1.51% of the span to the left of midspan. This is the location of the nearest finite element integration points to midspan and is shown in Fig. 4.1. Because of this, the peak value of an influence diagram is not directly above the particular girder.

The values of the influence diagrams for the six-girder bridge are smaller than those for the five-girder bridge. The values for the seven-girder bridge are only slightly smaller than those for the six-girder bridge. The largest differences occur at points far away from the point where the influence is determined and are the largest for the centre girder of the original five-girder bridge. This is fortunate because the influence values at points far away from the point where the influence is determined are small. Thus, the largest differences occur in small numbers which are not of much practical importance to the designer.

There is one difference between the influence diagrams for a right bridge and those for the skew bridge considered. In the skew bridge the influence values are reduced everywhere when the number of girders is increased. However, by increasing the number of girders in a right bridge, the influence values for the interior girders B and C become larger in the edge panel DE (Fig. 4.1) of the original five-girder bridge. This difference may be as a result of the slab which tends to span in the shortest diagonal direction when the slab is skew. The slab transfers some of the load which normally goes to the girders in right bridge directly to the supports and, therefore, always causes a reduction in the influence values. This slab effect is shown in Fig. 3.13.

The effect which an increase in the number of girders has on the girder bending moments when the bridge is subjected to two-lane traffic is determined by analysing four skew bridges with \( \alpha = 60 \) degrees. The bridges are loaded with two AASHTO HS20-44 trucks. The additional girders are spaced at the same spacing. The span of
the bridges considered is 60 ft. The girder spacings used are six and nine feet. The stiffness ratios used are $H = 5$ and $H = 30$. With these values, a large range of bridges is covered. The results are reported in Table 4.4.

In Table 4.4 the maximum girder bending moments in the six- and seven-girder bridges are expressed in terms of percentage differences from the five-girder bridge results. An increase in the number of girders always results in a reduction in the girder design moments. The table clearly shows that the moment differences become smaller when the stiffness ratio $H$ or the girder spacing $b$ increases. The largest differences are always those for the centre girder. This correlates with the differences in the influence diagrams in Fig. 4.1.

The maximum difference between the five- and seven-girder bridge results is 6.3%. This value for the bridge with $H = 5$ and $b = 6$ ft is reduced to 2.0% when $H$ increases to 30 and to 2.3% when the girder spacing increases to 9 ft. It should be noted that $H = 5$ is a low value for prestressed concrete girder bridges. Larger $H$-values are normally used in practice which means that differences smaller than 6.3% will occur. It should be noted that the differences are on the conservative side.

The differences between the six- and seven-girder bridges are much smaller than the differences between the five- and six-girder bridges. It is reasonable to assume that the differences which can be expected if the number of girders is further increased will be much smaller. Thus the results obtained from a five-girder bridge closely approximates the results for a bridge with more girders and are conservative.

The addition of girders at the same girder spacing increases the width of a bridge which provides space for more traffic lanes. Three and four lane traffic are not considered in the present study because both Badaruddin (8) and Sithichaikasem (112) showed that although the girder design moments are larger for more traffic lanes, the resulting moments are smaller after the AASHTO probability reduction factor is applied. It is very unlikely to have three HS20-44 trucks, each loaded to its maximum
capacity, and located at exactly the same instant such that each truck causes its maximum possible contribution to the maximum bending moment in the girder under consideration.

The increase in girder design moments in a skew bridge as the result of more lanes of loading is less than the increase for right bridges. This statement is based on the difference which exists between the bending moment influence diagrams of right and skew bridges as discussed in Section 4.4.

The conclusion reached is that the behaviour of skew bridges is similar to that of right bridges as far as the effect of the number of girders is concerned. The results of the present parameteric study for a five-girder bridge can conservatively be used for the design of bridges with more than five girders. They can also be used for bridges with more than two traffic lanes.

4.5. Influence of Girder Torsional Stiffness

In order to determine the effect and importance of girder torsional stiffness on the load distribution in a bridge, various bridges are analysed with and without girder torsional stiffness and with different angles of skew. The maximum girder bending moment results of these analyses are reported in Tables 4.5 and 4.6. The results without torsion indicated in the tables are those for steel I-beams since they have negligible torsional stiffnesses. The bridges are subjected to two AASHTO HS20-44 trucks located in various positions, each corresponding to the maximum bending moment in the girder under consideration.

Table 4.5 shows maximum girder bending moments for two bridges with a girder spacing of six feet. The spans of the two bridge are 40 and 80 ft, which are the minimum and maximum spans considered in this study. The H-value of both bridges is ten. In one particular case the torsional stiffness of the girders in the 40 ft span bridge is increased by 47%. The results for this case is marked with an asterisk. The
maximum difference in the maximum girder bending moments in this case is 1.3% which shows that the torsional stiffness has little influence as far as the design girder bending moments are concerned. This is fortunate because there are uncertainties regarding the true torsional stiffness of a prestressed concrete I-shaped girder.

For the cases considered, the maximum bending moments in the interior girders always *increase* when the girder torsional stiffness is ignored. This cannot be said of the exterior girders. Table 4.5 shows that it is possible to have a decrease in the design moment of the exterior girders of very short bridges if the torsional girder stiffness is ignored. The influence of girder torsional stiffness becomes gradually larger when the angle of skew increases. However, when \( \alpha = 60 \) degrees the differences between results with and without torsion are still in the order of five percent, which is small. The effect of girder torsional stiffness is larger when the bridge span increases because the torsional stiffness of girders used for the larger spans are also larger. A similar observation was made by Badaruddin (8).

The results of three other skew bridges are reported in Table 4.6. The angle of skew is 60 degrees. The contribution of each truck to the maximum girder bending moment is shown. The girder spacing, \( b \), is nine feet. For a 100% reduction in torsional stiffness the maximum influence on the girder bending moments is 6.5%. It is not likely that the maximum effect of girder torsional stiffness on the girder bending moments will be much larger than 6.5% in other practical slab-and-girder bridges.

Table 4.6 also reveals that a reduction in the girder torsional stiffness has the same effect on the behaviour of the bridge as an increase in the value of \( H \). This can be expected because the rotation of the slab above a supporting girder is resisted by the torsional rigidity of the girder. The slab is thus in effect stiffened in bending in the transverse direction. When the torsional stiffness of the girders is reduced, this torsional stiffening effect of the girders on the transverse bending stiffness of the slab is reduced, which means that the effective rigidity of the slab is less. The \( H \)-value
Increases when the slab rigidity decreases and thus also when the girder torsional stiffness is reduced.

Because the present parametric study is done for bridges which have precast prestressed concrete girders, a minor modification is necessary to apply the data to bridges with steel I-beam girders. The recommendation based on the reports of Badaruddin (8), Sithichaikasem (112) and the data in Tables 4.5 and 4.6 is that the design moments in steel I-beam slab-and-girder bridges must be increased to five percent above the girder design moments for the prestressed concrete girder bridges analysed in this report.

It is important to notice that girder torsion may have some effects other than the influence on girder bending moments which may be of much more importance. Surana (115), for instance, reported that the combination of torsional and bending shear stress in the girders at the obtuse corners can cause shear cracking.

4.6. Influence of the End Diaphragms

The support boundary conditions used in the parametric study are discussed in Section 2.2. An imaginary diaphragm which is rigid in bending in its own plane, is used at each abutment to tie the slab and girders together. The slab ends and girder ends are simply supported. A question that arises is how do the end diaphragms influence the maximum girder bending moments. No attempt is made to study this issue completely but two skew bridges with $\alpha = 60$ degrees are analysed to determine the effect of the end diaphragms.

The properties of the first bridge are: $a = 45$ ft, $b = 8.5$ ft and $H = 12$. The bridge is loaded with two HS20-44 trucks located in different positions such that maximum girder bending moments result in the edge and centre girder. When the end diaphragms are removed so that only the beam ends are ball supported and the slab ends are completely free and unstiffened, the edge girder maximum moment reduces
by 1% and the centre girder moment increases by 6%. The properties of the second bridge are: \( a = 76 \text{ ft}, \ b = 8.5 \text{ ft} \) and \( H = 20 \). When the end diaphragms are removed the maximum edge girder moment increases by 0.3% and the centre girder design moment increases by 2%.

The edge girders are thus not much effected. The most prominent effect of the end diaphragms is that when they are sufficiently stiff the maximum bending moments in the interior girders are reduced. The effect is larger when the span is short. It is expected that the effect will be large when the H-value is small and when the angle of skew is large.

Because the cost of the end diaphragms is small compared to the total cost of a bridge and because they are in any case needed to protect and stiffen the free edge of the slab, the designer can just as well see to it that they are proportioned to be very stiff.

### 4.7. Locations of the Trucks for Maximum Girder Bending Moments

The two-lane truck loading condition used in the parametric study is described in detail in Section 2.3.5.1. The transverse distance between the wheels of two adjacent trucks is fixed at four feet. The minimum transverse distance between an edge girder and a truck wheel is two feet.

The bridge girders considered are called A, B and C and are shown in Figure 2.1. A refers to the edge girder, B to the second girder and C to the centre girder. The two trucks are called Truck 1 and Truck 2. Truck 1 is always the one closest to girder A. Figure 2.1 is used to define the directions in which the trucks move.

When a single isolated beam carries an HS20-44 truck which moves from the left to right over the beam, the maximum static bending moment occurs under the middle axle at a point 2.333 ft to the right of midspan. It is found that some shift occurs in the location of the point of maximum girder bending moment in a right slab-and-
girder bridge. This is so because when a wheel load acts on the slab between two girders, the load is distributed over some distance by the time that it reaches the girders. In a right bridge the middle axle location which produces the maximum girder bending moment is normally more than 2.333 ft to the right of midspan and the point of maximum bending moment is closer than 2.333 ft from midspan.

The transverse truck locations which give the maximum girder bending moments is found to be the same for all girder spacings between six and nine feet. The transverse locations are independant of the angle of skew. For maximum bending moment in girder C the trucks are always spaced symmetrically with respect to the longitudinal centre line, that is, the wheels of the trucks are on both sides two feet away from girder C. When the wheels of Truck 1 are two feet away from girder A it gives the maximum moment in both girder A and girder B.

The location of the point of maximum bending moment in a girder depends on the parameters b, a and H. The truck locations which cause maximum bending moments also depend on these parameters. However, it is found that the span does not have an appreciable influence on the longitudinal location of the trucks with respect to midspan. With reference to Fig. 2.1 it is found that for all bridges analysed that the point of maximum bending moment in girder A is within 3% of the span to the left and 6% of the span to the right of midspan. For girder B this range is from 6% of the span to the left to 6% of the span to the right of midspan. The maximum moment in girder C always occurs at midspan.

The designer should realize that the bending moment envelope diagrams for the girders are noticeably flat in the region of maximum moment. This is not important if precast pretensioned concrete girders are used because the central 20% of these girders are normally designed to carry the same maximum moment. However, if cover plates are used to strengthen steel I-beams at their maximum bending moment locations, the shape of the girder bending moment envelope diagram is important in
calculating the lengths of these cover plates.

Referring to Fig. 2.1, the directions of truck movement which corresponds to maximum girder moments are as follows:

1. For all spans, girder spacings and angles of skew considered, Truck 1 always moves from the right to the left to obtain the maximum bending moment in girder A, and from the left to the right for the maximum moment in girder B.

2. Truck 2 moves from the left to the right to obtain the maximum bending moment in both girder A and girder B when the girder spacing is six feet. It is found that the maximum bending moment in girder A is never very sensitive to the direction in which Truck 2 moves.

3. To obtain the maximum bending moment in girder A when \( b = 9 \) ft, Truck 2 always moves from the left to the right when \( H = 5 \) and from the right to the left when \( H = 30 \) except for the case when \( \alpha = 60 \) degrees. For the maximum moment in girder B when \( b = 9 \) ft, Truck 2 always moves from the right to the left except for the single case when \( \alpha = 60 \) degrees and \( H = 5 \).

4. The maximum bending moment in girder C is always obtained when Truck 1 moves from the left to the right and Truck 2 in the other direction.

4.8. Results of the Parametric Study

4.8.1. General

The results of the parametric study consist of maximum girder bending moment coefficients as well as girder deflection coefficients. The girder bending moments reported are the maximum bending moments \( M_{cg} \) in the composite T-section girders and do not include the contribution of the longitudinal bending moments in the slab \( M_s \) for reasons explained in Section 4.2. The girder deflections reported are calculated
at midspan for truck locations corresponding to the maximum bending moment in the
girder under consideration.

The results of the 108 bridges analysed are reported in Tables 4.7 through 4.38. The maximum bending moment $M_{eg}$ in the composite girders can be obtained by multi­
plying the tabulated moment coefficients by both the rear wheel load $P$ and the span, a. The girder bending moment contribution of Truck 1 and Truck 2 are tabulated separately. Except for girder A, the exterior girder, the bending moment contribution of Truck 1 should not be seen as the maximum girder bending moment for single­
lane loading. The transverse locations of a single truck which produces the maximum girder bending moments in the interior girders are not necessarily the same as the locations when a second truck is present.

The maximum bending moment at midspan of girder C results when the nearest wheels of the trucks are two feet away on both sides of girder C. The longitudinal locations of the trucks, which move in opposite directions, are such that the middle axle of Truck 1 is just as far to the right of midspan as the middle axle of Truck 2 is to the left of midspan. Thus the trucks are antisymmetrically staggered with respect to the longitudinal and skew centre lines of the bridge. Tables 4.7 through 4.38 show that the bending moment contributions of the two trucks are not equal. The contributions should be equal, but they are slightly different because of the method used to interpo­late bending moment values at midspan from the surrounding finite element integra­
tion points.

The maximum girder bending moments are also presented in Figures 4.2 through 4.28 which are used to interpret the behaviour of the bridge. In these figures no dis­
tinction is made between girder B and girder C. The exterior girder bending moments and maximum interior girder bending moments are plotted.

The midspan deflections $\Delta$ can be obtained by multiplying the tabulated deflection coefficients in Tables 4.7 through 4.38 by $P a^3 / E_g I_{eg}$. 
4.8.2. Influence of the Stiffness Parameter $H$ and the Geometric Parameter $b/a$

The vertical stiffness at any point along a beam is a function of $k_i EI/L^3$

where,

$k_i =$ a constant depending on the boundary conditions and the location of the point under consideration.

$EI =$ the bending stiffness of the beam.

$L =$ the span of the beam.

The vertical stiffness of a composite girder in a slab-and-girder bridge is thus a function of $k_1 E_g I_{cg}/a^3$. Similarly, the vertical stiffness of a section of the slab which is effective in distributing load in the transverse direction is a function of $k_2 a D/b^3$, where $k_2 a$ is a fraction of the span, $a$, which is the width of the effective slab section spanning in the transverse direction and $D$ is the flexural stiffness of the slab per unit length.

The vertical stiffness ratio $R$ is defined as the ratio of the vertical stiffness of an interior composite T-section girder to the vertical stiffness of the section of the slab effective in the transverse direction and is thus proportional to:

$$\frac{E_g I_{cg}}{a^3 \begin{pmatrix} a D \end{pmatrix} b^3}$$

which can be rearranged as: $\frac{E_g I_{cg}}{a D} \left( \frac{b}{a} \right)^3$ which equals $H(b/a)^3$.

This vertical stiffness ratio $R$ is proportional to two terms. The first term combines the flexural bending stiffness of the girders and that of the slab and is introduced in Chapter 2 as the flexural stiffness parameter $H$. The second term is purely geometric and is the ratio between the girder spacing and span of the bridge. A large value of $H$ or $b/a$ implies that the flexural stiffness of the composite girders is large compared to the flexural stiffness of the slab. Because the $b/a$ ratio is raised to the power three, it
is obvious that a small change in the $b/a$ ratio has a more pronounced effect on the structural behaviour than an equivalent change in the $H$-value.

The influence of the vertical stiffness ratio $R$ on the structural behaviour is now studied. A bridge which has a very large $R$-value may either have very stiff supporting girders or a very flexible thin slab. Consider the theoretical case where a slab-and-girder bridge has an extremely thin flexible slab. A point load is applied directly above a girder. The particular girder deflects under the load while the other girders deflect a negligible amount, because the slab is too flexible to transfer large loads to them. This means that the loaded girder has to carry nearly all the load by itself. The flexible slab is unable to distribute the point load to the other girders. If the point load can move around the result is that all girders must be designed for very large bending moments. A large value of $H$ or $b/a$ corresponds to the above behaviour. The effect of $H$ on the exterior girders is not as described above if the bridge is subjected to truck loads. This is explained in Section 4.8.2.1.

On the other hand, a bridge which has a small $R$-value can be thought of as one in which the slab is thick enough to distribute an applied point load so that all the girders help to carry the load. A small $R$-value corresponds thus to a more uniform load distribution.

The effect of the $b/a$ parameter can alternatively be explained as follows: a very small $b/a$ ratio corresponds to a long span bridge with girders at a very close spacing. The cross-section of the bridge does not deform much and the bridge behaves like a single beam in which the load is distributed uniformly over the width. On the other hand, a large $b/a$ ratio corresponds to a bridge with a short span and a large girder spacing. The bridge behaves more like a slab in which the bending moments are nonuniformly distributed over the width.

To summarize, a reduction in either $H$ or $b/a$ corresponds to an increase in the ability of the slab to distribute the load. The behaviour of the bridge is more sensitive
to changes in $b/a$ than to $H$. The stiffness ratio $R$ served its purpose in this discussion and is not used as a variable in the parametric study. However, the two parameters that make up $R$ are two of the major parameters considered.

The two sections which follow deal with the effects which variations in the parameters $H$ and $b/a$ have on the behaviour of the bridge according to the data obtained from the parametric study.

4.8.2.1. Effect of Varying the Stiffness Parameter $H$

Figures 4.2 through 4.7 show the maximum girder bending moment $M_{eg}$ as a function of the stiffness parameter $H$ for different angles of skew, spans and girder spacings when the bridge is subjected to two HS20-44 trucks. The maximum bending moment in the interior girders always increases as $H$ increases whether the increase in $H$ is due to a decrease in slab thickness or due to an increase in the supporting girder bending stiffness.

The girder moments are more sensitive to changes in $H$ when the $H$-value is small. For instance, Fig. 4.2 which is for a bridge with $a = 40$ ft and $b = 6$ ft shows that for $\alpha = 0$ a 50% increase in the $H$-value from $H = 5$ to $H = 7.5$, results in a 5.8% increase in the maximum interior girder bending moment. A 50% increase in the $H$-value from $H = 20$ to $H = 30$ results in only a 2.4% increase in moment.

It is fortunate that the girder design moments are insensitive to moderate changes in the $H$-value because there are many uncertainties surrounding the true value of $H$. These uncertainties include the effect of cracks in the slab concrete, the true modulus of elasticity of the slab and girder concrete and the approximation of shear lag by an effective flange width.

The exterior girder behaves differently. An increase in $H$ results in a decrease in the maximum exterior girder bending moment. It is found that when the angle of skew is 60 degrees, there are in some cases a slight increase in the exterior girder
moment when \( H \) is increased between \( H = 5 \) and \( H \approx 15 \) whereafter the moment decreases again. The maximum moment in the exterior girder is extremely insensitive to changes in \( H \) over the whole range of \( H \) considered. The exterior girder design moment can of course not continue to increase as \( H \) is decreased because at \( H = 0 \), which corresponds to a bridge with a rigid slab, the moments in the girders are zero. It should be recalled that the girder moments \( M_{eg} \) do not include the longitudinal slab moments \( M_s \).

The difference in the behaviour of the interior and exterior girders can easily be explained with reference to Fig. 3.21 which is taken from Ref. 112. Figure 3.21 shows the midspan girder bending moment influence lines for a point load \( P \) moving transversely across a right bridge at midspan. The \( b/a \) ratio of the bridge is 0.05. Assume that the girder spacing is eight feet. To obtain the maximum bending moment in girder \( C \), Truck 1 is placed in panel \( BC \) and Truck 2 in panel \( CD \) with the nearest wheels two feet away from girder \( C \) on both sides. In panels \( BC \) and \( CD \) the values of the moment influence diagram for girder \( C \) increase when \( H \) increases. Therefore, the maximum bending moment in the interior girder \( C \) increases.

On the other hand, to obtain the maximum bending moment in the exterior girder, girder \( A \), the nearest longitudinal row of wheels of Truck 1 is placed two feet away from girder \( A \). The second row of wheels of Truck 1 falls directly on top of girder \( B \). All the wheels of Truck 2 fall in panels \( BC \) and \( CD \). The influence line for girder \( A \) shows that when \( H \) increases, only the first row of wheels of Truck 1 causes an increase in bending moment. All the other wheels of Truck 1 and 2 cause a reduction in moment when \( H \) increases. The sum of reductions is more than the moment increase produced by the first line of wheels of Truck 1. Therefore, the maximum exterior girder bending moment decreases when \( H \) is increased.

Figure 3.21 also demonstrates that a small \( H \)-value corresponds to a more uniform distribution of load, because the influence lines are flatter for smaller \( H \)-values.
Note that $H$ depends on the ratio $b/a$ because $H$ has the term $I_{cg}$ in its numerator and the span, $a$, in the denominator. $I_{cg}$ depends on the effective flange width which equals the girder spacing in most practical bridges. However, in the parametric study the values of the parameters are changed one at a time. Thus if $b/a$, for instance, is varied the necessary changes are made to the slab thickness and cross sectional girder properties to keep the value of $H$ the same.

4.8.2.2. **Effect of Varying the Parameters $b$ and $b/a$**

As discussed in Section 4.8.2, an increase in the $b/a$ ratio corresponds to a decrease in the ability of the slab to distribute the wheel loads in the transverse direction. In the case of a loading condition consisting of a single point load, the consequence is always an increase in the girder maximum bending moments.

Increasing the girder spacing means that each girder supports a larger slab area. Because more of the wheel loads can be applied to a larger slab area, the result of increasing the girder spacing is an increase in the girder design moments.

Figures 4.8 through 4.14 show the variation in the maximum girder bending moments $M_{cg}$ and girder midspan deflections $\Delta$ as a function of the girder spacing, $b$, for different spans, $H$-values and angles of skew. The exterior, second and centre girder results are marked in the figures as A, B and C respectively. In each case the span, $a$, and $H$-value are kept constant while $b$ is increased.

Forty-eight of the bridges analysed in the parametric study have a girder spacing of six feet, 48 have a girder spacing of nine feet and 12 bridges have girder spacings somewhere between these values. The purpose of Figures 4.8 through 4.11 is to demonstrate that it is possible to obtain acceptably accurate girder bending moment results for bridges with girder spacings between six and nine feet by interpolating linearly between the data for $b = 6$ ft and $b = 9$ ft.
Figure 4.8 gives results for a right bridge with a span of 40 ft. It shows that as the girder spacing \( b \) increases, the maximum bending moments in all the girders of the bridge increase. The abscissa also represents a variation in \( b/a \) since the span, \( a \), is kept constant as \( b \) is increased. The increase in girder bending moment with \( b \) is as the result of the larger slab area which each girder carries as well as the result of the larger \( b/a \) ratio which decreases the ability of the slab to distribute the load in the transverse direction.

Figures 4.8 through 4.11 show that the variation in maximum girder bending moment with \( b \) is very close to linear in the case of the exterior and centre girder. For girder B the path deviates more from a straight line. A possible explanation for this is that the largest contribution to the maximum moment in the girder under consideration comes from the wheels the nearest to the particular girder. The transverse locations which result in the maximum moment in the exterior girder is such that the nearest wheel of Truck 1 is two feet away from the exterior girder. As the girder spacing increases, this two-foot distance remains the same. Similarly, to obtain the maximum bending moment in girder C, the wheels of the two trucks are two feet away from the centre girder on both sides. As the girder spacing increases, these two-foot distances also remain the same. However, for girder B the transverse distance between girder B and the nearest wheels to girder B varies with the girder spacing. This is because the transverse locations of the trucks which give maximum bending moment in girder B is such that Truck 1 remains two feet away from the exterior girder.

The solid straight lines in the figures interpolate the maximum interior girder bending moments for girder spacings between six and nine feet. For all the bridges analysed it is found that when the girder spacing is six feet, the maximum interior girder bending moment always occurs in girder C. When the girder spacing is nine feet, the maximum interior girder bending moment always occurs in girder B. Using
linear interpolation between these two cases for the interior girders provides results for other girder spacings which are less than 1.8% in error.

Figures 4.9 and 4.11 show that this linear variation with \( b \) also holds when \( \alpha = 60 \) degrees. This linear variation also holds for the midspan girder deflections shown in Figs. 4.12 through 4.14. Because this is true for the minimum and maximum spans considered, the minimum and maximum \( H \)-values used and the minimum and maximum angles of skew, it is very unlikely that a bridge with parameters somewhere between these limits will behave differently.

Sithichaikasem (112) showed that when both \( b \) and \( a \) are increased such that the ratio \( b/a \) is kept constant, a similar nearly linear variation with \( b/a \) exists for the maximum girder bending moments in right bridges.

It was mentioned before that the result of an increase in the \( b/a \) ratio is always an increase in the maximum bending moments resulting from a single point load. However, this is not always true for truck loads. The discussion above is concerned with a variation in the \( b/a \) ratio by changing \( b \). In this case moment increases do result when \( b/a \) is increased. Figures 4.15 through 4.22 show the variation of girder bending moments \( M_{cg} \) as a function of \( b/a \) for different angles of skew, \( H \)-values, girder spacings and spans. The \( b/a \) ratio is now varied by changing the span, \( a \). The figures show that the girder bending moments now decrease when the \( b/a \) ratio increases. This happens even though the ability of the slab to distribute the loads decreases when \( b/a \) increases. This behaviour results because the 14 ft longitudinal axle spacing of the three axles of a truck remains the same when the span is reduced. As the span is reduced, the first and third axle get closer to the supports and contribute less to the maximum moment near midspan.

The variation in the girder bending moments when \( b/a \) is varied by changing \( a \) is not far from linear. Note that the vertical scale used for the bending moments is large. In all cases a straight line can replace each curve shown such that the
differences between the two lines are less than 2%.

4.8.3. Effect of Varying the Angle of Skew $\alpha$

The influence of the angle of skew on the distribution of wheel loads is the crux of the present study. A bridge built on skew alignment always has smaller girder bending moments than its right counterpart. The larger the angle of skew becomes, the smaller the girder design moments obtained. This holds for all girders in the bridge. However, it is found that when $b = 6$ ft, $a = 40$ ft and $H = 10$, there is a small increase in the exterior girder bending moments when $\alpha$ is increased. This increase above the value for a right bridge is less than 1.5% and occurs when $\alpha$ is between 0 and 45 degrees. Chen (14) reported a slight increase in the contribution of the front wheels of his H-type trucks to the maximum moments for spans shorter than 35 ft. This slight increase is of no importance and, indeed, is well within the uncertainties of the present analyses.

The reduction in bending moments in the girders of skew bridges result as a consequence of the following two effects:

1. With the abutments not perpendicular to the girders some of the wheels of the trucks are closer to the supports than in the corresponding right bridge. The total maximum static bending moment on the bridge is thus reduced.

2. In short span bridges with large skew angles there is a tendency for the slab to span in the shortest diagonal direction. This slab action which is indicated in Fig. 3.13 decreases the loads which are normally carried to the supports through the girders in right bridges. The slab transfers part of the load directly to the supports. There are corresponding changes in the bending moments in the slab. The effect of skew on the slab moments is not determined in this study.

Figures 4.23 through 4.28 show the maximum girder bending moments $M_{cg}$ as a function of the angle of skew for different $H$-values, girder spacings and spans.
Figure 4.23 reveals that the exterior girders are very insensitive to changes in the angle of skew for $\alpha$ between 0 and 45 degrees. The interior girders are also insensitive to changes in $\alpha$ for $\alpha$ between 0 and 30 degrees. Most of the reduction in girder bending moments occurs for angles of skew larger than 45 degrees. It is also obvious that the effect of skew is more pronounced when the H-value is small. This effect can be explained in terms of the tendency of very stiff girders to oppose the action of the slab in spanning in the shortest diagonal direction. Figures 4.23 through 4.28 show that the reduction in girder bending moment with $\alpha$ is large for a combination of a large girder spacing, a small span and a small H-value.

The reduction of maximum bending moments in the interior girders due to skew is always less than 5% for angles of skew up to 30 degrees. When $\alpha = 60$ degrees, a reduction as much as 38% is possible. The reduction of maximum bending moments in the exterior girders due to skew is always less than 8% for angles of skew up to 45 degrees. When $\alpha = 60$ degrees the maximum possible reduction is 25%.

Figures 4.2 through 4.7 show the variation in maximum girder bending moment with H for different angles of skew. It seems that the effect of skew is only a reduction in the girder moments since the shape of the diagrams remains nearly the same. This is especially true of the interior girders where the largest bending moment reductions take place.

There is a tendency for an edge girder to become the controlling girder in a skew bridge. This is because the bending moments in the interior girders are reduced much more by skew than those for the exterior girders. This tendency becomes more pronounced for a combination of a large angle of skew, a small H-value, a large span and a small girder spacing. Chen (14) made a similar observation. However, it is found that the edge girder controls in only two of the 108 bridges analysed. In these particular two cases the edge girder maximum moment is only 0.3% and 1.0% larger than the maximum bending moment in the interior girders.
It can then be concluded that it is possible to avoid the undesired condition of having the controlling moment in an edge girder by keeping the truck wheels at least two feet away from the edge girders. This undesired condition is discussed in detail in Section 2.2. This conclusion is limited to bridges with spans not exceeding 80 ft.

4.9. Comparison with the AASHTO Design Recommendations for Right Bridges

The current (1985) AASHTO Standard Specifications for Highway Bridges (5) permit the use of the following well-known method for the design of right slab-and-girder bridges subjected to truck loads. The maximum bending moment in a bridge girder can be obtained by applying half the load of one truck to a single isolated beam which has the same span as the girders in the bridge. Half the load of an HS20-44 truck is the loads from the wheels on say the left side of the truck. The three point loads resulting from the wheels on the left side of an HS20-44 truck is P/4, P and P for the front, middle and rear axle, where P is defined in Section 2.3.5.1. The maximum static bending moment in the beam obtained in this way is then multiplied by a wheel load fraction to obtain the corresponding maximum bending moment in the bridge girder. The wheel load fraction is the number of these rows of three wheels which the corresponding girder in the bridge has to carry. The very complex analysis of a slab-and-girder bridge is thus simplified to a beam subjected to a set of moving point loads.

The portion of the AASHTO specifications that relate to the distribution of truck loads among girders may be summarized briefly as follows. For steel I-beams or prestressed concrete girders, a wheel load fraction of b/5.5 is recommended for the interior girders of bridges subjected to two-lane traffic, where b, the girder spacing in feet, is in the range considered in this study. The bending moments in the exterior girders may be determined, by applying to the girder the reaction of the wheel loads, by assuming that the slab acts as if simply supported between adjacent girders.
Therefore, when the minimum distance between the edge girder and nearest wheels is two feet, the wheel load fraction for the edge girders, based on the above simply supported slab action, is \( \frac{2}{3} \) for \( b = 6 \text{ ft} \) and \( \frac{8}{9} \) for \( b = 9 \text{ ft} \). For the girder spacings considered, the AASHTO specifications further require that the wheel load fraction for the exterior girders must not be less than \( \frac{b}{4 + b/4} \) for steel I-beams. Another requirement is that the exterior girders must be designed to have at least the same load carrying capacity as the interior girders.

The maximum bending moment coefficients for interior and exterior girders obtained by using the AASHTO wheel load fractions are shown in Figs. 4.2 through 4.7. The bending moment coefficients for the exterior girders resulting from the wheel load fraction \( \frac{b}{4 + b/4} \) are not indicated. It is found that this fraction provides very conservative design moments. When the bridge span is 40, 60 and 80 ft this fraction gives girder design moments about 60\%, 40\% and 30\% too high, whichever girder spacing is used.

The comparisons in Figures 4.2 through 4.7 between the present results for right bridges without diaphragms and the current AASHTO specifications show that the AASHTO provisions result in bending moments for the interior girders that are in many cases too small. This is especially so for combinations of large H-value, short span and small girder spacing. For the range of parameters considered in this study the \( \frac{b}{5.5} \) interior girder wheel load fraction is between 12\% too small and 32\% too high. Culham (20) who analysed right bridges with intermediate diaphragms also found that the \( \frac{b}{5.5} \) fraction gives interior girder bending moment results which are too small for short spans and too large for large spans.

The figures show that the current AASHTO provision for exterior girders which is based on the assumption that the slab acts as if simply supported between adjacent girders is unconservative in most of the cases considered. It is more unsafe when \( H \) is small and the span is large. It is found that the girder spacing does not have any
significant effect. For the range of parameters considered in this study this specified AASHTO method for the exterior girders is up to 23% on the unsafe side. However, the AASHTO requirement that edge girders must have at least the same load carrying capacity as the interior girders governs in these cases. Culham (20) who analysed right bridges with intermediate diaphragms also found that this provision for the exterior girders underestimates the load carried by the exterior girder.

It is clear from Figures 4.2 through 4.7 that the AASHTO specification which requires the same load carrying capacity for all the girders in the bridge leads to over-conservative design of the exterior girders. For bridges with short spans, stiff girders and large girder spacings the design bending moments can be more than twice what they actually are. However, it may be feasible to make all girders identical. The designer should be more conservative in designing the exterior girders because monolithic curbs and parapets might possibly increase the design bending moment in the edge girder as discussed in Section 2.2.

To summarize, this comparison between the present results for right bridges and the current AASHTO wheel load fractions shows that some improvements in the analysis method are desirable. The b/5.5 wheel load fraction for the interior girders oversimplifies the behaviour of the bridge and can result in design moments up to 12% on the unsafe side. In other cases it is up to 32% too conservative. The recommendation for design of the exterior girders as if the slab is simply supported between adjacent girders is unconservative in most of the cases considered. On the other hand, the requirement to design exterior girders to have the same load carrying capacity as the interior girders is very conservative for cases where the outer wheel locations are restricted to be at least two feet away from the exterior girders.
CHAPTER 5
DESIGN CRITERIA FOR RIGHT AND SKEW SLAB-AND-GIRDER BRIDGES

5.1. General

It is shown in Section 4.9 that the current (1985) AASHTO specifications for the distribution of wheel loads in right slab-and-girder bridges are sometimes unsafe and often too conservative. It is unreasonable to expect that designers should have to carry out a complete computer analysis of a bridge each time that a skew bridge is to be designed. The need to have some sort of simplified analysis procedure for the girders of skew slab-and-girder bridges exists because the AASHTO design specifications provide no design recommendations regarding this matter.

The purpose of this chapter is to develop a reliable method of analysis for simply supported, skew slab-and-girder bridges. Such a method of analysis should be easy to use, should approximate the true behaviour of a bridge with acceptable accuracy and should preferably be in a form familiar to practicing engineers. It can also be used to obtain trial structural member sizes for a first computer analysis. If engineers use improved design criteria which also take advantage of the beneficial effect of skew, safe and more economic bridge designs will result.

The general format in which the criteria for the analysis of maximum girder bending moments are expressed is presented in Section 5.2. This is followed by Section 5.3 in which improved analysis criteria for the maximum girder bending moments of right bridges are developed. Section 5.4 deals with criteria for the analysis of maximum bending moments in the girders of skew slab-and-girder bridges. The proposed analysis procedure for slab-and-girder bridges is summarized in Section 5.5 in the form of a design algorithm which should be convenient to use in a bridge design office.
Sections 5.6 and 5.7 are concerned with girder midspan deflections and also with the distribution of certain dead loads. These matters are considered as of secondary importance in the present study and are, therefore, not elaborately investigated.

5.2. Design Criteria Format for Girder Bending Moments

An accurate method of analysis for the girder bending moments in skew bridges is possible by using a set of design graphs like those presented in Figs. 4.2 through 4.28. However, it is necessary to interpolate (sometimes quadratically) between the various graphs to obtain the design bending moments in the girders. Accurate results can be obtained in this way, but the successive interpolations are inconvenient.

Although the actual distribution of load to the girders in a slab-and-girder bridge is very complex, a fictitious load distribution which is characterized by the concept of a wheel load fraction can be used to account for the moments in the girders. The AASHTO wheel load fraction, \( b/5.5 \), for the interior girders of right bridges is based on research done by Newmark (75). The factor \( b/5.5 \) reflects the linear trend in \( b \) which is observed in the present study, but it does not include the effects of \( H \) and \( b/a \) directly. This wheel load fraction is an oversimplification of the design equation proposed by Newmark which does include all relevant parameters.

The expression of the distribution of wheel loads among girders by a wheel load fraction is a well established concept used in the design of slab-and-girder bridges. This concept is now expanded to cover skew bridges as well. Because the data from the present study show that the exterior girders behave differently from the interior girders, they are considered separately. First of all, improved wheel load fractions are determined for right bridges. Instead of developing independent expressions for wheel load fractions in skew bridges for each angle of skew, it is more convenient to incorporate the effect of skew by multiplying the wheel load fractions of a right bridge by a skew reduction factor.
Unlike Newmark's wheel load fractions, which are based on the distribution of load from only one axle of each truck, the current wheel load fractions are obtained directly from the maximum girder bending moments caused by two complete HS20-44 trucks. Because the wheel load fraction represents a fictitious distribution of load which approximates the actual very complex load distribution, it can be expected that some scatter in computed wheel load fractions will exist. It is necessary to search for a combination of bridge parameters which define the wheel load fractions with the smallest possible scatter.

The maximum bending moment in a girder is called the design bending moment $M_d$. The design bending moment coefficient can be expressed as:

$$M_d/Pa = (M_{static}/Pa)(b/Q)(Z)$$

with $Q$, $b$ and $a$ in feet where,

$$b/Q = \text{wheel load fraction}.$$  

$$Q = \text{a variable, currently 5.5 ft for interior girders according Ref. 5.}$$  

$$Z = \text{skew reduction factor for girder bending moments.}$$  

$M_{static}/Pa$ = maximum static bending moment coefficient which results when one row of three wheels ($P/4$, $P$ and $P$) of one HS20–44 truck is applied to an isolated beam which has the same span as the girders in the bridge.

When $a > 33$ ft the maximum static bending moment coefficient is:

$$M_{static}/Pa = 12.25/a^2 - 8.75/a + 9/16$$  

(5.2)
5.3. Criteria for Right Bridges

5.3.1. Exterior Girders

Figure 5.1 shows Q-values for the exterior girders in right bridges which should be used in Eq. 5.1. It should be noted that these Q-values apply only when the minimum distance between the edge girder and nearest truck wheels is two feet. Figure 5.1 shows a well-defined functional relationship between Q and \( H(b/a)^3 \). The quantity \( H(b/a)^3 \) is proportional to the vertical stiffness ratio \( R \) which is discussed in Section 4.8.2. The following two equations give conservative Q-values for exterior girders in right bridges:

For \( H(b/a)^3 < 0.0569 \),

\[
Q = 400H(b/a)^3 - 478[H(b/a)^3]^{1.1} + 6.7
\]  (5.3)

For \( H(b/a)^3 \geq 0.0569 \),

\[
Q = 5.24H(b/a)^3 + 8.74 \text{ ft}
\]  (5.4)

Differences between the Q-value data points and these lines are always conservative and by not more than 5%. Considering the complex behaviour of a slab-and-girder bridge, the above expressions for Q can be considered as very good. Q is measured in feet.

5.3.2. Interior Girders

Figure 5.2 shows Q-values for the interior girders in right bridges which should be used in Eq. 5.1. The variable \( a/(10\sqrt{H}) \) which were used by Newmark (75) gives less scatter of wheel load fractions than any other variable used in an attempt to find the best one. This variable originates from the thought that the bending moments in the girders should depend in some way on the relative deflections of the girders which are proportional to the quantity \( a^3/E_g I_{cg} \). For a particular slab the quantity \( a^2/H \) amounts to the same thing. If \( a/\sqrt{H} \) is used, a convenient linear relationship exists. Two well-defined Q-value data bands can be distinguished. One for the group of
bridges which has a girder spacing of six feet and one for the other group with a nine feet girder spacing. The two straight lines shown in Fig. 5.2 are conservative estimates for Q when \( b = 6 \, \text{ft} \) and \( b = 9 \, \text{ft} \). According to Figs. 4.8 through 4.11 the maximum girder bending moments can be considered to vary linearly with \( b \) when \( H \) and \( a \) are kept constant. This linearity is recognized and applied to obtain conservative Q-values for interior girders in right bridges as follows:

\[
Q = (0.01538 + b/150)\left(\frac{a}{\sqrt{H}}\right) + 4.26 + b/30 \, \text{ft} \tag{5.5}\]

 Unlike Newmark's expression for Q, this expression depends on both \( a/\sqrt{H} \) and \( b \). The differences between the Q-value data points and this expression for Q are always conservative and by not more than 8%. This includes the data points for the bridges with girder spacings other than six or nine feet.

Newmark (75) proposed a straight line for Q which intersects the Q-axes at 4.4 and 6.08. His Q-values are thus more conservative especially for bridges with large spans and smaller H-values. Newmark's Q-value gives 25% larger girder design moments than the present Q-value when the bridge has a nine feet girder spacing and the abscissa is four. As mentioned before, his Q-values are based on the distribution of a single axle load from each truck.

The different abscissas for interior and exterior girders which are necessary to ensure narrow Q-value data scatter bands point out the difference in the behaviour of interior and exterior girders.

5.4. Criteria for Skew Bridges

Chen (14) determined Q-values for skew bridges as a function of the quantity \( a/(10\sqrt{H}) \). The quantity \( a/(10\sqrt{H}) \) is currently used to define the Q-values of interior girders in right bridges. In Chen's work, the reduction due to skew is not expressed in terms of a skew reduction factor. Chen's Q-values, like Newmark's, are based on
the load distribution of a single axle load of each of two trucks. Chen's results reveal that as the angle of skew increases, the scatter in Q-values also increases. When the angle of skew is 60 degrees, the maximum difference between his Q-value data points and the conservative linear expression for Q which he proposed is 55%. The beneficial girder bending moment reduction as the consequence of skew was not effectively incorporated into the design equation because of the large scatter.

The skew reduction factor Z proposed in this study is defined as the ratio of the maximum girder bending moment in a skew bridge divided by the maximum girder bending moment in the corresponding right bridge. Figures 5.3 and 5.4 show the skew reduction factor Z for interior and exterior girders as a function of the variable $b/(aH)$. The parameter $b/(aH)$ seems to be a logical choice as variable because Figures 4.2 through 4.28 show that the reduction due to skew is large in bridges with a large girder spacing, a small span and a small H-value.

The Z-value data points for interior girders which are plotted in Fig. 5.3 lie in well-defined data bands for each angle of skew considered. The lines are linear expressions from which conservative skew reduction factors can be obtained for each angle of skew. The scatter in Z-values increases as the angle of skew increases. However, when $\alpha = 60$ degrees, the differences between the Z-value data points and the indicated straight line for Z are conservative by not more than 8.5%. Thus, the use of the variable $b/(aH)$ on which the present effect of skew is based results in much less scatter in data than the method used by Chen. The reduction for interior girder bending moments due to skew is always less than 5% for angles of skew up to 30 degrees. At 60 degrees skew, a reduction as much as 38% is possible.

The Z-values for the exterior girders which are plotted in Fig. 5.4 show similar data bands as the interior girders. Up to 30 degrees skew, nearly no reduction takes place. This is also very obvious from Figs. 4.23 through 4.28. The maximum percentage scatter is even less than that for the interior girders. Fig. 5.4 also shows a 1.5%
increase in exterior girder bending moment which is possible for small angles of skew. This is of no practical importance. The reduction for exterior girder bending moments due to skew is always less than 8\% for angles of skew up to 45 degrees. At 60 degrees skew the maximum reduction possible is 25\%.

The \( Q \)-value expression in Fig. 5.2 is used to determine the design bending moment for the interior girders in a right bridge. To determine the interior girder bending moment in the corresponding skew bridge, this conservative design value is multiplied by the skew reduction factor \( Z \) in Fig. 5.3 according to the conservative straight lines for \( Z \). This conservatism in design bending moments adds up as follows: for the interior girders a maximum of 8, 8 and 15\% for \( \alpha = 30, 45 \) and 60 degrees respectively. For the exterior girders a maximum of 8, 11 and 13\% for \( \alpha = 30, 45 \) and 60 degrees respectively.

Two changes in the determination of the skew reduction factor \( Z \) are suggested. At first, to obtain the correct girder design moments in a skew bridge by multiplying \( M_{\text{d}_{\text{rigth}}} = M_{\text{static}} (b/Q) \) by \( Z \), the \( Z \)-values should be calculated from:

\[
Z = \frac{M_{\text{d}_{\text{skew}}} \text{ from analysis}}{M_{\text{d}_{\text{rigth}}} = M_{\text{static}} (b/Q)}
\]

rather than from:

\[
Z = \frac{M_{\text{d}_{\text{skew}}} \text{ from analysis}}{M_{\text{d}_{\text{rigth}}} \text{ from analysis}}
\]

New \( Z \)-values obtained in this consistent way are plotted in Figs. 5.5 and 5.6. It is now possible to obtain \( Z \)-values for right bridges as well. Only now the factor \( Z \) should be considered as a correction factor and not a skew reduction factor.

The second change suggested is a matter of choice and judgement. The question is where should the straight lines for \( Z \) be drawn. The chosen \( Z \)-lines indicated in Figs. 5.5 and 5.6 are such that the following maximum percentage differences occur between the 'correct' bending moments obtained from the analyses and those obtained according to this proposed analysis procedure. Positive differences are conservative:
| DIFFERENCES IN GIRDER BENDING MOMENTS |
|-------------------------------|-----------------|-----------------|
| α degrees | EXTERIOR GIRDERS | INTERIOR GIRDERS |
| 0 | -0.7 to + 5.0 | -3.7 to + 3.7 |
| 30 | -3.4 to + 4.0 | -2.7 to + 3.3 |
| 45 | -4.5 to + 5.5 | -2.9 to + 3.3 |
| 60 | -6.1 to + 6.8 | -4.9 to + 6.1 |

The judgement reflected in these particular choices of the Z-lines in Figs. 5.5 and 5.6 is that an error of 6.1% on the unsafe side is not a matter of concern. The probability of having two HS20-44 trucks, each of them loaded to its maximum capacity, each located in exactly the correct position at exactly the same instant to produce the maximum bending moment in the girder under consideration is very small. Furthermore, these design moments are increased by 24 to 30%, a crude allowance for impact and durability effects. A more conservative designer can use the data in Figs. 5.5 and 5.6 to choose his own more conservative lines for Z.

5.5. Proposed Analysis Procedure for Slab-and-Girder Bridges

The analysis procedure for the maximum bending moments in the girders of slab-and-girder bridges can be summarized in the following design algorithm. Determine the maximum girder bending moments $M_d$ for a right bridge with the same girder spacing, span and stiffness ratio as the skew bridge and multiply this bending moments by the skew reduction factor $Z$ according to the following equation:

$$M_d = (M_{static})(b/Q)(Z)$$

where,

$M_d =$ design girder bending moment for prestressed concrete girders. Due to the lack of torsional stiffness in steel I-beams increase $M_d$ by 5% if steel I-beams are used as supporting girders.
\[ M_{\text{static}} = \text{maximum static bending moment in an isolated beam with the same span as the girders in the bridge. The load on the beam is half the load of one HS20-44 truck.} \]

For \( a > 33 \) ft: \[ M_{\text{static}} = (12.25/a^2 - 8.75/a + 9/16) \text{Pa} \]

\( Z = \) skew reduction factor as listed in the table below. In the case of a right bridge, \( Z \) is a correction factor. The use of linear interpolation between these \( Z \)-lines for other angles of skew is less than 2% on the unsafe side.

<table>
<thead>
<tr>
<th>( \alpha ) degrees</th>
<th>INTERIOR GIRDERS</th>
<th>EXTERIOR GIRDERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0 - 1.6 ( b/(aH) )</td>
<td>1.0</td>
</tr>
<tr>
<td>30</td>
<td>0.99 - 2.4 ( b/(aH) )</td>
<td>1.0 - 1.6 ( b/(aH) )</td>
</tr>
<tr>
<td>45</td>
<td>0.96 - 3.7 ( b/(aH) )</td>
<td>0.99 - 2.6 ( b/(aH) )</td>
</tr>
<tr>
<td>60</td>
<td>0.87 - 6.75 ( b/(aH) )</td>
<td>0.94 - 4.8 ( b/(aH) )</td>
</tr>
</tbody>
</table>

\( b/Q = \) wheel load fraction where, \( Q \) is determined from the following equations.

\( Q \) for interior girders is:
\[ Q = (0.01538 + b/150)(a/\sqrt{H}) + 4.26 + b/30 \]

\( Q \) for exterior girders is:
\[ Q = 400H(b/a)^3 - 478[H(b/a)^3]^{1.1} + 6.7 \quad \text{for} \quad H(b/a)^3 < 0.0569 \]
\[ Q = 5.24H(b/a)^3 + 8.74 \quad \text{for} \quad H(b/a)^3 \geq 0.0569 \]

Half of the load carried by one of the heavy axles of a truck equals \( P \). The span, \( a \), the girder spacing, \( b \), and \( Q \) in feet.

The above method of analysis is easy to use. With only a few calculations girder design moments can be obtained which are accurate enough for the purposes of design. For right bridges this method of analysis gives results which are within 5% of the data obtained from the sophisticated finite element analyses used in this study.
For skew bridges this method of analysis gives results which are within 7% of the data obtained from the analyses. Note that there will be a significant increase in the maximum exterior girder bending moment if the edge girder to truck wheel minimum distance of two feet is reduced. This method of analysis can also be used for bridges with more than five girders.

5.6. Girder Deflections due to Truck Loads

The deflection coefficients reported in Tables 4.7 through 4.38 give the midspan girder deflections when the trucks are located such that the maximum bending moment results in the girder under consideration. The girder deflections can be obtained by multiplying the tabulated coefficients by \( \frac{P a^3}{(E g I_{cg})} \), where \( E_g \) and \( I_{cg} \) are the modulus of elasticity and bending moment of inertia of an interior composite girder. Although the exterior composite girders are more flexible due to the short slab overhangs, their deflection coefficients are also expressed in terms of the bending moment of inertia of an interior composite girder.

The tables show that the general effect of skew is a decrease in the girder deflections which is similar to the effect of skew on the girder bending moments. There are exceptions. In some cases it is found that a slight increase occurs in the exterior girder midspan deflection when \( \alpha \) is increased between 0 and 45 degrees. These increases are less than 3%.

When \( H = 30 \) and \( \alpha = 60 \) degrees it is found that the relative deflection between the centre and edge girder is less than that for a right bridge. When \( H = 5 \), the relative deflection becomes more if \( \alpha \) is increased from 0 to 60 degrees, but not when the span is 40 ft. Thus, it can not be said that the deflections in a skew bridge are in general more nonuniform than those in a right bridge.

An approximate way to calculate the midspan girder deflections \( \Delta \) is to use the wheel load fraction \( b/Q \) and the skew reduction factor \( Z \) as follows:
\[ \Delta = (\Delta_{\text{static}})(b/Q)(Z) \]

where,

\[ \Delta_{\text{static}} = \text{the midspan girder deflection when half the load of one HS20-44 truck is} \]

applied to an isolated beam with the same span as the bridge girders in the location which produces the maximum static bending moment in the beam.

Deflections calculated in this way are, however, not very accurate, because the two multipliers are based on the maximum girder bending moments and not on girder deflections.

It should be pointed out that AASHTO provision 1.7.6 directly states that the girder deflections, which are limited to \( a/800 \), are the deflections computed in accordance with the assumption made for loading, that is, using the wheel load fraction \( b/Q \). This study shows that the exterior and interior midspan girder deflections obtained in this way can underestimate the true deflection by as much as 22%.

A more appropriate way to obtain girder deflections is to use separate wheel load fractions and skew reduction factors which are based on the deflection data. These new factors are determined and the results are shown in Figs. 5.7 through 5.10. The wheel load fraction for deflection is \( b/X \) and the skew reduction factor for deflection is \( Y \). Girder midspan deflections can be determined from:

\[ \Delta = (\Delta_{\text{static}})(b/X)(Y) \]

with \( \Delta_{\text{static}} \) as defined before.

The X-values for interior girders are shown in Fig. 5.7. A well-defined functional relationship exists between X and \( H(b/a)^3 \). For values of H up to about 20 the X-value data points fall along two rather smooth curves: one for \( b = 6 \) ft and one for \( b = 9 \) ft. Figures 4.12 through 4.14 show that linear interpolation can be used for girder spacings between these.

A designer can obtain a much better value for the interior girder midspan deflection in a right bridge by using Fig. 5.7 instead of the \( b/Q \) wheel load fraction for
bending moments.

The skew reduction factor $Y$ for interior girder midspan deflections is shown in Fig. 5.8 as a function of $b/(aH)$. A well-defined band of $Y$-values exists for each angle of skew. The maximum percentage scatter in $Y$-values is less than 10% and occurs when the angle of skew is 60 degrees. The straight lines indicated in the figure present conservative values for $Y$. If a designer uses his own less conservative $Y$-lines he can obtain deflections which are within 5% of those obtained from the finite element analyses on skew bridges.

The $X$-values for exterior girders are shown in Fig. 5.9. A well-defined functional relation for $X$ again exists with $H(b/a)^3$. Here, unlike the curves for interior girders, the curves do not have the same shape. It should be realized that the ratio $I_e^\text{exterior} / I_e^\text{interior}$ is not a constant for all the bridges. However, its variation is less than 6%. It is difficult to obtain a reliable deflection for the exterior girders because of the unknown stiffening effect of the curbs. Figure 5.9 is for bridges in which the effect of the curbs is ignored. It can be used to estimate the exterior girder midspan deflection in a right slab-and-girder bridge.

The skew reduction factor $Y$ for exterior girder midspan deflections is shown in Fig. 5.10 as a function of $b/(aH)$. The maximum percentage scatter in $Y$-value data points is even less than that for the interior girders. This figure can be used to estimate the exterior girder midspan deflection in a skew bridge in a similar way as what is done for the interior girders.

5.7. Girder Bending Moments due to Dead Load

Although the determination of the behaviour of a bridge under dead load is not a major goal of the present study, the following two dead load cases are considered. A good discussion of the various dead load effects which should be considered in the design of a slab-and-girder bridge can be found in Ref. 75.
5.7.1. Curbs and Parapets

Concrete curbs and parapets are normally constructed after the slab concrete is strong enough to act compositely with the supporting girders. The weight of the curbs is thus transversely distributed over the full width of the bridge. If the load per unit length of one of the curbs and parapets is \( c \), the total maximum static bending moment on the bridge equals \( (2/8)ca^2 \). It is assumed that the two line loads act directly above the edge girders.

The maximum girder bending moments \( M_{eg} \) which result from this loading condition are reported in Table 5.1. The longitudinal bending moments in the slab \( M_s \) are again ignored. The girder bending moments are expressed as fractions of the total static bending moment on the bridge.

Table 5.1 shows that the behaviour of the girders under this load is different from the behaviour when they are subjected to truck loads. When \( H \) increases the maximum bending moment in the edge girder increases while the maximum moments in all the interior girders decrease. The exterior girder bending moment influence line in Fig. 3.21 reveals why the behaviour is different. The transverse influence line for the midspan bending moment in the exterior girder may also be interpreted as a bending moment diagram due to a concentrated load on top of the exterior girder at midspan. This is due to the reciprocal relation between loads and longitudinal curvatures which is described by Newmark (72). The influence diagram clearly shows that when \( H \) increases, the bending moment in the edge girder increases while those in the interior girders decrease.

Table 5.1 further shows that when \( b/a \) increases, the maximum exterior girder bending moment fraction increases while those of the interior girders decrease. In the case of truck loads it is necessary to know \( H \), \( b \) and \( b/a \) to determine the distribution of loads. Now the behaviour is determined by only the \( H \) and the \( b/a \) ratio.
The effect of skew on the distribution of these line loads is less pronounced than when the bridge is subjected to truck loads. The effect of skew on the exterior girder is always a reduction in the maximum bending moment. The largest reductions occur when \( \alpha \) and \( b/a \) are large and when \( H \) is small. When \( \alpha \) is 60 degrees, the largest reduction is 14\%, whereas, the similar reduction for truck loads is 25\%.

In most cases considered the effect of skew on the interior girder bending moments due to the line loads, \( c \), is an increase in the maximum bending moments. However, for very flexible girders, \( H = 5 \), it turns out that the moments in the interior girders increase as \( \alpha \) is increased to 45 degrees, whereafter moment reductions occur for any further increase in \( \alpha \). When \( \alpha = 60 \) degrees and \( H = 5 \), the bending moments are still larger than those for the right bridge. In most cases the effect of skew is more pronounced when \( H \) and \( b/a \) are large.

For a right bridge each edge girder carries between 29 and 45\% of the total static bending moment. The second girder, girder \( B \), carries between 6 and 15\%. The moment in the centre girder is always less than 10\% of the total maximum static bending moment on the bridge.

It is not really worthwhile to consider the quite small effect of skew when the distribution of the two line loads are determined. For instance, the largest reduction in the maximum exterior girder bending moment is only about 5\% of the total maximum static bending moment. The largest increase in the maximum interior girder bending moment is also about 5\% of the total static bending moment.

There is a considerable shift in the point of maximum bending moment in girder \( B \) for large angles of skew. The location of the maximum bending moment is approximately where a transverse line which originates at midspan of the exterior girder intersects with girder \( B \).

AASHTO provision 1.3.1(B2a) states that the dead load from curbs and railings may be equally distributed to all the girders. This is obviously an unrealistic and
unsafe assumption.

5.7.2. Roadway Resurfacing Load

The second type of dead load considered is a uniformly distributed load of intensity \( \omega \) applied to the total deck area between the faces of the curbs. This type of loading results as the consequence of additional layers of roadway resurfacing material.

Girder bending moments expressed as fractions of the total maximum static bending moment on the bridge are listed in Table 5.2. The total maximum static bending moment on the bridge equals \((1/8)(4b\omega)a^2\).

Except for some cases when \( \alpha = 60 \) degrees an increase in \( H \) results in a decrease in the exterior girder bending moments. An increase in \( H \) results in an increase in the interior girder bending moments. However, it is possible to have a decrease in interior girder bending moment when \( H \) is increased between 20 and 30 in right bridges.

An increase in the \( b/a \) ratio always results in a decrease in the maximum exterior girder bending moment, but it can decrease or increase the interior girder bending moments.

The exterior girder bending moment is rather insensitive to the angle of skew. The maximum reduction when \( \alpha = 60 \) degrees is only 3\% of the total maximum static bending moment. A slight increase in moment is possible when \( \alpha \) is increased between 0 and 45 degrees.

The effect of skew on the interior girder bending moments is always a reduction in moment. The interior girders are more sensitive to the angle of skew than the exterior girders. However, the maximum reduction at 60 degrees skew is only 11\% of the total maximum static bending moment.
The assumption that each interior girder in a right bridge carries a width of load equal to the girder spacing and each exterior girder carries a width of load equal to half the girder spacing results in a distribution of load which is correct within 4.5% of the total static bending moment on the bridge.
CHAPTER 6
SUMMARY AND CONCLUSIONS

6.1. Summary

This study is concerned with the behaviour of simply supported, right and skew slab-and-girder bridges subjected to truck loads. A simplified procedure is proposed for the determination of live load moments in each girder which is sufficiently accurate for all practical purposes.

The abutments of skew slab-and-girder bridges are not perpendicular to the girders which span in the direction of the traffic. Many skew highway bridges have already been built in grade separations where the intersecting roads are not perpendicular to one another. They are also necessary where natural or existing man-made obstacles prevent a perpendicular crossing and consequently they are commonly found in mountainous areas. In many cases, the lack of space at complex intersections and in congested built-up areas may also require bridges to be built on skew alignment.

A literature survey shows that there is no information available which tells a bridge design engineer exactly how to take account of the effects of skew when designing a slab-and-girder bridge. In existing research papers the effects of skew are determined and explained, but are not presented in such a way that a designer knows quantitatively what to do. Therefore, research on skew slab-and-girder bridges with the goal to develop design criteria which include the effects of skew is desirable.

With this goal in mind a parametric study was done by analysing 108 different simply supported slab-and-girder bridges subjected to two AASHTO HS20-44 trucks. With the aid of a HARRIS-800 computer, the finite element method was used to analyse these bridges. The girders were modelled as eccentric stiffeners which cause shear lag in the slab. Only five-girder bridges were analysed, but it is shown that the results of a five-girder bridge can conservatively be applied to bridges which have
more girders. Only steel I-beam and precast prestressed concrete girders were considered. The torsional stiffness of the precast prestressed concrete girders used was taken into account and the difference between the effects of precast concrete and steel I-beam girders is demonstrated. Except for rigid diaphragms at the abutments no internal diaphragms were considered. The minimum distance between the edge girder and nearest truck wheels was taken as two feet. The stiffening effect of the curbs and parapets was ignored. The bridge spans considered were between 40 and 80 ft, the girder spacings between 6 and 9 ft and the angle of skew, $\alpha$, defined in Fig. 2.1, between 0 and 60 degrees. The typical bridge analysed is shown in Fig. 2.1.

The data from these analyses were used to determine the behaviour of a slab-and-girder bridge for different structural properties of the bridge members. The emphasis is on the maximum girder bending moments resulting from the distribution of truck loads. The present results for right bridges are compared with those according to the current (1985) AASHTO design recommendations. With the bridge behaviour known, the data from the analyses were interpreted to formulate a simple analysis procedure for right and skew slab-and-girder bridges. This analysis procedure can be used to obtain girder bending moments which are within 7% of the data obtained from the finite element analyses. Live load girder deflections and dead load girder bending moments resulting from the curbs and roadway resurfacing layers are also discussed.

No closed form exact solutions exist for skew slab-and-girder bridges with which results can be compared. Therefore, it was first necessary to determine if the nine-node Lagrangian-type shell element, which was used to model the skew bridge deck provided correct results when used in a skew configuration. Furthermore, it was necessary to perform a convergence study on a typical bridge to determine how much the mesh had to be refined to ensure reliable results. For the purpose of comparing results the finite element mesh selected was used to analyse slab-and-girder bridges.
for which solutions existed.

6.2. Conclusions

The most important conclusions drawn from this study are summarized below. They are grouped into the following categories: design criteria, behaviour of the bridge, method of structural analysis and errors that can be expected.

6.2.1. Conclusions Regarding Design Criteria

The method of analysis proposed in Chapter 5 can be used to determine the maximum girder bending moments in simply supported, right and skew slab-and-girder bridges subjected to two-lane truck loads. The design moments obtained in this way are within 7% of the moments obtained from the finite element analyses in this study.

If the wheel load fractions for girder bending moment are used to determine girder midspan deflections the results can underestimate the true deflections by 22%.

For the range of parameters considered in this study, the AASHTO wheel load fraction $b/5.5$ for interior girders gives results which are between 12% on the unsafe side and 32% too large. It is likely that the interior girder bending moments will be underestimated for bridges with a large $H$-value, a small span and a small girder spacing. The AASHTO method to determine the maximum exterior girder bending moment by assuming that the slab acts as if simply supported between girders underestimates the actual exterior girder bending moments in most of the bridges considered. It gives bending moments which are up to 23% too small. The AASHTO exterior girder wheel load fraction $b/(4 + b/4)$ for steel I-beams yields results which are between 30 and 60% too large.
6.2.2. Conclusions Regarding the Behaviour of the Bridge

The effect of skew is a reduction in the girder bending moments. The larger the angle of skew and the ratio $b/(aH)$, the larger the resulting reductions. The maximum interior girder bending moment reduction as a consequence of skew is always less than 5% for angles of skew up to 30 degrees, but the reduction is as large as 38% when $\alpha = 60$ degrees. The exterior girders are less affected by skew. The maximum exterior girder bending moment reduction as the consequence of skew is always less than 8% for angles of skew up to 45 degrees, but the reduction is as large as 25% when $\alpha = 60$ degrees. For all girders, the most significant reductions occur when the angle of skew is more than 45 degrees.

Because the exterior girders are less affected by skew than the interior girders there is a tendency for the edge girder to become the controlling girder in a skew bridge. This tendency is more pronounced in a bridge with a large angle of skew, a small $H$-value, a large span and a small girder spacing. However, by keeping the faces of the curbs directly above the edge girders, the maximum bending moment always occurs in an interior girder for spans up to 80 ft.

A study of a practical skew bridge in which the length of the slab overhang at the edge girders is increased from 19 to 39 inches shows that the resulting change in the edge girder bending stiffness has only a 3% effect on the maximum edge girder bending moment, while the interior girders are hardly affected at all. Although the length of the overhang is not important in itself, it does determine the location of the face of the curb, which is very important. A designer can successfully avoid having the controlling design moment in an edge girder by keeping the face of the curb directly above the edge girder. This applies only to bridges with spans up to 80 ft.

The results for five-girder bridges can conservatively be used for bridges which have more girders. This is true for both right and skew bridges. The differences in girder bending moments between bridges with different number of girders are smaller
when $H$ and $b$ become larger.

The maximum girder bending moments are insensitive to moderate changes in the girder torsional stiffness. The effect which an increase in the girder torsional stiffness has on the maximum girder bending moments resulting from truck loads is similar to the effect of increasing the slab thickness. When the girder torsional stiffness is reduced, the maximum bending moments in the interior girders increase. The effect of girder torsional stiffness becomes larger with increasing skew. Even for $\alpha = 60$ degrees the bending moment differences between girders with and without torsional stiffness are still in the order of five percent, which is small.

The presence of stiff end diaphragms can reduce the maximum bending moments in the interior girders of a skew bridge subjected to truck loads. This is especially noticeable in bridges with short spans, large angles of skew and small values for $H$. The edge girders are not significantly affected by the presence of end diaphragms.

In skew slab-and-girder bridges the point of maximum bending moment in the exterior and first interior girder can shift with as much as 6% of the span away from midspan. The bending moment envelope diagrams for all girders are noticeably flat in the region of maximum bending moment.

The maximum bending moment in the interior girders always increases when $H$ increases. The effect of a change in $H$ is larger when the $H$-value is small. However, the maximum interior girder bending moment is insensitive to moderate changes in $H$ which is fortunate because many uncertainties surround the true value of $H$. The edge girder bending moments are extremely insensitive to variations in $H$. An increase in $H$ normally results in a very small decrease in the edge girder maximum bending moment. The exception is when $\alpha = 60$ degrees when a small increase in the maximum exterior girder bending moment is possible if $H$ increases between $H = 5$ and $H \approx 15$. 
All maximum girder bending moments and midspan deflections due to truck loads increase when the ratio b/a is increased by varying the girder spacing, b. It turns out that this variation with the b/a ratio is almost linear. The maximum girder bending moments due to truck loads decrease when b/a is increased by varying the span, a.

6.2.3. Conclusions Regarding the Method of Structural Analysis

A nine-node Lagrangian-type isoparametric thin shell element behaves much better under skew distortion than a similar eight-node serendipity element. When a rectangular shell element which is used to model the skew deck is distorted into a parallelogram which fits into the skew network, the element becomes too stiff in bending as well as in membrane action. Element quality decreases with increasing skew and decreases rapidly for angles of skew larger than 40 degrees. However, sufficient accuracy can be maintained by refining the mesh. If the slab is modelled with finite elements which act too stiff, the girder bending moment results will be too small because the stiffer deck distributes the loads better than it should.

A convergence study on a typical slab-and-girder bridge shows that there is a limit to mesh refinement after which the increase in computational cost is not justified because it does not lead to more reliable results. Numerical problems may result when the girders in the bridge are very stiff compared to the bending stiffness of the slab.

Present solutions for right slab-and-girder bridges compare very well with an existing finite element solution by Mehrain (63) and with an exact solution for a non-composite bridge by Newmark (75). A present solution for a skew noncomposite five-girder bridge with $\alpha = 60$ degrees is in poor agreement with a finite difference solution by Chen (14). Differences as much as 42% exist. Chen used a very coarse finite difference grid. Certain selected important girder bottom fibre stresses in the present solution of a 40 degrees skew seven-girder bridge differ by as much as 7.5%
from a finite element solution by Powell (86). There is reason to accept the current analysis because the current shell element which is used to model the skew deck behaves much better in skewed configuration than the element used by Powell.

6.2.4. Conclusions Regarding Errors that can be Expected

Results from five right bridges subjected to truck loads and having member properties which cover a large range of bridges show that the use of the bending moment of inertia of a composite girder $I_{cg}$ to calculate the bottom fibre stress gives results which are less than 6% in error, which is quite acceptable. This is due to the approximation of shear lag in the slab by the use of an effective flange width. The contribution of the longitudinal bending moment $M_s$ in the flange of a composite girder to the total bending moment $M_{cg}$ acting on a composite girder can be ignored. Its inclusion does not ensure smaller stress errors when $I_{cg}$ is used to calculate girder bending stresses. The longitudinal bending moments in the slab $M_s$ are larger when $H$ becomes smaller. For the five right bridges considered, $M_s$ is less than 3.5% of $M_{cg}$.

Errors in bottom fibre stresses of girders calculated by using $I_{cg}$ are considerably larger when a bridge is subjected to a single point load instead of truck loads.

A comparison between two bridges with the same $H$-value, but with different girder properties and slab thicknesses shows that the maximum girder bending moments can differ by 2% when the bridge is subjected to truck loads. These small differences are due to the fact that $H$ depends on the effective flange width which approximates the effect of shear lag in the slab.
6.3. Recommendations for Further Research

Further research on skew slab-and-girder bridges is necessary, especially research with the goal to develop design criteria for aspects of bridge design not covered in this report. Designers still have the following questions regarding the design of skew slab-and-girder bridges:

1. How does skew affect the bending moments in the slab?
2. What are the magnitudes of the design forces in the diaphragms at the abutments?
3. How should the support bearing reactions be adjusted to compensate for skew?
4. Is torsion in the girders a problem at the obtuse corners of the bridge?
5. For what shear forces should the girders be designed?
6. Can the present analysis procedure be extended to cover continuous skew bridges?
7. Are internal transverse diaphragms worthwhile?
8. Are bridge-to-diaphragm connections effective?
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TABLES
Table 2.1  Properties of Supporting Girders Used in the Parameter Study

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<th>a</th>
<th>H</th>
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<th>$A_{sx}$</th>
<th>$J$</th>
<th>$I_{gx}$</th>
<th>$e$</th>
<th>$I_{cg}$</th>
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Notes:
1. PCI- = AASHTO-PCI standard prestressed concrete I-sections.
2. IL- = Illinois standard prestressed concrete I-sections.
Table 3.1  Element In-Plane Behaviour: Rectangular Cantilever Beam

<table>
<thead>
<tr>
<th>Element Name</th>
<th>Mesh No.</th>
<th>Total Number of Equations</th>
<th>Vertical Deflection at A (in)</th>
<th>Horizontal Stress at E (ksi)</th>
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Table 3.2  Element In-Plane Behaviour: Skew Cantilever Beam

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<th>Mesh 1 2X8</th>
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<th>Mesh 3 8X18</th>
<th>Mesh 4 8X24</th>
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Table 3.3  Deflection Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)

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<th>Deflection $\times 10^3$ (in) at Point</th>
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<td>G</td>
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<td>J</td>
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<td>J</td>
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<td>K</td>
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<td>Mesh 3</td>
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<tr>
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<tr>
<td>C</td>
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% Change Between Mesh 3 and Mesh 2

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<tr>
<td>C</td>
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<tr>
<td>H</td>
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% Change Between Mesh 2 and Mesh 1

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<td>J</td>
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Table 3.4  Girder Bending Moment Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)

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<th>Load Acting at Point</th>
<th>Bending Moments in Girders (in-lb) Near Point (Actual location is 10.778' left of ref. point)</th>
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Table 3.5  Girder Axial Force Convergence for $\alpha = 0$ degrees (Mesh 1,2,3)

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<th>Load Acting at Point</th>
<th>Axial Forces in Girders (Pounds) Near Point (Actual location is 10.778' left of ref. point)</th>
</tr>
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% Change

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<th>and</th>
<th>Mesh 1</th>
<th>J</th>
<th>K</th>
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Table 3.6 Deflection Convergence for $\alpha = 60$ degrees (Mesh 1,2,3)

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Table 3.7  Girder Bending Moment Convergence for $\alpha = 60$ degrees  
(Mesh 1,2,3)

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<td>E</td>
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<tr>
<td></td>
<td>F</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>K</td>
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<td></td>
<td>Mesh 2</td>
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% Change

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### Table 3.8 Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 1,2,3)

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<td>and</td>
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<tr>
<td>and</td>
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<tr>
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<tr>
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Table 3.9 Summary of the Maximum %–Change in Results Between Mesh 3 and Mesh 1,2

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<th>Mesh 2</th>
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<tr>
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<td>a = 60 deg.</td>
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**Notes:**
1. The values in brackets are the more realistic maximum differences.
Table 3.10  Deflection Convergence for $\alpha = 60$ degrees (Mesh 4,2,5)

<table>
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<th>Deflection x $10^3$ (in) at Point</th>
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Table 3.12  Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 4,2,5)

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<td></td>
<td>G</td>
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<td></td>
<td>H</td>
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<td>K</td>
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<td>J</td>
</tr>
<tr>
<td></td>
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Table 3.13 Summary of the Maximum %-Change in Results Between Mesh 5 and Mesh 2,4

### Deflections

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</tr>
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<td>Loads on Slab</td>
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### Bending Moments in Girders

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<td>Loads on Slab</td>
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### Axial Forces in Girders

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<td>Loads on Slab</td>
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<td>(1.8)</td>
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Notes:
1. The values in brackets are the more realistic maximum differences.
Table 3.14  Girder Bending Moment Convergence for $\alpha = 60$ degrees (Mesh 2,5)

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<tr>
<th>Case</th>
<th>Load Acting at $\alpha = 60$ deg.</th>
<th>Bending Moments in Girders (in-lb) Near Point (Actual location is 10.778' left of ref. point)</th>
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<td>185.4                                             39.98                                             6.502                                             .2030                                             177.6</td>
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<td>C</td>
<td>36.73                                             178.0                                             704.6                                             165.5                                             35.63                                             33.59</td>
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<tr>
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<td>F</td>
<td>291.3                                             114.9                                             22.19                                             3.681                                             .0121                                             544.1</td>
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<td><strong>Mesh 2</strong></td>
<td>G</td>
<td>94.61                                             209.5                                             154.8                                             32.26                                             4.265                                             117.0</td>
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<td>487.2                                             349.1                                             91.33                                             19.09                                             2.070                                             215.3</td>
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<td>58.38                                             250.6                                             478.6                                             121.1                                             22.84                                             53.19</td>
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<td>J</td>
<td>45.91                                             134.4                                             259.1                                             68.84                                             9.879                                             51.63</td>
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Table 3.15  Girder Axial Force Convergence for $\alpha = 60$ degrees (Mesh 2,5)

<table>
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<td>C</td>
</tr>
<tr>
<td></td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>J</td>
</tr>
<tr>
<td></td>
<td>K</td>
</tr>
</tbody>
</table>
Table 3.16 Example Problem: BRIDGE-2

<table>
<thead>
<tr>
<th>Load at Point</th>
<th>Displacements (in) at Point</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present Solution</td>
<td>Powell's GENDEK-5</td>
<td>Mehrain's COMDEK-CD</td>
</tr>
<tr>
<td>1</td>
<td>.0424</td>
<td>.0421</td>
<td>.0421</td>
</tr>
<tr>
<td>2</td>
<td>.0041</td>
<td>.0041</td>
<td>.0041</td>
</tr>
<tr>
<td>3</td>
<td>.0294</td>
<td>.0290</td>
<td>.0290</td>
</tr>
<tr>
<td>4</td>
<td>.0024</td>
<td>.0024</td>
<td>.0024</td>
</tr>
</tbody>
</table>

For Girder Bottom Fibre Stress (psi.) at Point 1:

<table>
<thead>
<tr>
<th>Load at Point</th>
<th>Girder Bottom Fibre Stress (psi.) at Point</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>155.4L (3.12)</td>
<td>150.7 (27.2)</td>
<td>149.7 (3.12)</td>
</tr>
<tr>
<td>2</td>
<td>4.762L (-26.7)</td>
<td>6.5 (1.91)</td>
<td>6.5 (1.91)</td>
</tr>
</tbody>
</table>

For Girder Bottom Fibre Stress (psi.) at Point 3:

<table>
<thead>
<tr>
<th>Load at Point</th>
<th>Girder Bottom Fibre Stress (psi.) at Point</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>208.4 (7.53)</td>
<td>193.8 (2.62)</td>
<td>193.7 (2.62)</td>
</tr>
<tr>
<td>4</td>
<td>3.393 (-.21)</td>
<td>3.4 (5.23)</td>
<td>3.3 (5.23)</td>
</tr>
</tbody>
</table>

Notes:
1. L = Just left of the diaphragm centroid at midspan.
2. R = Just right of the diaphragm centroid at midspan.
3. The values in brackets are the % difference from the GENDEK-5 solution.
### Table 3.17 Example Problem: BRIDGE-3

<table>
<thead>
<tr>
<th>Beam Name</th>
<th>Transverse Location of Load</th>
<th>Finite Difference Solution by Chen</th>
<th>Exact Solution by Newmark</th>
<th>Present Finite Element Solution x Exp-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.172</td>
<td>0.174</td>
<td>173.8</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>0.106</td>
<td>0.101</td>
<td>101.2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.056</td>
<td>0.055</td>
<td>54.93</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>0.029</td>
<td>0.028</td>
<td>28.11</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.013</td>
<td>0.013</td>
<td>12.64</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>0.004</td>
<td>0.004</td>
<td>3.757</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.903</td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-3.120</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-4.245</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.056</td>
<td>0.055</td>
<td>54.92</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>0.087</td>
<td>0.084</td>
<td>83.88</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.111</td>
<td>0.112</td>
<td>112.4</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>0.081</td>
<td>0.078</td>
<td>77.76</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.047</td>
<td>0.047</td>
<td>47.14</td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>0.029</td>
<td>0.029</td>
<td>28.62</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>0.016</td>
<td>0.016</td>
<td>16.33</td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td>0.007</td>
<td>0.007</td>
<td>6.953</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.903</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.013</td>
<td>0.013</td>
<td>12.64</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>0.028</td>
<td>0.028</td>
<td>27.90</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.047</td>
<td>0.047</td>
<td>47.14</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
1. The bending moments in the beams are obtained by multiplying the listed coefficient with the value of the load times the span.
Table 3.18  Example Problem: BRIDGE-4

<table>
<thead>
<tr>
<th>Beam Name</th>
<th>Transverse Location of Load</th>
<th>Finite Solution Difference</th>
<th>Present Finite Element Solution ( \times \exp(-3) )</th>
<th>Approximate Percentage Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>0.188</td>
<td>190.6</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>AB</td>
<td>0.084</td>
<td>68.72</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.032</td>
<td>28.86</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.016</td>
<td>15.16</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>AC</td>
<td>0.009</td>
<td>7.938</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>0.006</td>
<td>4.433</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.003</td>
<td>2.448</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DE</td>
<td>0.002</td>
<td>1.263</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>0.001</td>
<td>.4447</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>0.030</td>
<td>24.70</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>AB</td>
<td>0.060</td>
<td>43.81</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.100</td>
<td>109.2</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.060</td>
<td>43.09</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.028</td>
<td>23.33</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>0.016</td>
<td>13.41</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>0.010</td>
<td>7.568</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>DE</td>
<td>0.006</td>
<td>4.405</td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>0.003</td>
<td>2.469</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>0.008</td>
<td>7.741</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>AB</td>
<td>0.014</td>
<td>13.52</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.027</td>
<td>22.93</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>0.058</td>
<td>40.75</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.097</td>
<td>104.5</td>
<td>-7</td>
</tr>
</tbody>
</table>

Notes:
1. The bending moments in the beams are obtained by multiplying the listed coefficient with the value of the load times the span.
Table 4.1  Errors in the Bottom Fibre Stresses in Supporting Girders which Result from the use of the Effective Flange Width Concept

<table>
<thead>
<tr>
<th>Definition of the Bridge</th>
<th>Correct Bottom Fibre Stress (ksi)</th>
<th>Approximate Stress using $I_{cg}$ and the Total Moment $M_{cg}$ Acting on the Composite T-Section (ksi)</th>
<th>Error %</th>
<th>Approximate Stress using $I_{cg}$ but Ignoring the Contribution of the Slab Moment to $M_s/M_{cg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = 6 A</td>
<td>0.4262</td>
<td>0.4162</td>
<td>-2.3</td>
<td>0.4260</td>
</tr>
<tr>
<td>a = 60 B</td>
<td>0.4822</td>
<td>0.4694</td>
<td>-2.7</td>
<td>0.4839</td>
</tr>
<tr>
<td>H = 5 C</td>
<td>0.4819</td>
<td>0.4744</td>
<td>-1.6</td>
<td>0.4907</td>
</tr>
<tr>
<td>b = 6 A</td>
<td>0.1524</td>
<td>0.1432</td>
<td>-6.0</td>
<td>0.1444</td>
</tr>
<tr>
<td>a = 60 B</td>
<td>0.2075</td>
<td>0.2079</td>
<td>0.2</td>
<td>0.2096</td>
</tr>
<tr>
<td>H = 30 C</td>
<td>0.2219</td>
<td>0.2287</td>
<td>3.1</td>
<td>0.2310</td>
</tr>
<tr>
<td>b = 6 A</td>
<td>0.4222</td>
<td>0.4170</td>
<td>-1.2</td>
<td>0.4241</td>
</tr>
<tr>
<td>a = 80 B</td>
<td>0.4465</td>
<td>0.4362</td>
<td>-2.3</td>
<td>0.4462</td>
</tr>
<tr>
<td>H = 5 C</td>
<td>0.4341</td>
<td>0.4281</td>
<td>-1.4</td>
<td>0.4394</td>
</tr>
<tr>
<td>b = 6 A</td>
<td>0.1680</td>
<td>0.1604</td>
<td>-4.5</td>
<td>0.1614</td>
</tr>
<tr>
<td>a = 80 B</td>
<td>0.2118</td>
<td>0.2125</td>
<td>0.3</td>
<td>0.2139</td>
</tr>
<tr>
<td>H = 30 C</td>
<td>0.2209</td>
<td>0.2275</td>
<td>3.0</td>
<td>0.2293</td>
</tr>
<tr>
<td>b = 6 A</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>a = 80 B</td>
<td>0.5005</td>
<td>0.4942</td>
<td>-1.3</td>
<td>0.5008</td>
</tr>
<tr>
<td>H = 10 C</td>
<td>0.5012</td>
<td>0.5028</td>
<td>0.3</td>
<td>0.5107</td>
</tr>
</tbody>
</table>

Notes:
1. See Table 2.1 for the structural properties of the girders.
2. A, B and C refer to the edge, second and centre girder as shown in Fig. 2.1
3. Each bridge is loaded with two HS20-44 trucks such that the maximum bending moment results in the girder under consideration.
4. The value of the wheel load $P = 10$ kips.
5. The angle of skew is zero.
6. The values of $I_{cg}$ used to calculate stresses in the exterior girders are smaller than the values used for the interior girders. The size of the deck overhang is taken into account.
Table 4.2  Percentage Girder Bending Moment Differences Obtained from Three Bridges with the same H and b/a Ratios
Loading Condition: A Single Point Load

<table>
<thead>
<tr>
<th>Girder Under Consideration</th>
<th>Load Applied at Midspan of Girder</th>
<th>% Difference in Maximum Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.6</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>6.6</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.7</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 4.3  Percentage Girder Bending Moment Differences Obtained from Two Bridges with the same H and b/a Ratios
Loading Condition: Two AASHTO HS20-44 Trucks

<table>
<thead>
<tr>
<th>Girder under Consideration</th>
<th>Trucks Located for Large Moments in Girder C</th>
<th>Trucks Located for Large Moments in Girders A, B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>1.0</td>
</tr>
<tr>
<td>B</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>C</td>
<td>1.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Table 4.4  Effect of an Increase in the Number of Girders on the Girder Moments

Maximum Bending Moment Coefficients for two HS20-44 Truck Loads

<table>
<thead>
<tr>
<th>Bridge Properties</th>
<th>Five-Girder Bridge</th>
<th>Five-Girder Bridge</th>
<th>%-%CH Bridge</th>
<th>Seven-Girder Bridge</th>
<th>Seven-Girder Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td>b(ft) H Girder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 5</td>
<td>A 0.2987</td>
<td>0.2956</td>
<td>-1.0</td>
<td>0.2948</td>
<td>-1.3</td>
</tr>
<tr>
<td></td>
<td>B 0.3001</td>
<td>0.2918</td>
<td>-2.8</td>
<td>0.2910</td>
<td>-3.0</td>
</tr>
<tr>
<td></td>
<td>C 0.3040</td>
<td>0.2892</td>
<td>-4.9</td>
<td>0.2848</td>
<td>-6.3</td>
</tr>
<tr>
<td>6 30</td>
<td>A 0.2792</td>
<td>0.2791</td>
<td>&lt;0.0</td>
<td>0.2791</td>
<td>&lt;0.0</td>
</tr>
<tr>
<td></td>
<td>B 0.3819</td>
<td>0.3799</td>
<td>-0.5</td>
<td>0.3798</td>
<td>-0.6</td>
</tr>
<tr>
<td></td>
<td>C 0.4185</td>
<td>0.4111</td>
<td>-1.8</td>
<td>0.4103</td>
<td>-2.0</td>
</tr>
<tr>
<td>9 5</td>
<td>A 0.3660</td>
<td>0.3656</td>
<td>-0.1</td>
<td>0.3655</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>B 0.3831</td>
<td>0.3797</td>
<td>-0.9</td>
<td>0.3795</td>
<td>-0.9</td>
</tr>
<tr>
<td></td>
<td>C 0.3544</td>
<td>0.3474</td>
<td>-2.0</td>
<td>0.3464</td>
<td>-2.3</td>
</tr>
<tr>
<td>9 30</td>
<td>A 0.3658</td>
<td>0.3658</td>
<td>&lt;0.0</td>
<td>0.3658</td>
<td>&lt;0.0</td>
</tr>
<tr>
<td></td>
<td>B 0.5742</td>
<td>0.5742</td>
<td>&lt;0.0</td>
<td>0.5742</td>
<td>&lt;0.0</td>
</tr>
<tr>
<td></td>
<td>C 0.5547</td>
<td>0.5526</td>
<td>-0.4</td>
<td>0.5526</td>
<td>&lt;0.0</td>
</tr>
</tbody>
</table>

Notes:
1. Angle of skew $a = 60$ degrees.
2. Span $a = 60$ ft.
3. %-%Change = difference with respect to the five-girder bridge results.
4. $M_{cg} = (coefficient) \times Pa$
Table 4.5  Effect of Girder Torsional Stiffness on the Girder Bending Moments (1)

Maximum Girder Bending Moment Coefficients for two HS20-44 Trucks

<table>
<thead>
<tr>
<th>Angle of Skew</th>
<th>Girder A %-CH</th>
<th>Girder B %-CH</th>
<th>Girder C %-CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.240</td>
<td>0.364</td>
<td>0.399</td>
</tr>
<tr>
<td>+2.2</td>
<td>0.364</td>
<td>0.372</td>
<td>0.400</td>
</tr>
<tr>
<td>0.236</td>
<td>0.372</td>
<td>0.373</td>
<td>0.400</td>
</tr>
<tr>
<td>0.243</td>
<td>0.373</td>
<td>0.377</td>
<td>0.400</td>
</tr>
<tr>
<td>0.230</td>
<td>0.377</td>
<td>0.382</td>
<td>0.402</td>
</tr>
</tbody>
</table>

Notes:
1. The girder spacing $b = 6$ ft. $H = 10$.
2. The bridge span for the first group of data is $a = 40$ ft.
   The bridge span for the second group of data is $a = 80$ ft.
3. $^* = $ Girder torsional stiffness is increased by 47% for this case.
4. $^+= $ Change is the percentage difference between results.
5. $M_{cg} = (\text{coefficient}) \times \text{Pa}$
Table 4.6  Effect of Girder Torsional Stiffness on the Girder Bending Moments (2)

Maximum Girder Bending Moment Coefficients for two HS20-44 Trucks

<table>
<thead>
<tr>
<th>Bridge Properties</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With Torsion</td>
<td>No Torsion</td>
<td>With Torsion</td>
</tr>
<tr>
<td>a = 0 A</td>
<td>.4053</td>
<td>.4218</td>
<td>.0950</td>
</tr>
<tr>
<td>H = 10 B</td>
<td>.3344</td>
<td>.3422</td>
<td>.2818</td>
</tr>
<tr>
<td>H = 20 B</td>
<td>.3521</td>
<td>.3849</td>
<td>.0752</td>
</tr>
<tr>
<td>H = 40 B</td>
<td>.3075</td>
<td>.3194</td>
<td>.2530</td>
</tr>
<tr>
<td>a = 80 C</td>
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<td>.3135</td>
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</tr>
<tr>
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<td>.2906</td>
<td>.2664</td>
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<tr>
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<td>.3029</td>
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<tr>
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<td>.3123</td>
<td>.2469</td>
</tr>
<tr>
<td>a = 80 C</td>
<td>.2727</td>
<td>.2780</td>
<td>.2727</td>
</tr>
</tbody>
</table>

Notes:
1. The girder spacing b = 9 ft.
2. The % Change is the difference in the total girder bending moments.
3. \( M_{cg} = \text{(coefficient)} \times \text{Pa} \)
Table 4.7  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew \( \alpha = 0 \) degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.3953</td>
<td>0.1271</td>
<td>0.522</td>
<td>0.0606</td>
</tr>
<tr>
<td></td>
<td>B</td>
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<td>0.2547</td>
<td>0.564</td>
<td>0.0572</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2639</td>
<td>0.2639</td>
<td>0.528</td>
<td>0.0501</td>
</tr>
<tr>
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<td>A</td>
<td>0.4053</td>
<td>0.0950</td>
<td>0.500</td>
<td>0.0598</td>
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<td>B</td>
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</tr>
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<td>C</td>
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<td>0.2993</td>
<td>0.599</td>
<td>0.0546</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
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<td>0.0649</td>
<td>0.472</td>
<td>0.0592</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3639</td>
<td>0.3065</td>
<td>0.670</td>
<td>0.0648</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.3301</td>
<td>0.3301</td>
<td>0.660</td>
<td>0.0588</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.4095</td>
<td>0.0505</td>
<td>0.460</td>
<td>0.0580</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3815</td>
<td>0.3179</td>
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</tr>
<tr>
<td></td>
<td>C</td>
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<td>0.3452</td>
<td>0.690</td>
<td>0.0618</td>
</tr>
</tbody>
</table>

Table 4.8  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew \( \alpha = 30 \) degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
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<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.3825</td>
<td>0.1251</td>
<td>0.508</td>
<td>0.0586</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2971</td>
<td>0.2458</td>
<td>0.543</td>
<td>0.0546</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2547</td>
<td>0.2546</td>
<td>0.509</td>
<td>0.0478</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>0.3953</td>
<td>0.0961</td>
<td>0.491</td>
<td>0.0583</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3246</td>
<td>0.2744</td>
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<td>0.0587</td>
</tr>
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</tr>
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<td>0.0671</td>
<td>0.468</td>
<td>0.0583</td>
</tr>
<tr>
<td></td>
<td>B</td>
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<td>0.3002</td>
<td>0.657</td>
<td>0.0632</td>
</tr>
<tr>
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<td>C</td>
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<td>0.3222</td>
<td>0.645</td>
<td>0.0570</td>
</tr>
<tr>
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<td>0.0531</td>
<td>0.457</td>
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</tr>
<tr>
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<td>B</td>
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<td>0.3124</td>
<td>0.687</td>
<td>0.0659</td>
</tr>
<tr>
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<td>C</td>
<td>0.3383</td>
<td>0.3383</td>
<td>0.677</td>
<td>0.0605</td>
</tr>
</tbody>
</table>

Notes:
1. \( M \) = (coefficient) \( \times \) Pa
2. Deflection = (coefficient) \( \times \) \( \frac{\text{Pa}^3}{E \cdot I_{cg}} \)
Table 4.9  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew \( \alpha = 45 \) degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.3629</td>
<td>0.1210</td>
<td>0.484</td>
<td>0.0548</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2811</td>
<td>0.2269</td>
<td>0.508</td>
<td>0.0500</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2388</td>
<td>0.2367</td>
<td>0.475</td>
<td>0.0439</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>0.3849</td>
<td>0.0932</td>
<td>0.478</td>
<td>0.0565</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3115</td>
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<td>0.569</td>
<td>0.0550</td>
</tr>
<tr>
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<td>C</td>
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<td>0.2754</td>
<td>0.553</td>
<td>0.0499</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
<td>0.3867</td>
<td>0.0711</td>
<td>0.458</td>
<td>0.0564</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3421</td>
<td>0.2875</td>
<td>0.630</td>
<td>0.0601</td>
</tr>
<tr>
<td></td>
<td>C</td>
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<td>0.615</td>
<td>0.0547</td>
</tr>
<tr>
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<td>A</td>
<td>0.3921</td>
<td>0.0579</td>
<td>0.450</td>
<td>0.0557</td>
</tr>
<tr>
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<td>B</td>
<td>0.3626</td>
<td>0.3010</td>
<td>0.664</td>
<td>0.0632</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.3268</td>
<td>0.3239</td>
<td>0.651</td>
<td>0.0583</td>
</tr>
</tbody>
</table>

Table 4.10  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 9 ft; Angle of Skew \( \alpha = 60 \) degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.3222</td>
<td>0.1050</td>
<td>0.427</td>
<td>0.0482</td>
</tr>
<tr>
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<td>B</td>
<td>0.2486</td>
<td>0.1775</td>
<td>0.426</td>
<td>0.0396</td>
</tr>
<tr>
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<td>C</td>
<td>0.1980</td>
<td>0.1933</td>
<td>0.391</td>
<td>0.0342</td>
</tr>
<tr>
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<td>A</td>
<td>0.3401</td>
<td>0.0926</td>
<td>0.433</td>
<td>0.0500</td>
</tr>
<tr>
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<td>B</td>
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<td>0.2068</td>
<td>0.488</td>
<td>0.0456</td>
</tr>
<tr>
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<td>0.2300</td>
<td>0.466</td>
<td>0.0407</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
<td>0.3521</td>
<td>0.0752</td>
<td>0.427</td>
<td>0.0511</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3047</td>
<td>0.2530</td>
<td>0.558</td>
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</tr>
<tr>
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<td>C</td>
<td>0.2729</td>
<td>0.2664</td>
<td>0.539</td>
<td>0.0475</td>
</tr>
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<td>0.3620</td>
<td>0.0653</td>
<td>0.427</td>
<td>0.0510</td>
</tr>
<tr>
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<td>B</td>
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<td>0.2693</td>
<td>0.598</td>
<td>0.0561</td>
</tr>
<tr>
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<td>C</td>
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<td>0.2868</td>
<td>0.580</td>
<td>0.0519</td>
</tr>
</tbody>
</table>

Notes:
1. \( M_{cg} = (\text{coefficient}) \times Pa \)
2. Deflection = (coefficient) \( \times \frac{Pa^3}{E I_{cg}} \)
Table 4.11  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha$ = 0 degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A 0.3693</td>
<td>0.0894</td>
<td>0.459</td>
<td>0.0554</td>
<td></td>
</tr>
<tr>
<td>B 0.2985</td>
<td>0.2510</td>
<td>0.549</td>
<td>0.0579</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C 0.2656</td>
<td>0.2656</td>
<td>0.531</td>
<td>0.0533</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>A 0.3778</td>
<td>0.0565</td>
<td>0.434</td>
<td>0.0538</td>
<td></td>
</tr>
<tr>
<td>B 0.3304</td>
<td>0.2806</td>
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<td></td>
</tr>
<tr>
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<td>0.3017</td>
<td>0.603</td>
<td>0.0599</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>A 0.3783</td>
<td>0.0303</td>
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<td>0.0525</td>
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</tr>
<tr>
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<td>0.0656</td>
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</tr>
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<tr>
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<td>0.0698</td>
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</tr>
<tr>
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<td>0.3400</td>
<td>0.680</td>
<td>0.0656</td>
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</table>

Table 4.12  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha$ = 30 degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
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<td>A 0.3562</td>
<td>0.0901</td>
<td>0.446</td>
<td>0.0537</td>
<td></td>
</tr>
<tr>
<td>B 0.2860</td>
<td>0.2408</td>
<td>0.527</td>
<td>0.0549</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C 0.2533</td>
<td>0.2524</td>
<td>0.506</td>
<td>0.0503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>A 0.3687</td>
<td>0.0600</td>
<td>0.429</td>
<td>0.0543</td>
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</tr>
<tr>
<td>B 0.3210</td>
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</tr>
<tr>
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<td>0.0575</td>
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<td></td>
</tr>
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</tr>
<tr>
<td>B 0.3568</td>
<td>0.2958</td>
<td>0.653</td>
<td>0.0673</td>
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<td></td>
</tr>
<tr>
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</tr>
<tr>
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<tr>
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<td>0.3328</td>
<td>0.666</td>
<td>0.0642</td>
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<td></td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = $ (coefficient) x $P_a$
2. Deflection = (coefficient) x $P_a^3/(E g_{cg})$
### Table 4.13 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 45$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
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<td>5</td>
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<td>0.3360</td>
<td>0.0882</td>
<td>0.424</td>
<td>0.0508</td>
</tr>
<tr>
<td></td>
<td>B</td>
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<td>0.2204</td>
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<td>0.0495</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2315</td>
<td>0.2281</td>
<td>0.460</td>
<td>0.0451</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>0.3518</td>
<td>0.0651</td>
<td>0.417</td>
<td>0.0522</td>
</tr>
<tr>
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<td>0.3011</td>
<td>0.2542</td>
<td>0.555</td>
<td>0.0569</td>
</tr>
<tr>
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<td>C</td>
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<td>0.2672</td>
<td>0.538</td>
<td>0.0531</td>
</tr>
<tr>
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<td>0.0426</td>
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<tr>
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<td>0.2808</td>
<td>0.621</td>
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</tr>
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<td>C</td>
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<td>0.3017</td>
<td>0.607</td>
<td>0.0603</td>
</tr>
<tr>
<td>30</td>
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<td>0.3629</td>
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<td>0.3163</td>
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<td>0.0615</td>
</tr>
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</table>

### Table 4.14 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.2903</td>
<td>0.0757</td>
<td>0.366</td>
<td>0.0430</td>
</tr>
<tr>
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<td>0.2264</td>
<td>0.1567</td>
<td>0.383</td>
<td>0.0368</td>
</tr>
<tr>
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<td>C</td>
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<td>0.1744</td>
<td>0.354</td>
<td>0.0330</td>
</tr>
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<td>0.0664</td>
<td>0.378</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.366</td>
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</tr>
<tr>
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<td>B</td>
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<td>0.0562</td>
</tr>
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</tr>
</tbody>
</table>

**Notes:**
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. $\text{Deflection} = (\text{coefficient}) \times \frac{Pa^3}{E I_{cg}}$
Table 4.15  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
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<td>Truck 2</td>
<td>Total Moment</td>
<td>Deflection</td>
</tr>
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<td>0.0438</td>
</tr>
<tr>
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<td>0.0553</td>
</tr>
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</tr>
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<td>0.0428</td>
</tr>
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</tr>
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<td>0.0603</td>
</tr>
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<td>0.0678</td>
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<td>0.3031</td>
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<td>0.0678</td>
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</table>

Table 4.16  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 30$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>Truck 1</td>
<td>Truck 2</td>
<td>Total Moment</td>
<td>Deflection</td>
</tr>
<tr>
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<td>0.0465</td>
<td>0.335</td>
<td>0.0422</td>
</tr>
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</tr>
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<td>0.2292</td>
<td>0.457</td>
<td>0.0488</td>
</tr>
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<td>0.321</td>
<td>0.0420</td>
</tr>
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<td>0.2388</td>
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<td>0.0583</td>
</tr>
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<td>0.2617</td>
<td>0.524</td>
<td>0.0567</td>
</tr>
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<td>0.0098</td>
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</tr>
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<td>0.0417</td>
</tr>
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<td>0.0668</td>
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<td>C</td>
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<td>0.2973</td>
<td>0.594</td>
<td>0.0653</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = $ (coefficient) x $Pa$
2. Deflection = (coefficient) x $Pa^3/(E\ I_{cg})$
3. * = Negative moment occurs.
Table 4.17  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 45$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
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<td>5</td>
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<td>0.0477</td>
<td>0.316</td>
<td>0.0393</td>
</tr>
<tr>
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<td>B</td>
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<td>0.1910</td>
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<td>0.0449</td>
</tr>
<tr>
<td></td>
<td>C</td>
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<td>0.2019</td>
<td>0.407</td>
<td>0.0427</td>
</tr>
<tr>
<td>10</td>
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<td>0.2808</td>
<td>0.0317</td>
<td>0.312</td>
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</tr>
<tr>
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<td>0.0533</td>
</tr>
<tr>
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<td>C</td>
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<td>0.2394</td>
<td>0.482</td>
<td>0.0515</td>
</tr>
<tr>
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<td>A</td>
<td>0.2858</td>
<td>0.0175</td>
<td>0.303</td>
<td>0.0415</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3152</td>
<td>0.2394</td>
<td>0.555</td>
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</tr>
<tr>
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</tr>
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Table 4.18  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 9 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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<th>Truck 1</th>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
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<tbody>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.1427</td>
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</tr>
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<td>0.0319</td>
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</tr>
<tr>
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<td>0.1981</td>
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</tbody>
</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. $\text{Deflection} = (\text{coefficient}) \times \frac{Pa^3}{E I_{cg}}$
Table 4.19  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 6 ft; Angle of Skew \( \alpha = 0 \) degrees.

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<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>-</td>
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<td>0.0402</td>
</tr>
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<td>0.0424</td>
</tr>
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</tr>
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Table 4.20  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 6 ft; Angle of Skew \( \alpha = 30 \) degrees.

<table>
<thead>
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<th>Deflection</th>
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<td>0.0392</td>
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<td>0.2217</td>
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<td>0.2468</td>
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</table>

Notes:
1. \( M_{cg} = (\text{coefficient}) \times Pa \)
2. Deflection = (coefficient) \( \times \frac{Pa^3}{E_{cg}I_{cg}} \)
Table 4.21  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 45$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
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<td>0.375</td>
<td>0.0406</td>
</tr>
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<td>0.1421</td>
<td>0.395</td>
<td>0.0395</td>
</tr>
<tr>
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<td>C</td>
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<td>0.1920</td>
<td>0.387</td>
<td>0.0369</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>0.3053</td>
<td>0.0590</td>
<td>0.364</td>
<td>0.0409</td>
</tr>
<tr>
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</tr>
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<td>0.0369</td>
<td>0.348</td>
<td>0.0405</td>
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<tr>
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</table>

Table 4.22  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
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<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
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<td>0.0801</td>
<td>0.346</td>
<td>0.0370</td>
</tr>
<tr>
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<td>B</td>
<td>0.2199</td>
<td>0.1223</td>
<td>0.342</td>
<td>0.0335</td>
</tr>
<tr>
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<td>0.0369</td>
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</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{(E\ I_{cg})}$
Table 4.23 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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<th>Truck 1</th>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-</td>
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<td>0.0397</td>
</tr>
<tr>
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<td>B</td>
<td>-</td>
<td>-</td>
<td>0.401</td>
<td>0.0420</td>
</tr>
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<td>C</td>
<td>-</td>
<td>-</td>
<td>0.405</td>
<td>0.0404</td>
</tr>
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<td>0.0389</td>
</tr>
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<td>0.0440</td>
</tr>
<tr>
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<td>C</td>
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<td>-</td>
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<td>0.0436</td>
</tr>
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<td>A</td>
<td>-</td>
<td>-</td>
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</tr>
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<td>0.0455</td>
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<td>-</td>
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Table 4.24 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 30$ degrees.

<table>
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<tr>
<th>H</th>
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<th>Truck 1</th>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
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<td>0.340</td>
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</tr>
<tr>
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</tr>
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<td>0.1972</td>
<td>0.394</td>
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<td>0.321</td>
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<tr>
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</tr>
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<td>B</td>
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<td>0.0447</td>
</tr>
<tr>
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<td>C</td>
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<td>0.2413</td>
<td>0.482</td>
<td>0.0452</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{(E \ I_{cg})}$
Table 4.25 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 45$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
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<td>A</td>
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<td>0.0564</td>
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<td>0.0378</td>
</tr>
<tr>
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<td>B</td>
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<td>0.1265</td>
<td>0.365</td>
<td>0.0377</td>
</tr>
<tr>
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<td>0.1835</td>
<td>0.370</td>
<td>0.0361</td>
</tr>
<tr>
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<td>0.0358</td>
<td>0.318</td>
<td>0.0378</td>
</tr>
<tr>
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<td>0.1275</td>
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<td>0.0405</td>
</tr>
<tr>
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<td>0.2053</td>
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<td>0.0398</td>
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<tr>
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<td>0.0567</td>
<td>0.299</td>
<td>0.0332</td>
</tr>
<tr>
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<td>B</td>
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<td>0.1011</td>
<td>0.300</td>
<td>0.0302</td>
</tr>
<tr>
<td></td>
<td>C</td>
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<td>0.1497</td>
<td>0.304</td>
<td>0.0282</td>
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Table 4.26 Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 60 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
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<tr>
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<th>Deflection</th>
</tr>
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<tbody>
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<td>0.0332</td>
</tr>
<tr>
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<td>B</td>
<td>0.1990</td>
<td>0.1011</td>
<td>0.300</td>
<td>0.0302</td>
</tr>
<tr>
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<td>C</td>
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<td>0.1497</td>
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<td>0.1979</td>
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<td>0.0387</td>
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</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{E_{cg}I_{cg}}$
Table 4.27  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
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<th>H</th>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
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<td>--</td>
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<td>0.0381</td>
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<td>--</td>
<td>--</td>
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<td>0.0453</td>
</tr>
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</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{E I_{cg}}$
3. * = Negative moment occurs.

Table 4.28  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 30$ degrees.

<table>
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<tr>
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<th>Total Moment</th>
<th>Deflection</th>
</tr>
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</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{E I_{cg}}$
3. * = Negative moment occurs.
Table 4.29  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 45$ degrees.

<table>
<thead>
<tr>
<th>H</th>
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<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
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<td>0.0361</td>
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<td>0.0729</td>
<td>0.364</td>
<td>0.0399</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2083</td>
<td>0.2071</td>
<td>0.415</td>
<td>0.0427</td>
</tr>
</tbody>
</table>

Table 4.30  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.1892</td>
<td>0.0271</td>
<td>0.216</td>
<td>0.0245</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.1579</td>
<td>0.0708</td>
<td>0.229</td>
<td>0.0232</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1265</td>
<td>0.1217</td>
<td>0.248</td>
<td>0.0234</td>
</tr>
<tr>
<td>10</td>
<td>A</td>
<td>0.2033</td>
<td>0.0210</td>
<td>0.224</td>
<td>0.0264</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.1963</td>
<td>0.0777</td>
<td>0.274</td>
<td>0.0283</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1529</td>
<td>0.1475</td>
<td>0.300</td>
<td>0.0295</td>
</tr>
<tr>
<td>20</td>
<td>A</td>
<td>0.2079</td>
<td>0.0131</td>
<td>0.221</td>
<td>0.0269</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2333</td>
<td>0.0746</td>
<td>0.308</td>
<td>0.0324</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1752</td>
<td>0.1704</td>
<td>0.346</td>
<td>0.0350</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.2090</td>
<td>0.0090</td>
<td>0.218</td>
<td>0.0272</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2552</td>
<td>0.0687</td>
<td>0.324</td>
<td>0.0342</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1869</td>
<td>0.1825</td>
<td>0.369</td>
<td>0.0374</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = \text{(coefficient)} \times Pa$
2. Deflection = (coefficient) $\times \frac{Pa^3}{E\gamma_{cg}}$
### Table 4.31  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 7.5 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.3596</td>
<td>0.1031</td>
<td>0.463</td>
<td>0.0530</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2987</td>
<td>0.2034</td>
<td>0.502</td>
<td>0.0508</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2352</td>
<td>0.2352</td>
<td>0.470</td>
<td>0.0449</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.3653</td>
<td>0.0358</td>
<td>0.401</td>
<td>0.0502</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3669</td>
<td>0.2254</td>
<td>0.592</td>
<td>0.0572</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2994</td>
<td>0.2994</td>
<td>0.599</td>
<td>0.0535</td>
</tr>
</tbody>
</table>

### Table 4.32  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 80 ft; Girder Spacing = 7.5 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.2962</td>
<td>0.0926</td>
<td>0.389</td>
<td>0.0429</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2462</td>
<td>0.1505</td>
<td>0.387</td>
<td>0.0370</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1848</td>
<td>0.1804</td>
<td>0.365</td>
<td>0.0322</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.3269</td>
<td>0.0511</td>
<td>0.378</td>
<td>0.0443</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3185</td>
<td>0.1923</td>
<td>0.511</td>
<td>0.0483</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2595</td>
<td>0.2534</td>
<td>0.513</td>
<td>0.0453</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. $\text{Deflection} = (\text{coefficient}) \times \frac{Pa^3}{(E I_{cg})}$
Table 4.33  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6.75 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.2569</td>
<td>0.0238</td>
<td>0.281</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2542</td>
<td>0.1340</td>
<td>0.388</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1990</td>
<td>0.1990</td>
<td>0.398</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.34  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 7.5 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.2723</td>
<td>0.0296</td>
<td>0.302</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2652</td>
<td>0.1642</td>
<td>0.429</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2146</td>
<td>0.2146</td>
<td>0.429</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.2646</td>
<td>*</td>
<td>0.265</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.3548</td>
<td>0.1587</td>
<td>0.514</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2598</td>
<td>0.2598</td>
<td>0.520</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.35  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 8.25 ft; Angle of Skew $\alpha = 0$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>0.2860</td>
<td>0.0373</td>
<td>0.323</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2710</td>
<td>0.1937</td>
<td>0.465</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2285</td>
<td>0.2285</td>
<td>0.457</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = (\text{coefficient}) \times Pa$
2. Deflection = (coefficient) $\times Pa^3/(E \times I_{cg})$
3. * = Negative moment occurs.
Table 4.36  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 6.75 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>0.1985</td>
<td>0.0296</td>
<td>0.228</td>
<td>0.0264</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>0.1725</td>
<td>0.0812</td>
<td>0.254</td>
<td>0.0251</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1319</td>
<td>0.1260</td>
<td>0.238</td>
<td>0.0247</td>
</tr>
</tbody>
</table>

Table 4.37  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 7.5 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>0.2083</td>
<td>0.0321</td>
<td>0.240</td>
<td>0.0283</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>0.1771</td>
<td>0.1000</td>
<td>0.277</td>
<td>0.0272</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1385</td>
<td>0.1323</td>
<td>0.271</td>
<td>0.0262</td>
</tr>
<tr>
<td>30</td>
<td>A</td>
<td>0.2358</td>
<td>0.0146</td>
<td>0.250</td>
<td>0.0318</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.2927</td>
<td>0.1348</td>
<td>0.427</td>
<td>0.0448</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.2173</td>
<td>0.2123</td>
<td>0.430</td>
<td>0.0450</td>
</tr>
</tbody>
</table>

Table 4.38  Maximum Composite Girder Bending Moment and Deflection Coefficients: Span = 40 ft; Girder Spacing = 8.25 ft; Angle of Skew $\alpha = 60$ degrees.

<table>
<thead>
<tr>
<th>H</th>
<th>Girder</th>
<th>Truck 1</th>
<th>Truck 2</th>
<th>Total Moment</th>
<th>Deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>0.2150</td>
<td>0.0335</td>
<td>0.249</td>
<td>0.0296</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>0.1767</td>
<td>0.1175</td>
<td>0.294</td>
<td>0.0289</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.1446</td>
<td>0.1375</td>
<td>0.282</td>
<td>0.0274</td>
</tr>
</tbody>
</table>

Notes:
1. $M_{cg} = \text{coefficient} \times Pa$
2. $\text{Deflection} = \text{coefficient} \times \frac{Pa^3}{(E \cdot I_{cg})}$
### Table 5.1  Maximum Girder Bending Moments $M_{cg}$ for Dead Load: Curbs and Parapets

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>$H$</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 30$</th>
<th>$\alpha = 45$</th>
<th>$\alpha = 60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>9/80</td>
<td>5</td>
<td>0.295</td>
<td>0.153</td>
<td>0.096</td>
<td>0.291</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.335</td>
<td>0.135</td>
<td>0.057</td>
<td>0.328</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.369</td>
<td>0.118</td>
<td>0.025</td>
<td>0.360</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.385</td>
<td>0.108</td>
<td>0.011</td>
<td>0.378</td>
</tr>
<tr>
<td>9/60</td>
<td>5</td>
<td>0.331</td>
<td>0.133</td>
<td>0.057</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.375</td>
<td>0.113</td>
<td>0.018</td>
<td>0.365</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.407</td>
<td>0.095</td>
<td>-0.06</td>
<td>0.399</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.413</td>
<td>0.089</td>
<td>-0.07</td>
<td>0.405</td>
</tr>
<tr>
<td>9/40</td>
<td>5</td>
<td>0.369</td>
<td>0.109</td>
<td>0.018</td>
<td>0.358</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.411</td>
<td>0.088</td>
<td>-0.09</td>
<td>0.401</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.436</td>
<td>0.070</td>
<td>-0.18</td>
<td>0.428</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.448</td>
<td>0.059</td>
<td>-0.18</td>
<td>0.442</td>
</tr>
</tbody>
</table>

Notes:
1. Maximum total static bending moment on the bridge = $(2/8)a^2$.
2. The longitudinal bending moment in the slab is ignored as usual.

### Table 5.2  Maximum Girder Bending Moments $M_{cg}$ for Dead Load: Roadway Resurfacing

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>$H$</th>
<th>$\alpha = 0$</th>
<th>$\alpha = 30$</th>
<th>$\alpha = 45$</th>
<th>$\alpha = 60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>9/80</td>
<td>5</td>
<td>0.166</td>
<td>0.209</td>
<td>0.226</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.155</td>
<td>0.218</td>
<td>0.241</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.145</td>
<td>0.226</td>
<td>0.250</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.140</td>
<td>0.230</td>
<td>0.254</td>
<td>0.140</td>
</tr>
<tr>
<td>9/60</td>
<td>5</td>
<td>0.152</td>
<td>0.214</td>
<td>0.235</td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.141</td>
<td>0.225</td>
<td>0.250</td>
<td>0.142</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.132</td>
<td>0.234</td>
<td>0.257</td>
<td>0.133</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.131</td>
<td>0.238</td>
<td>0.256</td>
<td>0.131</td>
</tr>
<tr>
<td>9/40</td>
<td>5</td>
<td>0.136</td>
<td>0.219</td>
<td>0.241</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.127</td>
<td>0.234</td>
<td>0.252</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.122</td>
<td>0.244</td>
<td>0.254</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.119</td>
<td>0.249</td>
<td>0.253</td>
<td>0.119</td>
</tr>
</tbody>
</table>

Notes:
1. Maximum total static bending moment on the bridge = $(1/8)(4b\omega)a^2$.
2. The longitudinal bending moment in the slab is ignored as usual.
FIGURES
Figure 2.1  Geometry of the Typical Skew Slab-and-Girder Bridge Considered
a) THREE COMPONENTS OF THE TOTAL MOMENT ($M_{cg}$) ON THE T-SECTION

b) TRANSFORMED COMPOSITE T-SECTION

c) CROSS SECTION ACTING IN TORSION

Figure 2.2 Girder Properties
Figure 2.3 Relative Truck Locations
Figure 3.1 Compatibility Problem Between an Eccentric Beam Element and a Shell Element (1)
Figure 3.2  Compatibility Problem Between an Eccentric Beam Element and a Shell Element (2)
Figure 3.3  Nodal Degrees of Freedom and forces Acting on the QLSHELL Element
Figure 3.4  Eccentric Assembly of Beam and Shell Elements
Figure 3.5a  Plan View of two QLSHELL Elements Showing the Incompatibility Due to Differential V-displacements in the Beam Element

Figure 3.5b  Incompatibility due to $\theta_x$ Rotations in the Shell Elements
Figure 3.6  Rhombic Plate Subjected to a Uniformly Distributed Load: Deflections
Figure 3.7  Rhombic Plate Subjected to a Uniformly Distributed Load: Maximum Principal Moments
Figure 3.8 Rhombic Plate Subjected to a Uniformly Distributed Load: Minimum Principal Moments
Figure 3.9  Skew Cantilever Beam: Geometry and Mesh Layout
Figure 3.10 Skew Cantilever Beam: Vertical Deflection at Point A Relative to the Deflection Obtained From Mesh 4
Figure 3.11 Geometry and Structural Properties of the Bridge Used in the Convergence Study
Figure 3.12 Finite Element Mesh Models Used in the Bridge Convergence Study
Figure 3.13  Slab Action in Very Skew Short Bridges
Figure 3.14 Midspan Axial force in the Slab in the Longitudinal Direction
Figure 3.15  Midspan Bending Moment in the Slab in the Transverse Direction
Figure 3.16 Example Problem BRIDGE-1: Geometry, Member Properties and Mesh Layout (Taken from Ref. 63)
Figure 3.17 Example Problem BRIDGE-1: Deflection at the Location of the Load (Taken from Ref. 63)
Figure 3.18 Example Problem BRIDGE-1: Distribution of the Longitudinal Direction Axial force in the Deck (Taken from Ref. 63)
Figure 3.19 Example Problem BRIDGE-1: Strong-Axis Bending Moments in the Girders (Taken from Ref. 63)
Figure 3.20  Example Problem BRIDGE-2: Geometry and Member Properties
Figure 3.21 Influence Lines for Girder Bending Moment $M_{cg}$ at Midspan due to a Point Load $P$ Moving Transversely Across the Bridge at Midspan: $b/a = 0.05$ (Taken from Ref. 112)
Figure 3.22  Example Problem BRIDGE-3 and -4: Plan View and Cross Section
Figure 4.1  Midspan Girder Bending Moment Influence Lines for a Point Load P Moving Along the Skew Centre Line
Figure 4.2 Maximum Girder Bending Moment Variation with H:

\[ a = 40 \text{ ft}; \ b = 6 \text{ ft} \]
Figure 4.3 Maximum Girder Bending Moment Variation with $H$:
\[ a = 60 \text{ ft}; \; b = 6 \text{ ft} \]
Figure 4.4 Maximum Girder Bending Moment Variation with H:
$\alpha = 80$ ft; $b = 6$ ft
Figure 4.5  Maximum Girder Bending Moment Variation with H:
a = 40 ft; b = 9 ft
Figure 4.6  Maximum Girder Bending Moment Variation with H:

\[ a = 60 \text{ ft}; \] \[ b = 9 \text{ ft} \]
Figure 4.7  Maximum Girder Bending Moment Variation with $H$:
$a = 80 \text{ ft}; b = 9 \text{ ft}$
Figure 4.8  Maximum Girder Bending Moment Variation with b/a by Changing b:
a = 40 ft; \alpha = 0 degrees
Figure 4.9 Maximum Girder Bending Moment Variation with b/a by Changing b: 
\( a = 40 \text{ ft}; \alpha = 60 \text{ degrees} \)
Figure 4.10  Maximum Girder Bending Moment Variation with b/a by Changing b:
\( a = 80 \text{ ft}; \alpha = 0 \text{ degrees} \)
Figure 4.11  Maximum Girder Bending Moment Variation with b/a by Changing b:
a = 80 ft; α = 60 degrees
Figure 4.12 Girder Midspan Deflection Variation with b/a by Changing b:
\(a = 40 \text{ ft}; \alpha = 60 \text{ degrees}\)
Figure 4.13 Girder Midspan Deflection Variation with b/a by Changing b: 
\(a = 80\) ft; \(\alpha = 0\) degrees
Figure 4.14 Girder Midspan Deflection Variation with b/a by Changing b:
\( a = 80 \text{ ft}; \alpha = 60 \text{ degrees} \)
Figure 4.15  Maximum Girder Bending Moment Variation with $b/a$ by Changing $a$:  
$b = 6$ ft; $H = 5$
Figure 4.16 Maximum Girder Bending Moment Variation with b/a by Changing a:
b = 6 ft; H = 10
Figure 4.17  Maximum Girder Bending Moment Variation with b/a by Changing a:

b = 6 ft; H = 20
Figure 4.18 Maximum Girder Bending Moment Variation with \( b/a \) by Changing \( a \):

\( b = 6 \text{ ft}; H = 30 \)
Figure 4.19 Maximum Girder Bending Moment Variation with b/a by Changing α:
b = 9 ft; H = 5
Figure 4.20 Maximum Girder Bending Moment Variation with b/a by Changing a: b = 9 ft, H = 10
Figure 4.21  Maximum Girder Bending Moment Variation with b/a by Changing a:
b = 9 ft; H = 20
Figure 4.22 Maximum Girder Bending Moment Variation with b/a by Changing α:
b = 9 ft; H = 30
Figure 4.23 Maximum Girder Bending Moment Variation with $\alpha$: $a = 40$ ft; $b = 6$ ft
Figure 4.24 Maximum Girder Bending Moment Variation with $\alpha$:

- $a = 60$ ft; $b = 6$ ft
Figure 4.25  Maximum Girder Bending Moment Variation with $\alpha$:
$\alpha = 80$ ft; $b = 6$ ft
Figure 4.26 Maximum Girder Bending Moment Variation with $\alpha$:

$\alpha = 40$ ft; $b = 9$ ft
Figure 4.27 Maximum Girder Bending Moment Variation with $\alpha$:

- Exterior girders: $a = 60$ ft; $b = 9$ ft
Figure 4.28 Maximum Girder Bending Moment Variation with $\alpha$:

$a = 80$ ft; $b = 9$ ft
**Figure 5.1** Q-values for Exterior Girder Bending Moments in Right Slab-and-Girder Bridges
Figure 5.2 Q-values for Interior Girder Bending Moments in Right Slab-and-Girder Bridges
DESIGN BENDING MOMENT COEFFICIENT = $M_d/P_a$

$M_d/P_a = (M_{static}/P_a)(b/Q)(Z)$

$\alpha = 0 \text{ DEGREES}$

$\alpha = 30$

$\alpha = 45$

$\alpha = 60$

FOR SYMBOLS SEE GRAPH FOR $Q$-VALUES

Figure 5.3 Interior Girder Skew Reduction Factor Z for Bending Moments
DESIGN BENDING MOMENT COEFFICIENT \( M_d/P_a \)

\[
M_d/P_a = (M_{\text{static}}/P_a)(b/Q)(Z)
\]

\( Q = 0-30 \) DEGREES

Figure 5.4 Exterior Girder Skew Reduction Factor \( Z \) for Bending Moments
DESIGN BENDING MOMENT COEFFICIENT = \( \frac{M_d}{Pa} \)

\[ M_d/Pa = (\frac{M_{\text{STATIC}}}{Pa})(b/\Omega)(Z) \]

\( Z \) IS A CORRECTION FACTOR WHEN \( \alpha = 0 \)

Figure 5.5 Consistent Interior Girder Skew Reduction Factor \( Z \) for Bending Moments
DESIGN BENDING MOMENT COEFFICIENT = \( \frac{M_d}{P_a} \)

\[
\frac{M_d}{P_a} = (\frac{M_{\text{STATIC}}}{P_a})(\frac{b}{Q})(Z)
\]

\( a \) = 0 +
\( a \) = 30 *
\( a \) = 45 0
\( a \) = 60 X

Figure 5.6 Consistent Exterior Girder Skew Reduction Factor Z for Bending Moments
Figure 5.7 X-values for Interior Girder Midspan Deflections in Right Slab-and-Girder Bridges
MAXIMUM MIDSPAN GIRDER DEFLECTION = Δ
Δ = \( \Delta_{\text{STATIC}} \left( \frac{b}{X} \right) (Y) \)

\[ \begin{align*}
\alpha = 0 & \text{ DEGREES} \\
\alpha = 30 \\
\alpha = 45 \\
\alpha = 60
\end{align*} \]

Figure 5.8 Interior Girder Skew Reduction Factor Y for Midspan Deflections
MAXIMUM MIDSPAN GIRDER DEFLECTION = \( \Delta \)

\[ \Delta = (\Delta_{\text{static}})(b/X)(Y) \]

\( Y \) = SKREW REDUCTION FACTOR FOR DEFLECTION

\( a, b \) AND \( X \) IN FEET

\( \alpha = 0 \) DEGREES

FOR SYMBOLS SEE GRAPH FOR \( Q \)-VALUES

\( b = 6 \) ft  
\( b = 9 \) ft

Figure 5.9  X-values for Exterior Girder Midspan Deflections in Right Slab-and-Girder Bridges
MAXIMUM MIDSPAN GIRDER DEFLECTION = \Delta
\Delta = (\Delta_{\text{STATIC}})(b/X)(Y)

Figure 5.10 Exterior Girder Skew Reduction Factor Y for Midspan Deflections