RANDOM VIBRATION OF NONLINEAR BUILDING-FOUNDATION SYSTEMS

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A method is presented for evaluating the effect of soil-structure interaction on the dynamic response of nonlinear building-foundation systems subjected to random seismic excitations. Uncertainties in the building-foundation system and loading are summarized and included in the evaluation of the overall structural reliability.

The subsystem approach is adopted to model a soil-structure system. The superstructure is modeled as a shear-beam; the substructure is considered as a surface foundation on a halfspace. The nonlinear behavior in the coupled system is associated with the material nonlinearity in the structure and soil deposit, as well as the geometric nonlinearity caused by foundation uplifting. A smooth hysteretic model is used to represent the nonlinear behavior in the substructure as well as in the superstructure. For a structure with a large number of degrees of freedom, a DOF-reduction technique can be used to simplify the analysis. For structural reliability evaluation, the uncertainties in the dynamic modeling, the parameters of the coupled system, and the ground motion are also included. Using the available seismic hazard model, the lifetime safety of a building-foundation system can be evaluated.
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CHAPTER 1

INTRODUCTION

1.1 General

The effect of soil flexibility on the dynamic response of structures, particularly when subjected to seismic excitations, has been a subject of considerable interest and research in the last three decades. This effect may be important, especially when dealing with very stiff structures on relatively soft soils.

Structures under severe earthquake excitations are often loaded beyond the yield limit and exhibit nonlinear and hysteretic behavior. Effective methods have been developed (e.g., Baber and Wen, 1980) for random response analysis of multistory buildings with nonlinear hysteretic behavior. However, the effect of soil-structure interaction on the response of building-foundation systems including the nonlinear soil hysteresis and foundation uplifting has not been examined.

The purpose of this study is to extend the available nonlinear random vibration method to consider the dynamic response and structural safety of nonlinear building-foundation systems under random seismic excitations. Using this method, the effect of nonlinearities in the structural behavior, soil hysteresis, and foundation uplifting on the soil-structure interaction are examined.

1.2 Related Previous Studies

Two general methods are available to analyze the interaction effect: the direct method (e.g., Lysmer, 1979; Gomez-Masso et al. 1979), which treats the soil-structure system as an integral system, and the subsystem approach (Luco, 1982), which separates the system into
a superstructure and a substructure. Depending on the problem, a particular method may be more suitable. For instance, the subsystem approach permits a good engineering approximation of the soil-structure interaction effect. This approach is particularly convenient when the foundation is assumed to be rigid or when the deformation of the foundation is described by a small number of degrees of freedom. One of the advantages of the subsystem approach is that the analysis of each subsystem (superstructure, foundation and soil) can be performed by the analytical technique best suited to that particular part of the total problem. In addition, the subsystem approach provides intermediate results that may be useful in developing an understanding of the interaction effect and in testing the accuracy of the final results (Luco, 1982). Therefore, this approach is considered in this study.

In the subsystem approach, the interaction effect is usually considered into the analytical method through the foundation-soil impedance functions (force-displacement relationships). The impedance functions are generally obtained by a method in which the soil deposit is idealized as a halfspace and the foundation as a rigid circular plate. Numerical values of these functions for elastic or viscoelastic soil deposits are available in the literature (e.g., Parmelee, Perelman and Lee, 1969; Veletsos and Verbic, 1973; Wong and Luco, 1976; Rucker, W., 1982). To consider linear and nonlinear substructures in the study, the related work of elastic substructure, foundation uplifting, and soil hysteresis are described as follows:

**Elastic Substructure** — The dynamic response of a rigid circular plate on an elastic halfspace medium subjected to a harmonic excitation has been studied by many investigators. The main differences between the solutions are the consequence of assuming different boundary conditions between the plate and the elastic halfspace. Reissner (1936) studied the vertical translation of a rigid circular plate by assuming a uniform pressure distribution between the plate and the medium. Sung (1953) investigated the same problem but considered three different pressure distributions. In addition to vertical translation, Toriumi (1955) included horizontal translation and rotation of the plate about
its diameter; a uniform pressure distribution was assumed for two cases, and a linearly varying one for a third case. Bycroft (1956) considered these three modes in addition to a torsional mode by assuming stress distributions corresponding to static loading conditions.

Without making an assumption of the contact pressure distribution (relaxed boundary condition), Veletsos and Wei (1971) presented numerical results for the steady-state coupled rocking and sliding motions, over wider ranges of parameters than considered in the previous studies. Assuming that only a portion of the halfspace in the form of a semi-infinite truncated cone is effective in transmitting the energy imparted to the disk, Meek and Veletsos (1973) obtained an approximate expression for the impedance functions. On this basis, Veletsos and Verbic (1974) adjusted the coefficients in these equations so that the results obtained therefrom were in reasonable agreement with the available frequency-dependent data.

Normally, because of the frequency-dependence of the foundation impedance functions, most interaction problems are solved by the frequency domain approach that solves a set of simultaneous linear algebraic equation for each value of frequency. Since this type of analysis is based on the principle of superposition, it is limited only to linear systems. Some simple mechanical systems with frequency-independent elements have been used to approximate frequency-dependent impedance functions over a limited range of frequency (e.g., Richart, Hall, and Woods, 1970; Sarrazin, Roesset, and Whitman, 1972).

In 1977 Takemiya proposed a substructural system with frequency-independent parameters that fits the impedance functions (Veletsos and Verbic, 1974) over a wide range of frequency. Therefore, the soil-structure system can be analyzed in the time domain accurately using this method. This model is adopted in the study as a basis to include foundation uplifting and soil hysteresis.

Most of the above studies (except Sarrazin, Roesset, and Whitman, 1972), irrespective of the degree of sophistication, are deterministic. Considering linear soil-structure system in a random seismic
environment, Romo-Organista et al. (1977) developed an algorithm for the generation of the power spectra from the corresponding response spectra. Asano (1982) studied the random response of elastic-plastic soil-structure systems using the Markov-vector approach, but restricted his attention to lateral displacement of the foundation.

**Foundation Uplifting** — Under severe earthquake excitations, the foundation plate may partially separate from the soil. A number of studies have examined the problem of loss of contact between the soil and the foundation. Meek (1975) examined the effect of the base mat tipping considering a one DOF system resting on a rigid medium. Uchida, Miyashita and Nagata (1973) introduced a bilinear relationship between rocking moment and rotation, based on a finite element representation of the elastic halfspace. Using the same concept, Kennedy, et al. (1976) and Takemori, et al. (1976) approximated the rocking-spring characteristics using a linear elastic theory for the halfspace.

Assuming that only normal stress in compression and corresponding shear stress (friction stress) can occur in the contact area, Wolf (1976) developed a rigorous procedure for determining the impedance functions of a rigid plate that is partially in contact with an elastic halfspace, which is discretized with finite elements or with circular rigid subdisks. Using the static influence coefficients of the elastic halfspace, an approximate method to determine the contact area for a given overturning moment and normal force was proposed. This approximate method is considered in the study. Representing the foundation by two-spring model with viscous damping in which the nonlinear equations of motion can be linearized, Psycharis (1983) investigated the dynamic behavior of linear multi-story buildings. On the other hand, Yim and Chopra (1983) considered the foundation of linear multi-story buildings by a Winkler foundation and two-spring foundation to describe the flexibility and damping of the supporting soil.

**Soil Material Nonlinearity** — As the soil strain increases, the soil behavior becomes nonlinear and hysteretic. The soil hysteresis has been examined by many researchers. Using the results of different types of
laboratory tests, Hardin and Drnevich (1972) used a modified hyperbolic function to model the soil behavior; whereas, Streeter, Wylie and Richart (1974) used the Ramberg and Osgood hysteresis (1943) to determine the variation of shear modulus and equivalent viscous damping with the strain. Based on results of field tests, Prakash and Puri (1981) used the approximate method proposed by Barkan (1962) to determine the strain-dependent dynamic shear modulus. Veletsos and Verbic (1973) considered the soil deposit as a linear viscoelastic halfspace, idealized as a standard Voigt solid or a uniform hysteretic behavior, to account for energy dissipation. The smooth hysteretic model described in Chapter 2 was used by Pires, Wen and Ang (1983) to describe the soil shear stress-strain relationship and the parameters of the model were determined from test results.

The foundation of a structure would not experience a motion identical to the free field ground motion. The size and rigidity of the foundation modifies the high-frequency part of excitations due to actual spatial variation of the free field ground motion (Hall, Morgan, and Newmark, 1978). This phenomenon is called "kinematic soil-structure interaction", and is different from "inertial soil-structure interaction" as defined in the previous paragraphs. The effect of kinematic interaction is negligibly small for a surface foundation subjected to vertically propagating shear waves.

1.3 Objective and Scope

One main aspect of this study is to investigate the interaction effect on the nonlinear response of building-foundation systems. For this purpose, analytical methods extending the random vibration analysis of multistory buildings with nonlinear-hysteretic properties are developed to include dynamic soil-structure interaction. Throughout this investigation, the subsystem approach is adopted to analyze the soil-structure interaction problem. The superstructure is modeled as a shear-beam building; whereas the substructure is represented by a
surface foundation on a halfspace. The effect of nonlinearities in the coupled system on the soil-structure interaction are also examined by considering the structural material nonlinearity in the superstructure and the soil hysteresis and foundation uplifting in the substructure. The ground motion is assumed to be vertically incident shear waves, which is modeled as a zero-mean filtered Gaussian shot noise random process, and the effect of kinematic soil-structure interaction is neglected in the study. For superstructures with a large number of degrees of freedom (DOF), a DOF-reduction technique may be used, and the number of DOF will be investigated to satisfy the accuracy for engineering purposes.

Another phase of the study is to investigate the effect of interaction on the overall structural safety under seismic excitations. For this purpose, the statistics of the maximum structural response is estimated in the performance assessment of building-foundation systems. The uncertainties in the building-foundation system are also identified, quantified, and included in the evaluation of the overall structural reliability.

1.4 Organization

In Chapter 2, a model for calculating the probabilities and statistics of the response of nonlinear-hysteretic systems under random seismic loadings is summarized. The expected equivalent stiffness, statistics of maximum story displacement, and sensitivity coefficients are also studied. For structures with a large number of DOF, the DOF-reduction technique, which reduces the full DOF system to a reduced number of DOF model, is developed.

In the first part of Chapter 3, the superstructure is modeled as a shear-beam building. Then the substructural model of a rigid surface plate on an elastic halfspace is described and extended to include the foundation uplifting and/or soil hysteresis. The hysteretic parameters in nonlinear substructures are evaluated from experimental and
theoretical results. A soil-structure system consisting of a
superstructure and substructure is analyzed for its dynamic response.

Numerical results for four sample problems are presented in Chapter
4 to investigate the significance of soil-structure interaction. Single-DOF buildings are first used to investigate the effect of interaction on the dynamic response of linear and nonlinear building-foundation systems. The accuracy of the foundation rocking hysteretic parameters, $\beta$ and $n$, is also examined. A ten-story UBC steel building is examined to validate the DOF-reduction technique and to calculate the corresponding interaction effect. A four-story UBC steel building is also used to calculate the interaction effect. Finally, an idealized single-DOF reactor building is used to investigate the interaction effect of massive and stiff structures.

The methodology for seismic safety evaluation of the building-foundation system is described in Chapter 5. The uncertainties in the model, the parameters of the soil-structure system, and in the earthquake loading are defined and assessed. The ten-story UBC steel building described in Chapter 4 is used to demonstrate the DOF-reduction technique in the seismic safety assessment. The four-story UBC steel building is used to estimate the seismic safety, including the uncertainties in the coupled system. The idealized single-DOF reactor building is used to investigate the sensitivity of the structural response to various system parameters.

Chapter 6 contains the summary and major conclusions of the study.

1.5 Notation

The symbols and notations used in the text are summarized as follows (throughout the text, the time derivative of any quantity will be denoted with a dot over the symbol):

- $a$ = ground acceleration.
- $a_o$ = dimensionless frequency parameter.
- $b$ = ground displacement.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_1, b_2, b_3)</td>
<td>parameters dependent on Poisson's ratio in the substructural model.</td>
</tr>
<tr>
<td>(A, a, \alpha, \beta, \gamma)</td>
<td>parameters controlling the hysteretic loop shape and yielding level.</td>
</tr>
<tr>
<td>([B])</td>
<td>covariance matrix of the random seismic loading.</td>
</tr>
<tr>
<td>(c)</td>
<td>coefficient of viscous damping.</td>
</tr>
<tr>
<td>(C, K)</td>
<td>equivalent linear coefficients for hysteresis model.</td>
</tr>
<tr>
<td>(E[\cdot])</td>
<td>expected value.</td>
</tr>
<tr>
<td>(G)</td>
<td>system coefficient matrix or soil shear modulus.</td>
</tr>
<tr>
<td>(H(t))</td>
<td>interacting shear force.</td>
</tr>
<tr>
<td>(K_x, K_\theta)</td>
<td>static stiffnesses of foundation in translational motion and rocking motion, respectively.</td>
</tr>
<tr>
<td>(k)</td>
<td>story stiffness or spring constant in the substructure model.</td>
</tr>
<tr>
<td>(M(t))</td>
<td>interacting rocking moment.</td>
</tr>
<tr>
<td>(m)</td>
<td>structural story mass.</td>
</tr>
<tr>
<td>(m_r)</td>
<td>added mass due to rocking motion.</td>
</tr>
<tr>
<td>(N)</td>
<td>Bayesian correction variable for prediction errors.</td>
</tr>
<tr>
<td>(p)</td>
<td>general ground motion or structural system parameter.</td>
</tr>
<tr>
<td>(q)</td>
<td>total restoring force in each element.</td>
</tr>
<tr>
<td>(Q_x, Q_\theta)</td>
<td>foundation impedance functions in translational and rocking motions, respectively.</td>
</tr>
<tr>
<td>(r)</td>
<td>foundation radius.</td>
</tr>
<tr>
<td>(s)</td>
<td>white noise power spectral density.</td>
</tr>
<tr>
<td>(S)</td>
<td>response covariance matrix.</td>
</tr>
<tr>
<td>(t_d)</td>
<td>earthquake strong motion duration.</td>
</tr>
<tr>
<td>(u)</td>
<td>relative displacement of two consecutive lumped masses, or the deformation of the hysteretic spring.</td>
</tr>
<tr>
<td>(\text{Var}[\cdot])</td>
<td>variance.</td>
</tr>
<tr>
<td>(V)</td>
<td>soil shear wave velocity.</td>
</tr>
<tr>
<td>(W)</td>
<td>total load of superstructure.</td>
</tr>
</tbody>
</table>
\( x, \Theta \) = foundation relative displacements with respect to the ground in translational motion and rocking motion, respectively.

\( z \) = hysteretic displacement (the hysteretic-restoring force is given by \( k_z \)).

\( \omega, \beta \) = parameters of the Kanai-Tajimi power spectral density function.

\( \varepsilon \) = hysteretic energy dissipated.

\( \nu \) = Poisson's ratio of the halfspace material.

\( \omega \) = excitation frequency.

\( \rho \) = mass density of halfspace.

\( \rho_{ij} \) = correlation coefficient between parameter \( i \) and \( j \).

\( \sigma \) = standard deviation.

\( \delta \) = coefficient of variation representing inherent variability.

\( \Delta \) = coefficient of variation representing prediction error.

\( \Omega \) = total coefficient of variation equal to \( \sqrt{\delta^2 + \Delta^2} \).
CHAPTER 2

METHODS FOR RANDOM VIBRATION ANALYSIS OF INELASTIC SYSTEMS

2.1 Introduction

Structures and soil deposits subjected to strong earthquake loadings are likely to undergo response in the inelastic range. In addition, the behavior of materials is hysteretic and often the stiffness and/or strength of the structural components or soil deposit deteriorate. As the earthquake loading is realistically described by a random process, the resulting response may be defined by the statistics and probabilities obtained from a random vibration analysis. Exact solution for the random vibration response statistics of hysteretic degrading system, however, is generally not possible. Recently, Wen (1976, 1980) and Baber and Wen (1980) proposed a versatile hysteretic restoring force model. The analytical method can be extended for soil-structure interaction problems.

This chapter contains a description of the analytical method for random vibration analysis of inelastic systems. The smooth hysteretic restoring force model can be used to represent the nonlinear behavior in a superstructure or a substructure (described in chapter 3). For investigating the effect of soil-structure interaction, the expected equivalent stiffness is defined, and the statistics of maximum response and the sensitivity coefficients are also included. In addition, the DOF-reduction technique is used to simplify the analysis for structures with a large number of DOF.
2.2 The Smooth Hysteretic Restoring Force Model

The fundamentals of the proposed hysteretic model may be summarized by considering a single-DOF system in which the equation of motion is

\[ m\ddot{u} + c\dot{u} + q(u, t) = -ma \]  

(2.1)

where the restoring force \( q \) is given by

\[ q(u, t) = aku + (1-a)kz \]  

(2.2)

The nonlinear differential equation governing \( z \) is given by

\[ \ddot{z} = A\dot{u} - \beta|\dot{u}|z^{n-1}z - \gamma\dot{u}|z|^n \]  

(2.3)

in which

- \( u \) = the relative displacement of the story mass in structure, or relative displacement (translation or rocking) of foundation;
- \( z \) = the hysteretic component of the displacement;
- \( m \) = mass;
- \( a \) = base acceleration;
- \( c \) = the coefficient of viscous damping;
- \( k \) = the initial stiffness of linear system;
- \( a, A, \beta, \gamma, n \) = parameters describing the shape of the hysteresis loop.

The total restoring force, \( q(u, t) \), has a hereditary property because of the inclusion of the \( z \) term. A large number of hysteresis shapes can be described by varying the parameters \( A, \beta, \gamma \) and \( n \). Some of the possible combinations are shown in Fig. 2.1.

The skeleton curve, defined as the locus of the tips of the hysteresis loops with different amplitudes, is given by

\[ u_s(z) = \frac{1}{2} \left\{ \int_0^z \frac{dx}{A-(\gamma+\beta)x^n} + \int_0^z \frac{dx}{A-(\gamma-\beta)x^n} \right\} \]  

(2.4)
The incremental work done by hysteretic action is

\[ d\varepsilon(t) = (1-\alpha)kzdu \] (2.5)

The expected rate of hysteretic energy dissipation is

\[ E[\dot{\varepsilon}(t)] = (1-\alpha)kE[\dot{u}z] \] (2.6)

For softening system, \( z \) attains a maximum value which is obtained by setting \( dz/du \) in Eq. 2.3 to zero. For positive \( \dot{u} \) and \( z \), this gives

\[ z = \left[ \frac{A}{(\beta+\gamma)} \right]^{1/n} \] (2.7)

The yield resistance \( q_y \) (explained in Sect. 2.4.1), is given by

\[ q_y = [\alpha + (1-\alpha)A]kz/A \] (2.8)

Other important physical properties of softening systems worth noting are the initial stiffness, \( k_i \), and the post-yielding stiffness, \( k_f \), as follows:

\[ k_i = \alpha k + (1-\alpha)kA \] (2.9)

\[ k_f = \alpha k \] (2.10)

when \( A \) is equal to 1, the yield resistance \( q_y = kz \), and the ratio of post-yield stiffness to initial stiffness reduces to the value \( \alpha \). The sharpness of the transition from the linear to nonlinear range is governed by the parameter \( n \), with the hysteresis approaching bilinear behavior as \( n \) approaches infinite.
The model is also capable of reproducing degrading material behavior; this is obtained by introducing two additional parameters in Eq. 2.3, giving

\[ 
\dot{z} = \left[ A\dot{u} - \mu(\beta|\dot{u}|z|^{n-1}z + \gamma|z|^n) \right]/\eta \tag{2.11} 
\]

where \( \eta \) and \( \mu \) are increasing functions of time, that account for the stiffness and strength deteriorations, respectively. The parameter \( A \) may also be defined as a decreasing function of time. In this form, monotonic reduction in \( A \) will represent degradations in both the stiffness and strength.

An in-depth study of the model, and the extension to multi-DOF systems may be found in Baber and Wen (1980).

### 2.3 The Stochastic Equivalent Linearization

The required response statistics may be obtained by the method of equivalent linearization (Atalik and Utku, 1976). The special form of the nonlinear hysteretic model presented in Sect. 2.2 permits the linearization of the equation of motion in close form, without resorting to the Krylov-Bogoliubov (KB) approximation.

Examining the equations of motion of a single-DOF system, Eqs. 2.1 through 2.3, reveals that the differential equation for \( z \), Eq. 2.3, is the source of nonlinearity. Hence, only Eq. 2.3 needs to be linearized. The linearized form of Eq. 2.3 or Eq. 2.11 was obtained by Baber and Wen (1980) as follows:

\[ 
\dot{z} = C\dot{u} + Kz \tag{2.12} 
\]

where \( C \) and \( K \) are equivalent linear coefficients, chosen such that the resulting solution for \( \dot{z} \) is as "close" as possible to that obtained with the original nonlinear equation. The general expressions for \( C \) and \( K \) in terms of the response statistics, are given in Appendix A.
The nonlinear random vibration problem is, therefore, reduced to a linear one in which the coefficients of the linear system are response-dependent.

The random vibration solution of an equivalent linear system is relatively straightforward. The equations of motion are first decomposed into a system of first-order differential equation, written in matrix form as

\[ \dot{Y} + GY = F(t) \]  

(2.13)

For the current model, there will be three first-order equations for each DOF — two replacing the second order dynamic equilibrium equation (Eq. 2.1 for single-DOF case) and one for the linearized hysteresis equation (Eq. 2.12).

Postmultiplying Eq. 2.13 by \( Y^T \), and then taking expected values and adding the resulting equation to its transpose, gives the classical result

\[ \dot{S} + GS + SG^T = B \]  

(2.14)

in which \( S = E[YY^T] \), the covariance matrix; and

\[ B = E[FY^T] + E[YF^T] = 2\pi s F \]  

(2.15)

where \( s \) is the two-sided power spectral intensity of a white noise.

The desired response statistics are obtained by solving Eq. 2.14 for the zero time-lag covariance matrix \( S \).

The system of equations defined by Eq. 2.14 is a system of nonlinear ordinary differential equations, because the matrix \( G \) depends on the response statistics, and its solution requires numerical integration in the time domain. The stationary solution for nondeteriorating systems, i.e., \( \dot{S} = 0 \), may be obtained iteratively using the algorithm by Bartels and Stewart (1972) for the structural system with fixed-base or the soil
deposit (for the soil-structure system explained in Sect. 3.4).

For systems with deterioration governed by energy dissipation, the expected rate of energy dissipation, Eq. 2.6, is considered for every DOF. Note that $E[\dot{u}z]$ in Eq. 2.6 is an element of the zero time-lag covariance matrix $S$. Hence the hysteretic energy dissipation may be obtained directly from the matrix $S$.

2.4 Additional Statistics

Additional statistics of the dynamic response are necessary to investigate the effect of soil-structure interaction. The expected equivalent stiffness is defined to evaluate the interaction effect, and the statistics of maximum response and the sensitivity coefficients are necessary for assessing structural reliability.

2.4.1 The Expected Equivalent Stiffness

The expected equivalent stiffness is used to modify the mode shape in the DOF-reduction technique, and to estimate the expected soil shear modulus in the substructure during random vibration analysis.

The restoring force, $q$, in Eq. 2.2 is composed of two parts. The first part, $aku$, is a linear restoring force component $q_1$ (Fig. 2.2a). The second part, $(1-a)kz$, is a hysteretic restoring force component $q_2$. The skeleton curve of $q_2$ is shown in Fig. 2.2b. The ultimate restoring force $f$ in the hysteretic restoring force component is

$$f = (1-a)kz$$

(2.16)

The hysteresis model used herein exhibits smooth yielding and, therefore, does not have a clearly defined yield point. However, the yield displacement, $\delta_y$, in $q_2$ may be defined as the ultimate restoring force $f$ divided by the initial stiffness in $q_2$; thus,
Adopting $\delta_y$ as the yield displacement in the restoring force $q$ with the initial stiffness $k_i$ in Eq. 2.9, the yield resistance $q_y$ may be defined as

$$q_y = \delta_y k_y = [a + (1-a)A] \frac{ky}{A}$$  \hspace{1cm} (2.18)

On the basis of the above, the idealized bilinear skeleton curve of the restoring force $q$ can be obtained as shown in Fig. 2.2c. The equivalent stiffness of the hysteresis loop may be defined by the secant stiffness of the hysteresis loops at the peak as

$$k_{eq}(u) = \frac{q}{u}$$  \hspace{1cm} (2.19)

where $q$ and $u$ are the restoring force and displacement at the peak of the hysteresis, respectively. From Fig. 2.2c,

$$k_{eq}(u) = k_i$$  \hspace{1cm} for $0 \leq u \leq \delta_y$  \hspace{1cm} (2.20a)

$$k_{eq}(u) = k_i + \frac{(q - \delta_k)}{u_i}$$  \hspace{1cm} for $\delta_y < u \leq \delta_f$  \hspace{1cm} (2.20b)

where $k_i = \frac{ak}{y_i}$

In the case of stationary random response, the expected equivalent stiffness may be defined as a function of the root-mean-square (RMS) value of the response $u$ in the following form:

$$E[k_{eq}(\sigma)] = \int_{0}^{\infty} k_{eq}(u) p(u, \sigma) du$$  \hspace{1cm} (2.21)
where the probability density of \( u \) was proposed by Rice (1954) for narrow band response as

\[
p(u_p, \sigma_u) = \frac{u_p}{\sigma_u^2} \exp\left(-\frac{u_p^2}{2\sigma_u^2}\right)
\]

(2.22)

Substituting Eqs. 2.22 and 2.20 into Eq. 2.21 yields

\[
E[k_{eq}(\sigma_u)] = k_i \left[1 - (1-\alpha) \exp\left(-\frac{\delta}{\sqrt{2}\sigma_u}\right)^2\right] \\
+ \sqrt{\pi} (1-\alpha) A\left(\frac{\delta}{\sqrt{2}\sigma_u}\right) [1 - \text{erf}\left(\frac{\delta}{\sqrt{2}\sigma_u}\right)]
\]

(2.23)

where \( \text{erf} \) is the error function.

In the case of nonstationary random response, the probability density of the peak amplitude \( p(u, \sigma, \rho, \sigma_u) \) (Kobori and Minai, 1967) is a function of the RMS value of \( u \), \( \dot{u} \) and the correlation coefficient \( \rho \). Therefore, the expected equivalent stiffness may be defined as

\[
E[k_{eq}(\sigma, \sigma_u, \rho_u)] = \int_0^\infty k_{eq}(u) p(u, \sigma, \sigma_u, \rho_u) \, du
\]

(2.24)

in which,

\[
p(u_p, \sigma_u, \rho_{uu}, \rho_u) = \exp\left(-\frac{u_p^2}{2\sigma_u^2}\right) \left[ \frac{u_p^{\rho_{uu}^2}}{\sigma_u^2} \exp\left(-\frac{\rho_{uu}^2 u_p^2}{2(1-\rho_u^2)\sigma_u^2}\right) \right] \\
+ \frac{\rho_{uu} \sqrt{\pi}}{\sqrt{1-\rho_u^2}} \frac{u_p^2}{2\sigma_u^2} \left[ \text{erf}\left(\frac{\rho_{uu} u_p}{\sqrt{2(1-\rho_u^2)\sigma_u^2}}\right) - 1\right]
\]

(2.25)

Substituting Eqs. 2.20 and 2.25 into Eq. 2.24, the expected equivalent...
stiffness can be obtained by numerical integration.

2.4.2 Maximum Displacement Statistics

Exact solution of the distribution function for the maximum displacement has not been obtained to date. Many approximate solutions have been developed, including an asymptotic approximation by Yang and Liu (1981) which is based on the simulation results of Shinozuka and Yang (1971), indicating that the distribution of the nonstationary extremum in the time interval \((t_1, t_2)\) can be described with the Weibull distribution. Assuming that the extremes occurring in \((t_1, t_2)\) are statistically independent and the total number of extremes, \(n\), is large, Yang and Liu (1981) obtained the distribution of the maximum (absolute global) response, \(U\). The mean value and standard deviation of the maximum response are

\[
E[U_m] = (K + 0.5772 K^{1/2}) \sigma
\]

and,

\[
\sigma_{U_m} = \frac{\pi}{\sqrt{6}} \frac{\sigma}{K^{1/2}}
\]

where \(\sigma\) is the standard deviation of \(u\) and

\[
K = (\alpha \ln(n))^{1/\alpha} = [\alpha \ln(t_2) - 2 \lambda_0(t_1) \int_{t_1}^{t_2} 2 \lambda_0(t) dt]^{1/\alpha}
\]

The parameters \(\alpha\) and \(\sigma\) are dependent on \(t_1\) and \(t_2\), as well as the nonstationary characteristics of \(u(t)\). \(\lambda_0\) is the time-varying zero upcrossing rate obtained by assuming that the displacement, \(u\), and velocity, \(\dot{u}\), are jointly Gaussian (Rice, 1941, 1945); i.e.,

\[
\lambda_0^+(t) = \frac{\sigma_u(t) \sqrt{1 - \rho_{uu}(t)}}{2\pi \sigma_u(t)}
\]
In the stationary case, the Weibull distribution of the extremes reduces to the Rayleigh distribution with $\alpha = 2.0$ and $\sigma = \sigma(t) = \text{constant}$, and Eqs. 2.26 and 2.27 reduce to the classical result of Davenport (1964).

Under the assumption that the dynamic response process is Gaussian, the value of $\sigma_{m}$ may be underestimated (Baber and Wen, 1980). For the present study, the variance of the expected maximum is taken as $\sigma_{m}^{2} = [(0.15)(E[U])]_{m}$ (Sues, Wen, and Ang, 1983).

For reliability analysis, the derivative of the expected maximum with respect to a general system parameter, $p$, is required. The Weibull parameter, $\alpha$, was found to be insensitive to parametric changes; hence, its derivative may be neglected. Then,

$$\frac{\partial E[U_{m}]}{\partial p} = (K + 0.5772k^{1-\alpha}) \frac{\partial \sigma}{\partial p} + [1 + 0.5772(1-\alpha)k^{-\alpha}] \sigma \frac{\partial K}{\partial p} \quad (2.30)$$

where,

$$\frac{\partial \sigma}{\partial p} = \frac{1}{\alpha^{1/\alpha}} \frac{\partial E[\tilde{u}(t_{1},t_{2})]}{\partial p} \quad (2.31)$$

and

$$\frac{\partial K}{\partial p} = \frac{1}{n(\alpha^{n})^{1-\alpha}} \frac{\partial n}{\partial p} \quad (2.32)$$

in which

$$n = \int_{t_{1}}^{t_{2}} 2\lambda^{+}(t) dt \quad (2.33)$$

For the stationary case, $\sigma = 2.0$, $\partial \sigma/\partial p = \partial \sigma / \partial p$ and $\partial \lambda^{+}/\partial p$ are constants. Evaluation of the derivatives appearing on the right-hand-sides of Eq. 2.31 and Eq. 2.32 are straightforward once the derivative of the covariance matrix is obtained (see Sect. 2.4.3).
2.4.3 Sensitivity Coefficients

Sensitivity coefficients are the derivatives of the response with respect to specific parameters; these represent the contribution of the parameter uncertainty to the total response uncertainty. The method for evaluating the sensitivity coefficients was proposed by Sues, Wen and Ang (1983).

Letting $p$ represent the parameter of interest, the derivative equation is obtained by differentiating Eq. 2.14 with respect to $p$ and interchanging the time and parameter derivatives; thus, obtaining

$$\frac{\partial}{\partial t} \frac{\partial S}{\partial p} + G \frac{\partial S}{\partial p} + \frac{\partial S}{\partial p} G^T + S \frac{\partial G^T}{\partial p} = \frac{\partial B}{\partial p}$$

(2.34)

where

$$\frac{\partial S}{\partial p} = \frac{\partial}{\partial p} E[YY^T]$$

(2.35)

The elements of $\partial S/\partial p$ are the necessary response statistic derivatives (i.e., sensitivity coefficients). Since the equation in Eq. 2.34 contains the matrix $S$, the solution for the matrix $\partial S/\partial p$ will require knowledge of the response covariance matrix $S$, which is part of the solution of the random vibration analysis.

For the stationary case, $\partial S/\partial p$ is constant in time. Thus, after rearranging terms and equating the time derivatives to zero, Eq. 2.34 leads to the corresponding stationary derivative equation

$$G \frac{\partial S}{\partial p} + \frac{\partial S}{\partial p} G^T = \frac{\partial B}{\partial p} - \frac{\partial G}{\partial p} S - S \frac{\partial G^T}{\partial p}$$

(2.36)

Observe that the matrix $\partial G/\partial p$ appearing on the right-hand-side of Eq. 2.36 contains the unknown derivatives of the equivalent linear coefficients (functions of the unknown response derivatives). Since
Eq. 2.36 takes the form \( AX + XB = C \), discussed in Sect. 2.3, the algorithm described in Sect. 2.3 may be used to solve the unknown derivative matrix \( \partial S/\partial p \). Initial values are assumed for the derivatives of the equivalent linear coefficients in solving the equations iteratively.

For the nonstationary case, the equation is solved numerically for the response statistics. The evaluation of the matrices \( \partial B/\partial p \) and \( \partial G/\partial p \) in Eq. 2.34 for different parameter \( p \) (structure, ground motion) can be extended to include substructural parameters, as described in Appendix B.

### 2.5 DOF-Reduction Technique

Since the analysis of multi-DOF system is generally costly, a simpler system with smaller number of DOF is sometimes preferred.

There are many studies on the earthquake response of complex structures represented by simple (one DOF) model. The natural period and damping under small oscillations are usually assumed to be those of the first mode of the structural response. The mathematical relationships between a simple structural model and the fundamental mode of a structure was first discussed by Biggs (1964) for linear systems. Pique (1976) extended Biggs' technique to inelastic steel structures, and proposed an equivalent single-DOF model with a multilinear hysteretic force-deformation relationship, in which a steel structure is assumed to deform according to its first mode shape. Saiidi and Sozen (1979) used a similar technique, in which a simplified Q-hysteresis loop was used for the first mode to evaluate the seismic response of reinforced concrete structures. In these studies, the maximum displacement responses obtained with an appropriate single-DOF model were found to be satisfactory under a given earthquake motion.

Based on the above observations, a DOF-reduction technique is used in this study for structures with a large number of DOF. Thus the motion of a large superstructural system is described approximately by
spatial coordinates equal to the first few modes of vibration. Strictly speaking, normal modes no longer exist after a structure becomes nonlinear. An iteration technique may also be used to update the changes in the mode shapes of vibration by the expected equivalent story stiffnesses described in Sect. 2.4.1 to obtain more accurate results with reduced DOF.

Considering the multi-DOF building-foundation system shown in Fig. 3.14 (explained in Sect. 3.4) and using a model with reduced m DOF out of a total of n DOF (m < n), the deformation vector \([x]\) may be expressed as

\[
[x] = [\Phi] \{p\}
\]

(2.37)

where \([\Phi]\) is the truncated modal matrix, of order n x m, and \([p]\) is the generalized displacement, of order n x 1. The \(i^\text{th}\) floor deformation, \(x_i\), relative to the foundation can be obtained from the interstory displacement \(u_i\); and hence

\[
u = x_1\quad u = x_j - x_i \quad \text{for } i = 2, n
\]

(2.38)

The governing equations of motion of a soil-structure system in terms of interstory displacements may be rewritten as a system of first-order differential equations (explained in Sect. 3.4) as

\[
A(\frac{dY}{dt}) = DY + C
\]

(2.39)

where \(A, D,\) and \(C\) are coefficient matrices; and

\[
Y^T = (x, \dot{x}, z, \theta, \dot{\theta}, z, \theta, u, \dot{u}, u, \dot{u}, z, \ldots, u, \dot{u}, z)
\]

(2.40)

Substituting Eqs. 2.37 and 2.38 into Eq. 2.39, the matrix \(Y\) in Eq. 2.40 is represented by
\[ Y^T = (x, \dot{x}, z, \dot{z}, \theta, \dot{\theta}, u, \dot{u}, p, \dot{p}, \ldots, p, \dot{p}, z, \ldots, z) \] (2.41)

From the \( Y \) matrix, it may be observed that the number of equations of motion is significantly reduced without neglecting the hysteretic restoring forces of the structural components. The total number of unknowns in the covariance matrix, \( S \), is \((9+2m+n)(10+2m+n)/2\). For example, if a system with 10 stories \((n = 10)\), there will be 780 unknowns for the complete solution; the number of unknowns is reduced to 231 with the one-DOF approximation, and 276 with the two-DOF approximation. The technique can be used also to estimate the necessary statistics of a soil-structure system.
Fig. 2.1 Possible Combination of $\beta$ and $\gamma$
(a) Linear Restoring Force $q_1$

(b) Hysteretic Restoring Force $q_2$

(c) Total Restoring Force $q$

Fig. 2.2 The Relationship between Restoring Force and Displacement
3.1 Introduction

The subsystem approach, in which the structure (superstructure) and the foundation and soil (substructure) are analyzed separately, is adopted in this study. The superstructure is modeled as a simple shear-beam system, whereas the substructure is modeled by a halfspace with a surface foundation. In general, the superstructural model is based on the results of experimental investigation, but the substructural model is mostly based on the results of theoretical analyses. The models selected in the superstructure and substructure are intended to satisfy the following criteria: (a) should approximate the overall behavior of the soil-structural system, (b) the models should be capable of reproducing the behavior of a broad range of system parameters, and (c) should yield tractable solutions under random excitations.

This chapter begins with a description of the superstructure which is idealized as a shear-beam type structure. The hysteretic model described in chapter 2 is used to represent the nonlinear structural behavior. Thereafter the linear and nonlinear substructural systems are described. In the substructure, the surface foundation is idealized as a rigid circular plate and the soil deposit as a homogeneous halfspace. The relaxed boundary condition (Veletsos and Wei, 1971) is assumed between the foundation and the soil; also the coupling between the rocking and sliding motions is neglected. Three types of substructures are considered: linear substructure (linear halfspace with welded surface foundation), linear halfspace with foundation uplifting, and nonlinear halfspace with foundation uplifting. The model proposed by Takemiya (1977) (or Veletsos and Verbic, 1974) is adopted for linear
substructures and extended for nonlinear substructures. The differential equation model described in Chapter 2 is used to represent the nonlinear behavior in the systems, and the hysteretic parameters are evaluated from the theoretical results or experimental data. Finally the analysis of soil-structure system is presented.

3.2 Superstructure Model

3.2.1 Shear-Beam Model of Superstructure

Shear-beam type superstructures are investigated; i.e., each floor in the structure is considered to be rigid (no rotation) and has only one DOF (translation). In order to develop a simple shear-beam idealization of a structure and maintain reasonable accuracy, it is necessary to determine (for each story) an equivalent story lateral stiffness and equivalent story yielding strength.

Equivalent Story Stiffness — Based on the assumptions that the column shears above and below a joint are the same, the inflection points in the columns above and below a joint are located symmetrically with respect to the joint, and that the rotations of all joints in one floor are the same, Anagnostopoulos (1972) suggested the following approximate expression for the lateral story stiffness $k$

$$k = \frac{24E}{h^2} \left( \frac{1}{\sum k_c} + \frac{1}{\sum k_{gt}} + \frac{1}{\sum k_{gb}} \right)$$

(3.1)

where:

- $E$ = modulus of elasticity;
- $h$ = story height;
- $l$ = girder length;
- $I$ = moment of inertia;
- $k = I/h$, column stiffness;
- $k_c = I_c/h_c$, girder stiffness for the adjacent top and bottom
gt gb g
floors, respectively.

In the case of reinforced concrete structures, the moment of inertia, \( I \), must include the effect of cracking. Based on the work of Medland and Taylor (1971), Anagnostopoulos (1972) proposed a relation for determining the effective moment of inertia, as follows:

\[
\frac{I_{\text{eff}}}{I_{\text{gross}}} = \begin{cases} 
0.80 & \text{for Columns} \\
0.40 & \text{for Beams}
\end{cases}
\quad (3.2)
\]

The larger coefficient for columns is a result of axial compression counteracting the effect of cracking.

**Equivalent Story Yielding Strength** — Based on the assumption that plastic hinges will be developed at the two ends of all the columns and girders in a story, Anagnostopoulos (1972) proposed an upper bound for the equivalent story yielding strength, \( q_y \), as follows:

\[
q_y = \min\left\{ \frac{2 \sum M_{yc}}{h}, \frac{2 \sum M_{yg}}{h} \right\}
\quad (3.3)
\]

where \( \sum M_{yc} \) and \( \sum M_{yg} \) are the sums of all the column and girder plastic moment capacities in one floor, respectively; and \( h \) is the story height.

Eq. 3.3 is valid only if shear failure does not occur in the columns. For structures that are designed according to standard codes and are properly detailed, premature shear failure should not occur (Anagnostopoulos, 1972).

### 3.2.2 Hysteretic Model Parameters

In order to properly model the restoring force behavior for a real structure, it is necessary to determine appropriate values for the hysteresis loop shape parameters \( A, \beta, \gamma \) and \( n \). A system identification
technique is adopted for this purpose using actual experimental displacement data (Sues, Wen, and Ang, 1983). For steel structures, it was found that $\beta = \gamma$, $a = 0.04$, $n = 1$, and $A = 1$ should be used; whereas for reinforced concrete, $\beta = -3\gamma$ with $\gamma < 0$, $a = 0.02$, $n = 2$, and $A = 1$ are appropriate. Because the value of $A$ is equal to 1 for steel and reinforced concrete structures, the value of the parameter, $k$, in Eq. 2.9 is simply the initial stiffness of the restoring force, and the yield restoring force, $q_y$, in Eq. 2.8 is equal to $kz$. If the yield level of the material is known, the values of $\beta$ and $\gamma$ may be determined through Eqs. 2.7 and 2.8.

3.3 Substructure Model

In the subsystem approach, the effect of soil-structure interaction is normally introduced into the response analysis through the foundation-soil impedance functions. These impedances, in effect, model the soil as a system of springs and viscous dashpots which provide restoring and dissipative forces. In general, the values of these stiffnesses and damping coefficients are dependent on the soil properties, the geometry of the foundation, the nature of the contact between the foundation and soil, and the excitation frequency.

In this section, the impedance values derived from analytical solutions are first described for the response of a linear substructure under harmonic excitations. Thereafter the techniques are presented to include foundation uplifting or to consider foundation uplifting and soil hysteresis in the substructural model for random vibration analysis. The model considered is shown in Fig. 3.1.

3.3.1 Elastic Substructure

Model Adopted and Assumptions — The analytical expressions of foundation impedance functions developed by Veletsos and Verbic (1974) are used in this study. The assumptions in the model are:

(1) The foundation is a rigid circular plate and the soil medium is
a homogeneous elastic halfspace.

(2) The relaxed boundary condition is assumed for the stress distribution between the plate and the halfspace: Under a horizontal force, the normal component of contact pressure is assumed to be zero; whereas under a rocking moment, the horizontal component of the interface pressure also is assumed to be zero.

(3) The rocking and sliding motions of the foundation plate may be treated separately in the analysis. From the results of Veletsos and Wei (1971) as shown in Figs. 3.2 (c) and (d), it is observed that the coupling between the rocking and sliding motions is negligibly small.

(4) The analytical expression for the impedance functions used in the study is in reasonable agreement with the available frequency-dependent data (Fig. 3.2) as shown in Fig. 3.3.

**Force-Displacement Relationship** — Let $H(t)$ and $M(t)$, shown in Fig. 3.1, be the amplitude of generalized harmonic forces acting on the disk along the translational and rocking directions, respectively, and $x, \theta$ be the amplitude of the corresponding displacements. The relationship between the force and displacement amplitudes may be stated as

\[
H(t) = Q_x x_0 \quad (3.4a)
\]

\[
M(t) = Q_\theta \theta \quad (3.4b)
\]

where $Q_j$ ($j = x \text{ or } \theta$) is a complex-valued stiffness (impedance) function of the form

\[
Q_j = K_j (k_{jj} + j \omega c_{jj}) \quad (3.5)
\]

where $K_j$ represents the static stiffness of the disk, defined approximately as (Veletsos and Wei, 1971)
\[ K = \frac{8Gr}{2-\nu} \]  \hspace{1cm} (3.6a)

\[ K = \frac{8Gr^3}{3(1-\nu)} \]  \hspace{1cm} (3.6b)

and \( i=\sqrt{-1} \); \( k \) and \( c \) are dimensionless functions of Poisson's ratio of the halfspace material \( \nu \), and of the dimensionless frequency parameter \( a = \frac{\omega r}{\nu s} \) \hspace{1cm} (3.7)

where \( \omega \) = the circular frequency of the excitation and resulting motion; \( r \) = the radius of the disk; \( V = \sqrt{G/\rho} \) is the speed of shear wave propagation in the halfspace; \( G \) = the shear modulus of elasticity of the halfspace material; and \( \rho \) = mass density of the halfspace. In the equivalent spring-dashpot representation of the supporting medium, \( k \) is the effective stiffness of the spring, and \( c \) is the damping coefficient of the dashpot. The coefficients and impedance functions in Eq. 3.5 are as follows:

For horizontal motion:

\[ k = 1, \quad c = b \]  \hspace{1cm} (3.8)

\[ Q = K (1 + ib \omega) \]  \hspace{1cm} (3.9)

For rocking motion:

\[ k_{\theta} = 1 - b \left[ \frac{(b_2 a_2)^2}{1+(b_2 a_2)^2} - b_3 a_2^2 \right] \]  \hspace{1cm} (3.10a)
where \( b_1, b_2, b_3, \) and \( b \) depend on Poisson's ratio as given in Table 3.1.

**Takemiya's Model for Force-Displacement Relationship** -- In Eq. 3.8, the quantities \( k \) and \( c \) are constants. This is equivalent to approximating the supporting medium by a spring and a dashpot arranged in parallel, which are independent of the exciting frequency. However, the quantities \( k \) and \( c \) in Eq. 3.10 are frequency-dependent. To obtain a substructural system with frequency-independent parameters, Takemiya (1977) proposed a model, in which the soil reactions are represented by a proper arrangement of springs and dashpots, to yield the desired frequency-dependent impedance functions. This arrangement is shown in Fig. 3.4 in which the substructure is modeled by a two DOF systems represented by a third-order system for the rocking motion and a Voigt-type (first-order) system for the translational motion. Using these models, the corresponding impedance functions can be determined by comparing with those of Eqs. 3.9 and 3.11 (Veletsos and Verbic, 1974); on this basis, the system parameters are found as:

\[
\begin{align*}
   k &= K_x, \\
   c &= b \frac{(b_2 a_0)^2}{1 + (b_2 a_0)^2} \\
   m &= b \frac{(b_2 a_0)^2}{1 - b_2 \frac{1 + b_2 a_0}{a_3}} \\
   k &= K_\theta \\
   c &= -c_r = b \frac{(r/V) K}{s \theta} \\
   m &= \frac{2}{3} \frac{k r_3}{\theta \theta} \\
   k &= -b K \theta \\
   c &= b \frac{(r/V) K}{s \theta} \\
\end{align*}
\]
where \( k, k_1, k_2 \) are spring constants; \( c, c_1, c_2 \) are damping constants; and \( m \) is the added mass associated with the rocking motion. Because the above system parameters are frequency-independent, the direct time domain analysis can be performed for the soil-structure system. This model is the basis for an extension to the nonlinear cases described in the following sections.

**Equations of Motion of Massless Foundation** — In the above model, the interacting shear force \( H(t) \) and bending moment \( M(t) \) are

\[
H(t) = c \ddot{x} + k \dot{x} \quad (3.14)
\]

and,

\[
M(t) = m \ddot{\theta} + c \dot{\theta} + k_1 \theta + k_{r1} \theta + k_{r2} (\theta - \Theta) \quad (3.15)
\]

in which \( \theta \) is an additional displacement parameter (see Fig. 3.4b) introduced to obtain the frequency-dependent impedance function for rocking motion; \( \theta \) and \( \Theta \) are related by the following equation

\[
c \frac{\dot{\theta}}{r2} = k \frac{(\theta - \Theta)}{r2} \quad (3.16)
\]

### 3.3.2 Effect of Base Plate Uplift

Under severe earthquake excitations, large overturning moments may lead to tension in part of the contact area between the soil and the base of a structure. In Sect. 3.3.1, it is assumed that there is a perfect bond at the contact area. If no tensile capacity (e.g., through prestressed anchors, piles) is provided under the base, the foundation may partially separate from the soil. This phenomenon of partial separation (uplifting) of the structural base from the soil during strong ground shaking has been observed in many earthquakes (Psycharis and Jennings, 1983).

**Model Adopted and Assumptions** — The Takemiya's model described in the last section is extended to include the geometric nonlinearity due to foundation uplifting. The analytical expression of the rocking
moment–rotation relationship derived by Wolf (1976) and the differential equation model, Eq. 2.3, are used to evaluate the nonlinear hysteretic parameters and the expected values of equivalent circular radii as follows.

Wolf's Rocking Moment–Rotation Relationship — Consider the base plate with partial separation from the soil as shown in Fig. 3.5, in which

\[ W = \text{the vertical force (} W = W_s \text{)}; \]
\[ s \]
\[ x = \text{the distance from the center of gravity of the contact area to the center of the whole plate}; \]
\[ r_1 = \text{the equivalent radius for translational motion}; \]
\[ r_2 = \text{the equivalent radius for rocking motion}; \]
\[ M = \text{the overturning moment}; \]
\[ H = \text{the horizontal force (} H = H_s \text{)}; \]
\[ W, M, H = \text{the reaction forces of the soil on the base plate}. \]

Assuming that only normal stress in compression and corresponding shear stress (friction stress) can occur in the contact area, and using the static influence coefficients of the elastic halfspace, Wolf (1976) proposed an approximate method to determine the contact area for a given overturning moment and normal force. The separation occurs when the overturning moment \( M \) exceeds the product of the vertical force \( W \) and one-third of the radius \( r \). The actual irregular area of contact may be replaced by an equivalent circular plate with the same center of gravity. The radii of the equivalent circular plate are calculated by equating the area for the translational motion and the moment of inertia for the rocking motion. After uplifting, the equivalent radii \( r_1 \) and \( r_2 \) are functions of \( M, x, \) and \( W \) as shown in Fig. 3.6 and (for \( 1/3 < M/Wr < 1 \))

\[ r_1 = -1.074 \frac{M_o^2}{W^2r} - 0.068 \frac{M_o}{W} + 1.142r \]  
(3.17a)

\[ r_2 = 1.5r - 1.5 \frac{M_o}{W} \]  
(3.17b)
\[ x_s = 0.5r - 1.5 \frac{M}{W} \]  

(3.17c)

where \( M \) and \( W \) are the static forces, not including the complex and frequency-dependent parts. The impedance functions relative to the center of contact can be estimated by substituting an equivalent circular plate for the actual area of contact, and can also be represented with equivalent springs and viscous dashpots. Therefore, the nonlinear stiffnesses and dampings in the impedance functions of the base plate are derived by transforming the equivalent lumped system to the center of the plate.

Examining the system parameters in Eq. 3.13 (Takemiya's model), it is obvious that the spring constant \( k \) is equivalent to the static stiffness \( k \) in rocking motion. Hence, the spring constant \( k \) may be used to represent the nonlinear force-displacement relationship. From Fig. 3.5,

\[ M = k_0 \Theta + Wx - H x \tan \Theta \]  

(3.18a)

where \[ k_{rlu} = \frac{8Gr^3}{3(1-v)} \]  

(3.18b)

The last term in Eq. 3.18a, \( H x \tan \Theta \), is negligibly small compared to \( Wx \), as \( \Theta \) is usually very small. Therefore, Eq. 3.18a can be reduced to

\[ M = k_0 \Theta + Wx \]  

(3.18c)

Substituting Eqs. 3.17b and 3.17c into Eq. 3.18c yields

\[ \Theta = \frac{5W^3(1-v)(M_o - 0.2Wr)}{18G(Wr - M_o)^3} \]  

(3.19)
However, for $M / W_r < 1/3$ (without uplifting)

\[ M_o = \frac{8Gr^3}{3(1-v)} \theta \]  

(3.20)

or

\[ \theta = \frac{3M_o (1-v)}{8Gr^3} \]  

(3.21)

### Hysteretic Model Parameters and Expected Equivalent Radii

The nonlinear relationship between the rocking moment $M$ and rotation $\theta$ can be described by Eqs. 3.19 and 3.21 as shown in Fig. 3.7. The smooth hysteretic model presented in Sect. 2.2 is used to characterize this nonlinear elastic rocking behavior as follows.

\[ \dot{M}_o = a_k \theta \dot{\theta} + (1-a)k \dot{\theta} \theta \theta \theta \]  

(3.22a)

\[ \ddot{z}_\theta = \ddot{\theta} - y \dot{\theta} |z|_\theta^n \]  

(3.22b)

Eq. 3.22 is a nonlinear elastic equation (no hysteresis). After repeated testing and comparison, the best curve that fits the theoretical solution (Eqs. 3.19 and 3.21) is shown in Fig. 3.7, in which the yield strength $q$ in Eq. 2.8 is $q = 0.57W_r$, the post-yield stiffness is $a K_y = 0.010K_y$, and $n = 0.75$. Then the value of $y$ can be estimated by

\[ y = \left( \frac{K_y}{0.57W_r} \right)^{0.75} \]  

(3.23)

Therefore, the rocking moment described by Eqs. 3.18(c) and 3.20 can be represented with Eq. 3.22 which can be solved without difficulty by
the equivalent linearization technique. However, the equivalent radii \( r_1 \) and \( r_2 \), in Eq. 3.17 are nonlinear functions of the rocking moment \( M \). Hence, the expected values of \( r_1 \) and \( r_2 \) may be obtained from the distribution function of the rocking moment \( M \) in the random vibration analysis. The rocking moment \( M \) is assumed to be an approximating zero-mean Gaussian variate. The expected values of the equivalent radii are

\[
E[r_1] = \int_{-\infty}^{\infty} r_1 f_r(m) \, dm
\]

\[
= 1.142r \cdot \text{erf}(M) - 0.142r \cdot \text{erf}\left(\frac{M}{3}\right)
\]

\[
+ \frac{0.136\sigma}{\sqrt{2\pi}} m \left( \exp\left(-M^2\right) - \exp\left(-\frac{M^2}{9}\right) \right)
\]

\[
- \frac{2.148}{\sqrt{2\pi} \sigma} \int_{-\infty}^{\infty} m^2 \exp\left(-\frac{m^2}{2\sigma^2}\right) \, dm
\]

in which \( M = \frac{W r}{\sqrt{2} \sigma} \), \( f_r = \) density function of \( M \), and \( \sigma = m_{o} \) root-mean-square value of \( M_{o} \).

Using the smooth nonlinear elastic model in rocking motion and the expected equivalent radii \( r_1 \) and \( r_2 \) in translational and rocking motions, respectively, the Takemiya's model shown in Fig. 3.4 for an elastic substructure may be extended to characterize the nonlinear uplift effect shown in Fig. 3.8. The corresponding system parameters, therefore, are:
For translational motion:

\[ k = \frac{8Gr_1}{xu} \quad , \quad c = b (r_1/\nu) \frac{k}{xu} \quad (3.26) \]

For rocking motion:

\[ k = \frac{8Gr_1^3}{r_1} \quad , \quad k = -b k \quad , \quad m = b (r_1/\nu)^2 \frac{k}{ru} \quad (3.27a) \]

\[ c = -c = b b (r_1/\nu) k \quad , \quad k = \frac{8Gr_2^3}{3(1-\nu)} \quad (3.27b) \]

In the solution procedure, the equivalent radii \( r_1 \) and \( r \) in Eqs. 3.26 and 3.27 are updated by the expected values in Eqs. 3.24 and 3.25, respectively, in first-order approximation.

Equations of Motion of Massless Foundation -- From the model shown in Fig. 3.8, the interacting shear force \( H(t) \) and bending moment \( M(t) \), may be represented with

\[ H(t) = c \frac{\dot{x}}{x_0} + k \frac{x}{x_0} \quad (3.28) \]

\[ M(t) = m \frac{\ddot{\theta}}{r_1u} + c \frac{\ddot{\theta}}{r_1u} + k \frac{(\theta_1-\theta)}{r_2u} + a k \frac{\theta}{r_1} \]

\[ +(1-a)k \frac{z}{\theta} \quad (3.29) \]

in which;

\[ \frac{\dot{z}}{\theta} = \dot{\theta} - \gamma \dot{\theta} |z| |^n \quad (3.30) \]
\[ (b_1 r)_s + V \theta - V \theta = 0 \]  
\[ \frac{2}{2} \frac{2}{s 1} \ \ \ \mathrm{s} \]  
(3.31)

3.3.3 Nonlinear Halfspace System with Base Plate Uplift

The soil shear modulus in Sections 3.3.1 and 3.3.2 is assumed to be a constant. However, the behavior of the soil becomes nonlinear and inelastic, i.e., the shear modulus decreases and the hysteretic energy dissipation (material damping) increases as the strain increases. Moreover, under large rocking moments, the foundation may become partially separated from the soil. As uplifting occurs, a large toe pressure is induced in the remaining small area of contact. This confining pressure will easily exceed the elastic limit of the soil. The Winkler-type foundation (Wolf and Skrikerud, 1978; Fukuzawa, et al. 1981; Yim and Chopra, 1983) is often used to model the plastic flow of the soil in the area of the outer edge of the footing.

Model Adopted and Assumptions -- The Takemiya's model described in Sect. 3.3.1 is extended to include soil hysteresis and foundation uplifting. The Winkler-type foundation is adopted to estimate the foundation contact area (or equivalent circular radii) under rocking moments. The smooth hysteretic model is used to represent the rocking moment-rotation relationship, and the nonlinear hysteretic parameters are evaluated by available experimental data as follows.

The Winkler-Type Foundation -- In the Winkler-type foundation as shown in Fig. 3.9(a), the soil is represented by vertical, elastic-plastic springs which act in compression only. The spring constants in the elastic range may be obtained from an elastic halfspace solution or from the coefficient of subgrade reaction. The plastic spring force per unit area of contact is assumed to be equal to the ultimate bearing capacity of the soil, \( q \). Although the Winkler model is a discrete representation and does not account for the frequency dependence of the impedance functions, it may be regarded as a useful engineering approximation.

From the Winkler-type foundation, the rocking moments at beginning of uplifting and at ultimate condition, and the minimum equivalent
circular radii are derived as shown in Appendix C. Following the same procedures as described in the last section, the expected equivalent radii are evaluated as presented in Appendix C.

**Hysteretic Model** — Laboratory tests indicated that the energy dissipation of soils is independent of the frequency of vibration. Therefore, the smooth hysteretic model described in Sect. 2.2 may be used with Takemiya's model to include the material energy dissipation and local nonlinearity associated with the separation of the foundation from the soil. Because the system parameters $k$ and $k_1$ in the model are the static foundation stiffnesses in translational and rocking motions, respectively, they may be used to represent the smooth hysteresis as shown in Fig. 3.11 in which

\[ k_{xn} = \frac{8G_1}{2-\nu}, \quad c_{xn} = b \times \frac{r_1}{V_{sn}} \times \frac{k_{xn}}{G} \quad (3.32) \]

\[ k_{r1} = \frac{8G_3}{3(1-\nu)}, \quad k_{r2n} = -b_1k_{\theta n}, \quad m_{rn} = b_3\left(\frac{r_2}{V_{sn}}\right)^2k_{\theta n} \quad (3.33) \]

\[ c_{r1n} = -c_{r2n} = b_1b_2\left(\frac{r_2}{V_{sn}}\right)k_{\theta n}, \quad V_{sn} = \sqrt{\frac{G}{\rho}}, \quad k_{\theta n} = \frac{8G_3r_3^3}{3(1-\nu)} \quad (3.34) \]

where: $G =$ initial (static) shear modulus;

$G = $ displacement-dependent (or strain-dependent) shear modulus.

The hysteretic parameters in the model may be obtained from the force-displacement relationship and the ultimate restoring force.

**Hysteretic Parameters for Foundation Translation** — The shear stresses (frictional stresses) are developed between the foundation and the soil by foundation translational motion. The hysteretic behavior of the friction force-displacement may be approximated by the shearing stress-strain relationship of the soil deposit. Hence, the hysteretic parameters proposed by Pires, Wen and Ang (1983) for the soil may be used in foundation translational motion; i.e., $A = 1, \beta = \gamma, n = 0.50$
first-order approximation).

From the system shown in Fig. 3.11, the interacting forces, shear force $H(t)$ and rocking moment $M(t)$, are

$$H(t) = c \ddot{x} + a k x + (1-a)k x$$  \hspace{1cm} (3.36)

$$M(t) = m \ddot{\theta} + c \ddot{\theta} + a k \theta + (1-a)k \theta$$

$$+ k (\theta - \theta)$$

and

$$c \ddot{\theta} = k (\theta - \theta)$$  \hspace{1cm} (3.38)

$$\dot{z} = A \ddot{x} - \beta |\ddot{x}| z^{n_x-1} - \gamma |\ddot{x}| z^{n_x}$$

$$\dot{\theta} = A \ddot{\theta} - \beta |\ddot{\theta}| \theta^{n_\theta-1} - \gamma |\ddot{\theta}| \theta^{n_\theta}$$  \hspace{1cm} (3.39)

3.4 The Soil-Structure System

The model adopted for a coupled soil-structure system is shown in Fig. 3.14. The equations of motion of such a system with a $n$-story superstructure building may be written as follows:

$$m (\ddot{x} + \ddot{\theta}) + \{1\} \{M\} \{\ddot{y}\} + H(t) = 0$$  \hspace{1cm} (3.41)
\[ J \ddot{\theta} + [h][M][\ddot{y}] + M(t) = 0 \]  
(3.42)

\[ m \ddot{y} + q - q_{\text{i}i} (1-\delta_{i,i+1}) = 0, \quad (i = 1, n) \]  
(3.43)

where:

- \( m \) = the mass of the foundation;
- \( b \) = the free-field ground displacement in the absence of the building;
- \([1]\) = the unit vector of order \((n \times 1)\);
- \([M]\) = the diagonal mass matrix of order \(n\) of the superstructure,
- \( m_i \) = the mass of the \(i\)th floor;
- \( J_i \) = the sum of the mass moment of inertia of all floors including the foundation about their respective centroidal axes;
- \([h]\) = vector of the heights of the floor masses above the foundation;
- \( \delta_{i} \) = Kronecker delta;
- \( q_i \) = \( c_i \dot{u}_i + a_k u_i + (1-\alpha_i)k_i z_i \), the \(i\)th interstory inelastic restoring force including viscous damping; where \( z_i \) is the hysteretic restoring force, and \( u_i \) is interstory displacement;
- \([y]\) = the absolute displacement vector of order \((n \times 1)\), given by

\[ [y] = [x] + [1]x + [h]\theta + [1]b \]  
(3.44)

in which \([x]\) is the deformation vector of order \((n \times 1)\) relative to the foundation; \( x_i \) is the \(i\)th floor deformation, obtained from the interstory displacement \([u]\) as

\[ x_i = \sum_{j=1}^{i} u_j \]  
(3.45)

and
\[ u = x; \quad u = x - x \quad \text{for } i = 2, n \quad (3.46) \]

The ground excitation, \( a \), is modeled as a Kanai-Tajimi filtered white noise. The filter properties are obtained by assuming that a
\( \text{white noise rock-base excitation } a \) acts upon a single DOF system representing the soil layer. The relative displacement at the ground
level \( u \) satisfies the following equation,

\[ \ddot{u} + 2\beta \omega \dot{u} + (\omega^2) u = -a \quad (3.47) \]

in which \( \omega \) and \( \beta \) are filter parameters. The absolute ground
acceleration, \( a \), is

\[ a = \ddot{u} + a = -2\beta \omega \dot{u} - (\omega^2) u \quad (3.48) \]

The governing equations of motion of a linear or nonlinear
soil-structure system in terms of interstory displacements may be
obtained from Eqs. 3.41 through 3.48 by substituting the interacting
forces in Sect. 3.3, and may be rewritten as a system of first-order
differential equations as

\[ A \left( \frac{d}{dt} Y \right) = DY + C \quad (3.49) \]

where:

\[ Y = (x, \dot{x}, z, \Theta, \dot{\Theta}, z, \dot{z}, u, \dot{u}, u, \dot{u}, \ldots, u, \dot{u}, z); \quad (3.50) \]

\[ C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^{T} a; \quad (3.51) \]

\[ A = \text{the coefficient matrix, containing the masses and heights of} \]

\[ D = \text{the matrix of the system coefficients including the} \]

equivalent linear coefficients, $C_{ei}^-, K_{ei}^-$  

Premultiplying Eq. 3.49 by $A^{-1}$ yields

$$\frac{d}{dt}Y = GY + F$$  \hspace{1cm} (3.52)

where $G = A^{-1}D$, and $F = A^{-1}C$  \hspace{1cm} (3.53)

Because Eq. 3.52 has the same form as that of Eq. 2.13, the stochastic equivalent linearization described in Sect. 2.3 can be applied directly. However, the algorithm proposed by Bartels and Stewart (1972), which is efficient solving the structural system (fixed ground) or the soil deposit problem by itself, is not directly suitable for the soil-structure system. Because the structural stiffness is much smaller than that of the foundation, the algorithm proposed by Parlett and Reinsch (1969) is adopted to scale the matrix $G$ of Eq. 2.14 before solving the equations. From the matrix $G$, the corresponding permutation matrix $P$ and the non-singular diagonal matrix $D$ can be obtained to assure that $DPGPD^{-1}$ is a balanced matrix. Postmultiplying Eq. 2.14 (in which $S = 0$) by $Q = PD$, and then premultiplying by $Q^T$ yields

$$Q^T G(Q^T) Q SQ + Q^T SQQ^{-1} Q^T = Q^T BQ$$  \hspace{1cm} (3.54)

Let $S = Q^T SQ$, $G = Q^T G(Q^T)$ and $B = Q^T BQ$, then

$$G S + S (G^T) = B$$  \hspace{1cm} (3.55)

Eq. 3.55 can then be solved by the algorithm developed by Bartels and Stewart (1972) and the zero time-lag covariance matrix $S$ obtains from $S_{-1} = (Q^T)^{-1} S Q_{-1}$.
For the nonstationary case, the solution of Eq. 2.14 requires numerical integration. Since the differential equations contain the matrix G which is very stiff, the Chord method with an analytic Jacobian matrix (Hindmarsh, 1974) may be used to solve the equations effectively.

The RMS values of the relative story displacement \( x \), the expected accelerations of the foundation in translation and rocking, and the expected base shear can be estimated from the equations of motion of the soil-structure system and the covariance matrix \( S \). Also, the stochastic energy distribution in the building-foundation system can be evaluated from the covariance matrix \( S \) described in Appendix D.
Table 3.1 Values of $b_x$, $b_1$, $b_2$ and $b_3$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Poisson's Ratio, $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.</td>
</tr>
<tr>
<td>$b_x$</td>
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</tr>
<tr>
<td>$b_1$</td>
<td>0.525</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.80</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
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- $n = 0.50$
- $n = 0.75$
- $n = 1.00$

Hysteretic model

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CHAPTER 4

SIGNIFICANCE OF SOIL-STRUCTURE INTERACTION

4.1 Introduction

The principles and general procedures described in the earlier chapters can be used for different building-foundation systems to investigate the significance of soil-structure interaction on linear and nonlinear building-foundation systems. Single-DOF buildings with different properties (steel or reinforced concrete buildings with linear or nonlinear structural material; also including slender and squatty structures with large or small foundations) supported on different soil deposits, are analyzed first to investigate the interaction effect on linear and nonlinear coupled systems. The effect of nonlinearities (from structural and soil properties, and from foundation uplifting) on the soil-structure interaction is also discussed. The accuracy of the proposed models for nonlinear substructures is appraised.

A ten-story steel building designed according to the UBC (Uniform Building Code) is used to demonstrate the reduced degree-of-freedom (DOF) technique; a four-story steel building designed according to UBC is studied to evaluate the interaction effect on the nonlinear coupled system. These two buildings are assumed to be supported on a soft soil deposit. Finally an idealized single-DOF reactor building supported on an intermediate soil deposit is considered to evaluate the interaction effect for a massive and stiff structure.

4.1.1 Summary of Main Results

(1) For linear coupled systems, the effect of interaction significantly increases the response for slender structures, but decreases for squatty structures. Moreover, coupled systems with the same aspect ratio but different foundation size may have different
effect of soil-structure interaction.

(2) Each nonlinearity in the coupled system (structural material, soil hysteresis, and foundation uplifting) may increase or decrease the structural response relative to the corresponding linear coupled system. However, the structural material nonlinearity seems to dominate this effect. Considering all of the nonlinearities in a coupled system, the nonlinear effect of interaction on conventional buildings is generally small especially at high excitation levels.

(3) Because the structural response remains largely in the elastic range for a stiff structure at high excitation levels and the effect of foundation uplifting and soil material nonlinearity is small, the response of a massive and stiff building is essentially the same as that of a linear system; in particular, soil-structure interaction will significantly reduce the response even at high excitation levels.

4.2 Single-DOF Buildings

This section begins with a description of the structural systems and ground motion employed in this study. Thereafter, the response of linear coupled systems is first evaluated and compared with those of previous soil-structure interaction studies. The response of nonlinear coupled systems are investigated to illustrate the effect of nonlinearities (geometric and material nonlinearities) on the interaction.

4.2.1 Systems Description

The important characteristics of soil-structure interaction may be examined on the basis of single-DOF systems. For this purpose, the following systems were analyzed: (1) linear superstructure and linear substructure; (2) nonlinear superstructure and linear substructure; (3) nonlinear superstructure and linear soil deposit with foundation uplifting; and (4) nonlinear superstructure and nonlinear substructure (nonlinear soil deposit with foundation uplifting). The superstructure
may be viewed either as a model of a one-story building or as a one-DOF approximation of a multistory structure.

The following dimensionless parameters are normally introduced:

\[ \lambda_1 = \frac{h^2}{Kh/(\pi r G)} \quad \lambda_2 = \frac{h}{r}, \quad \lambda_3 = \frac{m}{m_0}, \quad \lambda_4 = \frac{m}{(\pi r h p)} \]

(4.1)

where:
\[ k = \text{stiffness of building}; \]
\[ h = \text{height of building}; \]
\[ m = \text{mass of building}; \]
\[ m_0 = \text{mass of foundation}; \]
\[ r = \text{radius of foundation}; \]
\[ G = \text{initial shear modulus of soil deposit}; \]
\[ \rho = \text{soil mass density}. \]

The parameter \( \lambda_1 \) measures the relative stiffness between the superstructure and the foundation medium; \( \lambda_2 \) is the aspect ratio of the superstructure; \( \lambda_3 \) is the ratio of the foundation mass to the structural mass; and \( \lambda_4 \) is a relative mass density of the structure to the supporting medium.

The aspect ratio \( \lambda_2 \) is assumed to be either 1 or 5, representing conventional squatty or slender structures, respectively. The foundation mass ratio \( \lambda_3 \) is assumed to be 1. The mass of the foundation, \( m_0 \), may be defined as the entire mass of a raft-type foundation plus rigidly attached machines, stored goods, etc.

The parameter \( \lambda_4 \) varies within a narrow range for conventional steel or reinforced concrete buildings. Multistory reinforced concrete buildings (Blume, Newmark and Corning, 1961) have an average density of 17 pcf; whereas multistory steel buildings (Pigue, 1976) have an average density of 7.5 pcf. Because of the significant difference in density,
reinforced concrete buildings and steel buildings are analyzed separately. The densities are taken to be 16 pcf for reinforced concrete buildings and 8 pcf for steel buildings in the study.

Therefore, steel and reinforced concrete buildings are considered separately in two sets of structures. In each set, four types of structures (of different aspect ratio and foundation size) are considered. The structural details are shown in Tables 4.1 and 4.2. The structural hysteresis parameters are \( A = 1, \alpha = 0.02, \xi = 0.04, \) \( n = 1, \) and \( \beta = -3\gamma = 66.66 \) for reinforced concrete buildings; whereas \( A = 1, \alpha = 0.04, \xi = 0.02, n = 1 \) and \( \beta = \gamma = 0.50 \) for steel buildings. In each type of structure, four coupled systems with linear or nonlinear behavior are considered.

Because the variation of the density ratio is small for conventional steel or reinforced concrete buildings, the significance of interaction is determined largely by the parameter \( \lambda. \) To evaluate this parameter, the unit weight of soil deposits is assumed to be 100 pcf for soft soil, 115 pcf for intermediate soil, and 130 pcf for hard soil. The shear wave velocity, \( V_s = \sqrt{G/\rho}, \) is about 500 fps for dense sand, 800 fps for medium clay, and 1500 fps for cemented sand (Whitman, 1972). Hence, the following ranges of soil shear wave velocity, \( V_s, \) are assumed:

\[
V_s (\text{ft/sec}) < 800 \quad \text{for soft ground} \quad (4.2a)
\]

\[
800 < V_s (\text{ft/sec}) < 1600 \quad \text{for intermediate ground} \quad (4.2b)
\]

\[
V_s (\text{ft/sec}) > 1600 \quad \text{for hard ground} \quad (4.2c)
\]

Eight different soil shear modulus, i.e., 2000, 300, 100, 30, 10, 3, 1 and 0.18 kip/in/in, are considered for each coupled system in the dynamic analysis. From the structural properties shown in Tables 4.1 and 4.2, the equivalent stiffness ratio, \( \lambda, \) for hard ground is less
than 0.00284 for steel buildings, and less than 0.0284 for reinforced concrete buildings; for soft ground, $\lambda_1$ is larger than 0.0144 for steel buildings, and larger than 0.144 for reinforced concrete buildings. For intermediate ground, the equivalent stiffness ratio, $\lambda_1$, is between those for hard and soft ground conditions.

When only foundation uplifting (with linear soil deposit) is considered, the foundation hysteresis parameters in rocking motion are $\alpha = 0.01$, $\beta = 0.0$, $n = 0.75$, and $A = 1$; whereas when the substructural system is treated as nonlinear (geometric and material nonlinearities), the foundation hysteresis parameters are $\alpha = 0$, $\beta = \gamma$, $A = 1$ and $n = 0.5$ for translation, and $\alpha = 0$, $\beta/\gamma = 0.005$, $A = 1$ and $n = 0.5$ for rocking.

4.2.2 Load Description

The ground motion at a given site is influenced by the properties and geologic features of the subsurface soil and rock. This site effect is included by modelling the ground as being hard, intermediate or soft soils (see Sect.5.4). For different ground conditions, the spectral parameters, the mean strong motion durations, and the peak factors are given in Tables 5.5 and 5.6. The relationships between the expected maximum ground acceleration and the power spectral intensity, $S_0$, are given by

$$E[a_{\text{max}}] = 24.7\sqrt{S_0} \quad \text{for soft ground} \quad (4.3a)$$

$$E[a_{\text{max}}] = 28.4\sqrt{S_0} \quad \text{for intermediate ground} \quad (4.3b)$$

$$E[a_{\text{max}}] = 29.2\sqrt{S_0} \quad \text{for hard ground} \quad (4.3c)$$

Six levels of excitation with maximum ground acceleration of 0.1g, 0.2g, 0.4g, 0.6g, 0.8g and 1.0g, respectively, are selected as input
ground motion for the coupled systems and the corresponding fixed-base structures. To investigate the effect of interaction, the stationary response statistics are calculated for each coupled system and corresponding fixed-base structure.

To isolate the interaction effect, the same ground motion conditions are used both in the coupled systems and in the corresponding fixed-base structures to calculate the dynamic response or the response ratio (the response of coupled system to the response of corresponding fixed-base structure).

4.2.3 Results of Linear Building-Foundation Systems

Previous Studies -- From previous studies of linear coupled systems under deterministic excitations, the effect of interaction has been found: (1) to decrease the resonant frequency of the system; (2) to modify the magnitude of the peak response by decreasing the value for squatty structures and increasing the value for slender structures. Sarrazin and Roesset (1972) studied the linear coupled system subjected to random excitations (white noise input), and obtained the RMS structural relative displacements which have the same trend as the peak response described above. Generally, the interaction effect can be measured by the change in the peak responses (deterministic analysis) or in the RMS responses (random response analysis). However, for the linear coupled systems with the same aspect ratio, the effect of different foundation size on the interaction has not been discussed.

RMS Structural Displacement -- The effect of interaction on the RMS structural relative displacement (with respect to the foundation) is obtained as shown in Figs. 4.1(a) and (b) for steel and reinforced concrete buildings, respectively, in which the RMS displacement is normalized with respect to that of corresponding fixed-base structure. It is apparent from Figs. 4.1(a) and (b) that the RMS structural relative displacement has the same trend described above, i.e., the interaction decreases the displacement of squatty structures ($\lambda = 1$) but increases the displacement of slender structures ($\lambda = 5$). Moreover, a squatty coupled system ($\lambda = 1$) with large foundation ($r = 432 \text{ in}$) has
smaller displacement ratio (or larger interaction effect) than a squatty system with small foundation \((r = 144 \text{ in})\). When a slender coupled system \((\lambda = 5)\) with large foundation \((r = 432 \text{ in})\) compared to a slender system with small foundation \((r = 144 \text{ in})\), the reinforced concrete building has smaller displacement ratio (or smaller interaction effect), but the steel building has larger displacement ratio (or larger interaction effect). Generally, a reinforced concrete building has a larger interaction effect for intermediate ground condition than a corresponding steel building. However, the effect of interaction on hard ground condition is very small and can be neglected for linear conventional buildings.

**Two Competing Mechanisms** -- The decrease or increase in the structural relative displacement associated with interaction, may be explained by the net result of two competing mechanisms (Veletsos and Meek, 1974). The energy dissipated by radiation into the soil medium tends to decrease the structural response. However, the rocking and lateral swaying of the foundation tend to increase the structural motion and inertia force. The first factor may be the controlling one for squatty structures; whereas the second factor would dominate the response of slender structures. From the equations of motion presented in Sect. 3.4, the RMS foundation accelerations in translational and rocking modes, and the RMS structural relative acceleration (with respect to the foundation) can be obtained as shown in Figs. 4.2, 4.3 and 4.4, respectively. It is observed that the foundation accelerations in translational and rocking modes are increased, but the structural relative acceleration is decreased by the interaction. Because of the foundation accelerations, the absolute structural acceleration and the associated inertia force may be increased by the interaction for slender structures.

**4.2.4 Results of Nonlinear Building-Foundation Systems**

The nonlinear behavior of building-foundation systems may be caused by the foundation uplifting (geometric nonlinearity) and/or by the inelastic response (material nonlinearity) of the structure or soil
To investigate the effect of nonlinearities on the interaction, the response of nonlinear coupled systems are evaluated and compared to corresponding linear coupled systems. The results of previous studies considering a part of nonlinearities in the coupled system are first described. Also the accuracy of rocking hysteresis parameters is demonstrated.

**Previous Studies for Material Nonlinearities** — For an elastic structure on a viscoelastic halfspace, Veletsos and Nair (1975) found that the viscoelastic action decreases the structural response in the high-frequency and medium-frequency regions of the response spectra, but increases the structural response in the low-frequency region. Bielak (1978) investigated the steady-state harmonic response of a simple bilinear hysteretic structure supported on a viscoelastic halfspace and observed that a nonlinear hysteretic structure may have larger displacement than that of a rigid base structure. Considering only translational mode of the foundation supported on an elastic boundary layer by using the Markov-vector approach, Asano (1982) found that the interaction may reduce the structural response for structures with relative large post-yielding stiffness, but may increase the structural response for structures with small post-yielding stiffness. From these studies, it is obvious that the material nonlinearity in the structure or soil deposit will affect the interaction effect.

**Previous Studies for Foundation Uplifting** — Examining the case of a linear single-DOF oscillator rocking on a rigid foundation, Meek (1975) observed that the foundation tipping leads to a favorable reduction in the maximum transverse deformation. Wolf (1975), and Wolf and Skrikerud (1978) obtained that allowing a linear structure to uplift leads to a reduction of the total horizontal acceleration, the overturning moment, and the lateral displacement within the structure. Psycharis (1983) observed that the apparent fundamental period increases with the uplifting, resulting in larger rocking motion of the linear multi-story buildings. Yim and Chopra (1983) observed that the foundation uplifting reduces the deformation and forces of linear multi-story buildings. So far, the analysis of structures with foundation uplifting is limited to
linear structures supported on linear or elastic-plastic foundations subjected to deterministic excitations.

**Nonlinear Solutions by Hysteresis Model** -- The results of four kinds of coupled systems are obtained as shown in Figs. 4.5 to 4.15. The first coupled system is a linear building-foundation system and the other three are nonlinear building-foundation systems. The nonlinearities include the structural material nonlinearity, the structural and geometric nonlinearities, and combinations of all nonlinearities.

It is observed that the material or geometric nonlinearity in the coupled systems may increase or decrease the interaction effect relative to the corresponding linear coupled systems as described follows:

1. **Structural material nonlinearity:** The expected structural stiffness of a coupled system may be obtained from the response statistics as described in Sect. 2.4.1 and shown in Figs. 4.5 and 4.6 for steel and reinforced concrete buildings, respectively. The expected structural stiffness decreases with the excitation level (the site effect is included in Figs. 4.5 and 4.6). However, the expected structural stiffness increases for squatty structures ($\lambda = 1$) but decreases for slender structures ($\lambda = 5$) by the interaction. The effect of geometric and soil material nonlinearities on the expected structural stiffness is very small. In this light, the structural material nonlinearity may decrease the expected structural stiffness, but may increase the energy dissipation in the system. Therefore, the structural material nonlinearity may increase or decrease the structural response.

2. **Geometric nonlinearity:** Foundation uplifting decreases the expected equivalent radii as shown in Figs. 4.7 and 4.8 (for linear and nonlinear soil deposits, respectively), but increases the foundation responses (e.g., displacement) (Wolf and Skrikerud, 1978). Therefore, the energy dissipated by foundation uplift is decreased by the reduction of the expected radii, but increased by the increment of the foundation response. Figs. 4.7 and 4.8 also show that the expected radii are increased by the interaction (or with the soil flexibility) for a
squatty coupled system but are decreased for a slender coupled system. Furthermore, including the soil material nonlinearity in the coupled systems decreases the expected equivalent radii. Hence, the net effect of foundation uplifting may decrease or increase the structural response. However, the reduction of the equivalent radii tends to dominate the interaction effect on structural response; hence, foundation uplifting usually decreases the structural response.

(3) Soil material nonlinearity: As shown in Figs. 4.9 and 4.10 (for steel and reinforced concrete systems, respectively), the expected soil shear modulus ratios (normalized by the initial soil shear modulus) decrease with the excitation level, but increase with the interaction effect (or with the soil flexibility). Also, a coupled system with large foundation \((r = 432 \text{ in})\) has larger soil shear modulus ratio than the coupled system with the same aspect ratio but small foundation \((r = 144 \text{ in})\). A reduction of the soil shear modulus may increase or decrease the structural response as described earlier for linear coupled systems; the energy dissipated by soil hysteresis may decrease or increase the structural response (Veletsos and Nair, 1975). Therefore, the soil material nonlinearity may increase or decrease the structural response.

A phenomena is observed that the increasing ratio of foundation translational response (shown in Fig. 4.11) (which is used to measure the expected soil shear modulus) by the interaction effect, is smaller than the increasing ratio of the equivalent stiffness ratio \(\lambda\) (or smaller than the decreasing ratio of soil shear modulus in the equivalent stiffness ratio). From Sect. 2.4.1, it is obvious that the softer soil deposit has larger expected soil shear modulus ratio as shown in Figs. 4.9 and 4.10. This phenomena is reasonable, but it is not considered in the equivalent soil shear modulus ratio described in ATC-3 Code (1978) as presented in Table 4.3 in which the equivalent soil shear modulus ratio is a function of the excitation level only.

Because each nonlinearity in the coupled system may have two counter acting effects on a structure, the actual behavior of the nonlinear coupled system depends on specific values of the structural and soil properties, and excitation level. The effect of nonlinearities on the
response generally increases with the excitation level. The responses shown in Figs. 4.12 to 4.15 are for a maximum ground excitation of 0.6g. The specific effect of nonlinearities on the structural response may be summarized as follows:

Effect on RMS Response -- Compared with the corresponding linear coupled system, a coupled system with structural material nonlinearity has larger RMS structural displacement ratio (normalized by that of the corresponding fixed-base structure) for squatty structures, but may have smaller or larger RMS structural displacement ratio for slender structures as shown in Figs. 4.12 and 4.13 for steel and reinforced concrete systems, respectively. Among the different nonlinearities, the effects on the interaction are small. A squatty structure with large foundation appears to be most affected by the nonlinearities in the system.

Effect on Hysteretic Energy -- The interaction effect on structural hysteretic energy is significant for a squatty structure, but is small for slender structures, as shown in Figs. 4.14 and 4.15 for steel and reinforced concrete systems, respectively. The structural hysteretic energy may be increased or decreased by the interaction. The effects on the hysteretic energy among the different nonlinearities appear to be small.

Based on above observations, the structural material nonlinearity seems to have larger effect on the structural response than the other nonlinearities. Because of the competing mechanisms of each nonlinearity in the coupled system, the net result of nonlinearities may increase or decrease the structural responses.

Accuracy of Rocking Hysteresis Parameters -- In the theoretical moment-rotation relationship for elastic halfspace with foundation uplifting as described by Eqs. 3.19 and 3.21, the value of n is chosen as 0.75. Values of n = 0.5, 0.75 and 1.00 (shown in Fig. 3.7), are used to estimate the RMS relative structural displacement for reinforced concrete structures, and the results are not sensitive to n as shown in Table 4.4. Hence, the value of n = 0.75 is appropriate. To fit the experimental hysteretic moment-rotation relationship for nonlinear
halfspace with foundation uplifting (as shown in Fig. 3.12), the value of $\beta/\gamma$ in the hysteresis model is chosen as 0.005. A similar sensitivity study shows essentially the same results (Table 4.5). Therefore, a $\beta/\gamma$ of 0.005 is used.

### 4.3 Ten-Story UBC Steel Building

#### 4.3.1 Building Description

The structural frame that was originally designed by Pique (1976) according to the 1973 edition of the Uniform Building Code for Zone 3 seismic requirement is examined. The frame is assumed to have adequate bracings to resist out-of-plane motions. The building with four bays in the seismic direction, covering approximately 70 ft by 90 ft in plan area, is considered in this study. The floor framing plan and typical frame dimensions are shown in Fig. 4.16; the corresponding shear-beam model parameters are given in Table 4.6.

The equivalent story stiffness shown in Table 4.6 was obtained using Eq. 3.1; the story strength shown was estimated from an inelastic analysis. The values of the hysteretic parameters $A$, $a$ and $n$ are taken as $1$, $0.04$ and $1$, respectively. $\beta = \gamma$, and $\xi = 0.02$. The specific values of $\beta$ and $\gamma$ are shown in Table 4.6. The soil shear modulus is $1.94 \text{ kip/in/in (V \approx 300 ft/sec)}$ in this analysis. The equivalent radii of the foundation are $537.37$ inch ($r_1$) in translational motion, and $510.05$ inch ($r_2$) in rocking motion; the corresponding aspect ratios are $h/r_1 = 2.75$ and $h/r_2 = 2.89$.

#### 4.3.2 Discussion of Results

The system was analyzed for six levels of excitation with maximum ground accelerations ranging from $1/12g$ to $11/12g$ in $1/6g$ intervals. As the structural restoring force is nondegrading, a stationary response analysis was performed. The reduced-DOF technique can be used to analyze the coupled system and fixed-base building. To demonstrate this technique, the response of the fixed-base building is calculated by
different reduced-DOF systems. Then the response of the nonlinear coupled system is evaluated to illustrate the interaction effect.

The Reduced-DOF Technique — To examine this technique, the ten-story steel building was analyzed by full DOF system and six reduced DOF systems: 1 DOF with or without mode shape modification, 2 DOF with or without mode shape modification, 3 DOF and 4 DOF without mode shape modification. The RMS drifts and expected structural hysteretic energy were obtained as shown in Figs. 4.17 (a) and (b), respectively, in which excitation level at 7/12g is presented. Observations may be made as follows:

(1) RMS story drift: The difference between the RMS story drifts obtained by reduced-DOF and that by full DOF in the top story is smaller than that in the first story; whereas the difference in the middle stories is smaller than that in the top story. For the first story, the drifts obtained by the reduced-DOF are smaller than that by the full DOF, and the larger number of DOF gets better results. Moreover, the modification of mode shape improves the accuracy of reduced-DOF.

(2) Expected hysteretic energy: The difference between the expected hysteretic energy obtained by full DOF to those by reduced-DOF is large in high and low stories. The larger number of DOF gives better results and the modification of mode shape improves the accuracy.

Nonlinear Building-Foundation System — With soil material nonlinearity and foundation uplifting in the coupled system, the RMS response and expected hysteretic energy are obtained and compared with those of the corresponding fixed-base structure as shown in Fig. 4.18, in which the excitation levels at 1/12g and 7/12g are presented. It is observed that the effect of interaction decreases the RMS drifts and expected structural hysteretic energy at low excitation level ($a_{max} = 1/12g$) but increases the corresponding responses at high excitation level ($a_{max} = 7/12g$). However, this interaction effect on structural responses is small.

Using the equations of motion described in Sect. 3.4 and the covariance matrix $S$, the expected base shears of the fixed-base structure and corresponding coupled system are evaluated as shown in
Fig. 4.19. The expected base shear is reduced by the interaction but this reduction is small.

4.4 Four-Story UBC Steel Building

4.4.1 Building Description

The building has nine bays in the direction of the principal motion, covering approximately 65 ft by 190 ft in plan area. The floor plan and typical frame dimensions are shown in Fig. 4.20; the corresponding shear-beam model parameters and hysteresis parameters are given in Table 4.7.

The equivalent story stiffness shown in Table 4.7 was obtained using Eq.3.1; the story strength shown were estimated from an inelastic analysis. The values of the hysteretic parameters A, α and n = 1, 0.04 and 1, respectively, \( \beta = \gamma \), and \( \xi = 0.02 \). The specific values of \( \beta \) and \( \gamma \) are shown in Table 4.7.

4.4.2 Discussion of Results

The coupled system was analyzed for six levels of excitation with maximum ground acceleration ranging from \( 1/12g \) to \( 11/12g \) in \( 1/6g \) interval. The material nonlinearities of the structure and soil deposit, as well as foundation uplifting are considered in the coupled system.

The expected drift and hysteretic energy are obtained for the coupled system and the corresponding fixed-base structure as shown in Fig. 4.21, in which the excitation levels at \( 1/12g \) and \( 7/12g \) are presented. It is observed that the expected drift and hysteretic energy are decreased because of interaction at \( 1/12g \) excitation level; however, this effect is greatly reduced for the higher excitation levels. The response of a coupled system may become larger than that of the corresponding fixed-base structure as the excitation level increases.

The expected base shears are obtained for the coupled system and the corresponding fixed-base structure as shown in Fig. 4.22. The effect of
interaction decreases slightly the expected base shear especially at low excitation levels, but this effect becomes less significant at high excitation levels.

From the above analyses, it is observed that the effect of interaction on the response of conventional buildings is small especially at high excitation levels.

4.5 Idealized Single-DOF Reactor Building

4.5.1 System Description and Modeling

The soil-structure interaction may be significant for a massive and stiff building. To investigate the effect of interaction on such types of structures (e.g., nuclear reactor building), an idealized single-DOF reactor building is analyzed. The same idealized single-DOF reactor building was used also by Wolf and Skrikerud (1978). The structural parameters are given in Table 4.8. The values of the hysteretic parameters $A$, $a$ and $n$ are assumed to be 1, 0.04 and 1, respectively, $\beta = \gamma = 1.0$, and $\xi = 0.02$. The soil shear modulus considered is 35.7 kip/in/in ($V = 1200$ ft/sec).

4.5.2 Discussion of Results

The coupled system was analyzed for six levels of excitation with maximum ground acceleration ranging from $1/12g$ to $11/12g$ in $1/6g$ increment.

The RMS structural displacement and hysteretic energy are evaluated for the coupled system and for the corresponding fixed-base structure as shown in Fig. 4.23. The effect of interaction on the response of massive, stiff building appears to be significantly larger than that for conventional buildings. The RMS displacement and expected hysteretic energy are reduced by the interaction. The RMS displacement ratio (the RMS displacement of coupled system divided by that of corresponding fixed-base system) is 0.36 at an excitation level of $1/12g$ and is 0.42 at $11/12g$. On the other hand, the ratio of the expected hysteretic
energy is 0.020 at an excitation level of 1/12g and is 0.036 at 11/12g. Although the nonlinearities in conventional buildings tend to decrease the interaction effect, the effect of nonlinearities is small on a massive, stiff building. This is due to the fact that the structural response remains largely in the elastic range for a stiff structure at high excitation level, and the effect of foundation uplifting and soil material nonlinearity is small.
Table 4.1 Four Types of Single-DOF Steel Building

<table>
<thead>
<tr>
<th>( \lambda_2 )</th>
<th>( k ) (kip/in)</th>
<th>( m ) (kip-sec(^2)/in)</th>
<th>( m_0 ) (kip-in/m)</th>
<th>( h ) (in)</th>
<th>( r ) (in)</th>
<th>Period (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>0.112</td>
<td>0.112</td>
<td>144</td>
<td>144</td>
<td>0.222</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>0.56</td>
<td>0.56</td>
<td>720</td>
<td>144</td>
<td>1.11</td>
</tr>
<tr>
<td>1</td>
<td>270</td>
<td>3.036</td>
<td>3.036</td>
<td>432</td>
<td>432</td>
<td>0.666</td>
</tr>
<tr>
<td>5</td>
<td>54</td>
<td>15.18</td>
<td>15.18</td>
<td>2160</td>
<td>432</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Table 4.2 Four Types of Single-DOF Reinforced Concrete Building

<table>
<thead>
<tr>
<th>( \lambda_2 )</th>
<th>( k ) (kip/in)</th>
<th>( m ) (kip-sec(^2)/in)</th>
<th>( m_0 ) (kip-in/m)</th>
<th>( h ) (in)</th>
<th>( r ) (in)</th>
<th>Period (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
<td>0.224</td>
<td>0.224</td>
<td>144</td>
<td>144</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>180</td>
<td>1.12</td>
<td>1.12</td>
<td>720</td>
<td>144</td>
<td>0.50</td>
</tr>
<tr>
<td>1</td>
<td>2700</td>
<td>6.072</td>
<td>6.072</td>
<td>432</td>
<td>432</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>540</td>
<td>30.36</td>
<td>30.36</td>
<td>2160</td>
<td>432</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 4.3 Equivalent Soil Shear Modulus in ATC-3 Code (1978)

<table>
<thead>
<tr>
<th>( A_v )</th>
<th>( \leq 0.10 )</th>
<th>0.15</th>
<th>0.20</th>
<th>( \geq 0.30 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_n/G )</td>
<td>0.81</td>
<td>0.64</td>
<td>0.49</td>
<td>0.42</td>
</tr>
</tbody>
</table>

where \( A_v \) = Effective Peak Velocity-Related Acceleration

\( G = \) The average shear modulus at small strain level

\( G_n = \) The average shear modulus at large strain levels
### Table 4.4 RMS Response of Reinforced Concrete Structures for Different $n$ Values (in rocking motion)

<table>
<thead>
<tr>
<th>$\lambda_2$, $r$</th>
<th>1, 144 in</th>
<th>5, 144 in</th>
<th>1, 432 in</th>
<th>5, 432 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\text{max}}$</td>
<td>0.2g</td>
<td>0.6g</td>
<td>0.2g</td>
<td>0.6g</td>
</tr>
<tr>
<td>$n$</td>
<td>0.50</td>
<td>0.0127</td>
<td>0.0552</td>
<td>1.231</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>0.0131</td>
<td>0.0577</td>
<td>1.232</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>0.0133</td>
<td>0.0592</td>
<td>1.233</td>
</tr>
</tbody>
</table>

($V_s = 450$ ft/sec)

### Table 4.5 RMS Response and Foundation Rocking Hysteresis Energy of Reinforced Concrete Structures for Different $\beta/\gamma$ Values (in rocking motion)

<table>
<thead>
<tr>
<th>$\lambda_2$, $r$</th>
<th>1, 144 in</th>
<th>5, 144 in</th>
<th>1, 432 in</th>
<th>5, 432 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{\text{max}}$</td>
<td>0.2g</td>
<td>0.6g</td>
<td>0.2g</td>
<td>0.6g</td>
</tr>
<tr>
<td>$\beta/\gamma$</td>
<td>0.0</td>
<td>0.1117</td>
<td>0.058</td>
<td>1.224</td>
</tr>
<tr>
<td>RMS .005</td>
<td>0.1117</td>
<td>0.057</td>
<td>1.223</td>
<td>6.61</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0116</td>
<td>0.056</td>
<td>1.158</td>
<td>6.38</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>RMS .005</td>
<td>0.04</td>
<td>0.9</td>
<td>0.76</td>
<td>6.2</td>
</tr>
<tr>
<td>1.0</td>
<td>2.16</td>
<td>32.5</td>
<td>44.7</td>
<td>258.5</td>
</tr>
</tbody>
</table>

($V_s = 450$ ft/sec)
Table 4.6 Parameters for Ten-story UBC Steel Building

<table>
<thead>
<tr>
<th>Story</th>
<th>Stiffness (kip/in)</th>
<th>Strength (kips)</th>
<th>Mass (kip-sec^2/in)</th>
<th>β (= y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>555.5</td>
<td>685.0</td>
<td>1.31</td>
<td>0.405</td>
</tr>
<tr>
<td>2</td>
<td>512.0</td>
<td>675.0</td>
<td>1.31</td>
<td>0.379</td>
</tr>
<tr>
<td>3</td>
<td>469.5</td>
<td>650.0</td>
<td>1.31</td>
<td>0.361</td>
</tr>
<tr>
<td>4</td>
<td>437.5</td>
<td>612.5</td>
<td>1.31</td>
<td>0.357</td>
</tr>
<tr>
<td>5</td>
<td>381.5</td>
<td>570.0</td>
<td>1.31</td>
<td>0.335</td>
</tr>
<tr>
<td>6</td>
<td>372.5</td>
<td>505.0</td>
<td>1.31</td>
<td>0.369</td>
</tr>
<tr>
<td>7</td>
<td>316.0</td>
<td>437.5</td>
<td>1.31</td>
<td>0.361</td>
</tr>
<tr>
<td>8</td>
<td>309.0</td>
<td>465.0</td>
<td>1.31</td>
<td>0.332</td>
</tr>
<tr>
<td>9</td>
<td>212.0</td>
<td>347.5</td>
<td>1.31</td>
<td>0.305</td>
</tr>
<tr>
<td>10</td>
<td>183.0</td>
<td>310.0</td>
<td>1.28</td>
<td>0.295</td>
</tr>
</tbody>
</table>

\[ r_1 = 537.37 \text{ in}, \ r_2 = 510.5 \text{ in and } m_0 = 7.337 \text{ kip-sec}^2/\text{in} \]

Table 4.7 Parameters for Four-story UBC Steel Building

<table>
<thead>
<tr>
<th>Story</th>
<th>Stiffness (kip/in)</th>
<th>Strength (kips)</th>
<th>Mass (kip-sec^2/in)</th>
<th>β (= y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1075.0</td>
<td>1060.0</td>
<td>2.36</td>
<td>0.507</td>
</tr>
<tr>
<td>2</td>
<td>748.0</td>
<td>965.0</td>
<td>2.36</td>
<td>0.388</td>
</tr>
<tr>
<td>3</td>
<td>659.0</td>
<td>780.0</td>
<td>2.36</td>
<td>0.422</td>
</tr>
<tr>
<td>4</td>
<td>609.0</td>
<td>700.0</td>
<td>2.33</td>
<td>0.435</td>
</tr>
</tbody>
</table>

\[ r_1 = 752.1 \text{ in}, \ r_2 = 582.08 \text{ in and } m_0 = 7.19 \text{ kip-sec}^2/\text{in} \]
Table 4.8 Idealized Single-DOF Reactor Building

<table>
<thead>
<tr>
<th>$k$ (kip/in)</th>
<th>$m$ (kip-sec$^2$/in)</th>
<th>$m_o$ (in)</th>
<th>$h$ (in)</th>
<th>$r$ (in)</th>
<th>Period (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>771526</td>
<td>542.2</td>
<td>171.23</td>
<td>984</td>
<td>1181</td>
<td>0.167</td>
</tr>
</tbody>
</table>
Fig. 4.1 RMS Structural Relative Displacement Ratio (Normalized by that of Fixed-base Structure) of Single-DOF Systems

(a) Linear Steel Systems

(b) Linear Reinforced Concrete Systems
Linear Soil Deposit

(a) Linear Steel Systems

(b) Linear Reinforced Concrete Systems

Fig. 4.2 RMS Foundation Acceleration in Translational Mode \( a_{\text{max}} = 0.1g \) of Single-DOF Systems
Fig. 4.3 RMS Foundation Acceleration in Rocking Mode ($a_{\text{max}} = 0.1g$) of Single-DOF Systems
Fig. 4.4 RMS Relative Structural Acceleration of Single-DOF Systems (Normalized by that of Fixed-base Structure)
Fig. 4.5 Expected Structural Stiffness Ratio (Normalized by the Initial Stiffness) of Steel Buildings
Nonlinear Structure with Linear Foundation
Nonlinear Structure with Foundation Uplifting
Nonlinear Coupled System

(c) $\lambda_2 = 1$, $r = 432$ in

(d) $\lambda_2 = 5$, $r = 432$ in

Fig. 4.5 (continued)
Fig. 4.6 Expected Structural Stiffness Ratio (Normalized by the Initial Stiffness) of Reinforced Concrete Buildings
Nonlinear Structure with Linear Foundation
Nonlinear Structure with Foundation Uplifting
Nonlinear Coupled System

---

(c) $\lambda_2 = 1, r = 432$ in

(d) $\lambda_2 = 5, r = 432$ in

Fig. 4.6 (continued)
Fig. 4.7 Expected Foundation Radius Ratio for Nonlinear Structures with Linear Soil Deposit ($a_{max} = 0.6g$)
Fig. 4.8 Expected Foundation Radius Ratio for Nonlinear Structures with Nonlinear Soil Deposit (\(a_{\text{max}} = 0.6\) g)
Fig. 4.9 Expected Soil Shear Modulus Ratio, $E_n / G$, for Nonlinear Steel Buildings
Fig. 4.10 Expected Soil Shear Modulus Ratio, $E[G_n/G]$, for Nonlinear Reinforced Concrete Buildings
Fig. 4.11 Expected Foundation Translational Displacement for Nonlinear Coupled Systems
Fig. 4.12 RMS Structural Displacement Ratio of Steel Buildings (Normalized by that of Fixed-base Structure)
Fig. 4.13 RMS Structural Displacement Ratio of Reinforced Concrete Buildings (Normalized by that of Fixed-base Structure)
Fig. 4.14 Expected Structural Hysteresis Energy Ratio of Steel Buildings
(Normalized by that of Fixed-base Structure)
Fig. 4.15 Expected Structural Hysteresis Energy Ratio of Reinforced Concrete Buildings (Normalized by that of Fixed-base Structure)
(a) Floor Framing Plan

(b) Typical Frame Section

Fig. 4.16 Ten-Story UBC Steel Building
Fig. 4.17 The responses of Ten-story UBC Steel Building with Fixed-base
Fig. 4.18 Effect of Interaction on Responses of Ten-story UBC Steel Building ($V_s = 300$ ft/sec)
Fig. 4.19 Effect of Interaction on Expected Base Shear of Ten-Story UBC Steel Building
(a) Floor Framing Plan

(b) Typical Frame Section

Fig. 4.20 Four-Story UBC Steel Building
Fig. 4.21 Effect of Interaction on Responses of 4-story UBC Steel Building ($V_s = 300$ ft/sec)
Fig. 4.22 Effect of Interaction on Expected Base Shear of 4-Story UBC Steel Building
Fig. 4.23 Effect of Interaction on Responses of Idealized Single-DOF Reactor Building ($V_s = 1200$ ft/sec)
CHAPTER 5

SEISMIC RELIABILITY ASSESSMENT

5.1 Introduction

The chapter begins with a description of the methodology for the seismic safety assessment of building-foundation systems. The statistics of story displacements are considered for seismic safety evaluation. The uncertainties in the modeling and parameters of the coupled system and in the ground motion are identified, quantified, and included in the reliability assessment. The effect of soil-structure interaction on the structural safety assessment is illustrated by the ten-story and four-story UBC steel buildings, and the idealized single-DOF reactor building described in Chapter 4. The reduced-DOF technique is demonstrated for the ten-story steel building in seismic reliability assessment.

5.1.1 Summary of Main Results

(1) The reduced-DOF technique can simplify the random vibration analysis for structures with a large number of DOF. Generally, the two-DOF approximation with mode shape modification yields satisfactory results.

(2) For conventional buildings, the contribution of the superstructural parameter uncertainty (i.e. story stiffness, damping, mass, and yield strength) to the total variance of the maximum story drifts increases with the excitation level; whereas the contribution of the substructural parameter uncertainty (i.e. soil shear modulus, foundation stiffness, and foundation strength in translational and rocking modes), decreases with the excitation level and is negligibly small. Also the effect of soil-structure interaction on the maximum story drift and corresponding standard deviation, and the story
ductility exceedance probability is very small.

(3) For massive and stiff buildings, the contribution of the superstructural parameter uncertainty remains fairly uniform throughout all response levels; whereas the contribution of the substructural parameter uncertainty significantly increases with the excitation level and is important at high response levels. Therefore, the effect of interaction on massive and stiff structures should be considered in the seismic reliability assessment.

5.2 Reliability Evaluation

Damage of a building-foundation system, is a function of structural response variables (e.g., acceleration, displacement, dissipated energy, etc.). A relationship between structural damage and ductility ratio was proposed by Blume and Monroe (1971). Banon, et al. (1981) suggested the impending collapse as a function of the ductility ratio and total energy dissipation. Recently, Algan (1982) used the interstory drift to evaluate damage intensity for reinforced concrete buildings.

Takizawa and Jennings (1980) discussed that failure of a structure can be the result of localized failures of individual members and/or global structural instability. For predicting the structural member failure, the maximum displacement and dissipated energy are used as possible failure indicators for reinforced concrete members (Banon and Veneziano, 1982) and steel members (Kato and Akiyama, 1982). However, in this study, only the general story response statistics, and not the detailed member response information, are considered which may be obtained from the random vibration analysis.

In the light of the above, the seismic safety during the lifetime of a building-foundation system may be expressed in terms of the probability of exceeding some critical level of damage based on the story drifts and/or dissipated energy. For steel coupled systems, safety may be evaluated in terms of the probability of exceeding some critical ductility ratio. Consistent with the above definition of
system safety, the pertinent seismic hazard curve must be a function of the earthquake duration, in addition to its amplitude and frequency content. In this regard, the available seismic hazard evaluation model (e.g., Der-Kiureghian and Ang, 1977) may be used to assess the hazard curves in terms of the maximum ground acceleration.

For a given expected maximum acceleration $A = a$, the required response $X$ for which exceedance probabilities are needed, can be evaluated as described in Chapter 2. The uncertainties in the coupled system are also included in the statistics of $X$ by the method described in Sect. 5.3. Then the conditional cumulative distribution function, $F_{X|a}(x)$ is obtained by fitting an appropriate probability distribution $X|a$ with the mean and variance of the response $X$. For example, the Type I extreme value distribution has been found to fit simulation results well for the maximum interstory drift (Baber and Wen, 1980). From the available seismic hazard model, the probability density function of the expected maximum acceleration in $T$ years, $f_T(a)$, can be obtained. Therefore, the probabilities that particular response or damage levels will be exceeded over a specified time duration may be evaluated as

$$P\left(X_T > x\right) = \int_0^\infty P\left(X_T > x|A = a\right)f_T(a)da$$

$$= \int_0^\infty \left[1 - F_{X|a}(x)\right] f_T(a)da \quad (5.1)$$

5.3 Uncertainties Involved in the Soil-Structure System

The major uncertainties associated with estimating the dynamic response of a soil-structure system are from two sources: (1) incomplete or inadequate information of the physical phenomena, (2) inherent variabilities of the physical process. The first source of uncertainty results from the imperfection of the mathematical model used to predict the dynamic response, whereas the second source results from the variabilities of the material properties.
Suppose that the true response variable (e.g., maximum response), $X$, is estimated by a model denoted as $\hat{X}$ and the parameters of this model is represented with $R$. A correction $N$, which is considered to be a random variable, is necessary to represent the correction for the error in the prediction (Ang, 1973). Thus

$$X = N \hat{X} \tag{5.2}$$

and

$$\hat{X} = g(R) \tag{5.3}$$

where $g$ is a functional representation of the model. Assuming statistical independence between $N$ and $\hat{X}$, the mean value and uncertainty (variance) of the dynamic response may be obtained by first-order approximation (Ang and Tang, 1975) as

$$E[X] \approx E[N] E[\hat{X}] \tag{5.4}$$

and

$$\text{Var}[X] \approx (E[\hat{X}])^2 \text{Var}[N] + (E[N])^2 \text{Var}[\hat{X}] \tag{5.5}$$

where $E[N]$ and $\text{Var}[N]$ represent the expected bias and variance of the dispersion error in $g(R)$, respectively. $E[\hat{X}]$ is the mean response obtained from the model using mean parameter values, and $\text{Var}[\hat{X}]$ is the variance of the response associated with the parameter uncertainties and the randomness of the loading history, which may be represented with first-order approximation as

$$\text{Var}[\hat{X}] \approx \text{Var}_{gr} [\hat{X}] + \text{Var}_{R} [\hat{X}] \tag{5.6}$$

where

$$\text{Var}_{R} [\hat{X}] = \sum_i \sum_j \left( \frac{\partial \hat{X}}{\partial R_i} \right) \left( \frac{\partial \hat{X}}{\partial R_j} \right) \sigma_R^i \sigma_R^j \tag{5.7}$$
and $\text{Var} \{ \hat{\mathbf{x}} \}$ is the variance of the response due to the random nature of the loading history and can be obtained from a random vibration analysis as discussed in Chapter 2; $\overline{\mathbf{R}}$ is the set of mean model parameters; $\rho_{ij}$ is the correlation coefficient between parameters $R_i$ and $R_j$; $\sigma_i$ is the standard deviation of parameter $R_i$, and the derivatives in Eq. 5.7 are the sensitivity coefficients.

5.3.1. Mathematical Idealization Error

In the subsystem approach, the structure is represented with a shear-beam building and the soil deposit is represented with a halfspace system. The error in the dynamic response estimates associated with this idealized model of a soil-structure system is evaluated through the Bayesian correction variable, $N$, as discussed above. The mean and variance of $N$ may be evaluated by comparing the responses predicted by this model with the corresponding actual building responses under earthquake loads. From the calculated and observed results for the Millikan Library Building (Wong, Luco, and Trifunac, 1977), it is determined that the substructural model (rigid foundation and halfspace soil deposit) gives satisfactory force-displacement relationship for the foundation. Hence, the variance of $N$ may be estimated mainly by considering the uncertainty from the structural model.

For the response prediction error caused by the shear-beam model for fixed-base structure and the hysteresis model for hysteretic force, Sues, Wen and Ang (1983) suggested that the C.O.V. of the response is 0.21 for steel structures with fixed-base and 0.22 for reinforced concrete structures with fixed-base. Actually, the real building responses estimated from earthquake records have already included the effects of soil-structure interaction. The C.O.V. of the maximum response evaluated from comparisons between actual building responses and corresponding responses predicted with idealized shear-beam model, therefore, contains the effects of soil-structure interaction. The dynamic analysis including soil-structure interaction by the subsystem approach should, theoretically, gives more accurate responses than that obtained with the shear-beam model for fixed-support. Therefore, the
response prediction error caused by the subsystem approach should be less than that by the shear-beam model with fixed-support. Based on the response prediction error caused by the shear-beam model for fixed-base structures (Sues, Wen and Ang, 1983), and considering any inadequacy of the subsystem approach for nonlinear soil-structure system (i.e., embedded foundation, nonvertically incident seismic waves, etc.), a c.o.v. of 0.25 is conservatively assumed for the response prediction error associated with the mathematical idealization of the soil-structure system. This is the error that would be expected if the physical parameters in the model were known exactly.

5.3.2 Model Parameter Uncertainties

The basic parameters of a superstructure are the story stiffness, mass, damping, and yield strength; whereas for a substructure, the basic parameters are the foundation properties (shape, flexibility), soil shear modulus, soil density, foundation ultimate bearing capacity, soil cohesion, and friction angle. Uncertainties in these parameters would include the inherent variability of the material properties as well as the approximations used to estimate the parameters.

Based on available data for reinforced concrete and steel structures (Portillo Gallo and Ang, 1976; Galambos and Ravindra, 1978; Lai, 1980; Haviland, 1976), Sues, Wen and Ang (1983) proposed the uncertainties in the story stiffness, damping, mass and story strength as shown in Table 5.1. Since the value of the soil density falls within a narrow band (Meyerhof, 1982), the effect of uncertainty in the soil density can be neglected. The uncertainties in the other parameters of a substructure are discussed as follows.

**Soil Shear Modulus** — It has been shown that modulus values for sands are strongly influenced by the confining pressure, the strain amplitude and the void ratio (or relative density). The (initial) shear modulus of the sand may be evaluated with the following equation (Martin and Seed, 1982):
where \( \sigma \) = the effective mean principal stress in lb/sq in, and \( D \) = the relative density. The form of this equation suggests that the uncertainty in the shear modulus of the sand strongly depends on the uncertainty in the relative density. The in-situ relative density of a sand deposit can be determined by either direct methods (laboratory determination), or indirect methods that relate the in-situ value of the relative density to the SPT blowcount. The c.o.v. of \( G \) for several values of the mean and c.o.v. of the in-situ relative density of the sand are shown in Table 5.2. Errors in the prediction of \( G \) with Eq. 5.8 should also be considered. Based on the study by Fardis (1979) for the equation proposed by Hardin and Drnevich (1972), an additional c.o.v. of 0.12 is used to account for the differences between laboratory and in-situ values of the shear modulus \( G \). The total c.o.v. of the shear modulus is shown in Table 5.2 for several values of the in-situ relative density of sand.

For cohesive soils, the shear modulus is related to the undrained shear strength, \( S \), and may be calculated with the following equation (Martin and Seed, 1982):

\[
G = 2050 \frac{S}{\mu}
\]

(5.9)

The undrained shear strength, \( S \), is depth-dependent (Asaoka and A-Grivas, 1982), and is commonly measured by the unconfined compression test or the field vane test. The c.o.v. of the undrained shear strength measured by the unconfined compression test is 0.3, whereas by the simple shear test is 0.10 (Wu, 1974). Assuming an additional c.o.v. of 0.12 to account for errors in Eq. 5.9, the total c.o.v. of the shear modulus for clay is shown in Table 5.3.

Foundation Stiffness -- In formulating the foundation impedance functions, it is assumed that the foundation is a rigid circular plate and the soil deposit is a uniform halfspace. Therefore, the
uncertainties in the foundation stiffness functions arise from the idealization of the foundation shape, foundation flexibility, and soil profile as described below. Only mat foundation is considered in this study.

(1) Foundation shape: The foundation of buildings are usually not perfectly circular slabs. In many cases, they may be polygonal or rectangular mats. For foundations whose dimensions are nearly equal in two orthogonal directions, using the solution for an equivalent circular mat with the same area will be quite acceptable. For rectangular foundations, it is common practice to derive equivalent radii for horizontal excitation (based on the same area) and for rotational motions (based on equating the appropriate moment of inertia). This approximation is generally accepted for aspect ratios up to 6:1 (the ratio of the larger to the smaller side). This is confirmed for the static case by comparing the results with the stiffness curves derived by Barkan (1962) and Gorbunov-Possadov (1961). Studies by Vardanega (1981) and Dominguez (1978) seem to indicate that the approximation is also valid for dynamic stiffness. From these results, the c.o.v. of the foundation shape is dependent on the aspect ratio and are listed in Table 5.4.

(2) Foundation flexibility: With the assumption of a rigid circular foundation, the interaction between the soil and foundation is limited to two degrees of freedom—one horizontal translation and one rocking motion. This may be sufficient for usual application in a seismic analysis of nuclear power plants and multi-story buildings. Comparing the calculated and experimental results for the Millikan library building, Wong, Luco and Trifunac (1977) found that the flexibility of the foundation has a major effect on the deformation and stress patterns at the soil-foundation contact; whereas the relation between the total forces acting at the contact and the average motion of the basement slab is practically independent of the flexibility of the foundation. Waas and Riggs (1983) evaluated the effect of the flexibility of the base mat on the seismic response of a PWR-reactor building, and found that this effect with respect to soil-structure interaction is small and can be
neglected for practical design purposes. These results imply that the rigid foundation assumption may be used in soil-structure interaction studies to obtain the overall motion of the system. Based on the above results, a c.o.v. of the foundation flexibility is about 0.07. It is also necessary to assess the uncertainty associated with any inadequacy of the hysteresis model for representing the nonlinear foundation behavior. Experimental data to evaluate the hysteretic parameters are scarce. To include this prediction error, a c.o.v. of 0.10 is assumed for the foundation flexibility.

(3) Soil profile: At most real sites, the soil deposit rarely has uniform properties and the shear modulus will increase more or less with depth. The increase may be continuous or discontinuous as in the case of layered systems. For practical purposes, an equivalent homogeneous halfspace is used to represent the inhomogeneous soil deposit. By averaging the stress variation with depth, Holzloher (1979) evaluated the equivalent material quantities from the actual variation of the material properties with depth as

\[
S(z) = \frac{\int_0^z \sigma(x) dx}{\sigma(x) dx}
\]

(5.10)

\[
\frac{1}{G_e} = \frac{1}{\sum_{i=1}^{n} G_i \Delta S_i}
\]

(5.11)

where \(\sigma(x)\) is the stress at depth \(x\); \(G\) is the shear modulus of layer \(i\); \(\Delta S\) is the suitable interval of \(S(z)\) and \(G\) is the equivalent shear modulus. Applying this method to evaluate the equivalent shear modulus for all twelve cases investigated by Luco (1974), it is found that a c.o.v. of the static compliances is about 0.10. Based on a limited number of parametric studies on strip footings (two dimensional model) for different earthquake levels, Jakub (1977) concluded that a reasonable approximation to the foundation stiffness could be obtained
by assuming a uniform soil deposit at a depth of 0.6B (if the soil deposit were originally uniform) or at a depth of 0.4B (if the original modulus increased smoothly with depth), where B is the half-width of the footing. From Jakub's study, the c.o.v. of the equivalent stiffness of the soil deposit is about 0.15. The larger c.o.v. is the result of including the dynamic effects.

The total uncertainty in the foundation stiffness, therefore, can be obtained from the above c.o.v. in the foundation model and soil deposit as shown in Table 5.4. If the foundation shape is circular, the prediction error associated with foundation shape should be removed from Table 5.4.

**Ultimate Bearing Capacity** — The ultimate bearing capacity of a surface footing on a homogeneous soil can be determined by modifying the capacity proposed by Terzaghi (1943) for a shallow, concentrically loaded strip footing as

\[
q = 0.5BwN_u N_c R_E I + cN_u N_c R_E I
\]

where \( q \) = ultimate bearing capacity; \( N_u, N_c \) = bearing capacity factors for vertical concentric loaded strip footing; \( B \) = foundation width; \( c, w \) = effective soil cohesion and unit weight; \( R \) and \( R_c \) are correction factors for foundation size; \( S \) and \( S_c \) for foundation shape; \( E \) and \( E_c \) for load eccentricity; \( I \) and \( I_c \) for load inclination. Theoretical solutions and recommended values for \( N_u, N_c \) and the correction factors, which differ widely and involve many simplifications, are summarized by Szechy and Varga (1978).

The c.o.v. of the ultimate bearing capacity is related to the uncertainties of the factors in Eq. 5.12. Based on statistical analyses of currently available experimental data for surface footing on sand, Ingra and Baecher (1983) suggested the c.o.v. of ultimate bearing capacity from 0.20 to 0.30; a value of 0.30 is used in this study.
Soil Cohesion and Friction Angle — Meyerhoff (1982) suggested that the c.o.v. of clay cohesion is between 0.20 and 0.30. The value of 0.30 is used herein. The c.o.v. of sand friction angle is between 0.10 and 0.20; the value of 0.20 is used in this study.

5.4 Earthquake Loading

The input motion acting at the foundation level in the subsystem approach depends on the frequency of the excitation, the geometry of the foundation, the characteristic of the soil deposit (structure and material behavior), and the wave composition of the free-field motion (Luco and Wong, 1982). Based on the assumption that the seismic excitation is caused by plane vertically incident shear waves, the foundation input motion for a rigid surface foundation is identical to the free-field motion on the surface of the soil. In the random dynamic analysis of soil-structure systems, the ground motion is modeled as a random process.

5.4.1 Ground Motion Model and Uncertainties

Earthquake-induced strong ground motion may be modeled as a zero-mean filtered Gaussian shot noise random process (Amin and Ang, 1968). The intensity of the earthquake loading is characterized by its expected maximum acceleration, $E[a_{\text{max}}]$, and the frequency content by its power spectral density function (PSD function). The PSD function of a Kanai-Tajimi filtered stationary shot noise (the Kanai-Tajimi spectrum) takes the form

$$S_a(\omega) = \frac{s^2}{\omega^2 + 4\beta \omega / g} \left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4\beta^2 \left( \frac{\omega}{\omega_g} \right)^2 \left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 \frac{1 + 4\beta^2 \left( \frac{\omega}{\omega_g} \right)^2}{\omega^2 + 4\beta \omega / g}$$

(5.13)

where $s$ is the intensity scale of the PSD function; $\omega$ and $\beta$ are $g$ and $g$
shape parameters determined by examining actual earthquake records. In general, the parameters \( \omega \) and \( \beta \) will be affected by the epicentral distance, earthquake magnitude, and the ground layer rigidity.

Based on the shape of an average pseudo-velocity response spectra for eight accelerograms, Housner and Jennings (1964) suggested the values of \( \omega = 15.6 \text{ rad/sec} \) and \( \beta = 0.64 \) for 'firm' ground condition. Lai (1982) used the method of spectral moments to determine the Kanai-Tajimi parameters for 140 strong-motion records from their Fourier amplitude spectra, and found \( \omega \) to vary from 5.7 rad/sec to 51.7 rad/sec, \( \beta \) between 0.10 and 0.90. For 'rock' site records, \( \omega \) had a mean of 26.7 rad/sec and c.o.v. of 0.40, and \( \beta \) had a mean of 0.35 and c.o.v. of 0.36. For 'soft' site records, the corresponding means and c.o.v.s are 19 rad/sec and 0.43 for \( \omega \), and 0.32 and 0.36 for \( \beta \). Based on the Fourier amplitude spectra for the strong motion phase, Moayyad and Mohraz (1982) obtained the power spectra for soft, intermediate, and hard grounds, as shown in Fig. 5.1. The soft ground spectrum was based on the Fourier analysis of 161 records, the intermediate ground spectrum on 60 records and the hard ground spectrum on 26 records.

Sues, Wen, and Ang (1983) evaluated the appropriate Kanai-Tajimi parameters for each of the three ground conditions using a least squares procedure, and applied 'scale factors' to ensure that the total area of the Kanai-Tajimi Spectrum (the mean square value of the process, \( \sigma_a^2 \)), is not unduly amplified by the high frequencies. The results which are used in this study are listed in Table 5.5 and the curves obtained are plotted in Fig. 5.2 along with the Moayyad and Mohraz empirical curves.

When the parameters of the Kanai-Tajimi PSD function are known, the mean square ground acceleration, \( \sigma_a^2 \), is evaluated as

\[
\sigma_a^2 = \frac{S \omega \pi}{2G \beta^2} \left(1 + 4\beta^2\right) \tag{5.14}
\]
where \( F \) is the scale factor for ground condition shown in Table 5.5.

Since the ground motion is modeled as a random process, the intensity of the loading is measured by the root-mean-square of the process, \( \sigma_a \). Results of the seismic hazard analysis, however, are in terms of the probabilities of exceeding given maximum accelerations. Therefore, it is necessary to relate the expected maximum acceleration, \( \mathbb{E}[a_{\text{max}}] \), to \( \sigma_a \). The correlation suggested by Vanmarcke and Lai (1980) may be used; namely,

\[
\frac{\mathbb{E}[a_{\text{max}}]}{\sigma_a} = \begin{cases} \sqrt{2} \ln \left( \frac{2 t_d}{t_0} \right) & t_d \geq 1.36 t_0 \\ \sqrt{2} & t_d < 1.36 t_0 \end{cases} \quad (5.15)
\]

where:
- \( r = \) peak factor;
- \( t = \) duration of the strong-motion phase of the ground excitation;
- \( t_d = \) predominant period of the ground motion;
- \( t_0 = \) predominant period of the ground motion.

Based on the results of Mosayyad and Mohraz (1982), and Vanmarcke and Lai (1980), Sues, Wen and Ang (1983) proposed the mean and c.o.v. of the strong motion duration, \( t_d \), for three soil conditions as presented in Table 5.6. As the frequency structure of the earthquake motion is modeled by the Kanai-Tajimi spectrum, the value of \( t \) depends on \( \omega_0 \) and \( \beta \), and may be calculated by the moments of the Kanai-Tajimi spectrum for the three ground conditions. Using this method and with the mean durations given in Table 5.6, Sues, Wen and Ang (1983) suggested the peak factors, \( r \), in Table 5.6. The peak factor is insensitive to the duration and predominant period of the ground motion.

**Nonstationarity** — It is well known that the frequency content and intensity of earthquakes vary with time and as such are really nonstationary processes. Nonstationarity of the loading intensity varying in time is accounted for through the use of a temporal multiplier. It means that the temporal variation of the root-mean-square or mean-square of the process is governed by a specific
function of time. Possible forms for the modulating function for earthquakes were suggested by Amin and Ang (1968) and Shinozuka and Sato (1967).

5.5 Illustrative Examples

The lifetime safety of the building can be assessed by incorporating the site seismic risk curves. The reduced-DOF technique is adopted for the ten-story steel building. The four-story steel building is studied to evaluate the sensitivity of the response to various system parameters. The idealized single-DOF reactor building is analyzed to evaluate the sensitivity of the response to various system parameters.

5.5.1 Load Description

The ground motion was modeled as discussed in Section 5.4. The seismic hazard assumed for the reliability analysis is that of Santa Barbara (California). Based on the data given by Kiremidjian and Shah (1975), the hazard curves are evaluated using the method of Der Kiureghian and Ang (1977). The value \( \beta = 1.75 \) (slope of the magnitude-recurrence curve) was used, which is a representative value for the faults in the area. Also, the parameters \( a = 2.095 \) and \( b = 10.64 \) of the magnitude-slip length equation were assumed (Patwardhan, Tocher, and Savage, 1975). Finally, the attenuation equation used was proposed by Esteva and Villaverde (1973),

\[
a \max = 5.71 e^{0.8m} (R+40)^{-2.0} \tag{5.16}
\]

where \( a \max \) is the maximum ground acceleration expressed as a fraction of gravity, \( m \) is the earthquake magnitude in Richter scale, and \( R \) is the shortest distance between the site and the slipped area in km. The annual, 10-year, and 50-year hazard curves obtained from the analysis are shown in Fig. 5.3.
5.5.2 Ten-Story UBC Steel Building

To demonstrate the reduced-DOF technique in the reliability assessment, the lifetime probabilities of exceeding specified ductility ratios are evaluated for the ten-story UBC steel building with fixed-base according to Sect. 5.2. Using the yield displacement of Eq. 2.17 and the maximum displacement statistics described in Sect. 2.4.2, the expected maximum ductility and the corresponding standard deviation can be obtained (considering only the uncertainty from random loading). Therefore, the conditional cumulative distribution function for the ductility ratio is obtained by fitting a Type I extremal distribution to the maximum ductility ratio statistics. Using the seismic hazard curve shown in Fig. 5.3, the annual exceedance probabilities are calculated for the first story and the fifth story as shown in Fig. 5.4 (a) and (b), respectively. It is observed the difference between the full DOF and the reduced-DOF approximation is decreased with increasing number of DOF. Generally speaking, the two-DOF approximation with mode shape modification, gives satisfactory results.

5.5.3 Four-Story UBC Steel Building

Response Variance -- To evaluate the variance of the maximum drift, the derivatives of the drift with respect to the superstructural parameters (structural stiffness, damping, mass, and strength), and substructural parameters (soil shear modulus, foundation stiffness, foundation strength in sliding and rocking motions) were calculated. The derivatives of the maximum drift of each story with respect to the filter parameters, \( \omega \) and \( \beta \), and to the strong motion duration were also evaluated. The c.o.v. of the foundation strength in translational and rocking motions, are 0.2 and 0.3, respectively.

The significance of each parameter uncertainty in the coupled system may be represented by the product of the absolute value of the derivative and the pertinent parameter standard deviation. For the first-story maximum drift, these results are presented by Fig. 5.5 for the coupled system and the corresponding fixed-base structure as
functions of the excitation level. It is observed that the uncertainties in the structural mass, stiffness, strength, and load duration tend to dominate the response uncertainty especially at the higher response level. However, the difference between the coupled system and the fixed-base structure is generally small; this is expected as the interaction effect is small.

In the uncertainty analysis, the structural stiffness and structural strength are assumed to be perfectly correlated; also the soil shear modulus, the foundation stiffness, and foundation strength in translational and rocking motions are assumed to be perfectly correlated. All other parameters, including the filter parameters and load duration, are assumed to be independent of each other.

The dispersive error in the mathematical idealization of the coupled system is assumed to have a coefficient of variation of 0.25 for the steel coupled system.

The relative contributions of each source of uncertainty to the total variance of the first-story maximum drift are illustrated in Tables 5.7 and 5.8 for the fixed-base structure and coupled system, respectively. The contribution of the superstructural parameter uncertainty increases with the excitation level in these two systems, since the response is much more sensitive to changes in the initial structural stiffness and structural strength as it reaches the nonlinear range. However, the contribution of the substructural parameter uncertainty decreases with the excitation level and is negligibly small compared to the other parameters. This behavior may be observed in Fig. 5.5. Tables 5.7 and 5.8 also show that the contributions from the filter parameters, loading duration, and the randomness of the loading history remain fairly uniform throughout all response levels. Besides, the contribution of the system model uncertainty decreases with the excitation level, but the total response c.o.v. increases with the excitation level for the fixed-base structure and the coupled system.

**Ductility Exceedance Probabilities** — The steel frame was designed according to the 1973 edition of the Uniform Building Code in which the zone 3 seismic risk characterization, equivalent peak acceleration of
1/3g and ductility ratio of 4, were used. Based on the story stiffness and strength given in Table 4.7, the yield displacement defined by Eq. 2.17, are 0.986, 1.288, 1.185 and 1.149 inches for the four stories, respectively.

Fig. 5.6 shows the expected ductility and the corresponding standard deviation (considering all three sources of uncertainty) of all four stories for the fixed-base structure and the coupled system as a function of the maximum ground acceleration. It is observed that the interaction effect on the maximum ductility and standard deviation is very small.

The lifetime exceedance probabilities may be evaluated using Eq. 5.1 (where X represents the ductility ratio). The seismic hazard curve is presented in Fig. 5.3, and the conditional cumulative distribution function for the ductility ratio is obtained by fitting a Type I extremal distribution to the maximum ductility ratio.

The annual and 50-year exceedance probability curves are shown in Fig. 5.7 considering the interaction effect and the uncertainty in the response. The effect of interaction on the ductility exceedance probability is also small for this building.

5.5.4 Idealized Single-DOF Reactor Building

The c.o.v. of the system parameters and their correlation coefficients are assumed to be the same as those in the above example. The derivatives of the displacement with respect to the system parameters were calculated for the coupled system and the fixed-base structure; also, the product of the absolute value of the derivative and the pertinent parameter standard deviation are shown in Figs. 5.8(a) to 5.8(k).

For the fixed-base structure, the response uncertainty is dominated by the uncertainties in the structural mass, stiffness, load duration, filter damping, and filter frequency for the massive, stiff structure. The derivatives of the response with respect to the structural parameters, load duration, filter damping, and filter frequency are reduced by soil-structure interaction as shown in Fig. 5.8. On the
other hand, the derivatives of the response with respect to the substructural parameters are increased, particularly at the higher response levels. This is due to the fact that the structural responses are significantly affected by the interaction or by the substructural parameters at the higher response levels.

The relative contributions of each source of uncertainty to the total variance of the maximum displacement are illustrated in Tables 5.9 and 5.10 for the fixed-base structure and coupled system, respectively. For the fixed-base structure, it is observed that the contributions from the uncertainties in filter damping, filter frequency, and load duration are the most important, which is different from that found in the conventional fixed-base building. Besides, the c.o.v. of the total response decreases with the loading level. This behavior is probably characteristic of massive, stiff structures with fixed-base.

The contribution from the structural parameter uncertainties remains fairly uniform throughout all response levels for the fixed-base structure and coupled system. This is because the structural response of a stiff structure remains largely in the elastic range even at high excitation levels. For the coupled system, the contributions from the uncertainties in the system modeling and randomness of the loading history, as well as the total response c.o.v., have the same trend as that of conventional buildings. Table 5.10 also shows that the contributions from the substructural parameter uncertainties increase with the response level and is important at high response levels; however, this contribution is negligibly small for conventional coupled system as described in the last example. Therefore, the effect of interaction and the uncertainty from the substructural parameters are important in the analysis and design of massive, stiff structures.
Table 5.1 C.o.v. of Superstructure Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient of Variation</th>
<th>Reinforced Concrete Structures</th>
<th>Steel Structures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>δ</td>
<td>Δ</td>
</tr>
<tr>
<td>Story Stiffness</td>
<td>0.23 0.22 0.32</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>0.60 0.25 0.65</td>
<td>0.60</td>
<td>0.25</td>
</tr>
<tr>
<td>Story Mass</td>
<td>0.05 0.11 0.12</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Story Strength: Eq. 3.3</td>
<td>0.12 0.22 0.25</td>
<td>0.11</td>
<td>0.20</td>
</tr>
<tr>
<td>Inelastic Analysis</td>
<td>0.12 0.12 0.17</td>
<td>0.11</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 5.2 C.o.v. of Small Strain Shear Modulus of Sands

<table>
<thead>
<tr>
<th>$\mu_{D_x}$</th>
<th>$\Omega_{D_x}$</th>
<th>$\delta_G$</th>
<th>$\Omega_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>0.0</td>
<td>0.0</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.10</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>60%</td>
<td>0.0</td>
<td>0.0</td>
<td>0.12</td>
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<tr>
<td></td>
<td>0.15</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.14</td>
<td>0.19</td>
</tr>
<tr>
<td>75%</td>
<td>0.0</td>
<td>0.0</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.15</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Table 5.3 C.o.v. of Initial Shear Modulus of Clays

<table>
<thead>
<tr>
<th>Type of Test</th>
<th>Ω_{S_u}</th>
<th>Ω_G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Compression Test</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>Direct Shear Test</td>
<td>0.10</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 5.4 C.o.v. of Foundation Stiffness

<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>Shape</th>
<th>Flexibility</th>
<th>Soil Profile</th>
<th>Total c.o.v.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.10</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>0.06</td>
<td>0.10</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td>0.10</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>4</td>
<td>0.14</td>
<td>0.10</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>0.18</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.21</td>
<td>0.10</td>
<td>0.15</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 5.5 Kanai-Tajimi Spectral Parameters and Corresponding c.o.v., and Scale Factors for Mean Square Values

<table>
<thead>
<tr>
<th>Ground Condition</th>
<th>ω_g</th>
<th>β_g</th>
<th>Scale Factor F_G</th>
<th>Mean Square σ_{a}^2</th>
<th>c.o.v. (ω_g)</th>
<th>c.o.v. (β_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>10.9</td>
<td>0.96</td>
<td>0.81</td>
<td>67.7 s_o</td>
<td>0.425</td>
<td>0.426</td>
</tr>
<tr>
<td>Intermediate</td>
<td>16.5</td>
<td>0.80</td>
<td>0.83</td>
<td>95.7 s_o</td>
<td>0.425</td>
<td>0.426</td>
</tr>
<tr>
<td>Hard</td>
<td>16.9</td>
<td>0.94</td>
<td>0.79</td>
<td>101.2 s_o</td>
<td>0.398</td>
<td>0.391</td>
</tr>
</tbody>
</table>
Table 5.6 Strong-Motion Duration Statistics and Peak Factors

<table>
<thead>
<tr>
<th>Ground Condition</th>
<th>Mean (sec)</th>
<th>c.o.v.</th>
<th>Peak Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>10.0</td>
<td>0.90</td>
<td>3.0</td>
</tr>
<tr>
<td>Intermediate</td>
<td>7.0</td>
<td>0.90</td>
<td>2.9</td>
</tr>
<tr>
<td>Hard</td>
<td>5.5</td>
<td>1.00</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 5.7 Percentage Contribution from Uncertainty Sources to Total Variance of First Story Maximum Drift of Four-story UBC Steel Building (Fixed-base Building)

<table>
<thead>
<tr>
<th>E[a_max] (g)</th>
<th>Structural Modeling</th>
<th>Structural Parameters</th>
<th>Filter and Duration Parameters</th>
<th>Randomness of Loading History</th>
<th>c.o.v. of Total Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/12</td>
<td>0.34</td>
<td>0.06</td>
<td>0.14</td>
<td>0.46</td>
<td>0.36</td>
</tr>
<tr>
<td>3/12</td>
<td>0.31</td>
<td>0.10</td>
<td>0.13</td>
<td>0.46</td>
<td>0.38</td>
</tr>
<tr>
<td>5/12</td>
<td>0.27</td>
<td>0.15</td>
<td>0.13</td>
<td>0.45</td>
<td>0.40</td>
</tr>
<tr>
<td>7/12</td>
<td>0.24</td>
<td>0.20</td>
<td>0.12</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>9/12</td>
<td>0.21</td>
<td>0.24</td>
<td>0.12</td>
<td>0.43</td>
<td>0.46</td>
</tr>
<tr>
<td>11/12</td>
<td>0.18</td>
<td>0.29</td>
<td>0.12</td>
<td>0.41</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Table 5.8 Percentage Contribution from Uncertainty Sources to Total Variance of First Story Maximum Drift of Four-story UBC Steel Building (Coupled System, $V_s = 300 \text{ ft/sec}$)

<table>
<thead>
<tr>
<th>$E[a_{max}]$ (g)</th>
<th>System Modeling</th>
<th>Super-structural Parameters</th>
<th>Sub-structural Parameters</th>
<th>Filter and Duration Parameters</th>
<th>Randomness of Loading History</th>
<th>c.o.v. of Total Response</th>
</tr>
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<tbody>
<tr>
<td>1/12</td>
<td>0.4058</td>
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<td>0.0068</td>
<td>0.1154</td>
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<td>0.39</td>
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<td>0.1072</td>
<td>0.0012</td>
<td>0.1101</td>
<td>0.4045</td>
<td>0.41</td>
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<td>0.1104</td>
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<td>0.1889</td>
<td>0.0003</td>
<td>0.1112</td>
<td>0.3997</td>
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</tr>
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<td>0.2345</td>
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</tr>
<tr>
<td>11/12</td>
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<td>0.2802</td>
<td>0.0002</td>
<td>0.1065</td>
<td>0.3784</td>
<td>0.52</td>
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</table>

Table 5.9 Percentage Contribution from Uncertainty Sources to Total Variance of Maximum Displacement of Idealized Reactor Building (Fixed-base Building)

<table>
<thead>
<tr>
<th>$E[a_{max}]$ (g)</th>
<th>Structural Modeling</th>
<th>Structural Parameters</th>
<th>Filter and Duration Parameters</th>
<th>Randomness of Loading History</th>
<th>c.o.v. of Total Response</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.22</td>
<td>0.47</td>
<td>0.15</td>
<td>0.52</td>
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<td>0.43</td>
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</tr>
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<td>0.17</td>
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</tr>
<tr>
<td>9/12</td>
<td>0.24</td>
<td>0.18</td>
<td>0.34</td>
<td>0.24</td>
<td>0.43</td>
</tr>
<tr>
<td>11/12</td>
<td>0.24</td>
<td>0.19</td>
<td>0.32</td>
<td>0.25</td>
<td>0.43</td>
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</tbody>
</table>
Table 5.10 Percentage Contribution from Uncertainty Sources to Total Variance of Maximum Displacement of Idealized Reactor Building (Coupled System, $V_s = 1200$ ft/sec)

<table>
<thead>
<tr>
<th>$E[a_{max}]$ (g)</th>
<th>System Modeling</th>
<th>Super-structural Parameters</th>
<th>Sub-structural Parameters</th>
<th>Filter and Duration Parameters</th>
<th>Randomness of Loading History</th>
<th>c.o.v. of Total Response</th>
</tr>
</thead>
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<td>0.08</td>
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<td>0.11</td>
<td>0.08</td>
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<td>0.43</td>
</tr>
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<td>7/12</td>
<td>0.31</td>
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<td>0.08</td>
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<td>0.19</td>
<td>0.07</td>
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<td>0.47</td>
</tr>
<tr>
<td>11/12</td>
<td>0.26</td>
<td>0.17</td>
<td>0.23</td>
<td>0.07</td>
<td>0.27</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Fig. 5.1 Earthquake Power Spectra
(after Moayyad and Mohraz, 1982)
Fig. 5.2 Kanai-Tajimi Spectra (----) and Empirical Power Spectra (-----) (after Sues, Wen and Ang, 1983)
Fig. 5.3 The Hazard Curves in Santa Barbara (Cal.)
Fig. 5.4 Annual Ductility Ratio Exceedance Probability Curves of Ten-story UBC Steel Building with Fixed-base
Fig. 5.5 The System Parameter Uncertainties in the First-story Maximum Drift of 4-story UBC Steel Building
Fig. 5.5 (continued)
Fig. 5.6 Maximum Ductility Ratio Statistics of 4-story UBC Steel Building
Fig. 5.7 Ductility Ratio Exceedance Probability Curves of
4-story UBC Steel Building
Fig. 5.7 (continued)
Fig. 5.8 The System Parameter Uncertainties in the Maximum Displacement of Idealized Single-DOF Reactor Building
Fig. 5.8 (continued)
CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 Overall Summary

The response statistics of single-DOF buildings, ten-story and four-story UBC steel buildings, and an idealized single-DOF reactor building were estimated for fixed-base systems and coupled systems subjected to random seismic excitations. The purposes of the study are to investigate the effect of nonlinear soil-structure interaction on the dynamic response and on the structural safety of nonlinear building-foundation systems. Uncertainties in the building-foundation system and loading are also included in the evaluation of the overall structural reliability.

The subsystem approach is adopted to model a soil-structure system. The superstructure is modeled as a shear-beam system; the substructure is considered as a surface foundation on a halfspace. The analytical methods for modeling foundation uplifting and soil hysteresis were developed to investigate the significance of soil-structure interaction. A smooth hysteretic model is used to represent the nonlinear behavior in the substructure (from foundation uplifting and/or soil hysteresis) as well as in the superstructure (from structural material nonlinearity). The proper values of the hysteretic parameters are inferred from available experimental and theoretical results. The expected equivalent stiffness of the hysteresis and the expected equivalent foundation radii can be obtained from the response statistics. For a structure with a large number of degrees of freedom, a DOF-reduction technique is used to simplify the analysis and reduce computation time.

The uncertainties in the dynamic modeling, the parameters of the coupled system, and the ground motion were identified and included for structural reliability evaluation. The uncertainties in the
substructure system include those from the idealization of the foundation shape, foundation flexibility, soil profile, and soil properties; whereas the uncertainties associated with a superstructure include those in the specification of the structural mass, stiffness, strength, and damping ratio. Using an available seismic hazard model, the lifetime safety of a building-foundation system can be evaluated.

6.2 Conclusions

The main results and conclusions of the study may be summarized as follows:

1. The proposed model for nonlinear building-foundation systems is a viable tool to investigate the interaction effect on the response of coupled systems and to evaluate the corresponding system reliability under seismic loadings. The analytical hysteresis model can represent the nonlinear behavior of both the superstructure and substructure under cyclic loadings. For structure with a large number of DOF, the two-DOF approximation with mode shape modification (reduced-DOF technique) gives satisfactory results.

2. For linear coupled systems, the effect of interaction is significant: it increases the displacement of slender structures but decreases the displacement of squatty structures. Moreover, for coupled systems with the same aspect ratio but different foundation size, the effect of interaction will depend on the aspect ratio and the structural material—steel or reinforced concrete.

3. The material nonlinearities of a structure and soil deposit, and the geometric nonlinearity (foundation uplifting) can be included in a nonlinear coupled system. Generally, the nonlinearities in conventional coupled systems will reduce the interaction effect compared to corresponding linear coupled systems, and the structural material nonlinearity appear to have larger effect than other nonlinearities for conventional buildings. Among the different nonlinearities, the effects on interaction are small. Because each nonlinearity may have two
counteracting effects on a structure, the actual behavior of a nonlinear coupled system depends on specific values of the structural and soil properties, and excitation level.

4. For conventional coupled buildings, the effect of interaction on the structural response and reliability is small. Uncertainties in the parameters of a structure and ground motion duration tend to dominate the response uncertainty especially at the higher response level for fixed-base structures as well as coupled systems. The contribution of the substructural parameter uncertainties to the total response uncertainty is small compared to those of the other parameters.

5. For idealized single-DOF systems representing massive and stiff structures, the structural response is significantly reduced by the interaction effect, but the interaction effect of nonlinearities in the coupled system is small. The derivatives of the structural response with respect to the structural parameters, load duration, filter damping, and filter frequency, are also decreased by the interaction effect. The contributions of the uncertainties in the system model and randomness of the loading history, as well as the total response c.o.v., have the same trend as that of conventional buildings. However, the contributions from the substructural parameter uncertainties increase with the response level and is important at high excitation levels. Therefore, the effect of interaction and the uncertainty in the substructural parameters should not be overlooked in the design and analysis of massive and stiff structures.
APPENDIX A

EQUIVALENT LINEAR COEFFICIENTS

For real $n > 0$ the coefficients in Eq. 2.12 are

$$C = [\bar{A} - \bar{\mu} (\beta F_1 + \gamma F_2)] / \bar{\eta}$$ \hspace{1cm} (A.1)

$$K = -\bar{\mu} (\beta F_3 + \gamma F_4) / \bar{\eta}$$ \hspace{1cm} (A.2)

where:

$$\bar{A} = A - \delta \bar{\varepsilon} / \bar{A}$$ \hspace{1cm} (A.3a)

$$\bar{\eta} = 1.0 + \delta \bar{\varepsilon} / \eta$$ \hspace{1cm} (A.3b)

$$\bar{\mu} = 1.0 + \delta \bar{\varepsilon} / \mu$$ \hspace{1cm} (A.3c)

$$F_1 = \frac{\sigma^n_z}{\pi} \frac{\Gamma \left( \frac{n+2}{2} \right) 2^{n/2} I_s}{\bar{n}^{2(n+1)/2}}$$ \hspace{1cm} (A.4a)

$$F_2 = \frac{\sigma^n_z}{\sqrt{\pi}} \frac{\Gamma \left( \frac{n+1}{2} \right) 2^{n/2}}{\bar{n}^{2(n+1)/2}}$$ \hspace{1cm} (A.4b)

$$F_3 = \frac{\sigma^n_z}{\pi} \frac{\Gamma \left( \frac{n+2}{2} \right) 2^{n/2} \left\{ \frac{\sigma^{n-1}_z}{n} \right\} \bar{n}^{(n+1)/2}}{\rho \cdot \bar{u} z I_s}$$ \hspace{1cm} (A.4c)

$$F_4 = \frac{\sigma^n_z}{\sqrt{\pi}} \frac{\sigma^{n-1}_z}{\bar{n}^{2(n+1)/2}} \frac{\Gamma \left( \frac{n+1}{2} \right) 2^{n/2}}{\rho \cdot \bar{u} z I_s}$$ \hspace{1cm} (A.4d)

and

$$I = 2 \int_{-\pi/2}^{\pi/2} L \sin \theta \, d\theta$$ \hspace{1cm} (A.4e)

$$L = \arctan \left( \frac{\sqrt{1-\rho_u^2}}{\rho_u} \right)$$ \hspace{1cm} (A.4f)

$$\Gamma(\cdot) = \text{gamma function}$$
Appendix B

Derivative Expressions for Building-Foundation Systems

For building-foundation systems, the coefficient matrices G and B in differential equation Eq. 2.14 are

\[
G = A^{-1} G_0 T \\
B = A^{-1} B (A^{-1})^T
\]

where \( B = 2\pi s F^T F \) for white noise excitation.

When the derivatives with respect to the parameter \( p \) must be evaluated in Eqs. 2.40 and 2.42, the matrices \( \frac{\partial G}{\partial p} \) and \( \frac{\partial B}{\partial p} \) are

\[
\frac{\partial G}{\partial p} = A^{-1} \frac{\partial G_0}{\partial p} + \frac{\partial A^{-1}}{\partial p} G_0
\]

\[
\frac{\partial B}{\partial p} = \left( \frac{\partial A^{-1}}{\partial p} \right)^T B_0 (A^{-1})^T + A^{-1} \left( \frac{\partial B_0}{\partial p} \right) (A^{-1})^T + A^{-1} B_0 \frac{\partial (A^{-1})^T}{\partial p}
\]

where:

\[
\frac{\partial}{\partial p} (A^{-1}) = -A^{-1} \frac{\partial A}{\partial p} A^{-1}
\]

\[
\frac{\partial}{\partial p} (A^{-1})^T = -(A^{-1})^T \frac{\partial A^T}{\partial p} (A^{-1})^T
\]

Since the elements of matrix \( A \) consist of masses and heights of the coupled system, Eqs. B.5 and B.6 exist only when the parameter \( p \) is a mass of the coupled system. The derivatives of equivalent linear coefficients \( C \) and \( K \) with respect to the parameter \( p \) are calculated by evaluating \( \frac{\partial E}{\partial p} \), \( \frac{\partial F}{\partial p} \), \( \frac{\partial F}{\partial p} \) and \( \frac{\partial F}{\partial p} \) described as follows.
B.1 Evaluation of $\frac{\partial F}{\partial p}$, $\frac{\partial F}{\partial p}$, $\frac{\partial F}{\partial p}$, $\frac{\partial F}{\partial p}$

When the parameter $p$ is not the hysteresis parameter $n$, the relevant expressions (obtained by differentiating Eqs. A.6-A.8) are

$$\frac{\partial F_1}{\partial p} = \frac{2n/2}{\pi} \Gamma\left(\frac{n+2}{2}\right) \left\{ o_n \frac{\partial s}{\partial p} + no_{n-1} \frac{\partial \sigma}{\partial p} \right\} I_s$$  \hspace{1cm} (B.7a)

$$\frac{\partial F_2}{\partial p} = \frac{2n/2}{\sqrt{\pi}} \Gamma\left(\frac{n+1}{2}\right) no_{n-1} \frac{\partial \sigma}{\partial p}$$  \hspace{1cm} (B.7b)

$$\frac{\partial F_3}{\partial p} = \frac{n^2n/2}{\pi} \Gamma\left(\frac{n+2}{2}\right) (\sigma_u(n-1) o_n^2) \sigma_z^\alpha \frac{\partial \sigma}{\partial p} + \frac{\partial \sigma_u}{\partial p} o_{n-1} \left\{ \frac{1-\rho^2}{\rho^2} \right\}^{(n+1)/2}$$

$$+ \rho \frac{\partial I_s}{\partial p} + \frac{\partial \rho}{\partial p} \frac{\rho}{\frac{\partial I_s}{\partial p}} \right\}$$  \hspace{1cm} (B.7c)

$$\frac{\partial F_4}{\partial p} = \frac{n^2n/2}{\sqrt{\pi}} \Gamma\left(\frac{n+1}{2}\right) (\rho \sigma_u(n-1) o_n^2) \sigma_z^\alpha \frac{\partial \sigma}{\partial p} + \rho \frac{\partial \sigma_u}{\partial p} o_{n-1}$$

$$+ \frac{\partial \rho}{\partial p} \sigma_z^\alpha \sigma_u o_{n-1}$$  \hspace{1cm} (B.7d)

where

$$\frac{\partial \sigma_z}{\partial p} = \frac{1}{2\sigma_z} \frac{\partial E[z^2]}{\partial p}$$  \hspace{1cm} (B.8a)

$$\frac{\partial \sigma_u}{\partial p} = \frac{1}{2\sigma_u} \frac{\partial E[u^2]}{\partial p}$$  \hspace{1cm} (B.8b)
\[
\frac{\partial I_s}{\partial p} = -2 \sin^n \frac{L \partial L}{\partial p}
\]  

(B.9)

and

\[
\frac{\partial L}{\partial p} = \frac{-1}{\sqrt{1 - \rho_{uz}^2}} \frac{\partial \rho_{uz}}{\partial p}
\]  

(B.10)

However, when the derivative with respect to the parameter \( n \) is required the expressions are

\[
\frac{\partial F_1}{\partial n} = \frac{1}{\pi} \left[ \sigma z \left( \frac{n+2}{2} \right)^{n/2} \frac{\partial I_s}{\partial n} + \sigma z \left( \frac{n+2}{2} \right)^{n/2} \frac{\partial \rho_{uz}}{\partial n} \frac{\partial \rho_{uz}}{\partial n} \right]
\]  

(B.11a)

\[
\frac{\partial F_2}{\partial n} = \frac{1}{\pi} \left[ \sigma z \left( \frac{n+1}{2} \right)^{n/2} \frac{\partial \rho_{uz}}{\partial n} + \sigma z \left( \frac{n+1}{2} \right)^{n/2} \frac{\partial \rho_{uz}}{\partial n} \frac{\partial \rho_{uz}}{\partial n} \right]
\]  

(B.11b)

\[
\frac{\partial F_3}{\partial n} = \frac{1}{\pi} \left[ n \sigma \sigma \sigma \left( \frac{n-1}{2} \right)^{n/2} \frac{\partial \rho_{uz}}{\partial n} + n \sigma \sigma \sigma \left( \frac{n-1}{2} \right)^{n/2} \frac{\partial \rho_{uz}}{\partial n} \frac{\partial \rho_{uz}}{\partial n} \right]
\]  

(B.11c)
\[ + \left( n \sigma_z \sigma_{z}^{-1} \right) \Gamma \left( \frac{n+2}{2} \right)^{2} \left( n/2 \right) \cdot \left( \frac{2}{n} \right) \frac{\partial \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2}{\partial n} \]

\[- \frac{2}{n} \left( 1 - \rho_{uz}^{2} \right) (n+1)/2 + \rho_{uz} \frac{\partial I_{S}}{\partial n} + \frac{\partial \rho_{uz}}{\partial n} I_{S} \right] \]

\[ \left( B.11c \right) \]

\[ \frac{\partial F_{4}}{\partial n} = \frac{1}{\sqrt{\pi}} \left[ n \sigma_{uz} \sigma_{z}^{-1} \frac{\partial \left( \frac{n+2}{2} \right)^{\frac{3}{2}}}{\partial n} + n \sigma_{uz} \sigma_{z}^{-1} \frac{\partial \left( \frac{n+2}{2} \right)^{\frac{3}{2}}}{\partial n} \right] \cdot \frac{\partial \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2}{\partial n} \]

\[ + n \sigma_{uz} \sigma_{z}^{-1} \frac{\partial \left( \frac{n+2}{2} \right)^{\frac{3}{2}}}{\partial n} \cdot \frac{\partial \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2}{\partial n} + \sigma_{uz} \sigma_{z}^{-1} \frac{\partial \left( \frac{n+2}{2} \right)^{\frac{3}{2}}}{\partial n} \cdot \frac{\partial \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2}{\partial n} \]

\[ \left( B.11d \right) \]

where

\[ \frac{\partial \sigma_{z}^{n}}{\partial n} = \sigma_{z} \left[ \frac{n}{\sigma_{z}} \frac{\partial \sigma_{z}}{\partial n} + \frac{\partial \ln \sigma_{z}}{\partial n} \right] \]

\[ \left( B.12a \right) \]

\[ \frac{\partial \sigma_{z}^{-1}}{\partial n} = \sigma_{z}^{-1} \left[ \frac{n-1}{\sigma_{z}} \frac{\partial \sigma_{z}}{\partial n} + \frac{\partial \ln \sigma_{z}}{\partial n} \right] \]

\[ \left( B.12b \right) \]

\[ \frac{\partial}{\partial n} \left[ \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2 \right] = \left( 1 - \rho_{uz}^{2} \right) \left( n+1 \right)/2 \left[ \frac{1}{2} \frac{\partial \ln \left( 1 - \rho_{uz}^{2} \right)}{\partial n} \right] \]

\[ - \frac{n+1}{\left( 1 - \rho_{uz}^{2} \right)} \frac{\partial \sigma_{uz}}{\partial n} \]

\[ \left( B.12c \right) \]
\frac{\partial (2^{n/2}}{\partial n} = 2^{(n-2)/2} \ln 2 \tag{B.13}

and \frac{\partial \sigma_z}{\partial n}, \frac{\partial u}{\partial n}, \text{ and } \frac{\partial \rho_z}{\partial n} \text{ are as defined in Eq. B.8 with } p = n.

The derivative of the Gamma function with respect to its argument is obtained as

\frac{\partial \Gamma(x)}{\partial x} = \Gamma(x) \psi(x-1) \tag{B.14}

where \psi(\cdot) is the Digamma function (see, e.g., Hildebrand, 1976). Thus, by letting \( X_1 = \frac{(n+2)}{2}, X_2 = \frac{(n+1)}{2} \) and using the chain rule, the derivative of the Gamma function in Eq. B.11 may be evaluated as

\frac{\partial \Gamma \left( \frac{n+2}{2} \right)}{\partial n} = \frac{\partial \Gamma (X_1)}{\partial X_1} \frac{\partial X_1}{\partial n} = \frac{1}{2} \frac{\partial \Gamma (X_1)}{\partial X_1} \tag{B.15}

and

\frac{\partial \Gamma \left( \frac{n+1}{2} \right)}{\partial n} = \frac{1}{2} \frac{\partial \Gamma (X_2)}{\partial X_2} \tag{B.16}

Finally,

\frac{\partial I_s}{\partial n} = 2 \int_{\frac{\pi}{2}}^{\pi} \frac{\partial (\sin^n \theta)}{\partial n} d\theta - 2 \sin^n L \frac{\partial L}{\partial n}

= 2 \int_{\frac{\pi}{2}}^{\pi} \sin^n \theta \ell \sin \theta d\theta - 2 \sin^n L \frac{\partial L}{\partial n} \tag{B.17}

where \frac{\partial L}{\partial n} \text{ is defined by Eq. B.10 with } p = n.
B.2 Derivatives with Respect to Hysteretic Strength

Including the parameter $\nu$ in the model, Eq. 2.7 becomes

$$z_u = \left[ -\frac{A}{\nu(H+\gamma)} \right]^{1/n}$$  \hspace{1cm} (B.18)

and the yield strength is given as

$$q_y = [\alpha + (1-\alpha)A]^{k}\left[ -\frac{A}{\nu(H+\gamma)} \right]^{1/n}$$  \hspace{1cm} (B.19)

Solving Eq. B.19 for $\nu$ and differentiating the result with respect to $q_y$, the proportionality relating the variation in the yield strength to variation in $\nu$ is seen to be

$$\frac{\partial \nu}{\partial q_y} = -\frac{n}{q_y}$$  \hspace{1cm} (B.20)

In general, $\nu = 1.0$; thus, the expression may be simplified to

$$\frac{\partial \nu}{\partial q_y} = -\frac{n}{q_y}$$  \hspace{1cm} (B.21)

The desired response statistic derivatives are obtained by the chain rule as

$$\frac{\partial S}{\partial q_y} = \frac{\partial S}{\partial \nu} \frac{\partial \nu}{\partial q_y} = -\frac{n}{q_y} \frac{\partial S}{\partial \nu}$$  \hspace{1cm} (B.22)

The derivatives with respect to foundation rocking strength $M_{\text{max}}$ or soil bearing capacity $q_u$ may be evaluated by the same procedures as above. Therefore,

$$\frac{\partial S}{\partial q_u} = \frac{\partial S}{\partial \nu} \frac{\partial \nu}{\partial q_u} \frac{\partial M_{\text{max}}}{\partial q_u} = \frac{n}{M_{\text{max}}} \frac{\partial S}{\partial \nu} \frac{\partial M_{\text{max}}}{\partial q_u}$$  \hspace{1cm} (B.23)

The foundation rocking strength is

$$M_{\text{max}} = (2/3)q_r \sin \alpha$$  \hspace{1cm} (B.24)
Hence,

\[ \frac{\partial H_{\text{max}}}{\partial q_u} = \frac{2}{3} r^3 \sin^3 \alpha + 2r^3 \sin^2 \alpha \cos \alpha \left( \frac{1}{2} \sin 2\alpha - \alpha \right) \]  
(B.25)

The derivatives with respect to foundation translational strength \( H \) or with respect to cohesion \( c \) and friction angle \( \phi \) between the soil max and foundation, can be evaluated by the chain rule.

\[ H = c\pi (r^2) + W \tan \phi \]  
\( \text{max} \)  
(B.26)

Then, the desired derivatives are:

\[ \frac{\partial S}{\partial c} = \frac{\partial S}{\partial v} \frac{\partial v}{\partial H_{\text{max}}} \frac{\partial H_{\text{max}}}{\partial c} = - \frac{n}{H_{\text{max}}} \frac{\partial S}{\partial v} \frac{\partial H_{\text{max}}}{\partial c} \]  
(B.27)

\[ \frac{\partial S}{\partial \phi} = \frac{\partial S}{\partial v} \frac{\partial v}{\partial H_{\text{max}}} \frac{\partial H_{\text{max}}}{\partial \phi} = - \frac{n}{H_{\text{max}}} \frac{\partial S}{\partial v} \frac{\partial H_{\text{max}}}{\partial c} \]  
(B.28)

where:

\[ \frac{\partial H_{\text{max}}}{\partial c} = \pi r_1^2 \]  
(B.29)

\[ \frac{\partial H_{\text{max}}}{\partial \phi} = W \sec^2 \phi \]  
(B.30)
APPENDIX C

THE WINKLER-TYPE FOUNDATION FOR NONLINEAR SUBSTRUCTURAL SYSTEMS

Rocking Moment at Beginning of Uplifting — The critical loading of the Winkler foundation, \( W \), in which the separation of the foundation from the soil and the yielding of the soil occur simultaneously, can be evaluated. In this case, \( q_1 = 0 \), and \( q_2 = q_u \) in Fig. 3.9 (b), and

\[
W = 0.5q \frac{r^2}{u} \quad (C.1)
\]

At the beginning of uplifting, the rocking moment \( M_s \) can be obtained under the following conditions:

when \( W < W_{cr} \), the pressures \( q_1 = 0 \) and \( q_2 \) is in the elastic range, so

\[
M_s = 0.25W_s; \quad (C.2)
\]

whereas, when \( W > W_{cr} \), the pressures in some contact area is equal to \( q_u \) as shown in Fig. 3.9 (c), from which

\[
W = \frac{q_u r^2}{(1+\cos\theta)} \left( \frac{\pi+\theta}{\cos\theta} - \sin\theta + \frac{1}{3} \sin^3\theta \right) \quad (C.3)
\]

where \( \theta \) is half the angle in which the contact pressure is \( q_u \). \( \theta \) can be obtained from Eq. C.3. The corresponding rocking moment is

\[
M_s = \frac{q_u r^3}{(1+\cos\theta)} \left[ \frac{1}{4}(\pi-\theta) + \frac{1}{4}\sin^2\theta \cos^2\theta + \frac{5}{12}\sin^3\theta \cos\theta \right] \quad (C.4)
\]

Rocking Moment at Ultimate Condition — At the ultimate condition, the rocking moment causes a shear failure of the soil underneath the
foundation and leads to the overturning of the structure. Thus, the bearing pressures in Fig. 3.9(d) is equal to the ultimate bearing capacity, \( q_u \), and the contact area is equal to \( W/q_u \); i.e., from Fig. 3.9(d), the contact area is defined by,

\[
\frac{W}{q_u} = r^2 (\alpha - \frac{\sin 2\alpha}{2})
\] (C.5)

where \( \alpha \) is half the angle that contains the contact area. The ultimate rocking moment, \( M_u \), is

\[
M_u = \frac{2}{3} q_u r^3 \sin^3 \alpha
\] (C.6)

Therefore, for translational motion, the minimum equivalent circular radius, \( (r_1)_{\text{min}} \), is obtained as

\[
(r_1)_{\text{min}} = \sqrt{\frac{W}{\pi q_u}}
\] (C.7)

whereas on the basis of equivalent moments of inertia, the equivalent minimum radius, \( (r_2)_{\text{min}} \), for rocking motion can be obtained from the follows.

\[
(r_2)_{\text{min}}^4 = \frac{r^4}{\pi} (\alpha - 0.25\sin 4\alpha) - \frac{4W}{\pi q_u} (x_s)_{\text{max}}^2
\] (C.8)

in which

\[
(x_s)_{\text{max}} = \frac{2}{3} q_u r^3 \frac{\sin^3 \alpha}{W}
\] (C.9)

Expected Equivalent Radii — Between the beginning of the foundation separation and the ultimate condition, the rocking moment and the equivalent radius \( r \) are assumed to have the same linear relationship as
that of the elastic halfspace. As shown in Fig. 3.10 (b), when \( M < M_s \),
\[
    r_2 = r;
\]  \( \text{(C.10a)} \)
whereas when \( M > M_u \),
\[
    r_2 = (r_{2 \min})^2;
\]  \( \text{(C.10b)} \)
and when \( M < M < M_u \),
\[
    r_2 = (r_{2 \min})^2 + \frac{r-(r_{2 \min})}{M_u-M_s}(M-M_u);
\]  \( \text{(C.10c)} \)

In the random vibration analysis, the rocking moment may be assumed to be a zero-mean Gaussian variate; then its standard deviation is the RMS value of the rocking moment, \( \sigma_m \). Following the same procedures as in Sect. 3.3.2, the expected equivalent radius for rocking motion can be estimated as,
\[
    E[r_2] = (r_{2 \min})^2 + [r-(r_{2 \min})]A M_u \text{erf}(\frac{M}{\sqrt{2}\sigma_m})
    + A M_u \text{erf}(\frac{M_u}{\sqrt{2}\sigma_m}) + 2 A \frac{\sigma_m}{\sqrt{2}\pi} \{\exp[-\frac{1}{2}(\frac{M_u}{\sigma_m})^2]
    - \exp[-\frac{1}{2}(\frac{M}{\sigma_m})^2]\}
\]  \( \text{(C.11)} \)
in which \( A = [r-(r_{2 \min})]/(M-M_u) \).

Between the beginning of the foundation separation and the ultimate condition, the rocking moment and the equivalent radius \( r \) are assumed to have the same curvature relation as that of the elastic halfspace. As shown in Fig. 3.10 (a), when \( M < M < M_u \),
\[
    r_1 = AM_1^2 + BM_1 + C
\]  \( \text{(C.12)} \)
where:

\[ A = -1.074 \frac{r}{M_u} \]
\[ B = \frac{r}{M_u} [1.074(1 + \frac{M_s}{M_u}) - \frac{M_u (r - (r_1)_{\text{min}})}{r(M_u - M_s)}] \]
\[ C = r[-1.074 \frac{M_s}{M_u} + \frac{r \cdot M_u - (r_1)_{\text{min}} \cdot M_s}{r(M_u - M_s)}] \]

Therefore, the expected equivalent radius in translational motion can be evaluated as,

\[ E[r_1] = 2A \int_{M_s}^{M_u} m^2 f_M(m) \, dm - \frac{2B \sigma_m}{\sqrt{2\pi}} \left[ \exp\left(-\frac{1}{2} \left(\frac{M_u}{\sigma_m}\right)^2\right) \right. \]
\[ \left. - \exp\left(-\frac{1}{2} \left(\frac{M_s}{\sigma_m}\right)^2\right) \right] + (r_1)_{\text{min}} + [\text{erf}\left(\frac{M_u}{\sqrt{2}\sigma_m}\right)](C - (r_1)_{\text{min}}) \]
\[ + [\text{erf}\left(\frac{M_s}{\sqrt{2}\sigma_m}\right)] \cdot (r - C) \]  

(C.13)

in which \( f_M(m) \) is the probability density function of the rocking moment.
STOCHASTIC ENERGY DISTRIBUTION IN BUILDING–FOUNDATION SYSTEMS

When a structure or soil-structure system is subjected to a base excitation, the energy is imparted to it. Part of the absorbed energy is stored in the system in the form of kinetic and strain energy; the rest is dissipated through damping and inelastic deformation in the components of the system. The equations of motion for a single story soil-structure system subjected to an earthquake ground excitation are:

\[
(m + m)\ddot{x} + m\ddot{\theta} + m\dot{\theta} + c \dot{x} + a_k x + (1-a)k z = -(m + m)\ddot{b} \quad (D.1a)
\]

\[
\frac{2}{m\ddot{x}} + (J + m + mh)\ddot{\theta} + mh\ddot{\theta} + c \dot{\theta} + a_k \theta + (1-a)k z + k \frac{\theta - \theta}{r_2 n} = -m\ddot{b} \quad (D.1b)
\]

\[
m\ddot{x} + mh\ddot{\theta} + m\ddot{\theta} + (1-a)kz + c\dot{u} + aku = -m\ddot{b} \quad (D.1c)
\]

Assume the system is initially at rest; i.e., \( u(0) = \dot{u}(0) = 0 \). Postmultiplying the first equation by \( dx = \dot{x} dt \), the second equation by \( d\theta = \dot{\theta} dt \), and the third by \( du = \dot{u} dt \), then integrating from 0 to \( t \), and taking the expected values to the sum of the three equations, yield

\[
\bar{W} + \bar{W} + \bar{W} + \bar{W} + \bar{W} + \bar{W} = \bar{E} \quad (D.2)
\]

The individual energy terms in Eq. D.2 can be defined as follows:

\[
\bar{W} = mE[(\dot{x} + h\dot{\theta} + \dot{u})^2] / 2, \quad \text{represents the expected kinetic energy of the structure at time } t;
\]
where \( \dot{x}_o \), \( \dot{\theta} \) and \( \ddot{u} \) are the velocities at time \( t \).

\[
\bar{W} = c\int_0^t E[\dot{u}] \, dt,
\]
represents the expected energy dissipated by viscous damping in the superstructure;

\[
\bar{W} = \frac{akE[u]}{2},
\]
is the expected potential energy of the superstructure;

\[
\bar{W} = (1-\alpha)k\int_0^t E[z\ddot{u}] \, dt,
\]
represents the expected hysteretic energy in the superstructure from the onset of the base motion until time \( t \);

\[
\bar{W} = 0.5m E[\dot{x}]^2 + c\int_0^t E[\dot{x}]^2 \, dt + 0.5\alpha k E[x]^2 + (1-\alpha)k\int_0^t E[z\dot{x}] \, dt,
\]
is the expected energy absorbed by the substructure through translational motion.

In the above equation, the terms are, respectively, the expected foundation kinetic energy, the expected energy dissipated by viscous damping (radiation damping), the expected potential energy, and the expected energy dissipated by hysteresis (material damping) in translation.

The last term, \( \bar{W} \), on the left hand side of Eq. D.2 represents the expected energy absorption of the substructure in rocking motion, or

\[
\bar{W} = 0.5(J+m)E[\dot{\theta}]^2 + 0.5\alpha k E[\theta]^2 + 0.5k E[\theta]^2
\]

\[
+ (1-\alpha)k\int_0^t E[z\dot{\theta}] \, dt + c\int_0^t E[\dot{\theta}]^2 \, dt - k\int_0^t E[\theta \dot{\theta}] \, dt
\]

In Eq. D.3, the individual terms represent, respectively, the expected kinetic energy, the expected potential energy of spring \( k_{rl} \) (frequency independent part), the potential energy in spring \( k_{r2n} \) (frequency dependent part), the expected energy dissipated by hysteresis (material damping), the expected energy dissipation due to radiation damping in the frequency independent and frequency dependent parts.

The term, \( \bar{E} \), on the right hand side of Eq. D.2 represents the expected input energy to the soil-structure system; i.e.
where the ground motion acceleration, $a$, is modeled as a zero-mean filtered Gaussian shot noise random processes and can be represented by Eq. 3.48. Therefore, all the expected energies in above equations can be obtained from the covariance matrix $S$ which is the solution of Eq. 2.14. The extension to multi-story systems is straightforward.
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