METHOD OF SEISMIC RELIABILITY EVALUATION FOR MOMENT RESISTING STEEL FRAMES

By
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Technical Report of Research
Supported by the
NATIONAL SCIENCE FOUNDATION
(Under Grant CES 88–22690)

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF ILLINOIS AT
URBANA–CHAMPAIGN
URBANA, ILLINOIS
SEPTEMBER 1991
Method of Seismic Reliability Evaluation for Moment Resisting Steel Frames

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The objective of this study is development of a method for evaluation of performance of moment resisting steel frames under seismic loads. The emphasis is on modeling and quantification of the large uncertainties associated with the excitation, details in design according to current code specifications (UBC), and the nonlinear inelastic response behavior of the structure. A site-specific seismic hazard analysis is carried out to identify and quantify the uncertainties associated with the source, the path and the site condition. Future earthquakes are treated as either characteristic (major event along a well-identified major fault segment) or non-characteristic (local events). Ground motions are modeled as nonstationary random processes with time varying amplitude and frequency content, whose parameters depend on the source, path and site conditions. A strong column - weak beam model is developed for the structural frame and with which response can be obtained with good accuracy and computational efficiency. The response statistics are obtained by method of random vibration based on an equivalent linearization solution procedure and a smooth differential equation model for the hysteretic restoring force. The accuracy of this method is verified by comparison with simulations. A fast integration technique is then used to evaluate the probability of limit state (interstory drift limit being exceeded) considering the uncertainties in the excitation parameters. The robustness of the proposed method is demonstrated in the numerical examples throughout this study. The method can be used in assessing the risk implied in current earthquake resistant design, and in developing reliability-based code procedures.

Keywords: Seismic Hazard, Ground Motion, Characteristic Earthquakes, Attenuation, Random Process, Nonstationarity, SMRSF, Strong Column Weak Beam, Hysteresis, Simulation, Reliability, Fast Integration, Code and Standard.

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OPTIONAL FORM 272 (4-77)
(Formerly NTIS-35)
Department of Commerce
ACKNOWLEDGMENTS

This report is based on the thesis of Dr. D. Eliopoulos submitted in partial fulfillment of the requirements for the Ph.D. degree in Civil Engineering at the University of Illinois at Urbana–Champaign.

This study is part of the on-going research on Reliability Evaluation of Current Design Procedures for Steel Buildings Under Seismic Loads, supported by the National Science Foundation under grants NSF CES-88–22690 and NSF BCS-91–06390. This support is gratefully acknowledged. Contributions from D. Foutch and C.-Y. Yu is also gratefully acknowledged.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Objective and Scope</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Outline</td>
<td>2</td>
</tr>
<tr>
<td>2 SEISMIC HAZARD AND GROUND MOTION MODELING</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Seismic Hazard Analysis</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Characteristic Earthquake Hazard Analysis</td>
<td>7</td>
</tr>
<tr>
<td>2.2.2 Non-Characteristic Earthquake Hazard Analysis</td>
<td>11</td>
</tr>
<tr>
<td>2.3 Ground Motion Model and Parameter Identification</td>
<td>13</td>
</tr>
<tr>
<td>2.4 Numerical Example</td>
<td>18</td>
</tr>
<tr>
<td>3 BUILDING DESIGN ACCORDING TO UBC-88</td>
<td>35</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>35</td>
</tr>
<tr>
<td>3.2 Description of Five Story Building</td>
<td>36</td>
</tr>
<tr>
<td>3.3 Equivalent Lateral Force Design Procedure</td>
<td>37</td>
</tr>
<tr>
<td>3.3.1 Determination of the Equivalent Lateral Forces</td>
<td>38</td>
</tr>
<tr>
<td>3.3.2 Ductility Requirements</td>
<td>40</td>
</tr>
<tr>
<td>3.4 Design of Special Moment Resisting Space Frames (SMRSF)</td>
<td>41</td>
</tr>
<tr>
<td>3.4.1 East-West Frame</td>
<td>41</td>
</tr>
<tr>
<td>3.4.2 North-South Frame</td>
<td>42</td>
</tr>
<tr>
<td>4 STRUCTURAL MODELING AND ANALYSIS</td>
<td>47</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>47</td>
</tr>
<tr>
<td>4.2 Structural Models used in Random Vibration Analysis</td>
<td>48</td>
</tr>
<tr>
<td>4.2.1 The Shear Beam Model</td>
<td>48</td>
</tr>
<tr>
<td>4.2.2 The Hybrid Model</td>
<td>49</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.2.3 The Discrete Hinge Model</td>
<td>49</td>
</tr>
<tr>
<td>4.3 The Strong-Column-Weak-Beam (SCWB) Model</td>
<td>50</td>
</tr>
<tr>
<td>4.3.1 Modeling Hysteretic Behavior</td>
<td>53</td>
</tr>
<tr>
<td>4.3.2 Incorporating Viscous Damping</td>
<td>55</td>
</tr>
<tr>
<td>4.4 Identification of the SCWB Model Parameters</td>
<td>56</td>
</tr>
<tr>
<td>4.4.1 Identification of Linear Stiffness Coefficients</td>
<td>57</td>
</tr>
<tr>
<td>4.4.2 Evaluation of Rayleigh Damping Coefficients</td>
<td>58</td>
</tr>
<tr>
<td>4.4.3 Nonlinear Parameter Estimation</td>
<td>59</td>
</tr>
<tr>
<td>4.5 Response Analysis in the Time Domain</td>
<td>62</td>
</tr>
<tr>
<td>4.6 Evaluation of the Response Statistics</td>
<td>64</td>
</tr>
<tr>
<td>4.7 Numerical Example – Five Story, Three Bay, SMRSF</td>
<td>65</td>
</tr>
<tr>
<td>5 RANDOM VIBRATION ANALYSIS</td>
<td>72</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>72</td>
</tr>
<tr>
<td>5.2 SCWB Model Formulation</td>
<td>73</td>
</tr>
<tr>
<td>5.3 Maximum Response Statistics</td>
<td>78</td>
</tr>
<tr>
<td>5.4 Numerical Examples</td>
<td>81</td>
</tr>
<tr>
<td>6 ASSESSMENT OF STRUCTURAL SAFETY</td>
<td>86</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>86</td>
</tr>
<tr>
<td>6.2 Definition of Failure</td>
<td>87</td>
</tr>
<tr>
<td>6.3 Fast Integration Technique for Time Variant Reliability Analysis</td>
<td>88</td>
</tr>
<tr>
<td>6.4 Seismic Reliability Evaluation of Moment Resisting Steel Frames</td>
<td>91</td>
</tr>
<tr>
<td>6.5 Numerical Examples</td>
<td>92</td>
</tr>
<tr>
<td>7 SUMMARY AND CONCLUSIONS</td>
<td>96</td>
</tr>
<tr>
<td>7.1 Summary</td>
<td>96</td>
</tr>
<tr>
<td>7.2 Conclusions</td>
<td>98</td>
</tr>
<tr>
<td>7.3 Future Study</td>
<td>99</td>
</tr>
</tbody>
</table>
APPENDIX

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>SCWB MODEL FORMULATION: A THREE STORY EXAMPLE</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>A.1 Matrices in Equations of Motion</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>A.2 Identification of Linear Stiffness Coefficients</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>A.3 Equilibrium Equations in Quasi–Static Test</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>A.4 Formulation for Random Vibration Analysis</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>A.5 Equivalent Linearized Coefficients</td>
<td>106</td>
</tr>
<tr>
<td>B</td>
<td>SCWB MODEL: NONLINEAR PARAMETER ESTIMATION</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>B.1 Integral Increments $\Delta I_{1ji}$ and $\Delta I_{2ji}$</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>B.2 Partial Derivatives of the Residual</td>
<td>109</td>
</tr>
<tr>
<td>C</td>
<td>SYSTEM IDENTIFICATION</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>C.1 General</td>
<td>111</td>
</tr>
<tr>
<td></td>
<td>C.2 The Gauss Method</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>C.3 Scaled and Inverse Scaled Spectral Decomposition</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>C.4 Directional Discrimination</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>C.5 Projection Methods</td>
<td>118</td>
</tr>
<tr>
<td></td>
<td>C.6 Optimization Algorithm</td>
<td>119</td>
</tr>
<tr>
<td>D</td>
<td>DETAILS OF DRAIN–2DX COMPUTER PROGRAM</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>D.1 Program Capabilities</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>D.2 Column Element</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>D.3 Beam Element</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>D.4 Semi–Rigid Connection Element</td>
<td>122</td>
</tr>
<tr>
<td>E</td>
<td>PROBABILITY OF CHARACTERISTIC EARTHQUAKE OCCurrences</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>E.1 Probability of No Occurrence</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>E.2 Probability of $k$ Occurrences, $k = 1, 2,$</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>LIST OF REFERENCES</td>
<td>126</td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Statistics of characteristic earthquakes on the San Andreas and Imperial faults</td>
<td>22</td>
</tr>
<tr>
<td>2.2</td>
<td>List of recorded accelerograms at stiff sites (less than 150 ft of stiff clay, sand or gravel on rock, after Dorby, Idriss and Ng, 1978)</td>
<td>23</td>
</tr>
<tr>
<td>2.3</td>
<td>Statistical data (after Trifunac and Brady, 1975) for stiff sites used in the development of Eq. 2.19</td>
<td>24</td>
</tr>
<tr>
<td>2.4</td>
<td>Frequency dependent regression coefficients for scaling of ground acceleration Fourier amplitude spectra in terms of magnitude, distance to source and geologic site conditions (after Trifunac and Lee, 1989)</td>
<td>24</td>
</tr>
<tr>
<td>2.5</td>
<td>Frequency dependent regression coefficients for scaling of ground acceleration Fourier amplitude spectra in terms of modified Mercalli intensity and geologic site conditions (after Trifunac and Lee, 1989)</td>
<td>25</td>
</tr>
<tr>
<td>3.1</td>
<td>Uniform dead loads</td>
<td>42</td>
</tr>
<tr>
<td>3.2</td>
<td>Member sections (East–West frame)</td>
<td>43</td>
</tr>
<tr>
<td>3.3</td>
<td>Member sections (North–South frame)</td>
<td>43</td>
</tr>
<tr>
<td>4.1</td>
<td>Restoring moment parameters of SCWB model – five story, three bay SMRSF</td>
<td>66</td>
</tr>
<tr>
<td>5.1</td>
<td>Statistics of maximum story drifts</td>
<td>82</td>
</tr>
<tr>
<td>5.2</td>
<td>Probabilities of exceedance of various drift levels at a story</td>
<td>82</td>
</tr>
<tr>
<td>6.1</td>
<td>Probability of 1.5 percent drift being exceeded for the time window 1991 to 2041 ($u_f = 0.015$ h)</td>
<td>94</td>
</tr>
<tr>
<td>6.2</td>
<td>Probability of 1 percent drift level being exceeded for the time window 1991 to 2041 ($u_f = 0.010$ h)</td>
<td>94</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Segments of the Central and Southern San Andreas fault (after Geological</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Survey Open File Report, 1988)</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>Probability of occurrence of at least one characteristic earthquake within T</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>years from 1991</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>Mean significant duration of characteristic earthquake as function of</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>magnitude and distance to source</td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>Probability of exceedance of modified Mercalli intensity in a non-characteristic earthquake</td>
<td>27</td>
</tr>
<tr>
<td>2.5</td>
<td>Mean significant duration (one standard deviation) of non-characteristic</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>earthquake as function of modified Mercalli intensity</td>
<td></td>
</tr>
<tr>
<td>2.6</td>
<td>Ground acceleration instantaneous power spectra at the Los Angeles site</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>due to a non-characteristic earthquake ($I=8.7$, $t_D=0$)</td>
<td></td>
</tr>
<tr>
<td>2.7</td>
<td>Frequency modulation functions $\phi(t)$ of recorded accelerograms at the</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Hollywood Storage Building</td>
<td></td>
</tr>
<tr>
<td>2.8</td>
<td>Identified parameters and functions of the ground motion model for the</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Los Angeles site due to a characteristic earthquake ($M=7.5$, $R=60$ km,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_D=33.88$ sec, $\epsilon_S=-0.1$)</td>
<td></td>
</tr>
<tr>
<td>2.9</td>
<td>Simulated time history of the characteristic earthquake of the Mojave segment</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>at the Hollywood Storage Building ($M=7.5$, $R=60$ km)</td>
<td></td>
</tr>
<tr>
<td>2.10</td>
<td>Acceleration response spectrum of the ground motion record in Fig. 2.9</td>
<td>30</td>
</tr>
<tr>
<td>2.11</td>
<td>Identified parameters and functions of the ground motion model for the</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Los Angeles site due to a non-characteristic earthquake ($I=8.7$,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_D=10.66$ sec, $\epsilon_S=0$)</td>
<td></td>
</tr>
<tr>
<td>2.12</td>
<td>Simulated time history of a non-characteristic earthquake at the Hollywood</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Storage Building ($I=8.7$, $t_D=10.66$ sec)</td>
<td></td>
</tr>
<tr>
<td>2.13</td>
<td>Acceleration Response Spectrum of the Ground Motion Record in Fig. 2.12</td>
<td>32</td>
</tr>
<tr>
<td>2.14</td>
<td>Identified parameters and functions of the ground motion model</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>at the El Centro Site due to a characteristic earthquake ($M=6.5$,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_D=15.13$ sec, $\epsilon_S=-0.1$)</td>
<td></td>
</tr>
</tbody>
</table>
Figure | Page
---|---
2.15 Simulated time history of a characteristic earthquake at the El Centro site \((M=6.5, \ t_D = 15.13 \text{ sec}, \ \epsilon_S = -0.1)\) | 34
2.16 Acceleration response spectrum of the ground motion record in Fig. 2.15 | 34
3.1 Typical floor plan | 44
3.2 Elevation view, East-West frame | 44
3.3 Elevation view, North-South frame | 45
3.4 Design spectrum | 45
3.5 Member numbers, East-West frame (in reference to Table 3.2) | 46
3.6 Member numbers, North-South frame (in reference to Table 3.3) | 46
4.1 Discrete hinge model of a five story, four bay plane frame | 67
4.2 (a) Strong-column-weak-beam (SCWB) model; (b) \(i\)-th floor translational equilibrium; (c) \(i\)-th floor rotational equilibrium | 67
4.3 Three story, one bay frame: quasi-static test | 68
4.4 Modal comparison between SCWB model and the original five story, three bay SMRSF | 68
4.5 Comparison between story drifts of five story, three bay frame obtained by DRAIN-2DX and by the SCWB model (Imperial Valley, 1979 earthquake, El Centro differential array) | 69
4.6 Comparison between story drifts of five story, three bay frame obtained by DRAIN-2DX and by the SCWB model (El Centro, 1940 earthquake) | 70
4.7 Comparison between r.m.s. interstory drifts evaluated using DRAIN-2DX and the SCWB model | 71
5.1 Functions and parameters of the ground motion model, identified from the El Centro differential array record of the 1979, Imperial Valley earthquake | 83
5.2 Comparison between nonstationary root mean square story drifts obtained by Monte Carlo simulations and by the statistical equivalent linearization method (Imperial Valley, 1979 earthquake) | 84
5.3 Comparison between nonstationary root mean square joint rotations obtained by Monte Carlo simulations and by the statistical equivalent linearization method (Imperial Valley, 1979 earthquake) | 85
<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Different states of a system in the n-dimensional space defined by the random vector $\mathbf{X}$</td>
<td>95</td>
</tr>
<tr>
<td>B.1 Evaluation of $\theta_{j[i-1]} \Delta t_i$ and $\dot{\theta}_{ji} \Delta t_i$</td>
<td>110</td>
</tr>
<tr>
<td>D.1 Column element yield interaction surface</td>
<td>122</td>
</tr>
<tr>
<td>D.2 Beam element yield interaction surface</td>
<td>123</td>
</tr>
<tr>
<td>E.1 Time intervals to future characteristic earthquakes</td>
<td>125</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1 Objective and Scope

Structural safety under earthquakes has always been a major concern to design engineers. Due to the uncertainties associated with seismic loadings, however, evaluation of structural safety is not an easy task. In code provisions for aseismic building design, these uncertainties are considered in the form of load factors based on engineering judgement and socioeconomic considerations. For example, the 1988 Uniform Building Code uses an equivalent lateral force procedure taking into account the seismicity of the region where the building is located by a design spectrum and the effect of the ductility of the structural frame by a response modification factor $R_w$. A linear static analysis is then performed to determine the required strength and stiffness of the building's structural components. The safety (or reliability) implied in the procedure, however, is not given in the code and unknown.

Recently, significant progress has been made in the areas of earthquake engineering, random vibration and reliability analysis that the methods are now available to evaluate the reliability of aseismic design and assess the risk under potential earthquakes during the lifetime of the structure. Also, as more data have become available from recorded earthquakes, the understanding of ground motion has been improved and more accurate analytical models have been developed for predicting future events. Building analysis and design has traditionally been a deterministic procedure. Because of the large uncertainties in earthquake ground motion and structural resistance, however, a non-deterministic approach to the evaluation of structural performance under future seismic loadings is needed in order to obtain a credible measure of the reliability of the earthquake resistant design.
The objective of this study is to develop an integrated method for the evaluation of the seismic reliability of buildings, in particular, moment resisting steel frames. Currently available geological and seismological information is used to characterize and quantify the uncertainty of future seismic events. Earthquake ground motions are modeled as nonstationary stochastic processes with time-varying amplitude and frequency content. A computationally efficient structural model is developed to describe the global response behavior of the structure in the linear as well as in the nonlinear, hysteretic range. A random vibration method is then used to evaluate the conditional probabilities of failure with known ground motion parameters. A fast integration reliability method is finally employed to evaluate the overall risk of a limit state being exceeded due to future earthquakes, including the variability of ground motion parameters. The method can be used in assessing the risk implied in the current earthquake resistant design and in developing reliability-based code procedure.

1.2 Outline

In Chapter 2 a seismic hazard analysis is performed for a given site where the structure is located. Potential future earthquakes are categorized as either characteristic or non-characteristic. The characteristic earthquake is a major event which occurs along a major fault. Its expected magnitude depends on the characteristics of the fault segment and its recurrence time is relatively well understood. Non-characteristic earthquakes are minor local events whose occurrence can be treated collectively as a Poisson process. Non-characteristic events may be more destructive than a distant characteristic earthquake due to local geological conditions or if their epicenter is near the site. On the other hand, if the site is located near a major fault, the characteristic earthquake hazard usually prevails. For the characteristic earthquakes the major parameters in the analysis are recurrence time, magnitude, distance to the site and attenuation. For non-characteristic earthquakes the
major parameters are occurrence rate and modified Mercalli intensity. In both cases, available information at the site of interest is used to identify the parameters of the ground motion model. The model is that of a non-stationary stochastic process whose intensity and frequency content vary with time. It can be used to generate artificial ground acceleration for time history response analysis, or it can be used directly as input in a random vibration analysis.

The aseismic design of a five-story office building according to the equivalent lateral force procedure of the 1988 Uniform Building Code (UBC-88) is presented in Chapter 3. The building is designed to be located in Southern California. Moment resisting steel frames at the perimeter of the building provide the necessary lateral resistance. These frames are designed as special moment resisting space frames (SMRSF) following the provisions of UBC-88. Proper joint connections allow these frames to behave as plane frames. An interactive computer program (IGRESS-2, 1989) developed at the University of Illinois at Urbana-Champaign is used for the design.

A strong-column–weak–beam (SCWB) model is developed in Chapter 4. The model can be used to describe the inelastic response behavior of moment resisting steel frames with a smaller number of degrees of freedom, therefore reducing significantly the computational effort required for the analysis. It is assumed that yielding is concentrated at the beams and at the base of the frame. The hysteretic behavior of the inelastic response is described by inelastic rotational springs and a smooth restoring moment–rotation relationship. A system identification method is developed to estimate the model parameters. For this purpose, DRAIN-2DX, a finite element program developed at the University of California, Berkeley (Allahabadi and Powell, 1988) is used to generate the inelastic response of the actual structures. The SCWB model is shown to be accurate for response analysis.

In Chapter 5 the ground motion model and the SCWB model are implemented in random vibration analysis. The method of statistical equivalent linearization is used to obtain the
response covariance matrix which contains the second order statistics of response quantities of interest. The results of random vibration analysis compare very well with those obtained from simulations. A Type I probability distribution is assumed for the maximum interstory drift and its statistical parameters are derived from the calculated response statistics.

The system reliability analysis is presented in Chapter 6. Structural failure is defined in terms of interstory drift. Uncertainties in ground motion parameters such as significant duration, intensity etc. are taken into account in the evaluation of the probability of failure of a frame during its design lifetime using a fast integration technique based on first order, second moment reliability analysis. This approach reduces significantly the computational effort generally required for time-variant reliability analysis. A summary of the results and some important conclusions are given in Chapter 7.
CHAPTER 2
SEISMIC HAZARD AND GROUND MOTION MODELING

2.1 Introduction

One of the most important goals in earthquake engineering research is the prediction of future earthquake activity at a given site so that buildings and structures may be designed accordingly. Unfortunately, even though the knowledge on ground motion has increased significantly, predicting future earthquakes accurately is still beyond the capabilities of current technology. Nevertheless, data are accumulated from the recording of seismic events worldwide which allows a better understanding of ground motion and more accurate modeling and quantification of the uncertainty. For example, seismotectonic and geological maps of recognized Quaternary faults\(^1\) are available showing the type and depth of surficial depository material such as alluvium for many regions of the world. Seismic intensity maps are also available displaying epicenters and intensities of historical earthquakes. Databases of recorded ground motions are rapidly increasing in size as more areas are instrumented and new records become available.

In seismic hazard analysis one uses available geological and seismological information at a site to predict the probability of occurrence of future events, to estimate source and ground motion parameters and to develop analytical models for future earthquake ground motion. Commonly used source parameters include magnitude (or seismic moment), focal distance and modified Mercalli intensity. Ground motion parameters of interest include duration, frequency content, Arias intensity and effective peak acceleration. Attenuation laws are used to correlate the ground motion parameters with site to source distance, site geology, source and path characteristics, source directivity effect and so on. A summary of attenuation laws is given by Campbell (1985). As more data are accumulated from recent

\(^{1}\) These are faults for which there is evidence of movement in Quaternary times
earthquakes, empirical estimates of ground motion parameters become more reliable and the uncertainties associated with attenuation are better quantified.

Methods have been developed recently for statistical estimation of ground motion parameters and simulation of future earthquake events. An overview of some of these methods is given by Joyner and Boore (1988). Since future earthquake motions can not be predicted with certainty, stochastic models are the most appropriate for describing future events. Earthquake motions are generally nonstationary with time-varying intensity and frequency content. Nonstationary random process models have been proposed by Lin and Yong (1987), Deodatis and Shinozuka (1988), Der Kiureghian and Crempien (1989) and others. Yeh and Wen (1989) recently proposed a filtered white noise model with intensity and frequency content modulation in time (details are given in Section 2.3). The advantages of this model are that it can be easily implemented in random vibration analysis and that its parameters can be identified from ground motion time history or from available seismological information at the site of interest. For these reasons the model proposed by Yeh and Wen is used in this study.

2.2 Seismic Hazard Analysis

The procedure of seismic hazard analysis developed in this study is site specific and is therefore heavily dependent on the seismological and geological information available at the site. In practice, however, if all information is not available at this site, information from other sites with similar characteristics may be used.

Recent research indicates that potential future earthquakes presenting a threat to a site can be categorized as either characteristic (major events which occur along a major fault), or non-characteristic (minor local events). If a site is located near a major fault, the hazard from the characteristic earthquake is dominant; otherwise, non-characteristic local events may contribute significantly to or even govern the analysis.
Non-characteristic earthquakes are assumed to be randomly distributed in time with their occurrence following a Poisson distribution. This assumption is consistent with historical occurrence of seismic shocks of engineering importance. Small shocks may depart significantly from a Poisson process, but they are of marginal interest in engineering applications. Characteristic events have an occurrence behavior which is better understood than that of non-characteristic earthquakes and their recurrence time interval can be modeled by a lognormal distribution. The major parameters of the characteristic earthquake for the analysis are, therefore, recurrence time, magnitude, attenuation and focal distance. The major parameters of non-characteristic earthquakes for the analysis are occurrence rate and modified Mercalli intensity.

2.2.1 Characteristic Earthquake Hazard Analysis

The characteristic earthquake model (Schwartz and Coppersmith, 1984), developed after a series of paleoseismological studies, postulates that individual fault segments tend to generate essentially the same size earthquakes, with a relatively narrow range of magnitude near the maximum, at relatively regular recurrence periods. These earthquakes are referred to as characteristic events for the specific fault segment. As a result, the probability of occurrence of a characteristic event along a fault segment is time dependent; i.e., it increases with time since the last event. The seismic gap theory (Kelleher et al., 1973) supports this hypothesis. This theory states that the potential for a future earthquake is greater along those active fault segments having large elapsed times since the last characteristic earthquake. A lognormal distribution best represents observed recurrence time behavior (Nishenko and Buland, 1987). The parameters of this lognormal distribution can be determined from available information on the specific fault segment, such as the average recurrence time and the expected magnitude of characteristic events. The average recurrence time can be calculated from the dates of repeated historical events that ruptured the same fault segment.
or from geologic dating of slip events inferred to represent seismic events on the same fault segment. Table 2.1 based on data from the U.S. Geological Survey open-file report 88-398 (Geological Survey, 1988) provides the date of the most recent event, the expected magnitude and the expected recurrence time with its coefficient of variation for segments of the San Andreas fault and the Imperial fault. Since the recurrence time of characteristic events follows a lognormal distribution, their occurrence can be modeled as a renewal process with lognormal recurrence time and the probability of \( k \) occurrences within a specified time period may be evaluated as shown in Appendix E.

The expected magnitude of the characteristic event is estimated based on seismic moment. It is evaluated from the dimensions and other physical characteristics of the fault segment or from the size of previous events. Segments of the Central and Southern San Andreas fault are shown on the map of Fig. 2.1. The probabilities of occurrence of at least one characteristic event within \( T \) years from 1991 in the Mojave segment of the Southern San Andreas fault and the Imperial fault are plotted versus \( T \) in Fig. 2.2. The magnitude of future characteristic events from a specific segment can be considered equal to the expected magnitude for the segment, given in Table 2.1.

Trifunac and Brady (1975) defined the significant duration of ground motion as the time interval required to build up between 5 and 95 percent of the Arias intensity of the record. Adopting their definition, the significant duration \( t_D \), associated with the strong part of the ground motion, is obtained as a function of magnitude \( M \) and source distance \( R \) (km) using the empirical relation

\[
\log_{10} t_D = -0.14 + 0.2M + 0.002R + \epsilon_D
\]  

(2.1)

where \( \epsilon_D \) is an error term following a normal distribution with \( E[\epsilon_D] = 0 \) and \( \sigma_{\epsilon_D} = 0.135 \). This relation was obtained by regression analysis using the data of Table 2.2. The mean significant duration \( t_D \), calculated from Eq. 2.1 (i.e., with \( \epsilon_D = 0 \)), is plotted versus site to
source distance for different magnitudes in Fig. 2.3. The Arias intensity (Arias, 1970) is a measure of the energy of the accelerogram given by

$$I_A = \int_0^{t_F} a^2(t) \, dt$$

(2.2)

where \(a(t)\) is the ground acceleration and \(t_F\) is the total duration of ground shaking.

The ground acceleration Fourier amplitude spectrum is used to describe the frequency content of the characteristic event. Using the empirical model developed by Trifunac and Lee (1989), the Fourier amplitude spectrum is scaled in terms of magnitude, site-to-source distance, and local site geology. The model introduces a frequency dependent attenuation function, \(\text{Att}(\Delta, M, T)\) and expresses the Fourier amplitude spectrum, \(\mathcal{F}(T)\), as

$$\log_{10} \mathcal{F}(T) = M + \text{Att}(\Delta, M, T) + \hat{b}_1(T)M + \hat{b}_2(T)s + \hat{b}_3(T) + \hat{b}_6(T)M^2 - \epsilon_S$$

(2.3)

where \(T\) is the period (sec), \(s\) characterizes the geologic site conditions \((s = 0\) for alluvium, \(s = 2\) for basement rock, \(s = 1\) for intermediate sites), \(\Delta\) is a measure of the site-to-source distance to be explained in the following, and \(\hat{b}_1(T), \hat{b}_2(T), \hat{b}_3(T), \hat{b}_4(T)\) are regression coefficients given in Table 2.4. The error term \(\epsilon_S\) models the uncertainty in this relationship, primarily due to attenuation and local site conditions and is assumed to follow a normal distribution, \(N(0, 0.205)\). Eq. 2.3 applies only in the range \(M_{\min} \leq M \leq M_{\max}\), where

$$M_{\min} = -\frac{\min(-\hat{b}_1(T), 0.)}{2\hat{b}_6(T)} \quad \text{and} \quad M_{\max} = \max\left[ -\frac{1 + \min(-\hat{b}_1(T), 0.)}{2\hat{b}_6(T)}, 15 \right]$$

(2.4)

Outside this range, Eq. 2.3 is modified and given by

$$\log_{10} \mathcal{F}(T) = M + \text{Att}(\Delta, M, T) + \hat{b}_1(T)M_{\min} + \hat{b}_2(T)s + \hat{b}_3(T) + \hat{b}_6(T)M_{\min}^2 - \epsilon_S \quad M \leq M_{\min}$$

(2.5)

$$\log_{10} \mathcal{F}(T) = M_{\max} + \text{Att}(\Delta, M, T) + \hat{b}_1(T)M_{\max} + \hat{b}_2(T)s + \hat{b}_3(T) + \hat{b}_6(T)M_{\max}^2 - \epsilon_S \quad M \geq M_{\max}$$

(2.6)
In the attenuation relation $\text{Att}(\Delta, M, T)$, $\Delta$ is a "representative distance" from the earthquake source to the site, defined as

$$\Delta = S \left( \ln \frac{S^2 + R^2 + H^2}{S_0^2 + R^2 + H^2} \right)^{-\frac{1}{2}}$$

(2.7)

where $R$ is the epicentral distance, $H$ the focal depth, $S$ the fault size "felt" at a period $T$, and $S_0$ the coherence radius of the source, taken to be a half of the wavelength $\lambda$ for radiation of period $T$:

$$S_0 = \lambda / 2 = C_s T / 2$$

(2.8)

where $C_s$ is the shear wave velocity (in this work $C_s$ is taken equal to 1 km/sec). The fault size $S$ is assumed to be a linear function of magnitude:

$$S = 0.2 + 8.51(M - 3)$$

(2.9)

The frequency dependent attenuation function $\text{Att}(\Delta, M, T)$ can then be written as

$$\text{Att}(\Delta, M, T) = \begin{cases} 
A_0(T) \log_{10} \Delta & R \leq R_0 \\
A_0(T) \log_{10} \Delta_0 - \frac{R - R_0}{200} & R > R_0 
\end{cases}$$

(2.10)

where $A_0(T)$ is an empirically determined function (parabolic for $T < 1.8$ sec, constant for $T > 1.8$ sec)

$$A_0(T) = \begin{cases} 
-0.732025 & T \geq 1.8 \text{ sec} \\
-0.767093 + 0.271556 \log_{10} T - 0.525641 (\log_{10} T)^2 & T < 1.8 \text{ sec}
\end{cases}$$

(2.11)

and

$$\Delta_0 = S \left( \ln \frac{S^2 + R^2 + H^2}{S_0^2 + R_0^2 + H^2} \right)^{-\frac{1}{2}}$$

(2.12)

For distances $R \geq R_0$, the attenuation relation is a linear function of distance with slope $-1/200$. The transition distance $R_0$ is given by
\[ R_0 = \frac{1}{2} \left[ -\frac{200A_0(T)(1 - S_0^2/S^2)}{\ln 10} + \sqrt{\left( \frac{200A_0(T)(1 - S_0^2/S^2)}{\ln 10} \right)^2 - 4H^2} \right] \] (2.13)

A more detailed description of the attenuation function is given by Trifunac and Lee (1987).

The Arias intensity is used to evaluate the total energy of the characteristic earthquake. It can be obtained from the area under the Fourier amplitude spectrum as

\[ I_A = \frac{1}{\pi} \int_{0}^{\infty} F^2(\omega) \, d\omega \] (2.14)

where \( \omega = 2\pi/T \) is the cyclic frequency (rad/sec). The significant duration, the Fourier amplitude spectrum and the Arias intensity are used to identify the parameters of the ground motion model as described in Section 2.3.

### 2.2.2 Non-Characteristic Earthquake Hazard Analysis

Seismic events not generated in major fault segments fall under the category of non-characteristic events. Such events may be more destructive than a characteristic earthquake depending on the proximity of their epicenter to the site and local geological conditions. Compared to characteristic events, the source and the occurrence of a non-characteristic earthquake are less predictable. Nevertheless, information based on local geology and previous events recorded at the site can be used in the seismic hazard analysis.

Since the source of a non-characteristic earthquake is generally unknown, modified Mercalli intensity is used. Data have indicated that the intensities are exponentially distributed and a Poisson process can be used to model the occurrence of non-characteristic earthquakes for intermediate and large events. Small events may depart significantly from a Poisson process but they are only of marginal interest to engineering applications.
Following the Poisson assumption, the probability of occurrence of $k$ non-characteristic earthquakes in a time period $t$ is given by

$$P(N_{nc} = k) = \frac{\nu t^k}{k!} e^{-\nu t} \quad (2.15)$$

In this equation, $\nu$ is the mean annual occurrence rate of non-characteristic events at a site, with intensity higher than a modified Mercalli intensity $I$. At a given site, $\nu$ can be evaluated by the log-linear relation

$$\log_{10} \nu = a - b(I - I_{min}) \quad (2.16)$$

where $a$ and $b$ are constants associated with the site (Algermissen et al., 1982) and $I_{min}$ is the lowest modified Mercalli intensity considered. Based on the above assumptions, the probability that $I < i$, given the occurrence of a non-characteristic event, is given by

$$P(I < i \mid I_{min} < I < I_{max}) = \frac{\nu(I_{min}) \cdot \nu(i)}{\nu(I_{min}) \cdot \nu(I_{max})} = \frac{1 - 10^{-b(i - I_{min})}}{1 - 10^{-b(I_{max} - I_{min})}} \quad (2.17)$$

in which $I_{max}$ and $I_{min}$ are the upper and lower limits of $I$. The cumulative distribution function of $I$ is written as

$$F_I(i) = \frac{1}{1 - e^{-b(I_{max} - I_{min}) \ln 10}} \left[ 1 - e^{-b(i - I_{min}) \ln 10} \right] \quad ; \quad I_{min} \leq i \leq I_{max} \quad (2.18)$$

The probability that $I < i$ is plotted versus modified Mercalli intensity (with $I_{min} = 5$ and $I_{max} = 11$) in Fig. 2.4.

Given the intensity of an event, the significant duration $t_D$ may be evaluated from the empirical relation

$$\log_{10} t_D = 1.96 - 0.123 I + \epsilon_D \quad (2.19)$$

The uncertainty in this relationship is modeled by a random variable $\epsilon_D$ following a normal distribution with $E[\epsilon_D] = 0$ and $\sigma_{\epsilon_D} = 0.205$. Eq. 2.19 is obtained by a regression analysis
using the statistical data of Table 2.3. The significant duration $t_D$ calculated from Eq. 2.19 is plotted versus modified Mercalli intensity in Fig. 2.5.

To identify the spectral properties of a non-characteristic earthquake, its Fourier amplitude spectrum is scaled in terms of modified Mercalli intensity and local site geology. Following the method proposed by Trifunac and Lee (1989) the Fourier amplitude spectrum $FS(T)$ is expressed as

$$\log_{10} FS(T) = \hat{b}_1(T)I + \hat{b}_2(T)s + \hat{b}_4(T) - \varepsilon_S$$

(2.20)

where $T$ is the period (sec), $s$ characterizes the geologic site conditions ($s = 0$ for alluvium, $s = 2$ for basement rock, $s = 1$ for intermediate sites) and $\hat{b}_1(T), \hat{b}_2(T), \hat{b}_4(T)$ are regression coefficients given in Table 2.5. The uncertainty term $\varepsilon_S$ in this empirical relation follows a normal distribution $N(0, 0.205)$. The Arias intensity can then be evaluated from the Fourier amplitude spectrum using Eq. 2.14. The above information is used to identify the parameters of the ground model as described in the following section.

2.3 Ground Motion Model and Parameter Identification

The ground motion is modeled as an amplitude and frequency modulated filtered Gaussian white noise, following the approach of Yeh and Wen (1989). Such a process is described by an intensity function $I(t)$, a frequency modulation function $\phi(t)$ and a power spectral density function $S(\omega)$ and can be expressed as

$$a(t) = I(t) \xi(\phi(t)) = I(t)\xi(t)$$

(2.21)

In this equation, $\xi(\phi)$ is a zero mean, unit variance, stationary filtered white noise in $\phi$. Since $I(t)$ and $\phi(t)$ are functions of time, $a(t)$ is a nonstationary process with a time-dependent power spectral density function.
The intensity function controls the amplitude of \( a(t) \). The following form is used:

\[
I^2(t) = A \frac{t^B e^{-Ct}}{D + t^E}
\]  

(2.22)

where \( A, B, C, D \) are parameters to be identified.

The frequency modulation function, controls the variation of the frequency content of \( a(t) \) in time and is given as

\[
\Phi(t) = \frac{\mu_0(t)}{\mu_\prime_0(t_0)}
\]

(2.23)

where \( \mu_0(t) \) is the mean number of zero crossings and \( \mu_\prime_0(t_0) \) its first time derivative evaluated at time \( t_0 \). The function \( \mu_0(t) \) is continuous, differentiable, monotonically increasing and is modeled as an \( n \)-th order polynomial:

\[
\mu_0(t) = r_1t + r_2t^2 + \ldots + r_nt^n
\]

(2.24)

where \( r_1, r_2, \ldots, r_n \) are coefficients to be identified based on data of zero crossings. In most cases, a third order polynomial is sufficient.

For \( \zeta(\phi) \) which is stationary, the power spectral density (PSD) function controls the frequency content of the process. For \( \zeta(t) \) which is nonstationary, the instantaneous power spectrum (Mark, 1985) can be used to describe the change of frequency content with time. The instantaneous power spectrum of \( \zeta(t) \) at time \( t \) is given by (Yeh and Wen, 1989)

\[
S_{aa}(t, \omega) = \frac{1}{\Phi'(t)} S_{CP} \left( \frac{\omega}{\Phi'(t)} \right)
\]

(2.25)

where

\[
S_{CP}(\omega) = S_0 \left[ \frac{\omega_4^4 + 4 \zeta^2 \omega_5^2 \omega_6^2 \omega_7^2}{(\omega_6^2 - \omega_7^2)^2 + 4 \zeta^2 \omega_5^2 \omega_6^2} \right] \left[ \frac{\omega^4}{(\omega_t^2 - \omega_7^2)^2 + 4 \zeta^2 \omega_7^2 \omega_t^2} \right]
\]

(2.26)
is the Clough and Penzien spectrum of the stationary process $\overline{\xi}(\phi)$ (Clough and Penzien, 1973). Spectrum parameters, $S_0$, $\omega_g$, $\zeta_g$, $\omega_f$ and $\zeta_f$, are to be identified. Note that both the scale and the shape of the spectral density change with time according to $\phi'(t)$, the first derivative of the frequency modulation function (see Fig. 2.6). The nonstationary process $a(t)$ given by Eq. 2.21 and with the above spectral properties can be obtained by the following filtering equations (Yeh, 1989):

$$\ddot{x}_g + \left(-\frac{\phi''(t)}{\phi'(t)} + 2\zeta_g\omega_g\phi'(t)\right)\dot{x}_g + [\omega_g\phi'(t)]^2x_g = -[\phi'(t)]^2 1(t) \xi(\phi(t)) \quad (2.27)$$

$$\ddot{x}_f + \left(-\frac{\phi''(t)}{\phi'(t)} + 2\zeta_f\omega_f\phi'(t)\right)\dot{x}_f + [\omega_f\phi'(t)]^2x_f = -2\zeta_g\omega_g\phi'(t)\dot{x}_g - [\omega_g\phi'(t)]^2x_g \quad (2.28)$$

$$a(t) = 2\zeta_f\omega_f\frac{\dot{x}_f}{\phi'(t)} + \omega_f^2x_f + 2\zeta_g\omega_g\frac{\dot{x}_g}{\phi'(t)} + \omega_g^2x_g \quad (2.29)$$

In these equations, $\xi(\phi)$ is a zero mean, unit variance, Gaussian white noise, stationary in $\phi$ with power spectral density $S_0$, and $x_g$, $x_f$ are auxiliary variables. Eqs. 2.27 and 2.28 are the Clough–Penzien filtering equations (Clough and Penzien, 1973), modified by a time scaling transformation depicted by $\phi(t)$.

Yeh and Wen (1989) identified the model parameters based on an actual record. In this study, an alternative approach is also proposed in which the model parameters are identified from the scaled Fourier amplitude spectrum, the Arias intensity and the significant duration associated with future earthquakes. Therefore, this method can be used for sites where direct ground motion data are unavailable.

It can be shown that if $a(t)$ in a stationary process, the power spectral density function has the same shape as the square of the Fourier amplitude spectrum $FS^2(\omega)$. The Fourier
amplitude spectrum $FS(\omega)$ is the absolute value of the Fourier transform of $a(t)$:

$$FS(\omega) = \left| \int_0^{t_F} a(t)e^{-i\omega t} \, dt \right|$$

(2.30)

where $t_F$ is the length of the ground acceleration record. It is known that the area under the PSD function, $S_{aa}(\omega)$, is equal to the mean square acceleration:

$$\sigma_a^2 = \int_0^{\infty} S_{aa}(\omega) \, d\omega$$

(2.31)

From the definition of mean square acceleration and Eqs. 2.2, 2.14 and 2.31 one obtains

$$\sigma_a^2 t_F = t_F \int_0^{\infty} S_{aa}(\omega) \, d\omega = \int_0^{t_F} a^2(t) \, dt = \frac{1}{\pi} \int_0^{\infty} FS^2(\omega) \, d\omega$$

(2.32)

Comparing 2.31 and 2.32, the PSD function of $a(t)$ is related to the Fourier amplitude spectrum as

$$S_{aa}(\omega) = \frac{1}{\pi t_F} FS^2(\omega)$$

(2.33)

Eq. 2.33 reveals that the PSD function differs from the square of the Fourier amplitude spectrum only by a multiplication factor. Therefore, even though $a(t)$ is nonstationary, as an approximation, $S_0$, $\omega_g$, $\xi_g$, $\omega_f$ and $\xi_f$ can be identified from $FS^2(\omega)$ using a nonlinear optimization procedure based on the Gauss method, described in Appendix C.2. After the spectrum parameters have been identified, the PSD function is normalized so that $\sigma_{CP}^2 = 1$.

By direct integration, one can show that the variance of the Clough–Penzien spectrum as

$$\sigma_{CP}^2 = \pi \, S_0 \, \frac{\omega_g^4(\xi_g \omega_f + \xi_f \omega_g) + 4\xi_g^2 \omega_g^2[\xi_g \omega_f^3 + \xi_f \omega_g^3 + 4\xi_g \xi_f \omega_g \omega_f(\xi_g \omega_f + \xi_f \omega_g) \omega_g]}{2\xi_g^2 \xi_f(\omega_g^2 - \omega_f^2)^2 + 4\omega_g^2 \omega_f^2 (\xi_g^2 + \xi_f^2) + 4\xi_g \xi_f \omega_g \omega_f (\omega_g^2 + \omega_f^2)}$$

(2.34)
Hence, to normalize the PSD function to unit variance, after the spectrum parameters have been identified, one needs divide $S_0$ by $\sigma^2_{CP}$, calculated from the above relation.

The coefficients of the intensity function are identified by considering the total energy of the record $E_T$ evaluated from Eq. 2.14, and the significant duration $t_D$ of the strong ground motion part of record, evaluated from Eqs. 2.1 or 2.19. An initial time $t_0$ is assumed at the beginning of the record, before the strong ground motion part. The total length of the record $t_F$ is estimated as $t_F = t_0 + 3t_D$. Using the definition of significant duration (see Section 2.2) the energy levels at times $t_F$, $t_0$ and $t_1 = t_0 + t_D$ are calculated as $E_T$, $E_0 = 0.05E_T$ and $E_1 = 0.95E_T$ respectively. One can use these data points to identify the intensity function parameters A, B, C, D and E from the energy function of the record

$$E(t) = \int_0^t I^2(\tau) \, d\tau \quad (2.35)$$

where $I^2(\tau)$ is given by Eq. 2.22. The Gauss algorithm is again used for the identification.

The frequency modulation function coefficients are identified from previously recorded accelerograms at the site. A comparison between accelerograms from different earthquakes recorded at the Hollywood Storage Building, located in Santa Monica Boulevard, Los Angeles, California, showed that their frequency modulation functions were similar and can be interpreted as characteristic of the site. Fig. 2.6 shows the results of such a comparison using records from the February 9, 1971, San Fernando earthquake (Jennings, 1971) and the October 1, 1987, Whittier Narrows earthquake (Shakal et al., 1987). The epicenter of the San Fernando earthquake was 34 km North of the site while the epicenter of the Whittier Narrows earthquake was located 21 km East of the site. An average $\phi(t)$ obtained from these records is used for the site. To take into account the length of a specific record, the frequency modulation function is stretched to the total duration of the record, $t_F$ so that
\( \phi'(t/t_F) \) remains unchanged. In this manner, the rate of change of the frequency content of a record at a certain time instant is the same for any record. For example, to obtain the frequency modulation function of total duration \( t_{F1} \) using the frequency modulation function based on a record of total duration \( t_{F0} \), the coefficients of \( \phi(t) \) in Eq. 2.24 should be scaled according to

\[
\frac{r_{i1}}{r_{i0}} = \left( \frac{t_{F0}}{t_{F1}} \right)^{i-1}
\]  

(2.36)

where \( r_{i0} \) and \( r_{i1} \) are respectively the \( i \)-th order polynomial coefficients in the frequency modulation functions of duration \( t_{F0} \) and \( t_{F1} \).

2.4 Numerical Example

The seismic hazard analysis is demonstrated for a site located on Santa Monica Boulevard in Los Angeles, 60km from the Mojave segment of the San Andreas fault, i.e., the Hollywood Storage building. The seismic hazard due to both characteristic and non-characteristic earthquakes is investigated. The soil at the site can be characterized as stiff and firm.

The expected magnitude of the characteristic earthquake at the Mojave segment is \( M = 7.5 \) with an average recurrence interval of 162 years (Table 2.1). The probability of occurrence of at least one characteristic event in the next fifty years (until 2041), given that the last characteristic event occurred in 1857, can be obtained as

\[
P[T < T_0 + t | T \geq T_0] = \frac{1 - \Phi \left[ \frac{\ln(T_0 + t) - \lambda_T}{\xi_T} \right]}{1 - \Phi \left[ \frac{\ln T_0 - \lambda_T}{\xi_T} \right]} = 0.495
\]  

(2.37)

where \( \Phi[ . . . ] \) is the standard normal probability distribution, \( t = 50 \) yrs., \( T_0 = 134 \) yrs., and
\[
\xi_T = \sqrt{\ln\left(1 + \frac{\delta_T^2}{\mu_T}\right)} = 0.4119 \quad \lambda_T = \ln\mu_T - \frac{1}{2} \xi_T^2 = 5.0028
\] (2.38)

with \( \mu_T = 162 \) yrs. and \( \delta_T = 0.43 \) (Table 2.1). The probability of a given number of occurrences can be also calculated according to a renewal process model (Appendix E). Given the occurrence of a characteristic event with magnitude \( M = 7.5 \), the significant duration \( t_D \) can be determined from Eq. 2.1 which also implies that \( t_D \) follows a lognormal distribution with \( E[t_D] = 35.74 \) sec and \( \delta_T = 0.33 \). The ground acceleration Fourier amplitude spectrum can be obtained from Eq. 2.3, as a function of magnitude (\( M = 7.5 \)), epicentral distance (\( R = 60 \) km), geologic site conditions (\( s = 1 \)) and focal depth (\( H = 5 \) km).

The uncertainty in significant duration is modeled in Eq. 2.1 by the random variable \( \epsilon_D \) which follows a normal distribution, \( N(0, 0.135) \). The uncertainty in the Fourier amplitude Spectrum is modeled in Eq. 2.3 by \( \epsilon_S \) which is also normally distributed, \( N(0, 0.205) \). Using their distribution functions, one can generate values of \( \epsilon_S \) and \( \epsilon_D \) and then calculate the significant duration and Fourier amplitude spectrum from Eqs. 2.1 and 2.3 respectively. Eq. 2.14 can then be used to evaluate the total energy of the record. Once \( t_D, FS(\omega) \) and \( ET \) are known, the parameters of the ground motion model can be identified following the procedure described in Section 2.3. The identified model parameters for a random realization of \( \epsilon_S = -0.1 \) and \( \epsilon_D = 0.05 \) are shown in Fig. 2.8, along with the corresponding intensity, frequency modulation and normalized power spectral density function. Fig. 2.9 shows the ground acceleration time history at the Hollywood Storage Building site due to a characteristic event of the Mojave segment with magnitude \( M = 7.5 \) and with a significant duration \( t_D = 33.77 \) sec (for \( \epsilon_S = -0.1 \) and \( \epsilon_D = 0.05 \)). The acceleration response spectrum of this record is plotted versus period in Fig. 2.10 for five percent damping. Notice the long significant duration of the distant, characteristic earthquake and the influence of travel path attenuation on the ground motion, which has become somewhat uniform, without
any high intensity sharp peaks. The response spectrum also reveals the low frequency content
of the record, which would cause significant response in flexible structures.

For non-characteristic events, the coefficients a and b in Eq. 2.16 are given by
Algermissen et al. (1982) for the site, namely, \( a = -0.9337 \) and \( b = 0.37 \) with \( I_{\text{min}} = 5 \) and
\( I_{\text{max}} = 11 \). Following the Poisson assumption for the occurrence of such events in time, the
probability of occurrence of \( k \) non-characteristic earthquakes in the next \( T \) years, with
modified Mercalli intensity equal or greater than 5, is

\[
P[N_{\text{nc}} = k] = \frac{(\nu T)^k}{k!} e^{-\nu T}
\]

(2.39)

where \( N_{\text{nc}} \) is the number of occurrences and \( \nu = 10^a = 0.1165 \) is the average number of
events per year with modified Mercalli intensity equal or greater than 5. Given the
occurrence, Eq. 2.17 can be used to describe the distribution of the modified Mercalli
intensity of a non-characteristic earthquake. For this site, the modified Mercalli intensity
has a mean value of 6.14 and a coefficient of variation of 0.175. The significant duration is
calculated from Eq. 2.19, and the ground acceleration Fourier amplitude spectrum is scaled
in terms of modified Mercalli Intensity and geologic site conditions \( (s = 1) \) using equation
2.20. The parameters of the ground motion model are then easily identified. The identified
model parameters for a non-characteristic earthquake with \( I = 8.7 \) and \( t_D = 10.66 \) sec (for
\( \epsilon_S = -0.1 \) and \( \epsilon_D = 0 \)) along with the model intensity, frequency modulation and
normalized power spectral density functions are presented in Fig. 2.11. A ground
acceleration time history of a non-characteristic event (with \( I = 8.7, t_D = 10.66 \) sec) at the
Hollywood Storage Building is shown in Fig. 2.12; the corresponding elastic response
spectrum is presented in Fig. 2.13 for five percent damping. Notice the short duration of the
local, non-characteristic event and the high peak ground acceleration of the record. The
acceleration response spectrum reveals the intermediate and high frequency content of the
event, which affects primarily the response of stiff structures (i.e., buildings with natural period less than 1 sec).

To compare the relative size of the two records shown in Figs. 2.9 and 2.12, one may use the empirical relation \( M = 1.3 + 0.6 I \) (Gutemberg and Richter, 1942) to translate the modified Mercalli intensity \( I \) to a corresponding magnitude \( M \). According to this simple relation, a modified Mercalli intensity of \( I = 8.7 \) roughly corresponds to a local magnitude of 6.5 in the Richter scale. As expected, the size of the characteristic event from the Mojave segment is larger than the size of non-characteristic events. However, due to attenuation, the intensity of the distant characteristic earthquake from the Mojave segment at the Los Angeles site is much lower than the intensity of the non-characteristic, local earthquake.

A second site is also considered, located at the town of El Centro, about 10 km from the Imperial Valley fault. At this site, the characteristic event of the Imperial fault governs the seismic hazard, since the distance from fault is very small. The expected magnitude of the characteristic earthquake at the Imperial fault is \( M = 6.5 \) with an average recurrence period of 44 years (Table 2.1). The probability of occurrence of a specific number of characteristic events in the next fifty years (the time window 1991–2041 is again considered) can be calculated from the procedure given in Appendix E. Notice, that this probability is much higher than the corresponding probability at the Mojave segment. The average significant duration of the characteristic event at the El Centro site is \( t_D = 15.14 \) sec, as calculated by Eq. 2.1. The duration is much lower than that at the Hollywood storage building due to the size of the characteristic earthquake but also due to the short distance from the El Centro site to the Imperial fault. Fig. 2.14 presents the ground motion model parameters, and the model intensity, frequency modulation, and PSD function for the El Centro site. The frequency modulation function of the El Centro Differential Array record of the Imperial Valley, October 15, 1979 earthquake (Geological Survey, 1982) has been used for the identification of the frequency modulation function of the model. A ground acceleration record at the El
Centro site due to a characteristic event of the Imperial Valley fault with magnitude $M = 6.5$ and with a significant duration $t_D = 15.14$ sec (for $\epsilon_S = -0.1$ and $\epsilon_D = 0.$) is presented in Fig. 2.15. The acceleration response spectrum of this record is plotted versus period in Fig. 2.16. Notice the significant low frequency content of the record which may cause damage to flexible structures. This low frequency content would be most important as far as the inelastic behavior of moment resisting frames is concerned.

**Table 2.1** Statistics of characteristic earthquakes on the San Andreas and Imperial faults (U.S.G.S. Open File Report 88–398)

<table>
<thead>
<tr>
<th>Fault Segment</th>
<th>Date of Most Recent Event</th>
<th>Expected Magnitude</th>
<th>Expected Recurrence Time (yrs)</th>
<th>Coefficient of Variation of Recurrence Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>San Andreas Fault</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Coast</td>
<td>1906</td>
<td>8</td>
<td>303</td>
<td>0.43</td>
</tr>
<tr>
<td>San Francisco Peninsula</td>
<td>1906</td>
<td>7</td>
<td>169</td>
<td>0.44</td>
</tr>
<tr>
<td>S. Santa Cruz Mtns.</td>
<td>1906</td>
<td>6.5</td>
<td>136</td>
<td>0.45</td>
</tr>
<tr>
<td>Parkfield</td>
<td>1966</td>
<td>6</td>
<td>21</td>
<td>0.24</td>
</tr>
<tr>
<td>Cholame</td>
<td>1857</td>
<td>7</td>
<td>159</td>
<td>0.57</td>
</tr>
<tr>
<td>Carrizo</td>
<td>1857</td>
<td>8</td>
<td>296</td>
<td>0.38</td>
</tr>
<tr>
<td>Mojave</td>
<td>1857</td>
<td>7.5</td>
<td>162</td>
<td>0.43</td>
</tr>
<tr>
<td><strong>San Bernandino Mtns.</strong></td>
<td>1812</td>
<td>7.5</td>
<td>198</td>
<td>0.66</td>
</tr>
<tr>
<td>Coachella Valley</td>
<td>1680 ± 20</td>
<td>7.5</td>
<td>256</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Imperial Fault</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Imperial</td>
<td>1979</td>
<td>6.5</td>
<td>44</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Table 2.2  List of recorded accelerograms at stiff sites (less than 150 ft of stiff clay, sand or gravel on rock, after Dorby, Idriss and Ng, 1978)

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Station Name</th>
<th>Comp.</th>
<th>Mag. (M)</th>
<th>Distance to Source (km)</th>
<th>Significant Duration (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957 San Francisco</td>
<td>State Building</td>
<td>S09E</td>
<td>5.3</td>
<td>17</td>
<td>5.7</td>
</tr>
<tr>
<td>1957 San Francisco</td>
<td>State Building</td>
<td>S81W</td>
<td>5.3</td>
<td>17</td>
<td>8.2</td>
</tr>
<tr>
<td>1957 San Francisco</td>
<td>Alexander Building</td>
<td>N09W</td>
<td>5.3</td>
<td>16</td>
<td>7.3</td>
</tr>
<tr>
<td>1957 San Francisco</td>
<td>Alexander Building</td>
<td>N81E</td>
<td>5.3</td>
<td>16</td>
<td>9.5</td>
</tr>
<tr>
<td>Parkfield</td>
<td>Cholame Shandon 2</td>
<td>N65E</td>
<td>5.6</td>
<td>0.1</td>
<td>11.7</td>
</tr>
<tr>
<td>Parkfield</td>
<td>Cholame Shandon 5</td>
<td>N05W</td>
<td>5.6</td>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>Parkfield</td>
<td>Cholame Shandon 5</td>
<td>N85E</td>
<td>5.6</td>
<td>5</td>
<td>6.7</td>
</tr>
<tr>
<td>1957 San Francisco</td>
<td>Oakland City Hall</td>
<td>N26E</td>
<td>5.3</td>
<td>26</td>
<td>10.2</td>
</tr>
<tr>
<td>1957 San Francisco</td>
<td>Oakland City Hall</td>
<td>S64E</td>
<td>5.3</td>
<td>26</td>
<td>11.7</td>
</tr>
<tr>
<td>Borrego Mountain</td>
<td>San Onofre SCE</td>
<td>N33E</td>
<td>6.5</td>
<td>122</td>
<td>31.3</td>
</tr>
<tr>
<td>Borrego Mountain</td>
<td>Power Plant</td>
<td>N57W</td>
<td>6.5</td>
<td>122</td>
<td>30.0</td>
</tr>
<tr>
<td>Lower California</td>
<td>El Centro</td>
<td>N-S</td>
<td>6.5</td>
<td>58</td>
<td>17.5</td>
</tr>
<tr>
<td>Lower California</td>
<td>El Centro</td>
<td>E-W</td>
<td>6.5</td>
<td>58</td>
<td>16.9</td>
</tr>
<tr>
<td>Imperial Valley</td>
<td>El Centro</td>
<td>N-S</td>
<td>6.6</td>
<td>8</td>
<td>24.4</td>
</tr>
<tr>
<td>Imperial Valley</td>
<td>El Centro</td>
<td>E-W</td>
<td>6.6</td>
<td>8</td>
<td>23.8</td>
</tr>
<tr>
<td>San Fernando</td>
<td>Castaic</td>
<td>N21E</td>
<td>6.6</td>
<td>21</td>
<td>12.0</td>
</tr>
<tr>
<td>San Fernando</td>
<td>Castaic</td>
<td>N69W</td>
<td>6.6</td>
<td>21</td>
<td>16.5</td>
</tr>
<tr>
<td>San Fernando</td>
<td>14724 Ventura</td>
<td>N78W</td>
<td>6.6</td>
<td>26</td>
<td>22.6</td>
</tr>
<tr>
<td>San Fernando</td>
<td>14724 Ventura</td>
<td>S12W</td>
<td>6.6</td>
<td>26</td>
<td>16.4</td>
</tr>
<tr>
<td>San Fernando</td>
<td>15250 Ventura</td>
<td>S09W</td>
<td>6.6</td>
<td>26</td>
<td>18.4</td>
</tr>
<tr>
<td>San Fernando</td>
<td>15250 Ventura</td>
<td>S81E</td>
<td>6.6</td>
<td>26</td>
<td>24.5</td>
</tr>
<tr>
<td>San Fernando</td>
<td>15910 Ventura</td>
<td>S09W</td>
<td>6.6</td>
<td>26</td>
<td>21.8</td>
</tr>
<tr>
<td>San Fernando</td>
<td>15910 Ventura</td>
<td>S81E</td>
<td>6.6</td>
<td>26</td>
<td>26.5</td>
</tr>
<tr>
<td>San Fernando</td>
<td>Hollywood Storage</td>
<td>N-S</td>
<td>6.6</td>
<td>35</td>
<td>13.0</td>
</tr>
<tr>
<td>San Fernando</td>
<td>PE Lot</td>
<td>E-W</td>
<td>6.6</td>
<td>35</td>
<td>12.5</td>
</tr>
</tbody>
</table>
Table 2.3  Statistical data (after Trifunac and Brady, 1975) for stiff sites used in the development of Eq. 2.19

<table>
<thead>
<tr>
<th>Modified Mercalli Intensity</th>
<th>E(t_D)</th>
<th>σt_D</th>
<th>no. of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>21.00</td>
<td>14.19</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>18.75</td>
<td>11.26</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>12.14</td>
<td>3.83</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 2.4  Frequency dependent regression coefficients for scaling of ground acceleration Fourier amplitude spectra in terms of magnitude, distance to source and geologic site conditions (after Trifunac and Lee, 1989)

<table>
<thead>
<tr>
<th>Period (sec)</th>
<th>(\hat{b}_1(T))</th>
<th>(\hat{b}_2(T))</th>
<th>(\hat{b}_5(T))</th>
<th>(\hat{b}_6(T))</th>
<th>(M_{\text{min}})</th>
<th>(M_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>-0.258</td>
<td>0.041</td>
<td>-1.373</td>
<td>-0.030</td>
<td>0.000</td>
<td>15.000</td>
</tr>
<tr>
<td>0.065</td>
<td>-0.019</td>
<td>0.042</td>
<td>-1.681</td>
<td>-0.052</td>
<td>0.000</td>
<td>9.599</td>
</tr>
<tr>
<td>0.11</td>
<td>0.222</td>
<td>0.033</td>
<td>-2.207</td>
<td>-0.072</td>
<td>1.542</td>
<td>8.476</td>
</tr>
<tr>
<td>0.19</td>
<td>0.433</td>
<td>-0.003</td>
<td>-2.965</td>
<td>-0.086</td>
<td>2.530</td>
<td>8.374</td>
</tr>
<tr>
<td>0.34</td>
<td>0.610</td>
<td>-0.057</td>
<td>-3.844</td>
<td>-0.094</td>
<td>3.240</td>
<td>8.550</td>
</tr>
<tr>
<td>0.50</td>
<td>0.706</td>
<td>-0.084</td>
<td>-4.394</td>
<td>-0.098</td>
<td>3.598</td>
<td>8.692</td>
</tr>
<tr>
<td>0.90</td>
<td>0.820</td>
<td>-0.102</td>
<td>-5.100</td>
<td>-0.103</td>
<td>3.985</td>
<td>8.845</td>
</tr>
<tr>
<td>1.60</td>
<td>0.883</td>
<td>-0.110</td>
<td>-5.487</td>
<td>-0.107</td>
<td>4.137</td>
<td>8.823</td>
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<tr>
<td>2.80</td>
<td>0.869</td>
<td>-0.122</td>
<td>-5.395</td>
<td>-0.109</td>
<td>3.976</td>
<td>8.551</td>
</tr>
<tr>
<td>4.40</td>
<td>0.712</td>
<td>-0.121</td>
<td>-4.741</td>
<td>-0.102</td>
<td>3.478</td>
<td>8.361</td>
</tr>
<tr>
<td>7.50</td>
<td>0.184</td>
<td>-0.086</td>
<td>-2.924</td>
<td>-0.069</td>
<td>1.340</td>
<td>8.608</td>
</tr>
<tr>
<td>14.00</td>
<td>-0.933</td>
<td>0.002</td>
<td>0.580</td>
<td>0.012</td>
<td>0.000</td>
<td>15.000</td>
</tr>
</tbody>
</table>
Table 2.5  Frequency dependent regression coefficients for scaling of ground acceleration Fourier amplitude spectra in terms of modified Mercalli intensity and geologic site conditions (after Trifunac and Lee, 1989)

<table>
<thead>
<tr>
<th>Period (sec)</th>
<th>$\hat{b}_1(T)$</th>
<th>$\hat{b}_2(T)$</th>
<th>$\hat{b}_4(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.040</td>
<td>0.184</td>
<td>0.121</td>
<td>-2.118</td>
</tr>
<tr>
<td>0.065</td>
<td>0.209</td>
<td>0.101</td>
<td>-1.774</td>
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<tr>
<td>0.11</td>
<td>0.253</td>
<td>0.065</td>
<td>-1.476</td>
</tr>
<tr>
<td>0.19</td>
<td>0.306</td>
<td>0.019</td>
<td>-1.397</td>
</tr>
<tr>
<td>0.34</td>
<td>0.352</td>
<td>-0.021</td>
<td>-1.540</td>
</tr>
<tr>
<td>0.50</td>
<td>0.368</td>
<td>-0.036</td>
<td>-1.659</td>
</tr>
<tr>
<td>0.90</td>
<td>0.369</td>
<td>-0.044</td>
<td>-1.709</td>
</tr>
<tr>
<td>1.60</td>
<td>0.347</td>
<td>-0.056</td>
<td>-1.542</td>
</tr>
<tr>
<td>2.80</td>
<td>0.310</td>
<td>-0.079</td>
<td>-1.225</td>
</tr>
<tr>
<td>4.40</td>
<td>0.263</td>
<td>-0.081</td>
<td>-0.911</td>
</tr>
<tr>
<td>7.50</td>
<td>0.179</td>
<td>-0.029</td>
<td>-0.519</td>
</tr>
<tr>
<td>14.00</td>
<td>0.045</td>
<td>0.109</td>
<td>-0.610</td>
</tr>
</tbody>
</table>

Figure 2.1  Segments of the Central and Southern San Andreas fault (after Geological Survey Open File Report, 1988)
Figure 2.2  Probability of occurrence of at least one characteristic earthquake within $T$ years from 1991

Figure 2.3  Mean significant duration of characteristic earthquake as function of magnitude and distance to source
Figure 2.4  Probability of exceedance of modified Mercalli intensity in a non-characteristic earthquake

Figure 2.5  Mean significant duration (± one standard deviation) of non-characteristic earthquake as function of modified Mercalli intensity
Figure 2.6  Ground acceleration instantaneous power spectra at the Los Angeles site due to a non-characteristic earthquake ($I = 8.7$, $t_D = 10.66$ sec, $\epsilon_S = 0.$)

Figure 2.7  Frequency modulation functions $\phi(t)$ of recorded accelerograms at the Hollywood Storage Building
Figure 2.8  Identified parameters and functions of the ground motion model for the Los Angeles site due to a characteristic earthquake \((M=7.5, R=60 \text{ km}, t_D = 33.88 \text{ sec}, \epsilon_S = -0.1)\)
Figure 2.9  Simulated time history of the characteristic earthquake of the Mojave segment at the Hollywood Storage Building ($M = 7.5$, $R = 60$ km)

Figure 2.10  Acceleration response spectrum of the ground motion record in Fig. 2.9
Intensity Function Parameters
- \( A = 996.74 \)
- \( B = 2.867 \)
- \( C = 0.124 \)
- \( D = 4.274 \)
- \( E = 4.119 \)

Frequency Modulation Function Parameters
- \( r_1 = 1.659 \)
- \( r_2 = -0.079 \)
- \( r_3 = 0.0019 \)

Normalized PSD Function Parameters
- \( S_0 = 0.0087 \)
- \( \omega_g = 18.16 \)
- \( \xi_g = 0.62 \)
- \( \omega_f = 1.35 \)
- \( \xi_f = 0.66 \)

Figure 2.11 Identified parameters and functions of the ground motion model for the Los Angeles site due to a non-characteristic earthquake (\( I = 8.7, t_D = 10.66 \) sec, \( \epsilon_S = 0 \))
Figure 2.12 Simulated time history of a non-characteristic earthquake at the Hollywood Storage Building ($I = 8.7$, $t_D = 10.66$ sec)

Figure 2.13 Acceleration Response Spectrum of the Ground Motion Record in Fig. 2.12
Intensity Function Parameters
A = 6302.67
B = 1.758
C = 0.286
D = 32.19
E = 1.431

Frequency Modulation Function Parameters
r1 = 1.340
r2 = 0.516
r3 = -0.171
r4 = 0.022
r5 = -0.15 \times 10^{-2}
\cdot
r6 = 0.56 \times 10^{-4}
r7 = -0.11 \times 10^{-5}
r8 = 0.86 \times 10^{-8}

Normalized PSD Function Parameters
S_0 = 0.0093
\omega_g = 17.04
\zeta_g = 0.71
\omega_f = 1.22
\zeta_f = 1.00

Figure 2.14 Identified parameters and functions of the ground motion model at the El Centro Site due to a characteristic earthquake (M=6.5, t_D = 15.13 sec, \epsilon_S = -0.1)
Figure 2.15 Simulated time history of a characteristic earthquake at the El Centro site
\((M=6.5, t_D = 15.13 \text{ sec}, \epsilon_S = -0.1)\)

Figure 2.16 Acceleration response spectrum of the ground motion record in Fig. 2.15
3.1 Introduction

The design philosophy that has been adopted by almost all current codes for earthquake resistant design is that a building should be able to: 1) resist minor earthquakes without damage; 2) resist moderate earthquakes without structural damage, but possibly suffer some nonstructural damage; 3) resist ground motion of the strongest earthquake that is credible for the site with major damage but without collapse.

Following this philosophy, the 1988 Uniform Building Code (International Conference of Building Officials, 1988) uses a design earthquake with 10 percent probability of exceedance in 50 years and an equivalent lateral force procedure to design a building for ground excitation. The inelastic behavior of the structure is accounted for by a reduction factor $R_w$ according to the ductility capacity of the lateral load-resisting framework. This method controls the maximum drift but does not consider the low cycle fatigue which depends on the duration of the ground motion and can cause frame connections to fracture. There is also no provision for damage control in various levels of shaking.

A typical five story office building is designed according to current practice. The building is designed to be located in Southern California and to be symmetric in plan (Fig. 3.1). The design for seismic forces is compliant with the provisions of the UBC–88. The lateral force resistance is provided by Special Moment Resisting Space Frames (SMRSF) placed on the perimeter (two on each direction). Proper connections are used so that the SMRSF behave as plane frames and the columns and beams in the interior of the building only carry gravity loads. The computer program IGRESS–2 (IGRESS–2, 1989) developed in the University of Illinois at Urbana–Champaign is used for the design.
3.2 Description of Five Story Building

A five-story office building is designed in accordance to the equivalent lateral force design procedure of the 1988 Uniform Building Code (UBC-88). Structural engineers in Southern California have been consulted for the dimensions, the plan view and the vertical design loads of the building, so that the design is consistent with the current state of practice for low-rise steel frame structures constructed in areas of high seismicity. The typical floor plan is shown in Fig. 3.1. The shaded area in the middle is a core housing elevators, stairs and electrical and mechanical services. The elevation views in the North and East directions are given in Figs. 3.2 and 3.3 respectively.

The story height is 13 ft with the exception of the first story which is 15 ft high. The perimeter frames have moment resisting connections to provide the structure with lateral resistance and stability. These frames are shown in thicker lines in Figs. 3.1, 3.2 and 3.3. The connections of the interior frames are assumed to be pinned so that these frames only carry vertical loads of their tributary areas. The same pinned connections are used on the beams of the outer bays in the North–South perimeter frames, so that perimeter frames in orthogonal directions act independently in the resistance of lateral loads, and can be considered as plane frames. Each perimeter frame is designed to behave as a special moment-resisting frame in compliance with the provisions of UBC–88, section 2722(f). The vertical dead loads are given in Table 3.1. The Live Load is assumed to be 50 psf for a typical floor and 20 psf for the roof.

All floor and roof slabs are assumed to extend one foot from the column centerlines at each edge of the building (dashed line in Figure 3.1). The floor and roof decks are also assumed to be rigid enough to transfer the inertia forces from the center of the building to the perimeter frames in the event of an earthquake. The weight of exterior walls is assumed to be uniform and carried by the perimeter beams. Exterior walls are extended 6.5 ft from the roof level to form parapets.
3.3 Equivalent Lateral Force Design Procedure

The equivalent lateral force design procedure implemented in the UBC-88 is followed in this study for the design of the five story steel building. The process consists of the following steps:

1. Evaluate the seismic weight of the building $W$.
2. Estimate the fundamental period $T$.
3. Calculate the design coefficient $C$ and the base shear $V$.
4. Distribute $V$ vertically through the height of the structure.
5. Distribute $V$ horizontally to the various components of the lateral force resisting system according to their rigidities (in this study, components are the special moment-resisting frames).
6. Perform a static analysis of each component loaded with the gravity forces and equivalent lateral forces, and calculate story drifts, member forces and overturning moments.
7. Check the results of the analysis for code compliance with code provisions for lateral stiffness and strength, and design the members so that these provisions are satisfied.
8. Evaluate the story lateral stiffness for each component.
9. Using the story lateral stiffness of each component, recalculate the equivalent lateral force in each of them, considering also torsional effects.
10. Redesign each component (repeat steps 6, 7).
11. Reevaluate the fundamental period of the structure, $T$, using a more accurate method than the one initially used (as for example, equation 12-5 in UBC-88, section 2312(e)2B, method B).
12. Repeat steps 3 to 7 to obtain the final design. If the new design coefficient $C$ is less than 80 percent of the original one, then design each component using the new $C$. 
to satisfy the lateral stiffness requirements and 80 percent of the old C to satisfy the strength requirements.

3.3.1 Determination of the Equivalent Lateral Forces

According to the UBC-88, section 2312(e)2A, the total design base shear in a given direction shall be determined by the following formula:

\[ V = \frac{ZIC}{R_w} W \]  \hspace{1cm} (3.1)

The seismic zone factor \( Z \) depends on the seismicity of the region where the building is located, and its value ranges from zero, for regions without seismic hazard, to a maximum value of 0.4 for regions of strong seismicity. Different zones are determined from the seismic map of the United States (UBC-88, Figure No. 2) and the corresponding seismic zone factors are given in UBC-88, Table No. 23-I. The seismic zone factor represents the effective peak acceleration (EPA) of the design earthquake (Building Seismic Safety Council, 1986).

The importance factor \( I \) depends on the type of occupancy of the building and its value ranges from 1.0 for standard occupancy structures to 1.25 for essential facilities. Different occupancy categories can be found in UBC-88, Table No. 23-K and the corresponding importance factors are given in UBC-88, Table No. 23-L. The importance factor raises the factor of safety of the structure by increasing its stiffness and strength, which results in smaller inelastic deformations during severe ground excitation.

The seismic weight \( W \) is the weight of the building mass that will induce inertial forces during an earthquake. According to the UBC-88, section 2312(e)1, \( W \) shall be taken equal to the total dead load and applicable portions of the live load and snow load. For office buildings only the dead load (including the weight of the partitions and permanent equipment) needs to be considered.
The response modification factor $R_w$ depends on the type of the structural system used to resist the lateral forces. Different structural systems are described in UBC-88, section 2312(d)6 and the corresponding $R_w$ values are given in UBC-88, table No. 23-0. The response modification factor reduces the design base shear, taking into account the inherent ductility and hysteretic energy dissipation capability of the lateral load resisting system at displacements that approach the ultimate load displacement of the system, as well as the additional strength of the nonstructural components which can not be otherwise predicted. The response modification factor increases as the ductility of the system increases.

The design coefficient, $C$, is defined by

$$C = \frac{1.25 S}{T^{2/3}}$$  \hspace{1cm} (3.2)

This coefficient can not exceed the value of 2.75 and may be used for any structure without regard to soil type or structure period. The UBC-88 also specifies that the ratio $C/R_w$ must be greater than 0.075. The reason for this restriction is to impose a lower limit for $C$ for long period structures.

The site coefficient $S$ depends on the soil profile of the site where the structure is located. It is equal to unity if the structure is founded on rock and ranges up to a value of 2.0 for a soil profile containing more than 40 ft of soft clay. Different soil profiles and the corresponding site coefficients are given in UBC-88, Table No. 23-J.

The fundamental period $T$ of the structure is the period of the first mode of vibration. Since it is generally unknown at the onset of the design procedure, the following equation given in the code provides an estimate for it:

$$T = C_t (h_n)^{3/4}$$  \hspace{1cm} (3.3)

where $h_n$ is the height in ft to the top level of the structure, and $C_t$ is equal to 0.035 for steel frames. An expression derived from the recorded response of instrumented steel frame buildings shaken during the 1971, San Fernando earthquake is similar to equation 3.3, except
that \( C_t = 0.049 \) is used instead of \( C_t = 0.035 \). The first estimate of \( T \) though should be a conservative one, i.e. smaller than the true period of the building.

In this study, the seismic zone factor \( Z \) is equal to the maximum value 0.4, since the structure is designed to be situated in Southern California. The importance factor \( I \) is taken to be unity so that the inelastic behavior is not reduced by forcing a more conservative design. The response modification factor \( R_w \) is equal to 12, since the lateral force-resisting system consists of special moment resisting space frames (SMRSF). Finally, the site coefficient \( S \) is assumed to be equal to 1.2, which corresponds to stiff foundation soil.

The design spectrum used in this study is drawn in Fig. 3.4. It is a graphical representation of equations 3.1 and 3.2 which shows the variation of the percentage ratio \( V/W \) (base shear over seismic weight) with the fundamental period \( T \), for \( Z=0.4 \) and \( S=1.2 \). The flat horizontal part of the spectrum at the short period region corresponds to the upper limit of \( C=2.75 \), where the flat part at the high frequencies corresponds at the lower limit given by \( C/R_w = 0.075 \). The spectrum shows that for short period buildings, the design base shear would be 9.2% of the building weight, while for long period buildings, it should be no less than 3%. For the current structure, the design base shear is calculated to be 602 kips (5.71% of the seismic weight).

### 3.3.2 Ductility Requirements

In the equivalent lateral force design procedure, a static elastic analysis of the structure is performed, loaded with proper combinations of gravity and equivalent lateral loads. According to the UBC-88, section 2312(e)8, the computed story drifts from this analysis shall not exceed \( 0.03/R_w \) nor 0.004 times the story height for buildings greater than 65 ft in height. For the present structure, the allowable story drifts are calculated to be \( \delta_1 = 0.45 \) in for the first story and \( \delta_1 = 0.39 \) in for all consecutive stories. It is implicit in the code that the actual interstory displacement is expected to be \( (3R_w/8)\delta_x \) and less than 1.5 percent of the story
height, where $\delta_x$ is the elastic drift computed by the equivalent lateral force method. This requirement is translated to a ductility factor $\mu = 3R_w/8$, which for a system with special moment resisting frames is equal to 4.5.

3.4 Design of Special Moment Resisting Space Frames (SMRSF)

The lateral force resisting system in each direction of the building consists of two perimeter frames which are designed to comply with the requirements of SMRSF. These two frames are identical and each of them carries half the horizontal force in the corresponding direction. Each frame is designed for the base shear acting on it, distributed properly through its height, and for the dead and live load of its tributary area. The base shear is distributed through the height of the frame linearly, according to UBC-88, section 2312(e)4.

Since the design of the frames is meant to be consistent to the current state of practice for low-rise steel frame buildings, the following assumptions are made as is common in office building construction:

1. column cross-sections change, if necessary, every two stories.
2. girders have the same cross-section in the same story.
3. only the girders of the interior frames are connected to the floor slab through shear connectors. The girders of the SMRSF are not connected to the slab; therefore, there is no contribution of the slab to the effective width of the compression flange of the girders.
4. lateral support is provided to both flanges of the girders by bracing where necessary.

The design of all the frames was governed by the seismic forces rather than the action of the gravity loads, which are carried mainly by the interior frames.

3.4.1 East–West Frame

The final design of the SMRSF in the East–West direction is summarized below. The fundamental period of the frame is $T = 1.52$ sec. The frame is checked to comply with the
strength requirements of the code, using a base shear of 241 kips, and with the lateral stiffness requirements, using a base shear of 198 kips. Wide-flange sections are used for all members. The sections used are shown on Table 3.2. The member numbers on this table refer to Fig. 3.5. The total weight of the frame is 94 kips.

3.4.2 North–South Frame

The final design of the SMRSF in the North–South direction is summarized below. The fundamental period of the frame is $T = 1.50$ sec. The frame is checked to comply with the strength requirements of the code, using a base shear of 241 kips, and with the lateral stiffness requirements, using a base shear of 199 kips. Wide-flange sections are used for all members. The sections used are shown on Table 3.3. The member numbers on the table refer to Fig. 3.6. The total weight of the frame is 89 kips.

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Table 3.2 Member sections (East-West frame)

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Table 3.3 Member sections (North-South frame)

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Figure 3.1 Typical floor plan

Figure 3.2 Elevation view, East-West frame
Figure 3.3 Elevation view, North-South frame

Figure 3.4 Design spectrum
Figure 3.5  Member numbers, East-West frame (in reference to Table 3.2)

Figure 3.6  Member numbers, North-South frame (in reference to Table 3.3)
CHAPTER 4
STRUCTURAL MODELING AND ANALYSIS

4.1 Introduction

One of the major tasks of structural engineering is the development of analytical models that are able to describe the behavior of a structural system subjected to a given loading condition. The value of such models depends on their accuracy and computational efficiency. As our understanding of structural behavior improves and as more information and data become available, the accuracy of analytical models increases, along with their complexity. On the other hand, new computationally efficient algorithms are developed which can be used to reduce the cost of analysis. The evolution of powerful digital computers also reduces computation time dramatically. Nevertheless, the simplicity and computational efficiency of a structural model remains important, especially when the size of the problem is large.

This is particularly true in random vibration analysis, where the behavior of the structural system under a stochastic load is described by the first and second order statistics of response quantities such as nodal displacements, joint rotations, member forces etc. (see Section 5). These response quantities are frequently referred to as state variables, since their values define the state of the structural system at a given point in the time domain. Solving for the covariance matrix of the state variables, which consists of their second order statistics, increases the size of the problem dramatically. It can be easily shown that the covariance matrix of n variables, contains n(n+1)/2 unknown terms. As an example, if the response of a system is described by 200 state variables, its covariance matrix involves 20,100 unknowns. Hence, the simplicity of the model for this type of analysis is essential.
4.2 Structural Models used in Random Vibration Analysis

Previous models for random vibration analysis of inelastic, multi-degree-of-freedom systems related to this study are described herein. Most of them aim at reproducing the behavior of the system with a small number of state variables. These variables are defined so that the model represents the overall response of the structure in an average sense. Park, Ang and Wen (1984) developed a multi-degree-of-freedom hybrid model (described in Section 4.2.2) to evaluate the structural damage of reinforced concrete buildings subjected to earthquake excitation. The most commonly used model though is the shear beam model, which will be discussed in further detail in Section 4.2.1.

An alternative approach is to represent the individual members of the structure by discrete elements which can account for the inelastic behavior. Such models have been widely used in deterministic time-history analysis of inelastic structures (Moazzami and Bertero, 1987, Allahabadi and Powell, 1988). The discrete hinge model was used by Baber and Wen (1980) in random vibration analysis of plane frames. The advantages and limitations of this model are discussed in Section 4.2.3.

4.2.1 The Shear Beam Model

The shear beam model is the simplest and most widely used model in random vibration analysis of inelastic structures. Floors are assumed to be rigid, connected with inelastic translational springs. An equivalent lateral interstory stiffness is necessary for the model to describe the response behavior of the actual structure with some reasonable accuracy. Sues, Wen and Ang (1983) solved the eigenvalue problem using the complete stiffness matrix (generally available from a static analysis of the actual structure under lateral loads) and determined the shear beam model stiffness matrix matching the first mode of the original system exactly.
The basic drawback of the shear beam model is its inability to reflect stiffness coupling between stories in strong-column-weak-beam type buildings. Neglecting this coupling effect can totally change the behavior of such a structure in the elastic, and even more so in the inelastic range. Therefore, despite its computational efficiency, the shear beam model is suitable only for strong-beam-weak-column buildings.

4.2.2 The Hybrid Model

The hybrid model, proposed by Park, Ang and Wen (1984), is an extension of the shear beam model that allows rotation at the floor levels and accounts for the yielding in both columns and beams of a frame. Three basic elements are considered in this model. Elastic beam elements are used to model columns that remain elastic throughout the excitation (i.e., strong-column-weak-beam type behavior). Columns weaker than the beams at both joints are modeled by conventional shear beam type hysteretic springs, assuming that floors are rigid. Hysteretic rotational spring elements are also used to model the yielding of the beams. The basic drawback of this model is that it is highly loading dependent, since it requires a priori information about column behavior; i.e., one should know where yielding will occur and which columns remain elastic throughout the analysis in order to define the parameters of the model and proceed to the solution. A detailed description of the model is given by Park, Ang and Wen (1984).

4.2.3 The Discrete Hinge Model

The discrete hinge model was originally used for random vibration analysis of plane frames by T. T. Baber and Y. K. Wen (1980). The model is similar to the discrete hinge model used in deterministic analysis of yielding structures by Chen and Powell (1982). However, in this case a smooth hysteretic model (of the type described in Section 4.3.1) is used for each
hinge element, and random vibration analysis is performed using the method of statistical equivalent linearization. The basic assumption of the discrete hinge model is that yielding within a member is confined to discrete hinge regions, located immediately adjacent to the beam-column joints. As a result, all frame members are considered to remain elastic and the inelastic behavior of the frame is described through hysteretic hinge elements located at their ends. In order to reduce the degrees of freedom, only translational masses are considered; these are lumped at the nodal interfaces. Following these assumptions, the model involves two first-order differential equations for each translational degree of freedom, and one more for each hysteretic hinge element. As a result, the response of an ns story plane frame with m discrete hysteretic hinges is described by $2ns + m$ state variables whose covariance matrix involves $(2ns + m) (2ns + m + 1)/2$ unknowns. Furthermore, if system degradation is to be included, an additional m equations must be included, one for the energy dissipation at each hinge element. Clearly, the number of unknowns increases rapidly, and the method becomes very costly even for relatively small structures. As an example, the five story, four bay plane frame of Figure 4.1 has 5,050 unknown covariance terms, without considering any strength or stiffness degradation.

A modal transformation can be used to reduce further the number of active degrees of freedom of the model, using only the first few lower modes of the frame. In stationary random vibration analysis the elimination of superfluous higher modes improves the convergence of the iterative solution reducing computation time significantly. However, in transient random vibration analysis, elimination of higher modes reduces the accuracy in the results (Baber, 1986).

4.3 The Strong-Column-Weak-Beam (SCWB) Model

One of the objectives of this study is to evaluate the response statistics of low rise, moment resisting steel frames under earthquake loading. A strong-column-weak-beam (SCWB)
model is developed for this purpose. This model significantly reduces the size of the problem, while stiffness coupling between adjacent floors is retained. The model response matches the response produced by nonlinear finite element analysis without loss in accuracy. The SCWB model, presented schematically in Fig. 4.2(a), is based on the following assumptions:

- Two degrees of freedom are considered at each story level: displacement $v_i$ relative to the ground, and rotation $\theta_i$ relative to the vertical axis, clockwise positive. Assuming a constant translation $v_i$ throughout the $i$–th floor is equivalent to the assumption of a rigid diaphragm. The single rotation $\theta_i$ at the $i$–th floor can be viewed as the average joint rotation at that level.

- A rotational degree of freedom $\theta_0$ is considered at the base to account for the plastic hinges formed at the lower ends of the first story columns.

- Story masses are lumped at floor levels and rotational inertia can be neglected.

- Columns at a story remain elastic and are represented by an equivalent linear member with moment of inertia equal to the sum of the moments of inertia of the columns at that story.

- Yielding is localized at the base and floor levels of the structure and can be modeled by the hysteretic restoring moment of rotational springs.

- Viscous damping proportional to mass and stiffness is used to model the non–hysteretic energy dissipation of the actual system.

In some cases, stiffness degradation of the soil under the foundation may alter the response of a frame to a cyclic load significantly. Structural response may be also affected by nonstructural components of a building, such as partition walls and cladding. However, due to computational constraints, these factors are not considered in this study. The effects of gravitational loads (e.g. P–$\Delta$ effect) are also neglected to simplify the analysis.

The translational equilibrium at the $i$–th level, shown in Fig. 4.2(b), can be written as
where \( f_i \) is the inertial force of the \( i \)-th floor mass, \( f_i^1 \) the reaction from the elastic column element below the floor and \( f_i^{1+1} \) the reaction from the elastic column above the floor. These are given by the following relations

\[
f_i = -m_i \left( \ddot{v}_i + \ddot{x}_g \right) \tag{4.2}
\]

\[
f_i^1 = \frac{6EI_i}{h_i^2} \left( \frac{2u_i}{h_i} - \theta_i - \theta_{i-1} \right) \quad ; \quad u_i = v_i - v_{i-1} \tag{4.3}
\]

The rotational equilibrium at level \( i \), shown in Fig. 4.2(c), can be written as

\[
M_{i+1}^{nl} + M_{i+1}^{nu} + M_{i+1}^{lo} = 0 \tag{4.4}
\]

where \( M_{i+1}^{nu} \) is the moment applied to the joint from the linear columns of story \( i \) below the floor, \( M_{i+1}^{lo} \) is the moment applied to the joint from the linear columns of story \( i+1 \) above the floor and \( M_{i+1}^{nl} \) is the restoring moment of the rotational spring at the \( i \)-th floor, given by

\[
M_{i+1}^{nu} = \frac{2EI_i}{h_i} \left( 2\theta_i + \theta_{i-1} - \frac{3u_i}{h_i} \right) \tag{4.5}
\]

\[
M_{i+1}^{lo} = \frac{2EI_i}{h_i} \left( 2\theta_{i-1} + \theta_i - \frac{3u_i}{h_i} \right) \tag{4.6}
\]

\[
M_{i+1}^{nl} = a_i G_i \theta_i + (1 - a_i) G_i Y_i \tag{4.7}
\]

where \( u_i \) is the interstory drift at the \( i \)-th story; \( v_i \) the relative to the ground translation of the \( i \)-th story; \( \theta_i \) the average joint rotation at the \( i \)-th floor; \( h_i \) is the height of the \( i \)-th story; \( E \) the modulus of elasticity; \( I_i \) the moment of inertia of the equivalent elastic column element at level \( i \); \( G_i \) the elastic stiffness of the \( i \)-th story rotational hysteretic spring; \( a_i \) is the
post-to-pre-yield stiffness ratio of the rotational hysteretic spring for the i-th story, and $Y_i$ the hysteretic component of the joint rotation at the i-th floor. At the base, the rotational equilibrium is written as

$$M_0^{nl} + M_1^{lo} = 0$$  \hspace{1cm} (4.8)

where

$$M_0^{lo} = \frac{2EI_1}{h_1} \left( 2\theta_0 + \theta_1 - \frac{3u_1}{h_1} \right)$$  \hspace{1cm} (4.9)

and

$$M_0^{nl} = a_0 G_0 \theta_0 + (1 - a_0) G_0 Y_0$$  \hspace{1cm} (4.10)

Equilibrium equations 4.1, 4.4 and 4.8 can be written in matrix form using the expressions in 4.2, 4.3, 4.5, 4.6, 4.7, 4.9 and 4.10 as follows:

$$[M]\ddot{\mathbf{y}} + [K]\mathbf{y} + [A]\mathbf{\theta} = -[M]\ddot{\mathbf{a}}_g$$  \hspace{1cm} (4.11)

$$[A]^T\ddot{\mathbf{y}} + [E]\mathbf{\theta} + [D]\mathbf{y} = 0$$  \hspace{1cm} (4.12)

Matrices $[M]$, $[A]$, $[K]$, $[E]$ and $[D]$, which are functions of $m_i$, $h_i$, $E_i$, $G_i$ and $a_i$, are shown explicitly for a three story example in Appendix A.1. For a smooth hysteretic model Eqs. 4.11 and 4.12 are augmented by $ns+1$ additional first order ordinary differential equations for the hysteretic components $Y_i$:

$$\dot{Y}_i = g(Y_i, \dot{\theta}_i) \hspace{1cm} i = 0,...,ns$$  \hspace{1cm} (4.13)

where $ns$ is the number of stories of the model. The smooth hysteretic model used in this study is described in the following section.

4.3.1 Modeling Hysteretic Behavior

Most structures exhibit nonlinear behavior when subjected to severe dynamic excitations such as those due to earthquakes. This behavior is characterized by energy dissipation and is often modeled phenomenologically by a restoring force–displacement hysteretic rule. One of the most convenient ways to model hysteretic behavior is the differential model proposed
by Bouc (1967) and generalized by Wen (1980). This model is capable of representing different hysteretic loops by varying the governing parameters of the model. It is also computationally suited to random vibration analysis, since it can be easily combined with the method of statistical equivalent linearization (see Section 6.3).

The model expresses the restoring force \( Q \) of a single-degree-of-freedom system with mass \( M \) and elastic stiffness \( K \) as

\[
Q = a K u + (1 - a) K z \tag{4.14}
\]

where \( u \) is the displacement, \( a \) is the rigidity ratio, also called pre-yield-to-post-yield stiffness ratio, and \( z \) is given by the first order differential equation

\[
\dot{z} = A \dot{u} - \beta |\dot{u}| |z|^{n-1} z - \gamma |\dot{u}| z^n \tag{4.15}
\]

The introduction of \( z \) as an additional state variable provides the system with what is in essence a memory mechanism sufficient to describe the hysteretic behavior. The model has been further generalized by Baber and Wen (1980), to include stiffness and strength deterioration and by Park, Wen and Ang (1986) to capture hysteretic behavior under bi-directional loading. The rate of energy dissipated in the system, which is equal to the work done by the hysteretic part of the restoring force, is given by

\[
\dot{e} = (1 - a) K z \dot{u} \tag{4.16}
\]

In this study, stiffness and stress degradation are not considered, since no significant deterioration was observed during actual dynamic tests of steel frames (Foutch et al., 1986). Furthermore, equations 4.14 and 4.15 were used as restoring moment-rotation relationships for the hysteretic springs of the system. For steel structures, previous identification experience has shown that the exponent \( n \) in Eq. 4.15 can be considered to be equal to 2. Therefore, Eq. 4.13 for the moment-rotation relationship becomes

\[
\dot{Y}_i = A_i \dot{\theta}_i - \beta_i |\dot{\theta}_i| Y_i - \gamma_i \dot{\theta}_i Y_i^2 \tag{4.17}
\]

where \( \dot{\theta}_i \) is the joint rotation at level \( i \), \( Y_i \) is the hysteretic moment component at level \( i \),
ns is the total number of stories, and $A_i$, $\beta_i$, $\gamma_i$ are model parameters for the i-th story which need can be identified from test or analytical results (see Section 4.4.3).

4.3.2 Incorporating Viscous Damping

The viscous damping in the SCWB model can be modeled based on Rayleigh's assumption (Clough and Penzien, 1973). The damping matrix is given by a linear combination of the mass and stiffness matrices as follows

$$[C] = a[M] + b[K]$$  \hspace{1cm} (4.18)

in which $a$ and $b$ are proportionality constants. In nonlinear systems the stiffness matrix $[K]$ changes with time as the system enters the inelastic range. Therefore, according to Rayleigh's assumption, $[C]$ is also time–dependent and damping is reduced as the structure yields. Often, this might not be the case in an actual structure; however, viscous damping is not a major factor in the inelastic range where hysteretic damping prevails. The use of modal damping ratios to describe viscous damping in the inelastic range is also questionable since the natural frequencies and mode shapes are not constant. In this study, viscous damping is used to represent the energy dissipation in the elastic range and the proportionality constants $a$ and $b$ in Eq. 4.18 are determined using the elastic stiffness matrix and remain unchanged throughout the analysis.

Both terms in Eq. 4.18 are necessary in modeling viscous damping. Damping proportional to mass produces critical damping ratios which are inversely proportional to the frequencies of vibration (i.e., higher modes possess lower damping). On the other hand, damping proportional to stiffness is directly proportional to the frequencies of vibration (i.e., it damps out higher modes). The contribution of higher modes is not of interest in the response of structures to earthquake excitation. Therefore, stiffness proportional damping is used to damp them out. If stiffness and mass proportional damping is used, the relationship
between the critical damping ratio $\xi_i$ of the $i$-th natural frequency $\omega_i$ and the proportionality constants $a$ and $b$ can be easily shown to be

$$\xi_i = \frac{1}{2} \left( \frac{a}{\omega_i} + b\omega_i \right)$$ (4.19)

Eq. 4.19 can be used to calculate $a$ and $b$ if the lowest two natural frequencies of the elastic system and the corresponding damping ratios are known (see Section 4.4.2).

In this study, the damping matrix $[C]$ of the SCWB model is assumed to be constant throughout the analysis, and equal to

$$[C] = a\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} + b\begin{bmatrix} [K] & [A] \\ [A]^T & [E] \end{bmatrix}$$ (4.20)

Matrices $[M]$, $[K]$, $[A]$ and $[E]$ are given in Appendix A.1 for a three story system. Notice that $[E]$ contains the post-yield stiffness of the hysteretic rotational springs. This way, no viscous damping is associated with the hysteretic components of the rotation, $Y$, in Eq. 4.12.

4.4 Identification of the SCWB Model Parameters

The SCWB model parameters need to be identified according to the observed behavior of the real structure. The following properties of the actual steel frame are used as input in this identification process: story masses, column moments of inertia, the modulus of elasticity of steel, the fundamental frequency and the corresponding translational mode shape, the second lowest natural frequency and the measured inelastic response to some specified lateral loading or base excitation. Response quantities that need to be measured are the average rotation at the base and the translational accelerations or displacements at the story levels. It is noted that the rotation at the base is a result of yielding at the lower end of the first story columns.
Story masses are assumed to be the same in both the SCWB model and the original frame. The modulus of elasticity also remains unchanged. The moment of inertia of the elastic column element at the j-th level is taken to be equal to the sum of the moments of inertia of all the columns in that level. The identification of the remaining parameters though is not straightforward. The linear stiffness coefficients $G_j$ ($j = 0,...,ns$) of the rotational springs are identified first. The damping proportionality factors $a$ and $b$ are identified next. Finally, the rigidity ratio $\alpha_j$ and the hysteretic parameters $A_j$, $\beta_j$, $\gamma_j$ of the rotational springs are identified for each floor.

4.4.1 Identification of Linear Stiffness Coefficients

The idea behind the algorithm used to identify the rotational stiffness coefficients is to match the first mode of the model with that of the original frame. The first step of the identification procedure is to assign a value for the linear stiffness coefficient $G_0$ of the base rotational spring. The equilibrium equations of the elastic model for free, undamped vibration are formulated in terms of story rotations and story displacements relative to the ground. The rotational equilibrium equation at the base is then solved for $\theta_0$

$$\theta_0 = -\frac{2EI_1}{G_0h_1 + 4EI_1} \theta_1 + \frac{6EI_1}{G_0h_1^2 + 4EI_1h_1} v_1$$

(4.21)

and $\theta_0$ is eliminated from the equilibrium equations which can be written in matrix form as

$$[M]\ddot{v} + [K]v + [A]\phi = 0$$

(4.22)

$$[A]^T [\phi] + [E] \phi = 0$$

(4.23)

where $\phi^T = \{\theta_1, \theta_2, ..., \theta_{ns}\}$. Static condensation is performed next by solving equation 4.23 for $\phi$ and substituting $\phi$ in Eq. 4.22. The result is

$$[M]\ddot{v} + [\ddot{K}]v = 0$$

(4.24)
where $[\bar{K}] = [K] - [A][E]^{-1}[A]^T$. The generalized eigenvalue problem equation is written for the first mode using the known fundamental frequency $\omega_1$ and translational mode shape $y_1$ of the original frame:

$$[\bar{K}] y_1 = \omega_1^2 [M] y_1$$

(4.25)

Equation 4.25 can be solved for the unknown linear stiffness coefficients $G_1, G_2, ..., G_{ns}$. Some algebraic manipulation is necessary. The algorithm is demonstrated in Appendix A.2 for a three-story example.

The assignment of an initial value to $G_0$ is necessary, since static condensation would not be feasible if $\theta_0$ were not eliminated from the equilibrium equation. It was observed from numerical examples that $G_0$ must always be selected greater than a minimum value $G_{\min}$, for all subsequent $G_j, j = 1,...,ns$, to be positive. The value of $G_{\min}$ is a function of the fundamental mode and the elastic properties of the original frame.

4.4.2 Evaluation of Rayleigh Damping Coefficients

The two lowest natural frequencies $\omega_1$ and $\omega_2$ of the actual frame, along with the corresponding critical damping ratios $\xi_1$ and $\xi_2$ are used to evaluate the damping proportionality factors $a$ and $b$ of Eq. 4.18. Using Eq. 4.19, one can express $a$ and $b$ as

$$a = \frac{2(\xi_1 \omega_2 - \xi_2 \omega_1)\omega_1 \omega_2}{\omega_2^2 - \omega_1^2}$$

(4.26)

$$b = \frac{2(\xi_2 \omega_2 - \xi_1 \omega_1)}{\omega_2^2 - \omega_1^2}$$

(4.27)

Since no experimental data are available on viscous damping in the original structure, damping is modeled to be consistent with the assumptions of the design procedure and five
percent damping is assigned for the two lowest modes of the structure. The proportionality factors are then calculated using equations 4.26 and 4.27.

4.4.3 Nonlinear Parameter Estimation

This section deals with the identification of the nonlinear restoring moment model parameters at each floor. For the \( j \)-th floor, these are the rigidity ratio \( a_j \) and the hysteretic parameters \( A_j, \beta_j, \gamma_j \). A recorded time history of the response of the actual structure to cyclic loading may be used as input to the identification procedure. Generally the measured response level should be large enough to cause significant inelastic behavior and energy dissipation; otherwise, the identification of the parameters would not be as accurate. Results of shaking table laboratory model experiment or field measurements of buildings can be used for this purpose. In either case, accelerations are first measured and then numerically integrated to calculate velocities and displacements. However, experimental results are not available for the frames designed in this study. Therefore, an alternative approach is followed.

It has been shown in the past from actual test data that the shape of the hysteresis curve obtained from dynamic tests is very close to that obtained from quasi-static cyclic tests (Hall et al., 1984). The term "quasi-static" stands here for a static cyclic test which is mapped in the time domain. The response of the structure is then transformed to a recorded time history. The two basic assumptions which allow this transformation are

- masses have constant velocities (i.e., inertial forces are neglected)
- viscous damping is not considered (i.e., velocities are assumed to very low)

Quasi-static tests are simulated numerically by performing an incremental nonlinear static analysis of the actual frame using DRAIN-2DX. Such a test is described below for the three story, one bay frame of figure 4.3. A lateral load pattern is applied at the floor levels. The
load pattern is selected so that it results in similar interstory drifts for all stories. An incremental nonlinear static analysis is performed, using constant force increments $\Delta F_i$ and a cyclic load path. At least one complete cycle of significant inelastic behavior is necessary for the identification of the hysteretic parameters. The story displacements $\hat{y}_i$ relative to the ground and the average rotation at the base $\hat{\theta}_0$ are recorded. Henceforth, a "^" indicates a measured quantity. Let $\phi^T = \{\theta_1, \theta_2, \ldots, \theta_{ns}\}$ for an ns-story system. The equilibrium equations of the undamped SCWB model, formulated using interstory drifts ($u_i = \hat{v}_i - \hat{v}_{i-1}$ at the i-th story), are

$$[K]u + [A]\phi + a\hat{\theta}_0 = F \quad (4.28)$$

$$[B]u + [E]\theta + [D]Y = 0 \quad (4.29)$$

where $u$, $F$, $[K]$, $[A]$, $[B]$ and $[E]$ are given in Appendix A.3 for a three story example. Floor level rotations $\theta_1$, $\theta_2$, ..., $\theta_{ns}$ can then be expressed in terms of the measured responses by solving Eq. 4.28 for $\phi$ as follows

$$\phi = [A]^{-1}[F - [K]u - a\hat{\theta}_0] \quad (4.30)$$

With $\theta^T = \{\hat{\theta}_0, \theta_1, \theta_2, \ldots, \theta_{ns}\}$ evaluated, Eq. 4.29 may be used to formulate the residuals and identify the hysteretic parameters of the rotational springs. It is important to notice that the identification can be performed independently for each level, since the residuals at different levels are not coupled with respect to the unknown parameters.

The residual for the j-th story at time $t_i$ is given as

$$f_{ji} = a_j G_j \theta_{ji} + (1 - a_j) G_j Y_{ji} + Q_{ji} \quad ; \quad j = 0, ..., ns \quad (4.31)$$

where $Q_{ji}$ is the part of the restoring force at level $j$ at time $t_i$ which can be directly
calculated from the measured response quantities:

\[ Q_{ji} = \frac{4EI_j}{h_j} \theta_{ji} + \frac{2EI_j}{h_j} \theta_{(j-1)i} - \frac{6EI_j}{h_j^2} u_{ji} + \frac{4EI_{j+1}}{h_{j+1}} \theta_{ji} + \frac{2EI_{j+1}}{h_{j+1}} \theta_{(j+1)i} - \frac{6EI_{j+1}}{h_{j+1}^2} u_{(j+1)i} \]  \hspace{1cm} (4.32)

Interstory drifts are used instead of the displacements relative to the ground, for convenience in calculations. The hysteretic component \( Y_{ji} \) in Eq. 4.31 is related to the measured parameters by the integral of Eq. 4.17:

\[ Y_{ji} = A_j \int_0^{t_i} \dot{Y}_{ji} \, d\tau - \beta_j \int_0^{t_i} |\dot{Y}_{ji}| \, d\tau - \gamma_j \int_0^{t_i} \dot{Y}_{ji}^2 \, d\tau \quad ; \quad j = 0, \ldots, ns \]  \hspace{1cm} (4.33)

in which

\[ Y_j(\tau) = -\frac{Q_j(\tau) + a_j G_j \theta_{ji}}{(1 - a_j) G_j} \quad ; \quad j = 0, \ldots, ns \]  \hspace{1cm} (4.34)

The numerical evaluation of the integrals in Eq. 4.33 is cumbersome and detrimental to the efficiency of the identification algorithm, since numerical errors are accumulated, unless one uses a sophisticated (and therefore computationally expensive) numerical integration scheme. To avoid this problem, the residuals are formed using an incremental form of Eq. 4.29 (Eliopoulos and Wen, 1991). Following this approach, the residual of the \( j \)-th floor at time \( t_i \) is expressed as:

\[ f_{ji} = a_j G_j \Delta \theta_{ji} + (1 - a_j) G_j \Delta Y_{ji} + \Delta Q_{ji} \]  \hspace{1cm} (4.35)

where

\[ \Delta Q_{ji} = \frac{4EI_j}{h_j} \Delta \theta_{ji} + \frac{2EI_j}{h_j} \Delta \theta_{(j-1)i} - \frac{6EI_j}{h_j^2} \Delta u_{ji} + \frac{4EI_{j+1}}{h_{j+1}} \Delta \theta_{ji} + \frac{2EI_{j+1}}{h_{j+1}} \Delta \theta_{(j+1)i} - \frac{6EI_{j+1}}{h_{j+1}^2} \Delta u_{(j+1)i} \]  \hspace{1cm} (4.36)

also, assuming that the system is initially at rest,

\[ \Delta Y_{ji} = A_j \Delta \theta_{ji} - \beta_j \Delta I_{1ji} - \gamma_j \Delta I_{2ji} \]  \hspace{1cm} (4.37)

The interstory drift increment \( \Delta u_{ji} \) of level \( j \) at time \( t_i \) is defined as the difference
The increment of the rotation $\Delta \theta_{ji}$ in level $j$ at time $t_i$ is defined as the difference $\theta_{ji} - \theta_{j(i-1)}$. The values of the integral increments $\Delta I_{jji}$ and $\Delta I_{2ji}$ are given in Appendix B.1. The partial derivatives of the residual with respect to the unknown parameters are given in Appendix B.2.

The objective function for the $j$-th story is defined as

$$F_j = \sum_{i=1}^{n} f_{ji}^2$$

(4.38)

where $n$ is the number of available response measurements. The identification of $a_j$, $A_j$, $\beta_j$ and $\gamma_j$ is performed using the Gauss Method, which is presented in Appendix C.2. A projection method (described in Appendix C.5) is used to treat linear constraints present in the problem (e.g. $0 < a_j < 1$).

4.5 Response Analysis in the Time Domain

The strong-column-weak-beam (SCWB) model is used to calculate the response time history of low-rise, moment resisting plane frames. The model represents the actual frame with a reduced number of state variables which approximate the response of the actual frame in an average sense. Three degrees of freedom are considered at each story: a story translation, a story rotation and a hysteretic rotational component which accounts for the inelastic behavior of the story; in addition, a rotation and a hysteretic rotational component at the base account for plastic deformation in the base of the first story columns (see Section 4.3). The results of the analysis are compared with those obtained from DRAIN-2DX (Allahabadi and Powell, 1988), an upgraded version of the well known finite element program DRAIN-2D (Kannaan and Powell, 1973) developed at the University of California, Berkeley, for static and dynamic analysis of inelastic plane structures. DRAIN-2DX computes the response of the frame using two types of elements, a discrete hinge
beam-column element and a semi-rigid connection element. In the finite element formulation, columns are represented by column elements, beams by beam elements (identical to column elements except for a constant yield moment, independent of the axial force acting on the member) and panel zones by semi-rigid connection elements. A brief description of DRAIN-2DX is given in Appendix D.

Comparison (see Section 4.7) between the results of the SCWB model and the DRAIN-2DX finite element model shows that the simple SCWB model can reproduce the response with accuracy and reduce the computation time required significantly.

The equations of motion of the SCWB model, subject to ground acceleration \( \ddot{a}_g \), are

\[
[M] \ddot{v} + (a + b \dot{v}) + [K] (v + b \dot{v}) + [A] (\theta + b \dot{\theta}) = - [M] \ddot{a}_g
\]

\[
[A]^T (v + b \dot{v}) + [E] (\theta + b \dot{\theta}) + [D] Y = 0
\]

\[
\dot{Y}_j = A_j \dot{\theta}_j - \beta_j (\dot{\theta}_j \dot{Y}_j | Y_j - \gamma_j \dot{\theta}_j Y_j^2
\]

Matrices \([M], [K], [A], [D]\) and \([E]\) are given in Appendix A.1 for a three story example. A state variable formulation is used to solve the above equations. The state vector is defined as

\[
\dot{Y}^T = \{ \dot{v}^T, \dot{v}^T, \dot{\theta}^T, \dot{\theta}^T \} = \{ \dot{Y}_1^T, \dot{Y}_2^T, \dot{Y}_3^T, \dot{Y}_4^T \}
\]

and Eqs. 4.39, 4.40 and 4.41 are written in terms of the state variables as

\[
\dot{Y}_1 = Y_2
\]

\[
\dot{Y}_2 = -aY_2 - [M]^{-1}[K](Y_1 + bY_2) - [M]^{-1}[A](Y_3 + bY_3) - \ddot{a}_g
\]

\[
\dot{Y}_3 = -\frac{1}{b} \dot{Y}_3 - \frac{1}{b} [E]^{-1}[A] \dot{Y}_1 + bY_2 - \frac{1}{b} [E]^{-1}[D] Y_4
\]

\[
\dot{Y}_4 = A_j \dot{Y}_3 - \beta_j (\dot{Y}_3 Y_4 j - \gamma_j \dot{Y}_3 Y_4 j)
\]

The nonlinear system of ordinary differential equations 4.43 to 4.46 is solved numerically.
using a backward differentiation scheme. The use of an implicit method was found computationally efficient in comparison to popular predictor-corrector methods, since the system appeared to be stiff (Press et al., 1989). Finally, notice that \( \dot{y}_3 \) appears in the r.h.s. of Eqs. 4.44 and 4.46. Hence, the value of \( \dot{y}_3 \) is first calculated from Eq. 4.45 and then used in 4.44 and 4.46 in the evaluation of the derivatives of the state variables.

### 4.6 Evaluation of the Response Statistics

The Monte-Carlo simulation method can be used to evaluate the response statistics of low rise, moment resisting steel frames to earthquake motion. Using DRAIN-2DX or the SCWB model formulation, a deterministic analysis in the time domain is performed for a large number of recorded accelerograms and from the results the second order response statistics are calculated. As an example, if the response of interest for story \( j \) is the story drift \( u_j \), the sample mean and sample standard deviation of \( u_j \) at time \( t \) are given by

\[
\mu_{u_j}(t) = E[u_j(t)] = \frac{1}{n} \sum_{i=1}^{n} u_j(t) \tag{4.47}
\]

\[
\sigma_{u_j}^2(t) = E\left[ (u_j(t) - \mu_{u_j}(t))^2 \right] = \frac{1}{n-1} \left( \sum_{i=1}^{n} u_j(t)^2 - n \mu_{u_j}(t)^2 \right) \tag{4.48}
\]

where \( n \) is the number of response time histories in the sample. Since \( \mu_{u_j}(t) \) is very small, the term \( n\mu_{u_j}(t)^2 \) in Eq. 4.48 can be neglected.

Comparison (see Section 4.7) between response statistics obtained using DRAIN-2DX for the time history analysis and statistics obtained using the SCWB model formulation, shows that the SCWB model produces almost identical results with only a fraction of the computational effort required by DRAIN-2DX. This computational efficiency makes the SCWB model a powerful tool for simulations.
4.7 Numerical Example – Five Story, Three Bay, SMRSF

In this example, the parameters of the strong-column-weak-beam (SCWB) model are determined for the five story–three bay special moment resisting frame (SMRSF) designed in Section 3.4 according to the provisions of UBC–88. The linear parameters of the SCWB are identified first. The first two lowest natural frequencies and the first translational mode shape of the original frame are calculated using DRAIN–2DX and used as input for the identification of the model's parameters. The two lowest modes are assigned a five percent critical damping ratio and Rayleigh's proportionality coefficients a and b are calculated from equations 4.26 and 4.27:

\[ a = 0.3067 \quad ; \quad b = 0.0062 \]

The linear stiffness coefficients \( G_j \) of the story level rotational springs are identified next. Following the approach of Section 4.4.1, with \( G_0 \) equal to 500,000,000 kips-in/rad, the linear stiffness coefficients \( G_j \) \((j = 1, ns)\) can be identified. It is noted that \( G_0 \) must be at least 41,370,000.273 kips-in/rad. for \( G_j \)'s in all consecutive stories to be positive.

Finally, the nonlinear model parameters, \( \alpha_j, \beta_j, \gamma_j \) and \( \phi_j \), are identified. A quasi-static test is performed using DRAIN–2DX. The force increments applied at floor levels are:

\[ \Delta F_1 = 0.416 \text{ kips} \quad \Delta F_2 = 0.832 \text{ kips} \quad \Delta F_3 = 2.912 \text{ kips} \]
\[ \Delta F_4 = 4.576 \text{ kips} \quad \Delta F_5 = 7.488 \text{ kips} \]

Since the structure has no stiffness or strength deterioration, one complete loading cycle is applied with 50 incremental loading steps in one direction, 100 steps in the opposite direction, 100 steps again in the original direction and finally 50 steps in the opposite direction. The average column rotation at the base of the frame and the story drifts are recorded and used for the identification of the SCWB model's nonlinear parameters. The linear constraint
A_j > 0.8 is used in the identification algorithm. The identified values of the rotational spring parameters of the model are given in Table 4.1. A comparison of the first three vibration modes between the model and the original frame is presented in Fig. 4.4. It can be seen that the SCWB model reproduces the lower modes of the frame almost exactly. Fig. 4.5 compares the response time histories of the SMRSF to the Imperial Valley, 1979 earthquake (El Centro Differential Array), evaluated using DRAIN-2DX and the SCWB model. Fig. 4.6 shows the same comparison for the El Centro, 1940 event. In both cases, the SCWB model matches the DRAIN-2DX response very well. Finally, the response statistics obtained from DRAIN-2DX are compared with those obtained using the SCWB formulation. A sample of twenty time histories is used in each case. The root mean square (r.m.s.) story drifts for the five stories of the frame are shown in Fig. 4.7. The results of the SCWB model are very close to those obtained using DRAIN-2DX, especially where the r.m.s. response reaches a maximum. Finally, it should be noted that the SCWB model requires only one tenth of the computational time of the DRAIN-2DX model.

Table 4.1  Restoring moment parameters of SCWB model – five story, three bay SMRSF

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>α</th>
<th>A</th>
<th>β</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>base</td>
<td>494337800.</td>
<td>0.0015</td>
<td>0.80</td>
<td>14424840.</td>
<td>12877660.</td>
</tr>
<tr>
<td>1st story</td>
<td>13495000.</td>
<td>0.2471</td>
<td>0.92</td>
<td>10834.</td>
<td>9253.</td>
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<tr>
<td>2nd story</td>
<td>14652000.</td>
<td>0.0219</td>
<td>1.14</td>
<td>14684.</td>
<td>12373.</td>
</tr>
<tr>
<td>3rd story</td>
<td>16099000.</td>
<td>0.1521</td>
<td>0.99</td>
<td>13658.</td>
<td>11122.</td>
</tr>
<tr>
<td>4th story</td>
<td>8674200.</td>
<td>0.0473</td>
<td>1.12</td>
<td>9861.</td>
<td>8532.</td>
</tr>
<tr>
<td>5th story</td>
<td>12426000.</td>
<td>0.1869</td>
<td>1.00</td>
<td>101072.</td>
<td>79241.</td>
</tr>
</tbody>
</table>
number of stories $n_s = 5$  
number of hinges $m = 90$  

\[
\text{total number of unknowns: } (2 \times 5 + 90)(2 \times 5 + 90 + 1)/2 = 5,050
\]

**Figure 4.1** Discrete hinge model of a five story, four bay plane frame

![Discrete hinge model](image)

**Figure 4.2**  
(a) Strong-column-weak-beam (SCWB) model;  
(b) $i$-th floor translational equilibrium;  
(c) $i$-th floor rotational equilibrium

![Figure 4.2](image)
Figure 4.3  Three story, one bay frame: quasi-static test

Figure 4.4  Modal comparison between SCWB model and the original five story, three bay SMRSF
Figure 4.5 Comparison between story drifts of five story, three bay frame obtained by DRAIN-2DX and by the SCWB model (Imperial Valley, 1979 earthquake, El Centro differential array)
Figure 4.6 Comparison between story drifts of five story, three bay frame obtained by DRAIN-2DX and by the SCWB model (El Centro, 1940 earthquake)
Figure 4.7 Comparison between r.m.s. interstory drifts evaluated using DRAIN-2DX and the SCWB model
5.1 Introduction

Random vibration methods can be used to analyze the response of structural systems subjected under stochastic dynamic excitation. Both the excitation and the response are modeled as stochastic processes which can be specified in terms of their moment functions, such as the autocorrelation function, or equivalently, the power spectral density function. Such processes can be viewed as an ensemble of infinite possible sample time histories or "realizations". Random vibration methods evaluate response statistics in terms of those of the excitation. Alternatively, one can use the Monte Carlo simulation method, which is powerful but computationally expensive.

Methods used in the analysis of inelastic systems under random excitation include the Fokker–Planck approach, the perturbation approach, the stochastic averaging approach, the semi–empirical approach and the statistical equivalent linearization approach. A recent review of these methods is given by Wen (1989). All these approaches involve some approximation, since exact analytical solution is extremely difficult for the response of an inelastic multi–degree–of–freedom (MDOF) system under stochastic excitation. The statistical equivalent linearization method replaces the nonlinear system by an equivalent linear one, minimizing the error in the response in a mean square sense. It is most versatile and can be applied to multi–degree–of–freedom systems without difficulty. A review of the method and its applications is given by Roberts and Spanos (1990). It was first applied to smooth hysteretic systems by Wen (1980) and since then, it has been widely used in random vibration analysis of inelastic MDOF systems under earthquake excitation. The solution obtained by this method is the covariance matrix which contains the second order response statistics of the state variables of the system.
The statistical equivalent linearization method is employed in this study to solve for the response statistics of the SCWB model; i.e., the nonlinear first order differential equations for the smooth hysteretic restoring moment of all inelastic springs are replaced by equivalent linear ones and the response statistics are evaluated via random vibration analysis. Since the ground excitation is nonstationary, the response covariance matrix is a function of time.

5.2 SCWB Model Formulation

The equations of motion of the strong-column-weak-beam (SCWB) model can be obtained as follows using interstory drifts as the translational degrees of freedom:

\[ \ddot{u} + a \dot{u} + [K]u + [A] \theta + b \dot{\theta} + e_1 \ddot{a}_g = 0 \]  
\[ [E][u + b \dot{u}] + [F][\theta + b \dot{\theta}] + [D]Y = 0 \]  
\[ \dot{Y}_j = A_j \dot{\theta}_j - \beta_j |\dot{\theta}_j|Y_j - \gamma_j \dot{\theta}_j Y_j^2 \]  
\text{; } j = 0, \ldots, ns \quad (5.3)

where \( u \) is the \( j \)-th story drift, \( \theta \) is the clockwise positive rotation of the \( j \)-th rotational spring, \( Y \) is the hysteretic component of the \( j \)-th rotation, \( a \) and \( b \) are Rayleigh damping coefficients and \( \ddot{a}_g \) is the ground acceleration. \( A_j, \beta_j \) and \( \gamma_j \) are the already identified parameters of the \( j \)-th rotational spring and \( e_1, [K], [A], [E], [F] \) and \( [D] \) are functions of the model parameters (given for a three story example in Appendix A.4). Eqs. 5.3 can be linearized and given by:

\[ \dot{Y}_j = C_{eq}^j \dot{\theta}_j + K_{eq}^j Y_j \]  
\text{; } j = 0, \ldots, ns \quad (5.4)

Atalik and Utku (1976) showed that if the state variables are jointly Gaussian with zero means and if the nonlinear equations are sufficiently smooth, the linearized coefficients can be obtained as the expectations of the first partial derivatives of the original equations with respect to the state variables, i.e.,
The linearized coefficients $C_{eq}^j$ and $K_{eq}^j$ are expressed in closed form in terms of the second order response statistics in Appendix A.5.

Since the ground excitation is a filtered white noise with amplitude and frequency modulation and frequency content described by a Clough–Penzien spectrum, the governing equations of the problem need to be augmented by Eqs. 2.27, 2.28 and 2.29. In these equations, $a(t)$ is the same as $\ddot{a}_g$, the ground acceleration in the r.h.s. of Eq. 5.1.

For the response analysis it is convenient to introduce a state variable representation:

$$\begin{align*}
    y^T &= \begin{bmatrix} y_1^T, y_2^T, y_3^T, y_4^T, y_5^T \end{bmatrix} \quad (5.7)
\end{align*}$$

in which $y_1 = u$, $y_2 = \dot{u}$, $y_3 = \theta$, $y_4 = Y$ and $y_5^T = \begin{bmatrix} x_g, \dot{x}_g, x_f, \dot{x}_f \end{bmatrix}$, where $x_g, \dot{x}_g, x_f, \dot{x}_f$ are the auxiliary variables and their time derivatives in Eqs. 2.27 and 2.28. Using this state variable representation, one can easily transform eqs. 5.1, 5.2, 5.4, 2.27 and 2.28 to a system of first order, ordinary differential equations written as:

$$\begin{align*}
    \dot{y}_1 &= y_2 & \quad (5.8) \\
    \dot{y}_2 &= -ay_2 + [K](y_1 + by_2) + [F]y_4 - \xi_1^{T}y_5 & \quad (5.9) \\
    \dot{y}_3 &= -\frac{1}{b}y_3 + [E](y_1 + by_2) + [D]y_4 & \quad (5.10) \\
    \dot{y}_4 &= -\frac{1}{b}[C_{eq}y_3 + [C_{eq}][E](y_1 + by_2) + [K_{eq}]y_4 + [C_{eq}][D]y_4 & \quad (5.11) \\
    \dot{y}_5 &= [L]y_5 + f & \quad (5.12)
\end{align*}$$
where \([C_{eq}]_n\), \([K_{eq}]_n\) are diagonal matrices consisting of the linearized coefficients \(C_{eq}\) and 
\(K_{eq}\) and

\[
\mathbf{L} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\omega_g\phi'(t)^2 & -\frac{2\zeta_g\omega_g}{\phi'(t)} & 2\zeta_g\omega_g\phi'(t) & 0 & 0 \\
0 & 0 & 0 & 1 \\
-\omega_g\phi'(t)^2 & -2\zeta_g\omega_g\phi'(t) & -\omega_r\phi'(t)^2 & -\frac{\phi''(t)}{\phi'(t)} & -2\zeta_r\omega_r\phi'(t)
\end{bmatrix}
\]

(5.14)

\[
\mathbf{f}^T = \left\{ 0 ; -[\phi'(t)]^2 I(t) \xi(\phi(t)) ; 0 ; 0 \right\}
\]

(5.15)

\[
[\overline{K}] = [A][F]^{-1}[E] - [K]
\]

(5.16)

\[
[\overline{F}] = [A][F]^{-1}[D]
\]

(5.17)

\[
[\overline{E}] - \frac{1}{b}[F]^{-1}[E]
\]

(5.18)

\[
[\overline{D}] = -\frac{1}{b}[F]^{-1}[D]
\]

(5.19)

Eqs. 5.8 to 5.12 can be rewritten in compact form as

\[
\dot{y} = [G]y + a
\]

(5.20)

In this equation,
and
\[\tilde{a}^T = \left\{ 0 ; 0 ; 0 ; 0 ; f^T \right\}\] (5.22)

Postmultiplying both sides of Eq. 5.20 by \(y^T\), adding the resulting relation to its transpose and taking expectations, results in
\[\hat{S} = [G][S] + [S][G]^T + [B]\] (5.23)

where
\[S = E[yy^T]\] (5.24)
\[\hat{S} = E[yy^T] + E[\dot{y}\dot{y}^T]\] (5.25)
\[B = E[ay^T] + E[y\dot{a}^T]\] (5.26)

In the above equations, \([S]\) is the covariance matrix of the state variables and \([B]\) the cross-covariance matrix between the state and the loading vector. Since the excitation \(\zeta(\phi(t))\) is a shot noise, which is uncorrelated to its past and can be considered as a unit impulse (i.e., as a sudden change in the velocity \(\dot{x}_g\)), the only non–zero term in \([B]\) is
\[B_{kk} = 2E[\dot{x}_gf_2] = -2\phi'(t)^2 I(t) E[\zeta(\phi(t))\dot{x}_g] \quad k = 4ns + 4\] (5.27)

where \(ns\) is the number of stories of the SCWB model. Using the definition of an impulse and Eq. 2.27 one obtains:
\[ \dot{x}_g = \phi'(t)^2 \int_{t-\epsilon}^{t} \xi(\phi(\tau)) d\tau \]  

(5.28)

The autocorrelation function of a shot noise may be expressed as

\[ \text{E}[\xi(\phi(t_1)) \xi(\phi(t_2))] = 2\pi S_0 \delta(\phi(t_1) - \phi(t_2)) \]  

(5.29)

where \( S_0 \) is the power spectrum of the shot noise and \( \delta(.) \) is the Dirac delta function.

Substituting \( \dot{x}_g \) from Eq. 5.28 in Eq. 5.27, and carrying out the integration using Eq. 5.29, one obtains

\[ B_{kk} = 2\pi \phi'(t)^3 I(t)^2 S_0 \quad ; \quad k = 4n_s + 4 \]  

(5.30)

Eq. 5.23 is a system of first order nonlinear differential equations which can be solved numerically with a backward differentiation scheme. The nonlinearity is due to the fact that the coefficients in \([G]\) are in general functions of \([S]\). As in the time history analysis (Section 4.5) the use of an implicit method was found computationally more efficient than the popular explicit schemes, largely because the system of covariance matrix differential equations is stiff (Press et al., 1989).

Matrix \([G]\) in Eq. 5.23 contains the linearized coefficients \( C_{eq}^i \) and \( K_{eq}^i \) \((i = 1, \text{no. of rotational springs})\) which are functions of the second order joint statistics of \( Y \) and \( \dot{\theta} \). The linearized coefficients need to be updated at every step of the solution. Since the covariance matrix \([S]\) does not contain the required statistics for this purpose, an additional covariance matrix \([S_v]\) needs to be calculated, where \( y^T = \{ \dot{\theta}, Y \} \). The vector \( y \) is related to the state vector \( \dot{y} \) through a transformation matrix \([T]\) as

\[ y = [T]\dot{y} \]  

(5.31)

where
\[ [T] = \begin{bmatrix} [E] & b[E] & -\frac{1}{b} [I] & [D] \\ 0 & 0 & 0 & [I] \end{bmatrix} \] (5.32)

Hence,

\[ [S_v] = [T][S][T]^T \] (5.33)

To start the analysis, the linearized coefficients may be initialized by setting \( C_{eq} = A_i \) and \( K_{eq} = 0 \).

The state vector covariance matrix \([S]\) contains the response statistics of the SCWB model. These statistics can also be obtained via the Monte Carlo simulation method (see Section 4.6). The latter, however, is far more computationally expensive as a large number of time history analyses is required. To test the accuracy of the statistical equivalent linearization method, the results of the two different methods are compared and shown in Figs. 5.2 and 5.3. It is seen that the accuracy of the method is quite satisfactory and within the range reported in previous studies (e.g., Baber and Wen, 1980, Yeh and Wen, 1989). If more accuracy is needed, an empirical correction formula as given in Yeh and Wen (1989) may be used.

### 5.3 Maximum Response Statistics

Maximum response statistics are necessary in the evaluation of the reliability of a structure. These statistics may be obtained from the response covariance matrix using the method developed by Yang and Liu (1981). This method is developed specifically for analysis of maximum response of a nonstationary process. It is based on the simulation results of Shinozuka and Yang (1971), which indicate that the distribution of the peaks \( u_p \) of a nonstationary random process \( u(t) \), \( F_{U_p}(u_p; T_1, T_2) \), in the time interval \( (T_1, T_2) \) can be expressed by the Weibull distribution, i.e.,
in which \( \alpha \) and \( \sigma \) are parameters depending on \( T_1 \) and \( T_2 \) as well as the nonstationary characteristics of \( u(t) \). These parameters can be identified assuming that the mean and variance of the peak at time \( t \), given that it has occurred, are (Yang, 1973)

\[
E[u_p(t)] = \sqrt{\frac{\pi}{2}} \sigma_u(t)
\]

(5.35)

\[
E[u_p^2(t)] = 2\sigma_u^2(t)
\]

(5.36)

where \( \sigma_u(t) \) is the root mean square story drift at time \( t \). The average peak value \( \bar{u}_p \) and the coefficient of variation \( \delta_p \) for the time interval \((T_1, T_2)\) can be calculated using equations 5.35 and 5.36 and assuming that peaks are statistically independent as follows

\[
\bar{u}_p = \frac{1}{n} \sum_{i=1}^{n} E[u_p(t_i)]
\]

(5.37)

\[
\delta_p = \frac{\left( \frac{1}{n} \sum_{i=1}^{n} E[u_p^2(t_i)] - \bar{u}_p^2 \right)^{1/2}}{\bar{u}_p}
\]

(5.38)

in which the interval \((T_1, T_2)\) has been discretized into \( n \) segments. Since the distribution of the peaks is Weibull, the mean and the coefficient of variation are related to the distribution parameters \( \alpha \) and \( \sigma \) as

\[
\delta_p = \frac{\left[ \Gamma\left(\frac{2}{\alpha} + 1\right) - \Gamma^2\left(\frac{1}{\alpha} + 1\right) \right]^{1/2}}{\Gamma\left(\frac{1}{\alpha} + 1\right)}
\]

(5.39)
Eq. 5.39 may be used to calculate \( a \) and Eq. 5.40 may then be used to calculate \( \sigma \).

The distribution of the maximum response \( U_m \) can then be obtained under the assumptions that peaks are statistically independent and that the total number of peaks \( n_p \) in \((T_1, T_2)\) is large. It can be shown that the distribution of \( U_m \) approaches asymptotically to Type I (Yang and Liu, 1981), i.e.,

\[
F_{U_m}(u_m; T_1, T_2) = \exp \left\{ -\exp \left[ -K^{\alpha - 1} \left( \frac{u}{\sigma} - K \right) \right] \right\}
\]

(5.41)

where

\[
K = \left( \alpha \ln n_p \right)^{1/\alpha} = \left[ \alpha \ln \left( \frac{T_2}{T_1} \right) \right]^{1/\alpha}
\]

(5.42)

and \( \nu_0^+ \) is the time-varying zero upcrossing rate. Assuming that the displacement \( u \) and the velocity \( u \) are jointly Gaussian, \( \nu_0^+ \) can be expressed as (Shinozuka and Yang, 1971)

\[
\nu_0^+ = \frac{\sigma \sqrt{1 - \rho_{uu}^2(t)}}{2\pi \sigma u(t)}
\]

(5.43)

The mean value and the standard deviation of the maximum drift are also given as

\[
E[U_m(t)] = (K + 0.5772K^{1-\alpha})\sigma
\]

(5.44)

\[
\sigma U_m = \frac{\pi}{\sqrt{6}} \sigma K^{1-\alpha}
\]

(5.45)

The approach described is compatible with the random vibration method used in this study. The covariance matrix of the system's response for a given set of ground motion
parameters can be used to evaluate the probability of the maximum drift exceeding a specified threshold at a certain story. This conditional probability may be combined with the uncertainty of the ground motion parameters to evaluate the overall seismic risk of the structure (see Chapter NO TAG).

5.4 Numerical Examples

The response statistics of the five story three bay SMRSF, designed in Chapter 3 and modeled as a SCWB model in Chapter 4, are evaluated by random vibration analysis via the statistical equivalent linearization method. In this example, the ground motion model is based on the El Centro Differential Array record of the 1979, Imperial Valley earthquake (Geological Survey, 1982). The ground motion model parameters and the corresponding intensity, frequency modulation and power spectral density function are shown in Fig. 5.1. Root mean square interstory drifts and joint rotations obtained from the equivalent linearization method are compared with those obtained from response time history analysis. In both analyses, the SCWB model is used, since the difference between the SCWB model and a full finite element model based on DRAIN-2DX has been found to be very small (see Section 4.6). The root mean square interstory drifts obtained from 40 simulation samples and those obtained via the equivalent linearization method are compared in Fig. 5.2. The same comparison is presented in Fig. 5.3 for root mean square joint rotations. The response statistics of the statistical equivalent linearization method agree very well with those obtained from simulations.

Assuming that the maximum response approaches a Type I distribution (Eq. 5.41), the second order maximum drift statistics can be evaluated at each story from Eqs. 5.44 and 5.45. The mean maximum drift, \( E[U_{\text{max}}] \), and the corresponding coefficient of variation, \( \delta U_{\text{max}} \), at each story, evaluated based on these relations, are presented in Table 5.1 along with those
evaluated from 40 samples of time history analysis. The error in the mean maximum drift is less than ten percent for all cases. Also, Table 5.1 indicates that the analysis slightly overestimates the coefficient of variation of the maximum story drift, although 40 samples of response time history are not enough to provide an accurate estimate for this coefficient of variation. The analysis also gives the probability of exceedance of a specified drift threshold at a given level. Table 5.2 shows the probability of exceedance of different drift levels at each story, in which, drift levels are expressed as percentages of the story heights.

Table 5.1 Statistics of maximum story drifts

<table>
<thead>
<tr>
<th>Story Number</th>
<th>Analysis (Eqs. 5.44, 5.45)</th>
<th>Simulations (40 Samples)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E[U_max] (in)</td>
<td>δ U_max (in)</td>
</tr>
<tr>
<td>1</td>
<td>2.40</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>1.96</td>
<td>0.32</td>
</tr>
<tr>
<td>3</td>
<td>1.78</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>1.63</td>
<td>0.29</td>
</tr>
<tr>
<td>5</td>
<td>1.75</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 5.2 Probabilities of exceedance of various drift levels at a story

<table>
<thead>
<tr>
<th>Drift Level (%)</th>
<th>1st Story</th>
<th>2nd Story</th>
<th>3rd Story</th>
<th>4th story</th>
<th>5th Story</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.745</td>
<td>0.721</td>
<td>0.613</td>
<td>0.494</td>
<td>0.600</td>
</tr>
<tr>
<td>1.5</td>
<td>0.301</td>
<td>0.227</td>
<td>0.139</td>
<td>0.079</td>
<td>0.118</td>
</tr>
<tr>
<td>2.0</td>
<td>0.089</td>
<td>0.050</td>
<td>(0.235)</td>
<td>0.010</td>
<td>0.017</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0242</td>
<td>0.0103</td>
<td>0.0038</td>
<td>0.0012</td>
<td>0.0023</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0064</td>
<td>0.0021</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0003</td>
</tr>
</tbody>
</table>
Intensity Function Parameters
- \( A = 8.77 \times 10^{11} \)
- \( B = 3.012 \)
- \( C = -0.311 \)
- \( D = 3.38 \times 10^{11} \)
- \( E = 13.265 \)

Frequency Modulation Function Parameters
- \( r_1 = 1.340 \)
- \( r_2 = 0.516 \)
- \( r_3 = -0.171 \)
- \( r_4 = 0.022 \)
- \( r_5 = -0.15 \times 10^{-2} \)
- \( r_6 = 0.56 \times 10^{-4} \)
- \( r_7 = -0.11 \times 10^{-5} \)
- \( r_8 = 0.86 \times 10^{-8} \)

Normalized PSD Function Parameters
- \( S_0 = 0.83 \times 10^{-3} \)
- \( \omega_g = 19.00 \)
- \( \zeta_g = 0.50 \)
- \( \omega_f = 3.20 \)
- \( \zeta_f = 0.50 \)

Figure 5.1 Functions and parameters of the ground motion model, identified from the El Centro differential array record of the 1979, Imperial Valley earthquake
Figure 5.2  Comparison between nonstationary root mean square story drifts obtained by Monte Carlo simulations and by the statistical equivalent linearization method (Imperial Valley, 1979 earthquake)
Figure 5.3 Comparison between nonstationary root mean square joint rotations obtained by Monte Carlo simulations and by the statistical equivalent linearization method (Imperial Valley, 1979 earthquake)
CHAPTER 6
ASSESSMENT OF STRUCTURAL SAFETY

6.1 Introduction

The reliability of a structural system during a time period of interest (e.g. design lifetime) is defined as \( R = 1 - P_f \) where \( P_f \) is the probability of failure. This chapter considers the problem of evaluating the seismic reliability of a building over a given time period such as lifetime of the structure. Previous chapters deal with identification and quantification of the uncertainties related to future earthquake motion. The probability of failure obviously depends not only on the randomness of the ground motion but also on the uncertainties in the ground motion and structural resistance. The uncertainties in the loading parameters such as occurrence, intensity and duration are usually large and play a dominant role compared to those in structural system parameters such as mass, stiffness and damping. Therefore, as an approximation, the uncertainties in structural resistance are not considered in this study. A fast integration scheme is used to include the uncertainties and evaluate the probability of failure of the building under future earthquakes.

To include the effect of parameter uncertainties, one can use the method of Monte Carlo simulation. A ground motion record is generated and a time history response analysis is performed to determine whether failure (i.e., exceedance of a given limit state) occurs for a given set of ground motion parameters. The process is repeated and the relative frequency is obtained as the probability of failure. Obviously, when the safety level is high (i.e., when the probability of failure is low), a large number of simulations is necessary to obtain results with high confidence, i.e., approximately 10 to 100 times the reciprocal of the failure probability. Alternatively, one can evaluate conditional probabilities of failure for given values of the ground motion parameters and then integrate over the ground motion parameters to find the unconditional probability of failure. If \( X \) denotes the ground motion
parameters, the unconditional probability of failure \( P_F \) is given as

\[
P_F = \int_X P_f(x) f_X(x) \, dx
\]  

(6.1)

in which \( P_f(x) \) is the conditional probability of failure for \( X = x \), and \( f_X(x) \) is the joint density function of \( X \). Again, computation becomes excessive when the dimension of \( X \) is larger than three. As a result, the use of an approximate method with good accuracy and numerical efficiency is necessary. Recently such a method, generally referred to as the fast integration technique, has been developed by Wen and Chen (1987). This method basically transforms the reliability problem given in Eq. 6.1 in a form to which the well known first order reliability method can be applied. The fast integration technique is used in this study.

6.2 Definition of Failure

Dictionaries define failure as \textit{nonperformance of what is required or expected}. By this definition, failure is a function of one's perspective of obligation or duty. Thus, in the assessment of structural safety, it is the engineer's duty to define failure for a particular structure in a responsible fashion. For example, one could consider failure as the exceedance of the yield stress of the material during loading; or, one could consider failure as the exceedance of the ultimate stress of the material. In reliability analysis, failure can be defined by the limit state concept. A state of a system can be characterized as safe if the system performs as expected, or as unsafe if the system does not perform satisfactorily. A limit state is the transition state between safety and failure. To visualize this concept, consider an \( n \)-dimensional vector \( X \) which represents the state (or design) variables of the problem. These variables are random and the performance (or state) of the system is described by a scalar function \( g(X) \). A hypersurface, defined by \( g(X) = 0 \), divides the \( n \)-dimensional space into a safety and a failure region. This hypersurface is known as the limit state surface (or
failure surface) of the system. The safe region is then defined by \( g(\mathbf{X}) > 0 \), and the unsafe (or failure) region by \( g(\mathbf{X}) < 0 \) (Fig. 6.1). The function \( g(\mathbf{X}) \) is usually referred to as the limit state function. In structural design, ultimate limit states (which correspond to severe to life threatening damage) and various serviceability limit states are commonly used in describing structural performance.

In building industry, failure of moment resisting steel frames is defined in terms of interstory drift, i.e., the exceedance of a certain drift threshold. This definition is consistent with the design of special moment resisting space frames (SMRSF) according to the 1988 Uniform Building Code, in which lateral stiffness requirements govern the design (see Chapter 3). It is implicit in UBC-88 that the allowable interstory drift limit is \( (3R_w/8)\delta_x \) and less than 1.5 percent of the story height, where \( \delta_x \) is the elastic drift computed by the equivalent lateral force method. This elastic drift is limited to \( 0.04/R_w \) or 0.005 times the story height, whichever is smaller. For SMRSF, \( R_w = 12 \) and the maximum allowable elastic drift \( \delta_x^{\text{max}} \) is equal to 0.0033 times the story height. This maximum elastic drift corresponds to an allowable inelastic drift of 1.5 percent of the story height. Hence, a drift of 1.5 percent of the story height is considered to be the failure threshold in this study. The probability distribution of the maximum drift obtained from the results of random vibration analysis is used to evaluate the probability of exceedance of this threshold at each story during the building’s design lifetime of 50 years. Lower drift thresholds can also be used to ensure serviceability.

### 6.3 Fast Integration Technique for Time Variant Reliability Analysis

The fast integration technique, in essence, uses conditional probabilities of failure evaluated for given sets of the system random variables to describe a computationally convenient limit state function which can be used to assess the overall reliability. The system
parameters \( \mathbf{X} \) (whose probability distributions are known) are transformed to standard normal random variates \( \mathbf{U} \) using the transformation \( \mathbf{U} = \mathbf{T}(\mathbf{X}) \). An auxiliary standard normal variate \( U_{n+1} \), independent of \( U \) in the transformed space is then introduced and the reliability problem is formulated in the transformed space with a limit state function

\[
g(\mathbf{U}, U_{n+1}) = U_{n+1} - \Phi^{-1}\left(P_f[T^{-1}(\mathbf{U})]\right)
\]

(6.2)

where \( P_f \) is the conditional probability of failure given the system parameters \( \mathbf{X} \) and where \( \Phi^{-1}[ . ] \) is the inverse standard normal distribution function. Using Eq. 6.2, failure corresponds to \( g < 0 \). It can be shown that the failure probability according to this formulation is equal to that given by Eq. 6.1 (Wen and Chen, 1987).

In the assessment of seismic reliability of moment resisting steel frames, the major uncertain parameters for characteristic earthquakes are the significant duration and the scale factor in the Fourier amplitude spectrum. The uncertainty in these parameters is modeled by the random variables \( \varepsilon_D \) and \( \varepsilon_S \) as given in Eqs. 2.1 and 2.3. For non-characteristic events, the major uncertain system parameters are the modified Mercalli intensity, the significant duration, and the scale factor in the Fourier amplitude spectrum. The uncertainty is modeled by the random variables \( I, \varepsilon_D \) and \( \varepsilon_S \) as given in Eqs. 2.18, 2.19 and 2.20. In both cases system parameters are statistically independent, therefore the transformation to standard normal variates can be performed as follows: If a system parameter \( X \) follows a distribution with cumulative density function \( F_X(x) \), then \( U \) is the reduced standard normal variate with

\[
\Phi_U(u) = F_X(x) \leftrightarrow u = \Phi^{-1}[F_X(x)]
\]

(6.3)

The first order reliability method may now be used to determine the design point and the corresponding probability of failure from Eq. 6.2. The design point \( \mathbf{x}^* \) which is the point on the limit state surface (\( g = 0 \)) with minimum distance to the origin of the reduced variates
is the most probable failure point (Shinozuka, 1983). This minimum distance is the reliability index $\beta$ which is related to the probability of failure as

$$P_F = \Phi(-\beta)$$

(6.4)

where $\Phi[ \ldots ]$ is the standard normal distribution function. The search for the design point is performed using the direct method, developed by Ang (1986). The algorithm is summarized below for the case of $n$ uncorrelated system parameters:

1. Assume initial values for $x^*$ and calculate the corresponding reduced standard normal variates $u^*$

2. Evaluate the probability of failure $P_f$ for these parameters and solve $g(u^*, u_{n+1}^*) = 0$ for $u_{n+1}^*$. The solution is simply $u_{n+1}^* = \Phi^{-1}(P_f)$

3. Calculate the gradient vector $G$ in the reduced coordinates, defined as

$$G_i = \left[ \frac{\partial g}{\partial u_i} \right]_{g=0} = -\left[ \frac{\partial \Phi^{-1}[P_f(u)]}{\partial u_i} \right]_{u^*} ; \quad i = 1, \ldots, n$$

and

$$G_{n+1} = \left[ \frac{\partial g}{\partial u_{n+1}} \right]_{g=0} = 1$$

4. Calculate $a$, a unit vector in the opposite direction of $G$ pointing towards the failure domain, and the reliability index $\beta$

$$a = -\frac{G}{\left[ G^T G \right]^{1/2}} ; \quad \beta = -\frac{G^T u^*}{\left[ G^T G \right]^{1/2}}$$

5. Evaluate $u^* = -\beta \cdot a$ and repeat steps (2) to (5) until convergence in $\beta$ is achieved.
The partial derivatives of the performance function $g$ with respect to the reduced variates at each step are evaluated using a finite difference scheme, since a closed form of $P_f$ in terms of the system parameters is not available. Each calculation of $P_f$ requires a random vibration analysis. Hence, the method requires only $2n+1$ random vibration analyses for each iteration step, where $n$ is the number of system parameters. In all cases examined, convergence was achieved within four or five iterations.

### 6.4 Seismic Reliability Evaluation of Moment Resisting Steel Frames

The fast integration technique is applied herein in the evaluation of the probability of failure given the occurrence of an earthquake which is then combined with the occurrence probability to arrive at the risk of failure over a given time period such as lifetime of the structure. Using the concept of characteristic and non-characteristic earthquakes (see Chapter 2), the lifetime probability of failure of a frame at a specific story can be expressed as

$$
P_F(U_{\text{max}} \geq u_f) = \sum_{k=1}^{\infty} \left[ 1 - \left( 1 - P(U_{\text{max}} \geq u_f | O_{\text{ch}}) \right)^k \right] P(N_{\text{ch}} = k) +$$

$$+ \sum_{k=1}^{\infty} \left[ 1 - \left( 1 - P(U_{\text{max}} \geq u_f | O_{\text{nc}}) \right)^k \right] P(N_{\text{nc}} = k)$$

(6.5)

In Eq. 6.5, $P_F(U_{\text{max}} \geq u_f)$ is the probability of failure at a story ($u_f$ is the failure threshold) over the design lifetime of the frame; $P_F(U_{\text{max}} > u_f | O_{\text{ch}})$ is the probability that the maximum story drift $U_{\text{max}}$ will exceed the failure threshold $u_f$ given that a characteristic event has occurred; $P_F(U_{\text{max}} > u_f | O_{\text{nc}})$ is similarly defined for a non-characteristic event; $P(N_{\text{ch}} = k)$ is the probability of $k$ occurrences of characteristic earthquakes during the design lifetime of the frame; and $P(N_{\text{nc}} = k)$ is the probability of $k$ occurrences of non-characteristic earthquakes during the same period. The occurrence of characteristic
events follows a renewal process with lognormal recurrence time. The probability distribution \( P(N_{ch} = k) \) associated with this process may be evaluated as shown in Appendix E.2. It is noted that for more than one occurrence, the computations become cumbersome because multiple integrals need to be evaluated. Finally, since the occurrence of non-characteristic events follows a Poisson process, the second term in the r.h.s. of Eq. 6.5, using Eq. 2.15, becomes

\[
\sum_{k=1}^{\infty} \left[ 1 - \left( 1 - P(U_{\text{max}} \geq u_f|O_{nc}) \right)^k \right] P(N_{nc} = k) = 1 - e^{-\nu t} P(U_{\text{max}} \geq u_f|O_{nc})
\]

in which \( t \) is the design lifetime of the frame and \( \nu \) is the occurrence rate per year of non-characteristic events.

### 6.5 Numerical Examples

Consider the five story, three bay SMRSF at Santa Monica Boulevard, in Los Angeles, 60 km from the Mojave segment of the San Andreas fault. The frame has a design lifetime of 50 years (Chapter 3). The probability of occurrence of one characteristic earthquake from the Mojave segment during this period is \( P(N_{ch} = 1) = 0.495 \) (evaluated using Eq. E.1.3). The probabilities of two or more characteristic earthquakes occurring at the Mojave segment in the same time window (1991-2041) are very small and can be neglected.

The probabilities of failure at a story given the occurrence of an event, i.e., \( P_F(U_{\text{max}} > u_f|O_{ch}) \) and \( P_F(U_{\text{max}} > u_f|O_{nc}) \), are evaluated by the fast integration method. A drift level equal to 1.5 percent of the story height is considered to be the failure threshold. Table 6.1 gives these conditional probabilities given occurrence in which \( \beta_{ch} \) and \( \beta_{nc} \) are respectively the reliability indices against characteristic and non-characteristic events. The conditional probabilities \( P_F(U_{\text{max}} > u_f|O_{ch}) \) and \( P_F(U_{\text{max}} > u_f|O_{nc}) \) are then obtained by
Eq. 6.4 and the overall probability of failure \( P_F(U_{\text{max}} \geq u_t) \) for the time window 1991 to 2041 is calculated according to Eq. 6.5. The mean annual rate of non-characteristic earthquakes at the Los Angeles site, with modified Mercalli intensity greater or equal to 5, has been calculated in Section 2.4; this value, \( v = 0.1165 \), is used in Eq. 6.6. Notice in Table 6.1 that in a characteristic earthquake the third story of the frame is the most likely to fail, while the first story is the most critical in non-characteristic events. This is due to the fact that ground motion properties of characteristic and non-characteristic earthquakes differ significantly, each affecting structural response in a different way. It is also clear from Table 6.1 that the risk of structural failure due to non-characteristic events is much greater than that due to the characteristic earthquake of the Mojave fault segment, primarily because of the large distance (60 km) from the fault to the site and more frequent occurrence of the non-characteristic earthquakes.

The same results are presented in Table 6.2 for a lower drift threshold equal to 1 percent of the story height. The second and third stories are in this case more likely to exceed this drift limit than the first story which was the critical one in the previous case. Notice that in this case, the probability of exceedance of the 1 percent drift limit is higher in a characteristic event than in a non-characteristic. Still though, the overall risk due to non-characteristic events prevails due to the high occurrence rate of such events. The results indicate the importance and sometimes the necessity of performing such a reliability analysis for different performance levels rather than predicting structural behavior by extrapolation.

As far as the efficiency of the fast integration technique is concerned, it should be mentioned that about 50 random vibration analyses are required to evaluate the foregoing probability of failure independent of the failure probability level. By comparison, if Monte Carlo method is used, at a drift level of 1.5 percent and a failure probability of the order of \( 10^{-2} \), approximately 1000 simulations would be required to obtain the probability of failure
with some level of confidence. Even more simulations are required if higher drift levels (hence smaller failure probabilities) are considered.

**Table 6.1** Probability of 1.5 percent drift being exceeded for the time window 1991 to 2041 ($u_f = 0.015 \text{ h}$)

| Story | $\beta_{ch}$ | $P_F(U_{max} > u_f|O_{ch})$ | $\beta_{nc}$ | $P_F(U_{max} > u_f|O_{nc})$ | $P_F(U_{max} \geq u_f)$ |
|-------|-------------|------------------|-------------|------------------|--------------------|
| 1     | 3.76        | 0.00009          | 2.23        | 0.0129           | 0.0723             |
| 2     | 2.87        | 0.00205          | 2.28        | 0.0113           | 0.0647             |
| 3     | 2.82        | 0.00240          | 2.34        | 0.0097           | 0.0558             |
| 4     | 3.13        | 0.00087          | 2.42        | 0.0078           | 0.0446             |
| 5     | 3.33        | 0.00043          | 2.32        | 0.0102           | 0.0577             |

**Table 6.2** Probability of 1 percent drift level being exceeded for the time window 1991 to 2041 ($u_f = 0.010 \text{ h}$)

| Story | $\beta_{ch}$ | $P_F(U_{max} > u_f|O_{ch})$ | $\beta_{nc}$ | $P_F(U_{max} > u_f|O_{nc})$ | $P_F(U_{max} \geq u_f)$ |
|-------|-------------|------------------|-------------|------------------|--------------------|
| 1     | 2.80        | 0.0026           | 2.08        | 0.0188           | 0.1048             |
| 2     | 1.83        | 0.0336           | 2.00        | 0.0228           | 0.1408             |
| 3     | 1.83        | 0.0336           | 2.00        | 0.0228           | 0.1408             |
| 4     | 2.11        | 0.0174           | 2.07        | 0.0192           | 0.1146             |
| 5     | 2.33        | 0.0099           | 1.99        | 0.0233           | 0.1318             |
Figure 6.1 Different states of a system in the n-dimensional space defined by the random vector $X$
CHAPTER 7
SUMMARY AND CONCLUSIONS

7.1 Summary

A method for the performance evaluation of moment resisting steel frames under future earthquakes is presented in this study. An integrated approach is proposed, which can be used to evaluate the probability of failure (limit states being exceeded) of such frames under seismic excitation during their design lifetime. State of the art seismic hazard, random vibration and system reliability methods have been employed for this purpose.

The ground motion is modeled by a random process with time-varying amplitude and frequency content. The model parameters are identified from available seismological and geological information at the site of interest. This information is processed through a seismic hazard analysis and source and ground motion parameters are modeled as random variables which describe the variability of the excitation at the region. Future earthquakes are categorized as either characteristic and non-characteristic. The former are generated from a major fault whereas the latter are minor, local events. For the characteristic earthquakes, uncertainties are considered in the recurrence time, significant duration, source to site attenuation and geologic site conditions. For non-characteristic events, uncertainties are considered in occurrence time, significant duration, intensity and geologic site conditions. In both cases, appropriate probability distributions are used for the governing random variables, and the ground motion model parameters are evaluated by system identification techniques. The concept of characteristic versus non-characteristic events facilitates the quantification of ground motion uncertainties and the identification of the parameters of the ground motion model.

A simplified structural model with a reduced number of degrees of freedom is proposed for the response analysis. The strong-column-weak-beam (SCWB) model, having only two
degrees of freedom per story, describes the behavior of a moment resisting steel frame efficiently. Model parameters are evaluated by a system identification technique from the response of the actual frame in a quasi-static test. This test is simulated using DRAIN-2DX, a finite element program designed for nonlinear dynamic deterministic analysis of plane structures. The identification is then carried out using the Gauss method, a powerful algorithm suitable for both constrained and unconstrained optimization problems. Comparison of story displacements with those obtained from DRAIN-2DX showed that the SCWB model reproduces the response time history satisfactorily; i.e., it requires only a fraction of the computation time of DRAIN-2DX without any significant loss in accuracy. The response statistics are obtained by method of random vibration based on a statistical equivalent linearization technique. The results agree well with those obtained from simulations. The statistical equivalent linearization method predicts the response statistics at a small fraction of the computational effort required by simulations. Maximum response statistics at different stories and the probability of failure of a story are obtained, conditional on a given set of ground motion parameters.

The lifetime probability of failure of a moment resisting steel frame under earthquake excitation can be evaluated by integrating the conditional probabilities of failure (obtained from the random vibration analysis) over the range of values of the ground motion parameters. The fast integration technique proposed by Wen and Chen (1987) based on the powerful first order reliability method is used for this purpose. This approach reduces considerably the amount of computational effort required for the reliability evaluation. In all cases examined, the first order method converged within four to five iterations.

The procedure described above has been demonstrated for a five story, three bay SMRSF, designed according to the 1988 Uniform Building Code. The frame is part of the lateral force resisting system of a five story office building designed to be located in Southern California. Geological and seismological information at the site has been gathered and used to identify
the parameters of the ground motion model. Random vibration analysis using the SCWB model provides response statistics of interest and the probability of failure for given values of the ground motion parameters. The unconditional probability of failure is then calculated by the fast integration technique. This method, in combination with the occurrence probability of future earthquakes, gives the 50 year probability of failure of the frame under earthquake excitation.

7.2 Conclusions

Based on the results of this study, the following conclusions may be drawn:

(1) A comprehensive methodology is developed for the evaluation of the performance of steel buildings in future earthquakes. The randomness in earthquake ground motion as well as the uncertainties in source and ground motion parameters such as duration and attenuation are incorporated in the analysis. The uncertainties in ground motion parameters are generally large and their proper treatment in the reliability study is essential, since ground motion parameters have a major impact on structural response.

(2) Deterministic methods traditionally used in earthquake resistant design rely on safety factors to achieve a safe design. However, the reliability implied in the procedure is unknown. The present method calculates the risk associated with different damage levels and can be used to obtain the reliability implied in current code provisions.

(3) A study of moment resisting steel frames located in Los Angeles, California, and designed according to the special moment resisting space frame (SMRSF) provisions of the 1988 Uniform Building Code indicates that the risk of exceedance of drift of 1.0 percent of story height in fifty years is about 10 percent which corresponds to approximately the risk of exceedance of design earthquake in most current procedures.
(4) The robustness and computational efficiency of the proposed method have been demonstrated in the numerical examples throughout this study. The method has been proven to be powerful and accurate in evaluating the structural reliability under future seismic excitation. The present approach may be used by engineers in design to assess the risk implied in earthquake resistant building design and revise the design accordingly so that a target reliability level can be achieved. It may also be used to evaluate the risk consistency of the current code provisions and to develop reliability-based code procedures.

7.3 Future Study

The proposed method can be applied to a wide class of structural systems by employing a suitable structural model, such as the strong-column-weak-beam model which has been developed herein for moment resisting steel frames. Additional ground motion parameters may be incorporated in the method if they have significant influence on future ground motion and structural response. For example, if a site is located close to a fault, the directivity effect may become a significant factor in seismic hazard analysis and can be easily accommodated in this model. Uncertainties associated with the structure can also be considered, e.g., those due to non-structural components. Other important elements that affect structural response, such as soil-structure interaction and the effect of gravitational loads (e.g. P–Δ effect) can also be incorporated in the analysis, although they may increase the complexity of the structural model and the computational effort required.

The present method may be also used to evaluate the risk associated with other limit states such as low cycle fatigue since the statistics of hysteretic energy dissipation which is a good measure of cumulative damage are part of the solution of the statistical equivalent linearization method.
A.1 Matrices in Equations of Motion

A three story SCWB model is shown in Fig. 4.3. The governing equations of motion of the system are given in Section 4.3 (undamped system, Eqs. 4.11 and 4.12) and in Section 4.5 (system with Rayleigh damping, Eqs. 4.39 and 4.40), in terms of relative to the ground story displacements and relative to the vertical axis, clockwise positive, joint rotations at the base and the floor levels.

\[
\begin{bmatrix}
  m_1 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_3
\end{bmatrix}
\]

\[
y^T = \{v_1, v_2, v_3\}
\]

\[
\bar{\theta}^T = \{\theta_0, \theta_1, \theta_2, \theta_3\}
\]

\[
\bar{y}^T = \{Y_0, Y_1, Y_2, Y_3\}
\]
\[ [A] = \begin{bmatrix} \frac{6E_I_1}{h_1^2} & \frac{6E_I_2}{h_2^2} & -\frac{6E_I_1}{h_1^2} & -\frac{6E_I_2}{h_2^2} & 0 \\ 0 & -\frac{6E_I_2}{h_2^2} & \frac{6E_I_3}{h_3^2} & -\frac{6E_I_2}{h_2^2} & \frac{6E_I_3}{h_3^2} \\ 0 & 0 & -\frac{6E_I_3}{h_3^2} & -\frac{6E_I_3}{h_3^2} & \end{bmatrix} \]

\[ [K] = \begin{bmatrix} \frac{12E_I_1}{h_1^3} + \frac{12E_I_2}{h_2^3} & -\frac{12E_I_2}{h_2^3} & 0 \\ -\frac{12E_I_2}{h_2^3} & \frac{12E_I_2}{h_2^3} + \frac{12E_I_3}{h_3^3} & -\frac{12E_I_3}{h_3^3} \\ 0 & -\frac{12E_I_3}{h_3^3} & \frac{12E_I_3}{h_3^3} \end{bmatrix} \]

\[ [E] = \begin{bmatrix} \frac{4E_I_1}{h_1} + \alpha_0 G_0 & \frac{2E_I_1}{h_1} & 0 & 0 \\ \frac{2E_I_1}{h_1} & \frac{4E_I_1}{h_1} + \frac{4E_I_2}{h_2} + \alpha_1 G_1 & \frac{2E_I_2}{h_2} & 0 \\ 0 & \frac{2E_I_2}{h_2} + \frac{4E_I_2}{h_3} + \alpha_2 G_2 & \frac{2E_I_3}{h_3} & 0 \\ 0 & 0 & \frac{2E_I_3}{h_3} + \frac{4E_I_3}{h_3} + \alpha_3 G_3 \end{bmatrix} \]

\[ [D] = \begin{bmatrix} (1 - \alpha_0)G_0 & 0 & 0 & 0 \\ 0 & (1 - \alpha_1)G_1 & 0 & 0 \\ 0 & 0 & (1 - \alpha_2)G_2 & 0 \\ 0 & 0 & 0 & (1 - \alpha_3)G_3 \end{bmatrix} \]
A.2 Identification of Linear Stiffness Coefficients

Consider the three story SCWB system shown in Fig. 4.3. The equilibrium equations of the elastic system are given for free, undamped vibration in Section 4.4.1 (Eqs. 4.22 and 4.23). In these equations,

\[
[M] = \begin{bmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3 \\
\end{bmatrix}
\]

\[
\phi^T = [\theta_1, \theta_2, \theta_3]
\]

\[
\psi^T = [v_1, v_2, v_3]
\]

\[
[K] = \begin{bmatrix}
\frac{12EI_1}{h_1^3} + \frac{12EI_2}{h_2^3} + \frac{36E^2\tilde{I}_1^2}{G_0h_1^4 + 4EI_1h_1^3} & -\frac{12EI_2}{h_2^3} & 0 \\
-\frac{12EI_2}{h_2^3} & \frac{12EI_2}{h_2^3} + \frac{12EI_3}{h_3^3} & -\frac{12EI_3}{h_3^3} \\
0 & -\frac{12EI_3}{h_3^3} & \frac{12EI_3}{h_3^3}
\end{bmatrix}
\]

\[
[A] = \begin{bmatrix}
\frac{6EI_2}{h_2^2} - \frac{6EI_1}{h_1^2} + \frac{12E^2\tilde{I}_2^2}{G_0h_1^4 + 4EI_1h_1^3} & -\frac{6EI_2}{h_2^2} & 0 \\
-\frac{6EI_2}{h_2^2} & \frac{6EI_3}{h_3^2} - \frac{6EI_2}{h_2^2} & \frac{6EI_3}{h_3^2} \\
0 & -\frac{6EI_3}{h_3^2} & \frac{6EI_3}{h_3^2}
\end{bmatrix}
\]

\[
[E] = \begin{bmatrix}
\frac{4EI_1}{h_1} + \frac{4EI_2}{h_2} + \alpha_1 G_1 - \frac{4E^2\tilde{I}_1^2}{G_0h_1^2 + 4EI_1h_1} & \frac{2EI_2}{h_2} & 0 \\
\frac{2EI_2}{h_2} & \frac{4EI_2}{h_2} + \frac{4EI_3}{h_3} + \alpha_2 G_2 & \frac{2EI_3}{h_3} \\
0 & \frac{2EI_3}{h_3} & \frac{4EI_3}{h_3} + \alpha_3 G_3
\end{bmatrix}
\]
Static condensation is performed solving equation \(4.23\) for \( \Phi \). The result is substituted in Eq. 7.1, and Eq. 4.24 is obtained. The generalized eigenvalue problem for the first mode \((\omega_1, y_1)\) is shown in Eq. 4.25. This equation can be solved for the unknown linear stiffness coefficients \(G_1, G_2\) and \(G_3\) as follows:

\[
[K] y_1 = \omega_1^2 [M] y_1
\]

\[
= [K] y_1 - [A][E]^{-1}[A]^T y_1 = \omega_1^2 [M] y_1
\]

\[
= [E]^{-1}[A]^T y_1 = [A]^{-1}[K] y_1 - \omega_1^2 [A]^{-1}[M] y_1
\]

\[
= [A]^T y_1 = [E][[A]^{-1}[K] y_1 - \omega_1^2 [A]^{-1}[M] y_1]
\]

Notice that the vector in curly brackets can be directly calculated as it depends only on \(m, h, EI, G_0, y_1, \omega_1\). In addition, matrix \([E]\) can be expressed as the sum of two matrices \([E_s]\) and \([E_d]\) where

\[
[E_d] = \begin{bmatrix}
\alpha_1 G_1 & 0 & 0 \\
0 & \alpha_2 G_2 & 0 \\
0 & 0 & \alpha_3 G_3
\end{bmatrix}
\]

and \( [E_s] = [E] - [E_d] \)

Let \( x = [A]^{-1}[K] y_1 - \omega_1^2 [A]^{-1}[M] y_1 \). Then Eq. A.2.1 becomes:

\[
[A]^{-1}[K] y_1 = [E_s] x + [E_d] x
\]

\[
= [E_d] x = [A]^{-1}[K] y_1 - [E_s] x
\]

\[
= [E_d] x = b
\]

(A.2.2)

where \( b = [A]^{-1}[K] y_1 - [E_s] x \). Since \( x \) and \( b \) are known and \([E_d]\) is a diagonal matrix, Eq. A.2.2 can be easily solved for \(G_1, G_2\) and \(G_3\).
A.3 Equilibrium Equations in Quasi-Static Test

The equilibrium equations of the three story undamped SCWB model, shown in Fig. 4.3, are given in Section 4.4.3 (Eqs. 4.28 and 4.29). The coefficient matrices and vectors in these equations are given analytically below:

\[
[A] = \begin{bmatrix}
\frac{6EI_2}{h_2} & \frac{6EI_1}{h_1} & 0 \\
-\frac{6EI_2}{h_2} & \frac{6EI_3}{h_3} & \frac{6EI_2}{h_2} \\
0 & -\frac{6EI_3}{h_3} & -\frac{6EI_3}{h_3}
\end{bmatrix}
\]

\[
a = \begin{bmatrix}
-\frac{6EI_1}{h_1} \\
0 \\
0
\end{bmatrix}
\]

\[
[K] = \begin{bmatrix}
\frac{12EI_1}{h_1^3} & -\frac{12EI_2}{h_2^3} & 0 \\
0 & \frac{12EI_2}{h_2^3} & -\frac{12EI_3}{h_3^3} \\
0 & 0 & \frac{12EI_3}{h_3^3}
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
0 & 0 & 0 \\
\frac{6EI_1}{h_1^2} & \frac{6EI_2}{h_2^2} & 0 \\
0 & \frac{6EI_2}{h_2^2} & \frac{6EI_3}{h_3^2} \\
0 & 0 & \frac{6EI_3}{h_3^2}
\end{bmatrix}
\]

Matrices [D] and [E] are the same as in Appendix A.1. The external force \( F_j^k \) at floor level \( j \) at the \( k \)-th step can be expressed as

\[
F_j^k = F_j^{k-1} \pm \Delta F_j
\]

where \( \Delta F_j \) is the lateral force increment at floor level \( j \). The positive sign indicates loading and the negative unloading (or loading in the opposite direction).
A.4 Formulation for Random Vibration Analysis

Eqs. 5.1, 5.2 and 5.4 are the governing equations of the SCWB model for random vibration analysis via the method of statistical equivalent linearization. For the three story example considered here, the coefficient matrices and vectors in these equations are

\[
[A] = \begin{bmatrix}
\frac{6EI_1}{m_1h_1^2} & \frac{6EI_2}{m_1h_1^2} & \frac{6EI_1}{m_1h_1^3} & 0 \\
\frac{6EI_1}{m_1h_1^2} & \frac{6EI_2}{m_1h_1^2} + \frac{6EI_1}{m_1h_1^3} & \frac{6EI_2}{m_1h_2^2} & \frac{6EI_3}{m_1h_2^2} \\
0 & \frac{6EI_2}{m_2h_2^2} & \frac{6EI_3}{m_2h_2^2} & \frac{6EI_3}{m_3h_2^2} \\
\end{bmatrix}
\]

\[
[D] = \begin{bmatrix}
(1-a_0)G_0 & 0 & 0 & 0 \\
0 & (1-a_1)G_1 & 0 & 0 \\
0 & 0 & (1-a_2)G_2 & 0 \\
0 & 0 & 0 & (1-a_3)G_3 \\
\end{bmatrix}
\]

\[
[E] = \begin{bmatrix}
\frac{6EI_1}{h_1^2} & 0 & 0 \\
\frac{6EI_1}{h_1^2} & \frac{6EI_2}{h_2^2} & 0 \\
0 & \frac{6EI_2}{h_2^2} & \frac{6EI_3}{h_3^2} \\
0 & 0 & \frac{6EI_3}{h_3^2} \\
\end{bmatrix}
\]

\[
[K] = \begin{bmatrix}
\frac{12EI_1}{m_1h_1^3} & -\frac{12EI_2}{m_1h_1^2} & 0 \\
-\frac{12EI_1}{m_1h_1^3} & \frac{12EI_2}{m_1h_1^2} + \frac{12EI_2}{m_2h_2^2} & -\frac{12EI_3}{m_2h_2^2} \\
0 & -\frac{12EI_2}{m_2h_2^2} & \frac{12EI_3}{m_2h_2^2} + \frac{12EI_3}{m_3h_3^2} \\
\end{bmatrix}
\]

\[
e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]
A.5 Equivalent Linearized Coefficients

One can carry out the differentiation in Eqs. 5.5 and 5.6 to obtain the following expressions for the j-th story equivalent linear coefficients $C_{eq}^j$ and $K_{eq}^j$:

$$ C_{eq}^j = A_j - \beta_j E \left[ \frac{\partial |\dot{\theta}_j|}{\partial J} |Y_j| Y_j \right] - \gamma_j E[Y_j^2] \quad (A.5.1) $$

$$ K_{eq}^j = -\beta_j E \left[ \frac{\partial |Y_j| Y_j}{\partial Y_j} |\dot{\theta}_j| \right] - 2\gamma_j E[Y_j \dot{\theta}_j] \quad (A.5.2) $$

where

$$ E \left[ \frac{\partial |\dot{\theta}_j|}{\partial J} |Y_j| Y_j \right] = \sigma_{Y_j}^2 \left( 1 + \frac{\sin \phi_j}{\pi} - \frac{\phi_j}{\pi} \right) \quad (A.5.3) $$

$$ E \left[ \frac{\partial |Y_j| Y_j}{\partial Y_j} |\dot{\theta}_j| \right] = \sigma_{\dot{\theta}_j} \sigma_{Y_j} \left[ \frac{4 \left( 1 - \phi_j^2 \right)^{3/2}}{\pi} + 2 \phi_j \dot{\theta}_j Y_j \left( 1 + \frac{\sin \phi_j}{\pi} - \frac{\phi_j}{\pi} \right) \right] \quad (A.5.4) $$
\[ \phi_j = 2 \arctan \left( \frac{\sqrt{1 - \theta_j^2 Y_j}}{\vartheta Y_j} \right) \]  \hspace{1cm} (A.5.5)

\[ \vartheta \theta_j Y_j = \frac{E[Y_j]}{\sigma_j \sigma Y_j} \]  \hspace{1cm} (A.5.6)

and \( \sigma_{Y_j}^2 = E[Y_j^2] \), \( \sigma_{\theta_j}^2 = E[\theta_j^2] \).
APPENDIX B

SCWB MODEL: NONLINEAR PARAMETER ESTIMATION

B.1 Integral Increments $\Delta I_{1ji}$ and $\Delta I_{2ji}$

The identification procedure is identical for all story levels, since the residual is always given by equation 4.35. Using a finite difference approximation, the integral increments in the residual $f_{ji}$ of level $j$ at time $t_i$ can be evaluated:

$$
\Delta I_{1ji} = \int_{t_i}^{t_{i+1}} \theta_j \dot{Y}_j \, dx = \frac{1}{2} \left[ Y_{j(i-1)} Y_{j(i-1)} \left( |\theta_{j(i-1)}| \Delta t_i \right) + Y_{ji} Y_{ji} \left( |\theta_{ji}| \Delta t_i \right) \right]
$$

$$
\Delta I_{2ji} = \int_{t_i}^{t_{i+1}} \theta_j Y_j^2 \, dx = \frac{1}{2} \left[ Y_{j(i-1)}^2 \left( \theta_{j(i-1)} \Delta t_i \right) + Y_{ji}^2 \left( \theta_{ji} \Delta t_i \right) \right]
$$

In the above formulas, $Y_j = Y_j(\tau)$, given by equation 4.34 and $\dot{\theta}_j = \dot{\theta}_j(\tau)$. The first assumption of the quasi-static experiment is now used to evaluate the terms $\dot{\theta}_{j(i-1)} \Delta t_i$ and $\dot{\theta}_{ji} \Delta t_i$. According to this assumption translational velocities at all levels are constant during the test, i.e., $|\Delta u_{ji}| / \Delta t_i$ is constant. It is obvious that since $|\Delta u_{ji}| = |u_{ji} - u_{j(i-1)}|$ does not remain constant at each step, $\Delta t_i$ will be changing in the same rate that $|\Delta u_{ji}|$ changes.

Refer now to Fig. B.1. Using a finite difference approximation, rotational velocities at times $t_{i-1.5}$, $t_{i-0.5}$ and $t_{i+0.5}$ are calculated

$$
\dot{\theta}_{j(i-1.5)} = \frac{\theta_{j(i-1)} - \theta_{j(i-2)}}{\Delta t_{i-1}} ; \quad \dot{\theta}_{j(i-0.5)} = \frac{\theta_{ji} - \theta_{j(i-1)}}{\Delta t_i} ; \quad \dot{\theta}_{j(i+0.5)} = \frac{\theta_{j(i+1)} - \theta_{ji}}{\Delta t_{i+1}}
$$

and rotational velocities at times $t_{i-1}$ and $t_i$ are expressed as

$$
\theta_{j(i-1)} = \dot{\theta}_{j(i-0.5)} + \frac{\left( \dot{\theta}_{j(i+0.5)} - \dot{\theta}_{j(i-0.5)} \right) \Delta t_i}{\Delta t_i + \Delta t_{i+1}} ; \quad \theta_{j(i-1)} = \dot{\theta}_{j(i-1.5)} + \frac{\left( \dot{\theta}_{j(i+1.5)} - \dot{\theta}_{j(i-1.5)} \right) \Delta t_{i+1}}{\Delta t_i + \Delta t_{i-1}}
$$
and increments $\dot{\theta}_{j(i-1)} \Delta t_i$ and $\dot{\theta}_{ji} \Delta t_i$ are given by

\[
\dot{\theta}_{j(i-1)} \Delta t_i = \theta_{j(i-1)} - \theta_{j(i-1)} + \left( \theta_{j(i+1)} - \theta_{ji} \right) r_1 r_2 - \left( \theta_{ji} - \theta_{ji(i-1)} \right) r_1
\]

\[
\dot{\theta}_{ji} \Delta t_i = \left( \theta_{j(i-1)} - \theta_{ji(i-1)} \right) r_3 + \left( \theta_{ji} - \theta_{ji(i-1)} \right) r_4 - \left( \theta_{ji(i-1)} - \theta_{ji(i-2)} \right) r_3 r_4
\]

where

\[
r_1 = \frac{\Delta t_i}{\Delta t_{i+1}} = \frac{|u_{ji} - u_{j(i-1)}|}{|u_{j(i+1)} - u_{ji}|} ; \quad r_2 = \frac{\Delta t_i}{\Delta t_{i+1}} = \frac{|u_{ji} - u_{j(i-1)}|}{|u_{j(i-1)} - u_{ji}| + |u_{j(i+1)} - u_{ji}|}
\]

\[
r_3 = \frac{\Delta t_i}{\Delta t_{i-1}} = \frac{|u_{ji} - u_{j(i-1)}|}{|u_{j(i-1)} - u_{ji}|} \quad \text{and} \quad r_4 = \frac{\Delta t_{i-1}}{\Delta t_i + \Delta t_{i-1}} = \frac{|u_{j(i-1)} - u_{j(i-2)}|}{|u_{j(i-1)} - u_{ji}| + |u_{j(i-1)} - u_{j(i-2)}|}
\]

**B.2 Partial Derivatives of the Residual**

The partial derivatives of the residual $f_{ji}$ of level $j$ at time $t_i$ with respect to the unknown parameters $a_j, A_j, \beta_j, \gamma_j$ are given below:

\[
\frac{\partial f_{ji}}{\partial A_j} = (1 - a_j) G_j \Delta \theta_{ji}
\]

\[
\frac{\partial f_{ji}}{\partial \beta_j} = -(1 - a_j) G_j \Delta I_{1ji}
\]

\[
\frac{\partial f_{ji}}{\partial \gamma_j} = -(1 - a_j) G_j \Delta I_{2ji}
\]

\[
\frac{\partial f_{ji}}{\partial a_j} = G_j \Delta \theta_{ji} - G_j \Delta Y_{ji} - (1 - a_j) G_j \left( \beta_j \frac{\partial \Delta I_{1ji}}{\partial a_j} + \gamma_j \frac{\partial \Delta I_{2ji}}{\partial a_j} \right)
\]

where

\[
\frac{\partial \Delta I_{1ji}}{\partial a_j} = Y_{j(i-1)} \frac{\partial \theta_{j(i-1)} \Delta t_i}{\partial a_j} + Y_{ji} \frac{\partial \theta_{ji} \Delta t_{ji}}{\partial a_j}
\]

\[
\frac{\partial \Delta I_{2ji}}{\partial a_j} = Y_{j(i-1)} \frac{\partial Y_{j(i-1)} \theta_{ji} \Delta t_{ji}}{\partial a_j} + Y_{ji} \frac{\partial Y_{ji} \theta_{ji} \Delta t_{ji}}{\partial a_j}
\]
and

$$\frac{\partial Y_{ji}}{\partial \alpha_j} = \frac{Y_{ji} - \theta_{ji}}{1 - \alpha_j}$$

Figure B.1 Evaluation of $\dot{\theta}_{ji} \Delta t_i$ and $\dot{\theta}_{ji} \Delta t_i$
APPENDIX C
SYSTEM IDENTIFICATION

C.1 General

In system identification a mathematical model is developed which can reproduce the behavior of an actual system. This model is represented by a set of equations containing parameters that need to be identified using the criterion that the behavior of the original system is approximated with minimum error. To identify the model parameters, the behavior of the real system needs to be recorded. An objective function $F(\mathbf{x}, \mathbf{y})$ can then be defined in terms of $m$ recorded response quantities, $\mathbf{y}^T = [y_1, y_2, \ldots, y_m]$, and $n$ unknown system parameters, $\mathbf{x}^T = [x_1, x_2, \ldots, x_n]$, as a measure of the absolute error between the original system’s and the model’s behavior. This function is minimized in terms of the unknown parameters in a global sense and the optimal values $\mathbf{x}_1^*, \mathbf{x}_2^*, \ldots, \mathbf{x}_n^*$ of the system parameters are identified. Optimization algorithms generate a sequence of points $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_k, \ldots$ in the $n$-dimensional parameter space such that

$$F(\mathbf{x}_0) > F(\mathbf{x}_1) > \ldots > F(\mathbf{x}_k) > \ldots$$

Any method that generates points that satisfy the previous inequality is called a descent method. Once a point $\mathbf{x}_k$ is chosen, two decisions need to be made before the next point can be generated: (i) a direction must be selected, along which the next point is to be chosen, and (ii) a step size must be used along the chosen direction. Any descent method generates $\mathbf{x}_{k+1}$ by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + s_k \mathbf{d}_k$$

where $\mathbf{d}_k$ is the normalized direction vector and $s_k$ is the positive step size. In constrained
optimization, both $x^k$ and $x^{k+1}$ must be in the feasible region which is defined by the problem constraints.

One should recall that the necessary condition for a function $F(x)$ to be a minimum at $x^*$ is that $\nabla F(x^*) = 0$, when the gradient vector $\nabla F(x)$ exists. Therefore the minimization of $F(x)$ can be viewed as finding the roots $x^*$, of $\nabla F(x) = 0$. To accomplish that, the Newton-Raphson method may be used, which approaches the solution using the iterative formula:

$$x^{i+1} = x^i - [H]^{-1} q$$

In this formula, $x^i$, $x^{i+1}$ are the $i$-th and $i$-th + 1 step estimates of the solution vector, $q = \nabla f(x)$ evaluated at $x^i$, and $[H]$ is the Hessian matrix evaluated at $x^i$. The Hessian matrix, which contains the second derivatives of $F(X)$ with respect to $x$ is defined as:

$$H_{ij} = \frac{\partial^2 f(x)}{\partial x_i \partial x_j}; \quad i,j = 1,\ldots,n$$

The Newton-Raphson method will diverge if the initial guesses for $x$ are not "sufficiently" close to the solution (the Hessian matrix must be positive definite for convergence). Also, the method might converge to a local minimum since $\nabla F(x^*) = 0$ is satisfied there as well. This could be avoided if a variable step size is used. Finally notice that in the Newton-Raphson method the evaluation of the second derivatives of the objective function is necessary in order to obtain the Hessian matrix.

The method described below, originally applied to least squares problems (Gauss, 1809) eliminates the drawbacks of the Newton-Raphson algorithm. The Hessian matrix is replaced by a first order approximation $[N]$ and a step size $s$ along the chosen direction is used to improve the efficiency of the algorithm. Directional discrimination insures that the inverse
of \([N]\) is positive definite. A scaled spectral decomposition of \([N]\) is performed to avoid possible ill-conditioning. Finally, a projection method accounts for linear constraints in model parameters.

C.2 The Gauss Method

In many nonlinear parameter estimation problems, the objective function can not be defined directly in terms of the system parameters. Instead, it depends explicitly on the model equations which contain the parameters. To compute the derivatives of the objective function, one must first differentiate with respect to the model equations and then differentiate those with respect to the parameters. If the model is complex and the derivatives are not continuous, differentiating, even numerically, may involve excessive computational difficulties. The Gauss method simplifies somewhat the procedure by replacing the Hessian matrix \([H]\) with a first order approximation \([N]\) which depends only on first derivatives of the objective function with respect to the parameters.

In essence, the method replaces the model equations by their tangents; that is, the original nonlinear equations with respect to \(x\) are now replaced by linear ones. For this linear problem, \([N]\) is the exact Hessian matrix and according to Newton–Raphson method, the solution is given by

\[
x = x^{i} - [N]^{-1} q
\]

where \(q\) is the gradient vector containing the first derivatives of the objective function with respect to the parameters. However, this is not the solution to the nonlinear problem. Yet, if one accepts \(x^{i+1} = x\), the Gauss method may be regarded as solving a sequence of linear problems. To improve the efficiency of the method and possibly avoid convergence to local minima, a step size \(s\) is added to the algorithm. An outline of the method is given below.
Let the objective function (also called sometimes error function) be

\[ F(\mathbf{x}) = \sum_{k=1}^{np} \left[ f_k(\mathbf{y}, \mathbf{x}) \right]^2 \]

where \( \mathbf{x} \): vector containing the \( n \) unknown parameters
\( \mathbf{y} \): vector containing the \( m \) measured quantities
\( f_k \): residual function corresponding to the \( k \)th measurement
\( np \): total number of available measurements

The gradient vector is given by

\[ q_i = \frac{\partial F}{\partial x_i} = 2 \sum_{k=1}^{np} f_k \frac{\partial f_k}{\partial x_i} ; \quad i = 1, 2, 3, \ldots, n \]  \hspace{1cm} \text{(C.2.1)}

The Hessian matrix is given by

\[ H_{ij} = \frac{\partial^2 F}{\partial x_i \partial x_j} = 2 \sum_{k=1}^{np} f_k \frac{\partial^2 f_k}{\partial x_i \partial x_j} + 2 \sum_{k=1}^{np} \frac{\partial f_k}{\partial x_i} \frac{\partial f_k}{\partial x_j} ; \quad i, j = 1, 2, 3, \ldots, n \]  \hspace{1cm} \text{(C.2.2)}

In order to find a first order approximation to \( [H] \), the first term in C.2.2 is neglected and the approximation is defined as

\[ N_{ij} = 2 \sum_{k=1}^{np} \left( \frac{\partial f_k}{\partial x_i} \right) \left( \frac{\partial f_k}{\partial x_j} \right) ; \quad i, j = 1, 2, 3, \ldots, n \]  \hspace{1cm} \text{(C.2.3)}

Note that the term neglected contains the residual \( f_k \) as a factor. Since the residuals near the minimum are generally small, this provides some justification for regarding \( [N] \) as a good approximation of \( [H] \). Let \( [R] = [N]^{-1} \). Then

\[ \mathbf{x}^{k+1} = \mathbf{x}^k - s_k [R]^{(k)} q^{(k)} \]  \hspace{1cm} \text{(C.2.4)}

where \( q^{(k)} \) is the gradient vector evaluated at \( \mathbf{x}^k \), \( [R]^{(k)} \) is the inverse of the Hessian approximation evaluated at \( \mathbf{x}^k \) and \( s_k \) is the \( k \)-th iteration step size, calculated as follows:
Assuming that an acceptable direction along the tangent of the objective function \( F(\mathbf{x}) \) has been chosen, there always exists a number \( \eta \), such that if \( 0 < s < \eta \), then \( \Phi(s) \equiv F(\mathbf{x}^k - s \ [R]^{(k)} q^{(k)}) < F(\mathbf{x}^k, y) \). An interpolation–extrapolation algorithm, developed by Bard (1974) is used for this purpose. The basic idea behind interpolation is that if the initial guess for \( s^{(0)} \) is such that \( \Phi(s^{(0)}) \geq F(\mathbf{x}^k, y) \), a smaller value of \( s \) is tried and so on until an acceptable value is found. The idea behind extrapolation is that if the initial choice is acceptable, it is worth trying at least one more guess to improve the result if possible. In both cases, the new trial value of \( s \) is chosen to minimize a quadratic approximation of \( F(\mathbf{x}^k, y) \). A complete flowchart of the interpolation–extrapolation scheme is given by Bard (1974).

C.3 Scaled and Inverse Scaled Spectral Decomposition

An inverse scaled spectral decomposition is used to calculate the inverse matrix of the Hessian approximation \([N]\) in the Gauss method. Possible ill-conditioning (large condition number) in \([N]\) can be cured by scaling the matrix properly before computing its eigenvalues and eigenvectors. The simplest scaling method is to reduce all diagonal elements to unit magnitude. This method is described below.

Given a matrix \([A]\), a diagonal matrix \([B]\) may be defined as

\[
B_{ii} = \begin{cases} 
|A_{ii}|^{1/2} & A_{ii} \neq 0 \\
1 & A_{ii} = 0
\end{cases} \quad (C.3.1)
\]

The scaled version of \([A]\) is then defined as

\[
[C] = [B]^{-1}[A][B]^{-1} \quad (C.3.2)
\]

1. Condition number of a matrix is the ratio of the largest to smallest (in absolute order) eigenvalues.
with elements \( C_{ij} = A_{ij} / \sqrt{|A_{ii}A_{jj}|} \) (except when \( A_{ii} \) or \( A_{jj} = 0 \)) and, in particular, \( C_{ii} = 1 \). Let the spectral decomposition of \( [C] \) be

\[
[C] = [U][\Pi][U]^T
\]

where \( [\Pi] \) is diagonal (\( \Pi_{ii} = \pi_i \) is the \( i \)-th eigenvalue of \( [C] \)) and \( [U] \) is an orthogonal matrix whose \( i \)-th column contains the \( i \)-th eigenvector of \( [C] \). Eqs. C.3.2 and C.3.3 may be combined to yield

\[
[A] = [B][U][\Pi][U]^T[B] = [F][\Pi][F]^T
\]

where \( [F] = [B][U] \). The relation \( [A] = [F][\Pi][F]^T \) is the scaled spectral decomposition of \( [A] \). Inverting both sides of Eq. C.3.4,

\[
\]

where \( [G] = [B]^{-1}[U] \). The relation \( [A]^{-1} = [G][\Pi]^{-1}[G]^T \) is called the inverse scaled spectral decomposition of \( [A] \). Evaluating the inverse of a matrix \( [A] \) in such a way, provides insight to the nature of the matrix. This allows one to generate "almost inverses" of \( [A] \), which may have certain desirable properties (e.g. positive definiteness).

### C.4 Directional Discrimination

Directional discrimination is used to evaluate a positive definite matrix \( [R] \) which is in some sense close to the inverse of \( [N] \). Positive definiteness of \( [R] \) is necessary for \( d = -[R]q \) to be an acceptable direction. Furthermore, a reasonable \( d \) should be obtained even if \( [N] \) is singular or nearly so. The idea behind the method is to compute the various components of \( d \) in a suitably chosen coordinate system. Generally, it is worthwhile to transform the coordinates so as to eliminate "interaction" among the parameters, i.e., so as to obtain a diagonal \( [N] \). In such a coordinate system, the effect of varying one component of \( d \) is approximately independent of any other term in \( d \). To obtain a suitable
transformation of coordinates, the inverse scaled decomposition of \([N]\) (Eq. C.3.5) is used:

\[
[N] = ([G]^{T})^{-1}[\Pi][G]^{-1}
\]  

(C.4.1)

where \([\Pi]\) is the diagonal matrix containing the eigenvalues of \([N]\), and \([G]\) is defined in Appendix C.3. The relation \(q = -[N]d\) can therefore be written as

\[
[\Pi][G]^{-1}d = -[G]^{T}q
\]  

(C.4.2)

Let \(\bar{x} = -[G]^{-1}x\) and \(\bar{d} = -[G]^{-1}d\). Then, following Eq. C.2.1,

\[
\bar{q}_i = \frac{\partial F}{\partial \bar{x}_i} = \sum_{k=1}^{n} \frac{\partial F}{\partial x_k} \frac{\partial x_k}{\partial \bar{x}_i} \quad ; \quad i = 1, 2, 3, ..., n
\]  

(C.4.3)

where \(n\) is the number of unknown parameters. But since \(x = -[G]\bar{x}\),

\[
\frac{\partial x_k}{\partial \bar{x}_i} = G_{ki}
\]  

(C.4.4)

and

\[
\bar{q}_i = \sum_{k=1}^{n} \frac{\partial F}{\partial x_k} G_{ki}
\]  

(C.4.5)

or in matrix form, \(\bar{q} = [G]^{T}q\). Hence, Eq. C.4.2 can be written in the \(\bar{x}\) coordinate system as

\[
[\Pi] \bar{d} = -\bar{q}
\]  

(C.4.6)

or, since \([\Pi]\) is diagonal with \(\Pi_{ii} = \pi_i\), \(\pi_i \bar{d}_i = -\bar{q}_i\), \(i = 1, ..., n\). Solving now for \(\bar{d}_i\),

\[
\bar{d}_i = -\gamma_i \bar{q}_i \quad ; \quad \gamma_i = \pi_i^{-1}
\]  

(C.4.7)

Directional discrimination can now be applied to some of the components of the direction vector in the transformed coordinates. First, to guarantee an acceptable step, all \(\gamma_i\) must be positive. To achieve that, negative eigenvalues are replaced by their absolute values, i.e., \(\gamma_i = |\pi_i|^{-1}\). The problem of nearly zero eigenvalues still remains (in numerical computations, an eigenvalue is almost never exactly zero). If a \(\pi_i\) is very small, i.e., if
$|\pi_i| \ll \max_k |\pi_k|$, then the following strategy (Neutral Method) is recommended

$$\gamma_i = \begin{cases} \max \left[ |\pi_i|^{-1}, \delta \right] & \text{if } |\pi_i| > \epsilon \\ \beta & \text{if } |\pi_i| \leq \epsilon \end{cases} \quad (C.4.8)$$

where $\beta, \delta, \epsilon$ are constants (Bard, 1974).

Let $[\Gamma]$ be the diagonal matrix with $\Gamma_{ii} = \gamma_i$. Then from Eq. C.4.6,

$$\overrightarrow{d} = -[\Gamma] \overrightarrow{q} \quad (C.4.9)$$

Replacing $\overrightarrow{d}$ and $\overrightarrow{q}$ using their definitions and premultiplying by $[G]$,

$$d = -[G][\Gamma][G]^T \ q \quad (C.4.10)$$

and $[R] \equiv [G][\Gamma][G]^T$ is the required positive definite, "almost inverse" of $[N]$.

### C.5 Projection Methods

Projection methods are incorporated in the optimization scheme to take into account the effect of linear constraints. The basic idea behind projection methods is the following: At each iteration, the "normal step" is computed using the Gauss method with the constraints ignored. If $x^k$ is located at the interior of the feasible region, apply the "normal step". If this results in an infeasible point, truncate the step so that $x^{k+1}$ lies on the boundary of the feasible region. If $x^k$ is already on the boundary, take the "normal step" or a fraction of it if this is feasible. Otherwise, treat some of the active constraints as equality constraints and take a step along these constraints. A quadratic programming technique is used to select the the constraint along which the next step is taken. The method is described in detail by Bard (1974).
C.6 Optimization Algorithm

An summary of the optimization algorithm implemented in this study is presented here. Assume that a point $\mathbf{x}^k$ is already generated in the feasible region. The next point $\mathbf{x}^{k+1}$ can be generated as follows:

At the starting point $\mathbf{x}^k$:

- assemble the objective function at $\mathbf{x}^k$
- evaluate the gradient vector
- assemble the approximate Hessian matrix $[N]$ of the Gauss method
- calculate the scaled version of $[N]$
- obtain the scaled spectral decomposition of $[N]$ and the inverse scaled spectral decomposition of $[N]$
- using directional discrimination, obtain $[R]$ in transformed coordinates
- evaluate the gradient vector in transformed coordinates
- evaluate acceptable direction in transformed coordinates
- transform acceptable direction to original coordinates
- check if $\mathbf{x}^k$ is in the feasible region or on the boundary
  - if $\mathbf{x}^k$ is on the boundary use a projection method to evaluate the acceptable direction
  - evaluate the maximum and minimum step size
  - use interpolation–extrapolation to find the optimal step size and check for convergence.
- evaluate $\mathbf{x}^{k+1}$ using Eq. C.2.4.
APPENDIX D
DETAILS OF DRAIN-2DX COMPUTER PROGRAM

D.1 Program Capabilities

DRAIN-2DX is a finite element code developed at the University of California, Berkeley (Allahabadi and Powell, 1988) for static and dynamic analysis of inelastic plane structures. In this study, DRAIN-2DX is used for inelastic time-history analysis of moment resisting steel frames. Three types of elements are used: column elements, beam elements and semi-rigid connection elements. Masses are considered lumped at the nodes (degrees of freedom) and a Rayleigh damping formulation is used for the damping matrix. Mass proportional damping is modeled by dampers at the nodes, whereas stiffness proportional damping is modeled by dampers parallel to the elements. The stiffness matrix is assembled from the material and geometric (P-Δ) stiffness contribution of each element. The stiffness matrix is formulated and triangularized at the beginning of an analysis. When a change in stiffness occurs between two time steps of an inelastic time-history analysis, the stiffness matrix is reformulated and then triangularized. Loading can be specified according to element loads, static nodal loads, ground acceleration records, acceleration response spectra, initial velocities, ground displacement records or dynamic force records. Results include selected nodal displacements, element section forces, mode shapes, frequencies, and those based on response spectrum analysis. DRAIN-2DX can also perform energy calculations for both static and dynamic analysis, evaluating the external work at the nodes, elastic-plastic work on the elements, kinetic energy and damping work.

D.2 Column Element

The three degrees of deformation of a column element are axial extension and flexural rotation at both ends. Shear deformations are optional; they can be taken into account if
they are important. Yielding may take place only in concentrated plastic hinges located at element ends. Specifying an end eccentricity, though, can translate the location of a plastic hinge along the centerline of the element. In steel frame analysis, vertical eccentricities at the column connections and horizontal eccentricities at the beam connections are specified to move the plastic hinge locations to the faces of the joint (edges of panel zone). DRAIN–2DX interprets an end eccentricity as a rigid and infinitely strong link between the node and the plastic hinge location within the element.

The moment–rotation relationship at the element ends is represented by a bilinear curve. Strain hardening is approximated by elastic and inelastic components acting in parallel. The hinges in the inelastic component yield under constant moment, but the moment in the elastic component may continue to increase. Yield moments and axial forces need to be specified at both element ends for both positive and negative bending. The yield moment of the elasto–plastic component is governed by an interaction surface between axial force and bending moment acting on the element. The shape of this surface for steel columns is given in Fig. D.1. In this figure, A is the cross-sectional area, f_y is the yield stress, and Z is the plastic section modulus of the column.

D.3 Beam Element

This element is used to model beams in steel frames. It is essentially the same as the column element and only the yield surface is different. Since the axial force in a beam is not important, the interaction between moment and axial force can be ignored and the yield surface is specified simply by the yield moment, \( M_y^\pm = \pm Z f_y \), where Z is the plastic section modulus and f_y is the yield stress of the beam (Fig. D.2).
D.4 Semi-Rigid Connection Element

Connection elements are used to represent deformable connections in steel building frames. A semi-rigid connection element is essentially a rotational spring connected to two nodes and influenced only by the relative rotational displacement between these nodes. In this study, such an element is used at each frame joint to account for the deformation in the panel zone. The vertical and horizontal translations of the element's two nodes are constrained to be identical, so that the columns and beams incident to the joint move together. As a result, one vertical, one horizontal and two rotational degrees of freedom exist at each joint. The moment-rotation inelastic relationship is represented by a bilinear curve. As in the previous elements, strain hardening is approximated by elastic and inelastic components acting in parallel. The connection element ignores the actual physical dimensions of the joint.

![Figure D.1](image_url)  
*Figure D.1  Column element yield interaction surface*
Figure D.2  Beam element yield interaction surface

\[ M_y^- = -Zf_y \]

\[ M_y^+ = Zf_y \]
APPENDIX E
PROBABILITY OF CHARACTERISTIC EARTHQUAKE OCCURRENCES

E.1 Probability of No Occurrence

The recurrence time \( T \) of characteristic events follows a lognormal distribution with a cumulative probability function

\[
F_T(t) = \Phi \left[ \frac{\ln(t) - \lambda_T}{\xi_T} \right]
\]

(E.1.1)

where \( \Phi[.\] is the standard normal distribution function. Parameters \( \xi_T \) and \( \lambda_T \) can be calculated using Eq. 2.38 if \( \mu_T \) and \( \delta_T \) are already known. Assuming that the last event occurred \( T_0 \) years from today, the probability of no occurrence during the next \( t \) years may be obtained as

\[
P(N_{ch} = 0) = P(T \geq T_0 + t | T \geq T_0) = \frac{P(T \geq T_0 + t)}{P(T \geq T_0)}
\]

\[
1 - \Phi \left[ \frac{\ln(T_0 + t) - \lambda_T}{\xi_T} \right]
\]

\[
\therefore P(N_{ch} = 0) = \frac{1 - \Phi \left[ \frac{\ln(T_0) - \lambda_T}{\xi_T} \right]}{1 - \Phi \left[ \frac{\ln T_0 - \lambda_T}{\xi_T} \right]}
\]

(E.1.2)

The complement of this probability, is the probability of at least one occurrence during \( t \) (see also Eq. 2.37). The evaluation of the probability of occurrence of a specific number of events in time \( t \), treated in the following, requires greater computational effort.

E.2 Probability of \( k \) Occurrences, \( k = 1, 2, \ldots \)

Referring to Fig. E.1, let \( T_0 \) denote the time since the last characteristic event, \( T_1 \) the time from the previous to the next event, \( T_2 \) the time interval between the next two events
and so on. The probability of one occurrence in t years from now can then be written as

\[ P(N_{ch} = 1) = P(T_1 \leq T_0 + t < T_1 + T_2 | T_1 \geq T_0) = \]

\[
= \frac{\int_{T_0}^{T_0+t} \left[ 1 - F_{T_1}(T_0 + t - t_1) \right] f_{T_1}(t_1) \, dt_1}{1 - \Phi \left( \frac{\ln T_0 - \lambda T_1}{\xi T_1} \right)}
\]

(E.1.3)

Similarly, the probability of two occurrences in t years from now can then be written as

\[ P(N_{ch} = 2) = P(T_1 + T_2 \leq T_0 + t < T_1 + T_2 + T_3 | T_1 \geq T_0) = \]

\[
= \frac{\int_{T_0}^{T_0+t} \left\{ \int_{T_0}^{T_0+t-T_1} \left[ 1 - F_{T_2}(T_0 + t - t_1 - t_2) \right] f_{T_2}(t_2) \, dt_2 \right\} f_{T_1}(t_1) \, dt_1}{1 - \Phi \left( \frac{\ln T_0 - \lambda T_1}{\xi T_1} \right)}
\]

(E.1.4)

and so on for three occurrences and more. Notice that the dimension of integrations increases along with the number of occurrences considered, making the calculations lengthy. However, the probabilities of more than one or two occurrences are normally very small and can therefore be neglected in reliability calculations.

![Figure E.1 Time intervals to future characteristic earthquakes](image)
LIST OF REFERENCES


