NEURAL NETWORKS AND FUZZY LOGIC FOR STRUCTURAL CONTROL

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THESIS
Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering in the Graduate College of the University of Illinois at Urbana–Champaign, 1994

Urbana, Illinois
WE HEREBY RECOMMEND THAT THE THESIS BY

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ENTITLED  NEURAL NETWORKS AND FUZZY LOGIC

FOR STRUCTURAL CONTROL

BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF  DOCTOR OF PHILOSOPHY IN CIVIL ENGINEERING

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† Required for doctor's degree but not for master's.
A new method for the active control of structures is proposed in this study. This method is based on the use of learning capability and adaptivity of neural networks and the high degree of flexibility and adjustability acquired through utilization of fuzzy logic. This method is classified as a “learning control method” to emphasize on the important role of the learning capabilities of the controller. However, it can be classified as a smart or an intelligent control method too. This method has been called the “neuro-fuzzy control method” and its corresponding controller, the “neuro-fuzzy controller”. Neuro-fuzzy controllers can theoretically cope with any nonlinearity, delay and imperfection in the controlled structure. Hence, they can be considered as general controllers for structural purposes.

In this method, a neural network called the “emulator neural network” is trained to learn to predict the response of the structure from the history of response and control signals. It learns about all the sources of nonlinearity and time delay, actuators capacity and any imperfections in the whole control system, implicitly. Then it is used in a preliminary control of the structure and the training of another neural network called the “neuro-controller”. Neuro-controller has all the required knowledge of controlling the structure. At last a supplementary fuzzy controller is constructed to improve on the performance of the “neuro-controller”. These two controllers which work in series, constitute the “neuro-fuzzy controller”.

In this study, the neuro-fuzzy control method is explained and its capabilities are numerically assessed through its application to the digital control of a three storey steel frame structure, subjected to different simulated earthquake excitations. Also for the sake
of comparison, the predictive optimal control method is used in the control of the same structure, subjected to the same excitations. Then the results of the neuro–fuzzy control and the predictive optimal control methods are compared to each other. It is shown that the neuro–fuzzy controller is able to provide better results than the predictive optimal controller.

Also, it is proposed to use as the criteria for the evaluation of capabilities of any control method, the three characteristics of adaptivity, prediction capability and simplicity of that method. It is discussed and demonstrated that the neuro–fuzzy control method satisfies these criteria better than the other proposed methods.

Neural network related issues have played important roles in the progress of this study. These issues such as improvements on the learning speed of the multi–layer feed–forward neural networks are discussed in this article too.
To My Family.
ACKNOWLEDGEMENTS

The author would like to thank his advisor, Professor J. Ghaboussi for his patient advise and support during the course of this study. Also the author’s thanks go to Professors W. R. Perkins, D. A. Pecknold and Y. K. Wen for serving in the thesis committee. The author would also like to acknowledge the friendship of his colleagues in the civil engineering department, K. Nikzad, Bijan Banan, M. Lordgoie, Parvis Banan, Eric Williamson, Soobong Shin, D. Sidarta, Fernando Fonseca and M. Zhang.
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CHAPTER 1
INTRODUCTION

Structures should be designed to withstand different loading conditions they face during their lifetime. These loads may be classified as static or dynamic. Static loads are not functions of time such as dead load of the structures. Dynamic loads, however, are functions of time. For the purpose of active control of structures, dynamic loads can be divided into two categories. The first category contains loads that do not have a prominent effect on the overall design of the structures like small changes in the dead load or very short amplitude ground motions, and also the loads that are usually present and can be characterized by functions of time with parameters that can be determined with enough accuracy, such as usual loads due to snow, wind, wave, operation of machinery and normal traffic. The second category contains the loads of highly random nature which have decisive effects on the design of the structures, such as severe earthquakes, storms, waves in rough sea and blasts.

Design of structures for static and dynamic loads of the first category is straightforward. The structure should be designed to tolerate these loads by its stiffness and internal damping. However design for the dynamic loads of the second category is more challenging. To this end, an appropriate lifetime should be selected for the structure. Also all the probable severe dynamic loadings that may be applied to the structure during its lifetime should be determined based on the available statistics. Then the structure should be designed to withstand safe, serviceable and comfortable if subjected to these unusual dynamic loadings. The problem of dealing with severe loadings to assure safety, serviceability and comfort, while satisfying economic constraints, has always been a challenging problem in the design of civil engineering structures. Specially, with the increasing trend towards more flexible and taller buildings, this problem has become more significant than ever. Recent conventional solution to this problem has been to reduce the effect of the external excitations by the installation of passive control mechanisms like isolators and dampers, and designing the structural skeleton to withstand these passively controlled loadings. The design of passive mechanisms and structural skeleton is in general a com-
bined procedure. However passive mechanisms can help the mitigation of vibrations to some extent. Hence the structure should still be overdesigned with regard to the usual loading conditions. Also in case of occurrence of an exceptional dynamic loading such as a severe earthquake or a tornado which is stronger than the design dynamic loadings, the structure may experience considerable damage.

With the advent of technology of active control of mechanical systems, and the research topics in the fields of engineering becoming more and more interdisciplinary, the idea of active control of structures has attracted significant attention since late 60's. In those years the control theory had been well developed and applied in the control of other engineering systems such as pressure and speed control systems.

Active control is based on the use of three fundamental elements: sensors, controllers and actuators. Sensors collect informations about both the response of the controlled system and the external excitations. These informations are transmitted to the controller which is the brain of the control system. Controller processes these informations, analyzes them, and determines the required controlling action with regard to the rules and criteria of control, and issues some control commands. The control commands are in the form of signals. Control signals are then sent to the actuators which enforce the control commands by the application of forces or modification of the architecture of the system, etc. As can be seen, there is more intelligence involved in the performance of active control mechanisms than the passive control mechanisms. Because the actuators use a source of energy supply, they can apply strong forces or modify the form of the system considerably. Consequently, the active control mechanisms can be very effective in the control of structures.

The main idea of active control of structures is to design the structures for the usual normal loading conditions, and then design active control mechanisms to render help to the structural systems in hazardous situations. It is obvious that in its best form, the design of both the structure and the active control mechanism should proceed interactively as parts of a unique system. Another fascinating application of the active control of structures is expected to be in the repair of the damaged structures and strengthening of the existing structures. This application is of considerable importance in practice.

Although active control mechanisms have been used in different fields of civil engineering implicitly, the first documented explicit applications go back to Freyssinet in 1960
who proposed the use of prestressed tendon control and Zetlin in 1965 who designed several tendon controlled tall buildings which were not built. Zuk (1968) brought about the concept of kinetic structures which was based on the idea of tendon control proposed by Freyssinet and Zetlin. Also in his book (1970), he discussed the subject mostly from an architectural point of view, where the structures could have flexible configurations, to cope with different situations. To this end he proposed the use of tendons, similar to human muscles which can stretch by jacks and provide resistance to external excitations or modify the shape of the structure to the desired limit. Due to the fact that the active tendon mechanism and other similar active control mechanisms are practically light, strong and can be installed in the required arrangements, the idea of using them for the sake of control of structures attracted the interest of more researchers in the field of structural design. The methods of system control have been modified to fit the structural control characteristics. Methods like pole assignment, independent modal space control, optimal control, predictive control and pulse control have been proposed, elaborated and numerically applied in the control of structures. Several control mechanisms have been designed and studied like active tendon control, active mass dampers and aerodynamic appendages (Soong 1990). Although the primary attention has been given to the control of frame structures, there are several studies on the active control of other types of structures like bridges and floating structures too (Yang and Giannopolous 1979, Sirlin et al 1986, Shinozuka et al. 1987, Yang and Vongchavalitkul 1993, and Warnitchai et al. 1993). The results of all of these studies show great potentials for the active control of structures. In all of the proposed methods, first a reduced model of the structure which is a distributed parameter system should be provided and the structural parameters should be identified with great precision. All the sources of delay and nonlinearity in the controlled system should be detected too. A mathematical model of the controlled structure and control system should be developed. A number of control criteria like optimality of a performance index or boundedness of response should be defined. Finally control rules should be found by the utilization of these mathematical models and the control criteria. These control rules should then be used in the determination of control signals during active control of the structure. The common characteristic of these methods is that they need a sound mathematical formulation of the controlled structure and all of the control mechanisms. Hence, for the matter of distinguishing these control methods from the method which is proposed in this study, the above methods can be called the “conventional control methods”, or
preferably the "formulated control methods".

The formulated control methods have been studied during the last two decades, tested and improved. However, the subject of active structural control is still in its infancy and there is plenty of room remained for the improvements and introduction of more adaptive and powerful methodologies. For example, most of the developed algorithms are suitable for linear models. However the structural systems are nonlinear in general. Hence, there has been some efforts to modify the proposed algorithms to cope with the nonlinearities. Also while the structures are distributed parameter systems, with infinite number of degrees of freedom, the controlled structures have been modelled as systems with a limited number of degrees of freedom, mostly one degree of freedom. Working on these simple models for comprehensive studies and assessment of the algorithms are acceptable but there are many practical problems associated with many degrees of freedom systems that can not be addressed by such studies. These problems like spillover effects and change in the characteristics of the system under the effect of control forces should be addressed, to make the proposed algorithms acceptable for real world applications. Simultaneous modelling of the actuators and the structure is another practically important issue that should be studied too. So far, the controlled structures have been identified separately from their controlling actuators. The actuators have been considered ideal and hence idealistic models have been studied. However actuators may introduce significant nonlinearities to the system. Another problem to be studied is the effect of time delays on the performance of the formulated control algorithms. The predictive control algorithms have been able to handle this problem to some extend, however, so far, the controlled structures have been simple and identified perfectly. The last but very important problem in formulated structural control is the validity of the mathematical model which may be used to characterize a real structure.

A limited number of experiments have been performed on experimental models by the use of a number of proposed control algorithms. The results have been promising in some cases but not acceptable in others (Soong 1990). Such experimental studies may provide foundations for future improvements on these methods. It is expected that future studies on the formulated control algorithms result in better and more powerful versions of these algorithms.
In this study, a new method for the active control of structures is proposed. This new method uses the neural networks and fuzzy logic in the construction of a powerful adaptive controller.

Neural networks are adaptive systems that can be trained to learn about the characteristics of a phenomenon. Then, they can provide answers to the questions about that phenomenon based on what they have learned about it. Neural networks are comprised of very simple processing elements like the neurons of the human's brain. However, the collection of these processing elements constitutes a complex system that can exhibit fascinating performance, most significantly the adaptivity and learning capability.

For centuries, scientists and engineers have tried to find mathematical formulations to characterize the physical phenomena from the motion of particles to the structural behavior. To this end, they collect rough data about that phenomenon by performing some experiments. Then mathematical models that can best fit the data are developed by trial and error, optimization techniques, etc. This has been the dominant trend in the construction of rules and identification of the systems, and has served the researchers with their endeavors successfully. These mathematical models have been in the form of differential, integral or other forms of equations and inequalities. For the application purposes, the mathematical models have been tried to be very limited in the number of variables and degrees of nonlinearities. Fortunately these models have been compatible with the essential needs for the development of science and technology. However, in the recent years, the subjects of research are getting more interdisciplinary, both in science and technology. All the fields of research are enjoying the benefits of improvements in the other fields. The problems to be solved are getting more complicated than before. The number of variables and parameters that should be considered in the design of systems is increasing. Finding suitable mathematical models by the use of regression and conventional analysis and synthesis techniques is not easy in general and impossible in many cases. Hence, the construction of adaptive trainable systems that can extract the characteristics of a set of rough data by themselves, is very helpful in the enhancement of knowledge. Construction of such systems has been studied under the title of intelligent systems. The subject of intelligent system design has been active from the time of advent of analogue computers. However, the major breakthrough came about with the salient work of McClelland, Rumelhart and their co-workers (1986). They put the idea of one of these intelligent systems which was called neural networks, in a clear way. They discussed the capabilities of these
systems through numerical examples and for simple applications. The neural networks soon became popular tools in all the fields of research in science and engineering and found applications in industry too. The literature on the theory and application of the neural networks and the number of direct and indirect conferences and journals on the subject has grown exponentially since 1986. Fortunately, civil engineering has been one of the research fields that has enjoyed the benefits of the advantages of the neural networks. For example Ghaboussi and his co-workers have applied these systems in the material modeling and geomechanics and have proposed its use in other areas too (Ghaboussi 1993). Although the application of neural networks in civil engineering is not yet extensive, it is attracting more attentions through time.

In this study, the neural networks and fuzzy logic have been used for the sake of control of structures. Hence this method can be called an “intelligent or smart control method” or preferably a “learning control method” which represents the use of learning capability of the neural networks better. In this method, there is no need to reduce the distributed parameter system of a structure to a many degrees of freedom model, or identify explicitly the structural parameters, sources of delay or nonlinearities in the structure and actuators. Also there is no need to find a linear model to characterize the controlled system. The neural networks can be trained to learn about all of these effects. This is significantly fascinating for the real applications. However, for the sake of simulation and numerical studies and also for the preliminary design of a controller, it is only necessary to provide a suitable model for the controlled system, no matter how nonlinear it is and how many actuators with different characteristics are involved. To show the capabilities of this method, it has been applied in the numerical control of a structure. The structure is a three storey one bay frame which has been proposed by the other researchers in the field to be a reference structure for testing different control algorithms (Soong 1990). However, the method is general enough to be applied in the control of other types of the structures like bridges, floating structures, etc. The results of this numerical study are reported in the following chapters. The results have been very successful. However there is still room to improve on this method. The new proposed method is now ready to be tested on an experimental model.

With the promising results of this new method of structural control by the neural network and fuzzy logic in structural control, the possibility of the active control of structures
has become more clear. By the inclusion of this new method to the formulated methods, the number of proposed methods for the active control of structures is now considerable. All of them have shown capabilities. The author believes that comparison between different methods is necessary for more improvements, but it does not result in the remaining of only one of them as the best control method and exclusion of the other methods. It seems that different control methods and the resulting algorithms should be applied to different structures with regard to factors such as size, degree of complexity, nonlinearity and importance of the structure. It is the job of the designer to select an appropriate control algorithm for each specific structure.

Although different control methods have been applied to the aeronautical and space structures, the subject of active control of civil engineering structures is for the time being a research topic only. It is too early to expect the active control approach be considered in the design of publicly used structures. However, as the very primary applications, it is expected that the active control approach be used in the improvement of the mitigation of sway in the existing tall buildings and in the vibration of bridges, ships and other structures, as well as high tech structure such as chimneys, offshore structures or transmission towers which have linkages to the research programs.

This research has been comprised of two main stages which are directly correlated to each other:

- Improvement of the available multi-layer feed-forward neural networks, during which the training algorithm has been dramatically improved, tested on different problems and proved to be very efficient.
- Development and improvement of the neural network and fuzzy logic based control method and its application to a typical frame structure and preparation of the foundations for the future experimental studies of the method.

These two stages have been completed as a chain and the progress in the neural network related issues has resulted in accelerating the progress in the second stage of the study.
1.1 ORGANIZATION

In the following chapters the details of this study and the relevant results are presented. Beside this chapter which is "chapter 1, Introduction", there are 8 other chapters which are divided into two parts: Part 1, "Essentials Of The Formulated Control Methods, Neural Networks And Fuzzy Logic" and Part 2, "Active Control Of Structures By Using Neural Networks And Fuzzy Logic".

**Part One**, "Essentials Of The Formulated Control Methods, Neural Networks And Fuzzy Logic" contains chapters 2-4.

In chapter 2 the fundamentals of the structural dynamics, classical control and modern control theories are covered. The control methods which are proposed for the active control of structures by the other authors are reviewed briefly.

In chapter 3 basics of the neural network theory are reviewed. The multi-layer feed-forward backpropagation neural networks which are the most widely used type of neural networks and are used in this study too, are explained. Also some of the improvements on the neural networks which have been obtained as a part of this study are discussed.

In chapter 4 the fundamentals of the fuzzy set and fuzzy logic theory are explained. Inference methods are reviewed and the application of fuzzy logic in the control of different systems are reviewed in brief.

**Part Two**, "Active Control Of Structures By Using Neural Networks And Fuzzy Logic" contains chapters 5-9.

In chapter 5 the methodology of using neural networks and fuzzy logic in structural control is explained. Main computer programs which have been developed for the sake of this study are introduced. Coupling of equations of structure and actuators dynamics is discussed and the three storey frame structure which should be used in this numerical study is introduced.

In chapter 6 the results of the first phase of this study are discussed. In this phase only neural networks have been used in the control of the frame structure.

In chapter 7 the results of the second phase of this study are discussed. This phase of study is comprised of improvements on the first phase and also use of fuzzy logic to build a complementary controller.
In chapter 8 one of the most adaptive formulated control methodologies, the “predictive optimal control method”, has been adopted for controlling the frame structure. The results are then compared to the results obtained in the previous chapters.

In chapter 9 a comprehensive discussion of the results of this study, comments on the future works and predictions about the application of active control methods in structural control are presented.

Each of these chapters are written to be self content. At the end of each chapter, a list of the references used in that chapter, and at the end of the article a bibliography is included.

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PART ONE

ESSENTIALS OF THE FORMULATED CONTROL METHODS, NEURAL NETWORKS AND FUZZY LOGIC
CHAPTER 2
FORMULATED CONTROL METHODS

In this chapter, essentials of the classical and modern control methods are discussed. These methods can be classified as the “conventional” or preferably the “formulated” control methods as used in this study, to distinguish them from the other control methods such as fuzzy control and neural network based control which are classified as the “intelligent”, “smart”, “data based”, or the “learning” control methods.

A system that collects informations about its environment (including informations about its own behavior), and makes use of these informations to improve on its own capabilities for the sake of reaching a goal, is called an “actively controlled” system, generally known as a controlled system. There are also systems, which their performance is improved by the addition of devices, or changing their designs. These systems do not collect informations directly, however the knowledge about their behavior have been considered in their design. Hence they can be called “passively controlled” systems. The third group of systems which have not been designed for a specific purpose, are called “uncontrolled” systems with respect to that specific purpose. In this article, for the abbreviation purposes, the term “control” will be used for “active control”, and “passive control” will be referred to by its complete name.

The above definition of controlled systems is broad, and covers all types of controlled systems, including the controlled structures. For example, the very well known “closed-loop” control systems can be considered as special cases of the controlled systems. A closed-loop control system, collects measurements of its own response for the determination of the future control signals. For the structural systems, these measurements are taken out of the state of the system which is comprised of displacements and velocities.

A control method is based on the use of three fundamental elements: sensors, controller and actuators. Sensors collect informations about the response of the controlled system and sometimes the external excitations too. These informations are transferred to the
controller which is the brain of the control system. Controller processes these informations, analyzes them, and determines the required controlling action with regard to the rules and criteria of control, and issues some control commands. Control commands are in the form of control signals. Control signals are then sent to the actuators which enforce the control commands by the application of forces or modification of the architecture of the system, etc. As can be seen, there is more intelligence involved in the action of active control mechanisms than the passive control mechanisms. Because the actuators use a source of energy supply, they can apply strong forces or modify the form of the system considerably. So, the active control mechanisms are supposed to be very effective in the control of structures.

Control theory has been very well developed for liner systems. The linear systems are defined as systems, the superposition principle holds for. Control theory can be classified into "classical", "modern", and "intelligent" control theories. The "classical control theory" deals with the single input/single output systems. Using this method, objective is to improve the stability of the system by increasing its damping properties. The "modern control" theory, however, can handle the problems of multi-input/multi-output systems. The "pole allocation" and the "optimal control" methods are the two main branches of the modern control theory. The pole allocation method can be considered as a generalization of the classical control approach, while the optimal control theory is based on the minimization principles. In the optimal control, objective is to minimize a "performance index" which contains information about both the cost of control and the penalty associated with the undesired response of the system. Many of the control algorithms such as "instantaneous optimal control", "independent modal space control (IMSC)", "predictive control" and "bounded state control" which have been proposed in the recent years for the control of structures, are all descendents of the pole allocation or optimal control theories (Soong 1990).

The literature on the structural control has been widely increased in the last two decades. Meirovitch (1990) provides a concise but informative review of the subject. A review of active control of civil engineering structures can be found in Soong (1990). The need for studying the active control of these structures, arises from three basic engineering criteria: safety, serviceability and economy. In the conventional design of the civil engineering structures, the structure should be designed to withstand all the external excitations which may occur during its design lifetime. This includes the huge loadings from earthquakes, heavy winds, etc., which have two prominent characteristics: they are probable and temporary. Since these loadings play decisive roles in the design of structural components, the results of a safe design is a heavy, overdesigned and expensive structure. Also, even though safety may be obtained, problems of serviceability of the structure and comfort of the residents may still remain unsolved. For example the top floors of a high rise building may exercise excessive deformations, which results in discomfort of residents. All of these factors boost the idea of using a light, economic and fast responding control system that can render both safety and comfort, whenever the structure is subjected to the huge non-permanent dynamic loadings. Active control of structures, is supposed to provide such capabilities.

Application of the control theory to civil engineering structures has been effectively considered since late 60’s. All of the theoretical and experimental studies have shown the potentials of this approach in improving the performance of the structures under severe dynamic loading conditions, like simulated gusty winds and earthquakes. On the other hand, since the civil engineering structures are used by public, proposing new methods for the design and construction of civil structures needs more research and strong evidence of reliability. Therefore it takes more time to bring these ideas from research offices and laboratories to the design offices and construction sites.

In the following sections, essentials of structural dynamics, linear system theory, both classical and modern, and a review of the application of control theory in the structural control are presented.
2.1 ESSENTIALS OF STRUCTURAL DYNAMICS

In this section the main concepts and formulations in structural dynamics which are necessary for the construction of control methods, are presented. Modelling of the structures, configuration space and state space equations, modal analysis and lumped and distributed parameter structures are discussed briefly.

2.1.1 Modelling The Structures

The first step in studying the dynamic behavior of a system, is to provide a sound mathematical model, which can explain the relationship between the inputs to that system, and their corresponding outputs. The Newtonian mechanics and the Analytical mechanics are the tools which can be used to construct such models for structural systems.

The Newtonian mechanics, which is a vectorial approach, looks at a mechanical system as a collection of small particles. The behavior of each of these particles can be modelled by the Newton's second law of motion.

The analytical mechanics, on the other hand, is a variational method. It considers a global scalar value for the mechanical system, called the “Lagrangian”, which is comprised of the kinetic and potential energies of the system. So, the Lagrangian is a function of the dynamic variables of the system. Then, the equations of motion can be obtained by the method of calculus of variations.

No matter which of the above modelling approaches are used, the result is a set of differential equations, which are considered as the governing equations of motion for the mechanical system. This set of differential equations, may be linear or nonlinear, depending on the complexity of the system. While the control theory is developed to deal primarily with linear systems, the discussions in the following sections are restricted to the structures, governed by a set of linear homogeneous differential equations with constant parameters. These kind of systems are called “time invariant” systems. Although time invariant systems do not represent the whole structural systems, majority of the structural systems fall in this category.
2.1.2 The Configuration Space

If \( n \) is the number of degrees of freedom of a structure, then the \( n \) dimensional space, each of which coordinates represents one of the degrees of freedom of the structure, is called the “configuration space”. The equations of motion in the configuration space can be written as:

\[
Mq(t) + Cq(t) + Kq(t) = B_qf(t)
\]

where:

\( (\cdot) \) = the ordinary differential of \((\cdot)\) with respect to time,

\( q(t) \) = the \( n \) dimensional vector of degrees of freedom of the system (displacements),

\( \dot{q}(t) \) = the \( n \) dimensional vector of velocities,

\( \ddot{q}(t) \) = the \( n \) dimensional vector of accelerations,

\( f(t) \) = the \( r \) dimensional force vector,

\( M \) = the \( n \times n \) mass matrix,

\( C \) = the \( n \times n \) damping matrix,

\( K \) = the \( n \times n \) stiffness matrix,

\( B_q \) = the \( n \times r \) location matrix.

So, the motion in the configuration space is characterized by a second order differential equation.

The tip of the vector \( q(t) \) traces a curve in the configuration space, which is referred to as the “dynamical path”. Once the vectors \( q(t_1) \) and \( \dot{q}(t_1) \), for some time \( t_1 \) and also the force vector \( f(t) \) for all the times \( t \geq t_1 \) are given, the dynamical path can be found for any time \( t \geq t_1 \). It is obvious that two dynamical paths corresponding to two different initial conditions may coincide with each other.
2.1.3 The State Space

Generally, it is not easy to obtain a direct solution for the second order equations of motion in the configuration space. The following computational task can reduce the order of the differential equations, while increasing the number of equations. Consider the identity equation:

\[ q(t) = q(t) \]  \hspace{1cm} (2.2)

Now, by introducing a new vector \( x(t) \), where:

\[ x(t) = [ q(t) | q(t) ]^T \]  \hspace{1cm} (2.3)

equations (2.1), (2.2) and (2.3) can be combined to form a set of first order linear differential equations in \( x(t) \);

\[ x(t) = A \ x(t) + B \ f(t) \]  \hspace{1cm} (2.4)

where \( A \) is a \( 2n \times 2n \) dimensional matrix and \( B \) is a \( 2n \times r \) dimensional matrix, defined as:

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix} \quad B = \begin{bmatrix}
0 \\
-M^{-1}B_q
\end{bmatrix}.
\]  \hspace{1cm} (2.5)

The \( 2n \) dimensional vector \( x(t) \) is called the “state vector”, due to the fact that it contains all the informations regarding the state of the system throughout the time.
The 2n dimensional "Euclidean space" of the state vector is called the "state space". The tip of the state vector traces a curve in the state space, known as the "trajectory". It is obvious that contrary to the dynamical paths, the trajectories do not coincide.

2.1.4 Modal Analysis

For the structures with large number of degrees of freedom, solving the state space equations is impractical. This is mainly because each of the differential equations is coupled to the others, through the non–diagonal matrix \( A \). By modal analysis, it is possible to decouple the differential equations of motion, and then study each of the equations independently. This can be accomplished through rotating the coordinate system of the state space by the matrix transformation methods. Another prominent advantage of modal analysis is the fact that each mode represents a frequency of vibration of the structure. In structural dynamics, mainly the modes corresponding to the low frequencies contribute to the response of the structure while the modes of higher frequency act as sources of noise to the response. Thus, it is practical to study only the lower modes of vibration, instead of the whole response of the structure.

To find the modes of vibration of a structure, characterized by equation (2.4), first the eigenvalue problem for the matrix \( A \) should be solved. The eigenvectors are the modal vectors and the eigenvalues provide us with the modal frequencies.

*The eigenvalue problem for the matrix \( A \):* If \( A \) is a \( m \times m \) matrix, then the eigenvalue problem for \( A \), is finding the set of nontrivial vector \( u_i \) and their corresponding constants \( \lambda_i \), such that the following matrix equation is satisfied:

\[
A \ u_i = \lambda_i \ u_i \quad \quad \quad \quad \quad \quad i = 1, 2, \ldots, m .
\]

(2.6)

where \( u_i \) and \( \lambda_i \) are known as eigenvectors and eigenvalues of mode number \( i \). The eigenvectors are "orthogonal" to each other. When they are normalized, they constitute a set of "orthonormal" vectors.
Solution of the above algebraic eigenvalue problem can be obtained by using a suitable computational method. These methods are all iterative. Some of the widely used eigensolution methods are "the Jacobi method", "the power method", "the inverse iteration method", "the deflation method", "the LR, QR and QL methods", and "the subspace iteration method". Methodologies for solving eigenvalue problems can be found in Meirovitch (1980).

After solving the eigenvalue problem and finding the eigenvalues and eigenvectors, the set of orthonormal eigenvectors can be considered as the new system of coordinates to be used in the modal analysis of the structure.

2.1.5 Lumped And Distributed Parameter Structures

The differential equations obtained in the previous section are ordinary differential equations with respect to time. This means that the variables are not functions of spatial coordinates. Such systems, the parameters of which do not depend on spatial coordinates and so, their governing equations are ordinary differential equations, are known as the "lumped parameter" systems. They have a limited number of degrees of freedom, which can be represented in vectorial form. Contrary to the lumped parameter systems, are the "distributed parameter" systems. The parameters of a distributed parameter system depend on spatial coordinates and its motion should be modeled by partial differential equations. Such systems have infinite number of degrees of freedom.

The majority of the structures fall in the category of the distributed parameter systems. Integrating the partial differential equations for infinite number of degrees of freedom is much harder than integrating the ordinary differential equations for limited dimensions. Due to this fact, discretization methods have been proposed to model a distributed parameter structure by an approximate discretized structure. The discretized structure can then be studied as a lumped parameter structure. Some of these discretization methods are: "the Rayleigh–Ritz method", "the method of weighted residuals", and "the finite element method".
2.2 ESSENTIALS OF LINEAR SYSTEM THEORY

2.2.1 Definition Of A Linear System

A system is said to be linear if the superposition principle holds for it. For a more detailed explanation, consider the system, represented by the block diagram of figure 2.1a. If \( f(t) \) is an input to the system which results in an output \( z(t) \), the block diagram of Figure 2.1b should be valid according to the superposition principle. Figure 2.1b is the equivalent diagrammatic form of the following mathematical expression, explaining the superposition principle:

\[
\text{if : } \quad f(t) = c_1 f_1(t) + c_2 f_2(t) \quad (2.7)
\]

\[
\text{then : } \quad z(t) = c_1 z_1(t) + c_2 z_2(t) . \quad (2.8)
\]

Figure 2.1 Linear systems.

Superposition principle plays an important role in the development of the linear control theory. According to this principle, first it is possible to study the dynamics of a linear system as a superposition of its homogeneous and particular response, and second it provides the basis to study the response of a linear system in the phasor space, as explained in the following paragraphs.
2.2.2 Single Input/Single Output (SI/SO) Systems

The linear control theory has been essentially developed for the SI/SO systems. The dynamics of the majority of these systems, is described by an \( n^{th} \) order linear ordinary differential equation with constant coefficients, in the following form:

\[
a_n \frac{d^n z(t)}{dt^n} + a_{n-1} \frac{d^{n-1} z(t)}{dt^{n-1}} + \ldots + a_0 \ z(t) = f(t) \tag{2.9}
\]

The coefficients contain information about the characteristics of the system. The above differential equation can be written in a more compact form as:

\[
D(t) \ z(t) = f(t) \tag{2.10}
\]

where:

\[
D(t) = a_n \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \ldots + a_0 , \tag{2.11}
\]

is a linear homogeneous differential operator and contains all the informations about the characteristics of the system.

The solution to the above differential equations can be expressed in the following symbolic form:

\[
z(t) = D^{-1}(t) \ f(t) \tag{2.12}
\]

**Response in the time domain:** Sometimes it is possible to solve equation (2.9) by working in the real space, and by the use of the well known methods for solving the linear differential equations. In such cases, the response can be directly studied in time domain. But in many situations, and in studying the stability of the system, it is very helpful to study the response of the system in the frequency domain or the s-domain.

**Response in the frequency domain (response to harmonic excitations):** The response of the
linear system $z(t)$, to a harmonic excitation of the form:

$$f(t) = f_0 \cos \omega t$$

(2.13)

can be obtained by finding the response of the system to an exponential excitation of the form:

$$f(t) = f_0 e^{i\omega t}$$

(2.14)

where

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

(2.15)

and $i$ represent the imaginary part of a complex variable. Then by the superposition principle, the real part of the response to excitation (2.14) is the same as $z(t)$.

The solution of equations (2.9) and (2.14) can be written in the following exponential form:

$$z(t) = Z(i\omega) e^{i\omega t}$$

(2.16)

where $Z(i\omega)$ is a function of the excitation frequency and is a constant with respect to time. By combining equations (2.16) and (2.12),

$$D(t) Z(i\omega) e^{i\omega t} = f_0 e^{i\omega t}$$

(2.17)

which results in:
So:

\[ Z(iw) \left[ (iw)^n a_n + (iw)^{n-1} a_{n-1} + \ldots + a_0 \right] e^{iw t} = f_0 e^{iw t} \quad (2.18) \]

So:

\[ Z(iw) = \frac{1}{\left[ (iw)^n a_n + (iw)^{n-1} a_{n-1} + \ldots + a_0 \right]} f_0 \quad (2.19) \]

where

\[ G(iw) = \frac{1}{\left[ (iw)^n a_n + (iw)^{n-1} a_{n-1} + \ldots + a_0 \right]} \quad (2.20) \]

is known as the "frequency response function" of the system.

By using the frequency response function of the system, it is possible to find the response of the system for harmonic excitations. Also, to find the system response to a general excitation, first the excitation functions is expanded in the form of summation of a series of harmonic functions. Then the response of the structure under the effect of each of these harmonic excitations is obtained. Finally, the total response of the structure can be determined by the use of superposition principle.

**Response in the s-domain (Laplace transform):** This approach is a strong and general method of analysis, while the frequency response approach can be considered as a special case of it. The method is based on the Laplace transformations.

The Laplace transform of a function \( f(t) \), which can be a complex function, is defined as:

\[ F(s) = \mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t) \, dt \quad (2.21) \]

where \( s \) can be a complex variable. Also the Laplacian of the \( m^{th} \) time derivative of the
function \( f(t) \) can be found from:

\[
L\left[ \frac{d^mf(t)}{dt^m} \right] = \int_{0}^{\infty} e^{-st} \frac{d^mf(t)}{dt^m} \, dt = s^mF(s)
\]  

(2.22)

where \( \frac{df(0)}{dt} \) means the initial value of the \( i^{th} \) derivative of \( f(t) \).

By applying the Laplace transform to both sides of equation (2.9), and utilizing equation (2.22), a relationship between the Laplace transform of excitation and response of the structure can be obtained as:

\[
( a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0 ) Z(s) + C(s) = F(s)
\]

(2.23)

where \( C(s) \) is a polynomial in terms of \( s \), and its coefficients are determined from the initial conditions of the system. By introducing:

\[
G(s) = \frac{1}{( a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0 )}
\]

(2.24)

which is called the "system transfer function" or simply the "gain of the system", the response can be written as:

\[
Z(s) = G(s) F(s) - G(s) C(s)
\]

(2.25)

In the special case of harmonic excitations, the system transfer function is the same as the frequency response function, by simply letting \( s = \omega \).

Once the initial conditions and excitation to the system is given, it is possible to find the functions \( C(s) \) and \( F(s) \). Also \( G(s) \) can be formed by knowing the parameters of the system. Then using equation (2.25) the Laplace transform of the response \( Z(s) \) can be formed. Finally, the response of the system can be obtained, by taking the inverse of the
Laplace transformation as:

\[ z(t) = \mathcal{L}^{-1}[Z(s)] \]  

(2.26)

Using the method of “partial fractions”, it is possible to obtain a simpler form of the system transfer function, as:

\[ G(s) = \frac{1}{(a_0 s^n + a_{n-1}s^{n-1} + \ldots + a_0)} = \sum_{i=0}^{i=n} \frac{b_i}{s - a_i} \]  

(2.27)

where \( a_i, i = 1,2,\ldots,n \) are the roots of the denominator of \( G(s) \) and \( b_i, i = 1,2,\ldots,n \) are the coefficients to be found. These values can be complex numbers in general.

The constants \( a_i, i = 1,2,\ldots,n \) are known as the poles of the system. Since the poles and the constants of the partial fractions, may be complex values, they can be written as:

\[ a_i = a_i^R + i a_i^\text{Im} \quad i = 1,2,\ldots,n \]  

(2.28)

\[ b_i = b_i^R + i b_i^\text{Im} \quad i = 1,2,\ldots,n \]

where the real parts and imaginary parts of these parameters are shown by superscript of \( R \) and \( \text{Im} \) respectively. Knowing that

\[ \mathcal{L}^{-1}\left[\frac{1}{s - a_i}\right] = e^{a_i t} \]  

(2.29)

it is possible to study the response of the system from:

\[ G(t) = \sum_{i=1}^{i=n} b_i e^{a_i t} = \sum_{i=0}^{i=m} b_i e^{a_i^R t} e^{i(a_i^\text{Im} t)} \]  

(2.30)
The term $e^{i(\alpha_i t)}$ results in a harmonic response, while the term $e^{\alpha_i t}$ has a decaying effect if $\alpha_i^R < 0$ and has an amplifying effect if $\alpha_i^R > 0$.

**Stability of a SI/SO system:** A SI/SO system is said to be stable if under the effect of bounded external excitations, its response remains bounded throughout the time, and does not increase indefinitely. Only if the excitation increases indefinitely, the system response may be indefinite too. Since the real parts of the poles of the system transfer function determine the stability of the response of the system, stability analysis of a system can be achieved by studying the real parts of the poles. Then:

$$\begin{align*}
\text{The system is:} & \\
& \begin{cases}
\text{asymptotically stable, if all the poles have negative real parts.} \\
\text{critically stable, if some of the poles have negative real parts while the remaining have no real parts.} \\
\text{unstable, if at least one of the poles has positive real part.}
\end{cases}
\end{align*}$$

(2.31)

### 2.2.3. Multi-Input/Multi-Output (MI/ MO) Systems

Real systems including structures are rarely linear in behavior. The "equilibrium point" of a system is defined as the point, where the system resides in, if the dynamic loads vanish. So, for a structure, it is the point where the displacement of the structure remains constant ($q_e = \text{constant}$) and the velocity and acceleration of the structure vanish ($\dot{q}_e = \ddot{q}_e = 0$). For example the equilibrium point of a framed structure can be considered as its deformation under its static loads.

Considering the equilibrium position of a nonlinear system, a linear model for its dynamic behavior can be constructed by a "Taylor series expansion" of the equations of mo-
tion around the equilibrium point. Also, by a shift of the coordinate system, the equilibrium position can be shifted to the zero position. Then in this new coordinate system, \( q_e = q_e = q_e = 0 \), which implies that \( x_e = 0 \). Hence, the null state can be considered as the equilibrium point for the study of the system response, without any loss of generality. All the other points in the state space are referred to as ordinary points. The motion of the system in the state space can then be characterized by equation (2.4) as:

\[
x(t) = A \times(t) + B f(t)
\]  

(2.32)

where \( A \) is a \( 2n \times 2n \) dimensional matrix and \( B \) is a \( 2n \times r \) dimensional matrix, as defined by equation (2.5).

Solution to the above set of coupled first order differential equations can be obtained by introducing the “transition matrix” as:

\[
\Phi(t) = e^{At}
\]  

(2.33)

And the result will be:

\[
x(t) = \Phi(t) x(0) + \int_0^t \Phi(t - \tau) B f(t) d\tau
\]  

(2.34)

The transition matrix can be computed by a series expansion or solving the eigenvalue problem for \( A \). In the latter case, first the eigenvalue matrix \( \Lambda \), and the left and right matrices of eigenvectors \( V \) and \( U \) are found. Then the transition matrix can be written as:

\[
\Phi(t) = U e^{\Lambda t} V^T
\]  

(2.35)

and finally, the response of the structure is:
Based on the above integration method which is developed for continuous time systems, a discrete time integration method which is suitable for numerical purposes has been proposed (Meirovitch 1990).

**Stability of the equilibrium state in terms of Liapunov:** This definition of stability is based on the fact that the response of the system should be bounded when the excitations are bounded. In terms of Liapunov, the equilibrium state is said to be stable if for any initial time $t_0$ and any $\epsilon > 0$, there exists a $\delta > 0$, such that if

$$
\| x(0) - x_e \| \leq \delta
$$

then:

$$
\| x(t) - x_e \| < \epsilon
$$

In the above definitions, $\| ( ) \|$ stands for the Euclidean norm of the vector $( )$, and $\delta > 0$, is a function of both $t_0$ and $\epsilon > 0$. It is obvious that $\epsilon > \delta$. Also the following definitions provide more specificity about stability of a system:

**Uniform stability:** if $\delta$ is not a function of $t_0$, then the equilibrium state is said to be uniformly stable.

**Asymptotic stability:** if the equilibrium state is stable and:

$$
\lim_{t \to \infty} \| x(t) - x_e \| \to 0
$$
then the equilibrium state is said to be asymptotically stable.

The above definitions are general and cover the special case of $x_e = 0$.

Noticing equation (2.35), it is clear that stability of the equilibrium point is determined by the eigenvalues of matrix $A$. The characteristic equation of matrix $A$, plays the same role as the denominator of the system transfer function $G(s)$. Showing the eigenvalues of $A$ by $\lambda_i$, $i = 1, 2, ..., 2n$, it can be said that:

\[
\text{asymptotically stable }, \text{ if all the } \lambda_i\text{'s have negative real parts.}
\]

\[
\text{critically stable }, \text{ if some of the } \lambda_i\text{'s have negative real parts, while the remaining have no real parts.}
\]

\[
\text{unstable }, \text{ if at least one of the } \lambda_i\text{'s has positive real part.}
\]

**Controllability of the system:** The linear time invariant system:

\[
x(t) = A x(t) + B f(t)
\]

is said to be completely controllable, if it is possible to find some piecewise continuous control vector $u(t)$, where $f(t) = u(t)$, to change the state of the system from any initial state $x_0$ at time $t_0$, to any desired final state $x_f$ at time $t_f$ within the time interval $t_f - t_0$.

For the system of equations (2.41) to be completely controllable, the following matrix

\[
C = [ B | AB | A^2B | \ldots | A^{n-1}B ]
\]

known as the “controllability matrix”, which is a $n \times nr$ matrix should be of rank $n$. 
Controllability expresses the relationship between the control vector and the state of the system.

**Observability of the system:** To control a system, specifically a structure, the whole state of the response should be measured. But in most of the practical problems, it is only possible to measure all of the components of the state vector. Hence to control the structure, it should be possible to estimate the whole state vector from the available measurements. This can be expressed as the observability of the system. The measurements of the response, which is known as the output vector of the system $y(t)$ can be considered as a linear function of the system state vector. In a more complicated form, $y(t)$ can be considered as a linear function of both the state and control vectors

$$y(t) = C \ x(t) + D \ u(t) \quad (2.43)$$

where $y(t)$ is the $m$ dimensional output vector and $C$ and $D$ are $m \times n$ and $m \times r$ dimensional transfer matrices, respectively.

A time invariant system, characterized by the state and output equations (2.41) and (2.43) is said to be observable at time $t_0$, if there exists a time $t_f$, such that the state of the system $x(t_0)$ at time $t_0$ can be determined from the knowledge of the excitation vector $f(t)$ and the output vector $y(t)$ over the whole finite time interval $t_0 \leq t \leq t_f$. If observability is true for all the times $t_0 \leq t_f$, then the system is said to be completely observable. So, observability expresses the relationship between the output vector and the state vector of the system.

For the system of equations (2.41) and (2.43) to be completely observable, the following matrix

$$O_T = \begin{bmatrix} C^T & (A^T)C^T & (A^T)^2C^T & \ldots & (A^T)^{n-1}C^T \end{bmatrix} \quad (2.44)$$

which is a $n \times nr$ matrix should be of rank $n$. The matrix $O$ is known as the “observ-
ability” matrix.

From the definition of a completely observable system, it can be concluded that it is possible to determine the state vector \( x(t) \) from the knowledge of the output vector \( y(t) \) right at time \( t \) for a completely observable system.

2.3 ESSENTIALS OF THE LINEAR CONTROL THEORY

In this section the linear control theory is reviewed in brief. The general idea behind linear control theory and classification of the structures to uncontrolled, actively controlled and passively controlled structures are explained. The general scheme for the control of structures and at last control methods for SI/SO and MI/MO systems, which are known as classical and modern control theories will be explained in brief.

2.3.1. General Idea

In the last section, the stability of SI/SO and MI/MO systems was studied. Stability is of primary concern in designing the systems. Sometimes a system is not stable, and sometimes although it is stable, more stability is required. In both of these cases, improvement of the stability is to be achieved. As explained in the previous section, real parts of the poles of a SI/SO system and the real parts of the eigenvalues of a MI/MO system should be negative for the system to be stable. The more negative these real parts are, the more stable the system is.

In active structural control, improvement of the structural behavior can be achieved through the application of control excitations. The control excitations are supposed to change the values of the poles or the eigenvalues of the system to new ones, resulting in a better performance of the response. Hence the system should be controllable so that it can reach the desired stability. Also when the information about the state of the system is needed for control purposes, but only a small number of measurements of the response of the system is available, it should be possible to estimate the state of the system based on these limited number of output measurements. Hence, the system should be observ-
able. From this discussion, it can be implied that both the controllability and observability of the system are required to make sure that it is possible to achieve the desired degree of stability through the application of control excitations.

2.3.2 Classification Of Systems For Control Purposes

The key point in system control is the use of informations about excitations and response of the system, or in other words the input and/or output measurement vectors. The general form of the linear measurement vector is as shown in equation (2.43). The following classification clears the meaning of the system control, specifically structural control.

**Uncontrolled systems:** Considering the response of a system, the system is said to be uncontrolled, if there has been no output measurements relevant to the response, and also there has been no use of any relevant measurements for the determination of the inputs to be used in its design, neither explicitly nor implicitly. Examples of this kind are numerous. A civil engineering structure which has not been designed to withstand earthquakes, is to be considered an uncontrolled structure against earthquakes, although it can resist earthquakes to some degree. An automobile by itself, without a driver is a totally uncontrolled system, if in motion.

**Actively controlled systems:** Considering the response of a system, the system is said to be actively controlled, if there are some on-line input and/or output (response) measurements relevant to the system behavior, and these relevant measurements are used in the determination of some parts of the inputs to the system, explicitly. This part of the inputs is called the “control vector”, while the remaining inputs are considered as “external excitations”. The control vector may be in the form of forces, different types of energy and other sources that may cause the required changes in the response or alter the effect of the external excitations. Approximately all the animals and plants, a thermostat, an automobile with a driver controlling it, and the new era of actively controlled structures fall in this category. However in most of the man made automatic control procedures, only the measurement of the response of the system (output measurement) is used in the con-
control of the system, known as the closed loop or feedback control.

**Passively controlled systems:** Considering the response of any system there is a class of systems, where the knowledge about the response of the system has been used in its design. There is no on-line measurement that can be used in the determination of the inputs to the system explicitly. Hence there is no control excitations, and all the excitations to the system can be considered as the external excitations. These systems can be called passively controlled systems. Great majority of the engineering systems fall in this category. All the traditional civil engineering structures, from small wood structures to dams, trains and rails together, and conventional base isolation systems are all examples of this type of systems.

**Hybrid controlled systems:** These systems enjoy the benefits of both active and passive control techniques. In practice, the structures which are designed to be actively controlled, should also be designed to withstand to some extend, different dynamic loading conditions by themselves. In a more sophisticated design, use of the passive control mechanisms in conjunction with the active control mechanisms may result in more reliability and better performance of the structures. Thus most of the actively controlled structures should be classified as hybrid controlled systems. To show how much active or passive the hybrid control mechanism is, it is possible to introduce new terminologies for the controlled structures, such as highly active–low passive, etc. Studies on the subject of hybrid control of structures have just begun (Yang and Vongchavalitkul 1993).

### 2.3.3 General Schemes For Active Control Of Systems

Figure 2.2 shows an uncontrolled system. This uncontrolled system is sometimes called a process or a plant too. To control this system, the desired response is introduced to the system in the form of an input. This input is called the “reference input”. For the civil engineering structures, this reference input is a null vector and the response of the structure is a vector comprised of the displacement and its time derivatives. But for most of the mechanical structures, this reference input is not null and should be given to the system. It is expected that the controlled system follows the reference input. Figure 2.3
shows a controlled system, which uses the measurements of the external excitations in the construction of a control input to the process. This is called a “forward path” control system. In this case, controller is a unit that sends a control signal to the process, based on the informations received from the measurements of the external excitations and the reference input. Figure 2.4 shows a controlled system, which uses the measurements of its response in the construction of a control input to the process. This is called a “feedback path” or a “closed-loop” control system. Here, the controller sends a control signal, based on the measurements of the response. The most complicated control system is a combination of the forward path and the feedback control systems. This is shown in figure 2.5. The feedback control system is the most widely used control system, and in the remaining sections, discussion will be devoted to this type of control systems.

2.3.4 Classical Control (SI/SO Control) For Lumped Parameter Structures

Since the structural systems are in general many degrees of freedom systems, this kind of control can have very limited applications in structural control. The detail of a typical classical feedback control system can be seen in figure 2.6. The desired output of the system is given to the controlled system as the reference input. The actual response of the process is measured by sensors, and is fed back to the controlled system, as the response feedback. The error between the desired and actual response is then calculated by an error detector. Because the sensor measures a quantity which is not exactly the same as the actual response, but has a relation to it (it is called output), first the measured quantity (output) is passed through a transducer to be converted into the actual response. The error is then given to the controller, which issues a control signal according to the value of error. The control signal and the external excitations constitute the total excitation to the process.

The controller, transducer and process, can be considered as sub-systems, each having a “transfer function” known as the “gain” of that sub-system. These are shown in figure 2.7, which demonstrates the Laplace transform of the block diagram of figure 2.6. The terms in figure 2.7 are explained in the following paragraphs.

Controller gain function $K$: Generally, a constant gain is considered for the controller.
In this situation, the controller acts as an amplifier for the error. This provides a good understanding of the effect of the controller on the improvement of system response. This type of controller is known as the “proportional controller”, abbreviated by P.

**Process gain function** \( G(s) \): As studied in the previous sections, this gain function has the form of the inverse of a polynomial in terms of the Laplace operator \( s \).
**Transducer gain function** $H(s)$: Can be considered to have the form of the inverse of a polynomial in terms of the Laplace operator $s$.

Also the Laplace transform of the reference input, error, control signal, external excitations and the actual output can be shown by $R(s)$, $E(s)$, $U(s)$, $F_x(s)$ and $Y(s)$, respec-
Based on simple calculations, the following gain functions are defined for the controlled system:

![Figure 2.6 Block diagram of a feedback control system in time domain.](image)

![Figure 2.7 Block diagram of the feedback control system of figure 2.6, in s-domain.](image)
**Forward path gain function** $G_f(s)$: Is defined as $Y(s)/E(s)$. Then:

$$G_f(s) = \frac{Y(s)}{E(s)} = K \cdot G(s) \quad (2.45)$$

**Open loop transfer function** $G_o(s)$: Is defined as $H(s)Y(s)/E(s)$. Then:

$$G_o(s) = \frac{H(s)Y(s)}{E(s)} = K \cdot H(s) \cdot G(s) \quad (2.46)$$

**Closed loop transfer function** $G_c(s)$: Is defined as $Y(s)/R(s)$. Obtaining $G_c(s)$ requires some mathematical manipulations. The error can be obtained from:

$$E(s) = R(s) - H(s) \cdot Y(s) \quad (2.47)$$

also from equation (2.45):

$$E(s) = \frac{Y(s)}{G_f(s)} \quad (2.48)$$

Combining equations (2.47) and (2.48),

$$G_c(s) = \frac{Y(s)}{R(s)} = \frac{G_f(s)}{1 + H(s)G_f(s)} = \frac{KG(s)}{1 + H(s)KG(s)} \quad . \quad (2.49)$$

Comparing equations (2.45) and (2.49), it is clear that closing the loop, results in another gain for the system, which depends on the value of $K$ and the transducer gain $H(s)$. Also there are terms in the numerator of the closed loop transfer function, which result in a polynomial, as a function of $s$. The roots of the numerator are called
the "zero's" of the system, because they result in diminishing the closed loop transfer function. These zero's may have improving effects on the behavior of the system. Also, the poles of the closed loop transfer function are different from those of the uncontrolled plant. Considering $K$ as the variable in the system gain function, it is possible to modify it, so that the poles of the system move to the left side of the s-plane, to achieve more stability. Studying the effect of changing $K$ on the stability of the system and its effective range, has been the subject of a series of researches which have led to some well developed methods for SI/SO control design procedures, known as the "classical control" methods. These methods are mostly graphical. With the advent of computer softwares and computer graphics, these methods can be used faster and more accurate than before. The logarithmic bode plots, Nichols chart, root-locus technique, and the Nyquist method and the Nyquist criteria constitute the main body of the classical control design methods.

*Other related issues in classical control:* Wide application of feedback control has resulted in the development of controllers and devices, that can render better control. The "proportional + integral" controller, abbreviated by PI, the "proportional + derivative" controller, abbreviated by PD and the "three term controller", abbreviated by PID, which is a combination of PI and PD controllers, are among the most widely used controllers.

Devices called "compensators" are secondary controllers, which can be considered as filters for the main controller. Because they are mostly added after the design of the controller, they are called compensators. Lead, lag and lead-lag compensators are some of these devices.

The effect of a time delay in the controlled system is another factor that affects the outcome of the control action. Time delay is simulated by the multiplication of the exponential term $e^{-j\omega T}$ in the system frequency response function, where $T$ is the time delay in the system.

2.3.5 Modern Control For The Lumped Parameter Structures

Modern control is a descendent of the classical control approach. With the progress of space technology, the need for control systems that are optimal, emerged. This resulted in the proposal of the optimal control theory. The Bellmann's dynamic program-
ming, followed by the Pontryagin's minimum principle, constitute the body of the optimal control approach. They are considered as modern control methods. The pole allocation method, also known as the pole assignment or pole placement method, which is an extension of the idea of pole placement from classical control to the MI/MO systems, is also considered as another modern control method (Meirovitch 1990). In the following sections, the general idea of modern control and then the main points of each of the modern control methods will be explained.

**The idea of modern control:** Consider equation (2.4) which is the general form of equations of motion in the state space:

\[
x(t) = A x(t) + B f(t)
\]  

(2.50)

where \( A \) is a \( 2n \times 2n \) dimensional matrix and \( B \) is a \( 2n \times r \) dimensional matrix, defined by equation (2.5). For developing the modern control methods, it is beneficial to divide the excitations into two parts: the control excitation applied by the controller, and the external excitations. These parts can be denoted by \( u(t) \) and \( f(t) \) respectively. Their corresponding location matrices can be shown by \( B_u \) and \( B_f \), which result in the following form of the equations of motion:

\[
x(t) = A x(t) + B_u u(t) + B_f f(t)
\]  

(2.51)

where \( B_u \) is \( 2n \times r_u \) and \( B_f \) is \( 2n \times r_f \). Since it is a linear control method, assume:

\[
u(t) = G(t) x(t)
\]  

(2.52)

where \( G(t) \) is a \( r_u \times 2n \) matrix. From equations (2.52) and (2.51),
\[ x(t) = [ A + B_u G(t) ] x(t) + B_f f(t) \]  \hspace{1cm} (2.53)

As shown in section 2.2.3, the response of the structure is governed by the eigenvalues of matrix \([ A + B_u G(t) ]\). Hence, while external excitations do not theoretically have any effect on the characteristics of system, it is possible to modify the system characteristics by the application of a linear control. The idea of modern control is to find a suitable matrix \( G(t) \), which is called the “control gain matrix”.

**Pole allocation method:** Assume that the eigenvalues of matrix \( A \) and its right and left eigenvectors are calculated and are shown by \( \lambda_i, u_i \) and \( v_i \) respectively, where \( i = 1, 2, \ldots, 2n \). Also assume that \( m \) of these eigenvalues may cause instability in the system response. It is desired to change these \( m \) eigenvalues so that the performance of the system improves. This can be shown by

\[
\lambda_j \rightarrow \mu_j \hspace{1cm} j = 1, 2, \ldots, m
\]  \hspace{1cm} (2.54)

where \( \mu_j \) are the desired eigenvalues. Also in the above statements, it has been assumed that numbering of the eigenvalues does not follow any order. It is required to accomplish this change in the eigenvalues by applying just one control force \( u(t) \). In this case the matrix \( B_u \) reduces to a \( 2n \times 1 \) matrix. In other words, \( B_u \) is a \( 2n \) dimensional vector.

Without any loss of generality, it can be assumed that the control force is a linear function of the controlled modes, eigenvalues of which should be modified. This can be shown by

\[
u(t) = \left( - \sum_{k=1}^{k=m} g_k v_k^T \right) x(t)
\]  \hspace{1cm} (2.55)
which means that the gain matrix is a constant vector. Also $B_u$ can by the expansion theory be written as:

$$B_u = - \sum_{k=1}^{k=2n} p_k \ u_k$$  \hspace{1cm} (2.56)$$

where the constants $p_k$, $k = 1,2,\ldots,2n$ can be computed easily. It is proven that:

$$g_j = -\frac{1}{p_j} \prod_{k=1}^{m} \frac{(\mu_k - \lambda_j)}{(\delta_{kj} + \lambda_k - \lambda_j)} \quad j = 1,2,\ldots,m$$  \hspace{1cm} (2.57)$$

where $\delta_{kj}$ is the Kronecker delta.

Three important conclusions can be drawn from the above result. The first is that, as expected, only the modes, their eigenvalues to be modified, contribute to the control signal. The second is that, when introducing the location vector $B_u$, it should be noticed to satisfy $p_k \neq 0$, $k = 1,2,\ldots,m$. The third is that by the application of only one control signal, it is possible to control the whole structure, if $B_u$ is introduced correctly.

Pole allocation method for more than one control input, follows the same procedure, but it needs other criteria such as optimization of control cost, to provide a unique solution.

**Optimal control methods:** These methods are based on the cost–benefit idea. The cost of the control system is the cost of control and the benefit is the reduction of response of the system. A compromise should be made. This is done by the introduction of a “performance index” shown by $J$ which contains information about the cost of control and the penalty corresponding to a bad control (or benefit of reduction in the response), and trying to minimize it. To this end, and to obtain a linear control, a quadratic form should be proposed for the performance index. This is done by introducing three constant weighting matrices, the $2n \times 2n$ dimensional $Q_f$ and $Q$, and the $r_u \times r_u$ dimensional $R$. $Q_f$ and $Q$ are real symmetric positive semidefinite matrices and $R$ is a real
symmetric positive definite matrix. The performance index $J$ is then defined as:

$$J = x^T(t_f) Q_f x(t_f) + \int_{t_0}^{t_f} \left[ x^T(t) Q x(t) + u^T(t) R u(t) \right] dt.$$  \hspace{1cm} (2.58)

It is desired to control the state of the system over the time interval $t_0 \leq t \leq t_f$ where $t_0$ and $t_f$ are the initial and final times of control. Also, while the initial state of the system is known, its final state is unknown. The minimization problem is then stated as:

$$\min. \; J = x^T(t_f) Q_f x(t_f) + \int_{t_0}^{t_f} \left[ x^T(t) Q x(t) + u^T(t) R u(t) \right] dt$$  \hspace{1cm} (2.59)

subject to:

$$x(t) = A x(t) + B_u u(t) + B_f f(t).$$  \hspace{1cm} (2.60)

To solve this minimization problem, first the Lagrangian is formed as:

$$L = \int_{t_0}^{t_f} \left\{ x^T(t) Q x(t) + u^T(t) R u(t) \right\} dt + \lambda^T(t) \left[ A x(t) + B_u u(t) + B_f f(t) \right]$$  \hspace{1cm} (2.61)

where the vector of Lagrange multiplier $\lambda(t)$ is a function of time. Then the performance index can be written as:
\[ J = x^T(t_f) Q_f x(t_f) + L \]  

(2.62)

The optimality of \( J \) requires \( \delta J = 0 \). By defining the Hamiltonian \( H \), as the integrand of equation (2.61) and taking the first variation of the Lagrangian with respect to the variables \( x(t) \), \( x(t_f) \), \( u(t) \) and \( \lambda(t) \), the requirement of \( \delta J = 0 \) results in the following necessary conditions for the minimization of performance index:

\[ \dot{x}(t) = A x(t) + B_u u(t) + B_f f(t) \]
\[ \frac{\partial H}{\partial u} = 0 \]  
\[ \dot{\lambda}^T + \frac{\partial H}{\partial x} = 0 \]
\[ \lambda(t_f) = Q_f x(t_f) \]  

(2.63)

By taking derivatives, the following necessary conditions are obtained for the minimization of \( J \):

\[ \dot{x}(t) = A x(t) + B_u u(t) + B_f f(t) \]  

(2.64)

\[ \dot{\lambda}(t) = -A^T \lambda(t) - 2Qx(t) \]  

(2.65)

\[ u(t) = -\frac{1}{2} R^{-1} B^T \lambda(t) \]  

(2.66)
subject to :

\[ x(0) = x_0 = \text{the initial state of the system, which is known.} \]  

(2.68)

From equation (2.66) it is implied that \( u(t) \) is a linear function of \( \lambda(t) \). Hence, there should be a linear relationship between \( \lambda(t) \) and \( x(t) \), to obtain a linear control. Imposing this condition,

\[ \lambda(t) = K(t) \, x(t) \]  

(2.69)

where \( K(t) \) is the proportionality matrix. Comparing equations (2.69) and (2.67),

\[ K(t_f) = Q_f . \]  

(2.70)

Using equation (2.69) and its time derivatives, equation (2.65) can be derived in \( x(t) \) and \( x(t) \). Then by substituting for \( x(t) \) from the right side of equation (2.64), an equation which is only a function of \( x(t) \) and \( f(t) \) will be obtained:

\[ [K(t) + K(t)A - \frac{1}{2} K(t)BR^{-1}B^T K(t) + A^T K(t) + 2Q]x(t) + K(t)B f(t) = 0 . \]  

(2.71)

Assuming \( f(t) \) is a white noise process, it is possible to neglect the second term, and then:
\[ K(t) + K(t)A - \frac{1}{2} K(t)BR^{-1}B^T K(t) + A^T K(t) + 2Q = 0 \]  

subject to:

\[ K(t_f) = Q_f \]  

should be solved. Equation (2.72) is known as the Riccati equation, and the matrix \( K(t) \) as the Riccati matrix. By solving this backward first order differential equation, \( K(t) \) will be found and the Gain of the closed loop system \( G(t) \) can be computed from equation (2.65) as:

\[ G(t) = -\frac{1}{2} R^{-1} B^T K(t) \]  

Therefore, the main step in obtaining the gain matrix, is solving the Riccati equation. Methods for solving the Riccati's equation can be found in Kwakernaak and Sivan (1972).

**supplementary notes on optimal control:** The fact that actuators may reach their capacity should be noticed in the formulation of optimal control method. The Pontryagine’s minimum principle considers this boundedness of the actuators capacities as a necessary condition for optimality (Meirovitch 1990).

Control by using observers is an important issue for the practical purposes. The output measurements of the system response is generally a subset of the total state vector. Since the optimal control theory has been developed based on feedback of the total state vector, then, it is necessary to construct the whole state vector from the output measurements. This can be accomplished by the use of “observers”. Observers provide an estimate of the actual total state of response, to be used as feedback to the system, based on the output vector. Luenberger observer (Luenberger 1971) is a deterministic observer, while the
Kalman–Busy filter (Kwakernaak and Sivan 1972) is a stochastic observer which has been developed based on the consideration of noise in the system.

Control and observation spillover, deal with problems, related to the contamination of unconsidered modes of response by the modes which are considered in control design (Meirovitch 1990). Spillover may result in the instability of controlled system.

Sources of time delay in the controlled system result in nonlinearity and even instability in the system response. Soong (1990) has a section on time delay compensation by using Taylor expansion of time dependent variables. Abdel–Rohman (1987) and Agrawal et al. (1993) have studied this problem too.

Instantaneous optimal control is a discrete time approach, the criteria of which is the optimality of Hamiltonian at each time step. By this approach the external excitations can be considered in the instantaneous optimization process. Also nonlinearities to some degree in the parameters of structure can be taken into account (Yang et al. 1987).

Independent modal space control “IMSC” is an optimal–modal control approach. First, the modal control force for each mode is assumed to be a function of the response of that mode only. This results in the despoiling of the equations of motion for each mode. Then, a performance index is introduced for each mode, and these modal performance indices are minimized by the optimal control method, as explained in the previous paragraphs (Meirovitch 1990).

Among other structural control algorithms, pulse control, bounded state control and predictive optimal control can be named. All of these methods are categorized as optimal control methods and are reviewed by Soong (1990). The most successful of these methods in coping with nonlinearities and delay is the predictive optimal control method. This method has been used in several recent studies and has attracted considerable interest (Rodellar et al. 1987 and 1989). This method is applied to the control of a three story frame structure in chapter 8 of this study. A review of this method is presented in chapter 8 too.
2.4 STUDIES IN THE ACTIVE CONTROL OF STRUCTURES

To follow the progress in the active control of structures, some kind of classification of current investigations in this field is helpful. This classification can be done with regard to two subjects: control methodologies and control mechanisms. In the last several years, both of these subjects have been studied by many researches. There is a tendency towards finding a category of better control algorithms and mechanisms, and every now and then, researchers report on the studies made on similar structures, for the sake of indirect comparison between different methods and mechanisms. In a recent attempt to provide a reliable foundation for such studies, Soong and his co-workers (1985) have constructed a standard three storey one bay frame structural model for the experimental assessment of different control algorithms and mechanisms. Because the properties of structural components of the frame and the results of its identification are reported, the model can also be used in the numerical studies and evaluation of control methods and mechanisms. This model has been used in this study too, where the results can be seen in part two of this article.

In the following sections, a brief review of the proposed structural control methods and mechanisms are presented.

2.4.1 Methods For The Active Control Of Structures

Both pole allocation and optimal control methods have been studied for the purpose of active control of civil engineering structures. However, optimal control has attracted more attention of the researchers in the field and many control methods and algorithms have been proposed based on the concepts from optimal control.

In a series of attempts to investigate the possibility of application of the pole allocation method, Leipholz and Abdel-Rohman (1978,1986) have provided some results and have addressed on issues such as controllability and observability of multi-degree of freedom systems.

Optimal control method, in its original form has been studied by several authors. Soong (1990) has collected some of the salient results of these studies. In each of these
studies, a linear model has been assumed for the structure. Because it is hard to modify the original optimal control to cope with delays and nonlinearities in the controlled systems, some new methods have been proposed such as “optimal pulse control”, “instantaneous optimal control” and “predictive optimal control” methods, which have been briefly discussed in the previous section. These methods have been studied extensively recently and the results of these studies have shown considerable flexibilities attached with these methods, for the control of structures with nonlinearities and delays (Masri et al. 1981 and 1982, Yang et al 1987, Rodellar et al 1987 and 1989).

There are also several other methods such as the independent modal space control (IMSC) which has been studied for space structures by Meirovitch and his co-workers (Meirovitch 1990), and the bounded state control (Lee and Kozin 1986).

So far, all of the studies have been made on simple, mostly one degree of freedom structures. The results of these studies have shown great potentials for the active control of structures. However, real structures are distributed parameter systems which may be simulated by many degrees of freedom models. Numerical study on such models and experimental studies on real models may raise new questions about the capability of these methods when applied to real structures.

2.4.2 Mechanisms For The Active Control Of Structures

Among the widely studied active control mechanisms, “active tendon control ATC” also called active bracing system ABS, “active mass damper AMD”, “aerodynamic appendages” and “pulse generators” have been studied more than the others. In a recent study, Soong and Reinhorn (1993) have reported the results of active controlling of several full scale structures, subjected to real wind forces and ground excitations. They have found that active tendon mechanism shows generally more capabilities than active mass damper mechanism, specially for controlling the higher modes of structural response.

Active Tendon Control: has been proposed in 60’s by Freyssinet (Soong 1990) and empha-
sized by Zuk (1968) and Yao (1972). In this strategy, control forces are applied by electro-hydraulic actuators to the structure, via a number of prestressed tendons. Active tendon control has been studied for controlling different types of structures. Roorda (1975) has applied it for the active control of tall buildings, like steel chimneys. Yang and Giannopoulos (1979) and Warnitchai et al. (1993) have studied it for controlling the cable-stayed bridges. Reinhorn et al. (1987) have studied the capabilities of combined base isolation and active tendon control as a hybrid control mechanism. Lopez-Almanza et al. (1987) have considered an active tendon control system for tall buildings and Chung et al. (1988) have studied it for seismic structures. Optimization of the structure and its tendon control mechanism together, has been the subject of some works by Soong et al. (1987) and Cha et al. (1988).

**Active Mass Dampers:** Chang and Soong (1980) have shown that the effectiveness of tuned mass dampers TMD, which are passive control mechanisms can greatly be enhanced by the addition of an active actuator force, changing the tuned mass damper into an active tuned mass damper ATMD which is a hybrid system. Samali et al. (1985 a,b) have studied the potential of AMD in the mitigation of wind and earthquake induced coupled lateral-torsional motion of tall buildings. The results of a number of theoretical, experimental and practical applications of the AMD and ATMD systems for tall buildings in Japan can be found in Izumi et al. (1993), Soong and Reinhorn (1993) and Nagase et al. (1993).

**Pulse Generator:** is a simple, yet effective control mechanism. Pulse control is capable of coping with nonlinearities in the system. By monitoring the system state response, whenever violation of a predefined threshold occurs, a pulse will be applied to correct the state. Masri et al. (1980, 1981 and 1982) have developed and applied the pulse control strategy for the control of frame structures. One of the reasons they have counted for proposing this method is to use pulse generators which can apply large control forces of relatively short duration to control the structures. Also Udwadia and Tabaei (1981), Prucz et al. (1985) and Reinhorn et al. (1987) have investigated the capabilities of this method in structural control. Miller et al. (1988) have applied this method in controlling of both linear and nonlinear structures. Pulses can be generated by the flow of compressed air, hydraulic and electromagnetic actuators (Soong 1990).
Aerodynamic Appendages: This mechanism has been proposed for the control of structures subjected to wind forces by Klein and Salhi(1980). The control of structure is achieved through the motion of appendages which are installed on the structure. Possibility of using this mechanism has been studied by other researchers too (Leipholz and Abdel–Rohman 1986).

Other Control Mechanisms: Sirlin et al. (1986) have studied the feasibility of active controlling of the floating structures, supported by open-bottom chambers, where the pressure of trapped air has been adjusted to control the heave motion of platform. Yang and Vongchavalitkul (1993) have used a hybrid control strategy for seismic excited bridges, by controlling the opening of oil flow in the dampers, in addition to a passive isolation system. Feng (1993) has developed a strategy for the active base isolation of the buildings, where the friction force produced in the structure is controlled by the application of a normal force to the sliding bearings.

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In this chapter the essentials of neural network theory and related issues as required for the application to the control problems are reviewed briefly. Artificial neural networks are man made systems that can perform some intelligent activities, similar to those of the human's brain. They can learn and acquire the knowledge about a phenomenon and can also be trained to respond to that phenomenon appropriately. Their quality of response improves, as they learn more and more. This characteristic of the artificial neural networks puts them in a place between the conventional computational devices and human brain. That is why recently the people in different fields of science and technology are attracted to the artificial neural networks. In the last several years (1986–1994), almost every field of research has utilized these systems to improve the quality of their outcomes.

Beside the wide range of the application of neural networks in other fields, it has been recently used as a new tool in the study of civil engineering problems. Some of these studies include the mathematical modelling of the nonlinear structural materials, geotechnical engineering, damage detection (Ghaboussi et al. 1991, Ghaboussi 1992, 1993, 1994) and control of structures as demonstrated in this article.

In the following sections, a brief history of the artificial neural networks and general features of the multi-layer feedforward backpropagation neural networks are presented. At last, the simulation of a neural network and ways for the improvement of its learning capabilities in accordance with the requirements of this study are discussed.

3.1 HISTORY OF THE ARTIFICIAL NEURAL NETWORKS

In this section, first the motivation behind endeavors for the construction of artificial neural networks is discussed. Then a history of developments in this field is presented.
in a chronological order.

3.1.1 Motivation

Understanding the natural intelligence and construction of artificial intelligent systems, are the main motivations and objectives for the researches in the field of neural networks.

*Knowledge about intelligent behavior:* The curiosity of human being in discovering the origins of knowledge and functions of the natural brain, returns to centuries ago. The performance of intelligent systems has always been fascinating for the scientists. From the time of ancient philosophers to the time of pioneers in modern neurophysiology, Ramon E. Kajal and William James, and until now, man has tried to collect informations about the mechanism and organization of the natural nervous system of all the animals, specially human beings.

*Construction of the artificial intelligent systems:* Besides the curiosity to understand the behavior of the brain, man has always been interested in the construction of systems that can render compensate for human points of weakness. Mechanical devices have been invented to perform an intelligent behavior in a passive way, such as wheels, carts, mills, buildings, trains, planes, etc. These devices are built to compensate for the human "physical points of weakness". Computational devices, mechanical and electrical, both analogue and digital, have been invented as remedies to the lack of speed and precision of computation task in the human beings. Abacus, Napier bones, Pascaline and Arithmometer are but just a number of the primary mechanical computational devices. The census tabulator of Herman Holerith which was invented in 1890 has been the first analog computer. His work was followed by the invention of modern digital computers. The technology of producing the information keeping devices has been greatly developed as a main necessity in the challenge for improvements on these computational devices. These information keeping devices include cards, tapes, disks, diskettes, etc. (Fuori and Aufiero 1989).

With the recent advancements in technology, the need for systems that show some de-
gree of intelligence has been increased. For example, these intelligent systems should be able to perform some sort of learning, generalization, adaptivity, and decision making which are correlated to each other. Since the biological systems possess these capabilities, the idea of using their characteristics has attracted the attention of the researchers to solve the complicated technological problems. As examples, problems in pattern recognition, classification and control can be named. This has resulted in the appearance of new fields like artificial intelligence, artificial neural networks, and a mixture of both fields called the “artificial intelligent neural networks”. All of these systems are inspired by the massively parallel processing behavior of the human nervous system.

3.1.2 Brief History Of Artificial Neural Networks

Works of Pavlov and Luria in the mid 19th century revealed some facts about the conditioning mechanism of behavior and dynamic nature of cognitive processes. Conditioning mechanism, as established by Pavlov, states that repetition increases the correlation between a stimulus and its corresponding response. His experimental studies played a major role in the subsequent developments in the physiological psychology.

It is considered that William James has been the founder of the current developments in knowledge engineering. He wrote his famous book “Principles of Psychology” in 1890. In that book, he mentioned a number of important ideas about physiology and organization of brain, like neural activity, weight connection and parallel processing in the brain.

Mc Culloch and Pitts came with a model about neural activity in 1943. They concluded some assumptions, to explain their model. They pointed to the all or non process of neural activity, latent addition of synaptic excitations, the synaptic delay as the only source of delay, and the overall inhibitory property of the inhibitory synapses. Also they assumed that the structure of the neural net is fixed. With this model, they proved that it is possible to construct neural networks, that can represent any logical expressions. Their model was a massively parallel processing system.

The Mc Culloch–Pitts model is not used in today’s artificial neural networks in its original form. However it provided a strong basis for the future steps.
Donald O. Hebb wrote his famous book "The Organization of Behavior" and proposed the rule of updating the synoptic weights in 1949. His rule is called the "Hebbian Rule". Hebbian rule states that repeated effect of a neuron $A$ on another neuron $B$ strengthens the connection between the two neurons. He called his model, a "connectionism" model, and made four postulates about it. These postulates are that, the information is kept in the synaptic weights, there is a connection learning rate which is proportional to neural activations, weights are symmetric and that simultaneous repetition of activation of neurons increases their synaptic connection weight.

Hebb's Postulates are now used in the construction of neural networks.

Frank Rosenblatt introduced the first simulated neural network, called the "Perceptron" in 1958. Perceptron was in fact a "self-organization" and "self-associative" learning system. Rosenblatt showed that the information is kept in the form of connection weights in the perceptron. Then the perceptron responds to the new stimuli directly, according to the information kept in its connections.

ADALINE was the first engineering outcome of the previous studies. In 1960, Bernard Widrow and Marcian Hoff built an adaptive switch circuit, and called it the "Adaptive Linear", abbreviated by ADALINE. It was an artificial hardware neural network, in the size of a lunch box, and is still working. They updated the connection weights by finding the "Least Mean Square", abbreviated by "LMS", using the gradient descent method. This method of updating is a very common method of updating the weights.

After Minskey and Papert wrote their book "Perceptrons", the support for the development of artificial neural networks was considerably reduced. These authors were working with traditional Artificial Intelligence (AI) groups. They proved that there is no general convergence for perceptrons. This meant that perceptrons were not capable of learning all the desired logical expressions.

Between 1969 and 1982, the field of research on neural networks was not very active. The important studies were those of T. V. Kohonen and J. Anderson who introduced the ideas of "associative memory" and "interactive memory" respectively. These two ideas
were mainly the same. Also, S. Grossberg developed his very complicated ART programs, to study physiological matters.

In the years 1982–1986, John J. Hopfield built a series of silicon artificial neural networks, which are known as “Hopfield Nets”. They were practically very important and interesting for companies. Those networks used the optimization method in order to minimize their error of prediction, when updating the connection weights. This opened a new era in neural network developments, and reactivated this field.

David E. Rumelhart and his co–workers (1986a), introduced the backpropagation method for updating the connection weights of a multilayer neural network. In the same year, David E. Rumelhart and James L. McClelland (1986b), edited the book of “Parallel Distributed Processing”. This book had a very prominent effect in the progress of research in artificial neural networks, and has been an important reference for the people who are working in this field. It included all the important issues related to the architecture, activation functions and learning algorithms, known at that time.

The theoretical aspects of neural networks have been developed in the recent years. Also special notice has been given to their applications. Compared to ten years ago, the number of books and journals and conferences on neural networks and their applications has been increased considerably.


Among the many different proposed models of artificial neural networks, the “multi–layer feedforward backpropation” neural networks have become the most popular ones in engineering applications. In the recent years, developments in this type of neural networks have resulted in more complicated learning rules and architectures. Quickprop learning algorithm of Fahlman (1988), modular architectures of Jacobs and Jordan (1991), and fuzzy neural systems of Hsu et al. (1992), are some of these developments.
This type of neural network is used in this research.

3.2 CAPABILITIES, FUNCTION AND CONSTRUCTION OF NEURAL NETWORKS

Neural networks have been utilized in solving many problems in different fields of research. The power of neural networks comes from the fact that they are highly adaptive nonlinear systems. A neural network contains several variables that can be determined during the adaptation process. Because of this adaptivity, neural networks can be trained to learn the intrinsic relationships between different characteristics of a phenomenon. This phenomenon can be the recognition of an object from the measurements of its properties, as in letter recognition and weather prediction, or it can be the process of decision making about sending a control signal to a control mechanism based on the measurements of the state of that mechanical system, etc. With the adaptivity characteristics of the neural networks, it is not necessary anymore to identify the systems by the conventional methods. Identification will be accomplished implicitly and automatically through training of the neural network.

Here, those features and capabilities of neural networks will be reviewed, which are important for engineering applications and specifically structural control. In such cases, neural networks are used as mapping devices which demonstrate regression-like functions. The reason behind using neural networks for these type of problems is that in most of the real world problems, it is not possible, or at least it is very hard to prepare good mathematical models to characterize the processes. Even it is hard to find good regression models to explain the processes approximately. For example, in control problems, there are many sources of nonlinearity. Also there are time delays due to actuators dynamics. In general, these types of problems, are mapping and regression problems, with many input and output variables. Because these variables may be correlated to each other it is hard to find a good regression model. In such situations, use of neural networks is very beneficial in preparing a model to simulate the system behavior. It is possible to train a neural network to learn the intrinsic relationships between the variables of the system, the inputs to it and its corresponding outputs. Then, if a new set of inputs is given to the system, its outputs is expected to be the same as the values obtained from the real system,
when the system is subjected to the same inputs and under the same conditions. Predicting the response of a structure when subjected to external excitations, predicting the desired control forces for the control of a structure in the next sampling rate, and the determination of nonlinear material characteristics for a finite element analysis, are some but not all of the problems one may face in structural control. These problems can be handled by well trained neural networks.

The primary steps in the construction of a suitable neural network are as follows:

1. **Representation:** Determination of enough and concise input variables. Output variables are dictated automatically by the requirements of the problem.

2. **Selection of a suitable type of neural network:** First, decision should be made about the type of neural network, whether it is a multi-layer feedforward neural network, a recurrent neural network, etc. Then, the architecture, training strategy and the required parameters which define the behavior of the network, such as the type of activation function, learning rate, etc. should be selected. This includes choosing or constructing an algorithm for updating the parameters of the neural network, growing mechanism, etc.

3. **Selection of a suitable training set:** Number of the training cases should be large enough to cover all the possible situations that may occur during the real process. Meanwhile, it should be as small as possible to avoid excessive unnecessary time of training.

In this study, multilayer feed forward neural networks will be used. The method of updating the weights is the backpropagation rule, with quickprop algorithm. These are explained in the following sections.

### 3.3 MULTI-LAYER FEED-FORWARD NEURAL NETWORKS (MFFNN)

A multilayer feedforward neural network is comprised of many processing units also called processing elements, nodes, neurons and neurodes. Each of these units has a simple behavior. The units receive signals from and send signals to the other processing
units via wire-like connections. The complexity of a neural network comes from the cooperation of these very simple processing units. This is the fascinating point about the neural networks. The arrangement of these simple units is an important issue. The Multilayer feedforward backpropagation neural networks (MFFNN) are practically very useful because of their predefined architectural form, where the arrangement of their units follows a simple pattern. In the following sections, processing units, connections, architecture of MFFNN, and other related issues will be discussed.

3.3.1 Units, Connections And Activation Function

Each processing unit acts as an amplifier. It receives signals from the other units, and issues a signal which is a function of the received signals. This function can be nonlinear.

Figure 3.1 is a schematic representation of a processing unit \( j \). It receives inputs from some units \( i = 1, 2, \ldots, n_l \). These units can be called the “lower units”. The inputs are added together algebraically, according to the following rule:

\[
Z_j = \sum_{i=1}^{n_l} w_{ji} O_i
\]

where \( w_{ji} \) represents the connection weight (connection strength) associated with the wire-like connection from unit \( i \) to unit \( j \), and \( O_i \) is the output of unit \( i \). A function \( f \), which is called the “Activation function”, acts on this summation. The result is the output of unit \( j \), which will be sent to some other units \( k = 1, 2, \ldots, n_h \). These units can be called the “higher units”.

Activation function: Many types of activation functions, have been proposed and used for different problems. The most widely used activation function is the “Sigmoidal Function”. Due to the net input \( Z_j \) to unit \( j \), it returns the output \( O_j \) according to the following equation:
Figure 3.1 A processing unit, also called element, node, neuron and neurode.

\[ O_j = F(Z_j) = \frac{1}{1 + e^{-\lambda Z_j}} \]  

(3.2)

where \( \lambda \) is a constant parameter and is a characteristic of the unit. Figure 3.2 shows the sigmoidal activation function. It is similar to the activation of a real neuron. A real neuron is supposed to be a threshold logic unit. Some other popular activation functions are shown in figure 3.3.

3.3.2 Architecture Of MFFNN

A MFFNN is comprised of layers of units. A unit receives only input from the units in the previous layer, and sends only signal to the units in the succeeding layer. Neither inter-layer connections nor cross-layer connections are not permitted. Also, each unit in a layer is connected to all the units in the succeeding layer. The result is a neural network which is fully connected in the layers. The first layer receives input from surrounding. The last layer sends outputs to the surrounding. In other words, the first layer is the input layer, and the last layer is the output layer of the neural network. Also there should be at least one layer in between input and output layers. Theoretically, one hidden layer is enough for learning a mapping problem. Practically however, two hidden layers
Figure 3.2 The sigmoidal activation function.

Figure 3.3 Some of the popular activation functions. (a) Sigmoidal function shifted (b) Step function (c) Ramp function.

are recommended for complicated problems. Since there is no input from a layer to its previous layers, and the direction of signal propagation is from the input layer to the output layer, these types of networks are called the “feedforward neural networks”. Simplicity of the architecture of MFFNN has led to provide strong theoretical support for its convergence (Hornik 1991, Blum and Li 1991). Figure 3.4 shows a MFFNN and its components.
3.4 TRAINING ALGORITHM

Training of the neural network means modifying its connection weights, until the network can predict the output variables with an acceptable accuracy. So, a criteria for evaluating the error is required too. Training of the network is based on an optimization method, such as the steepest descent.

To train a neural network three steps should be followed:

1. A number of input–output pairs, should be constructed from the measurements of the real process. These input–output pairs (i–o pairs) constitute the training set. The number of these i–o pairs can be shown by \( n_{io} \). So, if each input vector is shown by \( I_i \), and its corresponding desired output which is called the target output vector, by \( T_i \), \( i = 1, 2, \ldots, n_{io} \), then the block diagram representing this cause and effect or mapping relation is as shown in figure 3.5a.

2. Beginning with a small architecture of the neural network, its connection weights should be selected by a random number generation mechanism. Then feeding the in-
put \( I_i \) to the network, results in the predicted output \( O_i \), which generally differs from the target output \( T_i \), \( i = 1, 2, \ldots, n_{io} \). The block diagram representing this cause and effect or mapping relation is as shown in figure 3.5b.

![Block Diagram](image)

Figure 3.5 Mapping.

3. Calculating the error between the predicted output \( O_i \) and the target output \( T_i \), \( i = 1, 2, \ldots, n_{io} \) for each i-o pair. Euclidean norm is considered for the calculation of this error. The total error is the sum of all the Euclidean norms, according to the following formulas,

\[
E_i = 0.5 \left\| O_i - T_i \right\|, \quad i = 1, 2, \ldots, n_{io} .
\]

\[
E = \sum_{i=1}^{n_{io}} E_i .
\]

where \( E_i \) is the Euclidean norm for i-o pair number \( i \), and \( E \) is the total error.

4. Calculating the gradient or slope or the sensitivity of error with respect to the connection weights. This is done through the backpropagation of error. Hence, these types of neural networks are called backpropagation neural networks. In this method, the slope of error with respect to all the connection weights, which are connecting two succeeding layers are calculated at the same time. Calculation proceeds from the output layer to the input layer.
5. Updating the weights by using a learning rule. The learning rule, used in this study is based on the “quickprop” algorithm, proposed by Fahlman(1988).

3.4.1 Backpropagation Method

This method, has been developed by Rumelhart and his co-workers, as well as Werbos and Parker (Rumelhart et al. 1986a,b, Allman 1989 and Eberhart and Dobbins 1990). Consider the FFNN shown in figure 3.6. The layers are numbered from 1 to \( n \). Each layer has a number of units, denoted by \( i_1 \) for layer 1, to \( i_n \) for layer \( n \). An input vector \( I = [ I_1, I_2, ..., I_{i_1} ]^T \) is introduced to the input layer. The predicted output is \( O = [ O_1, O_2, ..., O_{i_n} ]^T \). The target output is \( T = [ T_1, T_2, ..., T_{i_n} ]^T \). The error due to this prediction is:

\[
E = 0.5 \sum_{i=1}^{i=n} (O_i - T_i)^2 .
\] (3.4)

While the predicted output is a function of the network parameters, the target output \( T \) is not a function of them. Since for a fixed network the only parameters to be adjusted are the connection weights shown by matrix \( W \), then:

\[
O = f( W ) .
\] (3.5)

Thus for each connection weight \( w_{jk} \) which connects two units \( j \) and \( k \) in two successive layers,

\[
\frac{\partial E}{\partial w_{jk}} = \sum_{i=1}^{i=n} (O_i - T_i) (\partial O_i / \partial w_{jk}) .
\] (3.6)
Figure 3.6 A typical multilayer feedforward neural network (MFNN) which has been trained to learn the input-output relationship for a real phenomenon.

Also, for each of the output units $i$,

$$\frac{\partial E}{\partial O_i} = (O_i - T_i) \quad (3.7)$$

From the above equation, the derivative of error with respect to the net input $Z_i$ received by the output unit $i$ can be calculated from:

$$\frac{\partial E}{\partial Z_i} = (\frac{\partial E}{\partial O_i}) (\frac{\partial O_i}{\partial Z_i}) \quad (3.8)$$

By introducing

$$F'(Z_i) = \frac{\partial O_i}{\partial Z_i} \quad (3.9)$$
equation (3.8) can be written as:

$$\frac{\partial E}{\partial Z_i} = \left( \frac{\partial E}{\partial O_i} \right) F'(Z_i)$$  \hspace{1cm} (3.10)

Since the activation function is not an explicit function of the connection weights, the terms on the right hand side of the above equation can be calculated from activation of the output units. The result for sigmoidal activation functions (equation 3.2) is:

$$\frac{\partial E}{\partial Z_i} = \left( \frac{\partial E}{\partial O_i} \right) \left( \lambda F \left( 1 - F \right) \right).$$  \hspace{1cm} (3.11)

Now, considering each two units $j$ and $k$ in two succeeding layers $m-1$ and $m$ respectively, which are connected by the connection weight $w_{jk}$, the following equations will be obtained:

$$\frac{\partial E}{\partial O_j} = \sum_{k=1}^{k=m} \left( \frac{\partial E}{\partial Z_k} \right) \left( \frac{\partial Z_k}{\partial O_j} \right)$$  \hspace{1cm} (3.12)

$$= \sum_{k=1}^{k=m} \left( \frac{\partial E}{\partial Z_k} \right) w_{jk}.$$

Also

$$\frac{\partial E}{\partial Z_j} = \left( \frac{\partial E}{\partial O_j} \right) \left( \frac{\partial O_j}{\partial Z_j} \right)$$  \hspace{1cm} (3.13)

$$= \left( \frac{\partial E}{\partial O_j} \right) F'_j.$$

From the above equations, it is obvious that the derivatives of error with respect to the output $O_j$ and the net input $Z_j$ for unit $j$, depend on the derivatives of error with respect to the net inputs to the higher layer. Thus, backpropagation flows from the output
layer to the input layer. Backpropagation begins with calculating equations (3.7) and (3.10) for the output layer where \( m = n \). Then the derivative of error with respect to the weight \( w_{j,k} \) can be obtained from:

\[
\frac{\partial E}{\partial w_{j,k}} = \left( \frac{\partial E}{\partial Z_k} \right) \left( \frac{\partial Z_k}{\partial w_{j,k}} \right)
\]

\[
= \left( \frac{\partial E}{\partial Z_k} \right) O_j
\]

Which depends on the derivatives calculated for the higher layer, and the activation of the lower layer which is known from the last forward-propagation of input.

**Backpropagation algorithm:** By introducing the term \( \delta_k \) defined as:

\[
\delta_j = \frac{\partial E}{\partial Z_j}
\]

for unit \( j \), the algorithm for backpropagation of error can be formulated as follows:

**step 1:** Calculate:

\[
\frac{\partial E}{\partial O_i} = (O_i - T_i)
\]

for each of the output units \( i = 1, 2, \ldots, n \) where \( n \) is the number of layers of the neural network.

**step 2:** For each layer \( m = 1, 2, \ldots, n \), beginning with the output layer, backpropagate the error, and calculate the following derivatives. In the following equations, \( j \) represents a node in layer number \( m - 1 \) and \( k \) denote a node in layer number \( m \). After finding
the following derivatives for all the nodes in layer $m$, go to layer $m-1$:

$$F'(Z_k) = \frac{\partial O_k}{\partial Z_k}$$  \hspace{1cm} (3.17)

$$\delta_k = \frac{\partial E}{\partial Z_k} = \left( \frac{\partial E}{\partial O_k} \right) \left( F'_k \right)$$  \hspace{1cm} (3.18)

$$\frac{\partial E}{\partial w_{jk}} = \delta_k O_j$$  \hspace{1cm} (3.19)

$$\frac{\partial E}{\partial O_j} = \sum_{k=1}^{i_m} \left( \frac{\partial E}{\partial Z_k} \right) w_{jk}$$  \hspace{1cm} (3.20)

### 3.4.2 The Generalized Delta Rule

Updating the weights to reduce the error of prediction can be achieved through the application of steepest descent method. In this approach, after the calculation of the slope of error with respect to the connection weights, $\frac{\partial E}{\partial w_{jk}}$, the weights can be updated from:

$$w_{jk} = w_{jk} - \gamma \frac{\partial E}{\partial w_{jk}}$$  \hspace{1cm} (3.21)

where $\gamma$ is the learning rate. The learning rate can be a variable of convergence rate or training cycle, etc. Also updating the weights can be accomplished through “batch training” procedure. To this end, it is preferred to train the neural network by gradually increasing the size of the training set when the number of training cases is large. The weights are then updated frequently.
3.4.3 The Quickprop Learning Algorithm

This algorithm has been proposed by Fahlman (1988) to achieve a faster convergence than with the original generalized delta rule. According to this algorithm, higher order derivatives of error with respect to the connection weights of the neural network are considered in the updating of weights. This algorithm has been adopted and used in the development of a MFFNN for the purpose of this study. The steps to be followed in this algorithm are;

1. Keep the information about the derivatives of error with respect to weights and the corresponding changes in weights $\Delta w_{j,k}$, as obtained in the previous step of backpropagation of error. Denoting the previous step by $t - 1$ and the recent step by $t$, these derivatives of error can be shown by

$$S_{j,k}(t - 1) = (\frac{\partial E}{\partial w_{j,k}})_{t-1}.$$  \hspace{1cm} (3.22)

Also changes in weights can be denoted by $\Delta w_{j,k}(t - 1)$.

2. Obtain the new derivatives of error for step $t$ from:

$$S_{j,k}(t) = (\frac{\partial E}{\partial w_{j,k}})_t.$$  \hspace{1cm} (3.23)

3. Calculate the required change in the weights from the following rule:

if $S_{j,k}(t - 1) \times S_{j,k}(t) > 0.0$, then:

$$\Delta w_{j,k} = - \gamma \, S_{j,k}(t).$$  \hspace{1cm} (3.24)

where $\gamma$ is the learning rate.
if $S_{jk}(t-1) \times S_{jk}(t) < 0.0$, then:

$$\Delta w_{jk}(t) = -S_{jk}(t) \frac{\Delta w_{jk}(t-1)}{S_{jk}(t) - S_{jk}(t-1)}.$$  \hspace{1cm} (3.25)

### 3.5 ARCHITECTURE GROWING

One of the effective ways of increasing the capacity of a neural network and escaping from local minima is by modifying its architecture. Many algorithms have been proposed for adaptive architecture determination. These algorithms are based on the criteria for growing and pruning of the neural networks. The importance of this issue has resulted in frequent studies and considerable amount of literature on the subject, including Mezard and Nadal (1989), Ash (1989), Fahlman and Lebier (1990), Tenorio and Lee (1990), Wu (1991), Nikzad (1991), Hirose et al. (1991), Sietsma and Dow (1991), Fujita (1992) and Hamamoto et al. (1992).

The development of a growing algorithm in accordance with the needs of this study, has been a part of this research. Hence, after the development of a computer program for the simulation of a MFFNN, called “SUNN”, a growing algorithm has been developed and implemented in the computer program. In spite of the other growing algorithms where the units are generated manually, they can be generated both manually and/or automatically in this algorithm, depending on the user’s preference. An expert subroutine, consisting of “if–then statements” controls the generation of units. The growing algorithm has played an important role in constructing more powerful neural networks for this study. A concise about this growing algorithm is presented in the following section. More details about the computer program “SUNN” can be found in chapter 5.
3.5.1 Automatic Architecture Growing Algorithm And Criteria Used In This Study

In this study, the general strategy for the construction of a neural network has been to begin with a small architecture, and then letting it grow when necessary. This idea has been studied by Wu and Ghaboussi (1990) and has provided successful results (Wu 1991). This is important from two points of view: first, to let the network learn the general characteristics of the problem and avoid localization, and second, to keep the training time, which is an increasing function of the number of nodes and connection weights, as less as possible.

To accomplish the above mentioned goals, an algorithm for the assessment of the learning speed of the neural network and addition of new units to the network has been developed. The following criteria have been used for this purpose:

Criteria for the evaluation of convergence speed: The training of the neural network is accomplished in a batch mode. To this end, the neural network is forced to learn a small number of training cases, which is a subset of the whole training set. This training subset can be called a sub-batch while the whole training set is called the batch. After the neural network learns the training cases in the sub-batch, a small number of new training cases are added to the sub-batch, to form a new and larger sub-batch. The neural network is then forced to learn the new sub-batch of train cases, and so on until the network learns the whole training cases. Hence the size of sub-batch of training cases increases gradually and the neural network is trained gradually too. However in cases the neural network is trapped in a local minima or the capacity of the neural network is reached, the neural network can not learn all the training cases in the sub-batch with the desired accuracy. Detection of such situations is done by recording the number of updating cycles of the connection weights for the sub-batch, and the total error, average error and maximum error for the training cases in the sub-batch as a function of updating cycles. Slopes of these errors with respect to updating cycles are calculated. The general criteria for the evaluation of convergence speed is that whenever the slopes are very small, convergence is very slow and new units should be added to the neural network. Addition of new units produces new adaptive connection weights in the neural network.

For this study, a number of “if-then” rules have been implemented in the computer
program to evaluate the speed of convergence based on the aforementioned criteria. Implementation of such rule based criteria has provided the possibility for the development of an automatic node generation mechanism. The automatic node generation mechanism introduces new nodes to the neural network whenever convergence speed is low.

After the addition of a number of new units, it is logical to expect the neural network converge to better results. So, more training cycles will be permitted for the network to learn the same training sub-batch, before the addition of more units, if required.

**Determination of the number of units to be added:** The number of new units that should be added is a variable which should be defined by the user as an input to the developed neural network simulation program. But it is obvious that the number of added units should be small. This is due to the fact that it is preferred to keep the previous results of training as intact as possible, which means that the previous architecture should not be ruined completely and also the capacity of the neural network should not be increased abruptly. In cases where the neural network is trapped in a local minima, the weights of the new connections are given high values, and for some first cycles of additional training, the previous weights are frozen. This freezing mechanism helps the neural network escape the undesired local minima. The variables, controlling the number of cycles of freezing, weights of the new connections, etc., can be introduced by the user as inputs to the neural network simulation program or be generated automatically.

### 3.6 NEURAL NETWORK BASED CONTROL

There is an extensive literature on the application of neural networks in control of different processes. Miller, Sutton and Werbos (1990), Warwick, Irwin and Hunt (1992), and White and Sofge (1992) have collected useful informations on the subject. While neural networks have been widely used in other disciplines, there has been few applications in the control of civil engineering structures. Besides some qualitative papers, showing the possibilities of using neural networks in the control of simple cases, a numerical study is due to Nikzad and Ghaboussi (1991) and Nikzad (1991), who have provided some results for the noise control of a rigid body structure and a flexible plate.
In this research, and its descendent papers submitted for publication, the first method and detailed application of neural networks in the numerical study of controlling the steel frame structures is presented.

REFERENCES


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In this chapter, the fundamentals of fuzzy subsets and fuzzy logic theory, which is useful for the sake of control of structures is reviewed. Also the application of this theory in the control of different engineering processes is reviewed in brief.

Formulated (conventional) automatic methods for process control are based on providing exact mathematical modelling of both the process and the control mechanism. These conventional methods of automatic control have been useful in handling many control problems. On the other hand, there may be many sources of uncertainty and ambiguity in the process itself and also in the know how about controlling the process. Sometimes, these sources of uncertainty and ambiguity are so complicated that control should be handed over to an intelligent system. This intelligent system is often a human expert. The expert, uses his/her available knowledge about the overall process, compensates for changes in the parameters of the controlled plant, improves his/her controlling abilities through more experience, and tries to optimize the control behavior towards a perfection. Because of this high quality of performance of the human experts, the idea of producing artificial systems that are intelligent enough to perform these controlling tasks, the way the human experts do, is fascinating.

One of the basis for constructing such intelligent systems, is by the use of theory of fuzzy sets. This theory, is mainly introduced and developed by Prof. L. A. Zadeh, in 1965 (Zadeh 1965a,b). In this theory, a new kind of sets have been introduced which have uncertain boundaries. These sets which are called fuzzy sets, may contain members that have different values of belonging, called “membership”. In his two next papers, Zadeh has pointed to the application of fuzzy set theory in control problems (Zadeh 1971, 1974). However, it was the works of Mamdani and his co—workers that demonstrated and enhanced the industrial application of fuzzy logic based control. They studied the application of the fuzzy theory in the control of dynamic plants (Mamdani 1974, Mamdani and
Assilian 1975). Thereafter, there has been vast application of fuzzy logic in the construction of knowledge based expert controllers, for different engineering problems. Control of a cement kiln by fuzzy logic has been studied and put to work by Holmnblad and Ostergaard, in Denmark (1982). Controlling a system of pumps for start/stop, determination of number of pumps in operation, and controlling the water level for drainage has been studied by Kokawa in Japan (1982). Interest in fuzzy control application has been greatly raised after the appearance of a collection of papers about the application of fuzzy control, edited by Sugeno (1985). These papers have been prepared by many authors from different parts of the world, and all of them have reported encouraging results. Automatic train operation by Yasunobo and Miamoto from Japan (1985), controlling the addition of chemicals for water purification by Yagishita et al. from Japan (1985), control of a multi-degree of freedom robot arm by Scharf and Mandic from England (1985), control of a casting plant by Bartolini et al. from Italy (1985), aircraft flight control by Larkin from U.S.A. (1985), and automobile speed control by Murakami and Maeda from Japan (1985) are but some of these papers. Articles on similar studies and applications have appeared afterwards, a concise review of which can be found in the work of H. Berenji (1992).

Application of fuzzy logic in structural control has begun very recently. The possibility of using this approach has been mentioned by Yao (1987), and Yao and Natke (1992). Also Reinhorn and his co-workers (1993) have reported on the use of hybrid passive and active control by the use of fuzzy logic. They have controlled a rigid block subjected to earthquakes. The results have been promising.

A part of this study has been to investigate the applicability of the fuzzy control method in controlling the civil engineering structures. Hence, in the following sections, basic definitions in fuzzy set and fuzzy logic theory, fuzzy reasoning and evaluation, and fuzzy control strategies will be explained briefly. More details can be found in the extensive literature on fuzzy sets and fuzzy control, among which the following can be mentioned: Terano et al. (1987), Gupta and Yamakawa (1988), Pedrycz (1989), Kandel (1986), Yager (1987), Dubois and Prade (1980) and Kaufmann (1975).
4.1 THE CONCEPT OF FUZZY SET AND FUZZY LOGIC

The binary classification is a very familiar mathematical problem. In such problems one should determine if an object belongs to a class or not. For example, is a light on or off, or is a number greater than 5 or not. In such problems, the borders for distinguishing the classes are precisely defined. It is easy to imagine more complicated cases for which there are many classes with precisely defined borders, where it is required to determine to which class a given object belongs. As a simple example the problem of determination of the type of a metal can be mentioned: is it copper, iron or zinc, etc.

Because the borders are fixed and not flexible, some authors call this type of sets, the “crisp sets” (Terano et al. 1987). In either case, both the information about the object and the borders of the classes are completely known and the judgement for classification is required to be certain. The ordinary set theory deals with such kind of classification problems.

Meanwhile there are many situations, judgement about which can not be precise and abstract. This includes cases where some kind of comparison between similar objects is involved, or when there is some ambiguity and uncertainty involved in the evaluation of a phenomenon. As typical examples, one can consider questions like: how fast is a car, how successful is a control algorithm, how intelligent is a system, and how suitable is a medical treatment. Such questions that require more flexible evaluation and judgement, are very common in the real world problems. In such cases, while the object is completely known, the borders can not be well defined for classification purposes. Specially the problem becomes much more complicated when there are more than two classes available, or there are many parameters to be considered in evaluation and judgement. In such problems, one can not define the borders precisely. Fuzzy set theory, has been proposed to deal with these types of problems. It is called fuzzy for the purpose of distinguishing it from the ordinary or crisp set theory. Although there is some kind of uncertainty and ambiguity in fuzziness, the concept and meaning of fuzziness is different from that of statistical. In statistics, the uncertainty is in identification of the object, while in the fuzzy set theory, ambiguity is in the definition of the boundary of the sets.

By the application of fuzzy set theory to classification problems, more flexibility in classification can be obtained, rather than the ordinary set theory. Hence, it is supposed
that by utilizing the fuzzy set theory, it is possible to construct systems that are more intelligent and adaptive. This characteristic of fuzzy sets and fuzzy logic is useful for the construction of structural intelligent controllers.

4.2 BASIC DEFINITIONS IN FUZZY SET THEORY

Consider $X$, as the space of a number of points or objects. Elements of $X$, can be denoted by $x$. This can be shown by $X = \{ x \}$.

A fuzzy set $A$ in $X$, is determined by a function called the “membership function”, denoted by $\mu_A(x)$, defined for all $x \in X$. $X$ is called the “support set”, too. The membership function $\mu_A(x)$ states to what grade, $x$ is a member of $A$. It is common to associate a number in the interval $[0,1]$ to $\mu_A(x)$. When membership is maximum, $\mu_A(x) = 1$, and when minimum, $\mu_A(x) = 0$. Figure 4.1, shows three membership functions for three fuzzy variables. In figure 4.1–a, the variable $x$ is a measure that says how red an apple is. Based on this measure, the ripeness of the apple is expressed in the form of the membership function $\mu_A(x)$. The more red the apple, the more ripe it is. In figure 4.1–b, $x$ is the measure of control force, and the membership function expresses how good the control performance is. The less the absolute value of the force, the better the control performance is. In figure 4.1–c, $x$ is the measure of slenderness of a column. In this problem, the membership function states how good the design has been. The closer the slenderness to a required minimum slenderness, here 220.0, the better the design is.

Assigning a membership function to a fuzzy object, is called “quantification of that fuzzy object”. In the above examples, the ripeness of an apple, the quality of a control task, and the quality of a design have been quantified.

If the membership function $\mu_A(x)$ is either 0 or 1 for all the points of $X$, then the fuzzy set becomes a crisp set, also called an ordinary, or a non-fuzzy set. In this case, the term “characteristic function” is used instead of “membership function”. The characteristic function is shown by the letter $\chi$. Figure 4.2 shows the characteristic function for
the above mentioned variables of figure 4.1, where the boundaries are defined without any ambiguity and uncertainty. In figure 4.2-a, an apple is ripe if its redness measure is over 4.3. In figure 4.2-b the control performance is good if control force is between $-40.0$ and $+40.0$. In figure 4.2-c design is good if slenderness is between 190.0 and 250.0.

As can be seen, fuzzy sets are extensions of ordinary sets. Also it can be said that while the ordinary sets are "objective", the fuzzy sets are "subjective".

In most of the practical situations, it is necessary to work with more than one class. For example in a control problem, one may want to evaluate to which of the following classes, the velocity of an object belongs: approximately zero (S), small negative (SN), medium negative (MN), big negative (BN), small positive (SP), medium positive (MP) or big positive (BP). Using ordinary sets, the classification shown in figure 4.3 may be proposed.
The boundaries are well defined. By measuring the velocity of the object, it falls in one of the classes, and that is the result of evaluation. For example, if velocity is 8.0 cm/sec., it is BP, if 0.2 cm/sec., S, and if -4.0 cm/sec., MN. But a preliminary knowledge about control, tells us that this is not an appropriate way of evaluation, since the velocities of 3.1 and 5.9 cm/sec. belong to MP, while the velocity of 6.1 cm/sec. which is very close to 5.9 cm/sec. belongs to another class, BP. By using fuzzy sets, a better evaluation of the velocity can be provided. According to the fuzzy set theory, all the velocities are members of different classes, however their amount of membership is different. This amount of membership, is sometimes called the “grade” or “extent” or “degree of inclusion” of the variable to the set or class. Figure 4.4 shows a fuzzy classification of the same problem of the controlled velocity. As can be seen, continuous smooth functions are used to define the membership function for the classes. The velocity of 5.9 is very MP, a little bit BP, and obviously not negative. The velocity of 6.1 has the same situation, and is much similar to 5.9 than 3.1 is. This provides a more realistic and helpful foundation for the control of velocity.

![Figure 4.3 Ordinary or crisp boundaries.](image-url)

The following definitions, explain basics for fuzzy set operation rules. These definitions are valid for crisp sets too.

**Empty Fuzzy Set A**: A fuzzy set $A$ is empty if and only if $\mu_A(x) = 0$, for all $x$ in $X$. 
Normal And Subnormal Fuzzy Sets: A fuzzy set is said to be normal if the maximum of its membership function is 1; either it is said to be subnormal.

Equal Fuzzy Sets $A$ and $B$: $A$ and $B$, are said to be equal if and only if $\mu_A(x) = \mu_B(x)$ for all $x$ in $X$.

Complement of Fuzzy Set $A$: is another fuzzy set $\overline{A}$, with membership function $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$, for all $x$ in $X$.

Fuzzy Subsets: Fuzzy set $A$ is a subset of the fuzzy set $B$, if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x$ in $X$.

Union: Union of two fuzzy sets $A$ and $B$, denoted by $A \cup B$, is another fuzzy set $C$, where its membership function $\mu_C(x)$ is defined as:

$$\mu_c(x) = \max. [\mu_A(x), \mu_B(x)] \quad \text{for all } x \text{ in } X.$$ (4.1)

For abbreviation purposes, the following notation will be used very often:
\[ \mu_c(x) = \mu_A(x) \lor \mu_B(x) \quad \text{for all } x \in X. \quad (4.2) \]

where \( \lor \) means the maximum of the two quantities on both of its sides. As can be imagined, the union of two fuzzy sets is the smallest fuzzy set that contains both of the fuzzy sets \( A \) and \( B \). Figure 4.5 is an example of the union operation.

\[ 1.00 \]
\[ \mu \]
\[ \mu_A(x) \]
\[ \mu_B(x) \]
\[ \mu_A \cup_B(x) \]

Figure 4.5 Membership function for the union of two fuzzy sets.

**Intersection:** Intersection of two fuzzy sets \( A \) and \( B \), denoted by \( A \cap B \), is another fuzzy set \( C \), where its membership function \( \mu_C(x) \) is defined as:

\[ \mu_c(x) = \min. [\mu_A(x), \mu_B(x)] \quad \text{for all } x \in X. \quad (4.3) \]

For abbreviation purposes, the following notation is used very often:

\[ \mu_c(x) = \mu_A(x) \land \mu_B(x) \quad \text{for all } x \in X. \quad (4.4) \]

where \( \land \) means the minimum of the two quantities on both of its sides. As can be imag-
ined, the intersection of two fuzzy sets is the smallest fuzzy set that is contained in both of the fuzzy sets $A$ and $B$. Figure 4.6 is an example of the intersection operation.

Figure 4.6 Membership function for intersection of two fuzzy sets.

4.2.1 Useful Notation

Consider an ordinary set $X$, which has a finite number of members, expressed as:

$$X = \{ x_1, x_2, \ldots, x_n \}.$$  \hfill (4.5)

Because each member can be considered as a subset of $X$, it is possible to imagine that $X$ is a union of its members. So, by convention, $X$ can be shown in the following form too:

$$X = \sum_{i=1}^{i=n} x_i = x_1 + x_2 + \ldots + x_n .$$  \hfill (4.6)

This notation has been extended to fuzzy sets too. When a fuzzy set $A$ is defined on the finite set $X$, then for every member $x$ of $X$, there is a membership or grade of inclusion $\mu_A(x)$, defined for $A$. It is convention to show the fuzzy set $A$, in the following form:
\[ A = \sum_{i=1}^{n} \mu_A(x_i)/x_i = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n. \] (4.7)

Also, when \( X \) is not finite, summation will be changed to integral, and the result is:

\[ A = \int_X \mu_A(x)/x, \] (4.8)

which is a notation only, and means union not integration.

4.3 SOME PROPERTIES OF FUZZY SETS

Some of the properties of fuzzy sets, regarding the above definitions are explained here:

_De Morgan’s laws:_ For two fuzzy sets \( A \) and \( B \),

\[ \overline{A \cup B} = \overline{A} \cap \overline{B} \] (4.9)

\[ \overline{A \cap B} = \overline{A} \cup \overline{B} \] (4.10)

_Distributivity laws:_ For three fuzzy sets \( A \), \( B \) and \( C \),

\[ C \cap (A \cup B) = (C \cap A) \cup (C \cap B) \] (4.11)
\[ C \cup (A \cap B) = (C \cup A) \cap (C \cup B). \quad (4.12) \]

These laws hold for crisp sets too, but the following "exclusion" and "contradiction" laws which are valid for crisp sets, are not true for fuzzy sets, in general:

\[ A \cup \overline{A} = X \quad \text{not true for fuzzy sets} \quad (4.13) \]

\[ A \cap \overline{A} = \emptyset \quad \text{not true for fuzzy sets}. \quad (4.14) \]

**Commutativity laws:** For two fuzzy sets \( A \) and \( B \),

\[ A \cup B = B \cup A \quad (4.15) \]

\[ A \cap B = B \cap A \quad (4.16) \]

**Associativity Laws:** For two fuzzy sets \( A \) and \( B \),

\[ (A \cup B) \cup C = A \cup (B \cup C) \quad (4.17) \]

\[ (A \cap B) \cap C = A \cap (B \cap C). \quad (4.18) \]

**Boundary conditions:** For a fuzzy set \( A \),
\begin{align*}
A \cup X &= X \\
A \cap X &= A \\
A \cup \emptyset &= A \\
A \cap \emptyset &= \emptyset.
\end{align*}

\section*{4.4 Algebraic Operations with Fuzzy Sets}

Some of the most important operations on fuzzy sets are the following ones:

*Algebraic product of two sets* \( A \) and \( B \): is another fuzzy set, shown by \( A \cdot B \), which its membership function is defined as:

\[ \mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x) \]  

Since \( \mu_{A(x)} \) and \( \mu_{B(x)} \) are both less than 1.0, their product is less than either of them. So, the following conclusions can be made:

\[ A \cdot B \subseteq A \]
Note that for ordinary sets, the algebraic product and $\bigcap$ are equivalent operations, since the membership function is either 0 or 1.

**Algebraic sum of two sets $A$ and $B$:** is another fuzzy set, shown by $A + B$, and has the following membership function:

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x)$$  \hspace{1cm} (4.27)

As can be seen, De Morgan's laws apply to the algebraic product and algebraic sum, i.e.:

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$  \hspace{1cm} (4.28)

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$  \hspace{1cm} (4.29)

**Bounded sum of two sets $A$ and $B$:** is another fuzzy set, shown by $A \oplus B$, and has the following membership function:

$$\mu_{A\oplus B}(x) = (\mu_A(x) + \mu_B(x)) \land 1.$$  \hspace{1cm} (4.30)

In his first important paper on the fuzzy sets, Zadeh has given other similar names to these
operations. He has used the name “algebraic sum” for the “bounded sum”, and “dual of the algebraic product” for the “algebraic sum”.

**Absolute difference of two sets A and B**: is another fuzzy set, shown by \(|A - B|\), and has the following membership function:

\[
\mu_{|A-B|}(x) = |\mu_A(x) - \mu_B(x)| .
\]  

(4.31)

**Bounded difference of two sets A and B**: is another fuzzy set, shown by \(A \ominus B\), and has the following membership function:

\[
\mu_{A \ominus B}(x) = (\mu_A(x) - \mu_B(x)) \vee 0 .
\]  

(4.32)

**Convex combination of three sets A, B and Λ**: In vector algebra, by convex combination of two vectors \(V_1\) and \(V_2\), using variable \(0 < \lambda < 1\), we mean another vector \(V_3\), which has the following relation to the two original vectors:

\[
V_3 = \lambda V_1 + (1-\lambda)V_2 .
\]  

(4.33)

By a similar definition for fuzzy sets, the convex combination of fuzzy sets \(A\) and \(B\), using another fuzzy set \(\Lambda\), is another fuzzy set, shown by \((A, B : \Lambda)\), which has the following membership function:

\[
\mu_{(A, B : \Lambda)}(x) = \mu_\Lambda(x) \mu_A(x) + [ 1 - \mu_\Lambda(x) ] \mu_B(x) .
\]  

(4.34)

The property of this fuzzy set is that, no matter what \(\Lambda\) is, the following conclusion is valid:
\[ A \cap B \subseteq (A, B : \Lambda) \subseteq A \cup B. \]  \hspace{1cm} (4.35)

**Convex fuzzy set** \(A\): A fuzzy set \(A\) which has support \(X\) in the real numbers, is said to be convex, if and only if for any interval \([a, b] \subseteq X\), the following statement holds:

\[ \mu_A(x) \geq \mu_A(a) \wedge \mu_A(b) \quad \forall x \in [a, b]. \]  \hspace{1cm} (4.36)

Examples of convex and non-convex fuzzy sets are shown in figures 4.7-a,b.

---

\[ \mu \]  \hspace{1cm} (a)

\[ \]  \hspace{1cm} (b)

Figure 4.7 Typical fuzzy sets  (a) Convex,  (b) Non-convex.

---

\(\lambda\) – **Complement of fuzzy set** \(A\): is another fuzzy set, shown by \(\overline{A}^\lambda\), and has the following membership function:

\[ \mu_{\overline{A}^\lambda}(x) = \frac{1 - \mu_A(x)}{1 + \lambda \mu_A(x)}. \]  \hspace{1cm} (4.37)

De Morgan’s Law applies to the \(\lambda\) – Complement of \(A\). i.e.

\[ (A \cup B)^\lambda = \overline{A}^\lambda \cap \overline{B}^\lambda \]  \hspace{1cm} (4.38)
(\overline{A \cap B})^\alpha = \overline{A}^\alpha \cup \overline{B}^\alpha . \quad (4.39)

**Weak and strong \( \alpha \)-cuts for a fuzzy set \( A \):** They are not fuzzy sets, but ordinary sets, called \( A_\alpha \) and \( A_{\bar{\alpha}} \) respectively, and have the following properties:

\[ A_\alpha = \{ x \mid \mu_A(x) > \alpha \} ; \quad \text{and} \quad 0 \leq \alpha < 1 . \quad (4.40) \]

\[ A_{\bar{\alpha}} = \{ x \mid \mu_A(x) \geq \alpha \} ; \quad \text{and} \quad 0 < \alpha \leq 1 . \quad (4.41) \]

The characteristic function for these ordinary sets are to be shown by: \( \chi_{A_\alpha} \) and \( \chi_{A_{\bar{\alpha}}} \), as follows:

\[ \chi_{A_\alpha}(x) = \begin{cases} 1 ; & x \in A_\alpha \\ 0 ; & x \notin A_\alpha \end{cases} \quad (4.42) \]

\[ \chi_{A_{\bar{\alpha}}}(x) = \begin{cases} 1 ; & x \in \overline{A_\alpha} \\ 0 ; & x \notin \overline{A_\alpha} \end{cases} \quad (4.43) \]

It is obvious that as \( \alpha \) gets smaller, \( A_\alpha \) and \( A_{\bar{\alpha}} \) get bigger. When the membership function is continuous, the difference between weak and strong \( \alpha \)-cuts is not necessary to be considered. The following properties hold for \( \alpha \)-cuts:
\[(A \cup B)_a = A_a \cup B_a\quad (4.44)\]

\[(A \cap B)_a = A_a \cap B_a\quad (4.45)\]

A typical $\alpha$-cut is shown in figure 4.8.

![Figure 4.8 A typical $\alpha$-cut for fuzzy set $A$.](image)

**Resolution principle:** This principle, is a direct consequence of the definition of $\alpha$-cuts. It says that for each value of $x$, its grade of inclusion $\mu_A(x)$ is the maximum value of $\alpha$, for which the $\alpha$-cut contains that $x$. In mathematical terms,

\[
\mu_A(x) = \sup. \ [ \alpha \land \chi_{A_a} ]
\]

where "sup." is the abbreviation for supremum. A similar equation can be written for strong cut, but there is no difference between weak and strong cuts for practical purposes.

Now, it is possible to define the whole fuzzy set $A$, based on its $\alpha$-cuts. First, the fuzzy sets $\alpha A_a$ are defined for different values of $\alpha$. The membership functions for these fuzzy sets are defined as:
\[ \mu_{\alpha A_a}(x) = \alpha \chi_{A_a}(x) . \] (4.47)

Since \( \alpha A_a \) are fuzzy sets, their membership functions are defined over the whole support set \( X \). Now, the whole finite fuzzy set \( A \), is the union of these \( \alpha A_a \) sets, i.e.:

\[ A = \bigcup_{\alpha \in [0,1]} \alpha A_a , \] (4.48)

or:

\[ A = \int_{a=0}^{a=1} \alpha A_a . \] (4.49)

Figure 4.9, represents an example of \( \alpha A_a \) sets. Figure 4.10 shows how the whole fuzzy set \( A \), is obtainable from \( \alpha A_a \) sets.

Figure 4.9 (a) Fuzzy set \( A \) (b) A typical \( \alpha \) -cut (c) The resulting \( \alpha \) set.
4.5 FUZZY NUMBERS

Fuzzy numbers are special forms of fuzzy sets. They are defined in the space of real numbers $R$, and have properties that are useful for application purposes. These properties define a more specific shape for their membership functions, as follows:

A fuzzy number $A$, is a fuzzy set in $R$, such that:

i) it is a normal fuzzy set,

ii) it is a convex fuzzy set,

iii) its $\alpha$-cuts are closed intervals of $R$.

iv) it has a bounded support.

The most widely used membership functions for fuzzy numbers are: triangular, trapezoidal and convex membership functions, as shown in figure 4.11. All of these forms, are compatible with the definitions of fuzzy numbers.

4.6 EXTENSION PRINCIPLE

This principle plays an important role in fuzzy logic applications. It allows to extend the domain of definition of a mapping from $X$ to $Y$, to a mapping from a fuzzy subset $A$ of $X$ to $Y$. To be more specific, if there is a mapping from $X$ to $Y$, by using a func-
Figure 4.11 (a) Triangular fuzzy numbers  (b) Trapezoidal fuzzy numbers  
(c) Convex fuzzy numbers.

function $f$, then:

$$f : X \rightarrow Y$$  \hspace{1cm} (4.50)

Now, if $A$ is a fuzzy subset of $X$,

$$A = \sum_{i=1}^{i=n} \frac{\mu_A(x_i)}{x_i} .$$  \hspace{1cm} (4.51)

Then according to the extension principle,

$$f(A) = \sum_{i=1}^{i=n} \frac{\mu_A(x_i)}{f(x_i)} .$$  \hspace{1cm} (4.52)

In the case, where $A$ is not finite, the summation should be changed into integral:

$$f(A) = f(\int_X \frac{\mu_A(x)}{x} ) = \int_Y \frac{\mu_A(x)}{f(x)} .$$  \hspace{1cm} (4.53)

The summation and integration, have the meaning of union. So, if there is more than one $x$ associated with the same $y$, then the membership function of that $y$ is the maximum membership of those $x$’s.

Noticing figure 4.12, the concept of the extension principle for ordinary and fuzzy sub-
sets becomes more clear.

**Inverse Image of A Fuzzy Set:** The inverse image of a fuzzy set $B$, defined on $Y$, is another fuzzy set $A$ on $X$. The membership of $A$ is defined based on the membership of $B$, according to the following relation:

$$
\mu_A(x) = \mu_{f^{-1}(B)}(x) = \mu_B(f(x))
$$  \hspace{1cm} (4.54)

**Extension Principle For The N–ary Functions:** Consider $f$, as a mapping from the $n$–dimensional Cartesian space $X_1 \times X_2 \times \ldots \times X_n$, to a universe $Y$;

$$
y = f(x_1, x_2, \ldots, x_n).
$$  \hspace{1cm} (4.55)

Assume fuzzy sets $A_1, A_2, \ldots, A_n$, are defined on $X_1, X_2, \ldots, X_n$, and have membership functions $\mu_{A_1}, \mu_{A_2}, \ldots, \mu_{A_n}$, respectively. The result of operation of $f$ on the Cartesian space of fuzzy sets can be shown by $B$, where:

$$
B = f(A_1, A_2, \ldots, A_n).
$$  \hspace{1cm} (4.56)

Then the extension principle can be used to find the membership function for $B$, according to the following rule:

$$
\mu_B(y) = \text{Sup. min.} \left[ \mu_{A_1}(x_1), \ldots, \mu_{A_n}(x_n) \right]
$$  \hspace{1cm} (4.57)

$$
\mu_B(y) = 0 \quad \forall \ f^{-1}(y) = \emptyset.
$$
Figure 4.12 (a) Mapping from $X$ to $Y$. (b) Mapping of an ordinary subset of $X$ to $Y$ is an ordinary subset of $Y$, too. (c) Mapping from a fuzzy subset of $X$ to $Y$ is a fuzzy subset of $Y$, too.
4.7 FUZZY RELATIONS AND FUZZY REASONING

In problems where more than one fuzzy variables are involved, fuzzy relations are proposed to be used in the construction of multi-variable fuzzy boundaries. In this section, first the concept of fuzzy relation and then basic definitions which are used in the construction of fuzzy relations are explained.

4.7.1 Definition Of Fuzzy Relations

A fuzzy relation between two support sets $X$ and $Y$, shown by $R$, is a fuzzy set in the two dimensional space of $X \times Y$. Its membership function is also defined for each ordered pair $(x, y)$, and is shown by $\mu_R(x, y)$. Using integral form, it is:

$$ R = \int_{X \times Y} \mu_R(x,y) / (x,y) $$

$$ 0 < \mu_R(x,y) < 1. $$

This concept can be generalized to an $n$-ary fuzzy relation between $n$ sets, $X_1, X_2, \ldots, X_n$. In this case,

$$ R = \int_{X_1 \times X_2 \times \ldots \times X_n} \mu_R(x_1,x_2,\ldots,x_n) / (x_1,x_2,\ldots,x_n) $$

$$ 0 < \mu_R(x_1,x_2,\ldots,x_n) < 1. $$

Composition of Fuzzy Relations: If two fuzzy relations $R_1$ and $R_2$ are defined in $X \times Y$ and $Y \times Z$ respectively, then the composition of $R_1$ and $R_2$ shown by $R_1 \circ R_2$ gives the membership function for $X \times Z$ according to the following rule:

$$ R_1 \circ R_2 \leftrightarrow \mu_{R_1 \circ R_2} = \bigvee_y \left[ \mu_{R_1} \land \mu_{R_2} \right]. $$
This is called “max-min composition”. In fact, it is similar to the multiplication of two matrices, one with a dimension of \(X \times Y\), and the other with dimension of \(Y \times Z\), where the result is a matrix of dimension \(X \times Z\). The difference is that matrix multiplication has been changed to fuzzy intersection \(\wedge\), and matrix summation to fuzzy union \(\vee\).

Other forms of composition can be considered too. The “min–max composition” can be formed by interchanging \(\vee\) and \(\wedge\) in the “max–min composition”. Another very useful form is called the “max–star composition”, which is shown as follows:

\[
R_1 \cdot R_2 \leftrightarrow \mu_{R_1 \cdot R_2} = \bigvee_y \left[ \mu_{R_1} \ast \mu_{R_2} \right]
\]  

(4.61)

where \(\ast\) can be any operation on fuzzy sets.

Generalizations to more complicated forms like \(R_1 \circ (R_2 \circ R_3)\), and “\(n\)-fold compositions” shown by \(R \circ R \ldots \circ R\) have been made, too.

### 4.7.2 Basic Definitions Used In The Construction Of Fuzzy Relations

All of the operations for the one dimensional fuzzy sets are valid for fuzzy relations too. If \(R_1\) and \(R_2\) are two fuzzy relations in \(X \times Y\), then the basic definitions are:

**complement set:**

\[
\overline{R}_1 \leftrightarrow \mu_{\overline{R}_1}(x,y) = 1 - \mu_{R_1}(x,y)
\]  

(4.62)

**inclusion:**

\[
R_1 \subseteq R_2 \leftrightarrow \mu_{R_1}(x,y) \leq \mu_{R_2}(x,y)
\]  

(4.63)
union:

$$R_1 \cup R_2 \iff \mu_{R_1 \cup R_2} = \mu_{R_1}(x,y) \lor \mu_{R_2}(x,y)$$ (4.64)

intersection:

$$R_1 \cap R_2 \iff \mu_{R_1 \cap R_2} = \mu_{R_1}(x,y) \land \mu_{R_2}(x,y) .$$ (4.65)

The following definitions are also used in fuzzy set theory. They are defined for a fuzzy relation $R_1$ in $X \times Y$:

Identity relation $I$:

$$\mu_I(x,y) = \begin{cases} 1 & ; x = y \\ 0 & ; x \neq y \end{cases}$$ (4.66)

Zero relation $0$:

$$\mu_0(x,y) = 0 .$$ (4.67)

Universe Relation $E$:

$$\mu_E(x,y) = 1 \; \forall \; x,y \in X \times Y .$$ (4.68)
4.8 FUZZY REASONING

Fuzzy reasoning is the heart of every fuzzy intelligent system. For fuzzy reasoning about a phenomenon, some knowledge about the phenomenon should be collected. This knowledge should then be used in the construction of rules, in the form of “if–then” expressions. Now for a given “if” part and based on the available knowledge, conclusion should be made about the “then” part.

To do a fuzzy reasoning, these steps should be followed:
1. Collection of data about the process, and determination of different variables which are effective in the reasoning procedure.
2. Construction of “if–then” expressions, which are called “premises”.
3. Quantification of meanings, by which, membership functions for variables are defined.
4. Translation of the premises into fuzzy relations by choosing appropriate “implication rules”.
5. Conclusion making, based on the projection of the given “if” expressions on the fuzzy relations, by using “fuzzy inference rule”.

The first step, collection of data is very important for the construction of a fuzzy reasoning system. However it is not limited to fuzzy logics and is a common step in all the data based problems. Hence, in the following sections, the other steps which are directly related to the fuzzy logic are discussed briefly.

4.8.1 Construction Of Premises

The “if–then” expressions, are called premises. The simplest form of premises, which we call it the “first form of premises”, is as follows

\[
\text{if}: \ x \text{ is } A \quad \text{then}: \ y \text{ is } C. \tag{4.69}
\]

where \( x \) and \( y \) are variables and \( A \) and \( C \) are fuzzy variables. Given this premise as
the available knowledge, and a questioner “if” statement, conclusion should be made. i.e.:

\[
\begin{align*}
\text{what}, \text{ if: } x & \text{ is } A' \\
\text{Conclusion: } \text{ then: } y & \text{ is } C'.
\end{align*}
\]

(4.70)

So, the question is that of finding another fuzzy set, \( C' \). In some of the problems, \( A \) and \( A' \) are the same. Hence, by common knowledge \( C \) and \( C' \) should be the same too, which is called “modus ponens”. But there are other problems for which \( A \) and \( A' \) are not the same, although related. As an example, a modus ponens problem is:

\[
\begin{align*}
\text{if: acceleration is large } & \text{ then: control force is small.} \\
\text{what, if: acceleration is large}
\end{align*}
\]

(4.71)

A non–modus ponens problem, may be as follows:

\[
\begin{align*}
\text{if: acceleration is large } & \text{ then: control force is small.} \\
\text{what, if: acceleration is very large}
\end{align*}
\]

(4.72)

Another very common form of premises, which is used in control problems very often, and can be called the “second form of premises”, is as follows:

\[
\begin{align*}
\text{if: } x_1 \text{ is } A & \text{ and } x_2 \text{ is } B \text{ then: } y \text{ is } C. \\
\text{what, if: } x_1 \text{ is } A' & \text{ and } x_2 \text{ is } B'
\end{align*}
\]

(4.73)

If \( A' \) and \( A \), and \( B' \) and \( B \) are the same, it is called “modus ponens”. 
A more complicated form of premises, which we call it the "third form of premises" is as follows:

\[
\begin{align*}
\text{if: } & \ x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } B_1 \quad \text{then: } y \text{ is } C_1, \quad \text{else} \\
\text{if: } & \ x_1 \text{ is } A_2 \text{ and } x_2 \text{ is } B_2 \quad \text{then: } y \text{ is } C_2, \quad \text{else} \\
\quad & \quad \ldots \\
\text{if: } & \ x_1 \text{ is } A_n \text{ and } x_2 \text{ is } B_n \quad \text{then: } y \text{ is } C_n, \\
\text{what, if: } & \ x_1 \text{ is } A' \text{ and } x_2 \text{ is } B' \\
\end{align*}
\]

Conclusion: \quad \text{then: } y \text{ is } C'.

For all the three forms, the fuzzy sets \( A', B', \) and \( C' \), and also all the fuzzy sets \( A_i, B_i \) and \( C_i \) belong to the support sets \( X, Y \) and \( Z \), respectively.

4.8.2 Quantification Of Meanings

Selecting suitable membership functions is the next important step. Selection of wrong membership functions may result in divergence from good results. The best point to start is by using very simple common knowledge about the process and the variables, and then elaborating the membership functions gradually.

4.8.3 Fuzzy Implication Rules

There are a number of widely used rules for translating the fuzzy premises into fuzzy relations. These rules have similar but different forms for the three above forms of premises. The most widely used fuzzy implication rules are due to Zadeh (1974) and Mamdani (1974). These rules are given here for each of the aforementioned premises.
Fuzzy implication rule for the first form of premises: This translation is shown by \( A \rightarrow B \).

Zadeh’s rule: is shown here by \( R_z \), and has the following property:

\[
R_z = A \rightarrow C = \int_{X \times Y} 1 \land (1 - \mu_A(x) + \mu_C(y))/(x,y) . \tag{4.75}
\]

Mamdani’s rule: is shown by \( R_m \) and has the following property:

\[
R_m = A \rightarrow C = A \land C = \int_{X \times Y} \mu_A(x) \land \mu_C(y)/(x,y) . \tag{4.76}
\]

Fuzzy implication rule for the second form of premises: This translation is shown by \( A \land B \rightarrow C \).

Zadeh’s rule: is shown here by \( R_z(A, B; C) \), and has the following property:

\[
R_z(A, B; C) = A \land B \rightarrow C = \int_{X_1 \times X_2 \times Y} 1 \land (1 - (\mu_A(x_1) + \mu_B(x_2)) + \mu_C(y))/(x_1, x_2, y) . \tag{4.77}
\]

Mamdani’s rule: is shown by \( R_m(A, B; C) \), and has the following property:

\[
R_m(A, B; C) = A \land B \rightarrow C = \int_{X_1 \times X_2 \times Y} \mu_A(x_1) \land \mu_B(x_2) \land \mu_C(y)/(x_1, x_2, y) . \tag{4.78}
\]
Fuzzy implication rule for the third form of premises: This translation is the union of \( n \) translations of the second form. First, each of the \( n \) given premises are translated into the second forms, using either Zadeh’s or Mamdani’s proposed rules. This results in \( n \) fuzzy relations, as follows:

\[
A_i \cap B_i \rightarrow C_i , i = 1, 2, \ldots, n .
\] (4.79)

Then, because the premises are connected by “else” terms, the union of these \( n \) fuzzy relations, provides the final implication rule. i.e.:

\[
R = ( A_1 \cap B_1 \rightarrow C_1 ) \cup ( A_2 \cap B_2 \rightarrow C_2 ) \cup \\
\ldots \\
\ldots \\
\cup ( A_n \cap B_n \rightarrow C_n ) .
\] (4.80)

4.8.4 Fuzzy Inference Rules

After the determination of fuzzy relation, conclusion can be made by using fuzzy inference method. Several rules of inference have been proposed, among which the max– \( \ast \) composition is the most studied one. The max– \( \ast \) composition, for the three above mentioned forms of premises is as follows:

\[
C' = A' \ast ( A \rightarrow C )
\] (4.81)

\[
= \bigvee_x \left[ \mu_{A'}(x) \ast \mu_{R}(x, y) \right]
\]

which is obtained for the first form of premises,
\[ C' = (A' \cap B') \star (A \cap B \rightarrow C) \]

\[ = \bigvee_{x_1,x_2} \left\{ \mu_{A'} \cap B'(x_1,x_2) \star \mu_{R}(x_1,x_2,y) \right\} \]

which is obtained for the second form of premises,

\[ C' = (A' \cap B') \star \left[ (A_1 \cap B_1 \rightarrow C_1) \bigcup \right. \]

\[ \left. (A_2 \cap B_2 \rightarrow C_2) \bigcup \ldots \bigcup (A_n \cap B_n \rightarrow C_n) \right] \]

which is used for the third form of premises.

The \( \star \) in the above equations can be any fuzzy algebraic operation, like \( \land \), for which it is called the “max–min composition rule of inference”. If a dot product is used, it is called “max–product composition”.

The above premises can be generalized to more complicated forms, where more than two variables are considered in the same premise. The above forms are sufficient for our application purposes,

As mentioned above, by common sense, the modus ponens should be satisfied for the above inference rules. When using the max–min compositional inference rule, the implication rule proposed by Zadeh does not satisfy modus ponens, while the implication rule proposed by Mamdani does. There are other rules of inference, like “max–bounded product” and “max–drastic product”, which can be applied to Zadeh’s implication rule to satisfy modus ponens. These composition rules are more complicated than the max–min.

4.9 FUZZY LOGIC IN CONTROL

One of the fields, to which the theory of fuzzy logic was first applied, is the control field. There are many factors and parameters involved in a complicated control problem.
In some of the control problems, the effect of these factors and parameters are almost known, but not clearly. The idea behind the fuzzy control is to use the available knowledge about the problem, to construct a number of fuzzy rules which can be used in the control of the plant. This knowledge can be obtained through different approaches like:

1. An expert who controls the plant, expresses how to control it. Thus, the control rules should to be found based on his/her expressions.

2. The expert, knows how to control, but cannot explain this knowledge. In other words, the expert has acquired an implicit knowledge about the control of the process. In this case, one should observe and monitor the expert when he/she is controlling the process, and then extract the fuzzy control rules from these observations.

3. Beginning to construct the rules, from the very small, but common knowledge, and finding the required fuzzy rules of control, by trial and error.

4. By providing and using a fuzzy model of the controlled process.

5. Self-organization of the fuzzy controller.

After finding a set of suitable rules, a “fuzzy inference method” should be used to extract the control commands from the set of fuzzy rules.

4.9.1 Fuzzy Control Rules

A fuzzy rule is a fuzzy premise. It is an “if–then” expression. The “if” clause is called “antecedent” and the “then” clause, the “consequent”. In most of the control problems, a set of fuzzy rules should be found. They constitute the knowledge of the controller. For this study, using the third form of premises, the fuzzy rules are:
if: \( x_1 \) is \( A_1 \) and \( x_2 \) is \( B_1 \) \text{ then: } y \text{ is } C_1, \text{ else}
\[
\text{if: } x_1 \text{ is } A_2 \text{ and } x_2 \text{ is } B_2 \text{ then: } y \text{ is } C_2, \text{ else}
\]
\[
\ldots \ldots
\]
\[
\text{if: } x_1 \text{ is } A_n \text{ and } x_2 \text{ is } B_n \text{ then: } y \text{ is } C_n.
\] (4.84)

where \( x_1 \) and \( x_2 \) are called the “input variables”, \( y \) is called the “output variable”, and the fuzzy sets \( A_i, B_i, \) and \( C_i \) are called the “fuzzy values”.

### 4.9.2 Fuzzy Inference Method

Subject to each ordered pairs of input variables \( (x_1^*, x_2^*) \), the required output \( y^* \), is to be determined, by using the available fuzzy control rules. To this end, many inference methods, have been proposed. The general algorithm is to:

1. Use the implication rules, to construct the whole space of fuzzy relation \( R \).
2. Find \( C^*(y) \), the cross section of \( R \), with \( (x_1^*, x_2^*) \) :

\[
C^*(y) = \mu_R(x_1^*, x_2^*, y).
\] (4.85)

3. Find the required output \( y^* \). This can be done by taking the first moment of inertia of \( C^*(y) \) :

\[
y^* = \int_y C^*(y) \, y \, dy / \int_y C^*(y) \, dy.
\] (4.86)
This algorithm requires the construction of a table, to keep the information about a three dimensional function \( R = R(x_1, x_2, y) \) which needs a lot of memory. For practical reasons, it is possible to compute the implication rules for the case where \( (x_1, x_2) = (x_1^*, x_2^*) \) and find \( C^*(y) \) directly from:

\[
C^*(y) = (A_1(x_1^*) \cap B_1(x_2^*) \rightarrow C_1(y)) \cup (A_2(x_1^*) \cap B_2(x_2^*) \rightarrow C_2(y)) \ldots \cup (A_n(x_1^*) \cap B_n(x_2^*) \rightarrow C_n(y))
\]  

(4.87)

which requires less memory. Thereafter, \( y^* \) can be computed from equation (4.86).

### 4.9.3 A Simple Fuzzy Inference Method

Simpler forms of fuzzy rules and inference methods are proposed for practical purposes. A suitable form can be obtained by assuming that the output \( y \) in each of the control rules is a constant value instead of a variable one. Also, if two rules return the same value for \( y \), it is theoretically possible to assume that they are different by a very small value. By this technique, the “else” terms which bind the rules to each other will disappear. The form of the control rules, will then be as follows:

\[
\begin{align*}
\text{if: } & x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } B_1 \quad \text{then: } \ y = y_1, \\
\text{if: } & x_1 \text{ is } A_2 \text{ and } x_2 \text{ is } B_2 \quad \text{then: } \ y = y_2, \\
& \ldots \ldots \\
& \ldots \ldots \\
\text{if: } & x_1 \text{ is } A_n \text{ and } x_2 \text{ is } B_n \quad \text{then: } \ y = y_n,
\end{align*}
\]  

(4.88)

Now, for each of the rules, the membership can be obtained from the fuzzy relations:
\[ R_i = A_i \rightarrow B_i \quad , \quad i = 1, 2, \ldots, n . \] (4.89)

It is obvious that according to the implication rules, each of these fuzzy relations return only one value for each ordered pair \((x_1^*, x_2^*)\). For example, using Mamdani’s implication rule, the result will be:

\[ \mu_{R_i} = \mu_A(x_1^*) \land \mu_B(x_2^*) \quad , \quad i = 1, 2, \ldots, n . \] (4.90)

And the result of inference will be:

\[ y^* = \sum_{i=1}^{n} \mu_{R_i} \frac{y_i}{\sum_{i=1}^{n} \mu_{R_i}} . \] (4.91)

This form of the fuzzy control, has been used in this thesis.

REFERENCES


Applications, North Holland.


PART TWO

ACTIVE CONTROL OF STRUCTURES
BY USING
NEURAL NETWORKS AND FUZZY LOGIC
CHAPTER 5
METHODOLOGY
AND PROVISIONS FOR A NUMERICAL STUDY

In the previous chapters, literature and foundations required for building new intelligent control methods were reviewed. Also the classical and modern control theories were reviewed. Because these methods are formulated mathematically, they were called the "formulated control methods", to distinguish them from the other methods which can be called the intelligent, smart or preferably the "learning control methods" which demonstrate the use of adaptive learning systems. The category of learning control covers both the neural network and the fuzzy logic based control methods and their combinations too.

In this chapter a method for using neural networks and fuzzy logic in structural control is proposed. Also the computer programs which have been developed for a series of numerical and simulation studies are explained. The following chapters cover details about the method and results of its application to the control of a typical frame structure.

Among the softwares which have been developed for the accomplishment of this study, there is a computer program for the simulation of a plane frame structure with actuators, called "FRAME_ACTUATOR_DYNAMICS", and also a computer program for the simulation of artificial supervised multi-layer feed-forward backpropagation neural networks, called "SUNN". Other computer programs can be considered as secondary. They have been developed to provide data for graphical purposes, arrangement of results, etc.

5.1 BASIC IDEA OF STRUCTURAL CONTROL BY USING NEURAL NETWORKS

The idea of using neural networks in control of structures arises from the fact that neural networks are capable of learning mapping and generalization problems. Multi-
layer feed-forward neural networks which are the most widely used neural networks have been utilized in these series of studies. These neural networks enjoy the added benefit of being supported by a strong and well established convergence theorem, which proves that they can be trained to learn any mapping problem to within a desired accuracy (Hornik 1991, Blum and Li 1991). So, these neural networks are considered as the universal approximators.

The basic concept in using neural networks in structural control is to train a neural network to act as a controller to replace the control algorithm. This neural network is called the “neuro-controller” and the method is called the “neuro-control” method. A typical neuro-controller is shown in figure 5.1 in a closed-loop control scheme. In this example, the neuro-controller should control the structure by using one actuator only. Sensors measure the response of the structure at a number of selected degrees of freedom. The sensor readings and the actuator signal are provided as the input to the neuro-controller. The input to the neuro-controller also includes the immediate past history of the response of the structure and the actuator signals. The output of the neuro-controller is the next value of the actuator signal which should be sent to the actuator to produce the required actuator force. Since there is only one actuator involved in these studies, the output layer contains only one unit whose activation is the control signal to the actuator. It is obvious that appropriate neuro-controllers can be trained to control structures by several actuators. In such cases, the number of units in the output layer of the neuro-controller is equal to the number of actuators.

The training of feed-forward neural networks takes place in a supervised fashion. This means that the training data set consists of input/output pairs. However, the output of the neuro-controller is the actuator signal and the correct value of this quantity is not known. Therefore, the direct training of the neuro-controller is not possible. A number of methods have been proposed for the training of the neuro-controllers in other control applications. These methods, which can also be used in structural control problems, are listed below.

1. The neuro-controller can learn the control function from another controller. This has some applications in robotics and autonomous vehicles, where the neuro-controller
learns the control task from a skilled operator. However, this is not a very attractive option in structural control problems. If a neuro-controller is trained with the aid of another control algorithm, then the neuro-controller can only do as well as the control algorithm and no better. The only application of this method in structural control problems is in the initial training of the neuro-controller which will be further trained with other methods.

2. The second option is to use an inverse emulator as the neuro-controller. First an inverse emulator neural network is trained. The input to this neuro-controller is the re-
sponse of the structure and the output is the actuator signal. The training cases are generated by actuating the structure and recording the sensor readings. When the trained inverse emulator is used as the controller, a part of its input is the desired response of the structure (reference input) while the output of the neural network is the actuator signal required to produce that result. This has similarities to the open loop control procedure. Some applications of this option can be found in Nikzad (1992) and Nikzad and Ghaboussi (1991).

3. The neuro-controller is trained with the direct aid of an emulator neural network. The emulator neural network provides a means of determining the rate of change of the actuator signal with respect to the response of the structure. This is used to determine the error at the output of the neuro-controller, which should be backpropagated to the controller neural network. This method has been adopted in this study and it will subsequently be discussed in more detail.

5.2 ADVANTAGES OF USING NEURAL NETWORK BASED CONTROL METHODS

Three characteristics of a structural control method should be noticed for the assessment of its capabilities. These three characteristics are: its degree of adaptivity, its prediction capability and use of these predictions for controlling the structure, and its simplicity.

Adaptivity: A control method which is more adaptive to different control situations is generally more capable than the other methods. A successful control method is able to cope with nonlinearities, delays and imperfections in the controlled structure. The formulated control methods are based on the provision of a simple mathematical model of the controlled structure, mostly a linear model. These methods, including different linear optimal control methods and the pole assignment method were briefly reviewed in chapter 2. Hence, a precise identification of the structure and also the control mechanism including all the possible nonlinearities and delays and imperfections are required when using these methods. Such identification may be very tedious and time consuming and may result in sophisticated mathematical models. Assumptions like linearity of the system may be suit-
able and realistic for simple structural control problems, but are obviously not compatible with the majority of practical structural control problems. Efforts have been made to provide more adaptive control methods for structural control. Methods such as predictive optimal control or pulse control have been proposed and studied for simple problems. They have shown good performance however the range of their applicability to more complicated problems is still a question which is under study.

It is supposed that by using the learning capability of neural networks, it is possible to construct more adaptive structural controllers. The emulator neural network replaces the mathematical model of the structure. It can be trained to learn about all the nonlinearities, time delays and imperfections implicitly. This knowledge of the emulator neural network is then transferred to the neuro-controller implicitly, by the use of the proposed method.

**Prediction capability:** A successful control method should be capable of using in the construction of control signals some sort of predictions about the future response of the controlled structure. Some of the formulated control methods use the mathematical model of the controlled structure for the sake of prediction. As mentioned in the last section, these models are generally not realistic for complicated structural control problems. Hence the prediction capability of the formulated control methods is vulnerable to the modelling errors. Because the emulator neural network learns about the real behavior of the controlled structure, it can provide more reliable predictions. Thus, it is expected that a neural network based control method is able to provide and use better predictions and consequently perform a better control job.

**Simplicity:** Simplicity plays an important role in the success of a control method. As mentioned before, emulator neural network replaces the mathematical model of the control system. This provides significant simplification when the structural behavior is complicated and identification of the system is cumbersome. Also while the proposed method is generally applicable to all the structural control problems, there is not such degree of generality associated with any of the formulated control methods. This generality of the control method is a suitable measure of its simplicity.

These features of the neuro-control method of structures are investigated in the fol-
5.2.1 Methodology Of Structural “Neuro-Control”

In this section, the proposed method for the construction of an appropriate neuro-controller for the control of structures is explained in more details. After the construction of the neuro-controller, it can be used in the control of the structure. However, it can frequently be trained for more improvements and also for adaptivity to the time dependent changes in the parameters of structure, control mechanism, etc. The following steps should be accomplished for the preparation of an appropriate neuro-controller:

First, training of an emulator neural network which is capable of predicting the future response of the structure from its immediate history of response and control signals.

Second, using this emulator to alleviate the undesired deformations of the structure, according to a suitable criteria, roughly.

Third, Training a neural network to extract the general knowledge about controlling the structure form the training cases obtained in the second step. This neural network is called the “neuro-controller”, as mentioned before.

Fourth, using controller and emulator neural networks together to improve the control results and provide new training cases for the training of a new neuro-controller.

Fifth, training a new neuro-controller (or retraining the old neuro-controller) to learn about the improved control action from the new training cases.

Steps four and five can be omitted or repeated for many times, depending on the required control quality.
5.2.2 Neuro–Fuzzy Control Of Structures

While the main controller is the neuro–controller, use of an additional fuzzy logic based controller can render help neuro–controller to overcome unexpected situations. This strategy has been studied in this research, and has been dramatically effective in improving the control results. The resulting controller, comprised of a neuro–controller and a fuzzy controller is called a “neuro–fuzzy controller” in this study. Chapter 7 contains detailed explanations of this method and its application to the control of the frame structure.

As can be seen in the following chapters, the proposed general control strategy requires special attention in the selection of control criteria and also the determination of the required training time of the neural networks. Hence the neural network related issues have been considered important in this study. Improvements on some of the neural network related issues, have been strongly effective in the enhancement of learning capabilities of the emulator and controller neural networks. These issues have been qualitatively studied in this research.

5.3 COMPUTER PROGRAM “FRAME_ACTUATOR_DYNAMICS”

A computer program has been developed for the dynamic analysis of plane frame structures which should be controlled by an active tendon control mechanism, actuated by electro–hydraulic servo–valve actuators. These type of actuators are mechanical devices, and they should be considered as parts of the dynamical system for a realistic simulation of the whole controlled system. So, in the numerical simulation of the dynamic response of the controlled structure, it was considered important to include the effects of the actuator dynamics, the sampling period and the inherent delays in the control loop. The dynamics of electro–hydraulic actuators are defined by two differential equations describing the dynamics of the valve and the ram. The input to these two differential equations is the control signals issued to the actuators. However dynamics of the actuators is coupled with dynamics of the structure. Therefore, it is necessary to solve the equations of motion of the structure and the equations of the dynamics of the actuators simultaneously. In addition to the external forces, the actuator signals should be considered as
input to these equations. The actuator signals are issued at regular time intervals, called the sampling period, which is related to the inherent time delay in the control loop. The signals issued to the actuators are in the form of step functions where step length is equal to the sampling period. For the purpose of this study, a computer program has been developed for the simulation of a frame structure, controlled by servo-valve actuators. This program has been called “FRAME_ACTUATOR_DYNAMICS”.

This program is a general frame dynamic analysis program and the beams and columns of the frame are treated as flexible members. So, the displacement vector at each point of the structure is comprised of a horizontal displacement, a vertical displacement and a rotation relative to the base of the structure which has been considered fixed. The time domain and s-domain block diagrams of the mechanical system, comprising of an actuator and the structure are shown in figures 5.2a and 5.2b respectively. The parameters and variables in this figure are:

\[
\begin{align*}
K_1 & = \text{amplifier gain (non-dimensional scalar)} \\
K_2 & = \text{servo-valve gain (discharge rate/voltage signal)} \\
K_3 & = \text{transducer gain (voltage signal/force)} \\
G(s) & = \text{servo-valve transfer function.} \\
v(t) & = \text{control signal (volts) in time domain.} \\
V(s) & = \text{control signal (volts) in s-domain.} \\
q(t) & = \text{discharge from servo-valve in time domain.} \\
Q(s) & = \text{discharge from servo-valve in s-domain.} \\
f(t) & = \text{force, applied by the actuator on the structure, in time domain.} \\
F(s) & = \text{force, applied by the actuator on the structure, in s-domain.}
\end{align*}
\]

5.3.1 Dynamics Of The Actuator

Electro-hydraulic servo-valve actuators have been selected for the force control purposes. An electrohydraulic servo-valve actuator has two parts, the servo–valve and the
Figure 5.2 Block diagram of the structure–actuator system (a) Time domain (b) s-domain.

In the following sections, these two parts will be studied in detail. More information can be obtained from Walters (1967), Guillon (1969) and Pippenger (1984).

**Dynamics of the servo–valve:** can be characterized by a first order linear ordinary differential equation:
\[ \nu(t) = a_1 \nu(t) + a_2 \nu(t) \quad t > t_0 , \] (5.1)

subjected to:

\[ \nu(t = t_0) = \nu_0 \] (5.2)

and

\[ \nu(t) = \nu_0 \quad t > t_0 , \] (5.3)

where \( a_1 \) and \( a_2 \) are two constant values representing properties of the servo-valve, and equation (5.3) states that the control signals are in the form of step functions. Solution of this equation results in:

\[ q(t) = \frac{\nu_0}{a_2} + \left( -\frac{\nu_0}{a_2} + \nu_0 \right) e^{-\frac{\nu_0}{a_1}} \] (5.4)

Since a first order differential equation governs the behavior of the servo-valve, the gain of the servo-valve should have the following form:

\[ G(s) = \frac{1}{s + \frac{1}{\tau}} \] (5.5)

where \( \tau \) is the time constant which is a characteristic of the servo-valve. It is easy to find the constants \( a_1 \) and \( a_2 \) in terms of gains of the system \( K_1 \) and \( K_2 \) and the time constant \( \tau \) of the servo-valve as follows. In digital control, the actuator signals are sent at regular time intervals during which the signal \( \nu(t) \) is held constant. Taking Laplacian
of equation (5.4), and noticing that for the constant signal \( v(t) = v_0 \),

\[
V(s) = L[v(t) = v_0] = \frac{v_0}{s} \tag{5.6}
\]

then:

\[
Q(s) = \frac{v_0/s}{a_2} - \left( \frac{v_0}{a_2} + q_0 \right) \left( \frac{1}{s + a_2/a_1} \right) = \frac{v_0}{a_2} \frac{a_2/a_1}{s + a_2/a_1} - \frac{q_0}{s + a_2/a_1} = \frac{V(s)}{a_2} \frac{a_2/a_1}{s + a_2/a_1} - \frac{q_0}{s + a_2/a_1} \tag{5.7}
\]

Also from equation (5.5) and figure 5.2b,

\[
Q(s) = K_1 K_2 G(s) \left( V(s) - K_3 F(s) \right) = K_1 K_2 V(s) \frac{1}{s + \frac{1}{\tau}} + \ldots \tag{5.8}
\]

By comparison of equations (5.7) and (5.8),

\[
a_1 = \frac{1}{K_1 K_2} \tag{5.9}
\]

\[
a_2 = \frac{1}{K_1 K_2 \tau} .
\]

**Dynamics of the ram:** can be characterized by the first order linear differential equation:

\[
q(t) = \frac{C}{A} \dot{x}_r + \frac{V}{\beta A} \dot{u}(t) \tag{5.10}
\]
where:
\( x_r(t) \) = ram displacement (length)
\( q(t) \) = discharge (volume/time)
\( u(t) \) = actuator force (force)
\( A \) = area of ram (area)
\( C \) = coefficient of leakage (length^5 / (force \cdot time))
\( V \) = volume of the chamber (volume)
\( \beta \) = compressibility (pressure)

Equation (5.10), relates the actuator force to the ram displacement and discharge of the servo-valve. Since the actuator is connected to the structure, the simulated ram displacement should be kinematically compatible with the structure displacement vector, which results in the coupling of structure and actuator dynamics. This coupling can be expressed by the following equation:

\[
x_r(t) = r_u^T z(t)
\] (5.11)

where \( r_u \) is a constant vector representing kinematical relation and \( z(t) \) is the structural displacement vector. Thus equation (5.10) can be written as:

\[
q(t) = A r_u^T z(t) + \frac{C}{A} u(t) + \frac{V}{\beta A} \dot{u}(t)
\] (5.12)

5.3.2 Dynamics Of The Structure

The structure dynamics can be simulated by:

\[
M \ddot{z}(x) + D \dot{z}(t) + K z(t) = p(t) + u(t)
\] (5.13)

where:
\( M \) = mass matrix
\[ D = \text{damping matrix} \]
\[ K = \text{stiffness matrix} \]
\[ z(t) = \text{displacement vector} \]
\[ p(t) = \text{external loading vector} \]
\[ u(t) = \text{control vector, applied by the actuators} \]

Equations of motion for the structure and the ram should be put together and solved by numerical methods simultaneously.

### 5.3.3 The Coupled Equations Of Motion

In this study, it has been possible to use the symmetrical characteristics of the model structure to reduce the problem to the control of a frame structure, controlled by one actuator only. Hence, this special case is explained here. The general formulation for the control of frame structures by many actuators is presented in the following sections, too.

**The case of one actuator:** When one actuator is involved, the coupled equation of motion can be obtained by considering \( u(t) \) as an additional degree of freedom of the system. In this case, dynamics of the structure is characterized by:

\[
M \ddot{z}(x) + D z(t) + K z(t) = p(t) + u(t) l_u
\]

where \( l_u \) is the location matrix. Hence the coupled equation is:

\[
\begin{bmatrix}
M & 0 \\
0 & 0
\end{bmatrix}
\begin{Bmatrix}
\dot{z} \\
\dot{u}
\end{Bmatrix} +
\begin{bmatrix}
D & 0 \\
A r_u^T & d
\end{bmatrix}
\begin{Bmatrix}
z \\
u
\end{Bmatrix} +
\begin{bmatrix}
K - l_u \\
0 & k
\end{bmatrix}
\begin{Bmatrix}
z \\
u
\end{Bmatrix} =
\begin{Bmatrix}
p \\
q
\end{Bmatrix}
\]

where:
\[
d = V/\beta A
\]
\[
k = C/A
\]
The program "FRAME_ACTUATOR_DYNAMICS" can solve the above set of algebraic equations of motion by the Wilson's $\Theta$ integration method.

**The case of many actuators:** In the case of many actuators, it is easier to define a kinematic vector and a location vector for each actuator. Then for $m$ actuators,

$$x^i(t) = r^i_u T z(t) \quad i = 1, 2 \ldots , m$$

(5.16)

where $r^i_u T , i = 1, 2, \ldots , m$ are the kinematical relation vectors. The above equation defines the relationship between $x^i_r$ the ram displacement of actuator number $i$ and the displacement vector of the structure $z(t)$.

The equation of motion of the structure is then:

$$M \mathbf{z}(t) + D \mathbf{z}(t) + K \mathbf{z}(t) = p(t) + \sum_{i=1}^{i=m} u_i(t) l^i_u$$

(5.17)

where $l^i_u, i = 1, 2, \ldots , m$ are the location matrices. Thus, the equations of motion of the whole structure and actuators will be:

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} D & 0 \\ R^T_u & D_u \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} + \begin{bmatrix} K - l^i_u \\ 0 & K_u \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

(5.18)

where $u(t) , R^T_u , D_u , K_u$ and $q(t)$ are the actuator force vector, kinematics matrix, compressibility matrix, likage matrix and servo valve discharge vector, respectively and defined as:

$$u(t) = [ u_1(t) \; u_2(t) \; \ldots \; u_m(t) ]^T$$

(5.19)
\[ R_u^T = [ A_1 r_{1u}^1, A_2 r_{2u}^2, \ldots, A_m r_{mu}^m ]^T \] (5.20)

\[ D_u = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix} \quad ; \quad d_i = \frac{V_i}{\beta_i A_i} \] (5.21)

\[ K_u = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix} \quad ; \quad k_i = \frac{C_i}{A_i} \] (5.22)

\[ q(t) = [ q_1(t), q_2(t), \ldots, q_m(t) ]^T \] (5.23)

where, for \( i = 1, 2, \ldots, m \),
- \( u_i(t) \) = actuator force number \( i \) (force)
- \( q_i(t) \) = discharge of servo-valve number \( i \) (volume/time)
- \( A_i \) = area of the ram of actuator number \( i \) (area)
- \( C_i \) = coefficient of leakage of actuator number \( i \) (length\(^5\) / (force . time))
- \( V_i \) = volume of the chamber of actuator number \( i \) (volume)
- \( \beta_i \) = compressibility of actuator number \( i \) (pressure).

### 5.4 COMPUTER PROGRAM “SUNN”

A computer program has been developed to simulate a multi-layer feed-forward backpropagation neural network. Because its training takes place in a supervised manner,
it is classified as a SU pervised Neural Network, abbreviated by “SUNN”. Details of this
type of neural networks have been explained in chapter 3. The quickprop learning algo­
rithm proposed by Fahlman (1988) which is an improved version of the backpropagation
by generalized delta rule (Rumelhart and McClelland 1986) has been utilized for updat­
ing the connection weights. Growing mechanism as explained in chapter 3 has been
adopted to let the structure grow, for the sake of escaping from local minima and improve­
ment of convergence.

Theoretically, there is no doubt in the capability of such neural networks in learning
a required mapping and generalization problem. However practically, the speed of the
neural network simulation program is the key to the success of the proposed control meth­
ods. So, in this study considerable effort has been devoted to the improvement of the
learning algorithm and the convergence speed of the program. Although the computer
program has been modified frequently and improved gradually to be faster and also ac­
quire more capabilities within its original framework, the prominent progress has been
achieved after implementing the following items:

1. A mechanism for the random selection of training cases: With this mechanism the training
of the neural network is acheived in a batch mode (chapter 3). The training sub–batch
increases gradually by the addition of a number of new training cases to the previous sub–
batch of training cases. The neural network should learn the training cases in the new sub–
batch. The training cases are selected randomly from the new sub–batch in each training
cycle of the neural network. By using this mechanism a faster search in the space of con­
nection weights is possible. The probability of the neural network to reside in a local mini­
mum reduces and a faster convergence is achieved.

2. A criteria for the evaluation of convergence behavior and the addition of new units: An expert
block, comprising of a number of “if–then” statements have been developed and imple­
m ented in the computer program. The basic idea behind this criteria is to assess the conver­
gence speed of the neural network in learning the training cases in the current training
sub–batch. In this approach, the convergence speed is evaluated by counting the number
of training cycles. If this number grows to a limit called the “permitted cycles”, a number
of new nodes are added to the neural network, introducing more connection weights to
it, increasing its capacity and releasing it from local minimum. The number of units to be added can be controlled by the user. Also a number, which is called the “basic permitted cycles” should be introduced by the user as the lower limit for the permitted cycles. Permitted cycles, however, varies during the training of neural network in accordance with the following criteria.

3. A criteria for the determination of permitted cycles: An expert block consisting of “if–then statements” controls the “permitted cycles”. Aim of utilizing this block is to let the neural network use its learning capability before the addition of new nodes, as much as possible. Since after the addition of new nodes the architecture of the neural network is modified, it may take more time (training cycles) for the neural network to reside into a new local minimum. Hence, the “permitted cycles” is increased through its multiplication by a number, greater than 1.0, which should be introduced by the user in the input file. Training proceeds and after the addition of new training cases to the sub–batch and increasing its size, the number of “permitted cycles” reduces to the basic permitted cycles gradually. By implementing this criteria, a better control has been achieved on the determination of situations where additional units were required.

The three above mentioned improvements dramatically changed the status of the research. The new version of the program has been tested for different problems, not only related to this research, but also in relation to other projects. Since the focus of this study is not on the preparation of information and reporting the results of the neural network related issues, a qualitative study and observation has been made. The following qualitative conclusions can be mentioned here:

1. The required training time has been reduced for all the test problems. For mapping problems of smaller number of training cases, this reduction has been more considerable. For problems, where an extraction of the general features of data was required, the effect of these improvements have been more obvious when the number of training cases has been increased. The results have shown a reduction in the training time by about $\frac{1}{60}$ to $\frac{1}{100}$ of the previous training time.

2. The final number of nodes in the architecture has been greatly reduced for all the prob-
lems. This reduction in the number of nodes has resulted in an exponential reduction of the number of connection weights. In fact reduction in the number of nodes is the main source of reduction in training time.

3. Testing of the trained networks has shown that the overall quality of the response of each of the neural networks has been improved, due to more generalization capability.

These modifications had been introduced in the final stages of the research. So, as can be seen in the following chapters, more concise networks have been obtained after using the new version of the program for learning similar problems.

Sigmoidal activation function has been chosen for the units of the neural networks, returning a value in the range of \([-1, +1]\). Generally, the emulator neural network has been required to be equipped with two hidden layers, while only one hidden layer has been considered for the controller neural network.

### 5.5 THE MODEL STRUCTURE

As explained in the previous sections, tendon control mechanism, powered by electrohydraulic servo-valve actuators has been selected as the control mechanism. This mechanism has been studied by several authors and is the most noticed control mechanism as explained in chapter 2. The structure under study has been selected to be the model of a three storey one bay steel frame structure, constructed by Soong, Reinhorn and Yang (1985), and has been proposed to be the standard structure for the purpose of structural control studies. This model is shown in figure 5.3. They have provided enough informations about its properties. Also, they have identified it. Thus, it has been possible to simulate this structure and study it numerically. The frame structure is comprised of two similar plane frames which are connected together by horizontal elements. Two similar actuators are installed on the ground floor for its control. Because of symmetry, it is possible to simulate and study half of this system comprising of one of these plane frames controlled by one actuator. This statement is only valid in the case of in–plane loading conditions. For the cases of two dimensional and torsional loadings, the structure should be
Figure 5.3 The structure selected for the purpose of studying the proposed intelligent control [After Soong, Reinhorn and Yang (1985)].
modelled as a three dimensional frame structure.

Because the vertical components of the displacement and actuator forces do not play an important role in this problem, the kinematics matrix \( r_u \) and the location matrix \( l_u \) are assumed to be the same, expressed as:

\[
l_u = r_u = \begin{bmatrix} (- \cos \alpha) & 0 & 0 \end{bmatrix}^T
\]

for a shear model, where the direction of displacements and force of both the actuator and structure are considered to be positive if in the selected positive horizontal direction. Also, the assumption of neglecting the effect of vertical components of control forces is compatible with the shear model studied by Soong and his co-workers (1987).

The structure is a low damping system. Thus, its control is sensitive to the abrupt changes in control forces. Abrupt changes in control forces may result in the excitation of the high frequency modes of the response of the structure, which results in the appearance of large amplitude accelerations. These large amplitude accelerations are undesirable with regard to factors such as the comfort of residents and/or effects on any stored sensitive equipments. Such low damping model provides a good test structure, because it reveals the capabilities of the control method in the alleviation of not only the displacements but the whole state and accelerations of the system. Also the intelligence of the controller to avoid situations that controller should control itself can be studied. Such situations are very undesirable but probable and can be called "controller as a source of noise itself".

REFERENCES


CHAPTER 6
NEURO-CONTROL OF STRUCTURES

In this chapter, the results of the first phase of these series of studies will be presented. In chapter 5, the fundamentals of the proposed method were discussed. Because each chapter has been decided to be self content, first a general of the control problem will be explained and then details of the study will be presented.

Based on the method which was proposed in chapter 5, an algorithm has been developed and tested in computer simulation of the active control of the three storey frame structure of figure 6.1. This structure was subjected to ground excitations. First, an emulator neural network was trained to forecast the future response of the structure from its immediate past history of response and control signal. The trained emulator was then used in both predicting the future response and evaluating the sensitivities of the control signal with respect to those responses. At each time step of simulation, the control signal was adjusted to induce the required control forces in the actuators, based on a control criterion which will be explained in the following sections. Then a controller neural network was trained to learn the relation between the immediate past history of response of the structure and control signal as the inputs, and the adjusted control signal as the output. Then the trained neuro-controller was utilized in controlling the same structure for different dynamic loading conditions. Results of this initial study which will be presented in the following sections, indicate that the neural network based control algorithms have the promise of evolving into powerful adaptive controllers.

6.1 OBJECTIVES

the objective of this phase of research has been to prepare a neuro-controller that is able to reduce the relative displacement of the first floor, as much as possible. No credit has been given to the acceleration mitigation. In fact, in this phase of study, the problem of safety has been considered important, while the comfort of the residents has been left
Figure 6.1 The structure, actuator and tendons.
unnoticed. This is not a good criteria for the control of residential structures, however it seems logical for the initial steps of algorithm development. A control algorithm, reducing the size of both displacements and accelerations will be studied in the second phase of studies (next chapter).

Also, it has been assumed that the whole state of response can be available through sensor measurements. This is an idealistic assumption, which is hard to achieve in reality. It is necessary to develop an algorithm to work with a limited selected number of state measurements. These measurements constitute what is called the output vector in the formulated control approaches. This problem will be addressed in the second phase of studies (next chapter) too.

6.2 NUMERICAL SIMULATION

Numerical simulation of the response of the controlled structure, was explained in chapter 5. It was considered important to include the effects of the actuator dynamics, the sampling period and the inherent delays in the control loop.

The sampling period: The sampling period, which is the period of measurements of response of the structure and the earthquake excitation and time interval for sending the command signals was chosen 0.02 seconds. The coupled equations of the structure/actuator were numerically integrated using Wilson’s–Ω method.

The integration time step: The integration time step, used in the numerical analysis was chosen to be 0.001 seconds which is a small fraction (one-twentieth) of the sampling period. This will allow for a realistic representation of the generation of the actuator forces during the sampling period as a result of the dynamics of the actuator and the interaction of the structure and the actuators. While the sampling period of the recorded earthquakes which were used in this study is 0.02 seconds, the intensity of earthquake, used in integration procedure was obtained by a linear interpolation in between sampling periods.

The time delay: A time delay, due to the conversion of digital to analog of the control sig-
nals and the computation time for the determination of control signals has been considered. This time delay has been assumed to be 0.02 seconds.

6.3 CONTROL ALGORITHM

The final design of the controlled system is shown in figure 6.2. This figure shows the controller neural network, performing the task of controlling the structure, through sending suitable signals to the actuators. The actuators follow the control signal and apply a control force to the first floor of the structure. To reach this final design, passing from several stages of design is necessary. This constitutes the control algorithm which will be explained in the following sections.

The emulator neural network is shown in figure 6.3. As mentioned in chapter 5, the emulator neural network learns how to predict the response of the structure from the past history of response and actuator signal. The input to the emulator neural network is very similar to the input to the neuro-controller. It consists of the current values of the immediate past history of response and control signals plus the last control signal which has already been sent to the actuators. The output of the emulator neural network is the predicted response of the structure at the next time step.

The method of training of the neuro-controller is schematically shown in figure 6.4. During the training of the neuro-controller the structure is subjected to external excitation due to earthquake base motion, and also the control forces. On the basis of the sensor readings the neuro-controller issues a signal to the actuators. Before the neuro-controller is fully trained, it cannot control the motion of the structure satisfactorily. Therefore, the response of the structure will exceed the desired limits. The structural response obtained by the sensors is taken into the box designated “the control criterion”, where the magnitude by which the structural response exceeds the desired limits (the control error) is determined. Control error is the error at the output of the controller neural network. This error is then back-propagated through the emulator neural network to the neuro-controller. Only the connection weights in the neuro-controller are modified in this process. The training of the neuro-controller is continued until the control error, determined
from the control criteria, becomes negligible.

6.4 CONTROL CRITERIA

Control criteria define the objectives of the structural control. A realistic objective of the structural control is to reduce but not completely eliminate the motion of the structure under external loads. Eliminating the motion of the structure may be achievable in a theoretical environment where simplifying assumptions are made and where the actuators respond instantaneously and there are no delays in the control loop. However, it is not considered practical to completely eliminate the structural motion. The main question in developing a control criterion is: which displacements to reduce and by how much?
Figure 6.3 Training of the emulator neural network.
Figure 6.4 Schematics of the training of the neuro-controller.
It is not a good idea to control the current response; since if the response is sensed, it is too late to control it. It has been decided to choose to control the predicted displacements determined from the emulator. It is also desirable to avoid strong control actions and prefer to gradually reduce the displacements. Rather than simply reducing the displacements at the next time step the aim of the control criterion is to reduce an averaged displacement computed over the next several time steps. This leads to a much smoother response of the controlled structure. The emulator is used to predict the displacements in the next several time steps. In developing a method of computing an average of the future displacements, the following points should be considered.

1. Emulator predictions will be less reliable with increasing the number of time steps into the future. This is due to the fact that current predictions of the emulator, which already contain some error, have to be used as input for predicting the displacements of the next time step, which, consequently, are subjected to more uncertainty.

2. External excitations, such as the earthquake ground motions are not known for the next several time steps.

To consider the effect of the prediction error, a function, called the “prediction validity” function has been introduced. This is a decaying function signifying the reduced reliability of the predictions with the number of time steps into the future.

\[ f_v(k) = \text{prediction validity function} \]

\[ k = 1, 2, \ldots, \text{number of predictions} \]  

The magnitude of the predicted displacements at the future time steps should also be considered. More control effort should be directed toward the larger displacements. This is accomplished by introducing an “importance function”, as follows
\[ f_i(y_k) = \text{importance function} \quad (6.2) \]

\[ y_k = \text{predicted horizontal displacement of the first floor} \]

\[ k = 1, 2, \ldots, \text{number of predictions} \]

The combined effect of both the prediction validity and the importance functions is applied through a weighting function defined as follows

\[ f_w(k) = \frac{f_v(k) f_i(y_k)}{\sum_k f_v(k) f_i(y_k)} \quad (6.3) \]

The average predicted first floor horizontal displacement, \( \bar{y} \) which is controlled in the next steps is determined from the following equation.

\[ \bar{y} = \sum_k f_w(k) y_k \quad (6.4) \]

The prediction validity function, importance function, weighting function and the number of predictions depend on the characteristic of each control problem. The simplest forms of these functions is a uniform function which leads to the following equation

\[ f_w(k) = \frac{1}{\text{number of predictions}} \quad (6.5) \]

Using this function is equivalent to averaging the future predicted displacements over the next several time steps. All the future predictions are then considered to have the same validity and importance. This uniform weighting function has been used in this study. Stu-
dying the effect of consideration of other prediction validity and importance functions on the results of this stage of control scheme has shown that a linear decaying function for the prediction validity function, and a uniform function for the importance function provides better results. However to avoid introducing special situations in control scheme, the very simple uniform function for the weighting function has been chosen. Hence, the control criteria has been to reduce the average predicted response below some specified limit, which is usually a small value for structural control problems.

6.5 TRAINING OF THE EMULATOR NEURAL NETWORK

Training of the emulator is a deliberate and challenging task. Adaptive architecture determination and automatic node generation has played a prominent role in our training procedure. It should be mentioned that the "random selection mechanism" and the "convergence evaluation criteria" which were explained in chapter 5, have not yet been implemented in the computer program "SUNN" for this phase of studies. Figure 6.3 shows the final architecture of the emulator neural network. The input layer has 15 units, 3 of which represent the current value of the actuator signal and its values at the last two time steps. The remaining 12 units represent the relative displacements and relative accelerations of the three floors for the past two time steps. The output layer consists of 6 units which represent the relative displacements and relative accelerations for the next time step. During the training process, each hidden layer started with six units. As the training proceeded, more units were added to the hidden layers automatically, as needed. At the end of the training period each hidden layer had 12 units.

The training cases have been obtained from the results of several analyses in which the structure/actuator system was subjected to external excitation and the actuator forces. The analysis was performed with a time step of 0.001 seconds. The training cases were formed with the data taken at the intervals of sampling period of 0.02 seconds. The emulator neural network was trained with the results of three analyses in which the structure/actuator system was subjected to the following excitations:
50 seconds of the El Centro earthquake with 25% of the amplitude

50 seconds of actuator forces generated by white noise signals

50 seconds of the El Centro earthquake with 25% of the amplitude + actuator forces generated by white noise signals

The trained neural network has learned about the behavior of the structure/actuator system and is therefore capable of predicting the response of the structure. Figures 6.5 and 6.6 show the prediction capability of the emulator neural network, when the structure has been subjected to 25% El Centro and 25% Taft earthquakes respectively. Note that the emulator has been trained based on 25% El Centro; however it has been able to do a good prediction job for the 25% Taft earthquake. This shows that the emulator has been able to learn about the response of the structure, regardless of the source of the ground excitation.

6.6 STRUCTURAL CONTROL USING THE EMULATOR NEURAL NETWORK

The emulator neural network can be used, in an iterative mode, to control the motion of the structure. Although it can be considered a legitimate method of structural control, it has only been used to generate the necessary training cases for training of the neuro-controller. In this control method the emulator plays the main role. Although it does not directly generate the control signal, it provides the new control signal by adjusting the previous control signal gradually, using the information about the history of control signals and the structure response. This is done in an iteration loop consisting of prediction, criteria checking and signal adjustment. In this iterative loop the emulator is not only used for prediction, but also for calculating the Jacobian (sensitivity) and inverse Jacobian of the predicted response with respect to a change in the proposed control force. As stated before, in this specific example, the criterion has been the reduction of the relative displacement of the first floor with respect to the ground.

The sensitivity of the control signal as a function of the structural response can be com-
puted in two ways. One method is to directly compute the changes in the response of the structure due to the changes in the control signal. A second method is by utilizing the available information about the internal representation of the emulator network, and backpropagating a small error in the output nodes, while holding the weights unchanged. This second approach seems more precise. But since the system behaves nonlinearity in general, direct calculation provides more information about sensitivity of the prediction to signal or vice versa, for practical purposes.

In this study the relationship between only one input unit (the signal to be sent) and one output unit (the predicted relative displacement of the first floor) of the emulator had to be considered. The direct calculation which could provide more numerical flexibilities has been utilized. With \( e \) as the proposed control signal which is assumed to remain unchanged for the next several time steps of prediction, the sensitivity of \( e \) with respect to output \( y_k \) for each of the next \( k \) time steps has been calculated from the following equation.

\[
s_k = \frac{\Delta e}{\Delta y_k}
\]  

(6.6)

Now the average predicted sensitivity \( \bar{s} \) can be calculated with the same method that the average predicted displacement was calculated, by using the same weighting functions, as in equation (6.4).

\[
\bar{s} = \sum_k f_w(k) \ s_k
\]  

(6.7)

The adjusted proposed control signal is then

\[
e = e - (\bar{s}) \ (\bar{y})
\]  

(6.8)
This new signal is fed to the emulator and new predictions for the next $k$ time steps are made. Next the sensitivities are obtained and the averages of the displacements and sensitivities are calculated. Using these averages, a new signal correction is made and fed to the emulator. This loop is repeated until the control criterion is satisfied or the actuator is saturated. Figure 6.7 shows the displacements and total accelerations of the three floors for uncontrolled and controlled structure subjected to 25% El Centro earthquake. Figure 6.8 represents the time history of the control forces and the total work done by the actuators on the structure during the control procedure.

6.7 TRAINING OF THE NEURO-CONTROLLER

A three layer neural network has been proposed for the controller. The input layer consists of 14 units, representing two previous controller signals to the actuator and the relative displacements and accelerations of the three floors for the most recent two time steps. The output layer has only 1 unit which represents the signal to be sent to the actuator. This controller is trained in a manner similar to the method shown in figure 6.4. However, the training cases were first generated by using the emulator neural network in an iterative control loop, as described earlier. The training cases generated with this method were then used to train the neuro-controller. Once again, the automatic node generation method has been used in the training of the neuro-controller. The hidden layer started with 6 units at the beginning of the training process and ended with 20 units at the termination of the training.

6.8 RESULTS OF ANALYSIS

The performance of the trained neuro-controller is illustrated through several numerical simulations. In the first example, the structure shown in figure 6.1 was subjected to the El Centro earthquake record with 25% amplitude.

The results are shown in figures 6.9 through 6.11. From these figures it can be seen
that the controller has been reasonably successful in performing the control task it has been trained for, namely, reducing the relative displacement of the first floor. Figure 6.10 shows the frequency response of the relative displacements and the accelerations. The neuro-controller has managed to considerably reduce the response of the structure in its first mode, while the response in the second and the third modes has increased somewhat, specially evident in the acceleration spectra. The accelerations in the second and the third modes have increased considerably. It must also be noted that in our study, the neuro-controller was not specifically trained to control the accelerations. Controlling the accelerations is a somewhat different control task than controlling the relative displacements. The question will be addressed in the next chapter.

The strong criterion of zero relative displacement causes the controller to continue attempting to reduce the relative displacement of the first floor even after the effect of the external excitations have diminished. In effect, the controller is attempting to eliminate its own effect on the structure. This is the reason for the almost steady state response, which can be observed after about 30 seconds. This effect can also be observed in the time history of control forces, shown in figure 6.11. These problems can be remedied by modifying the training procedures and the control criterion. In modifying the training procedures and the training data set, the neuro-controller can be trained to acquire the additional knowledge to avoid over-controlling the structure.

The effect of the weakening of the control criterion is demonstrated in the next analysis. The analysis described above was repeated with a weaker control criterion. Rather than requiring that the relative displacement of the first floor, shown by \( x \), be reduced to zero, it was required to be reduced to a small value, \( |x| < 0.05 \text{ cm} \). The results are shown in figures 6.12 - 6.13. It is obvious from these figures that by slightly weakening the control criterion, the acceleration response of the controlled structure in the higher modes has been considerably improved. To study the importance of control criteria in more details, the control criteria has been weakened more, to \( |x| < 0.10 \text{ cm} \). As expected, more reduction in accelerations and the required control forces have been observed. These are shown in figures 6.14-6.16. The required control work, done by both of the actuators on the structure is shown in figure 6.16 (bottom).
In the previous examples, the structure has been subjected to the El Centro earthquake record, which was the same earthquake record used in the training of the neuro-controller. The performance of the neuro-controller, when the structure is being subjected to a different earthquake is demonstrated in the next example. The structure has next been subjected to the Taft earthquake record of 50% amplitude, controlled with the neuro-controller which has been trained with the El Centro earthquake. The results are shown in figure 6.17. It can be seen that the neuro-controller is as effective in controlling the structure when it is subjected to the Taft earthquake record as it is in controlling the structure when it is subjected to the earthquake record which the neuro-controller was trained on. This demonstrates the fact that the neuro-controller learns to control the motion of the structure, regardless of the source of excitation. This is due to the generalization capability of the neuro-controller.

With the next example, the performance of the structure was explored when subjected to ground shaking with higher intensity than it was trained to control. Also the related issue of the effect of the actuator saturation was investigated here. In this example, A neuro-controller which was trained with the El Centro earthquake record with 25% amplitude was used. This neuro-controller is then applied to the control of the structure which was subjected to the El Centro earthquake with 200% amplitude. Forces needed to control this structure are clearly beyond the capacity of the actuator used in this example. The results of the analyses are shown in figures 6.18 and 6.19. From these figures it appears that the neuro-controller has done its best to control the structure and has used the maximum capacity of the actuators when needed.

6.9 CONCLUDING REMARKS

The new method of active control of structures using neural networks as proposed in chapter 5 has been numerically tested in this chapter. In the proposed control method, a neuro-controller replaces the control algorithm. The training of the neuro-controller is accomplished with the aid of an emulator neural network. The emulator neural network is trained to learn the mapping between the control signal and the response of the structure. The trained emulator then allows the calculation of the sensitivity of response of the
structure with respect to the control signal, which in turn, allows the development of the training cases for the neuro-controller. The neuro-controller learns to control the structure from these training cases. The control criterion plays an important role in preparing a suitable practical controller. This fact has been demonstrated through a series of examples. An essential point in performing the numerical simulations of the structural control, has been to include the effects of actuator dynamics and a realistic sampling period, representing the inherent time delay in the control loop.

The results of this phase of study indicate that neural networks are potentially powerful tools in structural control problems. The learning capabilities of the neuro-controllers put them in the place of adaptive controllers. Neuro-controllers are also good candidates for nonlinear control problems. The two areas of the control criterion and the training procedure, as the main steps in constructing neuro-controllers were identified. More improvements and more results are reported in the next chapter.
Figure 6.5 Test of emulator for 40 seconds. The structure has been subjected to 25% El Centro earthquake. Capability of the emulator in predicting the relative displacements and relative accelerations of the three floors for the next sampling period is compared to the results of analysis under this loading condition.
Figure 6.5 Continued.
Figure 6.6 Test of emulator for 20 seconds. The structure has been subjected to 25% Taft earthquake. Capability of the emulator in predicting the relative displacements and relative accelerations of the three floors for the next sampling period is compared to the results of analysis under this loading condition.
Figure 6.6 Continued.
Figure 6.7 Control by emulator. Objective has been to mitigate the relative displacement of the first floor. Relative displacements and absolute accelerations of the three floors are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.7 Continue.
Figure 6.8 Control by emulator. Objective has been to mitigate the relative displacement of the first floor. Control force applied by each of the actuators and total work done by the actuators on the structure are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.9 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to 0.0 cm. Relative displacements and absolute accelerations of the three floors are shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.9 Continued.
Figure 6.10 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to 0.0 cm. Fourier spectrum for the relative displacements and absolute accelerations of the three floors are shown for 40 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.10 Continued.
Figure 6.11 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to 0.0 cm. Control force applied by each of the actuators on the structure are shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.12 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within (-0.05 cm, +0.05 cm). Relative displacements and absolute accelerations of the three floors are shown for 40 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.12 Continued.
Figure 6.13 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within \((-0.05 \text{ cm}, +0.05 \text{ cm})\). Control force applied by each of the actuators on the structure is shown for 100 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.14 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within (-0.10 cm, +0.10 cm). Relative displacements and absolute accelerations of the three floors are shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.14 Continued.
Figure 6.15 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within \([-0.10 \text{ cm}, +0.10 \text{ cm}\). Fourier spectrum for the relative displacements and absolute accelerations of the three floors are shown for 40 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.15 Continued.
Figure 6.16 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within (-0.10 cm, +0.10 cm). Control force applied by each of the actuators and total work done by the actuators on the structure are shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 6.17 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within (-0.05 cm, +0.05 cm). Relative displacements and absolute accelerations of the three floors are shown for 40 seconds. The structure has been subjected to 50% Taft earthquake and control forces.
Figure 6.17 Continued.
Figure 6.18 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within (-0.05 cm, +0.05 cm). Relative displacements and accelerations of the three floors are shown for 40 seconds. The structure has been subjected to 200% El Centro earthquake and control forces.
Figure 6.18 Continued.
Figure 6.19 Control by neuro-controller. Objective has been to reduce the relative displacement of the first floor to within $(-0.05 \text{ cm}, +0.05 \text{ cm})$. Control force applied by each of the actuators on the structure are shown for 100 seconds. The structure has been subjected to 200% El Centro earthquake and control forces.
CHAPTER 7
NEURO-FUZZY CONTROL OF STRUCTURES

In this chapter, the results of the second phase of these series of studies will be presented. In chapter 5, the fundamentals of the proposed method were discussed. In chapter 6 the results of a preliminary study on the application of neural networks in the control of frame structures was presented. In this chapter, a more detailed and an improved version of the neural network based control algorithm of chapter 6, and then the application of fuzzy logic in the control of the same frame structure will be discussed.

Because each chapter has been decided to be self content, at first the objectives and the general features of the control problem will be explained. Then details of the study will be presented.

7.1 OBJECTIVES

The structure under study is shown in figure 7.1. This is the same structure studied in the previous chapter. It has been controlled for ground excitations. Despite the first phase of study where only the displacements of the floors had to be reduced, the accelerations of the floor have been reduced too. Also the algorithm has been required to do a smooth control job, resulting in small forces and hence control economy. While in the last chapter, the output of the structure was permitted to be the whole state, in this phase, the objective has been to control the structure by a limited number of output measurements. Hence the output has been considered to be the relative acceleration of the first floor only.

7.2 NUMERICAL SIMULATION

Numerical simulation of the response of the controlled structure was explained in chapters 5 and 6. As in the previous chapter, the sampling period which is the period of
Figure 7.1 The structure, actuator and tendons.

**Actuator Properties:**
- \( A = \text{area of ram} = 5.06 \text{ cm}^2 \)
- \( V = \text{chamber volume} = 151.80 \text{ cm}^3 \)
- \( C = \text{leakage coefficient} = 0.10 \text{ cm}^2/(\text{NT} \cdot \text{sec}) \)
- \( \beta = \text{compressibility} = 2.1 \text{ MN/cm}^2/(\text{NT} \cdot \text{sec}) \)
- \( \tau = \text{time constant} = 0.2 \text{ sec} \)
- \( q_{\text{max}} = \text{max. valve flow} = 616 \text{ cm}^3/\text{sec} \)
- \( u_{\text{max}} = \text{actuator capacity} = 3200 \text{ NT} \)
- \( k_1 = 1.0 \text{ cm}^2/\text{NT} \)
- \( k_2 = 0.15 \text{ cm}^3/\text{sec} \)
- \( k_3 = 200.0 \)
measurements of response of the structure and the earthquake excitation and time interval for sending the signal commands, has been chosen to be 0.02 seconds. The coupled equations of the structure/actuator has been numerically integrated using Wilson's θ method. The time step used in the numerical analysis has been chosen to be 0.001 seconds. A time delay of 0.02 seconds has been considered in the control loop.

7.3 CONTROL ALGORITHM

Training of the neuro-controller has been accomplished by the same method as in the previous chapter. However, dramatic changes in the control criteria have been introduced. There have been three major steps in developing our neuro-fuzzy controller: a preliminary control by using an emulator neural network, training of a neuro-controller, and improving the neuro-controller performance by utilizing a set of simple fuzzy rules.

To accomplish these steps, an emulator neural network, has been trained to predict the relative acceleration of the first floor, based on an input comprised of the history of relative accelerations and signals which have been sent to the actuators. At each computational time step, the future relative velocities and displacements of the first floor have been guessed by using the predicted acceleration, . This has been accomplished by using the Wilson’s θ integration procedure. Also by utilizing the same neural network emulator, the sensitivities of the future accelerations to the control signal have been predicted. Then by the use of these sensitivities, the control signal has been modified to reduce the future acceleration of the first floor and as a result, the future velocities and displacements of the first floor too.

Based on the results obtained from this preliminary control, a set of training cases has been formed. This training set has been used in the training of the neuro-controller. Although the preliminary control has not been completely successful and smooth, the neuro-controller has been able to extract just the necessary information for performing a good control job.

Since the neuro-controller has been trained to control the structure in a very smooth way, it has not been trained to control the structure for very short time duration, unusual excitations. To solve this problem, a set of very simple fuzzy rules have been developed to improve on the neuro-controller job. The fuzzy rules of control have been so con-
structed to require only the recently measured relative velocity and displacement of the first floor, for the determination of necessary changes in the control signals. Then, the signals proposed by the neuro-controller have been modified by this supplementary fuzzy controller, and sent to the actuators.

The final control system is shown in figure 7.2, schematically. Also, this cycle of training of the controller neural network and application of fuzzy criteria to improve the control performance gradually, can be continued to obtain a desired controller neural network. In the following sections, each of these steps will be explained in more details.

Figure 7.2 Control by a neuro-fuzzy controller.
7.4 PRELIMINARY CONTROL BY USING THE EMULATOR NEURAL NETWORK

In this section the informations and results regarding the preliminary control by the use of the emulator neural network is presented. This step in the control algorithm is comprised of the training of a suitable emulator neural network, and then its utilization in a preliminary control of the structure.

7.4.1 Training Of The Emulator Neural Network

The trained emulator has been a 4 layered neural network, comprised of an input layer with 8 units, two hidden layers each with 2 units, and an output layer with one unit, as shown in figure 7.3.

The first input node represents the force that should be sent to the actuator. The next two units represent the two most recent forces that have been sent to the actuator. The fourth input unit represents the relative acceleration of the first floor that has already been measured, and the remaining four input units represent the last four relative accelerations of the first floor that have been measured before.

The output unit is used for predicting the next relative acceleration of the first floor that should be measured in the next time step. In this study the emulator has been used to predict not only the next, but also the second next relative acceleration of the first floor. This has been done by feeding back the first predicted acceleration to the input layer, assuming that prediction has been correct. It is obvious that the prediction reliability reduces for the second future time step.

The emulator has been trained with the train cases, obtained from the following loadings:

50 seconds of El Centro earthquake with 25% of the amplitude
50 seconds of actuator forces generated by white noise
50 seconds of El Centro earthquake with 25% of amplitude + actuator forces generated by white noise.
Figure 7.3 Training of the emulator neural network.
After this off-line training of the emulator, it has been tested for several combinations of earthquake excitations and actuator forces, on-line. Results of a typical test is shown in figures 7.5 and 7.6. The structure has been subjected to the Taft earthquake of 100% amplitude and a random force applied by the actuators simultaneously.

7.4.2 Preliminary Control

After training the emulator, it has been put in the control loop to be used in controlling the response of the structure. At each sampling time step \( k \), the relative acceleration of the first floor \( \dot{x}_k \) has been measured. Also the last desired signal \( f_k \), which has been sent to the actuator, has been assumed to remain unchanged for the future time steps. The input to the emulator has been formed, and based on this input, the predicted accelerations for the next two future time steps, \( \ddot{x}_{k+1} \) and \( \ddot{x}_{k+2} \) have been obtained. Also the approximate derivatives of the desired force with respect to the predicted accelerations, \( S_1 \) and \( S_2 \) (the sensitivities) have been obtained by increasing the desired force by a small amount, and then calculating the derivatives directly.

At this stage, the control criteria has been selected to be the reduction of the observed relative displacement \( x_k \) by 50%, which was required to be obtained during two sampling periods. Study has shown that selection of control criterion which required the reduction of \( x_k \) up to about 80% provided suitable control results. However larger control forces were needed and more high frequency accelerations were observed in the response of the controlled structure. It has been tried to modify \( \dot{x}_{k+2} \) by assuming that \( \dot{x}_{k+1} \) is not controllable. By the utilization of an integration method, and the known values for the present state of the structure, \( x_k, \dot{x}_k \) and \( \ddot{x}_k \) and also \( \ddot{x}_{k+1} \) which has been obtained from emulator prediction, the second required future acceleration \( \ddot{x}_{k+2} \) has been calculated. The Wilson’s \( \Theta \) integration method has been used for this purpose. Wilson’s \( \Theta \) integration method assumes the following relationships between the displacement, velocity and acceleration of the structure for each two successive integration times:
where \( c_1 \) to \( c_9 \) are integration constants and \( \Delta u_k \) should be obtained from the governing equations of motion, which requires the knowledge about the external excitations too. But before the control forces are determined, it is not possible to find \( \Delta u_k \). However in our algorithm the equations of motion have been used to solve an inverse problem. First, it has been assumed that the first predicted acceleration \( \ddot{x}_{p,k+1} \) has been accurate enough to assume

\[
\ddot{x}_{k+1} = \ddot{x}_{p,k+1} \tag{7.4}
\]

Then by using equations (7.3), (7.1) and (7.2), \( \Delta u_{k+1} \), \( x_{k+1} \) and \( \dot{x}_{k+1} \) have been calculated respectively. By the control criteria, the following requirement should be satisfied:

\[
x_{k+2} \leq 0.50 \; x_k \tag{7.5}
\]

By changing \( k \) to \( k + 1 \) in equations (7.1), (7.2) and (7.3), and using equation (7.5) and the predicted values of acceleration, velocity and displacement of the previous step, \( \Delta u_{k+2} \) has been obtained from equation (7.1). Then the required acceleration \( \ddot{x}_{k+2} \) has been calculated from equation (7.3). This is the acceleration that should
have been observed instead of $x_{k+2}^p$. The required change in the control force requested from to the actuators for the modification of $x_{k+2}^p$ to $x_{k+2}$, has been calculated from:

$$\Delta f = S_2 (x_{k+2} - x_{k+2}^p).$$ (7.6)

This force has then been added to the previous desired force to provide the new desired force.

To avoid the undesired sudden changes in the desired force, which result in large accelerations, a limit for $|\Delta f|$ has been defined. After the determination of the new desired force, the procedure for the prediction of accelerations and sensitivities has been resumed, and a new $\Delta f$ has been determined. This $\Delta f$ has been added to the new desired force, and so on, until the displacement has fallen into a small acceptable region, or the number of this prediction and force correction cycles has reached a predefined limit. In our simulation, the absolute value of $\Delta f$ has been limited to 15.0 newtons, and 10 number of corrections have been permitted. Results of this preliminary control, the proposed forces and the work done by the actuators are shown in figures 7.7 and 7.8.

At this stage, enough informations have been collected to form the train cases for the training of the controller neural network. Figures 7.7 and 7.8, show that while the control forces are much less than those obtained in the previous chapter, they are not smooth yet, and still produce high frequency response in the structure. Also the preliminary control by the emulator neural network can not dissipate the energy, induced by the external excitation.

### 7.5 CONTROL BY USING THE NEURO-CONTROLLER

In this section, the training procedure of the neuro-controller, a simple "turn on-off" fuzzy criteria and results of control by using neuro-controller are presented.
7.5.1 Training Of The Neuro-Controller

The controller neural network should be able to do a good controlling job for almost all the expected earthquakes. So, it should be trained to learn the general features of a successful control and avoid learning special features of a certain earthquake. Such a controller will have a small architecture. In this phase of study, same as the first phase (chapter 6), a three layer neural network has been chosen. As shown in figure 7.2, the input layer has been equipped with 7 input units. The inputs have been exactly the same as those of the emulator, except that the first unit in the emulator which represents the desired control signal to be issued, has been omitted. The only hidden layer has been equipped with 2 units. One output unit has been considered for the output layer, which represents the proposed signal which should be sent to the actuators. This unit contains the same information as the first input unit of the emulator, and represents the desired proposed control signal, sent to each of the actuators.

After the off-line training of the controller neural network, it has been put into work. Training of the neuro-controller is schematically shown in figure 7.4.

7.5.2 Turn On-Off Fuzzy Criteria

A “turn on-off” fuzzy criteria has been implemented in the control path. The duty of this simple operator is to turn off the controller whenever the external excitation falls below a predefined threshold, and turn it on when it resumes. This turn on-off operator can play an important role in the control of very low damping structures. In such systems, the noise in the actuators due to sources of delay and nonlinearity may result in actuator induced vibrations even when there is no external excitations. Hence, the controller should control such noises too. To avoid this undesirable situation, use of a turn on-off operator is recommended. This puts the system in the category of closed-open loop control systems. Implementation of a turn on-off operator can be beneficial from two points of view. First, elimination of the above mentioned actuator induced noises, and second providing a supplementary control to the controller behavior. In this study, by opening a window on the last 10 measurements of the ground accelerations, a weighting factor $\zeta$ has been calculated for each sampling period. The value of this weighting fac-
Figure 7.4 Schematics of the training of the neuro-controller.
tor $0.0 \leq \zeta \leq 1.0$ has then been multiplied by the control signal to provide the required control signal. As the external excitation dies, $\zeta$ tends to zero, which results in the actuators to be turned off gradually. When the excitation dies completely and actuator forces reside to zero, $\zeta$ is automatically set to 1.0 to be ready for a new earthquake. As mentioned before, this operator does not have a significant effect on the main control procedure. It can be eliminated for the real structures which have some sort of internal damping. It may also be possible to train the neuro-controller or another separate turn on-off neural network to learn about the response of the structure when it is subjected to small noise introduced by the actuators, and let these kinds of noises die out gradually. However training of the neuro-controller to learn this task too, may impair the generalization capability and its quality of control.

7.5.3 Results Of Control By Using The Neuro-Controller

The neuro-controller has been utilized in the control of the structure both with and without the "turn on-off" fuzzy criteria. The results of its control action, and its proposed forces are shown in figures 7.9-7.15. Figures 7.9-7.11 show the control results without the turn on-off fuzzy criteria. Figures 7.12-7.15 show the response of the controlled structure with the turn on-off fuzzy criteria. As can be seen in figure 7.15(top), $\zeta = 1.0$ when control is required. So, the turn on-off operator does not have a significant effect on the main control procedure. However, a comparison between figures 7.9 and 7.12, and also figures 7.11(top) and 7.15(bottom) shows that the turn on-off operator is able to control the undesirable actuator induced noise. Figure 7.14(bottom) shows the time history of the total work done by the actuators on the structure. Neuro-controller has been able to issue very suitable signals, resulting in a smooth and monotonic absorption of energy. Comparison of this figure to figure 7.8(bottom) reveals the advantages of using a neuro-controller which is trained to learn the general features of the control task, from the informations obtained through the preliminary control by the emulator.
7.6 CONTROL BY USING BOTH NEURO-CONTROLLER AND FUZZY CONTROLLER

In this section the construction of fuzzy controller for the sake of improvement on the neuro-controller performance is explained.

7.6.1 Fuzzy Controller

The peaks like those at the beginning and at the fifth seconds in figure 7.13, are due to sudden shocks from the earthquakes, here the El Centro earthquake. These peaks are hard to control. Such peaks can be reduced by selecting a suitable fuzzy criteria to improve on the neuro-controller job. The neuro-controller reduces the response to a high degree and provides a suitable environment for a fuzzy controller to act as a secondary controller. The fuzzy controller should correct the consequences of weakness of the neuro-controller in the past, that have resulted in these undesirable responses. This can be done by proposing a small correction force that should be added to the force proposed by the neuro-controller.

Our fuzzy criteria could have been very complicated, to the level of an expert controller. But a good controller should be very simple. Hence, in our study, aim has been to use the fuzzy logic as a very simple tool, and avoiding the need to spend a lot of time on the preparation of a fuzzy controller.

The input for our fuzzy criteria has been based on the already known actual relative velocity and displacement of the first floor only, and not on the predictions of the neural network emulator. As shown in figure 7.16, the relative velocity and displacement of the first floor have been divided to seven overlapping fuzzy sets. These fuzzy sets have been called:

\[ A_i \quad , i = 1, 2, \ldots, 7 \]
\[ B_j \quad , j = 1, 2, \ldots, 7 \]
and the membership function for each of these fuzzy sets has been chosen to be a trapezoidal. Then, for each value of relative displacement $x$, the membership to the fuzzy set $A_i$ is $\mu_{A_i}$ and for the relative velocity $\dot{x}$, the membership to the fuzzy set $B_j$ is $\mu_{B_j}$. These fuzzy sets have divided the whole state space of displacement-velocity into 49 overlapping partitions. To each of these partitions $i,j$, a certain force:

$$\Delta f_{ij}, i,j = 1,2,\ldots,7$$

(7.8)

has been attributed.

The control rules have been constituted in the following form:

$$\text{if: displacement is } A_i \text{ and velocity is } B_j \text{ then: force is } \Delta f_{ij}$$

$$i,j = 1,2,\ldots,7.$$  

(7.9)

Then the compatibility membership $\omega_{ij}$, for each partition $i,j$ has been calculated from Mamdani's implication rule:

$$\omega_{ij} = \mu_{A_i}(x) \wedge \mu_{B_j}(\dot{x})$$

$$i,j = 1,2,\ldots,7.$$  

(7.10)

and the total correction force $\Delta f_t$ has been calculated from:

$$\Delta f_t = \sum_{ij} \omega_{ij} \Delta f_{ij} / \sum_{ij} \omega_{ij}, i,j = 1,2,\ldots,7.$$  

(7.11)

Finally, the desired force $f_d$, which should have been sent to the actuator has been calculated from:
The proposed correction forces for each fuzzy partitioning are represented in Table 7.1.

Table 7.1 Correction force $\Delta f_{ij}$ for each of the fuzzy subspaces of Figure 7.20.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400.0</td>
<td>300.0</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
<td>200.0</td>
</tr>
<tr>
<td>2</td>
<td>300.0</td>
<td>200.0</td>
<td>100.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>200.0</td>
<td>200.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>100.0</td>
<td>60.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>$-60.0$</td>
<td>$-100.0$</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>$-200.0$</td>
<td>$-200.0$</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>$-100.0$</td>
<td>$-200.0$</td>
<td>$-300.0$</td>
</tr>
<tr>
<td>7</td>
<td>$-200.0$</td>
<td>$-200.0$</td>
<td>$-200.0$</td>
<td>$-200.0$</td>
<td>$-200.0$</td>
<td>$-300.0$</td>
<td>$-400.0$</td>
</tr>
</tbody>
</table>

7.6.2 Results Of Control By Using The Neuro–Fuzzy Controller

Results obtained for this last stage of developments in the control strategy are shown in figures 7.17–7.27. As can be seen in figure 7.17, the peaks in displacements which have been observed in figure 7.13 are reduced by about 30%, while accelerations have been increased. Also, figure 7.18 shows larger forces compared to the forces of figure 7.14. This is because such peaks come after sudden changes in the pattern of ground shak­ings and the controller should respond to them very fast, which results in greater control forces. Response of the three floors and their Fourier spectrum are shown in figures 7.19–7.21. They show considerable improvements both in the displacement and accelera-
tion mitigation, compared to the results obtained for the first phase of studies, as presented in chapter 6.

In other figures, performance of the control algorithm, for the control of other earthquakes of different amplitudes and patterns, are shown. Figures 7.22–7.23 are related to the control of structure under Taft earthquake of 50% amplitude. Figures 7.24–7.25 are related to the control of structure under a severe earthquake of 200% El Centro amplitude. This severe earthquake needs stronger actuators. However, the controller has tried to use the capacity of the available actuators as much as possible, in accordance with its knowledge. It should be noticed that the controller has been trained based on 25% El Centro earthquake records. However, the improved algorithm, has performed successfully during all of these tests. It has been able to reduce the amplitude of the displacements, velocities and accelerations of all the floors, for all the cases. Also, the control performance has been much more economical and smoother than what had been observed in the first phase of the study (chapter 6), for the same earthquakes.

As mentioned before, this structure is a very low damping structure and requires a very smooth control. The structure is very sensitive to the introduction of any excitation. To study the effect of increase in damping of the structure on its response, damping factors have been increased by 500%. Even in this situation, the structure is still a low damping structure. However much better control performance can be observed. Results are shown in figures 7.26–7.27.

Table 7.2 contains concise informations about the results of control by using the neuro–controller and neuro-fuzzy controller, obtained in this phase of study. As can be seen, neuro–fuzzy controller has reduced the maximum displacement which is important for safety reasons, to the expense of increasing the maximum accelerations, and maximum control forces. As a result, the total absorbed energy has been reduced too. Noticing the fact that the increase in accelerations has been observed just for a short time, and that reduction of displacement is vital for safety reasons, the overall performance of the neuro–fuzzy controller shows improvement compared to the neuro–controller.

Table 7.3 contains results from the neuro–fuzzy control of the structure subjected to
Table 7.2 Comparison of results for uncontrolled, neuro-controlled and neuro-fuzzy controlled structure, subjected to 25% El Centro earthquake.

<table>
<thead>
<tr>
<th></th>
<th>uncont.</th>
<th>neuro cont.</th>
<th>neuro - fuzzy cont.</th>
</tr>
</thead>
<tbody>
<tr>
<td>max. relative displacement $x_{\text{max.}}$ (cm.)</td>
<td>0.95</td>
<td>0.43</td>
<td>0.31</td>
</tr>
<tr>
<td>$(x_{\text{max.}})<em>{\text{cont.}} / (x</em>{\text{max.}})_{\text{uncont.}}$</td>
<td>—</td>
<td>0.45</td>
<td>0.33</td>
</tr>
<tr>
<td>max. relative velocity $v_{\text{max.}}$ (cm/sec.)</td>
<td>11.98</td>
<td>5.38</td>
<td>5.99</td>
</tr>
<tr>
<td>$(v_{\text{max.}})<em>{\text{cont.}} / (v</em>{\text{max.}})_{\text{uncont.}}$</td>
<td>—</td>
<td>0.45</td>
<td>0.50</td>
</tr>
<tr>
<td>max. absolute acceleration $a_{\text{max.}}$ (cm/sec$^2$)</td>
<td>260</td>
<td>124</td>
<td>193</td>
</tr>
<tr>
<td>$(a_{\text{max.}})<em>{\text{cont.}} / (a</em>{\text{max.}})_{\text{uncont.}}$</td>
<td>—</td>
<td>0.48</td>
<td>0.74</td>
</tr>
<tr>
<td>max. control force (Nt.)</td>
<td>—</td>
<td>855</td>
<td>1083</td>
</tr>
<tr>
<td>control energy (Nt. cm.)</td>
<td>—</td>
<td>6391</td>
<td>5583</td>
</tr>
</tbody>
</table>

different earthquakes. Controller has been able to reduce the response of the structure for all the cases, considerably. The most interesting result is that of the energy absorbed from the structure. As can be seen, for 200% El Centro earthquake, the neuro-controller has put a great deal of effort to absorb energy from the structure, although the neuro-controller has been trained based on an earthquake 8 times weaker (25% El Centro).

The above results show the generalization capability of the neuro-fuzzy controller in controlling the structure, no matter what the pattern of the excitation has been.

7.7 CONCLUDING REMARKS

The new proposed method of neuro-fuzzy control of structures has been very successful in controlling a typical frame structure. It is clear now that it is possible to use the neural networks, as simple trainable adaptive systems to learn the complicated task of controlling a structure, no matter how many sources of uncertainty and nonlinearity
Table 7.3 Comparison of results for uncontrolled and neuro–fuzzy controlled structure, subjected to different earthquakes.

<table>
<thead>
<tr>
<th></th>
<th>25% El Centro</th>
<th>50% Taft</th>
<th>200% El Centro</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uncont. cont.</td>
<td>uncont. cont.</td>
<td>uncont. cont.</td>
</tr>
<tr>
<td><strong>max. relative displacement ( x_{\text{max}} ) (cm.)</strong></td>
<td>0.95</td>
<td>0.31</td>
<td>1.34</td>
</tr>
<tr>
<td>( \frac{x_{\text{max}}\text{cont.}}{x_{\text{max}}\text{uncont.}} )</td>
<td>—</td>
<td>0.33</td>
<td>—</td>
</tr>
<tr>
<td><strong>max. relative velocity ( \dot{x}_{\text{max}} ) (cm/sec.)</strong></td>
<td>11.98</td>
<td>5.99</td>
<td>17.75</td>
</tr>
<tr>
<td>( \frac{\dot{x}<em>{\text{max}}\text{cont.}}{\dot{x}</em>{\text{max}}\text{uncont.}} )</td>
<td>—</td>
<td>0.50</td>
<td>—</td>
</tr>
<tr>
<td><strong>max. absolute acceleration ( \ddot{x}_{\text{max}} ) (cm/sec^2)</strong></td>
<td>260</td>
<td>193</td>
<td>288</td>
</tr>
<tr>
<td>( \frac{\ddot{x}<em>{\text{max}}\text{cont.}}{\ddot{x}</em>{\text{max}}\text{uncont.}} )</td>
<td>—</td>
<td>0.74</td>
<td>—</td>
</tr>
<tr>
<td><strong>max. control force (Nt.)</strong></td>
<td>—</td>
<td>1083</td>
<td>—</td>
</tr>
<tr>
<td><strong>control energy (Nt. cm.)</strong></td>
<td>—</td>
<td>5583</td>
<td>—</td>
</tr>
</tbody>
</table>

are available in the system. Emphasizing on this capability of the neuro–controllers, fuzzy control idea has been utilized as a simple corrective controller. The idea has not been to use the fuzzy logic in a high level, but to prepare a very simple fuzzy controller, based on the very elementary concepts and knowledge about the structural behavior. This fuzzy controller has been a secondary controller, installed to cooperate with the neuro–controller to provide better control results. Simplicity and flexibility, is another fascinating feature of the neuro–fuzzy control method.
Figure 7.5 Test of emulator for 40 seconds. The structure has been subjected to 100% Taft earthquake (top) and actuator forces on the structure (middle), simultaneously. Capability of emulator in predicting the relative acceleration of the first floor for the next sampling period is compared to the results of analysis (bottom) under this loading condition.
Figure 7.6 More details of figure 7.5.
Figure 7.7 Control by emulator. First floor response is shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.8 Control by emulator. Control forces applied by each of the actuators and the total work done by the actuators on the structure are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.9 Control by the neuro-controller before the addition of the turn on-off fuzzy criteria. First floor response is shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.10 More details of Figure 7.9.
Figure 7.11 Control by the neuro-controller before the addition of the turn on-off fuzzy criteria. Control force applied by each of the actuators on the structure are shown for 50 seconds (top). More details are shown too (bottom). The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.12 Control by neuro-controller. First floor response is shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.13 More details of figure 7.12.
Figure 7.14 Control by neuro-controller. Control force applied by each of the actuators and total work done by the actuators on the structure are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.15 Control by the neuro-controller. Weighting factor $\zeta$ and control force applied by each of the actuators are shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.16 Fuzzy sets and partitioning of the input space.
Figure 7.17 Control by neuro-fuzzy controller. First floor response is shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.18 Control by neuro-fuzzy controller. Control force applied by each of the actuators and total work done by the actuators on the structure are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.19 Control by neuro-fuzzy controller. Relative displacements of the three floors are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.19 Continued.
Figure 7.20 Control by neuro-fuzzy controller. Fourier spectrum of the relative displacements and absolute accelerations of the three floors are shown. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.20 Continued.
Figure 7.21 Control by neuro-fuzzy controller. Fourier spectrum of the relative displacements and absolute accelerations of the three floors are shown in logarithmic scale. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.21 Continued.
Figure 7.22 Control by neuro-fuzzy controller. Relative displacements of the three floors are shown for 20 seconds. The structure has been subjected to 50% Taft earthquake and control forces.
Figure 7.22 Continued.
Figure 7.23  Control by neuro-fuzzy controller. Control force applied by each of the actuators and total work done by actuators on the structure are shown for 20 seconds. The structure has been subjected to 50% Taft earthquake and control forces.
Figure 7.24 Control by neuro-fuzzy controller. Relative displacements of the three floors are shown for 20 seconds. The structure has been subjected to 200% El Centro earthquake and control forces.
Figure 7.24 Continued.
Figure 7.25 Control by neuro-fuzzy controller. Control forces applied by the tendons and work done by the actuators on the structure are shown for 20 seconds. The structure has been subjected to 200% El Centro earthquake and control forces.
Figure 7.26 Control by neuro-fuzzy controller. Damping of the structure has been increased by a factor of 5. It is still a low damping structure. First floor response is shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
Figure 7.27 Control by neuro-fuzzy controller. Damping of the structure has been increased by a factor of 5. It is still a low damping structure. Control force applied by each of the actuators on the structure is shown for 50 seconds. The structure has been subjected to 25% El Centro earthquake and control forces.
CHAPTER 8
COMPARISON TO FORMULATED CONTROL METHODS,
THE PREDICTIVE OPTIMAL CONTROL METHOD

The new method of “neuro-fuzzy control” of structures was proposed in the previous chapters. In chapter 7 the capabilities of this new method was shown through its application to the control of a three storey frame structure. The results have been practically successful. However this study can not be considered completed without an assessment of the performance of the proposed method in comparison to the “formulated” control methods. In this chapter, the results of application of the “predictive optimal control method”, which is one of the most flexible and powerful of the formulated control methods is presented in controlling the same structure of the previous chapters. Also to provide suitable results for the sake of comparison, the structure has been subjected to the same ground excitations. The results are then compared to those obtained through the application of the proposed neuro-fuzzy control method, demonstrated in chapter 7. It is shown that, the results of the application of the neuro-fuzzy control method are essentially better than those of the predictive optimal control.

8.1 THE PREDICTIVE OPTIMAL CONTROL METHOD

The predictive optimal control method has been proposed for practical digital control of structures by Martin-Sanches, Rodellar and their co-workers (1987, 1989). It has been used in both numerical and experimental studies of the frame structures. This method is a descendent of the optimal control methods, and can be considered as a more general form of those methods. The advantage of this method over the other formulated control methods is its flexibility to control the structure when there are inherent delays and nonlinearities in the controlled system. Rodellar and his co-workers (1989) have tested the capabilities of this control method in controlling the structure of figure 8.1, which is the same model structure of this study. However they have braced the two top floors of the
2 (4.91 Nt s²/cm)

Actuator Properties:

\( A \) = area of ram = 5.06 cm²

\( V \) = chamber volume = 151.80 cm³

\( C \) = leakage coefficient = 0.10 cm²/(NT·sec.)

\( \beta \) = compressibility = 2.1 MN/cm²/(NT·sec.)

\( \tau \) = time constant = 0.2 sec.

\( q_{\text{max}} \) = max. valve flow = 616 cm³/sec.

\( u_{\text{max}} \) = actuator capacity = 3200 NT

\( k_1 \) = 1.0 cm²/NT.

\( k_2 \) = 0.15 cm³/sec.

\( k_3 \) = 200.0

Figure 8.1 The structure, actuator and tendons.
structure. So the structure has been reduced to a one degree of freedom structure. They have studied the behavior of the structure under the effect of ground excitations. The structure has been shaken by White noise and 25% El Centro earthquake, and controlled by a tendon control mechanism, similar to the control mechanism, used in this study. Sampling period has been selected to be 0.01 seconds and a time delay of two sampling periods has been introduced to the system. The results have been encouraging.

In this study the same method has been used in the control of the same structure studied by Rodellar and his co-workers. However, the unbraced structure has been controlled to provide results for the comparison to the results of the neuro-fuzzy control method of chapter 7. It is obvious that this case requires more controlling capability than for the unbraced structure.

**8.2 METHODOLOGY**

The general idea behind the optimal control methods was explained in chapter 2. In these methods, aim is to construct a control strategy such that a compromise between the cost of control and the quality of the controlled response can be achieved. To this end, a performance index which is a scalar and contains both the information about the control cost and control response is introduced. Then by minimization of this performance index, the control rules are obtained. As explained in chapter 2, application of this method requires the solution of a set of Riccati equations. Since the optimization should take place over the whole control time domain, the Riccati differential equations are solved backwards with respect to time.

**8.2.1 Construction Of The Optimal Control Rules**

The following three steps should be accomplished to obtain the optimal control rules:

1. *Identification of the control system*, including the structure, all the control devices like actuators and sensors, and also all the sources of delay and nonlinearity in the control sys-
tem. Because the control rules are sensitive to the identification results, this step should be done precisely.

2. Providing a suitable mathematical model of the controlled system, based on the results of the identification step. This model is then used in the simulation of the controlled system. Hence, the model should be able to represent all the aspects of the controlled system, including all the delays and nonlinearities. In general the model is reduced to a linear model.

3. Obtaining the control rules, which includes introduction of the performance index and then its minimization.

The above three steps are common among all the optimal control methods. However the third step can be handled much easier and more practical in the predictive optimal control method. This is due to the fact that the predictive control is a discrete time control method.

In predictive optimal control method, after providing a suitable mathematical model for the control system, it is used in predicting the future response of the structure. In fact, the emulator neural network which was used in the neuro-fuzzy control method replaces this mathematical model, as explained in the previous chapters. However the emulator neural network was trained to learn the overall system behavior, to become a better substitute for the mathematical model which can not generally represent the real characteristics of the system.

8.2.2 Prediction Strategy

By Assuming a linear model for the control system, the governing equations of motion of the system are as explained in chapter 2:

\[ x(t) = A \ x(t) + B_u \ u(t) + B_f \ f(t) \] (8.1)
where \( x(t) \), \( u(t) \) and \( f(t) \) represent the state, control forces and external excitations in the state space. For an \( n \) degrees of freedom system, which is controlled by \( r_u \) control forces, and subjected to \( r_f \) number of external excitation, \( A \) represents the \( 2n \times 2n \) characteristic matrix of the structure, \( B_u \) the \( 2n \times r_u \) control force location matrix and \( B_f \) the \( 2n \times r_f \) external excitation location matrix.

The solution to the aforementioned differential equation is

\[
\begin{align*}
x(t) &= \Phi(t) \ x(t_0) + \int_{t_0}^{t} \Phi(t-\tau) \ [ B_u \ u(\tau) + B_f \ f(\tau) ] \ d\tau \\
\end{align*}
\]

(8.2)

where:

\[
\Phi(t) = e^{A(t-t_0)}
\]

(8.3)

Hence, in the discrete time approach, where the sampling period is \( \Delta t \), the response of the structure \( x(t+\Delta t) \) at each time step \( t+\Delta t \) can be obtained from the knowledge of the response \( x(t) \) at the previous time step \( t \) and the control and external forces which have been applied to the structure during the past increment of time \( \Delta t \). It is common to assume that the control force remains constant during the increment of time, and then the following equation is obtained:

\[
\begin{align*}
x(t+\Delta t) &= \hat{A} \ x(t) + \hat{B} \ u(t) + \int_{t_0}^{t} \Phi(t-\tau) \ B_f \ f(\tau) \ d\tau \\
\end{align*}
\]

(8.4)

where \( \hat{A} \) and \( \hat{B} \) are constant matrices, defined by:
Equations (8.5) and (8.6) are used in the prediction of response of the structure for future time steps, knowing the present response and control forces which will act on the structure. However the nature of external excitations is not completely known. So, the external excitations should not play a role in the construction of the control rules. In this case, it is assumed that the sampling period is small and the response is substantially governed by the present state and the control forces. So, the response of the structure at time step \( t + i \Delta t \) is proposed to be approximated by:

\[
\dot{A} = e^{A\Delta t} \tag{8.5}
\]

\[
\dot{B} = A^{-1} ( \dot{A} - I ) B_u \tag{8.6}
\]

and \( I \) represents the \( 2n \times 2n \) identity matrix.

In the case that the control forces are held constant during the control procedure in the time interval \([ t, t + i \Delta t ]\), equation (8.7) reduces to the simple following form:

\[
x(t + i \Delta t) = \dot{A}^i x(t) + \sum_{j=0}^{j=i-1} \dot{A}^{i-j-1} \dot{B} u(t + j \Delta t) \tag{8.7}
\]

where \( T_i \) and \( S_i \) are constant matrices, calculated according to the following equations:

\[
T_i = \dot{A}^i \tag{8.9}
\]
\[ S_i = ( I + \hat{A} + \hat{A}^2 + \ldots + \hat{A}^{i-1} ) . \] (8.10)

It is obvious that as \( i \) increases, the reliability of the prediction reduces.

### 8.2.3 Performance Index

The performance index proposed for the predictive optimal control has the same form as the performance index of the original optimal control (chapter 2). However it is not in the form of an integral over the whole control process. Instead, it is defined for each time step, based on the predictions of the response at that time step. Assuming prediction of response has been used for the next \( k \) time steps, the performance index is defined as follows:

\[ J = x(k)^T Q x(k) + u(0)^T R u(0) \] (8.11)

where \( Q \) and \( R \) are weighting matrices and it has been assumed that the control force is held constant for the whole prediction horizon. Also the control criteria has been the reduction of \( x(k) \) to zero.

### 8.2.4 Derivation Of The Predictive Optimal Control Rules

Control rules are obtained by taking derivatives of the performance index \( J \) with respect to the control force. By taking derivative of equation (8.11) with respect to \( u(0) \) and equating it to 0,

\[ \frac{\partial J}{\partial u(0)} = \frac{\partial x^T(k)}{\partial u(0)} Q x(k) + R u^T(0) = 0 . \] (8.12)
By utilizing equations (8.7), (8.8), (8.9) and (8.10), control force can be calculated from the following equation:

\[ u(0) = -(R + C_1)^{-1} C_2 x(0) \]  

(8.13)

where \( C_1 \), \( C_2 \) and \( C_3 \) are constant matrices which are calculated from:

\[ C_1 = S_k^T Q S_k \]  

(8.14)

\[ C_2 = S_k^T Q T_k \]  

(8.15)

Equation (8.13) represents a linear feedback control. The constants \( C_1 \) and \( C_2 \) are independent of the system response and depend on the characteristics of the system, the weighting matrices and the sampling period.

### 8.2.5 Time Delay Compensation

When a time delay is identified in the system, it should be considered in the construction of control rules. Assume there is a time delay of \( d \) sampling periods in the control system. In this case it is desired to reduce \( x(d + k) \) to zero. To this end one should first determine \( x(d) \) from equation (8.7), by letting \( i = d \) :

\[ x(d) = \hat{A}^d x(0) + \sum_{j=1}^{d} \hat{A}^{d-1} \hat{B} u(-j) \]  

(8.16)

where \( u(-j) \) are the known control forces which have been applied before. Then this predicted value of \( x(d) \) can be used in equation (8.13) in lieu of \( x(0) \) to provide a new form
of the control rule:

\[ u(0) = G_1 x(0) + G_2 \]  

(8.17)

where the proportionality gain matrix \( G_1 \) and the constant gain matrix \( G_2 \) should be calculated from:

\[ G_1 = -(R + C_1)^{-1} C_2 \hat{A}^d \]  

(8.18)

\[ G_2 = -(R + C_1)^{-1} C_2 \left[ \sum_{j=1}^{d} \hat{A}^{j-1} \hat{B} u(-j) \right] \]  

(8.19)

It should be mentioned that in the above formulation, the whole state of the system has been used for the construction of the control rules. However in practical situations, only a number of output measurements are available. It is assumed that it is possible to construct the whole state vector from the output measurements. In the following sections, equation (8.17) will be used as the predictive optimal control rule.

8.3 APPLICATION EXAMPLE

The structure under study is as shown in figure 8.1. It is a three storey one bay steel frame model, studied in the previous chapters too, and should be controlled by two actuators of limited capacity. Knowing the material properties and the profiles used in the construction of this model, the eigenvalues and eigenvectors of the characteristic matrix of the structure have been numerically identified. The structure has been considered as a three degrees of freedom shear model, where one degree of freedom has been attributed to each of the floors. The structure is a very low damping system, and so its response is very sensitive to any source of excitation. As mentioned in the previous chapters, such a
structure can provide us with suitable informations about the capabilities of the controller, since the controller should be able to introduce only the required forces to the structure and avoid applying forces that should be controlled by itself later on. In other words, the controller itself, should not be a source of excitation to the structure. The mass, stiffness, eigenvalues and modal matrices, and the modal frequency vector of the three degrees of freedom shear model structure are shown in table 8.1. Also the eigenvalues and eigenvectors of the matrix $A$ in equation (8.1) have been calculated from the knowledge of the eigenvalues and eigenvectors of the structure. These matrices are shown in table 8.2. They have been used in the calculation of the matrix $\hat{A}$ and its higher powers from the following equations:

$$\hat{A} = U e^{(\Lambda \Delta t)} V^T$$

(8.20)

$$\hat{A}^k = U e^{(k\Lambda \Delta t)} V^T$$

(8.21)

where $\Lambda$, $U$ and $V$ are the matrices of eigenvalues, right eigenvectors and left eigenvectors of the matrix $A$ respectively. $\hat{A}$ and its higher powers are used in equations (8.4), (8.5) and calculation of gain matrices $G_1$ and $G_2$.

### 8.3.1 Numerical Simulation

The same computer program used in the simulation of control problem of the previous chapters has been used here. In this computer program the equations of motion of the structure and actuators are considered coupled. This accounts for the nonlinearities arising from the actuator response to the control signals, its stiffness and damping effects too. However elongation of the tendons are considered negligible compared to the structural deformations. The Wilson’s–$\Theta$ integration method has been used for the dynamic analysis of the structure/actuator system. The integration time interval has been selected
Table 8.1 Parameters of the shear building model of the structure of figure 8.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| mass matrix \( M \) (Nt⋅sec^2/cm) | \[
\begin{bmatrix}
9.82 & 0 & 0 \\
0 & 9.82 & 0 \\
0 & 0 & 9.82 \\
\end{bmatrix}
\] |
| stiffness matrix \( K \) (Nt/cm) | \[
\begin{bmatrix}
40654 & -28758 & 0 \\
-28758 & 57516 & -28758 \\
0 & -28758 & 28758 \\
\end{bmatrix}
\] |
| eigenvalue matrix (rad^2/sec^2) | \[
\begin{bmatrix}
321.0281 & 0 & 0 \\
0 & 3582.7859 & 0 \\
0 & 0 & 9022.1860 \\
\end{bmatrix}
\] |
| eigenvectors matrix (cm)        | \[
\begin{bmatrix}
0.4543 & 0.7532 & 0.4756 \\
0.5924 & 0.1433 & -0.7928 \\
0.6654 & -0.6419 & 0.3811 \\
\end{bmatrix}
\] |
| modal frequencies (Hz)          | \[
\begin{bmatrix}
2.85 & 9.53 & 15.12 \\
\end{bmatrix}
\] |

To be 0.001 seconds. However, the sampling period for the measurements and control update has been chosen to be 0.020 seconds. Also for the study of the effect of presence of time delay in the control system, a delay of 0.020 seconds has been considered due to digital/analogue conversion of the control signal. The structure has been subjected to the El Centro earthquake with 25% amplitude. One delay has been simulated and one delay has been considered for the calculation of control gains.
Table 8.2 Parameters of the shear building model of the structure of figure 8.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Matrix A</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>eigenvalue matrix of A</strong></td>
<td></td>
</tr>
<tr>
<td>17.9173i</td>
<td>59.8564i</td>
</tr>
<tr>
<td>94.9852i</td>
<td>-17.9173i</td>
</tr>
<tr>
<td>-59.8564i</td>
<td>-94.9852i</td>
</tr>
</tbody>
</table>

| **right eigenvectors of A** |                               |
| -0.01268i -0.00629i -0.00250i 0.01268i 0.00629i 0.00250i |
| -0.01653i -0.00120i 0.00417i 0.01653i 0.00120i -0.00417i |
| -0.01857i 0.00536i -0.00201i 0.01857i -0.00536i 0.00201i |
| 0.22717 0.37661 0.23782 0.22717 0.37661 0.23782 |
| 0.29620 0.07165 -0.39640 0.29620 0.07165 -0.39640 |
| 0.33266 -0.32098 0.19055 0.33266 -0.32098 0.19055 |

| **left eigenvectors of A** |                               |
| 8.14056i 45.08532i 45.17789i -8.14056i -45.08532i -45.17789i |
| 10.61406i 8.57704i -75.30460i -10.61406i -8.57704i 75.30460i |
| 11.92060i -38.42570i 36.19892i -11.92060i 38.42570i -36.19892i |
| 0.45434 0.75322 0.47563 0.45434 0.75322 0.47563 |
| 0.59239 0.14329 -0.79280 0.59239 0.14329 -0.79280 |
| 0.66531 -0.64196 0.38110 0.66531 -0.64196 0.38110 |
8.3.2 Calculation Of The Gain Matrices

The two actuators have the same characteristics and behavior. So only one control signal should be issued to both of the actuators simultaneously. In this case, the control force reduces to a scalar $u(t)$. Also the gain matrices $G_1$ and $G_2$ reduce to vector quantities. To calculate the gain matrices, the weighting matrix $Q$ has been chosen to be diagonal:

$$Q = \text{diag.} [1.0\ 1.0\ 1.0\ 0.0\ 0.0\ 0.0]$$ (8.22)

and the matrix $R$ which reduces to a scalar quantity $r$, has been given different values and the effect of changing $r$ on the control performance has been studied. Also to achieve the best control on the displacements, the prediction has been made for the next time step right after time delay.

8.4 RESULTS

Table 8.3 shows the summary of the results, obtained in this phase of the study. Maximum displacement, velocity, and acceleration of the first floor of the structure, $x$, $\dot{x}$ and $\ddot{x}$ controlled by the predictive optimal controller, and the corresponding maximum control force and also the work done by the actuators in 20 seconds are shown for different values of $r$. Also the results which have been obtained through the application of the neuro-fuzzy controller in chapter 7, and the uncontrolled response are reported. Increasing $r$, means increasing the cost of control. Hence as $r$ increases, the control force reduces and the response of the controlled structure increases. Figure 8.2 is a graphical representation of the results, reported in table 8.3.

For the case of $r = 1 \times 10^{-9}$, the actuators become saturated which violates the optimality of the control method. To avoid saturation, the capacity of actuators has been increased slightly. The response of the structure is reduced considerably, however the control forces are large and the controller introduces energy to the structure.
Table 8.3 Effect of different cost weights $r$ on the predictive optimal control results, and comparison to the results of neuro-fuzzy controlled and uncontrolled structure.

| $r$            | $|x|_{\text{max}}$ cm | $|\dot{x}|_{\text{max}}$ cm/sec | $|\ddot{x}|_{\text{rel. max}}$ cm/sec | $|\dddot{x}|_{\text{abs. max}}$ cm/sec | $|\text{Force}|_{\text{max}}$ Nt | $\text{Work}$ Nt.cm |
|---------------|----------------------|---------------------------------|----------------------------------------|----------------------------------------|-------------------|-------------------|
| $1 \times 10^{-9}$ | 0.17                 | 4.15                            | 162                                    | 196                                    | 3990              | + 1841            |
| $3 \times 10^{-9}$ | 0.36                 | 6.18                            | 134                                    | 145                                    | 2135              | - 1574            |
| $5 \times 10^{-9}$ | 0.44                 | 6.24                            | 146                                    | 179                                    | 1409              | - 1653            |
| $7 \times 10^{-9}$ | 0.40                 | 6.84                            | 153                                    | 161                                    | 908               | - 1516            |
| $10 \times 10^{-9}$ | 0.45                | 7.06                            | 165                                    | 166                                    | 735               | - 1250            |
| neuro-fuzzy     | 0.31                 | 5.99                            | 192                                    | 193                                    | 1083              | - 5583            |
| uncontrolled    | 0.95                 | 11.98                           | 228                                    | 260                                    | -                 | -                 |

For the case of $r = 3 \times 10^{-9}$, the best results are obtained. Control forces are still large, but the actuators are not saturated. Figures 8.3 to 8.5 represent the time history of response of the first floor, Fourier transform of the response of the three floors, control forces and work done by the actuators on the structure. These results seem similar to those obtained by the use of the neuro-fuzzy controller. However, except the accelerations, the maximum of relative displacements, velocities and control forces are greater, and the actuators have been able to absorb much less energy from the structure, compared to the neuro-fuzzy controller.

The results for greater values of $r$ are not comparable to the neuro-fuzzy control, where greater response can be observed. As expected, control forces have been reduced with increase in $r$, and hence the accelerations have been reduced too.

For a better comparison of the predictive and neuro-fuzzy control methods, several important points should be mentioned here:
While the whole response of the structure has been fed back and used in the calculation of the above predictive optimal control forces, the feedback to the neuro-fuzzy controller has only been the relative acceleration of the first floor.

The structure has been simulated as a linear system. The only source of nonlinearity has been the actuator dynamics, which has not been considerable. Also only one identified time delay has been considered in the control loop. However such assumptions and simple mathematical modelling result in an idealistic simulation of the control system. In real situations, identification can not be done precisely. There may be more sources of delay and nonlinearity in the system. Also the structures are distributed parameter systems which can generally not be simulated as simple shear buildings. Even if the structures are identified precisely and simulated accurately, it is hard to find a suitable control rule for them. Meanwhile, in the neuro-fuzzy control approach, no assumptions and limitations have been imposed on the neuro-fuzzy controller with regard to the complexity of control system.

8.5 CONCLUDING REMARKS

In this chapter, the application of the predictive optimal control method in the control of a three storey frame structure was studied. There are similarities between the predictive optimal control method and the proposed neuro-fuzzy control method. Both of these methods emphasize on the need for the construction of a controller, which works based on some sort of predictions about the future of the response. Prediction of the response is formulated in the predictive control approach and is based on the current state of the system only. No information about the external excitation is used in the construction of the control rule. However in the proposed neuro-fuzzy control approach, the emulator neural network learns to predict the future response of the structure, with regard to the history of response and control signals. This latter method of prediction is more precise because some kind of implicit knowledge about the external excitations is considered in the predictions. Through the comparison of results, it was shown that the overall perform-
ance of the neuro-fuzzy controller has been better than the predictive optimal controller.

REFERENCES


Figure 8.2 Effect of $r$ on the performance of the predictive optimal controller.
Figure 8.3 Control by ‘Predictive optimal control’ algorithm. First floor response is shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces. $r = 3 \times 10^{-9}$.
Figure 8.4  Control by 'Predictive optimal control' algorithm. Fourier transform of the relative displacement and absolute accelerations of the three floors are shown. The structure has been subjected to 25% El Centro earthquake and control forces.

\[ r = 3 \times 10^{-9} \]
Figure 8.4 Continued.
Figure 8.5 Control by 'Predictive optimal control' algorithm. Control forces applied by the tendons on the structure and the work done by both of the actuators on the structure are shown for 20 seconds. The structure has been subjected to 25% El Centro earthquake and control forces. $r = 3 \times 10^{-9}$. 
In the previous chapters, a new method for the active control of structures was proposed. This method is based on the use of neural networks and fuzzy logic. The main purpose of proposing this method is to use the high degree of adaptivity and learning capabilities of neural networks, and also the capabilities of the fuzzy logic as a supplementary tool, for the construction of more powerful adaptive controllers. In this method, a neural network learns to control the structure, based on a set of data which is collected about the control procedure. This neural network is called the “neuro-controller”. A set of simple fuzzy rules is then developed as the supplementary fuzzy controller to improve on the performance of the neuro-controller. The resulting controller is then called the “neuro-fuzzy controller”.

Because the neuro-controller learns from a set of rough data the general task of controlling the structure, this method can be classified as a “data based” or an “intelligent” or preferably a “learning” control method. Also to be more specific, this method was referred to as the “neuro-control method” or the “neuro-fuzzy control method” in this text, depending on the use of neural networks only or both the neural networks and fuzzy logic. This method was used in the simulated digital tendon control of a typical three storey one bay steel frame structure. Several earthquake loadings were applied to the structure and the structure was controlled by both the neuro-control and the neuro-fuzzy control methods. All the results of the study were satisfactory.

In the last one or two decades several control algorithms have been proposed. They have more or less been successful in their application to the control of simple structures. The common characteristic of all of these algorithms is that they need a sound mathematical modelling to characterize the controlled system. To this end, the parameters of the controlled system should be identified. These algorithms can be called the “conventional control methods” or preferably the “formulated control methods” to emphasis on this fact that they are based on mathematical formulations and be distinguished from the
9.1 METHODOLOGY OF NEURO-FUZZY CONTROL OF STRUCTURES

The objective of this method is the training of a neural network to learn to control the structure. This neural network is called the “neuro-controller”. Then, a fuzzy controller is developed to cooperate with the neuro-controller and improve on its performance. The controller, comprised of the neuro-controller and the fuzzy controller is called the “neuro-fuzzy controller”.

9.1.1 Neuro-Controller

Construction of the neuro-controller is accomplished in three major steps. These steps are the training of an emulator neural network, a preliminary control by the use of the emulator neural network and the training of the neuro-controller.

*Emulator neural network* Emulator neural network learns about the behavior of the structure. It is then used in the prediction of response of the structure and also computation of sensitivity of the future response vector with respect to the vector of control signal. To this end, a set of train cases is provided by the application of random excitations similar to the real external excitations and also randomly generated actuator forces to the structure. Response of the structure and actuator forces are recorded. Then train cases are constructed from these records. The input layer of the emulator neural network is comprised of nodes, representing the immediate history of response and the control forces, in addition to the control signals to be sent to the actuators. Sensitivity of the response with respect to changes in the control signals can then be calculated easily either by backpropagation method or by a direct change in the control signal and measurement of the change in the outputs of the emulator.

*Preliminary control* The emulator is then used in the control of the structure. At this step the appropriate control signal which should be sent to the actuators are not known yet.
Hence the control signals which have been sent to the actuators are assumed to be suitable for the next time steps too. Sensitivities of the response to the signals are then calculated, and by the use of these sensitivities the control signals are modified so that the predicted response satisfies the control criteria. The control criteria should be defined according to the specifications or other requirements.

**Training of the neuro-controller** The results of the preliminary control are not generally of good quality. However the overall control is acceptable. A neuro-controller is then trained to learn to control the structure from the data collected during the preliminary control. In fact the whole knowledge involved in the preliminary control strategy, including the emulator, the control criteria and the required control forces are concisely transferred to the neuro-controller through its training. Neuro-controller extracts the general knowledge of control task from the training cases to performs a smooth control job. Neuro-controller can then be put to work alone.

**9.1.2 Fuzzy Controller**

Although the neuro-controller performs a smooth control job, there are exceptional situations that sharp changes in the control signals are required to mitigate the response of the structure induced by unexpected excitations. Such situations for example occur at the onset of a strong earthquake. Hence, a simple fuzzy controller which acts on the proposed control signal of the neuro-controller and improves on it, renders help in the mitigation of such excitations. The fuzzy controller is proposed to be simple and general enough to work under different types of excitations.

**9.1.3 Neuro-Fuzzy Controller**

The neuro-controller and fuzzy controller are installed in series. The control signal vector which is issued by the neuro-controller is then modified and improved by the fuzzy controller.
9.2 ADVANTAGES OF USING THE NEURO-FUZZY CONTROL METHOD

To judge about the capabilities of a control method, three characteristics of that method should be put into scrutiny. These characteristics are the degree of adaptivity, the prediction capability and use of the predictions, and the simplicity of that method. These three aspects are explained in more details in the following paragraphs. Thanks to the high learning capability of neural networks and also the flexibility obtained by using fuzzy logic, the proposed neuro-fuzzy learning control method is much better than the other control methods in all of the above three aspects.

Adaptivity By adaptivity here, the capability of the control method in coping with the sources of nonlinearity and delays and also imperfections in the controlled system is meant. Nonlinearities may arise from the nonlinear material, the actuator dynamics or change in the properties of the controlled structure due to control forces. Delays may exist in the controlled system because of the digital to analogue conversion of the control signal and the computation time. Imperfections in the real models or structures may result in some nonlinearities in the controlled system too. Most of the control algorithms are developed for the linear systems. The assumption of linearity is generally not valid. Some of these methods are modified to cope with the above mentioned nonlinearities. So far, the "predictive optimal control method" has been the most adaptive control method among the formulated control methods. It has been tested by some authors in the control of numerical models. Also in this study, it was used in the control of the standard three storey frame structure for the sake of comparison of the results. However all of these methods need a detailed reliable identification of the controlled system including structure, actuators, tendons and other subsystems of the control system. This is practically cumbersome and problematic for the real world applications, where a structure with many degrees of freedom should be controlled by several actuators with different characteristics. Neural networks are theoretically universal approximators. They can be trained to learn the characteristics of the controlled system including all the nonlinearities, delays and imperfections from a set of relevant data. Theoretically, an emulator neural network can be trained to learn predict the future response of a structure, based on the knowledge of the previous response and excitations to the structure. To this end, the input layer of the neural network should contain the information about the history of response and exci-
The output should then represent the future response of the structure. By the provision of enough data and training of a neural network on that data, a sort of implicit identification is accomplished. Details of using this capability of neural networks in the neuro-fuzzy control method was explained in the previous chapters.

**Prediction capability** A good controller should be able to predict the future response of the structure and issue an appropriate control signal to satisfy control objectives for some future time steps. This capability is specially important in the presence of time delays. The formulated control methods have limitations in prediction or use of predictions in the construction of control signals. It is hard for the proposed formulated control methods to consider the effect of external excitations in the construction of control rules. The neural network based control method, uses the history of response to predict the future response of the structure. Since the previous response contains some informations about the external excitations implicitly, the prediction contains an implicit extrapolation of the previous external excitations too. The prediction reliability deteriorates for more futuristic predictions. However it is obvious that prediction can be better done in the new proposed method than its formulated counterparts.

**Simplicity** Providing a suitable mathematical model to characterize the control system is not easy. Even if such model is provided, it seems hard to prepare a suitable set of control rules based on this model. The neural network based method provides us with a very simple scheme, where a neural network learns about the rules of control without the need for any explicit formulation of the control problem. The proposed control method seems applicable to any type of structure. This generality of the control method may be considered as its basic characteristic of simplicity.

With regard to the above mentioned facts, it is possible to say that the neuro-controlled can at least learn to perform as well as the other control methods. In fact a neural network may be trained based on the data obtained from a control, performed by another controller and replace it. However such a strategy is far inferior than what has been proposed in this study.
9.3 APPLICABILITY OF THE NEURO-FUZZY CONTROL METHOD

Many issues should be addressed in this relation. These issues include but not limited to the following questions:

How to provide data for the training of the emulator neural network? The answer to this question is two fold. Firstly it should be said that the need for providing data about the response of the structure for the sake of identification is common among all the other control methods too. Secondly this is a regular system identification problem. To identify the system, it is required to provide enough data about the response of the structure under appropriately selected external excitations. This rough data may be used in the training of the neural networks too. Another option is to identify the controlled system and provide a mathematical model to characterize the controlled system based on the rough data. This model can then be used in the analysis of the system. The results of this analysis is then used in the training of the emulator neural network. It should be mentioned that just the provision of a suitable mathematical model does not mean that it is possible to construct a suitable formulated controller. The mathematical model may be complicated for that purpose. However the results of analysis can be used as data in the neuro-control method. Hence the neuro-controller shares with the other control methods the same problem of the need for a set of data about the response of the structure. However to use this data in the training of the neuro-fuzzy controller it is generally not necessary to identify the parameters of the system from this data. The question of how to provide data for system identification has been a challenging question for decades in the field of structural mechanics and there is a vast literature on this subject (Hart and Yao 1977, Liu and Yao 1978).

How fast can the neural network learn the data? Although this question is not directly related to the control method, it is practically important. A part of this study has been to develop a faster training algorithm for the multi-layer feed-forward neural networks. For the initial stages of the study, a computer program has been developed to simulate a multi-layer feed-forward neural network. Fahlman's quickprop algorithm has been selected for the training of the neural network. Also an architecture growing mechanism has been implemented in the computer program. Although the performance of the neural network
was acceptable at this stage, its learning speed was low. A training algorithm has been developed to increase the learning speed of the neural network considerably. The training algorithm is based on two main fundamentals: random selection of training data from the batch of data, and addition of new nodes based on the evaluation of convergence speed. Whenever a new node is added, more training cycles are permitted and as the size of batch increases, the number of permitted training cycles gradually reduces to a certain limit and the probability of the addition of new nodes increases. These features have been explained in chapters 4 and 5 of this thesis in more details. This new training strategy has been greatly effective in increasing the learning speed of the neural network. A comprehensive study about the speed of convergence by testing several problems both in structural control and other subjects has shown that the speed of convergence has been increased by a factor of 60 to 100, and the size of the architecture has been significantly decreased. In fact part of increase in the convergence speed comes from the reduction in the size of the neural network.

In this study, the construction of neuro-fuzzy controller was accomplished through an off-line training of the neural networks. For the time being and with the available neural networks, this is a better strategy than the on-line training method. Once the required data is collected, the neural networks can be trained to learn the data. There is less limitation on the training time when using off-line training, compared to the on-line training of the neural networks.

*How can the controller adapt itself to the modifications in the controlled system?* These modifications include items like changes in the mass or the stiffness of the structure which take place during time. The controller can be trained when it is operating. There is always excitations to the structure and data can be collected from the response of the structure when subjected to these excitations. The controller can be trained by this data to learn about the modifications in the structural parameters. This can be done according to the proposed general training scheme of the previous chapters. The only difference is that instead of training the controller from no controlling knowledge it should be updated to improve its available knowledge.
9.4 NUMERICAL STUDY OF THE METHOD

A neuro–controller has been trained to control a three storey one bay frame structure, subjected to different earthquakes. Only one output measurement, the relative acceleration of the first floor has been used as the output measurement of the structure. In this method, first an emulator neural network was trained based on the data collected from the analysis of the structure subjected to El Centro earthquake with 25% amplitude or white noise as the ground excitations, in addition to white noise as the control signal. A preliminary control was performed by the use of the emulator to reduce the response of the structure. The neuro–controller was trained based on the data collected from the preliminary control. A fuzzy controller was developed to correct the control signals issued by the neuro–controller based on the previous velocity and displacement of the first floor. The velocity and displacement of the first floor have been calculated from simple integration of the acceleration.

The neuro–fuzzy controller was then tested for El Centro earthquake of 25% amplitude, Taft earthquake of 50% amplitude and also El Centro earthquake of 200% amplitude. The results for all of these cases have been significantly successful. The power of the controller in understanding the severity of the situation and its selection of suitable control signals is obvious. The previous chapters include detailed discussion of results.

Another controller was then developed by the use of the predictive optimal control method. To prepare this controller, all the state of response of the structure was considered as the output. The controller was used in the control of the same structure, when subjected to the same earthquakes. Comparison showed that the results of control by the neuro–fuzzy controller is generally of better quality.

9.5 PLANS FOR THE FUTURE RESEARCHES

In this numerical study, enough documents was provided for the demonstration of the performance of the proposed neuro–fuzzy control method. It is proposed to begin an experimental study on the active control of the standard three story frame by the use of the proposed neuro–fuzzy control method. Such study may bring into focus the practi-
cal issues that should be answered for the application of this method in the control of real structures. Also the immediate numerical studies may include:

- Application of this method in the control of tall buildings by the same tendon control mechanism, when many actuators are involved.
- Control of framed structures for other types of dynamic loadings such as wind loadings.
- Use of other control mechanisms such as active mass dampers and appendages to replace the tendon control mechanism in the aforementioned control problems.
- Control of other types of structures such as bridges, towers and floating structures.
- Use of more complicated control criteria such as optimality and tracking criteria.
- Control of structures with material nonlinearity.

9.6 PANORAMA

In this study, it was demonstrated that the neuro-fuzzy control method shows more capabilities than the other methods. This was shown through several numerical studies and comparison between the neuro-fuzzy and the predictive optimal control methods, as discussed in chapters 7 and 8. Although such comparative studies are useful for the sake of collecting knowledge about different features of different methods, the author believes that all of the proposed control methods may be useful in the control of structures. For the simple structures where a linear mathematical model can be provided, it is wise to use a simple control method. Also if a structure satisfies all the assumptions which are required for the application of a specific control method, there is no reason to ignore that control method. For complicated structures however, the use of a more adaptive control method is recommended, where use of the neuro-fuzzy control method is expected to be considerably beneficial.

The recent approach to the active control of structures is to combine the use of passive and active control mechanisms. These kind of hybrid systems have been studied in some simple problems recently. Some of these studies are reported in Ang and Villaverde (1993). More studies of this kind is expected to be done in the near future.

Application of active structural control methods, including the neuro-fuzzy control
method to the real structures doesn't seem possible in near future. Some elementary applications of active mass dampers have been investigated by Japanese designers. They have installed these mechanisms on the top floor of the high rise buildings to mitigate the wind induced response of the structures (Nagase et al 1993). Meanwhile it is too early to see a real application of the active control methods in structural control. To attract more attention of the designers, justify and demonstrate the advantages of using the active control methods and mechanisms in the control of civil structures, more experimental studies should be carried out. It is expected that same as the other advancements in technology, the first application of the active control methods appear in the control of special structures, like towers, structures that contain machineries and floating structures, etc.

Also it is expected that the benefits of using neuro–fuzzy control method become more and more clear through its application to the control of complicated structures.

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VITA

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He was cooperating with different structural design and construction companies in Shiraz and Tehran, Iran from 1985 to 1991. This working experience included the design and construction of precast concrete structures, design of concrete dams and also design of off-shore structures.

His first publication is the book "Artificial Viewpoint" in Farsi which was written in 1985, during the last year of his B. S. studies. During his Ph. D. studies in the University of Illinois, he wrote several papers with his advisor and colleagues on the issues related to neural networks, fuzzy logic and structural control.