Strain Rate and Inertial Effects on Impact Loaded Single-Edge Notch Bend Specimens

By

Pedro M. Vargas
Exxon Production Research Company

and

Robert H. Dodds, Jr.
University of Illinois

A Report on a Research Project
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Many significant problems in fracture mechanics of ductile metals involve surface breaking defects (cracks) located in structures subjected to short-duration loading caused by impact or blast. When the severity of impact loads is sufficient to produce large inelastic deformations, the assessment of crack-tip conditions must include the effects of plasticity, strain rate and inertia. This work examines the interaction of impact loading, inelastic material deformation and rate sensitivity with the goal of improving the interpretation of ductile fracture toughness values measured under dynamic loading. We focus on shallow and deeply notched bend test specimens, SE(B)s, employed routinely to measure the static fracture toughness of a material. A thorough understanding of the test specimen’s dynamic behavior is a prerequisite to the application of measured fracture properties in structural applications.

Three-dimensional, nonlinear dynamic analyses are performed for SE(B) fracture specimens \((a/W=0.5, 0.15, 0.0725)\) subjected to impact loading. Loading rates obtained in conventional drop tower tests (impact load-line velocities of \(\approx 6\) m/sec) are applied in the analyses. An explicit time integration procedure coupled with an efficient (one-point) element integration scheme is employed to compute the dynamic response of the specimen. Strain-rate sensitivity is introduced via a new, efficient implementation of the Bodner-Partom viscoplastic constitutive model. Material properties for A533B steel (a medium strength pressure vessel steel) are used in the analyses. Static analyses of the SE(B) specimens provide baseline responses for assessment of inertial effects. Similarly, dynamic analyses using a strain-rate insensitive material provide reference responses for the assessment of strain rate effects. Strains at key locations on the specimens and the support reactions (applied load) are extracted from the analyses to assess the accuracy of static formulas commonly used to estimate applied \(J\) values. Inertial effects on the applied \(J\) are quantified by examining the acceleration component of \(J\) evaluated through a domain integral procedure.
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ABSTRACT

Many significant problems in fracture mechanics of ductile metals involve surface breaking defects (cracks) located in structures subjected to short-duration loading caused by impact or blast. When the severity of impact loads is sufficient to produce large inelastic deformations, the assessment of crack-tip conditions must include the effects of plasticity, strain rate and inertia. This work examines the interaction of impact loading, inelastic material deformation and rate sensitivity with the goal of improving the interpretation of ductile fracture toughness values measured under dynamic loading. We focus on shallow and deeply notched bend test specimens, SE(B)s, employed routinely to measure the static fracture toughness of a material. A thorough understanding of the test specimen’s dynamic behavior is a prerequisite to the application of measured fracture properties in structural applications.

Three-dimensional, nonlinear dynamic analyses are performed for SE(B) fracture specimens (a/W=0.5, 0.15, 0.0725) subjected to impact loading. Loading rates obtained in conventional drop tower tests (impact load-line velocities of ≈ 6 m/sec) are applied in the analyses. An explicit time integration procedure coupled with an efficient (one-point) element integration scheme is employed to compute the dynamic response of the specimen. Strain-rate sensitivity is introduced via a new, efficient implementation of the Bodner-Partom viscoplastic constitutive model. Material properties for A533B steel (a medium strength pressure vessel steel) are used in the analyses. Static analyses of the SE(B) specimens provide baseline responses for assessment of inertial effects. Similarly, dynamic analyses using a strain-rate insensitive material provide reference responses for the assessment of strain rate effects. Strains at key locations on the specimens and the support reactions (applied load) are extracted from the analyses to assess the accuracy of static formulas commonly used to estimate applied J values. Inertial effects on the applied J are quantified by examining the acceleration component of J evaluated through a domain integral procedure.
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Computational support was provided by the HP/Apollo Workstation Laboratory of the Department of Civil Engineering, and on a Convex C240 supercomputer operated by the Computing Services Office of the University of Illinois. Grants from the Hewlett-Packard Corporation contributed significantly to the computational resources.
1. Introduction

The interpretation of measured fracture properties for a material under impact loading requires a careful assessment of dynamic effects on specimen response. Impact testing introduces potentially three dynamic effects of interest: 1) stress waves and specimen vibration, 2) high strain rates at the crack tip, and 3) acceleration of material in the crack-tip region. Direct experimental evaluation of these effects and their individual influence on ductile fracture parameters in shallow and deeply notched SE(B) specimens does not appear possible. In this study, we use 3-D, nonlinear finite element analyses to examine separately each of these effects. Quasi-static analyses of the specimens provide reference responses from which inertial effects are assessed. Companion dynamic analyses using two different materials, first a strain-rate independent material and then a strain-rate sensitive material, enable the effects of strain rate alone to be examined.

Strains at key locations on the specimens and the support reactions (or total load), quantities which can be measured during tests, are extracted from the analyses and used in conventional quasi-static methods for \( J \) computation (\( J \) methods). These estimates for \( J \) are compared to those obtained with domain integral computations which use near-tip fields from the nonlinear, dynamic finite-element analyses. A key feature of the analyses involves estimation of the time following impact at which inertial effects diminish sufficiently for the conventional (static) \( J \) formulas to apply.

Numerical results are reported here for SE(B) specimens with dimensions \( W=B=50 \) mm and a span of 200 mm for which experimental impact results are available [12]. Stress-strain properties for A533B (pressure vessel) steel are adopted in all finite element analyses. Loading rate effects on the uniaxial stress-strain behavior of this material have been studied extensively. The finite element models employed in these analyses have sufficient mesh refinement for accurate evaluation of the \( J \)-integrals and Crack Tip Opening Displacement (CTOD) over the crack-front as the loading increases. However, the meshes provide only crude estimates of the strain and stress fields over distances of several CTODs from the tip. Even so, these models require several days of computation on a fast, desktop workstation.

Following a brief discussion of a typical arrangement for impact testing of SE(B) specimens, the three-dimensional finite element models developed for the impact analyses and the constitutive model for viscoplastic effects are described. The remaining sections provide a detailed description and assessment of strain rate and inertial effects on the global specimen response and \( J \)-integral values. The paper concludes with a summary of the most significant observations derived from the numerical analyses.

2. Impact Testing Procedures

Dynamic fracture testing is performed frequently with a drop tower arrangement as illustrated in Fig. 1. Specimens are fabricated with a through-thickness saw cut that is pre-sharpened with fatigue loading. Three-point loading is accomplished with the support arrangement indicated in the figure. Deeply notched specimens \( (a/W=0.5, \text{ where } a \text{ is the crack depth and } W \text{ is the total specimen width}) \) with 50 mm square cross-sections and 200 mm span have been successfully tested with a drop weight of 545 Kg with an impact velocity of 6 m/sec. This impact velocity corresponds to a drop height of approximately 1.2m. The elapsed time from initial impact to specimen fracture is in the range of 0.001-0.006 seconds for ferritic materials with yield strengths of 400-700 MPa. Numerical studies [17,18] have demonstrated that the applied load vs. time for deeply notched specimens is predicted accurately using the ordinary static bending formula keyed to longitudinal (bending) strains at the quarter-span locations as indicated on the figure. Two factors enable this accurate correlation of measured strain values with applied load: 1) the soft aluminum wedges minimize elastic rebounding of
the drop weight upon impact, and 2) the large $a/W$ ratio confines inelastic deformations to the remaining ligament on the crack plane.

Figure 1. Typical Drop Tower Arrangement

Figure 2 shows the measured displacement–time curve for a typical drop tower test [10,13]. The testing apparatus effectively generates a constant velocity response over two regimes: (1) during the initial elastic and small-scale yielding response and (2) following formation of a full plastic hinge. To provide the final regime of constant velocity loading, the kinetic energy of the tup must eventually overwhelm the internal energy absorbed by the specimen prior to fracture. The nearly constant velocity produces a terminal, linear displacement–time loading of the specimen, i.e., the 2.5 m/sec loading region shown in Fig. 2. An optical probe attached directly to the specimen is often used to measure the load-line displacement. Even at large deformations, the plastic zone in these specimens is confined to the remaining ligament, leaving a large portion of material remote from the crack plane elastic, with little deformation, which facilitates attachment of the optical light probe.

3. Finite Element Models

Figure 3 shows typical finite element models developed in this study. Models with two levels of mesh refinement, denoted coarse and refined, are shown. Due to symmetry, only one quarter of the specimen is actually modelled (the shaded region). All elements are trilinear hexagonal bricks (8-nodes), using a one point Gauss integration coupled with an effective procedure to control hourglass modes. The figures indicate the number of elements and nodes for each of the finite element models. The anvil supports are modelled by constraining the vertical displacement of the bottom surface nodes 25 mm from the specimen ends.

The Spectrom code [11] is used to compute the dynamic response of the SE(B) specimens subjected to impact loads. This code utilizes an explicit scheme to integrate through time, which makes it ideal for monitoring stress–wave effects. Element formulations accommodate finite–strains and finite–rotations using an Updated Lagrangian approach. The code provides a 4–node shell element and the three–dimensional 8–node brick element. Several nonlinear constitutive models are available in the standard version but not a strain–rate dependent plasticity model. The program runs efficiently on 32–bit workstation environments as well as supercomputers. Numerous modifications and enhancements were implemented to support the SE(B) analyses [23].
The application of time dependent external forces is the preferred method of loading a finite element model. However, the lack of detailed information on the applied loading requires a procedure to generate a loading that, when applied to the finite element model, predicts the displacement history measured in tests. The finite element loading to model the drop tower test is generated by first executing displacement controlled (dynamic) analyses in which the loaded area is uniformly displaced according to a measured displacement history (Fig. 2). The coarse models are used in these analyses. Application of the uniform displacement across the loading area effectively creates a rigid boundary that causes high frequency oscillations. Reactions at the displaced nodes are extracted and smoothed to generate an equivalent external loading history. These external loads are applied to the refined models over the loading area as an equivalent (time-dependent) uniform pressure. The computed displacement histories of the specimen under the smoothed loadings are compared for agreement with the measured displacement history, with adjustments in the process until a satisfactory loading is produced. Figure 4 summarizes the steps of this procedure.

4. Modeling of Viscoplastic Response

The Spectrom code was enhanced to include a rate-dependent plasticity model suitable for ductile metals. The selected Bodner–Partom constitutive model belongs to the family of viscoplastic theories of the “unified” approach, which combine both time-independent plasticity and time-dependent phenomena such as creep and stress relaxation into a single state variable [3]. Initially developed from a simple model attributed to Norton [20], the Bodner–Partom material model has since been modified to include isotropic and directional hardening effects [4]. The essential features of the Bodner–Partom model are: 1) the Prandtl–Reuss flow rule, 2) a kinetic equation that relates the strain rate to the stress and the hardening variable, and, 3) an evolution law for the internal hardening variable. Vargas and Dodds [23] describe an efficient implementation for the Bodner–Partom material model in Spectrom to support the SE(B) analyses.

The characterization of viscoplastic properties of a material with the Bodner–Partom material model requires multiple uniaxial tests of the material under widely varying strain rates. The extensive testing and associated expense have limited the number of materials for which constants are available.
Dexter and Chan [6] tested A533B steel to determine properties for the Bodner-Partom material model. Table 1 summarizes the mechanical properties and the Bodner-Partom constants at 50°C used in this study. The corresponding (tensile) stress–strain curves for different strain rates at 50°C are shown in Fig. 5. The yield stress increases significantly with strain rate. In quasi-static analyses and rate-insensitive dynamic analyses, we use the stress–strain curve for a strain rate of 0.001/sec. Other materials similarly characterized for the Bodner-Partom model include: A537 steel, X46 and X70 pipeline steels, and B1900+Hf, a nickel based super-alloy [5].

![Stress-Strain Curves for A533B Steel Predicted by Bodner-Partom Model](image)

### Table 1. Mechanical Properties for A533B Steel

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<th>Value</th>
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<tr>
<td>Young's Modulus, $E$</td>
<td>30,000 ksi (206.9 GPa)</td>
</tr>
<tr>
<td>Poisson's Ratio, $v$</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$</td>
<td>64.5 ksi (445 MPa)</td>
</tr>
<tr>
<td>Ultimate Stress, $\sigma_u$</td>
<td>86.6 ksi (597 MPa)</td>
</tr>
<tr>
<td>Elongation</td>
<td>24%</td>
</tr>
<tr>
<td>Area Reduction</td>
<td>69%</td>
</tr>
<tr>
<td>Density</td>
<td>$7.35 \times 10^{-4}$ lb·sec²/in⁴ (7850 Kg/m³)</td>
</tr>
</tbody>
</table>

Bodner Partom Constants at 122°F (50°C):

$m = 0.441$ ksi⁻¹ (0.064 MPa⁻¹)

$n = 1.75$

$\dot{\omega} = 10^8$ sec⁻¹

$Z_0 = 200$ ksi (1379 MPa)

$Z_1 = 262$ ksi (1804 MPa)

5. Dynamic Effects in SE(B) Specimens

Figure 6 shows the vertical displacement histories at mid-span for the deep crack SE(B) ($a/W=0.5$) specimen. Displacements at two locations on the remaining ligament are indicated. The minimum vertical displacement occurs at the crack tip on the longitudinal centerplane of the specimen; the maximum vertical displacement occurs at nodes on the loaded area at the (outside) free surface. Load-line displacements measured with an optical light probe during the drop tower test are also given. The good agreement between the finite element analyses and the drop tower test record indicate the success of the load generation scheme described previously. Similar agreement is obtained for the $a/W=0.15$, 0.0725 specimens.

Times after impact are normalized by the time required for an unbounded dilatational wave to travel the width of the specimen, $t_W$ [17]. Using the elastic properties for A533B steel, the unbounded dilatational wave speed, $c_1$, is $5.1 \times 10^6$ mm/sec. For a specimen width $W=51$ mm, $t_W$ is then:

$$t_W = \frac{W}{c_1} = 8.5 \times 10^{-6} \text{ sec} \quad (1)$$

The 0.006 sec duration of the analyses corresponds to approximately 700 wave traversals over the specimen width. Spatial diffusion after several traversals significantly diminishes discrete wave effects and they become negligible for most of the specimen response.

The vertical displacement histories shown in Fig. 6 exhibit clear periodic oscillations during the initial 0.003 seconds. Simple elastic vibration in the first dynamic mode produces these oscillations. The first two modes and their frequencies were computed for the finite element models using the POLO–FINITE system [7]. Due to symmetry conditions imposed in the analyses, the first two computed modes correspond to the first and third modes of the full specimen. The periods of the first mode and third mode of the deep crack specimen are $0.62 \times 10^{-3}$ sec and $0.12 \times 10^{-3}$ sec, respectively.

Nakamura, et al. [17,18] introduced the concept of a transition time, $t_T$, which defines the point in the response after which inertial effects diminish rapidly. Upon impact of the loading tup, the spec-
imen velocity and the kinetic energy both increase very rapidly. As the specimen begins to deform, the internal energy also increases at a rapid rate. For the range of loading rates, specimen sizes and material flow properties considered here, the total internal energy eventually overtakes the total kinetic energy of the specimen due to extensive plastic deformation. The transition time occurs when the total kinetic energy ($T$) of the specimen becomes less than the internal energy ($U$) of the specimen. After the transition time, the kinetic energy continues to increase, but at a much diminished rate relative to the rate of increasing internal energy. Nakamura proposed, and validated, a limit of $2 \times t_T$ as a time after which the evaluation of the fracture parameters ($J$-integral) using conventional static formulas based on areas under load-load line displacement curves yields acceptable accuracy.

Figure 7 shows the evolution of the energy ratio after impact for the deep crack specimen. The transition time occurs very early in the response while the specimen remains predominantly linear-elastic. Strain rate effects are negligible during this early stage, and both rate-sensitive and rate-insensitive material models predict identical energy ratios.

Nakamura, et. al [17,18] performed a dynamic finite element analysis of a deep notch SE(B) specimen using a higher loading rate (terminal velocity of 4.7 m/sec compared to 2.5 m/sec here and thickness $B=25$ mm rather than 50 mm used here). They report the evolution of energy ratio indicated in Fig. 7 with a transition time identical to that found for the deep crack SE(B) specimens of this study. For the present analyses, Fig. 7 shows that the energy ratio decreases very rapidly, approaching zero soon after the transition time of $0.24 \times 10^{-3}$ seconds, which also corresponds to approximately $28 \times t_W$. Nakamura also reports a similarly normalized transition time of $28 \times t_W$ for a deep crack specimen.

In our study, we conducted dynamic finite element analyses for specimens with $a/W$ ratios of 0.5, 0.15, 0.0725 for a wide-range of loading rates. In all cases, the transition time occurs at approximately $0.4 \times$ the first elastic vibration period of each specimen. A more complete discussion of this issue is given by Vargas and Dodds [23].

5.1 $J$-Integral with Inertia Loading Effects

Extensions of the $J$-integral to incorporate the effects of dynamic loading for non-growing cracks are developed by including the kinetic energy density of material at the crack tip in the same manner as the strain energy density [17]. Thus,

$$
J = \lim_{\epsilon\rightarrow 0} \int_R \left( (W + T)n_1 - P_{ij}\frac{\partial u_i}{\partial X_j} n_j \right) d\Gamma
$$

(2)
\[ W = |F| \int_0^{t_i} \sigma_{ij} \, de_{ij} \quad ; \quad F = \frac{\partial x}{\partial X} \quad ; \quad x = X + u \]  

(3)

\[ T = \frac{1}{2} \rho \left( \frac{\partial u_i}{\partial t} \right)^2 \]  

(4)

where \( W \) and \( T \) are the strain and kinetic energy densities relative to the undeformed volume at \( t=0 \), respectively; \( n_i \) denotes components of an outward unit vector to the contour, \( \Gamma \); \( \sigma_{ij} \) and \( de_{ij} \) are the Cauchy stresses and differential strains (rate of deformation tensor \( \times \) \( dt \)), respectively; the (non-symmetric) 1st Piola-Kirchhoff stresses are indicated by \( P_{ij} \); \( \rho \) is the material mass density; \( u_i \) are the displacements; \( t \) denotes time and \( X_i \) denotes coordinates in the undeformed specimen at \( t=0 \).

As indicated in Eqn. 3, \( F \) denotes the deformation gradient relative to \( t=0 \). The positive direction of the contour is shown in Fig. 8. The integral becomes applicable for arbitrary material response in the limit as the contour shrinks to a point on the crack front. In three dimensions the contour \( \Gamma \) is defined in a plane perpendicular to the crack front at point \( s \) as shown in Fig. 8.

![Figure 8. Local J–integral in 3–D](image)

![Figure 9. Finite Volume for Use in Domain Integral Formulation](image)

By using a weight function which may be interpreted as a virtual displacement field, the contour integral is converted into a volume integral in three dimensions [14,15]. The resulting expressions are:

\[ J_{a-c} = \int_{s_a}^{s_c} \left[ J(s)q_k(s) \right] \, dS = J_1 + J_2 + J_3 \]  

(5)

\[ J_1 = \int_{V_0} \left( P_{ij} \frac{\partial u_i}{\partial X_j} - W \frac{\partial q_k}{\partial X_k} \right) \, dV_0 \]  

(6)

\[ J_2 = -\int_{V_0} \left( \frac{\partial W}{\partial X_k} - P_{ij} \frac{\partial^2 u_i}{\partial X_j \partial X_k} \right) q_k \, dV_0 \]  

(7)

\[ J_3 = -\int_{V_0} \left( T \frac{\partial q_k}{\partial X_k} - q \frac{\partial^2 u_i}{\partial X_j \partial X_k} q_k + Q \frac{\partial u_i}{\partial X_j} \frac{\partial^2 u_i}{\partial X_k} q_k \right) \, dV_0 \]  

(8)

where \( q_k \) is the weight function in the \( k \) coordinate direction (\( q_k(s) \) represents weight function value at point \( s \) on the crack front), \( V_0 \) represents the volume of the domain surrounding the crack tip (in the \( t=0 \) configuration), and \( s \) denotes position along the crack front segment. Figure 9 shows a typical domain volume defined for an internal segment along a three–dimensional surface crack. Vargas and Dodds [8,23] outline details of computational procedures to evaluate these integrals.
Figures 10 and 11 show the \( J \)-values computed using the rate sensitive and rate insensitive material models for the deep notch \( (a/W=0.5) \) and shallow notch \( (a/W=0.0725) \) specimens. These thickness average \( J \)-values are normalized the flow stress, \( \sigma_f \), and the remaining ligament, \( b=W-a \) (\( \sigma_f \) is the average of the yield and ultimate stresses). Using material property data for A533B steel (see Table 1), \( \sigma_f = 520 \text{ MPa} \). The rate sensitive material model produces slightly larger \( J \)-values for the deep notch specimen with essentially no difference observed for the shallow notch \( J \)-values.

\[
\frac{J_{\text{rem}}}{\sigma_f b} = 0.04 \\
\frac{J_{\text{rem}}}{\sigma_f b} = 0.05 \\
\text{Rate Sensitive Model} \\
\text{Rate Insensitive Model}
\]

\[\text{Figure 10. } J_{\text{rem}} \text{ from Domain Integrals for the Deep Crack SE(B) Specimens}\]

\[\text{Figure 11. } J_{\text{rem}} \text{ from Domain Integrals for the Shallow Crack SE(B) Specimens}\]

5.2 \( J \) Evaluation in Test Specimens

Rice et al., [21] demonstrated that the \( J \)-integral is related closely to the work done by the ligament moment acting through the rotation angle for SE(B) specimens under static loading (see Fig. 12). For deeply notched specimens \( (a/W > 0.5) \), the \( J \)-integral for a SE(B) is given approximately by:

\[
J = \frac{2}{bB} \int_{0}^{\Omega} M \, d\Omega \tag{9}
\]

where \( M \) is the moment on the remaining ligament at the crack plane and \( \Omega \) is the relative angle between the specimen ends (see Fig. 12). This definition provides an average value of \( J(s) \) across the entire crack front.

\[\text{Figure 12. Idealized 3 Point Bend Specimen}\]

By assuming the two ends of the specimen undergo a simple rigid–body rotation about the crack plane, the angle \( \Omega \) is related directly to the load–line displacement \( \Delta_{\text{LLD}} \), \( \Omega = \Delta_{\text{LLD}}/(L/2) \), where \( L \) is the span between supports. From equilibrium, the moment at the specimen center, \( M \), is simply \( (PL)/4 \). Eqn. 9 can be rewritten as:

\[
J = \frac{\eta}{bB} \int_{0}^{\Delta_{\text{LLD}}} P d\Delta_{\text{LLD}} \tag{10}
\]

where \( \eta \) is the dimensionless parameter \( \approx 2 \) for deep notch SE(B) specimens.
Sumpter [22] separates the external work of the applied load into elastic and plastic components, $W_e$ and $W_p$, and then writes

$$J = J_e + J_p = \frac{\eta_e}{bB} W_e + \frac{\eta_p}{bB} W_p$$  \tag{11}

where $\eta_e$ and $\eta_p$ are the dimensionless constants that relate the elastic and plastic external work to the fracture driving force. $W_p$ denotes the external work of the applied load acting through the plastic component of the load line displacement. $\eta_p$ indicates the relative amount of total plastic deformation that contributes to crack-tip driving force rather than plasticity remote from the crack plane. By using the relationship between the $J$-integral and stress intensity factor $K_f$ for plane strain, Eqn. 11 is rewritten as:

$$J = J_e + J_p = \frac{K_f^2(1 - \nu^2)}{E} + \eta_p \int_0^{\Delta_{LDD}} P \, d\Delta_p.$$  \tag{12}

This form ensures compatibility between measured values of $J$ and $K_f$ when the deformation is predominantly linear–elastic.

Because direct measurement of the applied loading may be impractical or very difficult in a dynamic test such as the drop tower, the following indirect methods to infer applied loads are evaluated:

1. Applied loads are evaluated from the support reactions. Experimentalists have proposed to use instrumented supports to measure reactions.
2. Applied loads are evaluated from the quarter-span strains measured on the top and bottom surfaces of the specimen that are calibrated against a static linear–elastic analysis. Because the specimen is statically determinate, the moment at the quarter-span location is one–half the centerplane moment and is equal to the applied load $\times L/8$ (neglecting inertial effects). For deep notch specimens, plastic deformation remains confined to the centerplane region which leads to a linear–elastic response at the strain gage locations and a linear variation of bending strain over the specimen width. This approach fails when plastic deformation disturbs the through–width, linear strain variation.
3. Applied loads are evaluated from the moment computed at the crack plane using nodal reactions. This moment, which includes inertial effects, is compared to the crack plane moment for a simply supported beam with a statically applied mid–span load. This method predicts a quasi–static, equivalent load needed to achieve the same moment across the ligament that occurs under dynamic loading.

Sumpter [22] obtained values for $\eta_p$ using 2–dimensional, slip–line solutions for SE(B) specimens with pure moment loading on the crackplane. This approach yields $\eta_p$–values of 2.0 for the deep crack specimen and 0.97 for the shallow crack specimen. Figures 13 and 14 show the $J$–values (denoted $J_{n}$) computed using these $\eta_p$ values and the dynamic finite element, load–load line displacement curves. $J_{n}$ values are normalized by the full–field domain integral values (denoted $J_{fem}$) obtained from the dynamic analyses. $J_{n}$ values computed using the different inferred loads, together with those computed from the (known) applied loads in the analyses, are included in the figures. Deviations of the normalized $J$–values from unity indicate the relative error incurred in using static formulas for the evaluation of $J$. Errors in the computation of $J$ approach 10% toward the end of the analysis for the deep crack specimen and 20% for the shallow crack specimen.

The separation of $J$ into elastic and plastic components is somewhat arbitrary. Other separation techniques, such as deformation of the specimen without the crack and the additional deformation that occurs due to the crack, are equally valid [2]. The original derivation by Rice [21] does not assume any separation of the total $J$. The expression to compute a static $J$–value without prior separation into $J_e$ and $J_p$ becomes
where \( \eta_T \) denotes an \( \eta \) factor for the total work on the ligament and \( \Delta_T \) denotes the total load-line displacement. Static, 3-D finite element analyses of these specimens were performed to obtain estimates for \( \eta_T \) [23]. We find \( \eta_T = 1.91 \) for the deep crack specimen and \( \eta_T = 0.82 \) for the shallow crack specimen. Figures 15 and 16 compare \( J \)-values computed using Eqn. 13 for the deep crack and the shallow crack specimen, respectively, with the full-field domain integral \( J \)-values. Relatively large inertial effects are observed for the initial 0.0006 seconds of the deep crack specimen, and for the initial 0.0004 seconds of the shallow crack specimen. These are approximately 2.5 \( \times \) the transition time of each specimen. After this time, all three methods to infer the load (for ligament work calculation) produce less than 5% error in \( J \) for the deep crack specimen and less than 10% error for the shallow crack specimen. Consequently, \( J \) predictions based on a total ligament work rather than a conventional elastic-plastic separation approach are more accurate for a dynamic analysis. Strain-rate sensitivity of the material does not affect the accuracy of \( J \)-values obtained using Eqn. 13.

\[
J_{\eta(T)} = \frac{\eta_T}{bB} \int_0^{\Delta_{LLD}} P \, d\Delta_{LLD(T)}
\]  

(13)
6. Inertial Effects on $J$

Equations 6 through 8 describe contributions to the $J$–integral for dynamic loading. $J_1$ and $J_2$ are independent of direct inertial effects and their sum is the $J$–integral for static loading. Inertial effects enter the computation of $J$ directly through $J_3$. The first and third terms of Eqn. 8 arise from the kinetic energy and the explicit derivative of the kinetic energy over the integration domain, respectively. These two terms become significant for situations that occur with unstable crack propagation where large velocities and large velocity gradients exist near the crack tip [16]. For the non–propagating cracks investigated in this study, these two terms represent less than 0.1% of $J_3$. For the present analyses, the second term in Eqn. 8 (denoted $J_{acc}$) dominates the value of $J_3$:

$$J_{acc} = \int_{V_0} Q \frac{\partial^2 u_i}{\partial t^2} \frac{\partial u_i}{\partial X_k} q_k \, dV_0$$ (14)

Figure 17 shows the average through thickness value of $J_{acc}$ for the deep notch specimen using both a strain–rate sensitive and insensitive material response. $J_{acc}$ remains nearly zero for the initial 0.003 seconds in both cases, after which relatively large oscillations develop. However, the magnitudes of $J_{acc}$ are extremely small compared to the total $J$–integral (see Fig. 10). For all analyses, the contribution of $J_{acc}$ to $J_{ave}$ is less than 0.1% over most of the response. Thus for loading rates typical of those in drop tower tests, accurate computation of $J$–values does not require the $J_3$ term. This confirms the quasi–static nature of the experiment with respect to the computation of $J$ and explains the good agreement between $J$ computed with the static equations (Eqns. 12 and 13) and the finite element results from the dynamic analyses described here (Eqn. 5).

Figure 17. $J_{acc}$ for the SE(B) Specimens

![Figure 17. $J_{acc}$ for the SE(B) Specimens](image)

Figure 18. Large Domain for Deep Crack SE(B) Specimen

![Figure 18. Large Domain for Deep Crack SE(B) Specimen](image)

Very early in the response ($t < 10^{-4}$ s), when discrete stress waves are still prevalent, $J_{acc}$ contributes a significant part of the total $J$–integral. In particular, for times less than $t < 10^{-4}$ s, $J_{acc}$ is needed to obtain domain independence of the $J$–values as shown in Fig. 19. Ten domains, each with an increasingly larger in–plane radius, are defined to examine the domain dependence of the $J$ values. The vertical distance from the crack tip to the domain edge defines the domain radius (size). Figure 18 indicates the largest domain employed with the corresponding domain radius. A unit value of $q$ is specified at every node in the domain interior. The $q$–value for all other nodes is set to zero.

Figure 19 shows the normalized values of $J_1$, (defined in Eqn. 6), $J_{acc}$, and the sum $J_1 + J_{acc}$ as a function of normalized domain radius for the strain–rate sensitive, deep–crack SE(B) analysis. The values of $J_{acc}$ and $J_1$ correspond to response times of $2-7 \times 10^{-5}$ secs. The specimen remains predominantly elastic over these response times. The average of all the domains (excluding the do-
The $J_1$ exhibits a domain dependence at these early response times. At $2 \times 10^{-5}$ secs $J_1$ is negative for the larger domains, and approaches the domain independent value with decreasing domain radius. $J_{acc}$ exhibits the opposite behavior, and approaches zero with decreasing domain radius. The sum of these two values is domain independent. At $5 \times 10^{-5}$ secs, $J_1$ alone is equal to the total $J$–integral for all domains. Small domains defined near the crack tip provide accurate $J$ values using $J_1$ alone.

7. Effects of Strain–Rate Sensitivity

Figure 20 shows the distribution of strain rates for the three specimens computed using the strain–rate sensitive model ($\bar{\varepsilon} = (2/3)\bar{\varepsilon}_{ij}\bar{\varepsilon}_{ij}$). The visible surfaces include the crack plane, the vertical free surface and the top surface of the specimens. Results for the rate–insensitive analyses are indistinguishable from those in Fig. 20. The identical strain rates are expected since the loading is defined to produce nearly constant velocity response in all specimens.

The strain rate distributions shown in Fig. 20 correspond to two loading regimes of interest: 1) the strain rates from zero to 0.001 seconds after impact, and 2) the strain rates from 0.005 to 0.006 seconds after impact. In these two regimes, the displacement history of the specimens exhibit constant velocity (see Fig. 6): a constant load line velocity of 0.28 m/sec up to 0.004 seconds and a constant terminal velocity of 2.5 m/sec after approximately 0.005 seconds. The load–line velocity at the terminal loading regime is ten times larger than in the initial regime. The strain rates shown in Fig. 20 also reveal the same factor of ten in the strain rates. During the terminal velocity regime, strain rates larger than 50/sec are concentrated in the remaining ligament.

Figure 21 shows the distribution of Mises stress, normalized with respect to the static yield stress of 445 MPa. Results for both rate–sensitive and rate–insensitive analyses are shown in the figures. Rate sensitivity does not affect the Mises stress distributions prior to 0.003 seconds for each of the three specimens—the behavior remains predominantly linear–elastic for all three geometries prior to this time. At 0.004 seconds into the response, all three specimens show full plastic hinge develop-

<table>
<thead>
<tr>
<th>Time (msec)</th>
<th>$J_{ave}$ (lb/in)</th>
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<tbody>
<tr>
<td>0.02</td>
<td>2.3</td>
</tr>
<tr>
<td>0.03</td>
<td>3.5</td>
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<td>7.0</td>
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<tr>
<td>0.07</td>
<td>8.2</td>
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</tbody>
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($1$ lb/in = 0.15 kPa–m)
Figure 20. Strain Rates Shown on Quarter Symmetric Models

Figure 21. Mises Equivalent Stresses for the SE(B) Specimens

Figure 22. Strain Rates Shown on Quarter Symmetric Models

Figure 23. Mises Equivalent Stresses for the SE(B) Specimens

ment across the remaining ligament for both material models. (Figure 6 confirms that elastic oscillations of the deep crack specimen cease after this time). The plastic zone for the deep crack specimen is confined to the remaining ligament. For the shallow crack specimen, the plastic zone extends through the depth of the specimen. Once plastic deformation becomes extensive, the rate sensitive material model shows significant increases in the Mises stress levels for all three specimens. The plastic zones developed after initial ligament yielding parallel the formation of high strain rate zones of Fig. 20.
The quasi-static stress–strain curve, shown in Fig. 5 and used in the rate–insensitive dynamic analyses, is also used as the stress–strain curve for the Mises material model in the static analyses. Static analyses with this stress–strain curve provide the counterpart for the rate–insensitive dynamic analyses. In addition, static analyses are performed using a stress–strain curve indicative of the strain rates experienced by the specimen. Figure 22 shows the equivalent uniaxial stress–strain history experienced by a typical crack–tip element in the three rate sensitive specimens. The crack–tip element utilized for the stress–strain history in the figure is located on the longitudinal centerplane, directly ahead of the crack tip, i.e., on $\theta = 0$. Also shown in the figure is the equivalent uniaxial stress–strain curve for A533B steel at $50^\circ$C, at a strain rate of $50 \text{ sec}^{-1}$. This stress–strain curve closely matches the crack–tip stress–strain response shown and is used in a second set of static analyses to simulate the rate sensitive material behavior.

Figures 23 through 25 compare the finite element $J$–values for the dynamic analysis with $J$–values for the corresponding static analyses. For a given load–line displacement, the $J$–values for the static analyses performed with the quasi–static stress–strain curve are nearly identical to those for the rate–insensitive dynamic analyses in all three specimens. Global inertia effects on thickness average $J$–values are thus found to be negligible.

$J$–values for the second set of static analyses, labelled by the corresponding strain rate of the equivalent Bodner–Partom material model, are also shown in the figures. For the deep crack specimen, these static results closely match the rate–sensitive dynamic analyses. For the medium crack specimen, the static $J_{fem}$ based on the simulated rate sensitivity model exceeds the computed $J_{fem}$ of the dynamic analyses by approximately 4%. For the shallow crack specimen, the static $J_{fem}$ with the simulated rate sensitivity exceeds the computed $J_{fem}$ of the dynamic analyses by nearly 10%. The use of an elevated stress–strain curve in a static analysis predicts the rate–sensitive $J$–value well for
the deep crack specimen. For the shallow crack specimen however, the spatial variation of the strain rates ahead of the crack tip affects the applied $J$. Consequently, the use of a single, elevated stress–strain curve in a static analysis to predict the applied $J$ is not accurate for the shallow crack specimen.

Figures 26 and 27 show the normalized $J$ distribution across the crack front at the end of the dynamic analyses for the deep crack and the shallow crack specimen, respectively. Also shown on the figures are the static analysis distributions (using the baseline stress–strain curve) at the same, final load–line displacement. The relatively small differences between the three sets of results indicate that $J$ variation across the crack front is independent of both dynamic and strain rate effects.

8. Summary

Selected results for 3–D, nonlinear dynamic and static analyses have been presented for SE(B) specimens subjected to impact loadings characteristic of those developed in drop tower tests. The static analyses provide reference solutions to assess the relative importance of strain rate and inertial effects in the dynamic analyses. The following items summarize the important observations and conclusions obtained from these analyses:

1) A methodology is presented and verified to load the dynamic models in a manner which predicts load–displacement histories measured experimentally. The method involves two analyses: one in which a displacement response is directly applied to the specimen, and a second analysis in which the nodal reactions from the first analysis are smoothed and then applied as a pressure loading to the detailed model for the specimen. Fracture mechanics parameters are taken only from analyses of the detailed model.

2) The transition time at which internal energy exceeds kinetic energy occurs while the specimens remain essentially linear–elastic. The transition time is consistently given by $0.4 \times$ the first period of elastic vibration for each specimen.

3) Three techniques to infer the applied load are evaluated: measured quarter–span strains calibrated to a static analysis, end reactions and ligament moments. All three methods lead to similar predictions of the applied $J$ using $\eta$ concepts.

4) The static formula to compute $J$ from applied work with $\eta_p$ values derived from plane–strain models produces errors of 10–20%. Accuracy of the static formula is improved when modified to relate the total energy absorbed by the specimen to $J$. For response times after approximately $2.5 \times$ the transition time, inertial effects diminish sufficiently for the static formula to apply.

5) Strain rates of up to $50/\text{sec}$ occur near the crack tip in the specimens at the imposed loading rate of 2.5 m/\text{sec}. Strain–rate sensitivity of the material increases the applied $J$ and the crack front stresses. Rate sensitive effects for the deep crack specimen can be assessed in a static analysis through the use of an elevated stress–strain curve that corresponds to a strain rate of $50/\text{sec}$. A similar assessment is not possible for the shallow crack specimen due to the greater spatial variation of the strain rates ahead of the crack tip.

6) The crack–front distribution of $J$, normalized by the through thickness average $J$, is independent of strain–rate sensitivity for the SE(B) specimens in this study.
7) The material acceleration component of the $J$-integral is negligible except very early in the response. At response times less than $5 \times 10^{-3}$ s, the acceleration term of the $J$-integral is necessary to achieve domain independence.

9. References


† Available from public technical libraries.
§ Available for purchase from vendor.