EFFECT OF HYSTERETIC MODELS ON THE INELASTIC RESPONSE SPECTRA

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The design response spectrum has been widely used in seismic design to estimate force and deformation demands of structures imposed by earthquake ground motion. Inelastic Design Response Spectra (IDRS) to specify design yielding strength in seismic codes are obtained by reducing the ordinates of Linear Elastic Design Response Spectrum (LEDRS) by strength reduction factor (R). The factor R can be determined by three components which are ductility factor (R_d), overstrength factor (R_s) and redundancy factor (R_r). In this study ductility factor is only considered which is defined as the ratio of linear elastic response spectrum to inelastic response spectrum. Since a building is designed using reduced design spectrum (IDRS) rather than LEDRS in current seismic design procedures it allows structures behave inelastically during design level Earthquake Ground Motion (EGM). In this study inelastic response spectrum (IRS) and ductility factor (R_d) are investigated. Followings are two objectives of this study.

1) Establishment of the functional form of R factor in order to account for the dynamic properties, response level, and hysteretic characteristics. Using this functional form of R factor it is convenient to calculate the IRS from LERS.

2) Investigation of the effect of the dynamic properties, response level, and hysteretic characteristics on IRS.
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To our families
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CHAPTER 1
INTRODUCTION

1.1 General Remarks

The design response spectrum has been widely used in seismic design to estimate force and deformation demands of structures imposed by earthquake ground motion. Inelastic Design Response Spectra (IDRS) to specify design yielding strength in seismic codes are obtained by reducing the ordinates of Linear Elastic Design Response Spectrum (LEDRS) by a strength reduction factor (R).

Since a building is designed using reduced design spectrum (IDRS) rather than LEDRS in current seismic design procedures, it allows structures to behave inelastically during a design level Earthquake Ground Motion (EGM). Inelastic Response Spectra (IRS) depend not only on the characteristics of the expected ground motion at a given site, but also on the dynamic properties and nonlinear characteristics of a structure.

In current seismic design procedures, base shear is calculated by the elastic strength demand divided by the strength reduction factor. This factor is well known as the response modification factor, R, which accounts for ductility, overstrength, redundancy, and damping of a structural system.

ATC 3-06 proposed different R-values for each structural system. These values have been obtained based on the investigation of typical performances of building structures obtained from the past earthquakes and the assumed toughness of the structural system. Since the strength reduction factor was introduced as the K factor in the Blue Book (1959) by SEAOC, it has not been much improved (ATC-19, 1995), even with the introduction of the R factor approach. Several weaknesses are pointed out in ATC 19 (1995). For example, a single value of R factor is assigned to a structure according to the type of
structural system, which is irrespective of the height and configuration of a building. This factor needs to be developed based on more rational manner rather than engineering judgement and experience.

1.2 Objective and Scope

Inelastic Response Spectra (IRS) depend not only on the characteristics of the expected ground motion at a given site, but also on the dynamic properties and nonlinear characteristics of a structure.

One main objective is to investigate the effect of key parameters on IRS. Key parameters can be target ductility ratio, structural period, and hysteretic characteristics, etc. Inelastic response spectra is obtained by either using nonlinear dynamic time history analysis or using linear elastic response spectrum and ductility factor, $R_\mu$. This study uses both procedures to determine the effect of key parameters. Five different hysteretic models are used to investigate the effect of each hysteretic model, which are elasto-perfectly-plastic, bilinear, strength degradation, stiffness degradation, and pinching model. Also, six different levels of target ductility ratios are used ($\mu_t = 1~8$) and thirty different discrete structural periods are used (0~3 second).

Also, in order to obtain the IRS using ductility factor, $R_\mu$ conveniently, the functional form of $R_\mu$ factor is established. In this study, this factor is assumed to be a function of the characteristic parameters of each hysteretic model, target ductility ratio and structural period. In order to obtain this factor, statistical studies are carried out based on the results from the nonlinear dynamic time history analysis of SDOF system. Forty EGMs are used to carry out this statistical study, which are recorded at stiff or rock site. This site is classified as $S_1$ in UBC (1994).
A common approach for calculating the Inelastic Design Response Spectrum (IDRS) uses a strength reduction factor to reduce the Linear Elastic Design Response Spectrum (LEDRS) to the inelastic design strength level depending on the ductility capacity of a system. Several studies have been conducted over the years with the purpose of improving the knowledge of design response spectra. In general, these studies have been improved in time as a result of a rapid increase in the number of recorded EGMs. A brief summary of most relevant statistical studies on response spectra is as follows:

The first attempt to study the characteristics of an ensemble of LERS of recorded ground motions was made by Housner (1959), who computed the average LERS of eight ground motions recorded from four different earthquakes.

Newmark and Hall (1973) studied elastic and inelastic response spectra of a 5% damped SDOF system subjected to three recorded EGMs and pulse-type excitation. Based on statistical studies, they proposed the method to construct the inelastic response spectra using the elastic response spectra.

Riddell and Newmark (1979) performed the statistical studies for evaluating IRS using 10 different EGMs recorded at the rock and alluvium soil site. They considered three different hysteretic models such as elasto-perfectly-plastic, bilinear and stiffness degradation models. According to their studies elasto-perfectly-plastic model gives conservative IRS.

Riddell, et al. (1989) presented the average IRS of four sets of earthquake records. Most of EGMs used in this study were recorded in South America. Emphasis is given to the reduction factors for constructing SIDRS from LERS.

Nassar and Krawinkler (1991) evaluated the average IRS of bilinear and stiffness degrading systems subjected to 15 ground motions recorded on firm soil sites in the western United States. They proposed a functional form of R factor with respect to ductility, natural period and second slope of bilinear model.
More recently, Miranda (1993) performed studies similar to that of Nassar and Krawinkler (1991). He used more earthquake records and considered the effect of different soil conditions on IRS.

Based on the results obtained by the above studies, more knowledge and insights on LERS, IRS, and strength reduction factor are accumulated. However, more researches are still needed on this field, which clearly account for the effect of hystrectic models, high modes, P-Δ effect, and characteristics of EGM such as distance, intensity, magnitude, etc..

1.4 Organization

This report summarizes the results of evaluation of the effect of hysteretic models, structural period, and target ductility ratio. Chapter 2 introduces the functional form of ductility factor, $R_t$, which is able to account for the characteristics of different hysteretic models, structural period, and target ductility ratio. Five different hysteretic models are considered. These are the elasto-perfectly-plastic, bilinear, strength degradation, stiffness degradation and pinching models. Chapter 3 introduces the procedures to determine the inelastic response spectrum (IRS) for a given target ductility ratio using either nonlinear dynamic time history analysis or linear elastic response spectrum and ductility factor, R. The functional form of ductility factor obtained from Chapter 3 is adopted to calculate IRS. Also in this chapter the effect of hysteretic models on IRS is investigated. Finally, Chapter 4 summarizes the significant conclusions from the research and presents the recommendations for future research.
CHAPTER 2
DETERMINATION OF DUCTILITY FACTOR CONSIDERING DIFFERENT HYSTERETIC MODELS

2.1 Introduction

Most seismic design provisions allow structures to behave inelastically during a severe Earthquake Ground Motion (EGM). For this reason, the required elastic strength demand is reduced by a scaling factor which is known as either a strength reduction factor or a response modification factor, R. This factor accounts for ductility, overstrength, damping and redundancy inherent in a structure. The strength reduction factor, R, has been determined mainly based on engineering judgement and accumulated experiences from past earthquakes rather than theoretical background. Even if knowledge and lessons have been gained from past earthquakes, this factor has not been changed or reviewed substantively since the horizontal force factor, K, was introduced by the SEAOC "Blue Book" in 1959.

According to the research results in recent years, the assigned values for the R factor have been questioned (ATC-19 and ATC-34). Also the use of a single value for the R factor for a given structural system is questionable. In a draft form in ATC-19 and ATC-34, the R factor is split into three factors in order to account for the effects of ductility, damping, redundancy and overstrength, explicitly.

This study establishes the functional form of ductility factor, $R_u$, which is able to account for the characteristics of different hysteretic models. For this purpose, statistical studies are carried out. Forty earthquake ground motion records are used for this study. Five different hysteretic models are considered. These are the elasto-perfectly-plastic, bilinear, strength degradation, stiffness degradation and pinching models.
2.2 Evaluation of Ductility Factor, $R_u$

Under a given earthquake ground motion the inelastic deformation of a system generally increases as its yield strength level becomes lower. The ductility factor, $R_u$, is defined as the ratio of the elastic strength demand $F_y(\mu = 1)$ to the inelastic yield strength demand $F_y(\mu_t)$ for a given target ductility ratio ($\mu_t$), which is represented by the following equation:

$$R_u = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_t)} \quad (2.1)$$

The relationship between $F_y(\mu = 1)$ and $F_y(\mu = \mu_t)$ is shown in Figure 2.1. The iterative procedure is required for calibrating the $R_u$ factor to attain a given target ductility ratio, $\mu_t$. The process is shown in Figure 2.2. However, the ductility ratio, $\mu$, may not always increase monotonically as the yield strength decreases. In particular, more than one yield strength corresponding to a given target ductility ratio, $\mu_t$, is possible. In this case, the largest yield strength is selected in this study since it is more relevant for seismic design.

Ductility ratio, $\mu$, is defined as follows:

$$\mu = \frac{\max |u(t)|}{u_y} \quad (2.2)$$

where $\max |u(t)|$ is the maximum absolute value of relative displacement of the SDOF system with respect to the ground under a given earthquake ground motion, and $u_y$ is the yield displacement of a system.

In order to evaluate the yield strength of an SDOF system for a given target ductility ratio and earthquake ground motion, the following equation of motion is used.

$$m \ddot{u}(t) + c \dot{u}(t) + F(t) = -m \ddot{u}_g(t) \quad (2.3)$$

where $m$, $c$ and, $F(t)$ are mass, damping factor, and restoring force, respectively, and $u_g(t)$ is ground displacement. Over-dots indicate time derivatives.
In this study, the damping ratio is assumed to be 5% of the critical damping for all cases since seismic design provisions are normally based on the 5% damped system.

### 2.3 Hysteretic Models Used in this Study

The ductility factor, $R_u$, has been evaluated using either the elasto-perfectly-plastic or bilinear models because of their simplicity. In this study, five different hysteretic models are considered which are (1) elasto-perfectly-plastic, (2) bilinear, (3) strength degradation, (4) stiffness degradation, and (5) pinching models. These models are shown in Figure 2.3. Among these models the elasto-perfectly-plastic (EPP) model is used as a basis model in this study. Thus, the effect of other hysteretic models on the $R_u$ factor is compared with that of the EPP model. The characteristic parameters of each hysteretic model are shown in Table 2.1. The characteristic parameters of each hysteretic model are described by Kunnath, et. al. (1990) in detail.

### 2.4 Earthquake Records Used in this Study

For the statistical study, 40 earthquake ground motions are used which were obtained from the Earthquake Strong Motion CD-ROM by the National Geographical Data Center (1996) and the U.S. Geological Survey digital data series, DDS-7, CD-ROM (1992). The software BAP (1992) was used for correcting the earthquake records. Also the software SMCAT (1989) by the National Geographical Data Center was used for classifying the earthquake records according to soil type. The soil condition is classified into four types $S_1$, $S_2$, $S_3$, and $S_4$ according to the Uniform Building Code of 1994.

In this study the ground motions recorded in soil type 1 ($S_1$) are only considered. According to UBC 1994, soil type 1 ($S_1$) is classified as “A soil profile with either (a) a rock-like material characterized by a shear-wave velocity greater than 2,500 feet per
second or by other suitable means of classification, or (b) stiff or dense soil condition where the soil depth is less than 200 feet”. The inventory of selected earthquake records is given in Table 2.2.

### 2.5 Determination of $R_\mu$ Factor Considering Different Hysteretic Models

It usually requires large computational efforts to calculate the ductility factor. In order to reduce the computational efforts for determining the $R_\mu$ factor, this study establishes the functional form of the $R_\mu$ factor. According to the former research works (Newmark and Hall, Han and Wen, Osteraas and Krawinkler, Nassar and Krawinkler, and Miranda), the $R_\mu$ factor is the function of structural period, target ductility, and characteristic parameters of hysteretic models. This study assumes that the $R_\mu$ factor is also dependent on these parameters. Thus, the $R_\mu$ factor is denoted as the following functional form:

$$R_\mu = f (T, \mu, \alpha_1, \alpha_2, \alpha_3, \alpha_4)$$  \hspace{2cm} (2.4)

The type and parameter of the $R_\mu$ function can be obtained by regression analysis. To expedite the regression analysis, the effect of each hysteretic model is assumed to be independent to each other. Thus, Eq. (2.4) can be rewritten as follows:

$$R_\mu = R(T, \mu) \times C_{\alpha_1} \times C_{\alpha_2} \times C_{\alpha_3} \times C_{\alpha_4}$$  \hspace{2cm} (2.5)

where $R(T, \mu)$ is the functional format of the $R_\mu$ factor of the elasto-perfectly-plastic model which is treated as a basis model in this study. The factors, $C_{\alpha_1}$, $C_{\alpha_2}$, $C_{\alpha_3}$ and $C_{\alpha_4}$ are considered as correction factors accounting for the effects of bilinear ($\alpha_1$), strength degradation ($\alpha_2$), stiffness degradation ($\alpha_3$), and pinching ($\alpha_4$) on the $R_\mu$ factor obtained from the elasto-perfectly-plastic model.
2.5.1 The $R_\mu$ Factor for Elasto-perfectly-plastic Model

In order to establish the functional form of ductility factor $R(T,\mu)$ for the elasto-perfectly-plastic model, statistical studies are carried out. The 11,200 $R_\mu$ factors are calculated using a nonlinear dynamic analysis of the SDOF system considering the following permutations:

1) Target ductility ratios of 1 (elastic behavior), 2, 3, 4, 5, 6, and 8 (7)
2) Forty discrete natural periods of the SDOF systems from 0.05 seconds to 3.0 seconds (40)
3) Forty earthquake ground motions recorded at an $S_i$ site (40)

Two stage regression analysis is carried out in the two dimensional domain. In the first stage, the function for $R_\mu$ vs. the natural period of SDOF is regressed for the discrete values of ductility ratios, and then the effect of the ductility ratio is evaluated at the second stage. The following function is obtained for the elasto-perfectly-plastic model:

$$R_\mu = R(T,\mu) = A_0 \times \{1 - \exp(-B_0 \times T)\}$$

$$A_0 = 0.99 \times \mu + 0.15$$

$$B_0 = 23.69 \times \mu^{0.83}$$

where $T$ is the natural period and $\mu$ is the ductility ratio.

Figure 2.4 shows the fitness of the regressed function of the $R_\mu$ factor. In this figure, the solid line represents the values obtained from the regressed function and the dashed line represents the actual mean values of $R_\mu$ factors obtained from 12,800 nonlinear dynamic analyses. In Figure 2.5, the solid lines represent the values obtained from the regressed Eq. (2.7) and (2.8), and the circle represents the actual values. Also Figure 2.6 compares the values of the $R_\mu$ factors provided by several researchers.
2.5.2 The Effect of the Second Slope of the Bilinear Model

The characteristic parameter of the bilinear model is $\alpha_1$, which accounts for the second slope. As mentioned earlier, this study accounts for the effect of the second slope using correction factor, $C_{\alpha_1}$. This correction factor calibrates the ductility factor, $R_\mu$, which is obtained from the elasto-perfectly-plastic model in order to account for the effect of the bilinear model. Figure 2.7 shows the $R_\mu$ factor vs. the structural period with the different levels of the second slope ($\alpha_1 = 0, 2, 5, 7, 10$ and 15 %). From this figure, it is found that the larger $R_\mu$ factor is obtained as the second slope increases. Table 2.3(a) shows the ratio of the $R_\mu$ factor obtained from the bilinear model to that from the elasto-perfectly-plastic model for different levels of the second slope and the target ductility ratio. From this table, it is seen that the correction factor should be a function of both $\alpha_1$ and $\mu_1$. Based on this finding, the following functional form of correction factor $C_{\alpha_1}$ is obtained using two stage regression analysis. At the first stage, a correction factor $C_{\alpha_1}$ vs. $\alpha_1$ is regressed for the discrete values of the target ductility ratio. The effect of the target ductility ratio is evaluated at the second stage. For this regression analysis, a total number of 67,200 nonlinear dynamic analyses are performed for the number of permutations described in Section 2.5.1 with 6 different second slopes. The following regressed function is obtained:

$$R_\mu = R(T, \mu) \times C_{\alpha_1} \quad (2.9)$$

$$C_{\alpha_1} = 1.0 + A_1 \times \alpha_1 + B_1 \times \alpha_1^2 \quad (2.10)$$

$$A_1 = 2.07 \times \ln(\mu) - 0.28 \quad (2.11)$$

$$B_1 = -10.55 \times \ln(\mu) + 5.21 \quad (2.12)$$

Figure 2.8 shows the fitness of the $R_\mu$ factor which can account for the second slope effects to the mean values of $R_\mu$ factors. In this figure, the solid line represents the regressed function and the dashed line represents the mean values obtained from nonlinear dynamic analyses of the SDOF system.
2.5.3 The Effect of the Strength Degradation Model

Strength degradation may be important for reinforced concrete structures, particularly in the structure with a low shear capacity. In order to establish the functional form of the correction factor accounting for the effect of strength degradation ($\alpha_2$), two stage regression analysis is also carried out based on 56,000 values of $R_\mu$ factors obtained from nonlinear dynamic analysis. Six different levels of strength degradation ($\alpha_2 = 0$, 3, 6, 9, and 12 %) are considered, where $\alpha_2$ is defined in Figure 2.3. Mean values of the $R_\mu$ factor with the different levels of strength degradation are shown in Figure 2.9. As shown in this figure, the $R_\mu$ factor decreases with an increasing level of strength degradation. Also, from Table 2.3(b), the correction factor should be a function of the target ductility ratio. Based on this fact, the following functional form of correction factor $C_\alpha$ is obtained using a similar procedure with that for $C_\alpha_1$.

\[
R_\mu = R(T, \mu) \times C_\alpha_2
\]  
\[
C_\alpha_2 = \frac{1}{A_2 \times \alpha_2 + B_2}
\]  
\[
A_2 = 0.2 \times \mu + 0.42
\]  
\[
B_2 = 0.005 \times \mu + 0.98
\]  

Figure 2.10 shows the fitness of the values of the $R_\mu$ factor obtained from a regressed function to the actual mean values of the $R_\mu$ factor. In this figure, the solid line represents the regressed function and the dashed line denotes the actual values.

2.5.4 The Effect of the Stiffness Degradation Model

Stiffness degradation reduces the energy dissipation capacity of a system. Thus, it is expected that a lower ductility factor is required for the system with stiffness
degradation than for the elasto-perfectly-plastic system. Figure 2.11 shows that the lower ductility factor is obtained as the level of stiffness degradation increases. From Table 2.3(c), the correction factor $C_{\alpha_3}$ is also a function of the level of stiffness degradation. For statistical analysis, 67,200 $R_u$ factors are obtained using nonlinear dynamic analyses of the SDOF system. Six different levels of stiffness degradation ($\alpha_3 = 15, 4, 2, 1, 0.5$ and 0) are considered. Two stage regression analysis is also used to establish the functional form of $C_{\alpha_3}$. The following function is obtained:

$$ R_u = R(T, \mu) \times C_{\alpha_3} \quad (2.17) $$

$$ C_{\alpha_3} = \frac{0.85 + B_3 \times \alpha_3}{1 + C_3 \times \alpha_3 + 0.001 \times \alpha_3^2} \quad (2.18) $$

$$ B_3 = 0.03 \times \mu + 1.02 \quad (2.19) $$

$$ C_3 = 0.03 \times \mu + 0.99 \quad (2.20) $$

Figure 2.12 shows the fitness of the $R_u$ factor. In this figure, the solid line represents the regressed function and the dashed line represents the real mean values of the $R_u$ factor.

2.5.5 The Effect of the Pinching Model

The presence of open cracks in the compression zone of reinforced concrete members causes a marked pinching of its hysteretic behavior. Pinching narrows the hysteresis loops so that the energy dissipation capacity of a member or system becomes lower. As the level of pinching becomes higher, the $R_u$ factor is expected to be lower. The correction factor, $C_{\alpha_4}$ as defined in Figure 2.3(d), accounts for the effect of pinching. For regression analysis to establish the functional form of $C_{\alpha_4}$, 67,200 values of $R_u$ factors are calculated using nonlinear dynamic analyses with 6 different levels of pinching ($\alpha_4 = 1.0, 0.4, 0.3, 0.2, 0.1$ and 0.05). From Table 2.3(d), the factor $C_{\alpha_4}$ is the
function of the level of pinching and target ductility ratio. The following functional form of the \( C_{\alpha 4} \) factor is obtained using two stage regression analysis:

\[
R_{\mu} = R(T, \mu) \times C_{\alpha 4}
\]

\[
C_{\alpha 4} = \frac{1}{1 + 0.11 \times \exp(-C_{4} \times \alpha_{4})}
\]  

(2.22)

\[
C_{4} = -1.4 \times \ln(\mu) + 6.6
\]

(2.23)

Figure 2.14 shows the fitness of the \( R_{\mu} \) factor which accounts for the pinching effects to the mean value of \( R_{\mu} \) factors. In this figure, the solid line denotes the regressed function and the dashed line denotes the actual mean values of the \( R_{\mu} \) factor.

\[\text{2.6 Validity of Proposed } R_{\mu} \text{ Function for the System Having Combined Hysteretic Characteristics}\]

The functional form of the \( R_{\mu} \) factor established in this study is examined using several systems with combined hysteretic characteristics (e.g., the system with bilinear, and strength degradation). Figure 2.15 shows the fitness of the \( R_{\mu} \) factor obtained from the regressed function to the actual \( R_{\mu} \) factor for two systems having combined hysteretic characteristics.

This figure shows that the functional form of the \( R_{\mu} \) factor fits the actual \( R_{\mu} \) values with good precision. Therefore, the functional forms proposed in this study could be used to calculate the \( R_{\mu} \) factor. The difference between actual and predicted values of the \( R_{\mu} \) factor is tested using residual. In this study, residual \( (e) \) is defined as:

\[
e = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{R_{\mu} - R_{\text{actual}}}{R_{\text{actual}}} \right| \times 100 \quad (\%)
\]

(2.24)
Table 2.4 shows the average residual of $R_\mu$ factor for different target ductility ratios. The average residual varies within 2.7 to 4.8% of the actual $R_\mu$ factor. The target ductility ratio gives a small effect on the variation of the average residual.

2.7 Conclusions

The strength reduction factor (ductility factor), $R_\mu$, is defined as the ratio of elastic strength demand imposed on the SDOF system to inelastic strength demand for a given ductility ratio. The main objective of this study is to evaluate the $R_\mu$ factor which accounts for the effect of different hysteretic models. This study considers the soil profile with stiff soil or rock (classified as $S_i$ in UBC). Statistical studies are carried out to establish the functional form of the $R_\mu$ factor. According to the results of this study, the following conclusions are made:

1. For a given target ductility ratio, the ductility factor $R_\mu$ is strongly dependent on the change of the period of the structure, particularly in short period range.

2. As the level of the second slope of a bilinear system becomes higher, the value of the ductility factor $R_\mu$ becomes greater. Thus, the effect of the second slope needs to be accounted for when the $R_\mu$ factor is evaluated.

3. According to the results of this study, the $R_\mu$ factor is affected by the level of strength degradation, stiffness degradation, and pinching. A lower $R_\mu$ factor is obtained with an increase of the level of stiffness and strength degradation, and pinching. Therefore, it is concluded that the effects of these hysteretic characteristics should be accounted properly to evaluate the $R_\mu$ factor.

4. This study assumes that the effect of each hysteretic model on the $R_\mu$ factor is independent of those of other hysteretic models. Figure 2.15 and Table 2.4 show that this assumption is valid. Therefore, the proposed functional form can be used to calculate the $R_\mu$ factor of a system with combined hysteretic
characteristics such as bilinear, strength degradation, stiffness degradation, and pinching.

5. According to the results of this study, the ductility factor, $R_\mu$, is the function of the structural period, and the level of the target ductility ratio, second slope for a bilinear system, strength degradation, stiffness degradation, and pinching. The following proposed $R_\mu$ functions can account for these effects explicitly.

$$R_\mu = R(T,\mu) \times C_{a1} \times C_{a2} \times C_{a3} \times C_{a4}$$

$$R(T,\mu) = A_0 \times \left\{1 - \exp(-B_0 \times T)\right\}, \quad A_0 = 0.99 \times \mu + 0.15, \quad B_0 = 23.69 \times \mu^{-0.83}$$

$$C_{a1} = 1.0 + A_1 \times \alpha_1 + B_1 \times \alpha_1^2, \quad A_1 = 2.07 \times \ln(\mu) - 0.28, \quad B_1 = -10.55 \times \ln(\mu) + 5.21$$

$$C_{a2} = \frac{1}{A_2 \times \alpha_2 + B_2}, \quad A_2 = 0.2 \times \mu + 0.42, \quad B_2 = 0.005 \times \mu + 0.98$$

$$C_{a3} = \frac{0.85 + B_3 \times \alpha_3}{1 + C_3 \times \alpha_3 + 0.001 \times \alpha_3^2}, \quad B_3 = 0.03 \times \mu + 1.02, \quad C_3 = 0.03 \times \mu + 0.99$$

$$C_{a4} = \frac{1}{1 + 0.11 \times \exp(-C_4 \times \alpha_4)}, \quad C_4 = -1.4 \times \ln(\mu) + 6.6$$

where $R(T,\mu)$ is the functional form of the $R_\mu$ factor for EPP model, and the factors, $C_{a1}$, $C_{a2}$, $C_{a3}$ and $C_{a4}$ are correction factors which account for the effect of bilinear($\alpha_1$), strength degradation($\alpha_2$), stiffness degradation($\alpha_3$), and pinching($\alpha_4$) respectively.

6. In this study, the damping is assumed to be of viscous type with a fixed damping coefficient. Therefore, further study is needed to examine other damping characteristics such as instantaneous stiffness proportional damping. Also, this study only considers the SDOF systems. The $R_\mu$ factor for MDOF systems has to be studied systematically. Finally, only rock or stiff soil types are considered. Degrading systems are expected to be strongly affected for soft soil sites, which have relatively longer times of strong motion.
Table 2.1 Key Parameters of Hysteretic Models

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<tr>
<th>Hysteretic Model</th>
<th>Parameters</th>
<th>Effect</th>
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<tbody>
<tr>
<td>Elasto-perfectly-plastic Model</td>
<td>$K_0$</td>
<td>Initial Stiffness</td>
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<tr>
<td></td>
<td>$U_y$</td>
<td>Yield Displacement</td>
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<tr>
<td>Bilinear Model</td>
<td>$K_0$</td>
<td>Initial Stiffness</td>
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<tr>
<td></td>
<td>$U_y$</td>
<td>Yield Displacement</td>
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<tr>
<td></td>
<td>$\alpha_1$</td>
<td>Second Slope</td>
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<tr>
<td>Strength Degradation Model</td>
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<td>Initial Stiffness</td>
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<td></td>
<td>$U_y$</td>
<td>Yield Displacement</td>
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<td></td>
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<tr>
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<td>Initial Stiffness</td>
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<td></td>
<td>$U_y$</td>
<td>Yield Displacement</td>
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<td></td>
<td>$\alpha_3$</td>
<td>Stiffness Degradation</td>
</tr>
<tr>
<td>Pinching Model</td>
<td>$K_0$</td>
<td>Initial Stiffness</td>
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<td>$U_y$</td>
<td>Yield Displacement</td>
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<td>$\alpha_4$</td>
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Table 2.2 List of Earthquake Records for S\textsubscript{i} Soil Site

<table>
<thead>
<tr>
<th>Event Name</th>
<th>Station Name</th>
<th>Event Date</th>
<th>M</th>
<th>Distance (km)</th>
<th>Component PGA (cm/s\textsuperscript{2})</th>
<th>Component PGV (cm/s)</th>
<th>Component PGD (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offshore Eureka</td>
<td>Cape Mendocino</td>
<td>1994.9.1</td>
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<td>1949.4.13</td>
<td>7.1</td>
<td>39</td>
<td>356</td>
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<td>Pacoima - Kagel Canyon</td>
<td>1987.10.1</td>
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<td>Miyako Harbor Works, Ground</td>
<td>1970.4.1</td>
<td>5.8</td>
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<td>NS</td>
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<td>163.20</td>
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Table 2.3 Average Increment / Decrement of $R_u$ factor due to Different Hysteretic Model

<table>
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<th>Target</th>
<th>(a) $R_u$ factor of Bilinear Model to one of EPP Model (percentage)</th>
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<tr>
<td>Ductility</td>
<td>$\alpha_i=0%$</td>
<td>$\alpha_i=2%$</td>
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<tr>
<td>$\mu=2$</td>
<td>100</td>
<td>103</td>
</tr>
<tr>
<td>$\mu=3$</td>
<td>100</td>
<td>104</td>
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<td>$\mu=4$</td>
<td>100</td>
<td>105</td>
</tr>
<tr>
<td>$\mu=5$</td>
<td>100</td>
<td>106</td>
</tr>
<tr>
<td>$\mu=6$</td>
<td>100</td>
<td>107</td>
</tr>
<tr>
<td>$\mu=8$</td>
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<th>Target</th>
<th>(b) $R_u$ factor of Strength Degradation Model to one of EPP Model (percentage)</th>
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<td>$\alpha_i=3%$</td>
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<td>$\mu=2$</td>
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<td>96</td>
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<td>$\mu=5$</td>
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<td>$\mu=6$</td>
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<td>$\mu=8$</td>
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<th>(c) $R_u$ factor of Stiffness Degradation Model to one of EPP Model</th>
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<td>$\mu=2$</td>
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<td>99</td>
</tr>
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<td>$\mu=5$</td>
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<td>99</td>
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<td>$\mu=6$</td>
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<td>$\mu=8$</td>
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<table>
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<th>Target</th>
<th>(d) $R_u$ factor of Pinching Model to one of EPP Model (percentage)</th>
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<td>Ductility</td>
<td>$\alpha_i=100%$</td>
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<tr>
<td>$\mu=8$</td>
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Table 2.4 Evaluation of Average Residual between actual and predicted $R_\mu$ factors

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<tr>
<th>Target Ductility</th>
<th>$\alpha_1=7%$, $\alpha_2=3%$, $\alpha_3=1.0$, $\alpha_4=10%$</th>
<th>$\alpha_1=5%$, $\alpha_2=6%$, $\alpha_3=2.0$, $\alpha_4=30%$</th>
<th>$\alpha_1=0%$, $\alpha_2=9%$, $\alpha_3=4.0$, $\alpha_4=40%$</th>
<th>$\alpha_1=0%$, $\alpha_2=12%$, $\alpha_3=0.0$, $\alpha_4=5%$</th>
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<tr>
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<td>$\mu=8$</td>
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Figure 2.1 Yield Strength for a Given Target Ductility Ratio vs. Structural Period
START

Input Earthquake Record

Select the Natural Period of SDOF System

Select the Target Ductility Ratio

Set the System (calculate $K_0$, $c$, $\omega_n$)

Select the Hysteretic Model

Calculate $F_y$ ($\mu=1$)

$F_y = F_y (\mu=1) - \Delta F$

$U_y = F_y / K_0$

Calculate $U_{\text{max}}$

$\mu = U_{\text{max}} / U_y$

$\mu = \mu_{\text{target}}$

$R_\mu = F_y (\mu=1) / F_y (\mu=\mu)$

END

Figure 2.2 Overall Procedure for Calibrating $R_\mu$ factor
Figure 2.3 Hysteretic Models
Figure 2.4 Fitness of Regressed $R_u$ factor for the EPP Model
Figure 2.5 Coefficients of the $R_\phi$ factor for the EPP Model
Figure 2.6 Comparison of $R_{\mu}$ Functions ($\mu=6$)
Figure 2.7 Effect of the Second Slope on $R_\mu$ factor

Figure 2.8 Fitness of $R_\mu$ factor for the Bilinear Model ($\alpha_1=7\%$)
Figure 2.9 Effect of Strength Degradation

Figure 2.10 Fitness of $R_\mu$ factor for the Strength Degradation Model ($\alpha_2 = 6\%$)
Figure 2.11 Effect of Stiffness Degradation

Figure 2.12 Fitness of $R_\mu$ factor for the Stiffness Degradation Model ($\alpha_3=1.0$)
Figure 2.13 Effect of Pinching

Figure 2.14 Fitness of R4 factor for the Pinching Model ($\alpha_4 = 0.1$)
Figure 2.15 Fitness of $R_D$ factor for the Complex Hysteretic Model

(a) for $\alpha_1=5\%, \alpha_2=6\%, \alpha_3=2.0, \alpha_4=30\%$

(b) for $\alpha_1=0\%, \alpha_2=12\%, \alpha_3=0.0, \alpha_4=5\%$
CHAPTER 3

INELASTIC DESIGN SPECTRA CONSIDERING THE EFFECTS OF DIFFERENT HYSTERETIC MODELS

3.1 Introduction

The design response spectrum has been widely used in seismic design to determine the yield strength and deformation of the system necessary to limit the ductility demand imposed by the EGM. The current seismic design code has been developed based on the assumption that the structures designed according to the code will behave inelastically during design level EGM. For attaining this performance objective, a reduced design response spectrum (IDRS) is used for calculating the design base shear (equivalent to design yield level force \( V_y \)) rather than LEDRS.

In most current seismic design codes the strength reduction factor, \( R \) is used to calculate the IDRS using LEDRS. This factor is defined as the ratio of LEDRS to IDRS. However, many researchers have found the weaknesses of the \( R \) factor used in current seismic design codes (ATC 1995, Bertero et al. 1991, and Mahin et al. 1981). For example it is very questionable to use a single value of \( R \) factor for a given structural system irrespective of the height or period of a structure (ATC-19 1995). According to the investigation of the response obtained from instrumented structures during recent earthquake (Miranda and Bertero, 1991) as well as experimental studies of small scale building models (Uang and Bertero 1986, Whittaker et al. 1990), it is confirmed that there is a need for the \( R \) factor to be improved. The number of statistical studies for response spectra that have considered inelastic structural behavior is much smaller than that of the studies for Linear Elastic Response Spectra (LERS). Also, the studies have generally considered a small number of EGMs. Moreover, hysteretic characteristics of the inelastic
system are not explicitly accounted even if hysteretic characteristics are important for inelastic system.

Inelastic Response Spectra (IRS) depend on the characteristics of the expected ground motion at a site, dynamic properties of a system, response ductility level (μ), and the nonlinear characteristics of the structural system. The objective of this study is to investigate the effect of hysteretic models on IRS. For a given target ductility ratio and structural period IRS is obtained by either using nonlinear dynamic time history analysis or using LERS and ductility factor, Rμ. The ductility factor, which is one component of the strength reduction factor (R), is defined as the ratio of LERS to IRS. In this study five different hysteretic models are considered which are elasto-perfectly-plastic, bilinear, strength degradation, stiffness degradation, and pinching models. In order to investigate the effect of each model on IRS, forty EGMs are used which were recorded at stiff soil sites.

3.2 Review of Previous Study

Several studies have been conducted over the years with the purpose of improving the knowledge of design response spectra. In general, these studies have been improved in time as a result of a rapid increase in the number of recorded earthquake ground motions. A brief summary of most relevant statistical studies on response spectra is presented in this section.

Housner (1959) calculated the average LERS of eight ground motions recorded from four different earthquakes. His research was the first attempt to evaluate the characteristics of an ensemble of LERS of recorded ground motions. Response spectra of inelastic systems were first studied by Veletsos (1969) to pulse-type excitations using two recorded EGMs. Newmark and Hall (1973) studied elastic and inelastic response spectra of a 5% damped SDOF system subjected to three recorded EGMs and pulse-type excitations. Based on statistical studies, they proposed the method to construct the inelastic response spectra using the elastic response spectra.
Riddell and Newmark (1979) performed statistical studies for evaluating IRS using 10 different EGMs recorded at the rock and alluvium soil site. They considered three different hysteretic models such as elasto-perfectly-plastic, bilinear and stiffness degradation models. According to their studies elasto-perfectly-plastic model gives conservative IRS. Riddell, et al. (1989) presented average IRS of four sets of earthquake records. Most of the EGMs used in this study were recorded in South America. Emphasis is given to the reduction factors for constructing SIDRS from LERS. However, no information is given on the dispersion of the recommended reduction factors. Nassar and Krawinkler (1991) evaluated average IRS of bilinear and stiffness degrading systems subjected to 15 ground motions recorded on firm soil sites in the western United States. They proposed functional form of the R factor with respect to ductility, natural period and second slope of bilinear model. More recently, Miranda (1993) performed similar studies to that of Nassar and Krawinkler (1991). He used more earthquake records and considered the effect of different soil conditions on IRS. Based on the results obtained by above studies, more knowledge and insights on LERS, IRS, and strength reduction factor was developed.

### 3.3 Hysteretic Models and Earthquake Records Used in This Study

Many previous investigations on IRS have been evaluated using either elasto-perfectly-plastic or bilinear model because of their simplicity. In this study five different hysteretic models are considered which are (1) elasto-perfectly-plastic, (2) bilinear, (3) strength degradation, (4) stiffness degradation, and (5) pinching models. Figure 2.1 shows these models. Among these models, the elasto-perfectly-plastic (EPP) model is used as a basis model in this study. Thus, the effects of other hysteretic models on IRS are compared with that of EPP model. The characteristic parameters of each hysteretic model are shown in Table 2.1.

The second slope ($\alpha_1$) in the bilinear model represents the ratio of the second slope to the initial slope (stiffness). In this study, second slopes of 0 to 15% are considered.
Strength degradation, as indicated in Figure 2.1(b), is considered as a product of the parameter $\alpha_2$ and the attained ductility ratio; as either $\alpha_2$ or deformation level is higher, the effect of strength degradation becomes more serious. This study considers the range of 0 to 12% for $\alpha_2$. The parameter $\alpha_3$ reflects the stiffness degradation of the hysteresis loop. As shown in Figure 2.1(c), all unloading paths aim at a common point ($\alpha_3 \times F_y$) on the primary hysteresis curve. Thus, higher level of deformation or $\alpha_3$ results in severe stiffness degradation. The range of 0 to 15 is considered for $\alpha_3$. Pinching behavior is expressed as the parameter $\alpha_4$. Reloading paths, after crossing the zero force axis, approach the specific point ($\alpha_4 \times F_y$) and retain this smaller stiffness until the path exceeds the yielding deformation level. Higher pinching is applied when higher values of $\alpha_4$ are used. The values used for $\alpha_4$ are from 5% to 100%. The characteristic parameters of each hysteretic model are described by Kunnath et al. (1990) in detail.

For the statistical study, 40 earthquake ground motions are used which were obtained from the Earthquake Strong Motion CD-ROM by National Geographical Data Center (1996) and U.S. Geological Survey digital data series, DDS-7, CD-ROM (1992). The selected earthquake ground motions are; 1) free field ground motion, 2) horizontal components, 3) recorded at stiff soil site ($S_1$), and 4) wide range of earthquake ground motion records in terms of magnitude and epicentral distance. This study does not consider the effect of near field or far field EGMs. The software BAP (1992) is used for correcting the earthquake records. Also the software SMCAT (1989) by the National Geographical Data Center is used for classifying the earthquake records according to soil type. The soil type is classified into four types $S_1$, $S_2$, $S_3$, and $S_4$ according to Uniform Building Code (UBC-1994; ICBO 1994).

In this study, only ground motion records at soil type 1 ($S_1$) are considered. According to UBC 1994 soil type 1 ($S_1$) is as follows:

"A soil profile with either (a) a rock-like material characterized by a shear-wave velocity greater than 2,500 feet per second or by other suitable means of classification, or (b) Stiff or dense soil condition where the soil depth is less than 200 feet." (UBC-1994)
The inventory of selected earthquake records is presented in Table 2.2.

### 3.4 Determining the IRS for a Given Target Ductility Ratio

The response of a damped SDOF system subjected to earthquake ground motions is given by

\[
m \ddot{u}(t) + c \dot{u}(t) + F(t) = -m \ddot{u}_g(t)
\]

where \(m\), \(c\), and \(F(t)\) = mass, damping coefficient, and restoring force of the system, respectively; \(u(t)\) = relative displacement; \(u_g(t)\) = ground displacement; and the over-dot denotes the derivative with respect to time.

Eq. (3.1) can be rewritten by normalization as follows:

\[
\ddot{u}(t) + 2 \omega \xi \dot{u}(t) + \omega^2 \frac{F(t)}{F_y} = -\frac{\omega^2}{\eta} \frac{\ddot{u}_g(t)}{\max|\ddot{u}_g|}
\]

where \(F_y\) = system's yield strength; and \(\omega\), \(\xi\), and \(\eta\) = natural circular frequency, damping ratio, and nondimensional strength of the system, respectively. The latter three quantities are defined as

\[
\omega = \sqrt{\frac{k}{m}}
\]

\[
\xi = \frac{c}{2m \omega}
\]

\[
\eta = \frac{F_y}{m \times \max|\ddot{u}_g|}
\]

where \(k\) = initial stiffness of the system.

An IRS with constant displacement ductility, as shown in Figure 3.1, is a plot of the yield strength of an SDOF system (with period \(T\)) required to limit the displacement to specified displacement ductility ratio, \(\mu_1\). Displacement ductility ratio is defined as the
absolute value of the maximum relative displacement divided by the yield displacement. This type of spectra is also referred to as strength demand spectra (Krawinkler and Nassar 1990). In this study, IRS with constant displacement ductility were computed by calibrating the nondimensional strength $\eta$ of the system until the ductility ratio of the system reaches the specified target ductility ratio. In this study, the tolerance for calibration is assumed to be 2% of the target ductility ratio.

As mentioned earlier, there are two procedures to obtain the IRS for a given target ductility ratio. One is using the nonlinear dynamic time history analysis of SDOF system, and the other is using LERS and ductility factor, $R_d$. The following sections are the results obtained from carrying out these two procedures.

### 3.4.1 IRS Using Nonlinear Dynamic Time History Analysis

In order to obtain the IRS using nonlinear dynamic time history analysis, following combination is used:

1) 40 EGMs
2) 7 different target ductility ratio (1-6)
3) 40 different periods (0-3 sec.)
4) 5 different hysteretic models (EPP, Bilinear, Pinching, Strength degradation, and stiffness degradation models)

In all cases, the damping ratio is assumed to be 5%. Figure 3.2 shows the procedure to determine the IRS for a given target ductility ratio using this procedure. From this analysis, the statistical values of IRS are obtained such as mean, standard deviation and coefficient of variation (COV).

The resulting IRS are shown in Figure 3.3 and compared with LEDRS of NEHRP provision (BSSC 1994). The spectra are plotted for displacement ductility ratios of 1-6
(from top to bottom). As shown in Figure 3.3, the shapes of the inelastic spectra differ significantly from that of inelastic spectrum in the NEHRP provisions. The larger the ductility demand, the larger the difference is. Furthermore, a hysteretic model with degrading hysteresis affects the inelastic response, which results in higher ordinates than the elasto-perfectly-plastic model. For example, as strength degradation becomes more serious, it requires higher values of IRS. This can be seen more clearly in Figure 3.4, which compares the effects of key parameters of hysteretic characteristics on inelastic response spectra for target ductility ratio of 3. According to this figure, it can be concluded that inelastic spectra are dependent on the hysteretic model.

With mean IRS, it is also important to evaluate the dispersion of the IRS. Figure 3.5 shows the coefficient of variation (COV) of strength demands (IRS) normalized by PGA of EGMs. It shows that COVs are nearly the same for different levels of ductility for periods less than one second, which means that the dispersion on inelastic strength demand does not increase with increasing ductility demands. However, the COV becomes larger as the structural period is longer. When the period of structure is longer than 2 sec., the COV is almost 100%. This indicates that large uncertainties are involved in IRS for long period range. However, as noted in previous investigations (Miranda 1993), the use of acceleration parameters to normalize the spectra produces an increase in dispersion of IRS in the long-period range. Also the ordinates of IRS become smaller for long period range compared to those for short period range which may cause the higher COV.

### 3.4.2 Inelastic Response Spectra Using LERS and Ductility Factor, $R_μ$

In this study, IRS is also calculated using LERS and the ductility factor, $R_μ$, to alleviate the computational efforts to find IRS using nonlinear dynamic time history analysis. Since large numbers of repetitive calculations are required to find out the effect of each key parameter on IRS, a simple procedure for calculating IRS is necessary such as using LERS and the ductility factor. However, since it requires the nonlinear dynamic time
history analysis to calculate the ductility factor, this study uses the functional form proposed by Lee, et al. (1999).

Figure 3.6 shows the procedure to determine the IRS for a given target ductility ratio using this procedure. From this analysis, the effect of each hysteretic model on IRS is evaluated quantitatively. Also the IRS obtained using LERS and the $R_J$ factor proposed by Lee, et al. (1999) are compared to the values obtained from those of nonlinear dynamic time history analysis which are treated as exact values.

(1) Ductility Factor, $R_J$

Ductility factor, $R_J$ is defined as the ratio of the elastic strength demand $F_y(\mu = 1)$ to the inelastic yield strength demand $F_y(\mu_t)$ for a given target ductility ratio $\mu_t$, which is represented by the following equation.

$$R_J = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_t)} \quad (3.6)$$

The relationship between $F_y(\mu = 1)$ and $F_y(\mu = \mu_t)$ is shown in Figure 2.2. In order to establish the functional form of $R_J$ factor for each hysteretic model, statistical studies are carried out using the mean values of $R_J$ factor. The followings are the proposed functional form of strength reduction factor by Lee, et al. (1999):

$$R_J = R(T,\mu) = C_{\alpha 1} \times C_{\alpha 2} \times C_{\alpha 3} \times C_{\alpha 4} \quad (3.7)$$

$$R(T,\mu) = A_0 \times \{ 1 - \exp(-B_0 \times T) \} \quad A_0 = 0.99 \times \mu + 0.15 \quad B_0 = 23.69 \times \mu^{-0.83}$$

$$C_{\alpha 1} = 1.0 + A_1 \times \alpha_1 + B_1 \times \alpha_1^2 \quad A_1 = 2.07 \times \ln(\mu) - 0.28 \quad B_1 = -10.55 \times \ln(\mu) + 5.21$$

$$C_{\alpha 2} = \frac{1}{A_2 \times \alpha_2 + B_2} \quad A_2 = 0.2 \times \mu + 0.42 \quad B_2 = 0.005 \times \mu + 0.98$$
where \( R(T, \mu) \) is the functional form of the \( R_\mu \) factor for an EPP model, and the factors, \( C_\alpha_1, C_\alpha_2, C_\alpha_3 \) and \( C_\alpha_4 \) are correction factors which account for the effect of bilinear(\( \alpha_1 \)), strength degradation(\( \alpha_2 \)), stiffness degradation(\( \alpha_3 \)), and pinching(\( \alpha_4 \)) respectively. Readers can find a more detail description of the statistical studies in Lee et al. (1999).

This study shows that the \( R_\mu \) factor is, in the medium- and long-period region, only slightly dependent on the period, \( T \), and is roughly equal to the target ductility ratio \( \mu \). In the short-period region, however, the \( R_\mu \) factor depends strongly on both \( T \) and \( \mu \). Furthermore, the effect of hysteretic behavior can be observed in the whole period region.

### (2) Inelastic Response Spectra (IRS) using LERS and \( R_\mu \)

Current seismic loading for building structures is based on the reduction of linear elastic design spectra (LEDRS) through empirical and period-independent reduction factors. As previously discussed, the difference between the shapes of LERS and IRS becomes larger with increase in the target ductility ratio. Moreover, the ordinate of an IRS depends on structural period and target ductility ratio as well as hysteretic characteristics of the system.

Figure 3.7 shows comparison between the IRS using LERS and \( R_\mu \) factor and mean values of actual IRS obtained from nonlinear dynamic time history analysis. From this figure, an IRS obtained using LERS and \( R_\mu \) factor is comparable to the actual IRS. Therefore, this study uses LERS and the \( R_\mu \) factor to investigate the effect of hysteretic models on IRS.
Table 3.1 shows the effect of each hysteretic model on IRS. Table 3.1(a) shows the ratio of the average ordinate of the IRS obtained from the bilinear model to that from the EPP model for different levels of the second slope and the target ductility ratio. From this table, lower IRS ordinates are obtained for higher second slope. It is also found that the effect of this model becomes more serious as the level of the second slope and the target ductility ratio increase. Among the IRS ordinates of the bilinear model evaluated by this study, the lowest IRS ordinate is 73% of IRS ordinate of the EPP model (for the case of $\mu_t=8, \alpha_1=15\%$).

The ordinate of the IRS is also affected by the level of strength degradation, stiffness degradation, and pinching (Table 3.1 (b), (c), (d)). The ordinates of IRS increase with an increasing level of hysteretic parameters such as strength degradation, stiffness degradation, and pinching, and target ductility. The effect becomes more apparent for higher target ductility. Among the IRS ordinates of strength degradation, stiffness degradation and the pinching model evaluated by this study, the lowest IRS ordinate is $123\%$ ($\mu_t=8, \alpha_2=15\%$), $115\%$ ($\mu_t=8, \alpha_3=0\%$), $108\%$ ($\mu_t=8, \alpha_4=5\%$) of the IRS ordinate for the EPP model.

### 3.5 Conclusions

This study investigates the effect of hysteretic models on the inelastic response spectrum (IRS). For a given target ductility ratio, inelastic response spectrum can be obtained either directly from a calibration procedure using nonlinear dynamic time history analysis of a SDOF system or from linear elastic response spectrum divided by strength reduction factor (R). In this study, the former procedure is used to determine whether the hysteretic model affects the IRS, and the later is used to calculate this effect in terms of numerical values. The reason to use the strength reduction factor (R) for calculating IRS is its simplicity.

The following conclusions are based on the results of this study:
1. An inelastic response spectrum is strongly dependent on the target ductility ratio. Also, for a given target ductility ratio, the inelastic response spectrum is strongly dependent on the change of the period, particularly in short period range.

2. According to the results obtained in this study, the effect of each hysteretic model is mild compared to those of natural period and ductility (Figure 3.3).

3. As the level of the second slope of a bilinear system becomes higher, inelastic response spectrum becomes lower.

4. According to the results of this study, the inelastic response spectra are affected by the level of strength degradation, stiffness degradation, and pinching. As the level of stiffness and strength degradation, and pinching increase, higher values of inelastic response spectra are obtained. Therefore, it is concluded that the effects of these hysteretic characteristics need to be accounted properly to determine the inelastic response spectra.

5. The effect of each hysteretic model on IRS becomes more serious as the target ductility ratio is larger.

6. It is appropriate to use the functional form of the strength reduction factor, $R_s$, when the IRS is calculated from the linear elastic response spectrum since the ordinates of the IRS using this approach are comparable to actual IRS (Figure 3.7).

7. According to the results of this study, the inelastic response spectrum depends on the structural period, the level of the target ductility ratio, the second slope for bilinear systems, strength degradation, stiffness degradation, and pinching (Table 3.1, Figure 3.4). The variation of IRS (COV) becomes larger as the structural period is longer. For the structural period of 2 second or more, the COV is almost 100%. This variation needs to be considered for calculation of the IDRS in seismic design codes.
8. Current seismic loading for building structures is based on the reduction of linear elastic design spectra (LEDS) through empirical and period-independent reduction factors. The shape of inelastic strength response spectra differs significantly from the shape of elastic strength response spectra. This difference between the shape of LERS and IRS increases with increase in ductility. Thus, direct scaling by using a single strength reduction factor of elastic spectra to obtain inelastic strength demands is neither rational nor conservative.

9. In this study, the damping is assumed to be of viscous type with a fixed damping coefficient. Therefore, further study is needed to examine other damping characteristics such as instantaneous stiffness proportional damping. Also, this study only considers the SDOF systems. The $R_\mu$ factor for MDOF systems has to be studied systematically. Also this study does not consider the effect of near field or far field EGMs.

10. Systems that degrade in strength and/or stiffness may be highly dependent on the duration of motion. The length of the strong motion part of an accelerogram increases as the soil type becomes softer. Therefore, this study should be done for other soil types.
Table 3.1 Comparison of Average IRS Ordinate due to Different Hysteretic Model

<table>
<thead>
<tr>
<th>Target Ductility</th>
<th>(a) IRS Ordinate of Bilinear Model to one of EPP Model (percentage)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>for $\alpha_1=0%$</td>
</tr>
<tr>
<td>$\mu=2$</td>
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</tr>
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<td>$\mu=6$</td>
<td>100</td>
</tr>
<tr>
<td>$\mu=8$</td>
<td>100</td>
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</tbody>
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<table>
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<th>Target Ductility</th>
<th>(b) IRS Ordinate of Strength Degradation Model to one of EPP Model</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>$\mu=2$</td>
<td>100</td>
</tr>
<tr>
<td>$\mu=3$</td>
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<td>$\mu=4$</td>
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<tr>
<td>$\mu=6$</td>
<td>100</td>
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<tr>
<td>$\mu=8$</td>
<td>100</td>
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<thead>
<tr>
<th>Target Ductility</th>
<th>(c) IRS Ordinate of Stiffness Degradation Model to one of EPP Model</th>
</tr>
</thead>
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<tr>
<td>$\mu=3$</td>
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<td>$\mu=6$</td>
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<td>$\mu=8$</td>
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<table>
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<tr>
<th>Target Ductility</th>
<th>(d) IRS Ordinate of Pinching Model to one of EPP Model (percentage)</th>
</tr>
</thead>
<tbody>
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<td>for $\alpha_4=40%$</td>
</tr>
<tr>
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<td>$\mu=6$</td>
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<tr>
<td>$\mu=8$</td>
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Figure 3.1 Inelastic Strength Response Spectrum for a Given Target Ductility Ratio

Figure 3.2 Procedure to Calculate IRS for a Given Target Ductility Ratio
Figure 3.3 Inelastic Response Spectra for a Given Target Ductility Ratio
Figure 3.4(a) Comparison of Inelastic Response Spectra for $\mu=3$
(c) Stiffness Degradation Model

(d) Pinching Model

Figure 3.4(b) Comparison of Inelastic Response Spectra for $\mu=3$
Figure 3.5(a) COVs of Inelastic Response Spectra for Each Hysteretic Model
Figure 3.5(b) COVs of Inelastic Response Spectra for Each Hysteretic Model
Figure 3.6 Procedure to Calculate IRS using LERS and Ductility Factor, $R_{\mu}$
Inelastic Strength Demand Spectra
(S1, Bilinear Model ($\alpha_1 = 0.05$), Mean Value)

(a) Bilinear Model ($\alpha_1 = 0.05$)

Inelastic Strength Demand Spectra
(S1, Strength Degradation Model ($\alpha_2 = 0.12$), Mean Value)

(b) Strength Degradation Model ($\alpha_2 = 0.12$)

Figure 3.7(a) Prediction of Inelastic Response Spectra for Each Hysteretic Model
Figure 3.7(b) Prediction of Inelastic Response Spectra for Each Hysteretic Model
CHAPTER 4
CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

The design response spectrum has been widely used in seismic design to estimate force and deformation demands of structures imposed by earthquake ground motion. Inelastic Design Response Spectra (IDRS) to specify design yielding strength in seismic codes are obtained by reducing the ordinates of Linear Elastic Design Response Spectrum (LEDRS) by a strength reduction factor (R). The factor R can be determined by three components for ductility ($R_u$), overstrength ($R_s$) and redundancy ($R_r$). Since a building is designed using reduced design spectrum (IDRS) rather than LEDRS in current seismic design procedures it allows structures to behave inelastically during a design level Earthquake Ground Motion (EGM). In this study inelastic response spectrum (IRS) and ductility factor ($R_u$) are investigated. This study contains two parts as follows:

1) Establishment of the functional form of the R factor in order to account for the dynamic properties, response level, and hysteretic characteristics. Using this functional form of the R factor, it is convenient to calculate the IRS from LEDRS.

2) Investigation of the effect of the dynamic properties, response level, and hysteretic characteristics on IRS.

The following sections present the conclusions and recommendations obtained from this study:
4.1.1 Strength Reduction Factor

1. For a given target ductility ratio, the ductility factor $R_\mu$ is strongly dependent on the change of the period, particularly in short period range.

2. The ductility factor, $R_\mu$, is affected by each hysteretic model such as bilinear, strength degradation, stiffness degradation, and pinching models. The function of the $R$ factor with respect to the structural period, and the level of the target ductility ratio, hysteretic models are proposed as follows:

$$R_\mu = R(T, \mu) \times C_{\alpha_1} \times C_{\alpha_2} \times C_{\alpha_3} \times C_{\alpha_4}$$

$$R(T, \mu) = A_0 \times \{1 - \exp(-B_0 \times T)\}, \quad A_0 = 0.99 \times \mu + 0.15 \quad B_0 = 23.69 \times \mu^{-0.83}$$

$$C_{\alpha_1} = 1.0 + A_1 \times \alpha_1 + B_1 \times \alpha_1^2 \quad A_1 = 2.07 \times \ln(\mu) - 0.28 \quad B_1 = -10.55 \times \ln(\mu) + 5.21$$

$$C_{\alpha_2} = \frac{1}{A_2 \times \alpha_2 + B_2}, \quad A_2 = 0.2 \times \mu + 0.42 \quad B_2 = 0.005 \times \mu + 0.98$$

$$C_{\alpha_3} = \frac{0.85 + B_3 \times \alpha_3}{1 + C_3 \times \alpha_3 + 0.001 \times \alpha_3^2}, \quad B_3 = 0.03 \times \mu + 1.02 \quad C_3 = 0.03 \times \mu + 0.99$$

$$C_{\alpha_4} = \frac{1}{1 + 0.11 \times \exp(-C_4 \times \alpha_4)}, \quad C_4 = -1.4 \times \ln(\mu) + 6.6$$

where $R(T, \mu)$ is the functional form of the $R_\mu$ factor for the EPP model, and the factors, $C_{\alpha_1}, C_{\alpha_2}, C_{\alpha_3}$ and $C_{\alpha_4}$, are correction factors which account for the effect of bilinear($\alpha_1$), strength degradation($\alpha_2$), stiffness degradation($\alpha_3$), and pinching($\alpha_4$) respectively.
3. As the level of the second slope of a bilinear system becomes higher, the value of the ductility factor, $R_{\mu}$, becomes greater. Thus, the effect of the second slope needs to be accounted for when the $R_{\mu}$ factor is evaluated.

4. The ductility factor, $R_{\mu}$, is affected by the level of strength degradation, stiffness degradation, and pinching. A lower $R_{\mu}$ factor is obtained with an increase of the level of stiffness and strength degradation, and pinching. Therefore, it is concluded that the effects of these hysteretic characteristics should be accounted properly for evaluating the $R_{\mu}$ factor.

5. This study assumes that the effect of each hysteretic model on the $R_{\mu}$ factor is independent of those of other hysteretic models. Figure 2.15 and Table 2.4 show that this assumption is valid. Therefore, the proposed functional form can be used to calculate the $R_{\mu}$ factor of a system having combined hysteretic characteristics such as bilinear, strength degradation, stiffness degradation, and pinching.

4.1.2 Inelastic Response Spectrum

1. Inelastic response spectrum is strongly dependent on the target ductility ratio. Also, for a given target ductility ratio, the inelastic response spectrum is strongly dependent on the change of the period, particularly in the short period range.

2. According to the results obtained in this study the effect of each hysteretic model is mild compared to those of natural period and ductility (Figure 3.3).

3. As the level of the second slope of a bilinear system is higher, the inelastic response spectrum becomes higher.

4. The inelastic response spectrum is affected by the level of strength degradation, stiffness degradation, and pinching. As the level of stiffness and strength degradation
and pinching increase, higher values of inelastic response spectra are obtained. Therefore, it is concluded that the effects of these hysteretic characteristics need to be accounted properly to determine the inelastic response spectra.

5. It is appropriate to use the function form of the strength reduction factor when the IRS is calculated from linear elastic response spectrum since the ordinates of IRS using a strength reduction factor are comparable to actual values of IRS (Figure 3.7).

6. Current seismic loading for building structures is based on the reduction of linear elastic design spectra (LEDS) through empirical and period-independent reduction factors. The shape of inelastic response spectra differs significantly from the shape of elastic response spectra. This difference between the shape of LERS and IRS becomes larger with increase in ductility. Thus, it is neither rational nor conservative that the design force is calculated using IDRS obtained from scaled LEDRS by single value of the R factor.

4.2 Recommendations for Future Studies

The results of this study show the importance of accounting for dynamic properties, response level, and hysteretic characteristics when calculating the IRS. They also raise some issues that deserve further investigation:

1. In this study, the damping is assumed to be of viscous type with a fixed damping coefficient. Therefore, further study is needed to examine other damping characteristics such as instantaneous stiffness proportional damping.

2. This study only considers the SDOF systems. The $R_u$ factor for MDOF systems has to be studied systematically. As building become higher, the effect of higher modes are more significant. Thus, ductility factor, $R_u$ for MDOF systems can be the topic of future study.
3. This study does not classify the EGMs according to near field and far field EGMs. In future study, EGMs need to be classified according to distance, magnitude, intensity, etc. The effects of selection of EGMs on IRS needs to be investigated. The effects of soil type at a site on the IRS also needs to be investigated.
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