STRENGTH AND BEHAVIOR OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED CONCRETE SLABS SUBJECTED TO COMBINATIONS OF FLEXURAL AND TORSIONAL MOMENTS

by
ALEX CARDENAS
and
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UNIVERSITY OF ILLINOIS
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1. INTRODUCTION

1.1. Object and Scope

An experimental investigation of the flexural yield criterion for reinforced concrete elements subjected to biaxial bending has been in progress at the University of Illinois, Urbana, since 1965. The over-all objective of this report is to summarize the information obtained. The specific objectives are:

(1) To present the general formulation of the yield criterion for reinforced concrete slabs subjected to biaxial loading.

(2) To provide a graphical solution, in polar coordinates, of the yield criterion.

(3) To present procedures for predicting the behavior of reinforced concrete slabs subjected to different loading conditions and at various stages of loading.

(4) To develop a general method to determine the flexibility of slab elements subjected to different loading conditions.

(5) To describe the experimental studies carried out to test the yield criterion presented.

The yield criterion presented here is applicable to isotropically and nonisotropically reinforced concrete slabs subjected to any combination of external moments. Although emphasis is placed on the yield capacity, procedures are also presented for the determination of the response at any stage of loading.
1.2. Outline of the Experimental Investigation

Two types of specimens were used in this investigation.

(1) Circular Specimens. These specimens were used in the particular case of applied isostatic moment.* The characteristics of these "C" specimens are shown in Fig. A.5 and described in Appendix A.

All of the circular specimens were isotropically reinforced. The main variables were: the amount of reinforcement and the concrete strength. There were six specimens tested in this series. Their properties are summarized in Table 1.1.

(2) Rectangular Specimens. These specimens were used in conjunction with three different loading conditions: uniaxial bending, pure torsion, and combined bending and torsion. The characteristics of these "B" specimens are shown in Fig. A.6 and described in Appendix A.

A total of thirty-five "B" specimens were tested. Twenty-three of them were isotropically reinforced and twelve were nonisotropically reinforced. The main variables were: the amount of reinforcement, and the orientation of the reinforcement with respect to the principal moment axes. The properties of these specimens are summarized in Table 1.1.

The results of the tests have been presented in plots relating moment to curvature, moment to steel strains and moment to concrete strains.

Moment-curvature plots are presented in Appendix A for all 41 specimens, and detailed moment-curvature, moment-steel strains and

* See Appendix B for all definitions.
moment-concrete strains are presented for representative tests of each series in the text.
2. THE YIELD CRITERION

2.1. **Introductory Remarks**

The application of the theory of plasticity to reinforced concrete slabs was stated fundamentally by Ingerslev (1)* as early as 1921. The yield-line theory presented by Ingerslev was further developed on mathematical grounds by Johansen (2). Hognestad (3) has presented a review of the historical development of the yield-line theory as of 1952.

The basis of the yield-line theory is the yield criterion which describes the role of the material in the yield line theory. The applications of the yield-line theory to structures of a certain material have to be based on the strength and behavior of that material under a general loading condition. These characteristics are best described by the yield criterion.

In the theoretical and experimental research carried out on the yield criterion, questions have arisen concerning the strength of a slab element when stressed in more than one direction, and the reorientation of the reinforcement bars at a yield line.

Wood (4) reported that in tests carried out at the Building Research Station on one-way spanning slabs, the yield moment capacity for slabs with reinforcement inclined at ±45° with the span direction was up to 16 percent higher as compared to that of a slab with reinforcement parallel to the span direction.

*Numbers in parentheses refer to entries in the References.*
Baus and Tolaccia (5) claim that there is an increase in the yield moment when a slab element is subjected to biaxial stresses. This increase is maximum when the absolute magnitudes of the stresses is the same.

Along the same lines, Kwiecinski (6) has reported that

(a) No twisting moment can exist at a yield line, and
(b) Partial "kinking" of the reinforcement takes place at a yield line.

As a result of the above conclusions, Lenschow (7) started an investigation, both theoretical and experimental, of the yield criterion for reinforced concrete slab elements subjected to biaxial forces and moments. As a result of his investigations, Lenschow presented the yield criterion in a single algebraic statement embodying both the magnitude of the yield moment and the orientation of the yield line, described the limitations of the yield criterion, and presented a graphical method for the use of the yield criterion.

On the basis of his theoretical and experimental results he has shown that there is no increase in yield moment as a result of biaxial stresses and that the reorientation of a reinforcing bar across a yield line is so small that the increase in moment capacity is negligible.

Reference (7) also presents a well detailed review of theoretical and experimental investigations related to the yield criterion.

In this chapter, Lenschow's flexural yield criterion will be reviewed and its projection to elements subjected to combined bending and torsion will be emphasized.
2.2. The Yield Criterion

Figure 2.1a shows a nonisotropically reinforced slab element subjected to a set of principal unit moments $M_1^*$ and $M_2$ acting along the cartesian coordinate axes $u$ and $v$. (The direction of the arrows is based on the right-hand convention.) Figure 2.1b shows Mohr's circle for the applied external moments.

The direction of the reinforcing bars coincide with the cartesian coordinate axes $x$ and $y$, and $\beta$ is the angle between the $x$- and $u$-axes. The resisting unit moments in the $x$- and $y$-directions are $M_x$ and $M_y$ (positive) and $M_x^0$ and $M_y^0$ (negative). For convenience, $M_x$ and $M_y$ also refer to the governing resisting unit moments when the slab element is subjected to the applied principal unit moments $M_1$ and $M_2$.

Let $n$ and $t$ be a system of cartesian coordinate axes such that the $t$-axis coincides with the yield line, and $\gamma$ is the angle between the yield line and the $u$-axis.

The principle of least resistance (7) applied to determine the formation of the yield lines states: yielding of the element will occur at a location in the slab element where the ratio of the applied moment to the resisting moment is a maximum regardless of the absolute value of the external moment.

From Fig. 2.1c, the resisting moments at the yield line as expressed by Mohr's circle are:

$$M_n = M_y \cos^2(\beta+\gamma) + M_x \sin^2(\beta+\gamma)$$ (2.1)

* See Appendix B for all notation.
The components of the applied moments at the yield line are:

\[ ME = M_2 \sin^2(\beta + \gamma) + M_x \cos^2(\beta + \gamma) \] \hfill (2.2)

\[ M_{nt} = (M_x - M_y) \sin(\beta + \gamma) \cos(\beta + \gamma) \] \hfill (2.3)

The applied moment will not exceed the capacity of the slab element if:

\[ |ME_n| < |M_n| \] \hfill (2.7)

and

\[ |ME_t| < |M_t| \] \hfill (2.8)

and

\[ |ME_{nt}| < |M_{nt}| \] \hfill (2.9)

The applied moments equal the carrying capacity when any of the above three inequalities becomes an equality. Along a line of least resistance Eq. 2.7 and 2.8 become equalities simultaneously. Let Eq. 2.7 become an equality, then:

\[ ME_n = M_n \] \hfill (2.10)

For convenience let:
\[
M_y = \eta M_x \quad \text{(2.11)}
\]

and

\[
M_z = \omega M_z \quad \text{(2.12)}
\]

The principle of least resistance can be expressed as:

\[
\frac{d}{dy} \frac{M_{E_n}}{M_n} = 0 \quad \text{(2.13)}
\]

and

\[
\frac{d^2}{dy^2} \frac{M_{E_n}}{M_n} < 0 \quad \text{(2.14)}
\]

From Eq. 2.1, 2.3, 2.4, 2.6 and 2.13 the following can be derived:

\[
M_{nt} M_{E_n} + M_{nt} M_n = 0 \quad \text{(2.15)}
\]

Therefore, if \( M_{E_n} = M_n \), then \( M_{nt} = -M_{nt} \) which means that at the yield line, which is a line of least resistance, the component of the external moment normal to the yield line is equal to the moment capacity across the yield line, and the internal torsional moment is in equilibrium with the external torsional moment.

2.3: The Orientation of the Yield Lines in a Slab Element

The orientation, \( \gamma \), of the yield line with respect to one of the principal moment axes, as shown in Fig. 2.1a, can be determined from Eq. 2.1, 2.3, 2.4, 2.6, 2.11, 2.12 and 2.15 which has been shown (7) to give:
\[-\tan^2\gamma - \omega \tan\gamma + \omega = 0 \]  
(2.16)

where \[\omega \equiv \frac{(\eta - \omega) \cot^2\beta + 1 - \eta \omega}{(1 - \eta) \cot\beta} \]  
(2.17)

From Eq. 2.16 the following solutions can be obtained:

if \( M_2 = 0; M_1 \neq 0; \):

\[\tan\gamma = -\frac{1 + \eta \cot^2\beta}{(1 - \eta) \cot\beta} \]  
(2.18)

if \( M_1 = 0; M_2 \neq 0; \):

\[\tan\gamma = -\frac{(1 - \eta) \cot\beta}{\eta + \cot^2\beta} \]  
(2.19)

and in general

if \( \omega \neq 0; \):

\[\tan\gamma = -0.5 \omega \pm 0.5 \sqrt{\omega^2 + 4\omega} \]  
(2.20)

From Eq. 2.20, two values of \( \gamma \) will in general be obtained which satisfy Eq. 2.13. However, only one of these values will satisfy Eq. 2.14 and this value will determine the condition of the maximum absolute ratio of the applied normal moment at the yield line to the corresponding resisting moment. The other value will minimize this relationship. It is therefore to be emphasized that satisfying Eq. 2.14 is important in order to compute the actual carrying capacity of the slab.

Figures 2.2, 2.3 and 2.4 show the solution of Eq. 2.16 for the cases of uniaxial bending, pure torsion and a particular case of
combined bending and torsion. All of these plots show the variation of the yield line orientation, $\gamma$, for different orientations, $\beta$, of the reinforcement with respect to the principal moment axis. The signs of the angles are shown in Fig. 2.1.

The general features of Fig. 2.2 through 2.4 show that when the reinforcement is perpendicular to either of the principal moment directions the angle $\gamma$ is zero; and, also, the fact that the smaller the amount of nonisotropic reinforcement, the larger the deviation of the yield line with respect to the principal moment directions. Two values of $\mu$ have been chosen, $\mu = 0.5$, and $\mu = 0.25$.

2.4. Resisting Moments at the Yield Line

It has been shown (7) that, in general, for nonisotropically reinforced slab elements, $\mu \neq 1$, with resisting moments in the directions of the reinforcement, $M_x$ and $M_y$, of magnitude:

$$M_x = A_s f_y d (1 - k_1 \frac{A_s f_y}{\sigma_{ca}})$$

(2.21)

and

$$M_y = \mu A_s f_y d (1 - k_1 \frac{\mu A_s f_y}{\sigma_{ca}})$$

(2.22)

where $\sigma_{ca}$ is the average compressive stress in the concrete of the uncracked zone and $k_1$ expresses the position at the centroid of the compressive stress block. The resisting moment across a yield line perpendicular to the n-direction can be expressed by:
\[ M_n = M_x \sin^2(\beta + \gamma) + M_y \cos^2(\beta + \gamma) \]
\[ + \left( \frac{1}{\mu} M_y - M_x \right)(1 - \mu)\sin^2(\beta + \gamma)\cos^2(\beta + \gamma) \] (2.23)

and the torsional moment at the yield line is:

\[ M_{nt} = (M_x - M_y)\sin(\beta + \gamma)\cos(\beta + \gamma) \]
\[ + \left( \frac{1}{\mu} M_y - M_x \right)\cos^2(\beta + \gamma) \]
\[ + \mu\sin^2(\beta + \gamma)\sin(\beta + \gamma)\cos(\beta + \gamma) \] (2.24)

The last term on the right-hand side of Eq. 2.23 and 2.24 reflects the fact that the depth of the neutral axis varies as the yield line rotates. The distribution of shear stresses over the compression zone is assumed to be similar to that of the compression stresses.

However, in deriving the yield criterion, the resisting moments have been expressed on the basis of Mohr's circle (Eq. 2.1 to 2.3) which does not consider the change in depth of the neutral axis as the yield line rotates. The maximum error involved in computing the resisting moment, \( M_n \), by Eq. 2.1 as compared with Eq. 2.23 is less than five percent. The error for the torsional moment, \( M_{nt} \), is less than ten percent for Eq. 2.3 as compared with Eq. 2.24. These approximations are considered acceptable in view of the simplicity they provide to the formulation of the yield criterion.

2.5. Graphical Representation of the Yield Criterion

In the following, a representation of the yield criterion in polar coordinates will be presented.
From Eq. 2.1, 2.4 and 2.16 the carrying capacity of the slab element with respect to the absolute value of the maximum principal applied moment, $M_2$, is:

$$M_2 = \frac{M_\gamma \cos^2(\beta+\gamma) + M_x \sin^2(\beta+\gamma)}{(\cos^2\gamma + \frac{1}{w} \sin^2\gamma)}$$  \hspace{1cm} (2.25)$$

Figures 2.5 through 2.10 show the representation of the yield criterion as stated by Eq. 2.25. This polar representation of the yield criterion has been obtained as follows: the polar radii show the magnitude of either the applied moment or the resisting moment, and the polar angle shows the direction of either the applied moment or the resisting moment with respect to the $u$-axis.

Figure 2.5 shows the representation of the yield criterion for an isotropically reinforced slab subjected to uniaxial bending. Because the slab is isotropically reinforced, the resisting moment is plotted as a circle which indicates that the resisting moment is constant in any direction considered. The applied moment can be obtained from Mohr's circle and plotted as indicated above. As the applied moment is increased, the curve representing its magnitude will expand maintaining its original shape and will eventually touch the curve representing the resisting moment. At this stage, the yield capacity of the slab will have been reached. The magnitude of the carrying capacity will be given by the polar radii to the point of contact, and the angle $\gamma$ will give the orientation of the yield line with respect to the principal moment axis. In this case and all other cases of isotropic reinforcement, the moment capacity will be given by the magnitude of the maximum applied principal
moment and the yield line will coincide with the direction on which this maximum principal moment acts. Therefore, $\gamma$ will be zero.

Figure 2.6 shows the graphical representation of the yield criterion for a nonisotropically reinforced slab subjected to uniaxial bending. In this case it can be seen that the position of the point of contact of the applied and resisting moments does not coincide either with the principal moment directions or with the directions of the resisting moments $M_x$ and $M_y$. It can also be seen that the applied moment in the $v$-direction cannot reach the resisting moment in that direction and that the yield line deviates an angle $\gamma$ from the principal moment axis.

Figures 2.7 and 2.8 show plots for an isotropically and a nonisotropically reinforced concrete slab subjected to pure torsion. Figures 2.9 and 2.10 show plots for an isotropically and a nonisotropically reinforced slab subjected to a combination of bending and torsion moments. The features of these plots are similar to those described for Fig. 2.5 and 2.6.
3. BEHAVIOR AND STRENGTH OF SLAB ELEMENTS
SUBJECTED TO ISOSTATIC MOMENT

3.1. Isotropically Reinforced Slabs ($\mu = 1.0$)

The behavior of an isotropically reinforced concrete slab element under isostatic moment has rather simple and straightforward characteristics. Because of the multi-axial symmetry of the loading and the strength of the slab element, the problem reduces to that of finding the strength-deformation characteristics of a beam. However, it has to be realized that the concrete in compression at any cross section of a slab element under isostatic moment is in a state of biaxial compression. The condition of biaxial compression in concrete requires consideration of two factors that may influence the strength and behavior of the slab element:

(a) Increase in compressive strength of the concrete, and
(b) Effect of the Poisson's ratio.

Hilsdorf (8) has shown, Fig. 3.1, that the difference in compressive strength for concrete under equal biaxial stresses compared with that under uniaxial stress amounts to an increase of about 15 percent. Although this is not a universal factor and should change with variables such as the type of aggregate, it appears reasonable to assume that the increase in strength is going to be on the order of 15 percent rather than 100 percent or even 50 percent.

Poisson's ratio has negligible effect on the strength of a section but does have a measurable effect on the strains. In Fig. 3.2 Poisson's ratio was determined from measurements of longitudinal and
transverse strains using SR-4 electric strain gages cemented to the compression surface of specimens subject to uniaxial moment. The average value was found to be $\gamma = 0.1$.

In the following, the strength-deformation characteristics will be described at different stages of loading.

(a) Cracking

The assumed stress and strain distributions over a rectangular cross section are shown in Fig. 3.3b and 3.3c. Compressive stresses are shown below the neutral axis to be consistent with the test setup.

The strains in the concrete in the two mutually orthogonal directions $n$ and $t$ are:

\[
\varepsilon_n^o = \frac{\sigma_n^o}{E_c} - \nu \frac{\sigma_t^o}{E_c}
\]

\[
\varepsilon_t^o = \frac{\sigma_t^o}{E_c} - \nu \frac{\sigma_n^o}{E_c}
\]

\[
\varepsilon_n = \frac{\sigma_n}{E_c} - \nu \frac{\sigma_t}{E_c}
\]

\[
\varepsilon_t = \frac{\sigma_t}{E_c} - \nu \frac{\sigma_n}{E_c}
\]

The strains in the reinforcement are:

\[
\varepsilon_n = \varepsilon_n^o \frac{h - c_n}{h - c_n'}
\]

\[
\varepsilon_t = \varepsilon_t^o \frac{h - c_t}{h - c_t'}
\]
For isotropic reinforcement and with the assumption that both layers of reinforcement act at the same level

\[ \epsilon_{cn}^o = \epsilon_{ct}^o \quad \text{and} \quad c_n = c_t \]  

(3.7)

From equilibrium of forces in the n or t directions:

\[ \sum F_n = F_n^o + N_n^o - F_n = \frac{1}{2} \sigma_{cn}^o (h - c_n) + \epsilon_{cn}^o \frac{h - c_n - d'}{h - c_n} E_A s \cos^2 \alpha^o \]
\[ + \epsilon_{ct}^o \frac{h - c_t - d'}{h - c_t} E_A s \sin^2 \alpha^o - \frac{1}{2} \sigma_{cn}^o c_n = 0 \]  

(3.8)

From Eq. 3.7 and 3.8, the depth of the neutral axis at cracking in either the n or t directions becomes:

\[ \frac{c_n}{h} = \frac{c_t}{h} = \frac{0.5 + n(1 - \nu) A_s/h d/h}{1 + n(1-\nu)A_s/h} \]  

(3.9)

The bending moment at the cracking stage in either the n- or t-direction is:

\[ M_{cn} = M_{ct} = F_n^o \frac{2}{3} h + N_n^o (d - c_n/3) \]  

(3.10)

where

\[ F_n^o = \frac{f_c}{2} (h - c_n) \]  

(3.11)

\[ N_n^o = n f_r (1 - \nu) A_s \frac{d - c_n}{h - c_n} \]  

(3.12)
\[
\frac{M_{cn}}{f_r h^2} = \frac{M_{ct}}{f_r h^2} = \frac{1}{3} \left( 1 - 2 \frac{c_n^2}{h} + \frac{c_n^2}{h^2} \right) + \frac{n(1 - \nu)A_s}{h} \left( \frac{d^2}{h^2} - 2 \frac{d}{h} \frac{c_n}{h} + \frac{c_n^2}{h^2} \right)
\]

\[\frac{1}{2} \left( \frac{c_n}{h^2} \right) \left( \frac{c_n}{h} \right) \]

(3.13)

The curvature at cracking in either the n or t direction is:

\[\Phi_{cn} = \Phi_{ct} = \frac{f_r (1 - \nu)}{h E_c (1 - c_n/h)} \quad (3.14)\]

The effect of Poisson's ratio on the strength and deformation characteristics is discussed quantitatively below. Let:

\[p = \frac{A_s}{d} = 0.01 \quad f'_c = 5000 \text{ psi}; \quad f_r = 7\sqrt{5000} = 500 \text{ psi};\]
\[f_y = 50,000 \text{ psi} \quad E_s = 29 \times 10^6 \text{ psi}; \quad n' = 8\]
\[d = 0.85h \quad d' = 0.15h \quad h = 4 \text{ in.}\]

Calculations for \(\nu = 0.1\)

The effect of Poisson's ratio can be included in the definition of the modulus of elasticity of concrete subjected to equal biaxial stresses:

\[E'_c = \frac{E_c}{1 - \nu}\]

where \(E'_c\) = effective modulus of elasticity for Poisson's ratio \(\nu \neq 0\)
\[E_c = \text{modulus of elasticity of concrete for } \nu = 0\]
The modular ratio becomes:

\[ n' = \frac{\frac{E_s(1 - v)}{E_c}}{n(1 - v)} \]

where \( n' \) is the modular ratio for \( v \neq 0 \)

From Eq. 3.9 \( c_n = c_t = 2.08 \text{ in.} \)

From Eq. 3.13 \( M_{cn} = M_{ct} = 1.49 \text{ k-in./in.} \)

From Eq. 3.14 \( \Phi_{cn} = \Phi_{ct} = 6.2 \times 10^{-5}, \text{ in.}^{-1} \)

From Eq. 3.3 and 3.4 \( \epsilon_{cn} = \epsilon_{ct} = 11.8 \times 10^{-5} \)

Calculations for \( v = 0 \)

From Eq. 3.9 \( c_n = c_t = 2.09 \text{ in.} \)

From Eq. 3.13 \( M_{cn} = M_{ct} = 1.51 \text{ k-in./in.} \)

From Eq. 3.14 \( \Phi_{cn} = \Phi_{ct} = 6.9 \times 10^{-5}, \text{ in.}^{-1} \)

From Eq. 3.3 and 3.4 \( \epsilon_{cn} = \epsilon_{ct} = 13.1 \times 10^{-5} \)

From the above calculations it follows that the influence of Poisson's ratio is negligible in determining the cracking moment even if a limiting-strain rather than a limiting-stress criterion is used. The effect of Poisson's ratio on curvature follows directly from its effect on strains.

(b) Yielding

The strains in the reinforcement in both directions are assumed to have reached the yield strain. The strain and stress distributions are shown in Fig. 3.3d and 3.3e.
The equilibrium of forces in the n or t direction gives:

\[ 2A_s f_y = \frac{E_c \varepsilon_y c_n^2}{(d - c_n)(1 - v)} \]  

(3.15)

The depth of the neutral axis at yield is

\[ c_n = d \left[ \sqrt{[\rho_n(1-v)]^2 + 2\rho_n(1-v) - \rho_n(1-v)} \right] \]  

(3.16)

The moment capacity at yield is:

\[ M_{yn} = M_{yt} = A_s f_y (d - c_n/3) \]  

(3.17)

where \( c_n \) is given by Eq. 3.16

The curvature at yield is:

\[ \Phi_{yn} = \Phi_{yt} = \frac{\varepsilon_y}{d - c_n} \]  

(3.18)

The effect of biaxial compression and Poisson's effect is discussed quantitatively in the following paragraphs:

Calculations for \( v = 0.1 \)

From Eq. 3.16  \[ c_n = c_t = 1.02 \text{ in.} \]

From Eq. 3.17  \[ M_{yn} = M_{yt} = 5.20 \text{ k-in./in.} \]

From Eq. 3.18  \[ \Phi_{yn} = \Phi_{yt} = 72 \times 10^{-5}, \text{ in.}^{-1} \]

Calculations for \( v = 0 \)

From Eq. 3.16  \[ c_n = c_t = 1.12 \text{ in.} \]
From Eq. 3.17

\[ M_{yn} = M_{yt} = 5.15 \text{ k-in./in.} \]

From Eq. 3.18

\[ \Phi_{yn} = \Phi_{yt} = 75 \times 10^{-5}, \text{ in.}^{-1} \]

From the above numerical values, the conclusions to be drawn are that Poisson's ratio has negligible effect upon the yield moment and a very small influence on the yield curvature of a slab element subjected to equal orthogonal moments.

These analytical results are furthermore supported by the test results reported by Lenschow (7) who showed that, as in the case for beams, differences in concrete strength of more than one hundred percent had negligible effect on the yield moment of underreinforced slab elements subjected to isostatic moments.

As a result of the above comparisons, Poisson's ratio will be neglected in all strength calculations reported here, but its influence on strains will be considered when its magnitude may have a bearing on the final results.

(c) **Ultimate**

The ultimate moment capacity can be calculated using the expressions:

\[ M_{un} = M_{ut} = A f_{su} d (1 - 0.4 k_u) \]  \hspace{1cm} (3.19)

where

\[ k_u = p \frac{f_{su}}{f_{cu}} \]  \hspace{1cm} (3.20)

and
\[ f_{cu} = \frac{10,000}{1 + \frac{8000}{f'_c}} \]  

(3.21)

\[ f_{cu} = \text{average stress in the concrete at flexural failure of the section (9). All stresses are in psi.} \]

The curvature at ultimate can be estimated by assuming that the strain in compression in concrete reaches a value of 0.004 (9).

\[ \Phi_{un} = \Phi_{ut} = \frac{0.004}{k_u d} \]  

(3.22)

(d) **Comparison of Calculated and Measured Quantities**

In the following, a quantitative evaluation of the behavior and carrying capacity of reinforced concrete slabs subjected to isostatic moment will be presented. All calculated values were evaluated from Eq. 3.1 through 3.22 and are shown with broken lines in the diagrams. The measured values are shown in solid lines.

The behavior of two typical specimens (C2, \( p = 0.01 \) and C24, \( p = 0.005 \)) in terms of moment-curvature, moment-steel strains, and moment-concrete strains diagrams is presented. In addition, moment-curvature diagrams for all specimens are presented in Appendix A.

(1) **Moment-Curvature Relationships**

Table 3.1 presents the calculated and measured values for all six specimens tested with isostatic moment. The agreement between measured and calculated values is good.

Figure 3.4 shows the layout of the reinforcement in specimen C24 shown in an upside-down position with respect to its position in
the test rig. Figure 3.5 shows a moment-curvature plot for circular specimen C2 and Fig. 3.6 shows the same plot for circular specimen C24.

Figure 3.7 shows the yield-line pattern in circular specimen C24 and Fig. 3.8 shows circular specimen C2 after failure.

(2) Moment-Strain Relationships

Figures 3.9 and 3.10 show the measured and calculated strains in the reinforcement for circular specimens C2 and C24. The uniformity of the measurements confirms the existence of a truly isostatic-moment condition in the entire testing area. The same can be said from Fig. 3.11 and 3.12 which show the measured and calculated compressive strains in the concrete for circular specimens C2 and C24 respectively.

3.2. Nonisotropically Reinforced Slabs ($\mu \neq 1.0$)

Let $x$ and $y$ be the directions of the reinforcement in a reinforced concrete slab, and $M_x$, $M_y$, be the resisting unit bending moments in the $x$- and $y$-directions respectively.

Let the ratio between the resisting unit moments be

$$\eta = \frac{M_y}{M_x}$$  \hspace{1cm} (3.23)

The ratio of the amount of reinforcement in the $y$ and $x$-directions will be defined as:

$$\mu = \frac{\rho_y}{\rho_x}$$  \hspace{1cm} (3.24)
It is to be noted that if both layers of reinforcement are considered acting at the same level and the amounts are equal, then Eq. 5.23 and 3.24 give the same result.

In the case of isostatic moment, the behavior and strength characteristics of nonisotropically reinforced concrete slabs, $\mu \neq 1.0$, can be predicted on the basis of the results for isotropically reinforced slabs.

Because of the isostatic moment condition, the ratio of the applied moment to the resisting moment will be maximum across the section with the lower amount of reinforcement. Therefore, the capacity of the slab will be determined by the resistance of the cross section with the smaller amount of reinforcement. The effect of Poisson's ratio can be neglected.

The yield lines will be perpendicular to the direction of the sections with lower reinforcement. In the direction perpendicular to the sections with heavier reinforcement, cracking of the concrete may or may not take place depending on the ratio $\mu$.

As a result, the behavior can be explained using the same basic equations derived for isotropically reinforced slabs, with the condition that the carrying capacity of the slabs will be determined by the capacity of the cross section with the lower amount of reinforcement.
4. BEHAVIOR AND STRENGTH OF SLAB ELEMENTS SUBJECTED TO UNIAXIAL MOMENT

4.1. Isotropically Reinforced Slabs ($\mu = 1.0$)

A reinforced concrete slab element subjected to a uniaxial moment is one in which only one of the principal applied bending moments has a finite magnitude, the other being zero.

Figure 4.1 shows a representative isotropically reinforced slab element subjected to a uniaxial bending moment. In the following paragraphs, the load-deformation characteristics at different stages of loading will be discussed.

(a) Cracking

Let $n$ and $t$ be a set of orthogonal coordinate axes where $n$ is the direction in which the applied uniaxial unit bending moment, $M_E$, acts. Let also $x$ and $y$ be a set of orthogonal coordinate axes that coincide with the directions of the reinforcement, and $\theta$ the angle between the $x$ and $n$ axes. The superscript $(^0)$ refers to the top reinforcement and strains.

The strain and stress distributions in a cross section perpendicular to the $n$-axis at the cracking stage are shown in Fig. 4.2(b) and (c), where:

\[ \varepsilon_{cn}^0 = \frac{\sigma_{cn}^0}{E_c} \quad (4.1) \]

\[ \varepsilon_{cn} = \frac{\sigma_{cn}}{E_c} \quad (4.2) \]
The strains in the reinforcement with both layers being considered acting at the same level are:

\[ \varepsilon_{sx}^o = \varepsilon_n^o \cos^2 \alpha^o + \varepsilon_t^o \sin^2 \alpha^o + \gamma_{nt}^o \sin \alpha^o \cos \alpha^o \]  
\[ \varepsilon_{sy}^o = \varepsilon_n^o \sin^2 \alpha^o + \varepsilon_t^o \cos^2 \alpha^o - \gamma_{nt}^o \sin \alpha^o \cos \alpha^o \]  
(4.3)  
(4.4)

Equations 4.3 and 4.4 can also be represented by Mohr's Circle.

Because of the external moment conditions, there is no shear strain in planes perpendicular to the n- or t-axes and the strains in the t-direction result from Poisson's ratio which has already been shown to be negligible. Thus, the effective modulus of deformation of the reinforcement in any direction can be written as:

\[ E_s (\cos h \omega^0 + \sin h \omega^0) \]

which makes the effective modular ratio, \( n_u \), become

\[ n_u = n (\cos h \omega^0 + \sin h \omega^0) \]  
(4.5)

The depth of the neutral axis at cracking is:

\[ \frac{c_n}{h} = \frac{0.5 + n_u A \bar{d}/h^2}{1 + n_u A \bar{s}/h} \]  
(4.5a)

The bending moment capacity at cracking is:

\[ \frac{M_{cn}}{r^2 h^2} = \frac{1}{1 - c_n/h} \left[ \frac{n_u A \bar{d}}{h^2} \left( \frac{d - c_n}{h} \right) + \frac{1}{3} - 0.5 c_n/h \right] \]  
(4.6)
The curvature at cracking is:

\[ \Phi_{cn} = \frac{f_r}{E_c(h - c_n)} \]  \hspace{1cm} (4.7)

(b) Yielding

At this stage of loading three cases are possible in an underreinforced section:

(1) Only one layer of reinforcement reaches the yield strain.

(2) Both layers reach the yield strain at the same time.

(3) Both layers reach the yield strain at different times.

Case (1) occurs when the layers of reinforcement are oriented at angles, \( \alpha^o = 0^o \) and \( \alpha^o = 90^o \), respectively. In this case only the layer inclined at \( \alpha^o = 0^o \) will yield, and the perpendicular layer of reinforcement will not influence the strength of the slab element.

Case (2) occurs when both layers are symmetrically oriented with respect to the yield line, \( \alpha^o = \pm 45^o \). It is to be noted that in all cases the yield line will occur perpendicular to the n-axis, the axis of the applied external uniaxial moment.

Case (3) comprises all other inclinations of reinforcement.

In the following, an analysis of the moment-curvature relationship at stages between cracking and yielding will be presented and the limitations on the different cases will be shown.

From equilibrium of forces in the n-direction, as shown in Fig. 4.2(d) and (e)
The unit bending moment in the \( n \)-direction is:

\[
M_n = N_n^0 (d - c_n/3)
\]  

(4.10)

The curvature in the \( n \)-direction becomes:

\[
\Phi_n = \frac{\epsilon_n c_n}{E_n} = \frac{\epsilon_n}{E_n} 
\]  

(4.11)

The ratio \( \Phi_n/M_n \) expresses the flexibility of the slab element in the \( n \)-direction. From Eq. 4.8, 4.10 and 4.11 the flexibility at any stage between cracking and first yielding is:

\[
\Phi_n = \frac{1}{A_s E_s (\cos \alpha^0 + \sin \alpha^0)(d - c_n)(d - c_n/3)}
\]  

(4.12)

Figure 4.3 shows a plot of Eq. 4.12 for \( A_s/h = 0.005 \) and \( A_s/h = 0.01 \). The ratio \( A_s/h \) rather than \( p \) is used in order to simplify comparison with the case for torsion.

In the following, the limitations of the moment-curvature relationships at yield on cases (1), (2) and (3) are shown.
The total force in the n-direction due to the two layers of reinforcement acting in the x- and y-directions is:

\[ N_n^O = N_{xn}^O + N_{yn}^O \]  \hspace{1cm} (4.13)

where

\[ N_{xn}^O = A_s E_s e_n^O \cos \frac{1}{4} \alpha^O \]  \hspace{1cm} (4.14)

\[ N_{yn}^O = A_s E_s e_n^O \sin \frac{1}{4} \alpha^O \]  \hspace{1cm} (4.15)

From Eq. 4.13, 4.14, and 4.15, the following relationships can be derived:

\[ \frac{N_{xn}^O}{N_n^O} = \frac{\cos \frac{1}{4} \alpha^O}{\sin \frac{1}{4} \alpha^O + \cos \frac{1}{4} \alpha^O} \]  \hspace{1cm} (4.16)

\[ \frac{N_{yn}^O}{N_n^O} = \frac{\sin \frac{1}{4} \alpha^O}{\sin \frac{1}{4} \alpha^O + \cos \frac{1}{4} \alpha^O} \]  \hspace{1cm} (4.17)

Equations 4.16 and 4.17 are plotted in Fig. 4.4. From Eq. 4.10, 4.12 and Fig. 4.4 the following can be concluded:

Case (1). \( \alpha^O = 0^O \), the reinforcement in the y-direction has no effect on the moment capacity or on the flexibility of the slab element.

Case (2). \( \alpha^O = \pm 45^O \), both layers of reinforcement contribute equally to both the moment capacity and the flexibility of the slab element.
Case (3). $0^\circ < \alpha^\circ < 45^\circ$. From the plot it can be concluded that for $\alpha^\circ \leq 20^\circ$ the contribution of the reinforcement in the y-direction to the moment capacity and the flexibility of the element is of the order of 2 percent. By the same reason it can also be said that the yield moment for a specimen with orthogonal reinforcement oriented at $\alpha^\circ = 20^\circ$ will reach 98 percent of its yielding capacity when the more effective layer of reinforcement reaches the yield strain. At the stage when the reinforcement in the less effective layer reaches the yield strain, it is necessary to check whether the stresses in the concrete can still be considered linear, and whether the more effective layer has not reached the strain hardening. If both conditions are satisfied then the moment equations above are still applicable. If the conditions are not satisfied, then it is necessary to consider the actual stress-strain relationships for concrete and reinforcement.

(c) Ultimate

The ultimate moment capacity can be calculated using the expressions:

$$M_{un} = N_{nu}^c d(1 - 0.4k_u) \quad (4.18)$$

where $N_{nu}$ = tension force in the reinforcement at failure of the section.

The curvature at ultimate can be estimated as

$$\phi_{un} = \frac{0.004}{k_u d} \quad (4.19)$$
(d) **Comparison of Calculated and Measured Quantities**

This section presents a quantitative evaluation of the variables that influence the load-deformation characteristics of slab elements subjected to uniaxial bending. It has been shown in the previous chapter that concrete strength has a very small effect on the carrying capacity of underreinforced slab elements. Therefore, for a given yield stress of the reinforcement, the variable that has the most effect on the load-deformation characteristics is the orientation of the reinforcement with respect to the axis on which the uniaxial moment acts.

The behavior and strength of three typical specimens (B7, $\alpha^o_x = -45^o; B8, \alpha^o_x = -22.5^o$ and B10, $\alpha^o_x = 90^o$) will be described in terms of moment-curvature, moment-steel strains and moment-concrete strains diagrams. All calculated values were obtained on the basis of the equations derived in sections 4.1a through 4.1c and are shown with broken lines in the diagrams. Test results are shown in solid lines. In addition, moment-curvature diagrams for all specimens tested under uniaxial bending are presented in Appendix A.

1. **Moment-Curvature Relationship**

Table 4.1 presents the calculated and measured moment-curvature values for all specimens tested under uniaxial bending. It includes both isotropically and nonisotropically reinforced specimens. The isotropically reinforced specimens are discussed in this section, and the nonisotropically reinforced specimens are discussed in other sections of this chapter.

The moment-curvature relationship for specimen B7 is shown in Fig. 4.5, which also shows the orientation of the reinforcement and the
graphical representation of the yield criterion. The calculated values for moment and curvature are in good agreement with the test results. There is a well defined yield point indicating the simultaneous yielding of the two layers of reinforcement.

Figures 4.6 and 4.7 show the moment-curvature relationships for specimens B8 and B10. Because of the orientation of the reinforcement in specimen B8, there were two well defined yield stages, the first corresponding to the yielding of the reinforcement in the more effective layer and the second to the yielding of the reinforcement in the less effective layer. The test results of B10 shown in Fig. 4.7 require no further comment since it is essentially a beam test where the direction of the reinforcement and the applied uniaxial moment coincide.

A quantitative evaluation of the flexibility of specimen B7, B8 and B10 calculated on the basis of the test results is in good agreement with the calculated values on the basis of Eq. 4.12.

(2) Moment-Steel Strain Relationship

Calculated and measured values for strains in the reinforcement in specimens B7, B8 and B10 are shown in Fig. 4.8, 4.9 and 4.10.

In calculating the steel strains plotted in Fig. 4.8 the actual depths of the two layers of reinforcement were used.

Figure 4.9 shows the measured and calculated steel strains in specimen B8. There is good agreement between measured and calculated values for layer 1 while there is a distinct deviation for layer 2 after cracking of the concrete.

In contrast to the trend shown in Fig. 4.9, agreement was obtained between measured and calculated values for both layers of bars
in specimen B7 as shown in Fig. 4.8. In specimen B7, both layers of bars were oriented at 45° to the principal-moment axis. On the other hand, in specimen B8 layer 1 made an angle of 22.5° and layer 2, 67.5° with the principal-moment axis. In calculating the strains in an isotropically reinforced slab subjected to uniaxial moment, shifts in the principal curvature direction are ignored. Actually, the orientation of cracks in the specimen tend to rotate in the early stages of loading to "avoid" the more effective layer of reinforcement. Consequently, layer 2 at 67.5° is strained more than what the calculations indicate. The effect of this shift in crack orientation on strength is negligible, but it shows on the strain plots.

Test results and calculated values for specimen B10 are shown in Fig. 4.10. The compressive strains in the transverse layer of reinforcement were calculated for Poisson's ratio, \( \nu = 0.1 \) and a limiting tensile strain in the concrete 0.0002.

(3) **Moment-Concrete Strain Relationship**

Calculated and measured values of concrete strains on the compression side of the test specimens, in both of the directions of the principal applied moments, longitudinal and transverse directions, are presented in Fig. 4.11 through 4.15.

Longitudinal and transverse strains for specimen B7 are shown in Fig. 4.11 and 4.12. The effect of the orientation of the reinforcement, with respect to the axis of the uniaxial moment, on the concrete strain follows from its effect on the flexibility of the specimen. The tensile strains in the transverse direction of the compression face have been calculated for Poisson's ratio \( \nu = 0.1 \). In a similar fashion,
Fig. 4.13 and 4.14 present calculated and measured concrete strain values for specimen B8, and Fig. 4.15 presents the results for specimen B.10.

It can be said from these results that a linear variation of both moment and deformations between cracking and yielding provides a good picture of the behavior of these specimens.

4.2. **Nonisotropically Reinforced Slabs** ($\mu \neq 1.0$)

As defined here, a nonisotropically reinforced slab is one in which the amounts of reinforcement in two mutually orthogonal directions are different. If the slab has both top and bottom reinforcement, it will be considered that the isotropy ratio is the same on the top and bottom reinforcement layers; and, also, that the orientation of the heavier and weaker layers of reinforcement is the same on the top and bottom.

In the following paragraphs, the load-deformation characteristics of nonisotropically reinforced slabs subjected to a uniaxial moment is discussed.

(a) **Cracking**

It has been shown (7) that the initial cracking in a nonisotropically reinforced slab occurs in a plane perpendicular to the direction on which the maximum principal moment acts. The nonisotropy of the reinforcement in reasonable amounts has no effect on the orientation of the initial crack but does have an effect on the contribution of the reinforcement to the cracking moment. Accordingly, the cracking
moment can be computed on the basis of the equations derived for isotropically reinforced slabs subjected to uniaxial bending by introducing a modified modular ratio, \( n'_u \), which can be expressed as:

\[
n'_u = n(\cos \alpha + \mu \sin \alpha)
\]  

Under these conditions, the depth of the neutral axis at the cracking load is:

\[
c_n = \frac{0.5 + \frac{n'_u A_d}{h^2}}{1 + \frac{n'_u A_s}{h}} \tag{4.21}
\]

The bending moment capacity at cracking is:

\[
\frac{M_{cn}}{f_r h^2} = \frac{1}{1 - \frac{c_n}{h}} \left[ \frac{n'_u A_d}{h^2} \left( \frac{d - c_n}{h} \right) + \frac{1}{3} - 0.5 \frac{c_n}{h} \right] \tag{4.22}
\]

and the curvature is:

\[
\phi_{cn} = \frac{f_r}{E_c (h - c_n)} \tag{4.23}
\]

(b) Yielding

At this stage of loading the yield line orientation does not, in general, coincide with the direction on which the maximum principal moment acts or with the directions of the reinforcing bars, as has been already discussed in Chapter 2. Consequently, the carrying capacity at yield can be calculated on the basis of the principle of least resistance explained in Chapter 2.
In the following paragraphs, an evaluation of the flexibility characteristics of nonisotropically reinforced slabs subjected to uniaxial moment at any stage of loading between cracking and first yielding is presented.

Figure 4.16a shows a nonisotropically reinforced slab subjected to uniaxial moment. Figures 4.16b, c, d, and e show the cross section of a slab element, and the assumed normal, tangential, and shear strain and stress distributions at the crack. It will be assumed, as shown in Fig. 4.16a, that the ratio of the amounts of reinforcement in the y- and x-directions is \( \mu \), and that the area of reinforcement per unit width in the x-direction is \( A_s \).

The resisting moment in the direction normal to the crack is:

\[
M_{E_n} = M_n = N_n^O(d - c_n/3) \tag{4.24}
\]

The resisting moment in the tangential direction to the crack, when such a section is uncracked, is:

\[
M_{E_t} = M_t = N_t^O(d - c_t/3) + 2F_t^O h/3 \tag{4.25}
\]

If the section is cracked, then the second term in the right-hand side of Eq. 4.25, which is the contribution of the concrete in tension, becomes negligible.

In Eq. 4.24 and 4.25 the component forces of the reinforcement in the normal and tangential directions to the crack, as shown in Fig. 4.16, are:
\[ N_n^0 = A_s \varepsilon_n^o \left[ \varepsilon_n^o (\cos^4 \alpha + \mu \sin^4 \alpha) + \varepsilon_t^o \sin^2 \alpha \cos^2 \alpha (1 + \mu) \right. \\
\left. + \gamma_{nt}^o \sin \alpha \cos \alpha (\cos^2 \alpha - \mu \sin^2 \alpha) \right] \]  \hspace{1cm} (4.26)

and

\[ N_t^0 = A_s \varepsilon_n^o \left[ \varepsilon_n^o \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \varepsilon_t^o (\sin^4 \alpha + \mu \cos^4 \alpha) \right. \\
\left. + \gamma_{nt}^o \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \right] \]  \hspace{1cm} (4.27)

Similarly, the forces in the concrete are:

\[ F_n = \frac{1}{2} \varepsilon_n^o E_c \frac{c_n^2}{d - c_n} \]  \hspace{1cm} (4.28)

\[ F_t = \frac{1}{2} \varepsilon_t^o E_c \frac{c_t^2}{d - c_t} \]  \hspace{1cm} (4.29)

and

\[ F_t^o = \frac{1}{2} \varepsilon_t^o E_c \frac{(h - c_t)^2}{d - c_t} \]  \hspace{1cm} (4.30)

Because of the nonisotropy of the reinforcement a shear strain exists in the planes normal and tangential to the crack which affects to some extent the strains in the direction of the reinforcing bars. An estimate of the maximum shearing strain can be obtained on the basis of the elastic solution for rectangular members subjected to torsion (10).

The maximum shearing strain for a given unit torque \( M_{nt} \) and a width to depth ratio larger than 10 is assumed to be:

\[ \gamma_{nt} = \frac{3M_{nt}}{Gh^2} = \frac{6M_{nt}}{E_c h^2} \]  \hspace{1cm} (4.31)
where $G$ is the shearing modulus of the concrete which for $\nu = 0$, becomes $G = E_c/2$.

The shearing strain at the level of the reinforcement is:

$$\gamma_{nt}^{o} = \gamma_{nt} \frac{d - h/2}{h/2} \quad (4.32)$$

which in view of the above approximations can be taken as $\gamma_{nt}^{o} = \gamma_{nt}$.

The shear force in the reinforcement is:

$$N_{nt}^{o} = A_s E_s \sin \alpha \cos \alpha \left[ \varepsilon_n^{o} \left( \cos^2 \alpha - \mu \sin^2 \alpha \right) + \varepsilon_t^{o} \left( \sin^2 \alpha - \mu \cos^2 \alpha \right) + \gamma_{nt}^{o} \sin \alpha \cos \alpha (1 - \mu) \right] \quad (4.33)$$

From equilibrium of forces in the n-direction, Eq. 4.26 and 4.28, the depth of the neutral axis is:

$$c_n = d \left[ -p_n c_1 + \sqrt{p_n c_1^2 + 2 p_n c_1} \right] \quad (4.34)$$

where

$$c_1 = \cos \alpha + \mu \sin \alpha + \frac{\varepsilon_t^{o}}{\varepsilon_n^{o}} \sin^2 \alpha \cos^2 \alpha (1 + \mu)$$

$$+ \frac{\gamma_{nt}^{o}}{\varepsilon_n^{o}} \sin \alpha \cos \alpha \left( \cos^2 \alpha - \mu \sin^2 \alpha \right) \quad (4.35)$$

In the t-direction, two cases may be considered:

(a) If the section is uncracked, the depth of the neutral axis is very close to, $c_t = h/2$, and the strains in the t-direction are small compared to those in the n-direction.
If the section is cracked, then the contribution of the concrete in tension becomes negligible, and the depth of the neutral axis can be expressed as:

\[ c_t = d - \sqrt{2pC_2 + \sqrt{pC_2^2 + 2pC_2}} \]  

(4.36)

where

\[ C_2 = \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \frac{\varepsilon^0_t}{\varepsilon_n^0} (\sin^2 \alpha + \mu \cos^2 \alpha) \]

\[ + \frac{\gamma_{nt}^0}{\varepsilon_n^0} \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \]  

(4.37)

An estimate of the ratios \( \varepsilon_t^0/\varepsilon_n^0 \) and \( \gamma_{nt}^0/\varepsilon_n^0 \) can be obtained as follows:

The stiffness of the slab in the n-direction is:

\[ K_n = \frac{M_n (d - c_n)}{\varepsilon_n^0} \]  

(4.38)

where \( K_n \), on the basis of the results obtained for isotropically reinforced slabs and modified by the factor \( \mu \), can be taken as being proportional to:

\[ K_n = A_s E_s (\cos^2 \alpha + \mu \sin^2 \alpha)(d - c_n)(d - c_n/3) \]  

(4.39)

From Eq. 4.38 and 4.39, the strain at the level of the reinforcement in the n-direction becomes:
\[ \epsilon_n^o = \frac{M_n}{A_s E_s (\cos \alpha + \mu \sin \alpha)(d - c_n/3)} \quad (4.40) \]

Similarly, in the t-direction the stiffness is:

\[ K_t = \frac{M_t (d - c_t)}{\epsilon_t^o} \quad (4.41) \]

where \( K_t \) can be taken as:

\[ K_t = A_s E_s \left( \sin \frac{1}{h} + \mu \cos \frac{1}{h} \right)(d - c_t)(d - c_t/3) \quad (4.42) \]

From Eq. 4.41 and 4.42 the strain at the level of the reinforcement in the t-direction becomes:

\[ \epsilon_t^o = \frac{M_t}{A_s E_s \left( \sin \frac{1}{h} + \mu \cos \frac{1}{h} \right)(d - c_t/3)} \quad (4.43) \]

From Eq. 4.31 the shearing strain \( \gamma_{nt}^o = \gamma_{nt} \) is:

\[ \gamma_{nt}^o = \frac{6M_{nt}}{E_c h^2} \quad (4.44) \]

From Eq. 4.40 and 4.43 the ratio between \( \epsilon_t^o \) and \( \epsilon_n^o \) can be derived to be:

\[ \frac{\epsilon_t^o}{\epsilon_n^o} = \frac{M_t (\cos \frac{1}{h} + \mu \sin \frac{1}{h})(d - c_t/3)}{M_n (\sin \frac{1}{h} + \mu \cos \frac{1}{h})(d - c_t/3)} \quad (4.45) \]
If the sections normal to the t-axis are also cracked, then the depths of the neutral axis in both the t- and n-directions, \( c_t \) and \( c_n \), respectively, will be comparable. On the other hand, if the sections normal to the t-axis are uncracked, the ratio \( \frac{\varepsilon_t}{\varepsilon_n} \) should be negligible.

Considering that the sections normal to the t-direction are cracked, then from Eq. 4.45 and the principle of least resistance applied to the formation of the cracks, as stated in Chapter 2, the following relationships can be obtained:

\[
\frac{\varepsilon_t}{\varepsilon_n} = \tan^2 \gamma \frac{\cos \frac{h}{\pi} + \mu \sin \frac{h}{\pi}}{\sin \frac{h}{\pi} + \mu \cos \frac{h}{\pi}}
\]  
(4.46)

Similarly from Eq. 4.43 and 4.44 the ratio \( \frac{\varepsilon_{nt}}{\varepsilon_n} \) can be expressed as:

\[
\frac{\varepsilon_{nt}}{\varepsilon_n} = \frac{6 M_{nt} A_s E_s (\cos \frac{h}{\pi} + \mu \sin \frac{h}{\pi})(d - c_n/3)}{M_n E_n h^2}
\]  
(4.47)

On the basis of the principle of least resistance and assuming, that for normal amounts of reinforcement, the ratio \( (d - c_n/3)/h \) is between 0.8 and 0.9, Eq. 4.47, for \( p = 0.01 \) and \( n = 7 \), can be expressed as:

\[
\frac{\gamma_{nt}}{\varepsilon_n} = 0.3 \tan \gamma (\cos \frac{h}{\pi} + \mu \sin \frac{h}{\pi})
\]  
(4.48)

The curvature in the n-direction can be expressed as

\[
\phi_n = \frac{\varepsilon_n}{d - c_n}
\]  
(4.49)
From Eq. 4.24, 4.46 and 4.49 the expression for the flexibility becomes:

\[
\frac{\Phi_{A_s} E_s h^2}{M_n} = 1 \left[ \frac{d-c_n}{h} \left( \frac{d-c_n/3}{h} \right) \cos^2 \alpha + \mu \sin^2 \alpha \right]
+ \frac{\epsilon_t^0}{\epsilon_n^0} \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \frac{\gamma_{nt}^0}{\epsilon_n^0} \sin \alpha \cos \alpha (\cos^2 \alpha - \mu \sin^2 \alpha)
\]

\hspace{1cm} (4.50)

For moderate amounts of reinforcement, an approximation can be made by considering that the terms containing the ratios \( \epsilon_t^0/\epsilon_n^0 \) and \( \gamma_{nt}^0/\epsilon_n^0 \) have a negligible effect on the flexibility of the slab element. On this basis Eq. 4.50 can be simplified to

\[
\frac{\Phi_{A_s} E_s h^2}{M_n} = \frac{1}{\left( \frac{d-c_n}{h} \left( \frac{d-c_n/3}{h} \right) \cos^2 \alpha + \mu \sin^2 \alpha \right) \epsilon_n^0}
\]

\hspace{1cm} (4.51)

The variation in flexibility calculated from Eq. 4.50 and 4.51 as a function of \( \alpha \), the angle between the main layer of reinforcement and the major principal moment axis, has been plotted in Fig. 4.17 for \( p = 0.01 \) and \( \mu = 0.5 \). It can be observed in Fig. 4.17 that the differences between Eq. 4.50 and 4.51 are less than 5 percent. Furthermore, the smaller values of flexibility are given by Eq. 4.50, because a tensile strain in the t-direction, \( \epsilon_t^0 \), decreases the strains in the n-direction \( \epsilon_n^0 \).

Figure 4.18 shows the variation of flexibility calculated from Eq. 4.51 as a function of \( \alpha \), for \( p = 0.01 \) and \( \mu = 0.25 \).
Figure 4.19 shows the variation of the depth of the neutral axis as a function of $\alpha$. The values plotted have been calculated for isotropic reinforcement $p = 0.01$, $p = 0.005$, and nonisotropic reinforcement $p = 0.01$ and $\mu = 0.5$.

Figure 4.20 shows the variation of concrete compressive strains in the $n$-direction as a function of $\alpha$, in both isotropically and nonisotropically reinforced slabs subjected to uniaxial bending.

(c) **Ultimate**

The ultimate moment capacity of nonisotropically reinforced concrete slabs subjected to uniaxial bending can also be computed on the basis of the principle of least resistance by introducing the components of the ultimate moment capacity of the nonisotropic reinforcement in a direction normal to the yield line.

(d) **Comparison of Calculated and Measured Quantities**

The effect of the nonisotropy of the reinforcement on the relationships between moment-curvature, moment-steel strains, and moment-concrete strains for slabs subjected to uniaxial bending is discussed here.

There were three specimens tested with nonisotropic reinforcement: B9, B11, and B12. The characteristics of these specimens are presented in Table 4.1, which also shows the numerical values of the calculated and measured moment-curvature relationships at cracking, yielding and the calculated and measured values of bending moment capacity at ultimate.
In the following paragraphs, the results of one of these specimens \((B9, \alpha_x^o = 45^\circ)\) will be discussed in terms of the moment-deformation characteristics above mentioned. In addition, moment-curvature relationships are presented in Appendix A for all three specimens of this series.

(1) Moment-Curvature Relationship

The moment-curvature diagram for specimen B9, which had the main reinforcement oriented at \(45^\circ\) to the principal moment axis, is shown in Fig. 4.21, which also shows the graphical representation of the yield criterion. The calculated values for moment are based on the yield criterion and those of curvature are based on Eq. 4.51. Calculated values are shown in broken lines and measured values in solid lines. The agreement between calculated and measured values is good.

(2) Moment-Steel Strain Relationship

Calculated and measured values for steel strains in specimen B9 are shown in Fig. 4.22. In calculating the steel strains plotted in Fig. 4.22, the actual depths of the two layers of reinforcement were used.

(3) Moment-Concrete Strain Relationship

Calculated and measured values of compressive concrete strains in the longitudinal direction of specimen B9 are shown in Fig. 4.23. The calculated values are based on: a linear strain distribution, and the values of curvature and depth to the neutral axis, as derived in Section 4.2b of this chapter.
5. BEHAVIOR AND STRENGTH OF SLAB ELEMENTS SUBJECTED TO COMBINED BENDING AND TORSION

5.1. Isotropically Reinforced Slabs ($\mu = 1.0$):

A reinforced concrete slab element subjected to combined bending and torsion is one in which both of the applied principal moments have a finite magnitude. The sign of the ratio of these principal moments may be negative or positive.

Figure 5.1 shows an isotropically reinforced slab subjected to combined bending and torsion. The axes on which the principal moments $M_1$ and $M_2$ act are $u$ and $v$. The $x$- and $y$-axes coincide with the directions of the reinforcement, and $\alpha$ is the angle between the $v$-axis and the $x$-axis. Because the slab is isotropically reinforced, the yield line will coincide with the $u$-axis. Therefore, the $t$- and $n$-axes, which are the yield-line axes, coincide with the $u$- and $v$-axes.

In the following paragraphs, the load-deformation characteristics at different stages of loading will be discussed.

(a) Cracking

The capacity at cracking for an element isotropically reinforced top and bottom is the same as that of a beam with equal amounts of tension and compression reinforcement. Under these conditions, the neutral axis is at mid-depth of the section, and the forces in the concrete and reinforcement are as shown in Fig. 5.2.

The larger applied principal moment is chosen to be $M_{E_n}$:

$$|M_{E_n}| \geq |M_{E_u}|$$
Accordingly, cracking will take place first on the section subjected to the moment $M_{En}$, and the section under the moment $M_{Et}$ may or may not crack depending on the relative absolute magnitude of $M_{Et}$ in relation to $M_{En}$.

Only in the case $|M_{En}| = |M_{Et}|$, and if the effects of dead weight moment are negligible, will cracking take place simultaneously in both the n- and t-directions.

For convenience the principal moment acting in the v-direction is defined as positive, and that acting in the u-direction as negative. In general the sense of the moments and curvatures can be determined by inspection. The following sign convention has been adopted to maintain consistency in the derivations. Compressive strains and forces are negative. Distances are measured from the neutral axis for curvature and from the point about which the moments is calculated for moment. Downward distances are negative.

\[
\Phi_n = \frac{\varepsilon_n}{(c_n - d')} = \frac{\varepsilon_n^o}{d - c_n}
\]

\[
\Phi_t = \frac{\varepsilon_t^o}{c_t - d'} = \frac{\varepsilon_t}{(d - c_t)}
\]

The strains in the reinforcement (with $\gamma_{nt} = \gamma_{nt}^o = 0$) are:

\[
\varepsilon_{sx}^o = \varepsilon_n^o \cos^2 \alpha^o + \varepsilon_t^o \sin^2 \alpha^o
\]

\[
\varepsilon_{sy}^o = \varepsilon_n^o \sin^2 \alpha^o + \varepsilon_t^o \cos^2 \alpha^o
\]
\[ \varepsilon_{sx} = \varepsilon_n \cos^2 \alpha + \varepsilon_t \sin^2 \alpha \quad (5.5) \]
\[ \varepsilon_{sy} = \varepsilon_n \sin^2 \alpha + \varepsilon_t \cos^2 \alpha \quad (5.6) \]

The force components of the reinforcement in the n- and t-directions are:

\[ N_n^0 = A_s E (\varepsilon_n \cos^2 \alpha^0 + 2 \varepsilon_t \sin^2 \alpha \cos^2 \alpha^0 + \varepsilon_n \sin^2 \alpha^0) \quad (5.7) \]
\[ N_t^0 = A_s E (\varepsilon_t \cos^2 \alpha^0 + 2 \varepsilon_n \sin^2 \alpha \cos^2 \alpha^0 + \varepsilon_t \sin^2 \alpha^0) \quad (5.8) \]
\[ N_n = A_s E (\varepsilon_n \cos^2 \alpha + 2 \varepsilon_t \sin^2 \alpha \cos^2 \alpha + \varepsilon_n \sin^2 \alpha) \quad (5.9) \]
\[ N_t = A_s E (\varepsilon_t \cos^2 \alpha + 2 \varepsilon_n \sin^2 \alpha \cos^2 \alpha + \varepsilon_t \sin^2 \alpha) \quad (5.10) \]

The force components of the concrete in the n- and t-directions are:

\[ F_n^0 = \varepsilon_{cn} E (h-c_n)/2 \quad (5.11) \]
\[ F_n = \varepsilon_{cn} E c_n/2 \quad (5.12) \]
\[ F_t^0 = \varepsilon_{ct} E c_t/2 \quad (5.13) \]
\[ F_t = \varepsilon_{ct} E (h-c_t)/2 \quad (5.14) \]

From equilibrium of forces in the n-direction

\[ N_n^0 + N_n + F_n^0 + F_n = 0 \quad (5.15) \]

The bending moment capacity in the n-direction is:

\[ M_{cn} = N_n^0 d + F_n^0 (2h/3 + c_n/3) + F_n c_n/3 + N_n d' \quad (5.16) \]
At cracking the stress in the extreme concrete fiber in tension is assumed to reach the nominal modulus of rupture $f_r$.

Because of symmetric reinforcement, the depth of the neutral axis in both the n- and t-directions is

$$c_n = c_t = h/2$$  \hspace{1cm} (5.17)

For symmetrically oriented reinforcement ($\alpha = \alpha^0$), and from Eq. 5.1, 5.2 and 5.17:

$$\frac{\varepsilon_n^0}{\varepsilon_t^0} = \frac{\varepsilon_n}{\varepsilon_t} = \frac{\Phi_n}{\Phi_t}$$  \hspace{1cm} (5.18)

The cracking moment in the n-direction is:

$$M_{cn} = n_{bt} A^f_r \frac{d-h/2}{h/2} (d-d') + 0.166 f_r h^2$$  \hspace{1cm} (5.19)

where

$$n_{bt} = n (\cos^4 \alpha + 2 \frac{\Phi_t}{\Phi_n} \sin^2 \alpha \cos^2 \alpha + \sin^4 \alpha)$$  \hspace{1cm} (5.20)

The curvature at cracking is:

$$\Phi_{cn} = \frac{2f_r}{E_c h}$$  \hspace{1cm} (5.21)

(b) **Yielding**

At this stage of loading the section in the n-direction is cracked. The contribution of concrete in tension to the moment capacity may be ignored. In the t-direction the section may or may not be
cracked, depending on the ratio of the applied principal moments. Therefore, the concrete in tension in the t-direction may or may not contribute to the moment capacity in the t-direction.

From equilibrium of forces in the n-direction (Fig. 5.2c) with $F_n^O = 0$,

$$N_n^O + N_n + F_n = 0 \quad (5.22)$$

The bending moment capacity in the n-direction is

$$M_n = N_n d + F_n c_n / 3 + N_n d' \quad (5.23)$$

From Eq. 5.1, 5.2, the force-strain relationships for concrete and reinforcement, and for symmetrically oriented reinforcement, $\alpha = \alpha^0$, Eq. 5.22 and 5.23 become:

$$c_n^2 = \frac{2A_sE_s}{E_c} \left[ (h-2c_n)(\sin^2 \alpha + \cos^2 \alpha) - 2 \frac{\Phi_t}{\Phi_n} (h-2c_t) \sin^2 \alpha \cos^2 \alpha \right] \quad (5.24)$$

and

$$\frac{\Phi_n}{M_n} = 1 / \frac{A_sE_s}{E_c} \left[ (d^2 + d' ^2 - \frac{4c_n h}{3} + \frac{2c_n^2}{3}) (\sin^2 \alpha + \cos^2 \alpha) 

- \frac{2 \Phi_t}{\Phi_n} \sin^2 \alpha \cos^2 \alpha (2d' - c_t h - \frac{c_n h}{3} + \frac{2 c_n c_t}{3}) \right] \quad (5.25)$$

A similar analysis in the t-direction gives the following:

$$c_t^2 = \frac{2A_sE_s}{E_c} \left[ (h-2c_t)(\sin^2 \alpha + \cos^2 \alpha) - \frac{2 \Phi_n}{\Phi_t} (h-2c_n) \sin^2 \alpha \cos^2 \alpha \right] \quad (5.26)$$
and

\[
\frac{\phi_t}{M_t} = \frac{1}{A_s E_s} \left[ (d^2 + d'_t^2 - \frac{4c_t h}{3} + \frac{2c_t^2}{3})(\sin^4 \alpha + \cos^4 \alpha) \right.
\]

\[- \left( \frac{2\Phi_n}{\Phi_t} \sin^2 \alpha \cos^2 \alpha \right)(dd' - c_n h - \frac{c_h}{3} + \frac{2c_n c_t}{3}) \]

\[
\frac{\phi_t}{M_t} = \frac{1}{A_s E_s} \left[ (d^2 + d'_t^2 - \frac{4c_t h}{3} + \frac{2c_t^2}{3})(\sin^4 \alpha + \cos^4 \alpha) \right.
\]

\[- \left( \frac{2\Phi_n}{\Phi_t} \sin^2 \alpha \cos^2 \alpha \right)(dd' - c_n h - \frac{c_h}{3} + \frac{2c_n c_t}{3}) \]

\[
(5.27)
\]

In order to simplify Eq. 5.24 through 5.27 let:

\[
A = \cos^4 \alpha + \sin^4 \alpha \quad (5.28)
\]
\[
B = \sin^2 \alpha \cos^2 \alpha \quad (5.29)
\]

then:

\[
c_n^2 = 2nA_s \left[ A(h-2c_n^t) - \frac{2B\Phi_t}{\Phi_n^t} (h-2c_n) \right] \quad (5.24a)
\]

\[
\frac{\phi_n}{M_n} = \frac{1}{A_s E_s} \left[ (d^2 + d'_t^2 - \frac{4c_t h}{3} + \frac{2c_t^2}{3})(\sin^4 \alpha + \cos^4 \alpha) \right.
\]

\[- \left( \frac{2\Phi_n}{\Phi_t} \sin^2 \alpha \cos^2 \alpha \right)(dd' - c_n h - \frac{c_h}{3} + \frac{2c_n c_t}{3}) \]

\[
(5.25a)
\]

\[
c_t^2 = 2nA_s \left[ A(h-2c_t) - \frac{2B\Phi_t}{\Phi_n^t} (h-2c_n) \right] \quad (5.26a)
\]

\[
\frac{\phi_t}{M_t} = \frac{1}{A_s E_s} \left[ (d^2 + d'_t^2 - \frac{4c_t h}{3} + \frac{2c_t^2}{3})(\sin^4 \alpha + \cos^4 \alpha) \right.
\]

\[- \left( \frac{2\Phi_n}{\Phi_t} \sin^2 \alpha \cos^2 \alpha \right)(dd' - c_n h - \frac{c_h}{3} + \frac{2c_n c_t}{3}) \]

\[
(5.27a)
\]
In the derivation of Eq. 5.26 and 5.27 it has been assumed that the section in the t-direction is also cracked. This assumption is not too unrealistic since the difference in depth of the neutral axis for a section cracked or uncracked is small for a section with a moderate amount of reinforcement.

The results of Eq. 5.25 have been plotted in Fig. 5.3 for two different amounts of reinforcement, \( A_g/h = 0.01 \) and \( A_g/h = 0.005 \), which have been combined with two different ratios of principal moment, \( ME_t/ME_n = -0.14 \) and \( ME_t/ME_n = -0.45 \). These moment ratios are the same as those chosen for the experimental investigation. The general shape of these diagrams is the same as those presented for uniaxial bending which were discussed in Chapter 4. On the other hand, it can be seen in Fig. 5.3 that for small amounts of reinforcement the differences in flexibility for an increase in principal moment ratio is small as shown by curves 1 and 2. As the amount of reinforcement is increased then this difference becomes more evident as shown in curves 3 and 4 of Fig. 5.3. In the case of combined bending and torsion, as in the cases of uniaxial bending or pure torsion, the flexibility of an isotropically reinforced slab is maximum when the isotropic reinforcement deviates an angle of 45° from the principal moment axis.

In the following it will be shown that the equations above derived apply to the particular cases of isostatic moment, biaxial bending, uniaxial bending and pure torsion.

1. **Isostatic Moment.** Making use of the conditions of symmetry, assuming that there is no compression reinforcement, and introducing the following identities:
\[ \alpha = 0^\circ \text{ or } 90^\circ; \ \varepsilon_n^0 = \varepsilon_t^0; \ \varepsilon_{cn} = \varepsilon_{ct}; \ N_n = N_t = 0 \text{ and } \Phi_t/\Phi_n = 1; \] then the depth of the neutral axis in either the n or t directions becomes:

\[ c_n = c_t = d \left( \sqrt{\frac{2}{pn + 2pn - pn}} \right) \]  

(5.28)

and the flexibility is:

\[ \frac{\Phi_n}{M_n} = \frac{\Phi_t}{M_t} = \frac{1}{A_s E_s (d-c_n)(d-c_n/3)} \]  

(5.29)

Equation 5.28 is equal to Eq. 3.16 for Poisson's ratio \( \nu = 0 \) and Eq. 5.29 expresses that for a certain amount of reinforcement, \( p \), and a modular ratio, \( n \), the flexibility of the slab element is a constant value which does not depend on the inclination of the reinforcement, \( \alpha \).

2. Biaxial Bending. (0 < ME/tME < 1). For no compression reinforcement and assuming that the sections in both the n and t directions are cracked, the depth of the neutral axis in the n-direction becomes:

\[ c_n = d \left( \sqrt{\frac{2}{pn_b + 2pn_b - pn_b}} \right) \]  

(5.30)

where \( n_b = n(\cos^4 \alpha + 2 \frac{\Phi_t}{\Phi_n} \sin^2 \alpha \cos^2 \alpha + \sin^4 \alpha) \)  

(5.31)

The flexibility in the n-direction at any stage between cracking and first yielding is:

\[ \frac{\Phi_n}{M_n} = \frac{1}{\left[ A_s E_s (d-c_n)(d-c_n/3)(\cos^4 \alpha + 2 \frac{\Phi_t}{\Phi_n} \sin^2 \alpha \cos^2 \alpha + \sin^4 \alpha) \right]} \]  

(5.32)
3. Uniaxial Moment ($ME_t/ME_n = 0$). For no compression reinforcement and the following strain and force conditions

$$\varepsilon_t^0 = 0; \varepsilon_{ct} = 0; N_n = N_t = 0; \text{ and } \Phi_t/\Phi_n = 0$$

The depth of the neutral axis in the n-direction becomes

$$c_n = d(\sqrt{\frac{p_n}{2m}} + 2p_n - p_n)$$

and the flexibility is:

$$\frac{\Phi_n}{M_n} = \frac{1}{M_n} \left[ A_s E_s (\cos^4 \alpha + \sin^4 \alpha)(d-c_n)(d-c_n/3) \right]$$

Equations 5.33 and 5.34 are the same as Eq. 4.9 and 4.12.

4. Pure Torsion ($ME_t/ME_n = -1$). In this case:

$$c_n = c_t \text{ and } \Phi_t/\Phi_n = -1$$

From Eq. 5.24a and 5.26a:

$$c_t = c_n = \sqrt{(2nA_s)^2 + 2nA_s h - 2nA_s}$$

and from Eq. 5.25a or 5.27a

$$\frac{\Phi_n}{M_n} = \frac{\Phi_t}{M_t} = \frac{1}{M_t} \left[ A_s E_s \left( \left( \cos^2 \alpha + \sin^2 \alpha \right)^2 + \left( \sin^2 \alpha + \cos^2 \alpha \right)^2 - 4c_n h/3 + 2c_n^2/3 \right) \right]$$

Equations 5.35 and 5.36 are the same as Eq. 4.61 and 4.63 presented in Ref. (7).
The conditions at yielding can be input in Eq. 5.24a through 5.27a by considering that the strain in the reinforcement in the x-direction or in the y-direction, according to the orientation of the x-y system of axes with respect to the axes of principal moments, n-t, reach the yield strain.

Here again, for underreinforced sections, three cases are possible:

1. Only one layer of reinforcement reaches the yield strain.
2. Both layers reach the yield strain at the same time.
3. Both layers reach the yield strain at different times.

The procedure to solve these three different cases is similar to that explained in Section 4.1b.

(c) Ultimate

The ultimate moment capacity can be calculated using the expression

\[ M_{un} = N^o_{nu} d + N_{nu} d' - 0.4 F_{nu} k_d \]  \hspace{1cm} (5.37)

where

- \( N^o_{nu} \) = component force in the n-direction due to the top reinforcement at ultimate
- \( N_{nu} \) = component force in the n-direction due to the bottom reinforcement at ultimate
- \( F_{nu} \) = concrete force in the n-direction at ultimate

and

\[ k_d = \frac{N^o_{nu} + N_{nu}}{f_{cu}} \]  \hspace{1cm} (5.38)
The curvature at ultimate can be estimated as:

$$\phi_{un} = \frac{0.004}{k_d}$$  \hspace{1cm} (5.39)

(d) Comparison of Calculated and Measured Quantities

In the following, a quantitative evaluation of the effects of the reinforcement orientation on the load-deformation characteristics of slab elements subjected to combined bending and torsion is presented.

There were two series of specimens subjected to combined bending and torsion. The first series (specimens B26 through B33) consisted of specimens isotropically reinforced (B26 through B29) and non-isotropically reinforced (B30 through B33). These specimens were subjected to a combination of $M_{\text{torsion}}/M_{\text{bending}} = 0.45$, as shown in Fig. 5.1a, so that the principal moment ratio was $M_{\text{E}}/M_{\text{E}} = M_1/M_2 = -0.14$.

The second series (specimens B34 through B40) also consisted of specimens isotropically reinforced (B34 through B37) and nonisotropically reinforced (B38 through B40). These specimens were subjected to a combination of $M_{\text{torsion}}/M_{\text{bending}} = 1.25$, so that the principal moment ratio was $M_{\text{E}}/M_{\text{E}} = M_1/M_2 = -0.45$.

The behavior and strength of four of the isotropically reinforced specimens, two from each series, are discussed here. The ones from the first series are B27A* ($\alpha^o = -45^o$) and B28 ($\alpha^o = -22.5^o$). The ones from the second series are B34 ($\alpha^o = -45^o$) and B35 ($\alpha^o = 0^o$).

The selection of these specimens has been made on the basis of the

* B27A was a duplicate test of B27.
orientation of the reinforcement with the principal moment axis so that they can provide a good overall picture of the load-deformation characteristics.

All calculated values were obtained on the basis of the equations derived in sections 5.1a through 5.1c and are shown with broken lines in the diagrams. Test results are shown with solid lines. In addition, moment-curvature diagrams for all specimens tested under combined bending and torsion are presented in Appendix A.

(1) **Moment-Curvature Relationship**

Table 5.1 presents the calculated and measured moment-curvature values for all specimens of these series. It shows the results for all 16 specimens of both series. The results of the isotropically reinforced specimens are discussed in this section and those of the non-isotropically reinforced specimens are discussed in other sections of this chapter.

Figure 5.4 shows the moment-curvature relationship for specimen B27A. It also shows the orientation of the reinforcement, x- and y-axes, the principal moment axes, n and t, and the graphical representation of the yield criterion for this case. The measured and calculated values are in good agreement, even though the calculated values at cracking were smaller than those measured. A modulus of rupture of \( f'_r = 7 \sqrt{f'_c} \) was assumed for all cracking calculations. The broken lines at a curvature of about \( 470 \times 10^{-5} \), in.\(^{-1}\), indicate reset of the loading equipment to carry the test results all the way to crushing of the concrete. This ultimate stage was accomplished in only a few of the tests because the large deformations involved made it difficult to
continue loading without risking damage to the loading equipment. However, in all cases that the tests were discontinued the extent and the shape of the load-deflection curve were adequate to obtain a reliable measurement of the ultimate resisting moment.

Figure 5.5 shows the moment-curvature relationship for specimen B28. The layers of reinforcement were inclined at 43.5° and 46.5° to the major principal moment axis. Therefore, both layers of reinforcement yielded at almost the same time. The calculated and measured values are in good agreement.

Figures 5.6 and 5.7 show the calculated and measured moment-curvature relationships for specimens B34 and B35. Both of these specimens were subjected to a principal-moment ratio $\frac{M_{E_t}}{M_{E_n}} = \frac{M_1}{M_2} = -0.45$. In specimen B34 the layers of reinforcement were oriented at 11° and 79° with respect to the principal moment axis. Consequently, when the layer at 11° from the principal moment axis yielded the slab had almost reached its yield capacity. The layers of reinforcement in specimen B35 were oriented at 34° and 56° with respect to the principal moment axis. Consequently, when the layer of reinforcement at 34° yielded, the capacity of the slab was about 5.20 k-in./in., and at second yielding, or yielding of the layer at 56° from the principal moment axis, the capacity reached was 5.80 k-in./in. This effect is shown better in the moment-steel strain relationship described in the following paragraph.

(2) Moment-Steel Strain Relationship

Calculated and measured values for strains in the reinforcement
of specimens B27A, B28, B34 and B35 are shown in Fig. 5.8, 5.9, 5.10 and 5.11.

In Fig. 5.8, which shows the results of specimen B27A, the layers 1 and 2 of reinforcement are oriented at 66° and 24° with respect to the axis of principal moment (see Fig. 5.4). The change in stiffness of the curves at about 5 k-in./in. indicate first yielding of the reinforcement. This change is reflected by the rapid increase of the strains in the reinforcement.

In specimen B28, the two layers of reinforcement were oriented at 43.5° and 46.5° with respect to the principal moment axis (see Fig. 5.5). Consequently, both of these layers had about the same strains throughout the test and yielded at essentially the same load.

Figures 5.10 and 5.11 show the calculated and measured values for specimens B34 and B35. The compressive strains in layer 1 of specimen B34, which makes an angle of 79° with respect to the principal moment axis (see Fig. 5.6) are due to the applied negative moment which is large enough to overcome the tensile strains produced by the positive moment. However, once the slab yields under the effect of the positive moment, the strains in this layer increase rapidly in tension.

The results of specimen B35 are shown in Fig. 5.11. As it was pointed out in the discussion of the moment-curvature for this specimen, there is a change of the moment-steel strain relationship at the first yielding. This is shown in Fig. 5.11 where at a moment of 5.20 k-in./in. the strains in layer 2 increase at a faster rate. This is due to the change in stiffness produced by the yielding of the reinforcement in layer 1. All calculated and measured values of
moment-steel strain are in good agreement as shown in the plots above described.

(3) **Moment-Concrete Strain Relationship**

The effect of the orientation of the reinforcement on the concrete strains follows directly from its effects on flexibility.

For illustration purposes only Fig. 5.12 shows a moment-concrete strain relationship for specimen B27A. The agreement between the calculated and measured values is good in this diagram as it was in all other moment-concrete strain relationships observed throughout the tests of these series.

5.2. **Nonisotropically Reinforced Slabs (μ ≠ 1.0)**

In the following paragraphs, the load-deformation characteristics of nonisotropically reinforced slabs subjected to combined bending and torsion is discussed. The definition of a nonisotropically reinforced slab as used here has already been stated in Section 4.2.

(a) **Cracking**

On the basis of the statements in Section 4.2a, the cracking moment of nonisotropically reinforced slabs subjected to combined bending and torsion can be calculated on the basis of a modified modular ratio, \( n'_{bt} \), which can be expressed as:

\[
n'_{bt} = n(\cos^4 \alpha + 2\mu \frac{\phi_t}{\phi_n} \sin^2 \alpha \cos^2 \alpha + \mu \sin^4 \alpha) \tag{5.40}
\]

Because of the symmetry of the reinforcement, the depth of the neutral axis in the direction of the principal moment axis is \( c_n = h/2 \)
so that the bending moment capacity at cracking is:

$$M_{cn} = n' \text{bt} A_s f_r \frac{d/h^2}{h/2} (d - d') + 0.166 f_r h^2$$  \hspace{1cm} (5.41)$$

and the curvature at cracking becomes

$$\Phi_{cn} = \frac{2f_r}{E_c h}$$  \hspace{1cm} (5.42)$$

(b) **Yielding**

The carrying capacity at yield of nonisotropically reinforced slabs subjected to combined bending and torsion can be calculated on the basis of the principle of least resistance as explained in Chapter 2. The formulation of the yield criterion will provide both the yield line orientation and the yield moment capacity.

In the following, an evaluation of the flexibility characteristics of nonisotropically reinforced slabs subjected to combined bending and torsion is presented. The derivations are valid at any stage of loading between cracking and first yielding.

Figure 5.13 shows a nonisotropically reinforced slab element subjected to combined bending and torsion which produces moments $M_{E_n} = M_n$ and $M_{E_t} = M_t$ as shown in the figure. Furthermore, it is assumed that a linear relationship between moment and curvature exists at the cracked sections as shown in Fig. 5.14.

Figure 5.15 shows the assumed strain and stress distributions in sections normal to the cracks.

The component forces of the top reinforcement in the n- and t-directions are:
\[ N_n^o = A_n E_n \left[ \varepsilon_n^0 (\cos^4 \alpha + \mu \sin^4 \alpha) + \varepsilon_t^0 \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \gamma_{nt}^0 \sin \alpha \cos \alpha (\cos^2 \alpha - \mu \sin^2 \alpha) \right] \] (5.43)

\[ N_t^o = A_t E_t \left[ \varepsilon_n^0 \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \varepsilon_t^0 (\sin^2 \alpha + \mu \cos^2 \alpha) + \gamma_{nt}^0 \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \right] \] (5.44)

and the shear force is:

\[ N_{nt}^o = A_n E_n \sin \alpha \cos \alpha \left[ \varepsilon_n^0 (\cos^2 \alpha - \mu \sin^2 \alpha) + \varepsilon_t^0 (\sin^2 \alpha - \mu \cos^2 \alpha) + \gamma_{nt}^0 \sin \alpha \cos \alpha (1 - \mu) \right] \] (5.45)

Similarly, for the bottom reinforcement:

\[ N_n = A_n E_n \left[ \varepsilon_n (\cos^4 \alpha + \mu \sin^4 \alpha) + \varepsilon_t \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \gamma_{nt} \sin \alpha \cos \alpha (\cos^2 \alpha - \mu \sin^2 \alpha) \right] \] (5.46)

\[ N_t = A_t E_t \left[ \varepsilon_n \sin^2 \alpha \cos^2 \alpha (1 + \mu) + \varepsilon_t (\sin^2 \alpha + \mu \cos^2 \alpha) + \gamma_{nt} \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \right] \] (5.47)

\[ N_{nt} = A_n E_n \sin \alpha \cos \alpha \left[ \varepsilon_n (\cos^2 \alpha - \mu \sin^2 \alpha) + \varepsilon_t (\sin^2 \alpha - \mu \cos^2 \alpha) + \gamma_{nt} \sin \alpha \cos \alpha (1 - \mu) \right] \] (5.48)

From equilibrium of forces in the n- and t-directions, and considering that the tensile strength of the concrete has been exceeded in both directions:
\[ N^0_n + N_n = F_n = 0.5 \varepsilon_n^0 E_c \frac{c_n^2}{d-c_n} \] (5.49)

and
\[ N^0_t + N_t = F_t = 0.5 \varepsilon_t E_c \frac{c_t^2}{d-c_t} \] (5.50)

From equilibrium of moments in the \( n \) and \( t \)-directions:
\[ ME_n = M_n = (N^0_n - N_n)(0.5h - d') + (N^0_n + N_n)(0.5h - c_n/3) \] (5.51)
and
\[ -ME_t = -M_t = (N^0_t - N_t)(0.5h - d') + (N^0_t + N_t)(0.5h - c_t/3) \] (5.52)

From the assumed linear distribution of strains the following relationships can be found:
\[ \varepsilon_n = -\varepsilon_n^0 \frac{c_n - d'}{d - c_n} \] (5.53)

and
\[ \varepsilon_t = -\varepsilon_t \frac{c_t - d'}{d - c_t} \] (5.54)

where tensile strains are positive and compressive strains are negative.

From Eq. 5.43, 5.46, 5.49 and the strain relationships, Eq. 5.53 and 5.54 the depth of the neutral axis in the \( n \)-direction is:
\[ c_n = d((-pD_1 + \sqrt{pD_1^2 + 2pD_1}) \] (5.55)

where:
\[ D_1 = (1 - \frac{c_n - d'}{d - c_n})(\cos^2 \alpha + \mu \sin^2 \alpha) + \frac{\varepsilon_t}{\varepsilon_n} (1 - \frac{c_t - d'}{d - c_t}) \sin^2 \alpha \cos^2 \alpha (1 + \mu) \]

\[ + \frac{2\gamma nt}{\varepsilon_n} \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \sin^2 \alpha) \]  

(5.56)

In a similar fashion the depth of the neutral axis in the t-direction can be calculated from Eq. 5.44, 5.47, 5.50 and the strain relations, Eq. 5.53 and 5.54:

\[ c_t = d(-pnD_2 + \sqrt{pnD_2^2 + 2pnD_2}) \]  

(5.57)

where

\[ D_2 = \frac{\varepsilon_n}{\varepsilon_t} (1 - \frac{c_n - d'}{d - c_n}) \sin^2 \alpha \cos^2 \alpha (1 + \mu) + (1 - \frac{c_t - d'}{d - c_t}) (\sin^4 \alpha + \mu \cos^4 \alpha) \]

\[ + \frac{2\gamma nt}{\varepsilon_t} \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \]  

(5.58)

Let:

\[ A_1 = \cos^4 \alpha + \mu \sin^4 \alpha \]  

(5.59)

\[ A_2 = \sin^2 \alpha \cos^2 \alpha (1 + \mu) \]  

(5.60)

\[ A_3 = \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \sin^2 \alpha) \]  

(5.61)

\[ A_4 = \sin^4 \alpha + \mu \cos^4 \alpha \]  

(5.62)

\[ A_5 = \sin \alpha \cos \alpha (\sin^2 \alpha - \mu \cos^2 \alpha) \]  

(5.63)
From equilibrium of moments in the n-direction as expressed by Eq. 5.51, and making use of Eq. 5.43, 5.46, 5.59, 5.60 and 5.61 the resulting expression is:

\[
M_n = A_s E_s \left[ \varepsilon_n^0 \left( 1 + \frac{c_n - d'}{d - c_n} \right) A_1 - \varepsilon_t \left( 1 + \frac{c_t - d'}{d - c_t} \right) A_2 \right] (0.5h - d') \\
+ A_s E_s \left[ \varepsilon_n^0 \left( 1 - \frac{c_n - d'}{d - c_n} \right) A_3 + \varepsilon_t \left( 1 - \frac{c_t - d'}{d - c_t} \right) A_4 + 2\gamma_{nt} A_3 \right] (0.5h - c_n/3) 
\]

(5.64)

The curvature in the n-direction is

\[
\Phi_n = \frac{\varepsilon_n^0}{d - c_n} 
\]

(5.65)

From Eq. 5.64 and 5.65 the flexibility of the slab in the n-direction is:

\[
\frac{\Phi A_s E_s h^2}{M_n} = \frac{d - c_n}{h} \left( 0.5 - \frac{d'}{h} \right) \left[ \varepsilon_n^0 \left( 1 + \frac{c_n - d'}{d - c_n} \right) A_1 - \varepsilon_t \left( 1 + \frac{c_t - d'}{d - c_t} \right) A_2 \right] \\
+ \frac{d - c_n}{h} \left( 0.5 - 0.33 c_n/h \right) \left[ \varepsilon_n^0 \left( 1 - \frac{c_n - d'}{d - c_n} \right) A_3 + \varepsilon_t \left( 1 - \frac{c_t - d'}{d - c_t} \right) A_4 + 2\gamma_{nt} A_3 \right] 
\]

(5.66)

In a similar fashion, the moment in the t-direction from Eq. 5.44, 5.47, 5.52, 5.60, 5.62 and 5.63, becomes:

\[
-M_t = A_s E_s \left[ -\varepsilon_n^0 \left( 1 + \frac{c_n - d'}{d - c_n} \right) A_2 + \varepsilon_t \left( 1 + \frac{c_t - d'}{d - c_t} \right) A_4 \right] (0.5h - d') 
\]
The curvature in the t-direction is:

\[ \Phi_t = -\frac{\epsilon_t}{d-c_t} \]  
(5.68)

From Eq. 5.67 and 5.68 the flexibility of the slab in the t-direction is:

\[ \frac{\Phi_t A_s E_s h^2}{M_t} = \frac{1}{\left( h - \frac{d-c_t}{h}\right) \left( 0.5 - \frac{d_t}{h}\right)} \left[ \frac{\epsilon_o}{\epsilon_t} \left( 1 + \frac{c_n - d'}{d-c_n}\right) A_2 + \left( 1 - \frac{c_t - d'}{d-c_t}\right) A_4 + 2\gamma_{nt} A_5 \right] \]

\[ + \frac{\left( h - \frac{d-c_t}{h}\right) \left( 0.5 - 0.33c_t/h\right)}{h} \left[ \frac{\epsilon_o}{\epsilon_t} \left( 1 + \frac{c_n - d'}{d-c_n}\right) A_2 + \left( 1 - \frac{c_t - d'}{d-c_t}\right) A_4 + 2\gamma_{nt} A_5 \right] \]

(5.69)

An estimate of the ratios \( \epsilon_o/\epsilon_t, \gamma_{nt}/\epsilon_o \) and \( \gamma_{nt}/\epsilon_t \) can be obtained in a similar fashion as for the case of uniaxial bending discussed in Section 4.2.

For nonisotropically reinforced slabs subjected to combined bending and torsion, these relationships are:

\[ \frac{\epsilon_o}{\epsilon_t} = \frac{\omega \cos 2\gamma + \sin 2\gamma}{\omega \sin^2 \gamma - \cos^2 \gamma} \frac{4}{\cos \alpha + \mu \sin \alpha} \]

(5.70)

\[ \frac{\gamma_{nt}}{\epsilon_o} = 0.3(\cos 4\alpha + \mu \sin \alpha) \frac{\left( \omega - 1 \right) \sin \gamma \cos \gamma}{\omega \cos^2 \gamma + \sin^2 \gamma} \]

(5.71)
and \[
\frac{\gamma_{nt}}{e_t} = 0.3 \left( \sin \alpha + \mu \cos \alpha \right) \frac{(w-1)\sin \gamma \cos \gamma}{\omega \sin^2 \gamma - \cos^2 \gamma}
\]
(5.72)

Furthermore, for the amounts of reinforcement in slabs used in practice, the ratios \((c_n-\delta')/(\delta-c_n)\) and \((c_t-\delta')/(\delta-c_t)\) can be taken to be between 0.10 and 0.15.

The variation in flexibility calculated from Eq. 5.66 as a function of \(\alpha\), the angle between the main layer of reinforcement and the major principal moment axis, has been plotted in Fig. 5.16 for moment ratios \(M_t/M_n = -0.14\) and \(M_t/M_n = -0.45\). These moment ratios are applied on slabs with \(p = 0.01\) and \(p = 0.005\), and \(\mu = 1\) and \(\mu = 0.5\).

From Fig. 4.17 and 5.16, it can be observed that the application of the negative moment, \(M_t\), increases the flexibility of a slab element subjected to an orthogonal positive moment. This effect of the moments \(M_n\) and \(M_t\) can be visualized by having an element being "stretched" in the longitudinal direction, the \(n\)-direction, and "squeezed" in the transverse direction, the \(t\)-direction. This "squeezing" in the transverse direction increases the flexibility of the element in the \(n\)-direction. On the other hand, if the element is "stretched" in both directions, the flexibility in the \(n\)-direction will decrease because of the physical restraint produced by the stretching in the \(t\)-direction.

Figure 5.17 shows the variation of the flexibility as a function of \(\alpha\), for moment ratios \(M_t/M_n = -0.14\) and \(M_t/M_n = -0.45\). These moment combinations have been applied on slab elements with \(p = 0.01\), \(\mu = 1\); \(p = 0.01\), \(\mu = 0.25\), and \(p = 0.0025\), \(\mu = 1\).

Figure 5.18 shows the variations of concrete compressive strains in the \(n\)-direction as a function of \(\alpha\), for a moment ratio
$M_c/M_n = -0.45$ and amounts of reinforcement, $p = 0.01$, $p = 0.0025$, and ratios of $\mu = 1$ and $\mu = 0.25$.

(c) **Ultimate**

The ultimate moment capacity of nonisotropically reinforced concrete slabs subjected to combined bending and torsion can be calculated on the basis of the principle of least resistance by introducing the components of the ultimate moment capacity of the nonisotropic reinforcement in direction normal to the yield line.

(d) **Comparison of Calculated and Measured Quantities**

The effects of nonisotropy and reinforcement orientation on the carrying capacity of slabs subjected to combined bending and torsion is discussed here in conjunction with the test results.

As explained in Section 5.1d, there were two series of specimens subjected to combined bending and torsion. The results of four nonisotropically reinforced specimens, two from each series, are discussed here. The two from the first series are $B32(\alpha_x^o = 67.5^0)$ and $B33(\alpha_x^o = 0^0)$ and the two from the second series are $B38(\alpha_x^o = 45^0)$ and $B39(\alpha_x^o = 90^0)$.

All calculated values were obtained on the basis of the expressions developed in this chapter and the formulation of the yield criterion in Chapter 2. They are shown with broken lines in the diagrams. Test results are shown with solid lines.

(1) **Moment-Curvature Relationship**

Figure 5.19 shows the moment-curvature relationship for specimen $B32$ which had the main layer of reinforcement oriented at $46.5^0$ to
the major principal moment axis. The principal unit moment is plotted along the vertical axis, and the curvature in the principal-moment direction is plotted along the horizontal axis. It is to be noted that in nonisotropically reinforced slabs, the normal to the yield line, the major principal moment axis, and the principal curvature do not necessarily coincide.

Figure 5.19 also shows the orientation of the reinforcement in the slab and the graphical representation of the yield criterion.

Figure 5.20 presents a sequence of 12 photographs showing the reinforcement and the development of cracking in specimen B32. Figure 5.20a shows the reinforcement in casting position which is upside-down with respect to the test position, Fig. 5.20b shows the slab in testing position, Fig. 5.20c shows the initial cracks which form at an angle of about 20° with the transverse axis of the slab. The major principal moment acts in a direction normal to these initial cracks.

Figure 5.20d shows the crack pattern at the yield load (Load 11). Note that the orientation of the cracks has changed from that observed in load 6. The cracks tended to parallel the transverse direction at the yield load. Figure 5.20e shows a close-up of the crack orientation at the yield load, and Fig. 5.20f and g show the crack patterns at deformations well beyond yielding. Figure 5.20h shows a side view of the deflections at load 22, the load at which the test was discontinued. The wooden wedges that are seen underneath the loading channels were used to reset the equipment and continue the loading without bending the hangers. They were placed at load 16 and the test was discontinued at load 22.
Figure 5.20j shows a close-up of the final yield line orientation. The measured orientation of the yield line with respect to the transverse direction of the slab was -4° as shown in Table 5.1. The calculated value was -6°. The yield lines were on top of the specimen indicating that the slab yielded under the effect of a positive moment as defined in this report. Figure 5.20k shows the initial crack direction in solid crack lines and the axes of the reinforcement. Note that the final orientation of the yield lines tend to avoid the heavier reinforcement which was placed at 67.5° from the span direction. Figures 5.20m and 5.20n show the top and bottom sides of specimen B32 after being removed from the test rig.

The moment-curvature relationship for specimen B33 which had the main layer of reinforcement oriented at 21° from the major principal moment axis is shown in Fig. 5.21. This figure also shows the graphical representation of the yield criterion, and the orientation of the reinforcement.

Figure 5.22 shows the moment-curvature relationship, the orientation of the reinforcement, and the graphical representation of the yield criterion for specimen B38. It can be observed in the graphical representation of the yield criterion that the slab yielded in the region of negative moment which means yield lines in the bottom side of the test specimen.

Figure 5.23 includes a series of photographs of specimen B38 showing the reinforcement and the development of cracks. Figure 5.23a shows the reinforcement orientation in an upside-down position with respect to its final position in the test rig. Figure 5.23b shows a
detail of the transverse reinforcement at the edges of the specimen. Figure 5.23c shows the slab in the testing position before application of the load. Figure 5.23d shows the slab at the cracking load as determined from the load-deflection relationship plotted during the test. The cracks are not visible in the photograph. Figure 5.23e shows the top surface of the slab at the yield load and 5.23f shows the bottom surface at the same load. The slab yielded in the negative-moment side as indicated by the yield criterion. Figure 5.23g shows an end view of the slab where the curvature at load 18, when the test was discontinued, can be observed.

Figure 5.24 shows the test results and calculated values of the moment-curvature relationship for specimen B39. It also shows the orientation of the reinforcement and the graphical representation of the yield criterion.

(2) **Moment-Steel Strain Relationship**

The moment-steel strain relationships for specimens B32, B33, B38 and B39 are shown in Fig. 5.25, 5.26, 5.27 and 5.28. The good agreement between the calculated and measured moment-curvature relationship for these specimens as discussed in the preceding paragraphs is reflected in the results of measured and calculated steel strains.

(3) **Moment-Concrete Strain Relationship**

The measured relationships between compressive concrete strain and the major principal moment are shown in Fig. 5.29 and 5.30 for specimens B32 and B39. The calculated curves, based on connecting the coordinates computed for first cracking and first yielding by straight lines, agree with the measured values.
6. BEHAVIOR AND STRENGTH OF SLAB ELEMENTS SUBJECTED TO TORSION

6.1. Isotropically Reinforced Slabs ($\mu = 1.0$)

In reinforced concrete slab elements subjected to torsion the applied principal moments have the same magnitude but different signs.

The load-deformation characteristics at different stages of loading can be stated on the basis of the general equations for combined bending and torsion derived in Chapter 5, for the particular case of:

$$\mathbf{M}_n = -\mathbf{M}_t$$  \hspace{1cm} (6.1)

Because of the symmetry of loading and for equal amounts of reinforcement oriented in the same direction top and bottom ($\alpha = \alpha^0$), the following relationships can be obtained:

$$\epsilon_n = \epsilon_t^0 \text{ and } \epsilon_t = \epsilon_n^0$$ \hspace{1cm} (6.2)

Similarly

$$\epsilon_{sx} = \epsilon_{sy}^0 \text{ and } \epsilon_{sy} = \epsilon_{sx}^0$$ \hspace{1cm} (6.3)

Similar relationships can be derived for the forces in the concrete and reinforcement which become

$$F_n = F_t^0 \text{ and } F_t = F_n^0$$ \hspace{1cm} (6.4)

$$N_n = N_t^0 \text{ and } N_t = N_n^0$$ \hspace{1cm} (6.5)

The ratio of the curvatures in the $n$- and $t$-directions becomes:
\[
\frac{\Phi_t}{\Phi_n} = -1 \quad (6.6)
\]

(a) Cracking

At cracking the stress in the extreme concrete fiber in tension is assumed to reach the value of the nominal modulus of rupture, \( f_r' \).

From Eq. 5.19 and 5.20, the cracking moment in either the \( n \)- or \( t \)-directions becomes:

\[
M_{cn} = M_{ct} = n_t A_s f_r \frac{d-h/2}{h/2} (d-d') + 0.166 f_r h^2 \quad (6.7)
\]

where

\[
n_t = n (\cos^4 \alpha - 2 \sin^2 \alpha \cos \alpha + \sin^4 \alpha) \quad (6.8)
\]

The curvature at cracking in either the \( n \)- or \( t \)-directions is:

\[
\Phi_{cn} = \Phi_{ct} = \frac{2 f_r}{E_c h} \quad (6.9)
\]

(b) Yielding

From Eq. 5.35 the depth of the neutral axis at yielding is:

\[
c_t = c_n = \sqrt{(2nA_s)^2 + 2nA_s h} - 2nA_s \quad (6.10)
\]

The moment capacity at yielding can be computed by evaluating the forces in the top and bottom reinforcement and in the concrete at the time that the tension reinforcement reaches the yield strain (Fig. 5.2c).
\[ M_n = N_n^C d + F_n c_n/3 + N_n d' \quad (6.11) \]

similarly

\[ M_t = N_t^C d + F_t c_t/3 + N_t^0 d' \quad (6.12) \]

The flexibility of the slab element at any stage between cracking and first yielding is:

\[
\frac{\Phi_n}{M_n} = \frac{\Phi_t}{M_t} = \frac{1}{A_s E_s} \left[ (d \cos^2 \alpha + d' \sin^2 \alpha)^2 + (d \sin^2 \alpha + d' \cos^2 \alpha)^2 - 4 c_n h/3 + 2 c_n^2/3 \right] \quad (6.13)
\]

The results of Eq. 6.13 have been plotted in Fig. 6.1 for amounts of reinforcement \( A_s/h = 0.01 \) and \( A_s/h = 0.005 \). As in the cases shown before for uniaxial bending and combined bending and torsion, the flexibility of an isotropically reinforced slab element subjected to torsion is a maximum when the angle between the reinforcement and the principal moment axis is \( 45^\circ \). From Fig. 4.3 and 6.1 it can be concluded that for \( \alpha = 0^\circ \) or \( \alpha = 90^\circ \), the flexibility of a comparable slab element is the same regardless of the applied moment, and also the fact that for an increasing value of the orientation of the reinforcement, \( \alpha \), between \( 0^\circ \) and \( 90^\circ \) the flexibility of a slab element subjected to torsion is larger than that of a slab element subjected to uniaxial bending.

(c) **Ultimate**

The ultimate moment capacity can be calculated using the expressions:

\[ M_{un} = M_{ut} = N_n^0 d + N_n d' - 0.4 F_{nu' u} \quad (6.14) \]
The curvature at ultimate can be estimated from

$$\phi_{u_n} = \phi_{u_t} = \frac{0.004}{k_u d}$$

(6.15)

The notation used above has been explained in Section 5.1c.

(d) **Comparison of Calculated and Measured Quantities**

This section presents a quantitative evaluation of the effect of the orientation of the reinforcement with respect to the principal moment axis on the load-deformation characteristics of isotropically reinforced concrete slabs subjected to pure torsion.

The tests results of three typical specimens (B15, $\alpha_x^o = -45^o$), B16, ($\alpha_x^o = 90^o$) and B17 ($\alpha_x^o = -22.5^o$), will be described in terms of relationships between moment-curvature, moment-steel strains, and moment-concrete strains. It is to be noted that the angles, $\alpha_x^o$, have been measured from the longitudinal direction of the specimen and are positive in the counterclockwise direction. On this basis, the orientation of the reinforcement with respect to the principal moment axis is $90^o$ in specimen B15, $45^o$ in B16, and $-67.5^o$ in B17.

All calculated values are based on the equations derived in Chapter 5 as applied to the particular case of pure torsion and are shown with broken lines in the diagrams. Test results are shown with solid lines. In addition, moment-curvature diagrams for all specimens subjected to torsional moment are presented in Appendix A.

(1) **Moment-Curvature Relationship**

Table 6.1 presents the calculated and measured moment-curvature values for all specimens subjected to pure torsion. There
were a total of nine specimens tested in this series. Seven of them were isotropically reinforced in both the top and bottom sides, B14 through B20, and the remaining two were nonisotropically reinforced with a ratio $\mu = 0.25$ and equal amounts of nonisotropy in top and bottom. The heavier and weaker layers of reinforcement were similarly oriented in the top and bottom.

This section describes the results of the isotropically reinforced specimens. The results of the tests on the nonisotropically reinforced specimens are discussed in other sections of this chapter.

The moment-curvature relationship for specimen B15, which had the reinforcement oriented in the direction of the principal moment axes, is shown in Fig. 6.2. The graphical representation of the yield criterion and the orientation of the reinforcement are also shown in the figure. In terms of moment curvature, the behavior of this specimen is similar to that of a beam with reinforcement in the direction of the applied uniaxial moment.

Figure 6.3 shows the calculated and measured values of the moment-curvature relationship for specimen B16. The graphical representation of the yield criterion and the orientation of the reinforcement are also shown in the figure. The shape of the moment-curvature diagram indicates that this specimen was nearly "overreinforced." The ductility of the slab was limited to a smaller value than that obtained in other specimens subjected to torsion.

Figure 6.4 shows the calculated and measured values of moment-curvature for specimen B17. The graphical representation of the yield criterion and the orientation of the reinforcement are also
shown in Fig. 6.4. Because of the orientation of the reinforcement with respect to the principal moment axes, $\alpha = -67.5^\circ$, there were two stages of yielding. The difference between the calculated and measured values between the first and second stages of yielding originates from the shifts in principal curvature which increase the efficiency of the less effective layer. This phenomenon has been discussed in paragraph 2 of Section 4.1d. This same trend can be seen in Fig. 4.6 which shows the results of an isotropically reinforced slab subjected to uniaxial bending and with reinforcement orientations similar to those in specimen B17.

(2) Moment-Steel Strain Relationship

Calculated and measured values for strains in the reinforcement plotted against unit principal moments for specimens B15, B16 and B17 are shown in Fig. 6.5, 6.6 and 6.7.

As already stated, the behavior of specimen B15 is similar to that of a beam under uniaxial moment. This fact is reflected in the steel strain measurements shown in Fig. 6.5.

Figure 6.6 shows the calculated and measured values of strains in the reinforcement for specimen B16. The orientation of layers 2 and 3 with respect to the principal moment axes is $45^\circ$. Consequently, the strains in both layers are the same.

Figure 6.7 shows the results for specimen B17. The marked differences of the calculated and measured strain values for layer 3 are due to the shifts in principal curvature as explained in paragraph 2 of Section 4.1d.
(3) Moment-Concrete Strains

Calculated and measured values of concrete strains plotted against unit principal moments for specimens B15, B16 and B17 are shown in Fig. 6.8, 6.9 and 6.10.

Figure 6.8 shows the principal compressive strains on the bottom surface of specimen B15, and Fig. 6.9 shows the principal compressive strains on the top surface of specimen B16. The agreement between the calculated and measured values is good.

Figure 6.10 shows the principal tensile strains in the concrete on the bottom surface of specimen B17.

6.2. Nonisotropically Reinforced Slabs ($\mu \neq 1.0$)

The load-deformation characteristics of nonisotropically reinforced slabs subjected to pure torsion can be evaluated on the basis of the general equations for combined bending and torsion derived in Chapter 5 as applied to the particular case of principal moments of the same magnitude but different signs.

(a) Cracking

The cracking moment of nonisotropically reinforced slabs subjected to pure torsion can be calculated on the basis of a modified modular ratio, $n'_t$, which can be expressed as:

$$ n'_t = n(\cos \alpha - 2\mu \sin^2 \alpha \cos \alpha + \mu \sin \alpha) $$

The expression for the moment capacity at cracking is:

$$ M_{on} = n'_t A_s f_y \frac{d - h/2}{h/2} (d - d') + 0.166 f_y h^2 $$
and the curvature at cracking:

\[ \Phi_{cn} = \frac{2f}{r} \]

(6.18)

(b) Yielding

The carrying capacity at yield of nonisotropically reinforced slabs subjected to pure torsion can be calculated using the principle of least resistance as explained in Chapter 2. Its flexibility characteristics can be evaluated on the basis of the expressions derived in Chapter 5 for the particular case of \( M_h = -M_t \).

The variation of flexibility calculated from Eq. 5.66, for \( M_h = -M_t \), as a function of \( \alpha \) (the angle between the main layer of reinforcement and the axis on which the positive principal moment, \( M_h \), acts) has been plotted in Fig. 6.11 and 6.12.

The moment ratio, \( M_t/M_h = -1 \), has been applied to slabs with \( p = 0.01 \) and \( p = 0.005 \), and \( \mu = 1 \) and \( \mu = 0.5 \), and has been plotted in Fig. 6.11. The same moment ratio for \( p = 0.01 \) and \( p = 0.0025 \), and \( \mu = 1 \) and \( \mu = 0.25 \) has been plotted in Fig. 6.12.

Figure 6.13 shows the variation of concrete compressive strains in the n-direction as a function of \( \alpha \) for amounts of reinforcement \( p = 0.01 \) and \( p = 0.0025 \), and \( \mu = 1 \) and \( \mu = 0.25 \).

(c) Ultimate

The ultimate moment capacity of nonisotropically reinforced concrete slabs subjected to torsion can be calculated on the basis of the principle of least resistance as explained in Section 5.2c.
Comparison of Calculated and Measured Quantities

Two nonisotropically reinforced specimens, B21 and B22, were subjected to torsion. Both of these specimens had $p = 0.01$ and $\mu = 0.25$ with the larger amount of reinforcement being placed in the transverse direction. The only difference was that the effect of the dead load moment in specimen B22 was taken directly by adding two extra reinforcing bars in the bottom of the specimen and in its longitudinal direction. Consequently, in specimen B22 the applied moment was considered without including the dead load moment effect. The results of both of these tests were satisfactory, and one of them, B22, is discussed here in terms of its relationships between moment-curvature, moment-steel strain, and moment-concrete strain.

1. **Moment-Curvature Relationship**

Figure 6.14 shows the calculated and measured moment-curvature relationships for specimen B22. The graphical representation of the yield criterion and the orientation of the reinforcement are also shown in this figure. All calculated values are based on the expressions derived in Chapters 2 and 5, as applied to the case of pure torsion, $M_n = -M_t$. The agreement between calculated and measured values is good.

2. **Moment-Steel Strain Relationship**

Figure 6.15 shows the calculated and measured steel strains in the transverse reinforcement. Up to the cracking load, because of the orientation of the reinforcing bars with respect to the principal moment axis, the calculated steel strains are zero. In plotting the
calculated curve, the variation between cracking and first yielding has been considered to be linear.

(3) Moment-Concrete Strain Relationship

Figure 6.16 shows the compressive concrete strains in the direction of the applied positive principal moment. The calculated values are based on the results of the flexibility and depth to the neutral axis, as stated in other sections of this chapter.
7. RESPONSE OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLAB ELEMENTS SUBJECTED TO VARIOUS LOADING CONDITIONS

7.1. Introduction

This chapter presents an over-all view of the influence of the amount, distribution, and orientation of the reinforcement on the response of isotropically and nonisotropically reinforced slab elements subjected to various combinations of the two principal moments. The load carrying capacity at yield for different amounts and orientations of the reinforcement is shown in graphical representations for various loading conditions. The influence of the reinforcement orientation and amount (in the two orthogonal directions) on the curvature in the direction of the major principal moment is also discussed. Theoretical and experimental trends are compared.

7.2. Moment Capacity at Yield

The carrying capacity at yield of isotropically and nonisotropically reinforced concrete slabs can be calculated using the principle of least resistance stated in Chapter 2. It has also been shown that a graphical representation in polar coordinates of the yield criterion provides both the bending moment at the yield line and the orientation of the yield line.

In this chapter, the envelopes of carrying capacity at yield in terms of the applied major principal moment, \( M_2 \), as expressed by Eq. 2.25 are shown graphically. These plots are presented in polar coordinates where the polar radii represents the carrying capacity in
terms of the applied major principal moment \( M_2 \), and the polar angle, \( \alpha \), is the angle between the main layer of reinforcement (the x-axis) and the direction on which the moment \( M_2 \) acts (the v-axis).

The difference between these plots and those presented in Chapter 2 (Fig. 2.5 through 2.10), is that the plots in Chapter 2 present solutions of carrying capacity and yield line orientation for one particular orientation of the main layer of reinforcement with the major principal moment axis and a given condition of applied moments. The plots in this chapter provide the solutions of carrying capacity for all orientations of the main layer of reinforcement with the major principal moment axis and a given condition of applied moments. In other words, the plots in this chapter show the loci of the solutions for carrying capacity at yield obtained in Chapter 2.

For convenience, the following identifications will be used:

<table>
<thead>
<tr>
<th>Mark</th>
<th>Amount of Reinforcement in x-direction, ( p )</th>
<th>Reinforcement Ratio, ( \mu = \frac{p_y}{p_x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>( p )</td>
<td>1.0</td>
</tr>
<tr>
<td>(2)</td>
<td>( p )</td>
<td>0.5</td>
</tr>
<tr>
<td>(3)</td>
<td>( 0.5p )</td>
<td>1.0</td>
</tr>
<tr>
<td>(4)</td>
<td>( p )</td>
<td>0.25</td>
</tr>
<tr>
<td>(5)</td>
<td>( 0.25p )</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In the following discussion it is assumed that the amount of reinforcement does not exceed that required to have a flexural failure by simultaneous yielding of the steel and crushing of the concrete.
(1) Isostatic Moment \( (M_1/M_2 = 1.0) \)

Figure 7.1 shows envelopes of maximum carrying capacity for slabs subjected to isostatic moment. The solution is trivial due to the multi-axial symmetry of the applied moment. It can be concluded from Fig. 7.1 that the maximum carrying capacity for nonisotropically reinforced slabs subjected to isostatic moment is governed by the smaller amount of nonisotropic reinforcement. The test results shown in Table 3.1 agree with these maximum carrying capacity envelopes.

(2) Biaxial Bending \((0 < M_1/M_2 < 1.0)\)

The solutions for biaxial bending shown in Fig. 7.2 have been derived for the particular case of \( M_1/M_2 = 0.5 \).

Curves 1, 3 and 5 require no further comment since they show the results for isotropically reinforced slabs. Curve 2, which shows the carrying capacity for \( p_x = p \) and \( \mu = 0.5 \), indicates that for \( \alpha = 0^\circ \) the capacity of the nonisotropically reinforced slab reaches that of the isotropically reinforced slab with \( p_x = p \). As the angle \( \alpha \) is increased, the carrying capacity decreases rapidly. On the other hand, curve 4 plotted for \( p_x = p \) and \( \mu = 0.25 \) shows that when the main layer of reinforcement is in the direction of the major principal moment, \( \alpha = 0^\circ \), the capacity reached is smaller than that which can be provided by the main reinforcement. The minor principal moment, \( M_1 \), and the smaller amount of reinforcement, \( 0.25p \), control the carrying capacity.

At \( \alpha = 90^\circ \), the major principal moment, \( M_2 \), and the smaller amount of reinforcement, \( 0.25p \), control the carrying capacity.
(3) Uniaxial Bending \((M_1/M_2 = 0)\)

Figure 7.3 shows the envelopes of maximum carrying capacity for slabs subjected to uniaxial bending. Since the minor applied principal moment is \(M_1 = 0\), then for \(\alpha = 0^\circ\), both of curves 2 and 4 develop the capacity provided by the main reinforcement. Test results which are also shown in the figure are in good agreement with the calculated values.

(4) Combined Bending and Torsion \((-1.0 < M_1/M_2 < 0)\)

Figures 7.4 and 7.5 show envelopes of maximum carrying capacity for slabs subjected to combinations of bending and torsion moments. Two of these combinations give principal moments \(M_1/M_2 = -0.14\) and \(M_1/M_2 = -0.45\). Figure 7.4 is similar to Fig. 7.3 and Fig. 7.5 is similar to Fig. 7.2. The only difference is that in Fig. 7.4 and 7.5 the minor principal moment has a negative value. The cusps in curve 4 indicate the change from positive to negative yield lines.

(5) Pure Torsion \((M_1/M_2 = -1.0)\)

Figure 7.6 shows envelopes of maximum carrying capacity for slabs subjected to pure torsion. Because of the equal magnitude but different signs of the principal moments, the carrying capacity of non-isotropically reinforced slabs will in general be governed by the weaker layers of reinforcement. It can be concluded that the most efficient way to use nonisotropic reinforcement in torsion, as far as strength is concerned, is placing the reinforcement at \(45^\circ\) from the principal moment axes. However, it is to be remarked that the most effective way
to resist torsion is by using isotropic reinforcement. Test results are also shown in the figure.

Figure 7.7 shows an overall view of the effectiveness of isotropic and nonisotropic reinforcement, $p_x = p$, $\mu = 1$ and $\mu = 0.25$, in resisting various loading conditions. It can be seen that for $M_1/M_2 = 1$ the heavier reinforcement is practically useless since it does not contribute to increase the carrying capacity. Therefore, for this moment condition a slab with $p_x = 0.25p$ and $\mu = 1$ is as good as a slab with $p_x = p$ and $\mu = 0.25$. For other loading conditions the contribution of the nonisotropic reinforcement to the carrying capacity varies as shown in the figure.

7.3. **Flexibility Characteristics**

The flexibility characteristics of slabs subjected to various loadings are also a function of the loading conditions and the amount, orientation and nonisotropy of the reinforcement. However, up to the cracking load the slab can be considered to consist of a homogeneous and isotropic material, and its flexibility can be calculated on the basis of elastic plate theory.

Between cracking and first yielding, the flexibility characteristics depend primarily on the reinforcement.

Figure 7.8 shows the flexibility of isotropically reinforced slab elements subjected to various loading conditions. This figure shows that the flexibility of slab elements subjected to isostatic moment, $M_1/M_2 = 1$, is a constant for any orientation $\alpha$ of the reinforcement. This reflects the multi-axial symmetry of the applied moment.
It is also shown in Fig. 7.8 that the flexibility for various other loading conditions is a maximum when the orientation $\alpha$ of the isotropic reinforcement with respect to the major principal applied moment axis is $45^\circ$. It also shows that the flexibility is larger for a slab subjected to pure torsion, $M_1/M_2 = -1$, which reflects the "squeezing" effect produced by the minor principal moment. This effect has been discussed in Section 5.2b.

Figure 7.9 shows the flexibility characteristics of isotropically and nonisotropically reinforced slabs subjected to uniaxial bending, $M_1/M_2 = 0$. The test results which are included in the figure are in good agreement with the calculated curves which are based on the expressions derived in Chapter 4.

Figures 7.10 and 7.11 show the flexibility characteristics for isotropically and nonisotropically reinforced slabs subjected to combined bending and torsion. Figure 7.10 shows the results for slabs with, $M_1/M_2 = -0.14$, and Fig. 7.11 the results for $M_1/M_2 = -0.45$. The calculated curves are based on the expressions derived in Chapter 5.

Figure 7.12 shows the flexibility characteristics of isotropically and nonisotropically reinforced slabs subjected to torsion. The calculated curves are based on the expressions derived in Chapter 6 and are in good agreement with the test results.
8.1. Outline of the Investigation

The primary objective of this report is to summarize the information obtained on the behavior and strength of reinforced concrete slabs subjected to varying combinations of principal moments. The investigation has been in progress since 1965.

A total of 41 tests were carried out using circular and rectangular slab specimens (Fig. A.5 and A.6). The main variables were:

(a) the principal-moment ratio, \( M_1/M_2 \), which covered the range \(-1.0 \leq M_1/M_2 \leq 1.0 \),
(b) the amounts of reinforcement in the two orthogonal directions which ranged from 1.0 percent to 1/4 percent, and
(c) the orientation of the reinforcement. The characteristics of all test specimens have been summarized in Table 1.1.

8.2. Behavior of the Test Specimens

The behavior of the test specimens can be categorized in relation to the applied principal moments, the amounts of reinforcement in the two orthogonal directions, and the orientation of the reinforcement with respect to the applied major principal-moment axis. Five different ratios of applied principal moments were realized in the tests:

- isostatic moment \( (M_1/M_2 = 1.0) \), which was used in conjunction with the circular test specimens,
- uniaxial moment \( (M_1/M_2 = 0) \), combined bending and torsion \( (M_1/M_2 = -0.14 \text{ and } M_1/M_2 = -0.45) \), and pure torsion \( (M_1/M_2 = -1.0) \), which were used in conjunction with the rectangular test specimens.
Two types of reinforcing schemes were used: isotropically reinforced slabs (slabs reinforced to have equal moment capacity in all planar directions) and nonisotropically reinforced slabs which did not have equal moment capacity in all planar directions. When used, the bottom two layers of reinforcement were the same as the top two layers.

At cracking, the behavior of all test specimens was essentially the same. First cracking always occurred in planes normal to the major principal-moment axis. The amounts and orientation of the reinforcement had negligible effect in the behavior of the specimens up to cracking.

At yielding, there was a definite influence of the applied principal moments, the amounts of reinforcement and the orientation of the reinforcement. In general, there were two stages of yielding. First yielding occurred when the reinforcement in the more effective layer (the layer making the smaller angle with the major principal-moment axis) reached the yield strain, and second yielding, when the reinforcement in the less effective layer reached the yield strain. In the isotropically reinforced specimens, there were local changes of crack orientation between first cracking and full yielding which affected neither the carrying capacity nor the final yield line orientation which was always normal to the major principal-moment axis. In non-isotropically reinforced slabs the initial crack orientation changed continuously from cracking to yielding and its final orientation did have a definite effect on the carrying capacity.

Between cracking and first yielding, the flexibility of isotropically and nonisotropically reinforced slabs, with $M_1/M_2 = 1.0$, was a constant which did not depend on the orientation of the reinforcement.
As the ratio $M_1/M_2$ decreased, the flexibility increased and varied with the orientation of the reinforcement. The flexibility of isotropically reinforced slabs was a maximum when the reinforcement was inclined at $45^\circ$ to the major principal-moment axis (Fig. 7.8).

In nonisotropically reinforced slabs, the variation of the flexibility between cracking and first yielding was also influenced by the applied principal moments, the amounts of reinforcement in the two orthogonal directions and the crack orientation. Its maximum value also depended on these variables (Fig. 4.18, 5.17 and 6.12).

8.3. Strength of the Test Specimens

It was observed that, for "underreinforced" slabs, the unit moment capacity at the yield line of isotropically reinforced slabs depended only on the amount of reinforcement. As opposed to the definition of "underreinforced" beams, the limiting amount of reinforcement in an "underreinforced" slab depends not only on the amounts of reinforcement but also on the applied principal moments and the orientation of the reinforcement. The unit moment capacity at the yield line of isotropically reinforced slabs is independent of the applied principal moments and the orientation of the reinforcement (Fig. 7.7). In non-isotropically reinforced slabs, the unit moment capacity at the yield line depends on the applied principal moments, the amounts of reinforcement in the two orthogonal directions, and the orientation of the reinforcement with respect to the major principal-moment axis (Fig. 7.7).

No measurable effect of reorientation of the reinforcing bars at the crack was observed in any of the tests.
There was no measurable enhancement of moment capacity at the yield line as a result of biaxial compression of concrete.

8.4. Analysis of the Test Results

The yield criterion for reinforced concrete slabs has been presented in a single expression (Eq. 2.25) comprising both, the magnitude of the unit moment at the yield line, and the orientation of the yield line. This expression is based on the principle of "least resistance" (Section 2.2).

Two interrelated graphical representations, in polar coordinates, of the yield criterion have been presented. The first one (Fig. 2.5 through 2.10), presents the graphical representation for one particular applied principal-moment condition, $M_1/M_2$, one orientation of the main reinforcement with the applied major principal-moment axis, $\alpha$, and one ratio of the amounts of reinforcement, $\mu$. The second graphical representation (Fig. 7.1 through 7.6) presents the yield criterion for one particular applied principal-moment condition, $M_1/M_2$, and all values of $\alpha$ and $\mu$.

The yield criterion presented in this report is based directly on the conditions of equilibrium, geometry and the material properties of concrete and steel under uniaxial stressing. The experiments have confirmed this criterion to be valid over the complete range of biaxial moments varying from pure torsion to uniaxial bending to the case of equal biaxial moments for underreinforced slabs.
REFERENCES


APPENDIX A. THE EXPERIMENTAL INVESTIGATION

A.1. Introductory Remarks

This appendix presents a description of the materials used, the characteristics of the test specimens, the test rig, instrumentation, and the measurements taken. It includes moment-curvature diagrams of all 41 tests conducted as part of the investigation.

A.2. Materials

(a) Cement

Atlas brand high-early strength cement was used in all specimens (Type III Universal Atlas).

(b) Aggregates

Wabash River sand and pea gravel were used in all specimens. Both aggregates have been used in the Structural Research Laboratory of the University of Illinois, Urbana, for many previous investigations and have passed the usual specification tests. The maximum size of the gravel was 3/8 in.

The origin of these aggregates is an outwash of the Wisconsin glaciation. The sand consisted mainly of quartz, and the major constituents of the gravel were limestone and dolomite.

(c) Concrete Mixes

Mixes were designed by the trial-batch method. Three batches were used in each slab, batches one and three were placed in the ends of
the specimen and batch two was placed in the middle portion, which was the critical portion of the specimen in relation to the loadings and the measurements made. Table A.1 lists the proportions of the concrete batches used in each slab. It also lists the strength characteristics and the age of the concrete at the time of testing. All proportions are in terms of dry weights.

In Fig. A.1 and A.2, the modulus of rupture and the splitting strength are compared with the compressive strength of concrete. The test results for the modulus of rupture have been taken from a previous investigation (11). The splitting strength was found from tests on 6 by 6-in. cylinders loaded by a compressive force on opposite generators of the cylinder. Strips of stiff fiberboard, 1/8-in. thick and 1/2-in. wide, were placed between the head of the testing machine and the cylinder to distribute the load uniformly along the length of the specimen. Average loading speed was 1.75 psi per second.

For the magnitude of compressive strength that was obtained in the test specimens, the following expression was selected to represent the modulus of rupture of the concrete:

$$f'_r = 7\sqrt{f'_c}$$  \hspace{1cm} (A.1)

where both stresses are expressed in psi.

The compressive strength of the concrete was determined from tests on 6 by 12-in. control cylinders. Figure A.3 shows a typical concrete stress-strain diagram. The average loading speed was 35 psi per second.
(d) Reinforcement

Deformed reinforcing bars of 1/4 in. diameter were used in all specimens. The reinforcement was purchased from the Triangle Steel and Supply Company in California and annealed at 1200°F for two hours by the Fred A. Snow Company of Chicago.

The layers of reinforcement as used in the slab specimens were designated in sequence from top to bottom of the slab in the testing position. Layer 1 was on top, then layer 2, and layers 3 and 4, when used, were in the bottom side of the slab, with layer 4 being closer to the bottom surface.

For $p = 0.01$ the No. 2 bars in layers 1 or 4 were spaced at 1.5 in. and those in layers 2 and 3 were spaced at 1.375 in. This smaller space was used in order that the moment capacity of layers 2 and 3, which have smaller depth ($d$) from the compression surface, would be the same as that of layers 1 and 4. For amounts of reinforcement, $p = 0.005$ or $p = 0.0025$, the spacing of the bars was increased in multiples of 1.5 in. for layers 1 and 4, and 1.375 in. for layers 2 and 3.

All reinforcing bars in the rectangular slabs were welded at the ends to transverse reinforcement. Tension tests on individual welded specimens showed that the welding did not alter the strength of the reinforcing bars significantly.

Table A.2 lists the characteristics of the reinforcing steel, yield stress, strain at the beginning of strain hardening, and strength. These characteristics were determined from tensile tests using a five-in. gage length, carried out on a Tinius-Olsen testing machine equipped with a plotter giving the load-deformation characteristics. Additional tests
were made to determine slip of the jaws of the machine on the specimen in order to determine the actual stress-strain relationship.

Because of variations in yield stress of the different bundles, specimens were taken of all reinforcement used in this investigation, and they were recorded for each particular test. Figure A.4 shows a steel stress-strain diagram.

A.3. Description of Specimens

There were two types of specimens: the "circular" specimens, designated as "C" specimens, used in the isostatic-moment tests, and the rectangular specimens, designated as "B" specimens, used in the uniaxial, torsion, and the combined bending and torsion tests.

The "C" specimen is shown in Fig. A.5. The test area was within the 3-ft diameter circle. The floating supports were placed along the 3-ft-6-in. diameter circle, and the specimen was loaded with downward forces applied on the 6-ft diameter circle. The loading area contained six equally spaced slots to minimize the effect of membrane forces outside the test area.

The "B" specimen is shown in Fig. A.6. The test area was 3 ft 6 in. wide and 4 ft 6 in. long. The combined bending and torsion moment was introduced as shown in Fig. A.6.

The actual thickness of all specimens was measured. The measured values were within 4 percent of the intended value. A dial gage was attached to a measuring fork which was provided with bubble levels in the horizontal and vertical positions. The sensitivity of the dial gage was 0.001 in. The thickness was taken as the average
value of 15 measurement points equally spaced and located in the test area.

After each specimen was tested, measurements were taken of the depths of the different layers of reinforcement with respect to the concrete surface. This measurement was taken at the same points where the thickness of the slab had been measured.

A.4. Casting and Curing

The reinforcement was placed in the forms after being instrumented with electric strain gages. The bottom layer, which would be defined as layer one after the slab was turned over to be placed in the testing rig, was supported by small pieces of No. 3 deformed bars providing a minimum cover of 3/8-in. for the reinforcement. The top layer, which would be defined as layer four, was placed on chairs which also provided a minimum cover of 3/8-in. for the reinforcement on that layer.

All concrete was mixed in a nontilting drum-type mixer of six cu. ft capacity. A butter mix of one cu. ft preceded the three batches that were used in the slab, and the total mix quantity used was about eleven cu. ft.

All specimens were cast in forms with a plastic impregnated plywood bottom. The forms for the "C" specimens had steel-sheet sides and those for the "B" specimens had plywood sides. Pieces of steel pipe 4 in. long, and 2 in. in diameter were screwed to the form in order to preform holes for the loading and supporting system. These pieces of
pipe were removed after smoothing the surface and before the concrete began to harden appreciably.

To determine the compressive strength of the concrete, three 6 by 12 in. cylinders were cast from each batch. Two 6 by 6 in. cylinders were cast from each batch to determine the splitting strength.

All concrete in the slab specimens and in the control cylinders was vibrated with a high frequency internal vibrator. The top surface of the test specimens was troweled smooth and the control cylinders were capped with a cement paste two to four hours after casting. One day after casting, the test specimens and all control cylinders were covered with wet burlap. The sixth day after casting, the burlap was removed in order to instrument the test specimens.

A.5. Instrumentation

(a) Electric Strain Gages on Reinforcement

A total of eight electric strain gages were used on the reinforcement of each specimen. These gages were placed on eight different bars. There were two instrumented bars per layer of reinforcement. All of the gages used were Budd Metal Film strain gages of type HE-lll, which had a nominal gage length of 1/16 in. This type of gage should read up to about 4 percent strain, however, it was found repeatedly that the gages were not reliable once the yield strain was exceeded.

The surface of the reinforcement bar was prepared for the mounting of the gage by grinding down a spot on the bar, then this surface was sanded using fine emery cloth and cleaned with solutions of
trichloroethylene and acetone. Some gages were mounted using C-2 epoxy and then the curing procedure was carried out for two hours in an oven at 200°F. Other gages were mounted using Eastman 910 cement, and no difference was noted in the performance of the gages with either cementing agent.

Once the gage was ready, the leads were soldered and taped, then the gages were waterproofed with a synthetic rubber coating, "Gagekote No. 2," and tar.

All gages were placed within the test area and its particular location is indicated in the figures describing the test results.

(b) Electric Strain Gages on Concrete

SR-4 type AR-1-S-6, 45° rosette electric strain gages were used. Each of the three elements of the rosette-gage had a nominal gage length of 3/4-in. and a width of 3/8-in. Some specimens were instrumented with gages on both concrete surfaces and others with gages on one surface only, depending upon the state of cracking to be produced by the loading condition.

The concrete surface at the locations of the gage was smoothed using fine sand paper, all dust was removed using compressed air, and then the surface was cleaned with trichloroethylene and acetone. Once the surface was dry, a thin layer of Eastman 910 cement was applied to the concrete surface and 6A-1A accelerator to the gage. The gage was then pressed against the concrete surface until all excess cement and air bubbles were removed. The leads were then soldered and the gage was moisture-proofed using Budd GW-1 waterproofing compound.
The locations of all gages are given in the figures describing the test results.

(c) **Mechanical Dial Gages**

The deflections of all specimens were measured using dial gages with a sensitivity of 0.001 in. which were placed on light steel bridges. In all "B" specimens the deflections were measured in three directions so that a calculation of principal curvatures was possible. In the "C" specimens the deflections were measured in only two directions.

There were two types of dial gage bridges. The larger type, which was placed hanging from the specimens, had five dial gages on each direction spaced 7.5 in. apart. The smaller type was an equilateral triangle with three dial gages on each side and was supported at the corners. The spacing of the gage points was 3 in. This smaller type could be moved freely on top of the slab, at any stage of loading, to measure deflections. The larger type, however, could not be moved out of position once the test was started. Both dial gage bridges can be seen in position in Fig. A.9 and A.10.

A.6. **Loading and Supporting System**

The loading and supporting system used for this investigation had been previously designed and constructed during the first stage of the research project. The main feature of this freely-hanging system was to minimize any influence of the support and loading apparatus on the boundary conditions of the test specimens.
(a) **Loading and Supporting System of Specimens Subjected to Isostatic Moment**

Figures A.7 and A.8 show the loading and supporting system of a "C" specimen. The specimen was suspended from the three corners of an equilateral triangular frame placed on top of an 18 by 20-in. steel box beam, Fig. A.7. Three lever arms distributed the supporting forces to six steel inverted U-shape blocks placed in a circle of 3-ft 6-in. diameter. The blocks were 3-in. high, 3 in. wide, and 15 in. long and formed a hexagonal support around the test area. A one-in. thick rubber layer was placed between the supports and the concrete to distribute the reaction uniformly.

Loading of the specimen was carried out using the same type of U-shape blocks described above. The length of these blocks was 20 in. Six blocks were placed on the loading wings along a 6-ft diameter circle and loaded in pairs by means of three steel beams, which transferred the loading from three hydraulic center-hole rams of 30-ton capacity connected to one electric pump.

All steel rods used in the supporting and loading devices were 5/8-in. diameter high-strength Stressteel rods. Both ends of these rods were hinged by means of a spherical washer and a spherical seat. The 36 hinges insured an even distribution of the applied moment regardless of the deformation of the test specimen.

(b) **Loading and Supporting System of Specimens Subjected to Combined Bending and Torsion**

Figures A.9 and A.10 illustrate the system used. The specimens were suspended from an 18 by 20-in. steel box beam by two high-strength Stressteel rods of 5/8-in. diameter. Two lever arms distributed
the supporting forces to four supports 20 in. long, of the same characteristics as those described on part (a), which were placed under the test specimen.

The load to the specimen was transmitted by means of two channels 5 by 1-3/4 in. (6.7 lb/ft) which were clamped to either end by three 1.0-in. diameter bolts. A hard layer of rubber was placed between the channels and the test specimen. The loading rod was then placed through the channels and a preformed hole in the test specimen and connected to the loading jack.

Because of the problems encountered when large deflections took place and the difficulty to load specimens to failure, the bottom loading channel at either end of the test specimen was eliminated after the first three specimens, and the loading channel was bolted to the slab securely at three points closely spaced giving this the same degree of restraint as the original setup.

All loading and supporting rods were hinged at both ends. There were a total of 16 hinges which insured an even distribution of the applied moments.

(c) **Load Measurements**

The loads were measured by means of 30,000-lb. capacity ring dynamometers made for this investigation. The dynamometers were placed between the jacks and the loading frame constructed of heavy I-10 steel beams. The dynamometers consisted of a ring 4-in. in diameter of T-1 steel located between steel plates. The cross section of the T-1 steel ring was 0.6-in. in the radial direction, and the height was 0.825 in. The ring rested on three 3/4-in. steel balls.
located between the ring and the plates, 120° apart. Under loading, three other 3/4-in. steel balls located between the supporting balls subjected the steel ring to bending and torsion. The strains were registered by eight SR-4 Type A-7 gages evenly distributed around the ring. The strain gages were connected in a four-arm bridge system to a Baldwin strain indicator. The calibration of the dynamometers was in the order of 4 lb per division of the strain indicator. However, the calibration was not perfectly linear especially in the initial range of 0 to 5000 lb. Therefore, the actual calibration curves were used to convert strain indicator readings into loads. Periodic recalibration of the dynamometers were made, but no significant changes were observed.

A.7. Test Procedure

The yield load was reached between 8 to 12 loading increments, and the crushing of the concrete at the ultimate load, when reached, took from 18 to 20 loading increments. Only in a few tests was crushing of the concrete in compression reached. The other tests were discontinued at a large deflection when further loading presented serious reset problems and possible damage of the loading equipment.

Cracking was usually reached in about five load increments. Up to yielding, the criterion was to apply equal load increments and, subsequent to yielding, equal deflection increments.

After each load increment, the dynamometer readings were taken, the bending and torsion lever arms were checked and their corresponding angle changes were read. Deflections were read and the
concrete and steel readings were taken. Once all readings were recorded, the load in the dynamometers was checked and a new load increment applied.

Each test took three to four men working from 6 to 9 hours. All control cylinders were tested concurrently with the yielding of the slabs or at the conclusion of the test.

A.8. **Moment-Curvature Diagrams**

This section presents moment-curvature diagrams for the 41 tests carried out as part of the experimental investigation. Table 1.1 describes the characteristics of all 41 test specimens.

Figure A.11 shows moment-curvature diagrams for all six specimens subjected to isostatic moment.

Figure A.12 shows moment-curvature diagrams for all 10 specimens subjected to uniaxial bending and Fig. A.13 the diagrams for all 9 specimens subjected to pure torsion.

Figures A.14 and A.15 show the moment-curvature diagrams for all specimens subjected to combined bending and torsion. Figure A.14 shows the diagrams for all 9 specimens subjected to $M_1/M_2 = -0.14$ and Fig. A.15 the diagrams for all 7 specimens subjected to $M_1/M_2 = -0.45$. 
APPENDIX B. DEFINITIONS AND NOTATION

B.1. Definitions

Isostatic Moment, denotes equal moment in all planar directions.

Layer of reinforcement, indicates the reinforcing bars in one level and one direction.

Longitudinal or span direction, denotes the direction of the longer axis of the rectangular test specimens.

Transverse direction, denotes the direction of the shorter axis of the rectangular test specimens.

Isotropically reinforced slab, is a slab that has equal moment capacity in all planar directions.

Nonisotropically reinforced slab, is a slab that has different moment capacities in different directions.

First yield, stage of loading at which the reinforcement in the more effective layer (the layer that makes the smaller angle with the direction of the major principal-moment axis) reaches the yield strain.

Second yield, stage of loading at which the reinforcement in the less effective layer reaches the yield strain.

B.2. Notations

Notations which are defined in the text may not be included in this list.
\(A, A_1, A_2, \ldots A_5\) = constants defined in the text

\(A_s\) = area of reinforcement per unit width in isotropically reinforced slabs, or in the \(x\)-direction of nonisotropically reinforced slabs

\(A_{sx}\) = area of reinforcement per unit width in the \(x\)-direction

\(A_{sy}\) = area of reinforcement per unit width in the \(y\)-direction

\(B\) = constant defined in the text

\(B_1, B_5, \ldots\) etc. = designation of rectangular test specimens

\(c_n\) = distance from the extreme compression fiber to the neutral axis in a cross section normal to the \(n\)-direction

\(c_t\) = distance from the extreme compression fiber to the neutral axis in a cross section normal to the \(t\)-direction

\(C_1, C_2\) = constants defined in the text

\(C_1, C_2, \ldots\) etc. = designation of circular test specimens

\(d\) = distance from the extreme compression fiber to the centroid of the tension reinforcement

\(d'\) = concrete cover plus one-half reinforcing bar diameter

\(\text{deg.}\) = degrees

\(D_1, D_2\) = constants defined in the text

\(E_c\) = modulus of elasticity of concrete

\(E_c'\) = effective modulus of elasticity of concrete for \(\nu \neq 0\)

\(E_s\) = modulus of elasticity of steel

\(f'_c\) = compressive 7-day cylinder strength of concrete (6 by 12 in.)

\(f_{cu}\) = average concrete compressive stress at ultimate

\(f_p'\) = compressive prism strength of concrete (20 by 20 by 5 cm.)

\(f_r\) = nominal modulus of rupture of concrete
\( f'_{ct} \) = split-cylinder strength of concrete (6 by 6 in.)
\( f'_{cs} \) = tensile strength of the reinforcement
\( f_{su} \) = stress in the reinforcement at ultimate
\( f_y \) = yield stress of the reinforcement
\( F_n \) = resultant bottom concrete force per unit width in the \( n \)-direction
\( F^0_n \) = resultant top concrete force per unit width in the \( n \)-direction
\( F_t \) = resultant bottom concrete force per unit width in the \( t \)-direction
\( F^0_t \) = resultant top concrete force per unit width in the \( t \)-direction
\( x_{nu} \) = resultant top concrete force per unit width at ultimate in the \( n \)-direction
\( G \) = shear modulus of concrete
\( h \) = height of a cross section
\( k_u d \) = distance from the extreme compressive fiber to the resultant concrete force
\( k_u \) = depth of the neutral axis at ultimate
\( K_n \) = stiffness of a reinforced concrete slab in the \( n \)-direction
\( K_t \) = stiffness of a reinforced concrete slab in the \( t \)-direction
\( L \) = length
\( M_1 \) = applied minor principal unit moment
\( M_2 \) = applied major principal unit moment
\( M_{cn} \) = resisting unit cracking moment in the \( n \)-direction
\( M_{ct} \) = resisting unit cracking moment in the \( t \)-direction
\( M_{yn} \) = resisting unit yield moment in the \( n \)-direction
\( M_{yt} \) = resisting unit yield moment in the \( t \)-direction
\( M_{un} \) = resisting unit ultimate moment in the n-direction
\( M_{ut} \) = resisting unit ultimate moment in the t-direction
\( M_n \) = resisting unit moment in the n-direction
\( M_t \) = resisting unit moment in the t-direction
\( M_{nt} \) = resisting unit twisting moment in a cross section normal to the n-axis
\( M_x \) = resisting unit moment in the x-direction
\( M_y \) = resisting unit moment in the y-direction
\( M_{E_n} \) = bending moment component of \( M_1 \) and \( M_2 \) in the n-direction
\( M_{E_t} \) = bending moment component of \( M_1 \) and \( M_2 \) in the t-direction
\( M_{E_{nt}} \) = twisting moment component of \( M_1 \) and \( M_2 \) in sections normal to the n-direction
\( n \) = modular ratio, also axis normal to the yield line, and in flexibility calculations axis normal to the initial crack
\( n' \) = modular ratio for Poisson's ratio, \( v \neq 0 \)
\( n_b \) = modified modular ratio for isotropically reinforced slabs subjected to biaxial bending
\( n'_b \) = modified modular ratio for nonisotropically reinforced slabs subjected to biaxial bending
\( n_{bt} \) = modified modular ratio for isotropically reinforced slabs subjected to combined bending and torsion
\( n'_{bt} \) = modified modular ratio for nonisotropically reinforced slabs subjected to combined bending and torsion
\( n_t \) = modified modular ratio for isotropically reinforced slabs subjected to pure torsion
\( n'_{t} \) = modified modular ratio for nonisotropically reinforced slabs subjected to pure torsion
\( n_u \) = modified modular ratio for isotropically reinforced slabs subjected to uniaxial bending
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n' )</td>
<td>modified modular ratio for nonisotropically reinforced slabs subjected to uniaxial bending</td>
</tr>
<tr>
<td>( N_n )</td>
<td>resultant force per unit width of the bottom reinforcement in the n-direction</td>
</tr>
<tr>
<td>( N^o_n )</td>
<td>resultant force per unit width of the top reinforcement in the n-direction</td>
</tr>
<tr>
<td>( N_t )</td>
<td>resultant force per unit width of the bottom reinforcement in the t-direction</td>
</tr>
<tr>
<td>( N^o_t )</td>
<td>resultant force per unit width of the top reinforcement in the t-direction</td>
</tr>
<tr>
<td>( N_{nt} )</td>
<td>resultant shear force per unit width of the bottom reinforcement in a cross section normal to the n-direction</td>
</tr>
<tr>
<td>( N^o_{nt} )</td>
<td>resultant shear force per unit width of the top reinforcement in a cross section normal to the n-direction</td>
</tr>
<tr>
<td>( N_{xn} )</td>
<td>component force per unit width of the top x-reinforcement in the n-direction</td>
</tr>
<tr>
<td>( N^o_{yn} )</td>
<td>component force per unit width of the top y-reinforcement in the n-direction</td>
</tr>
<tr>
<td>( N^o_{nu} )</td>
<td>resultant ultimate force per unit width of the top reinforcement in the n-direction</td>
</tr>
<tr>
<td>( N_{nu} )</td>
<td>resultant ultimate force per unit width of the bottom reinforcement in the n-direction</td>
</tr>
<tr>
<td>( p = A_s/d )</td>
<td>amount of steel per unit width in one layer of reinforcement in an isotropically reinforced slab, or in the x-direction of a nonisotropically reinforced slab</td>
</tr>
<tr>
<td>( P_x )</td>
<td>amount of nonisotropic reinforcement per unit width in the x-direction</td>
</tr>
<tr>
<td>( P_y )</td>
<td>amount of nonisotropic reinforcement per unit width in the y-direction</td>
</tr>
<tr>
<td>( t )</td>
<td>direction of a yield line</td>
</tr>
<tr>
<td>( u )</td>
<td>axis of the applied minor principal moment</td>
</tr>
<tr>
<td>( v )</td>
<td>axis of the applied major principal moment</td>
</tr>
</tbody>
</table>
\[ \begin{align*}
\mathbf{x} &= \text{axis of the reinforcement} \\
\mathbf{y} &= \text{axis of the reinforcement} \\
\alpha^\circ &= \text{angle between the top reinforcement in the} \\
&\quad \text{x-direction and the v-axis} \\
\alpha &= \text{angle between the bottom reinforcement in the} \\
&\quad \text{x-direction and the v-axis. For symmetrically} \\
&\quad \text{oriented reinforcement in top and bottom, } \alpha = \alpha^\circ \\
\alpha_x^\circ &= \text{angle between the top reinforcement in the x-direction} \\
&\quad \text{and the span direction of the rectangular test} \\
&\quad \text{specimens} \\
\beta &= \text{angle of the reinforcement in the x-direction} \\
&\quad \text{and the u-axis} \\
\gamma &= \text{angle between the yield line and the axis normal} \\
&\quad \text{to the governing principal moment} \\
\gamma_{nt} &= \text{shear strain on the extreme concrete fiber in} \\
&\quad \text{cross sections normal to the n-axis} \\
\gamma_{nt}^\circ &= \text{shear strain at the level of the top reinforcement} \\
&\quad \text{in cross sections normal to the n-axis} \\
\varepsilon_{cn} &= \text{concrete strain in the extreme bottom fiber in the} \\
&\quad \text{n-direction} \\
\varepsilon_{cn}^\circ &= \text{concrete strain in the extreme top fiber in the} \\
&\quad \text{n-direction} \\
\varepsilon_{ct} &= \text{concrete strain in the extreme bottom fiber in the} \\
&\quad \text{t-direction} \\
\varepsilon_{ct}^\circ &= \text{concrete strain in the extreme top fiber in the} \\
&\quad \text{t-direction} \\
\varepsilon_{sh} &= \text{steel strain at the beginning of strain hardening} \\
\varepsilon_{sn} = \varepsilon_n &= \text{steel strain at the level of the bottom reinforce-} \\
&\quad \text{ment in the n-direction} \\
\varepsilon_{sn}^\circ = \varepsilon_n^\circ &= \text{steel strain at the level of the top reinforcement} \\
&\quad \text{in the n-direction} \\
\varepsilon_{st} = \varepsilon_t &= \text{steel strain at the level of the bottom reinforcement} \\
&\quad \text{in the t-direction} \\
\varepsilon_{st}^\circ = \varepsilon_t^\circ &= \text{steel strain at the level of the top reinforcement} \\
&\quad \text{in the t-direction}
\end{align*} \]
\[ \varepsilon_{sx} = \text{steel strain at the level of the bottom reinforcement in the x-direction} \]
\[ \varepsilon_{sy} = \text{steel strain at the level of the bottom reinforcement in the y-direction} \]
\[ \varepsilon_{sy} = \text{steel strain at the level of the top reinforcement in the y-direction} \]
\[ \varepsilon_{su} = \text{strain in the reinforcement at ultimate} \]
\[ \varepsilon_{y} = \text{yield strain of the reinforcement} \]
\[ \varepsilon_{u} = \text{limiting compressive strain in the concrete} \]
\[ \eta = \frac{M_y}{M_x} = \text{ratio of resisting moments in the y- and x-directions} \]
\[ \theta = \text{angle between the major-principal moment direction and the transverse direction of the rectangular test specimens} \]
\[ \mu = \frac{p_y}{p_x} = \text{ratio between the amounts of reinforcement per unit width in the y- and x-directions} \]
\[ \nu = \text{Poisson's ratio} \]
\[ \sigma_{cn} = \text{concrete stress in the extreme bottom fiber in the n-direction} \]
\[ \sigma_{ct} = \text{concrete stress in the extreme top fiber in the t-direction} \]
\[ \sigma_{nt} = \text{shearing concrete stress in sections normal to the n-axis} \]
\[ \Phi_{cn} = \text{curvature at cracking in the n-direction} \]
\[ \Phi_{ct} = \text{curvature at cracking in the t-direction} \]
\[ \Phi_{yn} = \text{curvature at yield in the n-direction} \]
\[ \Phi_{yt} = \text{curvature at yield in the t-direction} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
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<tr>
<td>$\Phi_{un}$</td>
<td>curvature at ultimate in the $n$-direction</td>
</tr>
<tr>
<td>$\Phi_{ut}$</td>
<td>curvature at ultimate in the $t$-direction</td>
</tr>
<tr>
<td>$\Phi_n$</td>
<td>curvature in the $n$-direction, and in flexibility calculations in the direction of the major principal-moment axis</td>
</tr>
<tr>
<td>$\Phi_t$</td>
<td>curvature in the $t$-direction, and in flexibility calculations in the direction of the minor principal-moment axis</td>
</tr>
<tr>
<td>$\omega = \frac{M_2}{M_1}$</td>
<td>ratio of principal moments</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>constant defined in the text</td>
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</table>
## TABLE 1.1

**SUMMARY OF THE EXPERIMENTAL INVESTIGATION**

<table>
<thead>
<tr>
<th>Applied Moment</th>
<th>Type of Reinforcement</th>
<th>Mark</th>
<th>Concrete Strength 6 by 12-in. Cylinder</th>
<th>Yield Stress of Reinforcement</th>
<th>Amount of Reinforcement</th>
<th>Reinforcement Orientation</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$f_c'$, psi</td>
<td>$f_y$, psi</td>
<td>$P_x$</td>
<td>$P_y$</td>
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<tr>
<td>Isostatic</td>
<td>Isotropic</td>
<td>C1</td>
<td>6410</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>C2</td>
<td>4580</td>
<td>50,000</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>C3</td>
<td>2700</td>
<td>50,000</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C21</td>
<td>5960</td>
<td>50,000</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C24</td>
<td>5120</td>
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<tr>
<td></td>
<td></td>
<td>C25</td>
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<tr>
<td>Uniaxial</td>
<td>Isotropic</td>
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<td>50,000</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>B5</td>
<td>4850</td>
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<td></td>
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<td></td>
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<td></td>
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<td>B8</td>
<td>4020</td>
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<td>0.01</td>
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<tr>
<td>Nonisotropic</td>
<td></td>
<td>B9</td>
<td>4240</td>
<td>47,500</td>
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<td>0.01</td>
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<td>Pure Torsion</td>
<td>Isotropic</td>
<td>B14</td>
<td>6060</td>
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<tr>
<td></td>
<td></td>
<td>B15</td>
<td>5250</td>
<td>47,900</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>B16</td>
<td>4730</td>
<td>48,300</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>B17</td>
<td>5510</td>
<td>50,800</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>B18</td>
<td>5040</td>
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<td></td>
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<tr>
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<td>0.01</td>
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<td>Nonisotropic</td>
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<td>B21</td>
<td>5180</td>
<td>47,800</td>
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<td>0.0005</td>
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<tr>
<td></td>
<td></td>
<td>B22</td>
<td>5460</td>
<td>53,800</td>
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<td>0.0005</td>
</tr>
<tr>
<td>Combined</td>
<td>Isotropic</td>
<td>B26</td>
<td>4210</td>
<td>46,200</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Bending and</td>
<td></td>
<td>B27</td>
<td>5350</td>
<td>45,200</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Torsion</td>
<td></td>
<td>B27A</td>
<td>5250</td>
<td>46,900</td>
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<td>0.01</td>
</tr>
<tr>
<td>$M_1/M_2 = 0$</td>
<td></td>
<td>B28</td>
<td>5620</td>
<td>47,600</td>
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<td>0.01</td>
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<tr>
<td></td>
<td></td>
<td>B29</td>
<td>6010</td>
<td>47,600</td>
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<tr>
<td>Nonisotropic</td>
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<td>44,800</td>
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<td>0.0005</td>
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<tr>
<td></td>
<td></td>
<td>B31</td>
<td>5600</td>
<td>44,800</td>
<td>0.01</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B32</td>
<td>5500</td>
<td>55,300</td>
<td>0.01</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B33</td>
<td>4010</td>
<td>45,900</td>
<td>0.01</td>
<td>0.0005</td>
</tr>
<tr>
<td>Combined</td>
<td>Isotropic</td>
<td>B34</td>
<td>5150</td>
<td>52,000</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Bending and</td>
<td></td>
<td>B35</td>
<td>4600</td>
<td>54,000</td>
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<td>0.01</td>
</tr>
<tr>
<td>Torsion</td>
<td></td>
<td>B36</td>
<td>5430</td>
<td>52,000</td>
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<td>0.01</td>
</tr>
<tr>
<td>$M_1/M_2 = -0.14$</td>
<td></td>
<td>B37</td>
<td>5150</td>
<td>54,000</td>
<td>0.005</td>
<td>0.005</td>
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<tr>
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</table>

*Denotes Duplicate Test Specimen*
### TABLE 3.1

**TEST RESULTS FOR CIRCULAR SPECIMENS**

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<tr>
<th>Mark</th>
<th>Thickness (h, in.)</th>
<th>Concrete Strength (f', psi)</th>
<th>Steel Strength (f_y, psi)</th>
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<th>Calcd.</th>
<th>Measured</th>
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<td>0.01</td>
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<td>90</td>
<td>5.20</td>
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*Positive Values Are Measured Counterclockwise From The Transverse Direction*
| Test No. | Thickness | Concrete Strength | Yield Stress of Steel | Reinforcement | Reinforcement Rat. | Measured | Cracking | Yield | Ultimate | Calculated | Cracking | Yield | Ultimate | Measured | Cracking | Yield | Ultimate | Measured | Cracking | Yield | Ultimate | Measured | Cracking | Yield | Ultimate | Measured | Cracking | Yield | Ultimate | Measured |
|---------|-----------|------------------|----------------------|---------------|-------------------|----------|----------|--------|----------|------------|----------|--------|----------|----------|----------|--------|----------|----------|----------|--------|----------|----------|----------|--------|----------|----------|----------|
| 836     | 0.04      | 4210             | 45,200               | 0.01          | 0.01              | 90       | 1.56     | -      | 4.90    | 110        | 5.60     | 27     | 1.42     | 6       | 4.84     | 80       | 5.42     | 21     | 1.01     | 1.03     |
| 827     | 0.04      | 5350             | 45,200               | 0.01          | 0.01              | -45      | 1.44     | 9      | 5.00    | 115        | 5.70     | 21     | 1.57     | 7       | 4.86     | 90       | 5.78     | 21     | 1.03     | 0.99     |
| 827A    | 0.04      | 5230             | 49,900               | 0.01          | 0.01              | -45      | 1.77     | 8      | 5.08    | 105        | 5.50     | 22     | 1.55     | 7       | 5.28     | 100      | 5.94     | 21     | 0.96     | 0.93     |
| 838     | 0.08      | 5620             | 47,600               | 0.01          | 0.01              | -22.5    | 1.48     | 9      | 5.40    | 140        | 5.90     | 23     | 1.46     | 7       | 5.18     | 130      | 5.76     | 21     | 1.06     | 1.02     |
| 839     | 0.08      | 6010             | 47,600               | 0.01          | 0.01              | 90       | 1.47     | 9      | 2.80    | 75         | 3.25     | 24     | 1.60     | 7       | 2.63     | 80       | 3.51     | 21     | 1.06     | 0.93     |
| 830     | 0.06      | 5570             | 44,800               | 0.001         | 0.01              | 90       | 1.40     | 9      | 1.64    | 80         | 2.00     | 10     | 1.40     | 7       | 1.52     | 80       | 2.10     | 7      | 1.06     | 0.95     |
| 831     | 0.06      | 5600             | 44,800               | 0.001         | 0.01              | -45      | 1.20     | 12     | 1.65    | 80         | 2.05     | 34     | 1.40     | 7       | 1.56     | 85       | 2.16     | 37     | 1.06     | 0.95     |
| 832     | 0.10      | 5500             | 55,900               | 0.001         | 0.01              | 67.5     | 1.42     | 10     | 2.70    | 115        | 3.10     | -4      | 1.45     | 6       | 2.65     | 130      | 3.27     | -6     | 1.06     | 0.95     |
| 833     | 0.07      | 6930             | 65,900               | 0.0025        | 0.01              | 0       | 1.85     | 10     | 4.18    | 100        | 4.60     | 47     | 1.42     | 7       | 4.10     | 80       | 4.44     | 48     | 1.06     | 1.04     |
| 834     | 0.05      | 5150             | 52,000               | 0.01          | 0.01              | -45      | 1.80     | 11     | 5.56    | 100        | 5.94     | 33     | 1.60     | 7       | 5.38     | 80       | 6.14     | 34     | 1.03     | 0.97     |
| 835     | 0.12      | 6600             | 64,800               | 0.01          | 0.01              | 0       | 1.75     | 10     | 5.80    | 150        | 6.00     | 31     | 1.47     | 6       | 5.80     | 135      | 6.24     | 34     | 1.00     | 0.96     |
| 836     | 0.06      | 5430             | 52,000               | 0.01          | 0.01              | -22.5    | 1.52     | 11     | 5.30    | 155        | 5.80     | 34     | 1.54     | 7       | 5.46     | 130      | 5.87     | 34     | 0.97     | 0.99     |
| 837     | 0.13      | 5150             | 54,900               | 0.001         | 0.01              | -45      | 1.40     | 12     | 3.95    | 85         | 3.86     | 34     | 1.49     | 7       | 3.00     | 90       | 3.92     | 34     | 1.02     | 0.98     |
| 838     | 0.05      | 5310             | 53,100               | 0.001         | 0.01              | 45       | 1.46     | 12     | 3.65    | 60         | 4.10     | -53     | 1.42     | 7       | 3.61     | 55       | 4.40     | -51     | 1.01     | 0.93     |
| 839     | 0.07      | 5340             | 51,300               | 0.001         | 0.01              | 90       | 1.51     | 9      | 2.35    | 125        | 2.65     | 23     | 1.40     | 7       | 2.14     | 115      | 2.66     | 14     | 1.10     | 1.00     |
| 840     | 0.06      | 5090             | 49,900               | 0.001         | 0.01              | -45      | 1.40     | 11     | 1.60    | 60         | 1.30     | 43     | 1.35     | 7       | 1.54     | 70       | 2.00     | 41     | 1.04     | 0.95     |
| Sl No | Test | Concrete | Reinforcement | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate | Crack | Yield | Ultimate |
|-------|------|----------|---------------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|-------|-------|----------|
| 814   |      | 4,09     | 6,000         | 47,900|           | 1.35  | 7     | 1.50    | 7     | -     | 5.80    | 45    | 1.01  | 0.99    | 0.97  | 10 Day strength                        |
| 815   |      | 4,09     | 5,560         | 47,900|           | 1.80  | 8     | 4.95    | 75    | 5.78  | 45     | 0.99  | 0.32  | 90°     | 45    | Yield Failure       |
| 816   |      | 4,04     | 4,750         | 48,100|           | 1.68  | 7     | 6.06    | 80    | 6.41  | 45     | 0.37  | 1.02  | 90°     | 45    | At Second Yield                  |
| 817   |      | 4,03     | 5,530         | 50,800|           | 1.62  | 7     | 5.51    | 140   | 5.98  | 45     | 1.06  | 0.98  | 90°     | 45    | Bond Failure                       |
| 818   |      | 4,08     | 5,040         | 56,100|           | 1.41  | 7     | 6.06    | 80    | 6.41  | 45     | 0.37  | 1.02  | 90°     | 45    | At Second Yield                  |
| 819   |      | 4,06     | 5,350         | 52,100|           | 1.57  | 7     | 2.87    | 55    | 3.13  | 45     | 0.68  | 1.04  | 90°     | 45    | Yield Failure       |
| 820   |      | 4,06     | 5,890         | 51,800|           | 1.46  | 7     | 2.96    | 115   | 3.34  | 45     | 1.02  | 1.02  | 90°     | 45    | At Second Yield                  |
| 821   |      | 4,03     | 5,180         | 47,800|           | 1.35  | 7     | 2.80    | 115   | 3.00  | 45     | 1.02  | 1.00  | 90°     | 45    | At Second Yield                  |
| 822   |      | 4,06     | 5,660         | 53,800|           | 1.42  | 7     | 3.20    | 130   | 3.40  | 45     | 1.03  | 1.03  | 90°     | 45    | At Second Yield                  |

**TABLE 6.1**

TEST RESULTS FOR RECTANGULAR SPECIMENS-SUBJECTED TO TORSION
### TABLE A.1
PROPERTIES OF CONCRETE MIXES

<table>
<thead>
<tr>
<th>Mark</th>
<th>Compressive Strength, $f'_c$ psi</th>
<th>Splitting Strength, $f'_t$ psi</th>
<th>C:S:G by weight</th>
<th>Water by weight</th>
<th>Slump in.</th>
<th>Age at Test Days</th>
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</thead>
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<td>1*</td>
<td>2*</td>
<td>3*</td>
<td>1° 2° 3°</td>
<td>1 2 3</td>
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</tr>
<tr>
<td>C23</td>
<td>5920</td>
<td>6000</td>
<td>-</td>
<td>485 390 280</td>
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<td>5070</td>
<td>-</td>
<td>340 260 280</td>
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<tr>
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<td>4630</td>
<td>4320</td>
<td>4000</td>
<td>360 385 410</td>
<td>1:2.7:3.0</td>
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<td>4260</td>
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<td>5360</td>
<td>5330</td>
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<td>5660</td>
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<td>5045</td>
<td>380 360 435</td>
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*Average of three 6 by 12-in. cylinders

oAverage of two 6 by 6-in. cylinders

+Denotes duplicate test
<table>
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<tr>
<th>Mark</th>
<th>Average Yield Stress, $f_y$ (psi)</th>
<th>Strain at Strain Hardening ($\varepsilon_{sh}$)</th>
<th>Average Strength, $f'$ (psi)</th>
<th>Strain at Ultimate Strain ($\varepsilon_s$)</th>
<th>Number of Samples Tested</th>
<th>Gage Length (in.)</th>
<th>Comments</th>
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<td>5</td>
</tr>
<tr>
<td>B35</td>
<td>54,000</td>
<td>2.5</td>
<td>72,900</td>
<td>28</td>
<td>13</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B36</td>
<td>52,000</td>
<td>2.1</td>
<td>73,800</td>
<td>26</td>
<td>12</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B37</td>
<td>54,000</td>
<td>2.5</td>
<td>72,900</td>
<td>28</td>
<td>13</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B38</td>
<td>53,100</td>
<td>2.2</td>
<td>72,600</td>
<td>27</td>
<td>12</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B39</td>
<td>51,300</td>
<td>2.1</td>
<td>71,900</td>
<td>26</td>
<td>12</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>B40</td>
<td>49,000</td>
<td>2.1</td>
<td>67,900</td>
<td>28</td>
<td>12</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

*Indicates Duplicate Test
(a) Nonisotropically Reinforced Slab

(b) Mohr's Circle for Applied Moments

(c) Resisting Moments at the Yield Line

FIG. 2.1 NONISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO PRINCIPAL MOMENTS $M_1$ and $M_2$
FIG. 2.2 YIELD LINE ORIENTATION IN SLAB ELEMENTS SUBJECTED TO UNIAXIAL BENDING, \((M_1/M_2) = 0\)
FIG. 2.3 YIELD LINE ORIENTATION IN SLAB ELEMENTS SUBJECTED TO PURE TORSION, $(M_1/M_2) = -1$
FIG. 2.4 YIELD LINE ORIENTATION IN SLAB ELEMENTS SUBJECTED TO COMBINED BENDING AND TORSION, \((M_1/M_2) = 0.5\)
Fig. 2.5 Isotropically reinforced slab element subjected to uniaxial bending; \((M_1/M_2) = 0\)
FIG. 2.6 NONISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO UNIAXIAL BENDING, \((M_1/M_2) = 0\)
FIG. 2.7 ISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO PURE TORSION, \((M_1/M_2) = -1\)
FIG. 2.8 MONISOTROPICALLY REINFORCED SLAB ELEMENT SUBMITTED TO PURE TORSION, \((M_1/M_2) = -1\)
FIG. 2.9 ISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO COMBINED BENDING AND TORSION, \((M_1/M_2) = -0.5\)
FIG. 4.1 ISOTROPICALLY REINFORCED CONCRETE SLAB ELEMENT SUBJECTED TO UNIAXIAL BENDING ($\mu = 1.0$)

(a) Cross Section  (b) Strain  (c) Stress  (d) Strain  (e) Stress

(f) Strain  (g) Stress

FIG. 4.2 STRAIN AND STRESS DISTRIBUTIONS IN AN ELEMENT SUBJECTED TO UNIAXIAL MOMENT
FIG. 3.1 COMPRESSIVE STRENGTH OF CONCRETE UNDER UNIAXIAL AND BIAXIAL STRESSES. (Ref. 8)

\[ \frac{f_p}{f_p} = 330 \text{ kg/cm}^2 \text{ (4700 psi)} \]

- concrete prism strength
FIG. 3.2 UNIAXIAL BENDING; MEASURED LONGITUDINAL AND TRANSVERSE CONCRETE STRAINS
FIG. 3.3 STRAIN AND STRESS DISTRIBUTIONS IN AN ELEMENT SUBJECTED TO ISOSTATIC MOMENT
FIG. 3.4 REINFORCEMENT LAYOUT FOR SPECIMEN C24
FIG. 3.5 MOMENT-CURVATURE PLOT FOR SPECIMEN C2, $\alpha_1 = 0^\circ$
Effectively Diameter of Specimen = 36.4 in.

![Graph showing Moment-Curvature relationship for Specimen C24, \( \sigma_1^0 = 0^\circ \)](image)

- **Measured**
- **Calculated**

- Moment Before Readings
- Moment After Readings

Isotatic Moment and Isotropic Reinforcement

\[ p = 0.005 \quad \mu = 1.0 \]

Curvature \( x 10^5 \), in.\(^{-1}\)

Unit Moment, k-in./in.

FIG. 3.6  MOMENT-CURVATURE PLOT FOR SPECIMEN C24, \( \sigma_1^0 = 0^\circ \)
FIG. 3.7 CRACK PATTERN ON SPECIMEN C24

FIG. 3.8 OVERALL VIEW OF SPECIMEN C2 AFTER TESTING
FIG. 3.9  MOMENT-STEEL STRAIN PLOT FOR SPECIMEN C2, $O_1 = 0^\circ$
FIG. 3.10 MOMENT-STEEL STRAIN PLOT FOR SPECIMEN C24, φ = 0°
FIG. 3.11  MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN C2, $\alpha_1 = 0^\circ$
FIG. 3.12 MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN C24, $\alpha_1 = 0^\circ$
FIG. 4.1 ISOTROPICALLY REINFORCED CONCRETE SLAB ELEMENT SUBJECTED TO UNIAXIAL BENDING ($\mu=1.0$)

(a) Cross Section  (b) Strain  (c) Stress  (d) Strain  (e) Stress

(f) Strain  (g) Stress

FIG. 4.2 STRAIN AND STRESS DISTRIBUTIONS IN AN ELEMENT SUBJECTED TO UNIAXIAL MOMENT
FIG. 4.3 FLEXIBILITY OF AN ISOTROPICALLY REINFORCED SLAB ELEMENT SUBJEC TED TO UNIAXIAL BENDING
FIG. 4.4 RATIOS OF FORCE COMPONENTS TO TOTAL FORCE ALONG THE $n$ AXIS
Effective Width of Specimen = 40.6 in.

FIG. 4.5 MOMENT-CURVATURE PLOT FOR SPECIMEN B7, $\Theta_x = -45^\circ$
Effective Width of Specimen = 40.5 in.

Effective Width of Specimen = 40.5 in.

Moments Before Readings
Moments After Readings

Calculated

Measured

Isotropic Reinforcement
p = 0.01 \( \mu = 1.0 \)

Curvature \( \times 10^5 \) in.\(^{-1}\)

FIG. 4.6 MOMENT-CURVATURE PLOT FOR SPECIMEN BB, \( \phi_x = -22.5^\circ \)
Effective Width of Specimen = 41.2 in.

Fig. 4.7 Moment-Curvature Plot for Specimen B10, $\phi_x = 90^\circ$
FIG. 4.8  MOMENT-STEEL STRAIN PLOT, TENSION SIDE OF SPECIMEN B7, \( \alpha_x = -45^\circ \)
FIG. 4.9 MOMENT-STEEL STRAIN PLOT, TENSION SIDE OF SPECIMEN B8, $\alpha_x^o = -22.5^o$
FIG. 4.10 MOMENT-STEEL STRAIN PLOT, TENSION SIDE OF SPECIMEN B10, $\theta_x = 90^\circ$
FIG. 4.11 MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN B7, $\phi_x = 45^\circ$
FIG. 4.12  MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN B7, $\alpha_x = 45^\circ$
FIG. 4.15  MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN B10, $\phi_x = 90^\circ$
FIG. 4.16 NONISOTROPICALLY REINFORCED SLAB SUBJECTED TO UNIAXIAL MOMENT
α, Angle Between The Main Reinforcement and The Principal moment axis

FIG. 4.17 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO UNIAXIAL MOMENT
FIG. 4.18 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO UNIAXIAL MOMENT
Figure 4.19 Depth to the Neutral Axis in Isotropically and Nonisotropically Reinforced Slabs Subjected to Uniaxial Moment.
FIG. 4.20 COMPRESSIVE CONCRETE STRAINS IN ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS
Effective Width of Specimen = 39.6 in.

Nonisotropic Reinforcement
\( p = 0.01 \), \( \mu = 0.5 \)

Moment Before Readings
Moment After Readings

FIG. 4.21 MOMENT-CURVATURE PLOT FOR SPECIMEN B9, \( \theta_x = 45^\circ \)
FIG. 4.22 MOMENT-STEEL STRAIN PLOT, TENSION SIDE OF SPECIMEN B9, $\phi_x = 45^\circ$
FIG. 4.23  MOMENT-CONCRETE STRAIN PLOT, COMPRESSION SIDE OF SPECIMEN B9, $\alpha = 45^\circ$
Fig. 5.1 ISOTROPICALLY REINFORCED SLAB SUBJECTED TO COMBINED BENDING AND TORSION
FIG. 5.2 STRAINS AND FORCES IN AN ELEMENT SUBJECTED TO COMBINED BENDING AND TORSION
(Positive Values of Forces are Shown)
FIG. 5.3 FLEXIBILITY OF AN ISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO COMBINED BENDING AND TORSION
Effective Width of Specimen = 39.2 in.

- Moment Before Readings
- Moment After Readings

Isotropic Reinforcement
\( p = 0.01 \quad \mu = 1 \)
\( \frac{M_{E_t}}{M_{E_n}} = -0.14 \)

**FIG. 5.4** MOMENT-CURVATURE PLOT FOR SPECIMEN B27A; \( \alpha_x = -45^\circ \)
Effective Width of Specimen = 39.5 in.

![Graph showing moment-curvature plot for specimen B28, $\alpha_x = -22.5^\circ$.]

- Isotropic Reinforcement
  - $p = 0.01$, $\mu = 1$
  - $\frac{M_{t}}{M_{n}} = 0.14$

**Fig. 5.5** Moment-Curvature Plot for Specimen B28, $\alpha_x = -22.5^\circ$
Effective Width of Specimen = 39.3 in.

- Moment Before Readings
- Moment After Readings

FIG. 5.6 MOMENT-CURVATURE PLOT FOR SPECIMEN B34, $\theta_x^0 = -45^\circ$
Effective Width of Specimen = 39.0 in.

FIG. 5.7 MOMENT-CURVATURE PLOT FOR SPECIMEN B35, $\alpha_x = 0^o$
Calculate (Layer 1) Measured (Layer 1)

Calculated (Layer 2)

Measured (Layer 2)

Layer 1
△ Gage 5
○ Gage 8

Layer 2
○ Gage 6
△ Gage 7

FIG. 5.8 MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B27A, θ = -45°
FIG. 5.9 MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B28, $\phi_x = -22.5^\circ$
FIG. 5.10 MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B34, $\alpha_x = -45^\circ$
FIG. 5.11 MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B35, $\alpha^2 = 0^\circ$
FIG. 5.12 MOMENT-CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B27A, \( \alpha^o = -45^o \)
FIG. 5.13 NONISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECT TO COMBINED BENDING AND TORSION

FIG. 5.14 MOMENT-CURVATURE RELATIONSHIP
FIG. 5.15 ASSUMED STRAIN AND STRESS DISTRIBUTION
FIG. 5.16 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO COMBINED BENDING AND TORSION ($\mu = 0.5$)

$\alpha$, Angle Between the Main Layer of Reinforcement and The Major Principal Moment Axis

<table>
<thead>
<tr>
<th>Case</th>
<th>$\frac{M_1}{M_2}$</th>
<th>$\rho$</th>
<th>$\mu$</th>
<th>$d/h = 0.85$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.14</td>
<td>0.01</td>
<td>1.0</td>
<td></td>
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<tr>
<td>2</td>
<td>-0.14</td>
<td>0.005</td>
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<td>3</td>
<td>-0.14</td>
<td>0.01</td>
<td>0.5</td>
<td></td>
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<tr>
<td>4</td>
<td>-0.45</td>
<td>0.01</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.45</td>
<td>0.005</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.45</td>
<td>0.01</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>
FIG. 5.17 FLEXIBILITY OF ISOtROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO COMBINED BENDING AND TORSION (\(\mu = 0.25\))

\(\alpha\), Angle Between The Main Layer of Reinforcement And The Major Principal Moment Axis

\[ n = 7 \]
\[ d/h = 0.85 \]
FIG. 5.18 CONCRETE STRAINS IN ISOTROPICALLY AND ANISOTROPICALLY REINFORCED SLABS SUBJECTED TO COMBINED BENDING AND TORSION
Effective Width of Specimen = 39.6 in.

Moment Before Readings
Moment After Readings

Nonisotropic Reinforcement
$\frac{M_1}{M_2} = -0.14 \quad p = 0.01 \quad \mu = 0.25$

Curvature in The Principal Moment Direction $\times 10^5$, in.$^{-1}$

FIG. 5.19 MOMENT-CURVATURE PLOT FOR SPECIMEN B32, $\alpha = 67.5^\circ$
FIG. 5.20 DEVELOPMENT OF CRACKING IN SPECIMEN B32, $\alpha_x^0 = 67.5$
FIG. 5.20 DEVELOPMENT OF CRACKING IN SPECIMEN B32, $\alpha_x = 67.5$
FIG. 5.21 MOMENT-CURVATURE PLOT FOR SPECIMEN B33, $\alpha_x = 0^\circ$
Effective Width of Specimen = 39.5 in.

Nonisotropic Reinforcement

\[ M_1/M_2 = -0.45, \quad \rho = 0.01, \quad \mu = 0.25 \]

Curvature in the Principal Moment Direction \( \times 10^5 \), in.\(^{-1} \)

FIG. 5.22 MOMENT-CURVATURE PLOT FOR SPECIMEN B38, \( \alpha = 45^\circ \)
FIG. 5.23 DEVELOPMENT OF CRACKING IN SPECIMEN B38, $\alpha_x = 45^\circ$
FIG. 5.23 DEVELOPMENT OF CRACKING IN SPECIMEN B38, $\alpha_x = 45^\circ$
Nonisotropic Reinforcement

$$M_1/M_2 = -0.45, \beta = 0.01, \mu = 0.25$$

Curvature in the Principal Moment Direction $\times 10^5$, in.$^{-1}$

FIG. 5.24 MOMENT-CURVATURE PLOT FOR SPECIMEN B39, $\alpha_x^0 = 90^\circ$
FIG. 5.25  MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B32, $\alpha_x = 67.5^\circ$
FIG. 5.26  MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B33, $\alpha_x = 0^\circ$
FIG. 5.27  MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B38, $\alpha^o = 45^o$
FIG. 5.28 MOMENT-STEEL STRAIN PLOT, TOP SIDE OF SPECIMEN B39, $\alpha = 90^\circ$
FIG. 5.29 MOMENT-CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B32, $\alpha_x^o = 67.5^o$
FIG. 5.30 CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B39, $\alpha_x^0 = 90^\circ$
FIG. 6.1 FLEXIBILITY OF AN ISOTROPICALLY REINFORCED SLAB ELEMENT SUBJECTED TO TORSION

$\alpha$, Angle Between the Reinforcement and the $n$-direction

$A_s/h = A_{s}/h = 0.01$

$A_s/h = A_{s}/h = 0.005$

$d/h = 0.85$

$E_s/E_c = 7$
FIG. 6.2 MOMENT-CURVATURE PLOT FOR SPECIMEN B15, $\alpha^0 = -45^\circ$
Effective Width of Specimen = 38 in.

- Moment before readings
- Moment after readings

Isotropic Reinforcement
\( p = 0.01 \), \( \mu = 1 \)

FIG. 6.3 MOMENT-CURVATURE PLOT FOR SPECIMEN B16, \( \alpha_x = 90^\circ \)
Effective Width of Specimen = 37 in.

- Moment before readings
- Moment after readings

FIG. 6.4 MOMENT-CURVATURE PLOT FOR SPECIMEN B17, $\alpha^o_x = -22.5^o$
FIG. 6.5 STEEL STRAIN PLOT, SPECIMEN B15, $\alpha_x = -45^\circ$
Fig. 6.6 Steel strain plot, specimen B16, $\phi_x = 90^\circ$
Layer 2
△ Gage 25
▼ Gage 26
Layer 3
□ Gage 23
○ Gage 24

Calculated
Layer 3

Calculated
Layer 2

Measured

Steel Strain $\times 10^5$

Principal Unit Moment, k-in./in.

FIG. 6.7 STEEL STRAIN PLOT, SPECIMEN B17, $\phi_x = -22.5^\circ$
FIG. 6.8 CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B15, $\phi_x = -45^\circ$
FIG. 6.9  CONCRETE STRAIN PLOT. TOP SIDE OF SPECIMEN B16, $\alpha_x = 90^\circ$
FIG. 6.10  CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B17, $\theta_x = -22.5^\circ$
Angle Between The Main Layer of Reinforcement And the Principal Moment Axis

FIG. 6.11 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO TORSION (μ = 0.5)
FIG. 6.12 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO TORSION
Fig. 6.13 Concrete strains in isotropically and nonisotropically reinforced slabs subjected to torsion
(Note that this ordinate axis does not start at zero)
Nonisotropic Reinforcement

\[ M_1 / M_2 = -1, \quad p = 0.01, \quad \mu = 0.25 \]

Effective Width of Specimen = 37.1 in.

Moment before readings

Moment after readings

FIG. 6.14  MOMENT-CURVATURE PLOT FOR SPECIMEN B22, \( \alpha_x = 90^\circ \)
FIG. 6.15 STEEL STRAIN PLOT, SPECIMEN B22, $\phi = 90^\circ$
FIG. 6.16  CONCRETE STRAIN PLOT, BOTTOM SIDE OF SPECIMEN B22, $\alpha_x = 90^\circ$
FIG. 7.1 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBJECTED TO ISOSTATIC MOMENT ($M_1/M_2 = 1.0$)
FIG. 7.2 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBMITTED TO BIArxAL BENDING ($M_1/M_2 = 0.5$)
FIG. 7.3 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBJECTED TO UNIAXIAL MOMENT ($M_1/M_2 = 0$)
FIG. 7.4 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBJECTED TO COMBINED BENDING AND TORSION ($M_1/M_2 = -0.14$)

Test Results
- $\Delta p = 0.01 \quad \mu = 1$
- $\Delta p = 0.005 \quad \mu = 1$
- $\Delta p = 0.01 \quad \mu = 0.25$
FIG. 7.5  ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBJECTED TO COMBINED BENDING AND TORSION ($M_1/M_2 = -0.45$)
FIG. 7.6 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF REINFORCED CONCRETE SLABS SUBJECT TO PURE TORSION \((H_1/H_2 = -1.0)\)

Test Results

- \(p = 0.01\) \(\mu = 1\)
- \(p = 0.005\) \(\mu = 1\)
- \(p = 0.01\) \(\mu = 0.25\)
FIG. 7.7 ENVELOPES OF MAXIMUM CARRYING CAPACITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED CONCRETE SLABS (\(\mu = 0.25\))
FIG. 7.8 FLEXIBILITY OF ISOTROPICALLY REINFORCED SLAB ELEMENTS

\[ \alpha, \text{ Angle Between The Reinforcement and The Major Applied Principal Moment Axis} \]

\[ M_1/M_2 = -1 \]
\[ M_1/M_2 = -0.45 \]
\[ M_1/M_2 = -0.14 \]
\[ M_1/M_2 = 0 \]
\[ M_1/M_2 = 1 \]

\[ n = 7 \]
\[ d/h = 0.85 \]
\[ p = 0.01 \]
FIG. 7.9 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO UNIAXIAL MOMENT
FIG. 7.10 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO COMBINED BENDING AND TORSION ($M_1/M_2 = -0.14$)
FIG. 7.11 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO COMBINED BENDING AND TORSION (M₁/M₂ = -0.45)
FIG. 7.12 FLEXIBILITY OF ISOTROPICALLY AND NONISOTROPICALLY REINFORCED SLABS SUBJECTED TO TORSION ($M_1/M_2 = -1.0$)
FIG. A.1 VARIATION OF MODULUS OF RUPTURE WITH CONCRETE COMPRESSIVE STRENGTH

\[ f_r = 7 \sqrt{f_c'} \]

\[ f_r = 6 \sqrt{f_c'} \]

- REF. 11
FIG. A.2 VARIATION OF SPLITTING STRENGTH WITH CONCRETE COMPRESSIVE STRENGTH

\[ f_t = 6 \sqrt{f_c} \]

\[ f_t = 5 \sqrt{f_c} \]

- REF. 11
- This Investigation
FIG. A.3 CONCRETE STRESS-STRAIN DIAGRAM FROM COMPRESSION CYLINDERS
No. 2 Deformed Bar
Gage Length = 5 in.

FIG. A.4 STEEL STRESS-STRAIN DIAGRAM FROM TENSION TESTS
FIG. A.5 TEST SPECIMEN "C"
PLAN VIEW

Thickness = 4 in.

SECTION 1-1

SECTION 2-2
LOADING ARRANGEMENT

FIG. A.6 TEST SPECIMEN 'B'
FIG. A.7 PLAN VIEW OF "C" SPECIMEN SUBJECTED TO ISOSTATIC MOMENT

FIG. A.8 "C" SPECIMEN, SIDE VIEW
FIG. A.9 PLAN VIEW OF "B" SPECIMEN SUBJECTED TO COMBINED BENDING AND TORSION

FIG. A.10 "B" SPECIMEN, SIDE VIEW
FIG. A.11 MOMENT CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO ISOSTATIC MOMENT ($M_1/M_2 = 1.0$)
FIG. A.11 (Cont.)  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO ISOSTATIC MOMENT \((M_1/M_2 = 1.0)\)
FIG. A.11 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECT TO ISOSTATIC MOMENT ($M_1/M_2 = 1.0$)
FIG. A.12  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO UNIAXIAL MOMENT ($M_1/M_2 = 0$)
FIG. A.12 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO UNIAXIAL MOMENT ($M_1 / M_2 = 0$)

(c) $86, \alpha_x = 67.5^\circ$

(d) $87, \alpha_x = 45^\circ$
FIG. A.12 (Cont.)  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO UNIAXIAL MOMENT (M₁/M₂ = 0)
Effective Width of Specimen = 41.2 in.

Curvature $10^5$, in.$^{-1}$

(g) B10, $\alpha^0 = 90^0$

Effective Width of Specimen = 39.7 in.

Curvature in the Principal Moment Direction $x 10^5$, in.$^{-1}$

(h) B11, $\alpha^0 = -22.5^0$

FIG. A.12 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO UNIAXIAL MOMENT ($M_1/M_2 = 0$)
Fig. A.12 (Cont.) Moment-Curvature plots for specimens subjected to uniaxial moment ($M_1/M_2 = 0$)

- Effective width of specimen = 41.2 in.
- (j) B12, $\phi_x = 67.5^\circ$
- (k) B13, $\phi_x = 90^\circ$
Effective Width of Specimen = 39.0 in.

Principle Curvature $= 10^5$, in.$^{-1}$

(a) B14, $c_x^0 = 90^\circ$

Effective Width of Specimen = 37.0 in.

(b) B15, $c_x^0 = 45^\circ$

FIG. A.13  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO PURE TORSION $(M_1/M_2) = -1.0$
FIG. A.13 (Cont.)  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO PURE TORSION \((M_1/M_2) = -1.0\)

Effective Width of Specimen = 38 in.

(d) 816, \(\phi = 90^\circ\)

Effective Width of Specimen = 37 in.

(d) 817, \(\phi = -22.5^\circ\)
FIG. A.13 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO PURE TORSION \( \left( \frac{M_1}{M_2} \right) = -1.0 \)
FIG. A.13 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO PURE TORSION $(M_1/M_2) = -1.0$
FIG. A.13 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO PURE TORSION \((M_1/M_2) = -1.0\)
FIG. A.14 MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \((M_1/M_2) = -0.14\)
FIG. A.14 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION ($M_1/M_2$) = -0.14
Effective Width of Specimen = 40.4 in.

Effective Width of Specimen = 39.3 in.

FIG. A.14 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION ($M_1/M_2$) = -0.14
FIG. A.14 (Cont.)  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \((M_1/M_2) = -0.14\)
FIG. A.14 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \( \frac{M_1}{M_2} = -0.14 \)}
FIG. A.15 MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \( (M_1/M_2) = -0.45 \)
FIG. A.15 (Cont.)  MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \( (M_1/M_2) = -0.45 \)
FIG. A.15 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION \((M_1/M_2) = -0.45\)
Effective Width of Specimen = 38 in.

Curvature in The Principal Moment Direction $\times 10^5$, in.$^{-1}$

(g) $B_{40}, \alpha_x = -45^0$

FIG. A.15 (Cont.) MOMENT-CURVATURE PLOTS FOR SPECIMENS SUBJECTED TO COMBINED BENDING AND TORSION $(M_1/M_2) = -0.45$