EFFECT OF STRESS-STRAIN CHARACTERISTICS OF HIGH-STRENGTH REINFORCEMENT ON THE BEHAVIOR OF REINFORCED CONCRETE BEAM-COLUMNS

by

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and

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A REPORT ON A RESEARCH PROJECT SPONSORED BY ERICO PRODUCTS, INC. Cleveland, Ohio

UNIVERSITY OF ILLINOIS
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1. INTRODUCTION

1.1. Object

Reinforcing bars of high-strength steel are commonly used in reinforced concrete beams and columns in the U.S.A. and abroad, since they result in a considerable saving in the cost of the structure. Two grades of steel meeting the ASTM Specifications A615-68 for high-strength reinforcing bars (Grade 60 and 75) are available in the U.S.A.

Ultimate strength design of reinforced concrete sections in accordance with the 1963 ACI Code (1)* is based on the assumption of an ideal elasto-plastic (flat-top) stress-strain relationship for the reinforcement. However, the actual stress-strain relationship for high-strength reinforcement is far from being elasto-plastic. Some reinforcing bars, particularly those meeting the ASTM specifications for Grade 75 steel, and some European and British steels do not always have a well-defined yield point and a flat-plateau. Moreover, even those steels which do have a well-defined yield point, including most Grade 40 and some Grade 60 steels, exhibit strain-hardening; some of these have a very short flat-plateau, and others a considerably long one.

If a reinforced concrete section is analyzed using strain compatibility and equilibrium of forces, steel strains at ultimate capacity may lie in such regions of the stress-strain curve that steel stresses either smaller or greater than the yield strength $f_y$, as assumed in the ACI Code, may be obtained. For under-reinforced beams, or for columns

* Numbers in parentheses refer to the corresponding numbers in the List of References.
subjected to small axial loads and large bending moments, steel stresses greater than $f_y$ may be obtained, whereas for columns subjected to large axial loads and small bending moments, steel stresses smaller than $f_y$ may be obtained if the steel has such a stress-strain curve that a part of it lies below the ideal flat-plateau in the region near the yield point. Consequently, different ultimate capacity of the section will be obtained as compared to that calculated according to the equations given in the ACI Code. Moreover, the steel strains depend on the value of ultimate concrete strain $\varepsilon_u$ in the extreme compression fiber. If a higher value of $\varepsilon_u$ than 0.003 assumed in the ACI Code is permitted, greater steel strains are obtained which result in greater steel stresses, unless the strains are on the flat-plateau, and hence in greater ultimate moments. Also, variations in the percentage of reinforcement and the concrete strength result in different steel strains and hence different stresses and moments. Furthermore, if the section is loaded slowly up to failure, or if the working load is sustained on the section for some time and then the section is further loaded to failure in a short time, creep and shrinkage of concrete change the stress-strain relationship for concrete and hence result in different concrete and steel strains, which in turn result in different ultimate capacities than for the short-time loading.

The object of this research is to investigate analytically the effects of the stress-strain curves of reinforcing bars of various types of steel on the strength and behavior of reinforced concrete sections subjected to various combinations of axial load and bending moment. This has been done by means of a method of analysis which considers realistic properties of the steel and concrete.
1.2. Scope

The effects of the stress-strain curve of reinforcement have been investigated for four different stress-strain curves for each of the two grades of steel, ASTM Grades 60 and 75. For each grade of steel, one of the chosen stress-strain curves corresponded to the ideal elasto-plastic relationship assumed in the 1963 ACI Code.

The effects have been studied by obtaining complete load-moment and load-curvature diagrams for short-time loading of a 15-in. square reinforced concrete section for three values of total steel ratio $p_t = 0.01, 0.04$ and $0.08$ and three values of concrete strength $f'_c = 3000, 4000$ and $5000$ psi for each of the eight stress-strain curves for steel.

Similar analyses assuming a slow loading have been made for the same section for two values of $p_t = 0.01$ and $0.08$ and one value of $f'_c = 3000$, for three stress-strain curves of each grade of steel including the flat-top steel. For the combination of sustained and short-time loading, analyses of the same section have been made for three stress-strain curves for Grade 60 steel and two curves for Grade 75 steel for the same values of $p_t$ and $f'_c$ as for slow loading.

The effects of typical stress-strain curves of Cadweld-spliced reinforcing bars of Grade 60 and Grade 75 steel have been studied for the three loading conditions and for the same three values of $p_t$ and $f'_c$ as stated above for the unspliced bars.

Most of the analyses have been made for one value of cover over the reinforcement. However, in order to study the effect of varying the cover, two values have been considered in analyses for two stress-strain curves of Grade 60 steel with $p_t = 0.04$ and $f'_c = 4000$ psi.
The effects of various stress-strain curves for the reinforcement have been studied by calculating ultimate capacities for an ultimate concrete strain $\varepsilon_u$ limited to 0.003, as specified in the 1963 ACI Code, and also by determining ultimate capacities by maximization of moment at a given axial load but with $\varepsilon_u$ limited to 0.010.

1.3. Outline of Report

Discussion of ultimate strength design, as specified in the ACI Code, is presented in Chapter 2 to point out the differences that could be obtained by using the realistic stress-strain curves for the reinforcement. A review of pertinent previous analytical and experimental work also is presented in this chapter.

The method of analysis for obtaining load-moment and load-curvature diagrams which will be used to study the effects of the stress-strain curves of the reinforcement is presented in Chapter 3.

In Chapter 4, some stress-strain curves of reinforcing bars and of concrete determined from tests in various laboratories are presented and, considering the ASTM specifications and the ACI Code requirements, realistic stress-strain curves for reinforcement and concrete are selected for analyses in subsequent chapters.

The use of the method of analysis presented in Chapter 3 is explained in Chapter 5 by obtaining a typical interaction diagram for a 15-in. square reinforced concrete section provided with ASTM Grade 60 steel.

In Chapter 6, validity of the method of analysis given in Chapter 3 is tested by comparing the analytical results with the available experimental results.
Analyses of a 15-in. square reinforced concrete section provided with ASTM Grade 60 steels are presented and discussed in Chapter 7. The section is assumed to have been loaded in a short-time so that the effects of creep and shrinkage of concrete are neglected. Detailed explanations of the effects of the stress-strain curves of these steels are presented for various combinations of total steel ratio and concrete strength.

Analyses and explanations similar to those given in Chapter 7 are presented in Chapter 8 for reinforcing bars of ASTM Grade 75 steel.

In Chapter 9, effects of creep and shrinkage of concrete are considered in the analyses and the effects of the stress-strain curves of ASTM Grade 60 and Grade 75 steels are explained. Two loading conditions, namely, slow and continuous loadings up to failure, and combination of sustained and short-time loading are considered. The stress-strain curves for concrete modified to suit these loading conditions are explained.

In Chapter 10, analyses using typical stress-strain curves for Cadweld-spliced reinforcing bars of ASTM Grade 60 and Grade 75 steels are presented and discussed for three loading conditions, namely, short-time loading, slow loading, and combination of sustained and short-time loadings.

1.4. Notation

\[ A_s = \text{area of tension steel} \]
\[ A'_s = \text{area of compression steel} \]
\[ A_{sc} = \text{area of compression steel} \]
\[ A_{st} = \text{area of tension steel} \]
\[ E_c = \text{modulus of elasticity of concrete} \]
\[ E_s = \text{modulus of elasticity of steel} \]
\[ M = \text{moment at any stage of loading} \]
$M_b$ = moment at balanced conditions
$M_0$ = ultimate moment for zero axial load
$M_{02}$ = $M_0$ for steel 2 according to the ACI Code
$M_{07}$ = $M_0$ for steel 7 according to the ACI Code
$M_u$ = ultimate moment at any level of axial load for any steel
$M_{u2}$ = $M_u$ for steel 2
$M_{u7}$ = $M_u$ for steel 7
$P$ = axial load at any stage of loading
$P_b$ = axial load at balanced conditions
$P_c$ = concrete force
$P_{sc}$ = compression steel force
$P_{st}$ = tension steel force
$P_0$ = ultimate axial load for zero eccentricity
$P_{02}$ = $P_0$ for steel 2
$P_{07}$ = $P_0$ for steel 7
$P_u$ = ultimate axial load for any steel
$b$ = width of compression face of section
$d$ = effective depth of section
$d'$ = distance of centroid of compression steel from extreme compression fiber
$e$ = eccentricity of axial load from plastic centroid of section
$e_b$ = $e$ at balanced condition = $M_b/P_b$
$f_c$ = concrete stress
$f'_c$ = 28-day cylinder strength of concrete
$f''_c$ = strength of concrete in member
$f_s$ = steel stress
$f_{sc}$ = compression steel stress
\[ f_{st} = \text{tension steel stress} \]
\[ f_{su} = f_{st} \text{ at ultimate capacity} \]
\[ f_y = \text{yield strength of tension steel} \]
\[ f'_y = \text{yield strength of compression steel} \]
\[ k = \text{coefficient which relates depth to neutral axis with } d \]
\[ k_1 = \text{coefficient which relates average concrete stress with } f''_c \]
\[ k_2 = \text{coefficient which locates centroid of concrete force} \]
\[ k_3 = \text{coefficient which relates } f'' \text{ with } f'_c \]
\[ p = \text{tension steel ratio in beam } = A_s/bd = A_{st}/bd \]
\[ p' = \text{compression steel ratio beam } = A'_s/bd = A_{sc}/bd \]
\[ p_t = \text{total steel ratio } (A_{st} + A_{sc})/bt \]
\[ t = \text{overall depth of section} \]
\[ \epsilon_c = \text{concrete strain} \]
\[ \epsilon_m = \epsilon_c \text{ for which } k_1 \text{ is maximum} \]
\[ \epsilon_s = \text{steel strain} \]
\[ \epsilon_u = \text{ultimate concrete strain} \]
\[ \epsilon_y = \text{yield strain of tension steel} \]
\[ \epsilon'_y = \text{yield strain of compression steel} \]
\[ \epsilon_1 = \text{strain in bottom fiber of section} \]
\[ \epsilon_2 = \text{tension steel strain} \]
\[ \epsilon_3 = \text{compression steel strain} \]
\[ \epsilon_4 = \text{strain in extreme compression fiber of section} \]
\[ \phi = \text{curvature of section at any stage of loading} \]
\[ \phi_u = \phi \text{ at ultimate capacity for any steel} \]
\[ \phi_{u2} = \phi_u \text{ for steel 2} \]
\[ \phi_{u7} = \phi_u \text{ for steel 7} \]
2. STATEMENT OF PROBLEM

2.1. Discussion of the Ultimate Strength Analysis in the ACI Code

In the 1963 ACI Code, equations are presented for calculating the ultimate moment of reinforced concrete beams, in Chapter 16, and for calculating the ultimate axial load and ultimate moment of reinforced concrete columns, in Chapter 19. These equations are based on the assumptions given in Chapter 15 of the Code.

For reinforced concrete beams provided with tension steel only, the equation for ultimate moment can be written in general form as:

$$M_u = A_s f_{su} d (1 - k_2 \frac{p f_{su}}{f_{cav}})$$  \hspace{1cm} (2.1)

where
- $M_u$ = ultimate moment capacity of the section
- $A_s$ = area of tension steel
- $f_{su}$ = stress in tension steel at ultimate capacity of beam
- $d$ = effective depth of the section
- $p$ = tension steel ratio = $A_s / bd$
- $b$ = width of the section
- $f_{cav}$ = average stress in the compression zone of concrete at ultimate capacity of beam
- $k_2$ = coefficient which locates the centroid of the concrete force

In the 1963 ACI Code, it is assumed that the steel has ideal elasto-plastic (flat-top) stress-strain characteristics and the steel ratio $p$ is so limited as to insure that the tension steel yields at ultimate
capacity. Consequently, $f_{SU}$ is equal to the yield strength of the steel $f_y$.

Also, the average concrete stress $f_{CAV}$ can be determined from any stress-strain curve for concrete with ultimate concrete strain $\varepsilon_u$ equal to 0.003 which results in predictions of ultimate moment in reasonable agreement with comprehensive tests, or an equivalent rectangular stress block may be used with the average stress equal to $0.85 k_1 f'_c$ and $k_2 = k_1/2$. The value of the coefficient $k_1$ is specified in Section 1503(g) of the Code and, for $f'_c \leq 4000$ psi, $k_1 = 0.85$. With the above assumptions, Eq. (2.1) can be rewritten as:

$$M_u = A_s f_y d (1 - 0.59 q) \quad (2.2)$$

where $q = pf_y/f'_c$

Equation (2.2) is similar to Eq. (16-1) of the 1963 ACI Code.

The coefficient 0.59 is a function of the distribution of concrete stress in the compression zone (stress block) and hence of the shape of the stress-strain curve for concrete up to a limiting ultimate concrete strain, which is 0.003 in the ACI Code.

It can easily be shown from strain compatibility and equilibrium of forces that, for an under-reinforced beam with an idealized flat-top steel, the tension steel strain corresponding to $\varepsilon_u = 0.003$ is always on the flat-plateau of the stress-strain curve and $f_{SU}$ is equal to $f_y$. It is also obvious that, allowing a greater value of $\varepsilon_u$ changes only the coefficient 0.59 which has little effect on the ultimate moment of the section. Therefore, tests of reinforced concrete beams (2) have shown that the Eq. (2.2) predicts $M_u$ reasonably well if the stress-strain curve for steel has a flat plateau and the value of $p$ is such that the tension steel strain lies on it.
However, if the steel does not have a well defined yield point (i.e. a round-house stress-strain curve), or if it has a stress-strain curve with a short flat-plateau followed by strain-hardening, the tension steel strain corresponding to $\epsilon_u = 0.003$ can be in that region of the stress-strain curve which gives steel stress $f_{su}$ greater than $f_y$, and thus the ultimate moment will be greater than that given by Eq. (2.2) above. Moreover, if a higher value of $\epsilon_u$ is permitted, the steel strain will increase considerably as compared to the case of $\epsilon_u = 0.003$, and considerably higher steel stress will be obtained. At the same time, the value of $k_2$ will increase while that of $k_\perp$ will decrease which, together with greater value of $f_{su}$ will decrease the term in parenthesis in Eq. (2.2), which gives the lever arm of the internal forces. The reduction in the lever arm is generally small in proportion to the increase in $f_{su}$ and hence greater ultimate moment will still be obtained.

If compression steel is also provided in the section, a different arrangement of forces will result. With $\epsilon_u$ limited to 0.003, and for a flat-top steel, the tension steel stress still equals $f_y$, and the tension steel force remains the same as without compression steel. The total compression force, which is the sum of the forces in the compression steel and in the concrete, also remains equal to the tension force $A_s f_y$. The lever arm of the forces is only slightly affected by providing compression steel. Consequently, the ultimate moment is only slightly changed. With a higher limit on $\epsilon_u$, the tension steel strain is increased. But, since the tension steel stress remains constant, the tension steel force and hence the total compression force remain constant. The concrete force is reduced because of the reduction in $k_\perp$ at higher value of $\epsilon_u$ but this is compensated
for by the increase in the compression steel force caused by the increase in the compression steel strain, since the compression steel strain is generally in the elastic region. The ultimate moment is only slightly affected because of the slight change in the lever arm of the compression force.

However, for a steel with a round-house stress-strain curve or with a stress-strain curve having a short flat-plateau followed by strain-hardening, and with $\varepsilon_u = 0.003$, the tension steel strain can be in that region of the stress-strain curve which is above the conventional flat plateau, especially for small values of total steel ratio, so that greater tension steel stress is obtained. Since the compression steel can provide enough compression force to match the increased tension force, greater tension and compression forces are obtained, which result in a greater moment. Furthermore, with increase in $\varepsilon_u$, the tension steel strain, stress and force are increased, and the compression force in concrete is reduced, but the compression steel provides enough increase in compression force to compensate for the reduced concrete force as well as to match the increased tension steel force. Thus, the total tension and compression forces continue to increase with increase in the concrete strain in the extreme compression fiber $\varepsilon_u$, and hence the moment continues to increase.

For a reinforced concrete section subjected to an axial load and a bending moment, the tension and compression steel strains at ultimate capacity depend on the value of the ultimate axial load $P_u$. For small values of $P_u$ below the balance point, the effects of the stress-strain curve of the reinforcement are similar to those obtained for the case of pure moment. For high levels of ultimate axial load, with $\varepsilon_u = 0.003$, the tension steel
strains are in the elastic region while the compression steel strains may be on the flat-plateau for a flat-top steel or in that region of the round-house stress-strain curve which lies below that of the conventional flat-plateau. In the latter case, the compression steel stress is less than $f_y$ and a smaller moment is obtained than that for the flat-top steel. However, for a higher value of $\epsilon_u$, the compression steel stress, and hence the force, remain constant for the flat-top steel but the reduction in the concrete force caused by the reduction in $k_1$ causes a reduction in the ultimate moment. Consequently, the limit of $\epsilon_u = 0.003$ is satisfactory for a flat-top steel. This is also true for a steel with a short flat-plateau of the stress-strain curve. But, for a round-house stress-strain curve, the increase in compression steel strain caused by the increase in the value of $\epsilon_u$ results in greater compression steel stress and force which compensate for the reduction in the concrete force. Consequently, the moment increases with increase in $\epsilon_u$. At high values of $\epsilon_u$, the reductions in the concrete force and its lever arm cause so much reduction in the moment that the compression steel cannot provide enough increase in its force and moment to compensate for this reduction. Thus, the moment decreases at high values of $\epsilon_u$. The maximum moment may occur at $\epsilon_u > 0.003$ but is generally less than that obtained by assuming a flat-top stress-strain curve and $\epsilon_u = 0.003$.

The above-mentioned effects of the stress-strain curve of the reinforcement are illustrated below by analyzing a 15-in. square reinforced concrete section subjected to axial load and bending moment. The section has the following properties:
The following cases are considered.

Case (1): In strict accordance with the 1963 ACI Code, i.e., flat-top steel 2 (Fig. 4.6), equivalent rectangular concrete stress block and $\varepsilon_u = 0.003$.

Case (2): Flat-top steel 2, realistic stress-strain curve for concrete given in Fig. 4.11 and with $\varepsilon_u = 0.003$.

Case (3): Realistic stress-strain curve for ASTM A615-68 Grade 75 steel (steel 3, Fig. 4.6), and the stress-strain curve for concrete given in Fig. 4.11 with $\varepsilon_u = 0.003$. The stress-strain curve for steel 3 meets the requirements of the ACI Code for the design assumption that it has a flat-top stress-strain curve with $f_y = 75$ ksi (see Section 4.1.3).

Case (4): Realistic stress-strain curve for steel 3 (Fig. 4.6) and for concrete (Fig. 4.11), but with a higher limit on $\varepsilon_u$ up to 0.010. The basis for using this limit on $\varepsilon_u$ is given in Section 4.2.6. The ultimate moment is determined from the moment-strain diagrams in accordance with the criteria explained in Section 3.3.

For each of the above cases, two levels of ultimate axial load $P_u = 100$ and 800 kips will be considered, and strain compatibility and equilibrium of forces will be used to calculate the ultimate moment.

Table 2.1 shows the results of the analyses for $P_u = 100$ kips and 800 kips. It can be seen from this table that, for the section analyzed here:
(1) The compression steel does not yield when $\epsilon_u$ is limited to 0.003. Consequently, the equations given in Chapter 19 of the ACI Code are not directly applicable but require modification based on the compatibility of strains.

(2) With $\epsilon_u$ limited to 0.003, and with flat-top steel, the ultimate moment at a lower level of ultimate axial load ($P_u = 100$ kips) is little affected by the assumption of rectangular stress block for concrete (Case 1) instead of the realistic concrete stress-strain curve (Case 2), but at higher level of ultimate axial load ($P_u = 800$ kips), this assumption results in an ultimate moment which is 7 percent greater than that calculated by using the realistic stress-strain curve for concrete.

(3) If the realistic stress-strain curves are used for both steel and concrete and if $\epsilon_u$ is limited to 0.003 (Case 3), the resulting ultimate moment is smaller than that calculated with a flat-top stress-strain curve for steel and the rectangular stress block for concrete (Case 1), by 4 percent at $P_u = 100$ kips, and by 15 percent at $P_u = 800$ kips. The greater difference at the higher load level occurs because of the higher values of the concrete and compression steel forces for Case 1. The greater value of $k_1$ for Case 1 results in greater concrete force, and that portion of the stress-strain curve for steel 3 which lies below that of flat-top steel curve gives smaller compression steel force for Case 3.

(4) If the realistic stress-strain curve for concrete is used for both steels 2 (Case 2) and 3 (Case 3), and if $\epsilon_u$ is limited to 0.003, a reduction of 3 percent in the ultimate moment is obtained for steel 3 at $P_u = 100$ kips and 10 percent at $P_u = 800$ kips.

(5) At $P_u = 100$ kips, greater ultimate moment is calculated by allowing a higher limit on $\epsilon_u$ with the realistic stress-strain curves for
steel and concrete (Case 4) than for any of the other cases. Table 2.1 shows that the ultimate moment for Case 4 is greater than that for Case 3 by 28 percent. This shows that the effect of strain-hardening of steel 3 is increased by allowing a higher limit on $\varepsilon_u$.

(6) At $P_u = 800$ kips, even if a higher value on $\varepsilon_u$ is permitted, the use of realistic stress-strain curves for steel and concrete (Case 4), results in a smaller ultimate moment than for Case 1 (ACI Code), by 5 percent. But, when the realistic stress-strain curves are used for both steel 3 and concrete, the higher limit on $\varepsilon_u$ gives a 12 percent greater moment than when $\varepsilon_u$ is limited to 0.003.

It is clear from the above discussion that the assumptions made in the ACI Code result in considerable error in the ultimate capacity of the section reinforced with high-strength steels of the types considered. Therefore, some other method of analysis based on more realistic assumptions for the properties of steel and concrete is required in order to make effective use of high-strength steels.

2.2. Previous Work

Hognestad (3) presented a thorough study of tests of reinforced concrete columns and proposed a method of analysis which gave good agreement between the analytical and experimental results. His method is based on the use of realistic stress-strain curves for concrete and steel with certain assumptions which were valid for the type of steel used in the column tests.

The reinforcing bars were made of structural and intermediate grade steels having a stress-strain curve with a long flat-plateau. Hognestad proposed the stress-strain curve for concrete as shown in Fig. 4.8,
with the ultimate concrete strain $\varepsilon_u$ limited to 0.0038. This curve agrees reasonably well with the stress-strain curves obtained from tests, as shown in Fig. 4.7. Since, with the type of steel used in the columns, steel stress seldom reaches the strain-hardening region except in the case of pure bending, no effect of the higher limit on ultimate concrete strain is obtained and thus the limit proposed by Hognestad is justified. However, for high-strength steel, the yield point may not be well-defined, particularly for ASTM Grade 75 steel, and the tension steel strains at ultimate axial load levels below the balance point are well into the strain-hardening region with resulting steel stresses greater than $f_y$, and ultimate moments greater than would be calculated if the steel stress is assumed equal to $f_y$, as was done by Hognestad. Also, greater ultimate moments are calculated if a higher limit on $\varepsilon_u$ is allowed, as was illustrated in Section 2.1. At ultimate axial load levels above the balance point, Hognestad assumed that the compression steel yields; but, for high-strength steels, this may not be true unless a higher limit on $\varepsilon_u$ is permitted. Consequently, Hognestad's method is not satisfactory for reinforced concrete sections with high-strength steels, the stress-strain curves of which do not have a long flat plateau.

Sahlin (4) has proposed a method of analysis of reinforced concrete beams based on obtaining moment-concrete strain $(M - \varepsilon_u)$ diagrams for the section and taking the maximum moment as the ultimate moment. He has idealized the stress strain curve for the reinforcement as consisting of three straight lines, one representing the elastic region, another the flat plateau, and the third the strain-hardening region. He has proposed an exponential form of equation for the stress-strain curve for concrete, the falling branch of which is too steep. No limit on $\varepsilon_u$ was proposed. The idealization of the stress-strain curve of the steel is not satisfactory
for high-strength steels, particularly for those steels which have a roundhouse stress-strain curve. Also, no procedure for obtaining load-moment and load-curvature diagrams was presented. Nylander and Sahlin (5) tested continuous reinforced concrete beams and simply-supported short control beams under two-point loads and compared the experimental results with the analytical results obtained from Sahlin's (4) method. The measured moments and concrete strains show that the concrete curve proposed by Sahlin underestimates the moments at higher values of concrete strain in the extreme compression fiber.

Pfrang et al. (6) have presented a method of analysis for obtaining load-moment and load-curvature diagrams for reinforced concrete sections. They have assumed a flat-top stress-strain curve for steel and Hognestad's curve for concrete with \( \epsilon_u \) limited to 0.0038. Strain compatibility and equilibrium of forces are used to obtain a set of values of ultimate moment \( M_u \), ultimate axial load \( P_u \), and ultimate curvature \( \Phi_u \). The depth to the neutral axis \( kd \) is varied to obtain several sets of \( P_u, M_u \), and \( \Phi_u \), one such set for each value of \( kd \). Pfrang has shown that allowing a higher limit on \( \epsilon_u \) does not affect the ultimate capacity of the section if the flat-top stress-strain curve is assumed for the reinforcement. But, this is not true for high-strength steels having a round-house stress-strain curve or a short flat plateau followed by strain-hardening, particularly at small values of ultimate axial load when the calculated moment increases with increase in the concrete strain in the extreme compression fiber. Consequently, this method of analysis is also not satisfactory for the reinforced concrete sections with high-strength steels.

Todeschini et al. (7) have reported a limited number of tests of reinforced concrete columns provided with high strength steels having
round-house stress-strain curve, and presented a method of analysis to check their test results. They have used a realistic stress-strain curve for concrete, but idealized the stress-strain curve of steel as consisting of two straight lines, one representing the elastic region with slope equal to the modulus of elasticity of steel $E_s$, and the other representing a strain-hardening region with a slope equal to $KE_s$, where $K$ is a constant. Their analytical results agreed well with the experimental results. However, since their tests were made at levels of ultimate axial load which were at or above the balance point, the steel strain did not reach far into the strain-hardening region and the assumption of the second straight-line portion of the stress-strain curve proved satisfactory. Also, since the ultimate axial load levels were quite high, the calculated ultimate concrete strains were quite low. The tests were conducted at three values of eccentricities 0, 1 and 5 in. Thus, the whole range of the interaction diagram was not studied.

Green (8) tested long reinforced concrete columns with intermediate grade steel under sustained load and proposed a method of analysis based on a flat-top stress-strain curve for steel and Hognestad's curve for concrete modified to consider creep effects. Moment-curvature relationship was obtained at the applied axial load considering various values of time for sustained load, and the maximum moment was taken as the ultimate moment. Because of the assumption of the flat-top stress-strain curve for steel, Green's method of analysis is not satisfactory for the sections reinforced with high-strength steels, having round-house stress-strain curve or a short flat-plateau followed by strain-hardening.

Evans (9,10) tested axially and eccentrically loaded reinforced concrete columns provided with square twisted (British) high-strength steel
having yield strength $f_y$ greater than 60 ksi. Concrete strength, total steel ratio, and eccentricity of applied load were varied over wide ranges. No analysis based on realistic stress-strain curves for steel and concrete was presented.

2.3. Statement of Problem

The discussion of the ultimate strength analysis of the ACI Code given in Section 2.1, and of the various existing methods of analysis given in Section 2.2, shows that rational analyses based on realistic representations of the properties of the materials in the form of stress-strain curves for the reinforcement and concrete are needed in order to understand more clearly the behavior of real reinforced concrete members.

There are various types of steel available in the U.S. and abroad which have different stress-strain characteristics, particularly in the strain-hardening region. Use of mechanical splices makes the stress-strain relationship for reinforcement further involved. Any single idealization of these curves in the form of straight lines or equations will result in errors in the calculation of steel stresses. Furthermore, the stress-strain relationship for concrete, particularly in the falling branch of the stress-strain curve, is quite important for section with high-strength steel having a round-house stress-strain curve or a short flat-plateau followed by strain-hardening. This stress-strain relationship for concrete is further modified by creep and shrinkage effects which further increase the concrete strains that can be attained. Consequently, it is desirable to consider realistic stress-strain curves for reinforcement and concrete.

In order to be able to consider any shape of stress-strain curve for steel and concrete in the analysis, a numerical method of analysis is
preferable, since it avoids the limitations imposed by attempts to express the stress-strain characteristics by mathematical equations. The method of analysis explained in Chapter 3 serves this purpose. It can be used to study the effects of any shape of the stress-strain curve of the reinforcement on the strength and behavior of reinforced concrete sections.

There are several other factors which modify the effects of the stress-strain curve for steel such as: (1) the eccentricity or the value of the applied axial load $P_u$. Different values of $P_u$ give steel strains which lie in different regions of the stress-strain curve for steel, and hence different effects of the stress-strain curve for reinforcement are obtained. Therefore, a complete interaction diagram of ultimate axial load and ultimate moment must be studied. (2) Different values of total steel ratio $\rho_t = (A_{st} + A_{sc})/bt$, cylinder strength of concrete $f'_c$, yield strength of steel $f_y$, and combinations of these variables in the form of $q_t = \rho_t f_y/f'_c$ change the tension and compression steel strains and hence modify the effects of the stress-strain curve for steel. Therefore, wide ranges of these variables must be investigated. (3) The variation of the ratio $d'/t$ also changes the strains and hence results in different effects of the stress-strain curve of steel.
3. METHOD OF ANALYSIS

3.1. Introduction

The analysis of reinforced concrete sections subjected to a combination of axial load and bending moment usually requires a trial-and-error procedure to obtain the equilibrium of internal and external forces as well as moments. The equilibrium of internal and external forces requires that the strain distribution through the depth of the section must be such that the algebraic sum of the compression force in the concrete $P_c$, the force in the compression steel $P_{sc}$, and the force in the tension steel $P_{st}$ must be equal to the applied ultimate axial load $P_u$. This is shown in Fig. 3.1. The sum of the moments of the internal forces $P_c$, $P_{sc}$ and $P_{st}$ about the plastic centroid of the section is the resisting moment of the section at the applied ultimate axial load.

The equilibrium of forces mentioned above depends upon the value of $\varepsilon_u$, the strain in the extreme compression fiber. Given the section properties, and the stress-strain curves for steel and concrete, it is possible to obtain equilibrium of forces with different values of $\varepsilon_u$, each corresponding to a different set of internal forces $P_c$, $P_{st}$ and $P_{sc}$ with different strain configurations but with the same total force $P_u$. However, each set of internal forces will give a different total moment $M$. This means that for different values of $\varepsilon_u$, different moment values are obtained although the sum of the internal forces equals the applied external ultimate load $P_u$. One particular value of $\varepsilon_u = \varepsilon_u$ will give $M = M_u$, the ultimate moment capacity of the section. Therefore, for a given value of axial load, a relationship can be obtained between $M$ and $\varepsilon_u$, and the ultimate moment
capacity $M_u$, the ultimate concrete strain $\epsilon_u$, and the ultimate curvature $\Phi_u$, can be obtained from this relationship. How the ultimate moment is selected from the $M - \epsilon_u$ relationship, is explained in Section 3.3.

3.2. Assumptions

The following assumptions are made in the analysis of a reinforced concrete section subjected to various combinations of axial load and bending moment.

(1) A realistic representation of the stress-strain curve is assumed for the compressed concrete. The maximum ordinate of this stress-strain diagram $f''_c$ at the strain $\epsilon_0$, is taken equal to $k_3 f'_c$, where $k_3$ is a constant whose value is chosen so as to relate realistically the strength of concrete in the member with $f'_c$, the strength of the same concrete as determined from cylinder tests.

(2) The stress-strain curve for the reinforcing bars in tension and compression is assumed so as to represent as closely as possible the actual stress-strain relationship for the steel used. The steel may be assumed to have different stress-strain relationships in tension and in compression.

(3) The reinforcement is assumed to be provided only in two faces parallel to the axis of bending, and only one layer of reinforcement is provided in each face. Different amounts of steel may be provided in each face.

(4) The strains in the loaded section are assumed to vary linearly over the depth of the section at all stages of loading including ultimate. The strain in the reinforcing bars is assumed to be the same as that in the concrete at the level of the reinforcing bars.
(5) The useful limit of ultimate concrete strain $\varepsilon_u$ is assumed to be 0.010 as discussed in Section 4.2.6.

(6) The tensile strength of concrete is neglected.

(7) The member is assumed to be short so that the effect of length of the member is neglected.

3.3. Ultimate Capacity

The ultimate capacity of a reinforced concrete section subjected to various combinations of axial load and bending moment has been studied for the following cases:

(a) **Short-time loading**: The section is loaded continuously up to failure in a short time so that the effects of creep and shrinkage are neglected. In this case, the short-time stress-strain curve of the concrete, as explained in Section 4.2.5, is used in the analysis.

(b) **Slow loading**: The section is loaded continuously up to failure so slowly that the creep and shrinkage of concrete take place continuously while the section is being loaded. In this case, the short-time stress-strain curve of concrete is modified so that the effect of creep and shrinkage is taken into account. Because of the continuous loading of the section, a continuous stress-strain curve, as explained in Section 9.2.1 and shown in Fig. 9.1, is used for analyzing the section.

(c) **Combination of sustained and short-time loadings**: The section is loaded continuously up to any given value of axial load $P$ and bending moment $M$ in a short time. These values of $P$ and $M$ are sustained for any desired period of time. The section is further loaded continuously up to failure in a short time. How the stress-strain relationship for concrete
is obtained for this case of loading, is explained in Section 9.3.1 and shown in Fig. 9.13.

The method of obtaining moment-strain diagrams for various values of ultimate axial load for the above cases is explained in Section 3.4. The ultimate moment which represents the failure of a section is obtained on the basis of the following criteria.

Criterion 1: The ultimate moment is the maximum moment obtained from the moment-strain diagram.

Criterion 2: If the maximum moment obtained from the moment-strain diagram occurs at the strain in the extreme compression fiber \( \epsilon_4 \) greater than the useful limit of ultimate concrete strain (0.010), as explained in Section 4.2.6, then the moment at \( \epsilon_4 = 0.010 \) is the ultimate moment.

Criterion 3: If the moment at which buckling of compression bars occurs is less than the ultimate moment determined from the criteria 1 and 2 above, then the ultimate moment at which buckling of compression bars occurs is the ultimate moment.

Buckling of the compression bars occurs as follows:

(a) When a reinforced concrete section is subjected to an axial load and/or a bending moment, the compression bars resist an axial compression force which in general will cause the bars to buckle at some stage of loading. At lower values of the load, the concrete surrounding the bars is still intact and prevents them from buckling. At higher values of load and concrete strain in the extreme fiber \( \epsilon_4 \), the concrete cover begins to spall off. At this stage, however, the curvature of the member prevents the bars from buckling outwards, that is towards the center of curvature, and the bars can not buckle away from the center of curvature because of the presence of
intact concrete behind them. Therefore, the bars tend to buckle sideways, as shown in Fig. 3.2. But, such buckling will not occur until the concrete at the level of the compression bars is so cracked or crushed that it offers little or no resistance to the movement of the bars sideways. In view of the tests reported by Barnard (11), it has been assumed that this occurs when the strain in the concrete at the level of the compression bars is equal to 0.005. However, when the specimen is loaded continuously and slowly as explained in Section 9.2.1, the strain $\varepsilon_0$ corresponding to the maximum stress $f''_c$ may be greater than 0.005. In this case, concrete will resist the buckling of compression bars up to a strain equal to at least $\varepsilon_0$. Since, no evidence is available for assuming a reasonable limit of concrete strain at the level of compression steel before buckling of the compression bars occurs, a conservative limit of $\varepsilon_0$ or 0.005, whichever is greater, is assumed in the analyses in Section 9.2.

(b) The critical stress in the compression bars at which buckling occurs is calculated by the tangent modulus theory:

\[ f_{cr} = \pi^2 E_t / (l/r)^2 \]

where $E_t =$ tangent modulus of the compression steel

$l =$ unsupported length of the bar between ties

$r =$ radius of the compression bars.

According to this formula, if the stress-strain curve of the compression steel has a flat-plateau, the bars should buckle when the strain or stress in the bars reaches the value at yielding. But, Yamashiro (12) found from his tests that, even the #4 bars in compression and 6-in. spacing of ties, $l/r = 24$, the compression bars buckled well into the strain-hardening region at a strain greater than 0.03. The stress-strain curve of the bars in
Yamashiro's tests had a flat plateau up to a strain of about 0.018. The compression bars did not buckle while on the flat plateau, because the concrete at the level of the compression bars was still sufficiently intact to restrain the bars from buckling. Moreover, even if the compression bars begin to buckle while on the flat plateau, the additional strain in the compression bars resulting from the local bending of the bar reaches the strain-hardening region, and the bars again begin to offer resistance to buckling. Therefore, when using the above formula, it is assumed that buckling of the compression bars occurs in the strain-hardening region and the corresponding value of \( E_t \) is used for calculating \( f_{cr} \).

(c) If the strain in the compression bars corresponding to \( f_{cr} \) is less than the limiting strain at the level of compression bars, as explained in (a) above for buckling of compression bars, it is assumed that buckling occurs at this limiting value of strain.

3.4. Interaction Diagram

In this method of analysis, complete interaction diagrams are obtained to give the load-moment and load-curvature relationships at ultimate capacity of the section for all values of axial load from zero (pure moment) to the axial load for zero eccentricity. The method consists of obtaining the relationship between the moment \( M \) and the concrete strain in the extreme compression fiber \( \epsilon_4 \) at a given value of ultimate axial load \( P_u \). Several values of \( \epsilon_4 \) are used and the value of \( M \) calculated for each value of \( \epsilon_4 \) to obtain moment - concrete strain \( (M - \epsilon_4) \) diagram. The ultimate moment \( M_u \) is obtained from the \( M - \epsilon_4 \) diagrams in accordance with the criteria explained in Section 3.3. The ultimate concrete strain \( \epsilon_u \) and ultimate curvature \( \phi_u \)
are obtained from the strain distributions corresponding to the ultimate moment. The above procedure for obtaining the $M - \varepsilon_4'$ relationship is repeated for various values of ultimate axial load to obtain ultimate load-moment and ultimate load-curvature diagrams. The steps involved in the analysis are explained below:

1. Locate the plastic centroid of the section in accordance with Section 1900(b) of the 1963 ACI Code. The distance of the plastic centroid of the section from the tension steel is given by

$$d_{pc} = \frac{A'f'(d' - d')}{{\frac{A'f'}{s'Y} + \frac{A_f}{s_Y}} + \frac{k_f'bt(d - t/2)}}$$

2. Choose a given value of ultimate axial load $P_u$.
3. Choose a suitable value of the strain in extreme compression fiber $\varepsilon_4'$.
4. Assume a value of curvature $\Phi$.
5. Calculate the depth to the neutral axis $kd = \varepsilon_4' / \Phi$.
6. Calculate the strain in the compression steel $\varepsilon_3 = \varepsilon_4' - \Phi d'$.
7. Corresponding to $\varepsilon_3$, obtain the stress in the compression steel $f_{sc}$ from the stress-strain relationship for the compression steel.
8. Calculate the force in the compression steel $P_{sc} = A_{sc}f_{sc}$.
9. Calculate the strain in the tension steel $\varepsilon_2 = \varepsilon_4' - \Phi d$.
10. Corresponding to $\varepsilon_2$, obtain the stress in the tension steel $f_{st}$ from the stress-strain relationship for the tension steel.
11. Calculate the force in the tension steel $P_{st} = A_{st}f_{st}$.
(12) Calculate the area $A_c$ under the stress-strain curve of concrete between the strains $\epsilon_5$ and $\epsilon_4$; where $\epsilon_5 = 0$, if $kd \leq t$ and $\epsilon_5 = \epsilon_4 - \Phi t$ if $kd > t$.

(13) Calculate the compression force in the concrete

$$P_{cc} = A_c b \frac{kd}{(\epsilon_4 - \epsilon_5)}.$$ 

(14) Calculate the stress $f_{cc}$ in the concrete at the level of the compression steel; i.e. corresponding to the strain $\epsilon_3$ in the compression steel.

(15) If the tension steel is in compression, calculate the concrete stress $f_{ct}$ at the level of the tension steel corresponding to the strain $\epsilon_2$. If the tension steel is in tension, $f_{ct} = 0$.

(16) Calculate the net force in the concrete $P_c$, after making allowance for the reduction of concrete area due to the presence of steel

$$P_c = P_{cc} - A_{sc} f_{cc} - A_{st} f_{ct}.$$ 

(17) Sum the forces in the steel and concrete to get the total force $P_t = P_c + P_{sc} - P_{st}$.

(18) Compare the total force $P_t$ with the given ultimate axial load $P_u$. If $P_u = P_t \pm$ the allowable error, then equilibrium of forces is obtained with the curvature $\Phi$ assumed in Step (4). If $P_u \neq P_t \pm$ the allowable error, assume a new curvature $\Phi$ and repeat Steps (5) through (18) until equilibrium of forces is obtained.

(19) Calculate the total moment $M$ of the tension steel force $P_{st}$, the compression steel force $P_{sc}$ and the concrete force $P_c$ about the plastic centroid of the section.
Thus, for a given value of the ultimate axial load $P_u$, the moment $M$, and curvature $\Phi$ are obtained for one value of $\epsilon_u$ chosen in Step (3).

(20) Assume a new value of $\epsilon_u$ and repeat Steps (4) through (19) to obtain a new set of values of $M$, $\Phi$ and $\epsilon_u$ for the ultimate axial load $P_u$, chosen in Step (2). In this way any desired number of sets of the values of $M$, $\Phi$, $\epsilon_u$ can be calculated for one value of $P_u$ and moment-curvature $(M - \Phi)$ and moment-concrete strain $(M - \epsilon_u)$ relationships can be obtained.

(21) Obtain the ultimate moment from the $M - \epsilon_u$ diagram in accordance with the criteria explained in Section 3.3. The values of $\epsilon_u$ and $\Phi$ corresponding to the ultimate moment will be the ultimate concrete strain $\epsilon_u$ and the ultimate curvature $\Phi_u$, respectively.

(22) Choose a new value of $P_u$ and repeat Steps (3) through (20) to get new $M - \Phi$ and $M - \epsilon_u$ relationships and a new set of values of $M_u$, $\epsilon_u$ and $\Phi_u$ for the new value of $P_u$. In this way a desired number of sets of the values of $P_u$, $M_u$, $\Phi_u$ and $\epsilon_u$ can be obtained and interaction diagrams of $P_u$ vs. $M_u$ and $P_u$ vs. $\Phi_u$ can be plotted.

3.5. Computer Program

The method of analysis explained in the previous section consists of a trial-and-error procedure for obtaining the equilibrium of forces. Several trials are necessary to achieve the desired degree of accuracy in Step (18). The number of arithmetic calculations involved in Steps (5)
through (18) for one trial is quite large. Furthermore, the arithmetic calculations must be repeated for several values of \( \varepsilon_4 \) and one value of \( P_u' \) as in Step (20), and further repetitions of the entire procedure are required for several values of \( P_u' \) as in Step (22). Consequently, the analysis to obtain the interaction diagram for one reinforced concrete section requires a very large number of numerical calculations. Therefore, a computer program was prepared for use with an IBM 7094 computer. This program was latter modified for use with an IBM 360-75 computer.

The method of analysis is based on the assumptions for the realistic representation of the properties of the materials. Therefore, actual shapes of the stress-strain curves of reinforcing bars and concrete are to be used in the analysis.

The stress-strain curves for steel vary considerably depending upon the type of steel, especially in the strain-hardening region. Also, the shape of the falling branch of the stress-strain curve of concrete is very important for the purpose of investigating the effect of the stress-strain curve for the steel. Furthermore, many different stress-strain curves for concrete have been proposed by various authors. It was not considered practicable or desirable to attempt to develop mathematical equations for all of these different shapes of stress-strain curves. Therefore, in order to be able to consider various shapes of stress-strain curve for both steel and concrete, these curves were represented in the computer analysis by a large number of discrete points. As per Steps (7), (10), (14) and (15) of the method of analysis in Section 3.4, the computer calculates the stress in the steel and concrete corresponding to any strain by linear interpolation. Similarly, for Step (12), the computer calculates
the area under the stress-strain curve of concrete between two values of strain by numerical integration.

The block diagram of Fig. 3.3 shows the working of the computer program and the use of various subroutines to perform the numerical calculations.
4. REALISTIC REPRESENTATION OF THE PROPERTIES OF MATERIALS

4.1. Reinforcement

4.1.1. Types and Stress-Strain Curves of Available Steels

High-strength reinforcing bars are available in the U.S. in two grades as per ASTM specifications A 615-68. These are Grades 60 and 75 having yield strengths of 60 and 75 ksi, respectively. Figure 4.1 shows stress-strain curves determined from tests on various sizes of reinforcing bars which satisfy the ASTM specifications for Grade 60 bars. It can be seen that all of the bars have a well-defined yield point, but a varying length of flat plateau of the stress-strain curve. Also, all the stress-strain curves have approximately the same shape in the strain-hardening region. These stress-strain curves show that, for a given yield strength, the early strain-hardening steel which has the smallest flat-plateau will give the highest stress for a given strain in the strain-hardening region.

Figure 4.2 shows stress-strain curves determined from tests on various sizes of ASTM Grade 75 bars. Most of these bars do not have a well-defined yield point, and the shapes of the stress-strain curves vary considerably in the strain-hardening region.

High-strength steels of various grades are available in the United Kingdom and in several countries in Europe. The square-twisted reinforcing bars available in the U.K. have a round-house stress-strain curve as shown by curves 1, 2 and 3, in Fig. 4.3. According to the ASTM specifications, these bars are equivalent to Grade 60 steel. In Sweden, high-strength steels are designated as KAM 40, KAM 60 and KAM 90, having yield points of 40, 60 and 90 kg/sq cm, respectively. These bars also have
a round-house stress-strain curve. The Austrian-German steels are also available in three grades: TOR 40, TOR 60 and TOR 80. These steels do not have a well-defined yield point. Danish steels called TENTOR have yield strength of at least 71,000 psi, and have a round-house stress-strain curve as shown by curve 5 in Fig. 4.3. The yield strength of European steels is defined as the proof stress at 0.2 percent offset strain.

4.1.2. ASTM and ACI Code Specifications

The physical requirements for high-strength deformed reinforcing bars satisfying ASTM A 615-68 are as follows:

<table>
<thead>
<tr>
<th>Type of Steel and ASTM Specifications</th>
<th>Size</th>
<th>Grade</th>
<th>Yield Strength</th>
<th>Ultimate Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billet Steel A615</td>
<td>3-11</td>
<td>60</td>
<td>60,000 psi</td>
<td>90,000 psi</td>
</tr>
<tr>
<td></td>
<td>14,18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11,14,18</td>
<td>75</td>
<td>75,000 psi</td>
<td>100,000 psi</td>
</tr>
</tbody>
</table>

The yield strength is defined as the stress corresponding to a strain of 0.005 for Grade 60 steel, and of 0.006 for Grade 75 steel. The specifications do not specify the stress-strain relationship below and above the yield point.

Section 301 of the 1963 ACI Code specifies that the yield strength or yield point be determined in accordance with the applicable ASTM specifications. In addition, Section 1505 of the Code reads as follows:

"(a) When reinforcement is used that has a yield strength, \( f_y \), in excess of 60,000 psi, the yield strength to be used in design shall be reduced to 0.85 \( f_y \) or 60,000 psi, whichever is greater, unless it is shown
by tension tests that at a proof stress equal to the specified yield strength, \( f_y \), the strain does not exceed 0.003.

"(b) Design shall not be based on a yield strength, \( f_y \), in excess of 75,000 psi. Design of tension reinforcement shall not be based on a yield strength, \( f_y \), in excess of 60,000 psi unless tests are made in compliance with Section 1508(b)."

It is clear from the above that the yield strength of a Grade 60 steel is defined by the ACI Code as that specified by the ASTM. However, the ACI Code and the ASTM specifications differ in the definition of the yield strength for Grade 75 steel. If the stress-strain curve of a reinforcing bar gives a stress of 88.2 ksi at a strain of 0.006, then that bar will be assumed to have \( f_y = 0.85 \times 88.2 = 75 \) ksi, according to the 1963 ACI Code. Furthermore, Section 1503 of the 1963 ACI Code provides that, for the purpose of ultimate strength analysis, the stress-strain relationship of the bar be taken as elasto-plastic as shown in Fig. 4.4.

From the above-mentioned specifications, it can be concluded that, whatever the actual stress-strain curve of the reinforcing bars, the stress-strain relationship shown in Fig. 4.4 can be assumed for ultimate strength analysis if:

(1) The steel stress corresponding to a strain of 0.005 is at least 60,000 psi for Grade 60 steel, and

(2) The steel stress corresponding to the strain of 0.006 is at least 88,200 psi for Grade 75 steel.

In view of the above, the stress-strain curves shown in Fig. 4.5 for steels 5, 6, 7 and 8 satisfy both the ASTM specifications and the ACI Code requirements for Grade 60 steel and, for the purpose of ultimate strength analysis, the stress-strain curve 7 is to be used. The stress-strain curves for steels 1, 2, and 3 shown in Fig. 4.6 satisfy the ACI Code requirements,
while all the steels 1, 2, 3, and 4 satisfy the ASTM specifications; but, for the purpose of ultimate strength analysis, stress-strain curve 2 is to be used only for steels 1 and 3, and a similar flat-top stress-strain curve with $f_y = 0.85 \times 75 = 63.75$ ksi is to be used for steel 4.

4.1.3. **Stress-Strain Curves of Steel Selected for Analyses**

The stress-strain curves obtained from tests of Grade 60 reinforcing bars have been shown in Fig. 4.1. These bars have different yield strengths as well as different lengths of flat plateau. If these stress-strain curves are replotted with ordinates represented as the ratios of steel stress to yield strength, and abscissas as the ratios of steel strain to yield strain it will be found that all of the curves shown in Fig. 4.1 will be included between two nearly parallel curves. Such bounding stress-strain curves are shown in Fig. 4.5 as curves 5 and 6. Therefore, for the purpose of investigating the effects of the stress-strain curves of Grade 60 reinforcing bars, the analyses have been made with the four stress-strain curves shown in Fig. 4.5. These curves are identified in the figure to represent steels 5, 6, 7, and 8. The stress-strain curve for steel 7 is the basic curve used for analysis and design in accordance with the provisions of the 1963 ACI Code. Curves 5 and 6 have been chosen to represent the extreme cases for Grade 60 steel. Curve 8 was so chosen as to satisfy the ASTM specifications but not have a well-defined yield point; thus it represents the square-twisted British steels described in Section 4.1. Therefore, the comparisons of the analyses made with these stress-strain curves will show the effects of the ACI Code assumptions for the stress-strain relationship for reinforcement with regard to essentially all possible stress-strain curves for Grade 60 steels.
Figure 4.6 shows the stress-strain curves that were selected for analyses of reinforced concrete sections with Grade 75 steel. As explained in Section 4.2, curves 1, 2, 3 and 4 all meet the ASTM A615-68 specifications for Grade 75 steel. Comparison of these curves with those obtained from tests, and given in Fig. 4.2, shows that, for those steels which have a yield strength in the neighborhood of 75 ksi, stress-strain curves 1 and 4 are the extremes which bound the test curves of Fig. 4.2, except those having very high yield strength. Any stress-strain curve that lies below curve 3 does not satisfy the provisions of the 1963 ACI Code. Therefore, curve 3 was chosen to represent the lower limit of the ACI Code requirements. Like stress-strain curve 7 for Grade 60 steel, curve 2 represents the assumptions of the ACI Code for the stress-strain relationship of reinforcing bars for the purpose of ultimate strength analysis of reinforced concrete sections with Grade 75 steel, that is, instead of curves 1 and 3, which are more typical of steels meeting the yield strength requirements of the ACI Code, curve 2 is assumed for analysis in accordance with Chapter 15 of the Code. Comparisons of the results of analyses using stress-strain curves 1 and 3 with those obtained for steel 2 will show the effect of the assumptions of the ACI Code. Similarly, comparisons of the results of analyses using steels 2 and 4 will show the differences that could be obtained if steels that meet the requirements of ASTM specifications (but not those of the ACI Code) are used, in comparison with those that have an elasto-plastic stress-strain relationship, as assumed in the ACI Code.
4.2. Concrete

4.2.1. Concrete Stress-Strain Curves Obtained from Tests by Various Authors

Figure 4.7 shows stress-strain curves for concrete obtained in various laboratories. The three curves shown by full lines were obtained by the U.S. Bureau of Reclamation (13) from tests on 3x6-in. concrete cylinders and represent concrete of three different strengths and ages. The curves shown by small broken lines were obtained by Rüsch (14). His specimen was a prism 4 x 6 in. in cross-section with thickened ends 6 in. square. This curve represents tests at a constant strain rate of 0.001 per hour. The gage length in the tests was 12 in. The stress-strain curves with long broken lines were obtained by Barnard (11) from tests on necked specimens of 2.52-in. diameter in the neck and 4.5 in. at the ends. He used a gage length of 4 in. It can be seen in Fig. 4.7 that all of the stress-strain curves have approximately the same shape--more or less parabolic--up to the maximum stress at a strain $\varepsilon_0$ which varies from 0.0015 to 0.0025. The stress-strain curves obtained by the U.S. Bureau of Reclamation and by Barnard have similar shapes in the falling branch beyond $\varepsilon_0$, but that obtained by Rüsch falls off rapidly. Perhaps this difference is due to the use of a long gage length of 12 in. by Rüsch, while Barnard used a much shorter gage length of 4 in. The height of Rüsch's specimen in the reduced cross-section was about the same as the gage length whereas that of Barnard's specimen was almost twice the gage length. The ratio of gage length to the least lateral dimension in Rüsch's tests was 3 but that in Barnard's tests was 1.56. The gage length used in the Bureau of Reclamation tests is not known but can not be more than 6 in.--the height of the specimen. Since the crushing or failure of a specimen is confined to a small
portion of the length of the specimen, and considerably larger strains are
developed in this region than at other portions of the specimen, longer
gage lengths in relation to the dimensions of the specimen will measure
average strains but not the maximum strains that can be measured over a
smaller gage length if the failure occurs within the gage length. Because
the stress is the same for both gage lengths, stress-strain relationship
obtained with shorter gage length will give a flatter falling branch of the
stress-strain curve than will a longer gage length. In reinforced concrete
members, failure occurs in the region of maximum strains, and the strain-
distribution over the depth in the failure region governs the strength
of the member. Consequently, it is assumed that the concrete stress-strain
curve having the shape of the falling branch similar to that obtained by
Barnard (11) is more representative of the behavior of reinforced concrete
members.

4.2.2. Concrete Stress-Strain Curves Proposed by Various Authors

Figure 4.8 shows stress-strain curves proposed for concrete by
various authors. The plotted curves have been obtained from equations
giving the stress-strain relationship for concrete having a cylinder strength
of 4000 psi. These equations and the corresponding curves are explained
below:

(1) Hognestad (3)

Based on the results of his tests on eccentrically loaded
reinforced concrete columns, Hognestad proposed the following equation for
the stress-strain relationship of concrete.

(a) For $0 < \varepsilon \leq \varepsilon_0$
where \( f_c \) = concrete stress
\( \varepsilon \) = concrete strain corresponding to \( f_c \)
\( f'' \) = maximum concrete stress or strength of the concrete in the member
\( \varepsilon_0 \) = the strain corresponding to \( f'' \).

Hognestad proposed also that
\[
\varepsilon_0 = 2f''/E_c
\]
where \( E_c \) = initial modulus of elasticity of concrete.
\[
= 1,800,000 + 460f''
\]
so that \( f_c = E_c(1 - E_c/(4f'')) \)

The ultimate concrete strain \( \varepsilon_u \) recommended by Hognestad is equal to 0.0038. His reinforced concrete columns contained mild steel having yield strengths varying from 38.7 to 47.1 ksi.

(b) From \( \varepsilon_0 \) to \( \varepsilon_u \), Hognestad proposed a straight line with \( f_c \) at \( \varepsilon_u = 0.0038 \) equal to 0.85 \( f'' \).

For \( f' = 4000 \) psi
\( f'' = 3400 \) psi
\( E_c = 3,365,000 \) psi
\( \varepsilon_0 = 0.00202 \)

Hognestad's curve for the above values of \( f'' \) and \( \varepsilon_0 \) is shown in Fig. 4.8 as curve 1.

(2) **Sahlin (4)**

Sahlin proposed an exponential form of equation for the stress-strain relationship of concrete. His equation is:
where \( \sigma = \text{stress in concrete} \)
\[
\sigma = \sigma_m (\varepsilon / \varepsilon_0) e^{(1-\varepsilon/\varepsilon_0)}
\]

where \( \sigma \) = stress in concrete
\( \sigma_m \) = maximum concrete stress
\( \varepsilon \) = concrete strain corresponding to \( \sigma \)
\( \varepsilon_0 \) = concrete strain corresponding to \( \sigma_m = 0.002 \)

For \( \sigma_m = f''_c = 3400 \text{ psi} \), Sahlin's curve is shown in Fig. 4.8 as curve 2.
This is a continuous curve, without any limit on \( \varepsilon_u \).

(3) **Liebenberg (15)**

Liebenberg proposed the following equation for the stress-strain relationship of concrete:

\[
f = A_s \sqrt{u_{cyl}} (1 - B \varepsilon^n)
\]

where \( f \) = concrete stress
\( A_s \) = 67,000 lb \( \text{l}^{1/2} / \text{in.} \)
\( u_{cyl} \) = cylinder strength of concrete
\( B \) = \( 1 / [(n + 1) \varepsilon_m n] \)
\( n \) = \( f_{\text{max}} / (A_s \sqrt{u_{cyl}} \varepsilon_m - f_{\text{max}}) \)
\( \varepsilon_m \) = 0.002
\( f_{\text{max}} \) = maximum concrete stress from experimental results
\( \varepsilon \) = concrete strain corresponding to \( f \)

Liebenberg's proposed curve for concrete having a cylinder strength of 4000 psi is shown in Fig. 4.8 as curve 3.
The following equation for stress-strain curve of concrete was proposed by Todeschini:

\[ f_c = 2f''(\varepsilon/\varepsilon_0)/(1 + (\varepsilon/\varepsilon_0)^2) \]

where \( f_c \) = concrete stress

\( f'' = \text{maximum concrete stress} = 0.85f' \)

\( \varepsilon = \text{concrete strain corresponding to} f_c \)

\( \varepsilon_0 = \text{concrete strain at maximum stress} \)

\( f' = \text{maximum concrete stress as determined from cylinder tests} \)

Todeschini's curve for \( f' = 4000 \text{ psi} \) is shown in Fig. 4.8 as curve 4.

It can be seen in Fig. 4.8 that the stress-strain curves for concrete proposed by Leibenberg and Sahlin fall off rapidly beyond the strain \( \varepsilon_0 \). Leibenberg's curve falls off more steeply than Sahlin's.

Nylander and Sahlin (5) tested reinforced concrete continuous beams and based on the comparisons of their analytical results using Sahlin's curve with the experimental results, it can be concluded that the falling branch of the curve proposed by Sahlin is too steep. This will be further explained in detail in Section 4.2.6. The differences in the maximum stress for Liebenberg's curve and for the other three curves shown in Fig. 4.8 are due to the different values of the constant \( k_3 \) which relates the concrete strength in the member with the cylinder strength of concrete \( f'_c \). Curves 1, 2, and 4 have \( k_3 = 0.85 \), so that the maximum stress \( f'' \) is equal to 0.85 \( f'_c = 3400 \text{ psi} \); Liebenberg, however, has taken \( f'' \) greater than 0.90 \( f'_c \) depending on the experimental results with which he compared his equation for the curve.
4.2.3. **Rectangular Stress Block Specified by the ACI Code**

Sections 1503 (f) and (g) of the 1963 ACI Code read as:

''(f) At ultimate strength, concrete stress is not proportional to strain. The diagram of compressive concrete stress distribution may be assumed to be a rectangle, trapezoid, parabola, or any other shape which results in prediction of ultimate strength in reasonable agreement with the results of comprehensive tests.

''(g) The requirements of (f) may be considered satisfied by the equivalent rectangular concrete stress distribution which is defined as follows: At ultimate strength, a concrete stress intensity of $0.85f'_c$ shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance $k_1c$ from the fiber of maximum compressive strain. The distance $c$ from the fiber of maximum compressive strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction $k_1$ shall be taken as 0.85 for strengths, $f'_c$, up to 4000 psi and shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi.''

Figure 4.9 shows the strain and stress distribution over a 15-in. square section. Figure 4.9(d) shows the equivalent rectangular stress block in accordance with Section 1503 (g) of the ACI Code, stated above. As per the Commentary on the ACI Building Code Requirements (16), this rectangular stress distribution does not represent the actual stress distribution in the concrete compression zone, but has been recommended only to enable the designer to calculate quickly the compression force in the concrete and its centroid. However, Section 1503 (f) permits the use of
any other concrete stress distribution in the compression zone if the results of ultimate strength analyses using the assumed stress-strain relationship for concrete are in good agreement with comprehensive tests. Therefore, in the analyses of a reinforced concrete section carried out herein, concrete stress-strain relationships other than the rectangular stress block have been assumed, as explained in Section 4.2.5.

4.2.4. Comparison of Equivalent Rectangular Stress Block with Other Stress-Strain Curves

Figure 4.10 compares the equivalent rectangular stress block, specified by the 1963 ACI Code for the purpose of calculating the compression force in concrete for ultimate strength analysis, with two stress-strain curves, one proposed by Hognestad and the other by Todeschini. Hognestad found good comparisons of analytical results using his curve with the experimental results obtained from a large number of tests on eccentrically loaded columns. Todeschini tested a limited number of reinforced concrete columns, and obtained a good correlation between analytical and experimental results. Comparisons of Figs. 4.7 and 4.8 show that the stress-strain curves proposed by Hognestad and by Todeschini are very close to the curves obtained from tests. Both of these authors have taken the value of $k_3 = 0.85$, as does the ACI Code. The values of the constants $k_1$, $k_2$, and $k_3$ for these curves are compared below in all cases for $f'_c = 4000$ psi:

<table>
<thead>
<tr>
<th></th>
<th>Hognestad</th>
<th>Todeschini</th>
<th>ACI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_u$</td>
<td>0.0030</td>
<td>0.0038</td>
<td>0.0030</td>
</tr>
<tr>
<td>$k_1k_3$</td>
<td>0.648</td>
<td>0.668</td>
<td>0.667</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.410</td>
<td>0.432</td>
<td>0.414</td>
</tr>
</tbody>
</table>
It is clear from the above that, at $\epsilon_u = 0.003$, the equivalent rectangular stress block of the ACI Code gives higher values of $k_1 k_3$ and $k_2$ than either of the curves proposed by Hognestad or Todeschini. At $\epsilon_u = 0.0038$, as proposed by Hognestad, $k_1 k_3$ from Hognestad's curve is less than that of the rectangular stress block. Similarly the maximum value of $k_1 k_3$ for Todeschini's curve, which occurs at a strain $\epsilon_m = 0.004$, as shown in Fig. 4.12, is less than that of the rectangular stress block. Both the curves proposed by Hognestad and Todeschini have an initial tangent modulus $E_c = 3,400,000$ psi as compared to the value of $E_c = 3,660,000$ psi calculated for $f'_c = 4000$ psi according to Section 1102 (a) of the 1963 ACI Code. Curve (d) in Fig. 4.10 was obtained by a trial-and-error procedure to give approximately the same values of $k_1 k_3$ and $k_2$ as given by the equivalent rectangular stress block. It gives $k_1 k_3 = 0.848$ and $k_2 = 0.414$. However, the initial tangent modulus of this curve is 5,600,000 psi, which is very high. Therefore, such a curve is not representative of the concrete stress-strain relationship.

If the stress-strain curve of concrete is taken as a parabola up to $\epsilon_0$, as proposed by Hognestad, and $k_3$ is taken equal to 0.85, then it is not possible to obtain a curve with a falling branch which would give the same value of $k_1 k_3$ as the equivalent rectangular stress block of the ACI Code. However, if it is assumed that the stress-strain curve consists of a parabola from $\epsilon = 0$ to $\epsilon = \epsilon_0$ and has a flat plateau from $\epsilon_0$ to $\epsilon_u$, then such a curve will give $k_1 k_3 = 0.7225$ at $\epsilon_u = 0.00322$. But, this stress-strain curve also is not possible for a normal unconfined concrete. Therefore, the equivalent rectangular stress block gives a higher value of $k_1 k_3$ than any realistic curve representing the stress-strain relationship for concrete.
4.2.5. Concrete Stress-Strain Curves Selected for Use in Analyses

Comparison of Figs. 4.7 and 4.8 shows that Todeschini's curve, shown in Fig. 4.8 as curve 4, compares quite well with the test curves obtained by Barnard (11). Todeschini's curve is also similar to Hognestad's curve up to the strain $\varepsilon_u = 0.0038$ proposed by Hognestad. However, Hognestad's curve was not selected for use in the analyses because, its falling branch is a straight line and $\varepsilon_u$ is limited to 0.0038. In the method of analysis explained in Chapter 3, a higher limit of ultimate concrete strain is considered and, although the straight line representing the falling branch of Hognestad's curve could be extended up to any value of ultimate concrete strain, a continuous curve representing both the ascending and descending branches of concrete stress-strain curve is preferable. The stress-strain curves proposed by Sahlin and Liebenberg also have not been used here because these curves are too steep in the falling branch, as explained in Section 4.2.2. Therefore, Todeschini's curve given by the equation in Section 4.2.2 has been selected for use in the analyses. Such curves for three values of $f'_c = 3000, 4000,$ and $5000$ psi are shown in Fig. 4.11. For these curves, Fig. 4.12 shows the curves for the values of constants $k_1, k_2$ and $k_3$ for various values of concrete strain.

4.2.6. Useful Limit of Ultimate Concrete Strain in Extreme Compression Fiber

In the method of analysis explained in Chapter 3, the ultimate capacity of a reinforced concrete section is determined from the moment-strain curves in accordance with the criteria stated and explained in Section 3.3. Depending on the value of ultimate axial load and the stress-strain relationship of the reinforcing bars and concrete, the moment-strain
diagram may continue to rise as the strain in the extreme compression fiber increases. This will be discussed in Section 5.2 and shown in Figs. 5.1 and 5.2. However, at higher values of $\varepsilon_4$, the concrete in the compression zone is "damaged"; the extent of damage depends on the concrete strains. At some value of $\varepsilon_4$, the concrete in the compression zone is so damaged that it is no longer able to provide the required compression force to resist the applied external forces, and failure of the section occurs. This value of $\varepsilon_4$ is considered to be the useful limit of ultimate concrete strain in the extreme compression fiber. In the method of analysis explained in Chapter 3, this limit of $\varepsilon_4$ is taken equal to 0.010. Such a high limit was put on $\varepsilon_4$, as compared to the value 0.003 specified in the 1963 ACI Code, because of the following considerations.

(1) From the stress-strain curves obtained by Barnard (11), which are shown in Fig. 4.7, it can be seen that considerably higher strains, beyond 0.010, can be measured on concentrically loaded specimens. Barnard's tests show that, even at a strain of 0.0106, the specimen did not collapse, although it was well cracked. The technique of testing the specimen and measuring concrete strains is important in obtaining the falling branch of stress-strain curve. Beyond the maximum stress, concrete undergoes strain at reduced stress. Consequently, if the testing equipment releases the energy suddenly, the cracked concrete will rupture suddenly and very small strains will be measured.

(2) Larger strains are developed in the crushing zone of the concrete than in the regions outside it; therefore, the extent to which concrete strains can be measured depends on the gage length in comparison with the height and cross-section of the specimen, and also on the location
of the gage with respect to the failure region. Thus, for the same specimen, a longer gage length will give an average strain over the gage length, and hence a smaller strain than will a shorter gage length if the failure or crushing occurs within the gage length. However, if the crushing occurs outside the gage length, a considerably smaller strain will be measured. In reinforced concrete members, the local strains in the crushing zone cause the collapse and are, therefore, important in analyzing the reinforced concrete sections.

(3) The ultimate concrete strain that can be measured in tests of beams and columns depends on (a) the moment-strain relationship for the cross-section of the member in the failure region, and (b) the equipment used for the measurement of strains. If the moment-strain diagram peaks at a strain less than 0.010, it will be difficult to measure the strains beyond the value at the peak moment, particularly in tests of columns, because the energy is suddenly released from the testing apparatus and instability occurs after reaching the peak moment. But, if the moment-strain diagram continues to rise with an increase in strain, instability should not occur, and the specimen should continue to resist additional moment while undergoing large deformations. In this case, the strain measured depends on the type of instrumentation used. Mechanical strain gages used to measure changes in length between two gage points do not work at large strains because the gage points are physically dislocated as concrete begins to spall at the surface. For the same reason, electric resistance strain gages also become inoperative when large local deformations or spalling occurs at the gage location. In the beam tests reported in Ref. (17) and (18), linear differential transformer gages were used with continuously
recording equipment, and ultimate concrete strains up to 0.058 were recorded from tests of beams under a single point load. The moment gradient obtained under this loading was responsible for some confining effect on the concrete, and this would account for the very large concrete strains measured. In reinforced concrete frames, there is a very sharp variation of moment near the joint of beams and columns where maximum moment occurs. Also, in this region, there is a large shear force for which a considerable amount of web reinforcement in the form of stirrups is provided. Both these factors produce some confining effect, and strains greater than 0.010 would be obtained in the failure region.

(4) Nylander and Sahlin (5) tested continuous reinforced concrete beams, and measured very large concrete strains in the compression zone, up to a maximum of about 0.050. The interior supports, to provide continuity in their tests, were 200 mm apart, and the widths of the test beams were 100 and 200 mm, while the depth of the beams was 120 mm. Concrete strains were measured over a gage length of 200 mm between the interior supports. These dimensions suggest that there must have been some confinement of concrete between the interior supports, which could have helped in permitting these large strains. However, such large strains were measured even when the moment-strain curves were falling off slightly, and the maximum moment had occurred at considerably lower strains. In most of the beams, premature failure had occurred due to shear or fracture of anchorages. In those beams where precautions were taken to prevent anchorage failures, very large concrete strains, ranging from 0.012 to 0.047, were measured in the main test beams as well as in the short control beams. Considering the measured moment-strain diagrams, it can be concluded that a limit of \( \varepsilon_u = 0.010 \) is on the conservative side.
(5) It will be shown in Chapters 5, 7 and 8 that it is only at low levels of ultimate axial load that the ultimate moment occurs at $\varepsilon_u = 0.010$. As the ultimate axial load increases, the depth to the neutral axis increases, and the value of ultimate concrete strain decreases. For Grade 60 steel, at ultimate axial load levels below the balance point, the depth to the neutral axis is very small; consequently, there is some confining effect in concrete and the limiting value of $\varepsilon_u = 0.010$ is justified. However, at ultimate axial load levels above the balance point, the ultimate concrete strain is less than 0.006 for all values of $p_t$ and $f'_c$ used in the analyses in Chapter 7. At higher loads, the greater depth to the neutral axis does not provide the confinement that is available at lower load levels, and smaller values of the ultimate concrete strain should be obtained which is what the analysis predicts. For Grade 75 steel, it will be explained in Chapter 8 that, as $q_t = p_t f'_y / f'_c$ increases, moment-strain curves continue to rise up to $\varepsilon_u > 0.010$ even at higher values of ultimate axial load. However, in such cases, buckling of compression bars governs the ultimate capacity rather than the useful limit of ultimate concrete strain.

In view of the above-mentioned considerations, it seems justified to assume that the useful limit of ultimate concrete strain $\varepsilon_u$ may be taken at least as high as 0.010.
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5. TYPICAL INTERACTION DIAGRAM

5.1. Introduction

In this chapter, the analysis and the behavior of a typical reinforced concrete section subjected to various combinations of ultimate axial load and bending moment are explained in detail. It is intended that this chapter will provide a basis for the explanations of the factors that influence the strength and behavior of the section when the strain-hardening region of the stress-strain curve of the reinforcing bars is considered in the analysis.

The method of analysis explained in Chapter 3 has been used to obtain a complete interaction diagram for the 15-in. square reinforced concrete section shown in Fig. 5.3. The section consists of three #11 reinforcing bars in each face parallel to the axis of bending. In order to satisfy the provisions of the 1963 ACI Code for lateral ties, Section 806(b), and for the cover over the lateral ties, Section 808(c), the values of \(d'\) and \(d\) were taken as 2.45 and 12.55 in., respectively. The stress-strain curve for steel 5 shown in Fig. 4.5 was selected for the reinforcing bars, with a yield strength \(f_y = 60\) ksi. The concrete was assumed to have a cylinder strength \(f'_c\) equal to 4000 psi. The stress-strain curve shown in Fig. 4.11 was selected for the concrete. The value of the coefficient \(k_3\), as explained in Section 4.2.2, was taken equal to 0.85. The above section properties give total steel ratio \(p_t = 0.0416\) and \(q_t = p_t f_y / f'_c = 0.625\).

5.2. Moment-Strain Diagrams

As explained in Section 3.4, the moment-concrete strain \((M - \epsilon_4)\) diagrams were obtained for 11 values of axial load, including zero axial
load (pure moment) for the reinforced concrete section described in Section 5.1. These $M - \varepsilon_4$ curves are shown in Fig. 5.1, with the axial load $P_u$ indicated for each curve. The peak of each moment-strain curve is marked by a small vertical line and a number which corresponds to that shown subsequently on the interaction diagram in Fig. 5.3.

It can be seen in Fig. 5.1 that for $P_u = 0$ and $P_u = 100$ kips, the $M - \varepsilon_4$ curves continue to rise as $\varepsilon_4$ is increased. These $M - \varepsilon_4$ curves are repeated in Fig. 5.2 for strains up to $\varepsilon_4 = 0.030$. The stress-strain curve for the concrete is also shown in Fig. 5.2. It has been extended to a strain of 0.030 using the equation proposed by Todeschini et al (7). This figure shows that $M$ continues to increase for $P_u = 0$ and 100 kips even if $\varepsilon_4$ is increased to 0.030. This is explained below:

At these values of ultimate axial load, equilibrium of forces occurs at such strain distributions that, at low values of $\varepsilon_4$, the compression steel strain $\varepsilon_3$ is in the elastic region, and the tension steel strain is in the strain-hardening region. As $\varepsilon_4$ is increased, the compression and tension steel strains are also increased, which increases the tension and compression steel forces and their moments about the plastic centroid. At $\varepsilon_4 > \varepsilon_m$, the value of $k_1 k_2$ decreases, which reduces the compression force in the concrete. A small increase in the depth to the neutral axis increases the area of concrete in compression, which increases the compression force in the concrete, but not as much as the decrease in the same force caused by the decrease in $k_1 k_2$. Thus the compression force in the concrete is reduced and this reduction is compensated for by the increase in the compression steel force explained above. The increase in the depth to the neutral axis, coupled with the increase in the value of $k_2$ at higher value of $\varepsilon_4$,
reduce the lever arm for the concrete force, which reduces the moment of the concrete force. But, the increases in the moments of the tension and compression steel forces explained above more than compensate for the reduction in the moment of the concrete force. Therefore, there is a net increase in the total moment and the \( M - \varepsilon_4 \) curve continues to rise as \( \varepsilon_4 \) is increased.

Figure 5.1 shows that, at an ultimate axial load of \( P_u = 200 \text{ kips} \), the maximum value of moment occurs at \( \varepsilon_4 = 0.0045 \) (point 3), but the \( M - \varepsilon_4 \) curve is quite flat. The reduction in moment at \( \varepsilon_4 = 0.010 \) is only 1 percent of the maximum moment at \( \varepsilon_4 = 0.0045 \). As shown in Fig. 5.2, if \( \varepsilon_4 \) is increased beyond 0.010, the moment continues to decrease up to \( \varepsilon_4 = 0.014 \), but increases again as \( \varepsilon_4 \) is increased further until, at \( \varepsilon_4 = 0.026 \), the moment exceeds the previous maximum value at \( \varepsilon_4 = 0.0045 \). This is explained below:

The depth to the neutral axis increases as the axial load \( P_u \) increases. This increases the area of concrete in compression as well as the concrete force. At the same time, the lever arm is reduced. As explained above for the smaller axial loads, the reduction in the concrete force caused by the reduction in the value of \( k_1 k_2 \) is compensated for by the increase in the compression steel force, until the latter yields. For \( P_u = 200 \text{ kips} \), the compression steel yields at \( \varepsilon_4 \geq 0.004 \), which requires that the depth to the neutral axis must increase to increase the area of concrete under compression and the concrete force. This reduces the lever arm of the concrete force which in turn reduces the moment of the concrete force. For \( 0.0045 < \varepsilon_4 < 0.014 \), the reduction in the moment of the concrete force is more than the total increase in the moments of the tension and
compression steel forces, which gives a net reduction in the total moment, though this reduction is very small.

At high values of $\varepsilon_u$, the compression steel goes into the strain-hardening region and its force increases as $\varepsilon_u$ is increased. At very high values of $\varepsilon_u > 0.014$, the increase in the compression steel force more than compensates for the decrease in the concrete force, and the total increase in the moments of the tension and the compression steel forces about the plastic centroid exceeds the decrease in moment of the concrete force. This gives a net increase in the total moment, and the $M - \varepsilon_u$ curve again rises.

It can be seen in Fig. 5.2 that, at a very high concrete strain $\varepsilon_u'$, the stress-strain curve for concrete tends to become flat, as do the curves for $k_1 k_2$ and $k_2$ shown in Fig. 4.12. Also, at very high values of $\varepsilon_u$, the compression steel strain is so high that it lies in that region of the stress-strain curve which tends to become flat. As a result, an increase in strain produces very small changes in the compression forces in the concrete and in the steel. The equilibrium of forces at very high values of $\varepsilon_u$ is obtained with very small changes in the depth to the neutral axis and the lever arm. Consequently, there is very little increase in the moment at very high values of $\varepsilon_u$. Furthermore, this increase in moment is reduced as the ultimate axial load $P_u$ is increased, and the $M - \varepsilon_u$ curves for higher ultimate axial loads tend to become flat at the higher values of $\varepsilon_u$.

For $P_u = 300$ kips, which is very close to the balance point, the $M - \varepsilon_u$ curve is almost flat at the very high values of $\varepsilon_u$ shown in Fig. 5.2. At higher values of ultimate axial load, instead of a net increase, there occurs a net decrease in the moment, as the concrete force and its moment
are reduced more and more. This is so because more area of the concrete is in compression and the concrete plays a greater part in bringing about equilibrium of forces. Since it is the concrete force that tends to reduce the total moment which must be compensated for by the compression steel, more and more reduction in the concrete force and moment takes place as \( P_u \) increases. When the compression steel is not able to compensate for such a large reduction in the concrete force and its moment about the plastic centroid, the \( M - \varepsilon_4 \) curves fall off at the higher concrete strains, as can be seen in Fig. 5.1.

Although, theoretically the \( M - \varepsilon_4 \) curves for low values of axial load \( P_u \) continue to rise as \( \varepsilon_4 \) increases, even to such high values as shown in Fig. 5.2, practically it is not possible to load the reinforced concrete section to such high strains because the concrete covering the reinforcement will crush and fall off and, with normal spacing of lateral ties, the compression bars will buckle at high strains. Therefore, a useful limit of concrete strain has been taken equal to 0.010. This has been explained in detail in Section 4.2.6. In view of this, the \( M - \varepsilon_4 \) curves have been limited to 0.010 as shown in Fig. 5.1.

5.3. Load-Moment and Load-Curvature Diagrams

The ultimate moment was obtained from the \( M - \varepsilon_4 \) curves of Fig. 5.1 for each of the ultimate axial load values shown in that figure in accordance with the criteria explained in Section 3.3. This gives a set of ultimate moments corresponding to the chosen ultimate axial loads. These values of ultimate axial load and the corresponding ultimate moment are plotted in Fig. 5.3. The numbered points relate the ultimate axial
compression steel forces, which gives a net reduction in the total moment, though this reduction is very small.

At high values of \(\epsilon_4\), the compression steel goes into the strain-hardening region and its force increases as \(\epsilon_4\) is increased. At very high values of \(\epsilon_4 > 0.014\), the increase in the compression steel force more than compensates for the decrease in the concrete force, and the total increase in the moments of the tension and the compression steel forces about the plastic centroid exceeds the decrease in moment of the concrete force. This gives a net increase in the total moment, and the \(M - \epsilon_4\) curve again rises.

It can be seen in Fig. 5.2 that, at a very high concrete strain \(\epsilon_4\), the stress-strain curve for concrete tends to become flat, as do the curves for \(k_1, k_2\) and \(\epsilon_4\) shown in Fig. 4.12. Also, at very high values of \(\epsilon_4\), the compression steel strain is so high that it lies in that region of the stress-strain curve which tends to become flat. As a result, an increase in strain produces very small changes in the compression forces in the concrete and in the steel. The equilibrium of forces at very high values of \(\epsilon_4\) is obtained with very small changes in the depth to the neutral axis and the lever arm. Consequently, there is very little increase in the moment at very high values of \(\epsilon_4\). Furthermore, this increase in moment is reduced as the ultimate axial load \(P_u\) is increased, and the \(M - \epsilon_4\) curves for higher ultimate axial loads tend to become flat at the higher values of \(\epsilon_4\).

For \(P_u = 300\) kips, which is very close to the balance point, the \(M - \epsilon_4\) curve is almost flat at the very high values of \(\epsilon_4\) shown in Fig. 5.2. At higher values of ultimate axial load, instead of a net increase, there occurs a net decrease in the moment, as the concrete force and its moment
are reduced more and more. This is so because more area of the concrete is in compression and the concrete plays a greater part in bringing about equilibrium of forces. Since it is the concrete force that tends to reduce the total moment which must be compensated for by the compression steel, more and more reduction in the concrete force and moment takes place as $P_u$ increases. When the compression steel is not able to compensate for such a large reduction in the concrete force and its moment about the plastic centroid, the $M - \epsilon_4$ curves fall off at the higher concrete strains, as can be seen in Fig. 5.1.

Although, theoretically the $M - \epsilon_4$ curves for low values of axial load $P_u$ continue to rise as $\epsilon_4$ increases, even to such high values as shown in Fig. 5.2, practically it is not possible to load the reinforced concrete section to such high strains because the concrete covering the reinforcement will crush and fall off and, with normal spacing of lateral ties, the compression bars will buckle at high strains. Therefore, a useful limit of concrete strain has been taken equal to 0.010. This has been explained in detail in Section 4.2.6. In view of this, the $M - \epsilon_4$ curves have been limited to 0.010 as shown in Fig. 5.1.

5.3. Load-Moment and Load-Curvature Diagrams

The ultimate moment was obtained from the $M - \epsilon_4$ curves of Fig. 5.1 for each of the ultimate axial load values shown in that figure in accordance with the criteria explained in Section 3.3. This gives a set of ultimate moments corresponding to the chosen ultimate axial loads. These values of ultimate axial load and the corresponding ultimate moment are plotted in Fig. 5.3. The numbered points relate the ultimate axial
load and moment plotted in Fig. 5.3 to the corresponding points on Figs. 5.1 and 5.2. The strain distribution across the section at which the ultimate moment occurs is also shown in Fig. 5.4 for each of the ultimate axial load levels. The curvatures obtained from these strain distributions are plotted against the corresponding ultimate axial loads in Fig. 5.4.

The load-curvature diagram of Fig. 5.4 shows that, below the balance point, very large curvature is obtained at ultimate capacity. In this region, the ultimate concrete strain $\varepsilon_u$ is quite large and the depth to the neutral axis $kd$ is quite small. Since the ultimate curvature $\Phi_u$ is the ratio $\varepsilon_u/kd$, large curvatures result.

5.4. Strain Distributions for Moment-Strain Diagram

As explained in Section 3.4, the $M - \varepsilon_4$ diagram at a given ultimate axial load is obtained by taking various values of $\varepsilon_4$ and finding equilibrium of forces at each. This gives different strain distribution over the depth of the section at each value of $\varepsilon_4$. These strain distributions are shown in Figs. 5.5 and 5.6 for two axial load levels, $P_u = 200$ and 500 kips, respectively. The lower axial load level is below the balance point while the higher is above. Both of these strain-distribution diagrams show that for equilibrium of forces for increasing values of $\varepsilon_4$:

(a) the compression steel strain $\varepsilon_3$ is increased,
(b) the tension steel strain $\varepsilon_2$ is increased,
(c) the curvature $\Phi$ is increased, and
(d) the depth to the neutral axis $kd$ is reduced until the compression steel yields; thereafter $kd$ is increased.
Items (a), (b) and (c) above are obvious from the strain distributions in Figs. 5.5 and 5.6, and need no explanation. The change in the depth to the neutral axis is explained below:

It can be seen in Figs. 5.5 and 5.6 that, before the compression steel yields, the value of $\varepsilon_4$ is less than or equal to $\varepsilon_m$, so that $k_1k_3$ increases as $\varepsilon_4$ is increased. This means that both the compression steel force and the concrete force increase as $\varepsilon_4$ is increased. The tension steel strain usually lies in that region of stress-strain curve for which the stress, and hence the force in the tension steel, do not increase rapidly enough to balance the increase in the total compression forces. Therefore, the depth to the neutral axis must decrease in order to slow the increase in the compression steel strain and hence its force, and at the same time, the area of concrete under compression is reduced which reduces the increase in the concrete force caused by the increase in $k_1k_3$. This further increases the tension steel strain and its force, to obtain equilibrium of forces.

After the compression steel has yielded, the value of $\varepsilon_4$ exceeds $\varepsilon_m$ so that $k_1k_3$ is reduced as $\varepsilon_4$ is increased, and thus the concrete force is reduced. The compression steel force remains constant, if $\varepsilon_3$ is on the flat plateau, or increases slowly, if $\varepsilon_3$ is in the strain-hardening region. In the former case, the reduction in the concrete force has to be fully compensated for by increasing the area of concrete in compression, while in the latter case, the reduction in the concrete force is partly compensated for by the increase in the compression steel strain and hence its force, and partly by the increase in the area of concrete in compression. In both cases, the depth to the neutral axis is increased to obtain more area of concrete in compression.
5.5. Variation of Steel and Concrete Strains with Ultimate Axial Load

Figure 5.7 shows strains and stresses in the tension and compression steel, and in the extreme compression fiber of the concrete at the ultimate capacity of the reinforced concrete section whose properties are given in Section 5.1. These stresses and strains are marked on the stress-strain curves for the tension and the compression steels and for the concrete by a small line and a number which identifies the ultimate axial load level at which the ultimate moment capacity was calculated.

The numbers 1 through 12 correspond to those identifying ultimate axial load levels in Figs. 5.1 through 5.4. For example, the number 3 on Fig. 5.7 indicates that, at the ultimate axial load level 3, corresponding to $P_u = 200$ kips, (in Fig. 5.1 and 5.2), the ultimate moment was reached at such a strain distribution that:

- The ultimate concrete strain $\varepsilon_u = 0.00450$
- The compression steel strain $\varepsilon_3 = 0.00256$ (comp.)
- The compression steel stress $f'_s = 60$ ksi (comp.)
- The tension steel strain $\varepsilon_2 = 0.00546$ (tens.)
- The tension steel stress $f_s = 63.44$ ksi (tens.)

It can be seen in Fig. 5.7 that the ultimate moment of the section at various axial load levels does not occur at a constant value of the ultimate concrete strain $\varepsilon_u$. This is contrary to the assumption of Section 1503 (c) of the 1963 ACI Code. The calculated ultimate concrete strain decreases as the ultimate axial load increases. At ultimate axial load levels 1, 2 and 3, below the balance point, the ultimate concrete strain is significantly greater than the value of 0.003 assumed in the ACI Code. Figures 5.1 and 5.2 show that, at small values of the ultimate
axial load, there is significant difference between the moment at $\epsilon_4 = 0.003$ and at $\epsilon_4 = \epsilon_u > 0.003$. This effect of the limiting ultimate concrete strain is discussed in detail for additional cases in Section 7.2.

Figure 5.7 shows that the tension steel strain is reduced as the ultimate axial load is increased. At ultimate axial load levels below the balance point, the tension steel strain lies in the strain-hardening region of the stress-strain curve. Between the balance point and an ultimate axial load approximately equal to two-thirds of the axial load capacity with zero eccentricity ($2/3 P_o$), the tension steel strain lies in the tensile elastic region, and at ultimate axial loads greater than about $2/3 P_o$ it lies in the compressive elastic region. The compression steel strain lies in the lower portion of the strain-hardening region at ultimate axial load levels below the balance point, but at and above the balance point the compression steel strain is close to the value at yielding. For a steel having a stress-strain curve with a long flat plateau, such as steel 6 (Fig. 4.5), the compression steel strain will lie on the flat plateau at all axial load levels. But, for a steel having a round-house stress-strain curve such as that for steel 8 (Fig. 4.5) the compression steel strains will lie in that region of the stress-strain curve which is below the curve for a normal flat-top steel.
6. COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

6.1. Introduction

In this chapter, reinforced concrete beams and columns tested in various laboratories are analyzed by the method of analysis explained in Chapter 3, and the analytical results compared with the experimental results. This will therefore test the validity of the assumptions made in the analysis.

As explained in Chapter 3, the method of analysis is based on the use of realistic representations of the properties of the materials. Therefore, the following properties must be available before any comparison can be made with the test results.

(a) Actual stress-strain curves for the reinforcing bars used in the specimens.

(b) Cylinder strength of concrete $f'_c$ used in the specimens or a reasonable estimate of the strength of concrete in the specimens.

(c) Dimensions of the specimens including area of the reinforcement.

(d) Loading conditions.

Although the method of analysis explained in Chapter 3 is general, and can be used for analysis of reinforced concrete sections with any type of steel, the purpose of this method of analysis is to consider and utilize the strain-hardening region of the stress-strain curve of the reinforcement, particularly for high-strength steels having a round-house stress-strain curve or a stress-strain curve with a short flat-plateau followed by strain hardening. Therefore, the comparisons that can be made with the experimental
results have been limited to only those tests in which the steel stresses at failure reached or could have reached the strain-hardening region of the stress-strain curve. The requirements (a) and (b) for the properties of materials further limit the comparisons to only a few available test programs, since the actual stress-strain curves for steel are sometimes not reported, and also since the cube strengths of concrete are sometimes available instead of cylinder strengths, especially for the tests conducted in the United Kingdom. Conversion of cube strengths into cylinder strengths may result in considerable error.

In view of the above-mentioned limitations, comparisons of the analytical results are made with results from four test programs which are presented and discussed in Sections 6.2 through 6.5.

6.2. Gaston's Tests (19)

Gaston tested 6x12-in. reinforced concrete beams under third-point loads. Two series of beams were tested. In the T-series, only tension reinforcement was provided, while in the C-series, both compression and tension reinforcement was provided. Various values of concrete cylinder strength $f'_c$, and of the areas and sizes of the tension and compression reinforcement $A_{st}$ and $A_{sc}$ were used. The stress-strain curves of the reinforcing bars of all sizes had long flat-plateaus followed by strain-hardening which began at strains varying between 0.01 and 0.02. The yield strengths of bars ranged from 40,600 to 56,100 psi. The tension steel strains were measured by mechanical gages while the concrete strains in the extreme compression fiber were measured by means of electric resistance strain gages.
The measured tension steel strains show that, at ultimate capacity, strain hardening of the reinforcing bars in tension was reached in all the beams provided with compression steel, except in those beams which had a high percentage of tension reinforcement, whereas strain-hardening was seldom reached in the beams of T-series. Consequently, in order to compare the analytical results with test results at steel strains well into the strain-hardening region, comparisons have been made here with all beams of C-series, but only six beams of T-series, selected at random and covering all ranges of $f'_c$, have been analyzed.

Tables 6.1 and 6.2 give the properties of these beams as well as the values of the ultimate moments calculated by the method of analysis explained in Chapter 3 and the measured ultimate moments. The information required for the analyses is available in Ref. 19, except the following modifications which were found to be necessary:

1. Although only one typical stress-strain curve was given for each size of the reinforcing bars, the yield strength, the yield strain and the strain at the beginning of strain-hardening were reported for the bars in each beam. Also, the shapes of the strain-hardening region of the stress-strain curves of the same size bars were similar. Therefore, the reported typical stress-strain curves of the bars were modified to represent, as closely as possible, the stress-strain curves of the bars in each beam.

2. The method of analysis assumes a value of the coefficient $k_3$ which would relate the strength of concrete in the beam to the cylinder strength $f'_c$ of the same concrete. The 1963 ACI Code takes the value of $k_3 = 0.85$. At first, a few beams with compression reinforcement were
analyzed using $k_3 = 0.85$, but the calculated ultimate moments were less than the measured ultimate moments by 8 to 10 percent. The method of analysis explained in Chapter 3 has shown that, when the strain-hardening region of the stress-strain curve of the reinforcement is utilized in the analysis, ultimate capacity of the section depends on the value of $f''_c = k_3 f_c'$. The value of $k_3$ can vary widely depending on various factors including the type and properties of the specimen. For columns, for which a greater portion of the cross-section is in compression, the value of $k_3 = 0.85$ seems to be justified. However, for beams--especially under-reinforced beams--the depth to the neutral axis may be small and thus there may be some confining effect on the concrete in the compressed zone. Consequently, the value of $k_3$ may be greater than 0.85. This is also true for the beams tested by Gaston. Gaston has recommended the following relationship between the values of $k_1 k_3$ and $f_c'$ which gave good agreement between the test results and the analytical results using the measured steel stresses at ultimate capacity.

$$k_1 k_3 = 0.625 + \frac{600}{f_c' - 1500}$$

where $k_1$ is the coefficient which gives average stress in the compression zone in the beams. For very small values of $f_c'$ the above equation gives unreasonably large value of $k_1 k_3$. Therefore, for the purpose of analyses here, it was assumed that the upper limit on the value of $k_1 k_3$ is 1.0, i.e. for $f_c' \leq 3100$, $k_1 k_3$ was taken equal to 1.0 and for $f_c' > 3100$, $k_1 k_3$ was calculated from the above equation.

In the method of analysis used herein, the stress-strain curve for concrete proposed by Todeschini et al (7), (Fig. 4.11), has been used with $f''_c = k_3 f_c'$. Also, the coefficient $k_1$ which is determined from this
stress-strain relationship depends on the value of ultimate concrete strain in the extreme compression fiber, \( \epsilon_u \). Thus, the only purpose of using Gaston's equation for \( k_1 k_3 \) is to determine the value of \( k_3 \). As has been explained in Chapter 5, the ultimate moment of an under-reinforced concrete beam with compression steel occurs at a large value of \( \epsilon_u \), for which \( k_1 \) is small (Fig. 4.12). If such a value of \( k_1 \) were used to obtain \( k_3 \) from Gaston's equation, an unreasonably high value of \( k_3 \) would be obtained. For the values of \( f'_c \) used in the beams of C-series, \( k_1 k_3 \) as determined from Gaston's equation varies between 0.80 and 1.0 (limit). If \( k_1 \leq 0.80 \), \( k_3 \geq 1.0 \) and this value of \( k_3 \) is too high. Consequently, it was assumed that for all values of \( f'_c \), \( k_1 = 0.80 \). This is the value of \( k_1 \) given by the stress-strain curve shown in Fig. 4.11 at a concrete strain of 0.004 (Fig. 4.12) which is the ultimate concrete strain recommended by Gaston. The value of \( k_3 \) determined by this procedure for the beams of C-series varied between 1.00 and 1.25.

Tables 6.1 and 6.2 give the ratios of measured ultimate moments to the ultimate moments calculated by the method of analysis explained in Chapter 3. For the beams of C-series, these ratios vary between 0.95 and 1.05 with an average of 0.99. Gaston's analysis gave the same ratios varying between 0.95 and 1.08 with an average of 1.03. For the six beams of T-series (Table 6.2) analyzed here, the moment ratios vary between 0.93 to 1.0 with the average value of 0.99.

The values of ultimate concrete strain \( \epsilon_u \) obtained from the analysis are also given in Tables 6.1 and 6.2. The analysis assumes a limiting value of 0.010, and in most of the beams with compression steel, this limit was reached. For most of the beams with tension steel only, the calculated value of \( \epsilon_u \) is 0.003 except for those with very small amount
of steel when higher values of $\varepsilon_u$ are calculated. The measured concrete strains in Gaston's tests are much smaller than the calculated values of $\varepsilon_u$, obviously due to the difficulties in measuring concrete strains, as explained in Section 4.2.6.

Figures 6.1 and 6.2 show plots of load vs. tension steel strains (measured and calculated) for four beams. There is good agreement between the measured and calculated tension steel strains even though the concrete strains considered in the analyses were allowed to reach values considerably higher than 0.003.

6.3. Hajnol-Konyi's Tests (20)

Hajnol-Konyi tested reinforced concrete beams under third-point loads. These beams were provided with twisted steel having a round-house stress-strain curve. The overall depth of the beams was 18 in. The width of the top 6-in. depth of the beams was 8 in., but in the remaining 12-in. depth, the width decreased to 4 in. at the bottom. The span of the beam was 11 ft. Although the method of analysis explained in Chapter 3 is applicable to rectangular cross-sections, it can be used for the beams tested by Hajnol-Konyi, because the neutral axis always lies in the top rectangular portion. The central 44-in. length of the span of the beam having constant moment had no compression reinforcement. Only one bar was provided in tension in all the beams of series 1 and 2.

Although Hajnol-Konyi tested 36 beams in Series 1 and 2, comparisons can be made for only four beams for which the stress-strain curves of the reinforcing bars have been given in Fig. 7 of Ref. 20. Todeschini's curve (7) as shown in Fig. 4.11 was used for the stress-strain relationship of concrete.
Table 6.3 gives the properties and the measured and calculated ultimate moments of the beams selected for analysis. The cylinder strength of concrete $f'_{c}$ was taken equal to 82 percent of the cube strength reported in Ref. 20. It can be seen from this table that good agreement is obtained between the measured and calculated ultimate moments in spite of the approximation involved in conversion of cube strengths of concrete to cylinder strengths. The ratio of measured to calculated ultimate moments ranges between 0.97 and 1.03 with an average of 1.00.

6.4. Evans's Tests (9,10)

Evans and Lawson tested axially and eccentrically loaded reinforced concrete columns containing square twisted steel (T.S.) which is available in the United Kingdom. Typical stress-strain curves of the reinforcing bars are shown in Fig. 4.3. For the purpose of comparisons here, only the tests of eccentrically loaded columns will be considered, since this will provide a good check of the method of analysis explained in Chapter 3.

The following variables were considered by Evans in the eccentrically loaded columns.

(1) Four values of total steel ratio, $p_{t} = 0.01, 2.25, 4.00$ and 7.60.

(2) Three values of eccentricity, $e' = 2.5, 7.5$ and 12.5. These eccentricities correspond to failure at ultimate axial load levels above, near and below the balance point, respectively.

(3) Concrete strength was determined from tests of 6-in. cubes and varied from a low value of 2240 psi to a high of 7500 psi. It was assumed that the cylinder strength of the concrete $f'_{c}$ was equal to 82 percent of the cube strength. The coefficient $k_{3}$ was taken equal to 0.85.
(4) Yield strength, given as proof stress at a strain of 0.005, varied from 63,000 to 69,000 psi. Two typical stress-strain curves up to a strain of 0.005 were reported by Evans (9), and one more typical stress-strain curve of TS steel was reported by Hajnol-Konyi (21). Since the method of analysis requires the use of a realistic stress-strain curve for steel up to large strains, particularly for the columns tested at axial load levels below the balance point, these curves were extended to strains beyond 0.008 by considering the typical stress-strain curves of other steels having round-house stress-strain curves. Also, in order to use the actual proof stress reported for the steel in each column, the stress-strain curve for the reinforcement in each column was modified at all strains so as to fit between the bounding typical curves reported in Ref. 9.

All columns were 10 in. square and had lateral ties spaced at 8 in. With this spacing of lateral ties, normal unconfined concrete was assumed and a stress-strain curve for concrete similar to those shown in Fig. 4.11 and proposed by Todeschini (7) was used in the analysis.

The tests were conducted by increasing the load at constant eccentricity e for each column. This eccentricity was increased by the deflection of the column. Therefore, the value of the eccentricity e at ultimate was taken as the sum of the original value of eccentricity at which the load was applied and the measured deflection at ultimate.

Table 6.4 gives particulars and comparisons of measured and calculated ultimate loads for 15 of the 35 columns tested by Evans and Lawson. These 15 columns were selected at random so that comparisons were made for one column for each value of \( p_t \) and e. Also, in each series of columns, those columns were selected which would cover low, medium and high values
of concrete strength. The ratios of measured to calculated ultimate load vary between 0.91 and 1.08 with an average of 1.02. The calculated values of $\epsilon_u$ corresponding to ultimate capacity, determined according to the three criteria explained in Chapter 3, vary from 0.0035 to 0.0095, whereas much smaller values of $\epsilon_u$ were measured in the tests. It is believed that this is due to the difficulties encountered in the measurement of concrete strain as discussed in Section 4.2.6.

It can be concluded from the comparisons of measured and calculated ultimate loads that the method of analysis explained in Chapter 3 predicts the ultimate loads reasonably well.

6.5. Todeschini's Test (7)

Todeschini et al tested 11-in. square reinforced concrete columns provided with high-strength steel meeting the requirements of ASTM A-615 Grade 75 steel. Typical stress-strain curves of the reinforcing bars were reported, and yield strengths at a strain of 0.005 were given for the bars in each column. Consequently, for the purpose of analysis, the typical stress-strain curves given for the reinforcing bars were modified as required to get the same shape of curve but with the actual yield strength. The stress-strain curve for concrete proposed by Todeschini et al was used with the smaller of the two values of $f'_c$ reported in Ref. 7. Since the effect of the stress-strain curve of the steel is enhanced at high values of eccentricity of applied load, comparisons of the analytical results are made for the columns tested at 3.5 and 5.5-in. eccentricities. The eccentricities of 0 and 1.5 in. would not produce the desired effects since the ultimate capacity will be reached at low steel and concrete strains.
Table 6.5 gives the properties, and the measured and calculated ultimate loads for the columns selected for analysis. A good agreement is obtained between the analytical and experimental results. The ratio of measured to the calculated ultimate load varies between 0.97 and 1.07 with an average of 1.00. Todeschini's analysis, as discussed in Section 2.2, gives values of the same ratio for these columns ranging between 0.89 and 0.99 with an average of 0.95.
7. ANALYSES OF REINFORCED CONCRETE SECTIONS LOADED IN A SHORT TIME--ASTM GRADE 60 STEEL

7.1. Introduction

In this chapter, the 15-in. square reinforced concrete section shown in Fig. 3.1 has been analyzed according to the method of analysis explained in Chapter 3 to obtain load-moment and load-curvature relationships at ultimate capacity for various levels of ultimate axial load including pure moment case. The following variables have been considered in the analyses:

(1) Total steel ratio $p_t$: Three values of $p_t = 0.01, 0.04$ and $0.08$ have been considered. The extreme values of $p_t$ are the minimum and maximum values permitted in the 1963 ACI Code. One-half of this steel is provided in each of the two faces of the section parallel to the axis of bending.

(2) Compressive strength of concrete $f'_c$: Three values of $f'_c = 3000, 4000$ and $5000$ psi have been used in the analyses.

(3) Ratio $q_t = p_t \frac{f'_y}{f'_c}$: For all the analyses in this chapter, the yield strength of steel $f'_y$ has been kept constant and equal to 60 ksi. Thus, the variations in $q_t$ have been obtained by varying $p_t$ and $f'_c$ as given below.

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$f'_c$ (psi)</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5000</td>
<td>0.12</td>
</tr>
<tr>
<td>0.01</td>
<td>3000</td>
<td>0.20</td>
</tr>
<tr>
<td>0.04</td>
<td>5000</td>
<td>0.48</td>
</tr>
<tr>
<td>0.04</td>
<td>4000</td>
<td>0.60</td>
</tr>
<tr>
<td>0.04</td>
<td>3000</td>
<td>0.80</td>
</tr>
<tr>
<td>0.08</td>
<td>5000</td>
<td>0.96</td>
</tr>
<tr>
<td>0.08</td>
<td>3000</td>
<td>1.60</td>
</tr>
</tbody>
</table>
The values of $q_t = 0.12$ and $1.20$ are the extreme values that could normally be obtained in practice with ASTM Grade 60 steel.

(4) Cover to reinforcement: In all of the Sections 7.2 through 7.4, one value of the ratio $d'/t = 0.20$ has been used. However, in order to study the effect of the variation of this ratio, two values of $d'/t = 0.15$ and $0.20$ have been used in the analyses in Section 7.5.

(5) Stress-strain curves for reinforcement: Steels 5, 6, 7 and 8, the stress-strain curves of which are shown in Fig. 4.5, have been used for the analyses. The bases for selection of these steels are explained in Section 4.1.3. All of these steels meet the requirements of the ASTM specifications and the 1963 ACI Code provisions, and the stress-strain curve for steel 7 is to be used for the purpose of design in accordance with the 1963 ACI Code.

(6) Stress-strain curve for concrete: Only one stress-strain curve for concrete shown in Fig. 4.10 has been used in this chapter. This curve was proposed by Todeschini et al (7) and is designated as curve A. The basis of selection of this curve is explained in Section 4.2.5.

(7) Coefficient $k_3$: The value of the coefficient $k_3$ which relates the cylinder strength of concrete $f'_c$ with the strength of concrete in the member $f''_c$ is taken to be equal to 0.85 for all the analyses in this chapter.

The analytical results have been presented in the form of figures and tables which are explained in the following sections. These figures and tables show comparisons of load-moment and load-curvature relationships obtained for steels 5, 6 and 8, according to the method of analysis explained in Chapter 3, with those for steel 7 in strict accordance with the provisions
of the 1963 ACI Code. The ultimate concrete strain $\epsilon_u$ is limited to 0.010 for steels 5, 6 and 8 (see Section 4.2.6) but to 0.003 for steel 7 in accordance with Section 1503 (c) of the 1963 ACI Code. However, in order to investigate the effect of varying the limiting value of $\epsilon_u$, comparisons of the analyses have been presented in Section 7.2 for $\epsilon_u$ limited to 0.010 and 0.003 for steels 5, 7 and 8.

In order to compare the ultimate moments and ultimate curvatures of the section for steels 5, 6 and 8 with those for steel 7, the ordinates of the load-moment and load-curvature diagrams have been expressed as the ratio $P_u/P_{07}$, whereas the abscissas of the load-moment diagrams only have been expressed as the ratio $M_u/M_{07}$, where $P_u =$ ultimate axial load for steels 5, 6, 7 and 8, $P_{07} =$ ultimate axial load of the section for zero eccentricity with steel 7, $M_u =$ ultimate moment corresponding to $P_u$ for any steel, and $M_{07} =$ ultimate moment for zero axial load (pure moment) for steel 7.

7.2. Effect of Stress-Strain Curve of Steel with Variation of Limit on Ultimate Concrete Strain

Figures 7.1 through 7.4 show comparisons of load-moment diagrams at ultimate capacity of the section for steels 5, 7 and 8 for three values of total steel ratio $p_t$ and three values of concrete strength $f'_c$ which give three values of the ratio $q_t = p_t f'_y/f'_c$ as shown below:

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$f'_c$</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5000</td>
<td>0.12</td>
</tr>
<tr>
<td>0.04</td>
<td>4000</td>
<td>0.60</td>
</tr>
<tr>
<td>0.08</td>
<td>3000</td>
<td>1.60</td>
</tr>
</tbody>
</table>

In all these figures, load-moment diagrams are compared for two limiting
values of $\epsilon_u = 0.003$ in accordance with the 1963 ACI Code, and 0.010 according to the method of analysis in Chapter 3.

Figures 7.5, 7.6 and 7.7 show comparisons of load-curvature diagrams for steels 5, 7 and 8 for the same values of $P_t$, $f'_c$ and $q_t$ and for the same values of $\epsilon_u$ as for the load-moment diagrams.

The comparisons that show the effect of the limiting value of ultimate concrete strain are explained below for various regions of the load-moment diagram for each of the steels 7, 5 and 8.

7.2.1. Flat-Top Steel 7

It can be seen from Figs. 7.1 through 7.4 that, for the flat-top steel 7, there is practically no difference between the ultimate capacities of the section obtained by limiting $\epsilon_u$ to 0.003, and by the maximization of moment at a given ultimate axial load level in accordance with the criteria explained in Section 3.3, except in that region of the load-moment diagram which lies between the balance point and $P_u = 0.45 P_{07}$. In this region, greater ultimate capacity is obtained by allowing a higher limit on $\epsilon_u$. This is true for all values of $q_t$, and the increase in the ultimate capacity for higher limit on $\epsilon_u$ increases as $q_t$ is increased. These effects of $\epsilon_u$ are explained below for various regions of the load-moment diagram.

(a) Ultimate axial load levels below the balance point

At low levels of ultimate axial load (below the balance point), the tension steel yields at ultimate capacity, and the tension steel strain $\epsilon_2$ lies on the flat-plateau of the stress-strain curve for steel 7. The compression steel strain $\epsilon_3$ depends on the value of the strain in the extreme compression fiber $\epsilon_4$. Generally $\epsilon_3$ lies in the elastic region of
the stress-strain curve for the compression steel. The general tendency of the effect of increase in the value of $\varepsilon_4$ is: (a) to increase $\varepsilon_2$, (b) to increase $\varepsilon_3$ which increases the compression steel stress and hence the force until the compression steel yields, (c) to decrease the depth to the neutral axis and increase the lever arm, (d) to increase the curvature of the section, and (e) to reduce the value of $k_1k_2$ (see Figure 4.12) if $\varepsilon_4$ exceeds the value of the strain $\varepsilon_m$ at which $k_1k_2$ is maximum. Both (c) and (e) reduce the compression force in concrete. The tension force in steel stays constant at $A_{st}f'_y$, although $\varepsilon_2$ increases. Therefore, for a given value of $\varepsilon_4 > \varepsilon_m$, the decrease in the concrete force must be compensated for by the corresponding increase in the compression force in the compression steel, as long as the latter has not yielded, so that equilibrium of forces is maintained. However, at very high values of $\varepsilon_4$ the compression steel will yield and the compression and tension steel forces will be equal in magnitude. Thus, any decrease in the concrete force caused by the reduction in the value of $k_1k_2$ is compensated for by the increase in the depth to the neutral axis so as to increase the area of concrete in compression. This results in reduction in the lever arm of the concrete force. In the first case, when the compression steel does not yield, the reduction in the concrete force, the lever arm of which is generally greater than the distance of the compression steel from the plastic centroid, causes more reduction in the moment of the concrete force than the increase in the moment of the compression steel force. The moment of the tension steel force about the plastic centroid remains constant. There is thus a net reduction in the total moment when $\varepsilon_4$ is increased beyond $\varepsilon_m$. In the second case, when both the tension and compression steels have yielded at high value of $\varepsilon_4$,
the forces in these steels and their moments about the plastic centroid remain constant. The reduction in the lever arm of the concrete force caused by lowering the neutral axis, as explained above, reduces the moment of the concrete force. The total moment is thus reduced. Consequently, in either case, the increase in \( \varepsilon_u \) reduces the total moment, and the maximum moment occurs at \( \varepsilon_u = \varepsilon_m = 0.004 \). Since the difference in the moment at \( \varepsilon_u = 0.003 \) and 0.004 is negligible, the limitation of \( \varepsilon_u \) to 0.003 has practically no effect on the ultimate capacity of the section with flat-top steel at ultimate axial load levels below the balance point.

However, since the curvature of the section increases as \( \varepsilon_u \) is increased, considerable reduction in the ultimate curvature of the section is obtained when \( \varepsilon_u \) is limited to 0.003 and the maximum moment occurs at \( \varepsilon_u = \varepsilon_m > 0.003 \). This can be seen in Figs. 7.5, 7.6 and 7.7. It can be noticed that, as \( \rho_t \) increases, the limitation of \( \varepsilon_u \) to 0.003 causes greater decrease in the calculated value of ultimate curvature \( \Phi_u \).

(b) **Ultimate axial load levels** \( P_b < P_u < 0.45 P_{07} \)

In this region of load-moment diagram, the compression steel generally yields and the compression steel stress and force remain constant at \( f_y \) and \( A_{sc} f_y \), respectively. The tension steel strain lies in the elastic region of the stress-strain curve. Figure 4.12 shows that \( k_{1} k_{2} \) increases with increase in \( \varepsilon_u \) up to \( \varepsilon_m = 0.004 \). As stated earlier, the increase in \( \varepsilon_u \) causes increase in both tension and compression steel strains. Therefore, the tension steel stress and force increase with increase in \( \varepsilon_u \), until the tension steel yields. Since the compression steel force remains constant, the concrete force must increase, which is accomplished either by increase in the value of \( k_{1} k_{2} \), if \( \varepsilon_u < 0.004 \), or by increase in the depth to the
neutral axis, if $\varepsilon_4 \geq 0.004$. In both cases, total tension and total compression forces increase until the tension steel yields. But, for $0.003 < \varepsilon_4 < 0.004$, the reduction in the depth to the neutral axis results in increase in the lever arm of the concrete force. Thus, there is some increase in the moments of the concrete and tension steel forces, and hence in the total moment. When $\varepsilon_4$ exceeds 0.004, the increase in the depth to the neutral axis decreases the lever arm, and hence the moment of the concrete force about the plastic centroid. Although, there is some increase in the total compression and tension forces, the decrease in the moment of the concrete force is more than the increase in the moment of tension steel force about the plastic centroid, and the total moment decreases as $\varepsilon_4$ increases beyond $\varepsilon_m = 0.004$. Therefore, the maximum moment occurs at $0.003 < \varepsilon_4 < 0.004$, the lower value of $\varepsilon_4$ is obtained for higher value of $P_u / P_0$. Consequently, in this region of the load-moment diagram, the ultimate capacity of the section is reduced if $\varepsilon_4$ is assumed to be equal to 0.003.

However, if the compression steel does not yield at $\varepsilon_4 = 0.003$, as is true at lower levels of $P_u$ above the balance point for the section analyzed here, any increase in $\varepsilon_4$ increases $\varepsilon_3$ and hence the compression steel stress and force. Consequently, greater increase in ultimate capacity is obtained by increasing $\varepsilon_4$ than when the compression steel yields at $\varepsilon_4 = 0.003$.

As explained in (a) above, the curvature of a section is increased by increase in $\varepsilon_4$. Since, for $P_b < P_u < 0.45 P_0$, $\varepsilon_u$ is greater than 0.003, the ultimate curvature is reduced by the limitation of $\varepsilon_u$ to 0.003. This is clearly shown in Figs. 7.5 through 7.7.
(c) Ultimate axial load levels above 0.45 \( P_{07} \)

In this region of the load-moment diagram, the compression steel yields, while the tension steel strain \( \varepsilon_2 \) is either a very small tensile strain or lies in the compressive elastic region. The latter case occurs at higher load levels. This has been explained in detail in Section 5.5.

At higher load levels, a large area of concrete is in compression, and the entire section may be in compression. Thus, the compression force in the concrete is large. The lever arm of the concrete force is equal to \( d - k_2kd \).

As shown in Fig. 4.12, \( k_2 \) increases as \( \varepsilon_4 \) is increased. Since the depth to the neutral axis \( kd \) is large, \( k_2kd \) increases considerably even if \( kd \) decreases slightly with increase in \( \varepsilon_k \) up to \( \varepsilon_m = 0.004 \), and thus the lever arm is reduced considerably. For \( 0.003 < \varepsilon_4 \leq \varepsilon_m \), \( k_1k_3 \) increases as \( \varepsilon_4 \) is increased, and greater compression force is obtained than for \( \varepsilon_4 \leq 0.003 \).

This disturbs the equilibrium of forces, and to restore it the curvature must increase so that the neutral axis is raised, which will reduce the area of concrete in compression and hence the force in concrete. Also, the increase in curvature will increase the tension steel strain if it is in tension, which will increase the tension steel force, or it will decrease the tension steel strain if it is in compression, which will decrease the compression force in tension steel. The compression steel force remains constant at \( A_{sc}f_y \). When equilibrium of forces is reached, this redistribution of forces will result in a small increase in the moment of tension steel force, but a larger decrease in the moment of the concrete force about the plastic centroid, due mainly to the decrease in the lever arm of the concrete force. Thus, there is a net reduction in the total moment for \( 0.003 < \varepsilon_4 < 0.004 \).
When $\epsilon_4 > 0.004$, $k_4 k_2$ decreases and hence results in reduction of the concrete force. Therefore, the depth to the neutral axis $k_d$ must increase to increase the area of concrete in compression and hence the concrete force. This will reduce the strain, stress, and force in the tension steel if it is in tension, or will increase the strain, stress, and force in the tension steel if it is in compression. The increase in $k_d$ coupled with the greater value of $k_2$ at higher values of $\epsilon_4$ causes a large reduction in the lever arm of the concrete force. Consequently, when equilibrium of forces is reached, the moments of the concrete force and the tension steel force about the plastic centroid are reduced so much that the total moment is reduced with increase in $\epsilon_4$. Therefore, the maximum moment at ultimate axial load levels higher than $0.45 P_07$ occurs at $\epsilon_4 = \epsilon_u \leq 0.003$, and there is no effect of limiting $\epsilon_u$ to 0.003 on either the ultimate capacity or on the ultimate curvature.

7.2.2 Steel 5

It can be seen in Figs. 7.1 through 7.4 that, for steel 5 at ultimate axial load levels below the balance point, there is a considerable difference between the ultimate capacities and curvatures of the section calculated by maximization of moment, according to the method of analysis explained in Chapter 3, and those obtained by limiting $\epsilon_u$ to 0.003 in accordance with the 1963 ACI Code. This is true for all values of $\alpha_4$. At ultimate axial load levels above the balance point, the effect of limiting $\epsilon_u$ to 0.003 is the same as for the flat-top steel 7 explained in Section 7.2.1. However, if $\epsilon_u$ is limited to 0.003, the ultimate capacity of the section for steel 5 is not different from that for steel 7 even at low levels of ultimate axial load below the balance point. This is true for all values
of $q_t$, except for the very small value of $q_t = 0.12$ in combination with low levels of ultimate axial load. The above conclusions are explained below.

(a) **Ultimate axial load levels below the balance point**

As explained for steel 7, at ultimate axial load levels below the balance point, the tension steel strain exceeds the yield value and, for steel 5, lies in the strain-hardening region. Thus, the tension steel stress and force increase with increase in its strain. The effects of increase in $\varepsilon_4$—as was true for steel 7—are: (1) to increase the tension steel strain $\varepsilon_2$, (2) to increase the compression steel strain $\varepsilon_3$, (3) to decrease the depth to the neutral axis $d$, and (4) to increase the curvature. Since the depth to the neutral axis is already small, the decrease in $d$ is also small, and $k_2d$ is increased only slightly because of the increase in $k_2$ with increase in $\varepsilon_4$. Thus, the lever arm of the concrete force is reduced very slightly. The compression force in the concrete is reduced as $\varepsilon_4$ is increased mainly because the value of $k_4k_3$ is reduced for $\varepsilon_4 > \varepsilon_m$. But, since the compression steel strain is in the compressive elastic region, the increase in $\varepsilon_3$ caused by the increase in $\varepsilon_4$ results in an increase in the compression steel force which is greater than the reduction in the concrete force. Thus, there is a net increase in total compression force to match the increase in the tension steel force caused by the increase in $\varepsilon_2$ in the strain-hardening region. For the flat-top steel 7, the tension steel force stays constant at $A_{st}f_y$, and the increase in the compression steel force is just enough to compensate for the reduction in the concrete force for $\varepsilon_4 > \varepsilon_m$. Thus, the total tension and total compression forces remain constant. Because of the net increase in the total tension and total compression forces for steel 5, and because the lever arm of the concrete force is almost
constant, there is a net increase in the moments of these forces about the plastic centroid, which increase does not occur for steel 7. Consequently, the moment-strain \((M - \varepsilon_4)\) curves continue to rise as \(\varepsilon_4\) is increased and the maximum moment occurs at higher values of \(\varepsilon_4\) at low levels of ultimate axial load below the balance point. Also, the maximum moment so obtained is considerably greater than that at \(\varepsilon_4 = 0.003\).

If \(\varepsilon_4\) is limited to 0.003, \(\varepsilon_2\) does not go so much into the strain-hardening region as it does for higher values of \(\varepsilon_4\). Also, \(\varepsilon_2\) decreases as \(q_t\) is increased; it lies on the flat-plateau for high values of \(q_t\), in which case the tension steel force is the same as for steel 7, or it goes into the lower portion of the strain-hardening region for the small value of \(q_t\), in which case some increase in the tension steel force is obtained as compared to steel 7. For \(\varepsilon_u = 0.003\), the tension steel strain is also reduced as \(P_u\) is increased from zero to the value at the balanced conditions \(P_b\). This has been explained in Section 5.5. Therefore, depending on the load level and on the value of \(q_t\), a tension steel stress either equal to or greater than \(f_y\) is obtained. In the former case, the moment is the same as for the flat-top steel, whereas in the latter case, the moment is greater for steel 5 than for steel 7.

Table 7.1 gives the ratios of moments and curvature for steel 5 to those for steel 7, for \(\varepsilon_u = 0.003\) and \(P_u = 0\), for each of the three values of \(q_t\) given in the table. It can be seen in this table that, for \(\varepsilon_4 = 0.003\), the increase in moment obtained by utilizing strain-hardening of steel 5 is accompanied by a small decrease in curvature. This is so because of the increase in the depth to the neutral axis for steel 5, as explained below.

When the tension steel strain is in the strain-hardening region for a given value of \(\varepsilon_4\), greater tension steel stress and force are obtained
for steel 5 than for the flat-top steel 7. For equilibrium of forces, this increase in tension force must be accompanied by a corresponding increase in the total compression force which can be obtained only by increasing the depth to the neutral axis kd so as: (1) to increase the area of concrete in compression and hence the concrete force, and (2) to increase the compression steel strain, and thus stress and force, unless the compression steel strain is on the flat-plateau. The increase in kd reduces the tension steel strain and hence the stress and the force. Consequently, equilibrium of forces for a given value of \( \varepsilon_4 \) for a strain-hardening steel is obtained at a slightly greater depth to the neutral axis than for the flat-top steel. Since the curvature \( \Phi = \varepsilon_y/kd \), slightly smaller curvature is obtained for steel 5 than for steel 7 when \( \varepsilon_y = 0.003 \), and the strain-hardening region of the stress-strain curve is utilized in calculating tension steel stress. This occurs at low levels of \( P_u \) and for small values of \( q_t \).

In view of the above discussion, it can be concluded that, when \( \varepsilon_u \) is limited to 0.003, the effect of strain-hardening of steel 5 is not properly utilized. It is only by allowing a higher limit on \( \varepsilon_u \) that greater ultimate capacity and curvature are obtained at ultimate axial load levels below the balance point.

(b) Ultimate axial load levels above the balance point

In this region of the load-moment diagram, steel 5 behaves the same as steel 7 because steel strains at ultimate capacity are not in the strain-hardening region of the stress-strain curve. The tension steel strain lies in the elastic tensile or compressive region. The compression steel yields but its strain is in the strain-hardening region only at high values of \( \varepsilon_y > 0.006 \). For these strains, \( k_2 \) is high (greater than 0.5), and \( k_1 k_3 \) is
so small that $kd$ must increase appreciably in order to compensate for the reduction in the concrete force caused by the reduction in $k_1k_3$. Even if the compression steel strain for steel 5 is in the strain-hardening region, the compression steel stress increases at such a small rate that it does not provide enough increase in the compression force without increasing $kd$. The increase in $k_2$ and $kd$ reduce the lever arm of the concrete force so much that the moment of the concrete force about the plastic centroid, and hence the total moment are reduced with increase in $\varepsilon_u$. This is clearly shown in the moment-concrete strain diagrams in Figs. 7.22 through 7.24. Therefore, the limitation of $\varepsilon_u$ to 0.003 for steel 5 has the same effect on ultimate capacity and ultimate curvature at ultimate axial load levels above the balance point as for steel 7.

7.2.3. Steel 8

It can be seen in Figs. 7.2 through 7.4 that the ultimate capacity of the section for steel 8 is considerably reduced if the ultimate concrete strain $\varepsilon_u$ is limited to 0.003. This reduction depends on the ultimate axial load level as well as on the value of $q_u$. The higher the load level, the smaller is the reduction in ultimate capacity, whereas the greater the value of $q_u$ the greater is the reduction in ultimate capacity. The load level up to which the ultimate capacity of the section is reduced by limiting $\varepsilon_u$ to 0.003 increases with increase in $q_u$. Similarly, Figs. 7.5 through 7.7 show that the ultimate curvature of the section is also reduced considerably by the limitation of $\varepsilon_u$. The effects of this limitation are discussed below for various regions of the load-moment diagram and for various values of $q_u$. 


(a) **Ultimate axial load levels below the balance point**

As was true for steel 5, the moment-strain diagrams shown in Figs. 7.19 through 7.21 show that, for steel 8, the maximum moment of the section at ultimate axial load levels below the balance point occurs at $\epsilon_4 > 0.003$. At $P_u = 0$, the moment-strain diagrams continue to rise with increase in $\epsilon_4$. This is true for all values of $q_u$. If $\epsilon_4$ exceeds $\epsilon_m = 0.004$, the value of $k_1k_3$ decreases, which reduces concrete force, but $k_2$ increases, which reduces the lever arm of the concrete force. Also, the increase in $\epsilon_4$ results in increases in curvature, and in the compression and tension steel strains. Thus, greater tension and compression steel stresses and forces are obtained for steel 8. The reduction in concrete force is partly compensated for by increase in the area of concrete in compression caused by lowering of the neutral axis, and partly by the increase in the compression steel force. At equilibrium of forces, the total compression and total tension forces are increased with increase in $\epsilon_4$. The moments of the tension and compression steel forces are increased due to the increase in their forces, but the moment of the concrete force is reduced due to the reduction in both the force and its lever arm. Since the increases in the moments of the tension and compression steel forces are greater than the decrease in the moment of the concrete force, the total moment increases with increase in $\epsilon_4$. Consequently, the ultimate capacity and curvature of the section are reduced by limiting $\epsilon_u$ to 0.003.

Figure 7.8 and Table 7.1 give comparisons of the tension and compression steel strains for steels 7 and 8, when equilibrium of forces occurs at $\epsilon_4 = 0.003$. These strains are given for three values of $q_u = 0.12, 0.60$ and 1.60. It can be seen that, for the smallest value of $q_u = 0.12$ obtained with $p_t = 0.01$ and $f'_c = 5000$ psi, the tension steel strain is in
the strain-hardening region for steel 8. Although the tension steel strain for steel 7 is greater than that for steel 8, the tension steel force is greater for steel 8 than that for steel 7. This increase in the tension force is matched by the corresponding increase in the total compression force caused by a slight increase in the depth to the neutral axis. This reduces the lever arm of the concrete force by a negligible amount. Thus, the greater tension and compression forces for steel 8 result in greater moment. This is shown in Table 7.1.

Figure 7.8 shows that, for $\epsilon_4 = 0.003$, an increase in the value of $\epsilon_t$ results in a reduction in the tension steel strain but an increase in the compression steel strain. For $p_t = 0.04$, $f'_c = 4000$ and $q_t = 0.60$, equilibrium of forces occurs at such steel strains that the tension and compression steel stresses for steel 8 are nearly the same as those for steel 7. Therefore, the total moments for these steels are almost the same, as given in Table 7.1 and shown in Fig. 7.3.

However, for $q_t = 1.60$, obtained with $p_t = 0.08$ and $f'_c = 3000$ psi, the tension steel strain is further reduced and lies in that region of the stress-strain curve for steel 8 which is below that for steel 7. Thus, smaller tension steel stress and force are obtained for steel 8 than for steel 7, although the tension steel strain for steel 8 is greater than that for steel 7. For equilibrium of forces, a smaller total compression force is obtained for steel 8 than for steel 7. Therefore, the total moment for steel 8 is less than that for steel 7.

It can be concluded from the above discussion that, as $q_t$ is increased from a very small value of 0.12 to a very high value of 1.60, the ratio of moment for steel 8 to that for steel 7 changes from more than 1.0
for the small value to less than 1.0 for the high value of $q_t$ if $\varepsilon_4$ is limited to 0.003 at $P_u = 0$. The opposite is the case for curvature of the section at $\varepsilon_4 = 0.003$ for both steels. This is explained below:

$$\text{Curvature} = \Phi = \frac{\varepsilon_4 + \varepsilon_2}{d} = \frac{0.003 + \varepsilon_2}{d}$$

It is clear from the above equation that, for a given value of $\varepsilon_4$, curvature is proportional to the tension steel strain $\varepsilon_2$. Figure 7.8 shows that, for a very small value of $q_t$, tension steel strain is in the strain-hardening region for steel 8. Thus, when equilibrium of forces is obtained for steel 7 with $\varepsilon_4 = 0.003$, let the tension steel strain $\varepsilon_2 = \varepsilon_2,7'$ and the compression steel strain $\varepsilon_3 = \varepsilon_3,7'$. Then, for steel 8, this same strain distribution will give greater tension steel stress and force for steel 8, and thus will disturb the equilibrium of forces. Therefore, the depth to the neutral axis for steel 8 must increase to reduce the tension steel strain, stress and force, and at the same time increase the area of concrete in compression to increase the concrete force, and also increase the compression steel strain to increase the compression steel stress and force. Thus, equilibrium of forces is obtained with tension steel strain for steel 8 as $\varepsilon_2,8 < \varepsilon_2,7$. Consequently, smaller curvature is obtained for steel 8 than for steel 7 for the small value of $q_t = 0.12$.

For the high value of $q_t = 1.60$, equilibrium of forces occurs for steel 7 with tension steel strain $\varepsilon_2,7$ in the region $\varepsilon_y < \varepsilon_2,7 < 0.005$. In this range of strains, the stress-strain curve for steel 8 lies below that for steel 7, and a smaller tension steel stress and hence force are obtained for steel 8 as compared to steel 7. Thus, equilibrium of forces is disturbed. To restore it, the neutral axis must rise to increase the tension
steel strain, stress and force for steel 8, and at the same time reduce the concrete and compression steel forces. Consequently, \( \epsilon_{2,8} > \epsilon_{2,7} \). This gives greater curvature of the section for steel 8 than for steel 7.

(b) **Ultimate axial load levels above the balance point**

Since steel 8 does not have a well-defined yield point, the balance point on the load-moment diagram, as defined in Section 1900(b) of the 1963 ACI Code, can be computed only by using an arbitrary value for the tension steel strain \( \epsilon_2 \). However, since in this investigation of the effect of the stress-strain curve of steel, comparisons of ultimate capacity for steel 8 are made with that for steel 7, the balance point used is that computed for steel 7.

Figures 7.22 through 7.24 show moment-strain diagrams at two levels of ultimate axial load above the balance point. It can be seen that, for small values of \( q_L \), the maximum moment occurs at \( \epsilon_4 \) close to 0.003. This is also the case for steel 7, although the ultimate moment for steel 7 is greater than that for steel 8. Figure 7.9 shows that, for \( \epsilon_4 = 0.003 \), the tension and compression steel strains for steel 7 lie in the elastic region. This is also true for the tension steel strain for steel 8, but the compression steel strain lies in that region of the stress-strain curve for steel 8 which is below that for steel 7, and a smaller compression steel stress is obtained for steel 8 than for steel 7. Thus, the strain distribution at which equilibrium of forces is reached for steel 7 will give smaller compression steel force for steel 8, and equilibrium of forces will be disturbed for steel 8. To restore it, the neutral axis must move down to increase the area of concrete in compression, which will increase the concrete force, and also to increase the compression steel strain, which will increase
the compression steel force. This will also reduce the tension steel strain and hence the stress and force. Also, the lever arm of the concrete force will be reduced. Consequently, when equilibrium of forces is reached for steel 8, smaller total tension and compression forces are obtained which result in smaller total moment. In order to increase the tension and compression steel forces for steel 8 so as to be equal to or greater than those for steel 7, steel strains must be increased which can be achieved only by increasing $\epsilon_4$. Since, the total steel ratio is small and the compression steel strains are such that the compression steel stress increases at a smaller rate, large increases in strains are required to obtain the required forces. This can be obtained by a large increase in $\epsilon_4$ which will result in so much reduction in $k_1k_2$ and so much increase in $k_2$ that the concrete force and its lever arm will be reduced very much, and thus the total moment will be reduced. Therefore, for small values of $q_t$, there is little effect of the limitation of $\epsilon_u$ to 0.003.

For high values of $q_t$, at lower levels of ultimate axial load above the balance point, ultimate capacity of the section for steel 8 is still less than that for steel 7 for the same reason as for small values of $q_t$ explained above, but the maximum value of moment occurs at a greater value of $\epsilon_4$ than that for small value of $q_t$. This value of $\epsilon_4$ increases with increase in $q_t$, but decreases with increase in ultimate axial load level. For higher values of $q_t$, the total steel ratios are so large that a small increase in steel strains caused by a small increase in $\epsilon_4$ results in so much increase in steel forces that they more than compensate for the reduction in the concrete force at higher values of $\epsilon_4$. However, this is accompanied by a reduction in the lever arm of the concrete force which
reduces the moment of that force. The total moment will continue to increase with increase in $\varepsilon_4$ beyond 0.003 until the reduction in the moment of the concrete force is more than the total increase in the moments of the compression and tension steel forces. The maximum moment for steel 8 for $\varepsilon_4 > 0.003$ is considerably greater than that for $\varepsilon_4 = 0.003$. Therefore, the limitation of $\varepsilon_u$ results in considerable reduction in ultimate capacity of the section for steel 8, at lower levels of $P_u$ above the balance point.

At high levels of ultimate axial load, the entire cross-section may be in compression, and the total moment is quite small, as is the lever arm of the concrete force. Since, any increase in $\varepsilon_4$ reduces the lever arm because of the increase in $k_2$, and also for $\varepsilon_4 > \varepsilon_m$, since $k_1k_2$ and hence the concrete force are reduced by increasing $\varepsilon_4$, the reduction in the moment of concrete force is quite large as compared to the total moment. Furthermore, since the tension steel is in compression at very high levels of ultimate axial load, any increase in the compressive strain and hence the compressive force in the tension steel caused by the increase in $\varepsilon_4$ reduces the total moment. It is then only the compression steel that can increase the total moment. Since, the compression steel strain is in the strain-hardening region for high values of $\varepsilon_4$, its stress and hence its force do not increase enough to provide as much increase in the total moment as is required to compensate for the reductions in the moment caused by the tension steel and concrete forces, except when $p_t$ is very large and $f'_c$ is small. Therefore, the moment strain curves peak at $\varepsilon_4 \leq 0.004$. Furthermore, there is a very small difference between the moment for $\varepsilon_4 = 0.003$ and the maximum moment at $\varepsilon_4 > 0.003$. This can be seen in Figs. 7.22 and 7.23. For a very
high value of $q_u = 1.60$, obtained with $p_t = 0.08$ and $f'_c = 3000$, the compression steel force is much greater than the concrete force. Thus, it compensates for the reduction in the concrete force at high values of $\epsilon_u$, and the moment-strain curves continue to rise up to $\epsilon_u$ considerably greater than 0.003. For very high values of $\epsilon_u$, the lever arm of the concrete force, and hence its moment, are reduced so much that the compression steel cannot compensate for this reduction, and the moment-strain curve drops. The maximum moment is considerably greater than that for $\epsilon_u = 0.003$.

In view of the above discussion, it can be concluded that, at high levels of ultimate axial load for steel 8 there is little effect of the limitation of $\epsilon_u$ to 0.003 on the ultimate capacity, except for the very high value of $q_u$.

However, since curvature of the section increases with increase in $\epsilon_u$, there is considerable reduction in ultimate curvature when $\epsilon_u$ is limited to 0.003 and the maximum moment occurs at $\epsilon_u > 0.003$, even though there is a very small difference between the moment for $\epsilon_u = 0.003$ and the maximum moment. It can be seen in Fig. 7.7 that for $q_u = 1.60$ this is true at all levels of $P_u$ above the balance point.

7.3. Effect of Stress-Strain Curve of Steel with Variation of Total Steel Ratio

In this section, the results of analyses of the 15-in. square reinforced concrete section are presented and discussed for the variables given in Section 7.1 in order to study the effects of the stress-strain curves for steels 5, 6 and 8, in comparison with steel 7, for various values of total steel ratio $p_t$. Since concrete strength could modify the effects, two values of $f'_c = 3000$ and 5000 psi have been used for $p_t = 0.01$ and 0.08,
whereas three values of $r'_c = 3000, 4000$ and $5000$ psi have been used for $p_t = 0.04$. Only one value of $d'/t = 0.20$ has been used in the analyses in this section.

The load-moment and load-curvature diagrams of the section for steels 5, 6 and 8, obtained according to the method of analysis explained in Chapter 3, have been compared with the corresponding diagrams obtained in accordance with the assumptions in Section 1503 of the 1963 ACI Code; i.e., for steel 7 with ultimate concrete strain $\varepsilon_u$ limited to 0.003. Such comparisons make it possible to study the effects of realistic assumptions for the properties of materials in relation to those assumed in Sections 1503 (c) and (d) of the Code. These effects are discussed and explained below for each of the three values of $p_t$ considered in the analyses.

7.3.1. Low Value of Total Steel Ratio

Section 913 of the 1963 ACI Code permits the use of a total steel ratio $p_t$ as low as 0.01. Therefore, $p_t = 0.01$ was chosen to study the effects of the stress-strain curve of steel with small values of $p_t$. This steel ratio makes the reinforced concrete section very much under-reinforced. Since the tension and compression steel forces are small, concrete plays a greater role in providing ultimate capacity of the section.

As explained in Chapter 5, the tension steel yields at ultimate axial load levels below the balance points, i.e. when the axial load is small and the bending moment is large. For equilibrium of forces, a small total compression force is required to match the small tension steel force, and thus a very small area of concrete is required to resist compression, or a very small depth to the neutral axis is obtained. This gives a large strain in the tension steel and a large curvature of the section. Also, the
compression steel strain is small, and depending on the value of the ratio d'/t, the compression steel may even be in tension.

If the actual stress-strain curve of steel, including the strain-hardening region, is considered in the analysis, the tension steel strain will be in the strain-hardening region, and greater tension steel stress and hence force will be obtained than when strain-hardening of steel is neglected. The increase in the tension steel stress depends on the shape of the stress-strain curve in the strain-hardening region, and on the strain at which strain-hardening begins. It can be seen in Fig. 4.5 that, for a given yield stress, that steel for which strain-hardening begins at the smallest strain will give the greatest stress corresponding to a given strain in the strain-hardening region. Since the compression steel strain and stress are very small, the greater tension steel force can easily be matched by greater compression steel force obtained by increasing slightly the depth to the neutral axis with a slight change in the distribution of strains. Thus, greater total tension and total compression forces are obtained for the early strain-hardening steel. Since the lever arm is little affected when the total steel ratio is small, greater total moment of the tension and compression forces is obtained for the early strain-hardening steel. This can be seen in the moment-strain curves in Fig. 7.19 for $P_u = 0$. This figure shows that, even for $\epsilon_u = 0.003$, the calculated ultimate moment of the section for steel 5--the early strain-hardening steel--is the maximum, whereas it is the minimum for the flat-top steel 7. The moment-strain diagrams for steels 6 and 8 lie between those for steels 5 and 7 as do their stress-strain curves.
The tension steel force can be increased if the strain-hardening region of the stress-strain curve is considered in the analysis, and to match this increased tension steel force the total compression force must also increase. This requires that the depth to the neutral axis and/or the compression steel strain must increase. Figure 5.5 shows that this can be achieved only by increasing $\varepsilon_4$. The method of analysis explained in Chapter 3 assumes a higher limit on the ultimate concrete strain $\varepsilon_u$ than the ACI Code permits, and thus properly utilizes the strain-hardening region of the stress-strain curve. Consequently, greater ultimate moment is obtained for steels 5, 6 and 8 as compared to that for steel 7 at low levels of ultimate axial load.

The load-moment diagrams in Fig. 7.10 and 7.11 show that there is a considerable difference between the ultimate moments of the section for steels 5, 6 and 8 and that for steel 7 for $P_u = 0$ (pure moment). Table 7.4 shows that as much as 31 percent increase in the ultimate moment is calculated for steel 5 as compared to steel 7 for both values of $f'_c = 3000$ and 5000 psi. The small differences between the ultimate moments for steels 5, 6 and 8, as given in Table 7.2, are due mainly to the small differences in the tension steel stresses at ultimate capacity, as shown in Fig. 7.25.

Figure 7.15 shows a large increase in the curvature of the section for steels 5, 6 and 8, at ultimate axial load levels below the balance point, if strain-hardening of these steels is considered and $\varepsilon_u$ as high as 0.010 is permitted in the analysis. It can be noticed in Fig. 7.15 and Table 7.4 that, for $P_u = 0$, there is more than 150 percent increase in the ultimate curvature of the section with steels 5, 6 and 8 for $f'_c = 5000$ psi and more than 170 percent increase for $f'_c = 3000$ psi. This increase in ultimate curvature occurs because of the following:
Curvature \( \Phi = \frac{\varepsilon_4}{kd} \)

Since \( k = \frac{\varepsilon_4}{(\varepsilon_4 + \varepsilon_2)} \)
\[ \Phi = \frac{(\varepsilon_4 + \varepsilon_2)}{d} \]

The moment-strain (\( M - \varepsilon_4 \)) diagrams in Fig. 7.19 show that \( M \) increases with increase in \( \varepsilon_4 \), and the ultimate moment for steels 5, 6 and 8 at \( P_u = 0 \) is taken at \( \varepsilon_4 = \varepsilon_u = 0.010 \). It has been explained in Chapter 5 that the tension steel strain \( \varepsilon_2 \) increases as \( \varepsilon_4 \) is increased. For steel 7, \( \varepsilon_4 = \varepsilon_u = 0.003 \). Therefore, \( \varepsilon_2 \) for steel 7 is considerably smaller than for steels 5, 6 and 8. This is shown in Fig. 7.25. Thus, for steels 5, 6 and 8, both \( \varepsilon_4 \) and \( \varepsilon_2 \) are considerably greater than for steel 7, and larger curvatures are obtained for steels 5, 6 and 8.

Similarly, if the ultimate moments for steels 5, 6 and 8 at other levels of \( P_u \), occur at \( \varepsilon_4 = \varepsilon_u > 0.003 \), greater ultimate curvatures will be obtained for these steels than for steel 7 for which \( \varepsilon_u \leq 0.003 \).

As explained in Section 5.5 and shown in Fig. 5.7, the tension steel strain decreases as the ultimate axial load increases. When the axial load is close to the value at balanced conditions, the tension and compression steel strains are close to the yield value for steel 7. In this region of the stress-strain curve, steels 5, 6 and 7 are all alike, and the same tension and compression steel stresses are obtained for steels 5 and 6 as for steel 7. Consequently, the same moment is calculated for steels 5, 6 and 7 at \( \varepsilon_4 = 0.003 \). The \( M - \varepsilon_4 \) diagrams in Fig. 7.22 show that the maximum moment for ultimate axial load close to the balance point occurs at \( \varepsilon_4 \) very close to 0.003. Why the maximum moment at high levels of ultimate axial load does not occur at large value of \( \varepsilon_4 \) is explained in Section 5.2.

Figure 7.22 shows that the difference between the moments calculated for
$\varepsilon_4 = 0.003$ and the maximum moment is very small. This small difference due to the limiting value of $\varepsilon_u$ is shown in the small region of the load-moment diagram near the balance point.

For steel 8, the tension and compression steel strains at ultimate axial load levels near the balance point for steel 7 lie in that region of the stress-strain curve which is below that for steel 7, and smaller tension and compression steel stresses and forces, and hence smaller moments, are obtained for steel 8 than for steel 7. But, since the percentage of reinforcement is small, the reduction in ultimate moment is also small. A part of this reduction in ultimate moment for steel 8 is compensated for by using the maximum moment from the moment-strain diagram as the ultimate moment which occurs at $\varepsilon_4 = \varepsilon_u > 0.003$ as compared to $\varepsilon_4 = 0.003$ for steel 7.

At ultimate axial load levels between $P_b$ and $0.45 P_{07}$, the difference between the ultimate moment for steels 5 and 6 and that for steel 7 occurs because of the limitation of $\varepsilon_u$ to 0.003 for steel 7. This has been explained in Sections 7.2.1 and 7.2.2. The tension and compression steel strains corresponding to the maximum moment lie in that region of the stress-strain curve which is the same for all of the steels 5, 6 and 7. Therefore, no difference in the ultimate moment for these steels is obtained because of the shapes of their stress-strain curves.

At ultimate axial load levels higher than $0.45 P_{07}$, the maximum moment occurs at $\varepsilon_4 \leq 0.003$ for all of the steels 5, 6 and 7, and the tension and compression steel strains are less than 0.003, up to which strain all of these steels have the stress-strain curve. Therefore, the same stresses and forces, and hence the same moments, are obtained for steels 5, 6 and 7.
The stress-strain curve for steel 8 lies below that for steels 5, 6 and 7 between the strains 0.00145 and 0.00500. At all levels of ultimate axial load above the balance point, including the case of zero eccentricity, the compression steel strains lie in this region of the stress-strain curve, whereas the tension steel strain varies from 0.0025 tensile to 0.0020 compressive. Therefore, in comparison with steels 5, 6 and 7, smaller compression steel stresses and forces are obtained for steel 8 at all levels of ultimate axial load above the balance point, and also smaller tension steel forces are obtained at very high levels of ultimate axial load. Since, in this region of the load-moment diagram, the compression steel plays a greater role in providing the ultimate capacity, the smaller tension and/or compression steel forces result in smaller ultimate moment for steel 8 than for any of the steels 5, 6 and 7. But, since the percentage of reinforcement is small, the reduction in steel forces and hence in the ultimate moment is small. This can be seen in Figs. 7.10 and 7.11.

Figures 7.10 and 7.11 show that the reduction in ultimate capacity for steel 8 is greater with $f'_c = 3000$ psi than with $f'_c = 5000$ psi. This is so because, at ultimate axial load levels above the balance point, a large area of concrete is in compression, and hence a large concrete force is obtained. Since, for greater $f'_c$, the ratio of the compression steel force to the concrete force is smaller than for smaller $f'_c$, the reduction in compression steel force as compared to the total compression force is smaller for the greater $f'_c$. Consequently, the reduction in the ultimate capacity is smaller for $f'_c = 5000$ psi than for $f'_c = 3000$.

As explained in Section 5.5, the tension steel strain $\epsilon_2$ decreases for any steel as the ultimate axial load $P_u$ increases, and for steels 5, 6
and 8, $\varepsilon_u$ also decreases with increase in $P_u$. Thus the ultimate curvature of the section decreases with increase in $P_u$. But, the reductions in both $\varepsilon_u$ and $\varepsilon_2$ for steels 5, 6 and 8 result in greater reductions in ultimate curvatures for these steels than for steel 7, for which $\varepsilon_u = 0.003$ and only $\varepsilon_2$ decreases. When the maximum moment occurs at $\varepsilon_u \leq 0.003$ for steels 5, 6 and 8, at high levels of $P_u$, practically the same curvature is obtained for all of the steels 5, 6, 7 and 8. This can be seen in Fig. 7.15.

### 7.3.2. Intermediate Value of Total Steel Ratio

Figures 7.12 and 7.13 show comparisons of load-moment diagrams for steels 5, 6, 7 and 8 with $p_t = 0.04$ and three values of $f_c' = 3000$, 4000 and 5000 psi. As for $p_t = 0.01$, the load-moment diagrams for steel 7 were obtained with $\varepsilon_u$ limited to 0.003, in accordance with the 1963 ACI Code. Similarly, the load-curvature diagrams are shown in Figs. 7.15 and 7.16. The moment-strain diagrams for these steels, at three levels of ultimate axial load, are shown in Figs. 7.20 and 7.23.

(a) **Ultimate axial load levels below the balance point**

As was true in the case of $p_t = 0.01$, at ultimate axial load levels below the balance point, the method of analysis in Chapter 3 predicts considerably greater ultimate capacity of the section for steels 5, 6 and 8 with $p_t = 0.04$, as compared to that for steel 7 in accordance with the ACI Code. Table 7.4 shows as much as 34 percent increase in the ultimate moment at $P_u = 0$ for steel 5 with $\varepsilon_u$ limited to 0.010. The ultimate moments for steels 6 and 8 are intermediate between those for steels 5 and 7. This increase in ultimate moment decreases with increase in $P_u$ until at the balanced conditions there is a very small difference between the ultimate
capacities for these steels. Steel 8 gives slightly smaller ultimate capacity, whereas steels 5 and 6 give slightly greater capacity than steel 7 at the balanced conditions. The reason for the greater ultimate capacity for steels 5, 6 and 8 is the same as explained for $p_t = 0.010$, in Section 7.3.1, namely; greater tension and compression steel strains obtained with greater values of $\varepsilon_t = \varepsilon_u$ for steels 5, 6 and 8 result in greater steel stresses and forces which in turn result in greater ultimate moments for these steels. However, the increases in ultimate capacities for steels 5, 6 and 8 with $p_t = 0.04$ are slightly different from those with $p_t = 0.01$. The reasons for these differences with different values of $p_t$ are similar to those explained in Section 7.4 for variation of the ratio $q_t$. Figures 7.16 and 7.17 show that, as was true for $p_t = 0.01$, large curvatures of the section are obtained for steels 5, 6 and 8 as compared to those for steel 7, at ultimate axial loads below the balance point. This is due to the greater value of $\varepsilon_u$ for steels 5, 6 and 8 than for steel 7 for which $\varepsilon_u = 0.003$. Comparison of Figs. 7.25 and 7.26 shows that the tension steel strains $\varepsilon_2$ for all of the steels are smaller with $p_t = 0.04$ than with $p_t = 0.01$. Also, for $P_u = 0$, $\varepsilon_u = 0.010$ for steels 5, 6 and 8 but 0.003 for steel 7, with both values of $p_t$. Thus, the smaller values of $\varepsilon_2$ result in smaller values of ultimate curvatures with $p_t = 0.04$. This is shown in Tables 7.2 and 7.3. However, the ratios of ultimate curvature for steels 5, 6 and 8 to that for steel 7 are greater for $p_t = 0.04$ than for $p_t = 0.01$. This is so because, although the curvature decreases with increase in $p_t$ for all of the steels, greater decrease in ultimate curvature is obtained for steel 7 than for steels 5, 6 and 8, and thus the ratios of ultimate curvature increases with increase in $p_t$. 
(b) Ultimate axial load levels above the balance point

In this region of the load-moment diagram, the same ultimate capacity is obtained for steels 5 and 6, as was true for \( p_t = 0.01 \), because the maximum moment occurs at such a value of \( \varepsilon_u \) that the tension and compression steel strains do not exceed 0.003 up to which strain steels 5 and 6 have the same stress-strain relationships. But, although steel 7 also has the same stress-strain relationship up to the strain 0.003, limitation of \( \varepsilon_u \) to 0.003 results in smaller ultimate capacity for steel 7, as explained in Section 7.2. The maximum increase of 8 percent in ultimate capacity is obtained for steels 5 and 6 at ultimate axial load level slightly higher than the balance point. This increase in ultimate capacity decreases as \( P_u \) increases until at \( P_u \geq 0.45 \, P_07 \), the same ultimate capacity is obtained for steels 5, 6 and 7. This increase in ultimate capacity for steels 5 and 6 also occurs for \( p_t = 0.01 \) but is very small due to the very small area of reinforcement.

The ultimate capacity of the section for steel 8 is considerably smaller than that for steels 5, 6 and 7 at all levels of ultimate axial load above the balance point. This is due to the effect of that portion of the stress-strain curve for steel 8 which lies below those for steels 5, 6 and 7 near the yield point. This is explained in detail in Section 7.3.1 for \( p_t = 0.01 \). However, since greater reduction in compression and/or tension steel forces occurs for \( p_t = 0.04 \) than for \( p_t = 0.01 \), greater reduction in ultimate capacity is obtained for \( p_t = 0.04 \). This can be seen by comparing Figs. 7.10 and 7.11 with Figs. 7.12 and 7.13, respectively. The reduction in ultimate capacity for steel 8 in comparison with steels 5 and 6 varies from 5 to 10 percent, the maximum reduction being at ultimate
axial load close to the balance point where both the tension and compression steel strains for steel 8 lie in that region of the stress-strain curve which is below those for steels 5 and 6.

The moment-strain curves of Fig. 7.23 show that the maximum moment occurs at $\epsilon_4 = \epsilon_u > 0.003$ for ultimate axial load levels up to about 0.45 $P_0$. Consequently, as explained for $p_t = 0.01$, greater ultimate curvatures are calculated for steel 5, 6 and 8 than for steel 7 for which $\epsilon_4 = \epsilon_u = 0.003$. This can be seen in Figs. 7.15 and 7.17. However, for steel 8, the maximum moment occurs at a greater value of $\epsilon_4$ than for steels 5 and 6, which results in greater ultimate curvature $\phi_u$ for steel 8. But this increase in $\phi_u$ is accompanied by some reduction in ultimate capacity for steel 8, as explained above.

7.3.3. High Value of Total Steel Ratio

The ACI Code permits the use of $p_t = 0.08$ as the maximum steel ratio that can be provided in a reinforced concrete column section. Therefore $p_t = 0.08$ was chosen for analyses in this section. Figures 7.14 shows comparisons of load-moment diagrams for steels 5, 6, 7 and 8 for two values of $f'_c = 3000$ and 5000 psi. As for $p_t = 0.01$ and 0.04, $\epsilon_u$ was limited to 0.003 for steel 7. Similarly, Fig. 7.18 shows the load-curvature diagrams for these steels. The moment-strain diagrams for all of these steels are shown in Fig. 7.21 for $P_u = 0$, and in Figs. 7.24 for two levels of ultimate axial load above the balance point. One of these levels is slightly above while the other considerably above the balance point.

(a) Ultimate axial load levels below the balance point

As was true for $p_t = 0.01$ and 0.04, at ultimate axial load levels below the balance point, considerably greater ultimate moments are calculated
for steels 5, 6 and 8, according to the method of analysis in Chapter 3, than for steel 7 in accordance with the 1963 ACI Code. The reason for this is the same as explained in Section 7.3.1 for $p_t = 0.01$. This increase in ultimate capacity for steels 5, 6 and 8 decreases with increase in $P_u$.

Table 7.4 shows that the increases in ultimate moment for steels 5, 6 and 8 as compared to that for steel 7 are smaller for $p_t = 0.08$ than for $p_t = 0.04$ and 0.01. This is due partly to the smaller tension steel stresses for $p_t = 0.8$, as can be seen by comparing Figs. 7.24, 7.25 and 7.26, and partly due to the effect of compression steel which does not provide enough compression force to compensate for the reduction in the concrete force, and to match the increased tension force, because the compression steel strains are either on the flat-plateau or in the lower portion of the strain-hardening region for $p_t = 0.08$. This effect of the compression steel is explained in detail in Section 7.4.2. Also, since the tension steel strains are smaller for $p_t = 0.08$ than for $p_t = 0.01$ and 0.04, smaller ultimate curvatures are obtained for all of the steels 5, 6, 7 and 8 for $p_t = 0.08$. But, since the decrease in ultimate curvature for steel 7 is greater than that for steels 5, 6 or 8, the ratios of ultimate curvatures for steels 5, 6 and 8 to that for steel 7 are greater for $p_t = 0.08$ than for $p_t = 0.04$ and 0.01.

(b) Ultimate axial load levels above the balance point

At high levels of ultimate axial load above the balance point, when the maximum moment occurs at $\varepsilon_u \leq 0.003$, the same ultimate capacity is obtained for steels 5, 6 and 7, as has been explained for $p_t = 0.01$ and 0.04. However, for steel 8 smaller ultimate capacity but greater ultimate curvature are obtained than for any of the steels 5, 6 and 7. The reason
for this is the same as explained for $p_t = 0.01$ and $0.04$. Comparison of Figs. 7.12, 7.13 and 7.14 shows that, while greater reduction in ultimate capacity for steel 8 is obtained with $p_t = 0.04$ than with $p_t = 0.01$, no more reduction is obtained with $p_t = 0.08$ than with $p_t = 0.04$. Also, there is considerably greater increase in ultimate curvature with $p_t = 0.08$ than with $p_t = 0.04$. This is explained below:

It can be seen from the moment-strain diagrams in Figs. 7.23 and 7.24 that the maximum moment for steel 8, at ultimate axial load levels above the balance point, occurs at a considerably greater value of $e_4 = e_u$ with $p_t = 0.08$ than with $p_t = 0.04$ whereas this is not true for steels 5, 6 and 7. Also, the peak value of the moment for steel 8 with $p_t = 0.08$ and a given $f'_c$ is greater than the moment at that value of $e_4$ at which the maximum moment occurs for steels 5, 6 and 7. Thus, a part of the reduction in ultimate capacity for steel 8 with $p_t = 0.08$ is compensated for by the higher value of $e_u$ than that obtained with $p_t = 0.04$, and no more reduction in ultimate capacity for steel 8 in comparison with that for steel 7 is obtained with $p_t = 0.08$ than with $p_t = 0.04$.

As has been explained in Chapter 5, curvature of the section increases with increase in $e_4$. Therefore, the greater value of $e_u$ for steel 8 with $p_t = 0.08$ results in greater ultimate curvature than with $p_t = 0.04$.

7.4. Effect of Stress-Strain Curve of Steel with Variation of $q = p_t f'_y / f'_c$

In this section, the results of analyses of the 15-in. square reinforced concrete section presented in Sections 7.2 and 7.3 will be discussed in such a manner as to show the effects of the characteristics
of the stress-strain curves as a function of the variation of the value of the ratio $q_t = \frac{p_t f_y}{f'_c}$. Seven values of $q_t$ were obtained by using various values of $p_t$ and $f'_c$. These values of $q_t$ are given in Section 7.1. The yield strength of all steels was 60 ksi, and one-half of the total steel was provided in each face.

When $q_t$ is varied by varying either $f'_c$ or $p_t$ or both, the distribution of the total compression force between the compression steel and concrete is varied. With increase in $q_t$, the compression steel plays a greater role in resisting the applied external forces. The general effect of the compression steel with variation of $q_t$ is explained in Section 7.4.1. In Sections 7.4.2, 7.4.3 and 7.4.4 the effects of the stress-strain curve of the reinforcement with variation of $q_t$ is explained for steels 5, 6 and 8, respectively, in comparison with steel 7.

7.4.1. Effect of Compression Steel

In this section, a general discussion will be presented to explain the effect of the compression steel in a reinforced concrete section when the value of $q_t$ is increased by decreasing the value of $f'_c$ with constant values of $p_t$ and $f_y$ or by increasing $p_t$ with constant values of $f'_c$ and $f_y$. The conclusions so derived will be used to explain the effect of varying $q_t$ in Sections 7.4.2, 7.4.3 and 7.4.4.

(a) Variation of $q_t$ by varying $f'_c$ only

When $q_t$ is increased by decreasing $f'_c$ for given values of $p_t$ and $f_y$, the compression force in the concrete is reduced which reduces the total compression force. For a given value of $e_4$, the total compression force can be increased to obtain equilibrium of forces by increasing the depth to the
neutral axis, which increases the area of concrete in compression and also
the strain and hence the stress and force in the compression steel. The
compression steel stress can be increased only if the compression steel
strains are in such a region of the stress-strain curve that the stress
increases with an increase in strain. But, if the stress-strain curve of
the compression steel has a flat plateau and the compression steel strains
lie on it, no increase in compression steel stress and hence in its force
will be obtained. In this case, the depth to the neutral axis must increase
appreciably to bring enough area of concrete in compression to compensate
for the reduction in the concrete force due to the reduction in $f'_c$. The
other effects of the increase in the depth to the neutral axis are to
derease the tension steel strain and the lever arm of the concrete force.
Therefore, if the tension steel strains are not on the flat-plateau, the
tension steel stress and hence the tension steel force will decrease as the
depth to the neutral axis is increased for a given value of $\epsilon_u$. The decrease
in the lever arm caused by the increase in the depth to the neutral axis
reduces the moment of the concrete force.

For steel 7 which has a flat-top stress-strain curve, and with
$\epsilon_u$ limited to 0.003, the tension steel strains are on the flat plateau
while the compression steel strains are in the elastic region at ultimate
axial load levels below the balance point for all values of $p_t$, as shown
in Table 7.2. Thus, for a given value of $p_t$, the tension steel stress is
equal to $f_y$, and the total tension steel force $A_{st} f_y$, for any reasonable
value of $f'_c$. Therefore, the total compression force will also be the same
for both values of $f'_c$. For the greater value of $f'_c$, the depth to the neutral
axis is smaller, and thus the compression steel strain, stress and force are
smaller than the corresponding quantities for the smaller value of $f'_c$.

But, the compression force in the concrete and its lever arm are greater for the greater value of $f'_c$ than for the smaller. The resulting total moment of these forces will depend on the relative lever arms of the concrete and compression steel forces. If the lever arm of the concrete force is longer, as is the case for the reinforced concrete section analyzed here and as shown in Table 7.2 for all values of $q_t$, the greater $f'_c$ will result in some reduction in the moment of the compression steel force but in a greater increase in the moment of the concrete force. Consequently, the total moment will be greater for the greater $f'_c$ with a given value of $p_t$.

It is shown in Table 7.2 that, at $P_t = 0$, for each of the three values of $p_t = 0.01, 0.04$ and $0.08$, there is some reduction in the total moment as $f'_c$ is reduced. However, as $p_t$ increases, the compression steel force is increased, and the ratio of the concrete force to the compression steel force or to the total compression force is reduced. Consequently, the effect of variation of $f'_c$ as explained above, is also reduced. With $p_t = 0.01$, the increase in the total moment obtained by increasing $f'_c$ from 3000 to 5000 psi is 10 percent, whereas the corresponding increases with $p_t = 0.04$ and $0.08$ are only 5 and 2 percent respectively.

For strain-hardening steels 5, 6 and 8, with $\varepsilon_u = 0.010$, as is the case at $P_u = 0$, and as shown in Table 7.2, the tension steel strains are in the strain-hardening region at all values of $q_t$. It has been explained in Section 5.4 that the compression steel strain increases as $\varepsilon_u$ is increased. Therefore, in comparison with steel 7 for which $\varepsilon_u = 0.003$, greater compression steel strains are obtained for steels 5, 6 and 8. Also, as $p_t$ increases, the depth to the neutral axis is increased, and thus the compression steel strains are further increased. Consequently, for steels 5,
6 and 8 it is only at low values of $q_t$ that the compression steel strains are in the elastic region and thus the compression steel compensates for a part of the reduction in the concrete force as $f'_c$ is decreased. Table 7.2 shows the following:

1. For steel 5, the compression steel strains are in the elastic region for $q_t \leq 0.60$, on the flat plateau for $0.60 < q_t < 0.80$, and in the lower portion of the strain-hardening region for $q_t \geq 0.80$.

2. For steel 6, the compression steel strains are in the elastic region for $q_t \leq 0.80$, and on the flat plateau for $q_t > 0.80$.

3. For steel 8, the limiting value of $q_t$ up to which the compression steel strains are in the elastic region is in between 0.20 and 0.48, and beyond this limit they lie in that portion of the stress-strain curve for steel 8 which is below that for steel 7 and in which the stress increases with strain at a smaller rate than when the strains are in the elastic region.

In view of the above, the extent to which the compression steel compensates for the reduction in the concrete force as $f'_c$ is reduced depends on the value of $q_t$. For high values of $q_t$, the compression steel provides very little or no additional force when $f'_c$ is reduced and $p_t$ is kept constant. In this case the increased area of concrete in compression provides for all or a major portion of the reduction in the concrete force. Consequently, there is a greater increase in the depth to the neutral axis, and thus a greater decrease in the lever arm of the concrete force, which causes a greater reduction in the moment of the concrete force. However, when the tension steel strain is in the strain-hardening region, the increase in the depth to the neutral axis reduces the tension steel stress and its force.
Therefore, for equilibrium of forces, when the depth to the neutral axis increases for the smaller $f'_c$ to increase the compression steel force and/or the concrete force, there is some decrease in the tension steel force, and equilibrium of forces is obtained with slightly reduced total tension and compression forces for the case of the smaller $f'_c$, and with a smaller increase in the depth to the neutral axis and thus a smaller decrease in the lever arm of the concrete force. These factors have opposing effects. Smaller total tension and compression forces cause a greater reduction in the total moment, but the smaller reduction in the lever arm causes a smaller reduction in the moment of the concrete force.

When the compression steel compensates for a part of the reduction in the concrete force, and the tension steel is in the strain-hardening region so that its force is reduced when the depth to the neutral axis is increased, equilibrium of forces will be obtained with a very small increase in the depth to the neutral axis, and thus with a very small decrease in the lever arm of the concrete force. This results in a smaller reduction in the total moment with reduced $f'_c$ than when the tension steel has a flat-top stress-strain curve. Consequently, when $q_t$ is increased by decreasing $f'_c$ only, the ratio of the ultimate moment for a strain-hardening steel to that for a flat-top steel will increase slightly or remain the same.

When the compression steel force remains constant or increases very little for higher values of $q_t$; that is, it does not provide enough additional force to compensate for a part of the reduction in the concrete force, a greater increase in the depth to the neutral axis is required, which further reduces the lever arm and hence the moment of the concrete force. With the reduced tension steel force because of the strain-hardening steel, there is a greater reduction in the total moment for a strain-hardening
steel than for a flat-top steel for which the compression steel is always in the elastic region and thus provides for an additional force to compensate for a part of the reduction in the concrete force. Consequently, the ratio of the ultimate moment for a strain-hardening steel to that for a flat-top steel decreases with decrease in \( f'_c \). As will be explained in Sections 7.4.2, 7.4.3 and 7.4.4, this occurs at high values of \( q_t \) and \( p_t \).

(b) Variation of \( q_t \) by varying \( p_t \) only

When \( q_t \) is increased by increasing \( p_t \) with constant values of \( f'_c \) and \( f_y \), the tension steel force can increase in about the same proportion as the area of the tension steel. Therefore, the total compression force must also increase as much in order that equilibrium of forces is obtained. Since \( f'_c \) is constant, the compression force in the concrete can increase only by increasing the depth to the neutral axis so as to increase the area of concrete in compression. This increases the compression steel strain, but decreases the tension steel strain. Hence, if the compression steel strains are in the elastic region, the compression steel stress will increase, and thus the compression steel force will increase faster because its area has also been increased due to the increase in \( p_t \). Also, if the tension steel strains are not on the flat plateau, the decrease in the tension steel strain caused by the increase in the depth to the neutral axis will reduce the tension steel stress, and hence the tension steel force will not increase in proportion to the increase in the area of the tension steel.

For the flat-top steel 7, the tension steel strains are on the flat plateau whereas the compression steel strains are in the elastic region, at ultimate axial load levels below the balance point, for all values of \( q_t \) and with \( \varepsilon_u = 0.003 \). Thus, the tension steel stress is equal to \( f_y \) for all
the values of $q_t$ considered here, and the increase in $p_t$ increases the tension steel force in the same proportion as the area of the tension steel. Therefore, for equilibrium of forces, the total compression force also increases in the same proportion as $p_t$. But, as explained above, the increase in the depth to the neutral axis increases the strain and hence the stress in the compression steel, which takes a greater share of the total compression force in comparison with the increase in $p_t$. Consequently, the depth to the neutral axis is not increased as much and the concrete force does not increase in the same proportion as the tension or compression steel forces. Thus, the lever arm of the concrete force is only slightly reduced, and the total moment does not increase in the same proportion as the value of $p_t$. But, since the compression steel is able to provide for a greater portion of the increase in the total compression force, the proportionate reduction in the total moment, which occurs because of the rearrangement of the compression forces and their lever arms, is not as much as would be obtained in a singly reinforced concrete section.

For the strain-hardening steels 5, 6 and 8, the tension steel strains are in the strain-hardening region at lower levels of ultimate axial load and for all values of $q_t$. As explained above, for the case of variation of $q_t$ by varying $f_c'$ only, the compression steel strains for steels 5, 6 and 8 are greater than those for steel 7. Furthermore, the compression steel strains increase as $q_t$ is increased.

For low values of $p_t$ and $q_t$, the compression steel strains are in the elastic region, and thus the compression steel is able to provide a greater share of the increase in the total compression force with increase in $p_t$ as does steel 7. Therefore, for strain-hardening steels, the proportionate increase in the total moment is about the same as that with
steel 7 or only slightly increased. This is true as long as the increase in $p_t$ and $q_t$ does not push the compression steel strains into the flat plateau for steels 5 and 6, or into that portion of the stress-strain curve for steel 8 which lies below that of steel 7, and in which the stress increases with strain at a smaller rate. This occurs at those high values of $q_t$ which have been discussed previously for the case of variation of $q_t$ by varying $f'_c$ only. When the compression steel strains are on the flat plateau for steels 5 and 6, the compression steel stress is limited to $f'_y$. Thus, the compression steel does not provide as much increase in its share of the total compression force with increase in $p_t$ as does steel 7. The increase in the depth to the neutral axis decreases the tension steel strain, and hence the stress and the force. This results in a smaller increase in the total tension and total compression forces in proportion to the increase in the value of $p_t$ than for steel 7. Consequently, a smaller increase in the total moment is obtained by increasing $p_t$ at high values of $q_t$. The same is true for steel 8, except that the compression steel stress does not stay constant at high values of $q_t$, but increases with strain at a smaller rate, and hence the compression steel does not provide enough increase in its share of the total compression force in proportion to the increase in $p_t$ as does steel 7. Consequently, the ratio of ultimate moment for steels 5, 6 and 8 to that for steel 7 is reduced as $q_t$ is increased by increasing $p_t$ at high values of $q_t$.

7.4.2. Steel 5

(a) **Ultimate axial load levels below the balance point**

As explained in Section 7.3, considerable increases in the ultimate moment and ultimate curvature of a reinforced concrete section are obtained
at ultimate axial load levels below the balance point for all values of $q_t$ when the strain-hardening portion of the stress-strain curve for steel 5 is considered in the analysis with $\epsilon_u$ limited to 0.010. Table 7.4 gives the ratios of the ultimate moment and ultimate curvature calculated for $P_u = 0$ and $0.05 P_0$ with steel 5 according to the method of analysis in Chapter 3, to the ultimate moment and ultimate curvature calculated with steel 7 in accordance with the provisions of the 1963 ACI Code.

It can be seen in Tables 7.3 and 7.4 that, at $P_u = 0$ when $q_t$ is increased by increasing $p_t$ at constant values of $f'_c$ and $f_y$, there is a slight increase in the ratios of ultimate moments up to $q_t = 0.60$. Also if $q_t$ is increased either by decreasing $f'_c$ at constant values of $p_t$ and $f_y$, or by changing both $f'_c$ and $p_t$ at a constant value of $f_y$, the ratios of ultimate moment either increase slightly or remain constant up to $q_t = 0.60$. For $q_t > 0.60$ any increase in $q_t$ results in a decrease in the ratio of ultimate moment with steel 5 to that with steel 7. This effect of variation of $q_t$ is obtained because of the effect of the compression steel in combination with the effect of the strain-hardening region of the stress-strain curve of tension steel as explained in Section 7.4.1. It is explained briefly below:

For all values of $q_t \leq 0.60$, the compression steel strains are in the elastic region, as shown in Table 7.2 and Figs. 7.25, 7.26 and 7.27. Thus, as explained in Section 7.4.1, the compression steel is able to provide the required increase in its share of the total compression force as does steel 7, and enough increase in the total compression force is obtained to match the corresponding increase in the total tension force for steel 5. It can be seen from the values of the lever arm of concrete force given in
Table 7.2. that, as $q_t$ increases up to 0.60, there is a smaller reduction in the lever arm with steel 5 than with steel 7, which results in proportionately smaller reduction in the moment of the concrete force with steel 5 than with steel 7. Because of these two factors, either the same or slightly higher ratio of ultimate moment for steel 5 to that for steel 7 is obtained up to $q_t = 0.60$.

When $q_t$ exceeds 0.60, the compression steel strains for steel 5 are on the flat-plateau or in the lower portion of the strain-hardening region. In the former case, the compression steel stress remains constant; in the latter case, the compression steel stress increases at much smaller rate than does that for steel 7 which is in the elastic region. Consequently, steel 5 provides proportionately less increase in its share of total compression force than does steel 7. Also, there is a greater reduction in the lever arm of the concrete force as $q_t$ increases beyond 0.60 with steel 5 than with steel 7 which results in proportionately greater reduction in the moment of the concrete force. Furthermore, equilibrium of forces occurs with proportionately smaller increases in the total tension and compression forces for steel 5 than for steel 7. All these factors result in a proportionately smaller increase in total moment due to the effect of the strain-hardening region of the stress-strain curve for steel 5. Consequently, the ratio of ultimate moment for steel 5 to that for steel 7 decreases as $q_t$ is increased beyond 0.60.

A similar effect of variation of $q_t$ is obtained at $P_u = 0.05 P_{07}$. However, as $P_u$ increases, the compression steel strain increases and thus, the limit of $q_t$ up to which the compression steel strains are in the elastic region is reduced. This can be seen by comparing the compression steel strains in Tables 7.2 and 7.3.
Table 7.2 shows that ultimate curvature decreases as $q_t$ increases for both steels 7 and 5. However, the ratio of the ultimate curvature for steel 5 to that for steel 7 increases as $q_t$ increases because there is more reduction in the ultimate curvature for steel 7. This has been explained in Section 7.3.

(b) **Ultimate axial load levels above the balance point**

Figures 7.10 through 7.14 show that, at ultimate axial load levels in the range $P_b < P_u \leq 0.45 P_{07}$, there is some increase in the ultimate capacity of the section with steel 5 calculated according to the method of analysis in Chapter 3, as compared to that with steel 7 calculated in accordance with the provisions of the 1963 ACI Code. This increase is a maximum at an ultimate axial load level slightly higher than the balance point, and decreases as $P_u$ increases until at $P_u > 0.45 P_{07}$ the same capacity is calculated with both steels 5 and 7. Table 7.6 gives the ratios of the ultimate moment for steel 5 to that for steel 7 for all values of $q_t$ at such levels of ultimate axial load that, for each value of $q_t$, the maximum increase in ultimate capacity is obtained for steel 5 in comparison with steel 7. It can be seen in this table that the maximum increase of 10 percent in ultimate capacity is obtained with $P_u = 0.08$. This table also shows that, when $q_t$ is increased by decreasing $f'_c$ at constant values of $P_u$ and $f_y'$, there is no effect of the variation of $q_t$ on the ratios of ultimate moment except for the very small value of $P_u = 0.01$, in which case the increase in the capacity with steel 5 is very small--only 2 to 5 percent. However, if $q_t$ is increased by increasing $P_u$ only at constant values of $f'_c$ and $f_y'$, the ratio of ultimate moment for steel 5 to that for steel 7 increases as $q_t$ is increased, indicating that a greater increase in ultimate moment is obtained.
for steel 5 as \( p_t \) or \( q_t \) is increased. A part of this increase is obtained because of the effect of the ratio \( d'/t \) which will be explained in Section 7.5, and a part occurs because the maximum moment, which is the ultimate moment for steel 5, is obtained at greater values of \( \epsilon_u \), as is the case at ultimate axial load levels below the balance point. However, at ultimate axial load levels above the balance point, the effect of the strain-hardening region of the stress-strain curve of the reinforcement is not utilized except at very high value of \( q_t = 1.60 \). This is so because, in this region of the load-moment diagram, the tension steel strains are in the elastic region while the compression steel strains are on the flat plateau or in the lower portion of the strain-hardening region, for \( q_t = 1.60 \) and at ultimate axial load levels slightly higher than the balance point. In view of this, it can be concluded that, at ultimate axial load levels above the balance point, the effect of variation of \( q_t \), as explained above, occurs because of the difference in the limit on \( \epsilon_u \), but not because of the difference in the stress-strain characteristics of steel 5. Even for \( q_t = 1.60 \), when the compression steel strains are in the lower portion of the strain-hardening region, that is, \( \epsilon_3 \leq 0.005 \), the compression steel stress is only slightly greater than the yield stress, and thus its effect on the ultimate moment is negligible. However, the ultimate moment occurs at a greater value of \( \epsilon_u \), and thus there is a greater increase in the ultimate curvature for \( q_t = 1.60 \).

As shown in Figs. 7.15 to 7.17, there is a considerable increase in the ultimate curvature for steel 5 at ultimate axial load levels slightly above the balance point for all values of \( q_t \). But, as is true for the increase in the ultimate moment, the increase in the ultimate curvature
occurs because of the greater value of ultimate concrete strain for steel 5 rather than because of the stress-strain curve for steel 5.

7.4.3. Steel 6

(a) Ultimate axial load levels below the balance point

Figures 7.10 through 7.14 show that, at ultimate axial load levels below the balance point, considerably greater ultimate moments and ultimate curvatures are predicted for the section with steel 6 than with steel 7, as was the case for steel 5. This is due to the effect of the strain-hardening region of the stress-strain curve for steel 6 in tension together with the higher limit on $\varepsilon_u$ permitted in the method of analysis in Chapter 3.

The ratios of the ultimate moment for steel 6 to that for steel 7 given in Table 7.3 indicate that a maximum of 23 percent increase in the ultimate moment is obtained for steel 6 at $P_u = 0$. This table also shows the following:

(1) When $q_t$ is increased by decreasing $f'_c$ at constant values of $P_t$ and $f_y$, there is negligible effect of the variation of $q_t$ on the ratio of ultimate moments for steels 6 and 7, except for very high values of $q_t$ obtained with $P_t = 0.08$. This is so because, at lower values of $q_t$ obtained with $P_t = 0.01$ and $0.04$, as shown in Table 7.2, the compression steel strains are in the elastic region for both steels 6 and 7. Consequently, as explained in Section 7.4.1, both steels are able to provide the required increases in their shares of total compression forces to match with corresponding tension forces, and proportionately the same decreases occur in ultimate moments due to the reduction in $f'_c$, so that practically the same ratios of ultimate moment are obtained. However, at higher values of
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$q_t$ with $p_t = 0.08$, the compression steel strains are on the flat plateau for steel 6 but in the elastic region for steel 7 and, as explained in Section 7.4.1, compression steel 6 does not provide the required increase in its share of the total compression force to compensate for the reduction in $f'_c$. Therefore, the proportionate reduction in the ultimate moment for steel 6 is more than that for steel 7, and the ultimate moment ratios for steels 6 and 7 decrease with increase in $q_t$ for $p_t = 0.08$.

(2) There is a very slight decrease in the ratio of ultimate moments for steels 6 and 7 when $q_t$ is increased from 0.12 to 0.20 by decreasing $f'_c$ from 5000 to 3000 psi with $p_t = 0.01$, whereas there is a very slight increase in the same ratio if $f'_c$ is reduced from 5000 to 3000 psi with $p_t = 0.04$ to increase $q_t$ from 0.48 to 0.80, although in both cases the compression steel strains are in the elastic region. This is so because, as shown in Table 7.2, in the former case with $p_t = 0.01$, the compression steel strains are in the tensile elastic region for both values of $f'_c$ for steel 7, whereas for steel 6, the compression steel is in tension for $f'_c = 5000$ psi but in compression for $f'_c = 3000$ psi. In the latter case with $p_t = 0.04$, the compression steel strains are in the compressive elastic region for both steels and with both values of $f'_c$. When the compression steel is in tension, the total compression force is provided by concrete; but when the compression steel is in compression, it shares a part of the total compression force. Since, for the section analyzed here, the lever arm of the concrete force is longer than that of the compression steel force, the total moment of the total compression force is less when the compression steel is in compression than when it is in tension. Consequently, the total moment for steel 6 for $q_t = 0.20$, when the compression steel is in compression,
is proportionately reduced as compared to that for steel 7 for which the compression steel is in tension, and thus the ratio of the ultimate moments for steels 6 and 7 is very slightly reduced for $q_t = 0.20$. This will be explained in more detail for steel 8 in Section 7.4.4.

(3) When $q_t$ is increased by increasing $p_t$ at constant values of $f'_c$ and $f_y$, there is again a negligible effect of the variation of $q_t$ on the ratio of ultimate moments for steels 6 and 7 up to $q_t = 0.80$ with $p_t \leq 0.04$; but with $q_t \geq 0.96$ obtained with $p_t = 0.08$, the same ratio decreases as $q_t$ increases. The reason for this is the same as for (1) above namely, the effect of the location of the compression steel strains on the stress-strain curve of the compression steel.

(4) When $q_t$ is increased by varying both $f'_c$ and $p_t$, there is negligible effect of the variation of $q_t$ for all values of $q_t < 0.96$. But, at higher values of $q_t$ there is a small reduction in the ratio of ultimate moments for steels 6 and 7 for the same reason as given in (1) above.

For $P_u = 0.05 P_{07}^c$, Table 7.3 shows that the limit of $q_t$ up to which the compression steel strains are in the elastic region for steel 6 lies between 0.20 and 0.48. Therefore, as explained in Section 7.4.1, the compression steel provides for enough increase in its share of the total compression force only up to this limiting value of $q_t$, and the ratios of ultimate moments for steels 6 and 7 increase with increase in $q_t$ up to $q_t = 0.48$. Since, for $q_t > 0.48$, the compression steel strains are on the flat-plateau for steel 6, but in the elastic region for steel 7, the compression steel 6 does not provide the required increase in its share of the total compression force whereas steel 7 does provide enough increase in the compression steel force. Therefore, as explained in Section 7.4.1, the
ratio of ultimate moments for steels 6 and 7 decreases with increase in \( q_t \) beyond 0.48.

The effect of variation of \( q_t \) on ultimate curvatures of the section with steel 6, as compared to those for steel 7, is the same as explained in Section 7.4.2 for steel 5.

(b) **Ultimate axial load levels above the balance point**

Figures 7.10 through 7.14 show that, at all levels of ultimate axial load above the balance point, the same ultimate capacity of the section is obtained with steels 5 and 6. This has been explained in Section 7.3. Therefore, the effect of variation of \( q_t \) on the ultimate moment ratios for steels 6 and 7 is the same as for steels 5 and 7, which has been explained in Section 7.4.2.

Figures 7.15 through 7.18 show that the same ultimate curvatures are obtained for the section with steels 5 and 6 at all levels of ultimate axial load above the balance point for all values of \( q_t \), except at \( q_t = 1.60 \) obtained with \( p_0 = 0.08 \) and \( f'_c = 3000 \) psi. Therefore, the effect of the variation of \( q_t \) on the ultimate curvatures for steel 6 is the same as for steel 5 up to \( q_t = 0.96 \) which has been explained in Section 7.4.2. However, for \( q_t = 1.60 \), Fig. 7.14 shows that, at ultimate axial load levels between the balance point and \( P_u = 0.45 P_{07}' \), there is practically no difference in the ultimate moments for steels 5 and 6, but Fig. 7.18 shows that, in the same region of the load-moment diagram, considerably smaller ultimate curvatures are obtained for steel 6 than for steel 5. This is so because, the ultimate moment for steel 5 occurs at greater value of \( \varepsilon_u \) than for steel 6 due to the effect of early strain-hardening of steel 5, since the compression steel strains are in the lower portion of the strain-hardening
region of the stress-strain curve for steel 5, that is $\varepsilon_3 \leq 0.005$. This gives slightly greater compression steel stresses which makes the moment-strain curves quite flat for steel 5, as shown in Fig. 7.24. This results in too small increases in the ultimate moment for steel 5 to be shown in Fig. 7.14, but considerable increases in the ultimate curvature. This effect of $\varepsilon_u$ on ultimate curvature has been explained in Section 7.2.

7.4.4. Steel 8

(a) At low levels of ultimate axial load

Figures 7.10 through 7.14 show that, at low levels of ultimate axial load, there is a considerable increase in the ultimate moment and ultimate curvature of a reinforced concrete section calculated for steel 8 according to the method of analysis in Chapter 3, in comparison with those calculated for steel 7 in accordance with the provisions of the 1963 ACI Code. The increase in the ultimate moment is maximum at $P_u = 0$ and decreases as $P_u$ increases. Tables 7.4 gives the ratios of ultimate moment and ultimate curvature for steel 8 to those for steel 7 at two levels of ultimate axial load, $P_u = 0$ and $P_u = 0.05 P_{07}$ for seven values of $q_t$.

For $P_u = 0$, Table 7.4 shows that the ratio of ultimate moment for steel 8 to that for steel 7 ($M_{u8}/M_{u7}$) is a maximum with $q_t = 0.12$; the increase in ultimate moment for steel 8 is 26 percent. As $q_t$ is increased, the ratio $M_{u8}/M_{u7}$ is decreased so that the increase in the ultimate moment for steel 8 decreases as $q_t$ increases. This is so because of: (1) the presence of the compression steel as explained in Section 7.4.1, and (2) the shape of the stress-strain curve for compression steel 8. Table 7.2 and Figs. 7.25, 7.26 and 7.27 show that the compression steel strains for steel 8 are in the elastic region only for very small values of $q_t = 0.12$.
and 0.20, that is with $p_t = 0.01$. But, for higher values of $q_t$, that is with $p_t \geq 0.04$, the compression steel strains are in that portion of the stress-strain curve which lies below that for steel 7, and in which stress increases with strain at a smaller rate. For steel 7, the compression steel strains are in the elastic region for all values of $q_t$. Therefore, as explained in Section 7.4.1, except for very small values of $q_t$, the compression steel 8 does not provide enough required increase in its share of the total compression force in comparison with steel 7, and there is proportionately less increase in the ultimate moment with steel 8 than with steel 7. Consequently, the ultimate moment ratio decreases as $q_t$ increases. However, for $p_t = 0.01$, although the compression steel strains for both steels 7 and 8 are in the elastic region and thus, both can provide the required additional compression forces, the ultimate moment ratio still decreases as $q_t$ increases because of the following:

Table 7.2 shows that, for $q_t = 0.12$, the compression steel is in the tensile elastic region for both steels 7 and 8. Therefore, the concrete force is equal to the sum of the forces in the tension and compression steels. The lever arm for the concrete force for steel 7 is longer than that for steel 8. The total moment is equal to the moment of the concrete force about the centroid of the tension steel minus the moment of the compression steel force about the same point. This is the case for both steels 7 and 8 for $q_t = 0.12$. But, for $q_t = 0.20$ with $p_t = 0.01$, the compression steel strain is in the tensile elastic region only for steel 7 but in the compressive elastic region for steel 8. Hence, for steel 7, the concrete force is again equal to the sum of the forces in the compression and tension steels, but for steel 8, the concrete force is equal to
the difference of the forces in the tension and compression steels. Thus, although the concrete force is reduced for both steels 7 and 8, as $f'_c$ is reduced to increase $q_t$, the reduction in the concrete force for steel 8 is more than that for steel 7. This greater reduction in the concrete force is compensated for by the compression steel which is in compression for $q_t = 0.20$ for steel 8 only. As Table 7.2 shows, the lever arm of the concrete force is longer than that of the compression steel force for both steels 7 and 8, and the transfer of compression force from concrete to the compression reinforcement for steel 8 results in a smaller moment of the total compression force about the centroid of the tension steel than for steel 7 for which concrete alone provides for the total compression force. Thus, if the ratio $d'/t$ is small enough to put compression steel in compression for steel 7 too, the ratio of ultimate moment for steel 8 to that for steel 7 would be the same, or would increase slightly with increase in $q_t$ from 0.12 to 0.20 with $p_t = 0.01$. For higher values of $q_t$, both compression steels 8 and 7 are in compression, but no increase in the ultimate moment ratio is obtained because of the location of the compression steel strains for steel 8, as explained above.

For $P_u = 0.05 P_{07'}$, the limit of $q_t$ up to which the compression steel strains are in the elastic region for steel 8 is between 0.20 and 0.48, as was true for $P_u = 0$, but for $P_u = 0.05 P_{07}$ it is closer to 0.20. Thus, it is only for small values of $q_t$, with $p_t = 0.01$, that the compression steel provides for enough increase in its share of total compression force as does steel 7. Also, Table 7.3 shows that, for $q_t = 0.20$, the compression steel is in compression for both steels 7 and 8. Therefore, as explained earlier, there is some increase in the ratio of ultimate moments for steels 8 and 7.
(from 1.13 to 1.17) when $q_t$ is increased from 0.12 to 0.20. For $q_t > 0.48$, the compression steel strains for steel 8 only are not in elastic region, as was true for $P_u = 0$, and hence the ultimate moment ratio decreases with increase in $q_t$.

The effect of variation of $q_t$ on the ultimate curvature of the section for steel 8 in comparison with that for steel 7 is the same as for steel 5 explained in Section 7.4.2.

(b) At high levels of ultimate axial load

Figures 7.10 through 7.14 show that the increases in the ultimate capacity of the section, which are obtained because of the strain-hardening region of the stress-strain curve for steel 8 in comparison with that for steel 7, decrease as the ultimate axial load increases. At higher levels of ultimate axial load, when the tension and/or compression steel strains are in that portion of the stress-strain curve for steel 8, which lies below that for steel 7, smaller ultimate capacity is predicted for steel 8 than for steel 7. This has been explained in Section 7.3. At ultimate axial load levels close to the balance point for steel 7, both the tension and compression steel strains are in this region of the stress-strain curve. Thus, smaller tension and compression steel stresses are obtained for steel 8 than for steel 7, and a greater reduction in the ultimate moment is obtained for steel 8. However, as will be explained in Section 7.5, the ultimate axial load level at balanced conditions for steel 7, $P_b$, decreases as $q_t$ increases. This brings $P_b$ towards that region (lower portion) of the load-moment diagram where the ultimate moment for steel 8 occurs at higher value of $e_u$, which increases the tension and compression steel strains, stresses and forces. Consequently, as $q_t$ is increased, smaller reduction in
ultimate moment is obtained for steel 8 at the balanced conditions for steel 7. At high values of $q_t \geq 0.96$ obtained with $p_t = 0.08$, $P_b$ is reduced so much that, at ultimate axial load levels close to the balance point for steel 7, the tension steel is in the strain-hardening region of the stress-strain curve for steel 8, which gives greater stress for steel 8 than for steel 7 and thus greater ultimate moment for steel 8. This can be seen from the ratios of ultimate moment for steel 8 to that for steel 7 in Table 7.6. The ultimate moments for steel 7 have been calculated for the balanced condition as defined in the 1963 ACI Code, with the values of $k_1 k_2$ taken at $\varepsilon_4 = \varepsilon_u = 0.003$ for the same stress-strain curve for concrete as has been used for steel 8 (Fig. 4.11). The ultimate moments for steel 8 have been taken from Figs. 7.10 through 7.14 at such levels of ultimate axial load that, for each value of $q_t$, the ratio of the ultimate moment to the ultimate axial load or the eccentricity of the load is the same for both steels 7 and 8. The ratios of ultimate moments given in Table 7.6 indicate that a reduction of about 6 percent occurs in the ultimate moment for steel 8 in comparison with that for steel 7, for $q_t = 0.12$, and that this reduction decreases as $q_t$ increases, until for $q_t = 1.60$, there is an increase of 7 percent in the ultimate moment for steel 8, as compared to that for steel 7. These conclusions are true for the section analyzed here with $d'/t = 0.20$. As will be explained in Section 7.5.1, the balance point is raised for a smaller value of $d'/t$, and thus these comparisons will be made at higher levels of ultimate axial load, and the effect of variation of $q_t$ will be similar to that explained below.

At higher levels of ultimate axial load, Figs. 7.10 through 7.14 show that smaller ultimate capacity is predicted for steel 8 than for
steel 7. The extent to which the decrease in ultimate capacity occurs depends on the value of $q_t$. Table 7.6 gives the ratios of ultimate moments for steels 8 and 7 at two levels of $P_u$ above the balance point for steel 8. These load levels are $P_u = 0.6 P_{07}$ and $0.8 P_{07}$ for steel 7; but for steel 8, ultimate moments have been taken from Figs. 7.10 through 7.14 at the same eccentricity as for steel 7, in the same way as was done at balanced conditions. It can be seen in Table 7.6 that, at these load levels, the ratio of ultimate moment for steel 8 to that for steel 7 decreases as $q_t$ increases, indicating that the decrease in ultimate capacity of the section with steel 8 increases as $q_t$ increases, except that at high values of $q_t > 0.96$ obtained with $p_t = 0.08$ smaller reduction in ultimate moment is obtained as $q_t$ increases. This is explained below:

When $q_t$ is increased either by decreasing $f'_c$ or by increasing $p_t$ or by both, the compression steel takes a greater portion of the total compression force and its effects are thus enhanced. Since the decrease in the ultimate capacity for steel 8, in comparison with that for steel 7, occurs at high levels of ultimate axial load because of the smaller compression steel stresses which are obtained from that portion of the stress-strain curve that lies below that for steel 7, the reduction in the ultimate capacity for steel 8 is increased with increase in $q_t$. However, it can be seen in Figs. 7.22, 7.23 and 7.24 that the ultimate concrete strain $\varepsilon_u$ increases as $q_t$ is increased. This has the effect of increasing compression steel strains and stresses, so that for values of $q_t > 0.96$ the compression steel strains at ultimate capacity are such that the compression steel stresses are only slightly less than the yield stress which is the stress in the compression steel 7. Consequently, there is a smaller reduction in the ultimate moment for steel 8 at values of $q_t > 0.96$. 

Although, the reduction in the ultimate moment for steel 8 at high levels of ultimate axial load, as given in Table 7.6, is not great, it can be concluded that the maximum reduction occurs at intermediate values of $q_t$, with $p_t = 0.04$.

Figures 7.15 through 7.16 show that, at ultimate axial load levels above the balance point, ultimate curvature of the section with steel 8 increases with increase in $q_t$. Also, the ultimate axial load level up to which the increase in ultimate curvature is obtained increases with increase in $q_t$. This is so because $\varepsilon_u$ for steel 8 increases with increase in $q_t$. This can be seen in the moment-strain diagrams in Figs. 7.22 through 7.24. Although this increase in ultimate curvature is considerable, it is accompanied by some decrease in ultimate capacity.

7.5. Effect of Stress-Strain Curve of Steel With Variation of $d'/t$

Section 808(c) of the 1963 ACI Code requires that a minimum cover of 1-1/2 in. be provided over the lateral reinforcement in a reinforced concrete column. According to Section 806(b) of the Code, the minimum diameter of lateral ties in a reinforced concrete column is 1/4 in. Thus, the minimum value of the distance from the extreme compression fiber to the centroid of the compression steel is $1.5" + 0.25" + D/2$, where D = diameter of the reinforcing bars in compression. The ACI Code permits the use of minimum size of bars as #5. Therefore, the absolute minimum value of $d'$ is 2.06 in., but the ratio of $d'$ to the overall depth of the section $t$ will vary depending on the size of the column and of the compression bars. For small columns, the minimum value of the ratio $d'/t$ will be large, and vice versa. In symmetrically reinforced concrete sections, such as the one
used for the analyses in the chapter, the effective depth of the section is \( d = t - d' \). Therefore, the variation of the ratio \( d'/t \) results also in variation of \( d/t \). When equilibrium of forces is obtained at a given value of the applied axial load and a given value of the concrete strain in the extreme compression fiber \( \epsilon_u \), the strains in the tension and compression steels will depend on the ratio \( d'/t \). With a smaller value of \( d'/t \), greater strains will be obtained in the reinforcement and, unless these strains are on the flat-plateau of the stress-strain curve, greater stresses will be obtained in the reinforcement, which will change the internal forces and thus the capacity of the section. In the following sections, the effects of the stress-strain curve of the reinforcement with variation of the ratio \( d'/t \) will be discussed.

The same 15-in. square reinforced concrete section, as selected for analyses in Sections 7.2, 7.3 and 7.4, was analyzed with two values of the ratio \( d'/t \) = 0.15 and 0.20 and the effects are discussed for the stress-strain curve for steel 5 in comparison with that for steel 7.

7.5.1. Effect of Variation of the Ratio \( d'/t \) on the Balance Point

According to Section 1900(b) of the 1963 ACI Code, the balance point on the load-moment diagram is to be obtained from the strain and stress distributions over the cross section such that \( \epsilon_u = 0.003 \) and \( \epsilon_2 = \epsilon_y \) which, for steels 5 and 7, is 0.00207.

Thus

\[
kd = \frac{0.003}{0.00207 + 0.003} \quad d = 0.592 \text{d}
\]

\[
\phi_u = \frac{0.00507}{d}
\]

and

\[
\epsilon_2 = 0.003 - \phi_u d'
\]
If $\epsilon_y = \epsilon_y' = 0.00207$, so that the compression steel yields at balanced conditions, then

$$d' = \frac{0.003 - 0.00207}{f_u} = 0.183d$$

For $d = t - d'$

$$d' = 0.155t$$

According to the 1963 ACI Code, the absolute minimum value of $d' = 2.06$ in. for #5 bars. Thus, the minimum value of $t$ for which #5 compression bars will yield is 13.3 in. For smaller reinforced concrete sections and/or larger bars, or for greater values of $f_y'$ and $\epsilon_y'$, the compression steel will not yield at the balanced conditions defined by the 1963 ACI Code.

The ultimate axial load at the balanced conditions, $P_b$, is calculated as:

$$P_b = 0.85k'f'bkd + A_{sc}f_{sc} - A_{st}f_y'$$

Since, for symmetrically reinforced concrete section, $A_{sc} = A_{st}$

$$P_b = 0.85k'f'bkd - A_{st}(f_y' - f_{sc})$$

If the compression steel does not yield, $f_{sc} < f'_{y} = f_y$, and $P_b$ will decrease as $A_{st}$ or $P_t$ is increased for a given value of $f_c'$. Also, $P_{07}$, the ultimate axial load for zero eccentricity with steel 7, is $0.85f_c'bt + 2A_{st}f_y$ which increases as $A_{st}$ or $P_t$ is increased at constant $f_c'$. Therefore, the ratio $P_b/P_{07}$ decreases as $P_t$ increases. Again, if $f_c'$ is decreased at constant values of $P_t$ and $f_y'$, $P_b$ will decrease faster than $P_{07}$, and the ratio $P_b/P_{07}$
will decrease as \( f'_c \) is decreased. Consequently, when the ratio \( d'/t \) is such that the compression steel does not yield, the ratio \( P_b/P_{07} \) decreases as \( q_t \) increases. This can be seen in Figs. 7.10 through 7.14.

If the ratio \( d'/t \) is such that the compression steel yields, \( f_{sc} = f_y \) and \( P_b = k_1 k_2 f'_c b d_k \). In this case, \( P_b \) depends only on the variation of \( f'_c \) but not on \( A_{st} \) or \( p_t \). The ratio \( P_b/P_{07} \) will decrease as \( f'_c \) is decreased or \( p_t \) is increased but not as much as when the compression steel does not yield.

In the method of analysis explained in Chapter 3, the ultimate concrete strain is not limited to 0.003, but is obtained from the moment-strain diagrams corresponding to the ultimate moment determined from the three criteria explained in Section 3.3. Thus, the balance point as defined in the ACI Code does not always lie on the load-moment diagram obtained by the method of analysis in Chapter 3. Figures 7.10 through 7.14 show that as \( p_t \) increases, the balance point lies further inside the load-moment diagram because, at higher values of \( p_t \) or \( q_t \), the ultimate moment at ultimate axial load levels close to the balance point occurs at ultimate concrete strain \( \varepsilon_u \) greater than 0.003, and the compression steel yields at ultimate capacity. But, for the value of \( d'/t = 0.20 \) for which the load-moment diagrams of Figs. 7.10 through 7.14 have been obtained, the compression steel does not yield at \( \varepsilon_u = 0.003 \). Thus, a smaller total compression force is obtained, which results in a smaller ultimate load and ultimate moment. For the case of \( d'/t = 0.15 \), as shown in Fig. 7.28, the compression steel yields when \( \varepsilon_4 = \varepsilon_u = 0.003 \) and \( \varepsilon_2 = \varepsilon_y \). Thus, the balance point lies almost on the load-moment diagram. This is so because, at ultimate axial load levels close to the balance point, when \( d'/t \) is small enough to cause
yielding of the compression steel, the moment-strain diagrams are quite flat and the ultimate moment occurs at $\varepsilon_4 = \varepsilon_u$ close to 0.003 so that, in this region of the load-moment diagram, negligible difference is obtained between the ultimate capacity calculated for steel 5 according to the method of analysis in Chapter 3 and that for steel 7 calculated in accordance with the ACI Code. This will be explained further in Section 7.5.3.

7.5.2. **Effect of Variation of $d'/t$ Below the Balance Point**

As explained in Section 7.3, considerable increase in the ultimate capacity is obtained with strain-hardening steels at ultimate axial load levels below the balance point when the strain-hardening region of the stress-strain curve is considered in the analysis and $\varepsilon_u$ is limited to 0.010. For symmetrically reinforced sections, decrease in the ratio $d'/t$ increases the ratio $d/t$. This has the following effects:

(1) At ultimate capacity, greater strains are obtained in both the tension and compression steels. Since the tension steel strain is in the strain-hardening region, greater tension steel strain, stress and force are obtained. Also the compression steel strains are either in the elastic region of the stress-strain curve, for low values of $\alpha_\nu$, or on the flat-plateau for high values of $\alpha_\nu$. In the former case, the compression steel stress and hence the compression steel force increases as $d'/t$ is reduced, and equilibrium of forces is obtained with greater total tension and total compression forces. Furthermore, for taking moments of the tension and compression steel forces about the plastic centroid, the lever arm is increased as $d'/t$ is decreased and $d/t$ is increased. Thus, greater ultimate moment is obtained. In the latter case, when the compression steel strains
are on the flat-plateau for high values of \( p_t \), there is no increase in the compression steel force, but with a slight increase in the depth to the neutral axis, concrete force can be increased and tension steel strain and stress can be decreased, so that equilibrium of forces is obtained with very little increase in total compression and tension forces. However, the increase in the lever arm of the tension and compression steel forces, as stated above, considerably increases their moments about the plastic centroid because of the large forces in the tension and compression steels due to high value of \( p_t \) and high stress. Thus, greater ultimate capacity is calculated when the ratio \( d' / t \) is small. Table 7.7 gives the ratios of ultimate moment for steel 5 to that for steel 7 for both values of \( d' / t = 0.15 \) and \( 0.20 \), at various levels of ultimate axial load for \( p_t = 0.04 \) and \( f'_c = 4000 \) psi. For this case, the compression steel strains are always on the flat plateau for steel 5. It can be seen in this table that, at lower levels of ultimate axial load, larger ratios of ultimate moment for steel 5 to that for steel 7 are obtained with smaller value of \( d' / t \), indicating that greater increase in ultimate capacity is obtained for steel 5 than for steel 7 with smaller values of \( d' / t \). As the ultimate axial load increases, this increase in ultimate capacity decreases.

### 7.5.3. Effect of Variation of \( d' / t \) Above the Balance Point

At ultimate axial load levels above the balance point, the compression steel strains are on the flat-plateau, while the tension steel strains are in the elastic region. With \( \varepsilon_u \) limited to 0.005, if the value of \( d' / t \) is not small, the compression steel strains may also be in the elastic region at ultimate axial load levels a little above the balance point. In this case, if a greater limit on \( \varepsilon_u \) is permitted, not only
greater concrete force is obtained because of the increase in the value of 
$k_1k_3$ up to $\varepsilon_u = \varepsilon_m = 0.004$, but the compression steel yields at ultimate 
capacity and a greater ultimate moment is calculated by the method of 
analysis in Chapter 3. However, if $d'/t$ is small enough, so that the 
compression steel yields at $\varepsilon_u = 0.003$, the compression steel force remains 
constant, and very little increase in the ultimate moment is obtained for 
steel 5 with $\varepsilon_u > 0.003$ in comparison with that for steel 7 with $\varepsilon_u = 0.003$. 
Table 7.7 shows that for $d'/t = 0.15$ a maximum of 3 percent increase in 
the ultimate moment is obtained for steel 5 by the method of analysis 
explained in Chapter 3; but for $d'/t = 0.20$, the maximum increase in the 
ultimate moment is 9 percent.

7.6. Summary

In this chapter, a 15-in. square reinforced concrete section has 
been analyzed to obtain load-moment and load-curvature diagrams to study 
the effects of the stress-strain curves of ASTM A615-68 Grade 60 reinforcing 
bars on the strength and behavior of the section. The variables considered 
in the analyses are described in Section 7.1. The results obtained and the 
discussions and explanations presented in Sections 7.2 through 7.5 are 
summarized below.

(a) Ultimate moment

If the section is analyzed with ultimate concrete strain $\varepsilon_u$ 
limited to 0.003, in accordance with the provisions of the 1963 ACI Code 
practically the same moment is calculated at all levels of ultimate axial 
load for steels 5, 6 and 7 for all values of $q_t$, except for small values 
of $q_t$ in combination with low levels of ultimate axial load. This is so
because, with $\varepsilon_u \leq 0.003$, the tension and compression steel strains lie in that region of the stress-strain curves which is the same for all of the steels 5, 6 and 7. Thus, the strain-hardening region of the stress-strain curve is not properly utilized. But, for small values of $q_t$ with $\varepsilon_u = 0.003$, the tension steel strain is in the lower portion of the strain-hardening region at low levels of ultimate axial load, and an increase in ultimate moment is obtained due to the greater tension steel stress. This increase in the ultimate moment decreases as $q_t$ and/or $P_u$ increases.

For steel 8 which has a round-house stress-strain curve, when $\varepsilon_u \leq 0.003$, a considerable decrease in the ultimate moment is obtained in comparison with steel 7, at all levels of ultimate axial load, and for all values of $q_t$, except at very low levels of ultimate axial load in combination with small values of $q_t$. This is so because, with $\varepsilon_u \leq 0.003$, depending on the load level, the tension and/or compression steel strains are in that portion of the stress-strain curve for steel 8 which lies below that for steel 7, and smaller steel stresses and thus smaller ultimate moments are obtained for steel 8. For small values of $q_t$ in combination with low levels of ultimate axial load, the tension steel strains are in that portion of the stress-strain curve which is above the flat-plateau for steel 7 and thus, as for steels 5 and 6, some increase in the ultimate moment is obtained for steel 8. This increase decreases as $q_t$ increases, and for high values of $q_t$ obtained with $p_t = 0.08$, smaller ultimate moment is calculated for steel 8 than for steel 7, even at $P_u = 0$.

For the flat-top steel 7, at ultimate axial load levels below the balance point and above $P_u = 0.45 P_{07}$, extending the limit of ultimate concrete strain beyond 0.003 has practically no effect on the ultimate
moment. At ultimate axial load levels between the balance point and $P_u = 0.45 P_07$, the maximum moment occurs at $\epsilon_u > 0.003$, and a maximum increase of about 10 percent in the ultimate moment is obtained if a higher limit on $\epsilon_u$ is permitted. However, this increase in ultimate moment decreases as the ultimate axial load increases or the ratio $d'/t$ decreases.

For steels 5, 6 and 8, at ultimate axial load levels below the balance point, the moment-strain curves continue to rise as $\epsilon_u$ is increased, owing to the effect of the strain-hardening region of the stress-strain curve, and thus a considerable increase in the ultimate moment is obtained if a higher limit on $\epsilon_u = 0.010$ is permitted in the analysis.

At ultimate axial load levels above the balance point, the maximum moment for steels 5 and 6 occurs with tension steel strains in the elastic region and compression steel strains on the flat-plateau, so that the strain-hardening region of the stress-strain curve is not utilized even if the higher limit on $\epsilon_u$ is permitted. Consequently, steels 5, 6 and 7 are all alike in this region of the load-moment diagram.

For steel 8, at ultimate axial load levels above approximately $0.20 P_07$, considerable decrease in ultimate moment is obtained in comparison with steel 7, even if the higher limit on $\epsilon_u$ is permitted for steel 8, because of that portion of the stress-strain curve for steel 8 which lies below that for steel 7.

(b) **Ultimate curvature**

If $\epsilon_u$ is limited to 0.003, there is practically no difference between the ultimate curvatures for steels 5, 6, 7 and 8 at all levels of ultimate axial load, and for all values of $q_u$. 

If the section is analyzed according to the method of analysis explained in Chapter 3, and if the ultimate moment occurs at $\varepsilon_u > 0.003$, a greater ultimate curvature is obtained. Therefore, for strain-hardening steels 5, 6 and 8, at ultimate axial load levels below the balance point, a large increase in ultimate curvature is obtained since ultimate moment occurs at $0.003 < \varepsilon_u < 0.010$. For steel 7 with high values of $q_t$, and at ultimate axial load levels below the balance point, maximum moment occurs at $\varepsilon_u > 0.003$ and thus there is some increase in the ultimate curvature, but not as much as for steels 5, 6 and 8, even though ultimate moment is little affected by the limitation of $\varepsilon_u$ to 0.003 in this region of the load-moment diagram.

At ultimate axial load levels in the region $P_b < P_u < 0.45 P_{07'}$, since the maximum moment occurs at $\varepsilon_u > 0.003$ for all of the steels 5, 6, 7 and 8, greater ultimate curvature is obtained by allowing a higher limit on $\varepsilon_u$. There is very little difference between the ultimate curvatures for steels 5, 6 and 8 with small values of $q_t$ for the case of a higher limit on $\varepsilon_u$. But, as $q_t$ increases, greater curvature is obtained for steels 5, 6 and 8 than for steel 7. For high values of $q_t$ obtained with $P_t = 0.08$, greater ultimate curvatures are obtained for steel 8 than for any of the steels 5, 6 and 7 even though there is some reduction in the ultimate moment for steel 8.

At higher levels of ultimate axial load, practically the same ultimate curvatures are obtained for steels 5, 6 and 7 because, the maximum moment occurs at $\varepsilon_u \leq 0.003$ for all of these steels. This is true also for steel 8 for small values of $q_t$ obtained with $P_t = 0.01$ but, as $q_t$ increases, greater ultimate curvatures are calculated for steel 8 than for any of the steels 5, 6 and 7 due to the greater value of $\varepsilon_u$ for steel 8.
(c) **Effect of compression steel**

The compression steel plays an important part in utilizing the strain-hardening region of the stress-strain curve of reinforcement and thus in increasing the ultimate moments and ultimate curvatures at low levels of ultimate axial load. As long as the compression steel strains are in the elastic region, the compression steel provides the required increase in the total compression force, not only to match the increase in the tension steel force but also to compensate for the reduction in the concrete force due to the reduction in the value of $k_3$ at high values of $\varepsilon_u$. Thus, the total tension and compression forces are increased with a consequent increase in the ultimate capacity. As the ultimate axial load level and/or the value of $q_t$ increases, the compression steel strains increase while the tension steel strains decrease so that, when the compression steel strains are on the flat-plateau for steels 5 and 6 or in that portion of the stress-strain curve for steel 8 which is below that for steel 7, the compression steel does not provide enough required increase in its share of the total compression force and thus smaller increase in the ultimate capacity is obtained for steels 5, 6 and 8. At high levels of ultimate axial load, the same ultimate capacity is obtained for steels 5 and 6 as for steel 7, but smaller ultimate capacity is obtained for steel 8.

(d) **Effect of the ratio $d'/t$**

The increases in the ultimate capacity and ultimate curvature of the section due to the effect of the stress-strain curve of the reinforcement depend also on the ratio $d'/t$. For symmetrically reinforced concrete sections, as is the case in normal practice, a smaller value of $d'/t$ results in a greater value of $d/t$. Therefore, for given values of $P_u$ and $\varepsilon_u$, greater
tension and compression steel strains are obtained with the smaller value of \(\frac{d'}{t}\). Thus, at ultimate axial load levels below the balance point, the tension steel strains are pushed farther into the strain-hardening region, and greater increases in ultimate capacity are obtained for the strain-hardening steels than for the flat-top steel.

However, with the smaller value of \(\frac{d'}{t}\), the compression steel strains are pushed into the flat-plateau for small values of \(q_t\) and into the strain-hardening region for high values. Therefore, the limit of \(q_t\), up to which the compression steel can provide enough required increase in the total compression force and hence can result in greater increase in the ultimate capacity with increase in \(q_t\), is reduced. Also, for high values of \(q_t\), the compression steel strains are increased so much that buckling of compression bars becomes the governing criterion for ultimate capacity rather than the limiting ultimate concrete strain of 0.010. This occurs at ultimate axial load levels below the balance point but not above it because, at ultimate axial load levels above the balance point, \(\epsilon_u\) is small which gives small compression steel strain.

Since the compression steel strain and stress increase with decrease in the ratio \(\frac{d'}{t}\), the ultimate axial load at balanced conditions, as defined in the Code, increases and raises the balance point on the load-moment diagram. Also, at ultimate axial load levels in the region \(P_b < P_u \leq 0.45 P_0\), the compression steel strain at \(\epsilon_u = 0.003\) approaches the yield value for steels 5, 6 and 7 and thus the increase in the ultimate capacity for these steels due to the higher value of \(\epsilon_u\) decreases with decrease in \(\frac{d'}{t}\).
8. ANALYSES OF REINFORCED CONCRETE SECTIONS LOADED IN A SHORT TIME--ASTM GRADE 75 STEELS

8.1. Introduction

The same 15-in. square reinforced concrete section (Fig. 3.1) which was used for the analyses in Chapter 7 has been analyzed in this chapter with ASTM A615-68 Grade 75 steel to obtain load-moment and load-curvature diagrams. The same variables considered in Section 7.1 have also been considered in this chapter. With values of the total steel ratio $p_t = 0.01, 0.04$ and $0.08$, and concrete strengths $f'_c = 3000, 4000$ and $5000$ psi, and with the yield strength $f_y = 75$ ksi, the following values of the ratio $q_t = p_t f_y / f'_c$ are obtained for the cases considered in this chapter:

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$f'_c$</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>5000</td>
<td>0.15</td>
</tr>
<tr>
<td>0.01</td>
<td>3000</td>
<td>0.25</td>
</tr>
<tr>
<td>0.04</td>
<td>4000</td>
<td>0.75</td>
</tr>
<tr>
<td>0.08</td>
<td>5000</td>
<td>1.20</td>
</tr>
<tr>
<td>0.08</td>
<td>3000</td>
<td>2.00</td>
</tr>
</tbody>
</table>

The stress-strain curves for the reinforcement used in the analyses in this chapter are shown in Fig. 4.6 and the basis for their selection is explained in Section 4.1.3.

The results of the analyses are presented in the form of figures and tables which are explained and discussed in Sections 8.2 through 8.5. These figures and tables give load-moment and load-curvature relationships at ultimate capacity of the section, and are presented in such a manner as to compare the results obtained for each of the steels 1, 3 and 4 with
those for the flat-top steel 2 for two limiting values of ultimate concrete strain $\varepsilon_u^u = 0.003$, as assumed in the 1963 ACI Code, and $\varepsilon_u^u = 0.010$ assumed in Section 3.2, in order to investigate the effects of these assumptions. The results for steels 1, 3 and 4 with $\varepsilon_u^u$ limited to 0.010 are also compared to those obtained in strict accordance with the provisions of the 1963 ACI Code; that is with a flat-top stress-strain curve (steel 2), and with $\varepsilon_u^u$ limited to 0.003.

The ordinates of the load-moment and load-curvature diagrams are expressed as the ratio $P_u^u / P_{02}^u$, where $P_u^u$ is ultimate axial load for any steel and $P_{02}^u$ is the ultimate axial load at zero eccentricity for steel 2; the abscissas are expressed as the ratio $M_u^u / M_{02}^u$, where $M_u^u$ is ultimate moment for any steel and $M_{02}^u$ is the ultimate moment at zero axial load (pure moment capacity) for steel 2.

Since steels 1, 3 and 4 do not have a well-defined yield point, the balance point, as defined in the ACI Code, can be computed only by using an arbitrary definition of the yield strength for these steels. However, since the ultimate moments and ultimate curvatures for these steels have been compared with those for the flat-top steel 2, the balance point referred to in Sections 8.2 through 8.5 is always that computed for steel 2 in accordance with the definition given in Section 1900(b) of the 1963 ACI Code.

The effect of the stress-strain curves for steels 1, 3 and 4 in comparison with steel 2 are explained in this chapter with frequent reference to the explanations and discussions of the effects of the stress-strain curves for ASTM A615-68 Grade 60 steels 5, 6, 7 and 8 given in Chapter 7.
8.2. Flat-Top Steel 2

Figures 8.1, 8.2 and 8.3 show the load-moment diagrams for steel 2 for the five values of $q_t$ given in Section 8.1. Similarly, Figs. 8.10 and 8.11 show the load-curvature diagrams. For each value of $q_t$, the load-moment and load-curvature diagrams are shown for two limits of $\epsilon_u = 0.003$ and 0.010. The balance point as defined in the 1963 ACI Code is marked by a line and a letter B.

It can be seen in Figs. 8.1, 8.2 and 8.3 that the ratio of the ultimate axial load at the balanced conditions to that at zero eccentricity ($P_b/P_{02}$) decreases as $q_t$ increases, for the same reason as explained in Section 7.5.1 for steel 7. As shown in Fig. 8.27, for $P_u = 0$ and $\epsilon_u = 0.003$, the tension steel strain decreases as $q_t$ increases. With $q_t = 2.0$, obtained with $p_t = 0.08$ and $f'_c = 3000$ psi, the tension steel strain is in the elastic region. Therefore, for the section analyzed here, with $d'/t = 0.20$, the strain distribution corresponding to the balanced conditions defined in the ACI Code will give a negative value of $P_b$, indicating that the balance point lies in the region of axial tension. It has been explained in Section 7.5.1 that, as $d'/t$ decreases, the balance point is raised. If $d'/t = 0.177$ for steel 2 and $q_t = 2.0$, the balanced condition corresponds to the pure moment case.

For those values of $q_t$ for which $P_b$ is positive, at ultimate axial load levels below the balance point, there is little difference between the ultimate moments calculated by limiting $\epsilon_u$ to 0.003 and by maximization of the moment with a higher limit of $\epsilon_u = 0.01$, as was the case for steel 7 (Chapter 7). This is so because, as shown in the moment-strain diagrams in Figs. 8.21, 8.22 and 8.23, the maximum moment occurs at $\epsilon_u$ close to 0.003.
and the moment strain diagrams are quite flat. This was explained in detail in Section 7.2.1 for steel 7. However, at a very high value of $a_t = 2.0$ since, at $\epsilon_4 = 0.003$, the tension steel is in the elastic region even at $P_u = 0$ (Fig. 8.27), increasing $\epsilon_4$ beyond 0.003 increases the tension steel strain and hence the stress and force. Also, since the compression steel strain is in the elastic region, its strain, stress and force are increased by increasing $\epsilon_4$ so that the compression steel provides enough compression force to match the increased tension force. Consequently, the increased tension and compression forces result in a greater moment as $\epsilon_4$ is increased until the tension steel yields. At $P_u = 0$, an increase of 8 percent in the ultimate moment is obtained by extending the limit of $\epsilon_4$ beyond 0.003. This increase in ultimate moment will decrease as $d'/t$ is reduced. When $d'/t = 0.177$ instead of 0.20, for this section at $P_u = 0$, $a_t = 2.0$ and $\epsilon_4 = 0.003$, the tension steel strain is equal to the yield strain and very little increase in ultimate moment will be obtained by increasing $\epsilon_4$ beyond 0.003.

At ultimate axial load levels at and above the balance point, when $\epsilon_u$ is limited to 0.003 yielding of the compression steel depends on the ratio $d'/t$. At the balanced conditions with $\epsilon_u = 0.003$, for $t = 15$ in., the compression steel strain will be equal to the yield strain for steel 2 if $d' = 1.25$ in. ($d'/t = 0.0833$) which is less than the absolute minimum value of $d' = 2.06$ in. according to the 1963 ACI Code, as explained in Section 7.5. Therefore, for the section analyzed here, at ultimate axial load levels in the greater portion of the load-moment diagram above the balance point, with $\epsilon_u = 0.003$, the tension and compression steel strains are in the elastic region of the stress-strain curve. As $P_u$ increases, the
the tension steel strain decreases while the depth to the neutral axis and the compression steel strain increase. At high levels of ultimate axial load (depending on \( q_u \)) the compression steel strains are on the flat plateau.

In that region of the load-moment diagram above the balance point where the compression steel strains are in the elastic region, increasing the value of \( \varepsilon_u \) beyond 0.003, increases both the tension and compression steel strains, as explained in Chapter 5, and thus results in greater tension and compression steel forces, which increase the moment until the compression steel yields at \( \varepsilon_u > 0.003 \). Therefore, extending the limit of \( \varepsilon_u \) beyond 0.003 at ultimate axial load levels above the balance point results in a considerable increase in the ultimate capacity. This was also the case for the flat-top Grade 60 steel 7, as explained in Section 7.2.1. However, for the Grade 75 steel, this effect is magnified. This is so because, at \( \varepsilon_u = 0.003 \), the ratio of the compression steel strain to the yield strain for the Grade 75 steel is smaller than that for the Grade 60 steel so that, when a higher value of \( \varepsilon_u \) increases the compression steel strain to the yield strain, a greater increase in the compression steel stress and force is obtained for the Grade 75 steel than for the Grade 60 steel, resulting in a greater increase in the ultimate capacity. Figures 8.1, 8.2 and 8.3 show that the reduction in ultimate capacity caused by limiting \( \varepsilon_u \) to 0.003 increases as \( q_u \) increases.

At high levels of ultimate axial load, when the compression steel strains are on the flat-plateau, the maximum moment occurs at \( \varepsilon_u \leq 0.003 \) so that no increase in ultimate capacity is obtained by extending the limit beyond 0.003. The reason for the maximum moment not occurring at \( \varepsilon_u > 0.003 \) at high levels of ultimate axial load are similar to those given in Section 7.2.1 for steel 7.
It was explained in Chapter 5 that ultimate curvature increases as $\varepsilon_u$ increases. Since, as explained above, in certain regions of the load-moment diagrams the maximum moment occurs at $\varepsilon_u > 0.003$, greater ultimate curvatures are obtained by allowing a higher limit of $\varepsilon_u$, particularly at ultimate axial load levels around the balance point. This can be seen in Figs. 8.10 and 8.11. These figures show that as $q_t$ increases a greater increase in ultimate curvature is obtained because at higher values of $q_t$ ultimate moment occurs at greater values of $\varepsilon_u$.

### 8.3. Steel 1

Stress-strain curve for steel 1 is shown in Fig. 4.6. It was explained in Section 4.1.3 that this curve is one of the best that can be obtained for Grade 75 steel and meets the requirements of the 1963 ACI Code as well as the ASTM A615 specifications. Although, as per ASTM specifications, this curve gives $f_y = 92.5$ ksi, according to the ACI Code, for design purposes, this curve is equivalent to the flat-top curve 2 shown in Fig. 4.6 with $f_y = 75$ ksi. Therefore, the comparisons of the analyses of the section with steels 1 and 2 explained and discussed below will show the effects of the stress-strain curves of those steels which have $f_y > 75$ ksi but for which the Code assumes $f_y = 75$ ksi.

#### 8.3.1. Ultimate Concrete Strain Limited to 0.003

(a) **Ultimate axial load levels below the balance point**

Figures 8.1, 8.2 and 8.3 show comparisons of load-moment diagrams for steel 1 with those for steel 2. These figures show that, at ultimate axial load levels below the balance point, there is a considerable increase
in the ultimate capacity of the section with steel 1 as compared to that with steel 2 even at $\varepsilon_u = 0.003$. This occurs because of the strain-hardening of steel 1, as was the case for steel 5 in Section 7.2.2. Figure 8.27 shows that, for all values of $q_t$ except 2.0, the tension steel strains for steel 1 are in the strain-hardening region, while those for steel 2 are on the flat-plateau. Therefore, greater tension steel stresses and forces are obtained for steel 1 than for steel 2. This increased tension force is accompanied by increased concrete and compression steel forces caused by an increase in the depth to the neutral axis, so that at equilibrium of forces greater moment is obtained for steel 1. This increase in moment decreases as $q_t$ or $P_u$ increases, as was true for steel 5, because the tension steel strain and hence stress decreases with increase in $q_t$ or $P_u$. Table 8.1 shows that with $\varepsilon_u = 0.003$ and at $P_u = 0$, a maximum increase of 29 percent in the moment occurs for steel 1 in comparison with steel 2 for $q_t = 0.15$. As $q_t$ increases beyond 1.20, there is little or no increase in ultimate moment for steel 1 because the tension steel strains are not in the strain-hardening region. Similarly, as $P_u$ increases to $P_b$, the tension steel strain decreases to the yield strain for steel 2, and the same moment is calculated for both steels 1 and 2 when $\varepsilon_u = 0.003$.

The increase in the moment for steel 1 at $\varepsilon_u = 0.003$, as explained above, is accompanied by a small decrease in the curvature of the section. This is shown in Figs. 8.10 and 8.11. The reason for this is similar to that explained in Section 7.2.2 for steel 5.

(b) **Ultimate axial load levels above the balance point**

If $\varepsilon_u$ is limited to 0.003, as is assumed in the ACI Code, at ultimate axial load levels above the balance point, and for all values of $q_t$
considered in the analyses here, there is no difference between the ultimate moments and ultimate curvatures for steels 1 and 2. This is so because, the tension and compression steel strains are in the elastic region, which is the same for both steels 1 and 2, and thus the same forces, moments and curvatures are obtained.

8.3.2. Ultimate Concrete Strain Limited to 0.010

(a) Ultimate axial load levels below the balance point

If the section is analyzed by the method of analysis explained in Chapter 3 with $\varepsilon_u$ limited to 0.01, greater tension and compression steel strains are obtained than when $\varepsilon_u = 0.003$. This has been explained in Chapter 5. Therefore, as for steel 5, at ultimate axial load levels below the balance point, greater ultimate moments and ultimate curvatures are obtained for steel 1 than for steel 2. These increases in ultimate moment and ultimate curvature decrease as the ultimate axial load increases. This is so because, for given values of $P_t$, $f'_c$ and $q_t$, as has been explained in Chapter 5, the tension steel strain decreases as $P_u$ increases, which reduces the tension steel stress and force. Also, as explained in Chapter 5, the maximum moment occurs at a smaller value of $\varepsilon_u$ as $P_u$ increases, which further reduces the tension and compression steel strains and hence stresses and forces. Furthermore, at high values of $q_t$, buckling of the compression bars becomes the governing criterion at $\varepsilon_u < 0.010$ which limits the steel strains and stresses. Consequently, the effect of strain-hardening of steel 1 is reduced as $P_u$ increases and smaller increases in ultimate moment and ultimate curvature are obtained.

As $q_t$ is increased by decreasing $f'_c$, by increasing $P_t$ or by changing both, the effect of strain-hardening of steel 1 is magnified, and
slight increases in the ratio of ultimate moments for steels 1 and 2, but large increases in the ratio of ultimate curvatures are obtained. This can be seen by comparing curves (a) and (d) in Figs. 8.1, 8.2 and 8.3 for moments, and in Figs. 8.10 and 8.11 for curvatures. The explanations for this are similar to those for steel 5 in Section 7.4.2. Table 8.3 gives the ratios of ultimate moments and ultimate curvatures for steel 1 to those for steel 2 at $P_u = 0$. It can be seen in this table that, when $q_t$ increases from 0.15 to 0.25 by decreasing $f'_c$ from 5000 to 3000 psi with $p_t = 0.01$, the ratio of ultimate moment for steel 1 to that for steel 2 ($M_{ul}/M_{u2}$) remains constant and equal 1.34. This is so because, the compression steel strains are in the tensile elastic region for steel 2 for both values of $q_t$, but for steel 1 the compression steel strain is in the tensile elastic region for $q_t = 0.15$ and in the compressive elastic region for $q_t = 0.25$. The effect of these compression steel strains on the relative moments for steels 1 and 2 is similar to that explained for steel 6 relative to steel 7 in Section 7.4.3. When $q_t$ is increased from 0.25 to 0.75 by changing both $f'_c$ and $p_t$, the ultimate moment ratio for steels 1 and 2 increases slightly from 1.3- to 1.38 because, at $q_t = 0.75$, as shown in Table 8.2, the compression steel is in compression for both steels 1 and 2, and provides the required increase in the total compression force, and thus helps to increase the ultimate moment caused by strain-hardening of the tension steel. This effect of compression steel is similar to that explained in detail for Grade 60 steel in Section 7.4.1.

When $q_t$ is increased further from 0.75 to 1.20, the ratio $M_{ul}/M_{u2}$ decreases from 1.38 to 1.37. This occurs because, at these values of $q_t$, the compression steel is in the strain-hardening region for steel 1, and
thus the compression steel stress increases at a smaller rate than that for steel 2 which is in the elastic region. Consequently, the compression steel 1 does not provide enough increase in total compression force and thus does not help in increasing the ultimate moment for steel 1 in the same proportion as does steel 2, and the ratio $M_{ul}/M_{u2}$ is reduced slightly. This effect of compression steel is also explained in detail in Section 7.4.1.

If $q_t$ is increased further, the ratio $M_{ul}/M_{u2}$ should decrease; but Table 8.3 shows that it increases as $q_t$ is increased from 1.20 to 2.0 by reducing $f_c'$ from 5000 to 3000 psi, with $p_t = 0.08$. In this case, Table 8.2 shows that the ultimate moments for steels 1 and 2 decrease because of the reduction in $f_c'$. But, with $\varepsilon_u = 0.003$ for steel 2, the tension steel strain is in the elastic region at $q_t = 2.0$, and thus the tension steel stress and force are reduced, which causes a greater reduction in ultimate moment for steel 2 than for steel 1, which is in the strain-hardening region at both values of $q_t = 1.20$ and 2.0 and its stress is reduced only slightly. Consequently, the ratio $M_{ul}/M_{u2}$ increases. If the ratio $d'/t$ is small enough ($\leq 0.177$ as explained in Section 8.2), the tension steel strain for steel 2 will be on the flat-plateau, and thus the reduction in ultimate moment for steel 2 will be less than that for steel 1, and the ratio $M_{ul}/M_{u2}$ will decrease with increase in $q_t$ from 1.20 to 2.0.

The ratio of ultimate curvature for steel 1 to that for steel 2 increases as $q_t$ increases for the same reason as explained in Section 7.4.2 for steel 5.

(b) **Ultimate axial load levels above the balance point.**

The moment-strain diagrams of Figs. 8.24, 8.25 and 8.26 show that, at ultimate axial load levels above the balance point, the maximum moment
for steel 1 occurs at $\varepsilon_4 = \varepsilon_u > 0.003$ for all values of $q_t$, and that $\varepsilon_u$ increases as $q_t$ increases. With steel 1 and $\varepsilon_u = 0.003$, the compression steel strains are in the elastic region, while the tension steel strains are in the tensile elastic region at lower levels of ultimate axial load above the balance point and in the compressive elastic region at higher load levels. It has been explained in Chapter 5 that the compression and tension steel strains increase as $\varepsilon_4$ is increased. Therefore, for steel 1 with $\varepsilon_4 > 0.003$, the compression and tension steel forces are increased. Also, since $k_1k_3$ decreases with increase in $\varepsilon_4$ beyond $\varepsilon_m = 0.004$, the compression force in the concrete is reduced at higher values of $\varepsilon_4$ which is compensated for by the increase in the compression steel force. Thus, the total tension and compression forces are increased. Furthermore, since the lever arm of the concrete force is less than that of the compression steel force at higher values of $\varepsilon_4$, the decrease in the moment caused by the reduction in the concrete force is more than compensated for by the increase in the moment of the compression steel force.

However, as has been explained in Chapter 5, the depth to the neutral axis is increased with an increase in $P_u$ and/or $\varepsilon_4$. Since $k_2$ increases with increase in $\varepsilon_4$, the lever arm of the concrete force is reduced more at high values of $\varepsilon_4$, and there is more reduction in the moment of the concrete force than the total increase in the moment of the compression and tension steel forces, particularly when the compression steel strain is in the strain-hardening region so that the compression steel stress increases with strain at a smaller rate. Consequently, the total moment is reduced at high values of $\varepsilon_4$ depending on the level of the ultimate axial load.
With increase in $q_t$, the compression and tension steels provide greater increase in the forces and moments relative to the reduction in the concrete force and its moment, and the maximum moment occurs at a greater value of $\epsilon_4 = \epsilon_u$. With increase in $P_u$, there is an increase in the concrete force and in the depth to the neutral axis, and a greater decrease in the lever arm at high values of $\epsilon_4$. All these factors result in a greater decrease in the moment of the concrete force than the increase in the moments of the steel forces. Consequently, the maximum moment occurs at a smaller value of $\epsilon_4 = \epsilon_u$.

For higher values of $q_t$, for which the maximum moment occurs at greater values of $\epsilon_4$, the compression steel strain is increased so much that buckling of the compression bars becomes the governing criterion for the ultimate moment. For $q_t = 2.0$, this is true at all levels of ultimate axial load except at $P_u = 0$. Even for the axial load with zero eccentricity, the increase in the steel forces with increase in the uniform strain ($\epsilon_4$) is greater than the decrease in the concrete force, and the total axial load increases with increase in the strain. Thus, the ultimate axial load is taken at the strain = 0.005 corresponding to buckling of the compression bars, as explained in Section 3.3, and the axial load capacity with steel 1 is greater than that with steel 2 by 5 percent as can be seen in Fig. 8.3.

The moment-strain diagrams shown in Figs. 8.24, 8.25 and 8.26 show that when the ultimate moment of the section is calculated for steel 1 according to the method of analysis explained in Chapter 3, and for steel 2 according to the provisions of the 1963 ACI Code, the increase in ultimate moment for steel 1 in comparison with steel 2 depends on the values of $q_t$. 
and $P_u$ as does the value of $\epsilon_u$ which is explained above. The moment increases with increase in $q_t$ but decreases with increase in $P_u$. This can also be seen in Table 8.4 which gives the ultimate moments and their ratios for steels 1 and 2 at two levels of ultimate axial load above the balance point: $0.35 P_0$ and $0.60 P_0$ for steel 2. The ultimate moments for steel 1 have been taken from Figs. 8.16, 8.17 and 8.18 at the same eccentricity (ratio of the ultimate moment to the ultimate axial load) as for steel 2. It can be seen in this table that, at higher load levels and with smaller values of $q_t$, no increase in ultimate moment is obtained for steel 1.

As explained above, the ultimate concrete strain $\epsilon_u$ corresponding to the ultimate moment for steel 1 is greater than that for steel 2, for which $\epsilon_u$ is limited to 0.003. Therefore, greater ultimate curvatures are obtained for steel 1 than for steel 2. This is shown in the load-curvature diagrams in Figs. 8.19 and 8.20. These figures show that the increase in ultimate curvature for steel 1 decreases as $P_u$ increases because, as explained above, $\epsilon_u$ decreases more with increase in $P_u$ for steel 1 than for steel 2. Comparisons of the load-curvature diagrams in Figs. 8.19 and 8.20 show that when $q_t$ is increased by decreasing $f'_c$ at constant value of $p_t$, the ultimate curvature of the section at ultimate axial load levels above the balance point is little affected. But, when $q_t$ is increased by increasing $p_t$, a large increase in ultimate curvature is obtained, particularly at the high value of $p_t = 0.08$. This is so because of the high values of $\epsilon_u$ for $p_t = 0.08$, as can be seen in the moment-strain diagrams in Figs. 8.24, 8.25 and 8.26.

8.4. Steel 3

As explained in Section 4.1.3, the stress-strain curve for steel 3 shown in Fig. 4.6 is typical of Grade 75 steels and gives a yield strength
for steel 3 of 88.2 ksi at a strain of 0.006 as per the ASTM A615-68 specifications for Grade 75 steel. But, as explained in Section 4.1.2, according to the 1963 ACI Code this curve is considered to have a yield strength of 75 ksi and, for the purpose of analysis and design, is considered equivalent to curve 2 shown in Fig. 4.6. Any stress-strain curve below that of steel 3 will not meet the requirements of the ACI Code. Therefore, in this section comparisons of the analyses will be made for steels 2 and 3 in order to investigate the effects of the realistic stress-strain curve of reinforcement as compared to that assumed in the ACI Code.

8.4.1. Ultimate Concrete Strain Limited to 0.003

(a) Ultimate axial load levels below the balance point

The load-moment diagrams in Figs. 8.4, 8.5 and 8.6 show that when \( \varepsilon_u \) is limited to 0.003 the ultimate moment of the section calculated with steel 3 is greater than that with steel 2 at small values of \( q_t \), but it is less than that for steel 2 at high values of \( q_t \). This is so because, as shown in Table 8.1 and Fig. 8.27, for small values of \( q_t \), the tension steel strains for steel 3 are in the strain-hardening region, which is above the flat-plateau for steel 2, and thus gives greater tension steel stresses and forces and hence greater ultimate moments than for steel 2, for which the tension steel strains are on the flat-plateau. But, since the tension steel strain decreases as \( q_t \) increases, for high values of \( q_t \) obtained with \( p_t = 0.08 \), the tension steel strains are in that region of the stress-strain curve for steel 3 which lies below that for steel 2. Thus, smaller tension steel stresses and forces are obtained which give smaller moments for steel 3 than for steel 2.
With small values of \( q_t \), the increase in the moment for steel 3 decreases as \( P_u \) increases because, as explained in Chapter 5, the tension steel strain decreases with increase in \( P_u \). At \( P_u = 0 \), Table 8.1 shows that the maximum increase of 27 percent occurs for the minimum value of \( q_t = 0.15 \). Since the stress-strain curve for steel 3 is similar in shape to that for steel 1 but lies slightly below it, slightly smaller tension steel stresses are calculated with steel 3 which result in slightly smaller increase in moment for steel 3 than for steel 1, as can be seen in Table 8.1.

As explained in Section 7.2.2 for steel 5, with \( \varepsilon_u = 0.003 \), the increase in moment for steel 3 as compared to that for steel 2 occurs with small decrease in curvature. This is shown by the load-curvature diagrams of Figs. 8.12 and 8.13 and by the values given in Table 8.1.

(b) Ultimate axial load levels above the balance point

Figures 8.4, 8.5 and 8.6 show that, at ultimate axial load levels above the balance point, smaller ultimate capacity of the section is calculated for steel 3 than for steel 2, when \( \varepsilon_u \) is limited to 0.003 for both steels. This occurs because the compression steel strains for steel 3 are in that region of the stress-strain curve which lies below that for steel 2 and thus smaller compression and/or tension steel stresses and forces are calculated which result in smaller moments for steel 3 than for steel 2. This has been explained in detail for steel 8 in Section 7.2.3. This reduction in the ultimate capacity with steel 3 is magnified as \( q_t \) is increased by changing \( p_t \) and/or \( f'_c \). By comparing the load-moment diagrams for steel 3 with those for steel 2 in Figs. 8.4, 8.5 and 8.6, it is found that the maximum decrease of 9 percent in the ultimate capacity occurs for
steel 3 with $\alpha_A = 2.0$. This comparison of ultimate capacity is made at the same eccentricity for both steels 2 and 3.

The load-curvature diagrams in Figs. 8.12 and 8.13 show that, at ultimate axial load levels above the balance point, there is little difference in the curvatures of the section with steels 2 and 3 when $\varepsilon_u$ is limited to 0.003. Since, as explained above, the compression steel stress for steel 3 is less than that for steel 2, there is a slight increase in the depth to the neutral axis for steel 3 which increases the compression steel and concrete forces, as explained in Chapter 5. Therefore, actually there is a slight reduction in the curvature of the section with steel 3, but this is too small to be shown on the load-curvature diagrams.

8.4.2. Ultimate Concrete Strain Limited to 0.010

(a) Ultimate axial load levels below the balance point

Comparisons of the load-moment diagrams in Figs. 8.16, 8.17 and 8.18, and of load-curvature diagrams in Figs. 8.19 and 8.20 show that, at ultimate axial load levels below the balance point, the effect of the stress-strain curve of steel 3 in increasing the ultimate moment and ultimate curvature of the section calculated according to method of analysis explained in Chapter 3, is similar to that of steel 1, which has been explained in Section 8.3.2. However, since the stress-strain curve for steel 3 lies slightly below that for steel 1 between the strains 0.00145 and 0.01500, when the steel strains lie in this region of the stress-strain curve, slightly smaller increase in the ultimate moment is obtained for steel 3 than for steel 1. This is shown in the moment-strain diagrams in Figs. 8.21, 8.22 and 8.23.
Figure 8.28 shows that, at $P_u = 0$ and $q_t = 0.15$ and 0.25, the tension and compression steel strains are in that region of the stress-strain curve which is the same for steels 1 and 3. Therefore the same increases in ultimate moment and ultimate curvature are obtained for both steels 1 and 3 in comparison with steel 2, as shown in Table 8.2. As $q_t$ increases, the tension steel strain decreases while the compression steel strain increases, so that at higher values of $q_t$ and/or $P_u$ both tension and compression steel strains lie in that region of the stress-strain curve in which steel 3 gives smaller stress than steel 1. Thus, in comparison with steel 2, slightly smaller increase in ultimate moment is obtained for steel 3 than for steel 1, as shown in Figs. 8.16, 8.17 and 8.18, and in Table 8.2.

The load-curvature diagrams given in Figs. 8.19 and 8.20 show negligible difference between the ultimate curvatures of the section for steels 1 and 3 because the ultimate moments for these steels occur at almost the same ultimate concrete strains. Therefore, the effect of the stress-strain curve for steel 3 in comparison with steel 2 is similar to that for steel 1 as explained in Section 8.3.2.

(b) Ultimate axial load levels above the balance point

The moment-strain diagrams for the section with steel 3 (Figs. 8.24, 8.25 and 8.26) show that, as was true for steel 1, the value of $\varepsilon_u = \varepsilon_u$ at which ultimate moment occurs, depends on the values of $q_t$ and $P_u$. At lower levels of ultimate axial load above the balance point, the ultimate moment occurs at considerably greater values of $\varepsilon_u$ than 0.003 so that, as for steel 1, greater steel strains, stresses and forces are obtained for steel 3 in comparison with steel 2 for which $\varepsilon_u = 0.003$. Consequently, greater ultimate
capacity is obtained for steel 3 as compared to steel 2. Table 8.4 shows that at \( P_u = 0.35 P_{02} \) for steel 2, the ratio of ultimate moment for steel 3 to that for steel 2 (\( M_{u3}/M_{u2} \)) is more than 1.0 for all values of \( q_t \) and this ratio increases with increase in \( q_t \), as was the case for steel 1 explained in Section 8.3.2. But, because the stress-strain curve for steel 3 lies below that for steel 1, smaller increase in ultimate capacity is obtained for steel 3 and the values of \( M_{u3}/M_{u2} \) are smaller than those of \( M_{u1}/M_{u2} \). The difference between these moment ratios increases as \( q_t \) increases because the tension and compression steel strains are pushed towards that region of the stress-strain curve where curve 3 deviates more from curve 1, and also because the effect of the stress-strain curve is magnified with increase in \( q_t \).

As \( P_u \) increases, \( \varepsilon_u \) decreases for steel 3 and thus the steel strains, stresses and forces are reduced and less and less increase in ultimate moment is obtained for steel 3 in comparison with steel 2. At higher load levels, since the steel strains are in that region of the stress-strain curve for steel 3 which lies below that for steel 2, smaller ultimate moment is obtained for steel 3 than for steel 2, and the ratio \( M_{u3}/M_{u2} \) is less than 1.0. As \( q_t \) is increased at higher levels of \( P_u \), the effect of reduced steel stresses is magnified and the ratio \( M_{u3}/M_{u2} \) should decrease. Table 8.4 shows that, at \( P_u = 0.60 P_{02} \) for steel 2, this is true when \( q_t \) increases from 0.15 to 0.25 with \( P_t = 0.01 \); however, for further increases in \( q_t \), the ratio increases as it did for lower levels of ultimate axial load. This is so because, at higher values of \( q_t \), greater values of \( \varepsilon_u \) are obtained which result in greater steel strains and stresses, and hence greater moments, for steel 3 than for steel 2 for which \( \varepsilon_u = 0.003 \).
At still higher values of $P_u$, $\epsilon_u$ is further reduced, and a
greater decrease in the ratio of ultimate capacities with steels 3 and 2
is obtained. This effect of the stress-strain curve in reducing the
ratio $M_{u3}/M_{u2}$ prevails up to greater values of $q_t$ with an increase in $P_u$.
Although the reduction in ultimate capacity is small, the maximum reduction
occurs at $P_u \geq 0.90 P_{02}$ and $q_t = 0.75$, and amounts to 5 percent as can be
seen by comparing Figs. 8.4, 8.5 and 8.6.

The load-curvature diagrams in Figs. 8.19 and 8.20 show that, at
ultimate axial load levels above the balance point, there is practically
no difference between the ultimate curvatures of the section with steels 1
and 3 when $\epsilon_u$ is limited to 0.010, as was also the case at ultimate axial
load levels below the balance point. Consequently, the effect of the
stress-strain curve for steel 3 is similar to that for steel 1 explained
in Section 8.3.2.

8.5. Steel 4

It has been explained in Section 4.1.3 that the stress-strain
curve for steel 4 shown in Fig. 4.6 represents the lower bound of the
curves that meet the ASTM A615-68 specifications for Grade 75 steel. The
specified minimum yield strength of 75 ksi is reached at the specified
strain of 0.006. But, as explained in Sections 4.1.2 and 4.1.3, this curve
does not meet the requirements of the 1963 ACI Code for use as a flat-top
steel having $f_y = 75$ ksi. Therefore, the analyses of the section with
steel 4, which are discussed and explained below in comparison with steels 1,
2 and 3, will show the effects of the stress-strain curves of the reinforcing
bars that meet the ASTM specifications but not the ACI Code provisions.
8.5.1. **Ultimate Concrete Strain Limited to 0.003**

Comparisons of the load-moment diagrams for the section with steels 4 and 2 in Figs. 8.7, 8.8 and 8.9 show that, when \( \varepsilon_u \) is limited to 0.003, the ultimate capacity of the section with steel 4 is smaller than that for steel 2 at all levels of ultimate axial load, and with all values of \( q_t \) used in these analyses, except for very small values of \( q_t \) in combination with very low levels of ultimate axial load. This is so because, the tension and/or compression steel strains for steel 4 are in that region of the stress-strain curve for steel 4 which is below that for steel 2 and which gives smaller steel stresses and forces and hence smaller moments for steel 4. However, as shown in Fig. 8.27, at \( P_u = 0 \) with \( q_t = 0.15 \) and 0.25, the tension steel strains are in that region of the stress-strain curve for steel 4 which lies above the flat-plateau for steel 2, while the compression steel strains are in the elastic region which is the same for both steels 2 and 4. Therefore, greater moments are obtained with steel 4 than with steel 2. Table 8.1 shows that the maximum increase of 12 percent in the moment of the section with steel 4 as compared to steel 2 occurs for \( q_t = 0.15 \) and \( P_u = 0 \).

As \( P_u \) or \( q_t \) increases, the tension steel strains decrease while the compression steel strains increase so that both are pushed towards that region of the stress-strain curve for steel 4 which deviates more from that for steel 2, and thus greater reductions in steel stresses are obtained with consequent greater reductions in the moments with steel 4. When \( q_t \) is increased by increasing \( p_t \), this effect of the stress-strain curve for steel 4 in reducing the capacity at \( \varepsilon_u = 0.003 \) is magnified. The maximum reduction in capacity is about 15 percent which occurs as the case of zero eccentricity is approached for \( p_t = 0.08 \) and \( f_c' = 3000 \) psi (\( q_t = 2.0 \)).
Comparison of the load-curvature diagrams shown in Figs. 8.14 and 8.15 shows that there is little difference in the curvature of the section with steels 2 and 4 because $\varepsilon_u$ is the same (0.003) for both steels. However, at very low levels of ultimate axial load, the increase in ultimate moment for steel 4 with very small values of $q_u$, as explained above, is accompanied by a very small decrease in the curvature for the same reason as has been explained for steel 5 in Section 7.2.2. Similarly, with higher values of $q_u$, the decrease in the moment at very low levels of ultimate axial load is accompanied by a very small increase in the curvature.

8.5.2. Ultimate Concrete Strain Limited to 0.01

The moment-strain diagrams in Figs. 8.21 through 8.26 show that the maximum moment of the section with steel 4 occurs at substantially greater values of $\varepsilon_u = \varepsilon_u$ than 0.003 and, as was explained for steels 1 and 3 in Sections 8.3 and 8.4, $\varepsilon_u$ increases with increase in $q_u$ but decreases with increase in $P_u$.

At lower levels of ultimate axial load, the moment-strain diagrams continue to rise with increase in $\varepsilon_u$ so that the higher limit on $\varepsilon_u$ results in greater ultimate moments with steel 4 than with steel 2 for which $\varepsilon_u = 0.003$, as was true for steels 1 and 3 as explained in Sections 8.3 and 8.4. However, this increase in ultimate moment is much smaller for steel 4 than that for steels 1 and 3, because the stress-strain curve for steel 4 lies well below those for steels 1 and 3 in the region above 75 ksi. Since stress-strain curve for steel 4 is similar in shape to that for steel 8 (Grade 60), the effect of stress-strain curve for steel 4 on the
ultimate moment and ultimate curvature of the section is also similar to that for steel 8 explained in Section 7.3.

At $P_u = 0$, the ratio of ultimate moment of the section with steel 4 to that with steel 2 ($M_{u4}/M_{u2}$) given in Table 8.3 decreases with increase in $q_t$, as was the case for steel 8 explained in Section 7.4. Table 8.3 shows that, for steels 1 and 3, the ultimate moment ratios increase slightly with increase in $q_t$ up to 0.75, but this is not true for steel 4 because the shape of stress-strain curve for steel 4 is such that at $q_t = 0.75$, the compression steel strains give smaller stresses for steel 4 than for steel 2 whereas greater compression steel stresses are obtained for steels 1 and 3. This reduction in the ratio $M_{u4}/M_{u2}$ because of the effect of the compression steel is similar to that explained in Section 7.4.1 for Grade 60 steels. However, the ratio $M_{u4}/M_{u2}$ for $q_t = 2.0$ is greater than that for $q_t = 1.20$ for the same reason as explained in Sections 8.2 and 8.3 for steels 1 and 3.

Table 8.4 shows that, at $P_u = 0.35P_{02}$, the ratio $M_{u4}/M_{u2}$ increases with increase in $q_t$, whereas at $P_u = 0.60P_{02}$, it decreases with increase in $q_t$, as was the case at $P_u = 0$. This is explained below.

The ultimate axial load level $P_u = 0.35P_{02}$ is in that region of the load-moment diagram (above the balance point) where the compression steel plays an important role in resisting the external forces. With $\epsilon_u$ limited to 0.003 for steel 2, the compression steel strain is in the elastic region so that the compression steel stress is less than $f_y$ and, as explained in Section 8.2.1, considerable reduction in ultimate moment is obtained for the section with steel 2, and this reduction increases with increase in $q_t$. For steel 4 also, the compression steel strains are such
that they give smaller steel stresses (less than \( f' \)) because of the shape of the stress-strain curve. However, the greater value of \( \varepsilon_u \) for steel 4 increases the steel strains and stresses so that the maximum (ultimate) moment is obtained with steel stresses still less than \( f'_y \) but slightly greater than those for steel 2, and thus greater ultimate moment is obtained with steel 4 than with steel 2. At the very small value of \( q_t = 0.15 \), \( \varepsilon_u \) is also small (close to 0.003) so that the steel strains and stresses are not increased enough for steel 4, and the ratio \( M_{u4}/M_{u2} = 0.99 \). As \( q_t \) increases, \( \varepsilon_u \) increases for steel 4 and thus steel strains and stresses increase, but for steel 2 the steel strains and stresses decrease rapidly with increasing \( q_t \) so that greater differences are obtained between the ultimate moments for steels 2 and 4, and the ratio \( M_{u4}/M_{u2} \) increases. However, if a higher limit on \( \varepsilon_u \) is allowed for steel 2 also, or if the ratio \( d'/t \) is small enough, the compression steel will yield and the compression steel stress for steel 2 will be greater than that for steel 4 and the ratio \( M_{u4}/M_{u2} \) will decrease with increase in \( q_t \), and will be less than 1.0.

Since the compression steel strain is increased as \( P_u \) increases, if \( \varepsilon_u \) is limited to 0.003 for steel 2, at high levels of ultimate axial load the compression steel stress is either equal to or only slightly less than \( f'_y \) and the limitation of \( \varepsilon_u \) to 0.003 results in no or only slight reduction in ultimate moment with increase in \( q_t \). But, for steel 4, not only is \( \varepsilon_u \) reduced with increase in \( P_u \) but also, at high levels of ultimate axial load, the compression steel strains are in that region of the stress-strain curve which gives smaller steel stresses than those for steel 2, and large reductions in steel stresses and forces are obtained with increase in \( q_t \) for steel 4 as compared to steel 2. Therefore, the ratio \( M_{u4}/M_{u2} \) decreases
with increase in $q_t$. Table 8.4 shows that, at $P_u = 0.60 P_{02}$, the ultimate moment of the section with steel 4 is smaller than that with steel 2 for all values of $q_t$. Although, at this load level the difference is small, it increases with increase in $P_u$ and $q_t$ so that the maximum difference of 14 percent is obtained at the ultimate axial load with zero eccentricity at $q_t = 1.20$.

Figure 8.9 shows that when $q_t$ is increased from 1.20 to 2.0 by decreasing $f'_c$ from 5000 to 3000 psi with $p_t = 0.08$, less reduction in the ultimate axial load for zero eccentricity is obtained with steel 4 ($P_{04}$), and the ratio $P_{04}/P_{02}$ for $q_t = 2.0$ is greater than that for $q_t = 1.20$. This is explained below.

The maximum (ultimate) axial load for zero eccentricity with steel 2 ($P_{02}$) occurs at a strain $= \epsilon_Y = 0.0025$. For steel 4, the shape of the stress-strain curve is such that the stress and hence the force in the steel continues to increase with increase in strain. But the force in the concrete decreases with increase in strain beyond $\epsilon_c = 0.002$. For $f'_c = 3000$ psi, the reduction in concrete force with increase in strain is smaller than that for $f'_c = 5000$ psi. For $q_t = 2.0$, the increase in the steel force is greater than the reduction in the concrete force and the total force continues to increase with increase in strain, so that the ratio $P_{04}/P_{02}$ continues to increase. The maximum value of total force occurs at a strain of 0.006. But $P_{04}$ is taken at a strain of 0.005 corresponding to buckling of the compression bars. However, for $q_t = 1.20$, because of the greater reduction in concrete force, the maximum value of total force occurs at a strain of 0.0035, and $P_{04}$ is not increased as much beyond the strain of 0.0025 for steel 2 as it is for $q_t = 2.0$. Consequently, the ratio $P_{04}/P_{02}$ for $q_t = 2.0$ is greater than that for $q_t = 1.20$. 
It can be seen from the load-curvature diagrams in Figs. 8.19 and 8.20 that a large increase in ultimate curvature is obtained for the section with steel 4 in comparison with steel 2, particularly at lower levels of ultimate axial load, because of the greater value of $\varepsilon_u$ for steel 4, as was the case for steels 1 and 3 explained in Sections 8.3 and 8.4. Figures 8.19 and 8.20 also show that there is practically no difference between the ultimate curvatures of the section with steels 1, 3 and 4 at all levels of ultimate axial load and for all values of $q_t$. However, Fig. 8.19 shows that, at $q_t = 0.15$ and at very low levels of ultimate axial load, steel 4 gives greater ultimate curvature than steels 1 and 3. This is so because, as can be seen in Fig. 8.28, at $P_u = 0$, $\varepsilon_u$ and $\varepsilon_2$ are greater for steel 4 than for steels 1 and 3 which results in greater ultimate curvature, as explained in Section 7.2.2. However, the moment-strain diagrams for steels 1, 3 and 4 shown in Fig. 8.21 become quite flat, particularly for steel 4, and the ultimate moments for steels 1, 3 and 4 could have been taken from the curves at the same value of $\varepsilon_4$, instead of at the point of "maximum" moment, with little noticeable effect on the values obtained. If this were done, the ultimate curvatures would be almost the same.

8.6. Summary

In this chapter, the analyses for a 15-in. square reinforced concrete section provided with ASTM A615-68 Grade 75 steel have been presented in the form of load-moment and load-curvature diagrams. The stress-strain curves for the reinforcement used in the analyses are given in Fig. 4.6. The variables considered in the analyses are given in Section 8.1. The effects of these stress-strain curves on the strength and
behavior of the section are discussed and explained in Sections 8.2 through 8.5. These are summarized below.

(a) **Ultimate Moment**

If the section is analyzed with the ultimate concrete strain $\varepsilon_u$ limited to 0.003 in accordance with the provisions of the 1963 ACI Code, at ultimate axial load levels below the balance point and for very small values of $q_t = p_u f_y / f'_c$, the tension steel strains for steels 1, 3 and 4 reach that portion of the stress-strain curve which lies above the conventional flat-plateau, and thus greater tension steel stresses are obtained which give greater moments than when the flat-top stress-strain curve (for steel 2) is assumed in the analysis. This increase in moment capacity is a maximum for steel 1 at $P_u = 0$ (pure moment), and amounts to 29 percent for $q_t = 0.15$. It decreases as the ultimate axial load and/or the value of $q_t$ increases. At high values of $q_t$, the tension steel strains are close to or less than the yield value for the flat top steel 2, and in this region the stress-strain curve for steel 1 gives very little or no increase in stress as compared to steel 2. Thus, very small or no increase in moment is obtained for steel 1. But, for steels 3 and 4, the tension steel strains lie in that region of the stress-strain curve which is below that for the flat-top steel 2. Thus, smaller steel stresses and moments are obtained with steels 3 and 4 than with steel 2. A similar effect is obtained when $P_u$ is increased. At the balance point, steels 1 and 2 give the same moment, but smaller moments are obtained with steels 3 and 4.

At ultimate axial load levels above the balance point, compression steel plays a greater role in resisting the ultimate axial load and bending moment. Since, with $\varepsilon_u = 0.003$, the compression steel strains are in the
elastic region or only slightly greater than the yield strain (for the flat-top steel 2), almost the same steel stresses are obtained for steels 1 and 2, and thus the same moments are calculated with these steels. But, for steels 3 and 4, and with $\varepsilon_u = 0.003$, the compression steel strains lie in that region of the stress-strain curve which is below that for the flat-top steel. Thus, smaller compression steel stresses and hence smaller moments are obtained with steels 3 and 4 as compared to the flat-top steel 2. Since the stress-strain curve of steel 4 lies further below that of steel 2 than does the curve for steel 3, greater reduction in the capacity is obtained with steel 4 than with steel 3. This reduction in ultimate capacity with steels 3 and 4 increases with increase in $q_t$ and/or $P_u$. The maximum reductions of 9 and 15 percent are obtained with steels 3 and 4, respectively, for $q_t = 2.0$.

With $\varepsilon_u$ limited to 0.003, steel 4 gives smaller ultimate capacity than the flat-top steel 2, for all values of $q_t$ and all levels of ultimate axial load, except for very low load levels in combination with very small values of $q_t$, in which case greater ultimate capacity is obtained with steel 4 as explained above.

If a higher limit on ultimate concrete strain $\varepsilon_u$ is permitted in the analysis, at ultimate axial load levels below the balance point, little effect is obtained on the capacity with steel 2, because the tension steel stress remains constant at $f_y$. But, for very high values of $q_t$, when the tension steel strains at $\varepsilon_u = 0.003$ are in the elastic region, increase in the value of $\varepsilon_u$ increases steel strains, stresses and forces, and thus results in greater ultimate capacity. However, for steels 1, 3 and 4 which have a portion of their stress-strain curve above the flat-plateau for
steel 2, the higher limit on $\epsilon_u$ results in considerable increase in steel stresses and hence in ultimate capacity. The maximum increase is obtained with steel 1 and the minimum with steel 4. This increase in ultimate capacity depends on the values of $P_u$ and $q_t$. With increase in $P_u$, the increase in ultimate capacity is reduced, but with increase in $q_t$ it is increased for steels 1 and 3 up to a certain value of $q_t$ when the compression steel can provide required increase in the compression force to compensate for the reduction in the concrete force and to match the increased tension force. However, for steel 4, the capacity decreases with increase in both $P_u$ and $q_t$. At $P_u = 0$, comparison of ultimate moments with $\epsilon_u = 0.010$ for steels 1, 3 and 4 with that for steel 2 and $\epsilon_u = 0.003$ (in accordance with the 1963 ACI Code) shows that maximum increases of 38, 36 and 19 percent in the ultimate moment are obtained for steel 1, 3 and 4, respectively, when the tension steel 2 yields at $\epsilon_u = 0.003$. But, if the tension steel 2 does not yield at $\epsilon_u = 0.003$ and $P_u = 0$, still greater increases in ultimate moment will be obtained for steels 1, 3 and 4.

At ultimate axial load levels above the balance point, the compression steel strain depends on the values of $q_t$ and $P_u$. For higher levels of $P_u$ and smaller values of $q_t$, the compression steel 2 will yield at $\epsilon_u = 0.003$. But for lower levels of $P_u$ and/or greater values of $q_t$, it may not yield at $\epsilon_u = 0.003$, in which case increase in the limit on $\epsilon_u$ increases the compression steel strain and stress, and hence the ultimate capacity until the compression steel yields. Similarly, for steels 1, 3 and 4, a higher limit on $\epsilon_u$ up to 0.010 gives greater ultimate moments at lower levels of $P_u$ than with $\epsilon_u = 0.003$. At higher levels of $P_u$, the increase in ultimate capacity due to increase in $\epsilon_u$ depends on the value
of $q_t$. With higher values of $q_t$, the ultimate capacity increases but little increase is obtained with small values of $q_t$.

Comparisons of ultimate capacities of the section at $P_u > P_b$, for steels 1, 3 and 4 at $\epsilon_u \leq 0.010$ with those obtained for the flat-top steel 2 at $\epsilon_u \leq 0.003$ (ACI Code) show that: (1) Steel 1 gives the same capacity as steel 2 at high levels of ultimate axial load in combination with small values of $q_t$, but gives greater ultimate capacity at lower levels of $P_u$ with all values of $q_t$, and at higher levels of $P_u$ only with higher values of $q_t$. (2) Steel 3 gives greater capacity than steel 2 at lower levels of $P_u$ but smaller capacity at higher levels of $P_u$. The level of $P_u$ above which steel 3 gives reduced capacity increases with increase in $q_t$. (3) Steel 4 gives smaller capacity than steel 2 at higher levels of $P_u$ for all values of $q_t$, and at lower levels of $P_u$ for small values of $q_t$, but for higher values of $q_t$ and lower levels of $P_u$ it gives greater capacity than steel 2.

(b) Ultimate curvature

When the ultimate concrete strain $\epsilon_u$ is limited to $0.003$, practically the same ultimate curvature is obtained for the section with all of the steels 1, 2, 3 and 4, except at very low levels of ultimate axial load in combination with small values of $q_t$, in which cases very slight reduction in ultimate curvature for steels 1, 3 and 4 accompanies the increase in ultimate capacity owing to the effect of that portion of the stress-strain curve which lies above the flat-plateau for steel 2.

For a higher limit on $\epsilon_u$, large increases in ultimate curvature are obtained for steels 1, 3 and 4 as compared to steel 2, because the ultimate moments with steels 1, 3 and 4 occur at greater values of $\epsilon_u$ than
with steel 2. Comparisons of the ultimate curvatures for steels 1, 3 and 4, with $\varepsilon_u \leq 0.010$, and for steel 2 with $\varepsilon_u \leq 0.003$ (ACI Code) show that little difference in ultimate curvatures is obtained with steels 1, 3 and 4 but in all cases these ultimate curvatures are much larger than those for steel 2, particularly at lower levels of ultimate axial load. The ultimate curvatures decrease with increase in $P_u$ for all steels; but greater decrease is obtained with steels 1, 3 and 4 because of the decrease in $\varepsilon_u$ for these steels. At lower levels of ultimate axial load, the increase in the value of $q_t$ results in a decrease in the value of ultimate curvature for all steels, but greater reduction is obtained for steel 2 than for the steels 1, 3 and 4. At higher levels of ultimate axial load, with increase in $q_t$ obtained by increasing $p_t$, the ultimate curvature again decreases for steel 2 but increases for steels 1, 3 and 4 due to increase in $\varepsilon_u$ for these steels, but the increase in $q_t$ obtained by decreasing $f'_c$ at constant value of $p_t$ has negligible effect on the ultimate curvature with all steels.
9. ANALYSES OF REINFORCED CONCRETE SECTIONS CONSIDERING CREEP EFFECTS

9.1. Introduction

In this chapter, the same 15-in. square reinforced concrete section which was selected for analyses in Chapters 7 and 8 is analyzed to obtain load-moment and load-curvature diagrams with a stress-strain curve of concrete so modified as to represent the effect of creep of concrete.

In Section 9.2, the member is considered to have been loaded continuously up to failure at such a slow rate that the creep of concrete occurs simultaneously as the loading progresses. Consequently, the short-time stress-strain curve for concrete is modified to consider the creep of concrete as explained in Section 9.2.1.

In Section 9.3, the member is considered to have been loaded up to working conditions in a short time, this load is sustained for a desired period of time to allow the concrete to creep, and then the member is further loaded in a short time up to failure. The modifications which are necessary to obtain the stress-strain relationship for concrete for these conditions of loading are explained in Section 9.3.1.

In each of Sections 9.2 and 9.3, the reinforcing bars which satisfy both the ASTM specifications and the ACI Code requirements for Grade 60 and Grade 75 steels are considered in the analyses. For the purpose of analyses in this chapter steels 5, 6 and 7 shown in Fig. 4.5 were selected for Grade 60 steel and steels 1, 2 and 3 shown in Fig. 4.6 for Grade 75 steel. The basis of selection of these steels is given in Section 4.1.3.

Two values of \( p_t = 0.01 \) and 0.08, and one value of \( f'_c = 3000 \text{ psi} \), have been considered. For the analyses with Grade 60 steels \( (f_y = 60 \text{ ksi}) \),
these values of $p_t$ and $f'_c$ give $q_t = p_t f_y / f'_c = 0.20$ and 1.60, and for Grade 75 steel ($f_y = 75$ ksi), they give $q_t = 0.25$ and 2.00.

The results of analyses are presented in the form of figures and tables which are discussed and explained in Sections 9.2 and 9.3.

9.2. Slow Loading

9.2.1. Stress-Strain Relationship for Concrete

In this section, analyses are presented for the member which is loaded continuously and slowly up to failure so that the concrete creeps as the loading progresses. Consequently, the stress-strain curve for concrete representing the short-time loading condition (Fig. 4.11) is modified on the basis of the following considerations:

(1) Because of the continuous loading of the section, a continuous concrete stress-strain curve is used.

(2) The general effects of creep of concrete are to increase concrete strain at a given stress, and to decrease the strength of concrete. The former reduces the modulus of elasticity of concrete and thus makes the stress-strain curve flatter, while the latter reduces the value of the coefficient $k_3$ for $f''_c = k_3 f'_c$.

(3) The above effects of creep depend on several factors such as the quality of concrete, types of aggregates and cement, water-cement ratio, stress level, amount of reinforcement in compression, and most important of all, the rate of loading.

(4) Risch (14) has reported stress-strain curves for concrete obtained from tests of prisms at various rates of loading. These curves show that as the rate of loading decreases the strain $\varepsilon_0$ corresponding to the maximum stress increases while $k_3$ decreases.
(5) Green (8) has proposed a time-dependent logarithmic equation for the stress-strain curve of concrete from zero strain to \( \varepsilon_0' \) and has assumed a straight-line relationship for strains beyond \( \varepsilon_0' \) which is similar to the short-time stress-strain curve used by him.

(6) The purpose of the analyses here is not to recommend a stress-strain relationship for concrete, which would consider all the factors that affect creep of concrete, but to utilize the available information and reproduce a reasonable stress-strain relationship for concrete, in order to investigate the effects of the stress-strain curve of reinforcement on the strength and behavior of reinforced concrete sections under various combinations of axial load and bending moment. Since the method of analysis explained in Chapter 3 utilizes any shape of the stress-strain curve, it is possible to use any stress-strain relationship which can be considered to represent reasonably well the actual loading rate and material properties.

The stress-strain curve shown in Fig. 9.1 has been used in these analyses. It gives very nearly the same stress-strain relationship as proposed by Green (8) for a loading period of one year, but with \( k_3 = 0.75 \) instead of 0.81 as assumed by Green. Since, the columns tested by Green were cast horizontally, the value of \( k_3 \) for short-time tests was assumed by him to be 0.95 instead of 0.85 assumed here for vertically cast columns. Consequently, a smaller value of \( k_3 \) has also been assumed here for the slow tests of vertically cast columns. The stress-strain curve shown in Fig. 9.1 can be represented approximately by the following equation which is similar to that proposed by Todeschini (7) for short-time loading, and used here for analyses in Chapters 7 and 8.
\[ f_c = \frac{(1+B)k_3f'_c(e_c/e_0)}{1 + B(e_c/e_0)^2} \]

where \( B = 1.0 \) and \( k_3 = 0.85 \) give the stress-strain curve "A" for short-time test with \( e_0 = 0.002 \) as shown in Fig. 4.11.

For a slow and continuous loading, the values of \( e_0, k_3 \) and \( B \) can be modified depending on the factors mentioned above. For one year period of loading, it was assumed that \( e_0 = 0.006 \) (three times the value for short time), \( k_3 = 0.75 \) and \( B = 0.8 \). For other loading rates, other reasonable values of \( e_0, k_3 \) and \( B \) could be used to obtain the stress-strain curve. Except for \( B = 1.0 \), the above equation gives maximum stress at \( e_c > e_0 \). However, since the method of analysis here uses a stress-strain relationship represented by straight lines connecting discrete points on the curve, it is assumed that a straight line connects the points on the curve given by the above equation between the strains \( e_0 \) and \( e_0/B \). This ensures that the maximum stress is obtained at \( e_0 \). The curve obtained by the above equation for slow loading will be designated as curve "B."

9.2.2. ASTM Grade 60 Steel

Figures 9.2 and 9.3 show comparisons of load-moment diagrams obtained for steels 5, 7 and 8 by the method of analysis explained in Chapter 3, in accordance with the criteria for ultimate capacity explained in Section 3.3. Similarly, Fig. 9.4 shows load-curvature diagrams at ultimate capacity.

It can be seen from the load-moment diagrams that, at low levels of ultimate axial load, considerable increase in ultimate moment is obtained for steels 5 and 8 in comparison with flat-top steel 7. This is so because
the tension steel strains at ultimate capacity are in such a region of the stress-strain curve that greater steel stresses are obtained for steels 5 and 8 than for steel 7. This was also true for the case of short-time loading of the specimen as explained in Chapter 7. However, greater increase in ultimate capacity is obtained particularly with a small value of $p_t$ by loading the specimen slowly than in the case of short-time loading (Chapter 7) in spite of the fact that the strength of concrete ($\frac{k_2 f'}{c}$) is reduced for the slow loading, and the same limit on $\varepsilon_u = 0.010$ is allowed for all these steels. This is explained below.

Comparison of Tables 9.1 and 7.2 shows that, at $P_u = 0$, almost the same ultimate moment is calculated for steel 7 with the short-time stress-strain curve for concrete (curve A) and the slow-loading curve (curve B), whereas greater ultimate moments are obtained for steels 5 and 8 with concrete curve B than with curve A. It can also be seen in these tables that the steel strains, particularly the tension steel strains, are such that there is very little difference between the steel stresses for the two concrete curves. However, there is a considerable increase in the lever arm of the concrete force for curve B because of the smaller value of the coefficient $k_2$, as can be seen by comparing Figs. 4.12 and 9.1. This results in a considerable increase in the moment of the concrete force for the cases of curve B with steels 5 and 8.

The moment-strain curves shown in Figs. 9.5 and 9.6 continue to rise as the concrete strain in the extreme compression fiber $\varepsilon_{\text{u}}$ is increased, particularly at low levels of ultimate axial load. This was also true for the case of concrete curve A as explained in Chapter 7. However, for concrete curve B, the value of $k_1 k_3$ continues to increase with increase in $\varepsilon_{\text{u}}$ so that
the concrete force increases with increase in $\varepsilon_4$. Since the compression and tension steel strains increase with increase in $\varepsilon_4$ (see Chapter 5), the tension and compression steel forces also increase unless the strains are on the flat-plateau of the stress-strain curve. Thus, the total tension and compression forces continue to increase. Also, since the coefficient $k_2$ increases with $\varepsilon_4$ at a smaller rate for curve B than for curve A, the reduction in lever arm of the concrete force with increase in $\varepsilon_4$ is much smaller for curve B. Consequently, the moment-strain curves continue to rise with increase in $\varepsilon_4$.

At higher levels of ultimate axial load, very small increase in ultimate capacity is obtained for steels 5 and 8 as compared to the flat-top steel 7. Contrary to this, as explained in Chapter 7 for short-time loading, the same ultimate capacity is calculated for steels 5 and 7 but a considerable decrease in ultimate capacity is obtained for steel 8. This is shown in the moment-strain diagrams in Figs. 9.5 and 9.6 for slow loading and in Figs. 7.22 and 7.24 for short-time loading. These figures show that, for slow loading, the moment-strain curves continue to rise up to large values of $\varepsilon_4$ and the maximum moment occurs at considerably greater values of $\varepsilon_4$ than for short-time loading. However, the ultimate capacity is governed by buckling of compression bars which is assumed to occur when the concrete strain at the level of compression bars is equal to $\varepsilon_0 = 0.006$ as explained in Section 3.3. At this strain in the compression steel, there is only slight increase in the compression steel stress for steels 5 and 8 as compared to $f_y$ for steel 7. Also, the tension steel is in the elastic region which is the same for all of the steels 5, 7 and 8. Since, at high load levels, the compression steel plays a greater role in resisting
the external forces, only slight increase in capacity is obtained for steels 5 and 8. This increase in capacity increases with increase in $p_t$. Table 9.3 gives comparisons of ultimate moments of various load levels for the following cases:

Case (1): Ultimate moment for steels 5, 7 and 8 obtained for slow loading according to the method of analysis explained in Chapter 3.

Case (2): Ultimate moment for steel 7 in accordance with the 1963 ACI Code, as presented and discussed in Chapter 7.

It can be seen in this table that, at low levels of ultimate axial load, there is negligible difference between the ultimate moments calculated for steel 7 for the two cases. But, for steels 5 and 8, considerable increase in ultimate moment is obtained for case 1 as compared to case 2, as explained above. However, at high levels of ultimate axial load, considerable decrease in ultimate moment is obtained for steel 7 for case 1 as compared to case 2. Also, greater decrease is obtained with a small value of $p_t$ than with a large value. This is so because the value of $k_{3}$ is smaller for case 1 than for case 2. It has been explained in Chapter 7 that, at high levels of ultimate axial load, since the compression steel stress remains constant at $f_y$, concrete plays a greater role in resisting the external forces. Therefore, a smaller value of $k_{3}$ gives a smaller concrete force and a smaller total compression force which results in a smaller moment. This effect is pronounced for a small value of $p_t$. The same is the case for steels 5 and 8 for a small value of $p_t$, since the increase in compression steel force due to the shape of the stress-strain curve is very small up to the strain of 0.006 at which buckling of the bars
is assumed to occur. But, for \( p_t = 0.08 \), considerable increase in compression steel force is obtained for steels 5 and 8 for case 1 which compensates partly or fully for the reduction in the concrete force, and thus brings the ultimate moment closer to that for case 2. Table 9.3 shows that at \( P_u = 1000 \) kips, almost the same ultimate moment is obtained for steel 5 (case 1) as for steel 7 (case 2), but the ultimate moment for steel 8 (case 1) is smaller than that for steel 7 (case 2) because steel 8 does not provide enough increase in the compression steel force whereas steel 5 did.

Figure 9.4 shows that almost the same ultimate curvatures are calculated for steels 5, 7 and 8, except for low levels of ultimate axial load in combination with a high value of \( p_t \), in which case considerable increase in ultimate curvature is obtained for steel 7 as compared to steels 5 and 8. This is so because, for a high value of \( p_t \), the ultimate moment occurs at almost the same value of \( \epsilon_u \) for all of these steels and, as has been explained in Chapter 7, the increase in ultimate moment obtained by utilizing the strain-hardening region of the stress-strain curve results in reduction in ultimate curvature, when \( \epsilon_u \) is the same for the flat-top and the strain-hardening steels. For a small value of \( p_t \), the ultimate moment at low levels of ultimate axial load occurs at a higher value of \( \epsilon_u \) for steels 5 and 8 which gives greater ultimate curvatures for these steels and compensates for the above reduction in curvature, and thus results in almost the same ultimate curvature for all of the steels 5, 7 and 8.

9.2.3. **ASTM Grade 75 Steel**

The comparisons of load-moment and load-curvature diagrams for steels 1, 2 and 3 obtained by the method of analysis explained in Chapter 3
using the concrete stress-strain curve B shown in Fig. 9.1 are given in Figs. 9.7 through 9.10. It can be seen from these figures that the effects of the stress-strain curves of these steels are similar to those explained in Section 9.2.2 for steels 5, 7 and 8. However, these effects are enhanced for Grade 75 steels and greater increases in ultimate capacity are obtained for steels 1 and 3, as was true for short-time loading as explained in Chapter 8. Figures 9.7 and 9.8 show that, not only at low levels of ultimate axial load but also at high levels, considerable increase in ultimate moment is obtained for steels 1 and 3 in comparison with steel 2. At high levels of ultimate axial load and for short-time loading of the column, as explained in Chapter 8, steel 3 gives smaller ultimate moment than steel 2. But, for slow loading, since the compression steel strains reach that region of the stress-strain curve which is above the flat-plateau of steel 2, considerably greater compression steel stresses are obtained which give greater ultimate moments for steel 3 than for steel 2. This is shown in Table 9.5.

Table 9.6 gives comparisons of ultimate moments at various load levels for the following cases:

Case (1): Steels 1, 2 and 3 for slow loading according to the method of analysis in Chapter 3 (concrete curve B).

Case (2): Steel 2 for short-time loading in accordance with the 1963 ACI Code (concrete curve A).

This table shows the following:

(1) For steel 2, with the small value of \( p_t \), almost the same moments are obtained at low levels of ultimate axial load for both cases, but at high levels of ultimate axial load, smaller ultimate moment is
obtained for Case 1 than for Case 2. This is due to the effect of $k_{1}k_{2}$ as explained in Section 9.2.2 for steel 7. However, for the high value of $p_{t}$, the limitation of $\varepsilon_{u}$ to 0.003 for Case 2 results in considerably smaller moments, except at very high levels of ultimate axial load, as was explained in Chapter 8. The generally greater ultimate moments obtained for Case 1 as compared to Case 2 result from the higher value of $\varepsilon_{u}$ permitted for Case 1. The opposite condition at very high load levels results from the smaller value of $k_{1}k_{2}$ obtained for Case 1.

For steels 1 and 3, considerably greater values of ultimate moment are obtained for Case 1 than for Case 2 for both values of $p_{t}$ and at all levels of ultimate axial load, except for the small value of $p_{t}$ in combination with high levels of ultimate axial load. This is so because the higher value of $\varepsilon_{u}$ for steels 1 and 3 gives greater steel stresses and hence greater moments. But, the reduction in the concrete force caused by a smaller value of $k_{1}k_{2}$ at high levels of ultimate axial load is not compensated for by the small area of compression steel when the value of $p_{t}$ is small. Thus, smaller ultimate moments are obtained for Case 1.

The effect of the stress-strain curves on the ultimate curvatures of the section for steels 1 and 3 in comparison with steel 2 is similar to that for steels 5 and 8 in comparison with steel 7, as explained in Section 9.2.2.

9.3. Combination of Sustained and Short-Time Loadings

9.3.1. Stress-Strain Relationship for Concrete

In this section, analyses are presented for the member loaded in the following stages, which are illustrated in Fig. 9.13.
Stage 1: A given axial load $P$ and bending moment $M$ (corresponding to the working-load conditions) are applied in a short time.

Stage 2: This load $P$ and this moment $M$ are sustained on the member for a desired period during which the concrete creeps. This results in an increase in concrete strain and a reduction in concrete stress. Consequently, the distribution of forces between steel and concrete is changed.

Stage 3: The member is then loaded to failure in a short-time

Stages 1 and 2 are assumed to be equivalent to loading the member slowly, as was done in Section 9.2, up to the axial load $P$ and bending moment $M$ corresponding to the working-load conditions. During this process, the concrete creeps as the loading is applied, and a different distribution of forces is obtained as compared to the short-time loading up to working conditions. Consequently, the same stress-strain curve for concrete is used in the analysis for loading the member up to working-load conditions as was explained in Section 9.2.1. Figure 9.13 shows the strain and stress distributions and the resulting stress-strain relationship for concrete for the above stages of loading. The cross-section of the member is shown in Fig. 9.13(a).

At the end of Stage 1, the strain distribution corresponding to equilibrium of forces is shown in Fig. 9.13 (b) by line 1, the short-time stress-strain curve used for this stage of loading is shown in Fig. 9.13 (d) by the curve marked 1, and the stress distribution over the section is shown in Fig. 9.13 (c) by curve 1. At the end of Stage 2, the strains are increased under reduced stress as shown by the lines e-g and f-h in Fig. 9.13 (d), resulting in a new stress-strain relationship (curve 2) which is assumed to be represented by the equation given in Section 9.2.1 (curve B,
Fig. 9.1) for slow loading. The strain distribution over the cross-section, obtained from equilibrium of forces, is shown by line 2 in Fig. 9.13 (b), and the corresponding stress-distribution in Fig. 9.13 (c). With the assumption of curve 2 to represent the end of Stage 2, it is not necessary to go through Stage 1.

When the member is loaded to failure in Stage 3 concrete strains increase according to the stress-strain relationship (curve 1) representing short-time loading. Accordingly, each section of the compression zone follows curve 1, but starting from a different point on curve 2 depending on the stress level reached at the end of Stage 2. Two such paths followed by points h and g on curve 2 are shown in Fig. 9.13 (d). These points correspond to strains 0.001 and 0.002 at the end of Stage 2, which were reached from strains corresponding to points f and e at the end of Stage 1, and which are increased to 0.005 and 0.010, respectively, at the end of Stage 3, as shown by points j and k. This process results in new stresses for the strains reached at the end of Stage 3, and thus gives a new "stress-strain" relationship as shown by curve 3 in Fig. 9.13 (d). The corresponding stress distribution in the compression zone is shown by curve 3 in Fig. 9.13 (c). This stress-strain relationship depends on the extent to which the strains are increased during the third stage of loading. Since it is not known in advance what value of $\varepsilon_4$ will represent the ultimate conditions for Stage 3, several trial values of $\varepsilon_4$ are necessary, as in Step 3 of the method of analysis explained in Section 3.4, and for each value of $\varepsilon_4$ a different stress-strain relationship is obtained for curve 3. Also, in order to obtain equilibrium of forces, several trials are required for the value of curvature $\phi$ which fixes the depth to the neutral axis, and
hence changes the extent to which the strains are increased, as was done in Step 4 in Section 3.4. Consequently, a new stress-strain relationship must be obtained for each value of \( \phi \). The computer program prepared for the analyses in Chapters 5 through 8 and in Section 9.2 was modified to generate the concrete stress-strain relationships for Stage 3 of loading using the concrete stress-strain curves A and B. Once this stress-strain relationship is obtained, the remaining procedure for obtaining the load-moment and load-curvature relationships is the same as explained in Section 3.4.

9.3.2. ASTM Grade 60 Steel

In this section analyses of reinforced concrete sections are presented for the combination of sustained and short-time loading using the concrete stress-strain relationship explained in Section 9.3.1. Two stress-strain curves for reinforcement have been considered, representing steels 5 and 8 shown in Fig. 4.5. As explained in Section 4.1.3, these steels meet the requirements of both the ASTM specifications and the ACI Code provisions.

Comparisons of load-moment diagrams are made in Figs. 9.14 through 9.17 for the following cases.

Case (1): In strict accordance with the 1963 ACI Code; i.e., flat-top steel 7 (Fig. 4.5), concrete stress-strain curve A (Fig. 4.11), and \( \varepsilon_u \leq 0.003 \). The load-moment diagrams for this case were presented and discussed in Chapter 7. These are reproduced in Figs. 9.14 through 9.17 and are identified as curves 1.

Case (2): According to the method of analysis explained in Chapter 3, i.e. steel 5 or 8, as the case may be, concrete stress-strain curve A, and the criteria for
ultimate capacity explained in Section 3.3. The load-moment diagrams for this case too were presented and discussed in Chapter 7. These are identified as curves 2 in Figs. 9.14 through 9.17.

Case (3): The load-moment diagram representing the working-load conditions. These values of load and moment are taken equal to one-half of the values for Case 2 above, and are sustained for any desired period of time (one year in this case). These are identified as curves 3 in Figs. 9.14 through 9.17.

Case (4): The load-moment diagrams representing the combination of sustained and short-time loading for steel 5 or 8. These values of load and moment have been obtained by further loading the section beyond the values of P and M for Case 3 above in a short-time at the same eccentricity as in Case 3. The ultimate capacity is determined in accordance with the criteria explained in Section 3.3.

It can be seen in Figs. 9.14 and 9.16 that, for steel 5, there is practically no difference between the load-moment diagrams for Cases 2 and 4. But, as can be seen in Fig. 9.15 and 9.16, a slightly greater ultimate capacity is obtained for steel 8 for Case 4 than for Case 2. This is explained below.

The stress-strain relationship for concrete for the combination of sustained and short-time loadings (Case 4), as explained in Section 9.3.1, is only slightly different from that for short-time loading (Case 2). Figure 9.13 shows that the stress-strain curve 3 used for Case 4 in this section lies
slightly below curve 1 used for Case 2, in the ascending branch, but slightly above it in the falling branch of the curve. The maximum stress $f''_c = k_2 f'_c$ is the same for both cases, but $\epsilon_0$ corresponding to $f''_c$ is greater for Case 4 than for Case 2. Thus, the concrete stress-strain relationship used for Case 4 is intermediate between those for slow loading (Section 9.3.2) and short-time loading (Chapter 7), and depends on the extent to which the strains are increased in Stage 3 of the loading, as explained in Section 9.3.1. If these strains are slightly greater than those at the end of Stage 2 of loading, curve 3 (Fig. 9.13) lies closer to curve 2, but if these strains are considerably greater than those at the end of stage 2, as shown in Fig. 9.13, the stress-strain curve 3 lies closer to curve 1. The former case occurs at high levels of ultimate axial load $P_u$ and the latter at lower levels of $P_u$. This flatter concrete stress-strain relationship for Case 4 gives slightly smaller value of $k_2$ and hence slightly greater value of lever arm as compared to the short-time stress-strain curve used for Case 2. Also, since $k_2$ is the same for both Cases 2 and 4, the stress-strain relationship for Case 4 gives slightly greater values of $k_1 k_2$ as compared to Case 2 at the strain $\epsilon_u > \epsilon_0$.

It has been explained in Chapter 7 that, for steel 5, the compression steel strains at ultimate capacity, at ultimate axial load levels at and above the balance point, are on the flat-plateau of the stress-strain curve. Thus the compression steel stress remains constant at $f_y$. The effect of using a flatter concrete stress-strain relationship for Case 4 with greater value of $\epsilon_0$ than for Case 2 is to obtain greater strains in the compression steel, as was the case for slow loading as explained in Section 9.2.2. Since these strains do not give greater stress, no increase in
compression steel force is obtained and thus there is no increase in ultimate capacity of the section with steel 5 for Case 4 as compared to Case 2.

At low levels of ultimate axial load, the compression steel strains for steel 5 are in the elastic region for small values of $p_t$ and $q_t$ and in the strain-hardening region for high values. In both cases, the increase in the compression steel strain results in greater compression steel stress and force. However, since the ultimate capacity occurs at high values of $\epsilon_u$, the concrete stress-strain relationship for Case 4 is nearly the same as for Case 2. Hence, little difference in ultimate capacity is obtained with steel 5 for both cases.

For the round-house stress-strain curve, as for steel 8, the compression steel stress increases with increase in strain at all levels of ultimate axial load. However, this increase in stress is small since the strains are in that region of the stress-strain curve in which stress increases at a smaller rate. Therefore, a small increase in ultimate capacity is obtained for Case 4 due to the increase in compression steel stress than for Case 2. For high values of $p_t$, when the ultimate capacity is determined from the criterion of buckling of compression bars, the compression steel strain and stress are the same for both Cases 2 and 4. In this case, for the same value of $\epsilon_u$, slightly greater value of $k_1k_3$ results in slightly greater concrete force for Case 4 than for Case 2. This, coupled with slightly greater lever arm of concrete force for Case 4, results in some increase in the moment of concrete force, and hence in the ultimate capacity for Case 4. It has been explained in Chapter 7 that, at ultimate axial load levels above the balance point, smaller ultimate capacity is
obtained for steel 8 (Case 2) than for steel 7, in accordance with the ACI Code (Case 1), because the compression steel strains lie in that region of the stress-strain curve for steel 8 which is below that for steel 7, and thus smaller compression steel stresses are obtained for steel 8. Since, for Case 4, the compression steel strains and stresses are increased, the difference between the compression steel stresses for steels 7 and 8 is reduced. Consequently, the reduction in ultimate capacity at high levels of $P_u$ for steel 8 (Case 2), as compared to that for steel 7 (Case 1), is reduced when the loading condition of Case 4 is used. At low levels of ultimate axial load, since the ultimate capacity for steel 8 (Case 2) is already greater than that for steel 7 (Case 1), the ultimate capacity for Case 4 is further increased by a very small amount.

9.3.3. ASTM Grade 75 Steel

In this section analyses of reinforced concrete sections are presented for the combination of sustained and short-time loadings for ASTM Grade 75 steel. Only steel 3 (Fig. 4.6) has been selected for analyses here since, as explained in Section 4.1.3, it is more typical of Grade 75 steels and meets the ASTM specifications as well as the ACI Code provisions. Two values of $f'_c$ and one value of $f'_c$ have been considered as explained in Section 9.1.

Comparisons of load-moment diagrams are given in Figs. 9.17 and 9.20 for the same four cases of loading conditions, as explained in Section 9.3.2 for ASTM Grade 60 steel. It can be seen from these figures that, at all levels of ultimate axial load $P_u$, greater ultimate capacity of the section is obtained for Case 4 (combination of sustained and short-time loading) than for either Case 1 (in accordance with the ACI Code) or Case 2
(short-time loading according to the method of analysis explained in Chapter 3). If the ultimate capacity of the section for Case 4 is compared with that for Case 2, slightly greater ultimate capacity is obtained for Case 4 because of the increase in compression steel force and/or concrete force for Case 4, as was explained in Section 9.3.2 for steel 8. But, if the ultimate capacity of the section for Case 4 is compared with that for Case 1, considerably greater increase in ultimate capacity is obtained at low levels of ultimate axial load for Case 4 because of the greater tension steel stresses for Case 4 than for Case 1, as was true for the short-time loading explained in Chapter 8. This increase in ultimate capacity decreases as $P_u$ increases and, at high levels of $P_u$, very small increase in ultimate capacity of the section is obtained for Case 4 than for Case 1 for both values of $P_t$ or $q_t$. Contrary to this, for the small value of $P_t$ or $q_t$, slightly smaller ultimate capacity of the section is obtained for Case 2 than for Case 1 at high levels of $P_u$ because, as explained in Chapter 8, the compression steel strains lie in that portion of the stress-strain curve for steel 3 which is below that of the flat-top steel 2. As explained in Section 9.2, the combination of sustained and short-time loading increases the compression steel strains, and thus gives greater compression steel stresses than $f_y$ for Case 1. This results in greater ultimate capacity for steel 3 for Case 4 of loading.

9.4. Summary

In this chapter, analyses of a 15-in. square reinforced concrete section with ASTM Grade 60 and 75 steels have been presented and discussed for slow loading and for a combination of sustained loading at a working load level followed by short-term loading to failure. The effect of creep
of concrete is considered by modifying the stress-strain relationship for concrete in such a manner that it represents as closely as possible the loading conditions.

Stress-strain curves for steels 5, 7 and 8 (Grade 60 steel) shown in Fig. 4.5 and for steels 1, 2 and 3 (Grade 75 steel) shown in Fig. 4.6 have been considered in the analyses. A continuous stress-strain curve for concrete with \( f''_c = k_3 f'_c = 0.75 f'_c \) and \( \varepsilon_c = 0.006 \) (three times the value for short-time loading), as explained in Section 9.3.1 and shown in Fig. 9.1, has been used in the analyses for slow loading of the section. The stress-strain curve for concrete for the combination of sustained and short-time loadings is determined from the stress-strain curves for slow and short-time loadings as explained in Section 9.3.1 and shown in Fig. 9.13. Two values of \( p_t = 0.01 \) and \( 0.08 \) and one value of \( f'_c = 3000 \) psi have been used.

(a) **Slow loading**

Comparisons of load-moment diagrams (Figs. 9.2 and 9.3) for Grade 60 steels 5 and 8 with those for steel 7 show that the ultimate capacity of the section for slow loading, at all levels of ultimate axial load, is greater for steels 5 and 8 than for steel 7. This increase in ultimate capacity for steels 5 and 8 is quite large at low levels of ultimate axial load. The effect of slow loading is to increase the ultimate concrete strain and thus the steel strains which result in greater steel stresses and forces as compared to short-time loading. Also, the smaller values of \( k_1 k_3 \) and \( k_2 \) for slow loading result in smaller concrete force but a greater lever arm. Greater compression steel force for steels 5 and 8 compensates for the reduction in the concrete force, and greater tension steel force at low levels of ultimate axial load together with greater lever arm of concrete.
force result in greater ultimate moment of the section for steels 5 and 8 as compared to steel 7. This is also true for short-time loading, but greater increase in ultimate moment is obtained for slow loading, particularly for small value of $P_t$. At high levels of ultimate axial load, the compression steel and concrete play greater role in providing the required ultimate capacity. Since, for slow loading, $k_i k_3$ is small and the small increase in compression steel stress and force is mainly utilized in compensating for the reduction in concrete force, only slight increase in ultimate capacity is obtained for steels 5 and 8, as compared to steel 7. Contrary to this, for short-time loading, considerably smaller ultimate capacity is obtained for steel 8 as compared to steel 7. This has been explained in Chapter 7.

If the ultimate moment of the section for slow loading is compared with that obtained according to the ACI Code for steel 7, it is found that, at low levels of ultimate axial load, the same ultimate moment is obtained for steel 7 for both cases, but greater ultimate moments are obtained for steels 5 and 8 for slow loading, as explained above. However, at high levels of ultimate axial load, smaller ultimate moment is obtained for steel 7 for slow loading because of the smaller value of $k_i k_3$. The same is the case for steels 5 and 8 for the small value of $P_t$, since the stress and the area of the compression steel are not enough to compensate fully for the reduction in the concrete force. For the high value of $P_t$, steel 5 compensates for all of the reduction in the concrete force and thus gives the same ultimate moment for both cases. But this is not true for steel 8 and the result is a smaller ultimate moment for slow loading.

For ASTM Grade 75 steel, similar effects of the stress-strain curves of steels 1 and 3 as compared to steel 2 are obtained as explained above for steels 5 and 8 in comparison with steel 7. However, these effects
are enhanced for Grade 75 steel, and thus greater increases in ultimate moment are obtained for steels 1 and 3 as compared to steel 2 at all levels of ultimate axial load. Also the compression steel provides greater increase in its force to compensate for the reduction in the concrete force. Consequently, the reduction in ultimate moment at high levels of ultimate axial load caused by smaller value of $k_{12}$ for slow loading, as compared to that calculated according to the ACI Code, is also smaller for steels 1 and 3 than for steels 5 and 8, for small value of $p_t$. But for the high value of $p_t$, steels 1 and 3 more than compensate for the reduction in the concrete force for slow loading and thus result in greater ultimate moment as compared to that calculated according to the ACI Code.

(b) **Combination of sustained and short-time loadings**

Figures 9.14 through 9.20 show comparisons of load-moment diagrams for the combination of sustained and short-time loadings with those for the short-time loading in accordance with the 1963 ACI Code and also according to the method of analysis explained in Chapter 3. These cases of loading conditions are explained in Section 9.3.2.

Practically the same ultimate capacity of the section is obtained for steel 5 for the combination of sustained and short-time loadings (Case 4) as for the short-time loading (Case 2). Consequently the effect of the stress-strain curve for steel 5 as compared to steel 7 is similar to that explained in Chapter 7. However, for the round-house stress-strain curves for steels 8 and 3, the combination of sustained and short-time loading results in greater compression steel stresses and/or slightly greater concrete force and its lever arm for Case 4 than for Case 2. Consequently, slightly greater ultimate capacities are obtained for steels 3 and 8 for Case 4 than
for Case 2 particularly at high levels of ultimate axial load. As has been explained in Chapters 7 and 8 that, at high levels of ultimate axial load, steels 3 and 8 result in smaller ultimate moments for short-time loading (Case 2) as compared to those calculated according to the ACI Code for steels 2 and 7, respectively (Case 1). The increase in ultimate capacity for Case 4 as compared to that for Case 2 compensates for a part of this reduction in ultimate capacity for steel 8 and all of the reduction for steel 3. Thus, the difference in the ultimate capacities for steel 8 and 7 is reduced by using the combination of sustained and short-time loading for steel 8, whereas greater ultimate capacity is obtained for steel 3 for the combination of sustained and short-time loading than for steel 2 according to the ACI Code.
10. ANALYSES OF REINFORCED CONCRETE SECTIONS WITH CADWELD-SPICED REINFORCING BARS

10.1. Introduction

In this chapter, analyses are presented for the 15-in. square reinforced concrete section shown in Fig. 3.1, to investigate the effects of the stress-strain curve of Cadweld-spliced reinforcing bars on the strength and behavior of reinforced concrete sections. These splices are manufactured by Erico Products Inc., Cleveland, Ohio, and consist of a metal sleeve of varying length in which the bars are inserted end to end. A powdered mixture is ignited and allowed to flow into the sleeve and create a mechanical bond between the sleeve and the reinforcing bars to transfer force from one bar to the other.

The stress-strain curves, in the region of the splice, for the reinforcing bars spliced in this manner depend on the length of the sleeve and on the gage length used for measuring strains. Away from the spliced region, of course, these bars have the same stress-strain relationship as the unspliced bar. It has been found from tests of reinforced concrete beams (23) that the strength and behavior of a member subjected to flexure only, and having all the reinforcement spliced in the region of maximum moment, could be predicted fairly well by averaging the stress-strain characteristics in the spliced region over a length of bar equal to at least the effective depth of the section. The stress-strain curves for the spliced bars used in the analyses in this chapter are shown in Figs. 10.1 and 10.2 for Grade 60 and Grade 75 bars, respectively. These are not actual measured curves but are idealizations based on tests of spliced bars of various sizes with strains
measured over a gage length of 10 in. including the splice. The bars tested all exhibited typical strain-hardening and this is reflected in the stress-strain curves shown for the spliced regions. The curves have been drawn for a bar having a yield strength exactly equal to the specified strength of 60 or 75 ksi for Grades 60 and 75, respectively. The stress-strain curves 7 and 2 in Figs. 10.1 and 10.2, respectively, are shown with no strain-hardening to represent the stress-strain relationship assumed in the ACI Code. Although the effective depth of the section analyzed (12 in.) is greater than the gage length of 10 in. for which these stress-strain curves were obtained, no adjustment has been made, partly in order to be conservative and partly because no information exists regarding a suitable effective gage length for members subjected to axial load as well as bending. In addition, it has been assumed in the analysis that all of the bars are spliced at the location of the cross-section being considered. This, too, is a conservative assumption, since the average stress-strain characteristics for a combination of spliced and unspliced bars (staggered splices) would be significantly better than shown in Figs. 10.1 and 10.2.

Since Cadweld-spliced reinforcing bars have different stress-strain relationships in tension and compression, different stress-strain curves are shown in Figs. 10.1 and 10.2 for a spliced region subjected to tension or compression. In Fig. 10.1, for ASTM Grade 60 steel, the curve labelled 9 is used for compression stresses, and the curve labelled 10 is used for tension stress. Similarly, in Figs. 10.2, for Grade 75 steel, curves 11 and 12 are used for compression and tension stresses, respectively.

The conventional flat-top stress-strain curves assumed for the purpose of design in the 1963 ACI Code, shown in Figs. 10.1 and 10.2,
represent steels 7 and 2 which were used in the analyses in Chapters 7 and 8, respectively. It can be noted in these figures that the stress-strain curves for spliced bars lie partly below and partly above those for the flat-top steels 2 and 7. While the tension splices have a portion of their stress-strain curves considerably below those for steels 2 and 7, the compression-splice curves lie only slightly below and in a very small range of strains near the yield point.

The same 15-in. square section selected for analyses in Chapters 7, 8 and 9 has been analyzed in this chapter to obtain load-moment and load-curvature diagrams, which are presented and discussed in Section 10.2 for short-time loading, in Section 10.3 for slow loading, and in Section 10.4 for a combination of sustained and short-time loadings.

Three values of $p_t$ and $f'_c$ have been selected to obtain three values of $q_t$ for each grade of reinforcing bar, as given below:

<table>
<thead>
<tr>
<th>$p_t$</th>
<th>$f'_c$</th>
<th>$q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Grade 60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_y = 60$ ksi</td>
</tr>
<tr>
<td>0.01</td>
<td>3000</td>
<td>0.20</td>
</tr>
<tr>
<td>0.04</td>
<td>4000</td>
<td>0.60</td>
</tr>
<tr>
<td>0.08</td>
<td>5000</td>
<td>0.96</td>
</tr>
</tbody>
</table>

For the purpose of comparisons with the 1963 ACI Code for the flat-top steels 7 and 2, the ordinates of the load-moment and load-curvature diagrams have been divided by $P_01$ and $P_02$ for Grade 60 and Grade 75 steels, respectively; whereas the abscissas of the load-moment diagrams have been divided by $M_01$ and $M_02$, as was done in Chapters 7, 8 and 9.
The stress-strain curves shown in Figs. 10.1 and 10.2 for spliced bars are for that portion of the bar which contains the splice. Away from the splice, the actual stress-strain curve of the unspliced bar is to be considered, the effects of which have been explained in Chapters 7, 8 and 9. Consequently, the discussions and explanations presented in Sections 10.2, 10.3 and 10.4 apply only to that region of the member which contains the splices, and the spliced bars referred to in these sections mean only that portion of the bar containing the splice.

10.2. Short-Time Loading

10.2.1. Splices for ASTM Grade 60 Reinforcing Bars

In this section, analyses have been made using the steel stress-strain curves 9 and 10 (Fig. 10.1) for compression and tension, respectively, for the spliced bars, and curve 7 for the flat-top steel for comparisons. The concrete stress-strain curves used are those shown in Fig. 4.11, and the limiting concrete strain was assumed to be \( \epsilon_u = 0.003 \) in accordance with the ACI Code or 0.010 according to the method of analysis explained in Chapter 3.

Figures 10.3 and 10.4 show comparisons of load-moment diagrams for spliced bars (9,10) with the flat-top steel (7) and for both cases of the limiting value of \( \epsilon_u = 0.003 \) (ACI Code) and 0.010.

It was explained in Chapter 7 that limitation of \( \epsilon_u \) to 0.003 results in considerable decrease in ultimate capacity of the section at ultimate axial load levels below the balance point, particularly for steel 8 which has a round-house stress-strain curve. This is also true for the spliced bars. However, the effects of this limit on \( \epsilon_u \) is greater for the
spliced bars than for steel 8, because the stress-strain curve for the
tension splice lies considerably below that for steel 7 for stresses less
than the yield strength $f_y$. It was also explained in Chapter 7 that, if a
higher limit of $\varepsilon_u$ is permitted for steel 7, little increase in ultimate
capacity is obtained, except at ultimate axial load levels slightly above
the balance point. But, as for steel 8, the ultimate capacity of the
section with spliced bars is considerably increased by allowing a higher
limit on $\varepsilon_u$. This is shown by the moment-strain curves in Figs. 10.7. This
increase in ultimate capacity for the spliced bars is obtained at low levels
of ultimate axial load because the tension steel strains at ultimate
capacity are in that region of the stress-strain curve which gives tension
steel stress greater than $f_y$ for the flat-top steel. Table 10.1 shows that,
at $P_u = 0$, an increase of 32 percent in ultimate moment is obtained for the
spliced bars for $q_t = 0.20$ as compared to steel 7.

As the ultimate axial load is increased, the tension steel strain
decreases while the compression steel strain increases. This has been
explained in Chapter 5. At ultimate axial load levels near the balance point
for steel 7, the tension and compression steel strains are in that region of
the stress-strain curves for spliced bars which are below that for steel 7,
and thus reduced steel stresses are obtained which give reduced ultimate
capacities for spliced bars. At higher levels of ultimate axial load, the
tension steel strains are further reduced and reach that region of the
stress-strain curve which results in a small reduction in tension steel
stress. Since the stress-strain curve for a splice in compression is only
slightly below that for the flat-top steel near the yield point, a very small
reduction in compression steel stress is obtained. Thus, the reduction in
ultimate capacity is also small. A part of this reduction in ultimate capacity is compensated for by allowing a higher limit on $\varepsilon_u'$, as shown in Figs. 10.3 and 10.4. At high levels of ultimate axial load, the tension steel strains are in the elastic compression region of the stress-strain curve which is the same as for the flat-top steel. Thus, the reduction in ultimate capacity is due only to the slight reduction in the compression steel stress. Consequently, a very small reduction in ultimate capacity is obtained. The variation of steel strains with the increase in ultimate axial load is explained in detail in Chapter 5.

Table 10.1 gives comparisons of ultimate capacities for the spliced bars (9,10) and for steel 7 at three levels of ultimate axial load for each value of $q_t$. The intermediate level of ultimate axial load corresponds to the balanced conditions for steel 7 in accordance with the 1963 ACI Code. These comparisons are made for the spliced bars at the same eccentricities as for steel 7. It can be seen in this table that the maximum reduction of 11 percent in ultimate capacity is obtained for the spliced bars with $q_t = 0.20$. This reduction decreases as $p_t$ or $q_t$ increases.

Figures 10.3 and 10.4 show that, if the comparisons are made for $\varepsilon_u \leq 0.010$ for both steels, the maximum reduction at any load level is still 11 percent for $q_t = 0.20$. The corresponding maximum reductions for $q_t = 0.60$ and 0.96 are 8 and 7 percent, respectively. All these reductions in ultimate capacity occur at ultimate axial load levels slightly above the balance point.

Ultimate curvatures of the section for the spliced bars are compared with those for steel 7 in Figs. 10.5 and 10.6. It can be seen in these figures that, for $\varepsilon_u \leq 0.003$, slightly greater ultimate curvatures are
obtained for the spliced bars at $P_u \leq 0.45 P_{02}$, but at higher levels of $P_u$, little difference in ultimate curvatures is obtained between the two steels. However, if a higher limit of $\epsilon_u \leq 0.010$ is allowed, large curvature is calculated for the spliced bars. This effect on the ultimate curvature for spliced bars is similar to that explained for steels 5, 6 and 8 in Chapter 7.

10.2.2. Splices for ASTM Grade 75 Reinforcing Bars

In this section, analyses have been made using the steel stress-strain curves 11 and 12 (Fig. 10.2) for compression and tension, respectively, for the spliced bars, and curve 2 for the flat-top steel for comparisons. The concrete stress-strain curves shown in Fig. 4.11 have been used with the ultimate concrete strain $\epsilon_u \leq 0.003$ in accordance with the ACI Code and $0.010$ according to the method of analysis explained in Chapter 3.

Comparisons of load-moment diagrams for Grade 75 spliced bars (11,12) with those for the flat-top steel (2) are shown in Figs. 10.8 and 10.9 for both values of $\epsilon_u = 0.003$ and 0.010. These figures show that, as was the case for Grade 60 spliced bars, limitation of $\epsilon_u$ to 0.003 rather than to 0.010 results in considerable decrease in the ultimate capacity of the section with Grade 75 spliced bars at low levels of ultimate axial load. But, for the flat-top steel 2, little effect of $\epsilon_u$ is obtained at ultimate axial load levels at or below the balance point. This can be seen also in Fig. 10.12, for zero axial load. Figures 10.8 and 10.9 show that, at low axial load levels, considerably greater ultimate capacity is obtained with spliced bars than with flat-top steel if $\epsilon_u$ is not limited to 0.003.

It was explained in Chapter 8 that, at ultimate axial load levels above the balance point and for a considerable region of the load-moment diagram, the ultimate capacity of the section with steel 2 is increased by
allowing a higher limit on $\varepsilon_u$. Figures 10.8 and 10.9 show that this is true also for the spliced bars, but a greater increase in ultimate capacity is obtained for the spliced bars than for the flat-top steel with the higher limit on $\varepsilon_u = 0.010$. This increase in ultimate capacity increases with increase in $q_t$. Consequently, the difference between the ultimate capacities of the section with spliced bars and with the flat-top steel decreases as $q_t$ increases.

Table 10.2 gives comparisons of ultimate moments and curvatures of the section for the spliced bars with the flat-top steel at three levels of $P_u$ for each of the three values of $q_t$. As was the case for the Grade 60 spliced bars, these comparisons have been made at the same eccentricity for the spliced bars and for the flat-top steel for each level of $P_u$. Also, $\varepsilon_u$ is limited to 0.003 for the flat-top steel in accordance with the ACI Code, but the ultimate moment for the spliced bars is determined from the criteria explained in Section 3.3. The intermediate level of the ultimate axial load for each value of $q_t$ corresponds to the balanced conditions for steel 2.

It can be seen in Table 10.2 that, at $P_u = 0$, the maximum increase of 20 percent in ultimate moment is obtained for the spliced bars as compared to steel 2, for $q_t = 0.25$. This increase in ultimate capacity decreases as $q_t$ increases, and the minimum increase of 15 percent in ultimate moment is obtained for $q_t = 1.20$. However, at balanced conditions, the maximum reduction of 8 percent in ultimate capacity is obtained for the spliced bars for $q_t = 0.25$ and this reduction in ultimate capacity decreases as $q_t$ increases. But, if $\varepsilon_u$ is limited to 0.010 for both the spliced bars and the flat-top steel, the maximum reduction in ultimate capacity for the spliced bars at any level of ultimate axial load occurs for $q_t = 0.25$ and amounts to 13 percent. For $q_t = 0.75$ and 1.20, the corresponding maximum reductions are
9 and 5 percent. For zero eccentricity, the maximum reduction of 8 percent in ultimate axial load occurs for $\theta_t = 0.75$. The corresponding reductions in $P_u$ for $\theta_t = 0.25$ and 1.20 are 5 and 7 percent, respectively.

Smaller reductions in ultimate capacity occur at higher levels of ultimate axial load, as was the case for Grade 60 spliced bars, when both steels are in compression and the stress-strain curve 11 is used for the spliced bars. Since, this curve is only slightly below that for the flat-top steel near the yield point, very small reduction in ultimate capacity is obtained for the spliced bars. A part of this reduction in ultimate capacity is compensated for by the greater value of $\varepsilon_u$ obtained for the spliced bars.

Figures 10.10 and 10.11 show comparisons of load-curvature diagrams for the spliced bars and the flat-top steel for two limits on $\varepsilon_u$ (0.003 and 0.010). For $\varepsilon_u \leq 0.003$, there is a very small difference between the ultimate curvature of the section for both steels. However, for $\varepsilon_u < 0.010$, considerably large curvatures are obtained for the spliced bars than for the flat-top steel because of the greater value of $\varepsilon_u$ for the spliced bars. This effect on ultimate curvature of the section for the spliced bars is similar to that for steels 1, 3 and 4 explained in Chapter 8.

10.3. Slow Loading

10.3.1. Splices for ASTM Grade 60 Reinforcing Bars

In this section, analyses are presented and discussed for the reinforced concrete section with Cadweld-spliced reinforcing bars, the stress-strain curves of which are shown in Fig. 10.1; i.e. curves 9 and 10 for compression and tension, respectively. Since the section is loaded
slowly and continuously up to failure, concrete stress-strain curve B shown in Fig. 9.1 is used for the analyses. In order to compare the load-moment and load-curvature relationships for the spliced bars with those for the flat-top steel assumed in the ACI Code, analyses are also presented for steel 7 with concrete curve B. The ultimate concrete strain $\epsilon_u$ is limited to 0.010 for both spliced bars and steel 7.

Figures 10.13 and 10.14 show the load-moment diagrams for spliced bars (9,10) and for steel 7 for three values of $p_t$, $f'_c$ and $q_t$, as given in Section 10.1. These figures show that, at low levels of ultimate axial load, considerably greater ultimate capacity is obtained for the spliced bars for slow loading of the section, as was true for short-time loading explained in Section 10.2.1. However, at high levels of ultimate axial load, instead of slightly smaller ultimate capacity of the section with spliced bars as compared to the flat-top steel for short-time loading, slightly greater ultimate capacity is obtained for slow loading of the section with spliced bars.

Table 10.3 gives comparisons of strains, moments and curvatures of the section at two levels of ultimate axial load for each value of $q_t$. At \( P_u = 0 \), the maximum increase of 41 percent in ultimate moment is obtained for $q_t = 0.20$, obtained with $p_t = 0.01$. This increase decreases to 31 percent for $q_t = 0.96$, obtained with $p_t = 0.08$. At high levels of ultimate axial load, ultimate capacity is governed by the compression steel. The moment-strain curves continue to rise with increase in $\epsilon_u$ due to the shape of concrete stress-strain curve, as was true for steels 5 and 8, as explained in Section 9.2.2. Also, the ultimate capacity is determined from the criterion of buckling of compression bars corresponding to a strain of 0.006
at the level of compression steel, as explained in Section 3.3. At this
strain, the compression steel stress and force for the spliced bars are
greater than those for the flat-top steel. For the same value of $P_u$ and $\epsilon_u$
for both spliced bars and steel 7, greater compression steel force results
in greater ultimate moment for the spliced bars. Contrary to this, for
short-time loading, ultimate moment occurs at a smaller value of $\epsilon_u$ which
results in compression steel strains in that region of the stress-strain
curve for the spliced bars which is below that for the flat-top steel, and
thus slightly smaller compression steel stress and force are obtained, which
give slightly smaller ultimate moment for the spliced bars.

Figure 10.13 shows that in a very small region of the load-moment
diagram for $q_t = 0.20$, near the transition from the tension to compression
failure, ultimate moment for spliced bars is slightly smaller than that for
the flat-top steel. This is similar to the case for short-time loading, but
the reduction in ultimate capacity for slow loading is only 4 percent. For
greater values of $q_t$ (Fig. 10.14), there is no reduction in ultimate moment
for the spliced bars for slow loading as compared to the flat-top steel.

Table 10.4 gives comparisons of ultimate moments for spliced bars
(9,10) and steel 7 for slow loading with those for steel 7 for short-time
loading in accordance with the 1963 ACI Code. It can be seen from this table
that, at $P_u = 0$, while almost the same ultimate moment is obtained for steel 7
for slow loading and according to the ACI Code, considerably greater ultimate
moment is obtained for spliced bars for slow loading as compared to that for
steel 7 according to the ACI Code. This is also true at other low levels of
ultimate axial load, as was the case for steels 5 and 8, as explained in
Section 9.2.2. However, at high levels of ultimate axial load, smaller
ultimate moments are calculated for spliced bars as well as for steel 7 for slow loading as compared to that for steel 7 according to the ACI Code. This is due to the effect of smaller value of $k_1 k_3$, as explained in Section 9.2.2. The reduction in ultimate moment for spliced bars is smaller than that for the flat-top steel 7 because the compression steel strains are such that the spliced bars in compression give greater stresses than the flat-top steel, and hence partly compensate for the reduction in concrete force caused by the reduction in $k_1 k_3$.

Figures 10.15 and 10.16 give comparisons of load-curvature diagrams for spliced bars (9,10) and flat-top steel 7. At low levels of ultimate axial load, the ultimate moment occurs at almost the same values of $\epsilon_u$ for both spliced bars and steel 7. Consequently, as explained in Chapter 7, smaller ultimate curvatures are obtained for the spliced bars than for steel 7 because tension steel strains are such that greater tension steel stresses are obtained for the spliced bars. At intermediate levels of ultimate axial load, $\epsilon_u$ for the spliced bars is greater than that for the flat-top steel, and also the tension steel strains are such that smaller tension steel stresses are obtained for the spliced bars than for steel 7. Both these factors give greater ultimate curvatures for spliced bars. At high levels of ultimate axial load, $\epsilon_u$ is almost the same for both spliced bars and steel 7. Also, the tension steel is in the elastic compressive region of the stress-strain curve which is the same for the spliced bars and steel 7. Consequently, almost the same ultimate curvatures are obtained for spliced bars and steel 7.
10.3.2. **Splices for ASTM Grade 75 Reinforcing Bars**

In this section, analyses are presented for slow loading of the reinforced concrete section with Cadweld-spliced reinforcing bars of ASTM Grade 75 steel (11,12) and the comparisons are made with the flat-top steel 2. Ultimate moment of the section is calculated by using the concrete stress-strain curve B shown in Fig. 9.1 according to the method of analysis explained in Chapter 3. Also, comparisons of ultimate moments for spliced bars (11,12) and flat-top steel 2 for slow loading are made with that for steel 2 for short-time loading (concrete curve A) in accordance with the 1963 ACI Code.

Figures 10.17 and 10.18 show comparisons of load-moment diagrams for spliced bars (11,12) and flat-top steel 2 for the three values of $p_t$ and $q_t$ given in Section 10.1. These figures show that the effects of the stress-strain curves of the spliced bars for ASTM Grade 75 steel are similar to those for the spliced bars for ASTM Grade 60 steel, as explained in Section 10.3.1. Table 10.5 gives comparisons of ultimate moments for spliced bars (11,12) with those for the flat-top steel at two levels of ultimate axial load for each value of $q_t$. It can be seen from this table that, as was true for the spliced bars of ASTM Grade 60 steel, considerable increase in ultimate moment is obtained for the spliced bars as compared to the flat-top steel 2. At low levels of ultimate axial load, greater tension steel stresses are obtained for the spliced bars as compared to the flat-top steel which result in greater ultimate moments for the spliced bars. Similarly, at high levels of ultimate axial load, greater compression steel stresses give greater ultimate moments for the spliced bars than for the flat-top steel. Table 10.5 shows that the maximum increase of 29 percent in ultimate moment is obtained for the spliced bars for $q_t = 0.25$. 
Comparisons of ultimate moments given in Table 10.6 for the spliced bars (11,12) and the flat-top steel 2 for slow loading with those for steel 2 for short-time loading, according to the ACI Code, show that, at low levels of ultimate axial load, considerable increase in ultimate moment is obtained for the spliced bars due to greater tension steel stresses for these bars than for the flat-top steel. However, at high levels of ultimate axial load, the smaller value of $k_1 k_3$ for slow loading results in smaller moments for both spliced bars and steel 2 than for short-time loading. As $p_t$ or $q_t$ increases, the compression steel for the spliced bars compensates for a greater part of the reduction in concrete force due to smaller $k_1 k_3$. Thus, the reduction in ultimate moment for spliced bars at high levels of ultimate axial load decreases with increase in $q_t$, and for high value of $q_t = 1.20$, greater ultimate moment is obtained for slow loading than for short-time loading according to the ACI Code.

The load-curvature diagrams for the spliced bars and the flat-top steel for slow loading in Figs. 10.19 and 10.20 show that the effects of the stress-strain curves of the spliced bars of ASTM Grade 75 steel are similar to those for the spliced bars of grade 60 steel, as explained in Section 10.3.1.

10.4. Combination of Sustained and Short-Time Loadings

10.4.1. Splices for ASTM Grade 60 Reinforcing Bars

In this section, analyses are presented for reinforced concrete sections with Cadweld-spliced reinforcing bars of Grade 60 steel under combination of sustained and short-time loadings. The stress-strain curves 9 (for compression) and 10 (for tension) shown in Fig. 10.1 have been used for the
spliced bars. The concrete stress-strain relationship for this loading condition is determined as explained in Section 9.3.1. For the purpose of comparisons with the short-time loading and also with the ACI Code, load-moment diagrams are presented in Figs. 10.21 and 10.22 for the same four cases as explained in Section 9.3.2 for ASTM Grade 60 steel. Three values of $p_t$ or $q_t$, which are given in Section 10.1 have been considered for each case of loading.

Figures 10.21 and 10.22 show the following:

(a) For the spliced bars (9,10), at all levels of ultimate axial load $P_u$, and for all three values of $p_t$ or $q_t$, greater ultimate capacity is obtained for Case 4 (combination of sustained and short-time loading) than for Case 2 (short-time loading). Although the increase in ultimate capacity for Case 4 is very small, greater increase is obtained at high levels of $P_u$ than at low levels. This is due to the effect of the increase in the compression steel forces and/or the concrete force for Case 4, as was explained in Section 9.3.2 for steel 8. It may be recalled here that the stress-strain curve for spliced bars in compression (Curve 9, Fig. 10.1) is similar to but better than that for steel 8 (Fig. 4.5).

(b) The ultimate capacity of the section with the spliced bars (9,10) for Case 4 is considerably greater than that for Case 1 for all values of $q_t$ at ultimate axial load levels below the balance point for steel 7, as was true for short-time loading explained in Section 10.2.2. At ultimate axial load levels above the balance point, except in a small region near the balance point for small value of $q_t$, either the same or very slightly greater ultimate capacity of the section is obtained for the spliced bars for Case 4. In this region of the load-moment diagram above the balance point, as explained in Section 10.2.1, the spliced bars give smaller ultimate capacity.
for Case 2 than the flat-top steel 7 (Case 1) because the compression steel strains lie in that region of the stress-strain curve 9 (Fig. 10.1) which is below that for steel 7, and smaller compression steel stresses are obtained for the spliced bars. But, as explained in Section 9.3.2, the concrete stress-strain relationship for Case 4 is such that greater compression steel strains, stresses and forces, and/or greater concrete force are obtained as compared to Case 2. This results in compression steel stresses close to or greater than $f_y$, and thus either the same or slightly greater ultimate capacity is obtained for Case 4 than for Case 1. For $q_t = 0.20$, obtained with $P_t = 0.01$, the maximum reduction of 7 percent in the ultimate capacity is obtained for spliced bars (Case 4) at balanced conditions as compared to the flat-top steel (Case 1). The corresponding reduction in ultimate capacity for Case 2 is 11 percent. This is so because at this load level, although the compression steel stress for Case 4 is equal to or slightly greater than $f_y$, the tension steel stress is much smaller than $f_y$, and thus the total moment is reduced.

10.4.2. Splices for ASTM Grade 75 Reinforcing Bars

Figures 10.23 and 10.24 give comparisons of load-moment diagrams for Cadweld-spliced reinforcing bars of Grade 75 steel and for the flat-top steel 2. The stress-strain curves for these steels are shown in Fig. 10.2. Three values of $p_t$, $f'_c$ and $q_t$ given in Section 10.1, and the four cases of loading conditions explained in Section 9.3.2 have been considered in the analyses.

It can be seen in Figs. 10.23 and 10.24 that the effects of the stress-strain curves for the spliced bars of Grade 75 steel under the combination of sustained and short-time loading (Case 4) in comparison with the
short-time loading for spliced bars (Case 2) and with the ACI Code for the flat-top steel (Case 1) are similar to those for the spliced bars of Grade 60 steel discussed and explained in Section 10.4.1.

10.5. Summary

In this chapter, analyses for load-moment and load-curvature relationships have been presented and discussed for the same 15-in. square reinforced concrete section which was analyzed in Chapters 7, 8 and 9. The section is provided with Cadweld-spliced reinforcing bars of ASTM Grades 60 and 75. The purpose of these analyses was to investigate the effect of the stress-strain curves of these spliced bars on the strength and behavior of reinforced concrete sections under short-time loading, slow loading, and the combination of sustained and short-time loadings. The steel stress-strain curves used in the analyses for the spliced bars in tension and compression are shown in Figs. 10.1 and 10.2, and explained in Section 10.1. These curves give a conservative representation of the stress-strain relationship for the spliced bar over a gage length of 10 in. including the splice, and it is assumed that all of the bars are spliced at the section considered in the analyses.

Comparisons of the load-moment and load-curvature diagrams for the spliced bars have been made with those for the flat-top steels 7 (Grade 60) and 2 (Grade 75) in accordance with the 1963 ACI Code. Although, the spliced bars are assumed to be typical high-strength steels having $f_y = 60$ and 75 ksi for Grades 60 and 75, respectively, rather than from flat-top steels like steels 7 and 2, the comparisons have been made with steels 7 and 2 which are considered to represent the assumptions made in the ACI Code for the purpose of design.
Three values of \( p_t \) and \( f'_c \) have been considered in the analyses for each grade of steel, as given in Section 10.1.

The concrete stress-strain curve A (Fig. 4.11) has been used for the analyses for short-time loading in Section 10.2, curve B (Fig. 9.1) has been used for slow loading in Section 10.3, and curve 3 (Fig. 9.13), determined as explained in Section 9.2.1, has been used for the combination of slow and short-time loadings in Section 10.4.

(a) **Short-time loading**

If the ultimate concrete strain \( \varepsilon_u \) is limited to 0.003 in the analyses, considerable reduction in ultimate capacity of the section is obtained for the spliced bars of both grades of steel, in comparison with the flat-top steels 7 or 2, as the case may be, particularly at low levels of ultimate axial load. However, if the higher limit on \( \varepsilon_u \) (0.010) is permitted and the ultimate moment is determined from the criteria explained in Section 3.3, greater ultimate capacity of the section is obtained for the spliced bars as compared to the flat-top steels at low levels of ultimate axial load. This occurs because the tension steel stresses obtained for the spliced bars are greater than \( f'_y \) for the flat-top steel. At ultimate axial load levels near the balance point, the tension and compression steel strains are in those regions of the stress-strain curves where smaller stresses are obtained for the spliced bars than for the flat-top steels, even if the higher limit on \( \varepsilon_u \) is permitted. Thus, smaller ultimate capacity is obtained for the spliced bars. At higher load levels, compression steel and/or concrete play a greater role in providing the ultimate capacity of the section. Since, the stress-strain curve for the splice in compression is only slightly below
that for the flat-top steel near the yield point, small reductions in ultimate capacity are obtained for the spliced bars of both grade steels.

Ultimate curvatures of the section for the spliced bars are only slightly increased at low levels of ultimate axial load as compared to the flat-top steel when $\varepsilon_u$ is limited to 0.003. But, for $\varepsilon_u \leq 0.010$, large curvatures are obtained for the spliced bars. At high levels of ultimate axial load, practically the same ultimate curvatures are obtained for the spliced bars as for the flat-top steels for $\varepsilon_u \leq 0.003$. This is also the case for $\varepsilon_u \leq 0.010$ for the small value of $P_t = 0.01$. But, for higher values of $P_t = 0.04$ and 0.8, slightly greater ultimate curvatures are obtained for Grade 60 spliced bars and considerably greater curvatures for Grade 75 spliced bars as compared to the flat-top steels.

(b) Slow loading

The use of concrete stress-strain curve B in the analyses for slow loading of the section results in a greater value of ultimate concrete strain $\varepsilon_u$ and hence in greater steel strains, stresses and forces for the spliced bars than for the flat-top steels. Also, the smaller values of $k_1 k_3$ and $k_2$ for concrete curve B result in smaller concrete force but slightly greater lever arm than for the short-time curve A. The reduction in concrete force is compensated for by the greater compression steel force for the spliced bars. Consequently, greater ultimate capacity of the section is obtained for the spliced bars than for the flat-top steels at ultimate axial load levels below the balance point, as was the case for short-time loading. At high levels of ultimate axial load also, slightly greater ultimate capacity is obtained for the spliced bars for slow loading of the section, because the compression steel strains are such that greater compression steel stresses
are obtained for the spliced bars than for the flat-top steels. This is contrary to the case of short-time loading when the compression steel strains give smaller stresses for the spliced bars and hence smaller ultimate capacity than for the flat-top steels. All the above comparisons of ultimate capacities are made for $\varepsilon_u$ limited to 0.010 for both the spliced bars and the flat-top steels.

Ultimate curvatures of the section for the spliced bars as compared to those for the flat-top steels are slightly smaller at low levels of ultimate axial load, slightly greater at intermediate levels, and almost the same at high levels.

(c) Combination of sustained and short-time loadings

The stress-strain relationship for concrete used in the analyses for the combination of sustained and short-time loadings is intermediate between that for slow loading and short-time loading. Compared to the short-time loading curve for concrete, it helps in increasing the steel strains, stresses and forces and also the concrete force and its lever arm particularly at high levels of ultimate axial load. Consequently, greater ultimate capacity of the section is obtained at low levels of ultimate axial load for the spliced bars for the combination of sustained and short-time loading, as compared to the short-time loading for the spliced bars or for the flat-top steels. Also, at high levels of ultimate axial load, the reduction in the ultimate capacity for the spliced bars for short-time loading caused by the smaller compression steel stresses, as compared to the flat-top steels, is compensated for by the increase in the compression steel force and the concrete force, when the section is loaded under the combination of sustained and short-time loadings. Consequently, either the same or very slightly
greater ultimate capacity is obtained for the spliced bars, as compared to that obtained for the flat-top steel in accordance with the 1963 ACI Code. However, for the small value of $q_t$ obtained with $p_t = 0.01$, the ultimate capacity of the section for the spliced bars near the balance point is still smaller than that for the flat top steels but this difference is small—only 7 percent.
11. SUMMARY

11.1. Introduction

High-strength reinforcing bars of various types and grades are available in the U.S.A. and abroad. The stress-strain curves for these bars are quite different from the ideal elasto-plastic (flat-top) stress-strain relationship assumed in the ACI Code for the purpose of design of reinforced concrete sections. In this report, the effects of the stress-strain curves of typical high-strength reinforcing bars on the strength and behavior of reinforced concrete sections have been investigated for various combinations of axial load and bending moment.

Method of analysis: The method of analysis used for obtaining load-moment and load-curvature diagrams to study the effects of the stress-strain relationships of reinforcement consists of obtaining the moment-concrete strain relationship at a given level of axial load with ultimate concrete strain in the extreme compression fiber limited to 0.010. The maximum moment from the moment-strain diagram, or the moment at which buckling of compression bars occurs, whichever is smaller, is taken to be the ultimate moment. The ultimate curvature is determined from the strain distribution corresponding to the ultimate moment. Any shapes of stress-strain curves for the reinforcement and concrete can be used in the analysis.

Stress-strain curves for reinforcement: Four stress-strain curves for reinforcing bars of Grade 60 steel were selected for analyses in Chapter 7. These are steels 5, 6, 7 and 8 (Fig. 4.5). All of these steels meet the requirements of the ASTM specifications as well as those of the ACI Code for \( f_y = 60 \text{ ksi} \). The stress-strain curve for steel 5 has a short flat-plateau
and that for steel 6 a long one, followed by strain-hardening, while that for steel 7 has an infinitely long plateau (flat-top). Steel 8 has a round-house stress-strain curve more typical of European and British steels. Similarly, four stress-strain curves for steels 1, 2, 3 and 4 (Fig. 4.5) have been selected for analyses for ASTM Grade 75 steel in Chapter 8. Steels 1, 3 and 4 have a round-house stress-strain relationship, whereas steel 2 has an ideal elasto-plastic stress-strain relationship. All of these steels meet the ASTM specification, but only steels 1, 2 and 3 satisfy the provisions of the ACI Code for \( f_y = 75 \text{ ksi} \).

**Stress-strain curves for concrete:** Concrete stress-strain relationships selected for analyses for short-time loading, slow loading, and combination of sustained and short-time loading of the section are shown in Figs. 4.10, 9.1 and 9.13, respectively.

### 11.2. Comparisons of Analytical and Experimental Results

Analytical results obtained using the method of analysis explained in Chapter 3 have been compared in Chapter 6, with experimental results obtained from four available test programs, and good agreement was obtained between the analytical and experimental results. Since the method of analysis requires the use of actual or realistic representation of the actual stress-strain relationship for the reinforcement, the comparisons were limited to only those test programs in which the stress-strain curves for the reinforcement have been reported.

### 11.3. Effect of Variation of Strain in Extreme Compression Fiber

**Below the balance point.**

When a reinforced concrete section provided with any of the high-strength steels 1, 3, 4, 5, 6 and 8 or other similar steels is subjected to
a small axial load and a large bending moment, i.e., at ultimate axial load levels below the balance point, the moment continues to increase with increase in the strain in the extreme compression fiber \( \epsilon_4 \) up to a relatively large value of \( \epsilon_4 \). The general effects of increasing \( \epsilon_4 \) are: (1) to increase the compression and tension steel strains, and hence their stresses and forces, unless the strains are on the flat-plateau of the stress-strain curve; (2) to increase the curvature of the section; and (3) to reduce the lever arm of the concrete force and the average stress in concrete compression zone, if the value of \( \epsilon_4 \) is greater than \( \epsilon_m \) for which the average stress is maximum, and thus to reduce the concrete force. Generally, the tension steel strains are in the strain-hardening region, whereas the compression steel strains are in the elastic region. Therefore, with increase in \( \epsilon_4 \), the compression steel provides enough increase in the compression steel force to compensate for the reduction in the concrete force as well as to match the increase in the tension force. Thus, the total tension and compression forces are increased with increase in \( \epsilon_4 \). The moments of the tension and compression steel forces about the plastic centroid of the section increase, and more than compensate for the reduction in the moment of the concrete force. Consequently, the total moment increases with increase in \( \epsilon_4 \), and the limitation of \( \epsilon_4 \) to 0.003, as assumed in the ACI Code, results in a considerable reduction in the ultimate capacity and curvature of the section with high-strength steels of the type considered.

However, for the flat-top steels 2 and 7, the tension steel stress and hence the force remain constant. The increase in the compression steel force is just enough to compensate for the reduction in the concrete force, and the total tension and compression forces remain constant. The reductions
in the concrete force and its lever arm, with increase in $\varepsilon_4$, result in considerable reduction in the moment of the concrete force and hence in the total moment. Consequently, the maximum moment occurs at $\varepsilon_4 \leq \varepsilon_m$, and the limitation of $\varepsilon_u$ to 0.003 has little effect on the ultimate capacity of the section with flat-top steel.

Above the balance point.

At ultimate axial load levels above the balance point, the tension steel strains generally are in the elastic region, whereas the compression steel strains are either on the flat-plateau or in the strain-hardening region. Since the ultimate axial load is quite large, compression steel and/or concrete play a greater role in providing the ultimate capacity. For the Grade 60 steel with any length of flat-plateau, the compression steel stress and force remain constant and the compression steel does not compensate for the reduction in the concrete force for $\varepsilon_4 > \varepsilon_m$, and thus the total compression and tension forces are reduced which, together with the reduction in the lever arm of the concrete force, result in reduction in the total moment. The compression steel can compensate for the reduction in the concrete force only when the compression steel strains are in the strain-hardening region. But, this could be reached only at very high values of $\varepsilon_4$ for which the concrete force and its lever arm are reduced so much that the compression steel can not provide as much increase in the moment. Consequently, the maximum moment occurs at $\varepsilon_u \leq \varepsilon_m$, and there is little effect on the ultimate capacity of limiting $\varepsilon_u$ to 0.003.

However, for those steels which have round-house stress-strain curves, the compression steel strain, stress and force increase with increase in $\varepsilon_4$, and the extent to which the compression steel compensates for the
reduction in the concrete force depends on the relative magnitudes of the forces in the concrete and compression steel. For small percentages of steel area, the compression steel force is small and the increase in the compression steel force is too small to compensate for the reduction in the concrete force, and thus the ultimate moment occurs at small values of $\varepsilon_4$. For large percentages of steel, particularly for Grade 75 steel, the compression steel provides enough increase in force to compensate for the reduction in the concrete force as well as to provide additional compression force. Thus, the moment-strain curves continue to rise with increase in $\varepsilon_4$ and the maximum moment occurs at values of $\varepsilon_4$ considerably greater than 0.003. But, in this case, if the compression steel strain is high, buckling of compression bars may occur before the maximum moment is reached at $\varepsilon_4$ still greater than 0.003. Consequently, the limitation of $\varepsilon_u$ to 0.003 results in reduction in ultimate capacity of the section.

Also, for a round-house stress-strain curve, when the compression steel strain at ultimate capacity lies in that region of the stress-strain curve which is below that for the flat-top steel, smaller compression steel stress is obtained for the round-house steel than for the flat-top steel, and hence smaller ultimate moment is calculated when the actual stress-strain curve is used in the analysis. This conclusion does not apply to steel 1, the stress-strain curve of which never lies below that of the flat-top steel 2.

11.4. Effect of Variation of the Ratio $d'/t$

Yielding of the compression steel depends on the ratio $d'/t$, and on $P_u$, $\varepsilon_y$, and $\varepsilon_u$. If $\varepsilon_u = 0.003$, as assumed in the ACI Code, the compression
steel strain may be in the elastic region at lower levels of ultimate axial load above the balance point, particularly for small columns (large ratio of d'/t) and/or for Grade 75 steel. In this case, increase in $\epsilon_u$ increases the compression steel strain and hence the stress and the force. Thus, the ultimate moment is increased with increase in $\epsilon_u$ even for the flat-top steel.

11.5. Results of Analyses for Short-Time Loading

Grade 60 steel.

For short-time loading of the section with steels 5, 6 and 7, practically the same ultimate moments and ultimate curvatures are obtained at all levels of ultimate axial load, if $\epsilon_u$ is limited to 0.003, except for very small values of the ratio $q_t = \frac{P_t f_y}{f_c'}$ in combination with low levels of ultimate axial load, in which case the tension steel strains reach the strain-hardening region of the stress-strain curve, and greater ultimate capacity is calculated for steels 5 and 6 than for steel 7. But, for steel 8, considerable decrease in ultimate capacity is obtained for $\epsilon_u \leq 0.003$, as compared to that for steel 7. For $\epsilon_u \leq 0.010$, at ultimate axial load levels below the balance point, large increases in ultimate capacities and curvatures of the section are obtained for steels 5, 6 and 8 in comparison with steel 7. This increase in ultimate capacity decreases with increase in $P_u$. At ultimate axial load levels above the balance point, steels 5, 6 and 7 are all alike, but considerable decrease in ultimate capacity is obtained for steel 8 as compared to that for steel 7. At ultimate axial load levels between the balance point and $0.45 P_{07}$, since the compression steel does not yield for $\epsilon_u = 0.003$, allowing a higher limit on $\epsilon_u$ results in greater ultimate moment and curvature for steels 5, 6 and 7. For this reason, the ultimate axial load at balanced conditions decreases with increase in $P_c$. 
Grade 75 steel.

For short-time loading of the section with the flat-top steel 2, at ultimate axial load levels below the balance point, when the value of $q_t$ is such that the tension steel yields at $\varepsilon_u = 0.003$, little increase in ultimate capacity is obtained by allowing a higher limit on $\varepsilon_u$. But at lower levels of ultimate axial load above the balance point, the compression steel does not yield at $\varepsilon_u = 0.003$, and the ultimate capacity is considerably increased by allowing a higher limit on $\varepsilon_u$. This increase in ultimate capacity decreases with increase in $P_u$ but increases with increase in $q_t$. Also, when the ultimate moment occurs at $\varepsilon_u > 0.003$, ultimate curvature is considerably increased as compared to that at $\varepsilon_u = 0.003$.

For steel 1, when $\varepsilon_u$ is limited to 0.003, practically the same ultimate moments and curvatures are obtained as for steel 2 for all values of $q_t$ and at all levels of ultimate axial load, except for small values of $q_t$ in combination with low levels of ultimate axial load, in which case a small increase in ultimate capacity is accompanied by a very small decrease in ultimate curvature for steel 1. But with $\varepsilon_u \leq 0.010$ for steel 1, considerable increases in ultimate capacities and curvatures are obtained as compared to those for steel 2 with $\varepsilon_u \leq 0.003$ in accordance with the ACI Code. This relative increase in ultimate capacity decreases with increase in $P_u$ but increases with increase in $q_t$ up to a certain value of $q_t$ when the compression steel can provide the required increase in the compression force. The level of ultimate axial load up to which the increase in ultimate capacity is obtained for steel 1 increases with increase in $q_t$.

For steels 3 and 4, the stress-strain curves of which lie partly below and partly above that for steel 2, when $\varepsilon_u$ is limited to 0.003, greater
ultimate capacities are obtained as compared to steel 2 at low levels of $P_u$ in combination with small values of $q_t$. But, at higher levels of $P_u$, considerably smaller ultimate capacities are obtained for steels 3 and 4. Also, there is little effect on curvatures for these steels. But, for $\epsilon_u \leq 0.010$, considerable increase in ultimate capacities are obtained for steel 3 and 4 at lower levels of $P_u$, as compared to steel 2 with $\epsilon_u \leq 0.003$. Greater increase in ultimate capacity is obtained for steel 3 than for steel 4. At higher levels of $P_u$, although the ultimate capacities for steels 3 and 4 are increased by allowing a higher limit on $\epsilon_u$, they are still smaller than those for steel 2. The decrease in ultimate capacity is greater for steel 4 than for steel 3. The levels of $P_u$ above which the ultimate capacities for steels 3 and 4 are smaller than those for steel 2 increase with increase in $q_t$. The effects of the stress-strain curves for steels 3 and 4 on the ultimate curvature are similar to those for steel 1.

11.6. Results of Analyses for Slow Loading

When a reinforced concrete section is loaded slowly and continuously, creep and shrinkage of concrete occur during the loading process. The concrete stress-strain relationship used in analyses for slow loading is shown in Fig. 9.1. Such a stress-strain relationship results in large values of $\epsilon_u$ and hence in large values of compression and tension steel strains. Therefore, greater steel stresses are obtained for those steels which have a portion of their stress-strain curves above the conventional flat-plateau, and greater ultimate capacities are obtained for strain-hardening steels than for flat-top steels at all levels of $P_u$. However, the increase in ultimate capacity is much larger at lower levels of $P_u$, when both tension and compression steels provide an increase in ultimate capacity, than at higher
levels when only compression steel plays the major role in providing the ultimate capacity. This is true for steels of both Grades 60 and 75. Also, at high levels of $P_u$, those steels which have round-house stress-strain curves do not give smaller ultimate capacities than the flat-top steel since the compression steel strains are large enough to give compression steel stresses at least equal to $f_y$ for the flat-top steels. Since $\varepsilon_u$ is almost the same for strain-hardening steels and for the flat-top steels, practically the same ultimate curvatures are obtained for these steels except at low levels of $P_u$ when greater curvatures are obtained for the flat-top steels.

11.7. Results of Analyses for Combination of Sustained and Short-Time Loadings

When a given axial load $P$ and bending moment $M$ are applied in a short time and sustained over the section for any desired period of time, creep and shrinkage of concrete occur which increase the concrete strains but reduce the concrete stresses. If the section is then further loaded up to failure in a short time, the stress-strain relationship for concrete is intermediate between those for short-time loading and slow loading, as shown in Fig. 9.13. As a result, steel strains greater than those for short-time loading but smaller than those for slow loading are obtained. Unless the steel strains are on the flat-plateau, greater steel stresses are obtained for the combination of sustained and short-time loading than for short-time loading. Consequently, greater ultimate moments are obtained for those steels which have round-house stress-strain curves. At low levels of $P_u$, the increase in ultimate moment is very small. However, at high levels of $P_u$, the increase in ultimate moment is at least enough to compensate for the
reduction in ultimate moment obtained for these steels for short-time loading as compared to the flat-top steels. Of course, this increase in ultimate capacity depends on the period of sustained load.

11.8. Results of Analyses for Cadweld-Spliced Reinforcing Bars

Cadweld-spliced reinforcing bars have round-house stress-strain curves for the region of the bar containing the splice. Their stress-strain relationship depends on the gage length used for measuring the strains, and is different for the splice in compression and in tension. Typical conservative stress-strain curves for such bars over a gage length of 10 in. are shown in Figs. 10.1 and 10.2 for Grade 60 and 75 bars, respectively. Like other strain-hardening steels, the ultimate capacity of the section with Cadweld-spliced reinforcing bars for short-time loading, at low levels of $P_u$ below the balance point, is considerably greater than that for the flat-top steel of the same grade when the higher limit on $\epsilon_u$ up to 0.010 is permitted. At ultimate axial load levels near the balance point, the shapes of the stress-strain curves of these bars result in smaller ultimate capacity as compared to that for the flat-top steel. At higher load levels, very small reduction in ultimate capacity is obtained for the spliced bars. Limitation of $\epsilon_u$ to 0.003 results in considerable decrease in the ultimate capacity of the section for the spliced bars, particularly at lower levels of $P_u$. The effect on the ultimate curvatures of the section with spliced bars is similar to that with steels 8 and 4. Ultimate capacity of the section with spliced bars, as compared to that of the section with flat-top steel, is increased by loading the section slowly, or by using the combination of sustained and short-time loading. The analyses for slow loading of the
section in one year, and for a one-year period of sustained load, have shown that greater ultimate capacity is obtained for the spliced bars than for the flat-top steel at all levels of $P_u$ and for all values of $q_t$, except for the small value of $q_t$ in a very small region of the load-moment diagram near the balance point, in which case slightly smaller ultimate capacity is obtained for the spliced bars.
LIST OF REFERENCES

1. American Concrete Institute, "Building Code Requirements for Reinforced Concrete" (ACI-318-63).


16. American Concrete Institute, "Commentary on Building Code Requirements for Reinforced Concrete," (ACI-318-63).


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### TABLE 6.2

**COMPARISON OF ANALYTICAL RESULTS WITH GASTON'S TESTS (T-SERIES)**

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TABLE 6.3
COMPARISON OF ANALYTICAL RESULTS WITH HAJNOL-KONYI'S TESTS
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TABLE 6.5

COMPARISON OF ANALYTICAL RESULTS WITH TODESCHINI'S TESTS

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<th>$P_{calc.}$</th>
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TABLE 7.1
COMPARISON OF STRAINS, MOMENTS AND CURVATURES FOR STEELS
5, 6, 7 AND 8 AT $P_u = 0$ AND $\varepsilon_4 = 0.003$

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<th>Lever Arm</th>
<th>Moment $M$</th>
<th>Curvature $\phi$</th>
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<th>$\varepsilon_2$</th>
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<th>Lever Arm</th>
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TABLE 7.4
COMPARISON OF ULTIMATE MOMENTS AND ULTIMATE CURVATURES
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<td>0.0</td>
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<td>1.21 4.26</td>
<td>1.19 3.64</td>
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<td>1.16 4.42</td>
<td>1.16 3.74</td>
</tr>
<tr>
<td>0.12</td>
<td>0.05</td>
<td>1.18 2.76</td>
<td>1.11 2.82</td>
<td>1.14 2.80</td>
</tr>
<tr>
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<td>0.05</td>
<td>1.21 3.06</td>
<td>1.13 3.13</td>
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</tr>
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<td>1.11 3.57</td>
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<td>0.05</td>
<td>1.13 3.70</td>
<td>1.06 4.25</td>
<td>1.08 3.70</td>
</tr>
</tbody>
</table>

M_u5 = ultimate moment for steel 5
M_u6 = ultimate moment for steel 6
M_u7 = ultimate moment for steel 7
M_u8 = ultimate moment for steel 8
TABLE 7.7
COMPARISON OF ULTIMATE MOMENTS FOR STEELS 5 AND 7 WITH DIFFERENT VALUES OF $d'/t$, $p_t = 0.04, f'_c = 4000, q_t = 0.60$

<table>
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<tr>
<th>$P_j/P_{07}$</th>
<th>$P_u$</th>
<th>$d'/t = 0.15$</th>
<th>$M_{u5}$</th>
<th>$M_{u7}$</th>
<th>$M_{u5}$</th>
<th>$M_{u7}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>$p$</td>
<td>in.-kips</td>
<td>in.-kips</td>
<td>in.-kips</td>
<td>in.-kips</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2946</td>
<td>4093</td>
<td>1.39</td>
<td>2641</td>
<td>3528</td>
</tr>
<tr>
<td>0.079</td>
<td>100</td>
<td>3422</td>
<td>4041</td>
<td>1.18</td>
<td>3048</td>
<td>3501</td>
</tr>
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<td>200</td>
<td>3826</td>
<td>3983</td>
<td>1.04</td>
<td>3389</td>
<td>3548</td>
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</table>

BELOW BALANCE POINT FOR $d'/t = 0.20$

<table>
<thead>
<tr>
<th>$P_j/P_{07}$</th>
<th>$P_u$</th>
<th>$d'/t = 0.20$</th>
<th>$M_{u5}$</th>
<th>$M_{u7}$</th>
<th>$M_{u5}$</th>
<th>$M_{u7}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$p$</td>
<td>in.-kips</td>
<td>in.-kips</td>
<td>in.-kips</td>
<td>in.-kips</td>
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<tr>
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<td>4072</td>
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<td>3648</td>
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<tr>
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<td>3811</td>
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<td>3175</td>
<td>3372</td>
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</table>

$M_{u5} = $ ultimate moment for steel 5
$M_{u7} = $ ultimate moment for steel 7
TABLE 8.1

COMPARISON OF STRAINS, MOMENTS AND CURVATURES FOR STEELS 1, 3 AND 4 WITH THOSE FOR STEEL 2 
AT $P_u = 0$ AND $\epsilon_4 = 0.003$

<table>
<thead>
<tr>
<th>STEEL</th>
<th>$p_t$</th>
<th>$f'_c$</th>
<th>$q_t$</th>
<th>$\epsilon_3$</th>
<th>$\epsilon_2$</th>
<th>DEPTH TO N. A.</th>
<th>LEVER ARM</th>
<th>MOMENT M</th>
<th>CURVATURE $\phi$</th>
<th>$\frac{M}{M_2}$</th>
<th>$\frac{\phi}{\phi_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>psi</td>
<td>in.</td>
<td>in.</td>
<td></td>
<td></td>
<td>in.</td>
<td>in.</td>
<td>in.-kips</td>
<td>$10^{-6}$/in.</td>
<td></td>
<td></td>
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<tr>
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<td>0.01</td>
<td>5000</td>
<td>0.15</td>
<td>-0.00091</td>
<td>-0.01264</td>
<td>2.30</td>
<td>11.05</td>
<td>995</td>
<td>1304</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>0.01</td>
<td>5000</td>
<td>0.15</td>
<td>-0.00046</td>
<td>-0.01084</td>
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<td>10.92</td>
<td>1282</td>
<td>1153</td>
<td>1.29</td>
<td>0.88</td>
</tr>
<tr>
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<td>0.01</td>
<td>5000</td>
<td>0.15</td>
<td>-0.00049</td>
<td>-0.01096</td>
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<td>1261</td>
<td>1163</td>
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<td>0.89</td>
</tr>
<tr>
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<td>-0.01191</td>
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<td>11.00</td>
<td>1114</td>
<td>1243</td>
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</tr>
<tr>
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<td>4000</td>
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<td>-0.00438</td>
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<td>10.05</td>
<td>3228</td>
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<td>1.00</td>
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<td>9.88</td>
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<td>571</td>
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<td>0.93</td>
</tr>
<tr>
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<td>4000</td>
<td>0.75</td>
<td>0.00122</td>
<td>-0.00411</td>
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<td>9.97</td>
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<td>592</td>
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<tr>
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<td>643</td>
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<td>2.00</td>
<td>0.00167</td>
<td>-0.00233</td>
<td>6.75</td>
<td>9.23</td>
<td>5711</td>
<td>444</td>
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<td>1.00</td>
</tr>
<tr>
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<td>0.08</td>
<td>3000</td>
<td>2.00</td>
<td>0.00167</td>
<td>-0.00233</td>
<td>6.75</td>
<td>9.23</td>
<td>5711</td>
<td>444</td>
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<td>1.00</td>
</tr>
<tr>
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<td>2.00</td>
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$\ast$- is for tension

$M_{u2} =$ moment for steel 2, $\phi_2 =$ curvature for steel 2
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<th>$p_0$</th>
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<th>$\epsilon_3$</th>
<th>$\epsilon_2$</th>
<th>$\phi_u$</th>
<th>$M_u$</th>
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<th>LEVER</th>
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<td>0.0030</td>
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<td>995</td>
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</tr>
<tr>
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<td>3000</td>
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### TABLE 8.2 (continued)

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<th>$q_t$</th>
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<th>$\epsilon_3$</th>
<th>$\epsilon_2$</th>
<th>$\Phi_u$</th>
<th>$M_u$</th>
<th>DEPTH TO N. A.</th>
<th>LEVER ARM</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10$^{-6}$/in. in.-kips</td>
<td>in.</td>
</tr>
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<td>2.0</td>
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</table>

* - tension

for...
TABLE 8.3

COMPARISON OF ULTIMATE MOMENTS AND CURVATURES

FOR STEELS 1, 3 AND 4 WITH THOSE FOR STEEL 2

<table>
<thead>
<tr>
<th>$q_t$</th>
<th>$\frac{p_u}{p_0}$</th>
<th>$\mu_1$</th>
<th>$\phi_1$</th>
<th>$\mu_3$</th>
<th>$\phi_3$</th>
<th>$\mu_4$</th>
<th>$\phi_4$</th>
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<td>1.36</td>
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<td>3.90</td>
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<td>1.38</td>
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<td>1.20</td>
<td>3.93</td>
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</table>

$M_{u1} = \text{ultimate moment for steel 1}$

$M_{u2} = \text{ultimate moment for steel 2}$

$M_{u3} = \text{ultimate moment for steel 3}$

$M_{u4} = \text{ultimate moment for steel 4}$

$\phi_{u1} = \text{ultimate curvature for steel 1}$

$\phi_{u2} = \text{ultimate curvature for steel 2}$

$\phi_{u3} = \text{ultimate curvature for steel 3}$

$\phi_{u4} = \text{ultimate curvature for steel 4}$
TABLE 8.4

COMPARISON OF ULTIMATE MOMENTS AND CURVATURES FOR STEELS 1, 3 AND 4 WITH THOSE FOR STEEL 2
AT ULTIMATE AXIAL LOAD LEVELS ABOVE THE BALANCE POINT

<table>
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<th>STEEL 1</th>
<th>STEEL 3</th>
<th>STEEL 4</th>
</tr>
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<td>$P_u$</td>
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<tr>
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</tr>
<tr>
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<td>0.600</td>
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## Table 9.1

**Comparison of Strains, Moments and Curvatures for Steels 5, 7 and 8**

For slow test—low levels of ultimate axial load

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<th>STEEL</th>
<th>$p_t$</th>
<th>$f'_c$</th>
<th>$q_t$</th>
<th>$\varepsilon_u$</th>
<th>$\varepsilon_3$</th>
<th>$\varepsilon_2$</th>
<th>DEPTH TO N.A.</th>
<th>LEVER ARM</th>
<th>$M_u$</th>
<th>$\phi_u$</th>
<th>$\frac{M_u}{M_{u7}}$</th>
<th>$\frac{\phi_u}{\phi_{u7}}$</th>
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<td></td>
<td>psi</td>
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<td>5878</td>
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* is for tension

$\phi_u$ is the ratio of the ultimate moment to the yield moment.

$\phi_{u7}$ is the ratio of the ultimate moment to the yield moment at 70%. 

---

21.8
### TABLE 9.2
COMPARISON OF STRAINS, MOMENTS AND CURVATURES FOR STEELS 5, 7 AND 8
FOR SLOW TEST--HIGH LEVELS OF ULTIMATE AXIAL LOAD

<table>
<thead>
<tr>
<th>STEEL</th>
<th>P_t (kips)</th>
<th>P_u (kips)</th>
<th>P_u/P_07</th>
<th>ε_u</th>
<th>ε_3</th>
<th>ε_2</th>
<th>DEPTH TO N.A. (in.)</th>
<th>LEVER ARM (in.)</th>
<th>M_u (in.-kips)</th>
<th>φ_u</th>
<th>M_u/M_u7</th>
<th>φ_u/φ_u7</th>
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<td>0.786</td>
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<td>0.00170</td>
<td>15.56</td>
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<td>484</td>
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<td>1.00</td>
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<td>0.00164</td>
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<td>649</td>
<td>486</td>
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<td>5.95</td>
<td>622</td>
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<td>1.02</td>
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* is for tension
TABLE 9.3

COMPARISON OF ULTIMATE MOMENTS FOR STEELS 5, 7 AND 8 (SLOW TEST) WITH THOSE FOR STEEL 7 AS PER ACI CODE

<table>
<thead>
<tr>
<th>STEEL</th>
<th>P_u</th>
<th>P_t</th>
<th>q_t</th>
<th>CONC. CURVE</th>
<th>e_u</th>
<th>M_u (in.-kips)</th>
<th>M_u (ACI Code)</th>
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<tr>
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* $P_u = 0$ for tension
### TABLE 9.5

**COMPARISON OF STRAINS, MOMENTS AND CURVATURES FOR STEELS 1, 2 AND 3**

**FOR SLOW TEST-HIGH LEVELS OF ULTIMATE AXIAL LOAD**

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<th>$P_{u02}$</th>
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<th>$\varepsilon_3$</th>
<th>$\varepsilon_2$</th>
<th>DEPTH</th>
<th>LEVER</th>
<th>M</th>
<th>$\phi_u$</th>
<th>M</th>
<th>$\phi_u$</th>
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<td></td>
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<td>in.</td>
<td>in.-kips</td>
<td>N.A.</td>
<td>ARM</td>
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<td>10^-6/in.</td>
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<td>500</td>
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* is for tension
TABLE 9.6
COMPARISON OF ULTIMATE MOMENTS FOR STEELS 1, 2 AND 3 (SLOW TEST)
WITH THOSE FOR STEEL 2 AS PER ACI CODE

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TABLE 10.1
COMPARISON OF ULTIMATE MOMENTS AND CURVATURES FOR GRADE 60 CADWELD-SPICED BARS WITH FLAT-TOP STEEL (ACI CODE)

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### TABLE 10.2

COMPARISON OF ULTIMATE MOMENTS AND CURVATURES FOR GRADE 75 CADWELD-SPLICED BARS WITH FLAT-TOP STEEL (ACI CODE)

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### TABLE 10.3

**COMPARISON OF STRAINS, MOMENTS AND CURVATURES FOR GRADE 60 SPLICED BARS WITH THOSE FOR FLAT-TOP STEEL—SLOW LOADING**

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<td>4000</td>
<td>0.60</td>
<td>0.010</td>
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<td>2236</td>
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<td>0.79</td>
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</table>

$P_u = 400$ kips, $P_u / P_{u7} = 0.630$

| 7     | 0.01  | 3000  | 0.20  | 0.0078       | 0.00600       | 0.006         | 13.00        | 6.96      | 1010  | 600    | 1.00           | 1.00            |
| 9,10  | 0.01  | 3000  | 0.20  | 0.0080       | 0.00620       | 0.0080        | 13.32        | 7.04      | 1078  | 600    | 1.07           | 1.00            |

$P_u = 800$ kips, $P_u / P_{u7} = 0.674$

| 7     | 0.04  | 4000  | 0.60  | 0.0078       | 0.00600       | 0.00079       | 13.35        | 6.74      | 1704  | 580    | 1.00           | 1.00            |
| 9,10  | 0.04  | 4000  | 0.60  | 0.0078       | 0.00597       | 0.00644       | 13.07        | 6.85      | 1950  | 598    | 1.14           | 1.03            |

$P_u = 1200$ kips, $P_u / P_{u7} = 0.646$

| 7     | 0.08  | 5000  | 0.96  | 0.0078       | 0.00597       | 0.00061       | 13.10        | 6.84      | 2880  | 594    | 1.00           | 1.00            |
| 9,10  | 0.08  | 5000  | 0.96  | 0.0078       | 0.00592       | 0.00043       | 12.80        | 6.94      | 3370  | 607    | 1.17           | 1.02            |

*—is for tension
### TABLE 10.4

COMPARISON OF ULTIMATE MOMENTS FOR SPLICED BARS (9,10) AND STEEL 7 FOR SLOW LOADING WITH THOSE FOR STEEL 7 AS PER ACI CODE

<table>
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<tr>
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<th>$q_t$</th>
<th>CONC. CURVE</th>
<th>$e_u$</th>
<th>$M_u$</th>
<th>$M_u$ (ACI Code)</th>
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<tr>
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## TABLE 10.5

**COMPARISON OF STRAINS, CURVATURES AND MOMENTS FOR GRADE 75 SPLICED BARS WITH THOSE FOR FLAT-TOP STEEL - SLOW LOADING**

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<th>$\varepsilon_2$</th>
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<th>LEVER ARM</th>
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**P_u = 0**

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<td>0.25</td>
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**P_u = 500 kips = 0.746 P_02**

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<td>0.00600</td>
<td>0.00149</td>
<td>14.96</td>
<td>6.17</td>
<td>500</td>
<td>750</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>3000</td>
<td>0.25</td>
<td>0.0075</td>
<td>0.00599</td>
<td>0.00145</td>
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<td>6.21</td>
<td>504</td>
<td>832</td>
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**P_u = 1000 kips = 0.757 P_02**

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<td>0.0075</td>
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<td>0.00145</td>
<td>14.84</td>
<td>6.21</td>
<td>505</td>
<td>1440</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
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**P_u = 1500 kips = 0.707 P_02**

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<td>3366</td>
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*is for tension
TABLE 10.6
COMPARISON OF ULTIMATE MOMENTS FOR GRADE 75 SPLICED BARS (11, 12) AND STEEL 2 FOR SLOW LOADING WITH THOSE FOR STEEL 2 AS PER ACI CODE

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<td>11,12</td>
<td>1000</td>
<td>0.04</td>
<td>0.75</td>
<td>B</td>
<td>0.0075</td>
<td>1736</td>
<td>0.99</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>0.08</td>
<td>1.20</td>
<td>A</td>
<td>0.0030</td>
<td>3067</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1500</td>
<td>0.08</td>
<td>1.20</td>
<td>B</td>
<td>0.0075</td>
<td>2784</td>
<td>0.91</td>
</tr>
<tr>
<td>11,12</td>
<td>1500</td>
<td>0.08</td>
<td>1.20</td>
<td>B</td>
<td>0.0078</td>
<td>3366</td>
<td>1.09</td>
</tr>
</tbody>
</table>
STRESS-STRAIN CURVES FOR STEELS SELECTED FOR ANALYSES
\[ P_u = P_c + P_{sc} + P_{st} \]
\[ M = P_c \left( \frac{t}{2} - k_2 kd \right) + P_{sc} \left( \frac{t}{2} - d' \right) + P_{st} \left( d - \frac{t}{2} \right) \]
\[ P_c = k_1 k_f f_c bkd \]
\[ P_{sc} = A_s f' = A_{sc} f_{sc} \]
\[ P_{st} = A_f = A_{st} f_{st} \]

p,c.: plastic centroid

FIG. 3.1 EQUILIBRIUM CONDITIONS
FIG. 3.2 BUCKLING OF COMPRESSION BARS
SUBROUTINE 'DATA' READS AND PRINTS THE STRESS AND STRAIN VALUES FOR STEEL AND CONCRETE

SUBROUTINE 'ASTSS' ARRANGES STRESS-STRAIN VALUES IN THE ASCENDING ORDER OF STRAIN

SUBROUTINE 'SPC' CALCULATES STRESS FOR ANY STRAIN IN STEEL OR CONCRETE

SUBROUTINE 'PHIM' OBTAINS MOMENT-STRAIN RELATIONSHIP

1) ASSUME CURVATURE AND CALCULATE STRAINS
2) CALL AFA
3) CHECK EQUILIBRIUM
PRINT OUTPUT

SUBROUTINE 'AFA' CALCULATES FORCES AND MOMENT

1) CALL SPC FOR STRESSES IN STEEL AND CONCRETE
2) CALL FORCE
3) CALL MOMENT

SUBROUTINE 'FORCE' CALCULATES CONCRETE FORCE

SUBROUTINE MOMENT CALCULATES MOMENT OF CONCRETE FORCE

CALL DATA
CALL ASTSS
CALL SPC
CALL PHIM
PRINT $u, \epsilon, P$, AND $M$

ALL LOAD LEVELS ?

YES

ALL INTERACTION ?

YES

OUT

NO

FIG. 3.3 BLOCK DIAGRAM FOR COMPUTER PROGRAM
FIG. 4.1 STRESS-STRAIN CURVES FOR ASTM GRADE 60 STEEL OBTAINED FROM TESTS
FIG. 4.2 STRESS-STRAIN CURVES FOR ASTM GRADE 75 STEEL OBTAINED FROM TESTS
FIG. 4.3 STRESS-STRAIN CURVES FOR EUROPEAN AND BRITISH STEELS OBTAINED FROM TESTS

1. Hajnal-Konyi (21)
2. Evans (9)
3. Sorets (21)
4. Hajnal-Konyi (20)
FIG. 4.4 STRESS-STRAIN CURVES FOR STEEL ASSUMED IN THE ACI CODE
FIG. 4.5 STRESS-STRAIN CURVES FOR GRADE 60 STEELS SELECTED FOR ANALYSES
FIG. 4.6 STRESS-STRAIN CURVES FOR GRADE 75 STEELS SELECTED FOR ANALYSES
FIG. 4.7 STRESS-STRAIN CURVES FOR CONCRETE OBTAINED FROM TESTS
FIG. 4.8 STRESS-STRAIN CURVES FOR CONCRETE PROPOSED BY VARIOUS AUTHORS FOR $f'_c = 4000$ psi
FIG. 4.9 EQUIVALENT RECTANGULAR CONCRETE STRESS BLOCK ASSUMED IN THE ACI CODE
Fig. 4.10 Comparison of rectangular concrete stress block of ACI code with other proposed curves for $f'_c = 4000$ psi.

- **a**: Hognestad (3)
- **b**: Todeschini (7)
- **c**: ACI Code (1)
- **d**: Curve with $k_1 = 0.348$ and $k_2 = 0.414$

**Note:**

- $E_c = 5.6 \times 10^6$ psi
- $E_c = 3.4 \times 10^6$ psi
TODESCHINI'S EQUATION

\[ f_c = \frac{2f'_c (\epsilon_c/\epsilon_o)}{1 + 2(\epsilon_c/\epsilon_o)^2} \]

\( f'_c = 5000 \text{ psi} \)

**Fig. 4.11** Concrete stress-strain curves "A" used for analyses for short-time loading.
FIG. 4.12 VALUES OF COEFFICIENTS $k_2$ AND $k_1k_3$ FOR CONCRETE CURVE "A"
FIG. 5.1 MOMENT - STRAIN ($M - \varepsilon_4$) DIAGRAMS
FIG. 5.2  MOMENT-STRAIN DIAGRAMS AND CONCRETE STRESS-STRAIN CURVE
FIG. 5.3 A TYPICAL INTERACTION DIAGRAM

STEEL CURVE 5
CONCRETE CURVE A

$A_s = A_s' = 4.68 \text{ in.}^2$

$p_t = 0.0416$

$f = 60 \text{ ksi}$

$f_c' = 4000 \text{ psi}$
FIG. 5.4 LOAD-CURVATURE DIAGRAM AT ULTIMATE
FIG. 5.5 STRAIN DISTRIBUTIONS FOR $M - \varepsilon_4$ DIAGRAM AT $P_u = 200$ kips.
(BELOW THE BALANCE POINT)
FIG. 5.6 STRAIN DISTRIBUTIONS FOR $M - \varepsilon$ DIAGRAM AT $P = 500$ kips.
(ABOVE THE BALANCE POINT)
FIG. 5.7 STEEL AND CONCRETE STRAINS AND STRESSES AT VARIOUS LEVELS OF ULTIMATE AXIAL LOAD
FIG. 6.1 COMPARISON OF CALCULATED AND MEASURED STEEL STRAINS FOR GASTON'S TESTS
FIG. 6.2 COMPARISON OF CALCULATED AND MEASURED STEEL STRAINS FOR GASTON'S TESTS

The diagrams show the comparison of calculated and measured steel strains for Gaston's tests. The graphs plot total load (kips) against tension steel strain at mid-span for two different beams.

For Beam C3ynb:
- $M_u$ (TEST) = 74.2 k-ft
- $M_u$ (COMP.) = 74.4 k-ft
- $p' = 0.0193$
- $p = 0.0322$
- $f'_c = 4860$ psi
- $f_y = 42,100$ psi
- $d = 10.37''$

For Beam T11b:
- $M_u$ (TEST) = 19.4 k-ft
- $M_u$ (COMP.) = 19.7 k-ft
- $p' = 0$
- $p = 0.0062$
- $f'_c = 2520$ psi
- $f_y = 46,000$ psi
- $d = 10.72''$

The strain at mid-span is given by $\varepsilon_u = 0.01$ for Beam C3ynb and $\varepsilon_u = 0.007$ for Beam T11b.
FIG. 7.1 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 7.2 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 8 WITH $\epsilon_u$ LIMITED TO 0.003 AND 0.010

- Steel 7, $\epsilon_u \leq 0.003$ and $\epsilon_u \leq 0.010$
- Steel 8, $\epsilon_u \leq 0.003$ and $\epsilon_u \leq 0.010$

- $P_t = 0.010$
- $f_c = 5000 \text{ psi}$
- $q_t = 0.120$
FIG. 7.3 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 WITH $\epsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 7.4 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 7.5 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 WITH $\epsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 7.6 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 7.7 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
Fig. 7.8 Comparisons of strains and stresses for steels 5, 6, 7 and 8 with $\epsilon_u = 0.003$ at $P_u = 0$
FIG. 7.9 COMPARISONS OF STRAINS AND STRESSES FOR STEELS 5, 6, 7 AND 8 WITH $\epsilon_u = 0.003$ AT $P_u > P_b$
FIG. 7.10 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 6, 7 AND 8

STEEL

7
5
8
6

$p_t = 0.01$
$f_c = 5000 \text{ psi}$
$q_t = 0.12$

$P_u/P_07$

$M_u/M_{07}$

$1.0$

$0.9$

$0.8$

$0.7$

$0.6$

$0.5$

$0.4$

$0.3$

$0.2$

$0.1$

$0.0$

$0.2$

$0.4$

$0.6$

$0.8$

$1.0$

$1.2$

$1.4$

$1.6$

$1.8$

$2.0$

$2.2$

$2.4$

$2.6$

$2.8$

$3.0$

$3.2$

$3.4$

$0$

$0.2$

$0.4$

$0.6$

$0.8$

$1.0$

$1.2$

$1.4$

$1.6$

$1.8$

$2.0$

$2.2$

$2.4$

$2.6$

$2.8$

$3.0$

$3.2$

$3.4$
FIG. 7.11 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 6, 7 AND 8
FIG. 7.12 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 6, 7 AND 8
FIG. 7.13 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 6, 7 AND 8
FIG. 7.14 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 6, 7 AND 8

$P_t = 0.08$
$f_c = 3000 \text{ psi}$
$q_t = 1.60$

$P_t = 0.08$
$f_c = 5000 \text{ psi}$
$q_t = 0.96$
FIG. 7.15 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 5, 6, 7 AND 8
Fig. 7.16 Comparison of load-curvature diagrams for steels 5, 6, 7 and 8.

- $p_t = 0.04$
- $f'_t = 4000$ psi
- $q_t = 0.60$

- $p_t = 0.04$
- $f'_t = 5000$ psi
- $q_t = 0.48$
FIG. 7.17 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 5, 6, 7 AND 8

\[ p_t = 0.04 \]
\[ f_c' = 3000 \text{ psi} \]
\[ q_t = 0.80 \]
FIG. 7.18 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 5, 6, 7 AND 8
FIG. 7.19 MOMENT-STRAIN (M - $\varepsilon_4$) DIAGRAMS FOR STEELS 5, 6, 7 AND 8
AT $P_u = 0$

- $p_t = 0.01$
- $f'_c = 5000$ psi
- $q'_t = 0.12$

- $p_t = 0.01$
- $f'_c = 3000$ psi
- $q'_t = 0.20$
FIG. 7.20 MOMENT-STRAIN \((M - \varepsilon_4)\) DIAGRAMS FOR STEELS 5, 6, 7 AND 8 AT \(P_u = 0\)
FIG. 7.21  MOMENT-STRAIN (M - ε₄) DIAGRAMS FOR STEELS 5, 6, 7 AND 8 AT Pᵤ = 0
FIG. 7.22  MOMENT-STRAIN ($M - \epsilon_u$) DIAGRAMS FOR STEELS 5, 6, 7 AND 8
AT HIGH LEVELS OF $P_u$
$p_t = 0.08$
$f'_c = 5000 \text{ psi}$
$q_t = 0.96$

$\rho_t = 0.74$
$f'_c = 3000 \text{ psi}$
$q_t = 1.60$

**FIG. 7.21** MOMENT-STRAIN ($M - \epsilon_4$) DIAGRAMS FOR STEELS 5, 6, 7 AND 8 AT $P_u = 0$
FIG. 7.22  MOMENT-STRAIN (M - $\varepsilon_4$) DIAGRAMS FOR STEELS 5, 6, 7 AND 8
AT HIGH LEVELS OF $P_u$
FIG. 7.23  MOMENT STRAIN (M-ε₄) DIAGRAMS FOR STEELS 5, 6, 7 AND 8
AT HIGH LEVELS OF $P_u$
FIG. 7.24 MOMENT-STRAIN (M-ε₄) DIAGRAMS FOR STEELS 5, 6, 7 AND 8 AT HIGH LEVELS OF P_u
FIG. 7.25 STRAINS AND STRESSES AT ULTIMATE FOR STEELS 5, 6, 7 AND 8
Fig. 7.26 Strains and Stresses at Ultimate for Steels 5, 6, 7 and 8

\[ p_t = 0.04 \]
FIG. 7.27 STRAINS AND STRESSES AT ULTIMATE FOR STEELS 5, 6, 7 AND 8
FIG. 7.28 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5 WITH DIFFERENT VALUES OF $d'/t$
FIG. 7.29 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5 WITH DIFFERENT VALUES OF $d'/t$
FIG. 8.1 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 1 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
FIG. 8.2 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 1 WITH $\epsilon_u$
LIMITED TO 0.003 AND 0.010
FIG. 8.3 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEEL 2 AND 1 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
FIG. 8.4 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 3 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
FIG. 8.5 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 3 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
FIG. 8.6 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 3 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 8.7 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 4 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010

$P_t = 0.01$
$f'c = 5000$ psi
$q_t = 0.15$
FIG. 8.8 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 2 AND 4 WITH $\varepsilon_u$

LIMITED TO 0.003 AND 0.010
Fig. 8.9 Comparison of load-moment diagrams for steels 2 and 4 with $\epsilon_u$ limited to 0.003 and 0.010.
FIG. 8.10 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 2 AND 1 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
Fig. 8.11 Comparison of load-curvature diagrams for steels 2 and 1 with $\varepsilon_u$ limited to 0.003 and 0.010.
FIG. 8.12 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 2 AND 3 WITH $\varepsilon_u$
LIMITED TO 0.003 AND 0.010
Fig. 8.13 Comparison of Load-Curvature Diagrams for Steels 2 and 3 with $\varepsilon_u$ limited to 0.003 and 0.010

- $P_t = 0.04$
- $f'_c = 4000 \text{ psi}$
- $q_t = 0.75$

- $P_t = 0.08$
- $f'_c = 5000 \text{ psi}$
- $q_t = 1.20$

- $P_t = 0.08$
- $f'_c = 3000$
- $q_t = 2.00$
FIG. 8.14 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 2 AND 4 WITH $\varepsilon_u$ LIMITED TO 0.003 AND 0.010
FIG. 8.15 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 2 AND 4 WITH \( \varepsilon_u \) LIMITED TO 0.003 AND 0.010
FIG. 8.16 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 1, 2, 3 AND 4

STEEL

$P_t = 0.01$

$f_c = 5000 \text{ psi}$

$q_t = 0.15$
FIG. 8.17 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 1, 2, 3 AND 4
FIG. 8.18 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 1, 2, 3 AND 4
FIG. 8.19 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 1, 2, 3 AND 4

$P_0 = 0.01$
$f_c = 5000 \text{ psi}$
$q_t = 0.15$

$P_u = 0.01$
$f_c = 3000 \text{ psi}$
$q_t = 0.25$
FIG. 8.20 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 1, 2, 3 AND 4
FIG. 8.21  MOMENT-STRAIN \( (M - \varepsilon_4) \) DIAGRAMS FOR STEELS 1, 2, 3 AND 4 AT \( P_u = 0 \)
FIG. 8.22  MOMENT STRAIN ($M - \epsilon_4$) DIAGRAMS FOR STEELS 1, 2, 3 AND 4
AT $P_u = 0$

$p_t = 0.04$
$f_c' = 4000$ psi
$q_t = 0.75$
FIG. 8.23  MOMENT-STRAIN ($M - \epsilon_4$) DIAGRAMS FOR STEELS 1, 2, 3 AND 4 AT $P_u = 0$
FIG. 8.24 MOMENT-STRAIN (M - $\epsilon_4$) DIAGRAMS FOR STEELS 1, 2, 3 AND 4

AT HIGH LEVELS of $P_u$
FIG. 8.25 MOMENT-STRAIN $(M - \epsilon_4)$ DIAGRAMS FOR STEELS 1, 2, 3 AND 4 AT HIGH LEVELS OF $P_u$

$P_u = 0.287 P_{02}$

$P_u = 0.574 P_{02}$

$P_t = 0.04$

$f'_c = 4000$ psi

$q_t = 0.75$
FIG. 8.26 MOMENT-STRAIN (M - \( \varepsilon_4 \)) DIAGRAMS FOR STEELS 1, 2, 3 AND 4 AT HIGH LEVELS OF \( P_u \)
FIG. 8.27 STRAINS AND STRESSES AT ULTIMATE FOR STEELS 1, 2, 3 AND 4 AT $P_u = 0$ AND $e_4 = 0.003$
FIG. 8.28 STRAINS AND STRESSES AT ULTIMATE FOR STEELS 1, 2, 3 AND 4
$f' = 3000 \text{ psi}$

$\epsilon_0 = 0.006$

$f_c = 1 + 0.8 (\epsilon_c / \epsilon_0)^2$

$k_1k_3$ and $k_2$

**FIG. 9.1** CONCRETE STRESS-STRAIN CURVE "B" FOR SLOW LOADING, AND COEFFICIENTS $k_1k_3$ AND $k_2$
FIG. 9.2 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 5, 7 AND 8 - SLOW LOADING

STEEL

\[ \frac{P_t}{P_{07}} \]

\[ \frac{f_c}{3000 \text{ psi}} \]

\[ q_t = 0.20 \]
Fig. 9.3 Comparison of load-moment diagrams for steels 5, 7, and 8 - slow loading.
FIG. 9.4 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 5, 7 AND 8 - SLOW LOADING
FIG. 9.5 MOTIENT-STRAIN (M - ε₄) DIAGRAMS FOR STEELS 5, 7 AND 8 - SLOW LOADING

STEEL 5

STEEL 7

STEEL 8

M, in.-kips

ε₄ 10⁻³

Pₜ = 0.01

fᵣ = 3000 psi

qₖ = 0.20

Pₚ = 500 kips

= 0.786 P₀7
FIG. 9.6  MOMENT-STRAIN \((M - \epsilon_4)\) DIAGRAMS FOR STEELS 5, 7 AND 8 - SLOW LOADING
FIG. 9.7 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 1, 2 AND 3 - SLOW LOADING
FIG. 9.8 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEEL 1, 2, AND 3 - SLOW LOADING
FIG. 9.9 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEEL 1, 2 AND 3 - SLOW LOADING
FIG. 9.10 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR STEELS 1, 2, AND 3 - SLOW LOADING

STEEL 2
STEEL 1
STEEL 3

$P_t = 0.08$
$f_c = 3000 \text{ psi}$
$q_t = 2.00$

$\frac{P_u}{P_o2}$
$\phi_u \times 10^{-4}/\text{in.}$
FIG. 9.11 MOMENT-STRAIN (M - \( \epsilon_4 \)) DIAGRAMS FOR STEELS 1, 2 AND 3 - SLOW LOADING
FIG. 9.12 MOMENT-STRAIN (M-$$\varepsilon_4$$) DIAGRAMS FOR STEELS 1, 2 AND 3 - SLOW LOADING
FIG. 9.13 CONCRETE STRESS-STRAIN RELATIONSHIP FOR COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS
FIG. 9.14 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS

**ACI CODE**

1. SHORT-TIME LOADING
2. SUSTAINED LOADING
3. SUSTAINED AND SHORT-TIME LOADINGS

**Graphical Representation**

- Key:
  - 1: SHORT-TIME LOADING
  - 2: SUSTAINED LOADING
  - 3: COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS

- Parameters:
  - $p_t = 0.01$
  - $f_c = 3000$ psi
  - $q_t = 0.20$
FIG. 9.15 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 8 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADING

- ACI CODE
- SHORT-TIME LOADING
- SUSTAINED LOADING
- SUSTAINED AND SHORT-TIME LOADING

$P_c = 0.01$
$f_c' = 3000$ psi
$q_f = 0.20$
FIG. 9.16 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR STEELS 7 AND 5, AND 7 AND 8 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS.
Fig. 9.17 Comparison of load-moment diagrams for steels 2 and 3 -- combination of sustained and short-time loadings.

- ACI Code
- Short-time loading
- Sustained loading
- Sustained and short-time loadings

$p_t = 0.01$
$f_c' = 3000$ psi
$q_t = 0.25$

$p_t = 0.08$
$f_c' = 3000$ psi
$q_t = 2.00$
FIG. 10.1 STRESS-STRAIN CURVES FOR CADWELD-SPLICED BARS OF GRADE 60 STEEL - 10 IN. GAGE LENGTH.
FIG. 10.2 STRESS-STRAIN CURVES FOR CADWELD-SPLICED BARS OF GRADE 75 STEEL - 10 IN. GAGE LENGTH
FIG. 10.3 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SHORT-TIME LOADING
FIG. 10.4 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7—SHORT-TIME LOADING
FIG. 10.5 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7—SHORT-TIME LOADING

\( P_u/P_{07} \)

\( \varepsilon_u \leq 0.003 \quad \leq 0.010 \)

STEEL 7

(a) b

(9,10)

(c) d

\( t_e = 0.01 \)

\( f_t = 3000 \) psi

\( q_t = 0.20 \)
FIG. 10.6 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SHORT-TIME LOADING

\( \varepsilon_u \leq 0.003 \leq 0.010 \)

STEEL 7

a, b

(9,10)

c, d

\( p_t = 0.04 \)

\( f'_c = 4000 \text{ psi} \)

\( q_t = 0.60 \)

\( p_t = 0.08 \)

\( f'_c = 5000 \text{ psi} \)

\( q_t = 0.96 \)
FIG. 10.7 MOMENT-STRAIN \((M - \epsilon_4)\) DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7
SHORT-TIME LOADING
FIG. 10.8 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2--
SHORT TIME LOADING

STEEL 2

\(\epsilon_u \leq 0.003\) \(\leq 0.010\)

\(p_t = 0.010\)

\(f_c = 3000\) psi

\(q_t = 0.25\)
FIG. 10.9 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 - SHORT-TIME LOADING
FIG. 10.10 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2—SHORT-TIME LOADING
\[
\varepsilon_u \leq 0.003 \quad \leq 0.010
\]

STEEL 2 \( a \) \( b \)

(11,12) \( c \) \( d \)

\( \frac{P}{P_{0.2}} = 0.08 \)
\( f'_c = 3000 \text{ psi} \)
\( q_t = 1.20 \)

\( \frac{P}{P_{0.2}} = 0.04 \)
\( f'_c = 4000 \text{ psi} \)
\( q_t = 0.75 \)

**Fig. 10.11** Comparison of load-curvature diagrams for spliced bars (11,12) and steel 2 - short-time loading
FIG. 10.12 MOMENT-STRAIN ($M - \varepsilon_4$) DIAGRAMS FOR SPLICED BARS (11, 12) AND STEEL 2—SHORT-TIME LOADING
FIG. 10.13 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SLOW LOADING

$P_t = 0.01$
$f'_c = 3000$ psi
$q_t = 0.20$
FIG. 10.14 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SLOW LOADING
FIG. 10.15 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SLOW LOADING
FIG. 10.16 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 - SLOW LOADING

STEEL (9,12)

$P_t = 0.04$

$f_c = 4000$ psi

$q_t = 0.60$

STEEL (9,10)

$P_t = 0.08$

$f_c = 5000$ psi

$q_t = 0.96$
FIG. 10.17 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 - SLOW LOADING

- $P_t = 0.01$
- $f'_c = 3000$ psi
- $q_t = 0.20$
FIG. 10.18 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 - SLOW LOADING
FIG. 10.19 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 - SLOW LOADING

STEEL (11,12)

$P_t = 0.01$

$f_c = 3000 \text{ psi}$

$q_t = 0.25$
FIG. 10.20 COMPARISON OF LOAD-CURVATURE DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 — SLOW LOADING

STEEL (11,12)

- $P_t = 0.04$
- $f_t = 4000$ psi
- $q_t = 0.75$

STEEL (11,12)

- $P_t = 0.08$
- $f_t = 5000$ psi
- $q_t = 1.20$
FIG. 10.21 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9,10) AND STEEL 7 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS

ACI CODE

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<td>4</td>
<td>Sustained and short-time loadings</td>
</tr>
</tbody>
</table>

Graphical representation showing load-moment diagrams with

- $p_t = 0.01$
- $f'_c = 3000$ psi
- $q_t = 0.20$
FIG. 10.22 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (9, 10) AND STEEL 7 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS
FIG. 10.23 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS
FIG. 10.24 COMPARISON OF LOAD-MOMENT DIAGRAMS FOR SPLICED BARS (11,12) AND STEEL 2 -- COMBINATION OF SUSTAINED AND SHORT-TIME LOADINGS

1. ACI CODE
2. SHORT-TIME LOADING
3. SUSTAINED LOADING
4. SUSTAINED AND SHORT-TIME LOADINGS

\[ \frac{P}{P_0} = 0.04 \]
\[ f_c' = 4000 \text{ psi} \]
\[ q_t = 0.75 \]

\[ \frac{P}{P_0} = 0.08 \]
\[ f_c' = 5000 \text{ psi} \]
\[ q_t = 1.20 \]