A LUMPED-PARAMETER MODEL TO SIMULATE THE RESPONSE OF REINFORCED CONCRETE FRAMES WITH FILLER WALLS

by
J. P. FEDORKIW
and
M. A. SOZEN

A Report to the
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OFFICE OF THE SECRETARY OF THE ARMY
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Contract DAHC 20-67-C-0136
Subcontract 12472 (6300 A-030) US
OCD Work Unit 1127D

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University of Illinois
Urbana, Illinois
June, 1968
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1. INTRODUCTION

1.1 Object and Scope

Experimental evidence on the behavior of frames subjected to lateral loading and containing a filler of a tension-weak material has indicated great differences in ultimate load and energy absorption capacity as distinguished from similar unfilled frames. This work is aimed toward a systematic evaluation of the energy capacity of filled frames by developing an analytical procedure for the determination of load-deflection response and crack formation in frame and filler.

The behavioral characteristics which are of prime importance are concerned with the energy absorbing capacities of the structure, and a clarification of the mode of failure. It was observed from tests which were available prior to the present investigation, that the inclusion of filler walls in an otherwise conventional structural frame had a drastic effect in altering the manner in which the frame resisted horizontal loads. From an analytical standpoint in engineering practice, the analysis of the coupled frame and shear-wall system has been limited to a superposition of the separate capacities of the two load-carrying systems. It has been desirable to verify this behavioral assumption experimentally and
analytically, and to derive a better understanding of the available resistance in the filled-frame structure so that the potentialities of this system, which is in such widespread use, may be more fully realized.

The analytical study has been based on results which were obtained from an analytical model. The primary advantage of the analytical technique is that it allows solutions to be readily obtained for a range of different structural properties. Experimental techniques are necessarily limited by time and expense in a comprehensive definition of behavior. The analytical approach can be used most advantageously to verify and to clarify the experimental results, and to generalize the behavioral phenomena.

The analytical model was chosen to avoid the introduction of presumptive assumptions regarding over-all behavior and thus yields solutions which are nonderivative in nature. It is essentially a discrete physical model which reduces the solution for a structural system with an infinite number of degrees of freedom to one with a finite number of degrees of freedom. The model behavior is governed by the laws of particle mechanics which allow straightforward treatment of partial loadings and complex boundary conditions. The model has been successfully used in static and dynamic behavioral studies for a wide range of problems in plane and solid continua.
For the requirements of the present study, the model has been modified to handle the existence of cracking within the structure. This required the assumption of a criterion to define the conditions under which cracking takes place in localized regions of the structure. Solution of the problem has been coded for the IBM 7094 digital computer.

1.2 Nomenclature and Notation

References to 'stress' within the text are to be considered synonymous with the concept of 'force'. The symbolic form: \( \max[\ ] \) represents the maximum value of the set of values contained within the brackets. Symbols are defined where they first appear, and are summarized herein for convenience.

- \( x, y \) directions of axes
- \( u, v \) displacement components at the same mass point in the analytical model in the \( x, y \) directions respectively with units of length
- \( E_w \) elastic modulus of deformation for the filler material in units of force per unit area
- \( E_f \) elastic modulus of deformation for concrete in the frame in units of force per unit area
- \( \nu \) Poisson's ratio
- \( I \) moment of inertia of frame cross section
A
i,j
X$_{ij}$,Y$_{ij}$
$\bar{X}, \bar{Y}$
$\bar{S}_A$
$\bar{S}_S$
$\bar{S}_F$
$\bar{S}_E$
F$_x$,F$_y$
S
$\sigma_x, \sigma_y$
t
h
$\lambda$
$\Delta_k$
area of frame cross section
indices for numbering purposes in the x,y directions, respectively
body forces applied to the model at mass point 'ij' in the x,y directions respectively with units of force
components of body force per unit area
stiffness of axial spring in units of force per unit length
stiffness of shear spring in units of force per unit length
stiffness of flexural spring in units of force per unit length
stiffness of extensional spring in units of force per unit length
axial force components at a stress point in the x,y directions
shear force component at a stress point
unit normal stress components in the filler
unit shear stress component in the filler
thickness of the filler in units of length
mesh size in the model in units of length
lateral displacement of the kth story in units of length
\( p \) steel reinforcement in the frame as a percentage of the gross area of the cross section

\( P \) magnitude of the influence loads applied to the structure in the analysis with units of force

\( c^k \) vector quantity containing the generalized displacement components as obtained in the kth solution

\( \mathbf{c}^k \) symbolic notation for the set of stresses which exist in the entire structure for the kth solution

\( \mathbf{F}^k \) symbolic notation for the set of principal stresses within the filler for the kth solution, being a subset of \( \mathbf{c}^k \)

\( \mathbf{F}^{k,t} \) symbolic notation for the set of principal tensile stresses within the filler for the kth solution, being a subset of \( \mathbf{F}^k \)

\( \mathbf{F}_{\text{max}} \) the set of maximum allowable stresses for all possibilities of failure within the structure as determined by the respective failure criteria

\( F_{\text{max}} \) maximum allowable tensile stress in the filler (\( F_{\text{max}} \) is an element of the set \( \mathbf{F}_{\text{max}} \))

\( f_c' \) compressive strength of concrete in the frame in units of force per unit area

\( P_o \) total horizontal load on the structure at ultimate, with units of force
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<td>$\alpha$</td>
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<td>story height with units of length</td>
</tr>
<tr>
<td>$\beta$</td>
<td>ratio of column spacing to story height</td>
</tr>
<tr>
<td>$n$</td>
<td>number of stories</td>
</tr>
<tr>
<td>$A_s$</td>
<td>area of frame reinforcement</td>
</tr>
<tr>
<td>$f_y$</td>
<td>yield strength of frame reinforcement, with units of force per unit area</td>
</tr>
</tbody>
</table>
2. METHOD OF ANALYSIS

2.1 General

The analytical solution of the frame with filler walls can be reduced to the consideration of two interacting structural elements. The filler is considered to be a plate subjected to in-plane forces, and the frame as the familiar assemblage of lineal structural elements which resist load by flexure, extension, and shear. In this context, a combination of membrane theory for the plate element and a general framework analysis for the frame seems appropriate. The present work is a study of the behavior of multistory infilled frames not only for the elastic uncracked structure, but also for the range where cracking has appeared in frame and filler. A prime consideration in the analytical scheme has been the development of a rational method of predicting the sequence of crack formation in the structure with increasing load.

2.2 Critique of Existing Analytical Methods

Existing studies of infilled frame behavior have been mainly concerned with the behavior of a single-panel structure. Two basic approaches are implicit in the majority of cases. The first approach considers the behavior of the shear panel to be affected by the presence
of the bounding frame which is considered to be a stiffening element along the panel edge. Thus, the basic problem is envisioned as one of "plate" behavior. The other approach concentrates attention on the behavior of the structural frame as affected by the panel infill, and attempts to define the manner in which the panel affects the stiffness of the frame. Behavior is thus considered synonymous with that of the "frame". This duality in the analytical approaches is understandable in light of the structural behavior ordinarily associated with plates and frames acting as isolated entities. It has lead to inconsistencies in the hypotheses regarding the mechanism of shear resistance. Investigations concentrating on plate behavior tend to base the criterion of failure on the formation of diagonal tension cracks. On the other hand, the frame-oriented investigations gravitate toward the compression-strut hypothesis and predict failure as determined by the infill acting as a compression member in the framework. The quasi-empirical assumptions regarding behavior in these analyses tend to obscure the actual mode of resistance in the structure and can be criticized for suffering from a lack of generality in explaining the resistive mechanism.

A variety of specific analytic methods has been employed in theoretical studies of frame filler structures
and of related problems. Rosenhaupt (1)* has used a finite-difference technique in determining values for the Airy stress function in a masonry wall supported on a reinforced concrete beam. Boundary conditions at the wall-beam interface are expressed as relationships between the stress function and the physical properties of the beam. The result is a solution to the plane stress problem of an edge-stiffened plate subjected to in-plane forces. From the computational standpoint, solutions involve a set of simultaneous equations in terms of the unknown values of the stress function throughout the plate element. This scheme presents formidable difficulties in formulating boundary conditions if it is to be extended to consider cracking in the plate or in the bounding beam.

Hinkley (2) has solved a similar problem of a masonry wall with edge beams by using the McHenry-Hrennikoff lattice analogy. The analysis provides the elastic stress distribution in the wall. Of particular importance is the necessarily large number of lattice segments and a seemingly prohibitive amount of computer time, due mainly to slow convergence. With the availability of high-speed computers, solutions for this type of

* Numbers in parentheses refer to the correspondingly numbered items in the List of References.
problem may be obtained more efficiently using finite-difference or finite-element formulations rather than the lattice analogy.

Benjamin and Williams (3,4) employed the lattice analogy in the analysis of one-story infilled frames in the post-cracking range. In constructing the load-deflection behavior up to ultimate, two separate solutions are required - the uncracked lattice solution which yields the elastic behavior up to the value of the load at first cracking, and the solution containing an idealized fully-developed crack pattern. The ultimate load was computed by an empirical formula for the case of shear failure in the compression column, or it was obtained directly from the analysis if failure was due to yielding of the tension column steel or to the formation of a diagonal tension crack. In order to obtain the behavior in the post-cracking range below ultimate, it was assumed that the full crack pattern was fully materialized at a load level halfway between the first-cracking and ultimate loads. This assumption gives one point on the load-deflection curve for the lattice solution with the idealized crack and defines the slope above first cracking. The intersection of this line with the ultimate load gives a value for the ultimate deflection.
In the cases studied by Benjamin and Williams, the crack disposition was known from test results or could be generalized from similar tests due to the simplicity of the structure. The possibility of extending the analysis to multi-story structures is complicated by the necessity of knowing the crack pattern to be used in the second stage of the analysis. A solution, on this account, would necessarily be an iterative one. Also, the assumption of a single cracked state may not be applicable. The multiplicity of cracked configurations would complicate the calculation of the ultimate loads if the same failure criterion were used.

Smith (5) has considered the problem of determining ultimate loads for a multi-story infilled frame by reducing the infill panel to a compression strut in an equivalent pin-jointed frame which is analysed by conventional methods to give upper bound values for lateral loads corresponding to failure in one of the struts. Failure criteria for each strut are considered to be diagonal cracking and compressive failure as derived from theoretical analyses based on relative stiffnesses of frame and infill. The main objection to Smith's behavioral model is its inherent lack of generality in reproducing behavior of actual multi-story structures. Failure of the frame members is not recognized.
2.3 **Criteria for an Analytical Model**

The infilled frame can be generalized as a planar element occupying a finite two-dimensional region which is also the domain of a conventional structural frame. External loads are resisted by the frame and plate in consort. A general method of analysis for the infilled frame must necessarily be based on the solution of the differential equations of equilibrium for a plate element subjected to in-plane forces. The existence of frame elements in the plate region complicates the plate solution by classical methods in elastic theory. Solutions for a masonry filler wall which must account for crack propagation are difficult to handle by stress functions or variational methods. Numerical procedures utilizing finite difference formulations for the governing differential equations and plate models have the disadvantage of requiring considerable computation which has been overcome by the availability of high-speed digital computers (6,7).

A lumped-parameter model developed by Harper and Ang (8) for the analysis of contained plastic flow in plates has the advantage of mathematical consistency with the differential equations of equilibrium (9), and can be supplemented with flexural-extensional elements to represent the frame. The present analysis utilizes this model
with modifications to admit progressive cracking with increasing load.

2.4 Equations of Equilibrium

Referring to Fig. 2.1, the equilibrium equations for an element of the filler in terms of membrane forces are:

\[ N_{x,x} + N_{x,y,y} + \bar{X} = 0 \]
\[ N_{x,y,y} + N_{y,y} + \bar{Y} = 0 \]  \hspace{1cm} (2.1)

where \( \bar{X} \) and \( \bar{Y} \) are components of body force per unit area, and commas indicate partial differentiation. Utilizing assumptions of elastic isotropy and small displacements, equations (2.1) may be expressed as:

\[ u,_{xx} + \frac{(1+v)}{2} v,_{xy} + \frac{(1-v)}{2} u,_{yy} + \bar{X} \frac{(1-v^2)}{E_w h} = 0 \]
\[ v,_{yy} + \frac{(1+v)}{2} u,_{xy} + \frac{(1-v)}{2} v,_{xx} + \bar{Y} \frac{(1-v^2)}{E_w h} = 0 \]  \hspace{1cm} (2.2)

where \( u,v \) are orthogonal displacement components corresponding to the \( x,y \) directions respectively, \( v \) is Poisson's ratio for the wall, \( E_w \) is the elastic modulus, and \( h \) is the wall thickness.

Following Ang's model configuration (8), and referring to Fig. 2.2, displacements are defined at 'mass points', and the two-dimensional stress tensor is defined
at the 'stress points'. The partial derivatives of the displacement components in equations 2.2 are transformed into the equivalent finite difference expressions:

\[ u'_{xx} = \frac{1}{2\lambda^2} (u_{i+2j} - 2u_{ij} + u_{i-2j}) \]

\[ v'_{xy} = \frac{1}{2\lambda^2} (v_{i+i+1} - v_{i-1j+1} - v_{i+1j-1} + v_{i-lj-1}) \]

\[ u'_{yy} = \frac{1}{2\lambda^2} (u_{ij+2} - 2u_{ij} + u_{ij-2}) \]

\[ v'_{yy} = \frac{1}{2\lambda^2} (v_{ij+2} - 2v_{ij} + v_{ij-2}) \]

\[ u'_{xy} = \frac{1}{2\lambda^2} (u_{i+1j+1} - u_{i-1j+1} - u_{i+1j-1} + u_{i-1j-1}) \]

\[ v'_{xx} = \frac{1}{2\lambda^2} (v_{i+2j} - 2v_{ij} + v_{i-2j}) \]

to yield the equilibrium equations for the model at mass point \(ij\) (equations 2.4).

\[ \frac{E_w h}{4(1-\nu^2)} \left[ (6-2\nu)(u_{ij}) + (-2)(u_{i+2j} + u_{i-2j}) \right. \]

\[ \left. - (1-\nu)(u_{ij+2} + u_{ij-2}) + (1+\nu)(v_{i-1j+1} - v_{i-1j-1} - v_{i+1j+1} + v_{i+1j-1}) \right] = X_{ij} \]  

\[ \frac{E_w h}{4(1-\nu^2)} \left[ (6-2\nu)(v_{ij}) + (-2)(v_{ij+2} + v_{ij-2}) \right. \]

\[ \left. - (1-\nu)(v_{i+2j} + v_{i-2j}) + (1+\nu)(u_{i-1j+1} - u_{i-1j-1} - u_{i+1j+1} + u_{i+1j-1}) \right] = Y_{ij} \]

(2.4)
These equations are shown symbolically in Fig. 2.3. Alternately, equations (2.4) may be obtained by specializing Mohraz's shell equations (10) for a flat plate. \( X_{ij} \) and \( Y_{ij} \) are body forces applied to the model, and are related to \( \bar{X} \) and \( \bar{Y} \) as:

\[
\begin{align*}
X_{ij} &= \lambda^2 \bar{X} \\
Y_{ij} &= \lambda^2 \bar{Y}
\end{align*}
\]  

(2.5)

Similar equations may be derived for mass points near the fixed edge or on the boundary, and would include terms representing the flexural and extensional stiffnesses of the frame. The equations for a boundary mass point are shown symbolically in Fig. 2.4.

2.5 The Analytical Model

If mass points are arranged in a square grid, a one-story structure may be represented in either of the two ways as shown in Fig. 2.5. The first arrangement is similar to Mohraz's shell model by using the diagonal grid of mass points. Displacement components are defined in directions parallel to the boundaries of the structure. A disadvantage in computation of internal stresses and in additional programming arises from the necessity of considering different stiffnesses for stress points on the boundary and those in the interior. The second illustration
in the figure with the vertical grid arrangement of mass points as suggested by Harper (8), has the advantage of requiring only one type of stress point since all stress points representing the filler are removed from the boundary. Computation of internal stresses across a horizontal section of the model is facilitated for the latter case since a section contains only mass points. In the present study, the vertical-grid mass point arrangement has been used exclusively.

The stress points are the deformable components of the model, and are considered to contain the entire strain energy in the deformed structural model. Three types of stress points are shown in Fig. 2.6. Flexural and extensional stress points represent the respective deformations of the frame, and the interior stress points represent the in-plane deformations of the filler. For the analysis of an elastic structure, it would be sufficient to consider the filler stress point as a deformable node without explicitly defining the manner of resistance. The node could simply be assigned extensional stiffnesses in two directions and a shear stiffness. Since the present analysis attempts to treat cracking in the model, it will be convenient to depict the stress point as composed of a system of springs which will be modified to conform to the existence of a crack as the analysis proceeds.
Thus, the occurrence of cracking within the model structure is defined by a reconstitution of stress point springs in a manner to be explained in section 3.2.

Frame stress points are idealized as flexural and extensional springs which represent the corresponding deformations in the frame. The flexural springs are located at the mass points as shown in Fig. 2.6, with the extensional springs contained in the space between mass points. Values for the frame stiffnesses which are used in equilibrium equations such as the ones represented in Fig. 2.4 are derived from the area and moment of inertia of the transformed section. The expressions are:

\[ \frac{SF}{\lambda^3} = \frac{E_f I}{\lambda^3} \]  

\[ \frac{SE}{\lambda} = \frac{E_f A}{\lambda} \]

where \( E_f \) is the elastic modulus of the concrete in the frame,

\( I \) is the moment of inertia of the transformed section,

\( A \) is the area of the transformed section,

\( \lambda \) is the grid spacing in the model.

The filler stress point is idealized as shown in Fig. 2.7. Three independent spring systems are considered to represent the two-dimensional state of stress.
in the wall. Two axial spring systems correspond to the two orthogonal stress components, while shear stresses are represented by the shear spring system. An axial spring system is shown to consist of two springs in series, whereas the shear spring system consists of four springs arranged in pin-wheel fashion around the stress block "A".

Computation of forces within the analytical model is facilitated by defining explicit stiffnesses for the stress point spring systems. If the axial stiffness is denoted as $SA$, and the shear stiffness as $SS$, the equilibrium equations (2.4) yield:

$$\frac{SA}{E_w h} = \frac{1}{2(1-\nu^2)}$$

$$\frac{SS}{E_w h} = \frac{1}{4(1-\nu^2)}$$

where $E_w, \nu$ are the elastic modulus and Poisson's ratio for the filler, and $h$ is the filler thickness. These expressions are valid for an elastic isotropic filler material, and correspond to the case of plane stress.

Forces are computed from the displacements of the analytical model as:
\[ F_x = \overline{SA} [u_{ij} - u_{i-2j}] \]
\[ F_y = \overline{SA} [v_{i-1j+1} - v_{i-1j-1}] \]  \hspace{1cm} (2.8)
\[ S = SS [(1-\nu)(u_{i-1j+1} - u_{i-1j-1}) - (1+\nu)(v_{ij} - v_{i-2j})] \]

where \( F_x, F_y, S \) are the two axial and the shear force components at a stress point, and \( u, v \) are displacements as defined in Fig. 2.7.

The corresponding unit stresses \((\sigma_x, \sigma_y, \tau)\) are expressed as follows:

\[ \sigma_x = \frac{2F_x}{\sqrt{2\lambda h}} \]
\[ \sigma_y = \frac{2F_y}{\sqrt{2\lambda h}} \]  \hspace{1cm} (2.9)
\[ \tau = \frac{2S}{\sqrt{2\lambda h}} \]

In subsequent chapters, the stress conditions within the filler will be presented as the principal force components which are obtained directly from the analytical model.
3. METHOD OF SOLUTION

3.1 General

The appearance of cracking in frame and filler during the loading process decreases the stiffness of the structure. In terms of the analytical model, cracking was recognized by modification of the constituent spring systems. Criteria for determining the occurrence of cracking are described, and an explanation of the procedure employed to obtain the load-deflection is presented.

3.2 Failure Criteria

In order to reduce the number of failure possibilities, simplifying assumptions were made regarding the behavior of the frame and filler. It was considered that these assumptions would not unduly mask the realistic behavior of the model.

For the frame, it was assumed that flexural behavior during the early stages of loading would be sufficiently inhibited by the presence of the filler so that the possibility of attaining high bending moments to cause yielding in flexure could be discounted. Failure possibilities for the frame elements were thus restricted to cracking of the concrete as a result of extensional...
deformation and subsequent yielding of the reinforcement. Compression failure in the frame elements was admitted as a failure possibility.

The conditions for frame failure in tension were dictated by specifying limiting strains for cracking of the concrete and for the elastic limit of the reinforcement. Strain hardening in the reinforcement was not considered. Frame failure in compression was specified by a limiting compressive strain corresponding to the ultimate deformation of a tied column.

Failure in the filler was determined by limiting principal tensile and compressive stresses. According to the theory of principal stresses for continuous media, for every set of stress components \((F_x', F_y', S)\) there exists a set of principal stresses \((F_1, F_2)\) which are oriented generally at some nonzero angle to the x-y coordinate system. In terms of the analytical model, the existence of principal stresses can be predicted on the basis of principal stress theory for continuous media even though the actual principal stresses do not occur within the model structure. For each set of stress components, \(F_x', F_y', S\), Mohr's stress circle was used to compute principal stresses and to determine the orientation of the principal axes.
3.3 Modification of the Analytical Model to Recognize Localized Failure

Tension failure in the frame was considered to occur in two stages: initial cracking of the concrete, and yielding of the reinforcement. Cracked frame sections were assigned revised flexural and extensional stiffnesses which were similar to the expressions derived in section 2.5 for the uncracked section except that area and moment of inertia were determined from the reinforcement alone. For frame sections in which the reinforcement had yielded, the reinforcement was assumed to have a flat-top yield range and thus would maintain a constant load between the two corresponding mass points irrespective of the relative deformations between the mass points. The program deleted the extensional springs between the mass points containing a yielded section, and introduced appropriate load terms into the load vector to simulate the constant load effect of the yielded portion on the remaining structure. In this manner, the necessity of handling localized inelastic behavior in an otherwise linearly elastic structure was sidestepped with considerable advantage in programming simplicity. Compression failure in a frame element was handled by deleting the extensional spring system between the respective mass points.
In the case of failure occurring in the filler material, the constituent spring systems of the corresponding stress points were deleted according to the type of failure, in a manner which deprived the model structure of resistance similar to that experienced by an actual structure with the same mode of failure. If an actual structure suffers localized failure in compression, it may be assumed that the filler in this region is extensively fractured, and, discounting the possibility of sustaining further load through mechanical interlocking of the fractured portions, the material is locally incapable of sustaining load by membrane action. Therefore, in order to reproduce the same behavior for a compression failure in the model structure, both axial spring components and the shear spring system were deleted from further participation. This is equivalent to introducing a "hole" in the filler at the point of failure.

A crack in the filler was assumed to have been caused by the principal tensile stress exceeding the tensile strength of the material. Considering the possible types of in-plane forces which could be carried by the filler in the vicinity of the crack as summarized in Fig. 3.1, it is seen from considerations of the statical equilibrium of the filler element with a crack that only two cases must be discounted: tensile force across the
crack, and shear force. On this basis, modification of the stress-point spring system for tensile cracking is dictated by the following rules.

The shear spring system is unconditionally deleted, since shear capacity in the vicinity of the crack is considered to be nonexistent. The axial spring system closest to the direction of the principal tensile stress is deleted conditionally, and may be reintroduced at some later stage in the analysis to carry compression. Participation of this spring in tension is not allowed. This feature corresponds to the possibility of a tensile crack closing up and carrying compression which could occur due to reorientation of the stresses within the filler. Tension and compression are admitted for the axial spring system in the perpendicular direction. In this manner, the model behavior conforms to the behavioral assumptions for a cracked filler element as summarized in Fig. 3.1.

3.4 Method of Obtaining Load-Deflection Behavior from Analysis

The decrease in the structural stiffness which is obtained in an actual solution with progressive cracking was duplicated in the analytical model by a gradual depletion of the deformable elements with successive solutions. All deformable elements which remained in the
model were assumed to exhibit linearly elastic behavior. With respect to the computed load-deflection response of the structural model, instantaneous unloading was implied in order to allow modification of the structure to admit the new failure location prior to reloading for the subsequent solution.

To illustrate the method by which load-deflection behavior has been obtained, Fig. 3.2 shows the response for a single-story structure with a filler modulus of 2,500,000 psi and 1.1 percent frame reinforcement. The same structure will be discussed in greater detail in the following chapter.

The initial solution for the uncracked structure was obtained for influence loads of magnitude $P$. The initial set of generalized displacement components and the set of principal stresses within the filler are designed symbolically as $C^1$ and $F^1$ respectively. The stiffness for the uncracked structure may be expressed as the value of $P/\Delta_1$ where $\Delta_1$ is a characteristic displacement of the structure and may be determined from the appropriate components of $C^1$. The initial stiffness is shown in the figure as the slope of the line from the origin through the point 'A'.

For the particular structure under consideration, first cracking was due to tensile cracking in the
filler. If the maximum value of the principal tensile stresses in the filler corresponding to the initial set of generalized displacement components $C^1$ and the influence loads $P$ be designated as $\text{max} \ [F^{1,t}]$, and if the maximum tensile stress in the filler as determined on the basis of an arbitrary failure criterion be designated as $F_{\text{max}}$, then the load level corresponding to initial cracking is computed by $(P)(F_{\text{max}})/\text{max}[F^{1,t}]$. This load is located on the initial slope as point 'A', and becomes the initial point on the load-deflection plot. Subsequent points corresponding to successive cracking in the structure are shown terminating the set of solid radial lines from the origin.

The recursive nature of the analytical process may be generalized as follows. For a particular solution of the generalized displacements $C^k$ due to the invariant influence loads $P$, there exists a critical stress within the structure which is designated as $\text{max}[F^k]$. From the maximum allowable value of the same stress which is imposed by the corresponding failure criterion, and which is designated as $F_{\text{max}}$, the load level for failure is obtained by $(P)(F_{\text{max}})/\text{max}[F^k]$. The stress point containing the value $\text{max}[F^k]$ was modified according to the discussion in section 3.3, equations were regenerated, and a new solution $C^{k+1}$ produced for the same influence loads $P$. Each computational cycle determines a structural stiffness, and a load
level for the particular failure point. A record of the failure locations is maintained so that at any stage of the analysis, the extent and the disposition of cracking is available.

With reference to the solution presented in Fig. 3.2, the initial solution indicates that first cracking occurs with a stiffness and at a load level as defined by point 'A'. The next solution which occurs at a load level greater than that at 'A' is designated as point 'B'. If the structure were loaded to level 'A', and if the load were increased by an infinitesimal amount, failures would be produced at the appropriate stress points for all the solutions which are shown between 'A' and 'B'. Load-deflection behavior subsequent to 'A' could be indicated by the light broken line in the figure, which is characterized by a flat-top portion extending from 'A' to the stiffness defined through the point 'B'. An increase in load would then be necessary to reach the failure load level of 'B'. Alternately, the same crack formation would have been obtained by assuming that the load-deflection behavior followed the straight-line joining 'A' and 'B' as shown by the heavy broken line in the figure. For simplicity, all subsequent computed load-deflection plots are shown for the latter assumption, and omit the stiffnesses and failure loads for the individual solutions as shown in Fig. 3.2.
4. ANALYTICAL RESULTS

4.1 General

Analytical results are presented for two basic structural types shown in Figures 4.1 and 4.2: single-story and five-story structures. Experimental results are available for similar small-scale structures (11, 12) and will be compared with the analytical results in the following chapter.

In all solutions presented in this chapter, each panel of the actual structure was represented by mass points arranged in a 4x8 grid pattern which covered the area defined by the centerlines of the frame (Fig. 4.1). The model thus extended the actual filler a distance equal to half the depth of the bounding frame member. This minor discrepancy should have little effect on comparisons between computed and experimental behavior.

The application of horizontal load was identical for the single- and the five-story structures; equal loads were applied to the quarter points in each beam. Details for the single-story structure loading are given in Fig. 4.1.

The effects on behavior of different relative stiffnesses between frame and filler were investigated by varying the filler modulus and the amount of frame
reinforcement. Table 1 presents the variables covered by the analytical solutions. In general, the single-story structure was investigated for the effect of variation in filler modulus, and the five-story structure, for variation in frame stiffness. Two additional solutions were included for the five-story structure to determine the effect of openings in the filler, and the addition of vertical load to the horizontal load system. Invariant quantities which are common to all solutions are listed in Table 2.

For simplicity, the filler was assumed to be a linearly elastic isotropic medium with a Poisson's ratio equal to zero. This simplification for the behavior of a masonry filler may be justified in view of the nature of the desired results, where the determination of the overall structural behavior was deeded to be of prime importance.

4.2 Uncracked Single-Story Structures

In all analytical solutions, equal horizontal loads were applied to the quarter points in the beam. For present purposes, structural stiffness is defined as the ratio \( P/\Delta_1 \), where \( P \) is the total load on the structure, and \( \Delta_1 \) is the lateral deflection at the intersection of the center lines for the beam the tension column.
The effect of variation in filler stiffness on structural stiffness is discussed with reference to Fig. 4.3. Since all solutions were performed with the same magnitude of $P$, the deflection $\Delta_1$ is representative of the stiffness of the structure. For a structure with invariant frame properties, values of $\Delta_1$ are shown for the range of filler moduli from 200,000 to 7,000,000 psi. The scale on the right-hand side of the figure relates the deflections of filled frames to the deflection of a frame without a filler.

Low values of filler modulus may be expected in masonry fillers as a result of such factors as poor workmanship, thick joints, and certain types of mortars. Analyses indicate that the inclusion of a filler with a very low modulus has a substantial effect on increasing the structural stiffness. For the lowest value shown in Fig. 4.3, the lateral displacement $\Delta_1$ was reduced to approximately 36 percent of the deflection for a comparable frame without a filler.

The effect of filler stiffness on the deflected shape of the tension column is shown in Fig. 4.4 for the two extreme values of filler moduli: 200,000 and 7,000,000 psi, and for the frame without filler. For purposes of direct comparisons, the deflection mode shapes are presented as percentages of the respective values.
of $\Delta_1$. Flexural analysis of the unfilled frame yields a higher column moment at the base than at the junction with the beam, and thus produces the pronounced curvature in the lower portion. Inclusion of a filler indicates that column curvatures are reduced, and thus flexural deformation of the frame is inhibited.

Horizontal loads applied to the beam require shear and overturning moment to be resisted by the frame-filler composite across the base of the structure. The base shear is distributed between frame and filler as shown in Fig. 4.5. For the case of an unfilled frame, as represented by a filler modulus equal to zero, the frame alone must resist the total applied shear. Introducing a filler shows that the portion of the total shear carried by the frame is sharply reduced, but that the reduction occurs at a lesser rate than the corresponding reduction in structural stiffness as shown in Fig. 4.3.

The resisting moment at the base of the structure consists of three components: base moments in the columns, the couple provided by axial loads in the columns, and the resultant couple due to forces normal to the base of the filler. For various filler moduli, Fig. 4.6 shows the relative values of these three components expressed as percentages of the total resisting moment. The figure is divided by two curves into three areas which correspond to
the three components. The lower curve representing base moments in the columns is plotted with respect to the left-side ordinate scale, whereas the upper curve, which represents the resisting couple from axial loads in the columns, is plotted for the right-side ordinate scale. The central region between the two curves thus represents the resisting moment as provided by normal forces in the filler. For a given value of filler modulus (abscissa), the ordinates to the curves define the contribution of the three components in resisting the overturning moment.

In an unfilled frame, the resisting moment was provided by the combination of column base moments and the axial load couple. Figure 4.6 shows that the base moments account for 63 percent of the total resisting moment. For values of filler modulus increasing from zero to 1,000,000 psi, the contribution of the base moments decreased rapidly and continued to decrease for values above 1,000,000 psi but at a lower rate. The axial-load couple increased rapidly for low values of filler modulus and attained a maximum at approximately 600,000 psi. The resisting moment in the filler increased rapidly for values below 1,000,000 psi and thereafter continued to increase at virtually a constant rate.

In general, the effect of introducing a filler into a single-story structure served to suppress the
flexural behavior of the frame and to increase the axial force in the columns. Except for the very low values of filler modulus, the resisting moment in the frame was mainly provided by the axial load component. For the range of filler moduli which were considered, the total resisting moment in the frame exceeded that of the filler. Base shear and resisting moment in the filler increased at the higher values of filler modulus.

The effect of varying the filler modulus on the state of stress within the filler is discussed with reference to Fig. 4.7. Stress concentrations were obtained in two general regions as indicated by the locations of stress points numbered 1, 9, and 30. Maximum principal tensile stresses at the three stress points are plotted with variation in the filler modulus. The largest stress at a given value of filler modulus determines the location of the initial crack and the actual cracking load.

A rigorous application of the failure criterion requires that a fine distinction be made between stresses in determining the location of initial cracking. For values of filler modulus below approximately 3,300,000 psi, the difference in stress levels between points 9 and 30 is slight - generally less than one percent. In a narrow range at 3,000,000 psi and for all values below 800,000 psi, cracking occurs at stress point 30; otherwise stress
point 9 governs. Above 3,300,000 psi, there is a definite divergence in stress levels at the three points, and cracking initiates at point 1.

4.3 Behavior of Single-Story Structures with 1.1 Percent Reinforcement

The computed load-deflection response and the associated crack development is presented for two values of filler modulus: 2,500,000 and 2,800,000 psi which are designated as solutions 11 and 12 in Table 1. The above solutions were chosen to illustrate the formation of two different crack configurations which were characteristic of all single-story solutions. A comprehensive evaluation of the effect of filler modulus on behavior is deferred to the following section.

Figure 4.8 summarizes the symbols which are encountered in all subsequent figures showing crack development. The chronological development of cracking for the two solutions is presented in Fig. 4.9. The numerals in the illustrations at the top of the figure indicate the sequence of the initial crack formation. The lowest illustration in each column shows the fully-developed crack configuration at ultimate.

In solution 11, the initial crack progressed laterally along the base of the filler toward the
compression column and upward along the edge of the tension column. This type of crack was obtained in many of the following solutions and will be referred to as the 'bottom crack'. In solution 12, the initial crack progressed laterally along the top of the filler toward the tension column and downward along the edge of the compression column with a short diagonal segment bypassing the corner. This type of crack will be referred to as the 'top crack'.

The load-deflection curves for these two solutions are shown in Fig. 4.10. Two sets of scales are provided - one set for the full-size structure which was used in the analysis, and the other for the one-eighth scale model structure which was used in the experiments. For the latter case, the load scale (ordinate) is 1/64 times the ordinate for the full-size structure, and the deflection (abscissa) is 1/8 times the deflection abscissa for the full-size structure.

Points 'A' denote the initiation of cracking and define the stiffness of the uncracked structure. Structural stiffnesses corresponding to the stage at which the initial top or bottom cracks have been fully developed are labelled as points 'B'. The stiffness of the structure with the top crack (solution 11) is shown to be less than the structure with bottom crack. The reasons for
this difference may be explained by considering the possible mode of resistance which is afforded by the uncracked portion of the filler. With both types of crack configurations, the filler is approximately equally effective in resisting lateral displacement of the tension column which is subjected to high shear at the upper end. The difference lies in the degree to which the filler stiffens the knee frame consisting of the beam and the compression column. For the structure with the bottom crack, the knee frame is effectively stiffened by the entire filler, whereas the structure with the top crack contains a relatively more flexible knee frame which has been separated from the major portion of the filler.

Ultimate load in both solutions was attained with initial yielding of the reinforcement in the tension column. At this stage, cracking had progressed across the base of the filler and thus had eliminated the remaining tensile component for resisting the overturning moment.

The preceding discussion of load-deflection behavior and crack formation neglected the possibility of failure in the frame by shear. This failure mode was not programmed into the solution, but was checked independently using the print-out of stresses and displacements. Due to the existing imprecise knowledge regarding the behavior of reinforced concrete members subjected to
combined axial load and shear, conditions for failure were assumed based on the following reasoning. Since the presence of compressive axial load at a section carrying shear serves to increase the shear failure load, and since the presence of tensile axial load is detrimental to the shear capacity, a shear failure envelope was assumed as shown in Fig. 4.11. The failure strength of the section in pure shear was based on experimental studies of beams without shear reinforcement, and was computed as \(6 \sqrt{f_c'}\) times the gross area of the cross section, where \(f_c'\) is the compressive strength of the concrete in the frame expressed in units of pounds per square inch. The cracking strength in pure tension was computed from the tensile strength of the transformed area of the cross section. Figure 4.11 shows a linear variation for the tension-shear failure envelope although a convex curve could be admitted for this portion. In view of the uncertainties implicit in the assumptions of shear failure, the failure envelope of Fig. 4.11 can only be taken as a guide in indicating the possibility of imminent shear failure in the frame.

With full development of the initial crack as represented by points 'B' in Fig. 4.10, a check on shear conditions in the frame indicated that critical points occurred in both solutions in the uppermost segment of the
tension column. Fig. 4.12 presents the shear and axial load conditions at this location for both solutions in relation to the assumed failure envelope. If shear failure should occur at this location, the subsequent failure would be entirely different from that produced by the analytical solution. It could then be expected that the large shear carried by the tension column would be transferred to the compression side of the structure and would cause failure in the compression column by shear or by flexural hinging.

4.4 Behavior of Single-Story Structures with 2.2 Percent Reinforcement

The effect of a wide variation in filler modulus on the behavior of a structure with invariant frame properties is considered in the present section. Solutions numbered 13 through 17 as summarized in Table 1 were obtained for values of filler modulus between 200,000 and 4,500,000 psi which were considered to be representative bounds for low and high quality masonry fillers.

The computed load-deflection curves are shown in Fig. 4.13 barring the possibility of tension column shear failure which was discussed in the previous section. In all solutions, the ultimate load was reached with yielding of the reinforcement in the tension column. Solutions 14 through 17 attained virtually the same level
of ultimate load whereas the ultimate value from solution 13 was approximately 85 percent below this value. Thus it is concluded that single-story structures will attain approximately the same level of ultimate load for widely varying values of filler modulus. This statement must be qualified for extremely low values of filler modulus (viz. solution 13 with 200,000 psi), and for the possibility of shear failure in the tension column.

Crack formation for solutions 14 through 17 are presented in Figures 4.14 and 4.15. Solutions 14 and 16 initially produced the characteristic top crack whereas solutions 15 and 17 produced the bottom crack. These initial crack configurations are dependent on the location of the stress point at which the initial failure occurs, which may be determined from the corresponding value of filler modulus in Fig. 4.7. At ultimate, cracking produced an essentially intact diagonal within the filler, and thus had reduced the behavior of the filler to a compression strut.

Since the structures with the higher values of filler modulus have correspondingly higher filler stresses, the amount of filler cracking is directly related to the value of the filler modulus. Solution 17 exhibited compression failures within the filler at ultimate which attested to the high filler stresses in this structure.
All solutions showed cracking in the tension column throughout the lowest three segments with yielding of the reinforcement occurring in the lowest segment.

The conditions for shear failure in the tension column with formation of the initial crack as discussed in the previous section are also pertinent to the current solutions. Figure 4.16 shows the shear and axial load conditions from solutions 14 through 17 as compared with the assumed failure envelope.

Stress conditions within the filler are altered as cracking is developed, and indicate the mode of resistance within the filler at any stage of the analysis. Since the computed crack patterns have indicated that two basic initial crack configurations may be expected in single-story structures, the stress conditions within the filler, before and after the development of the initial cracks, will be examined.

Figure 4.17 shows the principal compressive stresses from solution 16 for the uncracked structure as the set of inclined bars which are centered at the stress point locations in the filler and are oriented to correspond to the principal stress directions. The bar lengths are drawn to a force scale and indicate the magnitude of the principal stress at the corresponding stress point. For purposes of comparison, the stresses in this and the
The principal compressive stress field in the uncracked filler consists of relatively uniform values inclined to the horizontal at approximately 45 degrees. Due to symmetrical conditions of loading and structural geometry, the corresponding principal tensile stress field is a mirror image of the principal compressive stress field.

Stress conditions existing in the filler after formation of the top crack are shown in Figures 4.18 and 4.19. The shaded areas indicate the stress points at which previous tensile failures have occurred, and thus define the extent of cracking at this stage of the analysis. The constituent spring systems within these stress points have been modified according to the discussion of section 3.3: shear springs and the tensile axial springs have been deleted while the compressive axial springs have been retained.

Figure 4.18 shows compressive forces transmitted between the beam and the uncracked portion of the filler with a concentration of stress occurring across the crack near the tension column. In general, the existence of the top crack has increased the magnitudes of the principal compressive stresses within the uncracked portion of the
filler, and has realigned these stresses to reflect the pinching effect which is created by the containment of the filler between the base and the tension column. Figure 4.19 illustrates the effect of the top crack in drastically reducing the magnitudes and the extent of the principal tensile stresses within the uncracked portion of the filler.

A similar stress redistribution with a predominant principal compressive stress field is observed for a structure with a bottom crack (Fig. 4.20). The corresponding principal tensile stresses within the uncracked filler are concentrated immediately below the beam (Fig. 4.21).

4.5 Behavior of Five-Story Structures

Three solutions were obtained to determine the effect of variation in the amount of frame reinforcement on behavior of five-story structures. Table 1 contains the pertinent quantities for these solutions which are numbered 51, 52, and 53.

Since it was expected that the cracking zone in a multi-story structure would be confined mainly to the lower stories, a saving in computation time was realized by analyzing a truncated version of the actual structure.
All five-story structures as reported herein were analyzed as equivalent two-story structures.

To assure that the state of stress within the filler of the replacement structure would conform to that existing in the two lowest stories of the original structure, an auxiliary load system was introduced in addition to the usual quarter point beam loads. Details of the analytical models corresponding to both structures and the auxiliary load system are shown in Fig. 4.22.

The auxiliary loads were applied to the top row of mass points in the replacement structure as the equivalent set of forces which exist across the horizontal section at the second story level in the five-story model. These loads are shown in Fig. 4.22 as shears and normal forces, and correspond to the respective unit stress distributions as obtained from elementary beam theory.

The computed load-deflection behavior for solutions 51, 52, and 53 is presented in Fig. 4.23. In all cases, the ultimate load was attained with initial yielding of the reinforcement in the tension column. The lateral displacement of the tension column at the second story level ($\Delta_2$) was used as the characteristic deflection in all plots of load-deflection behavior of five-story structures.
Figures 4.24, 4.25, and 4.26 show the crack development for the above solutions. It is to be understood that crack formations as obtained from the two-story analytical model are assumed to correspond to those expected in the two lowest stories of five-story structures.

Structures with lesser amounts of frame reinforcement would be expected to exhibit more extensive filler cracking at ultimate since the filler is the relatively stiffer element and carries the greater portion of the load. This was verified for the two solutions with the highest amounts of reinforcement (solutions 52 and 53). Solution 51 with the lowest amount of frame reinforcement did not follow this trend since yielding occurred in the tension column prior to full development of cracking within the filler.

The effect of cracking in frame and filler on the lateral displacements of the structure will be discussed with reference to solution 53. Deflection mode shapes as obtained from the two-story analytical model are shown in Figures 4.27 and 4.28 for the tension and compression columns at five stages of loading which are numbered according to the sequence of cracking (Fig. 4.26). For purposes of comparison, both figures include the deflected shapes corresponding to beam theory.
deflections as computed on the separate bases of pure flexure and pure shear.

The tension and compression columns have identical deflections for the uncracked structure (crack '1'), and indicate a predominantly flexural type of mode shape. At crack '25', cracks occur in both filler panels and in the three upper segments of the first story tension column. The tension and compression columns show flexural behavior as indicated by the reversed curvatures occurring within each story. In the remaining three load stages, the deflection modes for both columns tend toward the pure flexural type, and indicate increased flexural behavior in the frame.

To illustrate the mode of resistance within the filler, the distribution of principal stresses before and after cracking is presented for solution 52. The following figures show stresses in the two lowest stories and indicate stress magnitudes by scaled bar lengths similar to the method used in Fig. 4.17 for the single-story structure. The initial principal compressive stress distribution (Fig. 4.29) corresponds to the state of stress which is assumed in elementary beam theory. Variation of normal stress across a horizontal section is essentially linear and symmetrical about the center line of the structure. The principal tensile stress
distribution is a mirror image of the compressive stresses shown in Fig. 4.29. With cracking in the filler as shown by the shaded areas in Figures 4.30 and 4.31, the principal compressive stress field dominates the resistance within the filler. The panels tend to act as compression struts within the concrete framework with a minimal contribution of the tensile stress field to the mode of resistance. Post-cracking behavior of the filler is similar to that observed in the single-story analyses.

Shears, axial loads, and bending moments in the frame were checked at various stages in the analyses to verify the assumption that extension deformation was the sole basis for frame failure. Stress conditions in the beams were generally found to be subcritical with respect to all modes of failure, and thus justified neglecting beam failure in the analysis. Forces in the first-story columns were examined in terms of the interaction between axial load and shear, and axial load and bending moment.

In the single-story structures, the axial load-shear conditions in the tension column were critical after formation of the initial crack (Figures 4.12 and 4.14). Shear force at the top of the tension column was approximately 60 percent of the axial load, and indicated that the assumed extensional failure criterion was unrealistic for the single-story structures. This drawback was
rectified by recognizing that localized failure of the tension column due to the effect of shear would cause failure of the entire structure with the transfer of load to the compression column. The analogous conditions were checked in the first-story columns for the five-story analyses. In general, very low shear-axial load ratios were obtained which justified the applicability of the extensional frame failure criterion. Figure 4.32 shows a typical shear-axial load condition in the top segment of the first-story tension column after formation of the initial crack.

The axial load-bending moment relationship at the base of the compression column is shown in Fig. 4.33 for solution 53. Initial increase of bending moment with increasing axial load was very small since the filler was uncracked in this range and the entire structure behaved predominantly as a cantilevered beam. With appearance of cracking in the tension column and within the filler, bending moment in the compression column increased at a relatively constant level of axial load. With cracking completed in the two lowest stories of the tension column, the increase of bending moment with axial load in the compression column continued approximately at a constant rate up to the ultimate load. Comparison of the axial load-moment curve in Fig. 4.33 with the ultimate interaction
diagram for the frame cross section is shown in Fig. 4.34, and indicates the extent to which flexural behavior occurs in the frame.

An indication of the effectiveness of the filler as a load resisting element at ultimate may be illustrated by considering the behavior of a similar five-story infilled frame which has the filler omitted in the lowest story. A 'cantilever' analysis of this partially filled structure would assume that the first-story columns resist the overturning moment by axial load alone. A 'portal' analysis would assume resistance by bending moment and axial load. The respective theoretical values of axial load and bending moment which would be required at the base of the first-story columns are shown in Fig. 4.33 for the same ultimate load as the completely filled frame. The values of ultimate axial load and bending moment from the solution of the completely filled frame indicate that the presence of the filler in the lowest story tends to force a predominantly 'cantilever' type of behavior for the frame.

In contrast to the preceding discussion of compression column behavior for solution 53, the analysis of the frame with a lower amount of frame reinforcement (solution 52) indicates that flexure is the dominant mode of resistance in the frame (Fig. 4.35). The reasons for this
difference may be seen from the effect of the respective crack locations in the first-story filler panel on the manner in which the resisting moment is developed across the base of the structure. For solution 52, cracking occurred along the base of the filler (Fig. 4.25), and eliminated the possibility of developing tensile resistance normal to the filler base. As a result, the compressive forces normal to the base of the filler and the force in the compression column which were required to balance the force in the tension column, were small. Therefore, the major resisting moment was provided by the bending moment in the base of the compression column. In the cracked region of the structure, the frame was thus constrained to act predominantly in a flexural mode which corresponds to the 'portal' type of behavior.

For solution 53, the base crack in the filler was located away from the support and allowed tensile forces to be developed across the base of the filler (Fig. 4.26). A relatively higher axial load was developed in the compression column to balance the resultant tensile forces across the base. Thus, the resisting moment across the base was provided mainly through the normal forces in the filler and axial load in the frame, with the resulting high axial load to moment ratio at the base of the compression column.
4.6 Behavior of a Five-Story Structure with Vertical Loading

A single solution was produced for the five-story structure with a vertical load component in addition to the horizontal loading system and is designated as solution 54 in Table 1. Vertical load was uniformly distributed in each beam with the total vertical load equal to twice the total horizontal load. Due to the formulation of the analytical process where the individual solutions are produced for the same set of influence loads, it was necessary to maintain a constant vertical-to-horizontal load ratio throughout the analysis.

Similar to the procedure employed in the analysis of five-story structures with horizontal loads alone, a single solution was performed for the entire five-story structure subjected to horizontal and vertical loads. From this solution, the auxiliary load system was determined and was applied to the replacement two-story structure in the actual analysis. Load-deflection response and crack formation is shown in Figs. 4.36 and 4.37.

With reference to the response of the same structure without the vertical load component (solution 52), initial crack formation is the same in both cases: laterally along the base of the filler in both panels and vertically at the interface between the filler and the
tension column. In solution 52, the initial crack formed instantaneously upon reaching the initial cracking load of 130 kips whereas, in solution 54, several increases of horizontal load were necessary to attain the same extent of filler cracking. In the latter case, cracking of the tension column occurred simultaneously with the formation of the initial filler crack.

The resistance of the structure after yielding of the tension column reinforcement may be generalized by considering the statical force equilibrium of a free body as shown in Fig. 4.38. At this advanced stage of cracking, the filler has separated from the base and the tension column. Thus, in addition to moments and forces at the base of the compression column, the free body is subjected to three force systems whose resultants are shown in the figure. The tension column force is equal to \( A_s f_y \) where \( A_s \) is the area of reinforcement and \( f_y \) is the yield strength. The resultants of horizontal and vertical load components are \( P_o \) and \( \alpha P_o \). Panel dimensions are \( H \times \beta H \). The number of stories is \( n \).

The expression for statical equilibrium about the base of the compression column (neglecting compression column moment) is:

\[
(A_s f_y \beta H) + \alpha P_o \left( \frac{\beta H}{2} \right) = \left( \frac{n+1}{2} \right) H P_o
\]

(4.1)
which reduces to:

$$P_0 = \frac{2 \beta A_s f_y}{(n+1-\alpha \beta)}$$

(4.2)

From equation 4.2, the horizontal load $P_0$ for the structural properties of solution 54 (see Table 2) is given by:

$$P_0 = \frac{2(2)(12.3)(42)}{(5 + 1 - 2(2))} = 1034 \text{ kips}$$

This value is a reasonable estimate of the ultimate load as obtained from the analysis (Fig. 4.36). For a similar structure without vertical load ($a=0$), equation 4.2 gives $P_0 = 345 \text{ kips}$ which agrees with the ultimate load for solution 52 in Fig. 4.36.

On the basis of these two calculations, equation 4.2 may be assumed to provide a reasonable estimate of the ultimate horizontal load capacity for single-bay multi-story structures with or without the presence of vertical loads.

A limitation in the applicability of this equation arises from the assumption that the centroid of the compressive forces across the base of the filler is located at the compression column. In actual structures, it is reasonable to expect that the centroid may be located within the filler and thus would invalidate the equilibrium statement of equation 4.1.
The calculation of $P_0$ by equation 4.2 for solution 53 would yield 690 kips whereas the analysis (Fig. 4.23) predicted an ultimate horizontal load of 501 kips. Direct use of equation 4.2 would be unconservative in this case.

The computation of ultimate load must also recognize the possibility of shear failure at the base of the compression column. In this region, the column is stiffened by the filler and would be expected to fail in direct shear. Determination of the load level for this mode of failure is complicated by the existence of axial load and bending moment.

4.7 Behavior of a Five-Story Structure with Filler Openings

Analysis of the five-story structure with filler openings was performed with the same formulation which was used for structures with solid fillers, and is designated as solution 55 in Table 1. The structure was initialized to admit filler openings by designating the appropriate stress points as locations of compression failure. This procedure effectively eliminated these stress points from subsequent participation in resisting load.

Openings were introduced into both stories of the replacement structure. Figures 4.39 and 4.40 show the
locations of the openings and the principal compressive and tensile stresses for the initial uncracked solution. The stresses correspond to the same influence value of applied horizontal load which resulted in the principal compressive stress distribution for the structure with solid filler panels as shown in Fig. 4.29. Thus, the relative stress magnitudes in these three figures are directly comparable.

The introduction of the relatively large openings in the tension side of the structure created compressive and tensile stress concentrations due to the necessary rerouting of the stress trajectories to bypass the openings. The principal tensile stress field is apparently as effective as the principal compressive stress field at this stage of loading. Tensile and compressive stresses were comparable in magnitude. This behavior is contrasted with the greatly reduced principal tensile stress fields (in relation to compressive stresses) which developed in structures with cracks at the edges of the filler panels.

Figure 4.41 presents a comparison of load-deflection response for the initial stages of loading between structures with and without filler openings (solutions 55 and 52 respectively). Due to the stress concentrations within the filler with openings, initial cracking occurred at a lower load level. For the range of loading
shown in the figure, the particular opening size considered had a minor effect on the energy absorption capability of the structure, and produced a response which would be expected from a structure with solid filler panels.
5. COMPARISON OF COMPUTED AND OBSERVED BEHAVIOR

5.1 General

The prime objective of this chapter is to correlate the results of analysis and experiment in order to demonstrate the degree to which the analytical solutions reproduce behavior as determined from tests, and to verify the assumptions which were necessary in the formulation of the analytical scheme. Comparisons are presented between computed and experimental load-deflection response and cracking in the frame and filler. Sources for the experimental results were a series of tests carried out at the University of Illinois by Yorulmaz (11) and Fiorato (12).

In evaluating the correspondence between computed and experimental results, allowance must be made for conditions which were present in the test specimens and which were difficult to incorporate into the analysis. These include such factors as imperfections of fit between frame and filler, variability of masonry properties due to inconsistent workmanship, and residual stresses caused by differential shrinkage between frame and filler.

5.2 Single-Story Structures

The available experimental results which are used in the following comparisons consist of Yorulmaz's six
tests on single-story structures with frame reinforcement of 1.1 and 2.2 percent, and one test by Fiorato of a single-panel frame with 1.1 percent reinforcement. The latter test was performed on the uppermost panel of a five-story structure which had been previously loaded to failure and, thus, had support conditions which differed from Yorulmaz' tests.

Load-deflection curves for the two values of frame reinforcement are shown in Figures 5.1 and 5.2. The computed behavior, as shown, neglects the possibility of shear failure in the tension column which was discussed in the previous chapter. If shear failure should occur after formation of the initial crack, which is designated as point 'A' in both figures, the subsequent computed behavior would be expected to maintain the load level of 'A' and develop large undetermined displacements with the formation of a failure mechanism in the portion contained by the beam and the compression column.

In Fig. 5.1, comparison of the initial stiffnesses from tests shows excellent agreement with the computed value for Yorulmaz's test F3 and Fiorato's top-story test. The initial stiffnesses for tests F7 and F8 are shown to be much higher, which can be explained by the possible insensitivity in the deflection gages at low displacements. In Fig. 5.2, excellent agreement is
observed for test F5, with the other two test values occurring to either side of the computed slope. Since the computed behavior in both figures was obtained for a filler modulus of 2,500,000 psi, the tests verified that the choice of this value was reasonable.

Considering the location of initial cracking, all tests showed the characteristic 'top crack' to occur first, which is contrary to the initial crack location for the computed plots. However, from the discussion of the effect of a change in filler modulus on the initial crack location which was presented in the preceding chapter, it is seen that, for the cases of 1.1 and 2.2 percent reinforcement, filler moduli of 2,800,000 and 3,100,000 psi respectively would have produced the top crack without significantly altering the load-deflection curve. In view of the uncertainty of the values for the filler moduli at the time of testing, it may be concluded that the analytical solution affords a reliable means of predicting the location and the initial crack in the filler.

Since all of Yorulmaz's tests as shown in Fig. 5.1 exhibited failure in the frame, after initial cracking in the filler, either through shear in the tension column or by developing an extensional hinge at one of the load points in the beam, the ultimate load corresponding to point 'A' shows good agreement with tests F7 and F8.
Fiorato's top-story test (Fig. 5.1) did not exhibit frame failure after initial cracking is in Yorulmaz's tests. Full development of the top crack was followed by development of the bottom crack concurrent with cracking of the tension column. This test thus verified the possibility of obtaining the computed behavior shown subsequent to points 'A' in Figures 5.1 and 5.2.

5.3 Five-Story Structures

The source for the experimental results are four tests by Fiorato on structures with 1.1 and 2.2 percent frame reinforcement. The tests are designated as FF1B, FF1C, FF2B, and FF2C where the numerals refer to the percentage of reinforcement.

Since the analysis of five-story structures was obtained with two-story replacement structures, comparisons between computed and experimental load-deflection response are presented in terms of the tension column displacement at the second-story level and the total horizontal load on the five-story structure.

Figure 5.3 compares the computed and experimental load-deflection plots for structures with 1.1 percent reinforcement. The plot for FF1C is shown with the initial portion omitted since no measurements were recorded for this range.
The major discrepancy between predicted and observed behavior is indicated for the location and the load level at which initial cracking occurs. Analysis predicted initial cracking in the filler at a load level of 124 kips, whereas test FF1B produced initial cracking in the tension column at an equivalent load of 51 kips.

Corresponding to the 51 kips load, a maximum flexural unit stress of 85 psi was computed at the base of the structure on the basis of elementary beam theory and the assumption of an uncracked cross section, which consisted of the columns as flanges and the filler as the web. In view of the tensile strength which could be expected from the high quality concrete used in the test specimens, initial cracking in the column cannot be justified solely on the basis of the computed unit stress of 85 psi. Since simple beam theory and the assumption of an uncracked section yields an unreasonably low stress in the tension column at initial cracking, the influence of extraneous factors on the behavior of the test specimen is indicated.

The assumption of an uncracked section is suspect due to the possibility that cracking could have been caused by lifting the structure from the casting bed and during erection in the test frame. The mortar joint between the frame and filler would be particularly vulnerable to
separation due to the smooth concrete surface. Visual inspection of the structure after erection did not reveal cracking; however, the existence of discontinuities on this basis is inconclusive.

The possibility of differential shrinkage in a structure composed of two dissimilar materials such as concrete and masonry would tend to create residual tensile stresses in the columns and balancing compressive stresses in the filler. Due to thin mortar joints in the filler, it can be expected that shrinkage of the filler would be very small or nonexistent, whereas shrinkage of the concrete frame would be relatively significant. Shrinkage effects are more pronounced in small-scale specimens on account of the rapid loss of moisture which can occur after curing has stopped.

During fabrication, the five-story structure was subjected to two periods of curing. The first period followed casting of the frame and lasted for five days. After the filler was in place, the structure was cured for one day. Following the second curing period, the structure was left exposed until the time of testing. Of the four structures tested, the shortest duration of the latter exposure period was 18 days, which allowed ample time for shrinkage effects to be set up.
A precise quantitative evaluation of residual stresses is beyond the scope of this study since it involves a timewise interrelationship of shrinkage with creep. Simplified calculations will suffice to indicate the magnitude of the stresses which may be expected in actual single- or five-story structures.

Shrinkage was considered to occur in a direction parallel to the columns and to produce a uniaxial state of stress. The particular structure for which the following stresses were obtained corresponded to solution 51. Creep of the concrete in the frame was tacitly recognized by substituting a reduced value for the elastic modulus of 2,000,000 psi.

Assuming a free shrinkage unit strain of 0.0005 for the frame concrete and no shrinkage for the filler material, the following stress values were computed.

- Stress in the concrete: 680 psi (tension)
- Stress in the filler: 400 psi (compression)
- Stress in the reinforcement: 4750 psi (compression)

With the additional assumption that filler shrinkage was \( \frac{1}{4} \) of the concrete shrinkage, the corresponding stresses were obtained as: 530, 280, 7100 psi.

The quantities listed above are, at best, crude approximations to the stresses caused by shrinkage in the concrete. However, they do indicate that the apparent
tensile strength of the concrete in the frame could have been reduced to the low level implied by the test results.

Cracking in tests FF1B and FF1C (Fig. 5.4, 5.5) was contained mainly in the two lowest stories and justified the use of the "two-story method" of analysis. The best correlation with the predicted crack formation (Fig. 4.24) was provided by test FF1B. It is difficult to assess the reasons for obtaining more extensive cracking and a higher ultimate load for test FF1C in comparison with FF1B or the predicted case. Variations in material properties (steel, concrete, masonry filler) are possible causes of these discrepancies.

The load-deflection curves for frames with 2.2 percent reinforcement are compared with solution 52 in Fig. 5.6. At large deflections, the ultimate load obtained in the two tests approached the predicted value. The apparent stiffnesses of both specimens were less than predicted by analysis, and indicated that the material properties which were assumed for solution 52 did not correspond with the actual test conditions.

An explanation for this discrepancy is presented with reference to Fig. 5.7 which shows the initial portions of the same three curves of Fig. 5.6 and includes an additional curve from solution 56 which was produced with reduced values for the cracking strengths of the
concrete in the frame and the filler material. The deflections predicted by the latter solution generally exceed the experimental values which indicates that the reduction in material properties has been overestimated. In solution 56, the cracking strength of the frame was assumed to be one-third of the value used in solution 52, and similarly the filler cracking strength was reduced to two-thirds of the original value.

These reductions were chosen to anticipate the effects of shrinkage on the behavior of the test specimens. Since shrinkage would create residual tensile stresses in the columns the reduction of the cracking strength for the frame members is reasonable. However, the corresponding residual compressive stresses in the filler would effectively increase the filler cracking stress. Therefore, the response as predicted by solution 56 should not be assumed to reflect the behavior as affected by shrinkage stresses, but to indicate a trend toward increased structural flexibility which is obtained by lowering the strength characteristics in the analyses.

A more realistic estimate of shrinkage effects may be obtained by considering an increase in the limiting value for filler cracking (150 to 200 psi) and a decrease for frame cracking (350 to 175 psi). Table 3 presents the computed loads which would cause cracking in frame and
filler for the single- and five-story structures. The lower value of the two loads for each case determines the location of the initial crack, and is shown underlined in the table. Cases 2 and 4 represent the effects of shrinkage.

For the particular variations in the limits as chosen, shrinkage effects are shown to shift the initial crack location into the frame in the case of the five-story structure, whereas initial cracking for the single-story structure would still occur in the filler. Since this trend was observed in the tests, the preceding discussion serves to indicate that shrinkage could account for initial cracking in the frame for five-story structures and in the filler for the single-story structures.
6. SUMMARY

6.1 Review

The main objective of this study was to develop a procedure for the analysis of reinforced concrete frames with masonry filler walls subjected to in-plane forces. Determination of the load-deflection response was obtained for the entire range of loading up to ultimate. The analytical scheme recognized the occurrence of cracking within the frame and filler with increasing load.

The method of analysis was based on a lumped-parameter model representation of the actual structure as shown in Fig. 2.5. The infilled frame was considered to be a plate with stiffening elements occurring on the boundary and within the plate itself. In-plane equilibrium equations were derived for the plane continuum and transformed into equivalent equilibrium equations in terms of the analytical model. The resulting equations were augmented with terms which recognized the existence of the frame, and were equivalent to the finite-difference representations of the in-plane behavior of a plate with stiffening flexural and extensional elements. Formulation of the analytical process required assumptions regarding the behavior of the frame and filler materials, but retained the generality of overall structural behavior.
since the progressive locations of cracking within the structure were all determined on the basis of successive solutions.

The deformable nodes within the analytical model were interpreted as consisting of a set of spring components whose stiffness characteristics were obtained from the derived equilibrium equations. A scheme was devised to modify the constituent spring components within the nodes to represent the existence of cracking.

To produce a complete solution for a given structure, the analysis was programmed for a digital computer and yielded stresses, displacements, and the sequence of cracking within the model structure as load was increased up to ultimate.

Two basic structures were analyzed: single-story and five-story single-bay frames with horizontal loads applied at two points in each beam (Fig. 4.1, 4.2). To conserve computation time, the five-story structure was analyzed as an equivalent two-story structure with an auxiliary load system which reproduced the load effect of the upper three stories on the two lower stories in the original structure.
6.2 **Comparisons Between the Computed and Experimental Results**

Shear strength in the frame was a critical factor in determining the ultimate load level in both single- and five-story structures. Tests of single-story structures indicated that failure occurred as a result of shear at the upper end of the tension column or by formation of an extensional hinge in the beam. The latter failure mode was obtained in structures with low percentages of reinforcement. Analysis verified the critical shear condition in the tension column as shown in Fig. 4.12 and 4.16.

The load-deflection curves for single-story structures as compared with experimental results are shown in Fig. 5.1 and 5.2. Points 'A' in both figures indicate the stage in the computed curves at which critical shear conditions existed in the columns.

Tests showed that five-story structures were prone to shear failure at the base of the compression column. Of the four structures tested, one structure (FF1C) has a premature shear failure as the applied load was increasing, whereas the other three developed extended deflections at ultimate load (Fig. 5.3 and 5.6).

The comparisons between solutions 52, 56 and the test results for the five-story structures (Fig. 5.7)
indicate that the apparent tensile strength of concrete in the frame is lower than would normally be expected. This decrease may be due to residual stresses as caused by shrinkage or undetected cracking which was introduced during fabrication of the specimen. Since shrinkage effects are aggravated as the size of the specimen is decreased, it is expected that the effect of shrinkage stresses would be less apparent in full-size structures than in small-scale models.

The effect of introducing a filler into a single-story structure is shown with reference to the computed load-deflection curves of Fig. 4.13. The plots show that the same ultimate load capacity may be generally expected, irrespective of the value of filler modulus. This is of particular significance in cases where a filler has a very low elastic modulus as a result of poor workmanship or low-quality materials.

Graphical representations of the role assumed by the filler in resisting shear and overturning moment in the uncracked structure are provided by Fig. 4.5 and 4.6. With increasing values of the filler modulus the shear and moment in the columns are sharply reduced for values below 1,000,000 psi and remain relatively constant above 3,000,000 psi. The percentage of the total moment supplied by axial forces in the frame reaches a maximum
at a low value of filler modulus (600,000 psi). For values greater than 1,000,000 psi, the increase in the resisting moment provided by the filler is approximately equal to the decrease in the resisting moment due to axial loads in the frame.

6.3 Conclusions

This study has demonstrated that a discrete physical model analysis may be successfully employed in determining load-deflection response and crack formation in reinforced concrete frames with masonry fillers. Procedures were developed for modifying the components of the model to reproduce the effect of cracking in the frame and filler of an actual structure. The model is intended for the analysis of structures which are composed of tension-weak materials such as concrete and masonry.

Model representations of structural systems are particularly useful in handling problems with partial loadings and complex boundary conditions. The numerical procedures which are involved in producing a solution must necessarily be programmed for a digital computer.

A general computer program based on the discrete physical model is a versatile tool for analysis. Different structural configurations may be defined (multi-bays, multi-stories, filler openings). Thus it would be
possible to evaluate the behavior of existing structures without resorting to time-consuming and expensive tests. By varying the input data, behavior could be studied for different relative stiffnesses between frame and filler, or for different failure conditions.

Tests with laboratory specimens have indicated that analytical studies may overlook such effects as residual stresses due to shrinkage, and cracking which was not caused by the application of external load. Laboratory investigations may be profitably employed in defining the significant parameters which should be considered in the analysis, and for correlating results from analysis.
LIST OF REFERENCES


<table>
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<tr>
<th>Mark</th>
<th>No. of Stories</th>
<th>Frame Reinforcement percent</th>
<th>Filler Modulus $\times 10^6$ psi</th>
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<tbody>
<tr>
<td>11</td>
<td>1</td>
<td>1.1</td>
<td>2.5</td>
</tr>
<tr>
<td>12</td>
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</tr>
<tr>
<td>14</td>
<td>1</td>
<td>2.2</td>
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<td>2.2</td>
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<td>4.4</td>
<td>2.5</td>
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<tr>
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<td>5</td>
<td>2.2</td>
<td>2.5</td>
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<tr>
<td>55 (b)</td>
<td>5</td>
<td>2.2</td>
<td>2.5</td>
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<td>56 (c)</td>
<td>5</td>
<td>2.2</td>
<td>2.5</td>
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(a) Solution with additional vertical loads
(b) Solution with openings in the filler
(c) Solution with decreased limiting unit tensile stresses for cracking - 117 psi (frame), 100 psi (filler)
## TABLE 2

### INVARIANT QUANTITIES FOR THE ANALYTICAL SOLUTIONS

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Quantity</th>
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<tr>
<td>$P$</td>
<td>Influence Load</td>
<td>400 kips per story</td>
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<tr>
<td>$\lambda$</td>
<td>Grid Spacing</td>
<td>33 in.</td>
</tr>
<tr>
<td>$h$</td>
<td>Filler Thickness</td>
<td>7 in.</td>
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<tr>
<td>-</td>
<td>Cross Section of Frame</td>
<td>24 in. x 24 in.</td>
</tr>
<tr>
<td>$f_Y$</td>
<td>Yield Strength of Reinforcement</td>
<td>42,000 psi</td>
</tr>
<tr>
<td>-</td>
<td>Elastic Modulus of Reinforcement</td>
<td>30,000,000 psi</td>
</tr>
<tr>
<td>-</td>
<td>Limiting Unit Tensile Stress for Cracking in the Frame</td>
<td>350 psi</td>
</tr>
<tr>
<td>$f_{C}^{'}$</td>
<td>Compressive Strength of Concrete in the Frame</td>
<td>3,500 psi</td>
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<td>$E_f$</td>
<td>Elastic Modulus of Concrete in the Frame</td>
<td>3,500,000 psi</td>
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<tr>
<td>-</td>
<td>Limiting Unit Tensile Stress for Cracking in the Filler</td>
<td>150 psi</td>
</tr>
<tr>
<td>-</td>
<td>Compressive Strength of Filler</td>
<td>3,750 psi</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's Ratio</td>
<td>zero</td>
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TABLE 3

COMPUTED LOADS FOR INITIAL CRACKING AS AFFECTED BY A VARIATION IN THE ASSUMED LIMITING VALUES FOR CRACKING

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of Stories</th>
<th>Assumed Limiting Stress Values for Cracking (psi)</th>
<th>Computed Failure Loads (kips)</th>
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<tr>
<td></td>
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<td>Frame</td>
<td>Filler</td>
</tr>
<tr>
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<td>5</td>
<td>350</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>175</td>
<td>200</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>350</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>175</td>
<td>260</td>
</tr>
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</table>
FIG. 2.1 STRESS RESULTANTS IN WALL ELEMENT
FIG. 2.2 ANALYTICAL MODEL FOR FILLER WALL

\begin{align*}
\lambda & \text{ Grid Spacing} \\
\gamma, j, v, y & \rightarrow x, i, u, x
\end{align*}
Equilibrium Equation in \( x \)-Direction

\[
\frac{\text{Eh}}{4(1-v^2)} \begin{bmatrix}
\nu & -\frac{(1+v)}{2} & \nu \\
(1-v) & (1+v) & (1-v) \\
1-2j & i-1j-1 & i+1j-1
\end{bmatrix} = x_{ij}
\]

Equilibrium Equation in \( y \)-Direction

\[
\frac{\text{Eh}}{4(1-v^2)} \begin{bmatrix}
\nu & -\frac{(1+v)}{2} & \nu \\
(1-v) & (1+v) & (1-v) \\
1-2j & i-1j-1 & i+1j-1
\end{bmatrix} = y_{ij}
\]

Fig. 2.3 EQUILIBRIUM EQUATIONS FOR MASS POINT \( ij \) IN THE FILLER
FIG. 2.4 EQUILIBRIUM EQUATIONS FOR MASS POINT $ij$ ON UPPER BOUNDARY
FIG. 2.5 MODELS FOR THE SINGLE-STOREY STRUCTURE
FIG. 2.6 DETAIL OF THE ANALYTICAL MODEL
FIG. 2.7 STRESS POINT IDEALIZATION
FIG. 3.1 POSSIBILITIES OF SUSTAINING MEMBRANE FORCES IN A REGION OF THE FILLER WHICH CONTAINS A CRACK
FIG. 3.2 COMPLETED LOAD-DEFLECTION PLOT FOR SINGLE-STORY STRUCTURE - SOLUTION II

Deflection \( \Delta_1 \) (in.)

Total Load (kips)
The Actual Structure

Arrangement of Mass Points In The Analytical Model

FIG. 4.1 THE SINGLE-STORY STRUCTURE
FIG. 4.2 THE FIVE-STORY STRUCTURE
FIG. 4.3 VARIATION OF DEFLECTION $\Delta_1$ WITH ELASTIC MODULUS OF FILLER
FIG. 4.4 DEFLECTION MODE SHAPES FOR TENSION COLUMN
FIG. 4.5 DISTRIBUTION OF BASE SHEAR BETWEEN FRAME AND FILLER, SINGLE-STORY STRUCTURE
FIG. 4.6 DISTRIBUTION OF RESISTING MOMENT IN THE BASE OF THE SINGLE-STORY STRUCTURE
FIG. 4.7 PRINCIPAL TENSILE STRESSES AT THREE STRESS POINTS
Numbers Indicate Chronological Order of Failure

Circled Numbers Indicate that an Increase in Load From the Previous Load Level Was Necessary to Attain Failure

Compression Failure in the Filler

Existing Crack From Previous Failures

Yielding of Reinforcement

Cracking of Concrete in The Frame

FIG. 4.8 SYMBOLS EMPLOYED IN FIGURES OF CRACK FORMATION
FIG. 4.9 CRACK DEVELOPMENT-SOLUTIONS 11 AND 12

Solution 11
$E_w = 2,500,000$ psi

Solution 12
$E_w = 2,800,000$ psi
Fig. 4.10 Computed load-deflection plots for single-story structure - p = 1.1%
FIG. 4.11 ASSUMED FAILURE ENVELOPE FOR FRAME MEMBERS SUBJECTED TO COMBINED AXIAL LOAD AND SHEAR
\[ E_w = 2,800,000 \text{ psi Solution 12} \]
\[ E_w = 2,500,000 \text{ psi Solution 11} \]

**FIG. 4.12 SHEAR AND AXIAL LOAD CONDITIONS IN THE TENSION COLUMN OF SINGLE-STORY STRUCTURES AFTER FORMATION OF FIRST CRACKING**

\( p = 1.1\% \)
FIG. 4.13 COMPUTER LOAD-DEFLECTION PLOTS FOR SINGLE-STORY STRUCTURES - $\rho = 2.2\%$
FIG. 4.14 CRACK DEVELOPMENT - SOLUTIONS 14 and 15
FIG. 4.15 CRACK DEVELOPMENT - SOLUTIONS 16 and 17

Solution 16
$E_w = 3,100,000 \text{ psi}$

Solution 17
$E_w = 4,500,000 \text{ psi}$
FIG. 4.16 SHEAR AND AXIAL LOAD CONDITIONS IN THE TENSION COLUMN OF SINGLE-STORY STRUCTURES AFTER FORMATION OF FIRST CRACKING - $p = 2.2\%$
Stress Scale: 1 in. = 100 k

Fig. 4.17 Principal Compressive Stresses - Solution 16
FIG. 4.18 PRINCIPAL COMPRESSIVE STRESSES WITH FILLER CRACK - SOLUTION 16
FIG. 4.19  PRINCIPAL TENSILE STRESSES WITH FILLER CRACK - SOLUTION 16

Stress Scale: 1 in. = 100 k
FIG. 4.20  PRINCIPAL COMpressive STRESSES WITH FILLER CRACK - SOLUTION 15
FIG. 4.21  PRINCIPAL TENSILE STRESSES WITH FILLER CRACK - SOLUTION 15

Stress Scale: 1 in. = 100 k
FIG. 4.22 ANALYTICAL MODELS OF THE FIVE-STORY STRUCTURE
FIG. 4.23 COMPUTED LOAD-DEFLECTION PLOTS FOR THE FIVE-STORY STRUCTURE
FIG. 4.24 CRACK DEVELOPMENT - SOLUTION 51
FIG. 4.25  CRACK DEVELOPMENT - SOLUTION 52
FIG. 4.26 CRACK DEVELOPMENT - SOLUTION 53
Fig. 4.27 Deflection modes of tension column, solution 53
Crack Sequence Numbers - Ref. To Fig. 4.26

Percentage of Deflection at The Second Story Level

FIG. 4.28 DEFLECTION MODES OF COMPRESSION COLUMN, SOLUTION 53
FIG. 4.29  PRINCIPAL COMPRESSIVE STRESSES - SOLUTION 52
FIG. 4.30  PRINCIPAL COMPRESSIVE STRESSES WITH FILLER CRACK - SOLUTION S2
FIG. 4.31  PRINCIPAL TENSILE STRESSES WITH FILLER CRACK - SOLUTION 52
FIG. 4.32 SHEAR AND AXIAL LOAD CONDITIONS IN THE TENSION COLUMN OF FIVE-STORY STRUCTURE
FIG. 4.33 AXIAL LOAD AND BENDING MOMENT IN COMPRESSION COLUMN - SOLUTION 53
FIG. 4.34 AXIAL LOAD AND BENDING MOMENT INTERACTION CURVE

- Cantilever Method
- Portal Method

Compression Column - Solution 53 (see Fig. 4.32)
FIG. 4.35 AXIAL LOAD AND BENDING MOMENT IN COMPRESSION COLUMN - SOLUTION 52, 53
FIG. 4.36 LOAD-DEFLECTION PLOTS - SOLUTIONS 52,54
FIG. 4.37 CRACK DEVELOPMENT - SOLUTION 54
FIG. 4.38 FREE BODY OF MULTI-STORY STRUCTURE AT ULTIMATE
FIG. 4.39 PRINCIPAL COMPRESSIVE STRESSES, SOLUTION 55

Stress Scale: 1 in. = 400 k
Stress Scale: 1 in. = 400 ksi

FIG. 4.40 PRINCIPAL TENSILE STRESSES, SOLUTION 55
FIG. 4.41 COMPUTED LOAD-DEFLECTION PLOTS - SOLUTIONS 52, 55
FIG. 5.1 COMPUTED AND EXPERIMENTAL LOAD-DEFLECTION PLOTS, SINGLE-STORY STRUCTURES - $p = 1.1\%$
FIG. 5.2 COMPUTED AND EXPERIMENTAL LOAD-DEFLECTION PLOTS, SINGLE-STORY STRUCTURES - p = 2.2%
Deflection $\Delta_2$, Model Structure - (in.)

Deflection $\Delta_2$, Full-Size Structure - (in.)

Solution 51
Test FF1B (Fiorato)
Test FF1C (Fiorato)

FIG. 5.3 COMPUTED AND EXPERIMENTAL LOAD-DEFLECTION PLOTS, FIVE-STORY STRUCTURES - $p = 1.1\%$
FIG. 5.4 CRACK PATTERNS, TEST FF1B
Direction of Loading

FIG. 5.5 CRACK PATTERNS, FF1C
FIG. 5.6 COMPUTED AND EXPERIMENTAL LOAD-DEFLECTION PLOTS, FIVE-STORY STRUCTURE - $p = 2.2\%$
FIG. 5.7 COMPUTED AND EXPERIMENTAL LOAD-DEFLECTION PLOTS, FIVE-STORY STRUCTURE - p = 2.2%
APPENDIX. THE COMPUTER PROGRAM

The computer program which was developed for the analytical solutions was written for the IBM 7094-1401 system at the University of Illinois Digital Computer Laboratory. Coding of the problem was done entirely in FASTRAN which is a fast-compile version of FORTRAN II.

Storage and computational requirements of the program were all contained within the computer core so that the inefficiencies of extra-core data storage devices were avoided. The program has the capacity to solve a full five-story structure (360 equations) of the type shown in Fig. 4.2. Solutions for the equivalent two-story structure (144 equations) required 8½ seconds for each computational cycle as shown in Fig. A.1. The single-story structure (72 equations) required 3½ seconds per cycle.

Since the analytical model considers the displacement components at each mass point to be the unknown quantities, generation of the equilibrium equations is identical to the generation of the stiffness matrix for the model structure. Two methods were employed to solve for the unknown displacements - a direct procedure from the Gauss elimination method, and the Gauss-Seidel iterative method. The Gauss elimination technique was used to
obtain the initial solutions. It was observed that as cracking in the structure became more extensive, continued use of Gauss elimination became impractical as serious round-off errors appeared. Subsequent solutions were obtained by the Gauss-Seidel method which will theoretically converge to the correct solutions provided that certain conditions regarding the sum of absolute values of the off-diagonal terms hold. Practical use of Gauss-Seidel requires a finite number of iteration cycles so that there is always a truncation error. The process may be improved for convergence in a finite number of iterations by use of over-relaxation, the exact magnitude of which can best be determined by experience with the particular system being solved. With regard to the present problem, the accuracy of the solution could be determined by performing an independent check for force equilibrium within the model from the print-out of model stresses.

In both methods, storage of the entire stiffness matrix was unnecessary since the individual equations were generated as required. The symmetry and banded nature of the stiffness matrix was recognized in the Gauss elimination technique by using a procedure which required storage for only half of the terms in the nonzero band.

The majority of solutions which were performed for structures consisting of materials with normal elastic
moduli, initial cracking occurred in the filler. Thus, the initial solutions were performed for stiffness matrices which differed only slightly from the immediately preceding ones. Displacements in the structure differed from one solution to the following by small monotonic increments. For this type of behavior, the Gauss-Seidel iterative method is ideally suited since the program contains the solution for the previous displacements which are close to the new values. Somewhat larger errors resulted in the later solutions when yielding occurred in the frame steel and produced large increments in displacements. The use of Gauss-Seidel at this stage was a drawback, but since the stiffness matrix had become increasingly sparse, the alternative of reintroducing Gauss elimination was impractical.
START

READ INPUT DATA

- PRELIMINARY COMPUTATIONS
- SET UP ORDER CODE TO DEFINE THE MASS POINT LOCATIONS
- INITIALIZE BOOKKEEPING SYSTEM FOR RECORDING STRESS POINT FAILURES

GENERATE LOAD VECTOR

LAST SOLUTION? 

YES  

STOP

NO

COMPUTE DISPLACEMENTS

COMPUTE STRESSES

PRINT OUT STRESSES AND DISPLACEMENTS

DETERMINE NEW CRACK LOCATION AND CORRESPONDING FAILURE LOAD

MODIFY STRUCTURE FOR EXISTENCE OF NEW CRACK

FIG. A.1 FLOW DIAGRAM
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<td>Coordinator, Marine Corps Landing Force Development Activities, Quantico, Virginia 22133</td>
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<tr>
<td>Chief, Bureau of Medicine and Surgery, Department of the Navy, Washington, D.C. 20390</td>
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A LUMPED-PARAMETER MODEL TO SIMULATE THE RESPONSE OF REINFORCED CONCRETE FRAMES WITH FILLER WALLS

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A discrete physical model (lumped-parameter) has been developed for the analysis of reinforced concrete frames with masonry filler walls. The entire analytical procedure has been programmed for solution by digital computer, and yields crack formation and the load-deflection response up to the ultimate load. Comparisons are presented between results as obtained from analysis and tests of single- and five-story single-bay structures subjected to lateral loads.
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