One Mathematician Looks at the Classification of Mathematics

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At the very outset I want to warn you that I am here in the role of a mathematician who is interested in books and that I am entirely innocent of library procedures and terminology, the theory of classification, or the actual classification of anything but mathematical books. I am not sure that words of wisdom have ever come from the mouths of infants, but I am very strongly relying on that possibility. If this hope proves wrong, then I can only apologize and point out that every carnival should have a freak show and that I am only trying to do my duty.

I am also aware of the extensive use of the vertical pronoun in my talk, but I know of no alternative. I have spoken with a number of my mathematical colleagues, but I do not pretend that my remarks are really an accurate statement of the ideas of the mathematical community.

I shall be more than satisfied if I can act as a gadfly and provoke some discussion. Many of my remarks are very frankly critical. However, it is my earnest hope that they will not be taken offensively, but that they might be turned to constructive use. If this can be done, I shall be most pleased.

MATHEMATICAL TERMINOLOGY AND THE RESULTING CONFUSION

I should like to make a few remarks about mathematical terminology which may distinguish mathematics from certain other fields. Unlike chemistry which has a large supply of artificial technical words which is constantly augmented, the tendency in mathematics is to use homely words and to attach a new, technical meaning to these words. Thus, for example, the nouns "group," "ring," "ideal," "lattice," "field," "neighborhood," "measure," "sheaf," "fiber bundle," "place," etc., denote definite mathematical concepts whose exact meaning cannot be guessed—in fact, it is not even apparent in which area of study these words are used. Also, modifiers such as "regular," "normal," "absolute," "proper," "analytic," etc., are used in a quite technical fashion. (The meaning is not necessarily unique, however--the words "regular" and "normal" have well over

a dozen totally different usages.) This is not to say that we mathematicians do not have our words such as "homeomorphic," "isometric," "automorphism," "eigenfunction," but I can think of nothing in mathematics as dramatic as chemistry's word "dichloro-diphenyl-trichloro-ethane" (DDT).

In addition to attaching technical meanings to old, familiar terms, mathematicians often take over proper names; thus we get "Euclidean geometry," "Riemannian geometry," "Riemann zeta function," "Riemann integral," "Riemann surface," "Hilbert space," "Fourier series," "Chebyshev polynomial," and many others.

I am sure that these terminological practices (which occur in mathematical writings in every European language), complicate the job of the non-specialist classifier. Nevertheless, this practice is not likely to be discontinued if the alternative to using the word "measure" is to employ the far more cumbersome equivalent phrase "non-negative extended real-valued countably additive set function which vanishes at the empty set," which, in addition to being unwieldy, itself employs many technical terms. As a matter of fact, the word "measure" is fairly descriptive if one realizes that it is intended to generalize the notion of length, area, volume, mass--in short, the measure. However, it is easy to see that a book entitled *Measure Theory*, by Paul R. Halmos, will cause difficulty to the average librarian. I should like to take an imaginary trip with this book as it leaves the publisher, in 1950, and finds its way into the mathematics library. There is a joke among mathematicians that this book was actually classified, in some unnamed library, with the books on carpentry.[1] A more likely classification would be to put it in 510 V2, since it is published by Van Nostrand in their "University Series in Higher Mathematics." This number does put it in the mathematics bracket, so certainly is to be preferred to carpentry, but I do not feel that it is a very good classification as I hope to make clear later. Let us suppose, then, that the book has managed to elude the Serials Department (which might be possible since the so-called series to which it belongs is not numbered and this is only the second one of the Van Nostrand to be bound in blue.) What is in store for the book now? The answer might be that the Library of Congress card is obtained and the book is classified 513.83, since the book is declared to be concerned with the subject of topology (which is not accurate) and since topology, according to the 14th edition of Dewey, is a subfield of non-Euclidean geometry (which is not accurate either). Although the classification is not correct, it is better than the other possibilities I mentioned, and I would far rather leave it there than to move it to 512.812, which is where the new, presumably more modern and accurate, 16th edition of the Dewey classification system would have it put. Why would they place it there? Because, I am sure, that the designers of this system are under the impression that "measure theory" deals with ideas connected with divisibility and the old-fashioned theory of measurement, sometimes called "mensuration."

They made a guess which sounded plausible, but they are wrong. It is certainly easy to be misled by the similarity of the words "measure theory" and "mensuration," but I regret that the Library of Congress found it necessary to guess not only about the classification of a single book, but about an entire subject.

Another error that it is easy to commit is to group together books dealing with subjects (or objects) identified in part by the name of a man. For example, the 16th edition of Dewey classifies together, in 517.81, books dealing with Riemann surfaces and the Riemann zeta function, even though the content of these books is quite different. Again, one not familiar with the technical nature of these two subjects could not know that they are so different, but I do not feel that one who does not have this familiarity should be revising the classification system without considerable advice.

I have chosen only two examples of errors of this type; others could be adduced if there were any point in doing so.

CROSS FIELDS

There is another phenomenon that occurs in mathematical terminology, although I am sure that it is probably present in most other fields, as well, I refer to the interplay between various subareas which makes difficulties for a linear system of classification. In a sense, mathematics can be broken into five main areas of specialization: algebra, geometry, analysis, a newer area called topology, and applied mathematics (including statistics, mathematical physics, etc.). (In making this division, I have ignored topics such as mathematical logic or the history of mathematics, since I regard these areas as applied logic and applied history.) In addition to these five main fields there are familiar cross fields such as analytic geometry, which is primarily geometry, and algebraic geometry, which was geometry in the past but has recently become primarily algebraic and should be called "algebra with geometric terminology" or simply "geometric algebra." Recently, the similar-sounding fields of "algebraic topology" and "topological algebra" have appeared on the scene. Unfortunately, it is the case that at the present time both terms are misnomers. What is presently done in the temple of "algebraic topology" is algebra, and I think no one disputes it. Worse yet, what most people now do under the name of "topological algebra" is neither algebra nor topology, but really analysis. Even mathematicians, who tend to be somewhat perverse in their humor, do not like this terminological mess and these two misleading terms are gradually being replaced by the more technical and temporarily more accurate terms "homological algebra" and "functional analysis."

I have gone into this fairly extended and relatively technical discussion, not primarily to amuse you with the quixotic character of mathematicians who can't say what they mean or to amuse myself by joking at librarians who can't guess what the mathematicians mean

by what they say. My point is that even carefully chosen words develop new and different meanings, that subject areas merge and change in content and in direction, and that the outsider has little hope of guessing correctly.

The remarks I have just made apply to myself just as much as anyone. Although I have spent some time studying mathematics and have a fairly good exposure to the kind of things studied in its various branches and specialties, I do NOT have the knowledge to classify accurately the mathematics books published today in a system as detailed as the Dewey or the Library of Congress systems. On a number of occasions when I have been consulted by our mathematics librarian, I have not been able to specify the classification without consulting one of my colleagues in the mathematics department. The subject of mathematics is entirely too large and complex for a single man, even a specialist in the field, to keep up in it and to have a detailed knowledge of its interconnections, let alone the main results. Not only has the universal scholar disappeared, but even the universal geometer has gone from the scene.

CLASSIFICATION BY SERIES

I have already noted, with disparaging tones, the practice of classifying books in series. Unquestionably this is appropriate in the case of journals and many of the publications of universities and learned societies. However, I have serious doubt as to its wisdom in the case of a sequence[2] (I purposely avoid the term "series") of books put out by a commercial publisher, unless there is a clear underlying principle or unless the books deal with the same subject. One of the absurd results of this method of classification is that a translation, or a later edition, of a book may be separated from the original. Surely this is a mistake!

I am aware of the greater simplicity and the routine nature of assigning a number to an incoming member of a serial publication. Nevertheless, I believe it to be a poor procedure to follow and an evasion of the problem of finding the proper classification. Perhaps one reason I object is that practically all of the publication of mathematical books is in sequences, but another reason is that I believe that this method is nothing more than a classification by color and design of the binding. One problem I have heard of is the inability of placing a standing order on a sequence of books without assigning the work to the serials division and thus accepting a serial classification. Although it may not be good economics, in most cases I would prefer to order the books separately than do this.

THE DEWEY SYSTEM IN MATHEMATICS

There are a few comments that I should like to make concerning the Dewey mathematical classification. The main one is that it is

about fifty years out of date. Nevertheless, I suspect that it is probably rather satisfactory in a small library, particularly one which does not contain many books in the newer areas of research. For instance, I think it would do quite satisfactorily for a teaching-oriented liberal arts college with only a thousand or so books. The trouble comes when one attempts to give a detailed classification of the books in the newer branches where there is considerable research activity, since the system does not take these areas into account. Even the new 16th edition does not take much cognizance of the extensive developments of the earlier decades of this century, so it is hardly possible to find a location for the diverse further investigations in these newer areas. As an example, the important new branch of topology is relegated to 513.83, which, in addition to being an obscure location, is also inexact, since topology is not a subfield of non-Euclidean geometry. To subdivide the books in the several new branches of topology, as might be desired, would cause the numbering system to become unwieldy. It would seem that a larger category must be assigned to this field *if* one wishes to maintain the present level of detail in the system.

The other side of the coin is that there is considerable waste in the Dewey mathematical classification as it stands. Let me recall the basic outline of the system. It is as follows:

510 Mathematics (including works on Mathematics in general, collections, dictionaries, journals, etc.)
511 Arithmetic
512 Algebra
513 Elementary Euclidean geometry (including non-Euclidean geometry)[3]
514 Trigonometry
515 Descriptive geometry and projections
516 Analytic geometry (including algebraic geometry[4])
517 Calculus
518 —Not assigned[5]
519 Probabilities[6]

The category 511, though needed for smaller libraries, is mostly wasted and should probably be consolidated with algebra in research libraries. I believe that the University of Illinois library has only about 200 books in this category, of which about one-half deal with commercial arithmetic and a large number are old textbooks which have mistakenly found their way into the stacks. Most of 512 is wasted in our library, only our subcategory 512.8 is available for modern mathematics, and it contains almost five times as many books as all the other subcategories combined (even though we do have a number of old algebra textbooks in these other divisions). The same situation occurs in 513 and 516, although to a lesser degree. Entry 514 is a dramatic waste, since trigonometry is such a tiny subject. We have

about 200 books in this category and would probably do just as well with one tenth as many. Still worse, from a mathematician's point of view, is 515; it is *all* waste, for the mathematical portion of "descriptive geometry" is a very small portion of "projective geometry" and the remainder (that is, the major portion of the subject) is not mathematics at all, but mechanical drawing. At the University of Illinois the category 517 is well used and, in fact, our local ground rules permit us to let it spill over into the unassigned category 518. In addition to a few textbooks on calculus, these categories contain many hundreds of books in mathematical analysis. As might be expected, 519 has a substantial number of entries, even though the applications of mathematics to physics and engineering are not included there. This may give an idea how uneven the system is in a large, up-to-date mathematical library.

I have already indicated that I think the Dewey system is fairly well suited for a small library which does not attempt to acquire modern research books in mathematics, but whose books are mostly those that would be needed for undergraduate instruction. The system is rather appropriate for books on this level, and was probably designed with these libraries in mind. But I also believe that for a library of this size and depth there is not much need to go beyond the ten categories 510 to 519. Some additional division might prove useful, particularly in the 510 group, but I doubt that much is really needed. In a large library where there will be several thousand books on mathematics more division is helpful--but only to the extent that it truly conforms to the nature of the subject. Obviously a classification system can never be up-to-date, for there are sudden spurts in the development of certain areas followed by long periods of inactivity. One must be conservative in changing the system and no change is worthwhile unless it is a basic and a fundamental change. Despite these remarks, I do feel that the Dewey system in mathematics needs to be updated *if* it is to provide a detailed system of classification, for it does not even get close to the frontier. However, one of the questions that must be decided is whether such a detailed system is really desirable and whether it is even possible at the present time.

To my mind, the 16th edition of Dewey does not solve any of the real problems. It corrects a few errors, but propagates most of the old ones plus a few new ones that would be unfortunate to introduce. It is certainly not a step forward, and I doubt that its good features are worth the cost and confusion that a change would cause.

POSSIBLE CHANGES AND MODIFICATIONS

There are many problems that must be solved by any new system and I am sure that everyone here is better acquainted with most of them than I, so I shall refrain from going into much detail. Still, let me list a few desiderata for any new system that occur to me.

1) It should accomodate small libraries easily.
2) It should be appropriate for large research libraries.
3) It should allow the classifier to assign class numbers to the books quickly and accurately.
4) It should be simple enough so the faculty can understand it.
5) It should permit future modifications.
6) It should not be too expensive to adopt.

There are certainly other desirable things that we might hope for, but we have already been somewhat optimistic. As you might expect, I am not going to present a completely-worked out solution to this problem today. I *do* believe that a thoroughly satisfactory system is possible. However, I believe that any such solution must be the product of joint thinking and arguing on the part of *both* librarians and mathematicians. I am convinced that neither group can reach a reall satisfactory solution without the other, for I believe that a non-speci: ist is unable to decide what the basic categories in a field are and is unable to determine how these categories are related without consulting a specialist. Further, I believe that the specialists are not sufficiently aware of library procedure and problems to anticipate all the difficulties that come up in practice.

Desired property 3, perhaps, can use some amplification. I cannot overemphasize the importance of quick and accurate classificatic In the mathematics of today (as in most fields) the first few years of most books' lives are the most useful ones. If it takes several month to obtain a book and then several weeks to classify it, much of its value has been dissipated. Also, if the actual classification of the book turns out to be inexact, it may not reach the hands of a user while it is of prime value. I should also like to note that there is still some indefiniteness about the nature of the classifier referred to in 3—it is obvious that the more detailed and specific the classification system is, the more specialized the classifier must be in orde to be quick and accurate in his work.

Desired property 4 is not to be overlooked, either. You know bett than I how well the average professor really understands the system he is using and complaining about. (I leave open the question of whet er he might complain more or less, if he understood it.)

Before I turn to a slightly different topic, I should like to make reference to a method used in the classification of research papers by the *Mathematical Reviews,* which is published by the American Mathematical Society. This system has almost no resemblance to either the Dewey or the Library of Congress system, partly because it is right up-to-date, partly because it was made by mathematicians partly because it is designed for papers and not books, and partly because it does not take into consideration many problems that a librar classification must consider. Nevertheless it is interesting and any of you who are concerned with this problem would do well to write to the editors of the *Mathematical Reviews* and get a copy.

HOW ELABORATE?

Before a more satisfactory system is created, there is a basic question that must be settled. It is to decide how elaborate and detailed the system is to be and, of course, this is intimately tied with who does the classifying. Clearly there is an advantage in having a system in which one knows exactly where the books on the Fredholm integral equation of the first kind are to be found. However, the advantages of such a refined system largely evaporate if, either (1) most books dealing with this topic also deal with another topic, or (2) the subdivisions are so small and numerous that they are frequently missed and the book shelved elsewhere, more or less by mistake. I believe that only a mathematician who specializes in the area can really determine whether (1) is apt to be the case, and to a large extent (2) is up to the classifier.

I maintain that a system is too elaborate for a given institution when most of the detailed categories have only a few entries. I believe it is too elaborate for the classifier in a given institution if he is unable to classify quickly and accurately most (say 95%) of the books. I would further say that the system is too elaborate for the faculty of the institution if they are not able to keep in mind the scheme used in classifying books in their area of specialization.

Although an updated system would be a great help, I do not believe that I would meet my own adequacy criterion on speed and accuracy for a system as detailed as the present Dewey or Library of Congress systems. Further, I do not think that any single person, be he librarian or mathematician, can meet this criterion—in any case there are not enough of them to go around. Therefore, unless each institution is to have a panel for the classification of mathematics—a situation I find somewhat difficult to imagine—I believe the alternatives are (1) to have the more technical books classified by some centralized bureau, (2) to encourage the classification to be done in part by the author and/or the publisher, and (3) to simplify the system of classification mostly by reducing the number of subdivisions. Actually I would like all three of these to be employed to some extent, but I think that the third is by far the most important and most practical.

It seems to me that the Library of Congress is the natural organization to attend to the more technical books, but it is my understanding that they do not always suggest classification and, as I have indicated, when they do make such suggestions in mathematics they are frequently wrong. Certainly they need more mathematical advice than they are presently getting. If they are not able to obtain technical advice directly, then they should turn to the various technical societies, such as the American Mathematical Society, the American Chemical Society, etc. Another possibility is that various of the reviewing organs (which appear to be staffed primarily by scientific personnel), might lend their aid in the classification of the more

technical books and/or the propagation of this information. In any case, I see no reasonable alternative to some type of collaboration between people trained in library science and people trained in the particular disciplines.

An elaborate system puts extreme demands on the classifier and on the user. The more detailed the system, the more difficult it is for both the classifier and the researcher to learn and to use, the more rapidly it goes out of date, the more sensitive it is to errors of classification and to shifts in the emphasis in the subject matter. My personal feeling is that a highly refined classification in mathematics is not practical at this time.

Since I have come out for a simple system, let me be specific as to how simple I would make it. I have in mind a system of basic categories that would be used by small non-research mathematics libraries with additional categories that would be of use to a more extensive library. For the smaller library, after giving items like mathematical tables, collected works, history of mathematics, and dictionaries and encyclopedias of mathematics their separate entries and adding 30% out of conservatism, I come up with the grand total of twenty. I think that even the largest research mathematics library does not really need more than fifty divisions in mathematics. (My real figure is thirty-two, but conservatism makes me jump to the larger figure. I have discussed this matter with a colleague at Northwestern University, and his suggested figure was seven, but I think he may be somewhat radical.) One of the best research mathematics libraries in the country, at the Institute for Advanced Study at Princeton, has found that it does nicely with two categories—books and journals. (It is only honest to admit that they are not at all concerned with elementary books and purposely want to keep the system simple, since most of their users are only there for a year or so.)

SUMMARY

Let me summarize my remarks.

1) I believe the present Dewey system in mathematics has profound drawbacks and should be changed to conform more to the present nature of the subject.

2) I suggest the Library of Congress obtain help from a panel of mathematical specialists both in regard to the system and the actual classification of individual books. Assistance might be forthcoming from its sister organization, the National Academy of Sciences, or from the editorial board of the *Mathematical Reviews*, or from the International Mathematics Union, or from the American Mathematical Society.

3) I believe it should be examined as to how detailed a mathematical classification system we need and can properly apply. My own

opinion is that we could reduce drastically the number of categories without harm and with a gain in simplicity.

4) I think the list of approved subject headings should be revised in the light of current mathematics. If a small number of classification entries is employed, a fuller list of subject headings might be useful. In any case a modernization is in order.

5) I feel that the author of a book has the most intimate knowledge of its content and is best qualified to indicate appropriate subject headings. To some extent, he could assist in the classification.

6) The publisher should be encouraged to print the classification number and the subject headings inside the book along with the number of the Library of Congress card which many of them now carry. Agreement on the classification number and the headings might be accomplished at the time of the application for copyright.

In conclusion, I would like to say that I am at least cognizant that there are many difficulties which would have to be surmounted in accomplishing these proposals and not so idealistic that I expect much to come of them. However, I believe that the cost of inaugurating and implementing these hastily sketched suggestions would be small compared to the present procedures. I believe that the salvation, at least of mathematical classification, lies in its simplification and in the use of specialists for consultation, and not in the use of library gimmicks such as classification by series.

Notes

1. Another joke is that a book entitled *Rings and Ideals* was classified as fiction.

2. The collection of numbers: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$, is a sequence. If we attempt to add it up, we get the famous "harmonic series," $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \ldots$, which fails to converge and so is better left as a sequence. It seems that mathematicians turn sequences into series by trying to add them whereas librarians do so by classification and binding them together. Sometimes they are best left alone.

3. This is a 16th edition heading; in the 14th edition the term is Geometry.

4. The Algebraic Geometry was added in the 16th edition.

5. No subject is assigned to 518 in the 14th edition, but the 15th edition assigned it to Special Functions.

6. The heading Probabilities was changed to Probabilities and Statistical Mathematics in the 16th edition.