CAN YOU TEACH IN A NORMAL WAY?
EXAMINING CHINESE AND US CURRICULA'S APPROACH TO TEACHING FRACTION DIVISIONS

BY

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DISSERTATION

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Abstract

This dissertation presents two studies designed to examine the topic of fraction division in selected Chinese and US curricula. By comparing the structure and content of the Chinese and *Everyday Mathematics* textbooks and teacher's guides, Study 1 revealed many different features presented in the selected curricula. Major differences include the number of lessons on this topic, the algorithms introduced, the type of examples and exercises provided in the textbooks, and the teaching strategy suggested by the teacher's guides. Study 2 examined whether a set of lessons that represented Chinese textbook features or a set of *Everyday Mathematics*-style lessons were more effective in promoting U.S. sixth-grade students' understanding of fraction division algorithms and their ability to apply the algorithms to solve word problems. Results indicated that the participating U.S. students lacked understanding of mathematics concepts that are relevant to fraction division, and the participants did not effectively learn about fraction division from either the Chinese-style or *Everyday Mathematics*-style lessons. These studies suggest that to apply features of Chinese mathematic curricula to teach US students, it is important to take into account US students' prior knowledge, learning experiences, and learning styles.

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Chapter 1

Introduction and Background

Introduction

Over the past 20 years, a series of large-scale studies has shown that U.S. students underperformed in various international mathematics tests when compared to their East Asian counterparts (Hiebert, Gallimore, Garnier, Givvin, Hollingwsworth, & Jacobs, 2003; Stevenson & Stigler, 1992; Stigler, Gonzales, Kwanaka, Knoll, & Serrano, 1999; Stigler & Hiebert, 1999). To examine possible causes for such achievement differences, many researchers (Perry, 2000; Stevenson, Chen, & Lee, 1993; Stigler & Hiebert, 1998) have studied cross-cultural differences in mathematics teaching and learning; these differences include both schooling and non-schooling factors, for example, the structures of school system and policies (Cohen & Spillane, 1992), teachers' knowledge and classroom practice (Ma, 1999; Stigler & Stevenson, 1991) , and parental expectations of children's achievement (Stevenson, Lee, Chen, & Lummis, 1990; Stigler & Hiebert, 1998). These studies reveal some possible causes of U.S. students' relatively low achievement and, through these efforts, they also introduce other countries' successful mathematics education experiences that may help improve mathematics education in the United States. The purpose of this study is to partially trace such achievement differences to mathematics curricula that are used in China and the United States, and to suggest ways to improve mathematics curricula in the United States.

Mathematics curricula and their impact on teaching and learning have received more and more research attention both in the United States and in international context

(Clements, 2007; Li, 2007; Remillard, 2005; Schmidt, Wang, & McKnight, 2005; Stevenson, 1985). In addition to mathematics curriculum research, many reform curricula have been published and implemented as an attempt to improve the quality of mathematics education in the United States (Carpenter, Fennema, Franke, Levi, & Empson, 2000). Some of these reform-based mathematics curricula are widely used in the United States. However, very few studies (Fuson, Carroll, & Drueck, 2000; Riordan & Noyce, 2001) have examined the effectiveness of these curricula through experimental studies after the curriculum's development stage. And even fewer studies (Putnam, 2003) have examined how many of these curricula actually meet the reform mathematics standards and goals. Even though Sims and her colleagues (Sims, Perry, Schleppenbach, McConney, & Wilson, 2008) have found that U.S. classes that used a reform curriculum demonstrated a teacher-student discourse pattern that was similar to Chinese classrooms, there has not been much study on the structure and content of these U.S. reform curricula, especially in terms how these curricula differ from or resemble the curricula used in countries where students have shown higher mathematics achievement than U.S. students.

Among many cross-cultural studies, Chinese students are consistently rated among the top mathematics performers in international comparisons; and Chinese teachers and students have often been chosen as a baseline for U.S.-East Asia comparisons (Fan & Zhu, 2007; Fuson, 1988; Jiang & Eggleton, 1995; Ma, 1999; Perry, 2000; Stevenson, Hofer, & Randel, 2000; Zhou, Peverly, & Xin, 2006). However, because the Chinese government has only recently begun to allow scholars to study its curricula, comparison of elementary mathematics curricula used in China and the United States is a relatively new research focus.

In this study, I choose two popular Chinese textbooks and one widely used U.S. reform-based textbook, *Everyday Mathematics*, as my sample curricula. First, I compared how these textbooks present the topic of fraction division to students by examining their lesson structures, examples, and exercises. Then I analyzed their accompanying teacher's guides, in terms of what assistance they provide to help teachers understand and teach this topic. To examine whether a set of Chinese-style or a set of *Everyday Mathematics*-style lessons are more effective in helping U.S. children understand this topic, I conducted an experimental investigation in Study 2.

Results indicated that Chinese textbooks introduce fraction division through word problems and encourage students to discover the algorithm on their own, but that *Everyday Mathematics* presents the algorithms to students directly. Word problems provided in Chinese textbooks exceeded the ones in *Everyday Mathematics* both in terms of quantity and variety. Analysis of the teacher's guides indicated that Chinese teacher's guides provide more detailed instructions for teaching the topic than the *EM* teacher's guide. However, results of the teaching experiment indicate that a set of Chinese style lessons are not more effective than a set of *Everyday Mathematics*-style lessons in helping U.S. students understand and apply fraction division algorithms. Implications of these studies and suggestions for future research are discussed.

Background

Fraction division and teachers' knowledge. Fraction division is often considered one of the most difficult and least understood topics in elementary and middle-school mathematics (Fendel, 1987; Payne, 1976). Children's achievement on tasks related to this topic is usually very low (Carpenter, Lindquist, Brown, Kouba, & Silver, 1988; Hart, 1981). Therefore, it is urgent to develop a more effective curriculum for teaching this topic to children.

Not only is this topic challenging for students, it is also difficult for teachers to understand fully. Studies have shown that many teachers hold misconceptions about this topic and need help to provide effective teaching (Ball, 1990; Rule & Hallagan, 2006; Tirosh, 2000). Therefore, it is also necessary to find ways to improve teachers' content knowledge and pedagogical content knowledge (Shulman, 1986) about this topic.

Another major reason that fraction division was chosen as the mathematical concept for curriculum comparison is that Ma (1999) demonstrated significant differences between Chinese and U.S. teacher's content knowledge and pedagogical content knowledge on this topic. Ma surveyed 72 Chinese teachers and 23 U.S. teachers on their understanding of fraction division. She asked participating teachers to solve one calculation problem ($1\frac{3}{4} \div \frac{1}{2}$) and create one real-word problem from this equation. Her results showed that all Chinese teachers completed the computation correctly, and 65 (90%) of them created a story problem representing the meaning of the problem presented to them in symbolic form. In addition, many of them clearly pointed out the inverse relationship between fraction multiplication and division to explain the rationale for their word problems. In contrast, only 9 (43%) of U.S. teachers completed the computation correctly, and none created a story problem that was both conceptually and pedagogically correct. Through further interviews, Ma found that while Chinese teachers demonstrated a profound understanding of fundamental mathematics on this topic, U.S. teachers demonstrated relatively little understanding. Such dramatic differences between

Chinese and U.S. teachers' understanding of the topic motivated me to investigate possible causes for such differences and to discover ways to help U.S. teachers to improve their understanding.

One reason that fraction division is so challenging is because many teachers and students find it difficult to understand the meanings of the algorithms. (Bray, 1963; Capps, 1962; Elashhab, 1978; Johnson, 1965; McMeen, 1962) There are two arithmetic algorithms to calculate a fraction division problem. One is called the common-denominator algorithm, in which both the dividend and the divisor should first be transformed into their equivalent fractions with a common denominator. In this way, the problem is transformed into a whole number division problem when the numerator of the dividend is divided by the numerator of the divisor. For example, $\frac{3}{4} \div \frac{1}{8} = \frac{6}{8} \div \frac{1}{8} = 6 \div 1 = 6$. The other algorithm is called the invert-and-multiply algorithm, or "multiply the reciprocal of the divisor" in Chinese textbooks. For example, $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{4 \times 3}{5 \times 2} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$.

Studies have found that the common denominator algorithm is more meaningful to students (Brownell, 1938; Capps, 1962; Miller, 1957) because of its close connection with whole number division (Johnson, 1965). Some students can even construct it themselves (Sharp & Adams, 2002). Therefore, some researchers (Gregg & Gregg, 2007b) suggest that students should first be introduced to the common denominator algorithm so that they can establish a meaningful understanding of fraction division. However, in terms of calculation, when division results in a remainder or the divisor is greater than the dividend, for example, $\frac{5}{3} \div \frac{1}{4} = \frac{20}{12} \div \frac{3}{12} = 20 \div 3$, the common denominator algorithm becomes difficult for many students to complete accurately (Bray, 1963; Johnson, 1965).

In such cases, the calculation can be done easily and quickly with the invert-and-multiply algorithm ($\frac{5}{3} \div \frac{1}{4} = \frac{5}{3} \times 4 = \frac{20}{3} = 6\frac{2}{3}$), although students often find little sense in it and have to learn it by rote memorization (Capps, 1962; Elashhab, 1978; McMeen, 1962).

Between these two algorithms, current studies suggest that the common denominator algorithm is easy to understand but inefficient in calculation, and that the invert-and-multiply algorithm is efficient in calculation but difficult to understand. However, most mathematics textbooks in the world adopt invert-and-multiply because of its efficiency and connectedness to algebraic thinking (Bergen, 1966; Capps, 1962; Krich, 1964). Therefore, most students are taught to use the more mechanical algorithm. Although it is difficult to find meaning in the invert-and-multiply algorithm, teachers and students from China demonstrate deep understanding of this algorithm, especially in terms of using contextualized representations to explain and model the algorithm (Ma, 1999). Therefore, it is possible that teaching the invert-and-multiply algorithm through real-life models might be an effective method to help students understand this concept. Thus it is important to examine the real-life models of fraction division used in Chinese classrooms.

Many researchers have proposed different ways to categorize mathematical word problems in terms of the type of numbers involved, different operations, and context (e.g., Barlow & Darke, 2008; Baroody 1998; Greer, 1994; Marshall, Barthuli, Brewer, & Rose, 1989; Peck & Wood, 2008; Perlwitz, 2005; Vergnaud, 1988). Because this study is focused on division of fractions, I argue that it is important to consider children's understanding of different real-life situations in terms of how these situations represent the meaning of fraction division. Therefore, I chose to base my word problem categorization on Baroody's (1998) summary of meanings of multiplication word problems. This categorization includes four real-life models. Because division is the inverse operation of multiplication, these four multiplication models can be mirrored to represent meanings of division word problems. I summarize the four types of models as follows:

- **Fair-Sharing (partitive and measure-out/quotative)**. This type involves equally dividing a total amount into a number of equal collections. The unknown number can either be the amount of each share (partitive), for example; or the number of shares (measure-out/quotative). For example, Scott has 5½ pounds of rice, he wants to divide it into ½ pound bags, how many bags of rice does he end up with? $5\frac{1}{2} \div \frac{1}{2}$. In the case of partitive problems, because the number of shares are always given as an integer (although the amount of whole collection can be represented by a fraction). Partitive problems are not provided as an example in this topic very often. Most fair-sharing fraction division word problems are measure-out problems.

- **Rate**. This type involves finding a rate or speed given the total distance/productivity and usage of time. For example, it took Josh 2/5 hour to walk 1 ½ miles, how many miles does he walk for an hour? $1\frac{1}{2} \div \frac{2}{5}$. A special sub-type of this meaning is area meaning, which involves calculating the area of a rectangle or the volume of a cube.

- **Comparison**. This type involves determining the size of a set, given the size of another set and how many times larger or smaller this set is compared to the unknown set. For example, a bag of brown sugar weights 3½ pounds, the brown sugar is 2/3 as heavy as a bag of white sugar, how much does this bag of white sugar weigh? $3\frac{1}{2} \div \frac{2}{3}$

- **Combinations**. This type involves all the ways of combing more than one context. For example, Josh can run ¾ miles within 1/5 hour, Josh runs $\frac{1}{8}$ faster than Mike, how many miles can Mike run within an hour? Josh: $\frac{3}{4} \div \frac{1}{5} = 3\frac{3}{4}$ (m/h), Mike: $3\frac{3}{4} \div \left(1 + \frac{1}{8}\right) = 3\frac{1}{3}$ (m/h)

Many studies have found that the measure-out/quotative model of fair-sharing is the easiest meaning of fraction division for children to grasp. Because of children's understanding of the fair-sharing situation, they not only can easily understand the measure-out model, many of them can even spontaneously solve fraction division problems in measure-out situations (Sharp & Adams, 2002). Therefore, many researchers (e.g., Gregg & Gregg, 2007b) suggested that this model should be introduced to children first so that they can start with an easy understanding of fraction division. However, no studies have examined teaching fraction division through the other models. We do not know whether we should teach these other models or how we should teach them.

Interestingly, many Chinese teachers in Ma's study (1999) created word problems that represented the other models. 62 out of a total of 80 stories created by Chinese teachers represented either the comparison or the rate model, but no US teachers created any comparison or rate model story. Such differences motivated me to study what models are included in the Chinese textbooks and teacher's guides. Therefore, an in-depth examination of the content of Chinese and U.S. curricula is necessary.

Studies on content of Chinese and U.S. textbooks. There are two types of studies that focus on specific content in mathematics textbooks. One type focuses on specific learning activities or problem types; the other type focuses on the specific mathematical topics in the curricula. For example, with the increasing effort to promote students' problem-solving ability and the NCTM Standard's requirement of problem-solving as an important learning goal (National Council of Teachers of Mathematics, 2000), problem-solving has become a research focus. Some studies show that although Chinese students outperform their U.S. counterparts in tests of general mathematics abilities, they tend to make similar mistakes as U.S. students do when they solve realistic word problems (Cai, 1998; Cai & Silver, 1995; Xin, Lin, Zhang, & Yan, 2007); and that when solving problems that require flexible and non-routine strategies, U.S. students even outperformed Chinese students in some questions because U.S. students tend to use visual representations and drawings when they do not know the routine algorithms and symbolic representations needed, but Chinese students tend to give up if they do not know the mathematical solutions (Cai, 2000). These studies show that there is discrepancy between Chinese students' low problem-solving ability and Chinese curriculum's focus on in-depth understanding of knowledge and flexible application ability. Many researchers (Cai, Lo, & Watanabe, 2002; Fan & Zhu, 2007) have started to look at how problem-solving is presented in Chinese and U.S. textbooks.

Another example of studies on specific problem types is Zhu and Fan's studies (Fan & Zhu, 2007; Zhu & Fan, 2006). After examining various problem types represented in one Chinese and one U.S. textbook, they found that an absolute majority of problems provided in both of the textbooks were routine and traditional; however, among the small amount of non-traditional problems, U.S. textbooks provided a larger variety of problems, especially problems with visual information. As for the difficulty level of the problems, they found that problems in Chinese textbooks were more challenging than those in the U.S. books.

Compared with the large number of studies on problems types, only a very limited number of studies have focused on how specific mathematical topics are taught in China and in the United States. Most of these studies have focused on very basic and elementary concepts. After examining some Chinese and U.S. textbooks, some researchers found that Chinese textbooks focus more on the essential meaning of mathematics concepts than U.S. textbooks. For example, Capraro, Ding, Matteson, and Li (2007) studied how the equal sign is presented in U.S. and Chinese textbooks. A common misconception of young students is that the equal sign is a signal for “doing something” rather than a relational symbol of equivalent or quantity sameness. They found that this misconception is manifest in the U.S. and that U.S. textbooks include few activities that address this misconception; but Chinese students are able to interpret the equal sign as a relational symbol of equivalence because Chinese textbooks introduce the equal sign together with the more-than and less-than signs as relational symbols. A further analysis of this topic was done by this research team (Li, Ding, Capraro, & Robert, 2008). To find out how this difference may be traced back to other teaching materials, they examined selected Chinese and U.S. teacher preparation materials, students’ textbooks, and teacher’s guides. They found that although U.S. teacher preparation textbooks rarely interpret the equal sign as equivalence, Chinese teachers’ textbooks typically introduce the equal sign in a context of relationships and interpret the sign as “balance,” “sameness,” or “equivalence.” Another example of examining basic concepts in textbooks is Zhou and Peverly’s (2005) research on the teaching of addition and subtraction. They found that Chinese textbooks, using only small number combinations, introduce the concepts of addition and subtraction together with a focus on the inverse relationships between the two operations; but U.S. textbooks usually introduce subtraction as a very different operation than addition after students have learned addition with sums up to 20.

Other studies (Lo, Cai, & Watanabe, 2001; Zhou, Peverly & Xin, 2006) have looked at how Chinese and U.S. textbooks introduce higher levels of mathematics concepts as new knowledge. They found that Chinese textbooks are very similar in terms of always using short word problems to contextualize a new concept; but U.S. textbooks vary a great deal in their ways of introducing new knowledge: some U.S. textbooks expect students to discover a new concept on their own through problem-solving; for others, they introduce the new concept as a very abstract concept without much context. For example, Lo and Cai (Lo, et al., 2001; Lo, Watanabe, & Cai, 2004) found that although all of their selected Chinese and U.S. textbooks used contextual problems to introduce a new concept, the types of word problems provided in the Chinese and U.S. textbooks were very different. Chinese textbooks only included word problems that were short and specific to the new concept. But U.S. textbooks varied in the types of word problems provided. They sometimes used very elaborated contextual problems with various sub-problems that were not always relevant to the new concept; but in other cases, for example, when introducing the concept of arithmetic average, some U.S. textbooks did not use any contextualized problems at all (Cai, Lo, & Watanabe, 2002). Another example is that while Chinese textbooks used "equal-sharing" and "per-unit-quantity" word problems to introduce the concept of average; U.S. textbooks introduced the concept as a measure of central tendency without any word problems, which may be abstract and difficult for elementary school students to understand.

In addition to studies that compared U.S. and Chinese textbooks, there are also some articles that focus on describing how a certain topic is introduced in a Chinese textbook. For example, Li (2008b) described what Chinese students are expected to learn about fraction division and how the Chinese textbook is structured so that students can be guided to achieve the expected understanding of the topic. He pointed out that students are expected to learn more than just the algorithm (Invert-and-Multiply) and that the Chinese textbook makes good use of a problem solving approach to help students construct the algorithm. After Li's description of the Chinese textbook, he encouraged educators in the United States to learn from the perspective of the Chinese textbook. However, he did not provide much analysis on how any why the structure and content of Chinese textbook might benefit students more than U.S. curricula that have been used currently, and there is also a lack of experimental evidence that the U.S. students would learn effectively if they were taught with the Chinese perspective.

No matter how much difference we can find between U.S. and Chinese textbooks, one reason that we should not rush into copying each other's textbook is that the successfulness of a textbook not only depends on the textbook's structure and content, but also depends on policies and structures of the whole mathematics curricula of a country. The reason for many existing teaching strategies in U.S. textbooks and those in Chinese textbooks may be traced to the policies and structures of polices and structures of Chinese and U.S. curriculum.

Policies and structures of Chinese and U.S. curricula. Chinese and U.S. mathematics curricula differ dramatically in terms of whether or not they follow a centralized structure. In the United States, curricula are usually developed by mathematics educators, researchers, and publishers (Remillard, 2005). Although many curriculum materials are developed to support the curriculum standards published by the National Council of Teachers of Mathematics (NCTM), there is no mandated requirement for publishing a curriculum. Curriculum decisions are made at state and local levels (Moy & Peverly, 2005), and school districts can mandate the use of a single curriculum or allow each school to choose its own curriculum.

In China, a national mathematics curriculum that specifies goals, content topics and requirements at each grade level is published by China's Ministry of Education (Li &

Fuson, 2002; Moy & Peverly, 2005; Wang & Paine, 2003). A series of textbooks that closely follows this curriculum was commissioned by the Ministry of Education to be used in all Chinese schools (Madell & Becker, 1984). Recently, the education ministry started publishing several different textbooks that interpreted this national math curriculum in ways that were more relevant to the needs of different regions in China (Ma, 1999). Because most Chinese students need to compete in city-, province-, and nation-wide admission exams to enter high schools and colleges, and because these exams closely follow the content topics and requirements in the national curriculum, all mathematics instruction in Chinese school closely follows this national curriculum (An, 2000).

Organizations of content topics in Chinese and U.S. textbooks also differ greatly. Jiang (Jiang & Eggleton, 1995) and Askey (1999) found that U.S. schools typically use a "spiral curriculum," which means that topics are briefly introduced one year and then reviewed in successive years to build on previous learning. In this structure, U.S. textbooks usually cover many topics in each semester (Fuson, 1988; Hook, Bishop, & Hook, 2007; Stigler & Hiebert, 1999) with much review and repetition of previous knowledge (Schmidt, Wang, & McKnight, 2005) but not much new knowledge (Flanders, 1987).

In contrast, Chinese textbooks introduce topics with a much more sequential and nonrepetitive approach (Askey, 1999; Jiang & Eggleton, 1995). Each topic is taught with elaborated details, and a thorough understanding of the topic is required as preparation for the future learning of other closely connected topics (Ma, 1999). Students learn a great deal about a small number of topics included in each semester (Fuson & et al., 1988; Zhou & Peverly, 2005).

Many researchers (Schmidt, Houang, & Cogan, 2002, 2004; Schmidt, et al., 2005), after looking at U.S. textbooks from an international perspective, have argued that because U.S. textbooks cover many topics that are not necessarily related in a short period of time, and ignore the sequential connections among mathematics content topics, the textbooks do not present content topics in a coherent way that promotes students' in-depth understanding of mathematics knowledge. Not only are the structures of curricula very different, but also teachers in the two countries approach their curricula in very different ways.

Chinese and U.S. teachers' approach to their curricula. With increased attention on the implementation of reform-oriented curriculum in the United States, more and more researchers are realizing that teachers play a very important role as they apply the curriculum in their classroom teaching; and many researchers have called for studies on the interaction between teachers and their curricula (Remillard, 2005; Remillard & Bryans, 2004; Smith & Star, 2007). Many cross-national studies (e.g. Ma, Lam, & Wong, 2006; Nicol & Crespo, 2006; Remillard & Bryans, 2004) have been done on teachers' approaches to understanding and implementing their curricula. Existing studies that compare Chinese and U.S. teachers' approaches to their curricula indicate that large differences exist in how teachers in the two countries study, understand, and implement their curriculum (Li & Fuson, 2002; Ma, Lam, & Wong, 2006; Moy & Peverly, 2005).

In the United States, both teachers' understanding of their textbooks and their implementation of their curricula vary greatly. Among teachers who use the same curriculum, their selection of topic, content emphasis, and sequences of instruction may be very different (Freeman & Porter, 1989). Such different approaches may result from teachers' different beliefs about teaching, their content knowledge, and the professional development and support they receive (Remillard, 2005; Remillard & Bryans, 2004). When introduced to teachers, some curricula do not include much support for teachers to understand the rationale of the curriculum design; in those cases, teachers find it very difficult to understand and implement the new curriculum (Remillard, 2005; Remillard & Bryans, 2004).

U.S. teachers' different approaches to their curriculum may also be a result of teacher education programs. During their teacher education program, preservice teachers do not know the curricula they will use in the future because the curricula will be determined by their future school districts; therefore, preservice teachers do no have a chance to study their future curricula. Also, U.S. teacher education programs usually do not provide any training on any specific curriculum. In general, because there is no systematic guidance on how to use any specific curriculum, even when preservice teachers have the chance to practice teaching a curriculum in their student-teaching, their understanding and implementation of the curriculum may vary dramatically (Collopy, 2003; Lloyd & Behm, 2005; Nicol & Crespo, 2006; Pehkonen, 2004; Remillard, 2005).

In contrast, Chinese teachers' approaches to their mathematics curriculum are relatively uniform. Along with the national curriculum and the commissioned textbooks, there are also teacher's guides accompanying the textbooks. These teacher's guides explain in detail the rationale of the organization of the topics, the purpose of the inclusion of specific examples and exercises, and common mistakes and misconceptions of students. All teachers study the national curriculum, the textbooks, and the teacher's guides very closely. Teachers' design of their lesson plans and their implementation of their lessons are very similar; they also use the national curriculum and teacher's guide as the most important resource for learning the mathematics as well as students' misconceptions (Askey, 1999; Li & Fuson, 2002; Ma, 1999; Wang & Paine, 2003). Other than classroom teaching, teachers also attend professional development workshops, group discussions, and observation of other teachers' teaching; all these activities are intended to help teachers improve their understanding and implementation of their curriculum (Ma, 1999).

In addition to the teaching practice, teacher education programs in China also play an important role in helping preservice teachers become familiar with the national curriculum (Moy & Peverly, 2005). In China, each elementary school teacher teaches only one subject; preservice teachers are enrolled in programs of a specific subject and take classes that focus on the teaching of that subject. Preservice teachers of elementary school mathematics need to complete a series of courses that focus on the teaching of different topics in elementary school and middle schools. One major objective of these courses is to help preservice teachers have a systematic and in-depth understanding of the national curriculum, textbooks, and teacher's guides, which are all required readings and learning materials in these courses (Li, 2002; Ma et al., 2006; Sun, 2000; Wang & Paine, 2003). Because the instructors of those teacher education courses are all experts in the curriculum materials, they provide a great opportunity for preservice teachers to develop their understanding of the curriculum that they will be using in the future.

Overall, both the structures of the curriculum and teachers' approaches to their curriculum are centralized in China; in the United States, both of these two aspects are localized and diverse.

Lastly, no matter how well a curriculum is designed, the effectiveness of a curriculum is greatly dependent on the teacher's interpretation of it. Teachers' beliefs about student's thinking, mistakes, and misconceptions may significantly affect how a curriculum is used. Therefore, we need to incorporate these beliefs about learning when we examine mathematics curricula.

Beliefs about learning and mistakes in mathematics. Many researchers recognize the value of using students' work and thinking for teachers' professional training, and some of them have helped teachers to improve their mathematics teaching by establishing an understanding of students' thinking (Franke & Kazemi, 2001; Philipp, Thanheiser, & Clement, 2002; Steinberg, Empson, & Carpenter, 2004). However, there has not been much research on whether mathematics curricula assist teachers to better understand students' thinking or how some curricula provide such assistance to teachers.

One thing we do know is that Chinese and U.S. teachers respond to students' thinking, especially mistakes and errors, very differently. Chinese teachers respond to errors by "dwelling on them" (Wang & Murphy, 2004)—they ask students to explain the reasons behind the errors and learn from them. Compared to U.S. teachers, Chinese teachers ask more follow-up questions about errors (Schleppenbach, Flevares, Sims & Perry, 2007), and they also tend to ask other students to provide judgments or alternative answers to errors (Feil, 2007). After examining 46 mathematics lessons in China and in the United States, Schleppenbach and her colleagues (2007a) suggested that there are four unique characteristics about Chinese teachers' approach to errors: (a) Chinese teachers provide a classroom environment that is open and supportive of making errors; (b) many Chinese teachers plan instruction so that students will make mistakes that are helpful for learning; (c) Chinese teachers ask students to constantly review the right concept; and (d) Chinese teachers allow students more chances to work and learn through errors. On the other hand, U.S. teachers' responses are more like quick corrections of students' mistakes—they tend to make more statements about errors (Schleppenbach, et al., 2007a) or provide correct answers to students (Feil, 2007).

Many researchers agree that teaching is a cultural activity (Stigler & Hiebert, 1998, 1999), and some of them attribute these teaching differences to people's beliefs toward learning in the two countries. For example, (Stigler & Perry, 1988a) reasoned that "For Americans, errors tend to be interpreted as an indication of failure in learning the lesson. For Chinese and Japanese, they are an index of what still needs to be learned" (p. 29). Such cultural differences in beliefs towards learning are further reflected by Chinese and U.S. children's perceptions of learning. For example, most Chinese children recognize the value of effort as the top cause for intelligence and achievement (Li, 2004; Li & Fischer, 2004) while U.S. children recognize more of the value of "personal characteristics" and "social contexts."

Although it is true that such cultural differences may result from many social interactions and history within each country, we should not be satisfied by simply recognizing these differences, especially if they may contribute to the achievement differences in children. Studies have shown that, compared with Chinese students and teachers, U.S. students have lower motivation for devoting effort to math and U.S.

teachers have lower interest in teaching math (Stevenson, Lee, Chen, & Lummis, 1990). Research also shows that Chinese teachers' approach to student mistakes may lead students to a more thorough understanding of mathematical topics (Schleppenbach, et al., 2007a; Schleppenbach, Perry, Miller, Sims, & Fang, 2007b). Therefore, it is necessary to identify the origins of these culturally different practices and beliefs toward mathematics learning, especially towards students' mistakes. Thus I use this study to trace such differences in beliefs and approaches towards student mistakes, at least partially, to information and instructions in mathematics textbooks and teacher's guides in the two countries. I examined whether the suggestions in the curricula are consistent with teachers' approaches to student mistakes in each country.

In conclusion, there are major differences between Chinese and U.S. textbooks' structures and content, between Chinese and U.S. teacher's approaches to their curricula, and between Chinese and U.S. teachers' beliefs about mathematics learning. However, we cannot draw a causal relation between these differences and achievement differences between Chinese and U.S. students. Experiments need to be done to confirm whether any particular design is more effective in promoting students' learning. Therefore, in this study, I first identified possible reasons for these differences by comparing Chinese and U.S. curricula on how they present division with fractions; then, based on the features represented by Chinese and U.S. curricula, I created experimental textbooks—one representing Chinese features and the other representing U.S. features; and, lastly, I used these experimental materials to teach U.S. students and then examined their effectiveness in improving students' understanding of the topic of fraction division.

Chapter 2

Study 1: Content Analysis of Chinese and U.S. Curricula

Introduction

To examine in detail how Chinese curricula and *Everyday Mathematics* introduce fraction division to students and teachers, it is necessary to analyze the structure and content of textbooks and teacher's guides so that we can identify features in the curricula that promote effective learning on this topic. In study 1, through content analysis, I examined how Chinese textbooks and *Everyday Mathematics* introduce fraction division to students and how their teacher's guides assist teachers to understand and use their textbooks.

Method

Textbook selection. I chose two Chinese and one U.S. textbook and their accompanying teacher's guides as my samples for curriculum investigation. Both Chinese textbooks are published by People's Education Press (PEP). They are *Compulsory Education: Mathematics* (People's Education Press [PEP], 2002) and *Compulsory Education Mathematics (standard-based experimental edition)* (People's Education Press [PEP], 2006). The U.S. textbook is *Everyday Mathematics* developed by the University of Chicago School Mathematics Project (University of Chicago. School Mathematics Project, 2008).

In China, there are several series of mathematics textbooks currently being used. All of these textbooks closely follow the national mathematics curriculum requirements and have been approved by the Ministry of Education. The majority of Chinese students (over 70%) use the textbooks published by People's Education Press (Bao, 2002). Therefore, PEP textbooks were chosen for this study mainly because of their popularity. Also, because the Chinese national mathematics curriculum has gone through major revisions since 2001, when the Ministry of Education issued the National Mathematics Curriculum Standards for Compulsory Education, I will include two Chinese textbooks in this study: the 2002 version and the 2006 version.

There are several reasons for including both versions of textbooks in this study. First, most international studies on Chinese students' mathematics learning were conducted when the old curriculum was being implemented; therefore, textbooks following the old curriculum are very valuable if I try to identify textbooks features that may promote effective learning, as already documented in cross-national comparisons. If I did not include the old curriculum in my study, I might miss some important old-curriculum features if they were removed in the revised curriculum. Second, the revised curriculum is based on Chinese mathematic education researchers' further understanding in this area; new ideas and practices in this curriculum may give us inspirations for developing effective learning materials. But because the revised curriculum has only been implemented for a short time, many of these new ideas and practices have not been fully examined and tested. I hope to use my study to further examine these ideas. Third, although a few studies have examined the similarities and differences between these two versions of textbooks (for example, Bao, 2004), most of these studies have only examined the general structure and overall styles of the textbooks. No studies have examined whether the two versions of textbooks present the topic of fraction division with similar or different approaches. Although the 2002 Chinese version focused on basic knowledge and skills, the 2006 Chinese version is a reform curriculum that emphasizes exploratory learning and problem-solving skills, which are also stated goals for *Everyday Mathematics.* By comparing the 2002, 2006 Chinese curricula, and *Everyday Mathematics*, I hope to find out whether the reformed Chinese curriculum is more similar to the old Chinese curriculum or to *Everyday Mathematics*.

According to the Chinese curriculum, the topic of fraction division is covered in the first semester of sixth grade textbook. In addition, each textbook is accompanied by one teacher's guide. Altogether, two Chinese textbooks and two teacher's guides will be included in this study.

As for the selection of a mathematics curriculum used in the United States, there are two widely used reform-based mathematics curricula that cover the topic of fraction division—*Everyday Mathematics* and *Connected Mathematics.* I chose to use *Everyday Mathematics* for this study for three reasons. First, *Everyday Mathematics* covers mathematics learning from grade 1 to 6, which is the same as the grade levels served by the Chinese elementary curriculum. Furthermore, the teaching of the invert-and-multiply algorithm is covered in sixth grade for both *Everyday Mathematics* and the Chinese curricula. But *Connected Mathematics* is a curriculum for middle school only. Although it also covers the topic of fraction division, this topic is taught in seventh grade. Second, both *Everyday Mathematics* and the Chinese curricula introduce the concept of fractions in fourth grade, and across fourth- through sixth-grade, these curricula introduce addition, subtraction, multiplication, and division of fractions. However, in *Connected Mathematics,* both the basic concept of fractions and operations of fractions are introduced in seventh grade in one chapter. Although such differences between the fraction lessons in the Chinese curricula and *Connected Mathematics* might be interesting, because the focus of this study is to examine the detailed content of the lesson, it might be very difficult to compare the lessons content on fraction division with so many major differences in the structures of the whole curricula. And third, *Everyday Mathematics* is not only one of the most widely used textbooks for K-6 students, used in over 15 states by over 2.8 million students in the United States (University of Chicago School Mathematics Project, 2008), it is also used by all public schools in Champaign, IL, where the experiments in this study were conducted. Therefore, by choosing *Everyday Mathematics* as a curriculum for this study, I could study not only how this curriculum approaches the topic of fraction division, but also how it has prepared its learners to study the topic of fraction division with mathematics concepts taught prior to fraction division.

In this study, I examined the current (2004) edition of *Everyday Mathematics.* In this series, fraction division is covered in both the second semester of fifth grade (unit 8.12) and the second semester of sixth grade (unit 6.2). Therefore, both the fifth-grade (volume 2) and sixth-grade (volume 2) materials were examined in this study. For each textbook volume, there is an accompanying student's reference book, one teacher's lesson guide, and one teacher's reference book. Altogether, there will be two *Everyday Mathematics* textbooks and all of their accompanying guides and reference books included in this study.

Procedures and analysis. For the content analysis of the textbooks, I first identified the *algorithms* that are introduced by each textbook. Then I examined the *organization* of lessons in each textbook in the following aspects: (a) numbers of lessons and how they are related to other lessons in the same unit; (b) number of topics covered in each lesson; (c) types and numbers of examples and exercises in the lessons. Lastly, based on the four meanings of fraction division that were summarized earlier, I identified the *fraction division meanings* presented in each textbook and calculated the percentage of how much each meaning was presented when fraction division is taught in each textbook.

For the teacher's guide analysis, I started by identifying the *number of books* included in each teacher's guide and the *organization* of these books. Then I compared the stated *goals* in each teacher's guide to find out whether the goals are focused on conceptual learning and problem solving skills or on rote memorization and procedural skills. I also compared how each teacher's guide assisted teachers to understand and to *teach the fraction division algorithm*. Lastly, I examined the information provided in the instructions on *teaching through examples, exercises, and students' mistakes*.

Results

Content analysis of textbooks.

Fraction division algorithms. As I summarized earlier, there are two arithmetic algorithms that can be used to calculate the answer to fraction division problems—the common denominator algorithm and the invert-and-multiply algorithm. One major difference between the Chinese textbooks and *Everyday Mathematics* is their selection of the algorithms and their strategies for teaching the algorithms. *Everyday Mathematics* formally introduces both the common denominator algorithm (fifth grade) and the invert-and-multiply algorithm (sixth grade) as algorithms to calculate fraction division problems.

When the common denominator algorithm is introduced in fifth grade, it is taught with contextualized problems (for example, dividing 6 pounds of candies into ¾ pound boxes). Students are encouraged to use manipulatives to construct the algorithm. When the invert-and-multiply algorithm is introduced in sixth grade, *Everyday Mathematics* does not provide contextualized problems. All the examples and exercises are calculation problems, and the algorithm is presented to students rather than being constructed by students.

On the other hand, both Chinese textbooks do not include the common denominator algorithm. Some researchers (e.g. Baroody, 1998) suggest that students should be encouraged to first use the common-denominator method because children can easily use it in a meaningful way. Even though the Chinese textbooks go against this suggestion, Chinese teachers seem to still be able to teach a meaningful understanding of fraction division with only the invert-and-multiply algorithm. This may be a result of the way this algorithm is presented in the textbooks and teacher's guides. In both Chinese textbooks, all of the 10 lessons are focused on understanding and applying the invert-and-multiply algorithm. However, unlike the rote presentation of the invert-and-multiply algorithm in *Everyday Mathematics*, the Chinese textbooks use many contextualized word problems to encourage students to construct the algorithm, achieve a meaningful understanding of it, and apply it to problems. Later I will explain in detail how Chinese textbooks encourage students to construct the invert-and-multiply algorithm in a meaningful way. Before I do that, it is worth understanding the general structure of the textbooks in terms of when fraction division is taught, how much time is allocated to it, and what teaching strategies are included.

Organization of lessons. Both of the two Chinese textbooks and *Everyday Mathematics* have similar numbers of chapters or units included in one semester. The 2002 Chinese textbook has 5 chapters; the 2006 Chinese textbook has 7 chapters; the second semester of fifth grade in *Everyday Mathematics* has 6 units; and the second semester of sixth grade in *Everyday Mathematics* has 5 units. However, these textbooks differ a great deal in terms of lesson organization within the unit. In both Chinese textbooks, there is one chapter that is mostly devoted to fraction division (10 lessons for fraction division and 3 lessons for ratio). But in *Everyday Mathematics*, for both fifth and sixth grade, fraction division is only 1 out of 12 lessons within the unit. The other lessons in the unit include various topics related to numbers, fractions, fraction multiplication, mixed numbers (in fifth grade), number systems, positive and negative numbers, and algebra concepts (in sixth grade).

Such a lesson organization means that the time required for teaching fraction division varies dramatically between the Chinese textbooks and *Everyday Mathematics*. Both Chinese textbooks require ten lessons for fraction division, but *Everyday Mathematics* requires only two: one lesson in fifth grade and another in sixth grade. Besides the differences in time requirements, the Chinese textbooks and *Everyday Mathematics* also differ in the major content presented for teaching fraction division as well as the organization of the content.

Both the new and old Chinese textbooks have a similar organization for the fraction division lessons. The chapter for fraction division is included in the first semester of sixth grade, following the chapter on fraction multiplication. The fraction division chapter is divided into two sections, the first one (4-5 lessons) is focused on understanding the meaning of fraction division and on the fraction division algorithm. The second section (5-6 lessons) is focused on solving contextualized word problems. Each lesson lasts about 45 minutes and includes examples, exercises, and a take-home assignment.

These 10 lessons are organized by the multiple representations of the meaning of fraction division, the difficulties of the examples, and the complexity of calculations. For example, students first learn to divide a fraction by a whole number, then they learn to divide a whole number by a fraction, followed by a fraction divided by a fraction, and lastly they learn to conduct mixed operations including fraction division, multiplication, addition, and subtraction. Through these 10 lessons, from an easy to an advanced level, students go through multiple representations of the meanings of fraction division, many types of in-class exercises, and intensive practice from homework, all of which will be described later.

Compared with the two Chinese textbooks, *Everyday Mathematics* requires much less teaching time on fraction division. There are only two lessons devoted to this topic, and these two lessons are spread out between fifth and sixth grade. The one lesson in fifth grade is focused on using the common denominator algorithm to solve fraction division problems and to understanding the meaning of fraction division. The second lesson (sixth grade), which is given one year after the first lesson, is focused on applying the invert-and-multiply algorithm to calculate fraction division problems.

The number of topics covered in the lessons also differs between the Chinese and the U.S. textbooks. Even though there are 10 lessons in the Chinese textbooks, all examples, exercise, and assignments in the 10 lessons include fraction division. However, within the 2 lessons in *Everyday Mathematics*, several required exercises are not related to fraction division. These exercises include the topics of geometry, number comparison, multi-digit multiplication, adding and subtracting mixed numbers, and percent.

Lastly, the examples and exercises in the two Chinese textbooks and *Everyday Mathematics* also differ in terms of included problem types and their quantity. These data are represented in Figure 1.

Types	Chinese 2002	Chinese 2006	*EM* (Total)	*EM* (fifth grade)	*EM* (sixth grade)
Examples					
Calculation	0	0	7	3	4
Word problem	11	6	9	4	5
Algebra problem	3	3	0	0	0
Total	14	9	16	7	9
Exercises/ Assignments					
Calculation (fraction division)	56	28	51	16	35

Figure 1. Types and number of problems in the Chinese textbooks and *Everyday Mathematics*

Figure 1 (continued)

Calculation (mixed operations)	4	18	0	0	0
Calculation (no fraction division included)	24	0	25	15	10
Word problem	44	24	3	3	0
Algebra problem	10	7	0	0	0
Others (fraction division related)	13	19	2	0	2
Others (not related to fraction division)	0	0	23	12	11
Total	151	96	104	46	58

In the two Chinese textbooks, all examples are either word problems or algebra problems. According to the explanation in the teacher's guide, word problems are provided to help students build a meaningful understanding of fraction division, and algebra problems are provided to help students deepen their understanding of equations and to summarize their contextualized understanding of fraction division to an abstract mathematical meaning. Within the 10 lessons, the average number of examples in each lesson is one or two problems.

Compared to the small number of examples per lesson in the Chinese textbooks, *Everyday Mathematics* includes 7 examples in the fifth grade lesson and 9 examples in the sixth grade lesson. Such a difference is consistent with other studies on Japanese, Chinese, and U.S. math lessons, where researchers have found that U.S. lessons tended to cover many more topics and problems than Japanese and Chinese lessons (Perry, 2000). In addition, among *Everyday Mathematics*' examples, more than half are calculation problems, which focus on the procedure of applying an algorithm without any explicit guide to teaching its meaning. Even for the word problems in *Everyday Mathematics*, all of them (in both the fifth and sixth grade lessons) are focused on the common-denominator method, which means that there is not a single word problem provided to help students understand the meaning of the invert-and-multiply algorithm.

As for exercises and homework, the average number of exercises in each lesson is much less in the Chinese textbooks (11.8 problems in the 2002 version, and 7 problems in the 2006 version) than in *Everyday Mathematics* (31 in fifth grade, and 45 in sixth grade). However, within the smaller number of exercises in the Chinese textbooks, there are a large variety of problems (e.g. word problems, mixed operations, algebra problems), but *Everyday Mathematics* provides mostly calculation type exercises.

It is worth noting that both the 2002 version of the Chinese textbook and *Everyday Mathematics* include some calculation problems that are not fraction division. I should point out that all the 24 non-fraction division calculations in the Chinese textbooks are mixed with fraction division calculations. For example, one exercise is: "Compare the difference between the operations on the following calculations: $\frac{5}{6}+\frac{2}{3}, \frac{5}{6}-\frac{2}{3}, \frac{5}{6}\times\frac{2}{3}, \frac{5}{6}\div\frac{2}{3}$". According to the teacher's guide, such exercises are to help students distinguish the fraction division algorithm from algorithms for other operations on fractions. On the contrary, all the non-fraction division calculations in *Everyday*

Mathematics are presented as a totally separate set of exercises, which do not include any fraction division calculations. One last and most surprising difference among the exercises in textbooks is that although all the exercises and homework provided in the Chinese textbooks are directly related to fraction division in this chapter, *Everyday Mathematics* provides 25 questions that are totally irrelevant to fraction division in the two lessons (e.g., geometry, multi-digit multiplication, and number comparison, etc.). These questions are mostly included in "Math Boxes," which are often provided at the end of each lesson and include questions that are from previous lessons.

While it is surprising to see such differences in the types and numbers of problems in the textbooks, it may be even more surprising to see the different meanings and contexts presented by these problems, especially the word problems. In the following section, I analyze the different representations of the meanings of fraction division provided in the textbooks.

Representation of the meanings of fraction division. Almost all educators agree that students should achieve meaningful understandings of mathematical concepts and procedures through learning. According to the NCTM curriculum focal points for sixth grade, students should "use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work" (NCTM, 2008). NCTM also requires mathematics curricula to enable all students to "understand meanings of operations and how they relate to one another." It also requires all students to be enabled to "build new mathematical knowledge through problem solving" and to "monitor and reflect on the process of mathematical problem solving." Earlier, I discussed the number of word problems provided in each textbook. Now, I will examine in detail the content of these word problems and how they are designed to facilitate students' understanding of this topic.

Earlier, I summarized the four types of meanings that are usually represented by multiplication word problems. They are (a) Fair-sharing, (b) Rate, (c) Comparison, and (d) Combinations. Based on these four types of meanings, I examine which of these is represented in the word problems in each of the textbooks; and if they are, in what order and to what level they are presented in the textbooks.

I found that both Chinese textbooks provided word problems that represented all four types of meanings of fraction division, and each type of meaning typically was presented in two word problem examples and 5-10 word problem exercises. The two Chinese textbooks also presented the four types of meanings in a similar order: (a) fair-sharing, (b) rate, (c) compassion, and (d) combinations.

In the Chinese textbooks, there are two sections in the fraction division chapter. The first section is focused on understanding the meaning of fraction division and the algorithm. In this section, the word problems are short with simple fractions. For example,

> a car travels 18 km in 2/5 hour, how far does this car go in 1 hour? $18 \div \frac{2}{5}$.

According to the explanation in the teacher's guide, these simple word problems are intended to help students understand the meaning of fraction division and to construct the algorithm. The second section of the chapter is focused on problem solving. In this section, there are more word problems with combinations of meanings, and the solutions require more mixed operations of fractions or algebra thinking. For example,

> There are 25 people in the art extra curriculum group, which is ¼ more than the science group, how many people are in the science group? $x + \frac{1}{4}x = 25$.

These more complex word problems are intended to help students apply their understanding of fraction division to real word problems.

Although the two Chinese textbooks both present all four types of meanings in their word problems, the total number of word problem examples and exercises is less in the 2006 edition than in the 2002 edition. The word problems also differ in their complexity. The 2006 edition includes more word problems with combinations of meanings than the 2002 edition; and it also requires more mixed operations on fractions as solutions to the word problems than the 2002 edition (Figure 2).

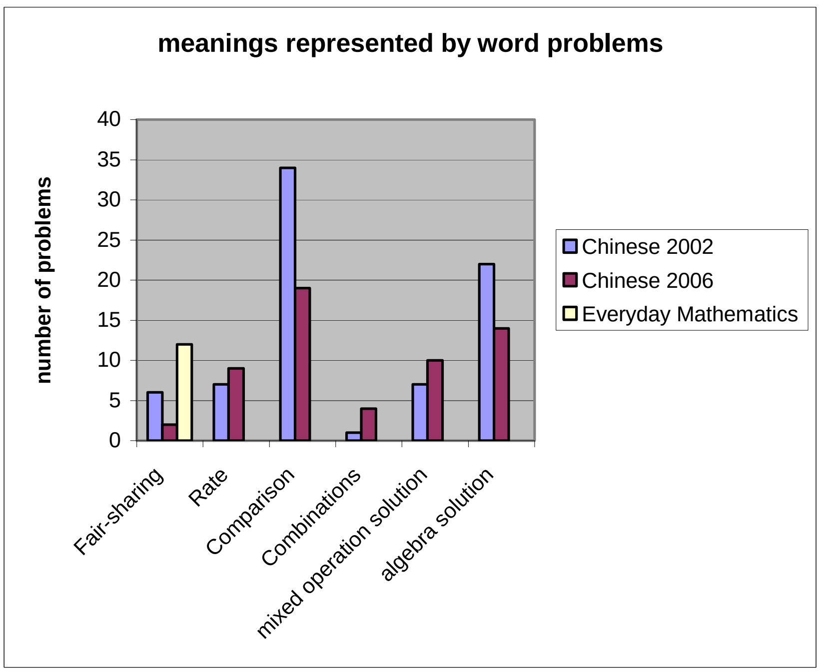

Figure 2. Quantities of each type of meaning of fraction division represented by word problems in the textbooks.

The differences between the two Chinese textbooks can be considered small compared to the difference between the Chinese textbooks and *Everyday Mathematics.* I have pointed out earlier that there are fewer word problems in *Everyday Mathematics* (9 in fifth grade, 3 in sixth grade) than in the Chinese textbooks (67 in the 2002 edition, 34 in the 2006 edition). Among the 12 word problems in *Everyday Mathematics*, there is only one meaning of fraction division represented by all of them, which is the measure-out meaning in the fair-sharing model. All of the 12 word problems for the measure-out meaning are intended to help students understand the common-denominator algorithm. This means that students do not have any contextualized examples or exercises to facilitate their learning of the invert-and-multiply algorithm.

Lastly, even for this measure-out meaning that is introduced by both Chinese and Everyday Mathematics textbooks, it is presented very differently by the Chinese and Everyday Mathematics textbooks. In both Chinese textbooks, the fair-sharing word problems are introduced with a focus on the inverse relationship between multiplication and division of fractions. For example, the first example of a fair-sharing problem actually includes three sub questions. They are:

1. If each person eats half of a moon cake, how many moon cakes altogether are eaten by four people? $\frac{1}{2} \times 4 = 2$;
2. To share two moon cakes fairly among four people, how much does each person have? $2 \div 4 = \frac{1}{2}$;
3. To share two moon cakes fairly, with each person getting half of a moon cake, how many people can share? $2 \div \frac{1}{2} = 4$.

After this example, the inverse relationship between fraction multiplication and division is explicitly summarized in the textbook, and it is further emphasized by exercises and homework. For example, the in-class exercise after the word problem example is: Based on the multiplication equation, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, fill in the blanks: $\frac{1}{6} \div \frac{1}{3} = \_$, $\frac{1}{6} \div \frac{1}{2} = \_$.

However, when *Everyday Mathematics* introduces measure-out word problems, the measure-out problem is provided alone. For example

1. How many 2-pound boxes of candy can be made from 10 pounds of candy?

2. How many ¾ -pound boxes of candy can be made from 6 pounds of candy?

In the *Everyday Mathematics* textbook, the inverse-relation between fraction multiplication and division is never mentioned.

After looking at these differences between how the Chinese textbooks and *Everyday Mathematics* present word problems, the differences found in Chinese and U.S. teachers' ability to construct word problems (Ma, 1999) is not terribly surprising.

Unfortunately, for teachers who use *Everyday Mathematics*, the limited number of lessons and representations of meanings in the textbook may not be the only reasons that they lack knowledge on teaching fraction division: the instructions and assistance from the teacher's guide is even scarcer when compared with those in the Chinese teacher's guides

Content analysis of teacher's guides. I find that the overall structures of Chinese and *Everyday Mathematics* teacher's guides differ a great deal in terms of the number of books, their organizations and functions, and how they are related to the textbooks.

Overall structures of teacher's guides**.** One major difference between the Chinese and Everyday Mathematics teacher's guides is the number of books provided. For each of the Chinese textbooks, there is only one book, called "teacher's manual," provided as a teacher's guide by the publisher. All the important information that teachers should know about their textbook is included in this teachers' manual. Chinese teachers only need to refer to this one book to use the textbook. But for teachers who use Everyday Mathematics, there are four books provided by the publisher as a teacher's guide—a teacher's lesson guide (the major guide), a teacher's reference book, a math masters book, and an assessment handbook.

What makes it confusing to use the four *Everyday Mathematics* books is that they do not share a common organization. Although each teacher's lesson guide covers one semester and is organized by units and lessons taught in that semester, each math master's book covers the content for one school year (two semesters), and is organized partially by units and lessons and partially by types of learning activities. The assessment handbook also covers one school year and is organized by units; however, within each unit, the instructions are not structured in the order of the lessons in the unit. Lastly, the teacher's reference book covers 4^{th} through 6^{th} grade, and it is organized by mathematical topics that are scattered across a total of six textbooks in the three grades. Because of the various organizations of these books, a large portion of the instructions in each book deals with locating relevant information in the other books.

In *Everyday Mathematics* guides, the instructions for teachers are not only scattered among several books, but also among many topics within a unit, which leads to another major difference between the Chinese and *Everyday Mathematics* teacher's guide. This difference is in the overview instructions provided in each chapter/unit. Because the Chinese textbook's chapter is mostly devoted to fraction division, the overview instructions and explanations for that chapter are all related to understanding and teaching fraction division. A major part of the overview in both Chinese teachers' manuals is the rationale for the structure of the lessons in the chapter—both conceptually and pedagogically. For example, the 2006 teacher's manual provides a flow chart diagram for the organization of concepts and skills covered in the lessons.

But for *Everyday Mathematics*, because the unit covers many topics that are irrelevant to fraction division, the overview instructions for the units in the teacher's lesson guides are mostly lists of information for individual topics. Most surprisingly, with so many topics covered in one unit, the guides provide no explicit reasons why these topics are grouped together as a unit or explanations on how these topics should be built upon each other. Such a difference shows that it is not just the structures of instructions that are different: the content of instructions provided in the Chinese and *Everyday Mathematics* teacher's guide also differ.

Stated learning goals in teacher's guides. Although both Chinese teachers' manuals and the *Everyday Mathematics* teacher's lesson guide provide learning objectives for fraction division, their objectives differ dramatically. *Everyday Mathematics*' objective is only focused on calculation: "practice the common denominator algorithm for dividing fractions and mixed numbers" (fifth grade) and to "learn a division algorithm for fractions and use it to divide fractions and mixed numbers" (sixth grade). There is no statement about either understanding the algorithm or problem solving.

In both Chinese textbooks, there are two explicitly stated learning goals in the teachers' manuals: (a) understand the meaning of fraction division; understand the algorithm and apply it to calculations fluently; (b) be able to use arithmetic and algebra methods to solve problems, especially the fraction division word problems with the comparison meaning. Because *Everyday Mathematics*' teacher's guide does not define understanding and application as goals, it would not be surprising if teachers did not emphasize these goals in their teaching.

But sometimes, even when teachers are told to teach for understanding and application, they may not know how to do it. Instructions in teacher's guides may be their most useful assistance in learning to teach for such learning goals.

Instructions for textbook examples and exercises. Both Chinese textbooks and *Everyday Mathematics* include examples, fraction division algorithms, and exercises as teaching content. Instructions for teaching this textbook content are also provided in all teachers' guides. However, the instructions differ in terms of the types of information they provide to teachers.

For each example in the Chinese textbooks, the procedures for solving the problem are also provided in students' textbooks. Therefore, instead of focusing on the problem-solving procedures, the accompanying teachers' manuals provide explicit explanations on the purpose of including the example, the focused concept in the example, and guiding questions to solve the example problem. In addition, the manuals often include students' prerequisite knowledge and common mistakes made when solving the problem, as well as how this example is related to other problems in following exercises and assignments. For example, example #3 in the 2006 version is a word problem with the rate meaning of fraction division. The example says:

> Ming walks 2km in 2/3 hour, and Hong walks 5/6 km in 5/12 hour. Who walks faster?

There are four pages of instructions in the teachers' manual on teaching this example. Following is a summary of the instructions:

Purpose and focus of this example:

1. This example examines situations when a number is divided by a fraction—both when a whole number is divided by a fraction and when a fraction is divided by another fraction.

2. This example involves the speed meaning of division: distance ÷ time = speed.

3. Students should be familiar with comparing speed.

4. The speed for Ming $2 \div \frac{2}{3}$ is the focus for this example. Its calculation should be constructed by students.

5. The calculation for dividing a fraction by another fraction, $\frac{5}{6} \div \frac{5}{12}$, should be built upon the calculation for dividing a whole number by a fraction.

6. The exercises on P31 are similar problems to this example and should help students further understand this example.

Suggestion for teaching this example:

1. Before teaching this example, the teacher may give a whole number division word problem with the speed meaning.

2. Teachers should encourage students to use different methods to solve this problem.

3. Guiding questions for students to create ways to calculate $2 \div \frac{2}{3}$.

4. Guiding questions to lead students to summarize the "invert and multiply" algorithm.

With such detailed explanations for each example, the Chinese teachers' manuals serve the function of providing teachers with chances to establish a thorough understanding of the examples. Compared to the thorough explanations in the Chinese teacher's manuals, the lesson guide for *Everyday Mathematics* seems to focus on providing procedural instructions for teaching lessons.

Unlike the Chinese textbooks, the students' *Everyday Mathematics* books do not include the procedures for solving the example problems. Therefore, a major part of the teachers' lesson guide is the answers for example problems. Other than that, the amount of instructions for teaching the examples in *Everyday Mathematics* is very limited. With no explanations on the structure of the lesson or the purpose of each example, *Everyday*

Mathematics' lesson guide provides mostly procedural instructions for teaching the examples—what to write on the blackboard, what to ask the students, and how to respond to students' answers. For only a few examples, the lesson guides instruct teachers to remind students of a concept or help students to clarify their understanding of a procedure; however, the guides never provide details for how teachers can provide such reminders or clarification.

There are also differences between the Chinese and *Everyday Mathematics*' instructions for the exercises. For most of the exercises in the Chinese textbooks (2/3 of all the problems in the 2002 versions, and 100% for the 2006 version), the teachers' manuals provide explanations for the purposes of the questions, the key concepts for solving the problems, and the relationship between the exercises and the examples. Interestingly, with all this information provided, the answers to the exercise problems are not included in the teachers' manual or anywhere else in the curriculum materials. In contrast, in the *Everyday Mathematics* lesson guide, *only* the answers to the problems are provided as instructions for the exercises; there are no explanations of the rationale for these problems. Such a difference indicates that Chinese teachers are steered to focus on what and how students learn through working on their exercises; teachers using *Everyday Mathematics* are steered to focus on the correct solutions to the exercise problems.

Such differences in teaching either for conceptual understanding or for procedural correctness may be further reflected by the instructions on teaching the fraction division algorithms in the teacher's guides.

Instructions on fraction division algorithms. As I pointed out earlier, while *Everyday Mathematics* introduces both common denominator and invert-and-multiply algorithms as fraction division algorithms, both Chinese textbooks only teach the invert-and-multiply algorithm. Accordingly, the instructions for teaching the algorithms differ between *Everyday Mathematics* and the Chinese teacher's guides. All instructions in the Chinese teachers' manual are focused on teaching the invert-and-multiply algorithm. But *Everyday Mathematics* includes information on teaching both algorithms. In addition, it also provides a thorough review article on these two algorithms, comparing their meanings, whether they are easy for students to understand, and their effectiveness in calculations. With such information, teachers are provided with opportunities to understand the rationale for teaching the common denominator algorithm before teaching the invert and multiply algorithm. They can also learn about why the invert-and-multiply algorithm is taught as the primary fraction division algorithm. Unfortunately, Chinese teachers are not provided with any of this information.

I suspect that if *Everyday Mathematics* provided instructions for teaching the invert-and-multiply algorithm that were as thorough as the review on the two algorithms, teachers would be knowledgeable and thus empowered to effectively teach the invert-and-multiply algorithm for conceptual understanding. Unfortunately, the lesson guide for *Everyday Mathematics* only provides simple procedural instructions for teaching the algorithm. The instructions say:

> Ask whether anyone knows how to solve the problem using the 'invert and multiply' rule. If so, have a volunteer demonstrate it. If no one is able to, have the class turn to the Division of Fraction Property (a proof of the algorithm) at the top of page 89 in the *Student Reference Book*.

Note that this important algorithm is presented to students without conceptual explanation of its meaning. According to these instructions, there is nothing that teachers need to do to guide students to construct an understanding of this algorithm. This stands in stark contrast to the instructions provided to the Chinese teachers.

According to both Chinese teachers' manuals, students are supposed to construct the invert-and-multiply algorithm as well as a deep understanding of it through solving problems. To assist teachers in helping students construct the algorithm, the teachers' manual instructions include a list of important steps for students' construction of the algorithm through problem solving; and they also provide a list of important points that should not be missed in students' summarizations.

Interestingly, there is a major difference between the two versions of Chinese teachers' manuals. The 2002 version does not provide any proof of the algorithm in the manual, but the 2006 version provides teachers with three proofs of the algorithm. However, even with the three proofs listed, the 2006 manual suggests that teachers teach this algorithm by letting students construct it. The reason for choosing constructing over proofing is that

> There is no need for contextual problems when teaching through proofs; they are very abstract and with no illustrations. Although most students may understand the proofs, the mathematical proofs do not reveal the concrete meanings of fraction division. Thus teaching the mathematics proofs may not benefit students on problem solving. Therefore, it is suggested that the algorithm be taught by students' discovery and construction.

Drawing on this statement, it is obvious that the learning goal of the algorithm is for students to derive it and apply it. Because teachers are not required to teach the proofs to students, the reason to include the three proofs in teachers' manuals seems to be to deepen teachers' understanding of the topic. Such a function of a teacher's guide may not be directly applied to classroom teaching, but it may be important for professional development of teachers because it provides teachers with chances to enhance their understanding of mathematics content.

Other than improving teachers' understanding, the functions of teachers' guides may also include improving teachers' pedagogical knowledge and pedagogical content knowledge by addressing students' common mistakes and confusions.

Instructions about students' mistakes and confusions. It is inevitable that students will sometimes have misconceptions or confusions when they learn about a topic, and it is also common that they make mistakes when solving problems. One common function of a teacher's guide is to help teachers to identify students' mistakes and strategically correct them.

Both Chinese teachers' manuals and *Everyday Mathematics*' lesson guide point out two specific common mistakes—students often take the reciprocal of the dividend instead of the divisor, or they do not change the division to multiplication. However, the Chinese teachers' manuals and *Everyday Mathematics*' lesson guide suggest very different approaches to help correct or avoid this mistake.

According to the Chinese teachers' manuals, students make these mistakes because they have not fully understood the meaning of fraction division and the algorithm. The manual further points out that there are three exercises in the textbooks designed to help students deepen their understanding of fraction division. These exercises are:

1. Paired multiplication and division calculation. For example, $15\times\frac{5}{6}$ and $15\div\frac{5}{6}$. The manual suggests that this exercise will help students to understand the difference and relationship between multiplication and division.

2. Without calculation, decide whether the given equation's (e.g. $\frac{6}{7}\div 3$, $9\div\frac{3}{4}$, $\frac{1}{2}\div\frac{2}{3}$) quotient is larger, equal to, or smaller than the dividend. The manuals point out

that this will help students apply their understanding of the concrete meaning of fraction division.

3. Comparison of operations with fractions. Students will calculate $\frac{5}{6}+\frac{2}{3}$, $\frac{5}{6}-\frac{2}{3}$, $\frac{5}{6}\times\frac{2}{3}$, and $\frac{5}{6}\div\frac{2}{3}$ at the same time.

Through these exercises, students should learn to distinguish the difference among the four operations with fractions.

It is quite clear that the Chinese teacher's manuals do not provide any quick fix to these common mistakes; instead, they suggest that teachers should understand the cause of the mistakes and then allow students time to learn by developing a better understanding of the meaning of fraction division. It is likely that during the development of these curricula, the editors had these common mistakes in mind and designed a few exercises specifically for helping students with this mistake.

Teachers who use *Everyday Mathematics* are not given such thorough assistance on how to approach these common mistakes. *Everyday* Mathematics does not provide any information on the mistake of not changing the division to multiplication, it only points out the mistake of taking the reciprocal of the dividend instead of the divisor: "A common error in using this algorithm is to take the reciprocal of the dividend instead of the divisor. Be sure to caution students to use the correct reciprocal." Other than these two sentences, no other instructions are given on helping students correcting the mistake. *Everyday Mathematics* seems to identify this mistake simply as a memorizing the algorithm incorrectly and thus provides a quick fix to address students' memory problems.

In summary, both Chinese textbooks require students to devote much more time to learning fraction division than does *Everyday Mathematics*. They provide word problem examples with multiple meanings of fraction division to guide students to construct the algorithm, and they also provide many exercises in various formats to help students apply their knowledge. As for *Everyday Mathematics*, there are much fewer word problems as examples and exercises than in the Chinese textbooks. Students are presented with the algorithm directly and are required to drill on calculation for most of their exercises. In terms of teachers' guides, both Chinese teachers' manuals provide teachers assistance to develop a thorough understanding of their textbooks. They also provide opportunities for teachers to deepen their understanding of the content knowledge of fraction division as well as their skills in teaching this topic. Compared to the Chinese teachers' manuals, *Everyday Mathematics* provides much less assistance to teachers to help them understand the textbook, the content knowledge, or the pedagogical knowledge. In addition, because instructions for teachers are scattered among four books, it is inconvenient and confusing to access information.

Discussion

Results of this textbook content analysis show that the two Chinese textbooks are very similar in their lesson structures, presentation of the invert-and-multiply fraction division algorithm, types and numbers of examples and exercises, and introduction of the multiple meanings for fraction division. *Everyday Mathematics* is dramatically different from the Chinese curricula in each of these aspects. The Chinese curricula feature textbooks that teach only the invert-and-multiply algorithm, provide multiple types of problems as examples and exercises, include many word problems, and introduce multiple meanings of fraction division. On the other hand, the *Everyday Mathematics* textbook teaches both the common-denominator and invert-and-multiply algorithms, only provides calculation and word problems as examples and exercises, provides no word problems in teaching the invert-and-multiply algorithm, and introduces only the measure-out meaning of fraction division.

Although these differences are striking, there is no convincing evidence that the noted differences contribute to differences in students' learning. No investigation to date has shown that either the Chinese textbook features or the *Everyday Mathematics* features are more effective in promoting students' learning of fraction division. On one hand, by introducing the common-denominator and then the invert-and-multiply algorithm, *Everyday Mathematics* follows the suggested teaching steps of many researchers. From this research, we would expect students to understand fraction division from a relatively easy-to-understand algorithm, then move to a more abstract algorithm. On the other hand, by providing students with multiple types of word problem for fraction division, the Chinese curricula should help students understand and apply the invert-and-multiply algorithm through meaningful problem solving. More importantly, when students are not restricted to solving fair-sharing word problems , as is the case with the Chinese curricula, they should have more opportunities to correct common misconceptions of division, for example, the belief that the dividend has to be bigger than the divisor and quotient (Fischbein et al., 1985). In addition, although each curriculum may have its own advantages for teaching fraction division, the different strategies may also carry their own disadvantages for students' learning. For example, teaching the common denominator algorithm may help students learn fraction division with an easier-to-understand meaning, but if only limited time is allotted to teach fraction division in a curriculum, students may be left with less time to study the invert-and-multiply algorithm. It is also possible that students will compare the two algorithms and find it more difficult to make sense of the invert-and-multiply algorithm than to do so with the common denominator algorithm, and become unwilling to learn about the invert-and-multiply algorithm. In the case of textbook examples and exercises, providing and mixing multiple types of exercises may help students construct multiple contextualized meanings of this topic, but it may also overload students' attention by providing more information than what students can comprehend. It seems critical that experiments are conducted to test whether the Chinese textbook features or the *Everyday Mathematics* features more effectively promote students' learning of fraction division.

Chapter 3

Study 2: Experimental Investigation of Chinese- and *Everyday Mathematics*-Style Textbooks

Introduction

As the results of Study 1 have shown, the Chinese textbooks and *Everyday Mathematics* differ in terms of the fraction division algorithms, numbers and types of examples and exercises, and contextualized meanings of fraction division. To determine whether a Chinese or *Everyday Mathematics* textbook is more effective in promoting students' learning, especially for U.S. students, I conducted a teaching experiment in Study 2. The purpose of this experiment was to compare the effectiveness of classroom teaching following a set of Chinese-style lesson compared to using a set of *Everyday Mathematics*-style lessons.

Method

Participants. I recruited 63 sixth-grade students (35 female, 28 male) from two public and one private school in Champaign, Illinois. 31 of the participants are white, 26 are Black, 5 are Asian American, and 1 is Latino. Before the experiment, all participants had formally studied addition, subtraction, and multiplication of fractions in their schools, but they had not formally studied the invert-and multiply algorithm for division of fractions in school.

Procedure. I designed two sets of lessons and two tests for this study. The two sets of lessons included one set of lessons based on the features of the Chinese textbooks and another set based on those of *Everyday Mathematics*. Each set included two lessons. The tests were one pretest and one posttest, both of which were designed to evaluate students' knowledge and attitudes about learning fraction division.

In addition to the teaching and testing materials, I also wrote teaching scripts for conducting the lessons. These scripts were designed to avoid any teaching bias that might result from the teacher's personal preference or teaching styles. The scripts included: guiding questions that the teacher should ask when teaching through each example and exercise; whether or not the teacher should draw a graphic demonstration of the problems for each example and exercise; the teacher's responses to students' answers; additional information related to fraction division that the teacher should provide to students; and instructions on monitoring the pretest and the posttest.

Participants were first grouped into small groups according to their regular classrooms; each small group contained 4-10 participants. Such a small group setting was arranged for two reasons. One reason was that I hoped to conduct the experimental teaching in a setting that closely resembled the students' usual classroom instruction, where students could discuss and share their thoughts with each other. Another reason was that, practically, I had to follow the schools' class schedule, which only allowed students to participate in this study during their regular mathematics class periods.

After the small groups were set up, I randomly assigned each group into one of the two conditions: condition I (*Everyday Mathematics*-style lessons) and condition II (Chinese-style lessons). To reduce the influences of school culture and learning styles, during this random assignment, I tried to balance the number of groups of each condition within each school.

During the experiment, each small group was taught two lessons, one on each of two consecutive days. The lessons took place either in participants' regular classrooms or in a separate area in the school library. For both conditions, each lesson lasted 45 minutes. The first lesson started with a 10-minute pretest, followed by 25 minutes of teaching and then a 10-minute assignment time when students worked on their assignment questions individually. The second lesson started with 25 minutes of teaching, a 10-minute assignment time, and ended with a 10-minute posttest. All tests and lessons were conducted by me, with another researcher observing my teaching. The additional researcher was present to observe whether I closely followed the lesson scripts when I conducted the lesson, and he also evaluated whether my teaching strategies were biased towards either of the conditions. Both the additional researcher and I kept detailed field notes of my teaching during the lessons. After each lesson, we discussed my teaching and the students' responses.

During the pretest and posttest, participants were told to work on their tests independently. They were also told that some of the questions may seem familiar and some others may be totally new to them; and if they were not sure about the answer, they could make a good guess or try using creative ways to solve the problem. In addition, they were allowed to skip a question if they would rather move on to the next question.

At the beginning of each lesson, each student was given a print out of the lesson that included examples, exercises, and assignments. I guided students through the problems in the order of problems listed on their print out. To control for experimental bias by the researcher, I kept my teaching strategy consistent between both conditions in terms of asking questions and responding to students' answers. When teaching through examples, I first read the example question, and then gave students one minute to think about the question, and then I asked for volunteers to provide answers to share their thoughts. When no student was able to provide the correct answer, I would either draw a pictorial illustration, when the question was a word problem, or reminded students of the algorithm, when the question was a calculation problem. Then I asked for volunteers to answer the question again. If still no student could provide the correct answer, I would demonstrate solving the problem to the group and then ended the question. If one or more students were able to answer the question correctly, the question would be ended with no further explanation. Such questioning and responding scripts were designed to prevent potential bias caused by discourse patterns that I might use for different lessons designs. Research (Schleppenbach et al., 2007a) shows that Chinese teachers tend to ask more follow-up questions regardless whether students give correct or incorrect answers than U.S. teachers. Although this practice of Chinese teachers is probably partially because of their beliefs in learning and teaching mathematics, there has not been any study that has examined whether the Chinese curriculum also contributes to the teaching style of Chinese teachers. In this study, whether the Chinese-style and *Everyday Mathematics*-style curriculum may lead to different teaching styles is not of interest. Therefore, the question-and-answer procedures were designed to be consistent between the two conditions. I strictly followed this design during the implementation of the lessons.

Lesson designs. Both sets of experimental lessons were focused on fraction division, and both contained two lessons. The design of the Chinese- and *Everyday Mathematics*-style lessons was based on the following five major differences I found from the textbook comparison:

1. All Chinese lessons were focused on the invert-and-multiply algorithm; *Everyday Mathematics* split the lessons between the common-denominator and the invert-and-multiply algorithms.

2. Chinese textbooks provided significantly more word problems, both as examples and exercises, than *Everyday Mathematics*; *Everyday Mathematics* provided mostly calculations as examples and exercises.

3. All assignment problems in Chinese textbooks were related to fraction division; assignments in *Everyday Mathematics* covered various topics that were not related to fraction division.

4. Word problems in the Chinese textbooks represented all four types of meanings of fraction division, the problems in *Everyday Mathematics* represented only the measure-out meaning of fraction division; the Chinese textbooks introduced the measure-out problems with a focus on the inverse relationship between fraction multiplication and division, but *Everyday Mathematics* textbooks did not mention this relationship.

5. Chinese textbooks guided students to construct the algorithm through solving word problem examples and all the word problem examples in the Chinese textbooks were solved by a combination of drawing pictorial illustrations and writing down equations. *Everyday Mathematics* presented the algorithms (with proof) to students directly.

The Chinese-style lessons. For the set of Chinese-style lessons, both lessons focused on the invert-and-multiply algorithm. Although the Chinese textbooks provided various meanings of fraction division, because I needed to restrict the experimental teaching to two lessons, I had to choose only two meanings of fraction division to include in the lessons: measure-out meaning and the comparison meaning. I chose the measure-out meaning because it is the most straightforward meaning of fraction division, and it is the easiest for students to understand. I chose the comparison for two reasons: (a) among all the meanings represented by problems in the Chinese textbooks, the comparison meaning is presented in most of the problems (see Figure 2); and (b) in Ma's (1999) study, most Chinese teachers constructed their fraction division word problems based on the comparison meaning.

The first lesson focused on the measure-out meaning of fraction division; it included four word problems as examples and whole-class exercises. Students were encouraged to construct solutions on their own for these problems. The lesson ended with a teacher-guided summary of the algorithm. The assignment for this lesson included two sets of calculation problems and one measure-out meaning fraction division word problem. The first set of calculation problems included two pairs of fraction multiplication and division problems, and it focused on the inverse relationship between multiplication and division. The second set of calculation included three fraction division problems.

The second lesson focused on the comparison meaning of fraction division. It included four comparison meaning word problems as examples and whole-class exercises. Students were encouraged to construct solutions with the teacher's guidance. After the word problems, the teacher guided students to summarize the algorithm again. Assignment for the second lesson included two comparison meaning fraction division word problems, one set of fraction division calculations, and one set of calculations that include all four operations on fractions.

The Everyday Mathematics-style lessons. The *Everyday Mathematics*-style lessons were also comprised of just two lessons. The first lesson focused on the common-denominator algorithm and the second lesson focused on the invert-and-multiply algorithm. The first lesson included three word problem examples, a hands-on demonstration of the common-denominator algorithm, and four calculation exercises. The in-class assignment included three sets of calculation problems. The first set included six fraction division problems; the second set included two fraction multiplication problems; and the third set included three problems that required students to fill out the blanks with “<” and “>”.

Lesson Two included direct presentation of the invert-and-multiply algorithm, one optional reading on a proof of the algorithm, four fraction division calculation problems as examples and whole-class exercises on applying the invert-and-multiply algorithm. The in-class assignment included two calculation problems on fraction division and three calculation problems on addition and subtraction of fractions.

Measurements. Participants were asked to complete a pretest and a posttest. Each test included a measurement of participants’ mathematics knowledge of fraction division and related topics, and they also included a measurement of participants’ beliefs about mathematics learning. To measure participants’ mathematics knowledge, three types of problems were included: a) division calculation problems; b) division word problems; and c) short answer problems that focused on the inverse relationship between multiplication and division. Both the pretest and posttest included whole number division calculation and fraction division calculation problems; they also both included word problems with the fair-sharing, speed, and comparison meanings.

It requires two pieces of knowledge to solve fraction division word problems—the general understanding of division word problems and the understanding of fraction operations. If students lack either of these understandings, they will not be able to solve fraction division word problems. To understand which piece of knowledge that participants had either achieved or still lacked, I designed the pretest to focus on evaluating participants’ general understanding of division word problems. Therefore, most of the pretest word problems were whole number division word problems. I focused the posttest on evaluating participants' ability to apply their knowledge of fraction division to solve word problems, so most of the word problems in the posttest were fraction division word problems.

Pretest. To evaluate participants' prior knowledge that is relevant to the learning of fraction division, the pretest included problems on concepts and skills that participants may have learned prior to this study. In Ma's (1999) study, when Chinese teachers were interviewed about teaching fraction division, many of them mentioned several concepts as important pieces of fundamental mathematics understanding of fraction division. These pieces included division with whole numbers, inverse operations between multiplication and division, the concept of fractions, and fraction multiplication. In Baroody's (1998) chapter on operations on fractions, he also implied that such knowledge is closely connected to knowledge of operations on whole numbers and the concept of fractions. To verify such a connection between knowledge sets, to measure participants' prior knowledge of related content, and to detect participants' existing knowledge of fraction division prior to the teaching, I assessed students on the following fraction division related knowledge on the pretest:

- Calculation of whole number division;
- Whole number division word problems;
- Inverse operations (multiplication and division) with whole numbers;
- Finding common denominators of fractions;
- Word problems on multiplication of fractions;

In addition, because some students may have learned about fraction division from their family or on their own, I also evaluated their existing knowledge of fraction division on the pretest:

- Calculation of fraction division when fractions have a common denominator;
- Calculation of fraction division when fractions have different denominators, which are both small numbers and easy to convert to common denominators;
- Word problems on fraction division with the measure-out meaning.

In addition to evaluating participants' mathematics knowledge, I was interested in finding out whether different styles of curriculum may affect students' interests and motivation. Prior research (e.g., Stevenson, Hofer, & Randel, 2000) has found that Chinese students were more interested in mathematics and were more motivated to learn mathematics than U.S. students. Therefore, I surveyed students on their beliefs about mathematics learning, especially about learning fractions. In the pretest, participants were asked the following questions:

- Do you like math?
- Do you like learning about fractions?
- Do you think that fractions are useful in your life?

Please refer to Appendix A for the detailed content of the pretest.

Posttest. After the two lessons, students were again evaluated on both their mathematics knowledge and their thoughts on mathematics learning. Fraction division knowledge was the focus of the posttest. In addition to evaluating how much students learned from their lessons, I also tested whether students were able to transfer their knowledge to solving new problems. Therefore, in the posttest, students were not only asked about questions that were covered in their lessons, but were also asked to answer questions that were not introduced in their lessons, for example, a word problem that represented the speed meaning of fraction division.

In addition to fraction division problems, I also hoped to evaluate whether students improved their understanding of knowledge related to fraction division, for example, the concept of division, through learning about fraction division. Therefore, the posttest included the following mathematics problems:

- Calculation of fraction division when fractions have a common denominator;
- Calculation of fraction division when fractions have different denominators that are both small numbers and easy to convert to common denominators;
- Calculation of fraction division when fractions have different denominators that are relatively difficult to convert to common denominators;
- Word problems with fraction division, with the measure-out meaning;
- Word problems with fraction division, with the speed meaning;
- Word problems with fraction division, with the comparison meaning.

I also included problems that I intended to tap knowledge related to raction division:

- Calculation of whole number division;
- Word problems with whole number division;
- Inverse operations (multiplication and division) with whole numbers;
- Finding common denominators of fractions;

After responding to all the mathematics questions, students were asked several questions about their general thoughts on mathematics learning, learning about fractions, and their preference for problem types and difficulties. These questions were designed to examine whether the teaching experiment influenced students' views on math learning. The questions are:

- Among all the questions you just saw, choose the most difficult problem.
- Among all the questions you just saw, choose the most fun problem.
- If you could choose a problem for homework, which one would you pick?
- Do you like math?
- Do you like learning about fractions?
- Do you think that fractions are useful in your life?

Results

Results of this study include three sections: (a) participants' proficiency in solving the mathematics problems in pre- and posttests, (b) their responses to the questions regarding their interest and motivation in learning mathematics, and (c) my field notes taken during instruction.

In the first section, I will present data on participants' performance on problems of prerequisite knowledge for fraction division that provide some indication of their readiness to learn the concept of fraction division. Next, I will show their learning in terms of their proficiency in solving fraction calculation problems and word problems. In the second section, I will show results of participants' choice for their favorite type of questions, for the questions they considered most difficult, and to what extent they like mathematics learning. In the third section, I will report my observations during my teaching of the experimental lessons; the data will include participants' responses to the instruction, participants' common mistakes and misconceptions during the instruction, classroom environments, and teachers' comments on the instruction.

Proficiency in solving mathematics problems. In this section, I will report student performance on the tests in four categories of problems: a) problems of prerequisite knowledge for fraction division; b) calculation; c) word problems; and d) common denominators. All the results reported were based on *Fisher's* exact tests, unless other tests are specified.

Problems of prerequisite knowledge for fraction division. Four types of questions were included in the pretest: whole number division calculations, whole number division word problems, common denominators of fractions, and the inverse relationship between multiplication and division. There was no significant difference between the two groups in terms of their performance on these questions. Despite the lack of significant differences, the quality of the participants' responses is worth noticing.

Among the six whole number division calculation problems, the two standard division questions, with whole number solutions and with the dividend bigger than the divider ($32 \div 8 =$ and $32 \div 4 =$), were answered correctly by most of the participants: 59 out of 63 participants answered $32 \div 8 =$ correctly, 51 participants answered $32 \div 4 =$ correctly. When the questions' answers were not whole numbers ($15 \div 4 =$ and $13 \div 4 =$), the number of participants who answered the questions correctly dropped to 23 (for $15 \div 4 =$) and 24 (for $13 \div 4 =$). When the problems contain a dividend that was larger than the divisor ($3 \div 12 =$ and $4 \div 12 =$), most participants did not answer correctly: only 3 participants answered $3 \div 12 =$ correctly, two did so by doing long division and gave the answer as a decimal number; and four participants answered the problem $4 \div 12 =$

correctly. The most interesting part of participants' responses to these two questions was that most participants gave the same incorrect answers. 50 out of 63 participants (79.4%) answered $3 \div 12 =$ with 4, as if the question were $12 \div 3 = 4$; and 43 out of 63 participants gave the answer 3 to the question $4 \div 12 =$. Switching the dividend and the divisor was also demonstrated in participants' responses to division word problems.

A total of four whole number division word problems were given in the pre- and posttest. There was no significant difference found between the two conditions in terms of their performance on these problems. However, participants' performance on the two fair-sharing word problems is very interesting. Across both conditions, the problem with a dividend that was larger than the divisor "cut a piece of ribbon that is 32 feet long into pieces that are 4 feet long, how many small pieces will you get?" was answered correctly by 44 participants, which is significantly more than the number of participants who answered any other word problems correctly ($p = .001$). On the other hand, for the question with a dividend that was smaller than the divisor "share 5 pizzas among 15 people fairly, how much pizza will each person get?" was only answered correctly by 2 participants. The most typical (n = 36, or 57%) incorrect answer was "3," as if the participants used the equation $15 \div 5 = 3$ to solve the problem (see Figure 3).

Word problem	Responses (N = 63)			
	Non-response	Correct	Partial	Incorrect
15 people share 3 pizzas fairly, how much pizza does each person get?	15	2	0	46
Cut a 32-feet-long ribbon into short pieces, each short piece is 4 feed long, and how many short pieces can you get?	10	44	0	9
Pour 5 ½ bottles of apple juice into glasses, each glass holds ¼ bottle of apple juice, how many glasses can you fill?	26	13	1	23
Pour 3 ½ gallons of orange juice into ½ -gallon bottles, how many bottles can you fill?	18	23	0	22
A car can travel 180 miles in 3 hours, how many miles can it travel in 1 hour?	29	21	0	13
Josh can swim for 1 ½ miles within 2/3 hour, how many miles can he swim in 1 hour?	26	7	0	30
Jose has 45 music CDs, his CDs are 3 times as many as what Jerry has. How many CDs does Jerry have?	25	17	0	21
There are 640 picture books in the library, there are also some science fiction books in the library, the number of picture books is 4/5 of the number of science fiction books. How many science fiction books are there in the library?	28	2	7	44

Figure 3. Participants' responses to word problems in pretest and posttest

A whole number word problem with the speed model and one with the comparison model were also given to participants. The results of participants' performance on these two questions will be reported in the section on word problems, together with their performance on the speed and comparison word problems that involve fractions.

In addition to calculation and word problems, two whole-number questions, focused on the inverse relationship between multiplication and division, were also given to participants: "given $139 \times 16 = 2224$, $2224 \div 139 =$;" and "given $15 \times 40 = 600$, $600 \div 40 =$." Most participants (n = 49 for the first one, n = 53 for the second one) were answered these questions correctly.

The last questions that examined participants' prerequisite knowledge for fraction division were two questions that asked participants to rename two fractions as their equivalent fractions with a common denominator. Only a very few participants completed these questions. 8 out of 63 participants were able to rename $\frac{7}{6}$ and $\frac{3}{8}$ with a common denominator on the pretest; and 14 of 63 participants were able to rename $\frac{3}{10}$ and $\frac{4}{15}$ with a common denominator on the posttest, which was not a significant increase (p = .24). With an understanding of participants' prerequisite knowledge for learning fraction division, it is interesting to look at their learning after receiving instruction on fraction division.

Calculation of fraction division. In the pretest, participants were asked to answer four fraction division calculation problems to show their existing understanding of fraction division. No significant difference was found between the two conditions in terms of their performance on these calculation problems on the pretests, both when participants' correctness on each individual question was compared and when participants' correctness on all four questions combined was compared. In general, this was because very few students solved these problems correctly: 5 participants in condition I (*Everyday Mathematics*-style) and 6 participants in condition II (*Chinese*-style) answered $\frac{3}{2} \div \frac{1}{4} =$ correctly. Among these 11 participants, 2 participants in condition I and 1 participant in condition II were able to answer 3 out of the 4 division calculation questions correctly; the other 8 among these 11 participants did not answer any of the other three questions correctly. Other than these responses, none of the other participants answered any of the division calculation problems correctly during the pretest.

As for the four fraction division calculation problems on the posttest, some participants did not answer any of these problems. There were significantly more cases when no answer was provided to a problem in condition II (77 cases) than in condition I (38 cases) ($p = 0.0009$). Because the reason for no response was ambiguous (participants were not able to answer the question, or not motivated to answer the question, or they ran out of time), it would be unfair to code all these cases as incorrect answers by the participants. Therefore, in the following comparison of students' performance on the calculation problems, these cases were discarded. Results of the following tests were based on cases when participants actually provided answers to the calculation problems.

Among the four fraction division calculation problems: $\frac{3}{5} \div \frac{1}{10} =$, $\frac{2}{15} \div 2\frac{7}{20} =$, $\frac{15}{26} \div \frac{25}{32} =$, $\frac{5}{13} \div \frac{7}{13} =$, across both conditions, participants were more likely to answer $\frac{3}{5} \div \frac{1}{10} =$ correctly than any other fraction division calculation problem, and participants in condition I were more likely than in condition II to answer this problem correctly (p = 0.019). No significant difference was found between condition I and II on the other three questions. But across the four questions, participants in condition I gave significantly more correct answers (24 correct answers) than condition II (10 correct answers; p=0.0153). It is worth noting that only 2 participants (1 from each condition) gave the correct algorithm for solving the problem $\frac{2}{15} \div 2\frac{7}{20} =$, and neither of them was correct in calculating the final answer. In addition, relatively few students provided the correct answer to the problem $\frac{15}{26} \div \frac{25}{32} =$ (3 in condition I, and 1 in condition II) or $\frac{5}{13} \div \frac{7}{13} =$ (7 in condition I, and 4 in condition II).

Participants' attempts to write down either the common-denominator or the invert-and-multiply algorithm were also examined. Among participants' answers to all four fraction division calculation problems on the posttest, there were significantly more cases when participants wrote the invert-and-multiply algorithm in condition I (31 cases) than in condition II (9 cases) (p = 0.0006); there were also significantly more cases of correct invert-and-multiply algorithms in condition I (23 cases) than in condition II (8 cases) (p = 0.0135).

Among students' attempts to apply the common-denominator algorithm, a typical mistake was $\frac{3}{5} \div \frac{1}{10} = \frac{6}{10} \div \frac{1}{10} = \frac{6}{10}$. It is worth noting because students commonly demonstrated this same error during their lesson (also see the section in which I discuss my field notes).

Students also tended to make mistakes when applying the invert-and-multiply algorithm. Some changed the division to multiplication without inverting, for example, $\frac{2}{15} \div 2\frac{7}{20} = \frac{2}{15} \times \frac{47}{20}$; others made mistakes in simplifying the final result after correctly applying the algorithm, for example, $\frac{15}{26} \div \frac{25}{32} = \frac{15}{26} \times \frac{32}{25} = \frac{480}{650} = 1\frac{170}{650}$.

Word problems. Other than the four whole number division word problems, a total of four fraction division word problems were given on the pre- and posttests. No significant differences were found between the two conditions on any of the word problems. However, significantly more participants in condition II (10) than in condition I (1) drew illustrations when they tried to solve the word problems on the posttest ($p = .024$).

As for the four fair-sharing word problems (two with whole numbers, two with fractions), across both conditions, participants were more likely to answer the whole number “cut a piece of ribbon that is 32 feet long into pieces that are 4 feet long” correctly than any other word problem ($p = 0.001$). Some participants answered the two fair-sharing fraction word problems correctly (n=23 for one question, n=13 for another question). Participants were least likely to answer “sharing 5 pizzas among 15 people fairly” correctly.

As for the speed model problems, significantly more participants answered the question correctly when the word problem only involved whole numbers (21 participants) than when the word problem involved fractions (7 participants) ($p = 0.0048$). 6 of the 7 participants who answered the speed problem involving fractions correctly on the posttest, also answered the speed problem in whole numbers correctly on the pretest.

Similarly, significantly more participants answered the comparison word problem correctly when the problem only involved whole numbers (17 participants) than when the problem involved fractions (2 participants) ($p = 0.0001$). Seven participants wrote down the correct equation for solving the comparison problem involving fractions, but they did not calculate the correct result. In addition, one common mistake in solving the comparison problem was that participants wrote down a multiplication equation rather than a division equation to solve the problem. For example, when trying to answer the question "Josh's 45 CDs are 3 times as many as Jerry's CDs, how many CDs does Jerry have?" 13 participants gave 135 ($45 \times 3 = 135$, rather than $45 \div 3 = 15$) as the answer, and two more participants wrote down 125, which probably resulted from a miscalculation of 45×3 .

Inverse relationship between multiplication and division. There was no significant differences between the two conditions in either pretest or posttest in terms of students' performance on the three questions: (a) given $139 \times 16 = 2224$, $2224 \div 139 =$; (b) given $15 \times 40 = 600$, $600 \div 40 =$; and (c) given $\frac{4}{5} \times \frac{7}{9} = \frac{28}{45}$, $\frac{28}{45} \div \frac{7}{9} =$. There was also no significant difference found between participants' answers to the question that involved whole numbers and the one that involved fractions. Most participants were able to answer these questions correctly (49 out of 63 for question 1; 53 out 63 for question 2; and 50 out of 63 for question 3).

Interest and motivation in learning math and fraction division. There was no significant correlation between participants' performance on the tests and their indicated interest in learning math generally or learning fraction division specifically. Participants in the two conditions also did not differ significantly in their choice of the most fun question, the most difficult question, and a question they would like to work on as homework. Although no differences were found in their responses to these questions, students' responses during the tests and the lessons indicated their interest and motivation in learning mathematics, which will be reported in the following session.

Field notes. After teaching each lesson, I discussed the lesson with the observer and, based on the lessons and discussions, took detailed field notes. In these notes, I attempted to capture the participants' responses to the instruction, common mistakes and misconceptions, teaching environment, and participants' math teachers' comments if they had observed my teaching.

Participants' responses to the instruction. During the lessons, participants were generally well behaved but did not necessarily participate actively in discussion or volunteer to answer questions, especially while studying word problems. Few of them asked me about why or how to solve a problem in the way I demonstrated, but many of them asked me, "what do you want me to copy down?" and copied my problem solving procedures and answers in their notes. After I read a word problem and asked for a volunteer to give an answer or to share ideas, usually no students volunteered. Some of them also expressed their feeling that they did not like being taught through word problems. For example, in condition I, after being taught about the common-denominator method through solving word problems, one participant asked me "can you teach in a normal way?" After I asked her to tell me what a "normal way" was, she replied "just tell us what to do, without these stories and stuff." Another participant, in condition II, after the first lesson, complained to me and said: "this is too hard. If you were our age, would you want to do this?"

Although some of these responses and complaints may have been caused by the difficult learning materials, some participants were explicit about not caring about trying their best during the lessons. For example, one participant had finished the posttest earlier than the time limit and turned in the answers to me. While he was sitting there waiting for his classmates, he told me that he realized he made a mistake on one of the problems. When I asked him whether he would like his paper back to make some changes since there was still time, he responded: “No, I don’t care. I know I am smart anyway.” Students’ responses on the pre- and posttest also reflected their attitude as indicated by skipping the questions about their interest in learning math or their choice of fun and difficult questions.

In addition to participants’ general responses to the teaching and tests, one specific case is worth noting. During the teaching of a condition I group, after I demonstrated how to apply the invert-and-multiply algorithm with the problem $320 \div \frac{2}{7} = \frac{320}{1} \times \frac{7}{2} = \frac{2240}{2} = 1120$ on the blackboard, I asked the group “does it make sense?” One participant told me that the $320 \div \frac{2}{7} = \frac{320}{1} \times \frac{7}{2}$ part made sense, but the $\frac{320}{1} \times \frac{7}{2} = \frac{2240}{2} = 1120$ part did not. And she told me that she did not understand how the two fractions were multiplied and then simplified. Such responses may be a result of participants’ lack of understanding of certain previously taught topics such as fraction multiplication and reducing to simplest terms.

Common mistakes and misconceptions. Students voiced several misconceptions. One common misconception was manifested in a belief that the dividend must be larger than the divisor. For example, during the instruction, one student asked “if a small number is divided by a big number, do you switch the two numbers?” And, during the test, when trying to solve $4 \div 12$, a few students complained that “you cannot divide 4 by 12, 12 doesn’t go into 4.” Many other students also claimed that this question was impossible to solve or did not have an answer. Most of students answered $4 \div 12$ by switching the dividend and the divisor, thereby indicating that they shared this misconception.

Another misconception voiced by the students was that the quotient must be smaller than the dividend. Several participants explicitly stated this idea during the instruction. For example, one student asked, "How do you divide a number by $\frac{4}{5}$ and get a bigger number?" Students' responses during the instruction also indicated that they lacked basic understanding of the concept of fractions, especially the meaning of numerators and denominators. And this lack of understanding also hindered them from learning fraction division.

The biggest challenge during teaching the common-denominator algorithm was to explain to students how the common denominator is sometimes removed from the final answer. For example, the answer for $\frac{3}{5} \div \frac{1}{10} = \frac{6}{10} \div \frac{1}{10} = 6$, rather than $\frac{3}{5} \div \frac{1}{10} = \frac{6}{10} \div \frac{1}{10} = \frac{6}{10}$. Some students mentioned that in fraction addition, the common denominator remains in the final answer. They failed to see how it was necessary to divide 10 by 10, resulting in 1, which allowed for the apparent removal of the denominator.

Participants were also confused by the invert-and-multiply algorithm. Some participants thought that when solving fraction subtraction problems, they should also change the subtraction to addition to solve the problems.

In addition to the confusion about the algorithms, many of the students' questions indicated that they did not remember certain previously learned information necessary for solving fraction division problems. For example, students asked the question: "What is an equivalent fraction?" "How do you reduce?" "How do you know what a common denominator is?" Many of the participants also indicated that they needed calculators or the multiplication tables to do fraction multiplication. Some participants remembered that multiplication is repeated addition and thus used repeated addition to solve multiplication problems, for example, $35 \times 7 = 35 + 35 + 35 + 35 + 35 + 35 + 35$; many others could not calculate the multiplication problems correctly.

Classroom environments and teachers' comments. Among all the participants, two groups were taught in their regular classrooms, and both teachers from the classrooms stayed and observed the lessons. Participants from Ms. L's class were assigned to condition I. After observing the first lesson, which was on the common-denominator method, she told me that the lesson did not provide enough procedure guidance and drill to help the participants remember the procedure. Then she told me about her way of teaching fraction division and suggested that it is the most effective way. She described her method as making a big table (see Figure 4), each row for one fraction division calculation problem, and each column for a step of the invert-and-multiply algorithm. By filling out the table, students would remember the procedures to solve fraction division problems.

	Change to improper fraction	Flip the second number	Change to multiplication	Multiply	Reduce	Change to proper fraction
$\frac{3}{5} \div \frac{1}{10} =$						
$\frac{2}{15} \div 2\frac{7}{20} =$						
$\frac{15}{26} \div \frac{25}{32} =$						

Figure 4. A table for practicing invert-and-multiply algorithm suggested by Ms. L

Participants from Ms. J's class were assigned to condition II. When participants were having a hard time understanding the fair-sharing word problems, which focus on the inverse relationship between division and multiplication, Ms. J tried reminding her students of the factor-family concept several times. Although she seemed confused by the example problems at the beginning, at the end of the lesson, she told her students that "this lesson is to help you understand the fraction division algorithm. You always forget some algorithms and procedures because you don't understand them. So this lesson is to help you understand. Once you understand it, you will not forget it so easily."

While it is encouraging to see that Ms. J realized that there may be some problems inherent in teaching by rote memorization and indicated that she valued teaching for deep understanding, data from this study show us that it is not easy to help students achieve good understand of fraction division, even when we tried to borrow features and content from curricula that lead to students' deep understanding in another country.

Discussion

Implications. The major aim of Study 2 was to examine whether U.S. sixth-grade students would learn about fraction division more effectively from lessons that followed Chinese-style lessons or from those that followed *Everyday Mathematics*-style lessons. Learning was measured with pre- and posttests that included problems on simple fraction division calculation, fraction division word problems, and related concepts including division, common denominators, and the inverse relationship between multiplication and division. Results indicated that after instruction, participants in condition I (the *Everyday Mathematics*-style lessons) improved in their simple fraction division calculations and tended to apply the inverse-and-multiply algorithm to solve calculation problems significantly more than those in condition II (the Chinese-style lessons). Otherwise, participants in the two conditions performed similarly, including no significant improvement in solving word problems, division, or common denominator problems.

Readiness to learn fraction division. Results of this study show that other than improvement in solving simple fraction division calculation problems, participants did not show evidence of much understanding of fraction division, nor did the instruction help students to correct their misconception about the relationship between dividend and divisor in division calculation. And students' understanding of common denominators did not improve after the instruction on the common-denominator algorithm. Although it is possible that the lack of improvement may be a result of inadequate instruction during the experimental lessons, observations by a second researcher indicate that the experimental teaching was adequate. Therefore, it is unlikely that the results of this study were caused by the researcher's inadequate teaching. Instead, given evidence from the students' pretests, the results are disappointing, but not surprising.

Data from this study indicate that the participants included in this investigation were not ready to learn fraction division because they lacked understanding of division and fractions. For example, most participants held misconceptions about division, when they saw a division question with a dividend that was smaller than the divisor; they solved the problem by switching the dividend and the divisor. And many of them believed that the quotient should always be smaller than the dividend—division makes a number smaller. Such misconceptions greatly hinder students from understanding the meaning of division of fractions.

Participants' performance on the common denominator fraction questions shows that they also lacked understanding of the concept of fractions. When the majority of the participants could not find a common denominator for two fractions and rename the two fractions to their equivalent fractions with that denominator, it is almost certain that they would not understand the common denominator algorithm for dividing fractions because this algorithm is totally based on the understanding of common denominator and equivalent fractions.

As for participants' readiness to learn from instruction on word problems, data also show that most participants in this study did not have a good understanding of division word problems, except for the fair-sharing model. Compared with the 44 participants who were able to solve the whole number fair-sharing problem correctly, only 21 participants answered the whole number speed problem correctly and even fewer were able to solve the comparison problem. Given that the majority of the participants were not familiar with speed or the comparison problems even with whole numbers, it is not surprising that they did not use these models to solve similar word problems with fractions. It was also unlikely that they would develop a good understanding of these types of problems if they first learned about them through problems with fraction division.

Another problem that hindered participants from learning fraction division in this study was the students' difficulty with performing simple calculations. In this study, all lessons and tests were given without allowing participants to use calculators or refer to multiplication tables. Many participants' responses show that they were not fluent with basic number facts for multiplication and division. For example, when solving $\frac{1}{3}\times\frac{1}{6}=\frac{1}{3\times6}$, many participants could not answer that $3\times6=18$. Some students were not sure whether the answer for $35\div7$ was 7 or 5. Because of such lack of fluency in number facts, students were focused more on finding the answer for simple whole number multiplication and division problems than on developing conceptual understanding of fraction division.

Lastly, participants in this study did not seem to be familiar with the instruction style where students are asked to activity construct problem solving solutions or build understanding of new knowledge. For example, many students' responses during the instruction indicated that they believed that it was most important to copy down teachers' writing in their own notebook, rather they actively respond to teachers' questions. Also, many students seemed to be more comfortable with following a procedure that they did not understand than asking for the meaning of that procedure. And students' calling instruction though word problems "not a normal way" indicates that they were rarely taught with word problems.

Although participants did not achieve much understanding of fraction division through the instruction in this study, students' performance on the tests and their responses to the instruction provided many interesting implications regarding teaching fraction division.

Applying the invert-and-multiply algorithm in calculation. Because the two lessons in condition I (*Everyday Mathematics* Condition) were both focused on presenting participants with fraction division algorithms, especially in lesson II, where all the examples, exercises, and assignment questions were about applying the invert-and-multiply algorithm to calculate fraction division equations, participants had more practice on calculation problems than participants in condition II. Therefore, it is not surprising that participants in condition I performed significantly better than those in condition II on the calculation problems. However, many researchers (Kamii & Dominick, 1997) have suggested that students benefit more when they are encouraged to explore or construct an algorithm on their own rather than presented with the algorithm directly. If this is true, why did participants in condition II, which required them to explore the invert-and-multiply algorithm through problem solving, fail to outperform participants in condition I, in which participants were presented with the algorithm directly?

Many mathematics educators (Gregg & Gregg, 2007a; Rittle-Johnson & Koedinger, 2001) proposed that because the fair-sharing model, which is associated with the common-denominator algorithm, is easy for students to understand, students should be introduced to fraction division through use of the common-denominator algorithm. Although these researchers provided evidence that it was easier for students to understand the common-denominator algorithm than to understand the invert-and-multiply algorithm, they did not provide any explanation of how students' understanding of the common-denominator could facilitate their learning of the latter. Results of Study 2 did not provide any such evidence. On the contrary, data from this study suggest that most participants in condition I did not demonstrate a good understanding of the common-denominator method and did not apply it in solving problems. As evidence, only three students in condition I even attempted to apply the common-denominator algorithm to the question $\frac{3}{5} \div \frac{1}{10}$ on the posttest. There is no evidence in this study that the teaching of the common-denominator algorithm helped participants in condition I outperform those in condition II in solving either fraction division calculation problems or word problems.

Next I consider why the constructing-algorithm group (Condition II, Chinese-style lesson 2) did not do as well as the presented-algorithm group (Condition I, *Everyday Mathematics*-style lessons). The data, especially the field notes, indicated that although the lessons in condition II were designed to encourage participants to construct the invert-and-multiply algorithm, participants were not able to actually construct the algorithm during the lessons. When I asked participants to solve word problems and to discover patterns in problem solving solutions, few students participated in the discussions or gave any response. Participants' performance on the posttest word problems indicated that most of the students in condition II did not solve the word problems after being taught about the same type of word problems in the lessons. Without understanding the word problems during instruction, it is unlikely that participants in condition II would construct and summarize the invert-and-multiply algorithm. In addition, because participants were engaged in word problems most of the time during the lesson, their chance to practice calculation problems was very limited. This lack of time to focus on calculations may have inhibited their ability to answer the calculation problems correctly.

Given the fact that participants in condition II spent much more time learning through word problems, they should have outperformed participants in condition I in terms of solving word problems. However, this was not the case.

Word problems. Most mathematics educators nowadays argue that word problems help students to achieve a conceptual understanding of mathematics concepts. Many studies (Barlow & Drake, 2008; Coy, 2001; Higgins, 1997; Kribs-Zaleta, 2008; Peck & Wood, 2008) have shown that students' understanding of fraction division would benefit from working with word problems. However, results from this study indicated that students who were taught through the use of more word problems did not outperform students who were exposed to fewer word problems during the lessons. These results seem to contradict conclusions from previous research. However, it is necessary to take a closer look at the type of word problems that were used in previous studies and to understand the kind of fraction division concept that students learned in those studies.

In most of the studies listed above, the word problems followed the fair-sharing model of fraction division. And, among the self-constructed solutions by students, all were based on the common-denominator method (Sharp & Adams, 2002). This means that students can construct their own solutions to solve fair-sharing fraction division word problems, and such problem-solving processes may help students understand the common-denominator algorithm. However, there is no evidence that the fair-sharing word problems help students to construct or understand the invert-and-multiply algorithm. In addition, no study has shown whether there are other types of word problems that enhance students' understanding of the invert-and-multiply algorithm.

Encouraged by the reality that Chinese students gain deep understanding of the invert-and-multiply algorithm through instruction on word problem, I also hoped to help U.S. students achieve such understanding in the same way. However, data from this study indicate that the U.S. participants in this study were not ready to learn fraction division through instruction on word problems.

Besides participants' lack of prerequisite knowledge, there were other obstacles to their learning. And a very important obstacle is students' motivation and interest in learning fraction division.

Motivation and interest in learning fraction division. A common argument for using problem solving as a mathematics teaching strategy is that good problems are motivational, and that they will promote students' interests to engage in mathematics, as summarized by Hiebert and his colleagues (Hiebert, Carpenter, Fennema, Fusion, & Piet, 1996). Higgins (1997) also showed that students who received one year of problem-solving instruction displayed greater perseverance in solving problems and more positive attitudes about the usefulness of mathematics than students who received traditional instruction where rote memorization was the focus of learning. Despite some positive results, Higgins did not find any correlation between participants' performance on solving mathematical problems and their interest in mathematics learning. Similar results were found in my study. In this study, compared with participants in *Everyday Mathematics*-style lessons, participants in the Chinese-style lessons did not show a stronger preference towards word problems over calculation problems, nor did they find word problems more fun than calculation problems. It may be reasonable to argue that because participants in condition II only received 2 lessons of teaching through word problems, they did not have enough time, as compared to the students in Higgins' year-long instructional experiment, to adjust to such a teaching method and benefit from it. Therefore, participants did not develop an interest in learning from word problems. However, other than this rather pragmatic explanation for the results, there might be another, deeper reason for participants' lack of interest and preference toward word problems.

Hiebert and his colleagues (Hiebert et al., 1996) argued that the source of interest and motivation in mathematics learning is not the problem-solving task itself. They suggested that students would develop interest and motivation in mathematics learning when the learning conditions allow students to apply their prior knowledge in completing mathematics tasks. Many studies (Ball, 1993; Cobb, Terry, Yackel, & McNeal, 1992; Fennema, Franke, Carpenter, & Carey, 1992) also have shown that only when students were able to apply their previous knowledge to the successful learning of new concepts were they able to achieve an increase in their confidence and interest in learning mathematics. This theory provides a possible explanation of why participants in this study did not demonstrate strong interest in learning fraction division. In this study, because of students' lack of prerequisite knowledge, they were not able to apply these concepts and skills to the learning of fraction division, and thus did not achieve the understanding of fraction division to solve calculation and word problems. Instead of providing participants with the excitement of building up new knowledge based on their previous knowledge, the lessons actually presented the participants with two rather disappointing facts: one was that they did not understand the concepts that they supposedly learned previously, and another was that they were not able to obtain understanding of the new concept. Such facts must be devastating rather than exciting to learners. Therefore, it is reasonable to argue that there is no reason for participants in this study, especially those who received the word problem-oriented lessons, to build up their interest and motivation in learning fraction division through word problems.

Future studies. One of the biggest differences between the Chinese textbooks and *Everyday Mathematics* that was not included in the design of this instructional experiment is the amount of teaching time suggested in the textbooks. According to *Everyday Mathematics*, only two lessons are provided for teaching fraction division—one on the common-denominator method in fifth grade and another on the invert-and-multiply algorithm in sixth grade. As for the Chinese textbooks, 10 lessons are provided to teach this topic in sixth grade. As many researchers (An, 2000; Li, 2000, 2008a; Stigler & Perry, 1988b; Zhou & Peverly, 2005) have pointed out, Chinese teachers tend to teach mathematics at a slow pace compared to U.S. teachers, and, at the same time, challenge students with a high level of mathematical thinking. Such teaching strategies seem to correspond well with the pace and content of the 10 lessons in the Chinese curricula. However, if I had included the difference of teaching time between the Chinese curricula and *Everyday Mathematics* in the design of current study, I would not be able to isolate the effect of different teaching time from the effect of lesson content when interpreting student learning. Because the focus of the current study was to examine whether the content of the Chinese curricula could help U.S. students achieve deep understanding of fraction division, especially through the instruction of multiple models of word problems, I chose to only include the differences between the Chinese and *Everyday* Mathematics lesson content in the design of this study and ignore other differences, especially amount of time spent teaching.

With the current design, the biggest limitation was that it did not take into consideration the amount of time needed for students to understand and learn from the Chinese lessons. There are three models of word problems covered in the Chinese curricula, and the same three models of word problems were also included in this study. However, the Chinese curricula suggest 8 lessons devoted to the learning of these word problems, yet participants in this study only received 2 lessons. Therefore, it is unlikely that they would achieve the same kind of understanding as Chinese students' from only 2 lessons. And with the participants' weak foundation on their understanding of division and fractions, such expected learning results become even less possible.

To further examine whether a Chinese-style curriculum can be effective in helping U.S. students learn fraction division through instruction with multiple models of word problems, a future study that provides the same amount of teaching time as suggested by the Chinese curricula is necessary. In future studies, not only the lesson content should reflect that in the Chinese curricula, the pace of learning should also follow the Chinese curricula.

Another major difference between the Chinese curricula and *Everyday Mathematics* that was not included in the design of this study is where the lessons of fraction division are located in the whole curriculum. As for *Everyday Mathematics*, one of the two fraction division lessons is taught in fifth grade and the other is taught in sixth grade, and each is grouped with lessons on various other topics (e.g. multiplication and division of whole numbers, decimals, and percentages) as a unit. But in the Chinese curriculum, the 10 lessons on fraction division are all located in a unit called “fraction division” in sixth grade, and this unit is taught immediately after the unit on fraction multiplication. In the current study, because of the limited time, such differences were not included in the design of the study.

However, many studies in cognitive psychology (Ambridge, Theakston, Lieven, & Tomasello, 2006; Litman & Davachi, 2008) have shown that when teaching is spaced in different ways, learners tend to achieve different results. To examine how different ways of arranging fraction division lessons in curricula can affect students’ learning, it is necessary to conduct future studies that include the element of how the lessons are spaced.

In addition to these two curricula differences that were not represented in the design of the experiment lessons; one major limitation of Study 2 is the researcher’s relationship with the participants. Because the researcher was not the teacher of the participants, she did not know those students prior to the study. As many educators and researchers have pointed out (e.g. Stigler & Hiebert, 1999), learning is a cultural activity, and the norms and expectations for learning can be very different in each classroom, school, and country. Therefore, the implementation of the experiment lessons might lead to different responses from the participants if the norms and expectations of learning of these participants were incorporated in the design of the lessons. Therefore, I suggest that in future studies, researchers should first become familiar with students by observing their regular classroom learning and even by test teaching a few lessons, and then the researchers can incorporate what he or she knows about the students into the design of the experiment lessons.

In addition to the implications from how the U.S. participants learned from the experimental lessons, this study, together with Study 1, also provide us with many insights of students' understanding of division and fractions, curriculum design, adopting curricula features from other countries, and teachers' understanding of fraction division. In the following chapter, these topics will be discussed in detail.

Chapter 4

General Discussion

Concept of Division

In two studies, I intended to focus on sixth-grade students' developing understanding of fraction division. Many researchers (Greer, 1994b) have argued that the understanding of fraction division is built on the understanding of division of whole numbers. In my investigation, specifically in Study 2, I found evidence of a lack of this basic understanding. Therefore, to understand the participants' performance in Study 2, and of their understanding of fraction division, it is first worth exploring their understanding of division.

Many mathematics educators have found that children's intuitive model for division is fair-sharing (Correa, Nunes, & Bryant., 1998; J. Mulligan, 1992; Mulligan & Mitchelmore, 1997; Squire & Bryant, 2002a). They have also suggested that between the two models of fair-sharing—partitive (when the number of shares are known, e.g., sharing 15 apples among 5 people) and quotative (when the amount of each share is known, e.g. sharing 15 apples so that each person has 3 apples), children always find it easier to understand and solve the partitive problems (Correa, et al., 1998; Squire & Bryant, 2002a, 2002b). Therefore, these researchers suggested that when teaching division, the curriculum should start with children's preferred partitive model and then introduce quotative problems. However, such suggestions were based on data from children in fourth grade or younger. It is unclear whether older children still find it easier to understand the partitive model than the quotative model.

Data from Study 2 show that participants were more likely to answer the quotative word problem correctly than the partitive question when the partitive question had a dividend that was larger than the divisor. Based on this result, it is reasonable to argue that sixth-grade students would not find the quotative model of fair-sharing difficult to understand. Therefore, although most of the fair-sharing problems included in the Chinese curricula and *Everyday Mathematics* are quotative word problems, there is no evidence that these quotative word problems would hinder sixth-grade students from extending their understanding of whole number division to fraction division.

Interestingly, it was a partitive word problem on the pretest that challenged most participants' understanding of division, and participants' answers to this question revealed their misconception about division. In answering the question "there are 5 pizzas, to share these pizzas fairly among 15 students, how much pizza will each student have," about half of the participants incorrectly answered "3." This incorrect solution is a reflection of the common misconception about division that "the dividend is always bigger than the divisor," (Fischbein et al., 1985; Graeber, 1993; Tirosh & Graeber, 1989), and it also indicates that students did not even think about what the problem asked: they just blindly calculated without attending to what they were calculating. I found this in both problems in which the dividends were smaller then the divisor.

One possible reason for participants' adhering to this misconception is the teaching of "factor family." During the teaching of one group when the regular classroom teacher was present, the teacher kept reminding students of the factor family, such as "2, 3, 6", "2, 5, 10", and "3, 6, 18." This "factor family" strategy is introduced in the fourth- and fifth-grade textbooks of *Everyday Mathematics* (University of Chicago School

Mathematics Project, 2008). For example, “fact triangle” exercises in fourth grade, unit 4, use a triangle of three numbers to practice the multiplication and division relationship among the three numbers (see figure 5a). And in fifth grade, with the stated purpose of helping students reinforce the relationship between multiplication and division, the textbook provides an exercise where students are required to fill out three numbers to complete the factor triangle (see figure 5b). If students have been constantly trained on factor families, it is possible that when they see the question $3 \div 12 =$, they reflexively respond with “4.” The teaching of “factor family,” although it might facilitate students’ rote memorization of simple multiplication and division facts, can be problematic because it not only fails to help correct children’s common misconception that “the dividend is always bigger than the divisor,” it may even reinforce such a misconception by only presenting cases when “the dividend is always bigger than the divisor.”

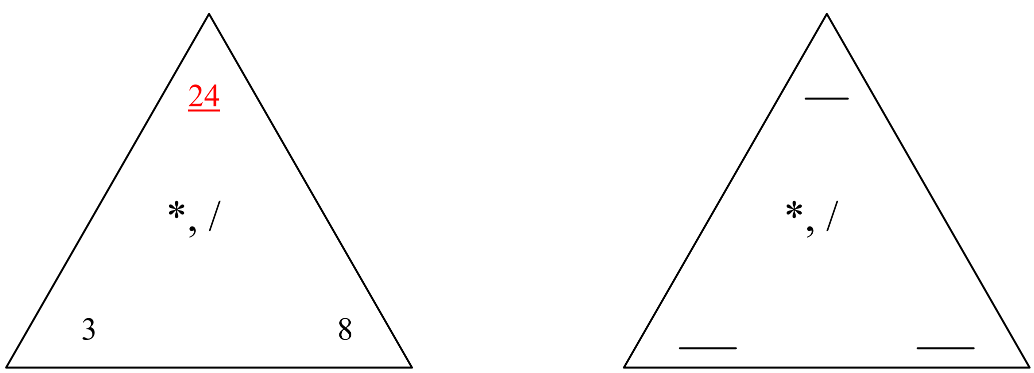

Figure 5a *Figure 5b*

Figure 5. Factor triangles, a multiplication and division exercise included in *Everyday Mathematics* fourth- and fifth-grade textbooks.

Another possible reason for participants’ misconception that the divisor should always be smaller than the dividend can be traced to children’s intuitive models of multiplication and division (Fischbein, Deri, Nello,, & Marino, 1985). Fischbein argued that children's (grade 5, 7, and 9) intuitive model for multiplication is repeated addition and their model for division is partition. Based on these models, children tend to believe that in multiplication the product is always bigger than the factors and "when multiplied, the number gets bigger;" as for division, children tend to believe that the dividend is always bigger than the divisor and the quotient, and "when you divide, the result gets smaller." When the number relation in a problem conflicts with these beliefs, children tend to change the numbers' relationships so that they will conform to these beliefs. Results from Fischbein's study (Fischbein, et al., 1985) show that although many educators hoped that students would automatically correct their misconceptions when they encounter situations that do not fit into these beliefs, by ninth grade, most students still did not reject such misconceptions.

Fortunately, research (Greer, 1994a; Tirosh & Graeber, 1989, 1990) shows that although these misconceptions can be deeply rooted in intuitive model of division, they can be corrected. When children (or pre-service teachers in Tirosh's study) were provided with models other than the fair-sharing models and with number relations that conflicted with their misconception, for example, a speed problem (Greer, 1987), they used a variety of methods to work out the answers and then rejected their previous misconception about division. Based on these studies, it is reasonable to argue that when the division word problems are based on models other than fair-sharing or when the divisor is larger than the dividend in the problems, by working on these problems, students may be prompted to correct their misconceptions about division and fractions. On the contrary, if students are focused only on the fair-sharing model when learning division, their misconception may be reinforced by frequent exposure to this intuitive partition model of division.

In this sense, the Chinese curricula that include multiple word problem models of division should, in theory, be more effective in helping children correct their misconceptions about division than *Everyday Mathematics*, which only provides the fair-sharing model of division. However, results of Study 2 do not show that more participants in the Chinese-style lessons overcame their misconceptions of division compared to the *Everyday Mathematics*-style lessons. Such results may be explained by at least three reasons. First, these deeply rooted misconceptions have been constructed by children since as early as first grade (Correa, et al., 1998; Mulligan & Mitchelmore, 1997). It is thus unlikely that they can be corrected after two lessons in sixth grade. Second, as pointed out in the discussion in Study 2, the design of the Chinese lessons were based on the Chinese textbooks and teachers' guides, with the assumption that students should have already built a strong foundation for the conceptual understanding of division and fractions before the lessons on fraction division, and should have rejected these misconceptions prior to these lessons. Therefore, the design of the lessons were not focused on correcting students' misconceptions of division. Third, Study 2 did not provide as much teaching time as suggested by the original Chinese curricula, therefore, participants might not have enough time to learn from the word problem instruction.

Although the participants in study 2 did not achieve deep understanding of fraction division through either the Chinese-style or the *Everyday Mathematics*-style lessons, nor did they reject their existing misconceptions about division after instruction, their performance in the pre- and posttests, as well as their responses to instruction, provide us with rich information that leads to further understanding of curriculum design, especially in terms of understanding curriculum design from the perspective of cognitive load theory.

Examine Curriculum Design With Cognitive Load Theory

Cognitive load theory has been used by many education researchers in their studies on curriculum design (Ayres, 2006; Sweller, 1988, 1989, 2006a; Sweller et al., 2007; Sweller et al., 1998; Van Merrienboer & Sweller, 2005). According to the theory, learning can be defined as a change in people's long-term memory, and this change is done by constructing understanding of a new concept based upon existing knowledge. And because people's working memory is limited, if a person's working memory is overloaded with unnecessary information, such overload will hinder the person from accessing useful information and learning the new concept. Therefore, the purpose of instructional design is to organize written, spoken, and diagrammatic information to optimize the learning process so that people can learn most efficiently. And such optimization means that people can make the best use their limited working memory and that people can quickly access the prerequisite knowledge needed for learning the new concept (Sweller, 2006a). So a curriculum should focus on helping students access prerequisite knowledge, reduce irrelevant thinking processes that may overload students' limited working memory, and construct understanding of the new concept.

Data from Study 2 show that the Chinese-style lessons were not effective in helping the participants construct understanding of fraction division. I will now use cognitive load theory to further examine this result. Based on participants' performance in the pre- and posttests, we can argue that although the design of the Chinese textbooks and the instructions on teaching provided in the Chinese teacher's guide may not overload Chinese students' cognitive load during learning, applying such design directly to U.S. students may be problematic for at least three reasons.

First, the design of the Chinese-style lessons assumes that students would extend their understanding of whole-number division to fraction division; however, data show that participants did not have an accurate conceptual understanding of whole-number division. For example, participants asked the question "how do you divide and get a bigger number?" and "if a small number is divided by a big number, do you switch the two numbers?" If the participants' working memory is consumed by their efforts to correctly understand and perform division, it is unlikely that they will have the capacity to solve the fraction division word problems and pay attention to the pattern of the solutions, which is necessary for learning to occur.

Second, I followed the instructions given in the Chinese teacher's guides in the design of the Chinese-style lessons. According to the teacher's guides, students should have had experience solving speed and comparison word problems that involve whole numbers, and they should be able to extend that experience to solving similar problems that involve fractions. This design indicates that the ability to solve speed and comparison word problem constitutes prerequisite knowledge for learning from the Chinese-style lessons. Therefore, the fraction division lessons do not introduce these problems as new concepts, but they focus on helping students access this knowledge and extend it to solve new problems. However, data from the pretest indicated that most participants in this study did not have the understanding of the speed and comparison word problem models prior to the fraction division lessons. In this case, it is reasonable to argue that when participants were asked to solve these word problems during the lessons, their working memory was occupied with their effort to understand the speed and comparison models for the first time.

Third, the participants in study should be considered inexperienced learners—learners with low understanding of relevant knowledge (Kalyuga, Chandler, & Sweller., 1998, 2001a; Kalyuga, Chandler, & Tuovinen, & Sweller, 2001b; Kalyuga & Sweller, 2004; Sweller, 2006b; Sweller & Cooper, 1985), because they lacked understanding of the concept of division and the skills to solve the speed and comparison word problems. According to the studies listed above, for inexperienced learners, teaching through worked examples is more effective than teaching through problem solving; but for learners who have a deep understanding of relevant knowledge and experience with similar problems, it is more effective to learn through problem solving than studying worked examples. The design of the Chinese textbooks and the teaching instructions given in the teacher's guide indicate that students are expected to build up their new knowledge through problem solving, which means that the Chinese curricula expect students to learn as an experienced learner with solid relevant knowledge and experience in solving similar problems. However, this teaching strategy might be problematic when applied to U.S. students, who do not have much of the knowledge and experience that Chinese students have.

By pointing out all these challenges in applying the Chinese textbooks to teaching U.S. students, I do not mean that any attempt to borrow from the design of Chinese curricula will be futile in the United States. But much attention and caution need to be paid to the underlying assumptions of the design of Chinese curricula, because a curriculum can only be effective when its audience is considered in its design. In addition to the consideration of learner's experience, we should also pay attention to the structure of a whole curriculum when examining the effectives of a particular lesson in the curriculum.

Textbook Structure

As discussed in an earlier section, a limitation of Study 2 was that the experiment did not incorporate two major differences between the Chinese and *Everyday Mathematics* curricula: the teaching time and how these lessons are located within their whole curricula. However, data from Study 2 provide insights for our understanding of curriculum design, both in terms of the length fraction division lessons and in terms of where they should be located in the curriculum.

First, results show that participants from both conditions neither achieved a conceptual understanding of fraction division algorithms nor the ability to apply the algorithms to solve problems. And, as discussed earlier, these learning outcomes were probably not due to the researcher's inadequate teaching or incorrect implementation of the designed lessons. Therefore, it is reasonable to argue that it is unlikely that students will obtain both conceptual understanding and skills to solve fraction division problems after receiving only two lessons of teaching, regardless of the textbook styles. Despite this conclusion, the number of lessons taught in this experiment is consistent with typical practices. With limited time devoted to one of the most difficult topics in elementary-middle school math, teacher L's fraction-division-procedure-table idea could probably be a very effective strategy to help students memorize the invert-and-multiply algorithm within a single lesson. But if we look at the Chinese curricula, although the task (learning through problem-solving) is more complicated and the expectation of students' competence is higher than those in *Everyday Mathematics*, teachers and students are given 10 lessons to explore and investigate in this topic. Research (Stevenson et al., 2000; Stevenson et al., 1990; Stigler & Fernandez, 1995; Stigler & Hiebert, 1998, 1999) has shown that the pace of teaching is much slower in China and Japan than in the United States, and the slow pace may allow students to develop a deeper understanding of mathematical concepts. Although allowing students enough time to investigate and develop deep understandings might sound like an overly simplified suggestion, it might be a critical component in helping students understand mathematical topics.

The second insight is that the fraction division lessons may be better taken up if they come at a different point in the curriculum. The data revealed that the lessons were designed with the assumption that students would have more prior knowledge than they actually did. Is it possible that this lack of prior knowledge can be traced to the curriculum design of the textbook that these participants have been using—*Everyday Mathematics*? Results of Study 1 show that the fifth-grade *Everyday Mathematics* lesson on fraction division is included in the unit called "Fractions and Ratios," which includes lessons of addition of mixed numbers, fractions of fractions, multiplication of fractions, multiplication of whole numbers, multiplication of mixed numbers, percentages, etc. However, the topic of common denominators, a topic closely related to the common-denominator fraction division method, is not included in the same unit. As for the invert-and-multiply lesson in the sixth grade, it is also located in a unit that includes many other topics of number operations. And the lesson on common-denominator algorithm and the lesson on the invert-and-multiply lesson are one year apart in *Everyday Mathematics.* When students need to access the prior knowledge to learn fraction division, they may find it difficult to do so, because the *Everyday Mathematics* textbook design does not provide a clear map of how the prerequisite knowledge is related to the fraction division concept.

It is clear that in *Everyday Mathematics*, although students had been introduced to the concepts of common denominators and reciprocals prior to learning about fraction division, these concepts were introduced far too long ago to be called up readily. Also, the lessons for these concepts were grouped with many other lessons that do not relate closely to fraction division. Therefore, students might not only experience the challenge of recalling knowledge learned a long time ago, they may also confuse several concepts that were grouped together in memory.

On the other hand, in the Chinese curricula, the unit on fraction division immediately follows the unit on fraction multiplication within the same semester; and the last part of the fraction multiplication unit is focused on the concept of reciprocal fractions. When students are asked to discover the invert-and-multiply pattern right after they learn the concept of reciprocal, it is reasonable to argue that many students would be able to notice the reciprocal fraction of the divisor in the algorithm and discover the invert-and-multiply pattern. In this sense, the structure of the curricula facilitates the implementation of the lesson content and makes it easy for students to connect their prior knowledge to new knowledge.

In addition to textbook structure, the teacher's guide is another important element that may affect how effectively the lessons can be conducted. The results of the curricula analysis and the classroom teachers' responses to the experimental teaching indicate that teachers' understanding of mathematics topics and their teaching strategies may also be greatly influenced by the curriculum they use.

Improve Teachers' Knowledge Through Mathematics Curricula

Although my two studies did not include any data on teachers' understanding of fraction division, research (Tirosh, 2000; Tirosh & Graeber, 1989, 1990) has indicated that many teachers also have misconceptions about division, such as "dividing a number always makes it smaller," or "the dividend has to be larger than the divisor." And Ma's 1999) study also show that many U.S. teachers lack deep understanding of fraction division. Other than factors like pre-service teacher education and professional development, mathematics curricula may also contribute to how teachers' develop understanding towards mathematics concepts.

Results from Study 1 showed that in teaching the invert-and-multiply algorithm, the Chinese textbooks provide 44 (2002 version) and 24 (2006 version) word problems as student exercises, and *Everyday Mathematics* does not include a single word problem as an exercise in that lesson. Such results indicate that, in their teaching, Chinese teachers are exposed to a lot more word problems than U.S. teachers. Ma's study also showed that the word problems given by Chinese teachers included many comparison problems and speed problems. This corresponds well with the analysis of the word problem types given in the textbooks. The Chinese textbooks include fair-sharing, speed, and sharing word problems as teaching examples; *Everyday Mathematics* only provides fair-sharing problems as examples. It is reasonable to argue that if teachers are required to teach the speed and comparison questions, as the Chinese teachers do, they need to understand these questions. But if teachers are not required to teach these questions, as is the case for U.S. teachers who use *Everyday Mathematics*, they are not obligated to study them. And even if the U.S. teachers wanted to learn about the speed and comparison problems, if no textbooks or teacher's guides provided any examples of speed or comparison word problems, it is unlikely that the teachers would even have a chance to learn about these types of problems. Therefore, it is important that multiple models of word problems be included in the textbooks not only to benefit the learners, but also the teachers.

These implications and insights indicate that we still lack good understanding of many factors relating to how curricula may affect students' learning. Therefore, in addition to all the existing efforts to improve students' mathematics achievement, for example, having a national standard, hiring better teachers, restructuring schools and classrooms systems, I see the need for more studies that can help examine the issues brought up in the discussion above.

Future Studies

Although these two studies revealed many important issues concerning the lessons on fraction division in Chinese and U.S. curricula as well as U.S. sixth-grade students' readiness to learn about this topic, the results of the studies are far from sufficient to provide a good solution for improving the fraction division lessons in U.S. mathematics curricula. Because these two studies were exploratory, they should serve as initial attempts to investigate features of specific lessons in different countries' mathematics curricula, and lead to further examination of these features and how they can be used in the reform of U.S. curricula.

Tackle students' prior knowledge. One limitation of study 2 is that the participants lacked prerequisite knowledge that is needed for learning fraction division, and, because of this, participants were not able to learn from the interventions. Future studies should investigating student learning of fraction division should anticipate this and first confirm that participants have the prior knowledge that is needed for learning fraction division.

Another possible reason that participants in Study 2 were not able to construct understanding of fraction division was that they were not familiar with division word problems that represent the meanings of speed or comparison. Because they lacked experiences with such word problems prior to the lessons, they had to learn about both these word problem types and the concept of fraction division as new concepts during the lessons. Therefore, the learning task was more demanding than what the Chinese curricula intended it to be. Such a difficult task may overload students' ability to learn and discourage them from actively participating in the lessons. To make this learning task applicable to U.S. students, I suggest that, in future studies, we first teach participants about multiple models of division word problems. Once students become familiar with the types of word problems, we can help them transfer this knowledge to fraction division learning. In that way, students only need to focus on learning one new concept at a time.

Based on the two suggestions above, I propose that future studies on Chinese-style and *Everyday Mathematics*-style lessons should include three parts: the first part should focus on the teaching and reviewing of the concepts of basic fractions, division, common denominators, and reciprocals so that participants would have a solid foundation of necessary prior knowledge before they receive lessons on fraction division; the second part should focus on introducing students to different models of division word problems; after confirming that students have a good understanding of the first two parts, researchers then can proceed to the third part—the Chinese-style experimental lessons on fraction division.

Experiments on curriculum structure. Other than the content of experimental lessons, data from the two studies indicate that it is also worth examining the structures of curricula. As discussed in Study 2, it is unclear whether a longer teaching time would help U.S. students better understand the Chinese-style lessons. Therefore, to examine the effectiveness of a Chinese-style curriculum, a future study that provides as much teaching time as suggested by the Chinese curricula is necessary.

In addition to teaching time, where the fraction division lessons are positioned in a curriculum is also worth investigating. Between the spiral structure of *Everyday Mathematics* and the linear structure of the Chinese curricula, it is unclear which structure is more effective in helping students learn fraction division. Also, students' learning can be evaluated in terms of many different aspects, such as a deep understanding of the concept, retaining the knowledge over a long period of time, flexibly applying the knowledge to solve problems, and transferring the knowledge to learning new concepts. Therefore, with these various aspects of learning, a series of studies can be done to evaluate the effectiveness of fraction division lessons when they are positioned in a spiral structured curriculum and when they are positioned in a linear structured curriculum.

Examine how curricula affect teachers' knowledge. Not only do students learn from their textbooks, teachers may also develop their content knowledge and pedagogical knowledge from an effective curriculum. Therefore, to understand the broader effectiveness of curricula, it is important to conduct an investigation into ways that curriculum can shape teachers' perceptions and understanding of mathematical concepts and teaching methods.

Many studies have shown that when teachers conduct their teaching by following a curriculum closely, the curriculum can support teachers' learning in terms of content knowledge and pedagogical knowledge (Remillard, 2000, 2005; Remillard & Geist, 2002). It will be interesting to see when teachers have to follow and teach fraction division with a Chinese-style curriculum, whether they will (a) develop deeper understanding of fraction division because they are required to teach the concept with multiple models of word problems; (b) ask more challenging questions instead of imposing rote memorization on students, because teachers are required to guide students' construction of the fraction division algorithms; (c) be more aware of students' misconceptions of division and provide more guidance to help students correct these misconceptions, or (d) think critically about existing teaching strategies on division and multiplications, for example, the factors family. These qualities of teachers are often found among teachers from China, Taiwan, and Japan, (Perry, 1993; Stevenson et al., 2000; Stigler & Hiebert, 1999; Stigler & Stevenson, 1991) and this previous research has suggested that these qualities help to promote students' deep understanding of mathematics concepts. Therefore, it is important to investigate whether teachers' can develop these qualities by teaching with a Chinese-style curriculum.

Although the current studies are exploratory, they provide much information on how we can prepare our students to learn fraction division, and they also lead to ideas of future studies, through which we can achieve a better understanding of designing fraction division lessons within mathematics curricula.

Conclusion

The purpose of these two studies was to investigate whether the Chinese curricula and *Everyday Mathematics* approach the teaching of fraction division differently, and if so, to determine which one is more effective in promoting U.S. students' understanding of fraction division in terms of their ability to solve fraction division calculation and word problems. The results of the studies, instead of giving us a direct answer about which one is the more effective curricula, reveals at least four critical issues around teaching and learning fraction division.

First, the two Chinese curricula and *Everyday Mathematics* differ dramatically in terms of the structure of the curricula, the types of problems included as examples and exercises in the lessons, the different types of word problems presented in the textbooks, and the process through which the students are expected to learn about the invert-and-multiply algorithm.

Second, although it is reasonable to expect that these different features cause students' learning outcomes to be different, data from the investigative experiment in this study did not indicate that students who received two Chinese-style lessons differed from those who received two *Everyday Mathematics*-style lessons in terms of their ability to apply fraction division algorithms to solve mathematics problems. However, data from the studies indicated that participants lacked the basic prior knowledge that is closely related to fraction division. Therefore, participants did not have a solid foundation from which to build new knowledge. In addition, data also showed that the learning style that the participants were used to is very different from the teaching strategies suggested by the Chinese curricula. Such unfamiliarity with the learning strategy used in the Chinese-style lesson may have further hindered participants in the Chinese-style condition from achieving an understanding of fraction division.

Third, based on participants' responses on the tests and during the lessons, the researcher found that most participants still held a common misconception about division, that "the dividend is always bigger than the divisor"; and they extended the misconception to the division of fractions.

Fourth, data from this study indicate that most participants had very limited experience with division word problems except for the fair-sharing model, which is the only type of word problem represented in *Everyday Mathematics*. This lack of experience with multiple models of division word problems, may hinder students from extending their ability to solve whole number division problems to fraction division problems, and may continue to reinforce the students' misconceptions about division that may have originated from students' intuitive understanding of division, which is based on the fair-sharing model.

Based on all these factors, to help students lay a solid foundation from which they can build an understanding of fraction division, mathematics curricula should strive to help students correct their misconceptions about division and provide multiple models of whole number division word problems prior to lessons on fraction division.

In conclusion, Study 1 revealed many differences between the Chinese and *Everyday Mathematics* curricula. Study 2 showed that to we cannot simply apply features of Chinese curricula to teaching U.S. students; when deciding what to teach, it is important to take into account U.S. students' prior knowledge and histories of learning experiences. Without consideration of all these factors in the design of a curriculum, students likely will not develop the sort of rich understanding of mathematics concepts that we hope all children will achieve.

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Appendix A

Study 2: Lesson Content

	Condition I *Everyday Mathematic-style*	**Condition II** **Chinese-style**
<u>Lesson I</u>	<u>Common-Denominator Algorithm</u>	<u>Inverse-multiply Algorithm (measure-out)</u>
Example 1	1. How many ½-inch segments are there in 4 inches? 2. How many $\frac{1}{3}$-inch segments are there in $\frac{1}{6}$ inch? 3. How many $\frac{3}{8}$-inch segments are there in $1\frac{1}{2}$ inches?	1. Each box of candies weighs ½ pound. How much do 8 boxes of candies weigh? 2. 8 boxes of candies weigh 4 pounds, how much does each box of candies weigh? 3. There are 4 pounds of candies, how many ½-pound boxes can you pack these candies into?
Exercise 1	1. $4 \div 8 =$ __ 2. $\frac{1}{6} \div \frac{1}{2} =$ __	given $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, 1. $\frac{1}{6} \div \frac{1}{3} =$ __ 2. $\frac{1}{6} \div \frac{1}{2} =$ __
Example 2	Calculation: $6 \div \frac{1}{3} =$ __	Word problem: Sam has 6 pounds of peanut brittle. He wants to pack it in 1/3 pound pages. How many packages can he make?
Exercise 2	Calculation: $7\frac{1}{2} \div 1\frac{1}{4} =$ __	Word problem: Abby has a piece of ribbon that is $7\frac{1}{2}$ feet long, she wants to cut it into shorter pieces that are $1\frac{1}{4}$ feet long, how many short pieces can Abby cut the ribbon into?
Others	Explanation: When using this algorithm, why can we ignore the denominators?	Summary question: Can you summarize a way to divide fractions?
Assignment 1	1. $\frac{7}{15} \div \frac{1}{15} =$ __ 2. $\frac{8}{9} \div \frac{2}{9} =$ __ 3. $\frac{1}{3} \div \frac{1}{9} =$ __ 4. $\frac{1}{2} \div \frac{3}{5} =$ __ 5. $\frac{5}{8} \div \frac{3}{7} =$ __ 6. $2\frac{1}{3} \div \frac{1}{6} =$ __	1. $\frac{1}{15} \times 7 =$ __ $\frac{7}{15} \div \frac{1}{15} =$ __ 2. $\frac{2}{9} \times 4 =$ __ $\frac{8}{9} \div \frac{2}{9} =$ __

Assignment 2	1. $\frac{1}{15}\times 7=$ __ 2. $\frac{2}{9}\times 4=$ __	1. $\frac{1}{3}\div\frac{1}{9}=$ __ 2. $\frac{1}{2}\div\frac{3}{5}=$ __ 3. $\frac{5}{8}\div\frac{3}{7}=$ __
Assignment 3	Write > or <: 1. 15+28 __ 10^2 2. $\frac{1}{2}+\frac{1}{2}$ __ $\frac{3}{4}$ 3. $\frac{19}{20}$ __ 0.6+0.3	Word problem: Mike wants to poor a jar of $2\frac{1}{3}$ gallons orange into $\frac{1}{6}$ gallon bottles. How many bottles can he fill up?
Lesson II	Inverse-Multiply Algorithm	Inverse-Multiply Algorithm (comparison)
	Present the algorithm $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ (reference: proof)	
Example 1	Calculation: $60\div\frac{4}{5}=$ __	Word problem: The water in Daniel's body weights 60 pounds, which is $\frac{4}{5}$ of his whole body weigh. How many pounds does Daniel weigh?
Exercise 1	Calculation: $320\div\frac{2}{7}=$ __	Word problem: There are 320 fiction books in the school's library, which are $\frac{2}{7}$ of all the library books. How many books are there in the library totally?
Example 2	Calculation: $2\frac{1}{2}\div\frac{2}{3}=$ __	Word problem: A bag of white sugar costs $2\frac{1}{2}$ dollars, which is $\frac{2}{3}$ of the cost of a bag of brown sugar. How much does a bag of brown sugar cost?
Exercise 2	Calculation: $35\div\frac{5}{7}=$ __	Word problem: Sam plants 35 acres of corns, $\frac{5}{7}$ of which are peas, how many acres of peas does Sam grow?
		Summary: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$
Assignment 1	Calculation: 1. $25\div\frac{2}{5}=$ __	Word problem: A pair of jeans cost 25 dollars, which is $\frac{2}{5}$ of the cost of a jean jacket. How much

	2. $111 \div \frac{3}{4} =$ __	does the jean jacket cost?
Assignment 2	Add or subtract: 1. $3\frac{2}{3} + 1\frac{4}{5} =$ __ 2. $8\frac{1}{7} - 3\frac{3}{4} =$ __ 3. $6\frac{1}{8} - 4\frac{5}{6} =$ __	Word problem: There are 111 students in 6^{th} grade, which is $\frac{3}{4}$ of the number of 7^{th} grade students. How many 7^{th} grade students are there in the school?

Appendix B

Study 2: Pretest Questions

1. Complete the equation by filling in the blanks.
   a. $32 \div 8 =$ ___
   b. $3 \div 12 =$ ___
   c. $15 \div 4 =$ ___

2. There are 5 pizzas, to share these pizzas fairly among 15 students, how much pizza will each student have?

3. Given that $139 \times 16 = 2224$, fill in the blank: $2224 \div 139 =$ ___

4. Rename $\frac{7}{6}$ *and* $\frac{3}{8}$ as their equivalent fractions with a common denominator.

5. Complete the equation by filling in the blanks.
   a. $\frac{3}{2} \div \frac{1}{4} =$ ___
   b. $\frac{5}{6} \div \frac{2}{15} =$
   c. $\frac{3}{10} \div \frac{2}{15} =$
   d. $\frac{5}{12} \div \frac{7}{12} =$ ___

6. There are $5\frac{1}{2}$ bottles of apple juice, if you need to fill each glass with $\frac{1}{4}$ bottle of juice, how many glasses can you fill?

7. Jose has 45 music CDs, his CD are 3 times as many as what Jerry has. How many CDs does Jerry have?

8. A car can travel for 180 miles within 3 hour on highway I-74, how many miles will this car travel in one hour?

9. Do you like math?
   □ I hate it. □ I don't care. □ A little bit.
   □ Sometimes. □ Most of the time. □ I love it all the time.

10. Do you think that fractions are useful in your life?
    □ No, never. □ Maybe. □ A little bit.
    □ Sometimes. □ Most of the time. □ Always.

Appendix C

Study 2: Posttest Questions

1. Complete the equation by filling in the blanks.
   a. $32 \div 4 = \_\_\_$
   b. $4 \div 12 = \_\_\_$
   c. $13 \div 4 = \_\_\_$

2. Jane has a piece of ribbon that is 32 feet long. She wants to cut it into smaller pieces, and each small piece will be 4 feet long. How many small pieces can Jane cut the ribbon into?

3. Given that $15 \times 40 = 600$, fill in the blank $600 \div 40 = \_\_\_$.

4. Given that $\frac{4}{5} \times \frac{7}{9} = \frac{28}{45}$, fill in the blank $\frac{28}{45} \div \frac{7}{9} = \_\_\_\_$.

5. Rename $\frac{3}{10}$ *and* $\frac{4}{15}$ as their equivalent fractions with a common denominator.

6. Complete the equation by filling in the blanks.
   a. $\frac{3}{5} \div \frac{1}{10} = \_\_\_$
   b. $\frac{2}{15} \div 2\frac{7}{20} = \_\_\_$
   c. $\frac{15}{26} \div \frac{25}{32} = \_\_\_$
   d. $\frac{5}{13} \div \frac{7}{13} = \_\_\_$

7. There are $3\frac{1}{2}$ gallons of orange juice, you need to pour the orange juice into small bottles, each small bottle can hold $\frac{1}{2}$ gallon of orange juice. How many small bottles can you fill up?

8. There are 640 picture books in the school library, there are also some science fiction books in the library, the number of picture books is $\frac{4}{5}$ of the number of science fiction books. How many science fiction books are there in the library?

9. Josh can swim for $1\frac{1}{2}$ miles within $\frac{2}{3}$ hour, how many miles can he swim in 1 hour?

10. Among all the questions you just saw, choose the most difficult problem.

11. Among all the questions you just saw, choose the most fun problem.

12. If you could choose a problem for home work, which one would you pick?