IMPROVED UNDERSTANDING AND PREDICTION OF THE HYDROLOGIC RESPONSE OF HIGHLY URBANIZED CATCHMENTS THROUGH DEVELOPMENT OF THE ILLINOIS URBAN HYDROLOGIC MODEL (IUHM)

BY

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DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Civil Engineering in the Graduate College of the University of Illinois at Urbana-Champaign, 2010

Urbana, Illinois

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Abstract

What happens to the rain in highly urbanized catchments? That is the question that urban hydrologists must ask themselves when trying to integrate the hydrologic and hydraulic processes that affect the hydrologic response of urban catchments. The Illinois Urban Hydrologic Model (IUHM) has been developed to help answer this question and improve understanding and prediction of hydrologic response in highly urbanized catchments. Urban catchments are significantly different than natural watersheds but there are similarities that allow features of the pioneering GIUH concept developed for natural watersheds to be adapted to the urban setting. This probabilistically based approach is a marked departure from the traditional deterministic models used to design and simulate urban sewer systems, and does not have the burdensome input data requirements that detailed deterministic models possess. Application of IUHM to the CDS-51 catchment located in the Village of Dolton, IL highlights the models ability to predict the hydrologic response of the catchment as well as the widely accepted SWMM model and in accordance with observed data recorded by the USGS. The model is further used to improve the understanding of urban catchment hydrology. It is shown that inlet storage and pressurized flow can have a significant impact on the hydrologic response in urban catchments. In addition, the unique structure and organization of urban sewer networks make it possible to characterize Horton’s Laws in urban catchments. Overall, the results provide invaluable insight into how the different hydrologic/hydraulic processes encountered in urban catchments effect the hydrologic response of the catchment.

The link between river network structure and hydrologic response for natural watersheds has been the subject of ongoing research for the past 30 years. In this research we investigate the link between sewer network structure and hydrologic response in urban catchments. It has been shown in natural watersheds that there are dispersion mechanisms that contribute to the impulse response function of the catchment: hydrodynamic dispersion, geomorphologic dispersion and hydrodynamic dispersion. We introduce a fourth dispersion mechanism, intra-state dispersion, that accounts for the variance in conduit (e.g. slope, length, diameter etc.) and overland
region input parameters (e.g. slope, area, imperviousness etc.) within an order. This dispersion mechanism is found to be the second largest contributor to the total dispersion in urban catchments, contributing less than hydrodynamic dispersion, but more than kinematic and geomorphologic dispersion. Furthermore, an uncertainty analysis is performed to help better understand the uncertainty in the predicted hydrologic response that is introduced by spatial variation in conduit and overland input parameters. It is identified that conduit slope and length are the greatest sources of uncertainty in the predicted direct runoff hydrograph for the CDS-51 catchment in the Village of Dolton, IL, and the CDS-36 catchment in the City of Chicago, IL.

The IUHM requires, as input, the mean and variance of parameters including conduit slope, overland slope, subcatchment area and imperviousness. Ideally the first two statistical moments of each of these input parameters would be calculated using the deterministic data for all conduits and subcatchments in the system. In reality, such detailed information is not always available, is uncertain, or time cannot be afforded to delineate all of the sub-areas within the catchment. IUHM was designed so that it could be used in such situations under the hypothesis that the mean and variance of the input parameters could be determined using only a sub-set of the full deterministic dataset. This hypothesis is tested and it is shown that a random sample capturing as little as 30% of the subcatchments and conduits in the CDS-51 catchment (located in the Village of Dolton, IL) can be used to generate the mean and variance in the ith-order conduit slope, overland slope, subcatchment area and imperviousness without significantly reducing the accuracy or increasing the uncertainty of the predicted hydrologic response.

IUHM provides an alternative to traditional deterministic models that maintains the non-linearity in the key physical processes at the urban scale and is capable of accounting for the uncertainty caused by spatial variation in the input parameters throughout the catchment. This model is still in its infancy and as such has the potential to be improved through ongoing research. The model has been setup to allow other hydrologic and hydraulic processes (e.g. stormwater best management practices, dual drainage) to be easily incorporated into the model.
This thesis is dedicated to the Cantone family
Acknowledgments

This research would not have been possible without ongoing funding and support from the Metropolitan Water Reclamation District of Greater Chicago as part of the University’s work on Chicago’s Tunnel and Reservoir Plan project. It also would not have been possible without the guidance and support of my advisor Research Assistant Professor Arthur Schmidt and the Environmental Hydrology and Hydraulic Engineering department at the University of Illinois at Urbana-Champaign. A special thanks to the members of the Tunnel and Reservoir Project (TARP) group led by Professor Marcelo Garcia and its supporting graduate and undergraduate students and staff, namely; Nathaniel Hanna Holloway, Daniel Christensen, Michelle Hollander, Waleska Echevarria, Nam Jeong Choi, Yong Won Seo, Andrea Zimmer, Pablo Cello, Arturo Leon, Nils Oberg, Yun Tang, Andrew Erickson, and Robin Ray. I also want to acknowledge my sponsor, the Australia-American Fulbright Commission, together with BHP Billiton, American Australian Association who supported me as a Murdoch Fellow and The University of Adelaide who, through the George Murray Scholarship, aided me in my research. Thank you to the members of my doctoral committee, Praveen Kumar, Murugesu Sivapalan and Yeou-Koung Tung for, for their guidance and direction. Their contributions are gratefully acknowledged. Finally, I must acknowledge my parents, Victor and Marisa, without whom I would not be where I am today. They have provided me with guidance and support my whole life and are responsible for me growing into the person I am today. To my siblings, Dominic, Vanessa, and Victor-Paul, their partners, Nicolle, Darren and Rosanna, and my two little nieces Maddison and Isabella, thanks for proving that family is still the most important thing even if you are thousands of miles apart.
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Chapter 1: Introduction

1.1 Background and Motivation

The dynamics of the urban landscape are changing in the face of urban sprawl and urban consolidation. Development has seen swamp lands turn into pastures, pastures into corn fields, and corn fields into concrete as developers seek land to induce residential and commercial development. Increased social awareness of the need to protect and manage our water resources has also had a significant impact in highly urbanized catchments. Residents are encouraged to employ best management practices (e.g., rain gardens, rainwater tanks etc.) and local and state water authorities are encouraging developers to engage in low impact development. These developments range in scale, but they share a common attribute; they increase the heterogeneity and non-linearity of the urban landscape. As a result, the path for a drop of rainfall has changed and is inherently more complex and difficult to predict. The primary objective of this research is to improve the understanding and representation of the hydrologic response in highly urbanized catchments.

A plethora of simulation packages exist for predicting the hydrologic response in urban catchments. However, most recently the focus has been on developing graphical user interfaces for the most commonly used existing hydrologic models, integrating these programs with the latest graphical information system software, and developing tools to manage data. Little work has been done to improve the underlying hydrologic and hydraulic models that drive these simulation packages. In fact, the most widely used model in the United States, the Stormwater Management Model (SWMM), which is endorsed by the United States Environmental Protection Agency, was developed by Huber et al (1981) some 30 years ago. SWMM is the base model used in the so-called state-of-the-art simulation packages (e.g., InfoSWMM, InfoWorks etc.) that are used most frequently by consulting firms worldwide. The SWMM runoff module has undergone little development over the past 30 years and is burdened by a number of problems. For example, the surface runoff calculation is based on a subcatchment width which has no real physical meaning and is often used as a calibration parameter. One of
the issues with deterministic models such as this one is that there is no way of quantifying the uncertainty introduced by parameters such as the subcatchment width, and inevitably calibration must be performed to ensure confidence in the predicted hydrologic response. One of the key objectives of this research is to develop a model that is capable of predicting the uncertainty in the hydrologic response due to uncertainties in its input parameters.

A further drawback of traditional deterministic models is the need for large volumes of input data to represent subcatchments, inlets, junctions, manholes and conduits in the catchment. For many large urban catchments in densely populated areas, sanitary and storm sewer infrastructure was installed over 50 years ago when documentation is not what it is today. As such, this critical input data is not always readily available. In some cases, the only available information is on the layout of the sewers themselves. This lack of input data forces engineers to engage in detailed survey of the system, which is expensive and time consuming, or more commonly forces them to make simplifying assumptions. Cantone and Schmidt (2009) investigated a number of commonly employed simplification techniques, including subcatchment aggregation and conduit skeletonization. This investigation highlighted that such simplification techniques can have a significant impact on the predicted hydrologic response and can inevitably introduce uncertainty into the predicted hydrologic response. A third objective of this research is to develop a method for predicting the hydrologic response in catchments that have limited or uncertain input data.

One of the primary motivations behind this research is the need to better understand and predict the hydrologic response of the combined sewer systems that contribute flow to Chicago’s Tunnel and Reservoir Plan (TARP), as part of an ongoing research effort between the Metropolitan Water Reclamation District of Greater Chicago (MWRDGC) and the University of Illinois at Urbana-Champaign. Chicago’s TARP collects storm and sanitary flows from a 971 km² service area spanning the City of Chicago and 51 suburbs. Within this area lie in excess of 400 combined sewer systems that each contains hundreds or thousands of pipes. These systems were originally designed to flow to combined sewer overflow (CSO) points scattered throughout the waterways in Chicago. In addition to the CSO network, there is a network of interceptor
sewers that conveys flow to the various water reclamation plants in the TARP service area. Combined, the CSO, interceptor and TARP networks form an extremely complex urban system. To try and simulate the entire myriad of hydrologic and hydraulic processes across the urban scale is impractical, particularly considering the majority of the system is ungauged and input data are limited and uncertain. This unique system has forced the District and the University to develop a set of models to simulate the response of the TARP system (and it’s contributing CSO and interceptor systems) that overcome the limitations of existing deterministic hydrologic and hydraulic models in a bid to better understand and operate the system. The practical goal of this research is to develop a hydrologic/hydraulic model capable of predicting the hydrologic response of the combined sewer systems that serve as inputs to the TARP system.

1.2 Specific Objective and Scope of Research

The purpose of this research is to improve the understanding and prediction of the hydrologic response of highly urbanized catchments. This objective will be achieved through the development of a robust probabilistic model for predicting hydrologic response based on the layout of the sewer network and a set of probabilistically generated hydrologic and hydraulic inputs. This model must also be capable of tracking the uncertainty, introduced by the probabilistically generated input parameters, through the model such that the uncertainty in the hydrologic response of the catchment can be quantified. In this way the model will be capable of simulating catchments that have uncertain input data.

The scope of this research is limited to developing a model that can simulate time-varying precipitation, infiltration overland flow, flow through inlet structures and the hydraulics of sewer systems. Despite singling out these key processes at the urban scale, the model developed will provide a framework that can be easily built upon and used to incorporate other hydrologic/hydraulic processes, such as subsurface flows, dual drainage, small scale processes (e.g., rain gardens), spatially varying precipitation and infiltration etc.
1.3 Dissertation Organization

This dissertation is the integration of three stand alone journal articles that address the three primary objectives of this research outlined in this introduction. Two of these papers have been submitted to the Water Resources Research Journal, whilst the other is to be submitted once the first two articles have been accepted. Chapter 2 summarizes a comprehensive review of the literature relevant to this research. Chapter 3 addresses the primary objective of this research by describing the methodology behind the Illinois Urban Hydrologic Model (IUHM), a model that improves the understanding and prediction of the hydrologic response in highly urbanized catchments. Chapter 4 focuses on the second of my research objectives, further developing the IUHM and enabling it to predict the uncertainty in the direct runoff hydrograph predicted by IUHM caused by uncertainty in the input parameters for the model. In Chapter 5, we seek to satisfy the final objective of this research by showing that the IUHM is capable of accurately predicting the mean and variance in the hydrologic response using a sub-sample of the full input data set, and hence broadening application of the model to catchments with uncertain input data. Chapter 6 draws some final conclusions before describing the research to be conducted in the future.

1.4 References


Chapter 2: Literature Review

The literature review presented below has been adapted from the review of the literature documented in my Preliminary Exam. It is the result of an exhaustive review of the relevant literature over the course of this research. It provides the foundation on which the research presented in subsequent chapter is built upon and helps highlight the gaps in the literature that this research aims to fill.

2.1 “Watershed” versus “Urban Catchment” Hydrologic Modeling

Penman (1961) defined hydrology as the science that attempts to answer the question, “What happens to the rain”? Although this sounds like a simple question, as Singh and Woolhiser (2002) explain, experience has shown quantitative description of the land phase of the hydrologic cycle may be complicated and uncertain. Sivapalan (2003) explains that difficulty in predicting the hydrologic response of a basin may be attributed to heterogeneity of the land surface, soils, vegetation, land use, etc. and variability in inputs over the scales of time and space. Singh and Woolhiser (2002) defined the term “watershed hydrology” as that branch of hydrology that deals with the integration of hydrologic processes at the watershed scale to determine watershed response. Similarly, let us define “urban hydrology” as that branch of hydrology that deals with the integration of hydrologic and hydraulic processes at the urban scale to determine catchment response.

Singh and Woolhiser (2002) identified what watershed hydrologic models are used for. A similar list can be compiled for urban catchment models like the one presented in this research. Urban catchment hydrologic models may be used in the planning, design and operation of stormwater, wastewater and CSO network projects. Such models may also be used for the assessment, development and management of stormwater, wastewater and CSO networks.

In distinguishing “watershed” and “urban catchment” hydrologic modeling, it is imperative to consider a number of the key scale issues identified by Blöschl and Sivapalan (1995). In their review of scale issues in hydrological modeling, Blöschl and
Sivapalan (1995) focus primarily on watershed or catchment hydrology. However, many of the issues they discuss are prevalent in hydrological modeling of urban catchments and provide a means for classifying the scope of this research. Blöschl and Sivapalan (1995) identify two types of models; predictive and investigative. This research falls into the second category, investigative modeling. This type of modeling traditionally has two key steps: (1) collecting and analyzing data and (2) calibrating and validating the model. This traditional approach is not so easily applied in this research because of the lack of input data and absence of calibration and validation data.

The key differences between “watershed” and “urban catchment” hydrologic models lie within their relative scales of time and space. Figure 2.1 paints an excellent picture of the range of characteristic space-time scales for different hydrological processes. When considering watersheds generally all of the hydrological processes shown in Figure 2.1 may be accounted for in the hydrologic model. In urban catchments, only subsets of these hydrological processes are generally considered, namely; precipitation, subsurface storm flow, infiltration excess overland flow, saturation excess overland flow and channel flow. In fact, many of the simulation packages used for urban catchment hydrologic modeling only account for precipitation, infiltration excess overland flow and channel flow.
While watersheds can have time scales ranging from minutes to years, urban catchments typically have time scales in the range of minutes to hours. Similarly, urban catchments typically encompass a smaller range of space scales, generally of the order of meters or kilometers. Given these differences in the time and spatial scales it is recognizable that urban catchments are effectively a portion of larger natural watersheds; the key physical distinction being in land surface characteristics. Urban catchments tend to be highly impervious, with the predominant land use being residential, commercial and industrial. In contrast, natural watersheds tend to be highly pervious, with the land use dominated by pasture, crop land, and other agricultural land cover. As a result of the greater portion of impervious area, the travel times in urban catchments are often significantly less than watersheds.

One of the key scale issues that Blöschl and Sivapalan (1995) identify is in relation to precipitation. Precipitation is one of the driving forces behind the hydrological cycle. One of the main challenges for modelers is deciding what
precipitation data to use. In the Chicago area, for example, precipitation data are available from a network of rain gages as well as from three ground-based radar sites. Rain gage data and NEXRAD data differ according to both their time and space scales. These differing time and space scales highlight a number of issues discussed by Blöschl and Sivapalan (1995). Firstly, precipitation generally does not exhibit any preferred scales or spectral gaps at the process scale, however as the authors identify this does not necessarily mean that such gaps do not appear at the observation scale. The rain gage data available for the Chicago area is generally hourly data and historical records are available for a period of 40 years. Conversely the NEXRAD data are available in 6 minute intervals but the period of record is around 10-15 years. So whilst precipitation shows variation on time scales of minutes to years we are unable to observe these scales.

Yet another scale that the Blöschl and Sivapalan (1995) define is the ever important modeling scale. The typical modeling scale for natural watersheds is 1 day; however, for urban hydrologic modeling a model time step of minutes is typically adopted to mirror the process scale. Having a modeling scale smaller than the observation scale means that downscaling must be performed (Blöschl and Sivapalan 1995). That begs the question: how do you go about converting an hourly record of rainfall into a record of minutes? Blöschl and Sivapalan (1995) explain that downscaling involves disaggregating and singling out. For example, a single hourly rainfall value may be assumed to be uniformly distributed over the entire hour. So if 1 inch of rain was recorded 1/60 inch of rainfall would be assumed to fall each minute. In reality this may or may not be the case, but without a finer observed scale there is no way to know.

Heterogeneity and variability in space and time have scale issues that plague all those performing hydrologic modeling. The heterogeneity of precipitation arguably exhibits discontinuity, periodicity and randomness. Bloschl and Sivapalan (1995) highlight that the intermittency of rainfall events make it a discontinuity, however, its diurnal and annual variations make it predictably periodic and statistical analysis of historical records allow its randomness to be predicted. The authors postulate that catchment and hydrological processes show organization in many ways, using the example of Horton’s laws. Horton’s laws of stream numbers, length, area and slope were derived for natural catchments. One of the questions in this research is: do these
laws hold up in the urban setting? Can a similar set of rules be developed? Is there a way of using the network layout of a CSO network to predict the outfall hydrograph? These are questions that will be answered in this research.

2.2 Key Processes in Urban Catchment Hydrologic Modeling

Chow et al (1988) defined hydrology as the study of the hydrologic cycle, that is, the endless circulation of water between the earth and its atmosphere. While, it is well documented that hydrologic cycle has no beginning or end, of primary concern in this research are the hydrologic processes involved from when precipitation occurs until it exits the combined sewer overflow system at either an overflow point or when it enters a deeper tunnel system. Once precipitation (in the form of rain, hail, sleet or snow) hits the ground it can do a combination of things, including (Chow et al, 1988, Maier, 2001):

- hit a water body directly;
- infiltrate into the ground;
- flow through the soil as subsurface flow;
- runoff the ground surface as overland flow from the point of impact until it reaches a water course or underground drainage system inlet;
- be intercepted by vegetation, where it is stored temporarily and then evaporated back into the atmosphere;
- be stored in surface depressions, where it is infiltrated into the ground or evaporated back into the atmosphere.

It is typical in modern hydrologic models to treat these processes within a hydrologic system. A hydrologic system may be defined as a structure or volume in space, surrounded by a boundary, that accepts water and other inputs, operates on them internally, and produces them as outputs (Chow et al, 1988). The hydrologic system of interest here is represented schematically in Figure 2.2. By representing the system in this way it allows laws of conservation of mass and energy, and continuity to be applied. In effect the models available take the given precipitation information and use different solution techniques and methods to determine the volumes and fluxes of surface runoff,
surface storage and infiltration. Although it may look simple on paper, the process of developing working equations and models of hydrologic phenomena required to produce the output are very complex. There is generally a high degree of approximation required in applying the physical laws because the systems are large and complex, and may involve several working media (Chow et al, 1988). It is the way in which these equations are formulated that the models available differ from each other. The different techniques for assessing overland flow and infiltration are discussed in Sections 2.2.2 and Section 2.2.3 respectively.

![Figure 2.2 Schematic of hydrologic system](image)

Figure 2.2 Schematic of hydrologic system

Building on the hydrologic system presented in Figure 2.2, further consideration is afforded to what happens to the surface runoff. Figure 2.3 highlights all of the processes that are possible for a CSO system connected to a deep underground tunnel and reservoir system, such as Chicago’s TARP system. As well as illustrating the
hydrological processes that occur, it goes one step further in identifying the possible feedbacks in the system. The primary feedback mechanism in this kind of hydrologic system is evaporation. Water evaporates from waterways, surface storage and surface reservoirs, and through transpiration evaporates from land and vegetation. The only slightly tricky part here is tracing how the groundwater flow connects back into the system. In places like Illinois the primary source of potable water is from groundwater. As such it is not too far of a stretch to connect groundwater flow with household, commercial and industrial sewage. Also in cases where groundwater is used for irrigation it may lead more directly into the water ways. Utilizing this knowledge it was possible to close all the loops in the system and the result is what is shown in Figure 2.3.

Modeling of groundwater flow and evaporation of surface storage are two processes that are not of concern in this research. Of primary concern is how the surface runoff is generated, captured and then conveyed. This is discussed in Sections 2.2.4 and 2.2.5. On the other end of the spectrum it is necessary to point out that probably the most important variable governing the output is precipitation. Although not a primary focus of this research, careful thought must be placed in determining the type of rainfall information to be used and its effect on the results. Inherently, the uncertainty of rainfall data presents a directly correlating uncertainty in the hydrologic simulation. Precipitation is addressed in the proceeding section.
2.2.1 Precipitation Data

Precipitation includes rainfall, snowfall, and other processes by which water falls to the land surface, such as hail and sleet (Chow et al, 1988). In most areas the greatest source of precipitation is rainfall and as such it is most important in modeling studies. Rainfall is highly variable in both space and time and hence is a non-linear phenomenon. Vaes et al (2001) highlight that ideally long historical spatially distributed rainfall series should be used in rainfall calculations, but at the same time the authors realize that performing long term simulations with such models would lead to very time consuming and practically infeasible calculations. In many cases, this can be avoided by simplifying
the rainfall input into uniform rainfall and single design storms. At the same time, the non-linear nature of rainfall may lead to a loss in accuracy when simplifications are applied. In any case, Vaes et al (2001) postulate that an optimum, between accuracy of the modeling results and calculation effort, must be found that reflects the degree of detail in the rainfall input. In their study of rainfall data from Uccle, Belgium, Vaes et al (2001) concluded that the optimum between model and input uncertainty is leading towards simplified models using continuous long term simulations, especially for capacitive systems, which behave non-linearly.

In Chicago, studies have shown that sizable and statistically significant increases in storm activity have occurred over the central portions of Chicago (which are of primary concern in this research) in the past 40 years (Changnon, 2001). A plethora of rainfall information is available for Chicago’s urban areas from networks of rain gages operated by different agencies. Perhaps the most important of these is a network of 25 recording gages operated by the Illinois State Water Survey (ISWS). The increase in storm activity in Chicago was highlighted in 2001 when the rain gage network recorded the highest number of heavy rainstorms in history. In that year, eight rainstorms of duration between 1 hour and 24 hours exceeded the 2 year or greater recurrence interval (Changnon and Westcott, 2002). It is well known that when subjected to heavy rainstorms Chicago is susceptible to a variety of problems, including flooding of viaducts and basements of businesses and residences, which can cause extensive property damage. In addition, Changnon and Westcott (2002) highlights that heavy rains act to disrupt and slow or stop traffic flow, at times affecting bus and rail routes, and the resulting lightning and high winds often knockout power systems.

Of the eight storms, two were reported to be one in 100-year events whilst the other six were one in 2- to 10-year events. The two larger storms caused major flooding, damaging property and transportation systems as well as causing severe overflows into Lake Michigan. However, for the other six storms, Changnon and Westcott (2002) highlighted the effectiveness of TARP in reducing flooding, reporting that only minor flooding occurred in those storm events.
As mentioned in Section 2.1 there a number of scale issues related to precipitation. Modeling scales for urban hydrologic models are in the order of minutes. The majority of rain gage stations collect hourly data and NEXRAD satellite data is collected at best in 6 minute intervals. There is evidence to suggest that even though NEXRAD data is available at 6 minute intervals, anything less than hourly data are thought to be of poor quality. As Blöschl and Sivapalan (1995) identify a modeling scale small than the observation scale will mean that downscaling must be performed. A further problem is created when trying to transfer information across spatial scales. Blöschl and Sivapalan (1995) highlight the difficulty in linking small scale and large scale parameters. Take for example the network of ISWS rain gages in Chicago. These rain gages are generally 5 miles apart. The majority of urban catchments within this area are less than two square miles in area, meaning that there is a good possibility that there will not be even one rain gage within the service area itself. What rain gage, collection of rain gages, or combination of rain gages and NEXRAD sites should be used to generate the precipitation? How should the information be distributed? This question has been tackled in the literature frequently with a wide variety of interpolation methods developed for the spatial estimation of rainfall from rain gage measurements. Such methods include the isohyetal method, optimum interpolation/kriging (Matheron 1973), spline interpolation (Creutin and Obled, 1982), inverse distance and others. More recently Cello et al (2008) have developed a method for combining rain gage and NEXRAD data to distribute rainfall spatially from a coarse grid to a series of finer grids.

2.2.2 Overland Flow/Surface Runoff Models

Horton (1933) described overland flow as follows:

“Neglecting interception by vegetation, surface runoff is that part of rainfall which is not absorbed by the soil by infiltration. If the soil has infiltration capacity, f, expressed in inches depth absorbed per hour, then when rain intensity i is less than f the rain is all absorbed and there is no surface runoff. It may be said as a first approximation that if i is greater than f, surface runoff will occur at the rate (i – f).”
Horton termed this difference \((i-f)\) “rainfall excess.” As shown in Figure 2.2, in addition to surface runoff there is also a surface storage component to the flow. As water flows overland it may become trapped in surface hollows as depression storage. If the underlying surface is impervious then this depression storage will be left to evaporate back into the atmosphere, on the other hand, if the underlying surface is pervious then water will be left to infiltrate into the soil to become groundwater flow or subsurface flow that may be released into a stream as a baseflow. Excess rainfall, that is neither retained on the land surface nor infiltrated into the soil, flows across the watershed surface as what may be termed direct runoff (Chow et al, 1988).

Determination of the volume of surface runoff and how it is routed through a subcatchment is one of the key features of modern day computer based hydrologic models. There are a plethora of methods available for performing these calculations ranging in complexity from something as simple as the Rational Method (Chow, 1962) to the more physically based Gridded Surface/Subsurface Hydrologic Analysis (GSSHA) model (Downer and Ogden, 2004). Each of the methods requires different subcatchment properties such as area, slope, length, land use, % imperviousness and connectedness. These parameters are combined with rainfall information and infiltration characteristics to assign the catchment a runoff coefficient and time of concentration. Runoff coefficient may be defined as the ratio of direct runoff to rainfall over a given period time and describes the quantity of water flowing over a subcatchment, while time of concentration is the time of flow from the farthest point of the watershed to the outlet. The way in which these two key parameters are derived is critical, and differs from model to model. It is not the objective of this research to explicitly compare all of the available models, this has been done in previous studies (Chow and Yen, 1976, Messner and Goyen, 1985, Singh and Woolhisser, 2002, Agbodo, 2005), however some of the differences are highlighted Section 2.2.6 and the model used in this study is described in more detail in Chapter 3.

2.2.3 Infiltration Methods

Infiltration is the process of water penetrating from the ground surface into the soil and is influenced by many factors including the condition of the soil surface and its vegetative cover, the properties of the soil, such as its porosity and hydraulic
conductivity, and the current moisture content of the soil (Chow et al, 1988). In many cases, infiltration is accounted for within the surface runoff model (e.g. rational method and SCS method) via the runoff coefficient; however in more detailed models infiltration is calculated separately using the properties of the soil. A number of different infiltration equations have been presented over the years including Horton (1933), Phillip’s (1957) and Green-Ampt (1911). Horton’s, Holton’s and Phillip’s equations are mathematical/empirical descriptions of frequency versus time whilst the Green and Ampt method uses a simplification of the more physically based Darcy’s Law.

Of particular interest in this research is the Green and Ampt approach. Green and Ampt (1911) proposed a physically based approximation that has an exact analytical solution in order to derive infiltration losses. By considering the simplified picture of infiltration and the vertical column of soil shown in Figure 2.4, Green and Ampt applied the continuity and momentum equations to derive the Green-Ampt equation:

$$F(t) - \psi \Delta \theta \ln \left(1 + \frac{F(t)}{\psi \Delta \theta}\right) = Kt$$

(2.1)

where $F(t)$ is cumulative infiltration, $\psi$ is wetting front suction head, $\Delta \theta = (1 - s_e) \theta_e$, $\theta_e$ is effective porosity, $s_e$ is degree of saturation, and $K$ is hydraulic conductivity.

![Figure 2.4 Schematic of Green-Ampt model](image)
When the infiltration rate exceeds the precipitation rate, i.e. \( f(t) > i(t) \), the cumulative infiltration is equal to the cumulative precipitation, i.e. \( F(t) = I(t) \), and the infiltration rate is given by:

\[
f(t) = K\left(\frac{\psi \Delta \theta}{F(t)} + 1\right)
\]  

(2.2)

When the infiltration rate, \( f(t) \), and precipitation rate (depression storage subtracted), \( i(t) \), are equal, i.e., \( f(t) = i(t) \), ponding will begin. The time to ponding, \( t_p \), can be determined by successive approximation using Equation (2.1) and (2.2). The cumulative infiltration at the onset of ponding, \( F(t_p) \), can then be used in the Green-Ampt equation to the critical time, \( t_{\text{crit}} \), and the subsequent time shift, \( t_{\text{shift}} \):

\[
t_{\text{shift}} = t_{\text{crit}} - t_p
\]  

(2.3)

If the precipitation rate exceeds the infiltration rate, i.e., \( i(t) > f(t) \), the method of successive approximation can be used to determine the cumulative infiltration from the Green-Ampt equation and the infiltration rate can be determined from Equation (2.2).

### 2.2.4 Inlet Models

One of the most neglected components of the available urban stormwater computer models is the inlet. Generally, inlets are not explicitly accounted for, but rather, must be represented as a junction or node. This is not ideal, as the hydraulics of inlets are important and may actually be the controlling factor in the hydraulic behavior of the system. For the most part, the hydraulic capacity of the pipe system exceeds the inlets, making inlet hydraulics extremely important. An inlet may be defined as a structure used to capture direct runoff and convey it to the underground sewer system. Inlets can be on-grade or sag and are most commonly rectangular or circular in shape, and feature a grated opening.
Hydraulic behavior of a sag gutter inlet may be described by a combination of weir and orifice flow (Neenah Foundry Co., 1987). In this way the flow through the inlet is a function of the upstream head, which in this case is assumed to be the depth of flow in the upstream gutter. Initially the behavior of the water through the inlet can be likened to a weir, whereby the discharge through the inlet, \( Q_{\text{weir}} \), is given by the general weir Equation 2.4:

\[
Q_{\text{weir}} = CPH^{3/2}
\]

where \( C \) is the weir coefficient, \( P \) is the perimeter (including only footage of sides subject to the flow) of the grate and \( H \) is the head in feet.

As the upstream head increases, the grate may become fully submerged, or ponded, and as such hydraulically the inlet will behave more like an orifice. Under orifice flow the grate’s open area becomes the determining parameter and discharge through the orifice, \( Q_{\text{orifice}} \), may be calculated using Equation 2.5:

\[
Q_{\text{orifice}} = CA\sqrt{2gH}
\]

where \( C \) is the orifice coefficient, \( A \) is the open area of the grate, \( g \) is the acceleration due to gravity and \( H \) is the head in feet.

Neenah Foundry Company (1987) is a leading manufacturer of inlet grates in the United States and they recommend that the weir and orifice flows from Equations 2.4 and 2.5 be compared and the equation with the lower flow value be allowed to predominate and taken as the inflow into the inlet.

For on-grade gutter inlets, inflow is governed by the flow velocity of the channel and inlet geometry. Velocity in the gutter is principally governed by the slope along the axis of the channel, the shape of the gutter cross-section, and the roughness of the surface in contact with the water, also known as the wetted perimeter (Neenah Foundry Company, 1987). Generally this type of gutter flow is simulated using Manning’s equation. The geometry of the inlet grate also governs the capacity of the system, with long narrow bars parallel to the direction of flow being the most efficient. Neenah Foundry Company (1987) has conducted tests for the grates which they supply using a
variety of longitudinal and cross-slopes. The data from the grate tests was compiled in graph form with values of “K” plotted versus the transverse gutter slope and a series of curves showing the “K” values for each longitudinal slope allows the selection of grate coefficients for the most generally used slopes. Flow, $Q_{grate}$, into the grate is then used by applying the selected K value in Equation 2.6.

$$Q_{grate} = KH^{5/2}$$

where $K$ is unique to the geometry of each grate and $H$ is the depth upstream of the grate in feet.

Guo (1997) developed a program called UDINLET for simulating street and inlet hydraulics. This program is capable of modeling a wide range of inlets differing according their size, position, grate opening etc. The methodology of Guo (1997) is the current state of the art in simulating inlet hydraulics.

2.2.5 Flow Routing Methods

Chow et al (1988) define flow routing as a procedure to determine the time and magnitude of flow (i.e. the flow hydrograph) at a point in the system from known or assumed hydrographs at one or more points upstream. Typically flow routing is characterized by either a lumped or distributed model. A lumped system model is calculated as a function of time alone at a particular location, while in a distributed system, routing of the flow is calculated as a function of space and time (Chow et al, 1988). The majority of models available utilize distributed flow models, as they best describe the passage of water through channels, accounting for flow rate, velocity and depth which vary in space and time throughout a system.

Distributed flow models are based on a set of partial differential equations, called the Saint-Venant Equations of for one-dimensional flow. The Saint-Venant equations were first developed by Barre de Saint-Venant in 1871 and describe one-dimensional unsteady open channel flow, which applies to the majority of combined sewer flows. A summary of the Saint-Venant equations, derived by Chow et al (1988) in reference to Figure 2.5 are shown below in Equations 2.7 to 2.10. These equations are in simplified
form and neglect lateral inflow, wind shear and eddy losses and assume a Boussinesq coefficient, $\beta$, of unity.

Figure 2.5 An elemental channel reach for derivation of the Saint-Venant Equations (Chow et al, 1988)

Continuity equation

Conservation form

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \quad (2.7)$$
where $\frac{\partial Q}{\partial x}$ is the rate of change of channel flow with distance and $\frac{\partial A}{\partial t}$ is the rate of change of area with time.

*Non conservation form*

$$V \frac{\partial y}{\partial x} + y \frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0$$  \hfill (2.8)

where $V$ is the velocity of channel flow, $\frac{\partial y}{\partial x}$ is the rate of change of channel flow depth with distance, $y$ is the flow depth, $\frac{\partial V}{\partial x}$ is the rate of change of channel flow velocity with distance and $\frac{\partial y}{\partial t}$ is the rate of change of channel flow depth with time.

*Momentum equation*

*Conservation form*

$$\frac{1}{A} \frac{\partial Q}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + g \frac{\partial y}{\partial x} - g \left( S_o - S_f \right) = 0$$  \hfill (2.9)

where $A$ is the area of the channel flow, $\frac{\partial Q}{\partial t}$ is the rate of change of channel flow with time, $Q$ is the channel flow, $g$ is acceleration due to gravity, $S_o$ is the bed slope and $S_f$ is the friction slope.

*Non conservation form (unit width element)*

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g \left( S_o - S_f \right) = 0$$  \hfill (2.10)

where $\frac{\partial V}{\partial t}$ is the rate of change of channel flow velocity with time.

The terms in the momentum equation describe the physical processes that govern the flow momentum. These terms are:

- local acceleration term, which describes the change in momentum due to the change in velocity over time,
• convective acceleration term, which describes the change in momentum due to change in velocity along the channel,

• pressure force term, proportional to the change in the water depth along the channel,

• gravity force term, proportional to the bed slope, and

• friction force term, proportional to the friction slope.

Invariably the distributed flow routing models that are available combine the full continuity equation with variable forms of the momentum equation. The three most common distributed models are:

1. **Kinematic Wave model**, which neglects the local acceleration, convective acceleration, and pressure terms in the momentum equation, effectively setting . The Kinematic wave model only requires one boundary condition (at the upstream end of the channel), but cannot account for downstream backwater effects, flow reversal, damping of flood peak or flow acceleration. This model is the simplest approximation of the Saint-Venant momentum equation, assuming that the water surface is parallel to the channel bed. Despite its deficiency of not accounting for the downstream backwater effects, Mays (2001) reports that because of its simplicity the Kinematic Wave model is the most extensively used and studied model among the dynamic wave approximations.

2. **Non-inertial Wave model**, which neglects the local and convective acceleration terms but includes the pressure term. Unlike the Kinematic Wave model, this model is able to account for backwater effects, flow reversal and peak attenuation. However, this model requires two boundary conditions and still cannot account for flow acceleration. Mays (2001) describes the Non-inertial model, also referred to as the Diffusion Wave Model approximation, as the most useful among the approximations of the dynamic wave equation, because it offers a balance between accuracy and simplicity for a large number of field situations.
Because the inertia terms are ignored, it doesn’t have the computational problems that are encountered in the Dynamic Wave model (Mays, 2001).

3. **Dynamic Wave model**, which considers the full momentum equation, adequately taking account of backwater effects that are described by the local and convective acceleration terms and the pressure term. Like the Diffusion Wave model, two boundary conditions are required and it accounts for flow reversal and damping of the flood peak. Unlike both of the other models described the Dynamic Wave model accounts for flow acceleration. Hydrodynamically it possesses two characteristic waves: for subcritical flows one travels downstream, while the other travels upstream (Mays, 2001).

Comparison of the application of the dynamic wave, diffusion wave, and kinematic wave models for flow routing in networks has been conducted and it has been found that the diffusion wave model generally agrees well with the dynamic wave solutions. Mays (2001) highlighted that for prismatic channels, including sewers, the diffusion wave equation provides a better compromise because the local and convective acceleration terms are of the same order and different sign, making it better to ignore both of them rather than just one.

One of the key differences in the available hydrologic models is in their solution of the Saint-Venant Equations. These equations are partial differential equations (PDE’s) that must be solved numerically. Numerical methods for solving PDE’s may be classified as either direct numerical methods, in which finite difference equations are formulated from the original PDE’s for continuity and momentum, or characteristic methods, whereby PDE’s are first transformed to a characteristic form and solved analytically or using a finite difference representation (Chow et al, 1988). Either way, the most common method of solution is to apply a finite difference approximation, which may are characterized as explicit or implicit.

Implicit schemes use finite-difference approximations for both temporal and spatial derivatives where as the explicit scheme uses a forward-difference scheme for the time derivative and a central difference scheme for the spatial derivative (Chow et al,
An explicit method uses only known information to calculate an unknown value at a new time step while an implicit method solves for an unknown value at a new time step based on known and unknown information (Agbodo, 2005). Explicit schemes are inevitably subject to instability and as such Courant and Friedrichs (1948) developed a necessary but insufficient condition for stability of an explicit scheme called the Courant condition. The Courant condition requires that the time step be less than the time for a wave to travel the distance $\Delta x$.

### 2.2.6 Comparison of Modeling Packages

As mentioned throughout the preceding sections there are a plethora of hydrologic/hydraulic models that are available for modeling watersheds and urban catchments. Singh and Woolhiser (2002) conducted an extensive review of how watershed hydrologic models have evolved over time. The earliest models include the Stanford Watershed Model (now HSPF) by Crawford and Linsley (1966) and HEC-1 (Hydrologic Engineering Center, 1968). Following the early development, emphasis was placed on more physically based models such as SWMM (Metcalf and Eddy et al., 1971), System Hydrologique Europeen (SHE) (Abbot et al., 1986a,b), TOPMODEL (Beven and Kirkby, 1979) and so on. Singh and Woolhiser (2002) identify the most commonly used watershed models in United States and elsewhere:

- **HEC-HMS** (Feldman, 1981, HEC, 1981) is the most commonly used model in the private sector for design of drainage systems in the United States.

- **NWS** (Burnash et al, 1973a,b) is the standard model for flood forecasting in the United States.

- **HSPF** and its extended water quality model are the standard models adopted by the Environmental Protection Agency (EPA).

- **MMS** (Dawdy et al, 1970) model of the USGS is the standard model for water resources planning and management works.

- **RORB** (Laurenson, 1964) model is commonly employed for flood forecasting, drainage design and evaluating the effect of land use change in Australia.
• TOPMODEL and SHE are the standard models for hydrologic analysis in many European countries.

• Xinanjiang (Zhao et al, 1980) model is a commonly used model in China.

Singh and Woolhiser (2002) identify in excess of 50 watershed hydrology models, distinguishing and classifying them according to different criteria. Models may be classified based on (Singh, 1995):

1. Process description: different models may be used for infiltration, surface runoff, evapotranspiration etc.

2. Time-scale: for watershed models time-scales can range from minutes to years.

3. Space-scale: maybe in the order of meters to kilometers.

4. Techniques of solution: depending on what form of the continuity and momentum equation is used solution may proceed using numerical methods such as finite difference, finite element, etc.

5. Land use: a plethora of land uses exist from rural to urban.

6. Model use: models may be predictive or investigative.

These different classifications are useful in comparing different watershed models, but it is also necessary to distinguish between the different water resources models that exist. Wurbs (1998) generalized that watershed models can be classified into (i) watershed models; (ii) river hydraulic models; (iii) river and reservoir quality models; (iv) reservoir/river system operation models; (v) groundwater models; (vi) water distribution system hydraulic models; and (vii) demand forecasting models. Under this classification system urban catchment hydrologic models would fall into the watershed model category. And in fact, many of the watershed models identified by Singh and Woolhiser (2002) are commonly used for urban catchment hydrologic modeling, most notably SWMM and HEC-HMS. There are, however, a number of other models that are used and as such warrant identification. A number of authors have compared the most commonly used urban catchment hydrologic models, Messner and Goyen (1985), Mays (1999) and Agbodo (2005).

Each of the investigations compares the various components (e.g. infiltration model, surface runoff model, routing procedure etc.) of each model, techniques of solution (e.g. implicit/explicit solution technique) and where the models are most commonly applied. A summary of these investigations is compiled in Tables 2.1 and 2.2 and highlight some of the key differences between some of the available models. Table 2.1 compares a selection of the most commonly used older models from around the 1970’s, whilst Table 2.2 compares some of the more advanced models that are used most commonly by engineers today. In addition, the key advantages and disadvantages of the different models, as highlighted in the literature, are presented in Table 2.3.
<table>
<thead>
<tr>
<th>Model</th>
<th>Rational Method</th>
<th>ILLUDAS</th>
<th>Illinois Urban Storm Runoff Method</th>
<th>Tr-55 (SCS Method)</th>
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<td>User-defined hyetograph or design storm</td>
<td>Hyetographs, allows aerial variation</td>
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<td>Subcatchment Properties</td>
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<td>Divided into strips with input length, width, slope and roughness</td>
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<td>Circular conduits. Based on length, size, slope and roughness of conduits, and size of manholes and junctions</td>
<td>Trapezoidal open channels</td>
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<td>Horton</td>
<td>SCS Curve Number</td>
</tr>
<tr>
<td></td>
<td>Initial Losses</td>
<td>Accounted for by runoff coefficient</td>
<td>Different constants for pervious and impervious areas</td>
<td>Depression storage</td>
</tr>
<tr>
<td></td>
<td>Surface Runoff</td>
<td>Time-area with Izzard’s time formula or kinematic wave</td>
<td>Non linear kinematic wave routing</td>
<td>Based on the SCS method</td>
</tr>
</tbody>
</table>
Table 2.1 (cont.)

<table>
<thead>
<tr>
<th>Model</th>
<th>Rational Method</th>
<th>ILLUDAS</th>
<th>Illinois Urban Storm Runoff Method</th>
<th>Tr-55 (SCS Method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Modeling</td>
<td>-</td>
<td>-</td>
<td>Combination of weir and orifice equation</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Accounts for upstream and downstream backwater effects as well as reverse flows, solving the St. Venant equations using a Y-segment technique</td>
<td></td>
</tr>
<tr>
<td>Pipe/Channel Routing</td>
<td>-</td>
<td>Steady Flow</td>
<td>SCS Method</td>
<td></td>
</tr>
<tr>
<td>Output Results</td>
<td>Peak discharge</td>
<td>Runoff hydrograph</td>
<td>Runoff hydrographs, and depth and velocity at inlets and at entrance and exit of all sewers.</td>
<td>Runoff hydrograph</td>
</tr>
</tbody>
</table>

Table 2.2 Comparison of modern day hydrologic/hydraulic models

<table>
<thead>
<tr>
<th>Model</th>
<th>HEC-HMS</th>
<th>InfoSWMM</th>
<th>GSSHA/SUPERLINK</th>
<th>InfoWorks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Urban</td>
<td>Urban</td>
<td>Watershed/Urban</td>
<td>Urban</td>
</tr>
<tr>
<td>Scale</td>
<td>All Sizes</td>
<td>All sizes</td>
<td>All sizes</td>
<td>All Sizes</td>
</tr>
<tr>
<td>Interface</td>
<td>WINDOWS</td>
<td>WINDOWS/ArcGIS</td>
<td>WINDOWS</td>
<td>WINDOWS/ArcGIS</td>
</tr>
<tr>
<td>Cost</td>
<td>$0</td>
<td>$15,000</td>
<td>$3,700</td>
<td>$55,000</td>
</tr>
<tr>
<td>Inputs Precipitation</td>
<td>Frequency storm, user-defined hyetograph, gridded precipitation, SCS</td>
<td>Constant and time-varying</td>
<td>Constant and time-varying and NEXRAD</td>
<td>Constant and time-varying</td>
</tr>
<tr>
<td>Model</td>
<td>HEC-HMS</td>
<td>InfoSWMM</td>
<td>GSSHA/SUPERLINK</td>
<td>InfoWorks</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Subcatchment Properties</td>
<td>Dependant on surface runoff method, but include area, slope, length, time of concentration, imperviousness</td>
<td>Dependant on surface runoff method, but divided into strips with area, slope, length, subcatchment width, imperviousness</td>
<td>DEM, land use, soil properties, area</td>
<td>Dependant on surface runoff method, but area, land use, soil type, slope, dimension</td>
</tr>
<tr>
<td>Inlet Properties</td>
<td>Rating Curve (Q v H)</td>
<td>Rating curve (Q v H)</td>
<td>Flow area</td>
<td>Rating curve (Q v H)</td>
</tr>
<tr>
<td>Pipe/Channel Properties</td>
<td>Dependant on method of routing but include geometry, slope, length, Manning’s roughness</td>
<td>All geometries. Dependant on method but usually, length, roughness, geometry</td>
<td>All geometries. Require geometry of link, length, inverts</td>
<td>All geometries. Dependant on method but usually, length, roughness, geometry</td>
</tr>
<tr>
<td>Infiltration Routines</td>
<td>Green-Ampt, SCS, Gridded Deficit Constant, Initial and constant</td>
<td>SCS Curve Number, Green-Ampt, Horton</td>
<td>Richards (see Downer and Ogden 2004), Green-Ampt, Green-Ampt w/ Redistribution</td>
<td>Fixed PR Model, Green-Ampt Model, Horton Infiltration Model, New UK PR Model, Wallingford Procedure Model, Constant Infiltration Model, US SCS Model</td>
</tr>
<tr>
<td>Initial Losses</td>
<td>Depression storage</td>
<td>Pervious and impervious depression storage</td>
<td>Depression storage</td>
<td>Depression storage</td>
</tr>
<tr>
<td>Model</td>
<td>HEC-HMS</td>
<td>InfoSWMM</td>
<td>GSSHA/SUPERLINK</td>
<td>InfoWorks</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------------------</td>
<td>------------------------------------</td>
<td>-------------------------------------</td>
<td>------------------------------------------------</td>
</tr>
<tr>
<td><strong>Surface Runoff</strong></td>
<td>Kinematic Wave,</td>
<td>EPA SWMM/Non linear</td>
<td>2D Diffuse Wave</td>
<td>Wallingford, Large Catchment, SPRINT, SWMM, Desbordes (see Wallingford 2004 for details of all the methods)</td>
</tr>
<tr>
<td></td>
<td>SCS Unit</td>
<td>reservoir, CUHP, NRCS</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit Hydrograph, Clark</td>
<td>Dimensionless and Triangular</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit Hydrograph,</td>
<td>Unit Hydrograph, Delmarva,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>User-specified Unit Hydrograph,</td>
<td>Clark and Snyder Unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Snyder Unit Hydrograph (see HEC 1989</td>
<td>Hydrographs (see MWHSoft 2005 for</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>for details of all the</td>
<td>details of all the methods)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>methods)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Inlet Modeling</strong></td>
<td>Diversion tool</td>
<td>Outlet tool, orifice or weir</td>
<td>Implicit scheme in Superlink (see Downer and Ogden 2004)</td>
<td>Orifice or weir</td>
</tr>
<tr>
<td><strong>Pipe/Channel</strong></td>
<td>Kinematic Wave, Lag,</td>
<td>Steady Flow, Kinematic Wave</td>
<td>Dynamic Wave. Explicit</td>
<td>Implicit solution to unsteady state St. Venant equation</td>
</tr>
<tr>
<td><strong>Routing</strong></td>
<td>Muskingum, Muskingum-Cunge (see US</td>
<td>or Dynamic Wave. Explicit</td>
<td>Finite Volume scheme</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Army Corps of Engineers for</td>
<td>solution of the unsteady state</td>
<td>for channels. Implicit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>details of all the</td>
<td>St. Venant equations with</td>
<td>Superlink scheme for pipes.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>methods)</td>
<td>variable time step</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Output Results</strong></td>
<td>Runoff hydrograph,</td>
<td>Runoff hydrograph, node and</td>
<td>Runoff hydrograph, dynamic movie</td>
<td>Graphical and Tabular results for all nodes and</td>
</tr>
<tr>
<td></td>
<td>node and junction reports, ArcMap</td>
<td>junction reports, ArcMap outputs</td>
<td>simulation</td>
<td>links and GIS outputs</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.3 Advantages and disadvantages of available hydrologic/hydraulic models

<table>
<thead>
<tr>
<th>Model</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational Method</td>
<td>Quick and easy</td>
<td>Yields only a peak discharge, and provides no information on the time distribution of storm runoff.</td>
</tr>
<tr>
<td></td>
<td>Requires very little input data</td>
<td>Is very dependant on selection of the runoff coefficient which is arbitrary</td>
</tr>
<tr>
<td></td>
<td>Free</td>
<td>Unable to model conduits and inlets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Only for small (&lt;200 acre) basins</td>
</tr>
<tr>
<td>ILLUDAS</td>
<td>Accounts for directly connected, supplementary paved and grassed areas individually</td>
<td>DOS based</td>
</tr>
<tr>
<td></td>
<td>Relatively simple to use</td>
<td>Availability limited</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does not account for backwater effects and does not explicitly account for pressurized flows.</td>
</tr>
<tr>
<td>Illinois Urban Storm Runoff Method</td>
<td>Adequately accounts for inlets</td>
<td>Can only model circular conduits</td>
</tr>
<tr>
<td></td>
<td>Accounts for backwater effects</td>
<td>Limited options for infiltration modeling</td>
</tr>
<tr>
<td></td>
<td>Uses efficient solution techniques</td>
<td></td>
</tr>
<tr>
<td>Tr-55</td>
<td>Easy to use</td>
<td>Unable to handle to complex pipe networks</td>
</tr>
<tr>
<td></td>
<td>Commonly used in the US</td>
<td>Does not account for backwater effects</td>
</tr>
<tr>
<td></td>
<td>Free</td>
<td>Limited options for infiltration modeling</td>
</tr>
<tr>
<td>HEC-HMS</td>
<td>Easy to use</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Free</td>
<td>Unable to account for backwater effects</td>
</tr>
<tr>
<td></td>
<td>Has a number of simulation options for surface runoff and pipe routing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nice graphical user interface</td>
<td></td>
</tr>
<tr>
<td>InfoSWMM</td>
<td>Great graphical user interface</td>
<td>Uses an explicit solution scheme to the Saint-Venant equations which can lead to instability</td>
</tr>
<tr>
<td></td>
<td>Easy to use</td>
<td>Expensive</td>
</tr>
<tr>
<td></td>
<td>Able to handle large systems with ease</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Variety of surface runoff, infiltration and conduit routing methods to choose from</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Utilizes SWMM as its underlying computational model which is the most used model in the US</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>Advantages</td>
<td>Disadvantages</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GSSHA/SUPERLINK</td>
<td>Very physically based model for surface runoff and infiltration</td>
<td>Long run times</td>
</tr>
<tr>
<td></td>
<td>Uses implicit solution scheme making it more stable</td>
<td>More complicated to setup than your average model</td>
</tr>
<tr>
<td></td>
<td>than other explicit solution schemes</td>
<td>Requires detailed input data (e.g. DEM etc)</td>
</tr>
<tr>
<td></td>
<td>Able to model inlets</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nice graphical user interface</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nice visual outputs</td>
<td></td>
</tr>
<tr>
<td>InfoWorks</td>
<td>Sophisticated simulation of hydrology and hydraulics</td>
<td>Expensive</td>
</tr>
<tr>
<td></td>
<td>Interfaced with GIS</td>
<td>Requires a lot of data</td>
</tr>
<tr>
<td></td>
<td>Able to analyze surcharge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Able to handle large networks due to its excellent data management system</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stable and robust</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Can model complex structures</td>
<td></td>
</tr>
</tbody>
</table>
Although the models described above are different in many ways they share one important common attribute – they require some degree of calibration. Take for example the SWMM model. In the runoff module in SWMM there exists a parameter called the subcatchment width which is utilized in calculating the time of concentration for the given subcatchment. This parameter has no real physical meaning and as recommended by the SWMM User Manual (Hubert and Dickinson, 1992), this parameter should be used as a calibration parameter for the model. A problem then arises when calibration data is not available for the area of interest. It is difficult to estimate parameters such as subcatchment width when they do not have a physical meaning. Furthermore, it is again necessary to emphasize that many of the commonly used models for urban catchment hydrology require a significant volume of input data. For existing systems that date back as far as the 1900’s, this data is simply not always available and survey of the systems is simply impractical or too expensive.

Outside of the suite of computer simulation packages (described above), which fall into the realm of deterministic models, there exist a number of watershed models that consist of one or more stochastic components. Inevitably watershed models are complicated with many input parameters, but frequently the information that they are required to provide is relatively simple (Singh and Woolhisser, 2002). In response to this simplicity, a number of statistical tools exist that may be useful in predicting watershed behavior. These tools include: regression and correlation analysis, time series analysis, stochastic processes and probabilistic analysis. Typically, these models are subject to uncertainty and reliability analysis due to the inherent uncertainty in the model structure and parameter values (Singh and Woolhisser, 2002).

Stochastic models have the potential to overcome the need for calibration data and large volumes of input data. Yen and Lee (1997) adapted the geomorphologic unit hydrograph method (GIUH) to allow derivation of the unit hydrograph for ungauged or inadequately gauged watersheds without the need of observed runoff and rainfall data and with knowledge of as little as the total watershed area and order of the catchment. The approach developed incorporated the geomorphic laws of stream order developed by Strahler (1957), a kinematic wave approximation of travel time (Lee and Yen, 1997)
and the general structure of the GIUH (Rodriguez-Iturbe and Valdez, 1979). More details of the GIUH concept are presented in Section 2.4.

### 2.3 Simplification of Urban Hydrologic Models

Although computer software has come along way and there now exist tools to model thousands of nodes and links, in many situations it remains impractical to model every node and link in the system. Time and budget constraints, combined with a lack of relevant or accurate input data, compel modelers to create a simplified model that “adequately” represents the hydrologic and hydraulic behavior of the network. The question then becomes; what is the best method for simplifying the network without introducing uncertainty and while maintaining an acceptable degree of accuracy? There is no universal answer to this question. Cantone (2007) investigated a number of the most commonly employed simplification techniques in hydrologic and hydraulic modeling. The aim of this research was to help answer the questions: How much network complexity needs to be included? Can pipes below a certain diameter simply be ignored because they are small in comparison with the rest of the system? Is it best to just gradually build up network complexity until no notable change in the hydrograph is observed? What are the dangers of applying traditional simplification techniques like conduit skeletonization and subcatchment aggregation?

Many different simplification techniques have been applied over the years to allow simulation of water systems. One example is skeletonization, which has been defined as the process of selecting for inclusion in a model only the parts of the network that have a significant impact on the behavior of the system (Haestad Methods, 2002). Skeletonization has primarily been investigated in relation to pressurized flow in water distribution systems (Eggener and Polkowski, 1976; Grayman and Rhee, 2000; Walski et al, 2004; Cantone et al, 2005). In these systems, skeletonization has typically involved modeling only those pipes that affect pressure in the system. Little work has been presented on skeletonization in stormwater or combined sewer systems that involve free-surface flow or a combination of free-surface and pressurized flow. The term conduit skeletonization, as introduced by Cantone (2007), refers to the omission of conduits in a combined sewer system to reduce model complexity. As an example, the
The Department of Water Management for the City of Chicago has recently undertaken hydrologic/hydraulic modeling of their combined sewer system. Their modeling protocol requires that only sewers greater than 42” in diameter and selected smaller diameter sewers that “have considerable impact on the hydraulics of the system” be modeled (City of Chicago, written communication, August 2007). Skeletonization of the network in this manner results in approximately 10% of the conduits in the system being modeled.

The literature identifies a number of different techniques for skeletonization, ranging from computer automated techniques to good old fashioned practical experience. In short, there is no way of prescribing a method of skeletonization guaranteed to produce accurate results. For a start every model is inherently different and the most logical, but perhaps not most practical, model to create is one including every component of the system. This is why in many cases, modelers skeletonize on the basis of their engineering judgment. The lessons learned from previous studies often guide engineers in deciding which pipes to include or not include in a skeletonized model. This of course makes it very difficult to establish a set of fixed rules for skeletonization.

Over the past 10 years increased computer power and more sophisticated data processing techniques has seen the emergence of hydraulic analysis programs with automated skeletonization techniques. Haestad Methods Inc. has been at the forefront of this technology with the development of Skelebrator. Skelebrator automatically skeletonizes water distribution systems using a combination of four skeletonization techniques (Haestad Methods, 2002):

1. **Data Scrubbing**: basically consists of simply removing all pipes that meet user-specified criteria such as diameter, roughness or other attributes. The primary drawback of this type of skeletonization is that there is no consideration of the hydraulic effects of a pipe’s removal.

2. **Branch Trimming**: a special case of pipe removal, in which the end branch of a tree shaped section of the system is removed and the demand is brought back to the last remaining node.
3. Series Pipe Removal: where pipes in series are combined into a single equivalent pipe with the same length and carrying capacity as the original pipes (intermediate nodes are removed).

4. Parallel Pipe Removal: where pipes in parallel are combined into an equivalent pipe with the same carrying capacity as the original pipes.

The most obvious benefit associated with a skeletonized model is the potential for cost savings. Costs generally increase with increasing level of detail in a network model, due to the following factors (Grayman and Rhee, 2000):

- More data must be collected, managed and inputted into the model
- Cost of many software packages increases with the increasing number of pipes
- Time required to analyze the results of larger models increases.

Consideration should also be given to the effort required in developing parameter values and calibrating large models. As the size of the model increases the parameterization and calibration become more difficult and labor intensive. Parameterization is in itself, a technique that is not completely understood and the determination of optimal parameter dimensions for water distribution network models is the subject of research scrutiny (Mallick and Lansey, 1994). It is possible for parameters to become a “garbage dump” for further error. More pipes inherently mean more input parameters, and if these parameters are not accurate or deterministic, the uncertainty in the model may be increased. As such, it is feasible that a skeletonized model could be more accurate than a full pipe model, if the additional detail used in the full pipe model incorporates non-deterministic input parameters.

McInnis and Karney (1995) described a method of obtaining a good indication of the sensitivity of the results to the skeletonization process. By beginning with the biggest and most dominant pipes in the system and then progressively adding details to refine their initially crude representation, they were able to grasp the sensitivity of the system with respect to transient pressures. This approach was also recommended by Cruickshank (2004), who recommended an approach that involved upgrading existing
trunk main model by adding smaller pipes, rather than building an entirely new model. Using this method, it was postulated that the integrity of the trunk main model could be preserved and smaller pipes could be imported into modeling software from GIS shape files. This article importantly highlighted that the use of GIS and SCADA systems could be incorporated into the development of a network model. Cruickshank (2004) utilized both of these systems in generating and importing accurate water consumption data for his model.

The most common simplification technique employed in urban stormwater modeling is subcatchment aggregation, sometimes referred to as catchment discretization, whereby a number of smaller subcatchments may be represented as one or more larger subcatchments. At the fine resolution end of the spectrum in a CSO model there would be one subcatchment for each inlet in the system – this can be thought of as a model without subcatchment aggregation. When applying this method one must also concern themselves with parameterization, which involves the determination of input parameters (e.g. subcatchment slope, % impervious, flow length, etc.) for the larger subcatchments that represent the physical processes of the smaller subcatchments combined. Often simple methods, such as the area-weighted average method, are employed, neglecting that many of the processes being linearly averaged are highly nonlinear. Alternatively, model parameters may be obtained by calibration if calibration data are available. There have been a number of studies (e.g. Zaghoul and Kiefa, 2001; Cheng et al, 2002; Cheng et al, 2005; Barco et al, 2008) conducted into the state-of-the-art techniques for calibrating hydrologic models, particularly in natural watersheds. However, in many catchments calibration data simply do not exist, forcing input parameters to be estimated from the best available data.

While several investigators have studied the minimum level of physical spatial scale required in hydrologic modeling to adequately represent the spatial heterogeneity of a catchment, there is surprisingly little work reported in the literature on the explicit effect of the catchment computational discretization size (Maizon and Yen, 1994). Zaghoul (1981) conducted one such study looking at the sensitivity of the main parameters in the Runoff-Transport Block of SWMM and the effects of aggregating their effects. This study highlighted that imperviousness, hydraulic width and conduit length
were the most sensitive parameters. Zaghoul (1981) identified that careful adjustment of
the hydraulic width was required to maintain accuracy in simulations. Maizon and Yen
(1994) studied the effect of spatial discretization using three different simulation
packages (Rational Method, RORB and HEC-1) by applying them to a hypothetical 829
ha 5th order natural catchment. This study was concerned with rainfall-runoff modeling
in natural watersheds where a network of streams is used to convey water rather than
an urban catchment with a network of combined sewers. It was found that the effects of
catchment discretization are model dependent, affect the entire shape of the outfall
hydrograph, and become more pronounced as discretization becomes coarser.

In the SWMM User Manual, Huber and Dickinson (1992) state that it is desirable
to represent the total catchment by as few subcatchments as possible, consistent with the
needs for hydraulic detail within the catchment. The basis of this statement is that the
required volume of input data (and personal time) decreases as the number of
subcatchments decreases. Smith (1975) and Proctor et al (1976) compared the effects of
lumping subcatchments and found that a single equivalent lumped catchment can be
formulated by proper adjustment of the subcatchment width. Reducing the value of
subcatchment width increases the storage of the catchment (Huber and Dickinson 1992).
The key to subcatchment aggregation is the replacement of lost storage and as such
when aggregating subcatchments the sum of the subcatchment widths should be
reduced accordingly (Huber and Dickinson, 1992). Studies have shown that the lumped
subcatchment width should be approximately twice the length of the main drainage
channel through the catchment in order to match hydrograph peaks.

Cantone and Schmidt (2009) applied different degrees of conduit skeletonization
and subcatchment aggregation to a small catchment (12.9 acre) in the Chicago area, the
Oakdale Avenue catchment, in order to grasp the effects of these different simplification
techniques. An investigation of three hydrologic/hydraulic simulation packages,
ILLUDAS, HEC-HMS and InfoSWMM, clearly showed that the effects of simplification
techniques like subcatchment aggregation and conduit skeletonization are dependent on
the simulation package being used. A base model and four levels of simplification were
tested and it was shown that depending on the model being used these four levels of
skeletonization have differing effects. The reasons for this lie in differences in the surface
runoff routing methods, infiltration methods and conduit routing methods used by the three simulation packages. ILLUDAS was found to be the most robust of the models, while greatest variation was found using InfoSWMM. Dynamic wave routing in InfoSWMM allowed storage and backwater effects to be investigated and highlighted the need to account for lost storage when skeletonizing conduits. Using each of the three packages, Cantone and Schmidt (2009) showed that there is a potential to underestimate the peak of the outfall hydrograph when skeletonizing and aggregating down to a simple lumped one subcatchment, one pipe model. This is a worrying conclusion, considering that modeling protocols such as the one suggested by the City of Chicago wouldn’t have included the largest pipe (750 mm) in this catchment. This indicates that an even greater degree of catchment discretization and subcatchment aggregation than the grossest simplification from this study would likely be used in simulating urban problems.

For a more detailed review of the literature on skeletonization and subcatchment aggregation together with further details of their potential dangers see Cantone (2007).

In watershed hydrologic modeling there have been a number of other methods that have been explored for simplifying network behavior and predicting hydrologic response, particularly in ungauged basins. Typically in watershed models, it is necessary to distinguish between distributed physically based models (i.e. based on theories of small scale processes, large data and computer requirements, large setup and computational times) and lumped conceptual models (i.e. arbitrary or inappropriate model structures, lack of physical basis, difficulties with calibration, faster in setup and computational times, more modest in terms of data requirements) (Sivapalan et al 2003). Debate over the pros and cons of these types of models has heightened in recent times, but as Sivapalan et al (2003) highlights this debate has not provided a way forward – rather there is a need for new approaches to enable the construction of hydrologic models that can be used for prediction in both gauged, and more importantly, ungauged catchments.

Bloschl and Sivapalan (1995) presented the generally accepted steps involved in the development of hydrological models of all types, in the context of a particular modeling goal:
a. collecting and analyzing data;

b. developing a conceptual model that, in the modeler’s mind, describes the important characteristics of a catchment;

c. translating the conceptual model into a mathematical model;

d. calibrating the model to fit a part of the historical data by adjusting the various coefficients; and

e. validating the model against remaining historical data.

While lumped models typically follow all of these 5 steps physically based models tend to replace the first two steps with detailed description of the physical processes involved. Sivapalan et al (2003) identifies that the approach taken to develop physically based models is an ‘upward’ or ‘bottom-up’ approach. SHE is an example of a physically based model. The alternative to this approach is the ‘downward’ or ‘top-down’ approach, introduced by Klemes (1983), which has been applied extensively in predicting hydrologic response using a simplified representation of the hydrologic processes involved. One example of the downward approach is the GIUH concept developed by Rodriguez-Iturbe and Valdes (1979). This method is discussed in detail in Section 2.4.

Reggiani et al (1998) developed a model integrating the micro-scale conservation equations for mass, momentum and energy over specially delineated regions called representative elementary watersheds (REWs). A REW includes all the basic functional properties of a watershed (channels, hillslopes) and constitutes a single functional unit, which is representative of other sub-entities of the entire watershed due to its repetitive nature (Reggiani et al, 1998). One of the unique attributes of the REW is that it can be assumed as being self-similar to the larger basin, in the sense that it reveals similar structural patterns independent of the observation scale. The REW itself is comprised of five subregions, namely: unsaturated zone, saturated zone, concentrated overland flow zone, saturated overland flow zone, and the channel reach zone. The self-similarity of the REWs allows there input parameters to be defined by watershed-scale parameters measurable in the field. This approach represents a middle ground between the physically based and lumped approaches, reducing the need for detailed distributed
inputs but maintaining a physical description of the hydrologic processes between the surface, subsurface and between REWs.

In 2002, the International Association of Hydrological Sciences (IAHS) initiated the Decade for Prediction in Ungauged Basins (PUB) (Hubert et al, 2002). This initiative was in recognition that an improved capability for predicting hydrologic responses in ungauged basins is essential for more informed management towards sustaining the world’s water and land resources (Littlewood et al, 2003). Sivapalan (2003) highlights that PUB without calibration is a difficult, unsolved problem, demanding urgent resolution, and requiring significant breakthroughs in data collection, process knowledge and understanding. Integral to the success of this initiative was a science-based approach that incorporated numerical (computer) models of gauged catchments, and a means for transferring information (i.e. model parameters) from gauged to ungauged catchments. Littlewood et al (2003) highlighted that parametrically parsimonious conceptual models (PPCMs), that require not more than six model parameters, can be useful in the development of schemes to estimate streamflow at ungauged sites. The advantage of such models is that require little input information but there are heavily reliant on the availability of reliable streamflow data. As previously demonstrated such calibration data is not available for the majority of catchments being considered in this research and the applicability of models like the PPCM are yet to be tested on highly urbanized catchments. Purely statistical ‘black box’ models do not readily apply to this model because of the absence of calibration data. This includes PPCM’s referred to by Littlewood et al (2002).

In discussing the challenges for predicting hydrologic response in ungauged basins, Sivapalan (2003) highlighted that fractals and modern stochastic techniques that utilize geometric and statistical self similarity to quantify the relationship between variables at different scales, have the power to describe complex heterogeneity with a minimum number of parameters. Such techniques provide a means for relaxing the need for detailed distributed input information. In watershed models Sivapalan (2003) postulated that fundamental to modeling framework for the “new theory of hydrology” would be a distributed representation of the river network connecting different parts of the basin, together with efficient parameterizations to represent sub-basin heterogeneity.
Snell and Sivapalan (1995) developed an approach for modeling watersheds, in which the network is modeled as a single channel entity using effective parameterizations that capture the spatial variation induced by the various components of the network. The meta-channel concept, as called by Snell and Sivapalan (1995), utilizes the width function of the catchment to describe network pattern and represents variation in channel geometry using the hydraulic geometry tools developed by Leopold and Maddock (1953). Effectively, the meta-channel is an individual channel that has varying cross-section according to the width function. Importantly, routing schemes based on meta-channel hydraulic geometry are explicitly nonlinear and spatially varying (Snell et al, 2004).

### 2.4 Geomorphologic Instantaneous Unit Hydrograph (GIUH)

The unit hydrograph is a surface runoff hydrograph resulting from one unit of rainfall excess uniformly distributed spatially and temporally over the watershed for the entire specified excess duration (Chow, 1964). Sherman (1932) introduced the theory of the unit hydrograph and suggested that it be derived from observed runoff data minus baseflow and records of rainfall minus abstractions. The geomorphologic instantaneous unit hydrograph (GIUH) was developed by Rodriguez-Iturbe and Valdes (1979) who postulated that the structure of the hydrologic response was intimately related to the geomorphologic parameters of a basin and could be used to generate the unit hydrograph without such observed data. Rodriguez-Iturbe and Valdes (1979) presented a method for determining the hydrologic response using the Strahler ordering procedure and Horton’s laws of stream numbers (R_B), stream lengths (R_L) and stream areas (R_A), together with a scale variable L_\lambda, and a dynamic parameter, v. In a companion paper, Valdes et al (1979) compared the geomorphologic IUH for a series of real basins with a physically based rainfall-runoff model of the same basins. The geomorphologic IUH’s generated were found to be remarkably similar in all basins analyzed. These two represent the foundation of the GIUH concept and paved the way for a plethora of subsequent research.

the concept of holding time, the time that any particle instantaneously injected into the basin remains in the basin. Each particle injected into the basin has an independent and identically distributed holding time because the water particles are assumed to be non-interacting and identical. Based on this concept, it follows that the IUH of the basin is equal to the probability distribution function (PDF) of the holding time. Gupta et al (1980) showed that the PDF of the basin holding time (i.e. IUH) is obtained by first determining the probability that a particle follows a certain path, amongst all other paths, to get to the outlet, multiplying it with the PDF of the random holding time for this path and then summing these products over all possible paths. This was the basic idea on which the probabilistic approach presented by Gupta et al (1980) was obtained. In their formulation, which compared favorably with field data for three Illinois river basins, the PDF of the holding times was assumed to have an exponential distribution. This assumption allowed for an analogy to be drawn between the GIUH approach and the classical approach of representing a basin in terms of linear storage elements and/or channels in series or parallel.

Rodriguez-Iturbe et al (1982) and Wang et al (1981) considered the assumption that channels of a given order have known linear response functions which are exponential distributions of time. Both studies looked at alternative parameterizations of the channel linear response functions, in particular the velocity terms in the exponential distribution adopted by Rodriguez-Iturbe and Valdes (1979). Kirshen and Bras (1983) also sought an alternative, deriving the response of individual channels by solving the continuity and momentum equations for the boundary conditions defined by the IUH. This formulation allows for the effects of both upstream and lateral inflow to the channels to be taken into account in the derivation of the basin’s IUH.

Since these initial pioneering papers, much debate has ensued on the best travel time distribution to use. Agnese et al. (1988) developed a travel-time formula from experimental data, Cheng (1982) combined the exponential and uniform distributions to generate the travel time PDF, while Jin (1992) suggested that the gamma distribution yields better results than the traditional exponential distribution. Lee and Yen (1997) refined the GIUH method by considering the travel time probabilistically and computing it hydraulically using the kinematic-wave approximation, avoiding the use
of empirical velocity equations. Lee and Yen (1997) highlighted that travel time depends on flow velocity, which is variable in both space and time along the channel network, and this variation needs to be accounted for. In particular, the authors were interested in extending the application of the GIUH concept into ungauged catchments. Yen and Lee (1997) achieved this by utilizing their kinematic-wave based GIUH scheme with Horton’s stream order laws to derive unit hydrographs for ungauged basins. Furthermore, Yen and Lee (1997) explored the impact of having different levels of geomorphic data. Level 1 required only the area and order of the project watershed to be known, with other parameters such as Horton’s ratios, $R_A$, $R_L$, and $R_B$ being assumed from data acquired from large region-averaged values. At the other end of the spectrum, Level 5 required that all input data required to drive the model be available for the project watershed. Yen and Lee (1997) tested all five model levels on four different catchments in the United States, showing that each model level generated hydrographs in good agreement with recorded hydrographs.

Closely related to the debate over which travel time distribution to use are a number of studies (Rinaldo et al, 1991, Snell and Sivapalan, 1994, Saco and Kumar, 2004) that have attempted to identify the key mechanisms contributing to the variance of travel times. Rinaldo et al (1991) identified two primary mechanisms contributing to the variance of travel times:

1. Hydrodynamic dispersion: dispersion introduced along the individual paths by hydrodynamic effects. If all paths had the same length this would constitute the only mechanism contributing to the variance since the rate of arrivals through different paths would coincide.

2. Geomorphologic dispersion: dispersion due to the heterogeneity of path lengths in the stream network which produces the spread in the arrival rates.

Saco and Kumar (2002) identified a third dispersion effect, namely kinematic dispersion, a mechanism that introduces spread in the travel time distribution due to spatially varying celerity. Saco and Kumar (2002) showed that kinematic dispersion’s contribution to the overall dispersion is comparable to geomorphologic dispersion and
significantly larger than hydrodynamic dispersion in natural watersheds. In order to account for kinematic dispersion, Saco and Kumar (2002) utilized Stall and Fork’s (1968) hydraulic geometry relations to allow for spatially varying celerity. The work performed by Rinaldo et al (1991) and Saco and Kumar (2002) allow the travel time distribution to be formulated from the advection-dispersion equation using kinematic wave celerity and relevant dispersion coefficients rather than computing the mean travel time explicitly. These formulations provide an alternative to the traditional travel distribution distributions based on a meant travel time and have the potential to be extended to account for additional mechanisms.

In recent years, literature on GIUH has extended beyond discussion of what the best travel time distribution to use is. A number of other studies have been conducted that have built on the original framework of Rodriguez-Iturbe and Valdes (1979). Lee and Chang (2005) extended the GIUH concept to account for the sub-surface flow mechanism. Previous GIUH models had only considered travel time associated with Hortonian overland flow and channel flow. Lee and Chang (2005) found that the recession limb of the simulated hydrograph could be improved by significantly because of the dominance of subsurface flow in flood recession. Kumar et al (2007) used the Clark (1945) and Nash (1957) models to simulate direct surface runoff and incorporated them into the GIUH framework. Kumar et al (2007) were successful in applying the GIUH based Clark and Nash models to a number of ungauged catchments in the Ajay catchment in southern India. Bhadra et al (2008) also introduced an infiltration component to the GIUH formulation, exploring three methods for calculating infiltration: Richards’ equation, Philip two term model and phi-index method. Lee et al (2008) considered the effects of variation in rainfall intensity during storms. Lee et al (2008) found that for concentrated storm events the introduction of a time delay for the selection of rainfall intensity could be used to avoid an overestimation of the peak discharge in the earlier kinematic wave GIUH formulation by Lee and Yen (1997).

2.5 GIUH in Urban Catchments

Since its original development, the GIUH framework has been applied extensively to natural watersheds. Little research, however, has been conducted on the
application of GIUH in urban catchments. Rodriguez et al (2003) developed a method for simulating runoff in urban catchments using Urban DataBanks (UDB) and the GIUH framework. Urban DataBanks are GIS geodatabases that contain a variety of layers, including cadastral parcels, building footprints, street sections, sewer systems, point elevations and rivers. Their method was used to derive Urban Unit Hydrographs (URBS-UHs) for three urban catchments in France with surface areas ranging from 18 to 180 ha. In their formulation Rodriguez et al (2003) made a number of key assumptions:

- Water flows over impervious surfaces and in sewer networks dominate the rapid response of urban catchments. In this way only impervious areas are modeled, Hortonian overland runoff from pervious areas is ignored.

- UDB’s contain enough geometric information to allow the path for the propagation of runoff to be identified and modeled. Surface runoff is assumed to initially collect on building roofs and other paved surfaces such as paring lots or streets, before becoming concentrated flow in gutters and underground pipes, and finally flowing into natural channels.

- The flow velocity field is a spatial variable depending on the considered flow conditions and available UDB information.

- Flow can be considered a monotonic response to a rain impulse.

- Downstream segments can reasonably be considered to accept the outflow of upstream segments.

- The entire area is assumed to be covered by cadastral parcels and street surfaces. Flow is parameterized over each parcel.

- Street flooding and sewage system overflow is not allowed.

- Each subcatchment can be represented by a hydrologic element (HE), composed of a cadastral parcel and its corresponding street portion, connected to a runoff branching structure (RBS), a vector representing the flow path along the street gutter and inside the sewer network.
• The flow distance for surface runoff is assumed to be the orthogonal projection of the center of gravity of the HE to axis of the center of the adjacent street segment.

• Street gutter flow is supposed to reach a priori the sewer system at each street network intersection and the hydraulic behavior of inlets has been ignored.

• Street gutters are modeled as small-sized pipes (250 mm diameter).

• Hydraulic dispersion along the flow path is ignored.

• Flow velocity along a given RBS segment is determined from Manning’s equation and flow velocity over HE’s was assumed to be a constant 0.5 m/s.

• Rainfall intensity over the catchment was derived using the time of concentration of the catchment using an iterative approach. This rainfall intensity is combined with a runoff coefficient, assumed to be the impervious fraction, and catchment area, and the rational formula is used to generate the flow in each segment.

A number of the assumptions made by Rodriguez et al (2003) make this approach difficult to apply to urban catchments to be simulated in this research. Firstly, the approach presented by Rodriguez et al (2003) is highly dependent on the quality and quantity of the information in the UDB’s. As previously mentioned this type of information is simply not available in many urban catchments. Secondly, Rodriguez et al (2003) ignore runoff from pervious areas. Runoff from pervious areas particularly for wet antecedent moisture conditions can be significant in shaping the surface runoff hydrograph.

Rodriguez et al (2005a) compared the method they developed in 2003 with two other methods usually applied in natural catchments (H2U and FDTF). The three unit hydrograph determination methods make use of varying degrees of the description of drainage network morphology and their combined application requires rainfall and runoff data collected from hydrological recording devices.
Rodriguez et al (2005b, 2008) built further on their distributed hydrological model based on UDB’s by considering additional hydrologic processes, including infiltration, evapotranspiration, interception by vegetation and groundwater intrusion into sewers. Rodriguez et al (2005b, 2008) postulate that the hydrological influence of the soil layer, in particular processes such as evapotranspiration and drainage of soil water through the sewer network, is a significant component of the urban water budget. They highlight that such processes are becoming increasingly important as best management practices, promoting rainwater infiltration, continue to be implemented. Rodriguez et al (2005b, 2008) model interception by trees using the model of Calder (1977), evaporation is based on recorded values of potential evapotranspiration using the Rutter et al (1971) formulation, sewer infiltration flux is modeled considering the pipe as an ideal drain (Cassan, 1986, and Gustafsson et al, 1996), and the influence of soil structure (incorporating the vadose and saturated zones) is modeled based on fundamental of the Top Model approach of Beven and Kirkby (1979). Simulations using the model were performed on small and homogenous urban catchment and a medium-sized heterogeneous urban catchment, and highlighted the importance of urban soil and soil-atmosphere interaction in urban catchments when modeling is performed over long periods, such as years or seasons.

Most recently, Gironas et al (2009) presented a morpho-climatic instantaneous unit hydrograph model for urban catchments based on the kinematic wave approximation. This work builds on the earlier work of Rodriguez et al (2003) and defines an individual flow path for each location in the catchment using detailed digital elevation models of the catchment. Gironas et al (2009) suggest that this explicit approach is more appropriate than an implicit approach (using Horton’s laws to characterize flow paths) in urban catchments because of the complexity of the flow path, possible deviation from Horton’s laws and increasing availability of drainage system data. We contend that the explicit approach is not necessarily the more appropriate approach for urban catchments. We agree that the flow path in an urban catchment is complex, but this does not mean the implicit approach can’t be used to represent the catchment. Using the implicit approach different components of the flow path can be represented using different states without the need for input information for every flow
path in the system. Secondly, Gironas et al (2009) do not provide any evidence to suggest that Horton’s laws can’t be adapted to urban catchments; such a conclusion would require significant testing. The first step towards this testing is presented in this paper. Finally, in relation to drainage system data, we would contest that level of detail required to employ an explicit approach is still widely unavailable in the United States. In a survey of over 50 municipalities in the greater Chicago area it was found that only two municipalities have a digital database of their sewer infrastructure. For older systems (combined sewer), that may be in excess of 100 years old, the data required to define each flow path simply are not available. If the data were available one might pose the question why not build a deterministic model?

Lhomme et al (2004) applied a GIS-based geomorphological routing model to the urban catchment El Batan in the city of Quito, Ecuador. Despite being described as an urban catchment the area of the catchment is some 52 km² and a significant portion of the network is made up of natural streams. The formulation presented by the authors of this paper is reliant on a detailed GIS database of input information and the model is calibrated using hydrologic rainfall and runoff data.

Few researchers have formulated models utilizing morphologic approaches for developing unit hydrographs in urban areas. The model developed by Rodriguez et al (2003) is heavily reliant on detailed catchment description provided through UDB’s and utilizes a number of simplifying of assumptions that distinguish it from the approach proposed herein. Similarly, the later work by Rodriguez et al (2005a, 2005b and 2008) and Lhomme et al (2004) present approaches that incorporate GIUH but are dependent on rainfall and runoff data and/or detailed catchment description. The approach presented in this paper is not dependant on detailed catchment parameters nor does it require the catchment to be gauged.

2.6 Uncertainty

One way to consider the accuracy of a model is by assessing its uncertainty. A model is a simplification of reality; it cannot describe all the relevant variables of the process precisely making model output uncertainty inevitable (Lei and Schilling, 1994). The overall uncertainty of a model is inherently dependant on the uncertainty of the
inputs, but may also be affected by uncertainty in model structure, model parameters and undetected numerical errors.

One of the most common shortcomings of a hydrologic/hydraulic model is the inability to collect all of the required input information. Without conducting a detailed field survey of the catchment it is very difficult to obtain input characteristics such as soil properties, connectedness, invert and inlet information and percentage imperviousness. Any uncertainty in these parameters will affect the overall accuracy of the model and its ability to portray the hydrologic/hydraulic behavior of the real system.

Schilling (1984) conducted a comparative study assessing uncertainties in runoff computations due to spatial resolution of rainfall input, loss assumptions, surface flow models and routing models. It was found that spatial resolution of rain data was the most limiting restriction to the reliability of the model, while the use of a less time consuming routing method such as the Kinematic Wave model is still reliable. This may be true in natural watersheds but the same may not necessarily be concluded in urban catchments where backwater effects are important (Mays, 2001).

Arnbjerg-Nielsen and Harremoes (1996) assessed the relative importance of individual contributions to the overall uncertainty of an urban storm drainage model using Monte-Carlo simulations. Based on an analysis of two small ungauged catchments, they found that the uncertainty in the description of the rainfall was generally the most important contributor to the overall uncertainty, followed by the uncertainty in the description of surface runoff and then uncertainty in the hydraulic resistance of the sewer system.

Wagener et al (2002) postulates that it may often be necessary to trade off performance and uncertainty to derive a suitable level of model complexity for the anticipated purpose. This is because the information content in the data is limited and potentially supports only a certain number of parameters to be identified (Young, 2001).

One of the most relevant studies found on uncertainty of hydrologic models was the one conducted by Melching et al (1987), in their research report entitled the “Incorporation of uncertainties in real-time catchment flood forecasting.” In their
discussion, Melching et al (1987) evaluated the effects of uncertainties in data, model parameters and model structure. Data uncertainties were described for four sources: rainfall, streamflow, evapotranspiration, and watershed morphology. Melching et al (1987) describe the sources of the uncertainties in this data and provide methods for handling such uncertainty. The authors found that there are eight primary sources of uncertainty for rainfall data (e.g. spatial and temporal variability), three for streamflow data (e.g. uncertainty in stage-discharge relation, uncertainty effects in evapotranspiration are negligible and that various aspects of watershed morphology (e.g. total area, impervious area, channel slope and length etc.) can introduce uncertainty.

In terms of parameter uncertainties, Melching et al (1987) distinguished between two types of unknown parameters, those that are fixed but unknown, and those that vary from rainfall event to rainfall event. Only the latter, were focused on, and were considered in relation to calibrated models, physical simulation models and forecast updating. Uncertainties in model structure stem from the model’s inability to truly represent the watershed’s physical runoff processes (Melching et al, 1987). The uncertainties introduced in model structure, through its routing and runoff and infiltration procedures, may affect the shape, volume and magnitude and timing of the peak of the predicted hydrograph, making it difficult to consider and account for such effects. Model structure uncertainty is most aptly assessed by the calibrating the model using measured flow data.

Wang and Tung (2006) highlighted that the Kinematic Wave GIUH developed by Lee and Yen (1997) is unavoidably subject to uncertainty, which results in uncertainty in the design flow hydrograph. The uncertainty is introduced in the derivation of rain drop travel times that are derived as functions of slope, roughness and channel geometry, which are all subject to uncertainty due to their spatial variability. Wang and Tung (2006) employ the modified Harr probabilistic point estimation (PPE) method (Chang et al, 1995), along with normal transform techniques, to quantify the uncertainty of the GIUH based flow hydrograph of specified rainfall events for a hypothetical watershed. Using the statistical features of the design flow hydrographs are used to stochastically generate flow hydrographs by the Gaussian linearly constrained simulation (Borgman

A number of methods exist for tracking uncertainty in multivariate problems (e.g. calculation of travel time). Harr (1996) splits these methods into three categories:

1. **Exact methods**: require that probability distribution functions of all component variables be known initially. Examples are numerical integration and Monte Carlo simulation. These methods have the advantage that you obtain a complete probability distribution of the random variable are disadvantaged by the considerable computer time that is necessary and the risk that the output may be no better than the input.

2. **First-Order Second-Moment (FOSM) methods**: the basis of these methods is the truncation of the Taylor series expansion of the functions. Inputs and outputs are simply expressed as expected values and standard deviations, meaning that knowledge of higher moments and computer power are not necessary.

3. **Point estimate methods (PEM)**: these methods assess model output uncertainty in terms of statistical moments by evaluating model output values at specified points with the parameter sample space. The points are selected to preserve finite statistical moments of random model parameters (Wang and Tung, 2009). These methods have been the subject of much research (Rosenblueth, 1975, Harr, 1989, Chang et al, 1995, Wang and Tung, 2009) and are quickly becoming the preferred method.

### 2.7 References


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Chapter 3: Understanding Urban Catchments

The research presented below has been adapted from the paper entitled; “Improved understanding and prediction of the hydrologic response of highly urbanized catchments through development of the Illinois Urban Hydrologic Model (IUHM).” This article, authored by Joshua Cantone and Arthur Schmidt, was submitted to the internationally renowned Water Resources Research journal on 17th March 2010 following review by the doctoral committee.

This paper is the pioneering paper of this research and introduces the Illinois Urban Hydrologic Model. It presents, in detail, the methodology used in developing the model and validates its ability to accurately predict the hydrologic response in highly urbanized catchments. The inaugural version of IUHM presented in this paper provided the platform for the dissertation research described in Chapters 4 and 5, and will be the basis for further development of the model in the future (see Chapter 6).

3.1 Abstract

What happens to the rain in highly urbanized catchments? That is the question that urban hydrologists must ask themselves when trying to integrate the hydrologic and hydraulic processes that affect the hydrologic response of urban catchments. The Illinois Urban Hydrologic Model (IUHM) has been developed to help answer this question and improve understanding and prediction of hydrologic response in highly urbanized catchments. Urban catchments are significantly different than natural watersheds but there are similarities that allow features of the pioneering GIUH concept developed for natural watersheds to be adapted to the urban setting. This probabilistically based approach is a marked departure from the traditional deterministic models used to design and simulate urban sewer systems, and does not have the burdensome input data requirements that detailed deterministic models possess. Application of IUHM to the CDS-51 catchment located in the Village of Dolton, IL highlights the models ability to predict the hydrologic response of the catchment as well as the widely accepted SWMM model and in accordance with observed data recorded by the USGS. The model is further used to improve the understanding of
urban catchment hydrology. It is shown that inlet storage and pressurized flow can have a significant impact on the hydrologic response in urban catchments. In addition, the unique structure and organization of urban sewer networks make it possible to characterize Horton’s Laws in urban catchments. Overall, the results provide invaluable insight into how the different hydrologic/hydraulic processes encountered in urban catchments effect the hydrologic response of the catchment.

3.2 Introduction

Urbanized catchments have changed complexion significantly over the past few decades and will continue to do so into the future. At the simplest level, urban consolidation and urban sprawl have seen the imperviousness of urban catchments increase. More complex landscapes are being developed to incorporate low impact development and include stormwater best management practices. Sewer systems built in the 20th century often have insufficient capacity to carry post-development flows and are constantly being retrofitted or replaced. Combined sewer overflow (CSO) systems are seldom built in preference to separated stormwater and sanitary sewer systems. Existing CSO systems that cannot be easily replaced are being intercepted by complex deeper tunnel systems in order to prevent CSO overflows to adjacent rivers and waterways in large storm events. All of these changes are unique in their own way but each introduces man-made heterogeneity into the catchment. Consequently these changes impact the behavior of water as it travels through the urban landscape. The question is: how does one represent the intricate physical processes that are associated with modern urban landscapes as a means of better understanding those processes?

Penman (1961) defined hydrology as the science that attempts to answer the question, “what happens to the rain”? Although this sounds like a simple question, as Singh and Woolhisser (2002) explain, experience has shown quantitative description of the land phase of the hydrologic cycle may be complicated and uncertain. Sivapalan (2003) explains that difficulty in predicting the hydrologic response of a basin may be attributed to heterogeneity of the land surface, soils, vegetation, land use, etc. and variability in inputs over the scales of time and space. Singh and Woolhisser (2002) defined the term “watershed hydrology” as that branch of hydrology that deals with the
integration of hydrologic processes at the watershed scale to determine watershed response. Similarly, let us define “urban hydrology” as that branch of hydrology that deals with the integration of hydrologic and hydraulic processes at the urban scale to determine catchment response.

Singh and Woolhiser (2002) compiled a list of uses for watershed hydrologic models. A similar list can be compiled for urban catchment models like the one presented in this research. Urban catchment hydrologic models may be used in the planning, design and operation of stormwater, wastewater and CSO network projects. Such models may also be used for the assessment, development and management of stormwater, wastewater and CSO networks. Current practices to characterize urban systems are predominantly based on concepts and tools developed for natural (undeveloped) watersheds. These concepts implicitly linearized the response and smooth the discontinuities and thresholds. Use of these often inadequate tools is largely because urban sewer systems have long been treated from a design perspective, in which implicit factors of safety and uncertainty from the design event concept mask errors from our lack of fundamental understanding of the processes controlling these systems. While the existing approaches provide adequate solutions for traditional stormwater management, they are sorely lacking in providing the understanding necessary for holistic approaches to urban stormwater management and sustainable redevelopment of urban systems. This chapter focuses on improving our understanding of the complex hydrologic and hydraulic processes of urban sewer systems.

In distinguishing “watershed” and “urban catchment” hydrologic modeling, it is imperative to consider a number of the key scale issues that were identified by Blöschl and Sivapalan (1995). The key differences between “watershed” and “urban catchment” hydrologic models lie within their relative scales of time and space. While natural watersheds can have time scales ranging from minutes to years, urban catchments typically have time scales in the range of minutes to hours. Similarly, urban catchments typically encompass a smaller range of space scales, generally of the order of meters or kilometers rather than tens to hundreds of kilometers. Given these differences in the time and space scales it is recognizable that urban catchments are effectively a portion of larger natural watersheds; the key physical distinction being in land surface
characteristics. Urban catchments tend to be highly impervious, with the predominant land uses being residential, commercial and industrial. In contrast natural watersheds tend to be highly pervious, with the land use dominated by meadows, forest, pasture, crop land, and other agricultural land cover. As a result of the greater portion of impervious area the travel times in urban catchments are often significantly less than in natural watersheds.

Bloschl and Sivapalan (1995, Figure 2) painted a picture depicting the hydrologic processes that are important at different time and space scales. A vast number of hydrologic processes affect the path that a drop of rainfall takes in an urban catchment. Figure 3.1 summarizes the hydrologic processes that can occur in a typical urban hydrologic system like the one existing in Chicago, IL. Each of these hydrologic processes has its own complexity and has been the subject of investigation over the past few decades. Given the time and length scales observed in urban catchments it can be identified that precipitation, infiltration excess overland flow and channel flow are the most important hydrologic processes at the urban scale. With this in mind, precipitation, overland flow, infiltration, depression storage, surface runoff and the combined sewer system were identified as the key hydrologic processes that must be understood and simulated in highly urbanized catchments.

Heterogeneity and variability in space and time are scale issues that plague all those performing hydrologic modeling. Consider the heterogeneity of precipitation which arguably exhibits discontinuity, periodicity and randomness. Bloschl and Sivapalan (1995) highlight that the intermittency of rainfall events make it a discontinuity, however, its diurnal and annual variations make it predictably periodic and statistical analysis of historical records allow its randomness to be predicted. Can the same conclusions be drawn for other catchment and hydrological processes? Natural watersheds have been shown to exhibit organization and in turn this organization has been used by authors to develop methods for predicting hydrologic response. For example, Horton’s laws of stream order, length, area and slope for natural catchments have been used extensively in simplifying the prediction of hydrologic response (see Yen and Lee, 1997). One of the questions in this research is do these laws hold up in the urban setting? Can a similar set of rules be developed? Is there a way of using the
network layout of a CSO network to predict the outfall hydrograph? These are some of the questions that we seek to answer in this chapter.

Figure 3.1 Flow chart describing the hydrologic and hydraulic processes that can be involved in urban catchments. This is based on the possible travel paths for a drop of water falling in a combined sewer area Chicago, IL. The processes highlighted (precipitation, overland flow, infiltration, depression storage, surface runoff, inlet/catch basin and combined sewer) were identified as being the most important at the urban scale and hence incorporated into IUHM.

The rest of this chapter is organized as follows. Section 3.3 summarizes the models currently available for modeling urban catchments. The Illinois Urban Hydrologic Model and its methodology are introduced in Section 3.4 along with its governing equations. Section 3.5 describes the CDS-51 catchment that is studied in this paper and results from its simulation are presented in Section 3.6. In Section 3.7 we use
IUHM to help better understand some of the intricacies in urban catchments, before a number of key conclusions are drawn in Section 3.8.

3.3 Existing Urban Hydrologic Models

Since the advent of computer technology, representation of urban catchments has fallen on computer models. Representation has evolved from simple back-of-the-envelope rational-method (Chow, 1962) calculations, to more sophisticated DOS based models like ILLUDAS (Terstriep and Stall, 1974), to complex Windows based models capable of solving the full St. Venant equations like InfoSWMM (MWHSoft, 2004) to coupled surface/subsurface models such as Mike-SHE (Abbott et al, 1986a and 1986b). There are now a plethora of widely used hydrologic simulation packages that are used to design and simulate urban stormwater and CSO systems. Over the past 30 years a number of authors have conducted studies comparing and classifying models based on a variety of criteria.

Singh (1995) postulated that hydrologic models could be classified considering six criteria: (1) process description, (2) time scale, (3) space scale, (4) techniques of solution, (5) land use, and (6) model use. Using these criteria Singh and Woolhiser (2002) conducted an extensive review of in excess of 50 existing watershed models. In 1985, Messner and Goyen compared the first stream of urban stormwater models which included ILLUDAS, SWMM (Huber et al., 1981), WASSP (U.K. National Water Council, 1981) and RATHGL (Goyen, 1980). Mays (1999) conducted an extensive review of the most commonly used public domain urban hydrologic models for stormwater design, which include: BASINS( USEPA 1998), FEQ (Franz and Melching 1997), HEC HMS (US Army Corps of Engineers, 1989), SWMM, TR-20 (NRCS, 1992), and TR-55 (NRCS, 1986). Agbodo (2005) compared seven of the most recently developed simulation packages, namely: HYDRA (Pizer, 1973), SewerCAD (Haestad Methods, 1999), InfoSWMM, InfoWorks (Wallingford, 2002), MOUSE (DHI, 2003), XP-SWMM (XP-Software, 1992) and SewerGEMS (Haestad Methods, 2004). These simulation packages represent a stream of models that have been adapted to include state-of-the-art data management and GIS technologies.
Although the models identified above are capable of modeling large urban systems, three key factors hinder their use. The first is that each of the models mentioned requires calibration, and for the most part adequate calibration data are not available. Secondly, the large amount of input data required for these models are often unavailable, and are uncertain when they are available. Finally, all of these models are deterministic. We have already established that there is a high degree of non-linearity and heterogeneity in urban systems. Simplifying assumptions commonly used with traditional deterministic approaches to make simulation of urban watersheds tractable linearized the response and smooth out discontinuities. This undesirable consequence of using existing models is discussed in more detail below.

In order to overcome the second problem, simplifying assumptions are typically made to allow the commercially available simulation packages to be used. Cantone and Schmidt (2009) explored a number of the commonly employed simplification techniques, including subcatchment aggregation and conduit skeletonization. Tests on a simple and a complex CSO network showed that such simplification techniques introduce error in the calculated outflow hydrograph and hence caution must be used in applying them.

Given the pitfalls of the currently available deterministic models and dangers associated with over simplifying such models, alternative stochastic modeling techniques were investigated. Yen and Lee (1997) showed that the geomorphologic instantaneous unit hydrograph (GIUH) approach, originally developed by Rodriguez-Iturbe and Valdes (1979), could be utilized to determine the hydrologic response of ungauged natural watersheds with limited or uncertain input data. The GIUH framework developed by Rodriguez-Iturbe and Valdes (1979) is a probabilistic framework that postulates that the hydrologic response of a watershed is intimately related to the layout of the stream network and the geomorphologic parameters of a basin. The GIUH approach has been the subject of a significant volume of research (see Gupta et al 1980, Kirschen and Bras 1983, Lee and Yen 1997 etc) over the past three decades looking at predicting the hydrologic response in natural watersheds, but has undergone little investigation in the urban setting.
Rodriguez et al (2003, 2005 and 2008) attempted to apply the GIUH concept in urban catchments using an approach that is highly dependent on the quality and quantity of information contained in Urban DataBanks (UDB). UDB’s are GIS geodatabases that contain a variety of layers, including cadastral parcels, building footprints, street cross-sections, sewer systems, point elevations and rivers. Such information is not always as readily available in the United States as it is in the European watersheds explored by Rodriguez et al (2003).

Most recently, Gironas et al (2009) presented a morpho-climatic instantaneous unit hydrograph model for urban catchments based on the kinematic wave approximation. This work builds on the earlier work of Rodriguez et al (2003) and defines an individual flow path for each location in the catchment using detailed digital elevation models of the catchment. Gironas et al (2009) suggest that this explicit approach is more appropriate than an implicit approach (using Horton’s laws to characterize flow paths) in urban catchments because of the complexity of the flow path, possible deviation from Horton’s laws and increasing availability of drainage system data. We contend that the explicit approach is not necessarily the more appropriate approach for urban catchments. We agree that the flow path in an urban catchment is complex, but this does not mean the implicit approach can’t be used to represent the catchment. Using the implicit approach different components of the flow path can be represented using different states without the need for input information for every flow path in the system. Secondly, Gironas et al (2009) do not provide any evidence to suggest that Horton’s laws can’t be adapted to urban catchments; such a conclusion would require significant testing. The first step towards this testing is presented in this chapter. Finally, in relation to drainage system data, we would contest that level of detail required to employ an explicit approach is still widely unavailable in the United States. In a survey of over 50 municipalities in the greater Chicago area it was found that only two municipalities have a digital database of their sewer infrastructure. For older systems (combined sewer), that may be in excess of 100 years old, the data required to define each flow path simply are not available. If the data were available one might pose the question why not build a deterministic model?
In this research we show that the framework introduced by Rodriguez-Iturbe and Valdes (1979) can be used as a basis for determining hydrologic response in large urban catchments. The framework has been necessarily altered to allow for simulation of the key hydrologic processes in the urban setting, including pervious and impervious overland flow and closed-conduit flow.

3.4 Illinois Urban Hydrologic Model (IUHM)

3.4.1 State Space Model

The Illinois Urban Hydrologic Model (IUHM) builds on the initial GIUH concept developed by Rodriguez-Iturbe and Valdes (1979). Using the Strahler ordering scheme (Strahler, 1957), an urban catchment of order $\Omega$ can be divided into different states. Each conduit in the sewer system is represented by a conduit state, $x_c$, and the overland flow region feeding a conduit is denoted an overland state, $x_o$, where $i = 1, 2, ..., \Omega$. The overland flow and conduit states allow simulation of infiltration excess overland flow and channel flow, which were earlier identified as the most important hydrologic processes at the urban scale of interest. Each raindrop falling on an overland region within the watershed is assumed to move successively from lower order to higher order conduits until it reaches the outlet.

In the traditional GIUH approach applied to natural watersheds, excess rainfall was used to generate the hydrologic response. IUHM allows user-defined rainfall intensity to drive the hydrologic response by sub-dividing the overland region into pervious and impervious regions. This division doubles the traditional number of possible paths in a watershed. This unique representation of the urban watershed allows for a finite number of possible flow paths to be modeled according to the probability, $P(w)$, of a drop of rainfall adopting a path $w$. This finite number of paths, $2^\Omega$, is significantly less than the number of paths that must be modeled using a deterministic approach. If $w$ denotes a specified path

$$x_{oi,perv} \rightarrow x_{oi,imp} \rightarrow x_{ci} \rightarrow x_{cj} \rightarrow \cdots \rightarrow x_{c\Omega} \rightarrow \text{outlet}$$
then the probability of a raindrop following that path is the probability, $P_{OA_i}$, of starting out in the initial overland state times the probabilities of making successive transitions to conduits of higher order along the path:

\[
    P(w) = P_{OA_i} \cdot P_{x_i,\text{per}} \cdot P_{x_i,\text{per},x_i,\text{imp}} \cdot P_{x_i,\text{imp},x_i} \cdot P_{x_i,\text{imp},x_i} \cdots P_{x_i,\text{imp},x_i}\quad \text{for odd paths}
\]

\[
    P(w) = P_{OA_i} \cdot P_{x_i,\text{imp}} \cdot P_{x_i,\text{imp},x_i} \cdot P_{x_i,\text{imp},x_i} \cdots P_{x_i,\text{imp},x_i}\quad \text{for even paths}
\]

(3.1)

where $i = 1, 2, \ldots, \Omega$ and $i \leq j \leq k \leq \Omega$, $P_{OA_i}$ is probability that a drop of excess rainfall will fall on an $i$th-order overland region (initial state probability), $P_{x_i,\text{per}}$ is the probability that a drop of excess rainfall will fall on a pervious region and is equal to the perviousness of the catchment, $P_{x_i,\text{imp}}$ is the probability that a drop of excess rainfall will fall on an impervious region and is equal to the imperviousness of the catchment, $P_{x_i,\text{per},x_i,\text{imp}}$ is the transitional probability of a raindrop traveling from a pervious region to an impervious region, which is equal to unity in the current version of the model, $P_{x_i,\text{imp},x_j}$ is the transition probability of a raindrop moving from an $i$th-order impervious overland region to an $i$th-order conduit and is equal to unity, and $P_{x_i,\text{imp},x_j}$ is the transition probability of a raindrop moving from an $i$th-order conduit to a $j$th-order conduit.

Note that in urban catchments there is a significant probability that a raindrop will transition from an $i$th-order conduit to another $i$th-order conduit. To account for the possibility of transitions within a given order would result in a significant increase in the total number of possible flow paths. Furthermore, convolution of a large number of paths with small probability could introduce numerical errors from rounding of small values. A Monte-Carlo simulation was performed on a 5th order catchment in Village of Dolton, IL (see Section 3.5 for details), allowing transitions to the same pipe order and it was found that the number of possible of flow paths exceeded 50,000. To account for such a large number of flow paths within the model would be intractable and take away from the intrinsic simplicity of the underlying concept. As such the distribution of possible successive flow paths within an order $i$ was used to generate a scaling factor, $a_i$, for the travel time within an $i$th-order conduit. This scaling factor represents the average
number of successive $i$th-order conduits that at drop of rainfall would flow through and is given by:

$$a_i = \sum_{n=1}^{N} nP_{x, a}$$  \hspace{1cm} (3.2)

where $n = 1, 2, ..., N$ is the number of successive $i$th-order conduits, and $P_{x, a}$ is the probability of a drop water travelling through $n$ successive $i$th-order conduits.

The total travel time of a raindrop of intensity, $i(t)$, moving through path $w$ to the watershed outlet, $\bar{T}_w(t)$, is given by:

$$\bar{T}_w(t) = \bar{T}_{x_{i,perv}}(t) + \bar{T}_{x_{i, pervimp}}(t) + a_i \bar{T}_{x_{i}}(t) + a_j \bar{T}_{x_{j}}(t) + ... + a_\Omega \bar{T}_{x_{\Omega}}(t)$$  \hspace{1cm} (3.3)

where $a_i, a_j, ..., a_\Omega$ represent the scaling factors accounting for transitions with a given order.

Unlike deterministic modeling, where it is necessary to find the component travel times for every path in the catchment, the GIUH formulation estimates the travel time collectively and probabilistically for each state by assuming the travel times of different states in the watershed are statistically independent and $f_{x_k}(t')$ is the travel time probability-density-function in state $x_k$, with a mean value $\bar{T}_{x_k}$ (Yen and Lee, 1997).

Using these assumptions Equations (3.1) and (3.3) can be combined to generate the network impulse response function of the catchment:

$$u(t', t) = \sum_{w \in W} \left[ f_{x_{i,perv}}(t') * f_{x_{i, pervimp}}(t') * f_{x_{i, imp}}(t') * f_{x_{j}}(t') * f_{x_{j}}(t') * \right. \left. ... * f_{x_{\Omega}}(t') \right] \cdot P(w)$$  \hspace{1cm} (3.4)

where $*$ denotes a convolution integral, and $w \in W$, $W$ being the path space $W = \{x_{oi,perv}, x_{oi, pervimp}, x_i, x_j, ..., x_\Omega\}$, ($i = 1, 2, ..., \Omega$). Equation (3.4) represents a network impulse response function rather than an instantaneous unit hydrograph because the travel time distribution is derived based on an intensity of rainfall rather than a unit of rainfall. This is an important departure from the traditional GIUH approach.
The direct runoff hydrograph for the catchment, $Q(t)$, is then given by

$$Q(t) = \sum_{j=1}^{\infty} \left[ u(t',t) \cdot q_{L_w}(t) \right] \cdot A$$  \hspace{1cm} (3.5)

where $A$ is the total area of the catchment, and $q_{L_w}(t)$ is the equivalent pervious excess rainfall, $q_{L_w,\text{perv}}(t)$, for the odd paths, i.e. $w = 1, 3, ..., (\Omega - 1)$, and equal to the equivalent impervious excess rainfall, $q_{L_w,\text{imperv}}(t)$, for even paths, i.e. $w = 2, 4, ..., \Omega$.

Many different forms of the travel time probability density function, $f_{t_x}(t')$, have been explored over the years by researchers (see Agnese et al., 1988, Cheng, 1982, Rinaldo et al., 1991, Jin, 1992). The two simplest, most commonly adopted distributions are the exponential and uniform distributions. In this inaugural version of IUHM, the travel time for both overland flow and conduit flow regions was assumed to follow an exponential distribution with a mean travel time, $\overline{T}_{t_x}$. In this way the catchment is conceptualized as linear reservoirs in series and/or parallel (Gupta et al., 1980), and is written:

$$f_{t_x}(t') = \frac{1}{\overline{T}_{t_x}(t)} \exp \left( \frac{-t'}{\overline{T}_{t_x}(t)} \right) \forall t'$$  \hspace{1cm} (3.6)

where $t$ relates to the rainfall intensity and $t'$ is the time associated with the travel time PDF.

The exponential distribution in Equation (3.6) has been chosen as the initial PDF for the travel time because of its simplicity and wide application in the literature. Using this distribution the convolution integral within Equation (3.4) can be easily solved by computing the Laplace transform for each travel time PDF (see Cheng, 1982 for details). As part of the ongoing development of the model, other PDF’s for the travel time have been explored. Such investigations have been performed and are the subject of the companion paper (Cantone and Schmidt, 2010, see chapter 4 of this thesis).
3.4.2 Travel Time Formulation

From the above formulation it is clear that in order to accurately characterize the network impulse response function, careful derivation of the mean travel time in each state is required. In the original formulation by Rodriguez-Iturbe and Valdes (1979) the mean travel time was derived using the mean length of the state and a constant velocity characteristic of the basin. It is well documented (Saco and Kumar, 2002) that the assumption of constant velocity does not mirror the non-linear flow characteristic that can be observed in the field. As such, many authors (Wang et al. 1981, Lee and Yen 1997 and Saco and Kumar 2002) have sought to investigate the effects of this nonlinearity by relaxing the constant velocity assumption. Lee and Yen (1997) refined the GIUH method by considering the travel time probabilistically and computing it hydraulically using the kinematic-wave approximation.

Lee and Yen (1997) showed that it was possible to simulate the surface-runoff process using the shape, length, slope and surface condition of a subcatchment. The travel time equations derived by Lee and Yen (1997) were based on the velocity of an individual rain drop. Subsequent formulations (Saco and Kumar, 2002 and Gironas et al, 2009) have derived the travel time based on the celerity of the flood wave. The kinematic wave or diffusive wave celerity, \( \mu_{x_k} \), of a flood wave based on the flow depth, \( h \), is given by (Cappelaere, 1997):

\[
\mu_{x_k} = \frac{Q_{x_k}K_{x_k}(h)}{B_{x_k}(h)K_{x_k}(h)}
\]

where \( B_{x_k}(h) \) is the width at the surface of the channel, \( K_{x_k}(h) \) is given by the expression \( K_{x_k}(h) = \frac{1}{n_{x_k}}A_{x_k}(h)R_{x_k}(h)^{2/3} \) (for S.I. units) with \( A_{x_k}(h) \) and \( R_{x_k}(h) \) being the area and hydraulic radius respectively, and \( K_{x_k}(h) \) is given by \( K_{x_k}(h) = \frac{\partial K_{x_k}(h)}{\partial h} \).

Using this celerity the mean travel time for a flood wave in a given state \( x_k \) can be calculated:

\[
T_{x_k} = \frac{L_{x_k}}{\mu_{x_k}}
\]
where \( L_{\text{m}} \) is the mean length the flood wave propagates within the state.

### 3.4.3 Conceptual Representation of IUHM

The flow path for a drop of water in the urban setting is significantly different than in a natural watershed, and, as such, the approach derived by Lee and Yen (1997) and others cannot be readily applied. Conceptually the hydrologic and hydraulic processes accounted for in IUHM are illustrated in Figure 3.2 and described herein.
Figure 3.2 Conceptual representation of how the flow paths through each hydrologic and hydraulic process are simulated in IUHM.
3.4.4 Derivation of Excess Rainfall

A hyetograph of rainfall intensity, \( i(t) \), versus time, \( t \), is used to drive IUHM. Rainfall is assumed to be uniformly distributed in space across the entire catchment. User-defined depression storage is abstracted uniformly in space from both pervious and impervious overland flow regions. Depressions are assumed to fill first such that the excess runoff on impervious areas, \( q_{o,\text{imp}}(t) \), will be zero until all depressions are filled and then equal to the rainfall intensity thereafter. For pervious regions, key runoff mechanism is infiltration excess rainfall (Bloschl and Sivapalan, 1995). The Green and Ampt method (Green and Ampt, 1911) is used to determine the infiltration rate, \( f(t) \), for the region. The infiltration excess rainfall for the pervious region, \( q_{o,\text{perv}}(t) \), is zero until all depressions are filled and then \( q_{o,\text{perv}}(t) = i(t) - f(t) \) thereafter. Infiltration is assumed to be uniform in space in the current version of the model and hence mean values of infiltration rate, wetting front suction head, effective porosity, and degree of saturation are required for the catchment.

3.4.5 Overland Flow

The kinematic wave travel time formulation derived in Section 3.4.2 (see Equation (3.8)) is used to derive the travel times for overland flow in IUHM and these are summarized in Table 3.1. A rain drop can fall on either a pervious or impervious region of the overland flow state, \( x_{o} \). This is a significant departure from the Lee and Yen (1997) and Gironas et al (2009) models where the overland state was considered as a whole. In the current version of IUHM excess rainfall from pervious regions is assumed to flow over impervious regions before reaching an artificial gutter, while flow from impervious regions contributes directly to the artificial gutter. The significance of this assumption is tested in this chapter and the results are discussed in Section 3.7.4. The total excess runoff from the overland flow state is

\[
Q_o(t) = Q_{o,\text{imp}}(t) + Q_{o,\text{perv}}(t)
\] 

(3.9)
### Inlet Hydraulics

An important component of urban sewer systems is the inlet or catch basin. Surprisingly this component of the hydrologic/hydraulic flow path is seldom modeled in the most commonly used deterministic models such as SWMM, TR-55 and HEC-HMS. Modelers are forced to make simplifying assumptions about the hydraulic capacity of the inlets/catch basins rather modeling them explicitly. In many urban systems, particularly those that are highly impervious, inlets can often form bottlenecks in the system, and hence prevent flow from reaching the underground conveyance system. As such, inlets are considered to be an important part of the hydrologic/hydraulic flow path and hence have been incorporated into IUHM.

Inlets can differ in shape and type. They can be rectangular, circular, on-grade, sag etc. Inlets are routinely simulated as part of hydraulic design, but are rarely modeled in studies of existing systems because their size, shape, type and other hydraulic characteristics are often not documented. The only way to obtain such information would be through field survey. Traditionally, sewer atlases will identify the location and shape (circular or rectangular) of the inlets in the system, but provide no further details. Realizing that any number of combinations of inlet configurations and layouts are possible, assumptions were made to allow their incorporation into IUHM.
At present, IUHM assumes that all inlets in the system are on-grade and are either circular or rectangular. All flow from the overland state is assumed to feed an artificial gutter. Flow from the gutter is distributed simultaneously into rectangular and circular inlets in proportion to their number in a given order. In addition to the overland flow from the given time step, the inlet may also receive surcharge flow from the previous time step due to insufficient inlet or conduit capacity. Thus, the total flow feeding the artificial gutter is

\[ Q_{\text{inlet}}(t) = Q_{\text{inlet}}(t) + Q_{\text{ex.inlet}}(t-1) + Q_{\text{ex.pipe}}(t-1) \]  

where \( Q_{\text{ex.inlet}}(t-1) \) is the flow in excess of the \( i \)th-order inlet capacity from the previous time step, and \( Q_{\text{ex.pipe}}(t-1) \) is the flow in excess of the \( i \)th-order conduit capacity.

Guo (1997) provides guidelines for simulating inlet hydraulics based on the flow feeding the inlet, gutter characteristics (longitudinal and transverse slope, roughness etc.) and inlet characteristics (open area, wetted perimeter, effective length etc.). In IUHM, both rectangular and circular inlets are assumed to be on grade grate inlets and their efficiencies are determined and then used to calculate the flow that will enter each \( i \)th-order inlet, \( Q_{\text{inlet}}(t) \), as well as the excess flow, \( Q_{\text{ex.inlet}}(t) \), that will be added to the overland flow for the next time step.

3.4.7 Conduit Flow

Flow into an \( i \)th-order conduit may be a combination of external inflow from the inlets and inflow from lower order conduits. In the traditional GIUH formulation channels were assumed to directly accept all flow from the overland state and were assumed to have infinite capacity as an open channel. In a sewer system there is a good chance that any given conduit in the system will become pressurized. In the vast majority of cases, conduits are circular in shape and as such IUHM is at present setup to account for circular conduits only. For a circular conduit it can be shown mathematically that the conduit reaches its pipe-full capacity when the depth is \( 0.82D \), where \( D \) is the diameter of the conduit (Chow, 1959). At depths above this the flow becomes unstable and as such this was chosen as the limiting depth above which surcharging will occur. The pipe-full discharge that can be conveyed by an \( i \)th-order conduit can be expressed as
\[ Q_{c_{i,full}} = \frac{0.31D_i^{8/3}S_{c_i}^{1/2}}{n_c} \]  

(3.11)

For first order conduits, there is no inflow from upstream conduits and as such the inflow to a first order conduit is simply

\[ Q_{c_{i,in}}(t) = Q_{\text{inlet}_i}(t) \]  

(3.12)

For higher order conduits, flow from lower order conduits must be accounted for. The flow entering an \( i \)th-order conduit from upstream can be expressed:

\[ Q_{c_{i,up}}(t) = \sum_{j=1}^{i-1} \frac{N_j Q_{c_{j,out}}(t) P_{x_{ji}}}{N_i} \quad \text{for } i \geq 2 \]  

(3.13)

where \( Q_{c_{j,out}}(t) \) is the outflow from the \( j \)th-order conduit. In this way the flow contributed from upstream conduits is a function of the number of conduits in any given order upstream, \( N_j \), and the probability of water transitioning from that order to the order of interest, \( P_{x_{ji}} \). Using this, the flow entering an \( i \)th-order conduit can be expressed

\[ Q_{c_{i,in}}(t) = Q_{\text{inlet}_i}(t) + Q_{c_{i,up}}(t) \]  

(3.14)

If the flow entering the conduit is greater than the pipe-full capacity, i.e. \( Q_{c_{i,in}}(t) > Q_{c_{i,full}} \), it is assumed to become pressurized and any flow in excess of the capacity is stored and added to the flow entering the \( i \)th-order inlet in the next time step, as described in Equation (3.22). The excess flow from the conduit is

\[ Q_{c_{i,pipe}}(t) = Q_{c_{i,in}}(t) - Q_{c_{i,full}} \]  

(3.15)

If the flow entering the conduit does not exceed the pipe-full capacity, then there will be a free-surface flow condition and the flow depth in the conduit must be determined. This flow depth was determined using the kinematic wave assumption to represent the flow and in turn allowed the conduit travel time to be determined.
\[
\bar{T}_{w_i}(t) = \frac{L_i B_i(t) \frac{1}{n} \bar{A}_i(t) \bar{R}_i(t)^{2/3}}{Q_i(t) \left[ \partial \left( \frac{1}{n} \bar{A}_i(t) \bar{R}_i(t)^{2/3} \right) / \partial h_i(t) \right]} \tag{3.16}
\]

where \(n_c\) is the Manning’s roughness coefficient for the conduit, \(\bar{h}_i(t)\) is the \(i\)th-order conduit flow depth, \(B_i(t)\) is the \(i\)th-order width at the surface of the conduit, \(\bar{R}_i(t)\) is the \(i\)th-order conduit hydraulic radius and \(\bar{A}_i(t)\) is the \(i\)th-order conduit area.

The travel times calculated in the overland and conduit regions are used in Equation (3.6) to formulate probability-density-functions for travel time in each state. The PDF’s are combined with path probabilities and excess rainfall in Equation (3.4) to generate the network impulse response function for the catchment, which is subsequently used to derive the direct runoff hydrograph described in Equation (3.5).

### 3.5 Study Catchment

A number of test simulations have been performed on the Calumet Drop Shaft 51 (CDS-51) catchment. The CDS-51 catchment is located in the Village of Dolton (see Figure 3.3), a southern suburb of Chicago, IL and contributes combined sanitary and storm flows to one of the drop shafts in the Calumet system of Chicago’s Tunnel and Reservoir Plan (TARP) (MWRDGC, 1999). The catchment contributing to CDS-51 captures storm and sanitary flows for a 316 ha service area. The combined sewer network feeding CDS-51 collects inflow from in excess of 800 inlets and conveys it to CDS-51 via a network of some 722 pipes ranging in size from 150 mm (6”) to 2150 mm (84”). All of the flow in the CSO system ends up in a 2150 mm pipe. Dry weather flows are intercepted by two MWRDGC Interceptor sewers at the corner of 158th Street and Ellis Avenue, as shown in Figure 3.4. In wet weather events, flows are initially intercepted by the MWRDGC interceptor sewers which convey flow to the Calumet Water Reclamation Plant. When the treatment plant reaches capacity, flow in the 2150 mm pipe is directed towards the CSO outfall and conveyed to TARP. If and when TARP reaches capacity the combined sewage overflows into the Little Calumet River. From April 2007 until April 2009, the United States Geological Survey (USGS) operated three
acoustic flow meters (see Figure 3.4) to monitor the flow entering CDS-51. Data from these meters were collected and processed by the USGS and these data will form the basis of comparison for IUHM for CDS-51.

Figure 3.3 CDS-51 catchment location plan. The CDS-51 catchment is used to test the performance of IUHM and better understand the hydrologic behavior in complex urban systems.
Figure 3.4 Schematic of CDS-51 hydraulic network connections. The hydraulics of the interceptor and TARP tunnels and their hydraulic structures impact the hydrologic response measured by the USGS at Meter A. Recorded flow conditions at Meter A are used to validate the IUHM model. The hydraulics of these regulating structures and tunnels were modeled using InfoSWMM rather than the IUHM model.

Characteristics (diameter, pipe length, upstream and downstream invert) of the combined sewer network in CDS-51 were obtained from the Village of Dolton sewer atlas developed in 1977. As part of the development of a detailed deterministic InfoSWMM model for CDS-51, subcatchments were delineated for each inlet using contours developed from Cook County’s LiDAR data, aerial photography, the sewer layout and engineering judgment. Imperviousness, infiltration parameters and overland
slopes were all obtained from the high resolution synthetic urban watershed developed by Crosa (2008). Manning’s roughness for the pervious and impervious overland flow regions were assumed to be 0.20 and 0.015 respectively, while a value of 0.016 was used for the conduits. The Green-Ampt infiltration parameters were taken to be homogeneous for the catchment and were derived by taking the weighted average of the parameters throughout the catchment. Depression storage of both pervious and impervious areas was assumed to be 0.381 cm (0.15 inches).

A geometric network, comprised of junctions and conduits, was created in ArcMap for CDS-51 by digitizing the sewer maps provided by the Village of Dolton. Using this geometric network a tool was created in ArcMap to assign each conduit an order based on the Strahler ordering procedure described in Section 3.4.2. Based on the conduit order, $N_i$, the number of $i$th-order pipes in each order was derived. In addition to assigning an order to each conduit, the GIS tool developed provides the data required to determine transition probabilities, $P_{ij}$, initial state probability, $P_{OA}$, path probabilities, $P(w)$, and scaling factors, $a_i$, which account for transitions from order $i$ to $i$.

CDS-51 is a 5th order catchment and has the subcatchment and conduit characteristics detailed in Table 3.2, while the transition characteristics for each order, transitional probabilities, and path probabilities are summarized in Tables 3.3, 3.4 and 3.5, respectively. For this catchment the majority of the required subcatchment and conduit data were available and as such the input parameters ($c_i$, $S_i$, $D_i$, $imp_i$, $N_{inlet,circ}$, and $N_{inlet,rec}$) were derived using all the available data. However, it is not necessary to use all of these data if they are not available. The parameters required for the model could be generated using a subset of the data or perhaps even parameters derived from nearby watersheds where such data are available. This is one of the key advantages of this model.
### Table 3.2 Input data for CDS-51 catchment

<table>
<thead>
<tr>
<th>Channel Order, $i$</th>
<th>$N_i$</th>
<th>$\bar{L}_{ci}$ (m)</th>
<th>$\bar{S}_{ci}$</th>
<th>$\bar{D}_{ci}$ (mm)</th>
<th>$\text{imp}_i$</th>
<th>$\bar{S}_{ci}$</th>
<th>$N_{\text{inlet, circ}}$</th>
<th>$N_{\text{inlet, rect}}$</th>
<th>$P_{\text{OA}}$</th>
<th>$a_i$</th>
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<tbody>
<tr>
<td>1</td>
<td>449</td>
<td>62</td>
<td>0.0048</td>
<td>330</td>
<td>57</td>
<td>0.0112</td>
<td>0.68</td>
<td>0.34</td>
<td>0.57</td>
<td>2.65</td>
</tr>
<tr>
<td>2</td>
<td>157</td>
<td>60</td>
<td>0.0036</td>
<td>463</td>
<td>58</td>
<td>0.0117</td>
<td>0.85</td>
<td>0.33</td>
<td>0.25</td>
<td>4.01</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>75</td>
<td>0.0019</td>
<td>718</td>
<td>58</td>
<td>0.0114</td>
<td>0.72</td>
<td>0.33</td>
<td>0.08</td>
<td>5.88</td>
</tr>
<tr>
<td>4</td>
<td>51</td>
<td>64</td>
<td>0.0015</td>
<td>1181</td>
<td>74</td>
<td>0.0104</td>
<td>1.20</td>
<td>0.18</td>
<td>0.09</td>
<td>19.25</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>99</td>
<td>0.0015</td>
<td>2058</td>
<td>60</td>
<td>0.0100</td>
<td>1.60</td>
<td>0.20</td>
<td>0.02</td>
<td>4.50</td>
</tr>
</tbody>
</table>

### Table 3.3 Details of pipe transitions for CDS-51

<table>
<thead>
<tr>
<th>Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Outlet</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>297</td>
<td>112</td>
<td>23</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>449</td>
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<tr>
<td>2</td>
<td>121</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>157</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>49</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>51</td>
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<tr>
<td>5</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 3.4 Transition probabilities, $P_{x_i|x_j}$, for CDS-51

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>0.737</td>
<td>0.151</td>
<td>0.112</td>
<td>0</td>
<td>-</td>
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<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>0.722</td>
<td>0.194</td>
<td>0.083</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3.5 Path probabilities, $P(w)$, for CDS-51

<table>
<thead>
<tr>
<th>Path</th>
<th>$P(w)$</th>
<th>Path</th>
<th>$P(w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1 : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.130</td>
<td>$w_2 : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.176</td>
</tr>
<tr>
<td>$w_3 : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0</td>
<td>$w_6 : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.047</td>
</tr>
<tr>
<td>$w_5 : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.035</td>
<td>$w_8 : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.050</td>
</tr>
<tr>
<td>$w_7 : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.037</td>
<td>$w_{10} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.104</td>
</tr>
<tr>
<td>$w_9 : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.074</td>
<td>$w_{12} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.020</td>
</tr>
<tr>
<td>$w_{11} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.015</td>
<td>$w_{14} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.020</td>
</tr>
<tr>
<td>$w_{13} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0</td>
<td>$w_{16} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.037</td>
</tr>
<tr>
<td>$w_{15} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.027</td>
<td>$w_{18} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0</td>
</tr>
<tr>
<td>$w_{17} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0</td>
<td>$w_{20} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.028</td>
</tr>
<tr>
<td>$w_{19} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.020</td>
<td>$w_{22} : x_{i,imp} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.043</td>
</tr>
<tr>
<td>$w_{21} : x_{i,perv} \rightarrow x_{i,perv} \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i \rightarrow x_i$</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
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</table>
Table 3.5 (cont.)

<table>
<thead>
<tr>
<th>Path</th>
<th>P(w)</th>
<th>Path</th>
<th>P(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{23} : x_{n,perv} \rightarrow x_{n,pervimp} \rightarrow x_i \rightarrow x_i)</td>
<td>0</td>
<td>(w_{24} : x_{n,sup} \rightarrow x_i \rightarrow x_i)</td>
<td>0</td>
</tr>
<tr>
<td>(w_{25} : x_{n,perv} \rightarrow x_{n,pervimp} \rightarrow x_i \rightarrow x_i)</td>
<td>0.009</td>
<td>(w_{26} : x_{n,sup} \rightarrow x_i \rightarrow x_i)</td>
<td>0.012</td>
</tr>
<tr>
<td>(w_{27} : x_{n,perv} \rightarrow x_{n,pervimp} \rightarrow x_i \rightarrow x_i)</td>
<td>0</td>
<td>(w_{28} : x_{n,sup} \rightarrow x_i \rightarrow x_i)</td>
<td>0</td>
</tr>
<tr>
<td>(w_{29} : x_{n,perv} \rightarrow x_{n,pervimp} \rightarrow x_i \rightarrow x_i)</td>
<td>0.022</td>
<td>(w_{30} : x_{n,sup} \rightarrow x_i \rightarrow x_i)</td>
<td>0.063</td>
</tr>
<tr>
<td>(w_{31} : x_{n,perv} \rightarrow x_{n,pervimp} \rightarrow x_i)</td>
<td>0.007</td>
<td>(w_3 : x_{n,sup} \rightarrow x_i)</td>
<td>0.011</td>
</tr>
</tbody>
</table>

3.6 Simulation of CDS-51

The input parameters generated for CDS-51 were used to simulate a series of storms that occurred in the greater Chicago area over the period April 2007 to February 2008. For brevity and clarity of presentation, results are presented for four of the storms simulated. Each of the storms presented were selected for different reasons. The January 2008 and April 2007 events caused the largest combined sewer overflow events for CDS-51 over the period simulated, while the July 2007 and August 2007 events represent high-intensity, short duration and low-intensity long duration events respectively. Precipitation estimates were obtained from a nearby United States Geological Survey (USGS) rain gage located on the southern boundary of the CDS-51 catchment on the Little Calumet River in the suburb of South Holland, IL. The characteristics of each storm are summarized in Table 3.6 and the rainfall hyetographs are provided in Figure 3.5.

Table 3.6 Summary of storm events simulated using IUHM

<table>
<thead>
<tr>
<th>Storm Event</th>
<th>Start Date and Time</th>
<th>Duration (hours)</th>
<th>Peak Intensity (mm/hr)</th>
<th>Total Precip (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 2007 Storm</td>
<td>24 April 2007 @ 23:00</td>
<td>14</td>
<td>8.63</td>
<td>49.8</td>
</tr>
<tr>
<td>July 2007 Storm</td>
<td>26 July 2007 @ 02:00</td>
<td>7</td>
<td>18.8</td>
<td>46.7</td>
</tr>
<tr>
<td>August 2007 Storm</td>
<td>5 August 2007 @ 05:00</td>
<td>15</td>
<td>5.08</td>
<td>19.3</td>
</tr>
<tr>
<td>January 2008 Storm</td>
<td>7 January 2008 @ 18:00</td>
<td>15</td>
<td>11.7</td>
<td>79.2</td>
</tr>
</tbody>
</table>
Figure 3.5 Comparison of hydrologic response of the CDS-51 catchment for a series of rain events during 2007-2008. (a) Rainfall hyetograph as recorded by the United States Geological Survey at a rain gage located on the Little Calumet River in the suburb of South Holland, IL. (b) Comparison of the direct runoff hydrograph (DRH) predicted by an all-pipe-all-subcatchment InfoSWMM model and IUHM. (c) Comparison of the inflow to CDS-51 predicted by InfoSWMM using the DRH’s in (b) with inflow recorded by the USGS.
The primary output of IUHM is the direct runoff hydrograph (DRH) derived in Equation (3.5). Observed results are available at three USGS monitored flow meters, A, B, and C, as shown in Figure 3.5. Each of these flow meters is downstream of a regulating structure that contains a weir regulating flow into the MWRDGC-operated interceptor sewers and CDS-51. Thus, in order to make meaningful comparisons between observed and predicted flows the hydraulics of the interceptor regulating structure and downstream interceptor sewers were modeled in InfoSWMM using the DRH from IUHM as an external inflow hydrograph. The results of these simulations, showing the inflow to drop shaft CDS-51 are shown Figure 3.5, along with the USGS observed hydrograph at Meter A. Further comparison is made to a detailed model developed using the commercially available InfoSWMM model (Version 8.0, MWH Soft 2009). InfoSWMM utilizes the commonly used and widely accepted SWMM model for overland flow and hydraulic routing computations, and hence was seen as providing a good metric for comparison to IUHM. The InfoSWMM model includes every pipe, junction and manhole within the CDS-51 catchment, with one subcatchment delineated for each inlet. This represents the kind of model that could be developed in practice if the extent of the modeling was small (e.g. to simply one drop shaft) and all input data were available. It is important to note that no calibration was performed for the IUHM or InfoSWMM model.

The Nash-Sutcliffe efficiency (NSE) coefficient (Nash and Sutcliffe, 1970) was used to compare the IUHM and InfoSWMM DRH predictions. Nash-Sutcliffe efficiencies can range from negative infinity to 1, with 1 indicating a perfect match between the predicted and observed data, while a value of zero indicates that the model predictions are as accurate as the mean of the observed data. Efficiencies of 0.86, 0.97, 0.76 and 0.89 were calculated for the January 2008, April 2007, July 2007 and August 2007 storms respectively. This together with the shape, timing and magnitude that can be observed in Figure 3.5 (b) highlights that IUHM is capable of predicting direct runoff hydrographs as well as the widely used and accepted SWMM model. Careful observation of the DRH predicted by IUHM reveals that it predicts a higher peak flow than InfoSWMM and also underestimates the magnitude of the tail of the hydrograph. This is most recognizable when comparing the results for the July 2007 storm which was a high-intensity short-
duration storm. The reasons for the differences in shape of the hydrograph are explored in Section 3.7 and can be explained by a number of the assumptions made in IUHM.

The inflow hydrograph for CDS-51 predicted by IUHM and InfoSWMM, together with the observed inflow hydrograph at Meter A are shown in Figure 3.5(c) for each of the storms analyzed. NSE coefficients were calculated to compare the inflow observed at USGS flow meter A with those predicted by IUHM. It was found that the NSE coefficients for IUHM were 0.91 and 0.7 for the January 2008 and April 2007 storms respectively. For these two storms IUHM provides a good prediction of shape, timing and magnitude of the inflow hydrograph for CDS-51. For the July 2007 storm, however, IUHM does a poor job of predicting the inflow hydrograph. There are many possible reasons for this discrepancy, some of which are discussed in Section 3.7 below.

One of the difficulties in making valuable comparisons between IUHM and the observed results is the unknown effect of the interceptor and deep tunnel systems. InfoSWMM was used to model the interceptor regulating structure and short portions of the interceptor sewers themselves but downstream boundary conditions were assumed for the interceptors (i.e. critical depth at the outfall of each interceptor). In reality, the reason for the CSO structures is because the interceptor system has insufficient capacity for large storms. Hence it is anticipated that for large storms backwater effects will limit flow to the interceptor to less than predicted based on the hydraulics of the regulating structure. The hydraulics of the interceptor and TARP systems is a subject of ongoing research at the University of Illinois at Urbana-Champaign. As this research continues there will be an opportunity to integrate the IUHM model with either the Illinois Transient Model (Leon et al, 2008) or Illinois Conveyance Analysis Program (Oberg et al, 2008) to obtain a better understanding of the combined hydrologic and hydraulic behavior of the unique Chicago system.

As a model for simulating the hydrologic response of highly urbanized catchments, Figure 3.5 highlights that IUHM is capable of predicting results as accurate as the more labor intensive InfoSWMM model. IUHM requires inputs for the equivalent of Ω conduits and contributing subcatchments, rather than for every conduit and subcatchment, which would be required in a deterministic model. This represents a
significant time and economic saving for modelers with little reduction in the accuracy of the predicted results.

### 3.7 Understanding Urban Catchment Hydrology Using IUHM

The primary objective of this chapter was to develop and apply a GIUH-inspired approach to urban systems so that we could better understand the way they function and decipher the relative controls of the various components of the system. In this section we fulfill that objective using IUHM to highlight the impacts of different components of the model and illustrate a number of key differences between watershed and urban hydrology.

#### 3.7.1 Spatial and Temporal Scale

Bloschl and Sivapalan (1995) identified that spatial and temporal scale were two of the key considerations when undertaking hydrologic modeling. We highlighted that the space and time scales in urban catchments were likely much smaller than in natural watersheds. It can be observed from Table 3.7 that the average $i$th-order conduit lengths for CDS-51 and three other nearby urban catchments range between 40 and 100 meters. In comparison, average $i$th-order channel lengths reported in the literature (Yen and Lee, 1997) for natural watersheds are rarely less than 1.5 km and in some cases are as high as 100 km. Similar observations can be made when considering total catchment area, where natural watersheds span hundreds of square kilometers compared to urban catchments that can service an area less than 100 hectares.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Area (ha)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDS-51</td>
<td>3.16</td>
<td>62</td>
<td>60</td>
<td>75</td>
<td>64</td>
<td>99</td>
</tr>
<tr>
<td>CDS-20</td>
<td>1.61</td>
<td>39</td>
<td>42</td>
<td>42</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>CDS-36L</td>
<td>0.65</td>
<td>37</td>
<td>41</td>
<td>56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS-36R</td>
<td>0.18</td>
<td>66</td>
<td>54</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Time scale is particularly important for GIUH based approaches because of the important role of travel time. For the urban catchments considered in this research it was observed that the mean $i$th-order conduit travel time was somewhere between one and
three minutes for a range of intensities. Overland travel times were found to be larger, in
the order of three to five minutes for impervious regions and 10 to 20 minutes for
pervious regions. In natural watersheds, channel travel times are observed to be slower
than those in urban catchments due to the larger length of travel and slower velocity
over rougher channel beds.

The question is: what is significant about these smaller spatial and temporal
scales? These smaller scales are particularly significant when trying to generate input
data such as precipitation. The time and spatial scales that we observe from the model
are representative of our observation scale. As Bloschl and Sivapalan (1995) identify
ideally we would like our modeling scale to be of the same order. The primary input
parameter for urban catchment modeling is precipitation. Typically precipitation data
are not available on the minute time scale and meter space scale. For example, the most
reliable source of rainfall data in Chicago is a network of 25 rain gages operated by the
Illinois State Water Survey (ISWS, 2009) that collect hourly rainfall on an 8 km by 8 km
grid. As a result of these much coarser space and time scales, downscaling must be
performed in order to ensure the modeling and observation scales are the same. For the
catchments presented in this paper, rainfall was assumed to be uniform in space and
hourly rainfall was downscaled to equivalent one minute periods. The effect of this
downscaling can be observed on CDS-51. A large volumetric difference between the
predicted and observed inflow hydrographs for the July 2007 storm can be observed (see
Figure 3.6). This high-intensity, short duration storm is typical of localized convective
storms that occur in the Chicagoland area. Our assumption of uniform rainfall in space
was justified on the basis that the spatial scale of the available precipitation data is
coarser than the spatial scale of the catchment itself. In reality the rainfall will not be
uniform and could potentially have a significant impact on the predicted results. This
impact is more significant than it would be in natural watersheds where the space and
time scales are as coarse, if not coarser, than the available precipitation data.
Figure 3.6 Comparison of the hydrologic response [(a) cumulative rainfall, (b) direct runoff hydrograph, (c) inflow to drop shaft CDS-51] predicted by IUHM using three different rain gages that are in close proximity to CDS-51. The three gages used were the USGS located on the Little Calumet River in South Holland, IL (thick black line), Cook County network Illinois State Water Survey (ISWS) gage 22 (thin black line) and the Cook County network ISWS gage 19 (thick grey line).

In addition to the USGS gage, the CDS-51 catchment is surrounded by four rain gages (ID’s 18, 19, 22 and 23) monitored as part of the 25 rain gage grid network in Cook County operated by the ISWS. The four rain gages surrounding CDS-51 are all within an eight km radius of the centroid of the catchment. The spatial resolution of these gages is relatively fine when compared to spatial distribution of rain gages usually encountered in the urban setting. For the July 2007 storm the rainfall hyetographs for the four ISWS gages and the USGS gage were compared. The recorded cumulative rainfall ranged
between 36 mm at ISWS Gage 22 and 87 mm at ISWS Gage 19, while 47 mm was recorded at the USGS gage. The impact of this heterogeneous rainfall on the hydrographs predicted by IUHM was analyzed and the results are presented in Figure 3.6. This analysis highlights that the shape, timing and magnitude of the predicted DRH and inflow hydrograph are highly dependent on the rain gage used. Furthermore, it is evident that for this particular storm our inability to describe the heterogeneity in the rainfall at the observed space and time scales across the CDS-51 catchment hinders our ability to accurately predict the hydrologic response of the catchment. This problem is not easily overcome in urban catchments and is the subject of ongoing research. In the companion paper, Cantone and Schmidt (2010, see chapter 4 of this thesis) present a methodology for tracking uncertainty in the hydrologic response caused by uncertainty in IUHM input parameters. It is foreseeable that such a framework could be used to quantify the effects of uncertainty in the rainfall hyetograph used to drive the model.

3.7.2 Network Organization and Horton’s Laws

The key hypothesis behind this research is that the hydrologic response of urban catchments is inherently linked to the structure of the sewer network conveying flow in the catchment. We have shown successfully that the developed model is able to predict the hydrologic response of urban catchments based on this hypothesis. The Strahler ordering scheme, developed for natural watersheds can be applied in the urban setting and provides a novel mechanism for representing the possible flow paths through the sewer network. The fact that this ordering scheme has been successfully applied is evidence that urban catchments follow an organizational structure, albeit different from the structure observed in natural watersheds. In natural watersheds, it has been shown by Horton (1945), Strahler (1950) and others that the geomorphology of natural stream networks follows a general set of laws. For example, the Horton-Strahler bifurcation ratio (the ratio of number of stream segments of one order to the number of the next higher), \( R_B \), in natural streams has been observed to range from 3 to 5 and the length ratio (ratio of average length of streams of order \( i \) to streams of order \( i-1 \)), \( R_L \), is usually between 1.5 and 3.5.

Using these same laws, the bifurcation and length ratios for four catchments in Chicago (CDS-51, CDS-20, CDS-36L and CDS-36R) were calculated and are shown in
Table 3.8. Bifurcation ratios for these catchments range between 0.87 and 7.35, while length ratios vary between 0.82 and 1.55. The first thing to note here is that it is possible for the bifurcation ratio to be less than two. In natural watersheds, all transitions were assumed to be from a channel of order to \(i\) to a higher order channel \(j\). This assumption was not adopted in this research. Rather, in urban watersheds, transitions from an \(i\)th-order pipe to another pipe of the same order \(i\) were permitted. As a result the possible range of bifurcation ratios for urban catchments is wider. The relatively wide range of bifurcation ratios is not surprising given that the four catchments analyzed are of different order. It is anticipated that as a larger number of catchments are analyzed, further patterns in the bifurcation ratio will be observed. The observed range for the length ratio is significantly smaller and is evidence of the human engineered structure of urban sewer networks. The lengths of conduits in urban sewer networks are much more uniform than in natural watersheds and as such we would expect a length ratio close to unity which is what is observed for the catchments tested in this research.

Table 3.8 Horton’s ratios for selected urban catchments

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>CDS-51</td>
<td>N</td>
<td>449</td>
<td>157</td>
<td>57</td>
<td>51</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>(R_B)</td>
<td>-2.86</td>
<td>2.75</td>
<td>1.12</td>
<td>6.38</td>
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<tr>
<td></td>
<td>(R_L)</td>
<td>0.97</td>
<td>1.25</td>
<td>0.85</td>
<td>1.55</td>
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<tr>
<td>CDS-20</td>
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<td>328</td>
<td>147</td>
<td>20</td>
<td>14</td>
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<tr>
<td></td>
<td>(R_B)</td>
<td>-2.23</td>
<td>7.35</td>
<td>1.43</td>
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<tr>
<td></td>
<td>(R_L)</td>
<td>1.08</td>
<td>1.00</td>
<td>1.02</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>CDS-36L</td>
<td>N</td>
<td>108</td>
<td>53</td>
<td>18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(R_B)</td>
<td>-2.04</td>
<td>2.94</td>
<td>-</td>
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<tr>
<td></td>
<td>(R_L)</td>
<td>1.11</td>
<td>1.37</td>
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<td>-</td>
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<td>CDS-36R</td>
<td>N</td>
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<td>15</td>
<td>-</td>
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<td>-</td>
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<tr>
<td></td>
<td>(R_B)</td>
<td>-0.87</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(R_L)</td>
<td>0.82</td>
<td>-</td>
<td>-</td>
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Although a relatively small subset of urban catchments has been assessed there is sufficient evidence to suggest that a similar set of ranges for Horton’s ratios could be developed for urban sewer networks. This is not surprising given that urban sewer systems are generally engineered to prescribed design standards resulting in inherent relations between the hydraulic properties of any point in the network and the characteristics of the catchment and network upstream from that point. The network
organization and similarity in these networks could prove invaluable and add value to models such as IUHM in generating input data for catchments that have uncertain deterministic inputs.

3.7.3 Effect of Accounting for Inlets and Pressurized Flow

One of the primary differences between natural and urban watersheds is the presence of inlet control structures and the possibility of pressurized flow conditions. These were issues not faced in the development of the GIUH approach for natural watersheds and have been largely ignored in the application of GIUH approaches to urban catchments. Neither Rodriguez et al (2003) or Gironas et al (2009) attempted to account for the possibility of pressurized flow or for the effect of inlet controls on flow entering the combined sewer system. In their evaluation of their model’s performance Gironas et al (2009) highlight their model’s inability to account for the pipe becoming surcharged and comment that some storage or flow detention in the system may not be adequately simulated by the model. IUHM attempts to overcome these problems by accounting for depression storage, inlet hydraulics and pipe surcharge. IUHM is largely based on the kinematic wave assumption precluding analytically accounting for backwater effects. In the present version of the model flow in excess of an inlet or conduit’s capacity is simply stored and re-introduced into the system in the following time step.

The impact of this assumption was tested on CDS-51 for a triangular (generated using the methodology of Yen and Chow, 1980) 100 year ARI storm event. The IUHM assumption—that the pipe reaches its pipe-full capacity at a depth 0.82D and any flow in excess of capacity is stored—was relaxed. In addition, inlet hydraulics were not modeled resulting in zero inlet storage. By relaxing the pressurized flow assumption and not allowing for inlet storage, all water falling on the catchment in a given time step is forced through the model resulting in larger conduit velocities and smaller mean conduit travel times in comparison to IUHM. This behavior (see Figure 3.7) is translated through into the direct runoff hydrograph where it is observed that the predicted DRH has a higher peak, shorter time to peak and shows less dispersion than the DRH observed using a version of IUHM that accounts for pressurized flow and inlet storage. This is an important observation and one that signifies the need to account pressurized
flow and inlet hydraulics, two phenomena that are not as prevalent in natural watersheds.

Figure 3.7 Effects of accounting for inlet hydraulics and pressurized flow in IUHM.
The “Inlets/Max” case refers to the runoff by predicted using IUHM when pressurized flow and inlet hydraulics are explicitly modeled. In the “No Inlets/Max” case, inlet hydraulics and pressurized flow are ignored so that all flow reaching an inlet is assumed to enter the conduit and flow in the conduit is allowed to increase beyond the conduit capacity. (a) Rainfall hyetograph for 100 year ARI triangular [Yen and Chow (1980)] storm. (b) DRH for CDS-51 catchment predicted by IUHM. (c) 1st order conduit travel time, $T_{xc1}$. (d) 2nd order conduit travel time, $T_{xc2}$. (e) 3rd order conduit travel time, $T_{xc3}$. (f) 4th order conduit travel time, $T_{xc4}$. (g) 5th order conduit travel time, $T_{xc5}$.
3.7.4 Effect of Overland Flow Routing Assumption

In urban catchments overland flow regions are comprised of both pervious and impervious regions and flow through these regions is often highly heterogeneous. Traditionally when combined sewer systems were built, flow was designed to flow from pervious to impervious regions. For this reason and because flow from pervious regions is generally much smaller in magnitude than flow from impervious regions, some authors (e.g. Gironas et al, 2009) have chosen not to include a travel time associated with flow over pervious regions. What is the impact of this omission? IUHM was used to quantify this omission by adapting the model to simulate three particular cases: (1) flow from only impervious areas (2) flow from pervious and impervious areas, routed from pervious to impervious regions, and (3) flow from pervious and impervious areas, routed from impervious to pervious regions. IUHM was then applied to CDS-51 for three different storm events (1 yr, 5 yr and 25 yr ARI) generated using the methodology of Yen and Chow, 1980. The results of this analysis are shown in Figure 3.8.
Figure 3.8 Effect of three different types of overland flow routing on hydrologic response of CDS-51 for a series of storm events. (a), (d) and (g) Rainfall hyetograph for 1, 5 and 25 year ARI triangular [Yen and Chow (1980)] storms. (b), (e) and (h) DRH predicted by IUHM. (c), (f) and (i) 1st order pervious overland travel time, $T_{xo1,perv}$.

For the relatively small 1 year ARI storm event all of the flow that initially falls on a pervious region infiltrates, resulting in the same DRH being observed for Cases 1 and 2. As such for small events one may conclude that assumption to not include pervious regions in the model are valid. When considering the third case it is clear that there would be significant benefit from routing flow from impervious regions to pervious regions, which is the same concept as many of the best management practices being encouraged worldwide. There is a significant reduction in the total volume of runoff, a reduction in the peak and a delay in the time to peak.
As the storm intensity increases there begins to be infiltration excess runoff from the pervious overland flow regions as shown (see Figure 3.8 (c), (f) and (g)) by the presence of a pervious overland travel time. If travel across these regions is ignored the peak of the hydrograph and total runoff volume may be underestimated as is evidenced by Figures 3.8(e) and 3.8(h). It is also clear that in larger storm events that the benefit of routing flow from impervious to pervious overland flow regions is much smaller.

3.8 Conclusions

Urban systems have evolved with mankind and developed into complex, non-linear, and highly heterogeneous systems made up of conduits, inlets, parking lots, driveways, gardens, rain water tanks and other appurtenances. The fact that these systems are predominantly manmade would tend to mark them deterministic, yet their complexity and evolution makes them indeterminate. Traditionally, deterministic modeling approaches have been adopted when trying to understand and predict the hydrologic response of highly urbanized catchments. These deterministic models vary in the hydrologic and hydraulic processes they simulate but all require detailed deterministic input data and are designed to be calibrated. The necessary data are often not available; it is impossible to measure all hydrologic variables and as-built data on existing infrastructure is not always readily available, particularly in older combined sewer systems. These limitations force engineers to make simplifying assumptions; assumptions that can significantly affect the predicted hydrologic response and that differ according to the simulation package being used (Cantone and Schmidt, 2009). In natural watersheds, these realizations were made three decades ago and prompted the development of alternative modeling approaches. The geomorphologic instantaneous unit hydrograph approach originally developed by Rodriguez-Iturbe and Valdes (1979) was modified and utilized by Yen and Lee (1997) to show that it could be used to improve the understanding of natural watersheds and aid in their simulation in circumstances where limited input data and calibration data were available. In this paper, we have developed and applied an alternative, GIUH inspired approach for simulating urban systems that improves the understanding of how these systems function. Using the developed Illinois Urban Hydrologic Model (IUHM) we were able to
draw a number of conclusions on the controls for different components and transformations in urban systems, by applying the model to the CDS-51 catchment located in the Village of Dolton, IL. Based on that analysis, the following conclusions were drawn:

- An urban catchment can be represented by a series of conduit and overland flow states based on the layout and structure of the sewer system collecting storm flows in the catchment. The Strahler ordering scheme can be applied to the combined sewer system allowing a finite number of flow paths to be defined. Each flow path can include a pervious and impervious overland flow state and a conduit state, and using the structure of the system a probability for each path can be defined. Flow within each state can be characterized using a kinematic wave approximation that allows the definition of travel time distributions which can be convoluted and combined with excess rainfall intensity to derive the network impulse response function for the catchment.

- Urban sewer networks exhibit organization and structure in ways similar to natural watersheds. In the same way that Horton’s laws were characterized for natural stream networks it is possible to characterize similar laws for urban sewer networks.

- There are many hydrologic and hydraulic processes that may affect the response of an urban system. In urban systems the key processes are rainfall, infiltration, infiltration excess overland flow, inlet hydraulics, and conduit flow hydraulics.

- The spatial and temporal scales in urban watersheds are significantly smaller than in natural watersheds. Rainfall data in urban catchments are often available
at a coarser scale than the length scale associated with the catchment itself.

Selection of the rainfall data to be used is critical to the prediction of the hydrologic response. Under certain storm conditions (e.g. high intensity short duration convective storms) recorded rainfall estimates may differ significantly between neighboring rain gages and hence introduce an uncertainty that translates into the predicted hydrologic response.

- One of the major differences between urban and natural watersheds is that there is a significant likelihood that conduits in the system may become pressurized. The hydraulic behavior of pressurized flow is complex and may have a significant impact on the hydrologic response.

- Another important difference between urban and natural catchments is that flows in urban catchments are often restricted by inlets and other appurtenances that have a limited flow capacity. Such devices can cause water to pond and follow different flow paths making it difficult to predict the hydrologic response without accounting for such effects.

- In urban catchments there is a high probability that water will travel from an \( i \)th order conduit directly into another conduit of the same order. In order to allow for this phenomenon a conduit travel time scaling factor, \( a_i \), was derived to represent the probability that such a transition can occur. This scaling factor acts to increase the travel time in any give conduit order, causing additional dispersion in the predicted hydrologic response.
• Typical overland flow paths for a drop of water before it reaches a sewer system are changing. Traditional engineered flow paths are designed to direct flow from pervious areas onto impervious areas and into the underground sewer system. In recent times these flow paths are increasingly being re-engineered to allow water to flow from impervious to pervious regions to allow additional infiltration and natural treatment. Existing GIUH formulations in the urban setting (e.g. Gironas et al, 2009) only account from flow from impervious regions. It has been shown in this paper that overland flow from pervious regions can make an important contribution to the hydrologic response, particularly as storm intensity increases. The impact of allowing flow to travel from pervious to impervious regions or vice-versa is highly dependent on storm intensity. For low intensity storms routing flow from impervious to pervious regions can significantly reduce the peak of the hydrograph and total runoff volume but for larger storms this effect is diminished.

Although water moves faster in urban systems than in natural systems, our understanding of these heavily engineered systems has been much slower in developing. The approach presented here and the recent work of others (Gironas et al, 2009) provides a foundation for better understanding the hydrology and hydraulics in complex urban systems. The development of IUHM is in its infancy but its probabilistic basis and simplicity gives it power as tool for prediction in urban catchments particularly when limited input and calibration data are available. There is much scope for further research using this tool. In the companion paper (Cantone and Schmidt, 2010) the dispersion mechanisms in urban catchments are explored and IUHM is adapted to determine uncertainty in the predicted hydrologic response induced by uncertainty in its probabilistic input parameters.
3.9 References


Metropolitan Water Reclamation District of Greater Chicago (MWRDGC), Tunnel and reservoir plan (TARP), 1999.


Chapter 4: Dispersion and Uncertainty in Urban Catchments

This Chapter presents the research adapted from the paper entitled; “Dispersion mechanisms and the effect of parameter uncertainty on hydrologic response in urban catchments.” This article, authored by Joshua Cantone and Arthur Schmidt, was submitted to the internationally renowned Water Resources Research journal on 17th March 2010 as a companion paper to the one presented in Chapter 3 following review by the doctoral committee.

The understanding of natural watersheds has been improved by considering the mechanisms that contribute to dispersion in the predicted hydrologic response. In this chapter we seek to gain similar understanding by considering the dispersion mechanisms that contribute to urban catchments. In addition, this chapter describes the methodology used to track uncertainty in the input parameters for IUHM through the model, culminating in the prediction of the uncertainty produced in the predicted direct runoff hydrograph. The probabilistic framework that IUHM is based provides a perfect platform for tracking uncertainty and it is one the key features of the model. Unlike traditional deterministic models for urban catchments IUHM allows the user to quantify the uncertainty in their prediction that is introduced by uncertainty in the input parameters for the model. The methodology for tracking uncertainty allows broader application of the model; particularly in situations when limited input data are available (see Chapter 5).

4.1 Abstract

The link between river network structure and hydrologic response for natural watersheds has been the subject of ongoing research for the past 30 years. In this chapter we investigate the link between sewer network structure and hydrologic response in urban catchments. It has been shown in natural watersheds that there are dispersion mechanisms that contribute to the impulse response function of the catchment: hydrodynamic dispersion, geomorphologic dispersion and hydrodynamic dispersion. We introduce a fourth dispersion mechanism, intra-state dispersion, that accounts for
the variance in conduit (e.g. slope, length, diameter etc.) and overland region input parameters (e.g. slope, area, imperviousness etc.) within an order. This dispersion mechanism is found to be the second largest contributor to the total dispersion in urban catchments, contributing less than hydrodynamic dispersion, but more than kinematic and geomorphologic dispersion. These dispersion mechanisms are incorporated in the Illinois Urban Hydrologic Model, which is a recently developed probabilistic approach for predicting the hydrologic response in highly urbanized catchments. Furthermore, an uncertainty analysis is performed to help better understand the uncertainty in the predicted hydrologic response that is introduced by spatial variation in conduit and overland input parameters. It is identified that conduit slope and length are the greatest sources of uncertainty in the predicted direct runoff hydrograph for the CDS-51 catchment in the Village of Dolton, IL, and the CDS-36 catchment in the City of Chicago, IL.

4.2 Introduction

In natural watersheds, the link between network structure and flow dynamics has been methodically explored during the past three decades in the literature (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980; Kirshen and Bras, 1983; Rinaldo et al., 1991; Yen and Lee, 1997; Saco and Kumar, 2002). Rodriguez-Iturbe and Valdes (1979) initialized the now commonly referred-to geomorphologic instantaneous unit hydrograph (GIUH) approach that links the hydrologic response of a catchment to its network structure using geomorphic stream-order information. In highly urbanized catchments, the GIUH approach has been seldom applied. Rodriguez et al. (2003) applied the GIUH approach in the urban setting for the first time, and since then a few others (Rodriguez et al 2005, Lhomme et al 2004, and Gironas et al 2009) have explored its application. The models developed in these studies are highly dependent on high resolution Urban DataBanks (UDB’s) containing detailed information on building footprints, street cross-sections, sewer systems etc. These detailed input data are not commonly available in the United States, particularly for existing combined sewer overflow (CSO) systems that in some cases are over 100 years old.
Most recently, Cantone and Schmidt (2010, see Chapter 3) adapted the traditional GIUH approach developed for natural watersheds so that it can be readily applied in highly urbanized catchments through development of the Illinois Urban Hydrologic Model (IUHM). Unlike the traditional GIUH approach, IUHM convolutes a series of non-linear network impulse response functions (rather than instantaneous unit hydrographs) to generate the direct runoff hydrograph for the catchment based on a time series of rainfall. For simplicity, Cantone and Schmidt (2010, see Chapter 3) assumed an exponential distribution for the travel time probability density function (PDF), with the mean travel time being calculated using mean values for catchment parameters in each order.

An important contribution to the GIUH literature was made by Rinaldo et al (1991) who derived a physically based model for the travel time distribution using the advection dispersion equation (Henderson, 1966) to describe flow through individual streams. As part of their work, Rinaldo et al (1991) found that under the assumption of spatially invariant celerity and hydrodynamic dispersion coefficient throughout the basin, hydrodynamic dispersion and geomorphologic dispersion are the two primary mechanisms contributing to the variance of travel times. Following on from this work, Saco and Kumar (2002) relaxed the assumption of spatially invariant hydrodynamic parameters and found that the presence of spatially varying celerities introduces a third mechanism, kinematic dispersion, which may contribute to the variance of travel times. Understanding these mechanisms and their relative contributions at different scales, allowed for improved understanding and prediction of the hydrologic response in natural watersheds.

In this research we explore the dispersion mechanisms that contribute to the variance of travel times in urban catchments, with an aim to improve our understanding and prediction of the hydrologic response of such catchments. The relative contributions of hydrodynamic, geomorphologic and kinematic dispersion in urban catchments are explored and a fourth mechanism allowing for variance in hydrodynamic parameters within an order is introduced. This fourth mechanism, referred to as intra-state dispersion, is found to be significantly larger than both hydrodynamic and kinematic dispersion in urban catchments.
One way to consider the accuracy of a model is by assessing its uncertainty. A model is a simplification of reality; it cannot describe all the relevant variables of the process precisely, making model output uncertainty inevitable (Lei and Schilling, 1994). The overall uncertainty of a model is inherently dependant on the uncertainty of the inputs, but may also be affected by uncertainty in model structure, model parameters and undetected numerical errors. Typically it is difficult to trace parameter uncertainty through deterministic models because of the need to describe the variance in each deterministic parameter. This task is made easier in stochastic models because it is not necessary to model every conduit, inlet and subcatchment, greatly reducing the number of inputs that contribute to the uncertainty. The stochastic nature of the IUHM provides a perfect platform for examining the effects of parameter uncertainty on the direct runoff hydrograph (DRH). Input parameters for the IUHM are generated probabilistically with an expected value and variance determined for each parameter in each order (based on the Strahler ordering scheme; see Strahler (1957)). In the inaugural version of IUHM, Cantone and Schmidt (2010, see Chapter 3) utilized only the expected value of each input parameter, but in this chapter we extend the model to utilize the variance, enabling IUHM to produce not only the expected value of each point on the DRH but also the variance. In this way the model will provide a means of quantifying the uncertainty in the DRH introduced by the spatial variation in input parameters such as overland slope, imperviousness, and conduit slope, length and diameter.

Few studies have looked at the uncertainty introduced by input parameters using the GIUH approach. The most relevant study was conducted by Wang and Tung (2006) who developed a framework for assessing the uncertainty of GIUH-based flow hydrographs introduced by spatial variability of overland/channel roughness and slope in natural watersheds. Wang and Tung (2006) used the modified Harr probabilistic point estimation method (Chang et al., 1995) to determine the statistical features of the design flow hydrograph. Gaussian linearly constrained simulations were then used for stochastic generation of design flow hydrographs so that the method could be applied to conduct reliability analysis on hydraulic structures.

Many authors have conducted studies on the causes of uncertainty in natural watershed models (e.g., Dawdy et al., 1972; Garen and Burges, 1981; Schilling, 1984;
Melching et al, 1987). The most comprehensive of these studies was that conducted by Melching et al (1987) who evaluated the effects of uncertainties in data, model parameters and model structure. Melching et al (1987) used three uncertainty analysis methods, Monte Carlo simulation, mean-value and advanced first-order second moment analyses, and tested them on the HEC-1 and RORB watershed models. In this chapter we present a methodology for quantifying uncertainty in prediction of the hydrologic response using the probabilistic approach employed by IUHM. This is marked departure from the kind of uncertainty analysis conducted by Melching et al (1987), which was based on assessing uncertainty in deterministic watershed models.

The rest of the chapter is organized as follows. Section 4.3 provides a review of the Illinois Urban Hydrologic Model, presenting the key concepts and equations that drive the model. In Section 4.4 we discuss the evolution of the travel time distribution in GIUH based approaches before reviewing the dispersion mechanisms that contribute variance into the predicted impulse response function in Section 4.5. Section 4.6 explores the possible sources of uncertainty in the IUHM and presents an approach for tracking such uncertainties through the model. In Section 4.7 we present a case study investigating the relative contribution of dispersion mechanisms and the primary sources of uncertainty for two urban catchments in Cook County, IL. Finally, we summarize the key findings of the paper and present some avenues for ongoing research in this area.

4.3 Review of Illinois Urban Hydrologic Model (IUHM)

Cantone and Schmidt (2010, see Chapter 3) postulated that the theory of Rodriguez-Iturbe and Valdes (1979) – that the hydrologic response of a natural watershed is inherently linked to the topological structure of the river network – could be adapted for urban sewer systems. An urban catchment of order, \( \Omega \), can be considered as a collection of paths, \( W \), that a water drop may follow from when it falls on the catchment until it reaches the outlet of the sewer system. Each path, \( w \), is defined by a sequence of states, either overland \( (x_{oi}) \) or conduit \( (x_{ci}) \), that represent the path that the drop of water may take. For each path there is a corresponding distribution of travel time that can be combined with the probability of a drop of water taking a certain
path, \( P(w) \), to derive the network impulse response function, \( u(t',t)[L/T] \), for the catchment:

\[
u(t',t) = \sum_{w \in W} \left[ f_{x_i,perv}(t') \ast f_{x_i,perv}(t') \ast f_{x_i,nep}(t') \ast f_{x_i}(t') \ast f_{x_i}(t') \ast \ldots \ast f_{x_i}(t') \right] \cdot P(w) \quad (3.4)
\]

where \( \ast \) denotes a convolution integral, \( f_{x_i}(t') \) is the travel time probability-density-function in state \( x_i \), and \( w \in W \), \( W \) being the path space \( W = \{x_{oi,perv}, x_{oi,perv}{imp}, x_i, x_j, \ldots, x_{\Omega}\}, (i = 1, 2, \ldots, \Omega) \). The direct runoff hydrograph for the watershed, \( Q(t) \), is then given by

\[
Q(t) = \sum_{t=0}^{\infty} \left[ u(t',t) \cdot q_{L_i}(t) \right] \cdot A \quad (3.5)
\]

where \( A \) \([L^2]\) is the area of the watershed and excess rainfall, \( q_{L_i}(t)[L/T]\).

Cantone and Schmidt (2010, see Chapter 3) assumed that the travel time PDF follows an exponential distribution with a mean travel time, \( \bar{T}_{x_i}(t) \), such that:

\[
f_{x_i}(t') = \frac{1}{\bar{T}_{x_i}(t')} \exp \left( \frac{-t'}{\bar{T}_{x_i}(t')} \right); \forall t'
\]

(3.6)

where \( t \) relates to the rainfall intensity and \( t' \) is the time associated with the travel time PDF.

Derivation of the mean travel time in each state is critical to the IUHM model. Cantone and Schmidt (2010, see Chapter 3) assumed that the travel time PDF follows an exponential distribution with a mean travel time, \( \bar{T}_{x_i}(t) \), such that:

\[
\mu_{x_i} = \frac{Q_{x_i} K_{x_i}^c(h)}{B_{x_i}(h) R_{x_i}(h)} \quad (3.7)
\]

where \( B_{x_i}(h) \) is the width at the surface of the channel, \( K_{x_i}(h) \) is given by the expression \( K_{x_i}(h) = \frac{1}{n_{x_i}} A_{x_i}(h) R_{x_i}(h)^{2/3} \) (for S.I. units) with \( A_{x_i}(h) \) and \( R_{x_i}(h) \) being the...
area and hydraulic radius respectively, and $K'_x (h)$ is given by 
$K'_x (h) = \frac{\partial K_x (h)}{\partial h}$.

Using this celerity the mean travel time for a flood wave in a given state $x_k$ can be calculated:

$$T_{x_k} = \frac{L_{x_k}}{\mu_{x_k}} \quad (3.8)$$

where $L_{x_k}$ is the mean length the flood wave propagates within the state.

In the formulation postulated by Cantone and Schmidt (2010, see Chapter 3), a rain drop can fall on a pervious or impervious rectangular overland region. Rain falling on a pervious region is assumed to flow onto an impervious region before reaching an artificial gutter channel that feeds the inlets to the underground sewer system. The travel time, $\bar{T}_{xo,perv}(t)$, corresponding to flow across the pervious region is

$$\bar{T}_{xo,perv}(t) = \frac{3}{5} \left( \frac{n_{o,perv}(1-\overline{imp})AP_{OA}}{N_{i}L_{x_i}S_{o,perv}\mu_{o,perv}(t)^{2/3}} \right)^{3/5} \quad (4.1)$$

where $n_{o,perv}$ is the Manning’s roughness coefficient for the pervious overland plane, $S_{o_i}$ is the mean $i$th-order overland slope, $\overline{imp}$ is the average imperviousness of the $i$th-order overland flow plane, $P_{OA}$ is probability that a drop of excess rainfall will fall on an $i$th-order overland region (initial state probability), $L_{x_i}$ is the average length of the $i$th-order conduit, $N_{i}$ is the number of $i$th-order conduits, and $q_{o,perv}(t)$ is the excess rainfall rate falling on pervious overland flow plane (zero until all depressions are filled and then equal to $i(t) - f(t)$ thereafter, where $f(t)$ is the Green and Ampt (1911) infiltration rate).

Similarly, the mean travel time, $\bar{T}_{xo,imp}(t)$, over the impervious region of the overland flow state is given by:

$$\bar{T}_{xo,imp}(t) = \frac{3}{5} \left( \frac{n_{o,imp}\overline{imp}AP_{OA}}{N_{i}L_{x_i}S_{o,imp}\mu_{o,imp}(t)^{2/3}} \right)^{3/5} \quad (4.2)$$
where $n_{o,imp}$ is the Manning’s roughness coefficient for the impervious overland plane, and, $q_{o,imp}(t)$, the excess rainfall rate falling on impervious overland flow plane (zero until all depressions are filled and then equal to the rainfall intensity thereafter).

For rain initially falling on the pervious portion of the overland plane there is an additional travel time, $T_{xo,perv}(t)$, associated with flow across the impervious portion of the overland plane

$$T_{xo,perv}(t) = \frac{3}{5} \frac{\bar{n}_{i,imp} \left(1 - \bar{n}_{i,imp}\right) AP_{OA}}{N_i L_{c, xo} S_{o, perv} q_{perv}(t)^{2/3}}$$

Flow from overland region that reaches the artificial gutter is assumed to be distributed simultaneously into rectangular and circular inlets in proportion to their number in a given order. In addition to the overland flow from the given time step, the inlet may also receive surcharge flow from the previous time step due to insufficient inlet or conduit capacity. Flow through the inlets is calculated according to the guidelines provided by Guo (1997) for simulating flow through on-grade inlets.

An $i$th-order conduit receives external flow from the inlets for the $i$th-order overland regions and also inflow from lower order conduits. Conduits are assumed to be circular in shape and convey flow until the conduit reaches its effective pipe-full capacity when the depth is $0.82D$, where $D$ is the diameter of the conduit (Chow, 1959). If the flow entering the conduit is greater than its pipe-full capacity, it is assumed to become pressurized and excess flow is stored and added to the flow entering the $i$th-order inlet in the next time step. If the flow entering the conduit does not exceed the pipe-full capacity, the flow depth is determined from Manning’s equation under the Kinematic Wave assumption ($S_o = S_f$). The flow through the conduit, $Q_{c, i}(t)$, and geometric properties of the conduit ($n_c$: Manning’s roughness coefficient for the conduit, $S_{c, i}$: average $i$th-order conduit slope, $D_{c, i}$: average $i$th-order conduit diameter) are used to determine the travel time in the $i$th-order conduit based:
\[
\bar{T}_{i_{ci}} (t) = \frac{T_{ci} B_{ci} (t) \frac{1}{n} \bar{A}_{ci} (t) \bar{R}_{ci} (t)^{2/3}}{Q_{ci} (t) \left[ \partial \left( \frac{1}{n} \bar{A}_{ci} (t) \bar{R}_{ci} (t)^{2/3} \right) / \partial \bar{h}_{ci} (t) \right]}
\]  

(3.16)

where \( \bar{h}_{ci} (t) \) is the \( i \)th-order conduit flow depth, \( B_{ci} (t) \) is the \( i \)th-order width at the surface of the conduit, \( \bar{R}_{ci} (t) \) is the \( i \)th-order conduit hydraulic radius and \( \bar{A}_{ci} (t) \) is the \( i \)th-order conduit area.

### 4.4 Travel Time Distribution

Since Rodriguez-Iturbe and Valdes (1979) first documented the GIUH approach, much debate has ensued on what form the travel time probability density function should take. Rodriguez-Iturbe and Valdes (1979) assumed the travel time distribution was exponential, with the mean travel time calculated based on a constant characteristic velocity for the entire basin. Gupta et al (1980) used a uniform distribution, before Cheng (1982) combined the exponential and uniform distributions. Agnese et al. (1988) developed a distribution from experimental results, while Jin (1992) suggested that the gamma distribution yields better results than the traditional exponential distribution.

Representing a significant departure from these traditional travel time distributions, Rinaldo et al (1991) developed a physically based model for the travel time distribution. They utilized an advection-dispersion equation to describe the flow through individual streams as derived by Henderson (1966):

\[
\frac{\partial h_{x_i}}{\partial t} + \mu_{x_i} \frac{\partial h_{x_i}}{\partial x} = C_{x_i} \frac{\partial^2 h_{x_i}}{\partial x^2}
\]

(4.4)

where \( h_{x_i}, \mu_{x_i}, \) and \( C_{x_i} \) are the flow depth, kinematic/diffusive wave celerity and coefficient of hydrodynamic dispersion for a given state \( x_i \).

The celerity can be computed from Equation (3.7) and the coefficient of hydrodynamic dispersion can be computed using:

\[
C_{x_i} = \frac{K_{x_i} (h)^2}{2B_{x_i} (h)Q_{x_i}}
\]

(4.5)
Rinaldo et al (1991) used Laplace transforms to derive a corresponding travel
time distribution for each state:

\[ f_{s_k}(t') = \frac{L_{s_k}}{\sqrt{4\pi C_{s_k}t'^3}} \exp \left\{ \frac{(L_{s_k} - \mu_{s_k}t')^2}{4C_{s_k}t'} \right\} \]  
(4.6)

This expression for the travel time distribution of individual states can be used to
obtain the Laplace transform for the network response but it has no analytical inverse.
For the special case of spatially invariant celerity and dispersion coefficient, Rinaldo et al
(1991) were able to derive an analytical expression for the network travel time
distribution. Saco and Kumar (2002) relaxed this assumption, and used the Path
Approximation Method to derive an expression for the network impulse response
function:

\[ f(t') = \sum_{w \in W} P(w) \frac{L_w}{\sqrt{4\pi C_w t'^3}} \exp \left\{ \frac{(L_w - \mu_w t')^2}{4C_w t'} \right\} \]  
(4.7)

where the mean length of path \( w \) is:

\[ L_w = \sum_{k \in w} L_{s_k} \]  
(4.8)

the equivalent celerity preserving the mean travel time for each path is:

\[ \mu_w = \frac{L_w}{E(T_w)} = \frac{L_w}{\sum_{k \in w} \frac{L_{s_k}}{\mu_{s_k}}} \]  
(4.9)

and the equivalent hydrodynamic dispersion coefficient, preserving the variance of
travel times over each path is:

\[ C_w = \frac{\mu_w^3 \text{Var}(T_w)}{2L_w} = \left( \sum_{k \in w} \frac{L_{s_k} C_{s_k}}{\mu_{s_k}^3} \right) \frac{\mu_w^3}{L_w} \]  
(4.10)

Saco and Kumar (2002) showed that this formulation for the travel time
distribution, incorporating the effects of spatially varying celerity within a path,
provided an improved representation of the impulse response function for natural
watersheds. This formulation also provided a basis for assessing the relative contributions of different dispersion mechanisms. The inaugural version of IUHM (Cantone and Schmidt, 2010, see Chapter 3) used the simple exponential distribution to describe the variance in travel time in each state. In this chapter we utilize the formulation of Saco and Kumar (2002) to determine the network impulse response function (see Equation 4.7) for urban catchments. Saco and Kumar (2002) used regression models to derive the spatial variation in reference flow velocity and depth throughout the basin. In contrast, we use Equations (3.7) and (4.5) to determine the dynamic wave celerity and hydrodynamic dispersion coefficient in each ith-order conduit and overland flow region based on statistical moments derived from a deterministic set of input parameters (e.g. conduit slope, length, diameter etc.). This removes the need for historical regression models and allows wide application of the model.

4.5 Dispersion Mechanisms

4.5.1 Review of Dispersion Mechanisms in Natural Watersheds

In natural watersheds, there is a body of literature (Rinaldo et al., 1991, Snell and Sivapalan, 1994, Saco and Kumar, 2002) that has explored the different mechanisms that contribute to the variance of the streamflow response. Saco and Kumar (2002) provide an excellent summary of the evolution of these concepts for natural watersheds. To the authors’ knowledge there have been no attempts to identify the different dispersion mechanisms that contribute to the variance of hydrologic response of urban catchments, which we address here. Saco and Kumar (2002) identified three dispersion mechanisms that contribute to the variance of streamflow response:

1. Hydrodynamic Dispersion (\(\Delta D\)): this dispersion arises due to differences in arrival times as a result of the difference in the coefficient of hydrodynamic dispersion in each state. The contribution of the variance of the catchment arrival time due to hydrodynamic dispersion may be represented by (Saco and Kumar, 2002):
\[ \Delta_D = \frac{\mu_n^3}{2} E_n \left( \text{Var}(T_w) \right) = \frac{\mu_n^3}{L(\Omega)} \sum_{w \in W} P(w) \frac{L_w C_w}{\mu_n^3} \]  

(4.11)

where mean path length is defined:

\[ \bar{L}(\Omega) = \sum_{w \in W} P(w) \bar{L}_w \]  

(4.12)

and the equivalent network celerity is given by:

\[ \mu_n = \frac{\sum_{w \in W} P(w) \bar{L}_w}{\sum_{w \in W} P(w) \frac{\bar{L}_w}{\mu_w}} \]  

(4.13)

2. **Geomorphologic Dispersion (\( \Delta_G \))**: this dispersion arises due to differences in arrival times as a result of the difference in path lengths to the outlet. The contribution to the variance in travel time due to geomorphologic dispersion is (Saco and Kumar, 2002):

\[ \Delta_G = \frac{\mu_n}{2\bar{L}(\Omega)} \left\{ \sum_{w \in W} P(w) \left( \bar{L}_w \right)^2 - \left( \sum_{w \in W} P(w) \bar{L}_w \right)^2 \right\} \]  

(4.14)

3. **Kinematic Dispersion (\( \Delta_K \))**: this dispersion arises due to differences in arrival times as a result of the differences in the celerities in each state along any given path to the outlet. The resulting kinematic dispersion is defined (Saco and Kumar, 2002):

\[ \Delta_K = \Delta_{KG} - \Delta_G \]  

(4.15)

where

\[ \Delta_{KG} = \frac{\mu_n^3}{2\bar{L}(\Omega)} \left\{ \sum_{w \in W} P(w) \left( \frac{\bar{L}_w}{\mu_w} \right)^2 - \left( \sum_{w \in W} P(w) \frac{\bar{L}_w}{\mu_w} \right)^2 \right\} \]  

(4.16)
These three dispersion coefficients can be used to determine the total variance in the network travel time for a natural watershed:

\[ \text{Var}(T_n) = \frac{2 L(\Omega)}{\mu_n^3} \left( \Delta_D + \Delta_o + \Delta_K \right) \]  

(4.17)

4.5.2 Dispersion Mechanisms in Urban Catchments

Cantone and Schmidt (2010, see Chapter 3) identified that one of the key differences between natural stream networks and urban sewer networks is that there is a high probability that water will transition between conduits of the same order. As a result, the number of conduits in each order in urban catchments is typically larger than in natural watersheds, which in turn introduces a greater variance in the input parameters used to characterize conduits and overland regions within an order. We postulate that this variance, due to spatially varying input parameters, introduces a fourth mechanism for dispersion, one that is significant and must be accounted for when predicting the hydrologic response of urban catchments.

Each of the three dispersion mechanisms described in Section 4.6 represent a contribution to the variance in travel time of the network. It follows that an additional variance in the travel time of the network, due to the variance in input parameters within a given state, must be derived. Before we derive this additional variance, it is necessary to make some small adjustments to the equations derived by Saco and Kumar (2002) for deriving the network impulse response function based on solution of the advection-dispersion equation. Cantone and Schmidt (2010, see Chapter 3) derived a factor, \( a_i \), to account for transitions within an order in urban sewer networks:

\[ a_i = \sum_{n=1}^{N} n P_{x_i,n} \]  

(3.2)

where \( n = 1, 2, ..., N \) is the number of successive \( i \)-th-order conduits, and \( P_{x_i,n} \) is the probability of a drop water travelling through \( n \) successive \( i \)-th-order conduits.

In the inaugural version of IUHM this factor was used to scale the travel time in each \( i \)-th-order conduit. Similarly, this factor needs to be incorporated when
determining the expected value, \( E(T_w) \), and variance, \( \text{Var}(T_w) \), of the travel time in a given path \( w \), to be used in Equations (4.9) and (4.10), such that:

\[
E(T_w) = \sum_{k \in w} a_i \bar{T}_{x_k} = \sum_{k \in w} a_i \frac{\bar{L}_{x_k}}{\mu_{x_k}} \tag{4.18}
\]

\[
\text{Var}(T_w) = \sum_{k \in w} a_i^2 \text{Var}(T_{x_k}) = 2 \sum_{k \in w} a_i^2 \frac{\bar{T}_{x_k} C_{x_k}}{\mu_{x_k}^3} \tag{4.19}
\]

Note here that \( x_k \) represents either a conduit or overland state. In overland states \( a_i \) is equal to unity.

The additional variance in the travel time for each path, caused by variance in the input parameters in each state, \( \sigma_{x_k}^2 \), can be incorporated into Equation (4.19) such that:

\[
\text{Var}_{I-S}(T_w) = \sum_{k \in w} \sigma_{x_k}^2 \tag{4.20}
\]

The variance, \( \sigma_{x_k}^2 \), can be computed for each overland and conduit state numerically using the mean-value First Order Second Moment (FOSM) method as described in Section 4.6. Using this variance, a new dispersion coefficient, \( \Delta_{I-S} \), representing the dispersion due to variation in celerity and coefficient of hydrodynamic dispersion within a state can be derived:

\[
\Delta_{I-S} = \frac{\mu_{x_k}^3}{2} \frac{E_w(\text{Var}_{I-S}(T_w))}{L(\Omega)} = \frac{\mu_{x_k}^3}{2L(\Omega)} \sum_{w \in W} P(w) \text{Var}_{I-S}(T_w) \tag{4.21}
\]

This fourth mechanism can be referred to as intra-state dispersion. In this way the combined effects of kinematic and hydrodynamic dispersion could be thought of as inter-state dispersion because they refer to the dispersion that result from variation in celerity and the coefficient of hydrodynamic dispersion between states.

Combining the expressions for the four dispersion coefficients yields the total variance in the network travel time for an urban sewer network:

\[
\text{Var}(T_n) = \frac{2L(\Omega)}{\mu_{x_k}} \left( \Delta_p + \Delta_{I-S} + \Delta_G + \Delta_K \right) \tag{4.22}
\]
4.6 Uncertainty in the Illinois Urban Hydrologic Model (IUHM)

4.6.1 Sources of Uncertainty

The probabilistic approach on which the IUHM is based provides a platform for tracking uncertainty through the model. There are a number of possible sources of uncertainty in the IUHM. Firstly, there is uncertainty due to the spatial variability in input parameters for each order. Input parameters such as conduit slope, length and diameter that are used to drive travel time calculations are based on average values for each order in IUHM. In reality these parameters vary in space throughout the catchment. The probabilistic approach lumps this spatial variability among all overland regions or conduits into a mean and variance. As a result there is an uncertainty introduced into the model. Secondly, there is uncertainty introduced from missing input data. IUHM has the advantage that a full deterministic set of input characteristics is not required for each conduit and overland region within the catchment. The average input parameters for each order may be determined from a subset of the entire deterministic input data set. For example, say that conduit slopes are only known for 100 of the 150 conduits in the first order of a given catchment. The average first order conduit slope can be determined using the 100 pipes with known slope. This is a sharp contrast to a deterministic model for which the slopes for the conduits with missing information would have to be assumed. At the same time an uncertainty is introduced by the smaller sample size used to generate the input parameter for IUHM. Thirdly, there is manufacturing uncertainty and the possibility of uncertainty in the reported ‘known’ input parameters themselves, because of imprecision in manufacturing, construction and measurement.

Each of the three types of uncertainty described above may be considered as forms of data/input uncertainty. The overall uncertainty of the model may also be affected by uncertainty in the model structure, model parameters, and/or undetected numerical errors (Melching et al., 1987). By thinking carefully about the critical hydrologic and hydraulic processes involved at the urban scale and, where possible, giving them a physical basis, it is hypothesized that the uncertainty in the IUHM structure will be minimized. There remains the possibility that uncertainty in the hydrologic response will be introduced vicariously through a number of the underlying
assumptions in the model. Most prevalent are the kinematic wave assumption, which means that backwater effects are not explicitly accounted for; inability of IUHM to account for the overland network of roads and alleys that may convey flow out of the catchment; and incapacity to account for any localized obstructions within the catchment (e.g. pump, diversion weir etc.). Model parameter uncertainty is often introduced into deterministic models via calibration parameters or other parameters that lack a physical meaning.

In this chapter, we are concerned with analyzing the effects of uncertainty introduced through the spatial variation of IUHM input parameters. The spatial variability in subcatchment overland slope, imperviousness and number of contributing inlets is considered, along with the spatial variability in the conduit input parameters length, diameter and slope. These parameters were chosen because information on the spatial variability of these parameters was readily available for the study basins. There are of course other sources of spatial variability in input parameters. These include rainfall intensity, Green and Ampt infiltration parameters, and Manning’s roughness. The effects of spatial variability in these parameters are being investigated as part of ongoing research. This chapter also ignores other sources of data uncertainty, including uncertainty introduced via sample size. Although only one primary form of uncertainty is being considered in this paper, the methodology presented for tracking uncertainty through the model could be readily applied to account for any of the sources of uncertainty discussed.

4.6.2 Methodology for Tracking Uncertainty

A number of methods exist for tracking uncertainty in multivariate problems (e.g. calculation of travel time). Harr (1996) splits these methods into three categories:

- **Exact methods**: require that probability distribution functions of all component variables be known initially. Examples are numerical integration and Monte Carlo simulation. These methods have the advantage that you obtain a complete probability distribution of the random variable but are disadvantaged by the considerable computer
time that is necessary and the risk that the output may be no better than the input.

- **First-Order Second-Moment (FOSM) methods:** the basis of these methods is the truncation of the Taylor series expansion of the functions. Inputs and outputs are simply expressed as expected values and standard deviations, meaning that knowledge of higher moments and computer power are not necessary.

- **Point estimate methods (PEM):** these methods assess model output uncertainty in terms of statistical moments by evaluating model output values at specified points with the parameter sample space. The points are selected to preserve finite statistical moments of random model parameters (Wang and Tung 2009). The number of computations required for the point estimation method follows $M^N$, where $M$ is the number of points (statistical moments – usually 2) and $N$ is the number of parameters. Thus, as the number of parameters increases, so does the number of computations.

The mean-value FOSM method was selected for tracking uncertainty in IUHM because of its simplicity, the need for only the first two statistical moments (which are readily available for the input parameters being considered), and its computational efficiency. In order to track the uncertainty in input parameters through IUHM, the FOSM method is first applied to determine the variance, $\sigma^2_{E(T_{u_i})}$, in the expected value of the travel time $E(T_{u_i})$ for all conduits and overland states that results from variance in input parameters:
\[
\sigma_{E(T_{x_0, perv})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial E(T_{x_0, perv})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial E(T_{x_0, perv})}{\partial S_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial E(T_{x_0, perv})}{\partial imp} \right)^2
\] (4.23)

\[
\sigma_{E(T_{x_0, imp})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial E(T_{x_0, imp})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial E(T_{x_0, imp})}{\partial S_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial E(T_{x_0, imp})}{\partial imp} \right)^2
\] (4.24)

\[
\sigma_{E(T_{x_0, pervimp})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial E(T_{x_0, pervimp})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial E(T_{x_0, pervimp})}{\partial S_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial E(T_{x_0, pervimp})}{\partial imp} \right)^2
\] (4.25)

\[
\sigma_{E(T_{x_1})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial E(T_{x_1})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial E(T_{x_1})}{\partial S_{o_i}} \right)^2 + \sigma_{E(D_{o_i})}^2 \left( \frac{\partial E(T_{x_1})}{\partial D_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial E(T_{x_1})}{\partial imp} \right)^2
\] (4.26)

Secondly, the variance, \( \sigma_{Var(T_{x_1})}^2 \), in the second moment of the travel time, \( \text{Var}(T_{x_1}) \), for all conduits and overland states that results from variance in input parameters must be determined:

\[
\sigma_{Var(T_{x_0, perv})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial \text{Var}(T_{x_0, perv})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial \text{Var}(T_{x_0, perv})}{\partial S_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial \text{Var}(T_{x_0, perv})}{\partial imp} \right)^2
\] (4.27)

\[
\sigma_{Var(T_{x_0, imp})}^2 = \sigma_{E(L_{c_i})}^2 \left( \frac{\partial \text{Var}(T_{x_0, imp})}{\partial L_{c_i}} \right)^2 + \sigma_{E(S_{o_i})}^2 \left( \frac{\partial \text{Var}(T_{x_0, imp})}{\partial S_{o_i}} \right)^2 + \sigma_{E(imp)}^2 \left( \frac{\partial \text{Var}(T_{x_0, imp})}{\partial imp} \right)^2
\] (4.28)
\[ \sigma^2_{\text{Var}(T_{\text{var}, \text{perimp}})} = \sigma^2_{T_{i}} \left( \frac{\partial \text{Var}(T_{\text{var}, \text{perimp}})}{\partial L_{i}} \right)^2 + \sigma^2_{S_{i}} \left( \frac{\partial \text{Var}(T_{\text{var}, \text{perimp}})}{\partial S_{i}} \right)^2 \]
\[ + \sigma^2_{\text{imp}, i} \left( \frac{\partial \text{Var}(T_{\text{var}, \text{perimp}})}{\partial \text{imp}, i} \right)^2 \]  

\[ \sigma^2_{\text{Var}(T_{i,c})} = \sigma^2_{T_{i,c}} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial L_{i,c}} \right)^2 + \sigma^2_{S_{i,c}} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial S_{i,c}} \right)^2 \]
\[ + \sigma^2_{D_{i}} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial D_{i}} \right)^2 + \sigma^2_{\text{imp}, i} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial \text{imp}, i} \right)^2 \]  

\[ + \sigma^2_{N_{\text{inlet,circ}}} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial N_{\text{inlet,circ}}} \right)^2 + \sigma^2_{N_{\text{inlet,rect}}} \left( \frac{\partial \text{Var}(T_{x,c})}{\partial N_{\text{inlet,rect}}} \right)^2 \]  

Thus, the variances in the input parameters, \( \sigma^2_{T_{i}} \), \( \sigma^2_{S_{i}} \), \( \sigma^2_{D_{i}} \), \( \sigma^2_{\text{imp}, i} \), \( \sigma^2_{S_{i,c}} \), \( \sigma^2_{N_{\text{inlet,circ}}} \) and \( \sigma^2_{N_{\text{inlet,rect}}} \) all become inputs to the IUHM model and are obtained from the sample data available for the conduits and subcatchments. These variances represent the uncertainty in the parameter which may be due to its spatial variability, sample size, or manufacturing/construction/measurement. In order to be able to solve Equations (4.23) through (4.30) it is necessary to differentiate the respective travel time equations (see Equations (4.1), (4.2), (4.3) and (4.4)) with respect to each varying input parameter. In some cases it is possible to determine these derivatives analytically but in other cases it is not. For example, determination of travel time in the conduits is dependant on solving for the pipe angle using the bisection method and as a result an analytical derivative doesn’t exist. To overcome this obstacle, and for simplicity and consistency, all derivatives were determined numerically using a central finite difference scheme such that:

\[ \frac{\partial T}{\partial z} = \frac{T_{z+\Delta z} - T_{z-\Delta z}}{2\Delta z} \]  

where \( T \) represents the travel time, \( z \) is the spatially variant input parameter and \( \Delta z \) is a small (~0.0001) change in \( z \).
Combining expressions (4.23) to (4.30) allows the variance in the network impulse response function (Equation (4.7)) to be determined:

\[
\sigma_f^2(t') = \sigma_{E(T_{w, perv})}^2 \left( \frac{\partial f(t')}{\partial E(T_{w, perv})} \right)^2 + \sigma_{E(T_{w, perv imp})}^2 \left( \frac{\partial f(t')}{\partial E(T_{w, perv imp})} \right)^2 + \sigma_{E(T_{w, imp})}^2 \left( \frac{\partial f(t')}{\partial E(T_{w, imp})} \right)^2 + \sigma_{E(T_{w, imp})}^2 \left( \frac{\partial f(t')}{\partial E(T_{w, imp})} \right)^2 + \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}^2}(T_{w, perv}) \left( \frac{\partial f(t')}{\partial \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}}(T_{w, perv})} \right)^2 + \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}^2}(T_{w, perv imp}) \left( \frac{\partial f(t')}{\partial \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}}(T_{w, perv imp})} \right)^2 + \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}^2}(T_{w, imp}) \left( \frac{\partial f(t')}{\partial \sigma_{\sigma_{\sigma_{\sigma_{\sigma_{\sigma}}}}}(T_{w, imp})} \right)^2.
\]

(4.32)

The derivatives in Equation (4.32) cannot be obtained analytically because of the convolution integral and as such were once again determined numerically using the central finite difference scheme described in Equation (4.31). This methodology for determining the variance in the network impulse response function allows the variance in the direct runoff hydrograph to be determined and can be used to determine confidence intervals for the DRH. This provides a means for modelers to quantify the uncertainty in the DRH produced by IUHM that is introduced by uncertainty in the input parameters themselves.

### 4.7 Case Study

#### 4.7.1 Catchment Description

The relative contribution of the identified dispersion mechanisms as well as the impact of uncertainty on the IUHM has been analyzed for two urban catchments in Cook County, namely, Calumet Drop Shaft 51 (CDS-51) and Calumet Drop Shaft 36 (CDS-36). The CDS-51 catchment (Figure 3.3) collects combined storm and sanitary flows from a 316 ha service area in the Village of Dolton, a southern suburb of Chicago, IL. The combined sewer network contributing to CDS-51 collects storm flows from in excess of 800 inlets and conveys this flow through a network of 722 circular conduits ranging in size from 150 mm to 2150 mm. Storm and sanitary flows for a 65 ha service
area are collected from 220 inlets and conveyed through conduits ranging in size from 200 mm to 1500 mm in the CDS-36 (Figure 4.1) catchment. This catchment is located in City of Chicago. Both of these catchments represent combined sewer systems that eventually contribute flow to the Calumet system of Chicago’s Tunnel and Reservoir Plan (TARP) (MWRDGC, 1999), and have been monitored over the past three years by the United States Geological Survey (USGS).

Figure 4.1 CDS-36 catchment location plan
CDS-51 is a fifth order catchment, while CDS-36 may be classified as third order. Individual pipe orders for each pipe in the catchment were determined using a tool developed in ArcGIS. This tool was also used to determine the transition probabilities (Table 4.1) for the conduits in each catchment. Characteristics (length, slope, diameter, number of inlets) of the combined sewer system were provided by the City of Chicago and Village of Dolton for CDS-36 and CDS-51 respectively, allowing the IUHM input parameters in Table 4.2 to be determined. Subcatchment characteristics (overland slope, imperviousness, Green and Ampt infiltration parameters etc.) for CDS-51 were obtained from a high resolution synthetic urban catchment developed by Crosa (2009) and from the Cook County and City of Chicago GIS databases for CDS-36.

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<td>-</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.1 Transition probabilities for CDS-51 and CDS-36 catchments
### Table 4.2 Input data for CDS-51 and CDS-36 catchments

<table>
<thead>
<tr>
<th>Catchment</th>
<th>CDS-51</th>
<th>CDS-36</th>
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</thead>
<tbody>
<tr>
<td>Order</td>
<td>1 2 3 4 5</td>
<td>1 2 3</td>
</tr>
<tr>
<td>$N_i$</td>
<td>449 157 57 51 8</td>
<td>108 53 21</td>
</tr>
<tr>
<td>$\bar{L}_{ci}$</td>
<td>$\text{Mean}$</td>
<td>62 60 75 64 99</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>769 899 803 1193 61</td>
</tr>
<tr>
<td>$\bar{S}_{ci}$</td>
<td>$\text{Mean}$</td>
<td>0.48 0.36 0.19 0.15 0.15</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>0.35 0.21 0.10 0.02 0.03</td>
</tr>
<tr>
<td>$\bar{D}_i$</td>
<td>$\text{Mean}$</td>
<td>330 463 718 1181 2058</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>10871 29812 63519 133017 6649</td>
</tr>
<tr>
<td>$\bar{imp}_i$</td>
<td>$\text{Mean}$</td>
<td>57 58 74 60</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>370 298 247 381 15</td>
</tr>
<tr>
<td>$\bar{S}_{oi}$</td>
<td>$\text{Mean}$</td>
<td>1.12 1.17 1.14 1.04 1.00</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>0.11 0.13 0.06 0.06 0.01</td>
</tr>
<tr>
<td>$N_{\text{inlet, circ}}$</td>
<td>$\text{Mean}$</td>
<td>0.68 0.85 0.72 1.20 1.60</td>
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<td>$\text{Variance}$</td>
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<td>$N_{\text{inlet, rect}}$</td>
<td>$\text{Mean}$</td>
<td>0.34 0.33 0.33 0.18 0.20</td>
</tr>
<tr>
<td></td>
<td>$\text{Variance}$</td>
<td>0.81 1.11 0.50 0.31 0.18</td>
</tr>
<tr>
<td>$P_{\text{QAI}}$</td>
<td>0.57 0.25 0.08 0.09 0.02</td>
<td>0.48 0.35 0.17</td>
</tr>
<tr>
<td>$a_i$</td>
<td>2.65 4.01 5.88 19.25 4.50</td>
<td>3.76 5.47 11.00</td>
</tr>
</tbody>
</table>

#### 4.7.2 Dispersion Mechanisms

To examine the relative contributions of the four different dispersion mechanisms in urban catchments identified in Section 4.5, an artificial rainfall event (see Figure 4.2(a)) precipitating 55 mm of rain in 120 minutes was applied to the study catchments. This rainfall event is equivalent to a 5 year ARI rainstorm event generated under the methodology of Yen and Chow (1980). Figures 4.2(b) and 4.3(b) show a comparison between the direct runoff hydrographs generated by IUHM for CDS-51 and CDS-36 using the; (1) exponential travel time distribution (Equation 3.6), (2) travel time distribution (Equation 4.6) presented by Rinaldo et al (1991) and Saco and Kumar (2002), and (3) newly adapted travel time distribution (after Rinaldo et al 1991) incorporating parameter dispersion. There is little difference observed in the DRH for the later two formulations, however there is a marked difference between the DRH’s generated using an exponential distribution and the advection-dispersion formulations. It can be seen that the predicted DRH for the exponential distribution has a higher peak, longer time to
peak and exhibits less dispersion than the DRH predicted using the IUHM advection-dispersion formulation. This is consistent with the behavior observed in natural watersheds (see Saco and Kumar, 2002). It is worth noting that the inclusion of the additional parameter dispersion mechanism doesn’t have a marked effect on the predicted DRH, however it is shown to be a significant contributor to the total dispersion.
Figure 4.2 Plots to analyze the relative contribution of different dispersion mechanisms for the CDS-51 catchment. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph predicted by IUHM with the 90% confidence interval. (c) Comparison of the contribution of hydrodynamic dispersion (DD), geomorphologic dispersion (DG), kinematic dispersion (DK) and parameter dispersion (DP).
Figure 4.3 Plots to analyze the relative contribution of different dispersion mechanisms for the CDS-36 catchment. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph predicted by IUHM with the 90% confidence interval. (c) Comparison of the contribution of hydrodynamic dispersion (DD), geomorphologic dispersion (DG), kinematic dispersion (DK) and parameter dispersion (DP).

Figures 4.2(c) and 4.3(c) compare the relative contributions of hydrodynamic, geomorphologic, kinematic and parameter dispersion to the total dispersion, for CDS-51 and CDS-36 respectively, over the duration of the rainfall event. It is clear from this analysis that hydrodynamic dispersion is the dominant dispersion mechanism in the urban catchments investigated in this study, accounting for 72% and 86% of the total
The variance observed in CDS-51 and CDS-36 respectively over the duration of the storm. The second largest contributor to the total dispersion is parameter dispersion contributing 17% and 10% of the total. Although parameter dispersion is not found to contribute as much as hydrodynamic dispersion, it is found to make a larger contribution to the total dispersion than kinematic (6% and 2%) and geomorphologic (5% and 2%) dispersion. Saco and Kumar (2002) found that in the Vermillion River Basin kinematic, geomorphologic and hydrodynamic dispersion contributed 35%, 60% and 5% of the total dispersion in the network response function respectively. Clearly, there is a significant distinction to be made between the relative contributions made to the total dispersion of the network response function for urban catchments and natural watersheds. The reasons for this lie in a number of key differences between urban sewer networks and natural river networks.

Firstly consider the relative spatial scales in between natural and urban networks. As Cantone and Schmidt (2010, see Chapter 3) concluded, the spatial scale in urban networks is smaller than in natural networks. For example, in Table 4.2 it can be observed that the average $i$th-order conduit length for the study basins is between 35 and 100 m, whereas the average length of $i$th-order streams for four basins studied by Yen and Lee (1997) was between 1.3 and 84 km. What effect does this difference in spatial scale have on the observed dispersion in the network response? The smaller spatial scale in urban networks means that there is a smaller range in the possible path lengths through the catchment when compared to natural watersheds. This smaller range in path lengths reduces the effect of spatially varying path length and celerity which are the causes of geomorphologic and kinematic dispersion respectively. Conversely the shorter path lengths and potentially higher celebrities result in an amplification of the hydrodynamic effects in the system.

### 4.7.3 Uncertainty in the Hydrologic Response

With the IUHM equipped to track uncertainty through to the DRH, IUHM was used to further analyze the hydrologic response of the study basins CDS-51 and CDS-36. Variances in the input parameters were determined for each order based on a sample for each catchment that included all conduits and subcatchments. In this way the variance in the input parameters represents the spatial variability of the input parameters within
the catchment. As the full sample was used the variance due to missing input data is negligible. In addition it was assumed that the variance due to manufacturing or measurement error is negligible. Variances for the input parameters for CDS-51 and CDS-36 are included in Table 4.2.

IUHM was used to generate the 90% confidence interval for the DRH’s for CDS51 (Figure 4.4(b)) and CDS-36 (Figure 4.5(b)). It can be observed that the largest uncertainties are introduced in the peaks of the hydrographs, which is often the most critical point on the hydrograph. These plots highlight the ability of IUHM to predict the uncertainty in the predicted hydrograph introduced by variability in the input parameters. The capability to describe the uncertainty in the predicted runoff hydrograph is a useful tool that can be utilized by modelers when trying to predict the hydrologic response in highly urbanized catchments. It provides a means for highlighting the uncertainty in the model results, which is often difficult to quantify in deterministic modeling.
Figure 4.4 Plots to analyze the sensitivity of the hydrologic response of the CDS-51 catchment to spatial variability in input parameters for a generic rainstorm. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph predicted by IUHM with the 90% confidence interval. (c) Comparison of the contribution to the overall variance in the network impulse response function of first and second moments of travel time for each state as determined from Equations (4.23) to (4.30). (d) Comparison of the contribution of variance of IUHM input parameters to the overall variance in the conduit travel time.
Figure 4.5 Plots to analyze the sensitivity of the hydrologic response of the CDS-36 catchment for a generic rainstorm. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph predicted by IUHM with the 90% confidence interval. (c) Comparison of the contribution to the overall variance in the network impulse response function of first and second moments of travel time for each state as determined from Equations (4.23) to (4.30). (d) Comparison of the contribution of variance of IUHM input parameters to the overall variance in the conduit travel time.
4.7.4 Quantifying Sources of Uncertainty

By considering the individual components of Equation (4.32) it is possible to quantify the largest sources of uncertainty in the network impulse response function predicted by IUHM. Using a simple triangular rainfall hyetograph, the uncertainty in the DRH (predicted by IUHM) for CDS-51 (Figure 4.4) and CDS-36 (Figure 4.5) due to spatial variability in the input parameters, $L_c, S_c, D_t, \text{imp}, S_o, N_{\text{inlet,circ}}$ and $N_{\text{inlet,circ}}$, was explored. Figures 4.4(c) and 4.5(c) highlight that the uncertainty in the DRH predicted by IUHM is primarily due to uncertainty in the conduit travel time. As such it was concluded that the greatest uncertainty in the IUHM is caused by the variability in the input parameters used in determining the conduit travel time. Figures 4.4(d) and 4.5(d) highlight that conduit length and conduit slope are the largest sources of uncertainty in the conduit travel time and hence in the IUHM.

It is anticipated that the uncertainty of the model will be dependant on the nature of the catchment and storm analyzed; however, it is clear that IUHM provides an easy method for determining this uncertainty and its contributors. It must be noted that a number of input parameters were excluded from this uncertainty analysis and may in fact be larger sources of uncertainty than those included in this analysis. In particular, uncertainties in the rainfall hyetograph are hypothesized as being the input parameters to which IUHM is most sensitive. At this time this evidence is only anecdotal, but further research is being undertaken to first quantify the uncertainties in these parameters and then track those uncertainties through the model. The key is that IUHM is an easy-to-use tool for conducting such analysis.

4.8 Summary and Conclusions

Cantone and Schmidt (2010, see Chapter 3) presented the Illinois Urban Hydrologic Model (IUHM) and showed that it could be used to analyze and predict the hydrologic response of highly urbanized catchments using a unique probabilistic approach. In the inaugural version of IUHM the travel time probability density function was assumed to follow an exponential distribution, as it had successfully utilized in natural watersheds in the GIUH approach (Rodriguez-Iturbe and Valdes, 1979). In this chapter, IUHM was modified to incorporate a physically based model for the travel time
distribution. This model utilized by Rinaldo et al (1991) and later by Saco and Kumar (2002) represents the flow through individual streams using Henderson’s (1966) advection-dispersion equation. This approach was accepted in the literature as a significant contribution to the evolution of the original GIUH approach. The approach presented by Saco and Kumar (2002) has been modified for urban networks to allow for the likelihood of transitions within a given order which was found by Cantone and Schmidt (2010, see Chapter 3) to be one of the major differences between urban and natural networks.

To allow for spatially varying celerity Saco and Kumar (2002) utilized historical regression models to predict reference velocity and depth throughout the river network. We calculate celerity and hydrodynamic dispersion coefficients for each $i$th-order conduit and overland region based on the physical characteristics of the urban sewer network. Deterministic datasets containing information about conduit diameter, slope and length, and overland slope and imperviousness in all (or a subset of) conduits and overland regions are used to calculate the mean and variance in these parameters in each order. This allows for greater flexibility in application of the model.

In this chapter we analyzed the relative contributions to the total dispersion of the network response function for two urban catchments, CDS-51 located in the Village of Dolton, IL, and CDS-36 located in the City of Chicago, IL. Rinaldo et al (1991) found that when using spatially invariant hydrodynamic parameters the total variance of the network response could be characterized by geomorphologic and hydrodynamic dispersion. Saco and Kumar (2002) relaxed the assumption of spatially invariant celerity and introduced a third mechanism called kinematic dispersion. In their study they found that kinematic dispersion made a significantly larger contribution to the total dispersion than hydrodynamic dispersion but a smaller contribution (about half) than geomorphologic dispersion. It was found in this study that in urban watersheds kinematic and geomorphologic dispersion contributed less than 10% of the total dispersion observed in the network response, while hydrodynamic dispersion accounted for 72% and 86% of the total variance observed in CDS-51 and CDS-36, respectively, over the duration of the storm analyzed. A fourth mechanism of dispersion, parameter dispersion, was introduced to account for the spatial variance in
input parameters used to generate the celerity and hydrodynamic dispersion coefficient in each ith-order conduit and overland region. We found that parameter dispersion accounted for 17% of the total dispersion in CDS-51 and 10% of the total dispersion in CDS-36. The addition of this fourth mechanism was not found to have a marked effect on the predicted DRH however its contribution to the overall variance in the network response should not be discounted.

The probabilistic nature of the IUHM provides a platform for tracking uncertainty in the hydrologic response of urban catchments. IUHM has been modified to account for the uncertainty in its input parameters. The mean-value first-order second-moment method was used, in conjunction with a central finite difference approximation for determining numerical derivatives, to track uncertainty in input parameters such as conduit slope, length and diameter, through the IUHM. As a result, IUHM is capable of predicting the uncertainty in the predicted DRH and can even be used as a tool for conducting sensitivity analysis. As an example, it was shown that for the CDS-51 and CDS-36 catchments in Cook County, IUHM is most sensitive to the spatial variability in conduit slope and conduit length. In addition, IUHM was utilized in predicting the 90% confidence interval for the inflow hydrographs for each catchment and provided insight when comparing the predicted response to that measure by the USGS.

By developing an approach for tracking uncertainty through the IUHM, this model not only provides a means for predicting hydrologic response in urban catchments, but also provides modelers with a tool for quantifying the uncertainty in that predicted response. Ongoing research is being conducted to explore the effects of using a sub-sample of the full deterministic input dataset to calculate the necessary inputs (see Table 4.2) needed to run IUHM. This type of analysis will become valuable when trying to predict the hydrologic response of urban catchments that have limited or missing input data. In a recent survey of over 50 municipalities in Cook County it was identified that many combined sewer atlases were missing the necessary upstream and downstream invert data and other input data required to build a deterministic model. If only a sub-sample of the full dataset is necessary to obtain a good prediction of the response, it will remove the need for time consuming and expensive survey or the need to make unqualified assumptions about the missing data. In addition, the ability of
IUHM to track uncertainty through the model will be used to quantify impact of the more uncertain dataset on the predicted hydrograph.
4.9 References


Metropolitan Water Reclamation District of Greater Chicago (MWRDGC), Tunnel and reservoir plan (TARP), 1999.


Chapter 5: IUHM and Uncertain Input Data

This chapter presents the research adapted from the paper entitled; “A probabilistic approach for predicting the hydrologic response of highly urbanized catchments with uncertain input data.” This article, authored by Joshua Cantone and Arthur Schmidt, is to be submitted to the internationally renowned ASCE Journal of Hydrologic Engineering in early 2010 following submission of the companion papers in Chapters 3 and 4.

This chapter logically builds on the uncertainty analysis presented in Chapter 4. The incorporation of variance into the travel time distribution and the adaptation of the IUHM to allow uncertainty to be tracked through the model, open the door for a plethora of new research. This chapter uses IUHM to test the hypothesis that the input parameters required to run IUHM may be determined using a sub-set of the deterministic subcatchment and conduit parameter sets without significantly affecting the mean or variance in the predicted hydrologic response. Showing that this hypothesis is true would allow IUHM to be applied when the input data for the catchment are limited or missing, and would reduce the need to perform expensive surveying of the system or make unqualified simplifying assumptions.

5.1 Abstract

The IUHM has been shown capable of accurately predicting the hydrologic response of highly urbanized catchments using a probabilistic approach that requires, as input, the mean and variance of parameters including conduit slope, overland slope, subcatchment area and imperviousness. Ideally the first two statistical moments of each of these input parameters would be calculated using the deterministic data for all conduits and subcatchments in the system. In reality, such detailed information is not always available, is uncertain, or time cannot be afforded to delineate all of the sub-areas within the catchment. IUHM was designed so that it could be used in such situations under the hypothesis that the mean and variance of the input parameters could be determined using only a sub-set of the full deterministic dataset. This hypothesis is tested in this chapter and it is shown that a random sample capturing as little as 30% of
the subcatchments and conduits in the CDS-51 catchment (located in the Village of Dolton, IL) can be used to generate the mean and variance in the \(i\)th-order conduit slope, overland slope, subcatchment area and imperviousness without significantly reducing the accuracy or increasing the uncertainty of the predicted hydrologic response.

5.2 Introduction

One of the most common problems hydrologic and hydraulic modelers face is lack of complete or certain input data. It is difficult to predict how an existing sewer system will behave if the input information required to run the model being used is uncertain. Engineers may choose to overcome this problem in a number of ways. Ideally a field survey could be designed and conducted to obtain the missing input data, whether it be as simple as lifting manhole covers to determine inverts or as complex as conducting a soil survey to determine relevant soil properties. No matter what type of survey is being conducted it is likely to be expensive and too often budget constraints limit the amount of surveying that can be performed. In such cases, engineers are forced to make simplifying assumptions. What is being modeled will typically drive the assumptions that are made. For example, consider an urban catchment being analyzed for a given storm event. Furthermore, assume that this catchment has limited information on conduit slopes and has yet to have its contributing service area delineated. Budget and time constraints are such that it is infeasible to complete a field survey to determine the missing conduit slopes and/or to delineate the subcatchment contributing to every inlet or catch basin in the system. Typically in this situation an engineer would simplify the sewer system to allow the model to be built in an efficient and timely manner. This may involve delineating subcatchments on a coarser scale, only modeling pipes of a certain diameter and higher, and/or assuming a typical slope for all conduits being modeled. All of these simplifications introduce uncertainty into the model (see Cantone and Schmidt, 2009) and using existing deterministic simulation packages (InfoSWMM (MWH Soft, 2004), HEC-HMS (US Army Corps of Engineers, 1989), InfoWorks (Wallingford, 2002), etc.) it is difficult or impossible to track the uncertainty from such assumptions through the model. The Illinois Urban Hydrologic Model (IUHM) developed by Cantone and Schmidt (2010a, see Chapter 3) overcomes
these limitations and provides an alternative to the commonly used simulation packages.

IUHM is a probabilistically based model for analyzing the hydrologic response of highly urbanized catchments. The model is capable of predicting the direct runoff hydrograph (DRH) at the outlet of a catchment for any given rainfall hyetograph. Cantone and Schmidt (2010b, see Chapter 4) further developed this model to track uncertainty in its input parameters through the model such that a variance is predicted for each point on the DRH. As a result, IUHM provides an ideal tool for assessing the effects that uncertain input data can have on the predicted hydrologic response of an urban catchment.

In this chapter we use IUHM to test a number of key hypotheses that will allow broader application of the model, particularly in cases where input data are uncertain. One of the key advantages of IUHM over traditional deterministic models is that input characteristics need not be known for every conduit and overland region in the catchment. It is hypothesized that a sub-sample of this information may be used to generate the mean and variance in the input parameters required to run the model. This hypothesis is tested in this chapter by modeling the CDS-51 catchment in the Village of Dolton, IL, using varying degrees of sub-sampling, ranging from the use of as little as 5% of the available input data to using all of the available input data. IUHM is used to predict the hydrologic response of CDS-51 using these different input datasets and to track the uncertainty introduced by the different degrees of sub-sampling. This analysis can be used to help answer the question; what degree of sub-sampling is required to produce an acceptable prediction of the DRH?

Sub-samples of the input data are generated randomly, which introduces an additional uncertainty. In order to understand the magnitude of this uncertainty a Monte Carlo Simulation is performed to determine how the sub-sampled input datasets vary due to randomness. This analysis will aid in answering the question; what percent of the input data uncertainty is due to sampling error and how does it compare to the uncertainty introduced by the spatial variation in the parameters themselves?
The rest of the chapter is organized as follows. Section 5.3 reviews the Illinois Urban Hydrologic Model before we introduce the CDS-51 catchment to be studied in this chapter, in Section 5.4. In Section 5.5, we present a methodology for sub-sampling the input data and trace the affects of this sub-sampling into the direct runoff hydrograph predicted by IUHM for eleven different degrees of sub-sampling. Section 5.6 analyzes the additional uncertainty introduced by the randomness of the sampling process and determines its impact on the predicted hydrologic response. Section 5.7 considers the effects of sub-sampling on prediction of the total catchment area before the paper is summarized in Section 5.8.

5.3 Review of the Illinois Urban Hydrologic Model

The roots of the Illinois Urban Hydrologic Model (IUHM) can be found in the pioneering work of Rodriguez-Iturbe and Valdes (1979) that linked the topological structure of river networks to hydrologic response of natural watersheds. Cantone and Schmidt (2010a, see Chapter 3) showed that the topological structure of sewer systems can be utilized in predicting the hydrologic response of highly urbanized catchments using the IUHM. IUHM is a probabilistic approach that is based on the likelihood that a drop of rainfall will follow one of a finite number of pre-defined paths through a network. Paths are defined as a sequence of overland and conduit states. The number of possible paths \(2^{\Omega}\) is dependent on the order, \(\Omega\), of the catchment, which is determined by applying the Strahler (1957) ordering scheme to all conduits in the catchment. Probabilities of a drop of water transitioning from one conduit to another are determined based on the topological structure of the network and are combined with initial state probabilities (i.e. the probability of a drop of water falling in an overland region contributing to an \(i\)th-order conduit) to determine the path probability, \(P(w)\).

Fundamental to the approach is derivation of the mean and variance of travel time in each overland and conduit state within the catchment. These travel times are determined based on the kinematic wave assumption following the theory postulated by Yen and Lee (1997) and built upon by Saco and Kumar (2002). Cantone and Schmidt (2010a, see Chapter 3) provide details on derivation of the mean travel time for pervious, impervious and pervious-impervious overland and conduit states. Cantone and Schmidt
(2010b, see Chapter 4) use the first-order second-moment method to determine the variance in travel time in each state based on the variance in input parameters; overland slope, imperviousness, conduit slope, conduit length, conduit diameter and the number of inlets contributing to each overland region.

Using the first two statistical moments of travel time in each state, a network impulse response function for the catchment can be derived:

\[
f(t') = \sum_{w \in W} P(w) \frac{\bar{L}_w}{\sqrt{4\pi C_w t'^3}} \exp \left( \frac{(\bar{L}_w - \mu_w t')^2}{4C_w t'} \right)
\]

(4.7)

where \( \bar{L}_w = \sum_{k \in w} \bar{L}_{k_w} \) is the mean length of path \( w \), \( \mu_w = \frac{\bar{L}_w}{E(T_w)} = \frac{\bar{L}_w}{\sum_{k \in w} \mu_{k_w}} \) is the equivalent celerity preserving the mean travel time for each path, and

\[
C_w = \frac{\mu_w^3 \text{Var}(T_w)}{2\bar{L}_w} = \left( \sum_{k \in w} \frac{\bar{L}_{k_w} C_{k_w}}{\mu_{k_w}^3} \right) \frac{\mu_w^3}{\bar{L}_w}
\]

is the equivalent hydrodynamic dispersion coefficient, preserving the variance of travel times over each path. The path celerity and hydrodynamic dispersion coefficient are derived based on the mean and variance of the travel time in each state. For details see Chapter 4.

Cantone and Schmidt (2010b, see Chapter 4) used the FOSM method to derive the variance in the network impulse response, in turn allowing the variance to be determined at each point on the direct runoff hydrograph:
\[ \sigma_f^2(t') = \sigma_{E(T_{x_0,perv})}^2 \left( \frac{\partial f(t')}{\partial E(T_{x_0,perv})} \right)^2 + \sigma_{E(T_{x_0,perv})}^2 \left( \frac{\partial f(t')}{\partial T_{x_0,perv}} \right)^2 + \sigma_{E(T_{x_0,imp})}^2 \left( \frac{\partial f(t')}{\partial T_{x_0,imp}} \right)^2 + \sigma_{V_{x0,perv}}^2 \left( \frac{\partial f(t')}{\partial \Sigma_{x_0,perv}} \right)^2 + \sigma_{V_{x0,imp}}^2 \left( \frac{\partial f(t')}{\partial \Sigma_{x_0,imp}} \right)^2 \]

This approach will be used in this paper to examine the variance in the DRH for different levels of sub-sampling within the catchment and to compare these to the variance caused by the heterogeneity of subcatchments and conduits of a given order.

### 5.4 Study Basin

The effects of using different degrees of sub-sampling to generate input data and the uncertainty introduced by the sampling will be analyzed for the Calumet Drop Shaft 51 (CDS-51) catchment located in the Village of Dolton, IL. CDS-51 (Figure 3.3) collects combined storm and sanitary flows from a 316 ha service area, and conveys them through a combined sewer network of 722 pipes ranging in diameter from 150 mm to 2150 mm. This catchment was selected because of the quality of the input data available. Conduit slopes, lengths and diameters were provided by the Village of Dolton for all 722 pipes, and overland sub-catchment characteristics (slope, imperviousness and soil properties) were derived for in excess of 800 sub-catchments utilizing the high resolution synthetic urban catchment developed by Crosa (2009). The quality of the input data is vital to this research as it reduces the uncertainty in the “known” parameter set. The key input parameters required for IUHM to model the CDS-51 catchment are provided in Table 4.2. The sample means and variances shown in Table 4.2 were determined from the input information available for all of the pipes and subcatchments in the catchment.
5.5 Sub-sampling of Input Data

A significant advantage of the IUHM is that it is not necessary to have data describing every conduit and overland flow region in the catchment of interest when determining the mean and variance for the input parameters. It is possible to use a subset of these data to determine the mean and variance of the input parameters. This is advantageous in a number of ways. Firstly, if the input characteristics are not known for every conduit and overland region in the catchment, IUHM could still be used to predict the hydrologic response based on the sub-set of information available. Secondly, there is potential to save time and money in not having to conduct surveys to determine the missing information. Thirdly, it may not be necessary to delineate the sub-catchment contributing to each inlet in the system, which can often be laborious. Similar potential benefits may be obtained using other deterministic models by aggregating subcatchments and skeletonizing the conduit network. However, these approaches linearly combine parameters describing non-linear processes, potentially introducing systematic error into the simulations. Cantone and Schmidt (2009) showed that such approaches can significantly change the predicted hydrologic response of the catchment. Finally, it would not be necessary to make unqualified assumptions (e.g. assume a ‘representative’ value for missing data) for missing input parameters, and avoiding the introduction of the uncertainty that such assumptions bring.

A series of sub-samples were created using different sub-sets of the full data set, ranging from the inclusion of 5% of the conduits in the systems to all of the conduits in the system. Note here that this sub-sampling is only used for generation of input parameters relating to conduits and subcatchments in the basin. The entire network is still used to generate transition and path probabilities which are fundamental to the model. Each sub-sample was defined by the number of conduits used to generate input parameters in the sample. Because each conduit has an inlet or junction at the upstream end and an overland flow region associated with that region, sampling a set of conduits implies sampling the corresponding overland flow regions. For example, the 5% sub-sample includes 22 first-order conduits, 8 second-order conduits, 3 third-order conduits, 3 fourth-order conduits and 2 fifth-order conduits. In contrast the full dataset is comprised of 449 first-order conduits, 157 second-order conduits, 57 third-order
conduits, 51 fourth-order conduits and 8 fifth-order conduits. Table 5.1 identifies the number of conduits included in each order for the range of sub-samples tested. A minimum of two samples were used in each order to ensure that a mean and variance in the sample could be determined.

### Table 5.1 Sub-sample characteristics for CDS-51

<table>
<thead>
<tr>
<th>Sub-sample</th>
<th>Number of Conduits in i-th order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>5%</td>
<td>22</td>
</tr>
<tr>
<td>10%</td>
<td>45</td>
</tr>
<tr>
<td>20%</td>
<td>90</td>
</tr>
<tr>
<td>30%</td>
<td>135</td>
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<tr>
<td>40%</td>
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<tr>
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</tr>
<tr>
<td>90%</td>
<td>404</td>
</tr>
<tr>
<td>100%</td>
<td>449</td>
</tr>
</tbody>
</table>

A random number generator was used to determine which conduits would be used to generate the mean and variance of the input characteristics in each order for the sub-sample being considered. It is important to note that by randomly selecting the conduits to be included in a sample we are introducing a sampling error. The sampling error reflects the fact that if the sample was to be randomly generated $N$ times, the result could be $N$ different datasets.

Careful thought was afforded to determining the input parameters that should be calculated using the sub-sample. Cantone and Schmidt (2010b, see Chapter 4) allowed for spatial variation in the following input parameters:

- $L_{si}$: $i$th-order conduit length
- $S_{si}$: $i$th-order conduit slope
- $D_i$: $i$th order conduit diameter
• \( \text{imp}_i \): imperviousness of \( i \)th order overland flow region

• \( S_{oi} \): slope of the \( i \)th order overland flow region

• \( N_{inlet} \): number of inlets contributing to an \( i \)th-order conduit

Based on examination of sewer maps for in excess of 50 municipalities in Chicago and its surrounding suburbs, of these input parameters, it is likely that the conduit length, conduit diameter and number of inlets contributing to each conduit will be known. On the other hand it is unlikely that all of the inverts or conduit slopes in the sewer system will be known. Thus, the only input parameters generated using the sub-sample are conduit slope, imperviousness, and overland slope. Furthermore, in order to determine the imperviousness and slope of the overland regions, subcatchments would need to be delineated for each inlet in the system. The fact that the subcatchments must be delineated introduces a further unknown input parameter, \( A_i \), the area contributing an \( i \)th-order conduit, which is critical in calculating the initial state probability \( P_{O Ai} \) and the total area of the catchment, \( A \). The initial state probability is defined:

\[
P_{O Ai} = \frac{\text{(total area draining directly into conduits of order } i)}{\text{(total basin area)}}
\]  

(5.1)

It is important to note that there is good possibility that the area contributing to the upstream junction of an \( i \)th-order conduit may be zero. This represents the situation where there are no inlets collecting flow at the junction upstream of the conduit of interest. In CDS-51 there are 258 (out of 449) non-zero first order areas, 92 (out of 157) non-zero 2nd order areas, 32 (out of 57) non-zero 3rd order areas, 35 (out of 51) non-zero 4th order areas and 6 (out of 8) non-zero 5th order areas. In order to prevent a bias being introduced into calculation of the first two statistical moments for the area of a given subcatchment by the zero values, such values were excluded from selection in the sub-sample for area. In that way, when only a sub-sample of subcatchments are delineated the initial state probability can be approximated:
\[ P_{OA} = \frac{N_{i,\text{non-zero}} \overline{A}_i}{\sum_{i=1}^{N} N_{i,\text{non-zero}} \overline{A}_i} \quad (5.2) \]

where \( \overline{A}_i \) is the average (sample mean) area contributing to a non-zero area overland region contributing to an \( i \)th-order conduit and \( N_{i,\text{non-zero}} \) is the number of non-zero areas contributing to conduits in the \( i \)th-order. It is important to note, that the denominator in Equation (5.2) is an estimate of the total catchment area, and hence we are effectively estimating the total area of the catchment. This has significant implications as it is possible that a by-product of this sub-sampling technique could be a useful means for estimating the total area of urban catchments (see Section 5.7).

Random input parameter sub-sets were generated for the 11 different degrees of sub-sampling, and the mean and variance of the conduit slope, overland slope and imperviousness for each order were determined. The initial state probabilities and total catchment areas were determined using Equation (5.2). These newly generated input parameters were used as inputs for the IUHM model, allowing a direct runoff hydrograph (DRH) for each degree of sub-sampling to be predicted (Figure 5.1). An artificial triangular rainfall hyetograph precipitating 55 mm of rainfall over a two-hour period was used to drive the model.

It is clear from Figure 5.1 that different degrees of sub-sampling can affect the shape and timing of the DRH and the predicted peak flow. The largest variation in the DRH can be observed when using sub-samples that include less than 50% of the available data set. However, it is evident that all degrees of sub-sampling less than 30% appear to converge around the DRH generated using all of the available data and fall within the 95% confidence interval predicted by IUHM for the 100% sample. On this basis, it could be concluded that it is possible to accurately predict the DRH for the CDS-51 catchment using as a sub-sample that includes as little as 30% of the available input data. This represents a significant saving in the time that would be required to develop the input parameters required to predict the hydrologic response of the catchment.
Figure 5.1 Comparison of the direct runoff hydrograph for CDS-51 for 11 different degrees of sub-sampling using one random sample. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph for the 11 different degrees of sub-sampling. Error bars represent the 95% confidence interval when using 100% of the available data. (c) Zoom to the peak of the DRH. (d) Zoom to the tail of DRH.
One of the key features of IUHM is the ability of the model to track the uncertainty in its input parameters through the model in order to predict the uncertainty in the DRH. The variances calculated for each input parameter are representative of the spatial variation in those parameters within an order. IUHM was used to track the uncertainty introduced by the spatial variation in the input parameters for each sub-sample. In order to compare how this spatial variation affects the DRH for each degree of sub-sampling, consider the mean and standard deviation of the predicted peak flow (Figure 5.2 and Table 5.2) for each of the 11 sub-samples. Figure 5.2 highlights that once 60% of the available input data are included in the sub-sample the predicted mean and standard deviation in the peak flow are within 5% of the peak flow predicted by IUHM using 100% of the available data. The coefficient of variation for the peak flow is similar across all degrees in sub-sampling despite the perturbation in the peak flow. This is an indication that the variance predicted in the DRH is highly dependent on the magnitude of the predicted flow.

Figure 5.2 Mean and 95% confidence for the peak flow predicted by IUHM using different fractions of the available input data for the rainfall hyetograph shown in Figure 5.1(a) and for one random sample
Table 5.2 Statistical moments for the peak flow for different degrees of sub-sampling

<table>
<thead>
<tr>
<th>Level of Sub-Sampling</th>
<th>Mean Peak Flow (m$^3$/s)</th>
<th>St. Dev. in Peak Flow (m$^3$/s$^2$)</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>14.62</td>
<td>2.02</td>
<td>13.82%</td>
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<tr>
<td>10%</td>
<td>18.15</td>
<td>1.98</td>
<td>10.91%</td>
</tr>
<tr>
<td>20%</td>
<td>19.51</td>
<td>1.54</td>
<td>7.87%</td>
</tr>
<tr>
<td>30%</td>
<td>17.94</td>
<td>2.05</td>
<td>11.42%</td>
</tr>
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<td>2.44</td>
<td>11.94%</td>
</tr>
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</tr>
<tr>
<td>80%</td>
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<td>2.70</td>
<td>12.91%</td>
</tr>
<tr>
<td>90%</td>
<td>21.01</td>
<td>2.72</td>
<td>12.95%</td>
</tr>
<tr>
<td>100%</td>
<td>20.69</td>
<td>2.69</td>
<td>13.00%</td>
</tr>
</tbody>
</table>

5.6 Effects of Sampling Error

The results presented in Section 5.5 are representative of one random sample for each sub-set of data. In reality the results could change significantly depending on the randomly generated sub-set. In order to quantify the uncertainty introduced by the sampling error associated with the sub-sampling, a Monte-Carlo analysis was used to generate 1000 sub-sets of input parameters for each degree of sub-sampling, and the contribution of uncertainty due to the sampling error was compared to the uncertainty caused by spatial variation in the selected input parameter for each order. Figures 5.3 to 5.6 quantify the relative contributions of variance in $imp_i$, $S_{oi}$, $S_{ci}$, and $A_i$, respectively, for each order. The contributors to the uncertainty are threefold:

- Variance in $imp_i$, $S_{oi}$, $S_{ci}$, or $A_i$ due to the spatial variability in the input parameters. Within any given order there is a range of possible values for conduit slope, overland slope etc. IUHM uses the mean of each parameter to describe the expected value among all overland regions or conduits of a given order. The variance due to spatial variability of the input parameters uses the variance of the parameter among all regions or conduits of a given order in a FOSM analysis to determine the uncertainty produced by using the mean to represent all conduits or regions of a given order.
• Variance in $imp_i$, $S_{ni}$, $S_{ci}$, or $A_i$ due to variability in the sample mean for each generation. This variance quantifies the variance in the sample mean calculated over the 1000 generations of the Monte-Carlo simulation. Each generation produces a sample mean and variance for the input parameters being investigated. This variance is representative of the variance around the sample mean. Elementary sampling theory has already established the effects of this kind of variance, but is presented here for completeness.

• Variance in $imp_i$, $S_{ni}$, $S_{ci}$, or $A_i$ due to variability in the sample variance determined for each generation. This variance quantifies the variance in the sample variance calculated over the 1000 generations of the Monte-Carlo simulation. It quantifies the uncertainty that sampling introduces into the variance due to spatial variation in the input parameters. This variance is inherently a part of the total variance used in the FOSM method to calculate the uncertainty from spatial variability. This estimated variance is affected by the degree of sub-sampling and needs to be considered for uncertainty analysis.

For each of the input parameters, across all orders and independent of the fraction of available input data used, it is evident that in the vast majority of cases the greatest source of uncertainty is caused by the spatial variation of the input parameters themselves. As expected the overall variance decreases with increasing sample size, but of that variance the variance caused by spatial variation appears to converge once 30% of the available data are used. The variance due to sampling error increases with increasing order, and is shown to be the largest for 5th order pipes. This is to be expected given the small sample size (8) available for this order.

Of the four input parameters tested it appears that the conduit slope introduces the greatest variance, with standard deviation being at least 1.5 times the mean. The second most variance is observed in the average area contributing to each i-th-order catchment followed by the imperviousness and subcatchment slope.
Figure 5.3 Comparison of the relative contributions of variance to prediction of the average imperviousness of an ith-order overland region. Numbers in each column represent the percent of the variance due to spatial variation in imperviousness in each order.
Figure 5.4 Comparison of the relative contributions of variance to prediction of the average slope of an \( i \)th-order overland region. Numbers in each column represent the percent of the variance due to spatial variation in overland slope in each order.
Figure 5.5 Comparison of the relative contributions of variance to prediction of the average slope of an \( i \)th-order conduit. Numbers in each column represent the percent of the variance due to spatial variation in conduit slope in each order.
Figure 5.6 Comparison of the relative contributions of variance to prediction of the average area of an $i$th-order overland region. Numbers in each column represent the percent of the variance due to spatial variation in area in each order.
To further test the effect of sampling, the mean and variances determined for each input parameter using the Monte Carlo simulation, were used in the IUHM to generate a DRH for CDS-51 for each level of sub-sampling. The predicted DRH’s are shown in Figure 5.7. Figure 5.7 highlights that repeating the sampling process 1000 times causes the predicted DRH for each degree of sub-sampling to converge to the DRH predicted using 100% of the available data. If we zoom in on the peak of the hydrograph (Figure 5.7(c)) it can be seen that there are slight differences in the peak of the hydrograph for each level of sub-sampling. This difference is reduced as the sample size increases, and it can be seen that once 20% of the available data are utilized the difference is almost indistinguishable. Even at the 5% level the difference in peak flows is only 0.05 times the standard deviation at the 100% level, and hence the difference in peaks is not statistically significant.

A further question is how does the additional variance due to the sampling error impact the uncertainty in the DRH predicted by IUHM? In order to visualize this, the mean of the peak flow and its 95% confidence interval are plotted in Figure 5.8 and statistical moments for the peak flow are tabulated in Table 5.3 for each degree of sub-sampling. Figure 5.8 and Table 5.3 highlight that the additional uncertainty introduced by randomness does not propagate through to cause increased uncertainty in the DRH.

Intuitively one might expect that because the overall variance in each input parameter has increased (as a result of randomness) there would be more variance in the predicted DRH for smaller degrees of sampling. This is observed (see Table 5.3) in the variances for each degree of sub-sampling. The predicted variance in the peak flow of the DRH decreases as the degree of sub-sampling increases.
Figure 5.7 Comparison of the direct runoff hydrograph for CDS-51 for 11 different degrees of sub-sampling using a 100 generation Monte-Carlo analysis. (a) Rainfall hyetograph for an artificially generated triangular storm event precipitating 55 mm of rain over a two-hour period. (b) Direct runoff hydrograph for the 11 different degrees of sub-sampling. Error bars represent the 95% confidence interval when using 100% of the available data. (c) Zoom to the peak of the DRH (error bars depicting 95% confidence interval are off the scale and are been clipped). (d) Zoom to the tail of DRH (error bars depicting 95% confidence interval are off the scale and are clipped).
Figure 5.8 Mean and 95% confidence for the peak flow predicted by IUHM using a 1000 generation Monte-Carlo simulation, using different fractions of the available input data for the rainfall hyetograph shown in Figure 5.7(a).

Table 5.3 Statistical moments for the peak flow for different degrees of sub-sampling using input parameters generated using Monte-Carlo analysis

<table>
<thead>
<tr>
<th>Level of Sub-Sampling</th>
<th>Mean Peak Flow (m$^3$/s)</th>
<th>St. Dev. in Peak Flow (m$^3$/s)$^2$</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
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<tr>
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<td>2.69</td>
<td>13.00%</td>
</tr>
</tbody>
</table>

5.7 Determination of Catchment Area

It was alluded to in Section 5.5 that the sub-sampling technique proposed for generating inputs for the IUHM could be used as a tool for determining the overall catchment area. In order to accurately determine the service area contributing to a storm/combined sewer it is necessary to delineate the sub-area contributing flow to every inlet or catch basin connected to the system. Typically this is done based on topographic maps describing the land surface and road layout, aerial photography, the layout of the sewer system itself and engineering judgment. This process can become laborious for large urbanized catchments. By randomly sampling a sub-set of the
conduits in each order it is possible to repeat the subcatchment area delineation process for a sub-set of the entire catchment. The total area of the catchment can then be approximated as:

\[ A = \sum_{i=1}^{\Omega} N_{i, non-zero} \bar{A}_i \]  

(5.3)

In order to quantify the effect of adopting such an approach, a Monte-Carlo simulation was performed for CDS-51 incorporating the sub-sampling methodology described in Section 5.5. For each degree of sub-sampling, 1000 random sub-sets were generated and the total catchment area corresponding to each generation was determined using Equation (5.3). The results of this analysis are shown in Figure 5.9, which displays the mean and 95% confidence interval in the catchment area for all 11 degrees of sub-sampling. The mean of the predicted catchment area stabilizes once 30% of the available data are used and 95% confidence interval shrinks as the sample size increases. On this basis it is possible to conclude that the catchment area can be predicted using Equation (5.3) by delineating subcatchment areas for as little as 30% of the system.

Figure 5.9 Mean and 95% confidence for the total catchment area predicted from Equation (5.3) using a 1000 generation Monte-Carlo simulation, using different fractions of the available input data.
5.8 Summary and Conclusions

The Illinois Urban Hydrologic Model (IUHM) developed by Cantone and Schmidt (2001a, 2010b) provides a unique probabilistic approach for determining the hydrologic response of highly urbanized catchments and is capable of quantifying the uncertainty in that response due to variance in its input parameters. In order to run the IUHM it is necessary to determine the mean and variance for input parameters (e.g. conduit slope, length and diameter, and overland, slope, imperviousness, and area) in each order. Calculations of the first two statistical moments for these parameters for the CDS-51 catchment were initially determined using the deterministic values available for every conduit and overland region in the catchment. We have shown that it is not necessary to use all of the available data to generate a reasonable prediction of the DRH. Instead, a sub-set of the available input may be randomly sampled and used to generate the mean and variance of the input parameters in each order. Eleven degrees of sub-sampling were tested and it was shown that as little as 30% of the available input data can be used whilst still predicting a DRH that is comparable to one produced using all of the available data. As a result it may be concluded that in catchments where there are limited or missing input data it is possible to simulate the hydrologic response of the catchment by randomly sub-sampling as little as 30% of the system. This will reduce the time needed to delineate subcatchments and could reduce the costs associated with surveying to collect the unknown data. Alternatively it could mean that if only 50% (for example) of the input data are known then it can be used to predict the hydrologic response of the system without having to survey for the missing data.

There is an additional uncertainty introduced into the input parameters due to sampling error. A Monte-Carlo simulation was performed to determine the magnitude of the variance introduced through sampling error and to compare this magnitude to the variance caused by spatial variation in the input parameters. This analysis was conducted for the CDS-51 catchment and it was concluded that the spatial variance in the input parameters themselves is the biggest source of uncertainty. It was shown that this uncertainty is dominant and that the uncertainty due to sampling error does not propagate significantly through to the DRH when at least 30% of the available input data are utilized. Of the four input parameters assessed, spatial variability in conduit
slope was shown to introduce the greatest degree of uncertainty into the model, followed by spatial variability in subcatchment area, overland slope and imperviousness. A by-product of this research was the realization that the total catchment area for the system being assessed can be accurately predicted by delineating as little as 30% of the subcatchments in the catchment.

5.9 References


Chapter 6: Final Conclusions and Future Research

The Illinois Urbana Hydrologic Model (IUHM) is a robust probabilistic approach for determining the hydrologic response in highly urbanized catchments. It has been shown that this model provides improved understanding of the hydrologic and hydraulic processes that are most important at the urban scale. IUHM is capable of predicting the shape, timing and peak of the direct runoff hydrograph at the outlet of urban sewer systems as well as the widely accepted SWMM model and is comparable to the response observed in the field. This is achieved using a fraction of the input data and catchment detail that is required by an all-pipe-all-subcatchment SWMM model. This model is not only capable of predicting the mean hydrologic response of an urban catchment, but can be further utilized to quantify the uncertainty in the predicted response. This feature of the model allows the greatest sources of uncertainty in the model to be identified and quantified. It was shown that for CDS-51 and CDS-36 catchments, the greatest source of uncertainty in the model is a result of spatial variation in the conduit slope and length. This uncertainty is a result of IUHM assuming that these heterogeneous input parameters can be represented by a mean and variance in each order. The ability of IUHM to track uncertainty in the hydrologic response caused by variance in input parameters was used to show that the IUHM is capable of accurately predicting the hydrologic response of catchments with limited or uncertain input data. It was shown that as little as 30% of the complete dataset of subcatchment and conduit input parameters could be used without significantly changing the predicted hydrologic response.

IUHM is a model with great potential and provides modelers with an alternative to traditional deterministic models when the goal is to predict the response at the outlet of the sewer system. Its ability to track uncertainty gives IUHM an advantage over deterministic models as it allows its users to quantify the uncertainty in the predicted response. The model, however, is only in its infancy and still has great scope for improvement and development. In the discussion that follows a number of ideas for improving IUHM are discussed.
At present the mean travel times for overland and conduit flows are calculated based on the kinematic wave equation. As a result, the model is incapable of accounting for backwater effects, ponding and other storage effects, which are often important in storm/combined sewer systems. Once conduits become pressurized in IUHM, and the maximum flow is reached, any excess water is stored and re-introduced into the system at the next time step. This is a simplification of reality. It has been observed that in some large storm events IUHM over predicts the peak discharge, and under predicts the time to peak and width of the tail of the DRH. It is hypothesized that this is due to the model’s inability to account for backwater, ponding and storage effects. Some anecdotal evidence has already been collected to support this theory. For some of the storms where this behavior has been observed, simulations have been run in InfoSWMM using both kinematic wave and dynamic wave approaches. Comparison of the DRH’s for IUHM, InfoSWMM (Kinematic) and InfoSWMM (Dynamic) illustrate that the predicted DRH from IUHM lies somewhere in between these two models. IUHM is unable to account for the ponding and backwater effects explicitly like the InfoSWMM (Dynamic) model does, but its artificial ponding mechanism once conduits become pressurized results in a better prediction than a purely kinematic wave approach.

Closely linked to the inability of IUHM to account for backwater, ponding and storage effects, is the inability of IUHM to account for overland flow along roadways. Typically, stormwater is collected from inlets strategically placed along curb and gutter systems that shape roadways. When these inlets or catch basins reach capacity, water is allowed to pond and may in fact be conveyed to an inlet or catch basin downstream or to nearby waterways or pervious areas. It is hypothesized that IUHM could be modified to account for the possibility of water transitioning into states other than the traditional overland and conduit states, through the introduction of a dual drainage system that represents the conveyance along roads and other overland conveyance paths. In order to incorporate a dual drainage system into IUHM careful consideration would need to be given to the path probabilities and it is likely that a conditional transition probability would need to be incorporated.

One of the biggest assumptions in IUHM is that rainfall and infiltration are uniform in space throughout the catchment. In reality, there is the possibility of
significant variability in precipitation and soil properties across the service area for a storm/combined sewer system. In Chapter 3, it was shown that the rain gage used to generate the rainfall hyetograph was critical to the response predicted by IUHM and has a significant impact on the shape of the DRH. This assumption was made because of the lack of detailed precipitation and soils data available to depict this spatial variation at the urban catchment scale. For example, precipitation data are available in Cook County at 25 rain gages on an 8 km x 8 km grid scale. The service areas for the sewer systems being considered are at a much finer scale. Even the finer grid scale that can be obtained using radar data will result in only one or two rainfall cells spanning typical service areas. As such, in order to be able to account for the effects of spatially varying rainfall new downscaling methodologies for determining the spatial variability of rainfall at the urban catchment scale must be developed. If such methodology is developed, IUHM provides a perfect platform for incorporating and quantifying the uncertainty effects of spatially varying rainfall. Similarly, the spatial variation in soils could be considered and the resulting uncertainty quantified if finer resolution soil data were readily available.

In Chapter 5 it was shown that the total catchment area could be predicted by delineating a sub-sample of the subcatchments in the catchment. At present, IUHM does not track the variance in this area through the model. In order to further quantify the uncertainty introduced by calculating the total area using a sub-sample, IUHM could be utilized to track the variance in area could through to the DRH in the same way variance in other input parameters, like overland slope and imperviousness, is already tracked through the model.

Perhaps the biggest scope for development of the model is in adapting the model to account for other hydrologic and hydraulic processes. As mentioned in the introduction the urban landscape is constantly changing and there is a need to better understand the impacts of these changes on the hydrologic response. For example, Chicago has undergone significant change in the past 5 years in an aim to make it a greener city. Across different scales in the greater Chicago area changes have been made to the urban landscape. Roof runoff that once fed directly into the sewer system is now stored in rainwater tanks, or forced through rain gardens in a bid to reduce impervious runoff. In addition, the City of Chicago implemented a ‘rain blocker’ program to restrict
flow that enters its combined sewers to minimize basement flooding and combined sewer overflows. What impacts do these changes have on the hydrologic response? IUHM provides the perfect framework for answering this question. IUHM could be adapted to account for the impact of these best management practices on the travel time through the system, by incorporating the possibility of a greater number of paths to the outlet. Perhaps instead of going from an overland region directly into a conduit there is the possibility of water flowing from an overland region into a rain garden, then through a rain blocker and then into the sewer system. All of these kinds of small scale processes could be implemented in IUHM using the robust framework that has been developed.