NON-LINEAR OPTICAL PROPERTIES OF SEMICONDUCTOR NANO-STRUCTURES FOR SLOW LIGHT AND WAVELENGTH CONVERSION

BY

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DISSERTATION

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Abstract

The use of semiconductor nanostructures for all-optical signal processing is investigated. We first examine theoretically the utilization of quantum dots for wavelength conversion via four-wave mixing. Our results show that quantum dots with only a single bound state are more efficient than both quantum wells and quantum dots with a large number of excited states. We compare experimentally quantum dots and quantum wells with results which are consistent with our theoretical analysis. We measure the small-signal conversion of both single and multiple optical channels, and compare the results to cross-gain modulation in the same device. Our results show that four-wave mixing provides efficient, high-speed wavelength conversion in up to four, independent channels, and at speeds up to 40 GHz. Using a pulsed laser, we also examine the signal-to-noise ratio for the converted signal with our measurements showing an excellent signal to noise ratio and no patterning effect for a 25 ps pulse. To examine the theoretical limit of four-wave mixing for short pulses, we perform numerical calculations using the finite-difference beam propagation method in both a quantum dot and quantum well semiconductor optical amplifier. These calculations indicate that the quantum dot device performs better at the powers and speeds of relevance to telecommunications, but that the faster spectral hole relaxation rate of quantum wells allows for more efficient conversion of pulses less than 1 ps.

We then examine how the cross-gain modulation response of the device can be increased and demonstrate that an additional pump field can create a cavity mode in the device which suppresses carrier oscillations and extends the XGM bandwidth.
from 1 GHz to greater than 25 GHz. Finally, we look at using a cavity mode for the purpose of slow- and fast-light and theoretically demonstrate that a fast- to slow-light transition occurs at the lasing threshold. These results compare well with previous measurements, and we present our own experimental investigations utilizing both a distributed-feedback laser and a ring laser. Utilizing a ring laser, we are able to achieve a delay bandwidth product of 10 for a 10 ps pulse in a single semiconductor device.
To my family
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Chapter 1

Introduction

1.1 Motivation and Research Goals

All-optical signal-processing is an important technology for the next generation of advanced, wavelength-division multiplexed networks. Current telecommunications networks perform signal processing entirely in the electrical domain, requiring that information transmitted optically be converted to electrical signals, processed, and then reconverted back to optical signals. This optical to electrical to optical (OEO) conversion process requires extensive overhead in terms of cost and equipment for detectors to receive the data, electronic circuitry to process the data, and then lasers and modulators to re-transmit the data as light. Furthermore, this conversion process limits the speed at which optical data can be processed as the limiting factor becomes the speed of the electronic circuitry performing the processing steps, which is much slower than the total bandwidth available in a typical fiber optic cable. Thus, to create more advanced, higher-speed networks, it is important to determine methods by which these signals can be processed entirely in the optical domain and avoid the equipment and cost overhead associated with OEO conversion. Furthermore, for these techniques to prove applicable outside the laboratory and useful for future telecommunications networks, they must demonstrate functionality at speeds sufficient for the next generation of telecommunications networks by pushing to limits beyond 40 Gb/s, toward 100 Gb/s and beyond.

The workhorse of the telecommunications industry has for several decades been the
quantum well. A quantum well works by confining the carriers into a two-dimensional sheet altering the density of states and improving optical performance by reducing the carrier density needed to achieve population inversion. The advent of high-quality quantum dots has provided new opportunities for advancement due to the discrete density of states, and the localization of carriers in individual dots. These two effects cause the carrier dynamics of quantum dots to differ significantly from quantum wells. As these carrier dynamics drive the susceptibility of the device, it is important to have a well developed theoretical basis for the non-linear susceptibilities of quantum dots to determine the advantages and disadvantages of quantum dots, specifically as they relate to all-optical signal processing.

Two important applications for all-optical signal processing are wavelength conversion and optical buffering. Modern telecommunications networks rely on wavelength division multiplexing (WDM) to utilize the full bandwidth of fiber optic cables. Wavelength conversion is an important functionality as it allows the data to be transferred from one wavelength channel to another. This allows for decentralized network management, simplified routing, and a reduction in the total number of wavelengths needed to operate a network. The buffering of optical data bits is another important functionality that allows for the synchronization of clocks for different channels, the operation of phased array antennas, and the prevention of packet collision. In the optical domain, this is achieved by creating a large refractive index dispersion in the media to create a large positive or negative group-index and change the group velocity at which the optical bits propagate. These bits may then be moved forward or backward to either synchronize the clocks of multiple data streams, or to allow routing without packet collision. While wavelength conversion and slow/fast-light vary drastically from each other in application, the underlying physics for each is fundamentally the same as they are both dependent on the non-linear properties of the device media. While wavelength conversion depends on the creation of new fields
through the imaginary part of the non-linear susceptibility, slow and fast-light rely instead on the real part to create phase shifts between the multiple optical components that compose the data stream so as to change the group velocity of the optical pulses.

In all of our examinations, we will focus on quantum dots and quantum wells which have been fabricated into semiconductor optical amplifiers (SOA), a diagram of which can be seen in Fig. 1.1. A substrate, usually InP, has multiple layers of quantum dots grown on it before being capped. The capping and substrate layers are doped to create a PiN diode. Forward biased, the injected carriers create population inversion allowing optical gain in the device. Optical confinement is provided by the active media and the ridge waveguide on top of the device. For an SOA this ridge waveguide is purposefully created at an angle to the facet to reduce facet reflections and prevent lasing. While the figure shows quantum dots as the active media, they can be replaced by stacks of quantum wells.

This dissertation focuses on utilizing quantum-dot and quantum-well SOAs in these two key areas of wavelength conversion and optical buffering. Specifically, it focuses on developing a theoretical model for the non-linear susceptibilities in quantum dots that allows for easy comparison to quantum wells, and then going beyond theoretical models to demonstrate these differences experimentally. It also examines novel ways of utilizing sharp changes that occur in the susceptibilities near the lasing threshold of a device due to resonant effects.

1.2 State-of-the-Art

1.2.1 Wavelength Conversion

The prominent methods for achieving all-optical wavelength conversion are cross-gain modulation (XGM) and cross-phase modulation (XPM). Each of these techniques rely
Figure 1.1: Diagram showing the structure of a quantum dot semiconductor optical amplifier. Wave guidance through the amplifier is provided by the index contrast in the center region and the ridge waveguide. The ridge waveguide is angled to reduce facet reflection and prevent lasing. Materials shown are for InAs dots grown on InGaAsP. Other material systems can be substituted.
on amplitude modulated pump light to modulate either the gain or refractive index of the material and for that change to pattern itself onto a probe beam that is sent through the device at the same time. These modulations rely on a change in the global carrier density, and are thus limited by the rate at which the global carrier density can be modulated. This modulation speed is determined by the relatively slow carrier generation and recombination rate ($\sim 1$ ns). Previous studies in quantum dots have demonstrated that they are also limited by this slow carrier lifetime [1] rather than the spectral hole recovery rate. Other studies in quantum dots at 1300 nm [2] have shown a greatly increased bandwidth over quantum well devices, achieving 40 GHz operation by utilizing the quick relaxation from the excited states of the quantum dots. Theoretical analysis has demonstrated [3] that these carrier dynamics of quantum dots can be significantly enhanced by utilizing a third pump field to deplete different states of the dots. Additionally quantum dash devices at 1550 nm have also been investigated [4, 5] and shown to exhibit significant inter-dash communication allowing for efficient cross-gain modulation over very wide pump-probe detunings.

Active media are not the only material system being investigated for wavelength conversion. To overcome the low bandwidth of XGM and XPM, other studies focus on four-wave mixing in silicon waveguides [6–9] as it is possible to utilize the Kerr effect to achieve broadband conversion. These studies have demonstrated wavelength conversion at speeds of 40 Gb/s with low bit-error rates [6]. They also possess an excellent signal to noise ratio as silicon is an indirect band gap material with very little background stimulated emission. The drawback however is that they suffer from low efficiency due to free carrier absorption caused by the necessarily large pump powers that must be utilized. As a result state-of-the-art converters are restricted to efficiencies of less than 100% with the record efficiency achievement being -5.5 dB [9] or 28% efficient.

To take the advantages of both of the techniques above, we have focused mainly
on four-wave mixing in an active media: quantum dots. Four-wave mixing takes advantage of frequency sum and difference generation inside the non-linear media to produce a new light field. In quantum dots and wells four-wave mixing relies on fast intra-band processes and thus should have a larger bandwidth than XGM or XPM, but a lower bandwidth than that provided by the Kerr effect in silicon waveguides. Previous studies have shown that four-wave mixing is inefficient in quantum well devices due to their small non-linearities [10–12]. Theoretical predictions [13] have shown that four-wave mixing in quantum dots should allow for more efficient conversion than both quantum wells and silicon waveguides due to the high device gain and enhanced non-linearities that arise due to the carrier dynamics of the quantum dot. Furthermore, the smaller linewidth enhancement factor of quantum dots should allow for symmetric conversion, equally converting signals to higher and lower frequencies [14], which is not possible in quantum wells. While some experimental studies have shown symmetric conversion [5,15,16], previous studies directly comparing quantum wells and quantum dots [17] have shown no clear advantage for quantum dots over quantum wells. This mismatch between theory and experiment shows two main limits of the current theory for wavelength conversion in quantum dot devices. First, the underlying theories rely on complex ladder-systems of states inside the quantum dots [13,18] which makes it difficult to pull out the time constants important to functionality, and thus, hard to make a comparison between devices. Second, most theoretical analyses have focused on generic quantum dots without taking into account the wide variation that exists between quantum dots of different materials and growth methods. Real dots can have varying carrier dynamics between themselves as the coupling to the continuum states and excited state structure can vary greatly. Thus, the properties of a particular dot system must be considered.
1.2.2 Slow and Fast Light

Slow-light and fast-light have both been investigated for use as optical buffers for a variety of applications including optical data storage, phased array antennas, and clock synchronization. Methods of achieving tunable delay include electromagnetically induced transparency (EIT) [19], coherent population oscillation (CPO) [20,21], four-wave mixing (FWM) [22], photonic crystal waveguides [23], and spectral hole burning among others [24].

In all of these listed methods of slow- and fast-light, the group velocity of light, $v_g$, is altered by utilizing the large dispersion that is caused by sharp absorption peaks and dips. This dispersion causes slow- and fast-light as the group index, $n_g$, includes a large contribution from the slope of the refractive index

$$v_g = \frac{c}{n_g} = n_0 + \omega \frac{\partial n}{\partial \omega}. \quad (1.1)$$

The relation between the material refractive index dispersion, Fig. 1.2(b), and the absorption, Fig. 1.2(a), of a material is related through Kramers-Kronig relations. Figure 1.2(c) shows the group velocity associated with a sharp dip in the absorption
of a material and caused by the sharp slope in the refractive index dispersion. A dip in the absorption characteristics of a material causes a steep slope in the refractive index and thus a large positive group index and slow-light. A peak in the absorption would do the opposite, create a large negative slope in the refractive index and result in a negative group index and fast-light. As creating tunable group delay or advance requires altering the absorption characteristics of the media, large variation in the group velocity is associated with large variations in the output power of the media.

EIT allows for large delays including stopped light [25], but has two limitations for practical applications. First, these techniques operate at temperatures far below room temperature in order to preserve the quantum coherence, and second, the material systems utilized are frequently incompatible, or difficult to integrate with other telecommunications devices. CPO, FWM and spectral hole burning, while not demonstrating as large a delay as EIT, have the advantage of working at room temperature, having large bandwidths, and being easily integrated with other photonic components as commercial optical amplifiers can serve as the slow-light media. However, these techniques result in large power variations as the group velocity is tuned requiring extensive post-processing [26]. Photonic crystals have demonstrated extremely slow group velocities through enhanced non-linear processes, however as the dispersion is built into the photonic lattice, the delay is not tunable and thus not usable in the applications previously mentioned. Recent experimental evidence [27] has demonstrated that fast- to slow-light switching occurs in a gain-clamped semiconductor optical amplifier and provided a large shift in the group velocity with a minimal change in the output amplitude. The underlying physical mechanism, however, is not understood.
1.3 Thesis Overview

This dissertation will first focus on a theoretical analysis of four-wave mixing in a quantum-dot semiconductor optical amplifier. We will begin by using the fundamentals of density-matrix theory to calculate the optical non-linearities for a quantum dot device, taking into account carrier density pulsation, spectral hole burning, and carrier heating. To simplify the analysis and allow for easier interpretation of the results, our analysis will focus on quantum dots which have only a single bound state. The calculated non-linearities will then be used to determine the theoretical conversion efficiencies, and compared to experiment in a quantum dot device.

Next, two different types of quantum dots are presented and compared for the application of wavelength conversion via four-wave mixing. Selecting the type of quantum dots best suited for efficient conversion, a quantum dot and quantum well device with the same device gain are directly compared allowing us to make a sound conclusion regarding the relative non-linearities and wavelength conversion efficiencies. In all cases, theory will be used to explain the fundamental differences for the conversion efficiency in each device. Once the best media for four-wave mixing has been experimentally verified, we examine the high-speed response of these devices for both cross-gain modulation and four-wave mixing.

The high-speed response will then be further investigated for four-wave mixing by utilizing multiple probe beams to demonstrate multi-channel wavelength conversion. Important cross-talk mechanisms will be investigated and discussed, along with how they limit the total number of channels that can be converted simultaneously. We will also examine ways of increasing the dynamic range of the wavelength tuning by examining dual-pump conversion and comparing the response to that of a single pump. To prove application beyond the laboratory setting, a pulsed laser will also be utilized to determine the signal-to-noise ratio of the converted signal along with
the possibility of any patterning effects that may arise for actual pulses rather than small-signal modulated fields.

Continuing the investigation of four-wave mixing in quantum dots we will go beyond what we can probe with our pulsed laser to examine the conversion efficiency of sub-picosecond pulses. These calculations will be performed for both a quantum well and quantum dot device, and will compare an array of pulse widths and pulse energies so that the advantages and disadvantages of each media can be determined. These calculations will be aided by the characterization of real devices to help determine the necessary parameters for the simulation, and to keep them firmly grounded in reality. These calculations will help us to understand the fundamental limits of each device for four-wave mixing.

Cross-gain modulation will then be revisited as a method of wavelength conversion, to determine methods by which the bandwidth can be enhanced. Specifically, we will examine additional pump lasers impinging on a quantum dot device to see how the internal carrier dynamics of the system can be changed. Our efforts will focus on using an SOA just beneath the lasing threshold to demonstrate a cavity mode created via external pumping. This cavity mode suppresses carrier oscillations, and increases the usable bandwidth for cross-gain modulation in the device from 1 to 25 GHz. Besides presenting experimental data, an analytical model will be presented as well to elucidate the underlying physical mechanisms and their limits.

This work on using cavity modes will then be continued to examine how it can be used to help alter the group-velocity of light for a field propagating through a media at the lasing threshold. A theoretical model will be developed, and the calculated results will first be compared to experimental data for a DFB laser. The same analytical model will then be used to explain the previously observed slow-light phenomenon in a gain-clamped SOA. Pushing the limits of this technique, we will then utilize a ring laser to examine the effect of cascading multiple passes through the active media on
achieving extremely large delay bandwidth products in a single semiconductor device. Our results indicate that tunable delay above the lasing threshold increases linearly with increasing passes through the media allowing us to achieve a 100 ps delay for a 10 ps pulse.

1.4 References


Chapter 2

Theory of Four-Wave Mixing in Quantum Dots

2.1 Introduction

To begin our examination of quantum dots for all-optical signal processing, we first focus on developing a theory for their non-linear susceptibilities. Using these susceptibilities, we then examine four-wave mixing for the purpose of achieving efficient, high-speed wavelength conversion. Previous work has already established the fundamental four-wave mixing theory in quantum wells [1], but theoretical examinations of quantum dots have mostly proved cumbersome, relying on complex ladder systems that make it difficult to determine the important underlying physical parameters that drive and limit the system [2,3]. To simplify this problem, and allow for a more physical understanding of the quantum dot system, we will focus our efforts on analyzing four-wave mixing in quantum dots with only a single bound state, but which are coupled to a continuum of states in the wetting and barrier layers. Our analysis will utilize density matrix theory to calculate the non-linear susceptibilities of these quantum dots. Using these susceptibilities, the four-wave mixing efficiency of a quantum dot device will be calculated and compared with experimental results.

Quantum dots prove significantly different than quantum wells due to carrier localization, and the coupling between the bound state and the continuum of states in the barrier or wetting layers. These different carrier dynamics result in large spectral holes and increased four-wave mixing efficiency. Our analysis also demonstrates that, while previous theories have focused on spectral hole burning in quantum dots, carrier
heating plays an important role in the four-wave mixing efficiency as well. This is true in quantum dots with a single bound state due to the slow thermalization of the localized quantum dot states.

2.2 Density-Matrix Theory for Nonlinear Susceptibility of Quantum Dots with Wetting Layers

Following the method of Uskov et al. [1], we used the density-matrix approach to calculate the susceptibilities responsible for four-wave mixing. To simplify our model, we have examined quantum dots with only one bound state, taking into account transitions between the bound state and the continuum of the associated wetting layer. Furthermore, as carrier heating relies primarily on carrier-lattice dynamics, and not carrier dynamics alone, it is not expected that the results should differ greatly for quantum wells and quantum dots. Thus we ignore carrier-heating in our QD theory, and assume QW like behavior for carrier heating when we perform our final calculations. A diagram of the theorized carrier dynamics can be seen in Fig. 2.1. Each dot contains a single bound state in the conduction band. Electrons can be captured from the continuum at a rate $\tau_c$, or they may escape from the dot at a rate $\tau_e$. They may also undergo non-radiative recombination at a rate $\tau_s$. The set of density-matrix equations that describes this system includes contributions from the continuum states coupled to the dots

$$
\dot{\rho}_{cw,k} = \sum_i \frac{\rho_{cd,i}(1 - \rho_{cw,k})}{\tau_{i,k}} - \sum_i \frac{\rho_{cw,k}(1 - \rho_{cd,i})}{\tau_{k,i}} - \frac{\rho_{cw,k}}{\tau_s} - \frac{\rho_{cw,k} - f_{cw,k}}{\tau_1} + \Lambda_{cw,k} \quad (2.1)
$$
There exists only a single bound state in the conduction band which fits two electrons due to spin degeneracy. Electrons may be captured to this state from the continuum at the rate $\tau_c$, or escape to the continuum at a rate $\tau_e$. They may also undergo non-radiative recombination at a rate $\tau_s$.

where $\rho$ is the occupation probability of the state. The subscript $cd$ indicates dot conduction states and the subscript $cw$ indicates wetting layer conduction states. $k$ indicates the wave-vector in the wetting layer and runs over the quantum-well like states therein, and $i$ runs over every state in the dot ensemble, including each dot twice to account for the spin degeneracy of the states. The first sum is the sum of all carriers escaping from the $i$ dot states into the $k$ wetting layer state at the rates $\tau_{i,k}$. The second term is the reverse, the total number of carriers lost from the $k$ wetting layer state into all possible dot states at the rates $\tau_{k,i}$. The third term represents non-radiative recombination. The fourth is spectral hole burning inside the wetting layer where the occupation probability relaxes back to the Fermi distribution, $f_{cw,k}$, at a rate $\tau_1$, and the final $\Lambda$ represents carrier injection.

The density matrix equation for the quantum dots is similarly

$$\dot{\rho}_{cd,i} = -\sum_k \frac{\rho_{cd,i}(1 - \rho_{cw,k})}{\tau_{i,k}} + \sum_k \frac{\rho_{cw,k}(1 - \rho_{cd,i})}{\tau_{k,i}}$$

$$- \frac{\rho_{cd,i}}{\tau_s} - \frac{i}{\hbar} (\mu_{vc,i}\rho_{cdvd,i} - \mu_{cv,i}\rho_{vdcd,i}) E(t)$$  \hspace{1cm} (2.2)
Here, the last term is the interaction with light and $\mu$ is the transition dipole moment, $\rho_{cdvd}$ is the coherence term of the density matrix equations and $E(t)$ is the electric field of the interacting light. Other, higher order effects such as spontaneous emission and Auger recombination have been ignored in our model.

The governing equation for the coherence terms is simply

$$\dot{\rho}_{cdvd,i} = -(i\omega_i + 1/\tau_2)\rho_{cdvd,i} - \frac{i}{\hbar} \mu_{cdvd,i}(\rho_{cd,i} + \rho_{vd,i} - 1)E(t).$$

(2.3)

Here, decoherence at a rate $\tau_2$ has been included phenomenologically to account for interactions with the outside system. Since the equations for the valence band states mirror those of the conduction band they need not be explicitly stated, and can be determined simply by interchanging the subscripts $c$ and $v$ in (1)-(3).

From these density-matrix equations, the general rate equations governing the carrier density in both the dots and wetting layer can be determined by summing over all states and dividing by the volume, $V$.

$$\frac{1}{V} \sum_k \rho_{cw,k} = N_w$$

(2.4)

$$\frac{1}{V} \sum_i \rho_{cd,i} = N_d$$

(2.5)

$N_w$ represents the carrier density in the continuum and $N_d$ is the carrier density trapped inside the dots.

To perform these summations, we assume the time constants are independent of $i$ (all dots release and capture carriers equally), but dependent on $k$ as continuum states closer to the bound state should relax more easily. This allows us to determine normalized expressions for the carrier escape time, $\tau_e$, and carrier capture time, $\tau_c$, as

$$\frac{\tau_e}{C_k} = \frac{\tau_{e,k}}{NV}$$

and

$$\frac{\tau_c}{C_k} = \frac{\tau_{c,k}}{DV}$$

respectively. Here, the $k$-dependence on the carrier dynamics has been isolated in $C_k$. $D$ is the total number of states in the quantum dots per
unit volume, twice the number of quantum dots due to spin degeneracy. Similarly, \( N = \frac{1}{V} \sum_{k} C_k \) is the effective number of wetting layer states per volume. There are, of course, an infinite number of states in the wetting layer if all \( k \) states are considered, but we expect \( C_k \) to fall off with larger \( k \) values such that \( N \) will be finite. However, we expect it to fall off slowly enough that it will be nearly equal to one for wetting layer states that have significant occupation levels, allowing us to approximate \( \frac{1}{V} \sum_{k} C_k \rho_{cw,k} \approx \frac{1}{V} \sum_{k} \rho_{cw,k} = N_w \).

By inserting these expressions into the summations we find

\[
\frac{1}{V} \sum_{k} \sum_{i} \rho_{cd,i} \frac{1 - \rho_{cw,k}}{\tau_{e,k}} = \frac{1}{V} \sum_{k,i} C_k \rho_{cd,i} \frac{1 - \rho_{cw,k}}{NV \tau_e} = \sum_{i} \rho_{cd,i} \frac{N - N_w}{NV \tau_e} = N_d(1 - \frac{N_w}{N}) \tag{2.6}
\]

Using the same approach the reverse process can be calculated

\[
\frac{1}{V} \sum_{k,i} \rho_{cw,k} \frac{1 - \rho_{cd,i}}{\tau_{c,k}} = \frac{N_w(1 - \frac{N_d}{D})}{\tau_c} \tag{2.7}
\]

Combining these results with our previous results we find the following rate equations

\[
\dot{N}_w = \frac{N_d(1 - \frac{N_w}{N})}{\tau_e} - \frac{N_w(1 - \frac{N_d}{D})}{\tau_c} - \frac{N_w}{\tau_s} + \frac{I}{qV} \tag{2.10}
\]

\[
\dot{N}_d = -\frac{N_d(1 - \frac{N_w}{N})}{\tau_e} + \frac{N_w(1 - \frac{N_d}{D})}{\tau_c} - \frac{N_d}{\tau_s} + 2a(N_d)E(t). \tag{2.11}
\]

Here, the sum over the coherence terms has been replaced by

\[
a(N_d) = -\frac{i}{\hbar} \frac{1}{2V} \sum_{i} (\mu_{vc,i} \rho_{cded,i} - \mu_{cv,i} \rho_{ced,i}) \tag{2.12}
\]
the material absorption of the system [4]. When normalized and written in terms of
the occupation probabilities \( f = N_d/D \) and \( w = N_w/N \) these equations become the
same rate equations which have already been extensively used and studied [3, 5–7]
validating our starting equations.

\[
\dot{w} = \frac{D}{N} \frac{f(1-w)}{\tau_e} - \frac{w(1-f)}{\tau_c} - \frac{w}{\tau_s} + \frac{I}{qVN} \tag{2.13}
\]

\[
\dot{f} = -\frac{f(1-w)}{\tau_e} + \frac{N}{D} \frac{w(1-f)}{\tau_c} - \frac{f}{\tau_s} + 2a_n(f)E(t) \tag{2.14}
\]

Here, \( a_n \) is the absorption renormalized for the occupation probability \( f \). Importantly,
in most circumstances the number of states in the continuum is very large compared
to the number of electrons; thus, we can achieve an excellent approximation by taking
the limit that \( N_w \ll N \), and find that the rate equations become

\[
\dot{N}_w = D \frac{f}{\tau_e} - \frac{N_w(1-f)}{\tau_c} - \frac{N_w}{\tau_s} + \frac{I}{qVN} \tag{2.15}
\]

\[
\dot{f} = -\frac{f}{\tau_e} + \frac{1}{D} \frac{N_w(1-f)}{\tau_c} - \frac{f}{\tau_s} + 2a_n(f)E(t). \tag{2.16}
\]

To calculate the four-wave mixing efficiency, we must determine the susceptibilities. To do this we assume an electric field of the form
\[ E(t) = E_0 e^{-i\omega_0 t} + E_1 e^{-i(\omega_0 + \delta) t} + E_2 e^{-i(\omega_0 - \delta) t} + c.c. \quad (2.17) \]

which is pictured in Fig. 2.2. Here \( \omega_0 \) is the pump frequency, \( \delta \) is the pump-probe detuning, \( E_0 \) is the slowly varying amplitude of the pump, \( E_1 \) is that of the probe, and \( E_2 \) is the conjugate formed through non-linear mixing. Together, these electric fields will create a polarization density of the similar form

\[ P(t) = P_0 e^{-i\omega_0 t} + P_1 e^{-i(\omega_0 + \delta) t} + P_2 e^{-i(\omega_0 - \delta) t} + c.c. \quad (2.18) \]

inside the material.

As the polarization density is directly related to the dipole terms

\[ P(t) = \frac{1}{V} \sum_{j=i,k} \mu_{vc,j} (\rho_{cv,j} + \rho_{vc,j}), \quad (2.19) \]

we expect the dipole terms to also follow the same form

\[ \rho_{cv,j} = \sigma_{j,0} e^{-i\omega_0 t} + \sigma_{j,1} e^{-i(\omega_0 + \delta) t} + \sigma_{j,2} e^{-i(\omega_0 - \delta) t}. \quad (2.20) \]

Here, \( j \) includes both the \( k \) continuum states and the discrete \( i \) states, but as we assume the light-fields are set to interact with the quantum dot states, the contributions of the continuum states will be ignored in our analysis.

Due to beating between the pump and probe, we expect both the state occupation probabilities and carrier densities to beat in time as

\[ \rho_{c,j} = \tilde{\rho}_{c,j} + \tilde{\rho}_{c,j} e^{-ist} + \tilde{\rho}_{c,j}^* e^{ist} \quad (2.21) \]

\[ N_j = \tilde{N}_j + \tilde{N}_j e^{-ist} + \tilde{N}_j^* e^{ist}. \quad (2.22) \]

Again, \( j \) represents both dot and continuum states, but this time both contributions
have to be considered as we expect both the dot and wetting layer populations to oscillate. Taking these assumptions, and putting them into the density-matrix equations for the quantum-dot states we can determine the polarizations to first order in $E_0$.

$$P_0 = \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{\chi}_i(\omega_0)(\overline{\rho}_{c,i} + \overline{\rho}_{v,i} - 1)E_0 \tag{2.23}$$

$$P_1 = \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{\chi}_i(\omega_1) \times \left[ (\overline{\rho}_{c,i} + \overline{\rho}_{v,i} - 1)E_1 + (\tilde{\rho}_{c,i} + \tilde{\rho}_{v,i})E_0 \right] \tag{2.24}$$

$$P_2 = \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{\chi}_i(\omega_2) \times \left[ (\overline{\rho}_{c,i} + \overline{\rho}_{v,i} - 1)E_2 + (\tilde{\rho}_{c,i}^* + \tilde{\rho}_{v,i}^*)E_0 \right] \tag{2.25}$$

where

$$\hat{\chi}_k(\omega) = \frac{1}{\omega - \omega_k + i/\tau_2} \tag{2.26}$$

is the Lorentzian lineshape determined by the decoherence time and is responsible for homogeneous broadening. Here, we have used our previously stated assumption that the electric field does not interact with continuum states to reduce the sums to include only dot states. Therefore, to solve for these polarizations and find the susceptibilities we must determine $(\overline{\rho}_{vd,i} + \overline{\rho}_{vd,i} - 1)$ and $(\tilde{\rho}_{cd,i} + \tilde{\rho}_{vd,i})$, which can be done by performing a steady-state and small-signal analysis of the density-matrix and rate equations.

For the steady-state solution we find

$$(\overline{\rho}_{cd,i} + \overline{\rho}_{vd,i} - 1) = \left( \frac{2}{D} \frac{\tau_d}{\tau_c} N_w - 1 \right) - \frac{2i|\mu_i|^2}{\hbar^2} \overline{\rho}_{cd,i} + \overline{\rho}_{vd,i} - 1)\overline{|E_0|^2}[\hat{\chi}_i(\omega_0) - \hat{\chi}_i^*(\omega_0)] \tag{2.27}$$

where

$$\tau_d = \left( \frac{1}{\tau_e} + \frac{1}{D} \frac{N_w}{\tau_c} \right)^{-1} \tag{2.28}$$

and $N_w$ is the steady state solution for $N_w$ from (2.10) and (2.11) determined by
setting $\dot{N}_w = 0$. An examination of these equations will show that the steady state value will be ultimately determined by the injected current and the carrier lifetime including contributions from both non-radiative recombination and stimulated emission. Thus, $\overline{N}_w$ is an external parameter that is controlled via the applied current and pump power. It is important to point out that in (2.27) we have assumed that the hole dynamics mirror the electron dynamics in the system.

By comparing our result in (2.27) with the results of the same calculations done for bulk [1], it is clear that $\tau_d$ is the equivalent of a spectral-hole burning time constant for quantum dots. Due to charge localization, electrons trapped in quantum dots have no direct interaction with each other, and thus cannot redistribute their energy via carrier-carrier interactions to return to thermal equilibrium. Instead, the energy exchange must occur through the continuum, with depleted dots capturing new electrons from the continuum, and dots which are populated ejecting electrons to the continuum. $\tau_d$ represents the rate at which the quantum dot ensemble will relax to thermal equilibrium via these capture and escape dynamics. At low wetting layer carrier densities, the relaxation is limited by how quickly electrons can escape from the populated dots; however, as the carrier density in the wetting layer increases, it is the rate of carrier capture that limits the relaxation rate. The above allows us to find a steady-state expression for the occupation probabilities as

$$\left( \rho_{cd,i} + \rho_{vd,i} - 1 \right) = \frac{\left( \frac{2}{D} \frac{\tau_d}{\tau_c} \overline{N}_w - 1 \right)}{1 + \frac{2|\mu_d|^2 |E_0|^2 \langle \hat{\chi}_i(\omega_0) - \hat{\chi}_i^*(\omega_0) \rangle}{\hbar^2 \overline{N}_w - 1}}.$$  \hfill (2.29)

When the pump is turned off we expect that the dot occupation probabilities should be the same as the occupation probability under thermal equilibrium, $f$, such that $\left( \rho_{cd,i} + \rho_{vd,i} - 1 \right) = (f_{cd} + f_{vd} - 1)$. By taking $E_0 = 0$ in (2.29) we find that

$$f_{cd} + f_{vd} - 1 = \left( \frac{2}{D} \frac{\tau_d}{\tau_c} \overline{N}_w - 1 \right)$$  \hfill (2.30)
showing that the occupation probability of the dots is completely dependent on the ratio of $\tau_d/\tau_c$ and the wetting layer filling factor. All dots have the same occupation probability under thermal equilibrium because we previously assumed that all dots captured electrons at the same rate. Furthermore, by taking the derivative of (2.30) it can be shown that

$$\frac{\partial f_c}{\partial N_w} + \frac{\partial f_v}{\partial N_w} = \frac{2 \tau_d}{D \tau_c} \left(1 - \frac{\tau_d}{\tau_c} N_w D\right).$$  

(2.31)

Similar to the steady-state analysis, we perform a small signal analysis as well, and find that to first order in $E_0$

$$(\rho_{cd,i} + \rho_{vd,i}) = \frac{1}{1 - i \delta \tau_d} \left\{ \tilde{N}_w \left[ (\rho_{cd,i} + \tilde{\rho}_{vd,i} - 1) \left( -\frac{1}{D \tau_c} \tau_d \right) + \left( \frac{1}{D \tau_c} \right) \right] 
- \frac{2i \tau_d |\mu_i|^2}{\hbar^2} (\rho_{cd,i} + \tilde{\rho}_{vd,i} - 1) \left[ (\tilde{\chi}_i(\omega_1) - \tilde{\chi}_i^*(\omega_0)) E_0^* E_1 + (\tilde{\chi}_i(\omega_0) - \tilde{\chi}_i^*(\omega_2)) E_0 E_2^* \right] \right\}. 
$$

(2.32)

This result leaves us with the need to determine $\tilde{N}_w$ in order to finalize our solution. For this we return to the rate equations, (2.13) and (2.14), and perform a small signal analysis to find that

$$\tilde{N}_w = \frac{-X(L_1 + L_2)}{WY - XZ}$$

(2.33)

where

$$L_1 = i \frac{1}{DV} \sum_i \frac{|\mu_i|^2}{\hbar^2} (\rho_{cd,i} + \tilde{\rho}_{vd,i} - 1) \times
[ (\tilde{\chi}_i(\omega_1) - \tilde{\chi}_i^*(\omega_0)) E_0^* E_1 + (\tilde{\chi}_i(\omega_0) - \tilde{\chi}_i^*(\omega_2)) E_0 E_2^* ]$$

(2.34)

$$L_2 = i \frac{1}{DV} \sum_i \frac{|\mu_i|^2}{\hbar^2} (\tilde{\rho}_{cd,i} + \tilde{\rho}_{vd,i}) |E_0|^2 [\tilde{\chi}_i(\omega_1) - \tilde{\chi}_i^*(\omega_2)]$$

(2.35)
\[ W = \frac{1 - \bar{f}}{\tau_e} + \frac{1}{\tau_s} - i\delta \] (2.36)

\[ X = \frac{D}{\tau_e} + \frac{N_w}{\tau_c} \] (2.37)

\[ Y = \frac{1}{\tau_e} + \frac{1}{D} \frac{N_w}{\tau_c} + \frac{1}{\tau_s} - i\delta \] (2.38)

\[ Z = \frac{1}{D} \frac{1 - \bar{f}}{\tau_c} \] (2.39)

While this expression may seem complicated, it is fundamentally an expression which takes into account the beating of the light field in \( L_1 \), saturation from the pump in \( L_2 \), and a bandwidth determined by the carrier lifetime in the quantum dot which can escape to or be captured from the wetting layer, or recombine non-radiatively. By taking (2.32) and substituting it into (2.33) we can find an expression for the varying wetting layer carrier density

\[
\tilde{N}_w = -iX \frac{1}{DV} \sum_i \frac{|\mu_i|^2}{\hbar^2} \left( \frac{2 \tau_d}{D \tau_c} N_w - 1 \right) \left[ (\hat{\chi}_i(\omega_1) - \hat{\chi}_i^*(\omega_0)) E_0^* E_1 + (\hat{\chi}_i(\omega_0) - \hat{\chi}_i^*(\omega_2)) E_0^* E_2 \right]
\]

\[
WY - XZ + Xi \frac{1}{DV} \sum_i \frac{|\mu_i|^2}{\hbar^2} \left[ \left( \frac{2 \tau_d}{D \tau_c} \right) \left( 1 - \frac{\tau_d}{\tau_c} \frac{N_w}{D} \right) \right] |E_0|^2 \left[ \hat{\chi}_i(\omega_1) - \hat{\chi}_i^*(\omega_2) \right]
\]

(2.40)

Again we have solved to first order by assuming that

\[ (\bar{\rho}_{cd,i} + \bar{\rho}_{vd,i} - 1) = \left( \frac{2 \tau_d}{D \tau_c} N_w - 1 \right) \] (2.41)

and

\[ (\bar{\rho}_{cd,i} + \bar{\rho}_{vd,i}) = \frac{2 \tau_d}{D \tau_c} \left( 1 - \frac{\tau_d N_w}{\tau_c D} \right) \tilde{N}_w. \] (2.42)

Taking these expressions and combining them with our earlier expressions for the polarization densities, we find the pump polarization density and linear susceptibility,
\( \chi^{(l)} \), to be

\[
P_0 = \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{x}_i(\omega_0) \left( \frac{\frac{2}{D \tau_c} \hat{N}_w - 1}{1 + \frac{2i|\mu_i|^2\tau_d}{\hbar^2} |E_0|^2 (\hat{x}_i(\omega_0) - \hat{x}_i^*(\omega_0))} \right) \tag{2.43}
\]

\[
\chi^{(l)}(\omega) = \frac{1}{\epsilon_0} \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{x}_i(\omega) \left( \frac{\frac{2}{D \tau_c} \hat{N}_w - 1}{1 + \frac{2i|\mu_i|^2\tau_d}{\hbar^2} |E_0|^2 (\hat{x}_i(\omega_0) - \hat{x}_i^*(\omega_0))} \right) \tag{2.44}
\]

Similarly, we solve for the probe polarization density

\[
P_1 = \epsilon_0 \chi^{(l)}(\omega_1) E_1 + \frac{1}{V} \sum_i \frac{|\mu_i|^2}{\hbar} \hat{x}_i(\omega_1) \left( \frac{1}{1 - i\delta \tau_d} \right) \left[ \frac{2}{D \tau_c} \left( 1 - \frac{\hat{N}_w \tau_d}{\tau_c} \right) \right] \hat{N}_w E_0
\]

\[
\times \left[ (\hat{x}_i(\omega_1) - \hat{x}_i^*(\omega_0)) E_0^* E_1 + (\hat{x}_i(\omega_0) - \hat{x}_i^*(\omega_2)) E_0 E_2^* \right] E_0 \tag{45}
\]

For \( P_1 \) the induced polarization density is split into three terms. The first is the linear polarization density associated with gain or absorption in the optical amplifier. The second terms represents the non-linear interaction between the pump and probe due to carrier density pulsation. The third term is the non-linear interaction due to spectral hole burning. The polarization density \( P_2 \) is identical to that of \( P_1 \) except with the subscripts 1 and 2 interchanged. We seek a way to simplify (45) and express it as

\[
P_1 = \epsilon_0 \chi^{(l)}(\omega_1) E_1 + \epsilon_0 \chi^{CDP}(\omega_1; \omega_0, \omega_1) E_1 + \epsilon_0 \chi^{SHB}(\omega_1; \omega_0, \omega_1) E_1
\]

\[
+ \epsilon_0 \chi^{CDP}(\omega_1; \omega_2, \omega_0) \frac{E_0^2}{|E_0|^2} E_2^* + \epsilon_0 \chi^{SHB}(\omega_1; \omega_2, \omega_0) \frac{E_0^2}{|E_0|^2} E_2^*, \tag{46}
\]

so that the underlying mechanism for the different contributions are more easily identifiable, and to allow us to develop generalized susceptibilities for each mechanism.
This is achieved by taking into account the linear interaction, and then breaking apart the non-linear interactions to separate the contributions from self-interactions and conjugate-interactions.

Taking this into account, we can determine generalized susceptibilities due to carrier density pulsation and spectral hole burning as

\[
\chi^{CDP}(\omega_1; \omega_2, \omega_3) = \frac{2\epsilon_0(c\eta)^2}{\hbar\omega_0\omega_1} \times \frac{dg(\omega_0)(\alpha + i)}{[1 + i(\omega_2 - \omega_3)\tau_d][D^r_s(WY - XZ) + \frac{2c\epsilon_0c_0\frac{d}{dN_w}\tau_s|E_0|^2}{\hbar\omega_0}]} \tag{2.47}
\]

\[
\chi^{SHB}(\omega_1; \omega_2, \omega_3) = -\frac{2i\tau_d}{\hbar^2} \left[ \frac{|E_0|^2}{1 + i(\omega_2 - \omega_3)\tau_d} \right] \times \frac{1}{\epsilon_0 V} \sum_i |\mu_i|^4 \hat{\chi}_i(\omega_1) \left( \frac{2}{D} \tau_d \frac{D}{\tau_c} N_w - 1 \right) \left[ \hat{\chi}_i(\omega_3) - \hat{\chi}_i^*(\omega_2) \right]. \tag{2.48}
\]

We have simplified the expression for \(\chi^{CDP}\) by applying the identities

\[
\frac{1}{V} \sum_i |\mu_i|^2 \frac{\tau_d}{\hbar} \hat{\chi}_i(\omega) \frac{2}{D} \left( 1 - \frac{\tau_d}{\tau_c} \frac{N_w}{D} \right) = -\epsilon_0 \frac{c\eta}{\omega} \frac{dg}{dN} (\alpha + i) \tag{2.49}
\]

\[
i\tau_s \frac{1}{V} \sum_i |\mu_i|^2 \left( \frac{2}{D} \tau_d \frac{D}{\tau_c} N_w - 1 \right) \left( \hat{\chi}_i(\omega) - \hat{\chi}_i^*(\omega) \right) = \frac{2c\eta\epsilon_0\tau_s}{\omega\hbar} g(w) \tag{2.50}
\]

which have been derived by taking the similar identities from [1] and substituting the equivalent values for \((f_c + f_v - 1)\) and \(\left( \frac{\partial f_c}{\partial N} + \frac{\partial f_v}{\partial N} \right)\) in the quantum dot system identified in (2.30) and (2.31).

These identities also introduce important parameters for comparison to experiment including the linewidth enhancement factor [8], \(\alpha\), the refractive index, \(\eta\), and
the material gain, $g(\omega)$, which is calculated from (2.44)

$$g(\omega) = -\frac{\omega}{\eta_c} \text{Im}[\chi^{(0)}(\omega)] \quad (2.51)$$

### 2.3 Model for Conversion Efficiency

The theoretical results developed in the previous section determined the non-linear susceptibilities $\chi^{CDP}$ and $\chi^{SHB}$ in addition to the linear susceptibility. For our purpose of examining four-wave mixing, we will use these susceptibilities to calculate the conversion efficiency. For wavelength conversion, efficiency, $\eta_{eff}$, is defined as the power out at the new wavelength divided by the power in at the original wavelength,

$$\eta_{eff} = \frac{|E_2(L)|^2}{|E_1(0)|^2}. \quad (2.52)$$

To calculate this efficiency, we use the analytical solution developed by [9] to determine the output power at the conjugate wavelength. The analytical solution for the output intensity of the light fields after propagating through a device of length $L$ is

$$E_0(L) = e^{\overline{G}/2(1-i\alpha)}[1 + F_-(L, \delta) \frac{|E_1(0)|^2}{E_{sat}^2}]E_0(0) \quad (2.53)$$

$$E_1(L) = e^{\overline{G}/2(1-i\alpha)}[1 + F_+(L, \delta) \frac{|E_0(0)|^2}{E_{sat}^2}]E_1(0) \quad (2.54)$$

$$E_2(L) = e^{\overline{G}/2(1-i\alpha)}F_-(L, \delta) \frac{E_0(0)^2}{E_{sat}^2}E_1^*(0) \quad (2.55)$$

In these equations, $\overline{G}$ is the steady-state, integrated device gain defined as the steady state solution to

$$\frac{dG}{dt} = \frac{G_0 - G}{\tau} - (e^G - 1) \frac{|E(0)|^2}{\tau} \quad (2.56)$$
where $\tau$ is the gain recovery time, $G$ is the integrated device gain

$$G = \int_0^L \Gamma g(z, t) dz \quad (2.57)$$

and $G_0$ is the unsaturated, integrated gain. $C$ is a phenomenological parameter used to compensate for the non-plane-wave nature of the waveguide modes as we are interested in light propagating through an SOA waveguide. This value has been taken to be 0.8 [9]. In (2.57) $g(z, t)$ is the material gain, and is multiplied by the confinement factor of the waveguide, $\Gamma$, to account for the fact that the entire light field does not overlap with active media. Since in four-wave mixing the dominant light field is the pump, we took the gain at the pump wavelength when determining $\bar{G}$.

In (2.55) the terms in brackets represent the non-linear interactions, with the first being CDP, and the sum over $x$ representing all other non-linear interactions, such as spectral hole burning and carrier heating whose strengths are determined by the normalized non-linear gain coefficients $\kappa_x$. Combining (2.52)-(2.54) the FWM efficiency becomes easy to derive as

$$\eta_{\text{eff}} = e^{\mathcal{G}} \left| F_-(L, \delta) \right|^2 \left| \frac{E_0(0)^2}{E_{\text{sat}}^2} \right|^2 \quad (2.58)$$

While originally derived for a simple quantum-well model, the above, (2.52)-(2.55), can be adapted to our rigorous quantum-dot model.

To begin this adaptation, we first define the saturation field for the QD system as

$$E_{\text{sat}}^2 = \frac{\hbar \omega_0}{2 \epsilon_0 c \eta \frac{dg}{dN} \tau_s} \quad (2.59)$$

Similarly the CDP term of $F_{\pm}$ must be rewritten to account for the more complicated dynamics. This is done by comparing the above expression with the solution
for the quantum well susceptibilities calculated in [1] and our derived quantum dot susceptibilities. From this comparison we find

$$F_{QD}^\pm (L) = -Ce^{\tau - 1} 2 \left[ \frac{1 - i\alpha}{D\tau_x (WY - XZ)} + \frac{|E_0(0)|^2}{E_{sat}^2} + \sum_x \kappa_x (1 - i\alpha_x) \right] (1 \pm i\delta\tau) \right) \tag{2.60}$$

Examining (2.55) it is important to note that the non-linear gain coefficient of the CDP term is normalized to be 1. Thus, we can determine the non-linear gain coefficient for spectral hole burning by normalizing the SHB susceptibility to the CDP susceptibility. The result of this normalization is

$$\kappa_{SHB} (1 - i\alpha_{SHB}) = \frac{i2\tau_d\omega_0}{\eta\epsilon_0\tau_s} \frac{dg}{dN} \times \left[ \sum_k \frac{|\mu_k|^4}{\hbar^2} \hat{\chi}_k(\omega) \left( \frac{2\tau_{ch} N}{\tau_s} - 1 \right) \left[ \hat{\chi}_k(\omega) - \hat{\chi}_k^*(\omega) \right] \right] \left[ \sum_k \frac{|\mu_k|^2}{\hbar} \left( \frac{2\tau_d N}{\tau_s} - 1 \right) \left[ \hat{\chi}_k(\omega) - \hat{\chi}_k^*(\omega) \right] \right]. \tag{2.61}$$

Carrier heating was included in our calculation by relying on the same formulation for the non-linear susceptibility as is found in quantum wells and bulk. As shallow quantum dots have the majority of their free carriers in the wetting and barrier layer this is considered a good approximation of the actual underlying physics. Keeping with the expression for $\chi^{CH}$ found in [1] and normalizing as we did to find $\chi^{SHB}$ we find that

$$\kappa_{CH} = \frac{\tau_{ch}}{\tau_s} \frac{\partial g/\partial T}{\partial g/\partial N} \frac{\Delta E}{\hbar_c} \left( 1 + \frac{\sigma N}{g(\omega)} \frac{\hbar\omega_0}{\Delta E} \right). \tag{2.62}$$

Here $\Delta E$ is the energy difference between the chemical potential, the energy needed to add one electron to the continuum, and the energy of an electron in a quantum dot bound state. $\tau_{CH}$ is the rate at which the electron gas cools back to the lattice temperature. $\hbar_c$ is the heat capacity of the free electrons assuming a 2-D electron gas
model

\[ h_c = \frac{\pi k_b^2 T m^*}{3 \hbar^2 l} \]  \hspace{1cm} (2.63)

where \( m^* \) is the effective mass for the electrons or holes, and \( l \) is the effective height of the quantum dot layer. For our calculations, it was considered to be the distance between adjacent quantum dot layers, which for our sample was 10 nm. The free carrier absorption cross section, \( \sigma \), was calculated from the Drude model,

\[ \sigma = \frac{q^3 \lambda^2}{4\pi^2 \epsilon_0 \eta m^* \mu}, \]  \hspace{1cm} (2.64)

but was found to be too small to have an impact on carrier heating due to the low carrier concentration at which gain can be achieved in quantum dots. Instead the primary carrier heating mechanism is not free-carrier absorption, but rather the removal of the lowest energy carries via stimulated emission while higher energy electrons are injected into the sample. The ratio \( \frac{\partial g}{\partial T} / \frac{\partial g}{\partial N} \) can be found analytically for the quantum dot system by observing that \( g \propto (f_c + f_v - 1) \), and that under large bias the majority of carriers actually reside in the barrier and wetting layers. Under these conditions the derivatives can be easily taken giving an analytical solution of

\[ \frac{\partial g}{\partial T} / \frac{\partial g}{\partial N} = -\frac{N_w \Delta E}{k_b T^2}. \]  \hspace{1cm} (2.65)

When combined with the assumption that carrier heating from free carrier absorption is insignificant, this results in the expression

\[ \kappa_{CH} = \frac{3\tau_{ch} N \Delta E^2 \hbar^2 L}{\pi \tau_s (k_b T)^3 m^* h_c}. \]  \hspace{1cm} (2.66)

This allows for an analytical calculation of the non-linear gain coefficient due to carrier heating in quantum dots.
Changes in temperature also have a linewidth enhancement factor associated with them as the varying occupation probabilities change both the real and imaginary parts of the susceptibility. In a quantum dot we expect the linewidth enhancement factor due to temperature changes, $\alpha_{CH}$, to be very close to the linewidth enhancement factor due to carrier density changes, $\alpha$, as the raising and lowering of the carrier temperature serves only to change the ratio between the dot and wetting layer occupation probabilities, and thus the number of carriers in the dots. Therefore, these values were set equal to each other.

2.4 Numerical Results

For theoretical calculations to have merit, it is important that they can be easily compared and matched with experiment. For this we have performed a simple four-wave mixing experiment in a semiconductor optical amplifier composed of 7 layers of InAs QDs grown on InGaAsP which was lattice matched to InP. The total device length was 2 mm. Importantly, gain and photoluminescence measurements showed no excited state in these dots allowing for a direct comparison to our derived model.

Figure 2.3 shows the gain spectra of the device at various bias currents. As can be seen in the plot, increasing the bias current has two effects. First, the peak gain increases, and second, the peak wavelength shifts toward shorter wavelengths. This blue shifting of the peak shows that not all dots fill at the same rate. Rather, lower energy dots fill first. Furthermore, this blue shifting will result in a large linewidth enhancement factor. Measurements on a similar quantum dot sample fabricated into a Fabry-Perot laser measured a linewidth enhancement factor of 5. Thus, for comparison to experiment, we used $\alpha = \alpha_{CH} = 5$. While this value is large for quantum dots, theoretical results have shown that shallow QDs, like those used, will have larger linewidth enhancement factors due to increased coupling between the
Figure 2.3: Gain of the QD-SOA for various bias currents. Higher current results in increasing gain, and in a shifting of the gain peak towards shorter wavelengths indicating a large linewidth enhancement factor and nonuniform dot filling.

bound state and barrier layer [10]. While the shifting gain peak at low bias goes against one of our initial assumptions, that all dots fill at the same rate, at high bias we can see the shift is greatly diminished. This is because at large bias current the high dot occupation probability causes the energy difference between the dots to become a minor factor in the carrier dynamics. This results in all dots filling at nearly the same rate as assumed in our model.

To perform four-wave mixing measurements, we sent both a strong pump and a weaker tunable probe into the QD sample. Though the gain peaks at 1480 nm, the limitations of our tunable lasers required that the pump laser be placed slightly off of the gain peak at 1490 nm so that we could scan both positive and negative pump-probe detunings. The tunable probe laser was then swept across the pump and the output spectrum measured on an optical spectrum analyzer (OSA). The amplified spontaneous emission was then subtracted and the efficiency calculated by comparing the power of the output conjugate to the input probe. Due to the resolution
Figure 2.4: Fit of gain data at 600 mA bias current showing good agreement at the experimental wavelengths of 1490 nm. Deviation at long wavelength is most likely due to free-carrier absorption which was not included in the fitting model.

limitations of our OSA, detunings of less than 150 GHz could not be measured as the strong pump would wash out the weaker conjugate signals.

To fit these experimental conditions to theory, we first fit the gain spectra of the device using a simple Gaussian approximation for the distribution of dot sizes. To do this we assumed that

\[
\Gamma g(\omega) = \Gamma g_0 e^{-\frac{(\omega-\omega_0)^2}{2\sigma^2}} - \alpha_i. \tag{2.67}
\]

Here, \(\omega_0\) is the peak-gain wavelength of 1480 nm. \(\alpha_i\) is the intrinsic loss assumed to be 5 cm\(^{-1}\). \(\sigma\) and \(\Gamma g_0\) were fitting parameters representing the width of the dot distribution due to inhomogeneous broadening, and the maximum modal gain of the sample respectively. The best fit can be seen in Fig. 2.4 where \(\sigma\) was found to be 26 meV and \(\Gamma g_0\) was 36.50 cm\(^{-1}\). While the fit shows excellent agreement near the gain peak, the absorption of long wavelength light is much higher than expected from this simple model. Attempts were made to correctly match the entire curve by
increasing the intrinsic loss, but this resulted in unphysically high values of the loss. This extra loss is most likely due to a deviation in the inhomogeneous broadening from a Gaussian profile. As our data were taken near the peak wavelength, and our theory is based on an assumption of operating near the peak wavelength as well, this variation from the theoretical model was not considered significant for the results presented here.

Once the gain was fit, the gain of the QD device, along with $\kappa_{SHB}$, $\tau_d$ and the dot occupation probability, $f$, were calculated using (2.44), (2.61), (2.28) and (2.30). To perform the summation over all states necessary for calculating $\kappa_{SHB}$, the material gain, and the quasi-Fermi levels in the wetting layer, we integrated over the density of states, $\rho(\varepsilon)$. This was assumed to have the form

$$\rho(\varepsilon) = \begin{cases} \frac{D}{\sqrt{2\pi}\sigma^2}e^{-(\varepsilon-E_b)^2/2\sigma^2}, & \varepsilon < E_b + \Delta E \\ \frac{m^*}{\pi\hbar^2}, & \varepsilon > E_b + \Delta E \end{cases}$$

This includes a single, inhomogeneously broadened bound state in the quantum dots, and a 2-D like continuum of states in the barrier layer. $m^*$ is the effective mass of the electrons, and $E_b$ represents the mean bound state energy in the dots and is equal to $\hbar\omega_0$.

Utilizing this density of states, calculations were performed at several current densities by re-calculating the quasi-Fermi level for each desired current density, and then calculating the desired parameters. Other physical parameters necessary for the calculations had to be determined as well. The differential gain, $\frac{dg}{dn}$, was determined from Fig. 2.3 to be $6.0 \times 10^{-16}$ cm$^2$. The carrier capture time was assumed to be 1 ps in agreement with previous experiments [11], the escape time was related through the Boltzman factor such that $\tau_e = \tau_c e^{-\Delta E/kT}$, and $\Delta E$ was assumed as 0.075 eV, a typical value for quantum dots. The device temperature corresponded to our experimental
condition of 288 K. The total number of states in the dots $D = 2 \times 10^{17} \text{ cm}^{-3}$ was determined from the areal dot density of $10^{11} \text{ cm}^{-2}$ per dot layer with each layer being 10 nm thick. The factor of 2 is as stated before from spin-degeneracy. $|\mu(\omega)|^2$ was calculated by equating the gain model of [4] with that of [1] to find that

$$|\mu(\omega)|^2 = \frac{e^2}{m_0^* w^2} |\hat{e} \cdot p_{cv}|^2. \quad (2.69)$$

For bulk, the momentum matrix element is known $|\hat{e} \cdot p_{cv}|^2_{\text{bulk}} = \frac{m_0}{6} E_p$. For quantum dots, we expect the result to be the same as a quantum well because self-assembled quantum dots are much wider than they are tall. For TE polarized light, we thus expect that $|\hat{e} \cdot p_{cv}|^2_{\text{dot}} = \frac{3}{2} |\hat{e} \cdot p_{cv}|^2_{\text{bulk}}$ for the conduction subband to the top heavy-hole subband transition and find that

$$|\mu(\omega)|^2 = \frac{e^2 E_p}{4m_0 \omega^2}. \quad (2.70)$$

Here, $E_p$ is the optical matrix parameter, and for InAs dots is 22.2 eV [4], and $m_0$ is the free electron mass.

The results of these calculations can be seen in Fig. 2.5 where instead of material gain, the integrated, modal gain $G_0(\omega_0) = \Gamma g_0 L$ has been plotted. These calculations show two expected trends. First, increasing the carrier density causes the dot occupation probability to increase from 0 to 1, with the integrated gain increasing proportionally. Second, $\kappa_{SHB}$ is proportional to $\tau_d$ and decreases with increasing carrier density. This is significant for two reasons. First, the proportionality between $\kappa_{SHB}$ and $\tau_d$ shows that slower carrier relaxation times allow for more efficient four-wave mixing providing a trade off between bandwidth and efficiency. Higher efficiency results in lower bandwidth, while large bandwidth reduces efficiency. This is also the fundamental reason quantum dots should be more efficient for telecommunications.
Figure 2.5: $\kappa_{SHB}$, $\tau_d$, $G_0(\omega_0)$, and the dot occupation probability plotted as a function of carrier density. Solid vertical line is the fitting condition.
applications than quantum wells at speed between 10-160 GHz. These speeds are slow enough that the 0.1-1 ps relaxation time of quantum dots can easily convert them. The faster, 50-10 fs [12, 13], relaxation times present in quantum wells result in less efficient conversion but with a much larger bandwidth.

Furthermore, the decrease in $\kappa_{SHB}$ with increasing bias is not unexpected. $\kappa_{SHB}$ is a measure of the creation rate of conjugate photons, and they are created through the simultaneous absorption of two pump photons and stimulated emission of a probe and conjugate photon. For this to occur, there must be unoccupied dots capable of absorbing pump photons. While this at first might cause the belief that the conversion is most efficient at low bias where the dot occupation is low, it is important to remember that the gain and absorption of the sample plays a large role as well. Once a conjugate beam is started, the gain of the sample will amplify it allowing a small conjugate to quickly grow. As the gain reaches a maximum and plateaus after all dots are filled, the non-linear gain-coefficient plateaus as well resulting in an optimal carrier density. This effect can be seen in Fig. 2.6 where the efficiency is plotted versus carrier density and shows a peak. It is important to point out that for comparison purposes gain saturation and pump power have not been considered in this plot. $P/P_{sat}$ was simply taken to be one for the calculation of $E_2$ but no saturation effects were applied to the gain. In general, saturation can play a large role in the ideal pump power [9]. This shows that for true optimization both pump power and carrier density must be considered.

To compare our four-wave mixing data to theory, we took the previous gain fit, calculated the integrated gain over the 2 mm long device, and compared it to the calculated integrated gain. With no good measurement of the confinement factor, how much the light field overlaps with the active media, it was allowed to drift over typical values for a quantum dot SOA with the best fit resulting in $\Gamma = 2.7\%$ for an integrated gain of 7.3. While this confinement factor is small, this is in the range for
Figure 2.6: Unnormalized efficiency vs. Carrier Density. Plot does not take into account carrier saturation or pump power so absolute values should not be considered correct.

A typical quantum dot device. The vertical line in Fig. 2.5 shows the carrier density, which provides the best fit and is in agreement with our previous gain fit. It shows calculated values for $\tau_d = 0.5 \text{ ps}$, $\kappa_{SHB} = 0.11$ with $\alpha_{SHB} = 0.013$ being found from the phase of $\kappa_{SHB}$; while $\alpha_{SHB}$ was included in our calculations, the small magnitude resulted in it having no significant effect on the outcome. The carrier heating effect included contributions from both holes and electrons for a total $\kappa_{CH} = 0.08$. Other important theoretical parameters were assumed including $\tau_s = 200\text{ps}$, an assumed value typical of semiconductor devices under large bias. $\tau_{CH} = 2.5 \text{ ps}$ in agreement with experimental measurements in similar quantum dots [14]. The input pump value was chosen to match experiment at $0.16P_{sat}$.

A comparison between our theoretical model and our experimental measurements can be seen in Fig. 2.7. The fit shows generally good agreement between theory and experiment, both in the magnitude of the conversion efficiency, and in the splitting between positive and negative detunings.
Figure 2.7: Experimental data with fit. Experiment is shown as points while matching theory is solid lines.

2.5 Discussion

The efficiency plot from our theory shows two plateaus. One with a bandwidth of a few GHz due to carrier density pulsation, and another that extends out to around 200 GHz before falling off. By utilizing the detuning range that lies on the second plateau it is possible to perform high-efficiency wavelength conversion at speeds greater than 160 Gb/s by utilizing the four-wave mixing effect. Calculations on typical quantum wells put the efficiency much lower [9] along with previous experimental measurements directly comparing quantum dots and quantum wells [15].

Importantly, the second plateau is determined more by carrier heating than by spectral hole burning. This becomes readily apparent when the individual contributions to four-wave mixing are plotted in Fig 2.8. While at first one might expect spectral hole burning to have a large contribution as $\kappa_{SHB} > \kappa_{CH}$, the large temperature linewidth enhancement factor increases the contribution from carrier heating above that of spectral hole burning. This result demonstrates that in shallow dots
Figure 2.8: Theoretical efficiency plots with individual contributions from carrier density pulsation, carrier heating, and spectral hole burning superimposed. All(-) indicates negative detuning while All(+) indicates positive detuning.

with a single bound state the primary four-wave mixing mechanism at large detunings, and for high-speed signals, is carrier heating. This is in contrast to most other theories which focus mainly on spectral hole burning [2, 3] in quantum dots. This large contribution from carrier heating is possible due to the slow thermal relaxation rate that occurs in these dots.

This slow relaxation is most likely due to the slow means by which carriers in the wetting layer can relax down into the quantum dots, which have been depleted through stimulated emission. Indeed, the measured thermal relaxation time of 2.5 ps is similar to the carrier capture time of 1 ps. As a result, we expect deep quantum dots with large energy offsets between the barrier layer and bound state to perform less efficiently as they have a reservoir of excited states which can quickly relax down and buffer the slow carrier capture. The drawback is that these shallow quantum dots, while being more efficient, cannot achieve the same symmetric conversion that has been reported in deeper quantum dots [16] due to their larger linewidth enhancement
factor caused by coupling to the continuum states. As both spectral hole burning and carrier heating are seen to be heavily reliant on a slow carrier capture time for high-efficiency, this factor becomes our limiting value in determining the maximum four-wave mixing efficiency and bandwidth in shallow quantum dots.

2.6 Summary

We have developed a theoretical model for the non-linear susceptibilities in quantum dots, and use them to calculate the FWM conversion efficiency. Our analysis has focused on dots with only a single bound state. The developed theoretical model demonstrates excellent agreement with our experimental measurements. The results indicate that carrier heating plays an important role in achieving efficient, high-speed conversion. Our results show that the limiting factor in both the conversion efficiency and the bandwidth is the carrier capture time of the quantum dots, as it drives both the spectral hole recovery rate, and the rate of carrier thermalization in the quantum dots. Faster carrier capture allows for a broader bandwidth, but decreased efficiency.

2.7 References


Chapter 3
Experimental Demonstration

3.1 Introduction

Our theoretical work presented in the previous chapter has shown that QDs with only a single bound state should provide efficient wavelength conversion due to their slow carrier dynamics. Previous theoretical work has also stated that quantum dots should be superior to quantum wells [4], but this has not yet been experimentally demonstrated. To date there have been studies demonstrating symmetric conversion in quantum dots at both 1300 nm and 1580 nm [5,6], which is made possible by the small linewidth enhancement factor of quantum dots. However, direct comparisons to quantum well devices have not demonstrated greatly improved efficiency [7]. In this chapter, we present an experimental comparison between differing quantum dots. Our comparison shows that quantum dots with multiple bound states demonstrate symmetric conversion, being equally efficient at up and down conversion, while those with a single bound state demonstrate asymmetric conversion. A rough comparison of the FWM efficiency between these two dot types will also be made and show superior conversion in shallow quantum dots. These quantum dots are then directly compared to a quantum-well device biased to have the same device gain. Our experimental results show that the quantum dot does provide superior conversion efficiency with a near 10 dB enhancement compared to a quantum well device.

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Once the continuous wave characteristics are established, we continue by performing an experimental analysis of the conversion efficiency for small-signal amplitude modulated light. We perform a direct comparison between FWM and cross-gain modulation (XGM) in the same quantum-dot device to demonstrate the increased bandwidth and format transparency provided by four-wave mixing. Our data show that four-wave mixing provides format transparent conversion at speeds in excess of 25 GHz while XGM is limited to a bandwidth of 1 GHz. We then continue to examine cross talk in FWM to investigate the feasibility of multi-channel conversion. We will present results that show the primary cross-talk mechanism is XGM, and that the cross-talk is 20 dB below the FWM signal level.

3.2 Quantum dot comparison

There are a multitude of different types of quantum dots. The two sets of dots that we will focus on are InAs dots grown on GaAs. These dots, which have a resonant energy around 1290 nm, have previously been measured for FWM and found to provide symmetric up and down conversion [5]. The other dots we will be focusing on are InAs dots grown on InGaAsP which is lattice matched to InP. These dots operate near 1500 nm. We will focus on these dots because they have one important difference between them: the number of bound states that exist in the quantum dot. InAs/InGaAsP dots have only a single bound state and do not possess any excited states. This feature can be seen in Fig. 3.1 where the amplified spontaneous emission spectra (ASE) of these dots is shown under different current bias. The data show only one peak even at high bias indicating that no excited states exist. This is possible due to the small conduction band offset that exists between the dot and barrier layer which does not allow higher energy bound states to fit inside the dot.

In contrast, InAs/GaAs dots have more than one bound state in the quantum
Figure 3.1: Amplified spontaneous emission (ASE) spectra at different current bias for InAs dots grown on InGaAs. Spectra show a single bound state with no excited state.

dot, this can be seen in Fig. 3.2 where large bias currents cause a clear second peak in the ASE spectra near 1200 nm. This second peak arises due to carriers filling the excited state of the dot. These excited states serve as carrier reservoirs that alter the dynamics of the quantum dot system.

While our developed theory focused on quantum dots with only a single bound state, it has helped to highlight the important factors which determine four-wave mixing efficiency in these systems. First, we expect that the excited states that exists in InAs/GaAs dots will reduce spectral hole burning. This is because the increased number of dot states decrease the carrier capture time as there are now more states capable of trapping carriers. An examination of (2.28) shows that $\tau_d$ increases linearly with the number of dot states when spectral hole burning is capture limited. Electrons trapped in excited states then serve as a buffer capable of quickly relaxing down to the ground state. This excited state to ground state relaxation has been measured to be extremely fast, around 150 fs [8]. The faster carrier capture time, coupled with the
Figure 3.2: Amplified spontaneous emission spectra at different current bias for InAs dots grown on GaAs. Spectra show a clear ground and excited state.

quick relaxation from the excited states suggests that the bandwidth of these samples should be increased, but at the cost of reduced efficiency as previously discussed.

These altered dynamics will change the contributions due to carrier heating as well. This has already been demonstrated experimentally. Experiments on InAs/GaAs QDs have shown no carrier heating [8]; however, InAs/InGaAsP dots have shown significant carrier heating with slow recovery times [9]. This can be physically understood by examining the possible thermal relaxation mechanisms. In quantum dots with multiple bound states the large number of carriers in a single dot allows for a quick return to thermal equilibrium via carrier-phonon-carrier collisions. However, in quantum dots with only a single bound state there are no other carriers in the dot with which to exchange energy requiring that thermal relaxation must occur through phonon collisions alone. This has been shown to be an inhibited process in quantum dots containing only single carriers due to phonon bottleneeking [10]. This slower thermal relaxation allows for increased carrier heating in InAs/InGaAsP dots, and should cause them to have higher efficiency.
Figure 3.3: The power of the conjugate produced via four-wave mixing in an optical amplifier comprised of InAs dots grown on InGaAsP. Results show that decreasing bias current has no effect on the conversion symmetry indicating that a large linewidth enhancement factor is intrinsic to the dots. Pump power was -0.9 dBm, and probe power was -12 dBm.

Figure 3.4: The power of the conjugate produced via four-wave mixing in an optical amplifier comprised of InAs dots grown on GaAs. Results show that decreasing bias current results in symmetric conversion indicating a large linewidth enhancement factor only for large current densities. Pump power was 5.1 dBm, and probe power was -9 dBm.
A side-by-side comparison of four-wave mixing in these two different dot systems can be seen in Figs. 3.3 and 3.4. It is important to note that due to different device and experimental parameters the conjugate powers at the same current can not be directly compared for determining conversion efficiency. This can be easily understood by examining Figs. 3.2 and 3.1 and noting that when both devices are biased at 600 mA there is much more ASE collected from the InAs/GaAs QDs as the device is longer and has higher gain at the same current. We can, however, make a rough comparison by examining the conjugate powers in instances where the collected ASE is nearly equal as the spontaneous emission is proportional to the gain. When the InAs/InGaAsP dots are biased at 600 mA the peak ASE output is -16 dBm, and the conjugate power for a near 50 GHz detuning is -35 and -30 dBm depending on positive or negative detuning. The InAs/GaAs dot sample has about the same ASE when biased at 250 mA, yet produces a -35 dBm conjugate at a 300 mA bias despite having higher pump and probe powers. From this rough analysis we can conclude that InAs/InGaAsP dots, which have only a single bound state, have a higher FWM conversion efficiency as predicted.

We can also compare conversion asymmetry between these two samples. In this case the difference is clear: as the current density is reduced in the samples the FWM efficiency becomes symmetric for InAs/GaAs QDs while remaining asymmetric for InAs/InGaAsP quantum dots. This is significant in highlighting the different linewidth enhancement factors of these two systems, including contributions from both the carrier density and carrier temperature. These results show that the InAs/GaAs dots have an intrinsically low linewidth enhancement factor, which only becomes significant for large carrier densities where significant populations are generated in the excited states. This is shown by the nearly symmetric conversion at low bias, which becomes asymmetric at higher bias current where the ASE spectra shows significant excited state filling. InAs/InGaAsP dots however have an intrinsically
large linewidth enhancement at room temperature due to the strong coupling to the continuum states [11]. This causes the FWM conversion to be asymmetric at all bias currents.

For the purpose of wavelength conversion, quantum dots have been previously noted as significant because of the possibility of achieving efficient and symmetric conversion. However, our theoretical results show that, while quantum dots can achieve both of these goals they can not achieve both at the same time. Optimization of conversion efficiency relies on utilizing the slow dynamics of carrier-capture which necessitates using a dot with only a single bound state and which thus closely couples to the continuum resulting in a large linewidth enhancement factor and asymmetric conversion. Symmetric conversion can be achieved as well, but only by utilizing dots which have a number of excited states to buffer the ground state from the continuum, and by operating at a bias where these excited states do not populate enough to affect the gain spectra of the device. As actual applications of wavelength conversion rely more on total efficiency than symmetry, our investigation will continue by focusing on InAs/InGaAsP dots.

3.3 Quantum dot and Quantum well Comparison

The quantum-dot SOA used was made of seven layers of InAs quantum dots grown on InGaAsP lattice matched to InP with a 7° tilt to reduce feedback [12,13]. Both photoluminescence and gain measurements showed only one bound state in the quantum dots. The device is 3.2 mm long and has a gain peak of 1490 nm. The quantum well SOA had six unstrained InGaAs quantum wells grown on InGaAlAs barrier layers, was 2.2 mm long and had a gain peak of 1560 nm. Similar to the quantum-dot device the waveguide was tilted to reduce feedback. Our investigation began by utilizing continuous wave light.
Figure 3.5: Experimental setup for measuring four-wave mixing using continuous wave light. The pump and probe lasers are combined using an 80/20 coupler, and then the two co-propagating light fields are sent into the SOA using a lensed fiber. Polarization controllers are utilized to ensure that both the pump and probe are TE polarized. The output of the device is then collected with a lensed fiber and sent to an optical spectrum analyzer to measure the spectral components.

The experimental setup can be seen in Fig. 3.5. A strong pump and weaker probe, were mixed together using an 80/20 coupler resulting in input probe and pump powers of -12 dBm and -1 dBm respectively. The light was coupled into and out of the device using lensed fibers with an estimated total loss of 28 dB due to intrinsic loss and coupling as determined by measuring the loss when biased at transparency. Due to the polarization sensitivity of the quantum dot and quantum well SOAs, polarization controllers were used to ensure that both input beams were TE polarized. The output signal was then measured on an optical spectrum analyzer.

The typical output spectrum can be seen in Fig. 3.6, where the input pump and probe are readily visible, along with the conjugate and secondary conjugate produced via four-wave mixing. The secondary conjugate is due to the weaker probe acting as the pump and converting the pump wavelength. The pump wavelength was set to the gain peak of the device, and the probe beam was swept across the pump. The ASE was subtracted from the conjugate peak to determine the conjugate power used in calculating the conversion efficiency. Due to the large pump power, data taken near the pump was ignored as both the probe and conjugate were hidden inside the pump peak. This limited the measurement to data with detunings larger than $\sim 250$ GHz due to the resolution of the optical spectrum analyzer. This experiment was performed on both the quantum dot and quantum well sample and the results can
Figure 3.6: Typical spectrum of the optical output caused by FWM in a QD. Spectrum shows clear pump, probe and conjugate signals, along with a small secondary conjugate.

Figure 3.7: Conversion efficiency for both the quantum well and quantum dot devices as a function of probe detuning. Results show increased efficiency for the QD sample, and asymmetric conversion for both samples.
be seen in Fig. 3.7.

The quantum-dot device was biased at 500 mA, and the quantum well at 350 mA such that each device had a net gain of 34 dB above transparency. The data show that the quantum dot device has an enhanced conversion efficiency over the quantum well device. Both results show a large asymmetry between up and down conversion. This highlights what has been previously mentioned, that while shallow quantum dots do provide an increase in efficiency as predicted by our theoretical model and several others they do not provide the symmetric conversion that is frequently attributed to quantum dots in other theories and experiments [5, 6, 14, 15].

3.4 High-Speed Response

While the continuous wave measurements are useful for comparing quantum wells and quantum dots, actual data transmission requires the efficient conversion of high-speed signals. The experimental setup was modified to allow high-speed modulation of the probe beam by adding a high-speed LiNbO$_3$ optical modulator driven by a network analyzer. Due to the high insertion loss of the modulator, an erbium-doped fiber amplifier (EDFA) was used to boost the signal after the 80/20 coupler. Use of the EDFA required a change of quantum dot sample to one with a gain peak of 1532 nm, closer to the gain peak of the EDFA. This device was 2 mm long and had a net small-signal gain of 20 dB above transparency when operated at a bias current of 500 mA, but was otherwise similar to the previously described sample. After passing through the SOA, a band-pass filter with a bandwidth of 0.5 nm was used to filter out all signals but the conjugate. Following the filter, another EDFA was incorporated to account for the loss through the filter. This experimental setup can be seen in Fig. 3.8. The pump was set to 1532 nm and had an input power of 9.52 dBm; the probe wavelength was swept from 1533.4 nm to 1530.6 nm and had an input power
Figure 3.8: Experimental setup for measuring the high-speed response of four-wave mixing. The probe laser is modulated using a LiNbO$_3$ mach-zender modulator driven by a network analyzer. The modulated probe is then combined with the pump laser using an 80/20 coupler. Polarization controllers are used to ensure that both light fields are TE polarized. The co-propagating light fields are then amplified via an erbium doped fiber amplifier (EDFA) and sent into the device using a lensed fiber. The output of the device is then collected using a lensed fiber, filtered using a band-pass filter and then amplified using another EDFA before being sent to a high-speed photo-detector. The detected response is then measured on the network analyzer. The conjugate’s response is then compared to the signal’s response to measure FWM efficiency.
Figure 3.9: FWM conversion efficiency as a function of wavelength and modulation. Results show a flat response out to 25 GHz indicating a greater than 25 GHz bandwidth. Results also show that a greater than 100% conversion efficiency is possible, but that there is a large asymmetry between up and down conversion.

of 2.38 dBm. The actual optical power coupled into the device was much less due to the coupling loss into the device. The conversion efficiency was determined by taking the ratio of the conjugate’s RF power to that of the input probe. The modulation frequency was swept and the results are shown in Fig. 3.9.

The conversion efficiency remains asymmetric similar to the continuous wave case, and has greater than 100% efficiency for detunings of \( \leq 1 \text{ nm} \). Moreover, the RF frequency response is nearly flat to 25 GHz demonstrating a > 25 GHz bandwidth. The reduced efficiency at small modulation frequencies is due to gain compression caused by the probe interacting with its sidebands through carrier density pulsation which becomes insignificant for probe-sideband separations greater than the inverse of the carrier lifetime \( (1/\tau_s) \). Modulation frequencies > 25 GHz could not be tested due to the limitations of our experimental setup.

The XGM response of the same device was also measured, and the results can
Figure 3.10: Normalized cross-gain modulation conversion efficiency. The response quickly falls off with a 3 dB bandwidth of 1 GHz in contrast to the FWM data that demonstrated a flat response out to 25 GHz. Asymmetry in up and down conversion is due to chirp cause by the linewidth enhancement factor.

Figure 3.11: Phase of the output signal at the converted frequency relative to the output at the original frequency. Results show that FWM has a flat phase response allowing for format transparent conversion. Cross-gain modulation is inverted, with a large phase variation, showing incomplete conversion of phase information.
be seen in Fig. 3.10. Here it is clear that the 3 dB bandwidth is greatly reduced to \( \sim 1 \text{ GHz} \). For the output signals to be undistorted, both the magnitude and phase of the frequency components must be preserved. The phase of the conjugate relative to the output signal can be seen in Fig. 3.11. The XGM signal is inverted (starts at \( \pi \) radians) and varies widely with modulation frequency, whereas the relative phase of the FWM signal is nearly flat at zero. This shows that the converted FWM signal will be undistorted out to at least 25 GHz while similar pulses converted with XGM will be highly distorted due to both phase and amplitude variations.

### 3.5 FWM Cross-Talk

Another key difference between XGM and FWM is the possibility of achieving multi-channel conversion using FWM. This is because XGM relies on a modulating pump field to create changes in the global carrier density; thus affecting all optical fields passing through the media. FWM, however, only requires that the weak probe field be modulated, minimizing changes in the global carrier density. Furthermore, the produced signal is limited by energy conservation to a narrow spectral range. However, there will still be some level of cross talk due to XGM in the device, for while the probe modulation produces minimal XGM it does still create an effect. To measure this inter-channel cross-talk and examine the feasibility of converting multiple channels simultaneously in quantum dots, two probes were sent in simultaneously. Due to the lasers at hand, the wavelengths were shifted such that the pump was at 1552.5 nm, probe one was at 1553.5 nm, and the modulated probe two was at 1554.5 nm. Due to FWM each probe produced a conjugate. The band-pass filter was set to allow conjugate one through and the RF response was measured, and used as the cross-talk signal. The modulation of the probes was then switched, so that probe one was modulated and probe two was not. Again conjugate one was measured, with its
Figure 3.12: Comparison between RF response for cross-talk and actual signal. Inset shows optical setup for each case, dashed lines indicate sidebands created due to modulation and filter indicates band-pass position. Data show that the FWM signal is about 20 dB larger than the cross-talk signal from cross-gain modulation except at low modulation speeds.

It is shown in the data that the cross-talk signal drops off sharply with modulation speed until it hits the noise floor of -40 dB. However, the FWM signal remains constant and is near 20 dB higher than the cross talk signal for high modulation frequencies. In comparing the cross-talk curve to Fig. 3.10, it is apparent that the primary cross-talk mechanism is cross-gain modulation between the second probe and the first conjugate, as the shape of the cross-talk response is similar to the XGM response. Charge localization in quantum dots should help to reduce global changes in the carrier population, but previous studies have demonstrated that this cross-talk can occur through the wetting layer [16]. This results in a measurable level of cross-talk due to interdot processes. Our data show however, that this cross-talk is well below the signal level, especially for high frequencies where XGM has a very small response. Thus, by restricting ourselves to high-speed signals whose important
frequency components lie outside the area of XGM cross-talk, efficient, low-noise, multi-channel conversion can still be realized.

3.6 Summary

In summary, we have experimentally demonstrated that wavelength conversion via FWM in quantum dots is more efficient than in quantum wells with our results indicating a 10 dB enhancement in efficiency. However, while attaining high efficiency conversion, the conversion is asymmetric due to the large linewidth enhancement factor present in quantum dots with a single bound state. The efficiency enhancement results in a greater than 100% conversion for both continuous wave light, and for small-signal amplitude modulated signals. The determined conversion bandwidth was found to be > 25 GHz limited by the experimental equipment, and much larger than the XGM bandwidth of 1 GHz in the same device. Measurement of the phase variation shows that wavelength conversion via four-wave mixing is format transparent as expected. An examination of the cross-talk between two signals propagating in the same SOA indicates that FWM is capable of converting multiple high-speed signals with low cross-talk due to XGM. This multi-channel conversion will be further investigated in the next chapter.

3.7 References


Chapter 4

Multi-channel four-wave mixing

4.1 Introduction

The previous chapter demonstrated that the low inter-channel cross-talk of FWM provides the possibility for efficiently converting multiple channels simultaneously. This is possible because the non-linear mechanisms rely not on a perturbation of the global carrier density, but only on a perturbation among the subset of carriers that are resonant with them. To further investigate multiple channel conversion and the limitations imposed by cross-talk, in this chapter we present data on four-wave mixing in quantum dots using four, small-signal modulated probes. Previous work utilizing FWM in quantum wells has demonstrated conversion of 40 Gb/s for two channels and with an efficiency of -15 to 2 dB utilizing pumps with powers as high as 1 W [2–4]. Using quantum dots, we achieve a > 40 GHz bandwidth for each individual channel with efficiencies ranging from 3 to 10.5 dB for an in-fiber pump power of 9.5 dBm.

Using this multi-probe experiment, we then examine the cross-talk mechanisms that limit multi-channel conversion. We examine the effects of both secondary conjugates overlapping with the primary conjugates, and cross-gain modulation. In both cases, our data demonstrate that the cross-talk is small for the four channels in our experiment. However, we identify that the limiting factor in the total number of channels that can be converted is the total power of the probe and pump lasers.

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4.2 Small-signal analysis

4.2.1 Experimental setup

The quantum dot device used in our experiment was comprised of seven layers of InAs dots grown on InGaAsP lattice matched to InP [5]. The device was approximately 4 mm long, and with the waveguide tilted 7° to the facet to prevent feedback into the gain cavity [6] similar to that used in Chapter 3. For all measurements the device was biased at 900 mA and held at a heat sink temperature of 15° C. This bias resulted in a small-signal gain of 38 dB at 1542 nm, and an output saturation power of 18 dBm. The experimental setup for these measurements is shown in Fig. 4.1.

Four, distributed-feedback (DFB) lasers served as the probe beams. They were temperature and current tuned to be separated by 0.63 nm (80 GHz) to ensure that as the sidebands were swept out they would not overlap. The four wavelengths of
the probes were 1528.66, 1529.29, 1529.93, and 1530.56 nm and had input powers of -3.04, -2.25, -3.37, and -1.37 dBm respectively. They were coupled together using three 50/50 couplers and fed into a Mach-Zender modulator driven by a network analyzer that swept the modulation speed from 2 to 40 GHz. Each probe had its own polarization controller that was individually adjusted to ensure maximum modulation for each beam. After exiting the modulator the probes were then coupled with the pump using another 50/50 coupler. The pump was blue detuned from the probes and was placed at 1527.83 nm with an input power of 9.5 dBm.

Similar measurements with the pump red detuned were attempted, but because the probe laser wavelengths were on the edge of the gain spectrum for both the EDFA and the SOA there was a large asymmetry in the system gain resulting in asymmetric conversion and not allowing clean data to be taken for a red detuned pump. While some asymmetry is expected due to the linewidth enhancement factor of the dots, it was impossible to separate the conversion asymmetry from the gain asymmetry.

Once coupled into a single fiber, all five laser beams were amplified with an EDFA and coupled into the device using a lensed fiber with a fiber to facet coupling loss of about 10 dB. This loss was determined by biasing the device at transparency, and measuring the fiber-to-fiber loss. This loss was then assumed to be evenly divided among both facets. The pump, probe beams, and four-wave mixing produced conjugates were then collected at the output facet of the device with a second lensed fiber. The polarizations of both the pumps and probes were adjusted to achieve the largest gain possible assuring that each was TE polarized.

The output signals were then fed into an optical spectrum analyzer (OSA) that could be used as a tunable, band-pass filter. The resolution bandwidth of the filter was set to 1 nm, and tuned to each probe and conjugate in turn. The filter bandwidth was intentionally chosen to be large so that we could be certain that the response was not modified by the filter. These filtered signals were then amplified with another EDFA.
to overcome the large insertion loss of the OSA when used as a bandpass filter. The signal was detected on a high-speed photodetector with its electronic output amplified using an electronic RF amplifier. The modulation of the probes was swept from 2-40 GHz, the working frequency range of the RF amplifier. Output probe and output conjugate responses were then compared to determine the efficiency. Prior to these measurements, the output probe response was compared to the input probe response and the results showed a completely flat response indicating that any deviation from the conjugate’s response compared to the output probe must be due to four-wave mixing and not the gain response of the device changing the probe output.

4.2.2 Results

Figure 4.2 shows the optical spectrum at the output facet of the device. The strong pump and four probes are clearly visible, along with the four conjugates mirrored on the other side of the pump. There are however, several other laser lines present as
well which require explanation. While the primary conjugates are produced through pump-pump-probe optical interactions, the presence of multiple probes allows for pump-probe(1)-probe(2, 3, or 4) optical interactions. As there are four probes, we expect for there to be three such secondary lines and they can be seen just to the right of the primary conjugates. The slight frequency mismatch between these conjugates and the primary conjugates is due to the fact that the pump-probe(1) separation is larger than the probe-probe separation. These secondary conjugates are a possible means of cross-talk between the multiple channels, but as later analysis and results will show, have no significant contribution on the signal. On the far right, three small peaks can be observed as well. These are due to probe-probe interactions and are expected to be quite small due to the weak strength of the probes compared to the pump. The spectrum also clearly shows the gain asymmetry present in the setup due to operating at the edge of the EDFA and QD-SOA gain peak, as the ASE is highly sloped, and the probe power increases from small to large detunings to a degree larger than the input power difference. The side-bands due to modulation cannot be seen in this spectrum as it was taken at the low modulation speed of 2 GHz which leaves the modulation sidebands too close to the center wavelengths to be resolved with the optical spectrum analyzer used. As the modulation speed was swept to 40 GHz, sidebands could be seen to sweep out from all four probes and conjugates. It is important to point out that all conjugates are produced inside the QD-SOA. Optical spectrum taken of the signals after being amplified by the EDFA, but before being sent into the SOA, showed no conjugates at all.

The response of all four signals can be seen in Fig. 4.3. It is apparent in the data that all four channels have a flat response out to the frequency limitations of our network analyzer and RF amplifier showing that all frequency components of a 40 GHz signal are well converted. The efficiencies were calculated by comparing the conjugate response to the probe output. The saturated gain of each probe in the
presence of the pump (∼21 dB) was then added to the results so that the output conjugate would be compared to the input probe as conversion efficiency is defined as the ratio between the conjugate’s output power and the probe’s input power. Finally, to account for the gain difference from the second EDFA going into the detector for each probe, the gain difference was determined relative to the pump and added to each. It should be noted that this is an underestimation as it does not take into account the EDFA’s different gain for each conjugate. The results, however, show that all four channels have greater than 100% chip conversion efficiency demonstrating that efficient, multi-channel conversion is possible. This high efficiency is a direct result of the unique carrier dynamics in quantum dots as described in Chapter 2. With four channels each capable of efficient, 40 GHz conversion, the device shows the possibility of converting even higher speed signals near 160 GHz. The drop in efficiency with increased pump-probe separation shows that this efficient conversion is, however, limited to a range of only a few nm.
4.2.3 Cross-talk

With any multi-channel approach however, it becomes necessary to examine the cross-talk between channels. In this case, cross-talk was examined by repeating the above experiment and measuring the response of the conjugates. Then all but one probe was turned off, and the response of that single conjugate measured. This was repeated for each conjugate, and the results of having a single probe were compared to the same probe when there were three other channels present. It should be noted that because all four channels are synchronized the cross-talk effects are maximized. Results for all four channels were similar, so probe 2’s magnitude and phase response are shown in Fig. 4.4 to be representative of all four channels.

Given the system under study, we expect two possible cross-talk mechanisms. The first is cross-gain modulation from the other probes. This can be identified from the frequency and phase behavior of cross-gain modulation. As cross gain-modulation has a bandwidth of only a few GHz, we would expect any cross-talk from this source
to start large and then fall off. Furthermore, as XGM is out-of-phase with the signal it should result in a phase variation from the phase response of a single channel, and a reduction in efficiency. The other cross-talk mechanism is secondary conjugates that overlap with the spectral regime of the conjugate we are examining. Such conjugates include the secondary conjugates visible in Fig. 4.2. It is expected that in our setup a large secondary contribution would result in an increase in the apparent efficiency as all of our signals are in phase with each other. Furthermore, they would not cause a change in the phase response as they are in-phase with the signal.

The data show the four-channel response to be 1.2 dB lower than the single channel. This reduction in magnitude goes against secondary conjugates providing a high level of cross-talk as they would cause an increase in efficiency as the detector would measure not only the primary but the secondary conjugate as well. Indeed the opposite seems true as increasing numbers of probes increase device saturation and reduce overall efficiency. Given the source of the secondary conjugates this is not unexpected. While the primary conjugate is proportional to $P_p^2 P_s$, the secondary is proportional to the $P_s^2 P_p$, so the power ratio between the primary and secondary conjugate is simply the ratio between the pump and the signal. As the signal is typically 10 to 20 dB lower than the pump, the expected cross-talk from secondary conjugates is 10 to 20 dB below the signal. Since only one secondary conjugate can overlap with a primary conjugate this results in a low level of cross-talk.

At low modulation speeds, the data show a drop in efficiency for multiple active probes. The drop in efficiency and the associated phase variation, both occurring at low frequency, allow us to conclude that this is due to the increased cross-gain modulation caused by all of the probes beating together. First thoughts would lead us to believe that cross-gain modulation should not be extensive in quantum-dots due to inhomogeneous broadening and each probe interacting with a different subset of the dot ensemble. However, our results indicate that strong interdot carrier dynamics
allow for cross-gain modulation cross-talk in the dot ensemble. From the data, we can see that its effect is limited to only a few GHz thus having a minimal effect on the conversion of high-speed signals. Increasing numbers of probe beams would increase the stimulated emission rate however, and therefore the bandwidth of cross-gain modulation [7], allowing it to introduce cross-talk at higher frequencies. This shows that the limiting factor in cross-gain modulation cross-talk is actually the total input power of the combined probes and pump. Higher input power results in stronger cross-talk extending to higher frequencies.

Our data show a definite cross-talk effect with four probes; however, if all channels were not synchronized we would expect much less cross-gain modulation so our results can be considered a worst case scenario. Decorrelated signals would result in less cross-talk as the channels would not all beat in time together reducing the gain modulation. Even in this extreme example, the magnitude and phase responses are only slightly perturbed showing good signal conversion and minimal cross-talk. From this cross-talk analysis, we can conclude that for multiple channel conversion, both cross-gain modulation and secondary conjugates provide only a minimal level of cross-talk. Therefore, multiple channels can be converted simultaneously limited only by the total input power and the saturation power of the device.

The varying phase response for both single and multiple probes is somewhat unexpected. The data presented in Chapter 3 show a completely flat phase response in a device with the same material system and design. The only important difference between the two experiments were the laser wavelengths utilized as our multiple probe lasers did not function at the gain peak of the device. The lasers utilized previously operated near 1532 nm, at the gain peak of the device, and where the gain of the post-amp EDFA is flat. In this experiment, the lasers operated near 1528 nm, on the edge of both the device’s and post-amp EDFA’s gain characteristics and where the gain varies sharply with wavelength. This gain variation may have caused the
sidebands responsible for the modulation to be amplified by different amounts amplifying the effect of chirp and skewing the output phase. Another possibility is self- and cross-phase modulation. Were these effects prominent however, we would expect that the multiple-channel phase response would vary greatly from the single-channel phase response as the increased power of the multiple probes would create a larger phase shift via cross-phase modulation than the single probe would cause to itself via self-phase modulation. Because the two phase responses are nearly identical we can conclude that self- and cross-modulation are minimal.

4.3 Summary

In this chapter, we have presented results achieving efficient, broadband conversion that was achieved in four, simultaneous, co-propagating wavelength channels. Our results also demonstrated a greater than 40 GHz bandwidth for each individual channel with efficiencies as high as 10.5 dB. Such efficient and broad-bandwidth wavelength conversion is possible due to the unique carrier dynamics of the quantum dot active media used. We also examined the cross-talk between the multiple channels and showed that the primary cross-talk mechanism is cross-gain modulation rather than secondary conjugates produced via probe-probe interactions. For the data presented, cross-talk proved minimal, but the underlying physics of XGM indicates that the limiting factor in the number of channels which can be converted simultaneously is the total optical power present in the device. This is because higher optical power results in higher saturation, increasing the XGM response and bandwidth such that sufficiently high powers will increase cross-talk to sufficient levels as to be detrimental.

4.4 References


Chapter 5
Toward Applications

5.1 Introduction

While our previous results have shown that high conversion efficiency is achievable in quantum dots, our results have also shown that this efficient conversion is limited to a narrow detuning range of a few nanometers. This narrow detuning range makes FWM impractical for real applications in which the actual wavelength may need to be converted over many tens of nanometers. To overcome this limitation, we will utilize a second pump laser to expand the wavelength range over which the information can be converted. The theoretical basis for dual-pump conversion has been previously establish [2, 3] and demonstrated in quantum wells using both co- and counter-polarized light [4–6]. Efficient dual-pump conversion requires that the pump-probe interactions modify the global carrier density so as to allow mixing with a second, far-detuned pump. As the physics describing the global carrier fluctuations are different in quantum dots due to charge localization, it is necessary to demonstrate this is still a viable means of extending the tunable range. Our results indicate that interdot communication inside the quantum dot sample is sufficient to allow for efficient conversion over the entire gain regime of the device, and that the high-speed characteristics of the conversion suffer no penalty as we are still able to accurately convert a 40 GHz signal.

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Furthermore, though conversion efficiency is an important metric for wavelength conversion, it is not the only important metric for actual applications. Real applications also require that there is a large signal-to-noise ratio (SNR) so that the created conjugate does not disappear into the background ASE. Unfortunately, small-signal modulation experiments can not give us information on the SNR as they return only the modulated component of the light. To overcome this limitation in our previous experiments, we present here data using a 25 ps pulsed laser and achieve a high SNR of 22 dB. Also, the pulsed data confirm our previous results by achieving greater than 100% efficiency while showing no patterning effect on the output pulse. Pump-probe detuning measurements on the pulse show an achievable single channel 3dB bandwidth of 100 GHz.

5.2 Dual-pump conversion

The presence of secondary conjugates in Fig. 4.2 indicates the clear possibility of using dual-pump conversion to expand the wavelength range over which four-wave mixing can be utilized. The localization of carriers in quantum dots might at first seem to prohibit dual-pump conversion as the pump and probe each interact only with the selection of dots which are in resonance with them and not with the entire ensemble. However, the presence of the secondaries demonstrates strong inter-dot processes allowing the beating of pump and probe light fields to be felt by the entire ensemble of inhomogeneously broadened dots. This is the same physical effect that allowed for cross-talk via cross-gain modulation when converting multiple probes simultaneously. To examine dual-pump conversion in quantum dots, we have investigated both continuous wave and small-amplitude modulated signals.

To allow us to sweep both the second pump and the probe, a separate experimental setup was used. A different quantum dot sample was utilized as well, which while
similar in composition was from a different processing batch resulting in a shifted
gain peak of 1533 nm, and was 3 mm long. The probe was a tunable laser, and so
was the first pump. The second pump was a DFB laser whose wavelength remained
static. The probe was then swept away from the static pump and the conjugate
response measured. The tunable pump was then moved away from the static pump,
the tunable probe reset to its original wavelength, and the entire process repeated.
These continuous wave results for different pump-pump detunings can be seen in Fig.
5.1.

When choosing the polarization for the second pump, we kept it copolarized with
the first pump and signal due to the large polarization dependence of the gain in the
quantum dots. While orthogonal polarization has been shown to provide a flattened
efficiency curve as the two pump light fields do not interact with each other [4, 5],
the polarization dependence of the gain would not allow for such an arrangement.
However, as we are only concerned with converting data over a very wide wavelength
range, where the pump-pump detuning would be very large compared to the pump-probe detuning the pump-pump interactions will be negligible. Furthermore, using copolarized pumps has been shown to provide maximum conversion efficiency [7] allowing for continued high-efficiency performance.

The solid lines are the optical spectra for several different pump spectra overlaid on top of each other showing the two pumps, with the static pump on the far right at 1553.04 nm. The marks show the conjugate power for the corresponding pump-pump detuning. The probe is not shown in the plots, and is red-detuned from the static pump. As is clear from the plots, the efficiency drops off as the probe moves farther from the static pump. However, by moving the second pump to larger detunings the efficiency for converting it to the new wavelength can be greatly enhanced. When the two pumps are nearly degenerate, conversion is more efficient due to pump-pump interactions; however, when the two pumps are well separated there is little change in the efficiency with increasing pump-pump detuning. This continues over the entire gain regime of the device showing that the inter-dot processes allow for dual-pump four-wave mixing to efficiently convert information from one wavelength to another over a wide range of wavelengths limited by the gain regime of the device.

While the continuous wave data show that the dual-pump setup is capable of creating highly efficient conversion over a large wavelength range, we must still determine if information can be as effectively converted as in the single pump case. To do this, the probe beam was placed at 1529.30 nm and modulated as before. One pump was set to 1528.68 nm while the second pump was turned off. The response was then measured of the conjugate produced at 1528.04 nm. Next, the second pump was turned on at 1527.37 nm and the response of the dual-pump conjugate at 1526.76 nm was measured. Both the phase and magnitude responses are shown in Fig. 5.2. The data show very little deviation between the single-pump and dual-pump responses demonstrating that dual-pump, four-wave mixing can provide
Figure 5.2: Comparison of the conjugate response for conjugates produced through single and dual-pump conversion. The inset shows the relative wavelengths for the two pumps, the probe, and the two produced conjugates $c_1$ and $c_2$. Results show no significant difference between the responses of $c_1$, produced through single-pump FWM, and $c_2$, produced through dual-pump FWM.

It is important to point out that this wide-band conversion must be achieved very differently in quantum dots than in quantum wells or bulk optical amplifiers. In both quantum well and bulk, the continuum of energies and absence of charge localization allow for the beating of the pump and probe, which only deplete resonant carriers, to affect all the carriers present in the device creating gain fluctuations at the second pump and providing wide-band conversion. Due to the discrete energy states in quantum dots, and the localization of the carriers in individual dots, the carrier beating among some fraction resonant with the pump and probe will not necessarily cause the population of all dots in the ensemble to beat. Were there no interdot communication there could be no wide-band conversion as the beating of the pump and probe at one energy would not effect the dots at the second pump’s energy. Our efficient, wide-band results indicate strong interdot processes. This is
most likely assisted by the fact that we chose quantum dots with a shallow conduction band edge as they would have only a single bound state and provide more efficient conversion. This shallow conduction band-edge allows for easier carrier capture and escape between dots providing an efficient mechanism for wide-band conversion.

Combined with our cross-talk results we can conclude that there is an inherent give-and-take in designing quantum-dot semiconductor optical amplifiers for wavelength conversion. Eliminating inter-dot communication would eliminate the primary cross-talk mechanism allowing nearly infinite channel performance, but it would also eliminate the possibility of wide-band conversion. It also provides the possibility that quantum dots with differing conduction-band edges could be engineered to optimize their performance for either multi-channel, low-cross-talk operation, or single-channel, wide-conversion-range operation.

5.3 Large-signal analysis

For effective wavelength conversion of telecommunication signals, it is not only important to have a large bandwidth, be efficient, and have tunability over a wide wavelength range, but the signal-to-noise ratio (SNR) of the output signal must be large as well to ensure that it is not lost in the background ASE. While our previous measurements have shown large bandwidth and high efficiency, small-signal experiments cannot effectively probe the SNR of the converted signal. Indeed with a highly biased SOA the amplified spontaneous emission may cause a large reduction in the SNR causing the conversion to be useless for real world applications. To determine the actual SNR of the converted signal, the experiment was modified to use a probe pulse provided by a gain-switched laser capable of producing 25 ps pulses. The setup can be seen in Fig. 5.3.

The gain-switched laser used created 25 ps pulses at a repetition rate of 1 MHz.
Figure 5.3: Experimental diagram of the large-signal experiment. A gain-switched laser is first filtered to choose a single mode, and then amplified to account for filtering loss. It is then coupled with the pump and sent into the quantum-dot SOA. An optical spectrum analyzer is used to filter the output signal allowing either the conjugate or probe pulses to be viewed on the oscilloscope.

As it utilized a Fabry-Perot diode laser, the output spectrum was broadband and multimode. To pick a single mode for frequency conversion, a pre-filter with a bandwidth of 50 GHz was utilized; this placed the probe wavelength at 1543.84 nm. This filtering slightly broadened and distorted the pulse but otherwise made no significant change in the pulse shape. Due to the power loss from filtering, an EDFA was used to increase the power by 6 dB to boost the output power back up to its original level. The pulse was then coupled with a continuous wave pump and the combined signal sent through another EDFA to boost the input signal into the quantum-dot SOA. The output of the SOA was then filtered using an optical spectrum analyzer with a resolution bandwidth of 1 nm allowing the conjugate and probe signals to be measured independently. A comparison between the output pulses of the probe and conjugate can be seen in Fig. 5.4.

The data show that there is no difference in the shape of the conjugate and probe pulses showing accurate wavelength conversion. Furthermore, to ensure accuracy of the zero level for measuring the SNR, both the conjugate and probe were measured, and the optical port to the oscilloscope was capped and the measurement redone. The capped measurement was considered the zero level and subtracted from both the probe and conjugate signals. Figure 5.4 shows a small background light level above
Figure 5.4: Comparison between output conjugate and probe pulses showing no pulse distortion and background light below the detection limit of the optical oscilloscope. Data show no change in pulse shape and a large SNR of 22 dB for the converted signal.

Figure 5.5: Comparison between output conjugate and probe output at the transparency current. Both curves are normalized by the same constant. Results show a much larger output than input with a conversion efficiency of 350%.
the detection level of the oscilloscope corresponding to an SNR of 22 dB for the converted signal. As the bandwidth of our pulse filter was larger than the bandwidth of our signal, this represents a lower bound for the SNR. To confirm that the conjugate pulse in Fig. 5.4 was due to the conjugate and not to some other background beating, the OSA filter was set to a wavelength away from where the conjugate is expected to form at which point the pulse disappeared. This demonstrates that the perceived conjugate pulse is not due to background ASE pulsations, or left over from incomplete pre-filtering, but is an actual conjugate produced from the non-linear mixing of the pump and probe.

To determine the efficiency of the pulsed measurement, the bias on the SOA was reduced to the transparency current so that the output power would be equal to the input power. The probe pulse was then measured from the exit facet. In this way it is possible to directly compare the conjugate output previously measured to the input probe without requiring estimates of coupling loss to calibrate the input to the output. Fig. 5.5 shows the results of this comparison. The output conjugate is much larger than the input probe yielding a 350% conversion efficiency. The pump-probe detuning for the above was 0.8 nm, or about 100 GHz.

To determine a 3 dB bandwidth over which this high efficiency conversion could be completed, the pump was moved away from the probe in 25 GHz steps, (0.2 nm) and the conjugate measured for each detuning. The results can be seen in Fig. 5.6. The largest wavelength separation for which the output power dropped 3 dB was found to be 0.8 nm, which corresponds to a 100 GHz, 3 dB bandwidth. For all of these detunings, efficiency was greater than 100%.
Figure 5.6: Comparison between the output conjugate for several detunings. Results show that the conjugate output power falls 3 dB over a range of 0.8 nm.

5.4 Summary

By utilizing dual-pump conversion and a pulsed laser, we have successfully demonstrated two more important characteristics a wavelength converter must possess to be applicable for real-world applications: wide wavelength tunability and high SNR. Our dual-pump results indicate that the interdot dynamics of the QD-SOA are sufficient for efficient dual-pump conversion over the entire gain regime of the device. However, as these same interdot dynamics cause cross-talk among multiple simultaneously converted channels our results also indicate that in designing a wavelength converter a necessary give-and-take exists between optimizing multiple-channel performance while allowing wide-wavelength tunability in the same device.

Furthermore, by using a pulsed laser we have also shown that the converted signal has a large signal-to-noise ratio of > 22 dB. The pulsed measurements also confirmed our small-signals results by achieving 350% efficiency while showing no patterning effects. Pump-probe detuning measurements for the pulsed laser also indicate that a
single channel 3 dB bandwidth of 100 GHz is achievable in the quantum dot device. Combining all of these results we can conclude that quantum dots provide an excellent medium for efficient wavelength conversion and show the possibility for continued high-efficiency at even higher speeds than those tested here.

5.5 References


Chapter 6

FWM of ultra-fast pulses

6.1 Introduction

In the previous chapters, we have presented experimental work that demonstrated quantum dots are more efficient for converting continuous wave signals. Furthermore, we have shown efficient wavelength conversion in quantum dots at up to 40 GHz using small-signal amplitude modulation and a 25 ps pulsed laser. However, in all cases the bandwidth of the quantum dot device could not be completely defined as it extended well past the regime which we could measure with our experimental apparatus. To go beyond this limitation, we continue with a theoretical investigation for the conversion of high-speed pulses in semiconductor, quantum-dot devices. In this work we theoretically investigate the efficient wavelength conversion of optical pulses from 1000 ps to 300 fs in a quantum-dot SOA. We also compare these results to identical calculations for a quantum-well SOA. While similar calculations have been previously done on quantum-well devices [1], the dynamics we have measured in the previous chapters, along with our theoretical results from Chapter 2 allow us to make the first calculations directly comparing quantum well and quantum dot devices at these ultra-fast speeds.

Our results show that the fast relaxation time constants of quantum wells make them superior at converting ultra-fast signals compared to quantum dots who are limited by much slower carrier capture. Furthermore, the advantages of quantum dots seem limited to a narrow energy and speed regime. Importantly though, this narrow
regime is that which is of use to telecommunications. So while quantum dots are not superior in all instances, for the speeds and energies utilized in telecommunications they show a distinct advantage.

6.2 Analytical Model

The modified nonlinear Schrödinger equation that governs the propagation of a fast pulse through a semiconductor media was derived in [2] to be

\[
\left\{ \frac{1}{2} g_N(\tau) \left[ \frac{1}{f(\tau)} - i\alpha_N \right] + \frac{1}{2} \Delta g_T(\tau) (1 - i\alpha_T) - \frac{1}{2} \left[ \frac{\partial g(\tau, \omega)}{\partial \omega} \right]_{\omega_0} \left. \frac{\partial}{\partial \tau} - \frac{1}{4} \left. \frac{\partial^2 g(\tau, \omega)}{\partial \omega^2} \right|_{\omega_0} \frac{\partial^2}{\partial \tau^2} \right\} V(\tau, z). \quad (6.1)
\]

On the left side of the equation, \(\beta_2\) is the dispersion of the media, \(\gamma\) is the intrinsic loss, \(\gamma_{2p}\) is the two-photon absorption coefficient, and \(n_2\) represents the non-linear Kerr effect. On the right side \(g_N(\tau)\) represents the gain of the sample at time \(\tau = t - z/v_g\) where the system is in the reference frame of the pulse of light and traveling at the group velocity of the pulse, \(v_g\). \(f(\tau)\) is the dynamic gain fluctuation due to spectral hole burning, and \(\alpha_N\) is the linewidth enhancement factor due to changes in the carrier density. \(\Delta g_T(\tau)\) is the gain change due to temperature changes of the electron gas via carrier heating inside the sample, and \(\alpha_T\) is the linewidth enhancement due to these changes in temperature. The two final terms arise from the gain profile and are due to gain dispersion in the active media. The envelope function of the optical electric field, \(V(\tau, z)\), is normalized so that \(|V(\tau, z)|^2 = P(\tau, z)\), the optical power at time \(\tau\).

The three dynamics terms, \(g_N(\tau)\), \(f(\tau)\) and \(\Delta g_T(\tau)\), are each expressed by inte-
grals to account for their dynamic changes due to the applied light field whose forms have been previously established [2, 3]. The first is

\[ g_N(\tau) = g_0 \exp \left( -\frac{1}{W_s} \int_{-\infty}^{\tau} e^{-\tau'/\tau_s} |V(\tau')|^2 d\tau' \right). \] (6.2)

Here \( g_0 \) is the gain in the absence of saturation, \( W_s \) is the saturation energy due to carrier depletion of the amplifier and \( \tau_s \) is the carrier lifetime. Spectral hole burning is accounted for in the factor \( f(\tau) \)

\[ f(\tau) = 1 + \frac{1}{\tau_{shb} P_{shb}} \int_0^{\tau} |V(\tau')|^2 e^{(\tau'-\tau)/(\tau_{shb})} d\tau' \] (6.3)

where \( \tau_{shb} \) and \( P_{shb} \) are the spectral hole relaxation rate and saturation power respectively. The saturation power is given in [4] as

\[ P_{shb} = \frac{1}{2\Gamma C_{nl}} \frac{\hbar^2/\tau_1^2}{\hbar \tau_2} \] (6.4)

where

\[ C_{nl} = \frac{6}{5} \langle |\hat{e} \cdot \hat{p}_{cv}|^2 \rangle \frac{e^2}{m_0^2 \omega_s^2 \eta \epsilon_0 A}. \] (6.5)

\( \tau_{shb} \) is the spectral hole burning recovery rate, \( \tau_2 \) is the decoherence time, \( \eta \) is the refractive index of the media, \( A \) is the cross-sectional area of the optical mode, and \( \Gamma_3 \) is the spatial overlap factor for the third-order processes. We have also applied the identity that \( \langle |\hat{e} \cdot \hat{p}_{cv}|^4 \rangle = \frac{6}{5} \langle |\hat{e} \cdot \hat{p}_{cv}|^2 \rangle \) [5]. The transition matrix element can be found to be \( \langle |\hat{e} \cdot \hat{p}_{cv}|^2 \rangle = (m_0/6)E_p \), where \( E_p \) is the optical matrix energy parameter for the material [6]. The gain change due to temperature changes is similarly formulated as

\[ \Delta g_T(\tau) = -h_1 \int_0^{\tau} |V(\tau')|^2 e^{(\tau'-\tau)/(\tau_{ch})} d\tau'. \] (6.6)

where the constant \( h_1 \) is a measure of how quickly the sample heats up due to free
carrier absorption and stimulated emission. An expression for $h_1$ in quantum dots and quantum wells will be derived in the following section.

The gain dispersion terms account for the shape of the gain as you move away from the gain peak and are determined by assuming a parabolic gain model near the pump wavelength, and by assuming a linear change with changing gain

$$\frac{\partial g_0(\tau, \omega)}{\partial \omega} \bigg|_{\omega_0} = A_1 + B_1 [g_0(0) - g_0(\tau, \omega_0)] \quad (6.7)$$

$$\frac{\partial^2 g_0(\tau, \omega)}{\partial \omega^2} \bigg|_{\omega_0} = A_2 + B_2 [g_0(0) - g_0(\tau, \omega_0)] \quad (6.8)$$

where the $A$ and $B$ coefficients are determined from the gain spectrum as

$$A_1 = \frac{\partial g_0}{\partial \omega} \quad (6.9)$$

$$B_1 = \frac{\partial A_1}{\partial g_0} \quad (6.10)$$

$$A_2 = \frac{\partial^2 g_0}{\partial \omega^2} \quad (6.11)$$

$$B_2 = \frac{\partial A_2}{\partial g_0} \quad (6.12)$$

### 6.2.1 Carrier Heating

To calculate the gain change from carrier heating, we must first determine $h_1$. To do this, we begin by looking at the change in the plasma’s energy density from thermal equilibrium, $\Delta U$, due to changes in the plasma’s temperature, $\Delta T$,

$$\Delta U = h_{cv} \Delta T. \quad (6.13)$$
$h_{cv}$ is the heat capacity of the electron gas. This expression can be transformed to produce the gain change due to temperature change as

$$\Delta g_T(t) = \frac{\partial g}{\partial T} \frac{\Delta U}{h_{cv}}. \quad (6.14)$$

The rate equation for $\Delta U$ is

$$\frac{\partial \Delta U}{\partial t} = -\frac{\Delta U}{\tau_h} + \left( \frac{\gamma_f}{A} + \frac{\Delta E g_0}{A \hbar \omega} \right) P(t). \quad (6.15)$$

The first term represents carrier-phonon collisions which cool the electron gas down to the lattice temperature at a rate $\tau_h$. The second term includes the contributions of free-carrier absorption with a free carrier absorption coefficient $\gamma_f$, and stimulated emission of carriers with an energy $\Delta E$ below the chemical potential due to an optical mode with power $P(t)$, and at frequency $\omega$. If $P(t) \ll P_{sat}$, where $P_{sat} = \frac{\hbar \omega A}{\partial g/\partial N_{\tau_s}}$ allowing you to ignore carrier density fluctuations over the time scale of carrier heating, then we can solve (6.15) to find that

$$\Delta U(t) = \int_0^t \frac{\gamma_f \hbar \omega + \Delta E g_0}{\frac{\partial g}{\partial N_{\tau_s}} P_{sat}} \frac{P(t')}{\frac{\partial g}{\partial N_{\tau_s}}} e^{(t-t')/\tau_h} dt' \quad (6.16)$$

which when combined with (6.14) yields

$$\Delta g_T(t) = \frac{\partial g}{\partial T} \frac{h}{h_{ch}} \int_0^t \frac{\gamma_f \hbar \omega + \Delta E g_0}{\frac{\partial g}{\partial N_{\tau_s}} P_{sat}} \frac{P(t')}{\frac{\partial g}{\partial N_{\tau_s}}} e^{(t-t')/\tau_h} dt' \quad (6.17)$$

Here, we have taken as our initial condition $\Delta U(0) = 0$, and thus, so too does $\Delta g_T(0) = 0$. 

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Quantum Dots

For quantum dots with a single bound state, the ratio between the carrier density and temperature differential gains was derived in Chapter 2 to be

$$\frac{\partial g}{\partial T} = -\frac{N\Delta E}{k_bT^2}. \quad (6.18)$$

It was also previously shown that the heat capacity of the electron gas is dominated by the two dimensional reservoir of states in the barrier and wetting layer, and is thus

$$h_{cv} = \frac{\pi k_b^2 T m^*}{3h^2 l} \quad (6.19)$$

where \(l\) is the thickness of the QD layer, \(m^*\) is the effective mass of the electron or holes, and \(T\) is the temperature of the plasma. From this, along with the assumption that free carrier absorption is minimal in quantum dots due to the low carrier density at which gain is achieved, we can find the general expression for the gain change due to carrier heating to be

$$\Delta g_T(t) = -\frac{3g_0N\Delta E^2h^2l}{\pi k_b^3 T^3 \tau_s m^*} \int_0^t \frac{P(t')}{P_{sat}} e^{(t'-t)/\tau_h} dt' \quad (6.20)$$

which allows us to find the value of \(h_1\) for QDs to be

$$h_1 = \frac{3g_0N\Delta E^2h^2l}{\pi k_b^3 T^3 \tau_s m^* P_{sat}}. \quad (6.21)$$

Quantum Wells

For quantum wells the ratio of \(dg/dT\) to \(dg/dN\) can not be easily simplified, and the dominant heating mechanism arises from free carrier absorption due to the typically
larger carrier concentrations. This results in an $h_1$ of

$$h_1 = \frac{3hL}{\pi k_B^2 T m^*} \frac{\partial g}{\partial T} \gamma f \hbar \omega_0 \tau_s P_{sat}, \quad (6.22)$$

where $\frac{\partial g}{\partial T}$ and $\frac{\partial g}{\partial N}$ must be determined through gain modeling.

### 6.3 Gain Model

To ensure that our comparison between quantum dots and quantum wells was grounded in reality, we first determined the necessary simulation parameters by fitting the gain curves of actual devices. The gain for a semiconductor device is

$$g(\omega) = -\frac{\omega}{\epsilon \epsilon_0 \eta} \times \int_{-\infty}^{\infty} \frac{|\mu(E')|^2}{\hbar} \Gamma(\hbar \omega - E') \rho_r(E') [f_c(E') + f_v(E') - 1] dE' \quad (6.23)$$

where $\Gamma(E)$ is a Lorentzian with width $1/\tau_2$ due to homogeneous broadening, $\rho_r(E)$ is the reduced density of states, and $f_c(E)$ and $f_v(E)$ are the Fermi-Dirac distributions for the conduction and valence bands respectively [6]. In modeling the quantum well and quantum dot devices, the key difference is the density of states.

For a quantum well, the reduced density of states is simply that for a two dimensional system

$$\rho_{qw}(E) = \begin{cases} 0 & \text{for } E < E_g + E_1 \\ \frac{m^*}{\pi \hbar^2 l} & \text{for } E \geq E_g + E_1 \end{cases} \quad (6.24)$$

Here, $l$ is the effective thickness of the well, $E_g$ is the electronic band gap of the material, and $E_1$ is the energy of the first bound state in the quantum well. Higher energy bound states have been ignored in these calculations due to the fact that the current density was not sufficient to provide non-negligible filling of those states.

For the quantum dot sample, the density of states is the same as that considered
Again, we have assumed a Gaussian distribution of dot energies due to inhomogeneous broadening, and included a continuum of states with a two dimensional density of states to account for coupling between the dots and the continuum of the wetting layer. Here $E_d$ is the average energy of the bound state in the quantum dot taking into account both the confinement energy and electronic band gap while $\sigma$ accounts for inhomogeneous broadening of the quantum dot ensemble. $D_d$ is the volume dot density.

To determine the gain, the quasi-Fermi levels must also be determined. For the quantum well this was done by setting the carrier density, $N$, and solving the expression

$$N = \int_{-\infty}^{\infty} f_k(E) \rho_k(E) dE$$

for the quasi-Fermi energy in $f_k$ where $k$ can represent either the conduction or valence band.

For the quantum dot, we can again take results from Chapter 2 to find the average dot occupation probability as

$$f(N_w) = \frac{1}{D_d} \frac{\tau_d(N_w)}{\tau_c} N_w$$

Where

$$\tau_d(N_w) = \left( \frac{1}{\tau_c} + \frac{1}{D_d} \frac{N_w}{\tau_c} \right)^{-1}$$

is the spectral hole recovery rate for the quantum dot sample already established. Using this average occupation probability, the quasi-Fermi level of the dot ensemble...
can be found in a manner similar to that of a quantum well.

Once the quasi-Fermi levels for the dot and quantum well were found, the gain could be calculated and compared to experiment for a device of each type. These gain fits then provide the necessary parameters to perform the high-speed pulse simulations.

6.4 Gain Fitting

For the purpose of our fitting, two different bias of the devices were chosen, and fitted together allowing only the current density to vary for each fit. All other values were held constant at either their calculated values, or at values found in literature. References for these can be found with their associated values in Table 6.1.

The quantum well device was six In$_{0.532}$Ga$_{0.468}$As wells with In$_{0.528}$Ga$_{0.257}$Al$_{0.215}$As barrier regions grown on InP. The device was 2 mm long and had the waveguide angled relative to the facets to avoid back reflection. The gain spectra was taken by sending in a continuous wave beam and measuring the output power and comparing to the input power. Coupling loss into and out of the device was taken into account by measuring the loss when the device was biased at transparency. The data and fit of the quantum well device can be seen in Fig. 6.1. In order to fit the data, the following parameters were allowed to vary: $\tau_2$, $N$, and $E_1$. Between the two curves however, only $N$ was adjusted with $\tau_2$ and $E_1$ both held constant.

The quantum dot device is the same one as utilized in Chapters 4 and 5. The gain was measured in the same manner as that of the quantum well device. The data fit can be seen in Fig. 6.2. Parameters that were allowed to vary were limited to $\tau_2$, $N_w$, and $E_d$. Again, for the two curves the only value that varied between them was the density in the continuum $N_w$. As can be seen in this fitting, there is a slight deviation between the shape of the fit and the data. This is due to our assumption
Figure 6.1: Comparison of data and gain fit for the quantum well device

Figure 6.2: Comparison of data and gain fit for the quantum dot device
of a Gaussian distribution of dot energies. The data instead show a slightly broader gain profile than can be accounted for with a Gaussian. We considered this deviation to be small however, and thus kept our gain model for simplicity rather than perform an interpolation of the data to find a more exact gain distribution.

Once these gain fits were made the temperature, carrier density, and other parameters were adjusted to allow for direct calculations of the parameters needed in Eq. (6.1).

### 6.5 Simulation

The table of parameters used in our simulation and fit can be found in table 6.1. This table includes the source of each number and the method in which we calculated it from our gain fit. Some basic numbers, such as the intrinsic loss of the device, had to be assumed based on reasonable parameters and are noted as such.

Looking at the simulation parameters, we see that one of the largest differences between the quantum dot and quantum well is the spectral hole relaxation rate, $\tau_{shb}$ and the spectral hole saturation power, $P_{shb}$. This difference in saturation powers is due to the larger $\tau_{shb}$ of quantum dots which causes the spectral hole to refill more slowly, and thus requiring weaker pump powers to create a significant hole. The other large difference lies in $\tau_2$, the decoherence time, with quantum wells dephasing significantly faster than quantum dots as carriers are not localized. Another important difference is in $h_1$ which shows that the quantum well device undergoes significantly more carrier heating than the quantum dot due to the larger free carrier absorption present in quantum wells. $P_{shb}$ is also larger in a quantum dot due to the smaller confinement factor. In our simulation, both devices were given carrier and temperature linewidth enhancement factors of 5, as that is typical for a quantum well, and similar for the quantum dot devices we have used in the previous chapters.
Table 6.1: Parameters used in pulsed calculations. $A_1$ is exactly zero for both the quantum well and quantum dot device because the pump is set to the gain peak.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>QW</th>
<th>QD</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Device Length</td>
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<td>Assumption</td>
</tr>
<tr>
<td>Intrinsic Loss</td>
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<td>Assumption</td>
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<td>Cross-Section</td>
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<td>Assumption</td>
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<td>Modal Gain</td>
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<td>Kerr Coefficient</td>
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<td>Ref. [3]</td>
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<tr>
<td>Device Dispersion</td>
<td>$\beta_2$</td>
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<td>Ref. [3]</td>
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<td>Pump Wavelength</td>
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<td>Carrier lifetime</td>
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<td>Carrier Saturation Energy</td>
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<td>176 pJ</td>
<td>Eqs. (6.21) &amp; (6.22)</td>
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<tr>
<td>Carrier Heating Coefficient</td>
<td>$h_1$</td>
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<td>5.46 x 10$^{12}$ m$^{-1}$J$^{-1}$</td>
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<td></td>
<td>$B_1$</td>
<td>2.0 x 10$^{-14}$ s</td>
<td>2.2 x 10$^{-14}$ s</td>
<td>Eq. (6.10)</td>
</tr>
<tr>
<td></td>
<td>$B_2$</td>
<td>0.515 x 10$^{-28}$ s$^2$</td>
<td>-0.882 x 10$^{-28}$ s$^2$</td>
<td>Eq. (6.12)</td>
</tr>
<tr>
<td>Carrier differential gain</td>
<td>$dg/dN$</td>
<td>1.23 x 10$^{-19}$ m$^2$</td>
<td>1.56 x 10$^{-19}$ m$^2$</td>
<td>Gain Fit</td>
</tr>
<tr>
<td>Temp differential gain</td>
<td>$dg/dT$</td>
<td>-383 m$^{-1}$K$^{-1}$</td>
<td>-161 m$^{-1}$K$^{-1}$</td>
<td>Gain Fit</td>
</tr>
<tr>
<td>Spectral Hole Saturation</td>
<td>$P_{shb}$</td>
<td>54 mW</td>
<td>3 mW</td>
<td>Eq. (12) in [4]</td>
</tr>
<tr>
<td>Confinement Factor</td>
<td>$\Gamma$</td>
<td>7.0 %</td>
<td>2.3 %</td>
<td>Gain Fit</td>
</tr>
<tr>
<td>Carrier LEF</td>
<td>$\alpha_N$</td>
<td>5.0</td>
<td>5.0</td>
<td>Assumption</td>
</tr>
<tr>
<td>Temp. LEF</td>
<td>$\alpha_T$</td>
<td>5.0</td>
<td>5.0</td>
<td>Assumption</td>
</tr>
</tbody>
</table>
One parameter change from the fits is that \( g_0 \) was set to 30 cm\(^{-1} \) for each device. This was done so that each device could be compared with identical gain. The actual \( g_0 \) from fits were slightly less than that for both devices. Also, in each case it was assumed that the pump wavelength was set exactly to the gain peak of the device so that \( A_1 \) is exactly 0.

For our simulation, we performed a finite difference beam propagation method following reference [3]. To compare compatible devices, we simulated each device as being 500 \( \mu \)m long, and subdivided into 400 section of equal length for the numerical calculations. The time step in each simulation was dynamically determined by the width of the pulse we were simulating. In each case, the total time period of the simulation was ten times the pulse full-width half maximum, and divided into 650 equal time divisions.

The two input pulses corresponding to the pump and probe were Gaussian in shape, and for each pulse width the detuning was set to twice the inverse of pulse’s full width half max so that the spectral components of the pump and probe would not overlap. The electric field for the two detuned pulses with the same pulse width was the same as used in [3]

\[
V(\tau) = P_0 e^{-\tau^2/(2\sigma^2)} + P_1 e^{-\tau^2/(2\sigma^2)} e^{i\delta \tau}
\]  

(6.29)

Where \( P_0 \) and \( P_1 \) are dependent on the pump and probe peak powers, \( \delta \) is the detuning between the pump and probe in angular frequency, and \( \sigma \) is the standard deviation determined by the desired pulse width \( \sigma = \frac{\text{Pulse Width}}{2\sqrt{\ln(2)}} \). In all of our simulations, the pulse energy of the probe was set to one tenth that of the pump. It is important to point out that in all of these simulations the pump was pulsed as well as the probe, unlike our previous examinations which focused on continuous wave pumps.

The four-wave mixing efficiency was then calculated by filtering the output and
input spectra of the amplifier, and comparing the power of the output conjugate wavelength with that of the input probe wavelength. Both the pump pulse energy and pulse width were then varied to compare between the quantum dot and quantum well device. Furthermore, to determine which components were contributing most significantly to four-wave mixing in the different power and pulse width regimes, we ran simulations which turned off all but one of the primary mixing mechanisms, and then compared the individual mixing components with the total mixing.

6.6 Simulation Results

Fig. 6.3 shows the time trace for a simulated co-propagating 10 ps pump and probe pulses. This is shown as an example of what all time traces for our simulations were, since we held the ratio between pulse width, detuning and total time trace constant. All of the input traces appeared exactly as in Fig. 6.3 although with different scales on the x- and y-axis dependent on what the specific values for the pulse energy and pulse width were. The time trace shows an oscillating pulse even though both the pump and probe pulses were Gaussian in shape. This is the combined envelope of the pump and probe pulse. The detuning difference between the wavelengths causes the two synchronous pulses to beat in time and create pulsations in the total power.

Fig. 6.4 shows a typical input and output spectra from our calculations. These spectra were obtained by performing a fast fourier transform on the input pulse shown in Fig. 6.3 and on the time trace output by the simulation. The input pulse clearly shows the pump and probe pulses with the probe pulse detuned by 200 GHz from the pump pulse. The output spectra on the same figure shows not only the amplified pump and probe due to gain in the semiconductor optical amplifier, but also has multiple conjugates produced via four-wave mixing. For our studies here, we have focused on the dominant first-order conjugate that is positioned at a detuning exactly
Figure 6.3: Sample time trace for input pulses. Beating between pump and probe pulses creates oscillations in the combined envelope function.

opposite that of the pump. In this case at -200 GHz. From this spectra figure it is simple to understand how the four-wave mixing efficiency was calculated. For the input probe, the input spectra was filtered in order to eliminate the pump pulse, and then inverted back to the time domain for the pulse energy to be calculated. Similarly, for the output conjugate, the output spectra was filtered to leave only the first conjugate, and again the FFT spectra was inverted back to the time domain and the pulse energy calculated. The ratio of the output conjugate energy to the input probe energy then became the four-wave mixing efficiency of our simulation.

The pulse widths we simulated were 1000 ps, 100 ps, 10 ps, 1 ps, 600 fs, and 300 fs. Each simulated pulse was sent into both a quantum dot and quantum well device with pump energies of 100 aJ to 100 $\mu$J. These same pulses were then sent into devices that had all but one of the non-linear effects removed to give us an of each effect’s contribution to four-wave mixing. It is important to note that for short pulses none of the non-linear effects exist independent of the other. Reduction in the total carrier density from CDP for example, reduces the possible magnitude
Figure 6.4: Input and output spectra for a simulated 10 ps pulse with a pump power of 1 fJ. Input spectra shows clear pump and probe peaks. Output spectra shows both gain of the original pump and probe, along with multiple conjugates produced through the non-linear interactions.

of the spectral hole that can be created. Thus the individual FWM contributions can not simply be added to give the total expected mixing. However modeling the individual mechanisms alone should give us a qualitative understanding of at what powers and pulse widths various mixing mechanisms are important. Furthermore, as we shorten the pulse width, we also increase the detuning as it is necessary to do so to keep the spectral components of the pump and pulse separate from each other.

Thus, changes in pulse width also model increasing detuning separation between the pump and probe. Fig. 6.5 shows the results for these calculations. Looking at the results for the 1000 ps pulse we can see that for low speeds and energies there is no significant difference between quantum wells and quantum dots. As pulse energy increases however, the quantum well becomes more efficient than the quantum dot device, for even higher energies the quantum dot device becomes more efficient. As pulses get faster, the quantum dot becomes more efficient at lower pulse energies until the pulse width gets to 1 ps, at which point the quantum dot efficiency quickly drops.
Figure 6.5: Comparison between QD and QW four-wave mixing for pulses of different widths and powers. All plots have the same axes to make comparison simpler. Vertical lines indicate average pump power for those energies. 1 W is maximum that can be transmitted in typical fiber optic cables, 1 mW is a typical power for telecom lasers, and 1 kW is a typical pulsed laser power. Results show that a QW is almost always superior, except for 100 and 10 ps pulses in the telecom energy regime.
Figure 6.6: Individual four-wave mixing components for a 100 ps pulse.

Figure 6.7: Individual four-wave mixing components for a 10 ps pulse.

down to the quantum well level again. The vertical lines on the plots indicate the average pulse power with faster pulses achieving the same pulse power at lower pulse energies. For a typical telecommunications signals powers above 1 W are not feasible due to non-linear losses in the fiber optic cables. Thus, typical powers are near 100 $\mu$W to 1 mW. Typical semiconductor lasers also have saturation powers around 18 dBm making the production of higher powers difficult.

Looking at Figs. 6.6 to 6.8 we can see the cause of the two peaks shown in the efficiency curves. The first peak arises due to spectral hole burning and its saturation, while the second arises from saturation of the total carrier density which affects both
carrier heating and carrier density pulsation. Because spectral hole burning has a lower saturation power it peaks at much lower pump pulse energies than the other two, which come about at higher energy due to the higher saturation power. For broad pulses, there is no discernible difference in how SHB behaves for quantum wells and dots, however as the pulse width shortens to 10 ps SHB becomes more significant resulting in the increased efficiency for low energy pulses. As the pulse width continues to be reduced, the speed goes below the response time of spectral hole burning in the quantum dot device, resulting in a reduction in the effect due to SHB and allowing quantum well devices to be more efficient overall. For higher pulse energies, quantum wells are more efficient than quantum dots because they have more significant mixing due to carrier density pulsation and carrier heating. We expect carrier heating to be more efficient due to the larger value of $h_{11}$. The CDP contribution for both a QW and QD are nearly equal, but with the CDP peaking at higher energies in the QD due to the high saturation energy $W_s$.

Carrier heating and carrier density pulsation seem almost unchanged for all pulse speeds. This is in contrast to the expectation that the much smaller value of $\tau_s$ should cause the CDP effect to drop off for high speeds. Especially as $\tau_s = 200$ ps we would
expect the effect of CDP to be insignificant for 1 ps pulses. This is due to the fact that while $\tau_s$ accounts for the carrier lifetime due to spontaneous and non-radiative recombination, it does not account for the stimulated emission. This can significantly shorten the carrier lifetime allowing CDP to function for much higher speed signals given sufficient stimulated emission. Indeed for a 1 ps pulse the CDP effect peaks at 100 pJ. A simple analysis results in a peak power of about 100 W which would allow for extremely fast stimulated emission rates. As a result these extremely high peak powers can create very short lifetimes through stimulated emission and allow CDP to continue to function at higher speeds.

For powers relevant to telecommunications, which are on the order of microwatt to milliwatt, the best efficiency is achieved in the quantum dot using 10 ps pulses due to the increased efficiency from spectral hole burning that arises at these pulse speeds. This is in agreement with what we have stated previously.

### 6.7 Summary

We have performed numerical calculations of fast pulses traveling through both a quantum dot and quantum well semiconductor optical amplifier using the finite-difference beam propagation method. Our results show that for pulses of less than 100 ps in width, there is no significant difference in efficiency, however for shorter pulses of 10 ps, quantum dots become significantly more efficient in the power regimes relevant to telecommunications due to enhanced spectral hole burning. Quantum wells in contrast become more efficient for pulses of less than 1 ps and for extremely large pulse energies. Our results are also compatible with our previous experiments which utilized average pump powers near 1 mW and showed higher efficiency in a QD device.
6.8 References


Chapter 7

Resonantly Enhanced Cross-Gain Modulation

7.1 Introduction

In the previous several chapters, we have taken an in-depth look at four-wave mixing for wavelength conversion in quantum-dot devices. As our data have shown though, there is a cross-gain modulation effect in these devices as well. While cross-gain modulation has also been used to achieve wavelength conversion, the data on our quantum dot samples presented in Chapter 3 show that the bandwidth is limited to 1 GHz. Previous work on increasing the XGM bandwidth has succeeded in achieving extremely high bit rates of 320 Gb/s [1], but required using extensive post-filtering to correct for the high-chirp and pulse distortion that arises at such high speeds. Recent work on QDs has shown that some quantum dots are capable of an intrinsic bandwidth near 40 GHz [2] by utilizing the fast relaxation of the excited states. Theoretical analysis has shown that the XGM bandwidth may be extended by using an additional pump field to alter the underlying carrier dynamics of the excited states [3]. Such dots do not work at the primary telecom wavelength of 1550 nm, however.

Our single bound state dots do operate near 1550 nm, but do not have a carrier reservoir of excited states. Thus, to increase the XGM bandwidth we examine using an additional probe to suppress carrier oscillations and take advantage of the enhanced non-linearities in quantum-dots. Previous work has established the fundamental understanding of cross-gain modulation in both an ideal SOA [4,5] and on the
lasing mode of a cavity [6]. The key difference between these models is that an ideal SOA contains no feedback, and in this situation the cross-gain modulation response is flat and limited in bandwidth by the carrier lifetime. A laser, however, experiences a large level of feedback due to facet reflection creating a relaxation oscillation inside the cavity which can suppress carrier oscillations and extend the XGM response. The strong facet reflection of a laser is not-ideal for wavelength conversion however, as the formed Fabry-Perot cavity limits the wavelengths which can propagate and be converted.

In this chapter, we present a simple analytical model for the XGM response of an almost ideal SOA which contains a level of reflection too small to create significant Fabry-Perot modes. Utilizing this small feedback level, we demonstrate that an additional pump field can create a relaxation oscillation inside the cavity and increase the XGM bandwidth. We then perform an experimental demonstration of this enhancement in which the XGM bandwidth is extended from 1 GHz to 25 GHz. We compare these experimental results to our theoretical model. Finally, we discuss how these effects should differ between quantum dots and quantum wells given their different carrier dynamics and non-linearities.

7.2 Theory

Our model, of the almost ideal SOA, will focus on three propagating light fields. The first is the signal field, $S_s(t)$, the light field which we desire to move information to. Next, is the unmodulated pump, $S_1(t)$, which is used to create the relaxation oscillation and extend the XGM bandwidth. The final light field is the modulated pump, $S_2(t)$, which contains the information we wish to move to the signal. A diagram of how we consider these fields and their interaction with the device is shown in Fig. 7.1. The cavity mode, $S_1(t)$, experiences a small level of feedback in the cavity,
Figure 7.1: Diagram of the theoretical model for XGM in an SOA. $S_1(t)$ has a low level of reflectivity and experiences feedback in the cavity. As the device is below lasing the population of $S_1(t)$ is maintained via the external pump $S_{1,\text{in}}(t)$. The system is driven by $S_2(t)$, and the signal field $S_s(t)$ is initially continuous wave, but becomes amplitude modulated at the exit facet due to cross-gain modulation.

but must be maintained via an externally injected pump beam as the gain does not overcome mirror loss. $S_2(t)$ drives the system, and patterns its amplitude modulation onto $S_s(t)$ via cross-gain modulation. The rate equations for carrier density, $N(t)$, and photon density, $S_1(t)$, have been previously established for a lasing cavity [6].

$$\frac{\partial N(t)}{\partial t} = \frac{I}{qV} - \frac{N(t)}{\tau_s} - v_g g(\lambda_1, t) S_1(t) - v_g g(\lambda_2, t) S_2(t)$$ (7.1)

$$\frac{\partial S_1(t)}{\partial t} = v_g g(\lambda_1, t) S_1(t) - \frac{S_1(t)}{\tau_p} + \frac{S_{1,\text{in}}}{\tau_p}$$ (7.2)

Here, $\tau_p = (\frac{2v_g}{L} \ln(1/R))^{-1}$ is the photon lifetime for a cavity of length $L$, facet power reflectivities $R$ on both sides, and a group velocity $v_g$. Above, as in the rest of this analysis, we ignore intrinsic loss as for low reflectivity cavities the dominant photon loss mechanism is the device facets. To account for the fact that the device is not a laser, and therefore, $S_1(t)$ must have external input to be non-zero in value, we have added an additional term, $S_{1,\text{in}}/\tau_p$, to account for the second, continuous-wave pump which feeds the cavity mode. We also assume that $S_2(t)$ is an external parameter caused by the modulated pump propagating through the cavity. Feedback effects which may alter $S_2(t)$ have not been considered.

For the signal beam propagating through the device, the output is related to the input via the gain of the device, $g(\lambda_s, t)$, and its length, $L$. Assuming that $g(\lambda_s, t)$
does not vary along the length of the device

\[ S_{s,\text{out}}(t) = S_{s,\text{in}}(t)e^{g(\lambda_s, t)L}. \]  

(7.3)

The gain of the device is in turn related to both the carrier density and the other optical fields present in the device.

\[ g(\lambda_i, t) = \frac{g_i + a_i \Delta N(t)}{1 + \epsilon_1 S_1(t) + \epsilon_2 S_2(t)} = \frac{g_i + a_i \Delta N(t)}{U(t)} \]  

(7.4)

Here, \( g_i \) is the steady-state gain, and \( a_i \Delta N(t) \) accounts for gain changes due to changes in the carrier density, \( \Delta N(t) \). The \( \epsilon_i \) terms are included phenomenologically to account for non-linear gain saturation due to spectral hole burning and carrier heating.

By taking the time derivative of (7.3) the time response of the output signal field can be shown to be

\[ \frac{\partial S_{s,\text{out}}(t)}{\partial t} = \frac{\partial(e^{g_s(t)L})}{\partial t} S_{s,\text{in}}(t) + e^{g_s(t)L} \frac{\partial S_{s,\text{in}}(t)}{\partial t}, \]  

(7.5)

To first order, this expression can be expanded and simplified by using (7.4) and remembering that \( S_{s,\text{in}} \) is an unmodulated signal field and does not vary in time.

\[ \frac{\partial S_{s,\text{out}}(t)}{\partial t} = S_{s,\text{in}}(t)e^{g_s(t)L} \left[ \frac{a_s \frac{\partial N}{\partial t}}{U(t)} - \frac{g_s \epsilon_1 \frac{\partial S_1}{\partial t}}{U(t)^2} - \frac{g_s \epsilon_2 \frac{\partial S_2}{\partial t}}{U(t)^2} \right] \]  

(7.6)

We assume that the modulated input pump will cause both the carrier density and the other light fields to oscillate as well so that we express

\[ N(t) = N + n(\omega)e^{-i\omega t} + \text{c.c.} \]  

(7.7)
where \( j \) can be 1, 2, or \((s, out)\). Adopting this notation, from now on symbols not explicitly stating their \( t \) dependence will denote the steady state value.

Solving for the steady state value of \( S_1 \) we find that

\[
S_1 = \frac{S_{1, in}}{1 - \mathcal{L}} \tag{7.9}
\]

where \( \mathcal{L} = g_1 v_g \tau_p \) and is equal to one if the device is a laser, and less than one for an SOA. The results show that a significant internal photon density can be created for \( \mathcal{L} \) near one for even modest input powers. It also shows that to create an effect in the cavity values of \( \mathcal{L} \) near but less than one should be investigated. This corresponds to a device that is biased just below the lasing threshold. In our case that is an SOA with high-gain but low facet reflectivities.

Substituting the (7.7) and (7.8) into (7.1), (7.2) and (7.6) we can express the rate equations to first order as

\[
-\,i\omega n(\omega) = -\,\frac{n(\omega)}{\tau_s} - v_g g_1 s_1(\omega) + \frac{v_g g_1 S_1(\epsilon_1 s_1(\omega) + \epsilon_2 s_2(\omega))}{U} - v_g g_2 s_2(\omega) - a_2 v_g S_2 \frac{n(\omega)}{U} + \frac{v_g g_2 S_2(\epsilon_1 s_1(\omega) + \epsilon_2 s_2(\omega))}{U^2} \tag{7.10}
\]

\[
-\,i\omega s_1(\omega) = s_1(\omega) \left(v_g g_1 - \frac{1}{\tau_p}\right) + v_g g_1 s_1(\omega) + \frac{v_g g_1 S_1(\epsilon_1 s_1(\omega) + \epsilon_2 s_2(\omega))}{U} - \frac{v_g g_1 S_1(\epsilon_1 s_1(\omega) + \epsilon_2 s_2(\omega))}{U^2} \tag{7.11}
\]

\[
s_{s, out}(\omega) = S_{s, in}(\omega) L e^{g_2 L} \left(\frac{a_2 n(\omega)}{U} - \frac{\epsilon_1 g_2 s_1(\omega)}{U^2} - \frac{\epsilon_2 g_2 s_2(\omega)}{U^2}\right) \tag{7.12}
\]

Here we have assumed that \( \lambda_s \approx \lambda_2 \) so that \( g_s = g_2 \) and \( a_s = a_2 \). These provide three equations with three unknown parameters \( n(\omega) \), \( s_1(\omega) \), and \( s_{s, out}(\omega) \) that can
be solved to give us the XGM response for $s_{s,\text{out}}(\omega)$

$$
\frac{s_{s,\text{out}}(\omega)}{s_2(\omega)} = -S_{s,\text{in}} e^{g_2 L} L \times \left[ \frac{g_2 \epsilon_2 (1 - \mathcal{L} - i\tau_p \omega)(1 - i\tau_s \omega) + \frac{U g_2}{P_2} (1 - \mathcal{L} - i\tau_p \omega)}{U^2 (1 - \mathcal{L} - i\tau_p \omega)(1 + \frac{S_2}{P_2} + \frac{S_1}{P_1} - i\tau_s \omega) + S_1 \mathcal{L} \epsilon_1 (1 - i\tau_s \omega) + \frac{S_1 \mathcal{L} \epsilon_1}{P_1}} \right] 
$$

(7.13)

where $P_1 = 1/(a_i v g \tau_s)$ is the photon saturation density for pumps 1 and 2. We have also assumed that the gain is linear with carrier density so that $a_1 g_2 = a_2 g_1$. Another important assumption we have made is that only one pump experiences feedback in the cavity, and therefore we have taken the limit that $g_2 v g \tau_p << 1$.

Examining (7.13), it is possible to see that for $S_1 = 0$ the response is that of a low pass filter with bandwidth determined by the carrier lifetime. Increasing values of $S_2$ extend the bandwidth as stimulated emission decreases the carrier lifetime. For $S_1 > 0$ though, a resonance appears as the contributions from the cavity no longer vanish. Increasing values of $S_1 \mathcal{L}$ magnify the resonance and move it to higher frequencies.

### 7.3 Experiment

To compare our theory to experiment, we measure the effect of an additional pump-field on the XGM response of a quantum dot SOA. The device used was the same as that discussed in Chapter 2. The experimental setup can be seen in Fig. 7.2. As pictured in the setup, the first pump is modulated using a LiNbO$_3$ MZM driven by a network analyzer. The modulated pump is then coupled with the probe laser using an 80/20 coupler, amplified with an EDFA, and then sent into the device using a lensed fiber. The second pump is not modulated, and is sent into the device in a direction counter-propagating to the first pump and probe. At the probe’s output facet, a circulator is used to separate the ingoing second pump from the outgoing first.
Figure 7.2: Experimental setup for measuring XGM with an additional applied pump. The first pump is modulated and coupled to the probe laser using an 80/20 coupler. The two light fields are then sent into the device using a lensed fiber. At the output facet of the device, they are similarly collected, and a circulator is used to separate them from the second pump field which is counter-propagating to the first pump and signal. All light fields are set to be TE polarized inside the device via manipulation of the polarization controllers (PC).

pump and probe. The output signal was then sent to an optical bandpass filter to remove the pump field from the probe. The filtered signal was then detected with a high-speed optical detector and the electrical signal read out on the network analyzer. The modulation speed of the network analyzer was swept from 0.1 to 25 GHz limited by the electrical amplifier which boosted the detected signal power. The bias current of the device was held at 800 mA.

For our experiment, the modulated pump was set to 1553.3 nm and had a power of 1 dBm. It was purposefully set away from the gain peak of the device to avoid the possibility of the feedback effects on this pump. The probe, at 1551.8 nm, was set close in wavelength to the first pump so that the device gain of the first pump and probe would be nearly equal as assumed in our model. The power was set to -20 dBm. The second pump was swept both in power and in wavelength to determine its effect on the probe’s response. At a power of 6.0 dBm, the sweep in wavelength can be seen in Fig. 7.3(a). For reference the gain of the device is plotted in Fig. 7.3(c). As the wavelength is swept from 1507 nm to 1557 nm, there is initially no visible enhancement, but as
Figure 7.3: (a) Cross gain modulation response with an additional unmodulated pump field. The unmodulated pump was set to a power of 6.0 dBm and its wavelength swept. Data show that as the second pump’s wavelength approached that of the device’s gain peak of 1532 nm a large bandwidth expansion is observed resulting in a 3 dB bandwidth of greater than 25 GHz. (b) Theoretical calculations for the response using a variety of different gains. Calculated results closely match experiment. All theoretical gains resulted in $L$ values of less than one so that the device would not function as a laser. (c) Gain of the QD device showing a peak gain near 1532 nm.
Table 7.1: Parameters for the theoretical calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Device Length</td>
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</tr>
<tr>
<td>Group Velocity</td>
<td>$v_g$</td>
<td>$c/3.5$</td>
</tr>
<tr>
<td>Facet Reflectivity</td>
<td>$R$</td>
<td>0.003</td>
</tr>
<tr>
<td>Non-radiative lifetime</td>
<td>$\tau_s$</td>
<td>1.2 ns</td>
</tr>
<tr>
<td>Pump 1 Saturation Density</td>
<td>$P_1$</td>
<td>$10^{16}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Pump 2 Saturation Density</td>
<td>$P_2$</td>
<td>$10^{16}$ cm$^{-3}$</td>
</tr>
<tr>
<td>Pump 1 Photon Density</td>
<td>$S_1$</td>
<td>$0.1 P_1$</td>
</tr>
<tr>
<td>Pump 2 Photon Density</td>
<td>$S_2$</td>
<td>$4 P_2$</td>
</tr>
<tr>
<td>Pump 1 non-linear saturation coeff</td>
<td>$\epsilon_1$</td>
<td>$0.200/P_1$</td>
</tr>
<tr>
<td>Pump 2 non-linear saturation coeff</td>
<td>$\epsilon_2$</td>
<td>$0.025/P_2$</td>
</tr>
</tbody>
</table>

The wavelength moves toward 1532 nm, the gain peak of the device, a large resonance appears, extending the 3 dB bandwidth. As the wavelength moves away from 1532 nm, the resonance again disappears as the gain decreases. A plot of the theoretical calculations for the XGM response for various different pump 2 gains can be seen in Fig. 7.3(b). These plots show excellent qualitative and quantitative agreement with the experimental data. The various theoretical parameters used in the calculation are listed in Table 7.1. As can be seen in (7.13), the absolute value of the optical powers is not important, only their ratio with the various saturation parameters matter. An examination of the table will show that $S_1 < S_2$ even though in our experiment the modulated pump had less power than unmodulated pump. This is because the given pump powers are the in-fiber powers for the two pumps, not the amount of power inside the cavity. Due to the experimental sensitivity to optical alignment, the alignment of the lensed fibers had to be adjusted to collect reliable data. Thus, the two facet couplings were not equal with the modulated pump’s input coupling having less loss than the unmodulated pump’s input coupling. The other various parameters are typical values for a semiconductor device chosen to give a close match between data and theory. The facet reflectivies are only 0.3% showing a small-level of reflection expected from a non-ideal SOA.
Figure 7.4: (a) Cross gain modulation response with an additional pump field set to 1517 nm. Increasing pump power shows an increase in resonance and an increase in the 3 dB bandwidth. (b) Theoretical calculations of the response with $L = 0.99$. Calculated curves show excellent agreement with experiment.

Experimentally it was observed that the response was extremely sensitive to the optical alignment of the second pump which made taking reliable data at the gain peak of the device difficult. Thus, to examine the power dependence the second pump was placed at 1517 nm, and the XGM response measured for increasing input power. These experimental results, and the theoretical results for $g_1 = 28.8 \text{ cm}^{-1}$ can be seen in Fig. 7.4. Again, the experimental results show a close qualitative and quantitative match with our theoretical calculations. Increasing $S_1$ results in the creation of a resonant peak and an increased bandwidth beyond 1 GHz.

### 7.4 Discussion

While our experimental work was limited to testing speeds less than 25 GHz, an examination of the theoretical model will allow us to predict the best performance than can be obtained. An examination of (7.6) shows that there are three contributing factors to the XGM response: carrier population oscillation, the pulsing cavity light field, and the non-linear saturation which represent spectral hole burning. The carrier
Figure 7.5: Comparison between different methods of increasing the XGM bandwidth. Dashed line shows traditional XGM which is dominated at low modulation speeds by carrier oscillations, and at high modulation speed by spectral hole burning. Typical bandwidth extension is shown as the dotted line which uses high pump powers to increase the stimulated emission rate and decrease the carrier lifetime. Solid lines shows the bandwidth extension possible using our resonant enhanced technique. The cavity mode is used to suppress carrier population to the response level of spectral hole burning. Vertical lines indicate the regimes in which carrier oscillations and spectral-hole burning dominate in typical XGM.

oscillations are limited by the carrier lifetime including both the stimulated emission rates and the non-radiative lifetime. The cavity field oscillations are similarly limited, but also have a contributing factor from the photon lifetime. The non-linear saturation has no limit in our model, but as it represents spectral hole burning in the device we expect the actual bandwidth to be limited by the spectral hole recovery rate.

In typical cross-gain modulation, the contribution from spectral hole burning is small compared to the contribution due to carrier population oscillations. This typical response can be seen as the dashed line in Fig. 7.5. The response is driven by carrier population oscillations for modulation speed of less than 1 GHz where the response is large. As the modulation speed becomes faster than the carriers can respond,
spectral-hole burning becomes the dominant XGM mechanism and is responsible for the much lower response plateau at frequencies greater than 3 GHz.

The typical method of increasing the bandwidth is to decrease the carrier lifetime. This can be achieved by increasing the pump power to increase the stimulated emission rate. Increasing the bandwidth in this manner however, requires ever increasing powers, which saturate the device and reduce the response as seen on the dotted curve of Fig. 7.5. In the figure, the pump power has been increased by a factor of 15, but the XGM bandwidth shows only a modest improvement to near 6 GHz. Furthermore, the additional gain saturation from the increased pump power greatly decreases the efficiency.

Using an SOA near the lasing threshold however allows for an additional pump to create a cavity mode which suppresses carrier oscillations. Modulations in the input pump are countered by corresponding oscillations in the cavity mode so that the global carrier density does not oscillate. This resonantly enhanced XGM is depicted as the solid line in Fig. 7.5. Here, rather than increasing the power of the non-resonant pump field, we have added an additional pump field $S_1 = 0.4$, a very small increase in the total optical power. However, the carrier oscillation response has been suppressed so that spectral hole burning dominates the dynamics and extends the 3 dB bandwidth to greater than 25 GHz. By suppressing carrier population oscillations, the XGM bandwidth becomes limited by the spectral hole recovery rate, and should allow for conversion of signals in the hundreds of GHz.

While our experimental results are for a QD device, our theoretical model is independent of the material used and thus applicable to both quantum wells and quantum dots. The key differences between quantum wells and quantum dots for this application are the saturation powers and the magnitude of the non-linear gain coefficients which allow for conversion at higher modulation speeds. The smaller confinement factor of quantum dots increases the saturation power and decreases the efficiency of
XGM due to carrier oscillations for a set pump power. This will reduce the amount of power needed in the cavity mode to counter the pump induced carrier oscillations. Furthermore, as we have shown previously in our four-wave mixing measurements, the quantum-dot non-linearities are significantly stronger than quantum-well non-linearities, increasing the XGM efficiency for higher-speed signals. This will result in higher XGM efficiency in quantum dots in the regime that are dependent on non-linear processes for cross-gain modulation.

7.5 Summary

In this chapter, we have presented a theoretical model for cross-gain modulation in a non-ideal SOA with a small level feedback. Our theoretical model shows qualitative and quantitative agreement with our experimental data. Using resonant enhancement the XGM bandwidth was increased from 1 GHz to 25 GHz with the addition of a second pump field used to create a cavity mode. This extension of the XGM bandwidth is possible due to the cavity mode suppressing carrier population oscillations so that spectral hole burning became the dominant XGM mechanism. From this, we predict that an extremely broad XGM response is possible with limits in the hundreds of GHz. While this effect is not dependent on the the properties of quantum dots, and is thus also possible in quantum-well devices, the increased non-linearities in quantum dots allow for easier carrier suppression and higher conversion efficiency.

7.6 References


Chapter 8

Slow-light and Fast-light at the Lasing Threshold

8.1 Introduction

In the previous chapter we demonstrated that the resonance of a cavity mode can provide a large increase in the XGM bandwidth. In doing this, we examined the magnitude response of the signal propagating in the cavity. Additionally, the phase response of such a cavity also provides a novel method of achieving slow-light.

Most techniques used to achieve a change in the group velocity rely on the dispersion characteristics of sharp absorption peaks and dips [1–3]. These sharp peaks and dips in absorption, cause sharp slopes in the refractive index via Kramers-Kronig relations. These large slopes in turn create large group delay as the group velocity $n_g = n + \omega \frac{\partial n}{\partial \omega}$. As a result however, the magnitude of the shifted and un-shifted pulses can vary widely requiring post processing to equalize the magnitudes. Using a single device to transition from slow-light in the absorption regime to fast-light in the gain regime, has already been demonstrated, but with large amplitude shifts accompanying the effect [4].

Recent experimental evidence has demonstrated that fast- to slow-light switching occurs in a gain-clamped semiconductor optical amplifier (GC-SOA) [5]. A spectral dip in a gain medium typically displays fast light characteristics, but when the lasing mode was present in the GC-SOA slow-light was observed instead. The stated explanation was the presence of anomalous gain at the onset of lasing due to spatial hole burning [6].
Figure 8.1: Diagram of the light fields considered in our analytical model. $S_1(t)$ is the lasing mode which experiences feedback at the device facets. An input photon flux $F_{2,\text{in}}(t)$ is injected into the device, and a flux $F_{2,\text{out}}(t)$ exits the other side of the device.

In this chapter, we investigate slow-light in a laser cavity. We begin by establishing an analytical model for the phase response of a modulated signal propagating in a laser cavity. This model shows that the underlying physical cause of the slow-light effect in the laser mode is the cross-gain modulation that occurs between the laser mode and the signal. We then test our theoretical model using a distributed-feedback (DFB) laser. Next, we examine the previous experimental data in a gain-clamped SOA and show that our theoretical analysis is sufficient to explain the phenomenon observed there as well. Finally, we experimentally demonstrate the effect in a ring-laser cavity for 10 ps pulses, and show that this method of optical delay can be easily cascaded to achieve increasing delay.

### 8.2 Analytical Model

The theoretical model is similar to that established in Chapter 7 except that we are interested in examining the effects above threshold, so we will assume that the cavity is lasing and thus $1/\tau_p = v_g g_1$ and that no pump beam is necessary to maintain the cavity mode. The theoretical model for our investigation is pictured in Fig. 8.1. We consider that a laser cavity with photon density, $S_1(t)$, and carrier density, $N(t)$, has an input photon flux, $F_{2,\text{in}}(t)$, and that a photon flux, $F_{2,\text{out}}(t)$, exits the cavity. For these assumptions, the rate equations for the carrier density and laser mode photon
In the above, \( I \) is the applied current, \( q \) is the electron charge, \( V \) is the volume of the active media, \( v_g \) is the group velocity inside the cavity, \( g(\lambda_i, t) \) is the gain of the device, \( L \) is the length of the device, and \( \tau_p \) is the photon lifetime for the lasing mode. These are similar to the rate equations used in Chapter 7 except in the treatment of the input field. As each new photon that exits the cavity comes from carrier recombination, the carrier population must change by the difference between the input and output flux. The total carrier population, \( VN(t) \), is reduced by the difference between photons in, \( AF_{2,\text{in}}(t) \), and photons out, \( AF_{2,\text{out}}(t) \) where \( A \) is the cross-sectional area of the light-field. Dividing by \( V \) to determine the average change on the current density we arrive at the final term in (8.1).

We will assume that \( g(\lambda_2, t) \) does not change along the length of the device so that

\[
F_{2,\text{out}}(t) = e^{g(\lambda_2, t)L} F_{2,\text{in}}(t). \tag{8.3}
\]

This should be a good approximation in a laser cavity as the gain of the device should be clamped due to the lasing mode. Taking the time derivative of the output flux we find that

\[
\frac{\partial F_{2,\text{out}}(t)}{\partial t} = \frac{\partial F_{2,\text{in}}(t)}{\partial t} e^{g(\lambda_2, t)L} + F_{2,\text{in}}(t) \frac{\partial g(\lambda_2, t)}{\partial t} \frac{\partial}{\partial t} L e^{g(\lambda_2, t)L}. \tag{8.4}
\]

For simplicity, we will also ignore non-linear saturation so that

\[
g(\lambda_i, t) = g_i + a_i \Delta N(t). \tag{8.5}
\]
To determine the output response, we perform a small-signal analysis by assuming that $F_{2,in}(t)$ is modulated causing both the carrier density and laser field to modulate as well.

$$F_2(t) = F_2 + f_2(\omega)e^{-i\omega t} + c.c. \quad (8.6)$$
$$S_1(t) = S_1 + s_1(\omega)e^{-i\omega t} + c.c. \quad (8.7)$$
$$N(t) = N + n(\omega)e^{-i\omega t} + c.c. \quad (8.8)$$

Adopting this notation, from now on symbols not explicitly stating their $t$ dependence will denote the steady state value.

By substituting (8.6)-(8.8) into (8.1), (8.2) and (8.4) we get a system of three equations and three unknowns

$$-i\omega n(\omega) = -\frac{n(\omega)}{\tau_s} - g_1 v g s_1(\omega) - (e^{g_2 L} - 1)\frac{f_{2,in}(\omega)}{L} - a_1 v g S_1 n(\omega) - F_{2,in}e^{g_2 L}a_2 n(\omega)$$

$$-i\omega s_1(\omega) = v g a_1 n(\omega)S_1 \quad (8.9)$$

$$f_{2,\text{out}}(\omega) = f_{2,\text{in}}(\omega)e^{g_2 L} + F_{2,\text{in}}e^{g_2 L}a_2 n(\omega)$$

(8.10)

(8.11)

allowing us to solve for the response of the output flux and the laser response

$$\frac{f_{2,\text{out}}(\omega)}{f_{2,\text{in}}(\omega)} = e^{g_2 L} \frac{i S_1 v g g_1 + \omega(1 + F_2 + S_1 - i\tau_s \omega)}{i S_1 v g g_1 + \omega(1 + e^{g_2 L}F_2 + S_1 - i\tau_s \omega)} \quad (8.12)$$

$$\frac{s_1(\omega)}{f_{2,\text{in}}(\omega)} = \frac{-i(e^{g_2 L} - 1)S_1/L}{i S_1 v g g_1 + \omega(1 + e^{g_2 L}F_2 + S_1 - i\tau_s \omega)}. \quad (8.13)$$

In these results, the responses are only dependent on the input flux and thus we have replaced $F_{2,in}$ with $F_2$. We have also re-expressed $S_1$ and $F_2$ in units of the saturation photon density, $1/(a_1 v g \tau_s)$, and saturation photon flux, $1/(a_2 \tau_s)$, respectively. It is
important to point out that the solution is a function of $v_g$, but that this is the group velocity of the laser mode, not the signal field. Thus, $v_g$ can be considered a static constant even though we are examining changes in the group velocity of the signal.

To begin, we use our response functions above to plot the magnitude and phase response of the signal field which can be seen in Fig. 8.2. For reference, we have also included the magnitude and phase response of the laser mode in the cavity. Fig. 8.2(a) shows the phase response of the output signal. As expected, and in agreement with previous theories and experiments [1,3,7], for no laser inside the cavity there is a fast light effect which causes a negative phase response. As the laser field is turned on however, a positive phase shift is observed for low modulation speeds, which corresponds to slow-light. For larger frequencies, the response transitions back to fast-light as indicated by the negative phase shift. As the laser power is increased, the magnitude of the phase shift is decreased, and the frequency at which the transition occurs is also increased. Examining Figs. 8.2(a) and 8.2(c), we are able to see the cause of this transition. The oscillating signal field modulates the carrier density, which in turn causes oscillations in the laser field via cross-gain modulation. These laser oscillations are out of phase with the signal field. As the modulation frequency approaches the relaxation frequency of the laser, the lasers response is amplified by resonant enhancement so that the laser field begins to dominate the carrier dynamics and drives the carriers in phase with the signal rather than out of phase, causing slow-light. As the modulation speed is increased beyond the relaxation frequency, the laser response decreases and becomes in phase with the signal. As a result, the signal field again dominates the carrier dynamics, driving them out of phase and returning to fast-light.

This can also be seen in the magnitude response of the curves where the laser-signal interaction also creates a dip in the signal magnitude at the relaxation frequency where the laser is most dominant. Kramers-Kronig relation then creates a slow-light
Figure 8.2: (a) Plot of the phase response for the output signal. With no lasing field present, the results show a negative phase response and the expected fast-light behavior. As the magnitude of the laser field is increased, slow-light appears at low frequencies and switches to fast-light at higher-frequencies. Increasing signal power broadens the region of slow-light while reducing the phase response. (b) The magnitude response of the output signal. Results show a dip in the gain (peak in absorption) that moves toward higher frequencies with increasing laser power. (c) Phase response of the laser field showing it to be initially out of phase with the signal as expected due to XGM, but becoming in phase for frequencies past the relaxation frequency. (d) Magnitude response of the laser field inside the device showing peaking due to relaxation oscillations that correspond to the slow-light to fast-light transition in the signal response. Parameters used in plots: $L = 800\mu$m; $v_g = c/3.5$; $g_1 = g_2 = 20$ cm$^{-1}$; $\tau_s = 1$ ns; $F_2 = 0.1$. 
Figure 8.3: A diagram of the gain (a) and refractive index (b) associated with the laser cavity. The center frequency corresponds to the signal frequency. Results show that the two gain dips in close proximity result in a slow-light effect for the signal at the center.

effect for the signal beam. A dip in gain is usually associated with fast light, but only if that dip is centered at the signal wavelength. In the above results the dip is not at zero modulation, but rather at higher frequencies and is thus not centered at the signal wavelength. To understand this, it is important to remember that the plots we have presented are the phase shift of the sidebands as they are swept away from the central signal frequency. Importantly there are two sidebands, and the gain dispersion curve is symmetric so that the calculations we show are for one half of the curve. Thus, what appears as one dip in our magnitude response, is actually two, one blue shifted from the signal field, and one red-shifted from the signal field. If Kramers-Kronig relation is applied to the two dips symmetric around the signal wavelength, we observe a central region of slow-light for small pump-probe detunings, and then regions of fast-light as the pump-probe detunings are increased. This is shown diagrammatically in Fig. 8.3.

In the figure, it shows that while each individual gain dip creates a negative slope in the refractive index, that at the central frequency, where the signal laser resides, there exists a positive slope resulting in slow-light. An important advantage of this method of slow- and fast-light is that the transition from slow- to fast-light can be achieved over a very narrow gain regime around the laser threshold. Thus, a large
phase shift can be obtained with a minimal change in the output power. This is in contrast to other methods which require moving the device from the absorption to gain regime, resulting in very large changes in the signal magnitude [4].

8.3 Laser Experiment

Our first experimental examination focused on a commercially manufactured DFB laser with quantum wells as the active media. The DFB laser purposefully has the two facets asymmetrically coated, with one coated to be highly reflective, and the other coated to be anti-reflective to ensure single-mode lasing. Due to the difficulty in coupling light through the highly-reflective facet, our signal beam was sent into the device at the anti-reflection coated facet, and then collected from the same facet using a circulator. The experimental setup is diagrammed in Fig. 8.4. We utilized a tunable laser diode (TLD) as our probe, which we passed through a Mach-Zender modulator (MZM) driven by a network analyzer to induce small-signal amplitude modulations. The polarization was then adjusted with a polarization controller (PC) so that the signal would be TE polarized inside the laser cavity. Next, it was amplified using an EDFA, and sent into the device through a lensed fiber focused on the anti-reflection facet, and then collected from the same fiber. The output signal was then filtered using an optical spectrum analyzer (OSA) that operated as a bandpass filter to eliminate the lasing modes from the DFB itself, so that our data would show only the response of the input signal beam. After filtering, the signal was again amplified before being sent to our detector connected to the network analyzer. The current of the DFB was then adjusted from below to above threshold so that the change in the phase response could be measured.

One complication is that increasing the current bias to a DFB also changes the refractive index. This causes the stop-band of the DBR structure internal to the
Figure 8.4: Experimental setup for testing fast-light to slow-light transition in a DFB laser.

laser to shift. This could cause the internal filter of the DFB to also provide an effect which would mask the changes we want to measure. For this reason, for each increase in current bias, the temperature of the heat sink was increased so that the filter response of the internal DBR structure would remain constant. The new heat sink temperature was chosen so that the side modes of the DFB laser would remain at the same wavelength for all bias currents rather than blue-shifting with increasing bias. By verifying that these side modes do not move, we can be certain that the internal DFB filter has also not changed, allowing for a fair comparison between different bias currents.

The DFB laser used had a threshold current of 10 mA, and produced a single lasing mode at 1543 nm. The input probe was set slightly offset at 1544 nm so that it could be filtered from the laser field, but still have significant gain in the device. The output spectrum for the applied biases can be seen in Fig. 8.5. The spectrum shows the clear lasing mode, and the input probe beam. Importantly, as the bias is raised the side modes of the DFB laser do not shift in wavelength. This demonstrates that our temperature correction is able to correct for the change in refractive index and maintain a constant filter response.

The phase response for the small-signal amplitude modulation can be seen in Fig.
Figure 8.5: The output spectrum of the DFB laser under different bias conditions. For increasing bias the temperature of the heat sink was increased as well to prevent the side modes from shifting to ensure that the internal optical filter response of the DFB was unchanged. The signal field can be seen at 1544 nm.

Figure 8.6: Phase shift of the signal beam below and above threshold. Reference data was taken at 6 mA, the lowest bias at which we were still able to take data. Phase response is similar to theoretical plots in Fig. 8.2(a) with an initial region of slow light followed by a quick transition to fast light at higher modulation speeds.
8.6. The reference data for our data was taken at 6 mA, which was determined to be above transparency, but was the lowest current at which small-signal data could still be measured. Below 10 mA, typical fast-light behavior is observed, with the light moving faster as the current is increased from 7 to 9 mA. As the current is further increased above threshold however, the fast-light switches to slow-light with a clear slow-light peaking that moves toward higher modulation frequencies for higher-bias. As the current bias is increased it is expected that the laser power and relaxation frequency will increase as well. This causes the slow-light effect to extend to higher modulation speeds as the bias increases in agreement with our theoretical model and plots shown in Fig. 8.2(a).

The data, however, do show one discrepancy with theory at low modulation speeds where the data show a small fast-light region not expected in theory. This is caused by the internal filter of the DFB laser. While our temperature correction kept the effect of the internal filter constant, it did not remove the effect. In taking the data, our signal beam had to be placed relative to the internal filter. The magnitude response for data taken at two different frequencies relative to the internal filter can be seen in Fig. 8.7.

In these two plots, the same experiment was performed but at different wavelengths. In Fig. 8.7(a) the signal was set as in Fig. 8.5, on top of a side mode where the internal filter allows propagation. In Fig. 8.7(b) the signal was set in the middle between two side modes and where the filter suppresses propagation. The two magnitude responses both show agreement with the theoretical curve 8.2(b), except at low modulation speeds. Specifically, at these low modulation speeds these two signals show an opposite response with one experiencing enhancement in gain while the other experiences gain suppression. As these two signals are identical in nature, except for their position relative to the internal grating, we can conclude that this difference is caused by the internal grating. This allows us to determine that the
Figure 8.7: The magnitude response for a signal field sent in at two different wave-lengths. (a) was set to have maximum transmission through the DFB filter. (b) was set to have minimal transmission through the filter. Both effects show a deviation from theory, but one which switches dependent on the position relative to the filter. This shows that the experimental discrepancy at low-modulation speeds is caused by the internal filter.

discrepancy in the phase response between theory and experiment, which occurs at these same low frequencies, is also due to the internal grating.

While these results show a small slow-light effect, the data were taken in a device designed to be a laser, and thus signal gain was limited by the quick onset of lasing. Studies in gain clamped SOAs, SOAs which have a built in laser field to clamp the gain, have shown much larger phase shifts are possible [5]. We will next focus on these devices.

### 8.4 Gain-Clamped SOA

A gain-clamped SOA is an SOA which has a built in distributed Bragg reflector (DBR) with a stop-band blue-detuned from the gain peak of the device. This DBR structure creates a lasing mode which pins the gain so that increasing signal power will receive the same level of amplification rather than diminishing gain due to gain saturation. If enough signal power is applied however, the device will saturate and
eliminate the lasing mode. In this region, experimental studies have demonstrated fast-light to slow-light switching, but no theoretical analysis has fully explained the phenomenon [5]. One initial explanation was that the gain anti-saturation which occurs in the device due to spatial hole burning could create the effect. This anti-saturation can be seen in Fig. 8.8(a). The gain saturation curve shows a very flat response due to the gain-clamping provided by the internal laser field. As the probe power is increased however, the increased saturation eventually lowers the gain enough that lasing action can no longer occur, at this transition an increase in the gain is noted due to the carriers redistributing themselves inside the device [6]. As signal power is further increased, gain falls off precipitously due to saturation.

Figure 8.8(b) shows the phase shift of an amplitude modulated beam sent through the device, and clearly shows fast-light at low currents, where there is no lasing field, which then quickly jumps to slow-light as the current is increased and the lasing threshold is passed. While these results show a passing similarity to the theoretical curves shown previously, the results do not show the exact same characteristics. First, the transition from slow- to fast-light is gradual compared to the laser’s response. The

Figure 8.8: (a) Gain saturation curves for the gain-clamped SOA used in [5]. Inset shows the saturation characteristics of an SOA without gain clamping. (b) Experimental curves from [5] showing a switch from fast-light to slow-light in a gain-clamped SOA.
phase shift is also much larger. However, our previous work focused on a laser, in which the gain of the signal is less than the gain of the device, and in which the signal field was weak. In a GC-SOA though, these two conditions are not true. The DBR structure inside a GC-SOA is purposefully placed far from the gain peak so that there will be larger signal gain in the device than laser gain. Furthermore, the above experiment utilized a fiber pig-tailed device allowing for more efficient coupling to the device facets. This will result in a larger value for $F_2$. Also, the device was operated well above threshold and it was the saturation due to the signal field that caused the gain to be reduced. Such a high level of saturation will result in a non-uniform carrier distribution inside the device so that the assumption that $g(z)$ is constant along the length of the device is no longer valid. The higher powers in the cavity also mean that non-linear saturation effects can not be ignored as before. Still, despite these differences, it is expected that our model should at least provide a qualitative agreement with the experimental results. Quantitatively, we expect that the analytical model will over-estimate the response, especially for low laser levels where there will be significant saturation due to the signal field which our model ignored.

To model the GC-SOA, we first set the device gain of the signal to that shown in the saturation curve of Fig. 8.8(a), 17 dB. 14 dB as shown in 8.8(a) plus 3 dB for insertion loss. Then the laser gain was reduced below the signal gain, and $F_2$ increased. The results of this calculation can be seen in Fig. 8.9. Due to the higher signal power in the cavity, effects due to non-linear saturation are included as well. As the non-linear saturation effects do not modify the underlying physics, we have not shown the solution for the response functions here, but they are provided in Appendix A.

The analytical calculation shows a good qualitative comparison to experiment, but over estimates the amount of phase shift when the laser field is turned off. From this
Figure 8.9: Analytical result for the gain-clamped SOA. Theoretical results are qualitatively similar to previous experimental measurements, but show a larger phase delay than experiment for when the laser field is turned off. This is due to the model assuming gain pinning due to the laser field which does not occur when the signal field is strong relative to the laser. Calculation parameters: \( L = 2 \) mm; \( v_g = c/3.5 \), \( g_1 = 8 \) cm\(^{-1} \), \( g_2 = 20 \) cm\(^{-1} \), \( \tau_s = 1 \) ns, \( F_2 = 1.5 \), and \( \epsilon_1 = \epsilon_2 = 0.03 \).
we can conclude that while our developed model is limited to power regimes where
the input signal is small compared to the laser field, that the underlying physical
mechanism in the GC-SOA is the same as in the DFB laser. This is not the anti-
saturation that occurs as the lasing mode of the GC-SOA is turned off as previously
thought, but rather the interaction between the lasing mode and the signal mode via
XGM as in the DFB laser.

8.4.1 Ring-Laser

As our theoretical analysis was not specific to any single type of laser cavity, it
is expected that this phenomenon should occur in all laser cavities. A ring laser
provides the unique ability to test how well this method of pulse time-shifting can
be cascaded with multiple passes, as we can input a pulse into the ring-laser cavity
and then monitor the output to see how it shifts after multiple passes through the
lasing media. For this case, our input pulse was a mode-locked laser which generated
600 fs pulses with a 25 MHz repetition rate. A diagram of the experimental setup
can be seen in Fig. 8.10. The mode-locked laser pulse was first filtered to broaden
it to a width of 10 ps as measured with autocorrelation. This broadened pulse was
then amplified through an EDFA before being sent into the ring cavity via a series
of 90/10 couplers. Inside the cavity, another bandpass filter was utilized to help
eliminate ASE and force the ring cavity to lase near the input wavelength. It should
be noted that due to the close proximity between the lasing wavelength and the signal
wavelength, the signal had a net loss as it progressed around the loop. Also in the
ring cavity were a polarization controller, and a LiNbO$_3$ MZM that was utilized for
its polarization dependence to filter out unwanted polarization modes. Next, the
pulse traveled through a quantum-well SOA whose current was tuned from below to
above the ring cavity threshold to control the time-delay in the cavity. An EDFA
was also placed in the cavity to adjust the round-trip loss, but its pump power was
Figure 8.10: Experimental setup for examining fast- to slow-light switching in a ring laser. Mode-locked fiber laser served as the laser source, with the short pulses filtered to create broader pulses. These pulses were then fed into the laser cavity, and the output pulse train due to the pulse propagating multiple times around the cavity were measured on the oscilloscope.

held constant during the experiment so that the only control knob was the current on the SOA. Finally, some portion of the input pulse would exit out via the same 90/10 couplers, and its time delay measured on the optical oscilloscope. A fiber delay line was utilized to adjust the round trip time of the ring cavity to make it easy to identify how many times a specific pulse had traversed the cavity. Isolators were also included to ensure propagation of the light fields in only one direction.

As seen in Fig. 8.10, we expect one pulse to go to the oscilloscope without passing through the ring-laser. This pulse was then used as our reference. Subsequent pulses would exit the ring-cavity after each round trip separated from the first pulse by an amount $\delta$, the time difference between the pulse round trip and the period of the mode locked laser. Each pulse would be weaker with each pass as the cavity had a net loss for the probe beam under all conditions. The position of these pulses measured with the oscilloscope under different SOA bias would then demonstrate both tunablity and cascadability of the setup as a single pulse had traversed the cavity multiple times.
Figure 8.11: (a) Autocorrelation trace of the input pulse showing a 12 ps width. This results in an actual pulse width of around 10 ps. (b) Oscilloscope traces of the output pulses for various numbers of loops. Increasing loops correspond with greater time shift showing excellent cascadability. Furthermore, increasing the bias causes the pulses to exit the loop later rather than sooner showing a slow-light effect.
Figure 8.12: Plot of time-delay vs. number of passes through the ring cavity. Shows a nearly linear response, with a shift from fast-light to slow-light. Reference time is determined by a 60 mA bias on the SOA.

Oscilloscope traces of the output for various input biases and different numbers of passes through the ring cavity can be seen in Fig. 8.11(b). From this data the increasing time delay with increasing bias is apparent showing that the slow-light effect of the lasing cavity continues to persist even for short 10 ps pulses. Increasing passes also demonstrate greater delay showing a high-level of cascadability for this setup. Pulses appear broader on the oscilloscope traces due as the optical detector used has a 10 GHz bandwidth which is much longer than the 10 ps pulses. A complete plot of time delay for all passes for which we could collect data can be seen in Fig. 8.12. The data show a nearly linear increase in the time-delay with increasing passes through the SOA indicating excellent behavior for cascading. The exception is at low loop numbers where the time delay seems to behave strangely with increasing loops. Indeed a single pass through the device shows not a time-delay, but instead a time-advance indicating fast-light. This can be understood by the fact that the first input pulse is strongest, and in this case was strong enough to saturate the SOA and
eliminate lasing so that the cavity performs as a typical SOA giving fast-light. As future passes through the loop cause the pulse amplitude to decrease, the ring laser turns on moving from a time-advance for the first loop, to a time delay for higher-number loops. The odd behavior for loops two to four can be understood as the naturally chaotic nature of a lasing cavity at threshold, when it is extremely sensitive to the power in the loop, causing contributions to the dynamics not just from the loop we are examining, but from other loop passes as well.

Importantly in this experiment we utilized 10 ps pulses, the equivalent of 100 GHz modulation. These pulses are much faster than the typical carrier lifetime showing that the effect is not limited to the oscillations in carrier number considered in our theory. For pulses as short as 10 ps the only reasonable physical effect to consider is spectral hole burning as the relaxation is extremely fast. These results indicate that the fluctuations in the laser field necessary to create slow-light are not limited to total-carrier depletion, but can also be caused by spectral-hole burning. This promises extremely fast high-speed capabilities for time-shifting signals even on the sub-picosecond time scale.

The excellent cascading characteristics of this slow-light method are a direct outgrowth of laser dynamics. While cascading multiple SOAs results in high levels of amplified spontaneous emission (ASE), increasing saturation and reducing the effect, and cascading absorbers causes ever-diminishing power and delay, cascaded lasers have neither of these problems. Lasers naturally suppress ASE via the lasing mode collecting all excess carriers through stimulated emission. Furthermore, as they operate in the gain regime, signal power is not necessarily diminished after passing through multiple devices. A final key advantage is that by moving from fast-light to slow-light at the lasing threshold, a large time-shift can be attained with only a small change in the output power, reducing the need for post processing to equalize powers after time-shifting. As many such post-processing techniques may themselves
introduce dispersion and alter the time delay, this greatly simplifies design of these devices.

8.5 Summary

In this chapter, we have presented a simple analytical model for the response of a signal field as it propagates through a laser media. Our results show that the signal field and laser field interact with each other via cross gain modulation. For low modulation speeds, those below the relaxation frequency of the laser, this interaction results in a slow-light response and delays the signal beam. For modulation speeds above the relaxation frequency of the laser the response returns to fast-light as is expected in a non-lasing device. We have experimentally tested this theory in a DFB laser with the results showing both qualitative and quantitative agreement. We then compared our analytical results with previous data taken for a gain-clamped SOA. Our model showed qualitative agreement, but overestimated the phase response. This is most likely due to the signal mode experiencing saturation inside the device. We then experimentally showed how well this technique of slow-light could cascade by using a ring-laser cavity. Our results showed excellent cascading with a linear increase in delay with increasing passes through the device. Using a 10 ps pulse, we were able to achieve a delay of 10 pulse widths in a single semiconductor device.

8.6 References


Chapter 9

Conclusion

9.1 Summary of key results

Work presented in this thesis has examined four-wave mixing in quantum dots for the purpose of wavelength conversion, and has also examined the use of laser resonances to create slow-light, and expand the cross-gain modulation bandwidth. In both cases the focus has been on all-optical processing for telecommunications applications.

We first presented a detailed theory for the non-linear susceptibilities in quantum dots, which focused on dots with only a single bound state coupled to a continuum of states via carrier capture and carrier escape. The model allowed us to determine the fundamental time constants that limit these processes in quantum dots, and to calculate the FWM efficiencies expected in a device. These calculated efficiencies were then compared to experiment showing qualitative and quantitative agreement. The theoretical analysis showed that the efficient conversion of high-speed signals relied equally on contributions from spectral hole burning and carrier heating.

An experimental study of FWM in QDs was also presented, including a direct comparison with QWs. The comparison showed that, as predicted by our theory, the QDs were more efficient at converting signals in the hundreds of GHz. Small-signal modulation measurements showed that the efficient conversion was not limited only to continuous wave signals, but continued to be true for modulated signals at speeds greater than 40 GHz. Furthermore, multi-channel conversion was demonstrated as well. Four, 40 GHz signals were simultaneously converted in a single device with each...
signal experiencing a greater than 100% conversion efficiency. The cross-talk between these channels was investigated and indicated that the primary cross-talk mechanism was cross-gain modulation. This cross-talk was well below the signal level for the high-frequency components, showing that it should not interfere with the efficient conversion of high-speed data signals.

Next, we examined other important factors that would allow FWM to be applied to real world applications. We demonstrated that by using two pumps, efficient four-wave mixing could be achieved over the entire gain regime of the device. Using a pulsed laser, we also showed that the conversion had no patterning effect as expected from our small-signal modulation data, and that the signal-to-noise ratio of the converted signal was > 22 dB, large enough to be practical for actual applications. Using pump-probe detuning measurements we also established that the single-channel, 3dB bandwidth of FWM in these devices was limited to near 100 GHz. Exploring even further we performed a numerical simulation between QWs and QDs for pulsed pump conversion for pump pulses between 1 ns to 300 fs. Our results showed that while quantum dots did provide superior conversion in the region of interest to telecommunications, that for higher speed signals the faster carrier relaxation of QWs allowed for more efficient conversion.

Examining other methods of wavelength conversion beyond FWM, we worked on extending the bandwidth of XGM using an imperfect SOA and an additional pump field to create a cavity mode inside the device. We established an analytical model for XGM in these devices, and then compared these results to our own experimental measurements. Data and theory showed both qualitative and quantitative agreement achieving a 3dB bandwidth of greater than 25 GHz for XGM. This was possible due to the laser-like mode suppressing carrier oscillations so that the dominant XGM mechanism became spectral hole burning. Importantly, this technique requires only a very small level of input pump light as this will create a large field inside the device
for an SOA near the lasing threshold.

Continuing our investigations into laser resonances we examined the behavior of the RF phase of an amplitude modulated signal beam as it passed through a laser cavity. An analytical model for the signal response was developed, and compared to experimental measurements we observed in a DFB laser. Our model showed that the signal field would create oscillations in the laser field through XGM. This laser field would then interact with the signal field to create slow- or fast-light depending on whether the modulation was above or below the relaxation oscillation of the laser. Again there was a strong agreement between experiment and theory. We then showed that by utilizing a gain clamped SOA, where the signal gain is much larger than the laser gain, that very large phase shifts could be achieved with a minimal change in the output power. We continued with an experiment on a ring-laser to explore how well this method of tunable slow-light could be cascaded by using 10 ps laser pulses. Our results showed a linear increase in delay with passes through the device indicating excellent cascading characteristics and allowing us to achieve a delay of 10 pulse widths using a single semiconductor device.

9.2 Future prospects

Moving forward, there are still many avenues of investigation. We demonstrated that the dominant cross-talk mechanism for multiple channels converted simultaneously via FWM was XGM. However, we did not examine how that cross-talk might be reduced. Our results on increasing the XGM bandwidth demonstrate that the magnitude of the XGM response can be reduced by using a cavity mode to suppress carrier oscillations. Thus, by combining these two techniques it may be possible to reduce inter-channel cross-talk inside the device. Furthermore, to provide a more simple package and setup, integration of the converter SOA and pump laser should
be attempted. Difficulties there include feedback into the laser from the SOA cavity, and temperature management of the total device. While we did compare different QD samples to examine their asymmetry when used for FWM, we did not have the opportunity for a more in-depth comparison due to the different wavelengths required for each type of quantum dot. A more in-depth comparison should be done, especially in how their differences allow for optimal dot selection for different applications.

Our examination of slow-light in lasing cavities should also be extended. This is especially true in the case of the ring laser where our theoretical analysis is not complete. While our analysis focused on carrier population oscillations, these processes should not be fast enough to effect a 10 ps pulse. Our experimental data, however, show that the phenomenon continues to be true for those quick pulses. While this hints at spectral-hole burning as a possible mechanism, a more formal theory should be developed.
Appendix A

Derivation of the signal response for a probe passing through a laser cavity with non-linear saturation

The rate equation for the total carrier number inside the device is

\[
V \frac{\partial N(t)}{\partial t} = \frac{I}{q} - \frac{N(t)V}{\tau_s} - v_g g_1(t) S_1(t) V - A (F_{2,\text{out}}(t) - F_{2,\text{in}}(t)) \tag{A.1}
\]

Where \( S_1(t) \) is the photon density of the cavity mode, \( F_2(t) \) is the photon flux entering and exiting the device, \( \tau_s \) is the non-radiative recombination rate, \( N(t) \) is the carrier density, \( V \) is the active volume, \( A \) is the cross-sectional area of that volume, \( v_g \) is the group velocity, \( g_1(t) \) is the gain for the cavity mode, \( I \) is the injected current, and \( q \) is the electron charge. Dividing by the device volume we return to a rate equation for the average carrier density

\[
\frac{\partial N(t)}{\partial t} = \frac{I}{qV} - \frac{N(t)}{\tau_s} - S_1(t) g_1(t) v_g - \frac{1}{L} (F_{2,\text{out}}(t) - F_{2,\text{in}}(t)) \tag{A.2}
\]

where \( L \) is the length of the cavity. The rate equation for the photon density of the lasing mode is

\[
\frac{\partial S_1(t)}{\partial t} = v_g g_1(t) S_1(t) - \frac{S_1(t)}{\tau_p}. \tag{A.3}
\]

Similar to the XGM analysis we relate the output flux to the input flux through the device gain

\[
F_{2,\text{out}}(t) = F_{2,\text{in}}(t) e^{g_2(t)L}, \tag{A.4}
\]

\[
\frac{\partial F_{2,\text{out}}(t)}{\partial t} = \frac{\partial F_{2,\text{in}}(t)}{\partial t} e^{g_2(t)L} + F_{2,\text{in}}(t) \frac{\partial g_2(t)}{\partial t} L e^{g_2(t)L}, \tag{A.5}
\]
but we no longer assume that $\frac{\partial F_{2,\text{in}}(t)}{\partial t} = 0$. Instead we assume that the input flux, $F_{2,\text{in}}(t)$ oscillates in time causing the lasing mode, carrier density, and output flux to also oscillate in time

\[ F_{2,\text{in}}(t) = F_{2,\text{in}} + f_{2,\text{in}}(\omega)e^{-i\omega t} + c.c. \]  

(A.6)

\[ F_{2,\text{out}}(t) = F_{2,\text{out}} + f_{2,\text{out}}(\omega)e^{-i\omega t} + c.c. \]  

(A.7)

\[ S_1(t) = S_1 + s_1(\omega)e^{-i\omega t} + c.c. \]  

(A.8)

\[ N(t) = N + n(\omega)e^{-i\omega t} + c.c. \]  

(A.9)

Adopting this notation, symbols not explicitly stating their $t$ dependence will now denote the steady state value.

The gain is again related to the carrier density, and light fields

\[ g(\lambda_i, t) = \frac{g_i + a_i \Delta N(t)}{1 + \epsilon_1 S_1(t) + \epsilon_2 F_{2,\text{in}}(t)} = \frac{g_i + a_i \Delta N(t)}{U(t)} \]  

(A.10)

Making the assumption that the device is lasing and thus $1/\tau_p = v_g g_1$ we can determine the equations for the small-signal amplitudes as

\[ -i\omega n(\omega) = -\frac{n(\omega)}{\tau_s} - g_1 v_g s_1(\omega) - \left( e^{g_2L} - 1 \right) \frac{f_{2,\text{in}}(\omega)}{L} 
- v_g S_1 \left( \frac{a_1 n(\omega)}{U} - \frac{g_1 (\epsilon_1 s_1(\omega) + \epsilon_2 f_{2,\text{in}}(\omega))}{U^2} \right) 
- F_{2,\text{in}} e^{g_2L} \left( \frac{a_2 n(\omega)}{U} - \frac{g_2 (\epsilon_1 s_1(\omega) + \epsilon_2 f_{2,\text{in}}(\omega))}{U^2} \right) \]  

(A.11)

\[ -i\omega s_1(\omega) = v_g a_1 n(\omega) S_1 \]  

(A.12)

\[ f_{2,\text{out}}(\omega) = f_{2,\text{in}}(\omega)e^{g_2L} + F_{2,\text{in}} e^{g_2L} \left( \frac{a_2 n(\omega)}{U} - \frac{g_2 (\epsilon_1 s_1(\omega) + \epsilon_2 f_{2,\text{in}}(\omega))}{U^2} \right) \]  

(A.13)
This again gives us three equations with our three desired unknowns allowing us to solve for the signal and laser response. As before in the case of XGM we make the assumption that \( a_1 g_2 = a_2 g_1 \). As the results are dependent only on the input steady state flux we also remove \( in \) from the subscript and let \( F_2 = F_{2, in} \). Furthermore, to simplify the expression the laser mode photon density and the input flux have both been normalized to the saturation density, \( 1/(a_1 v_g \tau_s) \), and the saturation flux \( 1/(a_2 \tau_s) \).

\[
\frac{f_{2, out}(\omega)}{f_{2, in}(\omega)} = e^{g_2 L} \times \frac{U^2(i g_1 v_g \frac{S_1}{U} + \omega(1 + F_2 \frac{S_1}{U} - i \tau_s \omega)) + (i g_1 v_g \epsilon_1 S_1 - g_2 L \epsilon_2 F_2 \omega)(1 - i \tau_s \omega)}{U^2(i g_1 v_g \frac{S_1}{U} + \omega(1 + e^{g_2 L} F_2 \frac{S_1}{U} - i \tau_s \omega)) + i g_1 v_g \epsilon_1 S_1(1 - i \tau_s \omega)} \]  \tag{A.14}

\[
\frac{s_1(\omega)}{f_{2, in}(\omega)} = \frac{-i U(e^{g_2 L} - 1) S_1 / L - i g_1 \epsilon_2 S_1 \frac{g_1}{g_2} (1 - i \tau_s \omega)}{U^2(i g_1 v_g \frac{S_1}{U} + \omega(1 + e^{g_2 L} F_2 \frac{S_1}{U} - i \tau_s \omega)) + i g_1 v_g \epsilon_1 S_1(1 - i \tau_s \omega)} \]  \tag{A.15}
Author’s Biography

David Nielsen was born in Albuquerque, NM in 1981, where he lived briefly before moving first to Mount Vernon, VA and then finally to Dayton, OH. In 2004, he received his B.S. degree in engineering physics from the Physics Department of Case Western Reserve University in Cleveland, OH. While there he participated in the Cryogenic Dark Matter Search (CDMS) including research on the superconducting properties of tungsten thin films for use as transition edge sensors. He received his M.S. degree from the Physics Department at the University of Illinois Urbana-Champaign, Urbana, IL in 2006. At Illinois, he first worked on developing novel methods of creating silicon nano-particles, before joining the Optoelectronics Group to investigate the non-linear properties of quantum dots and quantum wells. He is currently working towards his Ph.D. degree. His thesis focuses on using these devices for all-optical signal processing.