MODELING AND CONTROL OF HYDRONIC BUILDING HVAC SYSTEMS

BY

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THESIS

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Abstract

Energy requirements for heating and cooling of residential, commercial and industrial spaces constitute a major fraction of end use energy consumed. Centralized systems such as hydronic networks are becoming increasingly popular to meet those requirements. Energy efficient operation of such systems requires intelligent energy management strategies, which necessitates an understanding of the complex dynamical interactions among its components from a mathematical and physical perspective. In this work, concepts from linear graph theory are applied to model complex hydronic networks. Further, time-scale decomposition techniques have been employed to obtain a more succinct representation of the overall system dynamics.

The proposed model is then used to design predictive control strategies which are compared with traditional feedback control schemes using a simulated chilled water system as a case study. The advantages and limitations associated with these methodologies has been demonstrated. The cornerstone of this work is the development of a novel, distributed predictive scheme which provides the best compromise in the multidimensional evaluation framework of ‘regulation’, ‘optimality’, ‘reliability’ and ‘computational complexity’.
To my family, friends and Alma mater.
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Chapter 1

Introduction

1.1 Motivation

Throughout the last century and particularly in the last few decades, the human civilization has witnessed rapid strides in science, engineering and technology which have resulted in a significant impact on the society at large. While this impact has been mostly beneficial, making our lives more comfortable and in many ways easier, the price we had to pay in this process was being ignored for a long time until the dawn of this century when humankind realized that it could no longer afford to ignore certain realities. The impact of climate change and the rapid, irreversible depletion of natural resources is more visible today than at any time in the past and it is obvious that unless some affirmative and timely actions are taken, these problems would aggravate beyond control.

The issues of climate change and depletion of natural resources are closely related to energy. Irrespective of their size, type or popularity, all devices - from space ships to cars to a desk fan - consume energy. Therefore, even if they do not contribute to emissions directly, they certainly do so indirectly through the combustion engines, power plants and refineries that lie upstream in their energy chain. Also, it is precisely in this way that all devices are indirectly, if not directly, responsible for the consumption of some share of natural resources such as coal, natural gas and oil. Thus, reducing the global consumption of energy is a vital step in addressing the important environmental problems of this century. In this context, devices which are widely popular and which consume a significant amount of energy are of special interest because their impact on the climate and the consumption of natural resources is more pronounced than others, clearly because such an impact is strongly correlated with
energy consumption. Some examples of such devices are cars, computers and space heating and cooling systems.

Mitigating the environmental impact is the most important but not the sole motivation behind the thrust to reduce energy consumption. There are also substantial economic benefits associated with lesser energy consumption in the form of reduced costs both at the consumer and the supplier end. Reduction in energy demand leads to lower utility expenses at the consumer end. Likewise, it translates to lesser energy supply and therefore reduced operating costs at the supplier end. It can also obviate the need to build more energy infrastructure such as dams, power plants, mines and refineries, therefore leading to checks on the large capital costs that are associated with such projects.

The afore-mentioned environmental and financial aspects associated with energy have generated significant global interest in the pursuit of energy conservation in recent years. In this regard, as noted earlier, the focus is naturally on sectors which are responsible for a significant fraction of the overall energy consumption. Figure 1.1 shows sector-wise statistics on end use energy consumption in the USA. Since buildings account for around 41% of the total energy consumption, and contribute more than one-third in greenhouse emissions [20], the buildings sector presents significant opportunities for creating a meaningful impact on the global energy and emissions scenario. In this context, it is important to note that space heating and cooling together account for around one-fourth and one-third of the total end-use energy consumption in commercial and residential buildings as indicated by
Figure 1.2: End use energy usage in buildings [2]

The charts in Figure 1.2. This highlights the need for the development of energy efficient Heating, Ventilation and Air Conditioning (HVAC) technologies for buildings as a crucial step in the march towards achieving the energy, environmental and sustainability objectives set by governments throughout the world.

The problem of efficiency enhancement of building HVAC systems is inherently multidisciplinary and presents diverse opportunities from a research perspective in several different areas of technology such as design, architecture, alternative energy, modeling and control design. In this regard, the opportunities offered by the field of controls engineering are particularly important because its applicability is not limited to new and upcoming building technologies. Controls also has a significant ‘retrofit potential’ in the sense that it can be successfully applied to improve both the efficiency and performance of older, existing HVAC systems. Strong arguments for energy efficiency in the existing building stock have recently been made [21], therefore motivating the application of controls engineering in achieving such goals.

Centralized building and district hydronic\footnote{Systems which use circulating water for energy transport, see chapter 2 for details.} HVAC systems have become popular in recent years because of the operational and energy benefits associated with system integration. For instance, nearly 25% of commercial buildings in the USA with cooling infrastructure use centralized air-conditioning as their primary means for space cooling [22]. Similarly, the
rise in popularity of district heating for residential spaces in Denmark in a time window of 15 years can be gauged from Figure 1.3. Recognizing the critical importance of centralized hydronic HVAC building systems from a global energy consumption perspective and the potential of controls engineering as a tool with the ability to significantly alter the energy efficiency of existing devices, the present work attempts to explore meaningful, practical and effective modeling and control (systems engineering) tools for such systems. Apart from improvising the energy-efficiency of these HVAC systems, several other factors also motivate the development of such control technologies and have been discussed briefly in the next section and in detail in section 5.1.

![Figure 1.3: Dwellings according to type of heat installation in Denmark [3]](a) 1988

(b) 2003

This thesis undertakes a detailed study of hydronic building HVAC systems from a modeling and controls perspective. The main objectives and challenges in meeting these objectives are identified and novel solutions have been proposed and analyzed. The outcome is a set of modeling tools and control algorithms that appear promising when subjected to detailed simulation studies to examine their efficacy in meeting the objectives. The authors are optimistic that the contributions made by this work will be deemed technologically important and societally relevant, and thereby acknowledged by the scientific and engineering community with shared interests in the area of modeling, controls, building HVAC systems, energy efficiency and sustainability.
1.2 Research Objectives

The primary objective of this research, as outlined in the previous section is to aid the development of novel and promising modeling and control tools capable of satisfactorily addressing the energy related and other relevant issues associated with the operation of centralized, hydronic HVAC systems. The focus is on large scale, complex units serving existing commercial and residential buildings. The research objectives can be broken down into a set of four distinct objectives, each of which is briefly described below.

1.2.1 Identification of control objectives for hydronic systems

A list of control objectives is required as a starting step, based on which, the research needs and direction can be built upon. This necessitates a careful study of the issues concerning hydronic HVAC systems where controls engineering can be applied. Expectedly, this exercise shall also reveal important challenges such as conflicting objectives and the issue of optimal trade-offs.

1.2.2 Generic controls oriented model development

Model development is strongly tied to the target control objectives. Therefore, modeling requirements must be formulated based on the list of control requirements as discussed above. The modeling approach must prefereably be generic, and should also address the issue of complexity of such systems, therefore motivating the development of a model reduction procedure as an important part of the set of modeling objectives.

1.2.3 Synthesis of suitable control schemes

This is the main objective of the research presented in this work. Having identified the control objectives (see subsection 1.2.1), practical and efficient control schemes are to be developed to satisfactorily accomplish these objectives. Some of these objectives might be in conflict with one another and this is one of the few challenges that might need to be considered in the control design process. The control design procedure must be generic, so that it can be easily implemented on any existing hydronic HVAC system irrespective of their length, type or complexity. For that purpose, the idea here is to use a generic modeling framework for these systems, which is another important objective as outlined above.
1.2.4 Evaluation of proposed control schemes

It is important to verify the performance of the control schemes designed to address the underlying control objectives. For this purpose, a realistic testing scenario - either experimental or via simulations - needs to be considered. Also, traditional control strategies need to be designed against which the control schemes proposed in this research can be evaluated for a comparative analysis. It is expected that this exercise shall bring forth the advantages as well as the limitations of proposed control schemes and also serve as an example to elucidate the working details of these schemes.

1.3 Literature Survey

1.3.1 Important resources for building HVAC systems energy efficiency research

Some important resources providing information on the issues, statistics, and past and current research efforts concerning energy management of buildings are as follows:

1.3.1.1 ASHRAE

The American Society of Heating, Refrigeration and Air-Conditioning Engineers (ASHRAE) [23], founded in 1894 is an international organization of engineers, industrialists, scientists and researchers associated with the HVAC field. A few ASHRAE publications cater specifically to building systems such as High Performing Buildings (a quarterly magazine presenting case studies on exemplary buildings designed for sustainability), Building Information Modeling Guide (available for free online) and the Load Calculation Applications Manual. In addition to these, the ASHRAE Journal (a monthly magazine) often features articles which focus on issues and technologies related to the design, operation and control of building HVAC systems. The society also publishes four handbooks related to the field (Fundamentals, HVAC Systems and Equipment, HVAC Applications and Refrigeration) which are periodically updated. These provide detailed technical descriptions of the various HVAC components, together with general and component specific physical and modeling insights. ASHRAE also releases standards and guidelines from time to time to aid in the design, selection and operation of HVAC systems some of which are used by policymakers and
manufacturers both at the national and international level.

1.3.1.2 EIA

The Energy Information Administration (EIA) [24], created by the US Congress in 1977 is an independent statistical agency within the US Department of Energy. The following articles are periodically published by the EIA and made available online which contain both overall and sector-wise statistics regarding the national and international energy supply and demand:

- Short Term Energy Outlook: Energy projections for the next 18 months, updated monthly
- Annual Energy Outlook: Projection and analysis of U.S. energy supply, demand, and prices through 2030 based on EIA’s National Energy Modeling System
- International Energy Outlook: Assessment of the outlook for international energy markets through 2030
- Monthly Energy Review: Statistics on monthly and annual U.S. national energy consumption going back approximately 30 years, broken down by source
- Annual Energy Review: Primary report of historical annual energy statistics

The statistics are presented sector-wise and at various levels of detail. For the building sector, both heating and cooling data is made available based on geographical region, building type and building features.

1.3.1.3 Europe’s Energy Portal

Europe’s Energy Portal [25] is an independently run commercial organization rooted within the European Union (EU). It features numerous articles presenting statistics, issues and technological and policy initiatives concerning emissions and energy in Europe. It also publishes important EU directives related to energy and the environment. Detailed country-wise and sector-wise data, news and analysis are made available.


1.3.1.4 Other Resources

Some other general resources that provide useful background information and updates on activities related to this area are as follows:

- **USGBC [26]**: The U.S. Green Building Council (USGBC), founded in 1993, is a non-profit trade organization that promotes sustainability in how buildings are designed, built and operated. They provide useful online resources of research interest in energy efficient building systems such as technical articles, statistics, case studies, webcasts, videos and presentations.

- **facilitiesnet [27]**: This online portal contains a diversity of articles related to building technologies and building management strategies. It also includes some case studies and links to several other resources on energy efficient design and operation of buildings and data centers.

- **Building Technologies Program [28]**: The Building Technologies Program (BTP) is funded by the U.S. Department of Energy to promote research and technology development to reduce commercial and residential building energy usage. The program’s website features resources such as guidelines for best practices and also links to other agencies and online information repositories.

- **ENERGY STAR [29]**: It is a joint program of the U.S. Environmental Protection Agency and the U.S. Department of Energy. It provides free online resources such as useful strategies and guidelines for the design of energy efficient buildings and plants.

- **The Green Grid [30]**: The Green Grid is a consortium of IT companies and professionals seeking to improve energy efficiency specifically in data centers. Its website contains articles, survey findings, forum discussions and news updates.

1.3.2 Survey of important tools used in this work

In this work, controls oriented modeling presented in chapter 4 is based on a graph theoretical framework. Subsequently, model reduction is performed to yield a simple linear model.
Centralized and distributed model predictive control schemes are employed in chapter 6 based on this model. A literature review of these important tools and concepts used in this work is presented below. This background shall be useful and become more clear in the subsequent chapters of this thesis, when such tools are invoked.

1.3.2.1 Graph Theory based Modeling

Graph theory is a widely used, powerful system modeling and analysis tool and there are numerous examples in literature where its applicability has been demonstrated in the context of complex dynamical systems. The controls-oriented modeling framework presented in detail later in this work (section 4), is based on graph theory and is motivated by the past work in other fields which make use of this tool to achieve specific objectives. Some examples where graph theory is used in the context of deterministic (as opposed to stochastic) complex networks are briefly described in this section. This background serves as the motivation for using graph theory in this particular work.

Belykh, et al [31] have analysed the coupling graph with the aim of achieving stable synchronization in communication networks. In [32] Armbruster et al propose a graph based max-flow approach for the optimal placement and scheduling of power flow controllers in large scale power networks. Shukla and Radman [33] have described a graph connectivity analysis based technique to identify key buses for voltage scheduling in power networks. Vulnerability analysis of complex power systems has been studied by Oman et al [34] via a graph based model representation scheme. Li, et al [35] have applied graph theory to determine optimal communication topologies for information broadcast in wireless networks. A fast algorithm for the network observability problem has been proposed by Jain, et al [36] based on graph theory. Langari and Trefler [37] have used a graph transformation technique to accurately model communication protocols from a safety perspective. In [38], Scherrer and McPhee have developed an approach to obtain simplified models for a class of electromechanical multibody systems using tools from linear graph theory. Lastly, a vector graph theory framework has been proposed by Cannon et al [39] to develop a generic modeling approach for physical systems.
1.3.2.2 Model Reduction Methods

Model reduction has been employed in this work, as described later in detail in section 4.5, to simplify the ensuing control design task. An overview of the field of model reduction is presented below as a useful background.

Interest in model reduction has traditionally been motivated by the need to make simulations and control design less computationally expensive. Balanced model reduction is one of the most common model reduction approaches. For a historical perspective, the reader is directed to the articles by Kalman [40, 41], Moore [42], Enns [43], and Pernebo and Silverman [44]. Some other common techniques closely related to balanced model reduction are Hankel Norm Approximation [45] and Singular Perturbation Approximation [46, 47]. Other lines of research that have recently gained popularity are the so-called Moment matching based methods for model reduction and approximate balancing [48]. For a recent survey on model reduction techniques, the reader is directed to [49].

For systems with a large spectrum of time-scales, singular perturbation is a systematic and thus useful model reduction technique. Such an approach has been extensively employed for power systems (see [50, 51, 52, 53]) which exhibit multiple time-scales. In this thesis, model reduction for hydronic HVAC systems has been accomplished using a simple and straightforward time-scale decomposition scheme. Therefore, in principle, the approach presented can be thought of as a simpler implementation of the singular perturbation method applied specifically to the class of systems considered in this work.

1.3.2.3 Model Predictive Control

This section provides a background of developments in the area of Model Predictive Control (MPC), which is an important tool used in this work for control design presented in chapter 6. The development of modern optimal control theory can be attributed to Kalman [54, 55] who studied the linear quadratic regulation (LQR) problem and to the lineage of researchers who followed. However, industrial application of this technology has been limited because of the practical concerns that arise such as constraints, nonlinearities, model uncertainties and system complexity. This has led to the development of a more general and practical model
based optimal control methodology which is popularly known as Model Predictive Control. More details about the framework of MPC shall be presented in detail in section 6.2.

The history of MPC can be traced back to the period around late 1970s when various articles appeared showing an incipient interest in the technology, particularly in the process industry. Some of the important references in this context are Model Predictive Heuristic Control (Richalet et al [56] at Andersa Co.), Dynamic Matrix Control (Cutler and Ramaker [57] at Shell Oil Co.) and Quadratic Program Dynamic Matrix Control (Cutler et al [58] at Shell Oil Co.). While the first two cater to the control of unconstrained multivariable processes (using different approaches), the third algorithm provides for the explicit incorporation of input and output constraints. Another independent line of work in MPC arose around adaptive control ideas wherein some notable examples are Peterka’s Predictor-based Self Tuning Control [59], Extended Prediction Self Adaptive Control (EPSAC) by De Keysar et al [60], Generalized Predictive Control by Clarke et al [61], Multistep Multivariable Adaptive Control [62], Multipredictor Receding Horizon Adaptive Control [63], Predictive Functional Control [64] and Unified Predictive Control [65]. A significant challenge that the field of MPC faced was the lack of any underlying algorithms with guaranteed stability. This has led to some interesting theoretical developments such as the Constrained Receding Horizon Predictive Control [66], the stabilizing Input Output Receding Horizon Control [67] and Stable Generalized Predictive Control [68] where the focus in all these works is on stability. Some recent important results concerning stability of constrained MPC have been presented in [69, 70, 71, 72, 73].

MPC is quite popular in industry primarily because of its ability to handle constraints and the main sectors of its application are petrochemical refining and chemical processing. Important growth areas include pulp and paper, food processing, aerospace and automotive sectors. For an excellent survey on the industrial applications of MPC, the reader is directed to [74]. Some companies providing commercial MPC technology are DMC Corp, Adersa, Honeywell Profimatics, Setpoint Inc, Treiber Controls and SCAP Europa. For a detailed perspective on MPC, the books by Camacho and Bordons [75] and Rossiter [76] and the survey articles [77, 74, 78, 79] are useful resources.
1.3.2.4 Distributed Model Predictive Control

A Distributed MPC scheme is developed in section 6.9 of this work and therefore a background is presented here. Decentralized control is widely used for large-scale industrial systems owing to robustness, reliability, communication bandwidth and computational complexity considerations. Research on decentralized control design began around 1960s and some of the most celebrated works in this area include Siljak [80] on vector Lyapunov functions, Hovd and Skogestad [81] on sequential design and Iftar [82], Ikeda et al [83, 84] on overlapping decompositions.

The distributed control methodology is a variant of decentralized control where unlike the latter, communication is allowed between the local controllers. Distributed MPC is a recent area of research primarily motivated by the computational complexity required for the control of large scale systems with a traditional MPC structure. The review by Scattolini [85] provides an in-depth background of this field. Some publications that are worth mentioning in this regard are Camponogara et al [86], Jia and Krogh [87, 88], Du et al [89], Li et al [90], Venkat et al [91], Dunbar [92], Mercangöz and Doyle [93] and Alessio and Bemporad [94, 95]. These algorithms can be classified as fully or partially connected, noninteractive or iterative, independent or cooperative, nonlinear or linear model based and continuous time or discrete time where for the contextual definition of these adjectives the reader is directed to [85].

In this work, the focus is on ‘leader-follower’ or swarm type distributed MPC architectures (refer to section 6.9.1 for a discussion). Some examples where general control schemes based on such an architecture have been designed are multi-robot systems [96, 97], platoon of automotive vehicles [98], power plants [99], unmanned aerial vehicles [100] and large scale sensor networks [101]. In the MPC framework, such hierarchical control algorithms have been considered in [102, 103] and [104].

1.3.3 Overview of Research on Building Systems

This section summarizes past work in the area of modeling and control of building HVAC systems with focus on energy efficiency. The surveyed literature is presented below in five
different categories - (i) Background, (ii) Modeling and simulation, (iii) Optimal control (iv) Predictive Control and (v) Related publications.

1.3.3.1 Background publications on Building HVAC

The ASHRAE handbooks [105, 106, 107, 108] provide detailed descriptions of the various types of building HVAC systems, their physical architecture, their design aspects and standard modeling and control approaches. An example of a centralized chilled water system for a university campus has been described in [109]. The articles [110, 111, 112] discuss some interesting concepts associated with such systems. In [110], analytical models of pump power and evaporative temperature for a variable water flow chilled water system are developed and it is verified that such a system exhibits energy and other operational advantages when compared to a primary-secondary system. [111] undertakes a comparison of primary-only vs. primary-secondary chilled water systems. In [112], a NASA case study involving the selection of a chilled water plant has been presented. In [113, 114, 115], the authors review common control strategies that are used for building HVAC systems and also present their perspective on the role of control and optimization with regard to the underlying opportunities, problems and challenges. Some future trends in the control of such systems is studied in [116]. Energy use characteristics of variable primary flow chilled water systems and a cooling system in a semiconductor factory are presented in [117] and [118] respectively. The article [119] undertakes a comparative analysis of various operating strategies, viz. night purge, fan optimum start and stop, condenser water reset and chilled water reset for building HVAC systems.

1.3.3.2 Publications on Modeling and Simulation of Building HVAC

A review of papers on controls oriented modeling and simulation of building HVAC systems is presented here. Modeling and simulation of hydronic heating systems has been studied in [120, 121]. While [120] describes dynamic simulation of radiator based heating systems, [121] provides details of a MATLAB based simulator for hydronic heating systems. Network based modeling of building thermal characteristics is presented in [122, 123, 124, 125], which consider heating and cooling loads and HVAC system to zones, zone to zone and ambient to zones thermal interactions. Modeling and simulation work for testing the performance of specific control schemes is the subject of the papers [126, 127, 128]. In [126], the influence
of sensor position in building thermal control is analyzed. Dynamic simulation of variable air volume (VAV) air-conditioning systems is studied in [127]. A VAV model for simulation of off-normal operation and duty-cycling is presented in [128]. Details of the modeling and simulation framework used in this work shall be covered later in section 3.

1.3.3.3 Publications on Optimal Control of Building HVAC

The survey on the optimal control of building HVAC systems by Wang and Ma [129] provides a rich introduction and background about this field. While that article surveys almost all significant literature related to the subject, some specific papers are worth mentioning. Jian and Zaheeruddin [130] consider optimal on-off control of chilled water systems with storage. A set-point optimization scheme for supervisory control is presented in [131]. Ma and Wang [132] have studied strategies for energy efficient control of pumps for super high rise building systems. In addition to these, some interesting control strategies for building HVAC that appear in literature are reinforcement learning based optimal control [133], fuzzy logic control for VAV systems [134] and Complete Simulation-Based Sequential Quadratic Programming [135].

1.3.3.4 Publications on Predictive Control of Building HVAC

As a background for the predictive control schemes presented in this work in section 6.1, some interesting papers which use MPC for the control of building HVAC systems are mentioned here. Predictive control of complex district heating networks is considered by Sandou [136]. Yuan and Perez [137] have proposed an MPC scheme for temperature and ventilation control in the context of single duct VAV systems. Robust MPC schemes for air-handling unit control has been developed by Huang and Wang [138, 139]. Henze et al [140, 141] present both theoretical and experimental results on some MPC strategies that they have developed for whole building energy optimization via optimal zonal set-points and optimal charging and discharging strategies for thermal storage. In particular, the authors describe in detail the impact of weather forecasting accuracy on the controller performance. In a related work [142], the relationship between cost savings and energy savings is explored for systems with storage and under predictive control. Kolkotsa et al [143] have presented an MPC technique for obtaining the optimal zonal demand schedules. Lastly, application of MPC for the control of cogeneration systems has been described in [144, 145].
1.3.3.5 Other Related Publications

Some related publications from this area are cited in this section. The primary focus in most of these papers is not energy efficiency, however, they consider certain other important issues. Detailed studies on the interactions and control of thermo-hydraulic networks are presented by Franco et al [146, 147, 148] and Cai et al [149]. Modeling and Control work on heat exchanger networks (HENs) in process industries are described in [150, 151]. Both these papers use a static, graph-theory based modeling framework. In the context of district heating systems, the issue of hydraulic balance has been described in [152] and a multi-agent control approach for meeting heating loads is proposed in [153]. The paper [154] describes a comprehensive study on applying district cooling technology in Hong Kong and therefore is of practical interest.

1.4 Outline of the Thesis

The remainder of this work is organized as follows. A physical description of hydronic HVAC systems and its underlying components, together with its applications and some examples is presented in chapter 2. Chapter 3 presents nonlinear component models and details of a simulation test bed (THERMOSYS 3.1) based on these models. Controls oriented linear modeling which makes use of a graph theoretical framework is presented in chapter 4. Model reduction and performance evaluation of reduced order models for a test system are also included in this chapter. Chapter 5 deals with the traditional control design for these systems. In chapter 6, centralized and distributed MPC control schemes for the control of hydronic HVAC systems are presented in detail. The relative comparison of the traditional schemes and the centralized and distributed MPC schemes is studied in chapter 7 through simulations for a chosen test case. Finally, concluding remarks and directions of future research are put together in chapter 7. In addition to these chapters, two appendices have also been included in this thesis. Appendix A provides codes and other relevant details for the modeling work, whereas Appendix B discusses MATLAB implementation details of the control schemes developed. It should be noted that though the validity of the URLs cited for the online resources appearing in the bibliography has been verified for date of publication of this thesis, the author is not responsible in the event that these websites are moved or discontinued in future.
Chapter 2

Physical Details of Hydronic Systems

This chapter provides a general introduction to hydronic systems. A brief overview of the various types of hydronic systems and their general physical layout is presented in section 2.1. Section 2.2 lists and describes the typical components present in the hydronic systems considered in this work, i.e., centralized, closed water systems. Common applications of these systems are discussed in section 2.3 and some examples are then described in section 2.4. Most of the material presented in this chapter has been taken from the ASHRAE Handbook - HVAC Systems and Equipment [108].

2.1 General Introduction

Hydronic systems are defined as thermal management systems which use water (hot or chilled) to achieve the desired transfer of thermal energy to or from a space, process or device. This section presents a classification of these systems and describes their general physical layout.

2.1.1 Classification

Various classification criteria exist for hydronic systems such as operating temperature, layout or architecture, pumping arrangement, pressurization and mechanism for flow generation.

From the perspective of operating temperature, the first level of classification for these systems is heating or cooling systems. Heating systems can be further categorized as low-temperature water systems, medium-temperature water systems and high-temperature water systems. Low-temperature water systems consist of low-pressure boilers (see Figures 2.1 and 2.2) with working gage pressures and supply temperatures typically around 200 kPa and
120°C respectively. Sometimes boilers are replaced by simple steam-to-water (Figure 2.3) or water-to-water heat exchangers (Figure 2.4) for low-temperature water heating. For medium temperature water systems, the boiler pressure rating and supply temperature are usually about 1 MPa and 120 to 160°C respectively. High temperature water systems operate at boiler pressures close to 2 MPa and supply temperatures of the order of 200°C. For both small and large scale building heating systems, mostly low-temperature water heating is employed. Most hydronic cooling systems use a chiller to supply chilled water at a supply temperature ranging from 4 to 13°C and therefore are also known as chilled water systems. They have become very popular for building level and district level cooling. However, underground well
water can be sometimes used for applications with lower cooling requirements. In addition to conventional heating and cooling systems, another category of hydronic systems called dual-temperature water systems are also available which use the same apparatus to circulate chilled water in summer and hot water in winter. These systems result in lower equipment cost and are useful in places which require both summer cooling and winter heating.

Based on the layout of the piping, hydronic systems can be classified as series, parallel or series-parallel systems among which parallel piping networks are most commonly used for large scale systems (see section 2.2.1.2 for further details). Classified by flow generation, a hydronic system may be a gravity system or a forced system. The former uses the difference in density between the supply and return water temperatures to maintain flow and has now become obsolete, whereas, the latter uses a pumping mechanism for circulation of water. From a pumping arrangement perspective these systems may use primary-only pumping or primary-secondary pumping with either fixed speed or variable speed pump systems. Furthermore, the primary pumping architecture can be series, parallel or occasionally a
combination of series and parallel. Refer to section 2.2.1.1 for more details on the pumping mechanism. Hydronic systems can also be ‘once-through’ or recirculating, the latter being most commonly used.

In this thesis, the focus is on recirculating, forced flow, hydronic heating and cooling systems for building applications. This is because most building HVAC systems are of this type. No major assumptions on the layout or pumping arrangement has been made, which allows to incorporate a large variety of these systems within the modeling and control framework presented in this work.

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Figure 2.4: Water to water heat exchanger used in a solar heating system (Courtesy: homecents.com)

Figure 2.5: Schematic of a Hydronic Heating System [4]
2.1.2 General Physical Layout

A recirculating, forced hydronic system is essentially a network of several components working in tandem. Though these components have different roles in the operation of the overall system, their selection process is not independent of each other. At a fundamental level, the components of a hydronic system can be classified as thermal components and hydraulic components. The thermal components involve transfer of thermal energy between the hydronic system and a suitable environment (e.g. ambient or conditioned spaces). The hydraulic components are associated with the overall circulation and local distribution of water flow in the system. The control of the overall system is achieved through these hydraulic components, and therefore, they play an important role in their operation. More details on the control systems shall follow in section 5.1. Refer to Figures 2.5 and 2.6 for schematics of heating and cooling systems and Figure 2.7 for an illustration of the important underlying components.

An important distinction must be made here, between the HVAC system and the hydronic system. The latter is only a subsystem of the former and refers only to the water loop which is the core of the system. For example, a centralized cooling system is far more complex consisting of other appendages such as the condenser water loop serving the chillers and the air side loops for circulation and supply of conditioned air to the various service zones (see...
Figure 2.7: Components of a building HVAC system (Source: E Source)

Figure 2.8). Even the chiller consists of a complete vapor compression unit and is thus more complex than just one single component.

The thermal components can be further classified as source elements and sink elements which are described in detail in sections 2.2.2.1 and 2.2.2.2. The source elements provide the heating or cooling of the supply water, depending on the type of the system. In cooling systems, the source elements are the chillers, whereas, in heating systems they are typically boilers (Figures 2.1 and 2.2), but water to water heat exchangers (Figure 2.4) are also used sometimes. The sink elements are mostly liquid to air heat exchangers (Figure 2.9) or radiator panels (Figure 2.10) which transfer thermal energy to the conditioned spaces through forced convection or radiant heat transfer. Sometimes a cascaded set up is also used, wherein, the transfer of energy between the fluid in the hydronic system and air is accomplished via intermediate liquid to liquid heat exchangers (Figure 2.11). Apart from
Figure 2.8: Energy exchange semantics demonstrated using a simple building cooling system

Figure 2.9: Schematic of a liquid to air heat exchanger in a cooling system [6]
these source and sink elements, expansion chambers and thermal storage tanks are other devices which can also be included in the category of thermal components. Details of these are presented in sections 2.2.2.3 and 2.2.2.4. The most important hydraulic components in a hydronic system are pumps, flow valves and piping (section 2.2.1). Some other devices such as relief valves, gage cocks and strainers are also present but not discussed here in detail since they do not affect the dynamics of the system significantly.

Some physical and design details about the important thermal and hydraulic components

Figure 2.10: Illustration of a radiator panel in a heating system [7]

Figure 2.11: Example of a building heating system with a cascaded (intermediate) heat exchanger
in a hydronic system are presented in the next section. A brief overview of the other subsystems in the overall HVAC system is also presented.

2.2 Details of Components

2.2.1 Hydraulic Components

2.2.1.1 Pumps

Pumps lie at the core of the operation of hydronic heating and cooling systems. The choice of pumps, pumping architecture and their control and scheduling schemes strongly govern the operating characteristics of the overall system. Therefore, to achieve satisfactory performance in terms of thermal demand matching, energy efficiency, safety and reliability, proper attention must be paid to all aspects concerning pumps.

The choice of pumps is governed by the desired operating point of the hydronic system, and involves detailed comparative analysis of the pump characteristics such as head-flow rate dependency, efficiency curves and limits imposed by cavitation. Theoretically, the achieved
operating point is at the intersection of the pump and the system characteristics (see Figure 2.12) and therefore, accurate estimation of the system hydraulic characteristics is required to determine the most appropriate choice for the pump. However, that estimation is never error-free and thus, to mitigate the effect of such errors, it is desirable to use pumps with flat pressure-flow characteristics. Another common guideline which designers usually follow is to choose pumps with the maximum efficiency point to the left of the design operating point.

Hydronic systems can be classified as primary-secondary systems and primary-only systems based on the pumping architecture used. The difference between these architectures is schematically shown in Figure 2.13. The main advantage of using primary-secondary pumping over primary-only pumping is that it decouples the system into hydraulically independent subsystems which are easier to analyze, control and troubleshoot. However, flow
mixing which occurs at the junctions A and B as shown in Figure 2.13 (B) may lead to inefficient system operation which is an important limitation of this architecture. In large scale hydronic networks, both primary-secondary and primary-only systems may use banks of pumps instead of single pumps at the various pumping locations to boost flow. The arrangement of pumps in a particular bank may be either in series, parallel or series-parallel. In all these situations, the pumps deployed must be compatible with one another, meaning that inconsistencies such as a large pump overflowing a small pump in series with it or a small pump causing cavitation in a large pump parallel to it must be avoided. The underlying analysis is often more complex in the case of primary-only architecture when compared to the primary-secondary architecture because of the hydraulic decoupling facilitated by the latter.

Variable speed pumping (Figure 2.14) is a novel technology that is becoming popular for hydronic systems used in buildings. During the course of the day, the system flow rates vary in accordance with the variation in thermal demand. Therefore, the pump speeds can be adjusted to the most efficient levels, thus, possibly leading to a more energy efficient operating point when compared to traditional fixed speed pumping. However, proper realization of this energy-savings potential is subject to the efficacy of the control system designed for speed manipulation. Another advantage associated with the use of variable speed pumping in primary-secondary systems is that they can mitigate undesirable mixing at the junctions via proper coordination of primary and secondary pumping mechanism speed control. Despite these advantages, variable pumping technology is expensive and requires detailed analysis to
Figure 2.15: Series distribution circuits

ensure that in even the worst case control scenario, problems such as cavitation and overflow do not occur.

2.2.1.2 Piping

The underlying fluid distribution system is important from cost, energy and performance aspects. The key considerations in the design of the piping are the configuration of the building, the nature of the loads and, installation and maintenance costs. The design problem is twofold, essentially consisting of sizing the pipes to handle the required thermal demand and arranging the piping circuit so as to ensure appropriate distribution of the flow. A
frequently appearing metric in hydronic system piping design literature is $\delta t$ defined as the temperature difference between the return and supply streams. Higher $\delta t$ amounts to reduction in the flow rate required to meet the load and thus translates into less costly piping infrastructure. Therefore, piping arrangements which lead to higher $\delta t$ are preferred. In addition to this, it is also desired to keep the arrangement simple enough for ease of analysis and maintenance.

Four basic distribution circuits as shown in Figures 2.15 and 2.16 are most commonly used. The full series circuit is simple, inexpensive and leads to high $\delta t$, however it does not allow local capacity control of individual sink elements. An alternative to this is the diverting series circuit where instead of connecting the sink elements to the main circuit, bypass (diverting) lines are used. Because only a fraction of the main flow is diverted
to the sinks, the margin for capacity control via flow rate adjustment is small. Due to such limitations, the full series and diverting series configurations are used only for small residential or commercial buildings with insignificant load variability. The parallel piping networks - direct return and reverse-return - are the ones most commonly used for large scale systems. The main advantage with these networks is that they ensure that the supply water temperature to all the sink elements is the same. Individual capacity control of the sinks is facilitated and the interactions among the components is small. The reverse-return architecture provides nearly equal lengths for all the branches and thus is hydraulically more balanced than the direct-return scheme.

2.2.1.3 Valves

Most of the valves present in hydronic systems are control valves with the ability to modulate the flow rate to cater to changes in the thermal demands. Sizing of the control valves is an important design consideration so that the pressure drop caused by them is optimal (high pressure drop increases pumping work and low pressure drop makes the control difficult to achieve). Most control valves are generally two-way valves or three-way mixing valves as shown in Figure 2.17. The former provides a variable flow load response, whereas, the latter provides a constant flow load response. The choice and sizing of the valves is also dependent on the control scheme used for flow control and therefore valves sizes used for PID control
are different from those for on/off control. In recent years, however, due to the availability of variable speed pumping for control related tasks, the use of control valves is gradually waning since the elimination of valves leads to lower pressure drops and thus energy savings. Apart from control valves, relief and safety valves are also present in hydronic systems.

2.2.2 Thermal Components

2.2.2.1 Source Elements

The most commonly used source devices in hydronic heating systems are boilers (Figures 2.1 and 2.2). However, other devices such as steam-to-water (Figure 2.3) and water-to-water heat exchangers (Figure 2.4) are also used for small scale systems. Based on the fuel used, boilers may be designed to burn coal, wood, fuel oil, fuel gas, or use electricity. Based on the working pressure, boilers can be classified as low-pressure and high-pressure.
Another important characterization for these devices is whether they are condensing or noncondensing, with reference to the flue gas. The former is more efficient but the latter is more durable and therefore the choice depends on the application. Boiler selection often involves a thorough consideration of performance, first cost and space requirements. It must have a capacity range which is large enough to meet the varying load requirements of the system. It is also important to take into account their efficiency versus capacity characteristics.

The most common cooling source devices are chillers, although, water-to-water or air-water heat exchangers may be found in specific applications. Electric compressions chillers, which work on the vapor compression cycle (Figure 2.18) and absorption chillers (Figure 2.19) are popular in building systems. Electric chillers are classified based on the compression technique used and may consist of reciprocating, rotary vane, centrifugal, screw or scroll compressors depending on the particular application (Figures 2.20 and 2.21). Reciprocating compressors use pistons driven by a crankshaft whereas screw compressors use two meshed rotating helical screws to force the gas into a smaller space. Rotary vane compressors consist of a rotor with a number of blades inserted in radial slots in the rotor, which is mounted offset in a larger housing. As the rotor turns, blades slide in and out of the slots keeping contact with the outer wall of the housing, thus generating a series of decreasing volumes. A scroll compressor uses two interleaved spiral like vanes to compress the fluid. Reciprocating, screw, rotary vane and scroll compressors are all positive displacement devices. Centrifugal compressors (Figure 2.22) are continuous devices which use a rotary disk or impeller in a
Figure 2.21: Screw and scroll compressors

Figure 2.22: Centrifugal chiller cutaway (Source: York International)
housing to force the gas to the rim of the impeller, increasing its velocity. A diffuser then converts the kinetic energy of the gas to pressure energy. Reciprocating compressors are noisy and expensive and are becoming obsolete in chilled water systems. Comparatively, screw compressors are less noisy and are commonly used for small and medium load applications. For large scale hydronic systems, however, centrifugal compressors are widely used. Scroll compressors, on the other hand, have a large operation range and are best suitable for use in small to medium scale systems with high load variability. Multiple chiller systems (Figure 2.23) are often employed for large building systems because they allow better load control, energy efficient performance and availability of standby capacity in case of failures. Just like boilers, the choice of chillers is also dependent on capacity range, space constraints and efficiency characteristics.

2.2.2.2 Sink Elements

The sink elements in a typical hydronic cooling system are water-to-air finned coil heat exchangers available as packaged devices called air-handling units (AHUs) (see Figure 2.24). These devices consist of a bank of multirow coils with fins for enhanced heat transfer area. The most common practice is to use Aluminum fins on copper tubes. The flow configuration is typically cross-flow or counter-flow for the highest possible mean temperature difference between the air and water streams. Coil selection involves consideration of the job requirements (cooling with or without dehumidification), desired operating temperatures on both
liquid and air side, heat transfer characteristics, and, space and dimensional limitations. In the case of dehumidification, the underlying analysis must involve both latent and sensible heat transfer. Fouling is a critical maintenance issue associated with the operation of these devices, and therefore the economics of maintenance is also an important consideration in their choice.

In most hydronic heating systems, the sink elements provide direct radiant heat transfer between water and the conditioned space and typically consist of floor or ceiling mounted radiant panels (Figure 2.10) or heating loops provided underneath the floor in radiant-floor systems. Baseboard convectors or radiators are also widely used. The difference between the hydronic heating and cooling systems, in terms of the sink elements is further explained in section 2.3.1.

2.2.2.3 Thermal Storage

Thermal storage systems store thermal energy in a suitable storage medium for use at a later time. The main benefits of using thermal storage are reduced utility costs via proper storage scheduling, reduced size of thermal source elements, provision of back up thermal capacity, the potential to extend the system capacity, and load shaving. In chilled water systems, the two most popular storage technologies are (i) use of a well insulated, external,
stratified storage tank (Figure 2.25) and, (ii) ice-making mechanism (Figure 2.26) where ice is made at night which is used during the day to provide chilling of water. Likewise, for heating systems, water storage tanks or brick storage heaters (Figure 2.27) are the most common storage media. Another passive storage concept which has become popular in recent years in the HVAC industry is building mass thermal storage, where, the thermal storage capabilities of the building structure are utilized via precooling or preheating [155]. There are five operating modes associated with the use of thermal storage, viz., charging storage, charging storage while meeting loads, meeting loads from storage only, meeting loads from storage and source elements, and, meeting loads from source elements only. Proper scheduling of these operating modes is critical to realize the energy and cost savings potential that is available with thermal storage in place.

2.2.2.4 Expansion Chamber

The expansion chamber (Figure 2.28) serves both as a thermal and a hydraulic device. As a thermal device, it provides a space into which the non-compressible liquid can expand or contract with changes in temperature. This is allowed by providing an interface in the tank between water and a suitably chosen compressible gas. As a hydraulic device, the expansion tank serves as the reference pressure point in the closed hydraulic system, analogous to the ground in an electric circuit. The fluid pressure in the tank is set to the air pressure in the
tank plus or minus any pressure difference due to elevation of the column of fluid.

### 2.2.3 Other Components

As noted earlier, in the context of cooling systems, the term ‘hydronic’ essentially refers to the water loop in the commonly used air-water systems. Apart from the hydronic loop, the overall HVAC system consists of two other subsystems - the air distribution circuitry and the condenser water loop (Figure 2.8). The air distribution network (Figures 2.29 and 2.30) primarily consists of ducts with supply air outlets with or without diffusing equipment, and, return or exhaust air inlets. Other air handling equipment are fans for creating airflow and air filters. In some systems, temperature control in the conditioned spaces of a building is achieved through control of the circulated air volume. The condenser water loop is provided in large cooling systems to supply cold water to the chiller condensers for refrigerant condensation. It consists of cooling towers (Figure 2.31) and additional components such as pumps, valves, suction strainers and water temperature controllers. For more details on the air distribution and condenser water components, the reader is directed to [108]. Hydronic heating systems are mostly all-water and thus do not require air ducts like hydronic cooling systems.

Centralized HVAC systems also consist of apparatus providing centralized or decentralized ventilation for air-quality control, discussion of which is beyond the scope of this thesis. The
control system is another important part of hydronic systems, and a discussion of the current state or the art is reviewed in section 5.1.

2.3 Applications

2.3.1 Centralized Building Heating and Cooling Systems

Centralized heating and cooling systems generate cooling or heating in one location for distribution to multiple locations in one building or a neighbourhood. Systems of the latter kind are called district systems and are described in the next section, whereas, the discussion in this section pertains specifically to centralized building heating and cooling systems.

Centralized building heating systems are mostly of two types - all-air and hydronic (all-water); although air-water systems are also used in some situations. As is evident, all-air heating systems (Figure 2.32) use a central furnace to generate hot air which is then forced through ducts to the various conditioned spaces to provide convective heating. On the other hand, hydronic heating systems are non-ducted systems (see Figure 2.5 for a schematic), where hot water is generated by a boiler, circulated to the conditioned spaces for radiant
heat transfer, and, subsequently returned back to be boiler, thus resulting in a closed system. Such systems employ baseboard convectors, radiators, or, low temperature radiant panels to provide the desired heat transfer between the water and the zonal spaces (see section 2.2.2.2). Hydronic heating systems were popular in the early twentieth century but began to lose ground to all-air forced systems after World War II. However, in the past few decades, due to the development of district heating, thermal storage and other technologies, hydronic heating has regained popularity and can now be found in most medium and large scale residential and commercial facilities in North America and Europe. Compared to all-air systems, hydronic heating systems provide several benefits such as savings in cost, energy and space due to absence of ductwork and fans, less thermal leakage and quieter operation.

As opposed to heating systems which are mostly all-water, central building cooling systems are mostly air-water or all-air. All-water or purely hydronic cooling is an upcoming technology, and its use is confined to some modern buildings only. Therefore, in the context of building cooling systems the term ‘hydronic’ refers to the chilled water loop in air-water systems (see Figure 2.8). Nevertheless, a majority of large-scale, centralized cooling systems are air-water and thus analysis of the underlying hydronic subsystem is important. These systems use one or multiple air handling units (Figure 2.24) which generate cold air via heat exchange with the supply chilled water for circulation to the conditioned spaces through ductwork. The hydronic circuit consists of a chiller, pumps, control valves and piping. The
complete cooling system also includes a condenser water loop to provide the cooling necessary for condensation of the superheated, compressed refrigerant in the chiller condensers (see Figure 2.6 for a schematic). The details of the underlying components were already presented in section 2.2.

2.3.2 District Heating and Cooling

District heating and cooling systems are steam based or hydronic (water-based), large scale, centralized, systems for energy management serving large industrial complexes, densely populated urban areas or high density building clusters with large thermal loads (see Figures 2.33 and 2.34 for a schematic). They consist of three primary components - the production plant, the distribution network, and the consumer systems. The production plant consists of
conventional thermal source elements such as boilers or electrical chillers, but may also be a combined heat and power (CHP) plant (Figure 2.35) which promotes high energy efficiency. The heat in the CHP plant can be used directly to generate hot water in a heating system or may be used to operate absorption chillers in a cooling system. The distribution system is the most expensive portion of a district hydronic system and consists of supply and return piping, insulated by placing them underground or in concrete tunnels. The consumer systems are the in-building hydronic subnetworks which are fed from the main district network either directly or through heat exchangers.

District hydronic systems have become popular in recent years due to benefits such as higher energy efficiency (particularly if based on CHP), easier emissions monitoring and
Figure 2.32: Schematic of an all air centralized heating system for a small residential unit [15]

Figure 2.33: Architecture of a solar district heating network with storage [16]

control, and the maintenance and space related advantages resulting from fewer mechanical equipments when compared to local, facility level hydronic systems. Most of the heating requirements in several European countries is met using district heating systems, notably Iceland, Denmark and Finland where the market penetration of district heating is more than 50%. In North America, most university campuses in the United States and Canada use district heating and cooling networks with CHP [109].
2.3.3 Other Applications

Hydronic systems are also widely used in several applications other than residential and commercial building heating and cooling. For example, hydronic heating is becoming popular for radiant floor heating and snow melting. Apart from their use in residential and commercial buildings, radiant floor heating technology finds its application in greenhouses, dairy barns and other special purpose facilities. Hydronic snow melting is often used to remove snow and ice from driveways, sidewalks, parking lots, patios, airplane hangars, galleries and stadiums.

Hydronic systems are also used to meet thermal requirements in automotive components. Hydronic cooling technology has traditionally been used for cooling internal combustion
engines (Figure 2.36). The radiator which is essentially a forced convection, air-water heat exchanger is used as the source of cold water in this situation. In more modern systems, analogous to a cogeneration scheme, the heat generated from the engine is used to heat the passenger compartment in cars with an aim to improve the overall energy efficiency. Conventional hydronic heating systems are sometimes also used for large truck and trailer systems. Another important automotive application of centralized, water based heating and cooling is in maritime vehicles, where sea water itself is used for that purpose. In these vehicles, hot or cold air is hydronically generated for air conditioning of the passenger and crew compartments and equipment chambers.

Today, hydronic systems technology is also becoming popular for the cooling of electronic equipment. Data center hardware manufacturers such as IBM and HP now provide blade server racks with integrated chilled water tubes (Figure 2.37 shows an example). Similarly, water cooling technology is also provided by some companies inside the computer case for cooling of the processor core. Water cooling technology is several orders of magnitude more efficient than conventional methods which use forced air cooling or heat sinks, and is particularly effective in the elimination of local hot spots. The main challenge, however, due to space limitations is in designing the water channels to be sufficiently close to the hottest components. Therefore, water based cooling of electronic circuits is still an active area of research. Hydronic systems are also used for the cooling of machines and processes in refineries,
power plants, foundries, vehicle assembly lines and other manufacturing applications.

2.4 Test Systems Used in Present Work

In this work, two test systems are used for demonstration of the proposed modeling and control schemes and for validation of these schemes via simulations. Schematic of the first system is shown in Figure 2.38, which emulates a small scale residential hydronic heating system. It consists of two Liquid to Air Heat Exchangers (LAHXs) for convective transfer of heat to two zones in the house. The second system, which represents the hydronic loop of an air-water cooling system for an office building with six zones is shown in Figure 2.39. Note that in this case, a single zone may consist of a cluster of office rooms, and thus, the number of rooms in the building is more than six. This system uses a primary-secondary pumping scheme (without a decoupler, compare Figure 2.13 (B)) and two chillers acting as thermal source elements.
Figure 2.38: Schematic of a test heating system

Figure 2.39: Hydronic loop of a test chilled water system
The first system is used as a modeling example in section 4 to demonstrate the graph based modeling procedure and the model reduction algorithm. The second system, which is more complex, is used for performance evaluation of the reduced order model in section 4.7 and for the relative evaluation of the various control schemes in section 7. Details on the choice of the design operating conditions for this system have been provided in Appendix A.1.
Chapter 3

Nonlinear Component Models

The physical modeling of hydronic system components is presented in this chapter in detail. These models have been implemented in MATLAB in the form of an interactive Simulink toolbox called THERMOSYS (version 3.1)[156]. The modeling assumptions, governing equations and the corresponding THERMOSYS implementation of the various thermal and hydraulic components in hydronic systems have been presented in section 3.1. Section 3.2 enlists the procedure to connect the models together and run a nonlinear simulation of the overall system. In this chapter, unless otherwise mentioned, the term ‘liquid’ refers to the fluid circulating in the hydronic system which in most cases is aqueous propylene glycol solution (APGS) of a suitable concentration, which is more resistant to freezing than 100% water. It must also be noted that modeling of the expansion tank element is not described in this work because its effect on the thermal and hydraulic dynamics of the overall system is insignificant.

3.1 Summary of Component Models

3.1.1 Pumps

Pumps are hydraulic devices in the hydronic system. The pump model presented in this section is assumed to be causal such that given the inlet and outlet liquid pressures, or equivalently the head gain across the pump, the model computes the flow rate through it. This model is applicable to both constant and variable speed pumps. Note that the model is static and does not include its driving mechanism (prime mover, etc.)
3.1.1.1 Modeling assumptions

The pump model presented is based on the following assumptions.

1. The flow rate versus head characteristic curves of the pump are assumed to be available. They are usually provided by the pump manufacturer or can be generated using test data (see Figure 3.1 for a schematic).

2. The isentropic efficiency characteristic curves of the pump are also assumed to be available (see Figure 3.2 for a schematic).

3. The liquid is assumed to be incompressible.

3.1.1.2 Nomenclature

The following notations are used for the pump model presented:

- \( N \): Pump speed (RPM)
- \( p_{\text{out}} \): Pressure at outlet (kPa)
- \( p_{\text{in}} \): Pressure at inlet (kPa)
- \( Q \): Volumetric flow rate (m\(^3\)/s)
- \( h_{\text{in}} \): Specific enthalpy at inlet (kJ/kg)
- \( T_{\text{in}} \): Temperature at inlet (deg C)
- \( \rho_{\text{in}} \): Density at inlet (kg/m\(^3\))
- \( \dot{m}_{\text{in}} \): Mass flow rate at inlet (kg/s)
- \( \dot{m}_{\text{out}} \): Mass flow rate at outlet (kg/s)
- \( \dot{m} \): Mass flow rate inside the pump (kg/s)
- \( \eta \): Isentropic efficiency of pump
- \( s_{\text{in}} \): Specific entropy at inlet (kJ/K)
- \( h_{\text{out},s} \): Specific enthalpy at outlet under isentropic conditions (kJ/kg)
- \( h_{\text{out}} \): Specific enthalpy at outlet (kJ/kg)
- \( T_{\text{out}} \): Temperature at outlet (deg C)
Figure 3.1: Example of head rise versus flow rate characteristics for a pump

Figure 3.2: Example of efficiency characteristics for a pump
3.1.1.3 Governing equations

Using assumption 1, the volumetric flow rate through the pump can be obtained as follows:

\[ Q = Q(N, p_{out} - p_{in}) \]  \hspace{1cm} (3.1)

Next, the density of the liquid at the pump inlet can be found from its thermodynamic properties with the inlet temperature acting as an intermediate variable as shown below:

\[ T_{in} = T(p_{in}, h_{in}) \]  \hspace{1cm} (3.2)

\[ \rho_{in} = \rho(p_{in}, T_{in}) \]  \hspace{1cm} (3.3)

Using the volume flow rate and liquid density values found above and applying the principle of conservation of mass to the control volume consisting of the liquid contained in the pump at any instant, we obtain:

\[ \dot{m}_{in} = \dot{m} = \dot{m}_{out} = \rho_{in}Q \]  \hspace{1cm} (3.4)

Using assumption 2, the isentropic efficiency, \( \eta \) of the pump can be obtained as:

\[ \eta = \eta(N, p_{out} - p_{in}) \]  \hspace{1cm} (3.5)

From the liquid’s thermodynamic properties, the inlet entropy can be found:

\[ s_{in} = s(p_{in}, h_{in}) \]  \hspace{1cm} (3.6)

Under isentropic conditions, the enthalpy of the liquid at the outlet can now be obtained as follows:

\[ h_{out,s} = h(p_{out}, s_{in}) \]  \hspace{1cm} (3.7)
From the definition of the isentropic efficiency \([157]\) and using the values from Equations 3.5 and 3.7, the actual enthalpy of the liquid at the outlet is given by the following expression:

\[
h_{\text{out}} = h_{\text{in}} + \frac{(h_{\text{out,s}} - h_{\text{in}})}{\eta}\tag{3.8}
\]

Knowing the outlet enthalpy and pressure, the outlet liquid temperature can be found as follows:

\[
T_{\text{out}} = T(h_{\text{out}}, p_{\text{out}}) \tag{3.9}
\]

3.1.1.4 THERMOSYS implementation

The pump flow rate versus head and efficiency characteristics are implemented using data maps. A MATLAB data structure called PumpProp is used to store this data. The fields of PumpProp are vectors and matrices which contain look-up tables and permit pump characteristics to be estimated by interpolation or extrapolation. These fields are as follows:

1. \(rpm\): A vector of pump rotational speeds (in rotations per minute). There is a speed line in the pump characteristics associated with each such speed (see Figures 3.1 and 3.2).

2. \(DP\): A matrix containing the head rise values (in \(kPa\)) for points along the speed lines. For example, \(DP(i, j)\) corresponds to the head rise for the \(j^{th}\) point along the \(i^{th}\) speed line.

3. \(Q\): A matrix containing the volumetric flow rate (in \(m^3/sec\)) for points along the speed lines. For example, \(Q(i, j)\) corresponds to the flow rate for the \(j^{th}\) point along the \(i^{th}\) speed line.

4. \(Eff\): A matrix containing the isentropic efficiency for points along the speed lines. For example, \(Eff(i, j)\) corresponds to the flow rate for the \(j^{th}\) point along the \(i^{th}\) speed line.
While creating PumpProp, the following rules must be followed:

1. The number of points on each speed line must be the same.

2. The entries in the vector \( rpm \) must be in increasing order.

3. The data on each speed line must be in the order of increasing head rise.

4. Along each speed line, the slope of head rise with respect to flow rate must always be negative. This ensures that there is only one flow rate for each rotational speed - head rise pair.

5. The map must have at least two speed lines.

A visual schematic of the PumpProp data structure has been shown in Figure 3.3. To handle the interpolation and extrapolation functions, custom routines are built into THERMOSYS.

The THERMOSYS pump element is called ‘Variable Speed Pump’ and has 4 inputs and 2 outputs. In addition to these, it writes 3 variables to the MATLAB workspace. A conceptual block diagram representation of the THERMOSYS pump element is presented in Figure 3.4. The equations that are used to obtain the output variables are depicted beside them in parentheses. Note that the pump model requires inlet and outlet pressures as inputs. It must therefore be connected to junctions at the inlet and outlet. If in the physical system, a junction is not present at any of these locations, one must be artificially added. To reduce the effect of such ‘dummy’ junctions on the physical system, they must be assigned a very small physical volume.

3.1.2 Flow Junctions

Flow junctions refer to piping components in the hydronic system such as tees, wyes and crosses (see Figure 3.5) where two or more flows meet, split or redistribute. From a physical perspective, these components are associated with the redistribution of mass flow and thermal energy. Strictly speaking, the thermal redistribution is a result of the flow redistribution
Figure 3.3: Schematic of PumpProp data structure

Figure 3.4: Pump Element Representation in THERMOSYS
and thus junctions are considered as hydraulic components. Schematic of a general junction element is shown in Figure 3.6.

3.1.2.1 Modeling assumptions

The following assumptions have been made for the junction model presented:

1. The liquid is assumed to be ‘nearly incompressible’. It means that the variation in pressure causes a small change in density; the relationship between them is expressed by the bulk modulus. However, the effect of temperature on density is neglected.

2. The junction volume is small leading to negligible external heat and work transfer to the liquid. For the same reason, it can also be assumed that the thermodynamic state of the liquid inside the junction is uniform. Further, the thermodynamic state of all the outlet streams is the same as the state of the liquid inside the junction.

3. Flow redistribution in the junction creates no additional pressure drop.

3.1.2.2 Nomenclature

The following notations are used for the junction model presented:

\[ m_{\text{junc}} \] : Mass of liquid inside junction (kg)

\[ n_{\text{in}} \] : Number of inlet flows

\[ n_{\text{out}} \] : Number of outlet flows

\[ \dot{m}_{i-\text{in}} \] : Mass flow rate of \( i^{th} \) inlet stream (kg/s), \( i = 1, 2, \ldots, n_{\text{in}} \)

\[ \dot{m}_{i-\text{out}} \] : Mass flow rate of \( i^{th} \) outlet stream (kg/s), \( i = 1, 2, \ldots, n_{\text{out}} \)

\[ V_{\text{junc}} \] : Volume of the junction (mm³)

\[ \rho \] : Density of liquid inside the junction (kg/m³)

\[ \beta \] : Bulk modulus of liquid (kPa)

\[ p_{\text{junc}} \] : Pressure of liquid inside the junction (kPa)

\[ h_{i-\text{in}} \] : Specific enthalpy of \( i^{th} \) inlet stream (kJ/kg), \( i = 1, 2, \ldots, n_{\text{in}} \)
Figure 3.5: Typical junction elements in hydronic system piping

\[ h_{\text{junc}} : \text{Specific enthalpy of liquid inside junction (kJ/kg)} \]

\[ U_{\text{junc}} : \text{Total internal energy of liquid inside junction (kJ)} \]

\[ H_{\text{junc}} : \text{Total enthalpy of liquid inside junction (kJ)} \]

\[ T_{\text{junc}} : \text{Temperature of liquid inside junction (degC)} \]

### 3.1.2.3 Governing equations

Conservation of mass applied to the control volume defined by the liquid contained in the junction at any given instant yields:

\[
\frac{dm_{\text{junc}}}{dt} = \sum_{i=1}^{n_{\text{in}}} \dot{m}_{i-\text{in}} - \sum_{i=1}^{n_{\text{out}}} \dot{m}_{i-\text{out}} \tag{3.10}
\]

Since the junction has a constant volume, any change in the mass of the liquid \( m_{\text{junc}} \) inside the junction is a consequence of change in its density:
Figure 3.6: Schematic representation of a flow junction

\[
\frac{dm_{junc}}{dt} = V_{junc} \frac{d\rho}{dt} \tag{3.11}
\]

Though the density of liquids depends on both pressure and temperature, the effect on density due to change in pressure is small but more pronounced than the effect due to change in temperature (see assumption 1). Hence the definition of bulk modulus of elasticity of the liquid can be used to approximate the relationship between change in pressure and density, in the following way:

\[
\beta = \rho \frac{\partial p_{junc}}{\partial \rho} \bigg|_T \approx \rho \frac{dp_{junc}}{d\rho} \tag{3.12}
\]

Using, Equation 3.12 in 3.11, we can write:

\[
\frac{dm_{junc}}{dt} = V_{junc} \frac{d\rho}{dt} = V_{junc} \frac{dp_{junc}}{dp_{junc}} \frac{dp_{junc}}{dt} = V_{junc} \frac{\rho}{\beta} \frac{dp_{junc}}{dt} \tag{3.13}
\]

Finally, substituting \(dm_{junc}/dt\) from Equation 3.13 in Equation 3.10, we get the following relationship for the conservation of mass equation:

\[
V_{junc} \frac{\rho}{\beta} \frac{dp_{junc}}{dt} = \sum_{i=1}^{n_{in}} \dot{m}_{i-in} - \sum_{i=1}^{n_{out}} \dot{m}_{i-out} \tag{3.14}
\]

Next, conservation of energy applied to the said control volume yields:
\[
\frac{dU_{\text{junc}}}{dt} = \sum_{i=1}^{n_{\text{in}}} \dot{m}_{i-\text{in}} h_{i-\text{in}} - h_{\text{junc}} \sum_{i=1}^{n_{\text{out}}} \dot{m}_{i-\text{out}} \tag{3.15}
\]

We also have:

\[
\frac{dU_{\text{junc}}}{dt} = \frac{d(H_{\text{junc}} - p_{\text{junc}} V_{\text{junc}})}{dt} = \frac{d(m_{\text{junc}} h_{\text{junc}} - p_{\text{junc}} V_{\text{junc}})}{dt} = m_{\text{junc}} \frac{dh_{\text{junc}}}{dt} + h_{\text{junc}} \frac{dm_{\text{junc}}}{dt} - V_{\text{junc}} \frac{dp_{\text{junc}}}{dt} = \rho V_{\text{junc}} \frac{dh_{\text{junc}}}{dt} + h_{\text{junc}} \left( \sum_{i=1}^{n_{\text{in}}} \dot{m}_{i-\text{in}} - \sum_{i=1}^{n_{\text{out}}} \dot{m}_{i-\text{out}} \right) - V_{\text{junc}} \frac{dp_{\text{junc}}}{dt} \tag{3.16}
\]

Eliminating \(dU_{\text{junc}}/dt\) between Equations 3.15 and 3.16 results in the following equation representing conservation of energy for the flow junction:

\[
\rho V_{\text{junc}} \frac{dh_{\text{junc}}}{dt} = \sum_{i=1}^{n_{\text{in}}} m_{i-\text{in}}(h_{i-\text{in}} - h_{\text{junc}}) + V_{\text{junc}} \frac{dp_{\text{junc}}}{dt} \tag{3.17}
\]

In the above equation, the quantity \(dp_{\text{junc}}/dt\) is obtained using the conservation of mass Equation 3.14. The liquid temperature inside the junction can then be found out from the thermodynamic properties of the liquid:

\[
T_{\text{junc}} = T(p_{\text{junc}}, h_{\text{junc}}) \tag{3.18}
\]

Note that the density \(\rho\) appearing in the above equations depends on the thermodynamic state of the liquid in the junction and can be found from the liquid properties as follows:

\[
\rho = \rho(p_{\text{junc}}, T_{\text{junc}}) \tag{3.19}
\]
3.1.2.4 THERMOSYS implementation

The junction volume, $V_{\text{junc}}$, is typically very small and therefore from Equation 3.17, it can be seen that the rate of energy accumulation in the junction, $\dot{h}_{\text{junc}}$, is large. Also, since the bulk modulus, $\beta$, is practically a very large quantity, the ratio $V_{\text{junc}}/\beta$ is very small causing $\dot{p}_{\text{junc}}$ in Equation 3.14 to be quite large. Hence, while performing a simulation of the entire system, the fast junction dynamics will lead to very small time steps and thus prolong the simulation time. However, the overall dynamics of the system does not undergo any appreciable change in the span of a small time step. This is because the dynamics of the thermal components (heat exchangers, chillers and boilers) are much slower than the junction dynamics and thus retard the evolution of the overall system (see Figure 3.7 for an illustration).

To avoid the above-mentioned problem of numerical inefficiency, the junction dynamics is artificially slowed down through the introduction of factors (called time-scale multipliers) - $F_p$ and $F_h$, in Equations 3.14 and 3.17 respectively, as shown below:

$$F_p V_{\text{junc}} \rho \frac{dp_{\text{junc}}}{dt} = \sum_{i=1}^{n_{\text{in}}} m_{i-\text{in}} - \sum_{i=1}^{n_{\text{out}}} m_{i-\text{out}}$$  \hspace{1cm} (3.20)

$$F_h V_{\text{junc}} \rho \frac{dh_{\text{junc}}}{dt} = \sum_{i=1}^{n_{\text{in}}} m_{i-\text{in}} (h_{i-\text{in}} - h_{\text{junc}}) + V_{\text{junc}} \frac{dp_{\text{junc}}}{dt}$$  \hspace{1cm} (3.21)

Note that $F_p, F_h \geq 1$. Their choice is a key consideration in using the THERMOSYS
junction model. They must be large enough to ensure acceptable numerical efficiency but small enough so that the accuracy of the solution is not compromised. It is recommended that arbitrary values be assigned to these parameters at the beginning and then adjusted on-line during the simulation till a tradeoff between efficiency and accuracy is achieved.

In addition to the above modifications, the THERMOSYS model for the junction computes normalized derivatives which can be used to determine whether or not equilibrium has been attained. This can also be useful in deciding the appropriate values of the time-scale multipliers , $F_p$ and $F_h$, since they directly provide information on the effect of these multipliers on the rate of the dynamics. The normalized derivatives $\epsilon_p$ and $\epsilon_h$ corresponding to pressure and enthalpy are defined in Equations 3.22 and 3.23 respectively.

$$\epsilon_p = \left| \frac{1}{p_{\text{junc}}} \frac{dp_{\text{junc}}}{dt} \right|$$  \hspace{1cm} (3.22)

$$\epsilon_h = \left| \frac{1}{h_{\text{junc}}} \frac{dh_{\text{junc}}}{dt} \right|$$  \hspace{1cm} (3.23)

These parameters represent the percentage change in pressure and enthalpy per second. When they are small, the simulation is near equilibrium.

The THERMOSYS junction element block is called ‘Flow Junction’ and has 3 inputs and 5 outputs. A conceptual block diagram representation of the element is described in Figure 3.8.

![Figure 3.8: Junction element representation in THERMOSYS](image-url)
3.8. The equations that are used to obtain the output variables are depicted beside them in parentheses.

3.1.3 Piping

A pipe refers to the section of the liquid flow path in the hydronic system between two flow junctions (see Figure 3.9 for a schematic). It includes all valves, bends, elbows and other appendages that lie in that section of the flow path. From a physical perspective, if the inlet and outlet pressures at the junctions which are the endpoints of a pipe are known, the mass flow rate of liquid through the pipe is governed by its resistance characteristics. The pipe wall and the other components in a pipe all contribute to this resistance and there are various models in literature to represent their resistance characteristics. The development of a pipe model essentially involves ‘combining’ all such individual resistances to obtain the overall resistance characteristics of a complete pipe. In principle, this is analogous to the process of obtaining the equivalent impedance in a section of an electrical circuit from the impedance characteristics of individual elements.

3.1.3.1 Modeling assumptions

1. The flow in the pipe is one dimensional and unidirectional.

2. Flow is incompressible.

3. Flow is uniform, i.e. the flow velocity, $u$ depends only on time.

4. At any cross section of the pipe, the fluid pressure is uniform.

5. Body forces acting on the liquid are small compared to surface forces.

6. For simplicity, the functions and coefficients used to describe the resistance characteristics of the pipe components at steady flow conditions are assumed to be applicable under transient conditions of flow as well.
7. The thermodynamic state of the fluid inside the pipe is uniform and same as that at its outlet.

8. There is no external work or heat transfer involved with the fluid in the pipe.

3.1.3.2 Nomenclature

The following notations are used for the pipe model presented:

- \( \Delta p_{ff} \): Friction factor model pressure drop (\( kPa \))
- \( \rho \): Liquid density (\( kg/m^3 \))
- \( u_{ss} \): Steady state flow velocity (\( m/s \))
- \( f \): Friction factor (dimensionless)
- \( L_{eq} \): Equivalent length of pipe (\( m \))
- \( D_h \): Hydraulic diameter of pipe (\( mm \))
- \( Re \): Reynold’s number of liquid (dimensionless)
- \( \mu \): Liquid viscosity (\( Ns/m^2 \))
- \( \epsilon \): Mean pipe wall roughness (\( mm \))
- \( \Delta p_{hf} \): Head loss factor model pressure drop (\( kPa \))
- \( K_t \): Head loss factor (dimensionless)
- \( Q_{ss} \): Steady state volumetric flow rate (\( m^3/s \))
- \( A_I \): Isentropic area (\( mm^2 \))
- \( A_c \): Flow cross section area (\( mm^2 \))
- \( \Delta p_r \): Generic hydraulic resistance model pressure drop (\( kPa \))
- \( R \): Hydraulic resistance constant (\( m^3/s \))

Figure 3.9: Schematic of a pipe consisting of a coupling and valve
3.1.3.3 Governing equations

As a starting step, the various models used for describing the resistance characteristics of the components in a pipe are first presented. There are four models that are typically used in industrial applications: (i) The friction factor model, (ii) the head loss factor model, (iii) the isentropic area model, and (iv) the generic hydraulic resistance model. Details of these are presented below. Note that these models correspond to steady flow conditions.

(i) The Friction Factor Model

The friction factor method is used to model the pipe wall resistance effects as per the
following expression:

\[ \Delta p_{ff} = \frac{1}{2} \rho u_{ss}^2 f \frac{L_{eq}}{D_h} \]  

(3.24)

The friction factor \( f \) in this model is computed from a correlation to Reynold’s number, \( Re = \frac{\rho u_{ss} D_h}{\mu} \). If \( Re > 4000 \), the flow is assumed to be turbulent and in that case, Haaland’s correlation [158] is used as described by Equation 3.25. Otherwise, for the laminar case (\( Re < 4000 \)), Equation 3.26 is used.

\[ \frac{1}{\sqrt{f}} = -0.782 \ln \left[ \frac{6.9}{Re} + \left( \frac{e}{3.7D_h} \right)^{1.11} \right] \]  

(3.25)

\[ f = \frac{64}{Re} \]  

(3.26)

In Equation 3.24, note that the equivalent length, \( L_{eq} \) equals the physical length of a pipe in the case of fully developed flow in a straight, constant area tube. In other situations, expressions for \( L_{eq}/D_h \) such as those given in [159] must be used.

(ii) The Head Loss Factor Model

The head loss factor method is used to model resistance effects of a variety of pipe components such as valves, bends, diffusers and manifolds. It is described by the following equation:

\[ \Delta p_{hf} = \frac{1}{2} \rho u_{ss}^2 K_t \]  

(3.27)

[160] is a useful reference for obtaining the head loss factor \( K_t \) of a wide range of components with different geometries in the case of turbulent flow.

(iii) The Isentropic Area Model

The isentropic flow area of a component is a fictitious quantity defined as the area, which
assuming the flow is isentropic, gives the same pressure drop from total to static conditions, as the measured total pressure drop from the inlet to the outlet of the component. Using this definition, it can be shown that the pressure drop across a component with isentropic area $A_I$ is given by:

$$\Delta p_{ia} = \frac{1}{2} \rho \left( \frac{Q_{ss}}{A_I} \right)^2$$  \hfill (3.28)

The isentropic area of a component, $A_I$ is analogous to its head loss factor $K_t$ and can be obtained from the latter using the following relation:

$$A_I = \frac{A_c}{\sqrt{K_t}}$$  \hfill (3.29)

(iv) The Generic Hydraulic Resistance Model

The resistance characteristics of components which cannot be accurately described by any of the previous three models can be represented using a correlation of the form stated in Equation 3.30, called the generic hydraulic resistance model.

$$\Delta p_r = RQ_{ss}^n$$  \hfill (3.30)

The hydraulic resistance constant $R$ and the hydraulic resistance exponent $n$ that appear in the above equation can be estimated by fitting test data.

Consider the control volume shown in Figure 3.10 which corresponds to the fluid volume enclosed in the pipe at any instant. Application of the principle of conservation of mass leads to the following equation:

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$  \hfill (3.31)

Next, we write the momentum conservation equation for the control volume under the
stated assumptions. The integral form of the momentum conservation equation in the
direction of flow (X-direction) is given by the following equation (see [159]):

\[ F_{SX} + F_{BX} = \frac{\partial}{\partial t} \int_{CV} u \rho \, d\mathcal{V} + \int_{CS} u \rho \vec{V} \cdot \vec{d}A \]  
(3.32)

Due to assumption 5, we have the following:

\[ F_{SX} + F_{BX} \approx F_{SX} \]  
(3.33)

Furthermore, assumptions 1, 2 and 3 imply:

\[ \int_{CS} u \rho \vec{V} \cdot \vec{d}A = 0 \]  
(3.34)

\[ \frac{\partial}{\partial t} \int_{CV} u \rho \, d\mathcal{V} = \frac{\partial}{\partial t} (u \rho \nabla) = \rho \nabla \frac{du}{dt} \]  
(3.35)

The use of Equations 3.33, 3.34 and 3.35 in Equation 3.32 results in:

\[ F_{SX} = \rho \nabla \frac{du}{dt} \]  
(3.36)

But, \( F_{SX} = \text{Pressure force} - \text{Resistive force} \). The pressure force can be obtained by using
assumption 4. The resistive force is a function of the flow rate and we denote it by \( f(Q) \).
Therefore, the surface force \( F_{SX} \) can be expressed by the following equation:
\[ F_{SX} = (p_{in} - p_{out}) A_c - f(Q) \]  

(3.37)

At this point, the only task that remains is to determine the expression for the resistive force, \( f(Q) \). For this, we use assumption 6 in conjunction with the various resistance models described earlier. At steady state, we have \( F_{SX} = 0 \) and therefore from Equation 3.37:

\[ (P_{in} - P_{out}) A_c = f(Q_{ss}) \]  

(3.38)

However, at steady state, the overall resistance characteristics of the pipe are determined by the various resistance models, and thus we have:

\[ (P_{in} - P_{out}) = \text{overall head loss} = \Delta p_{ff} + \Delta p_{hf} + \Delta p_{ia} + \Delta p_r \]  

(3.39)

Thus, using Equations 3.24, 3.27, 3.28 and 3.30 in the above expression we obtain:

\[ (P_{in} - P_{out}) = \frac{1}{2} \rho u_{ss}^2 fL_{eq} \frac{1}{D_h A_c^2} + \frac{1}{2} \rho u_{ss}^2 K_t + \frac{1}{2} \rho \left( \frac{Q_{ss}}{A_I} \right)^2 + RQ_{ss}^n \]  

(3.40)

Using the fact that \( u_{ss} = Q_{ss}/A_c \), Equation 3.40 becomes:

\[ (P_{in} - P_{out}) = \frac{1}{2} \rho Q_{ss}^2 fL_{eq} \frac{1}{D_h A_c^2} + \frac{1}{2} \rho Q_{ss}^2 K_t A_c + \frac{1}{2} \rho \left( \frac{Q_{ss}}{A_I} \right)^2 + RQ_{ss}^n \]  

(3.41)

Comparing the above equation with Equation 3.38, we obtain the following expression for \( f(Q_{ss}) \):

\[ f(Q_{ss}) = \frac{1}{2} \rho \left( f \frac{L_{eq}}{D_h A_c} + K_t + \frac{A_c}{A_I^2} \right) Q_{ss}^2 + A_c RQ_{ss}^n \]  

(3.42)

Now, assumption 6 implies that the mapping \( f(.) \) from \( Q \) to \( f(Q) \) is the same as the mapping from \( Q_{ss} \) to \( f(Q_{ss}) \). Thus, the function \( f(Q) \) can be expressed by the following equation:
\[ f(Q) = \frac{1}{2}\rho \left( f \frac{L_{eq}}{D_h A_c} + \frac{K_i}{A_c} + \frac{A_c}{A_t^2} \right) Q^2 + A_c R Q^n \] 

(3.43)

Substituting \( f(Q) \) from Equation 3.43 into Equation 3.37 and then using Equation 3.36 together with the facts that \( \forall = A_c L_{eq} \) and \( u = Q/A_c \), we obtain the following final form of the momentum conservation equation for the pipe element:

\[
\frac{dQ}{dt} = \frac{A_c}{\rho L_{eq}} (p_{in} - p_{out}) - \frac{1}{2} \left( f \frac{1}{A_c D_h} + \frac{K_i}{A_c L_{eq}} + \frac{A_c}{A_t^2} \right) Q^2 - \frac{A_c R}{\rho L_{eq}} Q^n
\]

(3.44)

The liquid mass flow rate through the pipe is then given by:

\[ \dot{m} = \rho Q \] 

(3.45)

Lastly, due to assumptions 7 and 8, the conservation of energy for the control volume results in the following:

\[ h_{in} = h_{out} = h \] 

(3.46)

Note that the liquid properties - density and viscosity, which appear in the equations above are obtained from the thermodynamic properties at the inlet as follows (using \( T_{in} \) as an intermediate variable).

\[ T_{in} = T(p_{in}, h_{in}) \] 

(3.47)

\[ \rho = \rho(p_{in}, T_{in}) \] 

(3.48)

\[ \mu = \mu(p_{in}, T_{in}) \] 

(3.49)
3.1.3.4 THERMOSYS implementation

An order of magnitude analysis performed using practical values of the various pipe parameters suggests that the dynamics governed by the Equation 3.44 is fast compared to the dynamics of the thermal components. Therefore to boost numerical efficiency just as in the case of the junction elements, the pipe dynamics is artificially slowed through the introduction of a time-scale multiplier, denoted by $F_m$. The resulting modified governing equation is given by 3.50.

$$F_m \frac{dQ}{dt} = \frac{A_c}{\rho_l L_{eq}} (p_{in} - p_{out}) - \frac{1}{2} \left( \frac{f}{A_c D_h} + \frac{K_t}{A_c L_{eq}} + \frac{A_c}{A_f^2} \right) Q^2 - \frac{A_c R}{\rho_l L_{eq}} Q^n$$  \hspace{1cm} (3.50)

Once again note that $F_m \geq 1$. It is recommended that arbitrary values be assigned to it at the beginning and then adjusted on-line during the simulation till a satisfactory balance between efficiency and accuracy is achieved.

The THERMOSYS pipe element block is called ‘Dynamic Hydraulic Resistance’ and it has 4 inputs and 2 outputs. A conceptual block diagram representation of the element is presented in Figure 3.11. The equations that are used to obtain the output variables are depicted beside them in parentheses. It must be noted that the isentropic area, $A_I$ is treated
as an input to the model, rather than a parameter. This is to accomodate the fact that it can vary with time when control is achieved through manipulation of valves.

### 3.1.4 Heat Exchangers

As discussed in section 2.2.2.2, the thermal sink elements in hydronic building cooling systems are typically Liquid to Air Heat Exchangers (LAHXs). In heating systems, however, the most common sink elements are radiator panels and baseboard units, though LAHXs are also used sometimes. A nonlinear model of the LAHX dynamics is presented in this section. Modeling of radiator panels is an easy extension of the LAHX model but has not been pursued in this work and is an area of future development.

A LAHX is used to cool and dehumidify air in a cooling system or heat and humidify air if used in a heating system. The conditioned air is then distributed to the various spaces, such as rooms in a building via an air-distribution duct network (see section 2.2.3 for details). The LAHX is usually a shell and tube heat exchanger with crossflow arrangement and fins attached to the outside of the tubes for enhanced heat transfer area (see Figure 2.9). The liquid, i.e. chilled or hot water flows inside the tubes and air flows over the outside of the tubes and the fins. In most building systems, the LAHX is part of a packaged unit called the Air Handling Unit (AHU) as shown in Figure 2.24.

#### 3.1.4.1 Modeling assumptions

The following assumptions apply to the LAHX model presented in this section:

1. For simplicity, it is assumed that no humidification or de-humidification of air is involved in the LAHX, and thus only sensible heat transfer takes place.

2. There is no pressure drop on the liquid or air-side.

3. The liquid is assumed to be incompressible.

4. The thermodynamic state of the liquid inside the LAHX is assumed to be uniform. Furthermore, it is assumed to be the same as that of the liquid exiting the LAHX.
5. The temperature of the coil surface exposed to the air (external coil surface) is assumed to be uniform. Similarly, the temperature of the coil surface exposed to the liquid (internal coil surface) is also assumed to be uniform. However, there is a linear temperature distribution from the internal to the external surface of the coil due to conduction.

6. The air is assumed to be incompressible with constant specific heat.

7. The thermal capacity of the air is considered negligible (air temperatures follow quasi-steady distributions).

8. Fin heat transfer is one dimensional.

3.1.4.2 Nomenclature

The following notations are used for the LAHX model presented:

\( \dot{m}_{l-in} \) : Liquid mass flow rate at inlet (kg/s)
\( \dot{m}_{l-out} \) : Liquid mass flow rate at outlet (kg/s)
\( \dot{m}_l \) : Liquid mass flow rate inside LAHX (kg/s)
\( \dot{m}_{a-in} \) : Air mass flow rate at inlet (kg/s)
\( \dot{m}_{a-out} \) : Air mass flow rate at outlet (kg/s)
\( \dot{m}_a \) : Air mass flow rate inside LAHX (kg/s)
\( w_{a-in} \) : Inlet air humidity ratio (kg of water/kg of dry air)
\( w_{a-out} \) : Outlet air humidity ratio (kg of water/kg of dry air)
\( p_{l-in} \) : Liquid pressure at inlet (kPa)
\( p_{l-out} \) : Liquid pressure at outlet (kPa)
\( p_{a-in} \) : Air pressure at inlet (kPa)
\( p_{a-out} \) : Air pressure at outlet (kPa)
\( m_l \) : Mass of liquid inside LAHX (kg)
\( u_l \) : Specific internal energy of liquid inside LAHX (kJ/kg)
\( h_{l-in} \) : Specific enthalpy of liquid at inlet (kJ/kg)
$h_{l-out}$ : Specific enthalpy of liquid at outlet ($kJ/kg$)

$\dot{Q}_{lw}$ : Heat transfer rate between liquid and coil structure ($kW$)

$T_{l-out}$ : Temperature of liquid at outlet ($degC$)

$T_l$ : Temperature of liquid inside LAHX ($degC$)

$\rho_l$ : Density of liquid ($kg/m^3$)

$k_l$ : Thermal conductivity of liquid ($W/m - K$)

$Pr_l$ : Prandtl number of liquid (dimensionless)

$\mu_l$ : Liquid viscosity ($Ns/m^2$)

$Re_l$ : Reynold’s number of liquid (dimensionless)

$D_{hl}$ : Hydraulic diameter of liquid flow passages ($m$)

$A_{cl}$ : Total cross sectional area of liquid flow passages ($m^2$)

$L$ : Length of liquid flow passages ($m$)

$m_w$ : Mass of the solid walls of the LAHX ($kg$)

$c_w$ : Specific heat of the solid walls of the LAHX ($kJ/kg - K$)

$T_w$ : Temperature of the solid walls of the LAHX facing air side ($degC$)

$\dot{Q}_{wa}$ : Heat transfer rate between air and coil structure ($kW$)

$F_{fin-1}$ : Fraction of the total surface area between liquid and structure that is on finned surfaces (dimensionless)

$A_{sl}$ : Surface area between liquid and structure ($m^2$)

$A_{sl-1}$ : Surface area between liquid and structure that is on finned surfaces ($m^2$)

$A_{sl-2}$ : Surface area between liquid and structure that is not on finned surfaces ($m^2$)

$\dot{Q}_{lw-1}$ : Portion of heat transfer between liquid and coil via fins ($kW$)

$\dot{Q}_{lw-2}$ : Portion of heat transfer between liquid and coil not via fins ($kW$)

$\alpha_l$ : Convection coefficient between liquid and structure ($kW/K - m^2$)

$T_{wl}$ : Temperature of the solid walls of the LAHX facing liquid side ($degC$)

$\eta_{fl}$ : Liquid side fin efficiency (dimensionless)

$k_{wall}$ : Thermal conductivity of structure and fins ($W/m - K$)

$t_{wall}$ : Thickness of the structure ($m$)
\(U_{lw}\) : Overall heat transfer coefficient between liquid and structure (\(kW/K - m^2\))

\(L_{\text{fin-l}}\) : Length of the fin between liquid and structure (\(m\))

\(t_{\text{fin-l}}\) : Thickness of the fin between liquid and structure (\(m\))

\(j_l\) : Colburn modulus for liquid-structure heat transfer (dimensionless)

\(c_{pa}\) : Specific heat of air (\(kJ/kg - K\))

\(T_a\) : Air temperature inside LAHX (\(degC\))

\(U_{wa}\) : Overall heat transfer coefficient between air and structure (\(kW/K - m^2\))

\(A_{sa}\) : Surface area between air and structure (\(m^2\))

\(T_{a-in}\) : Temperature of air at inlet (\(degC\))

\(T_{a-out}\) : Temperature of air at outlet (\(degC\))

\(\alpha_a\) : Convection coefficient between air and structure (\(kW/K - m^2\))

\(\eta_{fa}\) : Air side fin efficiency (dimensionless)

\(L_{\text{fin-a}}\) : Length of the fin between air and structure (\(m\))

\(t_{\text{fin-a}}\) : Thickness of the fin between air and structure (\(m\))

\(j_a\) : Colburn modulus for air-structure heat transfer (dimensionless)

\(k_a\) : Thermal conductivity of air (\(W/m - K\)) (dimensionless)

\(D_{ha}\) : Hydraulic diameter of air flow passages (\(m\))

\(Re_a\) : Reynold’s number of air (dimensionless)

\(Pr_a\) : Prandtl number of air (dimensionless)

\(\mu_a\) : Air viscosity (\(Ns/m^2\))

\(A_{cf}\) : Total cross sectional area of air flow passages (\(m^2\))

\(h_{a-out}\) : Specific enthalpy of air at outlet (\(kJ/kg\))

\(h_{a-in}\) : Specific enthalpy of liquid at inlet (\(kJ/kg\))

3.1.4.3 Governing equations

The liquid mass flow rate must be conserved in the LAHX, and therefore the following continuity relationship holds:
\[ \dot{m}_l = \dot{m}_{l-out} = \dot{m}_{l-in} \quad (3.51) \]

Also, due to assumption 1, a similar equation can be written for the air mass flow rate:

\[ \dot{m}_a = \dot{m}_{a-out} = \dot{m}_{a-in} \quad (3.52) \]

The following is a direct consequence of assumption 1:

\[ w_{a-out} = w_{a-in} \quad (3.53) \]

Next, the following equations are a consequence of assumption 2:

\[ p_{l-out} = p_{l-in} \quad (3.54) \]

\[ p_{a-out} = p_{a-in} \quad (3.55) \]

The conservation of energy applied to the control volume defined by the liquid present in the LAHX at any instant leads to the following:

\[ m_l \frac{du_l}{dt} = \dot{m}_{l-in}(h_{l-in} - h_{l-out}) + \dot{Q}_{lw} \quad (3.56) \]

From the thermodynamic property relations for the liquid and using assumption 4, we have:

\[ T_{l-out} = T_l = T_l(p_{l-out}, u_l) \quad (3.57) \]

\[ \rho_l = \rho_l(p_{l-out}, T_l) \quad (3.58) \]

\[ h_{l-out} = h_l(p_{l-out}, T_l) \quad (3.59) \]
\[ k_l = k_l(p_{l-out}, T_l) \]  
\[ \text{Pr}_l = \text{Pr}_l(p_{l-out}, T_l) \]  
\[ \mu_l = \mu_l(p_{l-out}, T_l) \]

The Reynolds number of the liquid, \( Re_l \), by definition is:

\[ Re_l = \frac{\dot{m}_l D_{hl}}{\mu_l A_{cl}} \]  

The mass of liquid contained in the coils at any instant, \( m_l \), can be expressed as:

\[ m_l = \rho_l A_{cl} L \]

The Equation 3.56 is used to obtain the internal energy, \( u_l \), of the liquid inside the coils at any given instant. It must be noted however that the quantities \( m_l \) and \( h_{l-out} \) that appear in that ODE are dependent on the state of the liquid inside the coils, in particular \( u_l \). That dependence is expressed by Equations 3.64 and 3.59 via the intermediate variables \( T_l \) and \( \rho_l \) obtained from Equations 3.57 and 3.58.

The conservation of energy applied to the system defined by the solid structure comprising of the coil and fin surfaces involved in the heat transfer between the liquid and the air leads to the following:

\[ m_w c_w \frac{dT_w}{dt} = \dot{Q}_{wa} - \dot{Q}_{lw} \]  

Now, the expressions for the heat transfer rates \( \dot{Q}_{wa} \) and \( \dot{Q}_{lw} \) that appear in Equations 3.56 and 3.65 will be obtained from the basic concepts of heat transfer and the use of some standard correlations.
(i) Heat Transfer between Liquid and Structure

First, we calculate $\dot{Q}_{lw}$, the heat transfer rate between the LAHX structure and the liquid inside the tubes. For the sake of generality, it is assumed that the liquid-structure heat transfer surface is finned. The heat transfer between the liquid and the structure essentially occurs in two stages (see Figure 3.12).

The first stage consists of convection heat transfer between the liquid and the area of the structure, $A_{sl}$ in direct contact with the liquid. Part of this area, $A_{sl-1} = A_{sl}F_{fin-l}$ is on fins and the remaining part, $A_{sl-2} = A_{sl}(1 - F_{fin-l})$ is directly on the inside surface of the coils. Denoting the temperature of the coil surface exposed to the liquid side by $T_{wl}$ (see assumption 5), the following expression can be written for the heat transfer, $\dot{Q}_{lw-2}$ that occurs via the coil surface, $A_{sl-2}$:

$$\dot{Q}_{lw-2} = \alpha_l A_{sl-2}(T_{wl} - T_l)$$ (3.66)

Similarly, using the definition of fin efficiency [161], the heat transfer, $\dot{Q}_{lw-1}$ that occurs via
the finned surface, \( A_{sl-1} \) can be written as:

\[
\dot{Q}_{lw-1} = \alpha_l A_{sl-1} \eta_{fl} (T_{wl} - T_i) \tag{3.67}
\]

Therefore, the net heat transfer rate between the liquid and the structure given by the sum of heat transfers in equations 3.66 and 3.67, is as follows:

\[
\dot{Q}_{lw} = \alpha_l A_{sl} [(1 - F_{fin-l}) + \eta_{fl} F_{fin-l}](T_{wl} - T_i) \tag{3.68}
\]

In the second stage, the heat transferred from the liquid to the structure (Equation 3.68) penetrates through the thickness of coil and is conducted from its inside surface to its outside surface. Hence the following equation results from the theory of 1-D conduction:

\[
\dot{Q}_{lw} = \frac{k_{wall}}{t_{wall}} A_{sl} (1 - F_{fin-l})(T_w - T_{wl}) \tag{3.69}
\]

From Equations 3.68 and 3.69, the elimination of \( T_{wl} \) leads to Equation 3.70 as the desired final expression for the liquid-structure heat transfer rate \( \dot{Q}_{lw} \):

\[
\dot{Q}_{lw} = U_{lw} A_{sl} (T_w - T_i) \tag{3.70}
\]

Here, \( U_{lw} \) is a short-hand notation for the overall heat transfer coefficient between the liquid and the structure and is given by:

\[
U_{lw} = \frac{1}{t_{wall}} \frac{1}{k_{wall} (1 - F_{fin-l}) + \frac{\alpha_l [1 - F_{fin-l} (1 - \eta_{fl})]}{1}} \tag{3.71}
\]

The fin efficiency, \( \eta_{fl} \) appearing in Equation 3.71 is specific to the fin geometry. However, as a starting point, the expression for a planar fin with adiabatic tip [161] can be used as follows:

\[
\eta_{fl} = \frac{\tanh(\beta_l L_{fin-l})}{\beta_l L_{fin-l}} \tag{3.72}
\]
Where,
\[
\beta_l = \sqrt{\frac{2\alpha_l}{k_{wall}t_{fin-l}}}
\]  

(3.73)

The convection coefficient, \(\alpha_l\) is computed from the Colburn modulus, \(j_l\) as:
\[
\alpha_l = j_l \frac{k_l}{D_{hl}} Re_l Pr_l^{1/3}
\]  

(3.74)

Here, the Colburn modulus, \(j_l\) is obtained through correlations [162] as indicated in Equation 3.75 whereas the other variables namely \(k_l\), \(Re_l\) and \(Pr_l\) are as expressed by Equations 3.60, 3.63 and 3.61.

\[
j_l = f_c(Re_l)
\]  

(3.75)

(ii) Heat Transfer between Structure and Air

Next, we proceed to compute \(\dot{Q}_{wa}\), the heat transfer between the LAHX coil structure and the air forced over it. Consider the differential control volume on the air side shown in
Figure 3.13. Application of the first law of thermodynamics results in the following:

\[ \dot{m}_{a-in}c_{pa}(-dT_a) = dq \]  

(3.76)

Assuming \( U_{wa} \) to denote the constant average coefficient of heat transfer between the LAHX structure and the air, we can write:

\[ dq = U_{wa}dA_{sa}(T_a - T_w) \]  

(3.77)

Combining Equations 3.76 and 3.77 results in the following differential equation in \( T_a \):

\[ \frac{dT_a}{T_a - T_w} = -\frac{U_{wa}dA_{sa}}{\dot{m}_ac_{pa}} \]  

(3.78)

Integrating from inlet to outlet leads to:

\[ T_{a-out} = T_w + (T_{a-in} - T_w)exp\left(\frac{-U_{wa}A_{sa}}{\dot{m}_ac_{pa}}\right) \]  

(3.79)

The expression for \( U_{wa} \) (Equation 3.80) is similar to that for \( U_{lw} \) (Equation 3.71), except that there is no conduction term. Note that conduction was already accounted for in the computation of the liquid to structure heat transfer described earlier.

\[ U_{wa} = \alpha_a[1 - F_{fin-a}(1 - \eta_{fa})] \]  

(3.80)

The fin efficiency, \( \eta_{fa} \) appearing in Equation 3.80 can be expressed by an expression similar to Equation 3.72

\[ \eta_{fa} = tanh\left(\frac{\beta_aL_{fin-a}}{\beta_aL_{fin-a}}\right) \]  

(3.81)

Where,

\[ \beta_a = \sqrt{\frac{2\alpha_a}{k_{wall}t_{fin-a}}} \]  

(3.82)
The convection coefficient, \( \alpha_a \) is computed from the Colburn modulus, \( j_a \) as:

\[
\alpha_a = j_a \frac{k_a}{D_{ha}} Re_a Pr_a^{1/3}
\]  

(3.83)

Here again, the Colburn modulus, \( j_a \) is obtained through correlations [162] as indicated in Equation 3.84 and the Reynold’s number, \( Re_a \) is defined by Equation 3.85.

\[
j_a = f_c(Re_a)
\]  

(3.84)

\[
Re_a = \frac{\dot{m}_{a-in} D_{ha}}{\mu_a A_{ca}}
\]  

(3.85)

Note that the air properties \( k_a, Pr_a \) and \( \mu_a \) that are required in Equations 3.83 and 3.85 can be computed from the air inlet conditions, but can also safely be assumed to be constant.

Having obtained the value of the outlet air temperature \( T_{a-out} \) from Equation 3.79, the desired heat transfer rate from the structure to air can be expressed as:

\[
\dot{Q}_{wa} = \dot{m}_{a-in} c_p a (T_{a-in} - T_{a-out})
\]  

(3.86)

The values of \( \dot{Q}_{wa} \) from Equation 3.86 and \( \dot{Q}_{lw} \) from Equation 3.70 can now be used to complete the conservation of energy Equations 3.56 and 3.65.

The enthalpy of air entering the LAHX can be obtained using the air property tables as follows:

\[
h_{a-out} = h(p_{a-out}, T_{a-out}, w_{a-out})
\]  

(3.87)

Similarly, the temperature of air entering the LAHX, which appears in the above equations, can be obtained as:
3.1.4.4 THERMOSYS implementation

The function $f_c(.)$ which represents the correlation between Reynold’s number and Colburn modulus is implemented using a subroutine embedded in THERMOSYS. It assumes offset strip fins, geometrical data for which in the form of three dimensionless parameters is required as an input.

The THERMOSYS LAHX block element is called 'Liquid to Air Heat Exchanger' and has 7 inputs and 10 outputs. A conceptual block diagram representation of the element is presented in Figure 3.14. The equations that are used to obtain the output variables are depicted beside them in parentheses. It must be noted that the THERMOSYS LAHX block was developed for the general case which involves humidification or dehumidification of air. It first determines whether humidification or dehumidification occurs. If it does not occur,
governing equations presented in section 3.1.5.3 are used. Otherwise, a more complicated model is invoked.

### 3.1.5 Thermal Source Elements

As discussed in section 2.2.2.1, the typical source elements that are used in hydronic systems are chillers and boilers. Physically, these components are heat exchangers and the modeling procedure is similar to that presented for LAHXs. Figure 3.15 shows the heat transfer semantics in a typical source element. In the model presented in this section, the heat transfer between the heating/cooling source and the coils carrying water, which is often referred to as the operating capacity of the element is not modeled but is treated as a specified external input. It is practical to make such an assumption because chillers and boilers are typically set to run at prescribed operating capacities achieved via feedback capacity control, e.g., by changing the compressor speed in an electric vapor compression chiller or by manipulating the resistance of the variable-resistance filament in an electric boiler.
3.1.5.1 Modeling assumptions

Following assumptions have been made for the development of a model for thermal source elements:

1. As mentioned earlier, the external heat transfer in the source elements (e.g. from refrigerant to the water coil in a chiller unit or from the hot gas to the water coil in a gas-fired boiler unit) is assumed to be given as an input (time-varying in general)

2. There is no pressure drop in the liquid from the inlet to the outlet.

3. The thermodynamic state of the liquid within the coil is uniform and is the same as that at the outlet.

4. For simplicity, fin effects have been ignored when considering heat transfer involving the coil structure.

5. The temperature of the coil structure is assumed to be uniform.

3.1.5.2 Nomenclature

The following notations are used for the source element model presented:

\( \dot{m}_{in} \) : Liquid mass flow rate at inlet (kg/s)

\( \dot{m}_{out} \) : Liquid mass flow rate at outlet (kg/s)

\( \dot{m} \) : Liquid mass flow rate inside the coils (kg/s)

\( p_{in} \) : Liquid pressure at inlet (kPa)

\( p_{out} \) : Liquid pressure at outlet (kPa)

\( m_i \) : Mass of liquid inside the source element (kg)

\( u \) : Specific internal energy of liquid inside the source element (kJ/kg)

\( h_{in} \) : Specific enthalpy of liquid at inlet (kJ/kg)

\( h_{out} \) : Specific enthalpy of liquid at outlet (kJ/kg)

\( \dot{Q}_{lw} \) : Heat transfer rate between liquid and coil structure (kW)

\( T \) : Temperature of liquid inside the source element (deg C)
\( \rho \): Density of liquid (kg/m\(^3\))
\( k \): Thermal conductivity of liquid (W/m – K)
\( Pr \): Prandtl number of liquid (dimensionless)
\( \mu \): Liquid viscosity (Ns/m\(^2\))
\( Re \): Reynold's number of liquid (dimensionless)
\( D_h \): Hydraulic diameter of liquid flow passages (m)
\( A_c \): Total cross sectional area of liquid flow passages (m\(^2\))
\( L \): Length of liquid flow passages (m)
\( m_w \): Mass of the coils carrying liquid (kg)
\( c_w \): Specific heat of the coils carrying liquid (kJ/kg – K)
\( T_w \): Wall temperature of coils carrying liquid (deg C)
\( \dot{Q}_{ext} \): External heat transfer rate (kW)
\( \alpha \): Convection coefficient between liquid and coil walls (kW/K – m\(^2\))
\( A_s \): Surface area between liquid and coil walls (m\(^2\))
\( j \): Colburn modulus for liquid-coil structure heat transfer (dimensionless)

3.1.5.3 Governing equations

The conservation of mass applied to the control volume defined by the liquid present in the coils at any instant implies the following:

\[
\dot{m}_{in} = \dot{m}_{out} = \dot{m} \tag{3.89}
\]

As a result of assumption 2, we have:

\[
p_{in} = p_{out} \tag{3.90}
\]

Next, the conservation of energy applied to this control volume leads to:

\[
m \frac{du}{dt} = \dot{m}(h_{in} - h_{out}) + \dot{Q}_{tw} \tag{3.91}
\]
Also, from liquid thermodynamic properties and making use of assumption 3, we have:

\[ T = T(p_{\text{out}}, u) \]  \hspace{1cm} (3.92)

\[ \rho = \rho(p_{\text{out}}, T) \]  \hspace{1cm} (3.93)

\[ h_{\text{out}} = h(p_{\text{out}}, T) \]  \hspace{1cm} (3.94)

\[ k = k(p_{\text{out}}, T) \]  \hspace{1cm} (3.95)

\[ Pr = Pr(p_{\text{out}}, T) \]  \hspace{1cm} (3.96)

\[ \mu = \mu(p_{\text{out}}, T) \]  \hspace{1cm} (3.97)

The Reynold’s number, \( Re \), by definition is:

\[ Re = \frac{\dot{m} D_h}{\mu A_c} \]  \hspace{1cm} (3.98)

The mass of liquid contained in the coils at any instant, \( m \), can be expressed as:

\[ m = \rho A_c L \]  \hspace{1cm} (3.99)

The Equation 3.91 is used to calculate the internal energy, \( u \), of the liquid inside the coils at any given instant. It must be noted however that the quantities \( m \) and \( h_{\text{out}} \) that appear in that ODE are dependent on the state of the liquid inside the coils, in particular \( u \). That dependence is expressed by Equations 3.99 and 3.94 via the intermediate variables \( T \) and \( \rho \) obtained from Equations 3.92 and 3.93.
Applying conservation of energy to the control volume defined by the structure of the coil results in:

\[ m_w c_w \frac{dT_w}{dt} = \dot{Q}_{ext} - \dot{Q}_{tw} \]  

(3.100)

As noted in assumption 1, the external heat transfer rate, \( Q_{ext} \) in this model is considered to be a prescribed input. However, the heat transfer rate between the structure and liquid, \( \dot{Q}_{tw} \), appearing in Equations 3.91 and 3.100 is calculated from Newton’s law of cooling with fin effects neglected (assumption 4):

\[ \dot{Q}_{tw} = \alpha A_s (T_w - T) \]  

(3.101)

In the above equation, the heat transfer coefficient, \( \alpha \) is obtained from the Colburn modulus, \( j \):

\[ \alpha = j \frac{k}{D_h} Re Pr^{1/3} \]  

(3.102)

The quantities \( k, Re \) and \( Pr \) that appear above were already obtained in Equations 3.95, 3.98 and 3.96 respectively, whereas, the Colburn modulus, \( j \) can be obtained from \( Re \) through correlations [162] as shown below:

\[ j = f_c(Re) \]  

(3.103)

### 3.1.5.4 THERMOSYS implementation

The function \( f_c(.) \) which represents the correlation between Reynold’s number and Colburn modulus is implemented using a subroutine embedded in THERMOSYS, as was done in the case of LAHXs.

The THERMOSYS source element block is called ‘Heat Source’ and has 5 inputs and 5
outputs. A conceptual block diagram representation of the element is presented in Figure 3.16. The equations that are used to obtain the output variables are depicted beside them in parentheses. It must be noted that the THERMOSYS heat source block was developed for the general case which involves internal generation of heat inside the liquid and this must be set to zero in its current implementation here.

### 3.2 Hydronic System Simulations with THERMOSYS

The procedure to carry out simulations of a hydronic system using THERMOSYS is outlined in this section. Certain data structures are required to be defined in MATLAB as preliminaries before any simulation can be carried out. One of these is the PumpProp which pertains to pump characteristics and was explained in section 3.1.1.4. Apart from PumpProp, two other data structures called LiquidProp and MoistAirProp, corresponding to the liquid and moist air properties respectively are also needed. For convenience, a program called ‘FluidProp_5050APGS.mat’ to generate the LiquidProp data structure for APGS with 50% glycol by volume has been included as a part of THERMOSYS. Similarly, another program called ‘FluidProp_MoistAir.mat’ to generate the MoistAirProp data structure is also provided.

The step by step procedure to run a THERMOSYS simulation is as follows:

1. Declare PumpProp, LiquidProp and MoistAirProp as global variables.
2. Load the pump, liquid and moist air properties into the MATLAB workspace. In most cases, this would involve loading the MATLAB files ‘FluidProp_5050APGS.mat’ and ‘FluidProp_MoistAir.mat’ which automatically create LiquidProp and MoistAirProp data structures into the workspace. The PumpProp data structure, however, must be created based on the manufacturer’s catalog or test data as described in section 3.1.1.4. An easy way to accomplish this is to use the program ‘PumpProp_demo.mat’ to generate an example data structure PumpProp and then scale the underlying values to match the properties of the actual pump being modeled.

3. Create a SIMULINK model by dragging and dropping elements from the THERMOSYS library. Connect the various elements to represent the connectivity in the actual hydronic system. Typically, the outputs from one component such as mass flow rates, enthalpies and pressures shall serve as inputs to another component or a set of components. The inputs extrinsic to the system such as valve opening factors (isentropic areas) and external heat transfer rates in the source elements need to be specified separately.

4. For each element in the model, specify the GUI entries which include physical parameters and operating conditions. The operating condition entries need not be accurate and ‘guess values’ can be used to begin with. Note that the purpose of running the THERMOSYS simulation is to obtain the exact operating condition of the system and
therefore the operating condition values specified in the GUIs are not of much impor-
tance, except that they are used to estimate the initial conditions in some situations. 
Set the time-scale multipliers in the junction and hydraulic resistance elements to unity. 
Click ‘Apply’ which sets the initial conditions and then click ‘OK’ to close the GUI. 
This important step must be repeated for all the elements in the model. See Figures 
3.17 and 3.18 for an illustration.

5. Set the simulation time to the desired value, which would at least be in the order of 
hundreds of seconds, given the inherently slow dynamics of these systems. Based on 
experience, a variable step stiff solver such as ode23tb is recommended for efficient 
simulations.

6. Begin the simulation. There is a high possibility of the simulation being terminated 
initially due to the very stiff nature of the system. The time-scale factors in the junctions 
and the hydraulic resistances must be increased in suitable steps till the simulation run 
is successful.
3.3 Concluding Remarks

In this chapter, the THERMOSYS toolbox was introduced as a platform for modeling the nonlinear dynamics of hydronic systems. The governing equations used to model the various components were presented in detail. These will be used in the next chapter to derive linear models, which will eventually be required for the control design methodologies presented in the ensuing chapters.
Mathematical models for describing hydronic system components are well known and extensively reported in the literature. One such tool, THERMOSYS was introduced in chapter 3. However, a formal procedure to integrate these models into a generic framework for understanding and controlling the overall system dynamics is not very well developed. A significant attempt towards that has been reported in [150], where the authors propose a graph based procedure to obtain a static matrix representation of the behavior of heat exchanger networks (used in process industries). Static models have also been developed to optimize the production and distribution schedules in district heating networks [136] and for distributed control of such networks [153]. Though a static representation of the system is useful for estimating its steady state response and designing static controllers, real time control design - which is necessary for efficient transient performance leading to significant life-cycle energy savings - shall require modeling of the dynamical behavior of the system. This motivates the development of a generic and scalable modeling approach which is simple and accurate enough to facilitate design of robust, practically implementable control algorithms.

This objective of this chapter is to present a modeling approach which meets the requirements stated above. Its organization is as follows. Section 4.1 discusses the objectives and challenges involved in the modeling process and also provides an overview of the proposed framework. Sections 4.2 to 4.5 describe in detail the constituents of this framework, with appropriate examples, followed by some model validation results in section 4.6. Lastly, some concluding remarks are included in section 4.7.
4.1 Modeling Objectives

4.1.1 Goals

The development of control oriented models of hydronic systems was undertaken with the following requirements in mind:

1. Simple: Hydronic systems are complex on two accounts, viz., complexity of the dynamics of the individual components and complexity of the network architecture. This motivates the development of a modeling framework which incorporates simple, preferably linear component models and facilitates easy integration of these models to represent the dynamics of the overall system.

2. Capable of quantifying interactions: Though it is desirable to have a simple model of the system, it is important to accurately represent the thermal and flow interactions among the various system components. The accuracy with which these interactions are represented has a strong bearing on the accuracy and robustness of the controllers designed using the model.

3. Dynamic: Hydronic systems are characterized by their inherently slow dynamics due to large heat exchanger thermal capacities. To ensure that the various control objectives such as thermal comfort and energy optimality are still attained during the significantly long transient periods of operation, a dynamic model must be used for control design as opposed to a static model which characterizes only steady state operation.

4. Generic: It is preferred that the model be representative of a wide variety of both heating and cooling hydronic systems, irrespective of their spatial footprint and architecture.

5. Modular: It is also desired that the model be easily augmentable to accommodate any changes made to the actual physical system such as addition or removal of components. It is expected that flexibility in the model shall render the ensuing controllers easily augmentable to incorporate such changes in the physical system.
4.1.2 Challenges

The development of a generic, control oriented, dynamical model for hydronic systems faces the following challenges, which need to be considered:

1. Complexity: As noted earlier, one of the desirable characteristics of the model is that it must be simple enough to enable ease of control design but accurate enough to guarantee robustness during actual operation. The conflicting nature of these two requirements poses a challenge in the model development and to address it a piecewise linear modeling approach with subsequent model reduction has been used in this work, details of which will follow in this chapter.

2. Generality and modularity: The fact that the system in general can have an arbitrary architecture and arbitrary number and type of components implies that incorporating the attributes of generality and modularity in the model is non-trivial. To handle this problem, a graph based approach has been used which naturally allows the desired modeling flexibility.

3. Stiffness: It is a common observation that in thermo-fluid systems, the hydraulic dynamics is practically a few orders of magnitude faster than the thermal dynamics. This makes any dynamic model stiff which potentially leads to numerical inefficiency while using these models for validation or for online control schemes such as MPC. This issue has been exploited in this work for time-scale decomposition of the system, hence leading to significant reduction in the order of the model.

4.1.3 Overview of modeling procedure

A piecewise linear modeling framework, which uses graph theory to address the requirements of generality and modularity, and model reduction to handle complexity and stiffness, has been adopted. In this approach (Figure 4.1), the entire operating regime of the system is partitioned into smaller regimes, and a linear state space model is obtained to represent the
Figure 4.1: Illustration of the piecewise linear modeling framework

Figure 4.2: Summary of the control oriented modeling procedure
dynamics in each of these regimes. Each of these models are developed based on appropriately chosen ‘nominal’ operating conditions lying in the corresponding regimes.

The procedure involved in the development of any such linear model is summarized in Figure 4.2. A graph representation is used to quantify the network connectivity in terms of certain connectivity matrices. Also, linear models for the individual components are obtained about the nominal operating condition corresponding to the regime under consideration. The coefficients appearing in these models are then lumped into suitably defined coefficient matrices. The next step in the modeling involves concatenation of the connectivity and coefficient matrices to construct the matrices for the state space representation of the overall system. This concatenation is automated via a code. Subsequently, time-scale analysis is employed to simplify this model and obtain a reduced order representation of the system dynamics. The underlying details of each of these steps have been presented in the remainder of this chapter, complemented with a modeling example and model validation results at the end.

4.2 Connectivity Quantification

4.2.1 Graph representation

The literature review presented in section 1.3.2.1 indicates that graph theory is a powerful modeling and analysis tool in the context of complex dynamical systems. Therefore, a graph representation is used to picture and quantify the connectivity among the various components of the system. The details of the proposed representation are as follows:

**Vertices** : The vertices of the graph have been categorized as *energy flow vertices* and *mass flow vertices*. The former represent the thermal elements (both sink and source) while the latter correspond to flow junctions. Note that as mentioned in chapter 3, additional proxy flow junctions are introduced at the inlet and outlet of each pump to ensure that the model is consistent from a causality point of view.

**Edges** : The edges correspond to the flow paths between the various vertices. The
Table 4.1: Explanation of various graph elements

<table>
<thead>
<tr>
<th>Graph element</th>
<th>Inputs</th>
<th>States</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow vertex</td>
<td></td>
<td>Junction temperature, pressures</td>
<td></td>
</tr>
<tr>
<td>Energy flow vertex (source element)</td>
<td>External heat transfer rate</td>
<td>Liquid temperature, Structure temperature</td>
<td></td>
</tr>
<tr>
<td>Energy flow vertex (sink element)</td>
<td>Air mass flow rate and inlet temperature</td>
<td>Liquid temperature, Structure temperature</td>
<td>Heat transfer achieved</td>
</tr>
<tr>
<td>Pipe (pump)</td>
<td>Pump speed</td>
<td>Mass flow rate</td>
<td></td>
</tr>
<tr>
<td>Pipe (hydraulic resistance)</td>
<td>Isentropic area (valve opening factor)</td>
<td>Mass flow rate</td>
<td></td>
</tr>
</tbody>
</table>

The proposed representation is a digraph and the edges are directed along the fluid flow direction in the corresponding flow paths that they represent.

**Pipes**: In the graph, special elements called pipes are defined which represent directed paths that originate and end at the mass flow vertices. Therefore, a pipe is an aggregation of one or multiple edges and energy flow vertices. Each pump is a pipe because proxy flow junctions (mass flow vertices) are present at its inlet and outlet. Similarly, each piping element in the physical system (or hydraulic resistance, refer to section 3.1.3) is also a pipe in its graph. The fact that pipes decompose the graph is easy to verify.

Together, the vertices and pipes are representative of the dynamics of the complete system, whereas its connectivity information is characterized by the edges. Table 4.1 enlists the various input, state and output variables of the system which are associated with the vertices and pipes.

### 4.2.2 Notation and numbering

In the graph representation presented above, the following notation is used:

- \( n_l \): Number of pipes
- \( l \): Index to label pipes \((1 \leq l \leq n_l)\)
- \( n_k \): Number of edges
- \( k \): Index to label edges \((1 \leq k \leq n_k)\)
- \( n_j \): Number of mass flow vertices
\( j \) : Index to label mass flow vertices (1 ≤ \( j \) ≤ \( n_j \))

\( n_i \) : Number of energy flow vertices, i.e. thermal components

\( i \) : Index to label energy flow vertices (\( n_j + 1 \) ≤ \( i \) ≤ \( n_j + n_i \))

\( n_v \) : Number of control valves

\( v \) : Index to label control valves (1 ≤ \( v \) ≤ \( n_v \))

\( n_p \) : Number of pumps

\( p \) : Index to label pumps (1 ≤ \( p \) ≤ \( n_p \))

\( n_c \) : Number of thermal source components

\( c \) : Index to label energy flow vertices which represent thermal source components

(\( n_j + 1 \) ≤ \( c \) ≤ \( n_j + n_c \))

\( n_h \) : Number of thermal sink components

\( h \) : Index to label energy flow vertices which represent thermal sink components

(\( n_j + n_c + 1 \) ≤ \( h \) ≤ \( n_j + n_c + n_h \))

A few important features of the above notation must be pointed out. Firstly, note that \( n_i = n_c + n_h \). In fact, an energy flow vertex can either be labeled by ‘\( i \)’ or one of ‘\( c \)’ and ‘\( h \)’.

Secondly, the vertices must be numbered in the following order: (i) Mass flow vertices (denoted by \( j \)), (ii) Energy flow vertices representing thermal source components (denoted by \( c \)), and (iii) Energy flow vertices representing thermal sink components (denoted by \( h \)).

The edges can be numbered arbitrarily in any order and the same is true for pumps and control valves.

4.2.3 Connectivity matrices

The following matrices are defined based on the graph representation to quantify the required connectivity information:

(i) Pipe-junction incidence matrix
\[ A_{(n_j \times n_i)} = \{ a_{pq} \} \]  \hspace{1cm} (4.1)

Where,

\[
a_{pq} = \begin{cases} 
-1 & \text{if pipe } q \text{ exits junction } p \\
1 & \text{if pipe } q \text{ enters junction } p \\
0 & \text{otherwise}
\end{cases} \hspace{1cm} (4.2)
\]

(ii) Semi-incidence matrix and submatrices

The semi-incidence matrix, \( B \) is constructed using the flow semi-incidence matrix, \( B_f \) and thermal semi-incidence matrix, \( B_t \) as follows:

\[
B_{[n_k \times (n_l + n_i)\times (n_l + n_i)\times (n_l + n_i)]} = \begin{bmatrix} B_f_{(n_k \times n_j)} & B_t_{(n_k \times n_i)} \end{bmatrix} = \{ b_{pq} \} \hspace{1cm} (4.3)
\]

Where,

\[
b_{pq} = \begin{cases} 
1 & \text{if vertex } q \text{ is the tail of edge } p \\
0 & \text{otherwise}
\end{cases} \hspace{1cm} (4.4)
\]

(iii) Pipe decomposition matrix

\[
C_{(n_k \times n_i)} = \{ c_{pq} \} \hspace{1cm} (4.5)
\]

Where,

\[
c_{pq} = \begin{cases} 
1 & \text{if edge } p \text{ is contained in pipe } q \\
0 & \text{otherwise}
\end{cases} \hspace{1cm} (4.6)
\]
(iv) Mass-flow incidence matrix

\[ D_{(n_j \times n_k)} = \{d_{pq}\} \]  \hspace{1cm} (4.7)

Where,

\[ d_{pq} = \begin{cases} 
1 & \text{if edge } q \text{ enters junction } p \\
-1 & \text{if edge } q \text{ exits junction } p \\
0 & \text{otherwise}
\end{cases} \]  \hspace{1cm} (4.8)

(v) Energy-flow semi-incidence matrix

\[ E_{(n_i \times n_k)} = \{e_{pq}\} \]  \hspace{1cm} (4.9)

Where,

\[ e_{pq} = \begin{cases} 
1 & \text{if edge } q \text{ enters energy flow vertex } p \\
0 & \text{otherwise}
\end{cases} \]  \hspace{1cm} (4.10)

It is important to use the sparseness of these matrices to minimize memory and computation requirements during implementation.

4.2.4 Example

The graph based connectivity modeling procedure presented above is explained below using an example of a simple heating system that was introduced in chapter 2 (Figure 2.38). It is reproduced again in Figure 4.3. This example was chosen because it is simple enough to demonstrate the procedure to the reader interested in using it. The graph for this system constructed as per sections 4.2.1 and 4.2.2 is shown in Figure 4.4, and its elements have
Figure 4.3: Schematic of an example heating system

Figure 4.4: Graph of the example heating system

Table 4.2: Explanation of graph elements for example heating system

<table>
<thead>
<tr>
<th>Element number</th>
<th>Physical Explanation</th>
<th>Inputs</th>
<th>States</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Vertices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Junction 1</td>
<td>$T_1, p_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Junction 2</td>
<td>$T_2, p_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Junction 3</td>
<td>$T_3, p_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Junction 4</td>
<td>$T_4, p_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Boiler</td>
<td>$Q_{ext,5}$</td>
<td>$T_{L,5}, T_{w,5}$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>LAHX 1</td>
<td>$\dot{m}<em>{a-in,6}, T</em>{a-in,6}$</td>
<td>$T_{L,6}, T_{w,6}$</td>
<td>$Q_{out,6}$</td>
</tr>
<tr>
<td>7</td>
<td>LAHX 2</td>
<td>$\dot{m}<em>{a-in,7}, T</em>{a-in,7}$</td>
<td>$T_{L,7}, T_{w,7}$</td>
<td>$Q_{out,7}$</td>
</tr>
<tr>
<td><strong>B. Pipes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Edge 1</td>
<td>$\omega_1$</td>
<td>$\dot{m}_1$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Edges 2 and 3</td>
<td>$A_{f,1}$</td>
<td>$\dot{m}_2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Edges 4 and 5</td>
<td>$A_{f,2}$</td>
<td>$\dot{m}_3$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Edges 6 and 7</td>
<td>$A_{f,3}$</td>
<td>$\dot{m}_4$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Edge 8</td>
<td></td>
<td></td>
<td>$\dot{m}_5$</td>
</tr>
</tbody>
</table>
been explained in Table 4.2. The connectivity matrices defined in section 4.2.3 that were obtained for this system are as follows:

\[
A = \begin{pmatrix}
-1 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 \\
0 & 1 & -1 & -1 & 0 \\
0 & 0 & 1 & 1 & -1 \\
\end{pmatrix}
\]

\[
B_f = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}^T
\]

\[
B_t = \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}^T
\]

\[
C = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \\
\end{pmatrix}
\]

\[
E = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

### 4.3 Component linearization

#### 4.3.1 Operating condition specifications

The objective of this section is the development of linear models for the system components to represent their dynamics in any given regime of operation with reference to the piecewise linear modeling paradigm described in section 4.1.3. This requires the specification of a suitably chosen nominal operating point in the regime of operation under consideration. Any such operating point is defined by the following information, categorized component-wise:
**Pumps**: speed, inlet and outlet head, and volumetric flow rate

**Flow junctions**: inlet and outlet stream mass flow rates and temperatures

**Valves**: opening factors

**Thermal source elements**: liquid mass flow rate, inlet and outlet liquid temperatures, structure temperature, and external heat transfer rate

**Thermal sink elements**: liquid and air mass flow rates, liquid and air inlet and outlet temperatures, and structure temperature

It is possible to obtain the above information by a variety of techniques such as direct experimental measurements, estimation via governing physical equations or numerical simulation of satisfactorily accurate component models (lumped or finite element). In this work, an operating condition is obtained by simulation of a THERMOSYS model of the complete system (refer to chapter 3), with the system inputs such as pump speeds, valve opening factors, etc. set to their nominal values corresponding to the operating regime under consideration.

### 4.3.2 Linear component models

The following notations are used for the linear component models presented. Note that all the variables used below represent deviation of the corresponding physical quantity (state, input or output) from its nominal value explained in section 4.3.1.

- \( \dot{m}_l \): Liquid mass flow rate in pipe \( l \)
- \( \omega_l \): Speed of pump, if any, contained in pipe \( l \)
- \( p_{in,l} \): Liquid pressure at inlet of pipe \( l \)
- \( p_{out,l} \): Liquid pressure at outlet of pipe \( l \)
- \( A_{I,l} \): Isentropic area of valve, if any, contained in pipe \( l \)
- \( p_j \): Liquid pressure inside mass flow vertex \( j \)
- \( m_{inlets,j} \): Vector of mass flow rates for streams entering mass flow vertex \( j \)
- \( m_{outlets,j} \): Vector of mass flow rates for streams exiting mass flow vertex \( j \)
\( T_j \): Liquid temperature inside mass flow vertex \( j \)
\( T_{\text{inlets},j} \): Vector of liquid temperatures for streams entering mass flow vertex \( j \)
\( T_{\text{outlets},j} \): Vector of liquid temperatures for streams exiting mass flow vertex \( j \)
\( T_{L,i} \): Liquid temperature inside energy flow vertex \( i \)
\( T_{L-\text{in},i} \): Temperature of liquid entering energy flow vertex \( i \)
\( \dot{m}_{L-\text{in},i} \): Mass flow rate of liquid entering energy flow vertex \( i \)
\( T_{w,i} \): Structure temperature for energy flow vertex \( i \)
\( \dot{m}_{a-\text{in},i} \): Mass flow rate of air entering energy flow vertex \( i \)
\( T_{a-\text{in},i} \): Temperature of liquid entering energy flow vertex \( i \)
\( \dot{Q}_{\text{ext},i} \): External heat transfer rate for energy flow vertex \( i \) (applicable only to thermal source elements)
\( \dot{Q}_{\text{out},i} \): Liquid to air heat transfer rate achieved in energy flow vertex \( i \) (applicable only to thermal sink elements)

Linear models of the components can be represented in terms of the variables defined above, together with suitable scalar and vector coefficients. Details are provided below.

(i) Pipes

As noted earlier, pipes correspond to pumps or physical piping in the system. Equation 4.11 represents the linear version of Equation 3.1 (mass flow rate for pumps) and Equation 3.50 (mass flow rate for piping). Note that for each pump, its static model in chapter 3 has to be replaced by an approximate first order dynamic model of very small time constant (typically between 0.001 to 0.01 seconds) to fit the general structure of Equation 4.11.

\[
\frac{d\dot{m}_l}{dt} = a_1^l \dot{m}_l + a_2^l \omega_l - a_3^l p_{\text{in},l} + a_4^l p_{\text{out},l} + a_5^l A_{I,l}
\]  

(4.11)

(ii) Mass flow vertices

As was stated earlier, mass flow vertices represent both actual flow junctions present in the physical system and proxy junctions at pump inlet and outlets. The volume of the proxy
junctions must be assigned an arbitrary small value for modeling purposes. Equations 4.12 and 4.13 are the linear analogues of Equations 3.20 (Mass conservation for junctions) and Equations 3.21 (Energy conservation for junctions).

\[
\frac{dp_j}{dt} = b_j \left(||\dot{m}_{\text{inlets},j}||_1 - ||\dot{m}_{\text{outlets},j}||_1\right) \tag{4.12}
\]

\[
\frac{dT_j}{dt} = \langle d^j, \dot{m}_{\text{inlets},j} \rangle - \langle e^j, \dot{m}_{\text{outlets},j} \rangle + \langle f^j, T_{\text{inlets},j} \rangle - \langle g^j, T_{\text{outlets},j} \rangle \tag{4.13}
\]

Note that \(d^j, e^j, f^j\) and \(g^j\) are vectors of coefficients. Here, \(||.||_1\) and \(\langle ., . \rangle\) denote 1-norm and vector inner product respectively.

(iii) Energy flow vertices

The linear model for energy flow vertices is represented by Equations 4.14 and 4.15. The former represents conservation of energy for the liquid (see Equations 3.56 and 3.91) while the latter represents conservation of energy for the structure (compare Equations 3.65 and 3.100).

\[
\frac{dT_{L,i}}{dt} = q_{1}^i T_{\text{in},i} + q_{2}^i \dot{m}_{\text{in},i} + q_{3}^i T_{L,i} + q_{4}^i T_{w,i} \tag{4.14}
\]

\[
\frac{dT_{w,i}}{dt} = r_{1}^i \dot{m}_{\text{in},i} + r_{2}^i T_{L,i} + r_{3}^i \dot{m}_{a-in,i} + r_{4}^i T_{a-in,i} + r_{5}^i T_{L,i} + r_{6}^i T_{w,i} + r_{7}^i Q_{\text{ext},i} \tag{4.15}
\]

If the energy flow vertex \(i\) under consideration corresponds to a thermal sink element with index \(h\), the following algebraic relationship represents the linear analogue of the heat transfer rate (Equation 3.86) between the liquid and air:

\[
\dot{Q}_{\text{out},h} = s_{1}^h \dot{m}_{a-in,h} + s_{2}^h T_{a-in,h} + s_{3}^h T_{w,h} \tag{4.16}
\]
4.3.3 Linear coefficient matrices

The following matrices are now defined using the coefficients appearing in Equations 4.11 to 4.16:

(i) \( A_1 = \text{diag} \{ a_1^l \} \). Similarly define \( A_3, B_1, Q_1, Q_2, Q_3, Q_4, R_1, R_2, R_5, R_6, S_1 \) and \( S_2 \).

(ii) Construct \( A_2(n_l \times n_p) \) algorithmically using the following logic:

- All entries in row \( l \) of \( A_2 \) are set to zero if pipe \( l \) does not have a pump.
- If pipe \( l \) has a pump whose index is \( p \), then all entries of row \( l \), except the element \( A_2(l, p) \) are set to zero.
- The element \( A_2(l, p) \) is then assigned the value \( a_2^l \). In a similar way, the matrix \( A_4(n_l \times n_v) \) is constructed for the valves.

(iii) In row \( j \) of \( D \) defined in Equation 4.7, replace all \( 1^s \) by the elements of \( d^j \) and all \( -1^s \) by the elements of \( -e^j \). \( W_1(n_j \times n_k) \) is obtained by repeating this for all \( j \). Similarly construct \( W_2(n_j \times n_k) \) using \( f^j \) and \( -g^j \).

(iv) \( R_3(n_i \times (n_i-n_c)) \) is obtained by eliminating the first \( n_c \) columns from \( \text{diag} \{ r^3 \} \). Similarly construct \( R_4(n_i \times (n_i-n_c)) \).

(v) \( R_7(n_i \times n_c) \) is obtained by retaining the first \( n_c \) columns in \( \text{diag} \{ r^7 \} \) and eliminating the others.

(vi) Define \( S_3(n_h \times n_i) = \left[ 0 \right]_{(n_h \times n_c)} \text{diag} \{ s^h_3 \} \).

4.3.4 Example

The heating system shown in Figure 4.3 is used as an example to demonstrate the linear component modeling procedure that was presented above. For a chosen set of physical parameters and operating inputs, the operating values of the states and outputs of this system were obtained by simulation of its THERMOSYS model. The nominal operating condition
Table 4.3: Linearization techniques employed for example heating system components

<table>
<thead>
<tr>
<th>Component</th>
<th>Linearization method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pump</td>
<td>Simulations</td>
</tr>
<tr>
<td>Piping</td>
<td>Analytical derivation</td>
</tr>
<tr>
<td>Flow junctions</td>
<td>Analytical derivation</td>
</tr>
<tr>
<td>Boiler</td>
<td>System ID in THERMOSYS</td>
</tr>
<tr>
<td>LAHXs</td>
<td>Analytical derivation + simulations</td>
</tr>
</tbody>
</table>

Table 4.4: Pipe coefficients for heating system (read along rows)

<table>
<thead>
<tr>
<th>l</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1^l$</td>
<td>-1000</td>
<td>-28.62</td>
<td>-22.73</td>
<td>-22.74</td>
<td>-16.55</td>
</tr>
<tr>
<td>$a_2^l$</td>
<td>48.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_3^l$</td>
<td>33.9</td>
<td>0.083</td>
<td>0.083</td>
<td>0.083</td>
<td>0.083</td>
</tr>
<tr>
<td>$a_4^l$</td>
<td>0</td>
<td>0.135</td>
<td>0.114</td>
<td>0.113</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.5: Mass flow vertex coefficients for heating system (read along rows)

<table>
<thead>
<tr>
<th>j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1^j$</td>
<td>$9.9 \times 10^7$</td>
<td>$9.9 \times 10^7$</td>
<td>$9.9 \times 10^7$</td>
<td>$9.9 \times 10^7$</td>
</tr>
<tr>
<td>$d^j$</td>
<td>58.12</td>
<td>58.14</td>
<td>62.96</td>
<td>(58.1, 58.1)</td>
</tr>
<tr>
<td>$e^j$</td>
<td>58.14</td>
<td>58.08</td>
<td>(62.9, 62.9)</td>
<td>58.12</td>
</tr>
<tr>
<td>$f^j$</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>(0.42, 0.42)</td>
</tr>
<tr>
<td>$g^j$</td>
<td>0.83</td>
<td>0.83</td>
<td>(0.42, 0.42)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 4.6: Energy flow vertex coefficients for heating system (read along rows)

<table>
<thead>
<tr>
<th>i</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1^i$</td>
<td>10.43</td>
<td>1.36</td>
<td>1.36</td>
</tr>
<tr>
<td>$q_2^i$</td>
<td>-15.7</td>
<td>9.22</td>
<td>9.22</td>
</tr>
<tr>
<td>$q_3^i$</td>
<td>-164.5</td>
<td>-8.59</td>
<td>-8.59</td>
</tr>
<tr>
<td>$q_4^i$</td>
<td>154.07</td>
<td>7.24</td>
<td>7.24</td>
</tr>
<tr>
<td>$r_1^i$</td>
<td>-5.71</td>
<td>1.76</td>
<td>1.76</td>
</tr>
<tr>
<td>$r_2^i$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$r_3^i$</td>
<td>0</td>
<td>-1.91</td>
<td>-1.91</td>
</tr>
<tr>
<td>$r_4^i$</td>
<td>0</td>
<td>0.036</td>
<td>0.036</td>
</tr>
<tr>
<td>$r_5^i$</td>
<td>19.43</td>
<td>1.88</td>
<td>1.88</td>
</tr>
<tr>
<td>$r_6^i$</td>
<td>-19.43</td>
<td>-1.92</td>
<td>-1.92</td>
</tr>
<tr>
<td>$r_7^i$</td>
<td>0.43</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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is thus given by the combination of the relevant operating inputs, states and outputs as was discussed in section 4.3.1. Thereafter the component dynamics were linearized by employing appropriate techniques listed in Table 4.3. The coefficients that appear in Equations 4.11 to 4.15 that were obtained in the process are shown in Tables 4.4, 4.5 and 4.6. Using these coefficient values, a few of the somewhat complicated matrices defined in section 4.3.3 have been presented below. Note that the remaining matrices are either trivial or similar to these matrices in construction and have not been presented.

\[ A_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0.135 & 0 & 0 \\ 0 & 0.114 & 0 \\ 0 & 0 & 0.113 \\ 0 & 0 & 0 \end{pmatrix} \]  

\[ R_3 = \begin{pmatrix} 0 & 0 \\ -1.92 & 0 \\ 0 & -1.92 \end{pmatrix} \]  

\[ R_7 = \begin{pmatrix} 0.43 \\ 0 \\ 0 \end{pmatrix} \]

\[ W_2 = \begin{pmatrix} -0.83 & 0 & 0 & 0 & 0 & 0 & 0 & 0.83 \\ 0.83 & -0.83 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.83 & -0.42 & 0 & -0.42 & 0 & 0 \\ 0 & 0 & 0 & 0.42 & 0 & 0.42 & -0.83 \end{pmatrix} \]

### 4.4 Full Order State Space Representation

The full order state space model consists of the following state, input and output variables (also consult Table 4.1). Note that these variables represent deviation of the corresponding physical quantities from their nominal operating values.

### 4.4.1 States

The state vector consists of the following variables in the order specified.

(i) Liquid mass flow rates corresponding to the pipe elements, \( \dot{m}_l \)
(ii) Liquid pressures in mass flow vertices, $p_j$

(iii) Liquid temperatures in mass flow vertices, $T_j$

(iv) Liquid temperatures in energy flow vertices, $T_{L,i}$

(v) Structure temperatures in energy flow vertices, $T_{L,i}$

The number of states equals $n_l + 2(n_j + n_i)$.

4.4.2 Inputs

The input vector consists of the following variables in the order specified:

(i) Control valve isentropic areas, $A_{I,v}$

(ii) Pump speeds, $\omega_p$

(iii) External heat transfer rates corresponding to thermal source elements, $\dot{Q}_{ext,c}$

(iv) Air mass flow rates corresponding to thermal sink elements, $\dot{m}_{a-in,h}$

(v) Inlet air temperatures corresponding to thermal sink elements, $T_{a-in,h}$

Note that in non-VAV systems, inputs (i), (ii) and (iii) are typically used to control the system, whereas, (iv) and (v) can be treated as disturbance variables which are not actively manipulated. In VAV systems, (iv) and (v) are also actively controlled, whereas (i), (ii) and (iii) may or may not be altered. The number of inputs equals $n_v + n_p + n_c + 2n_h = n_v + n_p + n_i + n_h$.

4.4.3 Outputs

The output vector consists of energy exchange rates with air (heating/cooling) achieved by the thermal sink elements, $\dot{Q}_{out,h}$.

An important distinction must be made in this context. The outputs here correspond to quantities which are of practical usefulness and do not match their usual definition from a controls perspective as measurable quantities. In fact, it is interesting to see that in this situation it is the states which are directly measurable. The number of outputs equals $n_h$. 

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4.4.4 Model structure

The state space representation relates the states, inputs and outputs via the connectivity and coefficient matrices that were defined in sections 4.2.3 and 4.3.3 respectively. Equations 4.17 and 4.18 represent these relationships.

\[
\begin{align*}
\dot{\mathbf{m}}_l &= \left( \begin{array}{cccc}
A_1 & -A_3 & 0 & 0 \\
B_1 & 0 & 0 & 0 \\
W_1 & 0 & W_2 B_f & W_2 B_t \\
Q_2 E & 0 & Q_1 E B_f & Q_1 E B_t + Q_3 & Q_4 \\
R_1 E & 0 & R_2 E B_f & R_2 E B_t + R_5 & R_6 \\
0 & 0 & 0 & 0 \\
0 & 0 & R_7 & R_3 & R_4
\end{array} \right) \mathbf{m}_l \\
\dot{T}_j &= \left( \begin{array}{cccc}
A_4 & A_2 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & R_7 & R_3 & R_4
\end{array} \right) \mathbf{m}_{a-in,h} \\
\end{align*}
\]

Two important observations follow from Equation 4.17. Firstly, the ‘hydraulic dynamics’, which corresponds to mass flow rates and pressures, is completely decoupled from the
‘thermal dynamics’ but not vice versa. This lack of two-way coupling between these domains exists because of the fluid incompressibility assumption. Secondly, the state space matrix is singular as each loop in the system architecture results in a redundant mass balance equation (Equation 4.12) which is linearly dependent on other such equations. The singularity can be removed at this stage by eliminating all such redundant equations from the representation. However, this task has been integrated with the algebraic steps for model reduction that follow in section 4.5.

### 4.4.5 Example

The full order state space model for the example system in Figure 4.3 has $5 + 2(4 + 3) = 19$ states, $3 + 1 + 1 + 2(2) = 9$ inputs, and, 2 outputs. This example is further used in the model reduction scheme presented below.

### 4.5 Model Reduction

#### 4.5.1 Purpose and philosophy

The full order state space representation developed in the previous section is complex in the sense that the dimension of the underlying state space is typically very large. A case in point is the example heating system of Figure 4.3 which has a very simple architecture with few components only, but its full order model has 19 states. Models of such high dimensions are not suitable for control design purposes, because of the associated computational complexity. In an online control scheme such as MPC which involves frequent computations, such as at the beginning of a specified finite time horizon, a large computational time can cause undesirable time-delays which can even render the closed loop system unstable. This motivates the development of a system representation which is simple enough for control design but still captures the important transient and steady state characteristics of the dynamics.

In hydronic systems, the thermal states are most significant, particularly the structure temperatures in the thermal components which directly affect the useful outputs of the
system, i.e. the heat transfer rates achieved by the thermal sink components. A time-scale analysis for the states of the example heating system (Figure 4.3), whose results have been presented in Table 4.7, clearly verifies the above intuition. It is observed that the ‘slowest’ states of the system are the energy flow vertex structure temperatures and thus have the most significant effect on the overall system dynamics. Such observation is true for a general thermo-fluid system because the structure heat capacities are usually much higher than the other intrinsic capacities in these systems. This fact allows the reduction of the full order state space representation to a more concise description by treating the faster modes as static. The model reduction procedure presented in the remained of this section is based on this premise.

4.5.2 Algorithm

Based on the arguments presented above, a time-scale decomposition of the full order model presented in section 4.4.4. was performed, where all states with the exception of the energy flow vertex structure temperatures, $T_{w,i}$ were treated as quasi-steady. This results in the reduced order state space representation described by Equations 4.19 and 4.20.
\[
\frac{d}{dt} (T_{w,i}) = A_{ro} (T_{w,i}) + B_{ro} \begin{pmatrix} A_{I,v} \\ \omega_p \\ \dot{Q}_{ext,c} \\ \dot{m}_{a-in,h} \\ T_{a-in,h} \end{pmatrix}
\] (4.19)

\[
(\dot{Q}_{out,h}) = C_{ro} (T_{w,i}) + D_{ro} \begin{pmatrix} A_{I,v} \\ \omega_p \\ \dot{Q}_{ext,c} \\ \dot{m}_{a-in,h} \\ T_{a-in,h} \end{pmatrix}
\] (4.20)

The state space matrices, \(A_{ro}, B_{ro}, C_{ro}\) and \(D_{ro}\) in the above equations are obtained using the steps described below. For the derivation, the reader is directed to appendix A.2.

**Step 1**: Obtain \(Z_1\) and \(Z_2\)

\[
Z_1 = [Q_1 E - Q_1 E B_f (W_2 B_f)^{-1} W_2] B_t + Q_3
\] (4.21)

\[
Z_2 = [Q_2 E - Q_1 E B_f (W_2 B_f)^{-1} W_1] C
\] (4.22)

**Step 2**: Obtain \(Y_1\) and \(Y_2\)

\[
Y_1 = (W_2 B_f)^{-1} [W_2 B_t Z_1^{-1} Z_2 - W_1 C]
\] (4.23)

\[
Y_2 = (W_2 B_f)^{-1} W_2 B_t Z_1 Q_4
\] (4.24)

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**Step 3:** Obtain $Z_3$ as follows:

- Create $A_{fd}$ from \[
\begin{pmatrix}
A_1 & -A_3 A^T \\
B_1 A & [0]_{(n_j \times n_j)}
\end{pmatrix}
\]
by deleting the last row and column.

- Create $B_{fd}$ from \[
\begin{pmatrix}
A_4 & A_2 \\
[0]_{(n_j \times n_v)} & [0]_{(n_j \times n_p)}
\end{pmatrix}
\]
by deleting the last row.

- $P = \left( I_{(n_l \times n_l)} \ [0]_{(n_l \times (n_j-1))} \right)$

- $Z_3 = -PA_{fd}^{-1}B_{fd}$

**Step 4:** Obtain $A_{ro}$ and $B_{ro}$

\[
A_{ro} = R_6 + R_2 E B_f Y_2 - (R_2 E B_t + R_5) Z_1^{-1} Q_4
\]

(4.25)

\[
B_{ro} = \left( \{ R_1 E C + R_2 E B_f Y_1 - (R_2 E B_t + R_5) Z_1^{-1} Z_2 \} Z_3 \ R_7 \ R_3 \ R_4 \right)
\]

(4.26)

**Step 5:** Obtain $C_{ro}$ and $D_{ro}$

\[
C_{ro} = S_3
\]

(4.27)

\[
D_{ro} = \left( [0]_{(n_h \times n_v)} \ [0]_{(n_h \times n_p)} \ [0]_{(n_h \times n_c)} \ S_1 \ S_2 \right)
\]

(4.28)

Note that deletion of the rows and columns in step 3 above ensured that the state space matrix $A_{ro}$ in the final reduced order representation is non-singular. The issue of singularity of the state space matrix in the full order representation was discussed in section 4.4.4.
4.5.3 Implementation and example

An interactive MATLAB program was developed to formalize the process of obtaining the reduced order state space representation for any general hydronic system. This program sequentially implements all the steps involved in the modeling that were described in this chapter, viz., connectivity quantification, linear component modeling, full order state space representation and model reduction. In the future, this code shall be incorporated as a part of THERMOSYS. The code is presented in Appendix A.4.

The final, reduced order model representing the state evolution for the heating system of Figure 4.3 is presented as an example below (Equation 4.29). Comparing with the full order model in section 4.4.5, we observe a significant reduction (19 to 3) in the state space dimension.

\[
\frac{dx}{dt} = A_{ro}x + B_{ro}u 
\]

(4.29)

Where,

\[
x = \begin{pmatrix} T_{w,5} & T_{w,6} & T_{w,7} \end{pmatrix}^T,
\]

\[
u = \begin{pmatrix} A_{I,1} & A_{I,2} & A_{I,3} & \omega_1 & \dot{Q}_{ext,5} & \dot{m}_{a-in,6} & \dot{m}_{a-in,7} & T_{a-in,6} & T_{a-in,7} \end{pmatrix}^T,
\]

\[
A_{ro} = \begin{pmatrix} -1.05 & 0.52 & 0.52 \\ 0.28 & -0.33 & 0.008 \\ 0.28 & 0.008 & -0.33 \end{pmatrix},
\]

\[
B_{ro} = \begin{pmatrix} -0.016 & -0.007 & -0.007 & -0.014 & 0.43 & 0 & 0 & 0 & 0 \\ 0.004 & 0.011 & -0.008 & 0.004 & 0 & -1.91 & 0 & 0.036 & 0 \\ 0.004 & -0.008 & 0.011 & 0.004 & 0 & 0 & -1.91 & 0 & 0.036 \end{pmatrix}.
\]
Figure 4.5: Schematic of the test chilled water system used for model validation

4.6 Performance Evaluation of Model

4.6.1 Test System

In this chapter, the performance of the reduced order model is analyzed using a test system. The hydronic cooling system introduced in section 2.4 (Figure 2.39) is used for this purpose. The schematic is reproduced here in Figure 4.5 and its graph representation is shown in Figure 4.6. The modeling procedure described in the above sections and summarized in Figure 4.2 was applied to generate its reduced order space space model about the chosen nominal operating condition (see Appendix A.1).
4.6.2 Test Cases

The performance of the reduced order model for the test system was evaluated against its nonlinear THERMOSYS model via simulation experiments conducted for three test cases. In each of these experiments, nominal inputs are applied until the first 5000 seconds of simulation, and at that point one or more inputs are perturbed by small amounts. Thereafter, the linear response using the reduced order model and the nonlinear response using THERMOSYS are obtained and plotted for comparison. Details of these test cases are as follows:

**Test case 1**: Effect of disturbance

The temperature of the inlet air to LAHX 1 was reduced from its nominal value of 35°C to 30°C. The effect on the heat transfer achieved through LAHX 4 is studied.

**Test case 2**: Effect of perturbations to hydraulic control inputs
The valves feeding the LAHXs 1, 2 and 3 were simultaneously opened more by 5%, 10% and 15% respectively about their nominal opening factors (isentropic areas). The effect on the heat transfer achieved through LAHX 5 is studied.

**Test case 3**: Effect of perturbations to thermal control inputs and disturbances

The inlet air mass flow rate to LAHX 6 was increased by 20% about nominal and simultaneously, the external heat transfer rate (operating capacity) of chiller 1 was increased by 10% above nominal. The effect on the heat transfer achieved through LAHX 2 is studied.

4.6.3 Results

The simulation results corresponding to the afore-mentioned test cases have been presented in Figures 4.7, 4.8 and 4.9. Analysis of these results is presented below.

4.6.4 Analysis

Following observations were made from the study performed above:

1. The reduced order model resulted in a significant reduction of the state-space dimension from 55 to 8.

2. The differences between the linear and nonlinear responses at steady state were within 20% in all three case studies. This is true for the input perturbations considered here which were all within 20%.

3. Similarly, the transient characteristics of the linear and nonlinear responses, characterized by their time constant measurements were within 10% of each other.

It can be concluded, therefore, that the proposed reduced order state space model is both simple and accurate enough to be used for control design purposes.
Figure 4.7: Test case 1 simulation results

Figure 4.8: Test case 2 simulation results

Figure 4.9: Test case 3 simulation results
4.6.5 Suitability for control design

The reduced order model presented in this chapter provides some significant advantages when used for designing controllers to meet the heating and cooling demands of building zones. Firstly, this model directly predicts the effects of variations in control inputs such as valve opening factors on the energy exchange achieved through the LAHXs and therefore eliminates the need to consider any hydraulic analysis. This reduces the complexity involved in designing the controllers. Secondly, the reduced order model allows the use of full state feedback as the states correspond to heat exchanger structure temperatures which are easily measurable quantities.

Energy efficient operation of the system can be achieved through controllers designed using the principle of optimal control. An important consideration while designing controllers for such complex systems is the possibility of using simpler control architectures such as decentralized, block-decentralized or hierarchical. For this, an analysis of the (reduced-order) model can be performed for the identification of the dominant information structures in the system.

4.7 Concluding Remarks

In this chapter the details of a piecewise, linear, state-space modeling framework for hydronic systems was presented. The proposed approach yields simple, low order models which are particularly suitable for control design. It was verified through simulations that the transient and the steady state performance of the reduced order model in its associated operating regime is similar to that of the nonlinear model which was used for its development. Furthermore, the use of graph theory in the modeling approach renders it intrinsically generic and modular. The modeling procedure was easily formalized in the form of an interactive program for implementation in MATLAB.
Chapter 5

Design of Traditional Control Schemes

This is the first chapter in this thesis that is dedicated to the control of hydronic building HVAC systems. The underlying key control objectives are introduced in section 5.1. Section 5.2 discusses the key elements in the traditional control of such systems, followed by sections 5.3 and 5.4 which describe the most basic control methodologies that are typically employed. The limitations of these schemes are explained in section 5.5 and concluding remarks are made in section 5.6.

5.1 Control Objectives and Requirements

The control system is an indispensable part of any hydronic system. The most important control objectives for a building HVAC system are as follows:

1. **Thermal demands**: The zonal cooling loads in a building change during the course of the day, mainly depending on the occupancy and ambient conditions. This leads to variations in the energy demand that is required from each of the sink elements. The heat exchangers are subjected to these demand variations either implicitly such as in conventional systems where the building occupants manually adjust the temperature setpoints depending on their comfort requirements or more explicitly in ‘smarter’ systems where the demand setpoint is dictated by supervisory controllers mostly on a periodic (such as hourly) basis. Therefore, the primary goal associated with the control of the hydronic system is that the heat exchangers must satisfactorily achieve their implicitly or explicitly prescribed energy demands at all instants of time.
2. **Energy efficiency**: Section 1.1 explained the importance of energy efficiency in buildings and how HVAC systems are particularly significant in that context. The goal of more energy efficient HVAC systems is multifaceted in the sense that it mandates improvised design, rigorous commissioning and better controls. The role played by control is particularly important since it has the potential to affect the operational efficiency of the overall system. Therefore, it is strongly desirable that the control system employed seeks to satisfy the varying thermal demands in an energy efficient manner.

3. **Complexity**: As discussed in chapter 2, building hydronic systems can be arbitrarily complex and therefore the associated control tends to be complex as well. This is particularly true in the context of central control schemes, which are mostly designed with the intent of energy efficiency. Complex control is undesirable because it causes difficulty in fault detection and diagnostic maintenance and, may require a high amount of hardware and computational resources, therefore increasing the capital cost. Thus, the control must preferably be easily comprehensible and simple enough to implement.

4. **Component reliability**: Together with the afore-mentioned objectives, enhanced lifespan of the HVAC system is also an important consideration so as to maximize the return on the initial capital investment. This mandates that the employed control should result in minimal fatigue loading of the mechanical components while in operation.

Traditionally, building HVAC control systems were designed with the sole objective of meeting the thermal demands, with little or no consideration to energy efficiency and component reliability. This was motivated by the fact that simple on-off or Proportional-Integral-Derivative (PID) controllers could be used. Such controllers are readily available, simple, easily implementable and also do not require intricate models precise and physical understanding of the system dynamics. However, more advanced control schemes are necessary to accommodate the multi-objective control requirements and the underlying challenges.
5.2 Traditional Control Architectures

The key elements pertaining to the traditional control of building hydronic HVAC systems are as follows ¹:

5.2.1 Capacity control

An illustration of the traditional boiler and chiller control logic is shown in Figure 5.1. The control objective is to supply water at a prescribed set-point temperature. This is achieved by modulating the operating capacity based on feedback from a supply temperature sensors.

Most hydronic heating systems employ gas fired boilers. Depending on the nature of control (on-off or PID), the fuel supply to the burners is either turned on or off or continuously varied to achieve the desired modulation of heating capacity.

Cooling systems often use electric chillers working on the vapor compression cycle (VCC). The modulation of cooling capacity in these systems is generally achieved by turning the compressor on or off (on-off control) in fixed speed systems or variation of the compressor speed (PID control) in systems with a variable speed drive.

5.2.2 Supply temperature reset

In most hydronic systems, the common operating practice was to set the boilers and chillers to supply water at a fixed temperature, typically between 4 to 13°C for cooling systems, and around 120°C (under pressurized conditions) for heating systems. In this scheme, during off-peak operation, the supply water would be diluted with a bypass stream of the return

¹Note that the condenser water and air-side control is not covered in this section as it does not fall under the purview of hydronic system control
water to meet the varying demand. As is evident, this strategy is highly inefficient from an energy standpoint and therefore a new principle called supply temperature reset has been advocated in recent years [163, 164]. In this scheme, the supply temperature set-point is adjusted periodically during the course of the day based on the variable net energy requirements of the building. In this work, a chilled water temperature reset strategy has been assumed for the evaluation of on-off and PID control on the test system of Figure 2.39.

5.2.3 Valve control

Figure 5.2 shows an illustration of the traditional flow control logic employed in hydronic systems. In cooling systems, the chilled water flow rate through the cooling coils in each Air Handling Unit (AHU) is modulated to achieve a desired supply air temperature. The control system is often built into the AHU. In hydronic heating systems which mostly employ radiator panels (see section 2.2.2.2), the hot water flow rate through the panel is controlled to meet the desired room temperature set using the thermostat. In both heating and cooling systems, the flow control is accomplished by turning the valve completely on or off (on-off control) or via continuous variation (PID control) in Electronic Expansion Valves (EEVs).

5.3 On-off Control Scheme

A localized on-off scheme is one where the control of each source (capacity control) and sink element (valve control) is achieved by a local on-off controller. In this scheme, as explained in Figure 5.3, if the error with respect to the setpoint of the quantity that is being regulated (e.g. supply water temperature or room temperature), exceeds a specified threshold, the corresponding actuator is set to the ‘ON’ state (e.g. the compressor in the chiller or the...
The main advantage of the on-off scheme is that it obviates the need for a model and therefore is simple to implement. This makes it popular for use in many HVAC applications.

### 5.4 Decentralized PID control scheme

The decentralized Proportional Integral (PID) control architecture is similar to on-off, in the sense that it involves local control of the source and sink elements, but differs in the fact that the control is continuous. The error with respect to the setpoint of the quantity that is being regulated is used to generate a feedback signal as shown in Figure 5.4. The integral
action ensures that regulation is precise, without any steady state error. The derivative action, on the other hand facilitates smoother input signals, but is sometimes not used at all. The proportional, integral and derivative gains are usually tuned online using appropriate tuning rules until the closed loop dynamics is satisfactory in terms of transient characteristics such as rise time, overshoot, settling time etc. Furthermore, due to safety and stability considerations, saturation limits are usually imposed on the control inputs and controlled outputs.

5.5 Limitations of traditional schemes

As mentioned before, the traditional control schemes, i.e. on-off and PID are easily implementable. However, there are certain inherent limitations associated with them which are described below. These will also be revisited in the analysis presented in chapter 7 where the performance of these schemes will be evaluated against the relatively more advanced optimal control algorithms.

1. Non-optimality: Since the traditional schemes target thermal comfort alone, the potential for energy efficient performance is overlooked. This limitation is critical in the context of large scale building or district systems where energy consumption has become a primary concern.

2. Control interference: Local control of source and sink elements is subject to mutual-interference which can affect the efficacy of the control. An example of such an interference is a situation where the decision of turning a valve on or off solely on the basis of the temperature inside one room has an effect on the other rooms that has to be compensated by the corresponding controllers. In particular, such interference can lead to undesirable levels of fatigue loading of the actuators which is detrimental to the life cycle of the system.
3. Instability: Since local control guarantees only the stability of the corresponding subsystem, there is a possibility of the overall system being unstable. Therefore, a significant amount of trial runs and tuning may be necessary to establish the control parameters (such as feedback gains) under which the system is ‘well-behaved’.

In summary, it follows that the traditional control schemes are not well suited to cater to all the objectives that were discussed in section 5.1.

5.6 Concluding Remarks

In this chapter, the important control objectives with respect to hydronic building heating and cooling systems were outlined. The elements of traditional control schemes were presented. The important limitations of these schemes were highlighted and this motivates the contents of the next chapter which describes predictive control strategies for such systems with the aim of addressing these limitations.
Chapter 6

Predictive Control Schemes

6.1 Motivation

The important control objectives for hydronic building HVAC systems were outlined in chapter 5. The limitations of traditional P/PI and On-off schemes in satisfactorily achieving these objectives was also discussed. The objective of this chapter to present more ‘advanced’, alternative control schemes to overcome the limitations of such traditional schemes. These schemes employ an optimal control framework with the predictive control methodology used for optimization.

Model Predictive Control (MPC) schemes are becoming increasingly popular for a wide variety of processes, which can be attributed to their ability to handle constrained multi-variable problems and the fact that they are intuitively tunable. A building HVAC system is a particularly suitable candidate for the application of predictive control methodologies because of multiple control objectives, inherent complexity due to coupled and multivariable nature of the problem, and presence of constraints.

There has been significant interest lately in using MPC for various aspects of building HVAC control (see the literature survey in section 1.3.3.4). Most research efforts have focused on optimal operating strategies in the context of VAV systems (optimal air flow rate and air temperature set-points) [137, 139], thermal-storage (optimal charging and discharging schemes) [140], and load-side analysis (optimal zone temperature and ventilation set-points) [140, 143]. The common underlying theme in these efforts is the supervisory control of the HVAC system or its constituent subsystems. Control of the chilled/hot water flow and temperature in the hydronic loop to achieve the setpoints dictated by supervisory controller(s)
is still assumed to be conventional (local on-off or P/PI schemes). In this work, we extend the MPC framework to this ‘inner loop’ control problem, which pertains to control of flow rate in the hydronic loop subsystem together with chiller/boiler control, in order to meet the various control objectives. Two versions of model predictive scheme, viz. (i) centralized and, (ii) distributed have been proposed in this work, whose details have been provided in the rest of this chapter. However, we shall discuss some preliminaries in sections 6.2 to 6.5 before presenting these details.

6.2 Model Predictive Control

6.2.1 Overview

Model Predictive Control (MPC) is a form of control in which control action at the current time is obtained by solving a finite time horizon, open-loop, optimal control problem, using the current state of the plant as the initial state. The optimization yields an optimal sequence of inputs and first element in the sequence is applied to the plant while the rest are discarded. This procedure is repeated for each time instance. A historical and industrial perspective on MPC was provided in section 1.3.2.3 where its development and applications were outlined. An important advantage of this control methodology, which renders it practically very useful, is its ability to explicitly take into account hard constraints on controls and states. Therefore, MPC has been widely applied in petro-chemical and related industries where satisfaction of constraints is very important since the most efficient operating points typically lie within or close to the intersection of such constraints.

6.2.2 Mathematical Framework

In this work, MPC has been presented in a discrete framework which is the usual way of implementing it. Let the finite-time horizon consist of $N$ time samples. We denote the current sample by $k$, and the future values of the input lying in the time horizon beginning at this current time by $u(k + i|k)$ where $i = 0, 1, \ldots N - 1$. Similarly, the future values of
the state in the time horizon are denoted by \( x(k + i|k) \) where \( i = 1, 2, \ldots N \). The \( |k| \) that appearing in these notations is used to indicate that the future values are predicted from the knowledge of the state at the current time \( k \). The objective function to be minimized at the current time is a function of the future input and state values in the time horizon and is denoted by \( J_N(u(k + i|k), x(k + i + 1|k)) \), where \( i = 0, 1, \ldots N - 1 \).

Using the above defined notations, the optimization problem is as follows:

\[
    u_i^* = \arg \min_{u_i} J_N(u(k + i|k), x(k + 1|k)) \quad i = 0, 1, \ldots, N - 1. \tag{6.1}
\]

A plant model \( g \) is used to predict\(^1\) the future states \( x(k + i + 1|k) \) using the initial state \( x(k) \) and the future inputs \( u(k + i|k) \) as shown below.

\[
    x(k + i + 1|k) = g(x(k + i|k), u(k + i|k)) \quad \text{where} \quad i = 0, 1, \ldots, N - 1. \tag{6.2}
\]

Using the above relationship, the objective function in Equation (6.1) can be re-expressed as a function of the future inputs \( u(k + i|k) \) and the current state \( x(k) \) only. Note that the quantity \( x(k|k) \) and \( u(k|k) \) that appears in Equation (6.2) refer to the current state \( x(k) \) and the current input \( u(k) \) respectively.

The state and input constraints are usually expressed as box and slew rate constraints of the form represented by the following equations:

\[
    u_L \leq u(k + i|k) \leq u_H \quad \text{where} \quad i = 0, 1, \ldots, N - 1. \tag{6.3}
\]

\[
    x_L \leq x(k + i + 1|k) \leq x_H \quad \text{where} \quad i = 0, 1, \ldots, N - 1. \tag{6.4}
\]

\[
    |u(k + i + 1|k) - u(k + i|k)| \leq \Delta u_{max} \quad \text{where} \quad i = 0, 1, \ldots, N - 2. \tag{6.5}
\]

\[
    |x(k + i + 1|k) - x(k + i|k)| \leq \Delta x_{max} \quad \text{where} \quad i = 1, 1, \ldots, N - 1. \tag{6.6}
\]

In Equations (6.3) to (6.6), \( u_L \) and \( u_H \) are respectively, the lower and upper bounds on

---

\(^1\)Hence the terminology model predictive control
the input. Similarly, $x_L$ and $x_H$ are the lower and upper bounds on the state. The rate limits on the input and the state are denoted by $\Delta u_{max}$ and $\Delta x_{max}$ respectively. It must be noted that though the framework presented in this section corresponds to a system with a single input and single state, the extension to higher dimensions is straightforward. A multivariable formulation for the hydronic HVAC system shall be described in sections 6.6 and 6.7.

6.3 Distributed Control

6.3.1 Overview

Distributed control is a control philosophy based on the concept of distributed intelligence. In this scheme, the controller elements are not central in location but are distributed throughout the system with each component (sub-system) controlled by one or more controllers. The entire system of controllers is connected by networks for communication and monitoring\(^2\). In some systems - particularly those manifesting a natural hierarchy in their dynamics or structure - a central regulator may be used to dictate appropriate coordination among the various distributed controllers. This type of distributed control architecture is referred to as hierarchical control. See Figure 6.1 for illustrations of distributed (hierarchical and non-hierarchical), centralized and decentralized control architectures.

Distributed control is advantageous over centralized and decentralized control in several aspects. Some of these are listed below:

1. Most large scale systems consist of multiple interacting components (sub-systems). Use of distributed control provides flexibility in the sense that components can be easily removed or added without significantly affecting the control hardware and algorithm.

2. Distributed control is potentially less computationally complex than centralized control because in the former, the problem of computing the control signal for the overall system

\(^2\)Compare with decentralized control where each of the local controllers are independent
is divided into simpler, local-level problems. This translates to less processing power and memory requirements and therefore less costs.

3. Distributed control is more robust to hardware failures such as sensor/actuator faults than centralized control. The performance of the overall system can be severely affected in the event of faults in the central controller in the latter architecture. This claim will be examined in the future using a suitable case study.

4. Controller tuning is easier in a distributed scheme because tuning needs to be done at the local (sub-system) level only.

5. Since distributed control allows communication among the individual controllers (with or without a central regulator), its performance is expected to be better than a decentralized scheme where the local controllers are completely isolated from each other. This was indeed found to be the case in the simulation case study results presented
in the next chapter where the performance of decentralized PI and distributed MPC schemes have been compared.

6.3.2 Applications

In recent years, the increased availability of cheaper microprocessors, sensors and digital communication hardware, together with theoretical advancements have rendered distributed control easily implementable. The most common examples in industry include manufacturing processes (continuous or batch oriented), oil-refining plants, power generation and distribution networks, chemical process plants and the pulp and paper industry. Building automation is another important and upcoming distributed control application. In this context, the present work deals with the application of distributed control to underlying hydronic loops in building HVAC systems (section 6.9).

6.4 Quadratic Programming

6.4.1 General QP Problem

In this section, a brief background on Quadratic Programming (QP) is discussed, which is important in the context of the predictive control schemes developed in the remainder of this chapter. QP is a special type of mathematical optimization problem, where a quadratic function of several variables is optimized (maximized or minimized) subject to linear constraints on these variables. The mathematical formulation of the problem is as follows:

\[
\begin{align*}
\text{minimize} \quad & \frac{1}{2} x^T P x + q^T x + r \\
\text{subject to} \quad & G x \leq h \\
& A x = b
\end{align*}
\]

Here, \( P \in \mathbb{S}^{n} \), \( G \in \mathbb{R}^{m \times n} \) and \( A \in \mathbb{R}^{p \times n} \). Therefore, there are \( m \) inequality and \( p \) equality
constraints, which are all affine and hence convex. If $P$ is zero, the above problem becomes a linear Program (LP). If $P$ is positive definite, the objective function of the QP becomes convex and in that case every minimum is a global minimum.

6.4.2 Solution methods

If only equality constraints exist, the QP can be solved by using appropriately modified versions of gradient based methods such as the conjugate gradient method or the Newton’s method. In case of inequality constraints with convex objective function, interior point methods can be used. If the objective function is not convex, the active-set method is generally employed. The reader is directed to [165] for details on the conjugate gradient, Newton’s and interior point methods, and to [166] for a discussion of the active-set method.

6.5 Model for Predictive Control Design

The predictive controllers presented in this chapter are based on the modeling framework introduced in chapter 4. A linear, reduced order model for hydronic systems was obtained in Equations 4.19 and 4.20 in section 4.5. In section 4.4.2, it was remarked that among all the inputs appearing in the model, the valve opening factors, pump speeds and external heat transfer rates corresponding to the thermal source elements are the usual control variables in VAV systems. Here, we assume that the pumps are operating at fixed speeds unless there is a drastic change in the building thermal load. Therefore, within an operating regime (see Fig. 4.1), the only manipulated variables are the external heat transfer rates for the thermal source elements and the valve opening factors. The former are used to regulate the amount of thermal energy entering or leaving the system in order to meet the net thermal demand at any instant. The latter are manipulated to regulate the distribution of this energy to meet the thermal demands of individual zones in the building. In the light of these assumptions, the resulting model for control design is as shown in Equation 6.7
\[
\dot{x}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t)
\] (6.7)

Here, \(A = A_{ro}\) and \(C = C_{ro}\), whereas, \(B\) is given by the following Equation (refer to the nomenclature in section 4.2.2).

\[
B = B_{ro} \left[ (e_{(n_v+n_p+1)} e_{(n_v+n_p+2)} \cdots e_{(n_v+n_p+n_c)}) (e_1 e_2 \cdots \cdots e_{n_v}) \right]
\] (6.8)

In the above Equation, \(e_i\) is the \(i^{th}\) unit vector of dimension \((n_v + n_p + n_c + 2n_h)\), where \(i = 1, 2, \ldots (n_v + n_p + n_c)\). This is because the control inputs (Equation 6.7) consist of selected inputs (\(\dot{Q}_{ext,c}\) and \(A_{I,v}\) in that order) from the original input vector (Equation 4.19). The states and outputs in the model are the same as in Equations 4.19 and 4.20, viz. the thermal element structure temperatures and energy transfer (with air) achieved by the sink elements respectively.

In particular, a discrete version of the model, with sample time \(T_s\), as shown in Equation 6.9 will be used:

\[
x(k + 1) = Ax(k) + Bu(k) \\
y(k) = Cx(k)
\] (6.9)

### 6.6 Cost Functional

#### 6.6.1 Basic form

As was described in section 6.2.2, the first step in the design of a predictive control algorithm is the specification of a suitable cost functional which is to be minimized. For hydronic systems, the various control objectives were discussed in section 5.1, in accordance with which, the following cost function is proposed:
\[ J_k = \sum_{i=0}^{N-1} \sum_{j=1}^{N_u} \alpha_j u_j(k + i|k) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{N_y} |y_j(k + i|k) - y_{j,ref}(k + i|k)|^2 + \psi \sum_{i=1}^{N-1} \sum_{j=1}^{N_u} [u_j(k + i|k) - u_j(k + i - 1|k)]^2 \]  

(6.10)

The following nomenclature is used in the above equation, based on the nomenclature in section 4.2.2:

- **N**: Number of time samples in the control horizon
- **N\_u**: Number of control inputs = \( n_c + n_v \)
- **N\_y**: Number of outputs = \( n_h \)
- **u\_j**: \( j^{th} \) control input, \( j = 1, 2, ..., N_u \)
- **y\_j**: \( j^{th} \) output, \( j = 1, 2, ..., N_y \)
- **y\_j,ref**: Reference signal for \( j^{th} \) output, \( j = 1, 2, ..., N_y \)
- **\( \alpha_j \)**: Weight corresponding to \( u_j \) in the energy term
- **\( \gamma \)**: Penalty associated with regulation error term
- **\( \psi \)**: Penalty associated with slew rate term

The thermal source elements and pumps are the primary energy consuming components of a hydronic system. Energy consumption by the source elements, i.e. chillers and boilers is assumed to be linearly dependent on their operating capacities. On the other hand, manipulation of the valves affects the pressure difference across the pumps and therefore the energy consumption by the pumps is dependent on the valve opening factors. In the operating regime under consideration (with reference to the piecewise linear modeling framework of Fig 4.1), this dependence is assumed to be linear. As a result of such assumptions, the first term in Equation 6.10, which seeks the minimization of energy consumption over the control horizon, is linear.

The second term in this Equation penalizes the 2-norm of the regulation errors corresponding to the system outputs over the control horizon and is included to satisfy the
thermal comfort requirements. The last term penalizes the 2-norm of the slew rates for the system inputs over the control horizon and therefore arrests abrupt changes in the actuation signals.

### 6.6.2 Augmentation

To remove any steady state regulation errors in the outputs, i.e. to ensure perfect demand matching in the building zones, the cost functional presented in Equation 6.10 was augmented to penalize the 2-norm of the integral of the regulation errors in the outputs. The integration is performed in a discrete manner as per the following recursive relation, where $z_j$ is the integral of the regulation error for output $y_j$.

$$z_j(k + i + 1|k) = z_j(k + i|k) + T_s (y_j(k + i|k) - y_{j,ref}(k + i)) \quad (6.11)$$

The augmented cost functional is as follows, $\beta$ being the penalty associated with the augmentation term introduced:

$$J_k = \sum_{i=0}^{N-1} \sum_{j=1}^{N_u} \alpha_j u_j(k + i|k) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{N_y} \left[ y_j(k + i|k) - y_{j,ref}(k + i|k) \right]^2$$

$$+ \psi \sum_{i=1}^{N-1} \sum_{j=1}^{N_u} \left[ u_j(k + i|k) - u_j(k + i - 1|k) \right]^2 + \beta \sum_{i=1}^{N} \sum_{j=1}^{N_y} \left[ z_j(k + i|k) \right]^2 \quad (6.12)$$

### 6.6.3 Choice of parameters

A few remarks on the various parameters corresponding to the cost functional presented above are made here. It is desired to keep the sampling time, $T_s$ used in the system model (Equation 6.9) small enough to ensure sufficient accuracy relative to the corresponding continuous model (Equation 6.7). At the same time, it is also desired to choose $N$, the size of the control horizon as small as possible to reduce the problem size and thus the computational complexity of the associated optimization. However the product $N \times T_s$, which is the actual
size of the control horizon in seconds must be large enough to sufficiently address the typical, long transient dynamics of the system. Thus, there exists a natural trade-off in the relative choices of $N$ and $T_s$.

The values for $\alpha_j$ are obtained by analyzing the dependence of the energy consumption on the control inputs in the operating regime under consideration, estimated using appropriate models or experiments. In general, the source elements have different efficiencies. Also, each of the control valves affect the pressure difference across the pumps by different amounts depending on their location, size and other factors. To account for these differences, the weights $\alpha_j$ ($j = 1, 2, ..., N_u$) are different from each other, as opposed to the other weighting parameters in the objective function. In this work, these other weights - $\beta, \gamma$ and $\psi$ are chosen intuitively at first and then refined based on the simulation results. For more insight on the choice of parameters, the reader is directed to Section 7.3.1 of the next chapter which presents an example.

## 6.7 Constraints

As was discussed in section 6.2.1, a significant advantage of MPC over other control methodologies is that it has the ability to satisfy explicitly prescribed hard constraints on the states and inputs. In the control framework presented in this work, only input constraints are prescribed. Hydronic systems are inherently stable in the input-state sense (BIBS stability) and therefore, constraints on states are not deemed necessary. Two kinds of constraints are imposed on the inputs - saturation and slew rate, details of which are presented below.

### 6.7.1 Saturation constraints

A piecewise linear modeling approach was discussed in chapter 4, where the entire operating regime of the system is decomposed into smaller regimes, each with a corresponding reduced order, linear state space model. The control approach based on that modeling framework is essentially switched, with one controller designed and tuned for each operating regime. A
transition between regimes would therefore mandate a switch from one to another controller. To ensure robust performance of each such controller, it is necessary to ensure that the input signals generated by the control algorithm lie in the corresponding regime. Hence, suitable saturation limits are required on the inputs. Saturation limits may also be dictated by the physical limitations on the hardware due to safety and performance considerations. The form of the saturation constraints is described by Equation 6.13.

\[
\begin{align*}
    u_{j,min} & \leq u_j(k + i|k) \leq u_{j,max}, \quad \text{where, } i = 0, 1, \ldots N - 1, \ j = 1, 2, \ldots, N_u
\end{align*}
\] (6.13)

Note that the upper and lower limits in the above equation depend on how the operating regime was partitioned in the modeling framework, and may not be equal in magnitude.

6.7.2 Slew rate constraints

In addition to constraining the magnitude of the control signals, it is also important to impose appropriate rate limits on them. Abrupt changes in the control can damage the mechanical components such as valves and compressors (in the chiller units) and must be avoided. The mathematical form of the saturation constraints is as follows:

\[
\begin{align*}
    \Delta u_{j,min} & \leq u_j(k + i + 1|k) - u_j(k + i|k) \leq \Delta u_{j,max} \\
    \quad \text{where, } i = 0, 1, \ldots N - 2, \ j = 1, 2, \ldots, N_u
\end{align*}
\] (6.14)

6.8 Centralized Predictive Scheme

The details of a centralized MPC scheme for the control of hydronic systems are presented in this section. The model, cost functional and constraints used in this formulation were described in sections 6.5 to 6.7 of this chapter.
6.8.1 Control Architecture

A switched control framework based on the piecewise linear modeling framework (see chapter 4) is assumed here, as was explained in section 6.7.1. Therefore, in this context, the objective is to design an MPC controller for each operating regime, for which the reduced order, discrete state space model of the form shown in Equation 6.9 is available using the modeling procedure explained in chapter 4, (summarized in Fig. 4.2). Fig. 6.2 illustrates the semantics of the control scheme for any such predictive controller. At each time instant $k$, the corresponding state information, $x(k)$ is fed to the controller, which performs an online, finite time horizon optimization (based on the logic described in section 6.2.1) to yield the optimal input signal, $u(k)$, which is then applied to the plant. The details of the optimization problem are described in the remainder of this section.
6.8.2 Transformation to a QP Problem

The discrete state space model of Equation 6.9 is used to predict the future values of the states and outputs based on the current state and future inputs as shown below:

\[
x(k + 1|k) = Ax(k) + Bu(k)
\]

\[
x(k + 2|k) = Ax(k + 1|k) + Bu(k + 1) = A(Ax(k) + Bu(k)) + Bu(k + 1)
\]

\[...
\]

\[
x(k + i|k) = A^i x(k) + \sum_{j=0}^{i-1} A^{i-1-j} Bu(k + j|k)
\]

\[
y(k + i|k) = Cx(k + i|k) = C \left[ A^i x(k) + \sum_{j=0}^{i-1} A^{i-1-j} Bu(k + j|k) \right]
\] (6.15)

As explained in section 6.2.1, the objective functional must be expressed in terms of the future inputs \(u(k + i|k)\) and the current state \(x(k)\) only. In order to do that, the above expression for \(y(k + i|k)\) is substituted directly in the third term of the objective functional (Equation 6.12), and indirectly in the second term through substitution in Equation 6.11. The resulting objective function, shown in Equation 6.16 is a quadratic function of the inputs.

\[
J_k = v_k^T H_k v_k + f_k^T v_k
\] (6.16)

The vectors and matrices appearing in the above equation are explained below:

**Lifted input vector**

\[
v_k = [\hat{u}_{1,k} \hat{u}_{2,k} \ldots \hat{u}_{N_u,k}]^T
\] (6.17)

Where, for each \(i = 1, 2, \ldots N_u\),

\[
\hat{u}_{i,k} := [u_i(k|k) u_i(k + 1|k) \ldots u_i(k + N - 1|k)]
\] (6.18)

**Hessian Matrix**
The Hessian matrix, \( H_k \) is generated as shown below:

For \( p, q = 1, 2, ..., N_u \) and \( r, t = 0, 1, ..., N - 1 \):

\[
H_k((p-1)N + r + 1, (q-1)N + t + 1) = \gamma \sum_{i=(\max(r,t)+1)}^{N} \sum_{j=1}^{N_y} (c_jA^{i-r-1}b_p)(c_jA^{i-t-1}b_q) + \beta T^2 \sum_{i=(\max(r,t)+1)}^{N} \sum_{j=1}^{N_y} \left[ c_j \left( \sum_{s=0}^{i-r-1} A^s \right) b_p \right] \left[ c_j \left( \sum_{s=0}^{i-t-1} A^s \right) b_q \right] + \theta(p, q, r, t) \tag{6.19}
\]

Here \( \theta(p, q, r, t) \) is defined as follows:

For diagonal terms:

\[
\theta(p, q, r, t) = \begin{cases} 
\psi^p & \text{if } r = 0 \text{ or } N - 1 \\
2\psi^p & \text{otherwise}
\end{cases} \tag{6.20}
\]

For off-diagonal terms:

\[
\theta(p, q, r, t) = \begin{cases} 
-\psi^p & \text{if } p = q \text{ and } |N - 1| = 1 \\
2\psi^p & \text{otherwise}
\end{cases} \tag{6.21}
\]

The vector \( c_j \) in Equation 6.19 is the \( j^{th} \) row of \( C \). Similarly, \( b_p \) and \( b_q \) are the \( p^{th} \) and \( q^{th} \) columns of \( B \) respectively.

**Vector \( f \)**

For \( p = 1, 2, ..., N_u \) and \( r = 0, 1, ..., N - 1 \):
\[ f_k((p - 1)N + r + 1) = \alpha_p + 2\gamma \sum_{i=(max(r,t)+1)}^{N} \sum_{j=1}^{N_u} \left( c_j A_{i-r-1} x(k) - y_{j,ref}(k+i) \left( c_j A_{i-r-1} b_p \right) \right) \]
\[ + 2\beta T_s \sum_{i=(max(r,t)+1)}^{N} \sum_{j=1}^{N_u} \left[ c_j \left( \sum_{s=0}^{i} A^s \right) x(k) s - iT_s y_{j,ref}(i) \right] \left[ c_j \left( \sum_{s=0}^{i-r-1} A^s \right) b_p \right] \]

(6.22)

The constraints described in section 6.7, can be re-written in the following form:

\[ G_k v_k \leq w_k \]  

(6.23)

Here, \( G_k \) is given by:

\[ G_k = \begin{bmatrix} G_1^T & G_2^T & G_3^T \end{bmatrix}^T \]  

(6.24)

The matrices \( G_1, G_2 \) and \( G_3 \) are defined in Equations 6.25 to 6.27:

\[ G_1 = \left[ I_{(N \times N_u)} - I_{(N \times N_u)} \right]^T \]  

(6.25)

\[ G_2 = \begin{pmatrix} E_N & \vdots & E_N \end{pmatrix} \text{ (with } N_u \text{ blocks)} \]  

(6.26)

\[ G_3 = -G_2 \]  

(6.27)

In the above Equations, \( I_{(N \times N_u)} \) is the Identity matrix of dimension \( N \times N_u \). On the other hand, \( E_N \) is the matrix defined as:

\[ E_N = \begin{bmatrix} g_1 & g_2 & \cdots & g_{(N-1)} \end{bmatrix}^T \]  

(6.28)

The vectors \( g_i \) appearing in Equation 6.28 are defined as follows:

\[ g_i = -\hat{e}_i + \hat{e}_{i+1} \]  

(6.29)
Here, $\hat{e}_i$ refers to the $i^{th}$ unit vector of dimension $N$.

The vector $w_k$ in Equation 6.23 is given by Equation 6.30 below:

$$w_k = [z_1 \ z_2 \ z_3 \ z_4]^T$$  \hspace{1cm} (6.30)

The vectors $z_1$, $z_2$, $z_3$ and $z_4$ appearing in this Equation are defined in Equations 6.31 to 6.34 as follows:

$$z_1 = [u_{1,\max} h \ u_{2,\max} h \ \ldots \ u_{N_u,\max} h]$$  \hspace{1cm} (6.31)

$$z_2 = -[u_{1,\min} h \ u_{2,\min} h \ \ldots \ u_{N_u,\min} h]$$  \hspace{1cm} (6.32)

$$z_3 = \begin{bmatrix} \Delta u_{1,\max} \hat{h} \\ \Delta u_{2,\max} \hat{h} \\ \ldots \\ \Delta u_{N_u,\max} \hat{h} \end{bmatrix}$$  \hspace{1cm} (6.33)

$$z_4 = -\begin{bmatrix} \Delta u_{1,\min} \hat{h} \\ \Delta u_{2,\min} \hat{h} \\ \ldots \\ \Delta u_{N_u,\min} \hat{h} \end{bmatrix}$$  \hspace{1cm} (6.34)

In the above Equations, $h$ is the $N$ dimensional row vector with all entries 1. Similarly, $\hat{h}$ is the $N-1$ dimensional row vector with all entries 1.

The QP formulation of the optimization problem, in light of the above discussion, is as follows:

$$v_k^* = \arg \min_{\{v_k \mid G_k v_k \leq w_k\}} (v_k^T H_k v_k + f_k^T v_k)$$  \hspace{1cm} (6.35)

### 6.8.3 Optimization

To solve the above optimization problem (Equation 6.35), if $H_k$ is positive definite, i.e. $H_k \in S^n_+$, interior point methods such as the barrier function method [165] are useful. If $H_k$ is indefinite, the active-set method - which is generally more computationally complex than the interior point method - can be availed for optimization. The matrices $H_k$ and $G_k$, and the vector $w_k$, as given by Equations 6.19, 6.24 and 6.30 are independent of the time instant $k$ and therefore can be evaluated offline. However, the vector $f_k$ (Equation 6.22) is dependent on $k$ through the state $x(k)$ and therefore must be evaluated online for optimization at each
time instant. The computational complexity of the optimization, using the interior point or active set method is monotonically dependent on the problem size, i.e. the dimension of the vector $v_k$ which is $N \times N_u$.

At each time instance, in accordance with the MPC philosophy, the optimal input $u(k)$ is extracted from the result of the optimization, as the first vector in the optimal sequence of vectors (given by $v_k^*$). The rest of the entries in $v_k^*$ are discarded. This extraction process is mathematically described by the following Equation:

$$u(k) = [v_k^*(1) \ v_k^*(N + 1) \ v_k^*(2N + 1) \ \ldots \ v_k^*((N_u - 1)N + 1)]^T$$

(6.36)

### 6.9 Decentralized Predictive Scheme

#### 6.9.1 Purpose

The advantages of a distributed control scheme over a centralized scheme were discussed in section 6.3.1. In the particular context of MPC, where the optimization is performed online at each time instant, the computational complexity associated with any centralized implementation is an important concern. This motivates the development of a more computationally efficient, distributed MPC design. A literature survey of the field of distributed control and in particular distributed MPC was presented in section 1.3.2. In this section, these ideas have been extended for the optimization problem presented in sections 6.6 to 6.8. The details of this proposed distributed MPC scheme have been presented in the rest of this section.

#### 6.9.2 Coupling architecture in hydronic systems

As was indicated in chapter 2, the focus of this work is on hydronic systems with a ‘parallel distribution’ architecture (see Fig. 2.16). An analysis of the state space matrix, $A_{ro}$ in the reduced order model developed for these systems (Equation 4.29) reveals useful information about the nature of interactions among the states (structure temperatures). Most impor-
tantly, it is observed that the interaction between the state of a given sink element (LAHX structure temperature) and the states of the other sink elements in the system are relatively ‘weak’ when compared to its interactions with the states of any of the source elements (boiler/chiller structure temperatures). This can be verified by observing the state space matrices $A_{ro}$ and $B_{ro}$ for the test systems introduced in section 2.4 (see section 4.5.3 and Appendix A.4). This behavior can be attributed to the parallel distribution architecture of these systems.

Based on the above observations, the coupling architecture of such systems can be described by the paradigm of a leader-follower (master-slave) dynamical network such as ant-colonies and bird-flocks [167]. Here, the role of the leader and followers are played by the source and sink elements respectively. The communication based distributed MPC scheme called ‘Cooperative Iteration’, introduced in [86] was modified to cater to this type of coupling framework. The resulting algorithm is presented below.

### 6.9.3 Communication based Distributed MPC algorithm for a leader follower network

#### 6.9.3.1 Notation and assumptions

It is assumed that the system consists of a single master agent and $n_s$ slave agents. This means that all the source elements are lumped together as a single master agent whereas the slave agents are in general $n_s$ appropriately chosen clusters of the $n_h$ sink elements ($n_s \leq n_h$). The subscripts $m$ and $i$ are used to denote the leader (master) agent and the $i^{th}$ follower (slave) agent respectively. In what follows, the subscript $k$ is dropped from the quantities $v_k$, $H_k$, and $f_k$ presented in section 6.8.2. The input vector corresponding to the master agent states, which consists of external heat transfer rates for the sink elements in the system is lifted in accordance with the structure presented in Equation 6.17. It is denoted by $v_m$. Similarly, the lifted vector corresponding to the inputs corresponding to the $i^{th}$ slave agent, which consists of the valve opening factors (isentropic areas) of the underlying sink elements
in that agent, is constructed and denoted by \( v_i \). Based on this partitioning of the vector \( v \), the matrices and vectors\(^3\) in the optimization problem (Equation 6.35) are partitioned as shown in the following modified representation of the objective function:

\[
H = \begin{pmatrix}
    v_m & v_1 & \cdots & v_n \\
    H_{m,m} & H_{m,1} & \cdots & H_{m,n_s} \\
    H_{1,1} & H_{1,1} & \cdots & H_{1,1} \\
    \vdots & \vdots & \ddots & \vdots \\
    H_{n_s,m} & H_{n_s,1} & \cdots & H_{n_s,n_s}
\end{pmatrix} \begin{pmatrix}
    v_m \\
    f_m
\end{pmatrix}
\]

The constraint matrix \( G \) and vector \( w \) in the optimization problem are easily partitioned in similar way. The constraint matrix and vector for the master agent, obtained through that particular process are denoted by \( G_m \) and \( v_m \) respectively. The corresponding quantities for the \( i^{th} \) slave agent are obtained similarly and denoted by \( G_i \) and \( v_i \).

6.9.3.2 Algorithm

Steps (At time step \( k \)):

1. Initialization: The optimal lifted vectors of inputs corresponding to the leader agent, \( v_m^* \) and all the follower agents, \( v_i^* \) are initialized to feasible values lying within the prescribed constraints.

2. Master optimization: The following local optimization problem is solved for currently known values of \( v_i^* \):

\[
v_m^* = \arg \min_{\{v_m \mid G_m v_m \leq w_m\}} \left[ v_m^T H_{m,m} v_m + \left( \sum_{i=1}^{n_s} v_i^* H_{m,i} + f_m^T \right) v_m \right]
\]  

(6.38)

3. Slave optimization: For each follower, \( i \), the following \( n_s \) local optimization problems are solved (in parallel, independent of each other) for currently known value of \( v_m^* \):

\[
v_i^* = \arg \min_{\{v_i \mid G_i v_i \leq w_i\}} \left[ v_i^T H_{i,i} v_i + \left( \sum_{i=1}^{n_s} v_m^* H_{m,i} + f_i^T \right) v_i \right]
\]  

(6.39)

\(^3\)Remember that we have dropped the subscript ‘\( k \)’ while referring to these quantities. Also note that \( H \) is symmetric, so that \( H_{m,1} = H_{1,m} \), etc.
4. Cooperative iteration: Steps 2 and 3 are repeated in sequence, until convergence⁴.

6.9.4 Implementation

An illustration of the distributed scheme presented above is shown in Figure 6.3. Each slave communicates with the master and the master communicates with all the slaves. The rationale behind this scheme is the intuition that due to relatively weak coupling between the states of the slave agents, the matrices $H_{i,j}$ ($i \neq j$) in Equation 6.37 are ‘small’ compared to $H_{i,i}$ in the sense of a suitable metric (matrix 2-norm). As an example, in the simulation case study presented in the next chapter, for the D-MPC$_2$ scheme, $\|H_{1,1}\| \approx 4.0 \times 10^4$ whereas $\|H_{1,2}\| \approx 27.0$. As a result, such matrices are removed in step 3 of the algorithm, therefore allowing parallel optimization of the underlying slave optimization problems, without the need for any communication between them. This ‘decentralization’ at the slave level results in significant complexity reduction when compared to centralized MPC.

We denote the number of sink elements in the $i^{th}$ slave agent by $N_i$ (Thus $\sum_i N_i = N_h$). The computational complexity of the overall scheme is dependent on the number of iterations $N_{iter}$, size of the master level optimization problem $N \times n_c$ (see nomenclature in section 4.2.2), and the size of the most complex slave level problem $N \times \max N_i$. Therefore, the use of a large number of iterations can jeopardize the computational advantage gained by parallel optimization in step 3. It is therefore, recommended to terminate the algorithm after

---

⁴A proof of convergence is not included in this thesis, but will be part of a future publication
few iterations, which renders the proposed distributed scheme sub-optimal in practice. The remarks made in section 6.8.3 on solution methodology for the larger optimization problem (Equation 6.35) apply to the optimization of the smaller optimization problems in steps 2 and 3 of the algorithm as well. The control input $u(k)$ at any time instant is extracted from the optimal solution $v^*$ by the same process that was described earlier in Equation 6.35.

Finally, it is important to note that the proposed distributed MPC scheme is particularly suitable for large scale hydronic systems on two accounts. Firstly, in such systems, due to the presence of a large number of slave agents, the coupling between them is expected to be sufficiently small, rendering the scheme close to optimal (relative to the centralized MPC scheme). Secondly, increase in number of slave agents results in an increase in the number of parallel slave-level optimization problems in step 3, which might render the distributed scheme more computationally efficient with respect to the corresponding centralized scheme. Further analysis is required to verify these claims which shall be undertaken in the future.

6.10 Concluding Remarks

In this chapter, centralized and distributed MPC control schemes were developed with the aim of satisfactorily meeting the control objectives for hydronic HVAC systems. The motivation, mathematical formulation and important attributes of these schemes were presented in detail. Together with chapter 5, where traditional control schemes were described, this chapter concludes the control design task. In the next chapter, each of these schemes shall be implemented on a simulated test system for relative evaluation of their performance with respect to the important control objectives.
Chapter 7

Control Evaluation

In this chapter, a simulation case study is presented which attempts to evaluate the control schemes presented in this thesis. The performance of both traditional (chapter 5) and predictive (chapter 6) schemes are tested in the context of the control objectives given in section 5.1. The important findings have been reported and general conclusions drawn from the same. If required, the Appendices provided at the end of this thesis contain the information necessary to reproduce the test case and the results.

7.1 Test Case

7.1.1 Test system

The system considered in this case study is the chilled water system that was introduced in section 2.4. and is reproduced in Figure 7.1. This system emulates the chilled water loop architecture of a 2 storeyed building having three clusters of zones in each story. On the thermal side, it has two thermal source elements (chillers) and six thermal sink elements (liquid -air heat exchangers). On the hydraulic side, it includes a pair of primary pumps, a pair of secondary pumps and eight control valves - one for each thermal component. In addition to the nine physical flow junctions, two ‘dummy’ junctions (junction 10 and 11) at the primary pump outlets are also included in the simulated system to satisfy the causality requirements discussed in chapter 3. In terms of the terminology introduced in chapter 2, this system belongs to the class of recirculating, forced-flow, primary-secondary systems (without a decoupler) with direct-return parallel piping. In chapter 4, this system was
modeled about a set of chosen operating conditions using the modeling procedure developed there. The ensuing reduced order model was tested for satisfactory performance in section 4.6. (number of inputs, states, outputs)

The study presented in this chapter is based on nonlinear simulations using THERMOSYS v 3.1 - the simulation test bed described in chapter 3.

### 7.1.2 Operating conditions

The chilled water system operating conditions chosen for the test case presented are the same as in Appendix A.1 that were used to obtain its linear model in section 4.6. In what follows, we shall refer to these conditions as *nominal operating conditions*. A subset of these conditions, consisting of some of the key operating conditions, have been shown in tables 7.1, to 7.3, i.e. chiller capacities, Liquid to Air Heat Exchanger (LAHX) operating loads and control valve opening factors. In accordance with the control framework presented in chapters 5 and 6, the control variables are the operating capacities of the chillers and the control valve opening factors. Of the eight control valves present in the system, only those that correspond to the liquid feed lines in the LAHXs are manipulated (valves 1-6 in Fig. 7.1. Therefore the system has eight control inputs. The number of states is eight (structure temperature of each thermal element) and the number of outputs is six (sink element heat transfer rates).

The nominal operating condition for the test system is one where both the chillers are in operation - one at close to full capacity and the other at partial capacity (see Table 7.1). In practice, this situation corresponds to peak or near-peak cooling loads, which generally occurs from 10 am to 6 pm during the course of a normal summer day. Therefore, with respect to the piecewise linear modeling framework shown in Figure 4.1, the nominal operating condition corresponds to the ‘peak load operating regime’. In this test case, we consider the operation of the system confined to this operating regime. Therefore, switching control considerations (in the context of the predictive schemes developed in chapter 6),
considerations are ignored for simplicity. \(^1\).

To estimate the energy consumed by the chillers, under the action of the various control schemes, their coefficient of performance (COP) data is required. For the test system under consideration, the chillers in this system are assumed to incorporate twin-screw compressors. The corresponding performance curves, at part load conditions, are assumed to be governed by Fig. 5(b) of [168]. As can be seen from this figure, at the specified ambient temperature of 35 deg C, the relationship between the COP and the part load ratio of these chillers is very close to linear. Using this observation, the COP of these chillers is modeled by the following linear function of their operating capacities.

\[
COP_i = a_i + b_i \dot{Q}_{ext,i} \quad i = 1, 2. \tag{7.1}
\]

Using the nominal operating conditions from Table 7.1, the values of the constants in the above equation for the two chillers are as follows:

\[
a_1 = 1.4 \quad ; \quad b_1 = 0.0148
\]
\[
a_2 = 1.4 \quad ; \quad b_2 = 0.0073 \tag{7.2}
\]

7.1.3 Cooling load profile

The general trend of cooling load variation during the course of a summer day is shown in Figure 7.2, inspired by Figure 2.14 in [169]. A discretized load profile for the peak load operating conditions (between 10 am to 6 pm), inspired by this trend, is shown in Figure 7.3 and is assumed to be uniformly applicable to all the six sink elements in the test system. This profile is normalized about the nominal cooling load of the sink element (Table 7.2), which corresponds to 100% load\(^2\). The discretization is performed on an hourly basis.

\(^1\)Such considerations shall be a part of future work
\(^2\)The fact that the nominal loads for the six heat exchangers are different from each other (Table 7.2) implies that the 100% load value in the load profiles for these exchangers are different. Therefore, the load profiles for these heat exchangers are similar in trend only, not in actual load values.
Figure 7.1: Test chilled water system used in the case study

Table 7.1: Nominal Chiller Operating Conditions

<table>
<thead>
<tr>
<th>Chiller Number</th>
<th>Operating Capacity (kW)</th>
<th>Part Load Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>107.9</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>87.7</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 7.2: Nominal Heat Exchanger Operating Conditions

<table>
<thead>
<tr>
<th>LAHX Number</th>
<th>Operating Capacity (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.2</td>
</tr>
<tr>
<td>2</td>
<td>24.38</td>
</tr>
<tr>
<td>3</td>
<td>37.47</td>
</tr>
<tr>
<td>4</td>
<td>33.49</td>
</tr>
<tr>
<td>5</td>
<td>26.79</td>
</tr>
<tr>
<td>6</td>
<td>37.96</td>
</tr>
</tbody>
</table>

Table 7.3: Nominal Control Valve Operating Conditions

<table>
<thead>
<tr>
<th>Valve Number</th>
<th>Opening factor (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>
This discrete profile could be interpreted as the result of some supervisory set-point optimization in a ‘smart’ building system or as a simplified version of the actual load profile, implicitly acting upon the heat exchangers through the varying set-point temperatures in the various rooms of the building. The thermal comfort objective for the chilled water system, therefore, is for all the LAHXs to achieve the demands prescribed by this normalized load profile during the peak-load operating regime (10 am to 6 pm).

7.1.4 Supply temperature reset

In the context of traditional control schemes (chapter 5), the importance of a load-modulated chilled water set-point strategy called supply temperature reset, from an efficiency consideration, was discussed in section 5.2.2. In the test case presented, the supply temperature
strategy shown in Figure 7.4 is used, which was decided based on the reference load profile of Figure 7.3. In actual practice, the supply temperature reset strategy is decided based on guidebooks available for a particular system. In this case, the chilled water set-points were decided based on the feasibility of meeting the demands during all time intervals, assessed using THERMOSYS simulations of the nonlinear model of the system. This procedure is legitimate since guidebook data is also obtained by simulations or actual tests on the system.

7.1.5 Objectives

The performance of the traditional (on-off and PI) and predictive (centralized and distributed) control schemes is evaluated in the light of the important control objectives discussed in section 5.1. They are recapitulated below and interpreted with respect to the present test case.

1. Thermal comfort: Determined by how well the demands prescribed the reference load profile are achieved by the LAHXs.

2. Power consumption: The sum of the average chiller and pump power consumption during the time window of interest (10 am to 6 pm).

3. Reliability: Gauged by the frequency content present in the control signals.
4. Computational complexity: Quantified by the average computation time for one run of the code for the control algorithm.

7.2 Parameters for traditional control design

The features of traditional control schemes were described in chapter 5 (esp. see Figures 5.1 and 5.2). In particular, two particular schemes - localized on-off and decentralized PID were discussed. In the test case under consideration, on-off and PI schemes have been considered. The chilled water set-point temperatures follow the supply temperature reset scheme shown in Figure 7.4. The sink element (LAHX) thermal demands are assumed to follow the load profile shown in Figure 7.3.

7.2.1 Localized On-off

With reference to the On-off control logic shown in Figure 5.3, the allowable output error window to decide if the corresponding control input should be on or off was set to ±0.1 units. Furthermore, appropriate rate limits were also applied on the actuators.

7.2.2 Decentralized PI

In accordance with the discussion in section 5.4, the controller gains were obtained by tuning on the simulation test-bed (THERMOSYS), until satisfactory response characteristics were achieved in terms of stability, overshoot and settling time:

- **Chiller control**: P-gain = 30; I-gain = 1
- **Valve control**: P-gain = $7.5 \times 10^2$; I-gain = 4

Analogous to MPC, saturation limits of ±25% were imposed on the actuation signals on account of the robustness considerations made in section 6.7.1.
Table 7.4: Power Consumption Coefficients for Control Inputs

<table>
<thead>
<tr>
<th></th>
<th>Explanation of $u_j$</th>
<th>$\lambda_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Valve 3 opening factor</td>
<td>1.69 x 10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>Valve 4 opening factor</td>
<td>2.36 x 10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>Valve 5 opening factor</td>
<td>1.07 x 10^{-3}</td>
</tr>
<tr>
<td>4</td>
<td>Valve 6 opening factor</td>
<td>2.29 x 10^{-4}</td>
</tr>
<tr>
<td>5</td>
<td>Valve 7 opening factor</td>
<td>5.76 x 10^{-4}</td>
</tr>
<tr>
<td>6</td>
<td>Valve 8 opening factor</td>
<td>1.80 x 10^{-4}</td>
</tr>
<tr>
<td>7</td>
<td>Chiller 1 operating capacity</td>
<td>1/3.0</td>
</tr>
<tr>
<td>8</td>
<td>Chiller 2 operating capacity</td>
<td>1/2.0</td>
</tr>
</tbody>
</table>

### 7.3 Parameters for predictive control design

#### 7.3.1 Parameters

The augmented cost functional proposed in Equation 6.12 and reproduced in Equation 7.3 was used for the predictive schemes designed in this test case. The weights for the various terms in that functional were decided on the basis of the arguments presented in section 6.6.3, details of which are discussed below.

\[
J_k = \sum_{i=0}^{N-1} \sum_{j=1}^{N_u} \alpha_j u_j(k+i|k) + \gamma \sum_{i=1}^{N} \sum_{j=1}^{N_y} [y_j(k+i|k) - y_{j,ref}(k+i|k)]^2 \\
+ \psi \sum_{i=1}^{N-1} \sum_{j=1}^{N_u} [u_j(k+i|k) - u_j(k+i-1|k)]^2 + \beta \sum_{i=1}^{N} \sum_{j=1}^{N_y} [z_j(k+i|k)]^2 \quad (7.3)
\]

The linear dependence of the instantaneous power consumption, $P(t)$ on the control inputs is can be formulated as shown below:

\[
P(t) = \lambda_j u_j(t) \quad j = 1, 2, \ldots 8. \quad (7.4)
\]

For the test system, the values of the coefficients $\lambda_j$ for the operating range under consideration have been presented in Table 7.4. Note that the coefficients corresponding to the chiller capacities ($u_1$ and $u_2$) are the inverse of their nominal COPs, obtained using Equation 7.1. On the other hand, the coefficients corresponding to the valve opening factors ($u_3$ to $u_8$) represent their contribution to the total pump work done and were obtained via simulations.
The weighting coefficients, $\alpha_j$ that appear in the first term of Equation 7.3 are now decided by a simple and intuitive order of magnitude analysis. We choose $\alpha_j = \lambda_j$ for the chiller inputs and, since the coefficients for the valve inputs are about three orders of magnitude smaller than those of the chiller inputs, we choose $\alpha_j = 10^3 \lambda_j$ for the valve inputs. This is equivalent to saying that the pump power consumption is penalized $10^3$ times more than the chiller power consumption which is reasonable as the total power consumption by the pumps is small compared to the power consumed by the chillers at the nominal operating conditions (Appendix A.1). The other weights in the cost functional were decided by choosing appropriate values to begin with and then refining them in the course of the simulation. These are given as follows:

$$
\begin{align*}
\beta &= 5 \times 10^{-2} \\
\gamma &= 1 \\
\psi &= 1 \times 10^{-2}
\end{align*}
$$

The spectral value of the continuous state space model (Equation 6.7) for the system under investigation, $\lambda_{max}(A)$ was found to be $\approx 20$ s. Therefore, the sample time $T_s$ for obtaining the corresponding discrete model (Equation 6.9) was assumed to be 5 s, which is ‘small enough’ (see the discussion in section 6.6.3). The size of the predictive control horizon was chosen to be $N = 15$ time samples, for which the execution time of the centralized MPC algorithm was within 1 second on a desktop computer with a 2.0 GHz processor and 960 MB RAM. With these choices, the control horizon corresponds to $N \times T_s = 75$ s, and is large enough to address the long transient dynamics of the system (see Figures 4.7-4.9).

Saturation constraints of $\pm 25\%$ of the nominal value and slew rate constraints of $\pm 1$ units per sample interval were forced on all input channels. The physical basis and the mathematical formulation of the constraints were discussed in section 6.7.
7.3.2 Predictive schemes

The centralized and distributed MPC schemes for the afore-mentioned objective function and constraints are based on the framework presented in chapter 6. The discrete model used for control design was obtained from the reduced order, continuous time model about the prescribed operating conditions\(^3\) as per section 6.5, using the sample time chosen.

Implementation of the centralized scheme as a quadratic programming problem was presented in section 6.8. Two distinct distributed schemes corresponding to two different leader-follower control architectures were developed. The two chillers taken together constitute the master agent in both these schemes. However, the slaves are different as shown in Figure 7.5. The first architecture has two followers corresponding to two clusters of heat exchangers: LAHXs 1-3 and LAHXs 4-6. Therefore the sink elements serving a floor are clustered together which represents an intuitive choice. The second architecture, however, corresponds to an extreme case where each sink is treated as an independent follower. In the simulation experiments performed, the number of iterations used, \(N_{iter}\) were varied from 2 to 6.

\(^3\)See section 7.1.2 and Appendix A.1
Table 7.5: Controllers compared in the simulated test case

<table>
<thead>
<tr>
<th>Control logic</th>
<th>Abbreviation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Localized On-off</td>
<td>L-OF</td>
<td>11.9</td>
<td>11.95</td>
<td>12</td>
<td>12.05</td>
<td>12.1</td>
</tr>
<tr>
<td>Decentralized PI</td>
<td>D-PI</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
<td>41</td>
</tr>
<tr>
<td>Centralized MPC</td>
<td>C-MPC</td>
<td>42</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributed MPC scheme 1</td>
<td>D-MPC1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distributed MPC scheme 1</td>
<td>D-MPC2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.6: Demand response comparison (LAHX 6)

7.4 Comparative results

The above controllers (Table 7.5) were implemented on the THERMOSYS simulation test bed for the present test case, details of which were presented in the preceding sections. Based on the results obtained, the various control schemes were evaluated with respect to the control objectives outlined in section 7.1.5. The important findings from this exercise have been reported below. Note that in three iterations, the distributed MPC schemes had almost converged and therefore the results in sections 7.4.1 - 7.4.3 for D-MPC1 and D-MPC2 correspond to $N_{iter} = 3$. 

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7.4.1 Demand matching

Figure 7.6 shows the cooling capacity achieved by LAHX 6 under the action of L-OF, D-PI, C-MPC and D-MPC\textsubscript{2} for the step change in the reference at noon (see Figure 7.3). Similar observations were made for the five other heat exchangers in the system and also during the entire 8 hour time-window considered. The behavior for D-MPC\textsubscript{1} was observed to be similar to D-MPC\textsubscript{2} and therefore is not shown. It is evident that satisfactory steady state regulation was achieved for all these schemes with tight error bounds. The transient characteristics exhibit differences, but transient behavior of the response is of little significance here. This is because, the time-constants for the thermal dynamics inside the room are almost an order of magnitude larger than that for the HVAC system and the transience in the latter is not of much significance.

7.4.2 Valve loading

The control signals acting on valve 6, under the various control schemes, during the time window around noon are shown in Figure 7.7. A Lomb Periodogram is analogous to a Discrete Fourier Transform but for variable time steps. The periodogram of these signals
have been plotted in Figure 7.8. Once again, the behavior for D-MPC$_1$ was observed to be very similar to D-MPC$_2$ and therefore is not shown. It is evident that the critical frequencies for L-OF and D-PI schemes are around 2000 and 10 times higher, respectively, than C-MPC and D-MPC schemes, and hence adverse effects on the life-cycle performance of the system can be expected through traditional control strategies. The possible reason for this observation is that localized feedback is incapable of counteracting the effects of dynamical interactions (which act as a disturbance from a local perspective) that occur between the various components of the system. This highlights the importance of dynamic models in the design of controllers for hydronic systems over the common practice of using static models. Note that the control signals for the other valves were observed to have similar time and frequency domain characteristics.

### 7.4.3 Energy consumption

<table>
<thead>
<tr>
<th>Table 7.6: Comparison of average power consumption (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Overall subsystem</td>
</tr>
<tr>
<td>Chillers</td>
</tr>
<tr>
<td>Pumps</td>
</tr>
</tbody>
</table>
Table 7.6 and the bar-chart in Figure 7.9 provide a comparison of the average energy consumption by the chillers, pumps and the overall liquid loop subsystem for the various schemes. It is evident that the C-MPC scheme is the most energy efficient, resulting in 7.1% and 5.8% reduction in the chiller and pump power consumption respectively over the traditional D-PI scheme, despite the fact that a modern supply temperature reset strategy was employed for the latter. Note that pumps consume only a small fraction of the net energy for the system studied here, which might not be the case for very large scale building systems. For such systems, the combined effect of chiller and pump energy savings may be much higher than for this example.

The distributed schemes are observed to be suboptimal. The chillers consume almost the same energy as in C-MPC, but the pump energy performances are significantly different. This asserts that dynamic coupling among the sink elements, even though small, has a bearing on the hydraulic (pump) energy performance. However, the observation that D-MPC$_2$ is less optimal than D-MPC$_1$ with regard to pump energy consumption, suggests that the
choice of the leader-follower architecture for distributed controllers can be important from an energy efficiency perspective. This presents an interesting research problem where tools such as combinatorial and cluster analysis can be applied.

An analysis of energy consumption for the L-OF scheme was not performed, because of the possibility of high prediction errors considering their large operation regime. More advanced prediction models can be used to that effect, as a future improvement of this work.

### 7.4.4 Computational complexity

![Figure 7.10: Comparative study of computational complexity](image)

Due to their trivial control logic and decentralized architectures, the D-PI and L-OF schemes shall have negligible real-time computational complexities when implemented on microprocessors, compared to the predictive schemes where an optimization has to be performed at each time step. For C-MPC, the computational effort required for the solution of the optimization problem (Equation 6.35) is determined by its dimension, i.e. $N \times N_u$.

Figure 7.10 shows a computational complexity comparison of C-MPC and D-MPC$_2$ with different values of $N_{iter}$, using a desktop computer with a 2.0 GHz processor and 960 MB RAM. In this case, to demonstrate the effect of variation in problem size, the value of $N$ was varied with $N_u$ fixed.

It is interesting to observe that the difference in the computation times between the two
schemes scales almost exponentially with the problem size. Therefore, for large scale HVAC systems, C-MPC can lead to significant time delays which can be avoided by the use of a D-MPC scheme. Furthermore, higher computational and memory requirements for the C-MPC scheme results in costlier hardware, which may offset the cost benefit obtained by reduced energy consumption. It was also observed (by zooming in the plot) that the computational complexity of D-MPC schemes scales linearly, as expected, with the number of iterations. The corresponding plot for the D-MPC\(^1\) scheme has not been presented because it closely follows that for D-MPC\(^2\).

For the simulated system, both the D-MPC schemes converge to within 1% of their optimal values for \(N_{\text{iter}} = 4\), which obviates the need for a large number of iterations for the particular system considered. If this is not true, the D-MPC algorithm can be forced to terminate after a suitable number of iterations, which would render the scheme suboptimal in practice. Therefore, a tradeoff between optimality and computational complexity may be involved. In this case, the Hessian, \(H_k\) was observed to be singular and thus the underlying QP problem is indefinite. Therefore, the \textit{quadprog} solver of MATLAB which invokes the Active-set method was used for optimization. Use of barrier function methods are expected to result in faster convergence of the C-MPC and D-MPC schemes.

### 7.5 Analysis of Results

Based on the above results, the following general conclusions can been arrived at:

1. The localized on-off scheme is simple and easy to implement, but may be disadvantageous from long term reliability considerations due to significant fatigue loading of the mechanical components (chiller compressors and valves).

2. The Traditional PI schemes perform better than on-off, in terms of mechanical reliability but clearly consume higher energy when compared to more advanced predictive strategies. This difference in the energy consumption between traditional and predictive schemes may vary depending on the characteristics of system being considered, i.e.
Table 7.7: Case study summary

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Demand matching</th>
<th>Reliability</th>
<th>Power consumption</th>
<th>Computational Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-off</td>
<td>satisfactory</td>
<td>severely critical</td>
<td>not considered</td>
<td>extremely simple</td>
</tr>
<tr>
<td>PI</td>
<td>good</td>
<td>poor</td>
<td>high overall</td>
<td>simple</td>
</tr>
<tr>
<td>D-MPC</td>
<td>good</td>
<td>good</td>
<td>appreciable reduction overall</td>
<td>critical</td>
</tr>
<tr>
<td>D-MPC1</td>
<td>good</td>
<td>good</td>
<td>appreciable reduction for chillers and overall</td>
<td>significant reduction</td>
</tr>
<tr>
<td>D-MPC2</td>
<td>good</td>
<td>good</td>
<td>appreciable reduction for chillers and overall</td>
<td>significant reduction</td>
</tr>
</tbody>
</table>

its size, relative efficiencies of the various source elements and pumps, etc.

3. The centralized MPC scheme is the most optimal in terms of energy consumption, and yields ‘smoother’ actuation signals but tends to be computationally ill-posed, particularly for large scale problems.

4. Distributed MPC strategies offer a compromise in terms of the energy consumption, reliability and computational effort. However, proper choice of the distributed architecture is important to achieve the best tradeoff.

The above observations have been summarized in Table 7.7 for quick reference.

7.6 Controller selection guidelines

In the light of the observations made in this test case, a set of proposed guidelines for hydronic HVAC system controller selection have been presented below. However, they must be interpreted only in the context of the specific list of objectives in section 5.1.

1. For small scale systems, the ‘fighting’ between the various actuators (valves) is expected to be relatively small and therefore the traditional on-off and P/PI schemes, preferably with supply temperature reset can be used if complexity is the main concern. However, if energy efficiency is more important, centralized MPC may be used. The computational complexity of MPC for such systems is expected to be within acceptable limits and therefore a distributed MPC architecture might not be particularly advantageous.

2. For medium scale systems, such as the one considered in the test case, due to reliability concerns, the traditional schemes are not recommended. Centralized MPC may still be
used instead of distributed control, depending on whether the computational complexity is within tolerable bounds, particularly so if more efficient numerical techniques such as the barrier function method can be used.

3. For large scale systems, it appears imperative to use the distributed, optimal MPC scheme or any other distributed control architectures, owing to computational and mechanical reliability concerns which are particularly relevant for such systems.

7.7 Concluding Remarks

In this chapter, the traditional and more advanced control methodologies described and developed in chapters 5 and 6 were implemented on a test chilled water system. They were evaluated with respect to the important control objectives outlined previously. This exercise demonstrated the advantages and limitations associated with these methodologies. In particular, it was observed that the novel, distributed predictive scheme provides the best compromise in the multidimensional evaluation framework of ‘regulation’, ‘optimality’, ‘reliability’ and ‘computational complexity’. This is an important cornerstone of this work.

To conlude this thesis, the next chapter summarizes the findings from the work presented and also discusses some candidate problems for future work that naturally arise in the course of the modeling and control efforts pursued.
Chapter 8

Conclusions and Future Work

In this final chapter, the work presented in this thesis has been summarized in section 8.1 followed by concluding remarks in section 8.2. The contributions made by this work and future avenues of research have been discussed in sections 8.3 and 8.4.

8.1 Summary

A chapterwise summary of this thesis is presented below:

1. Chapter 1 motivates the problem of building HVAC modeling and control and provides a literature survey of this area and the tools that have been used in this work.

2. A general introduction to hydronic systems was presented in chapter 2. A classification of these systems in terms of their physical layout, underlying thermo-hydraulic components and other factors was provided. The common applications of such systems were also discussed.

3. Chapter 3 presented the details of THERMOSYS - a simulation test-bed for hydronic systems based on nonlinear models. These models were described, together with the procedure for running simulations using this test-bed.

4. In chapter 4, control oriented modeling was attempted. A piecewise-linear modeling framework was proposed which consists of reduced order, linear, state space models. The modeling procedure presented uses graph theory to quantify the topology of these systems, which makes it generic. Using a medium-scale chilled water system as example,
it was verified that the reduced order model has acceptable fidelity with respect to the nonlinear model.

5. The important control objectives for hydronic systems were presented in chapter 5. Details of traditional control strategies employed for these systems were provided.

6. Chapter 6 develops more advanced control methodologies based on the concept of MPC. Centralized and distributed control schemes were presented. The control task was posed as an optimization problem. The distributed schemes exploit the underlying coupling architecture for cooperative iterations.

7. The various control schemes were tested for conformity to the control objectives using a case study on a medium-scale chilled water system. It was found that traditional control schemes result in mechanical reliability issues and consume more energy when compared to predictive schemes. The distributed MPC scheme was found to provide a good balance between simplicity, reliability and optimality.

8.2 Conclusions

The important conclusions from this work are as follows:

1. Apart from thermal comfort, there are other important objectives, viz., energy efficiency, mechanical reliability and computational complexity which pertain to the control of hydronic systems.

2. To satisfy these control objectives, model based optimal control approach is useful. The underlying model must be simple but sufficiently accurate and dynamic to meet the control goals of energy efficiency and mechanical reliability. In particular, a graph based, linear, reduced order modeling framework was presented.

3. Practical considerations such as constraints encourage the use of MPC as the optimal control approach of choice. A distributed MPC scheme is found to be a useful alternative to a centralized scheme from computational complexity considerations.
4. Simulations reveal that distributed MPC incorporates the good features of both traditional schemes and centralized MPC. Improvements were obtained in energy efficiency and mechanical reliability with respect to traditional schemes and in computational complexity with respect to centralized MPC.

8.3 Contributions

An important problem which is relevant to society at the present time and in future is the subject of this work. To address that, a systematic approach is developed and evaluated.

This work highlights the importance of a systems based approach for improvising the performance of an important class of HVAC systems - centralized, building hydronic systems. In this approach, these systems are studied within the paradigm of a complex network of interacting sub-systems. Adhering to that view-point, a graph theoretical modeling framework is presented using dynamic models, wherein a time-scale based model reduction methodology is implemented to yield a representation which is simple, generic, modular and is able to quantify interactions. Based on the literature survey reported in this work, the proposed modeling approach is an important attempt to address the lack of a modeling methodology which balances all such desired attributes.

From a controls perspective, the application of MPC for control of the ‘inner’ hydronic loop in building HVAC systems is another contribution. This was motivated by the fact that though MPC has been successfully applied and demonstrated in the general area of buildings system control, hydronic loop control is still based on traditional on-off or PID schemes. One of the important highlights of this work is the development of a novel distributed MPC scheme for these systems. This scheme exemplifies the convergence of physical understanding, advanced control and network tools to achieve important control objectives.

In short, this thesis contributes some significant steps, concepts and ideas towards the development of an integrated approach for building system technologies. The thrust is to provide a practical solution to industry in the form of a toolbox, with the capability to
support the design of smart, practical and reliable building controllers, using the principles of network theory.

Lastly, another contribution of the present work is a rich of set of problems in building system modeling and control which emanate from it and are discussed briefly in the next section.

8.4 Future Work

This thesis identifies the following as important thrust areas of future research to build upon the work presented in this thesis.

8.4.1 Extension to air-side, zonal and other dynamics

The present work considers the hydronic system in isolation. In reality, the dynamics of the hydronic system are coupled to the dynamics of the other HVAC subsystems and the zones. For example, in the chilled water system that was used for case study in chapter 7, the vapor compression and condenser loop dynamics in the chillers, the air-side dynamics in the ducts, and the thermal dynamics of the conditioned spaces are all connected together. Therefore, to generate a more complete and practical picture, it is important to extend the modeling and control framework to include the overall thermal dynamics represented by such interactions.

This would clearly render more complexity to the problem because of additional components, coupling and time-scales. To meet this challenge, the graph based modeling approach must be suitably modified to encompass this more complete dynamics. Accordingly, the control methodology will also undergo changes. As a specific example, the thermal comfort as an objective might be quantified by room temperatures instead of heat transfer rates and MPC predictions would be based on the expanded model. The distributed MPC scheme might also change due to the modified coupling architecture. The scope of THERMOSYS must also be enlarged to include models for these new subsystems.
8.4.2 Additional degrees of freedom

Modern building HVAC systems, and in particular those at the large scale, may feature thermal storage components, combined heat and power source elements and variable speed pumps. From a controls perspective, this translates to additional degrees of freedom which may be exploited to improvise the efficiency and performance of the overall system. Besides augmenting the modeling framework, this might also mandate the use of more sophisticated control machinery such as hybrid and supervisory control with set-point optimization.

8.4.3 Robustness

In practice, one of the most important challenges facing the building system control problem is uncertainty in thermal loads. It is difficult to accurately characterize these loads because they depend on a multitude of factors such as ambient conditions, occupancy and lighting levels. A more practical approach to handle this issue is to make the controller robust to such uncertainties. A network approach using graphs to analyse the propagation of these uncertainties and disturbances can be explored. Accordingly, the control architecture might need to be modified to address this issue. Ideally, control design must be based on the intersection of optimality and robustness considerations and tradeoffs may be involved.

8.4.4 Refinement to existing approach

To make the modeling and control approach presented more efficient, it shall be useful to explore graph based methodologies from which the control architecture can be selected quickly. In the current approach, the distributed MPC architecture is based on the coupling information from the state space matrices, which in turn are generated by an algorithmic modeling and model reduction process. If the dynamical interactions could be represented directly on a graph, the afore-mentioned indirect route can be “short-circuited”. Tools such as graph partitioning, data clustering or combinatorics can be used on such a graph to identify appropriate control modules for a distributed, decentralized or hierarchical scheme.
The overall thrust, as highlighted earlier, is to develop a generic automated procedure for the analysis and control of building thermal dynamics.

### 8.4.5 Experimental validation

Experimental evaluation of the proposed tools is important for completeness and bringing out their true value. To that effect, collaboration with industrial partners or the development of an in-house testing facility needs to be undertaken in future.
Appendix A

A.1 Operating Conditions for the Chilled Water System

In the context of the nonlinear component models presented in chapter 3, the physical parameters for the chilled water system are as described in Tables A.1 to A.6. For the pumps, the ‘PumpProp’ data structure corresponds to the characteristics shown in Figures A.1 and A.2. The nominal inputs for the system have been shown in Tables A.7 and A.8. The nominal operating condition refers to the steady-state operating condition resulting from the application of these inputs to the system. The states and outputs that represent the nominal operating condition, as obtained from THERMOSYS simulations have been reported in Tables A.9 to A.11. Note that the default ‘MoistAirProp’ and ‘LiquidProp’ data structures in THERMOSYS were used. The set of speed lines ‘PumpProp’ is \{2972, 3714, 4456, 5200, 5944, 6687, 7431, 8173, 8915, 9658, 10400, 11266, 12132\}.

Figure A.1: Head vs flow rate characteristics for the pumps
Figure A.2: Efficiency vs flow rate characteristics for the pumps

Table A.1: Physical Parameters for Flow Junctions (c.f. Fig. 3.8)

<table>
<thead>
<tr>
<th>Junction number</th>
<th>$V_{junc}$ ($m^3$)</th>
<th>$\beta$ (kPa)</th>
<th>$F_p$</th>
<th>$F_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.138 \times 10^4$</td>
<td>$2.152 \times 10^6$</td>
<td>$3 \times 10^5$</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>$2.138 \times 10^4$</td>
<td>$2.152 \times 10^6$</td>
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<td>$2.152 \times 10^6$</td>
<td>$2 \times 10^5$</td>
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<td>$3.5 \times 10^4$</td>
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<td>$2.138 \times 10^4$</td>
<td>$2.152 \times 10^6$</td>
<td>$4.5 \times 10^4$</td>
<td>70</td>
</tr>
<tr>
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<td>$2 \times 10^5$</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>$6 \times 10^4$</td>
<td>$2.152 \times 10^6$</td>
<td>$2 \times 10^5$</td>
<td>27</td>
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<td>9</td>
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<td>$5 \times 10^5$</td>
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<td>$2.152 \times 10^6$</td>
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Table A.2: Physical Parameters for Piping elements (c.f. Fig. 3.11)

<table>
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<th>Pipe number</th>
<th>Order of edges</th>
<th>$L_{eq}$ (m)</th>
<th>$D_h$ (mm)</th>
<th>$A_c$ ($mm^2$)</th>
<th>$\epsilon$ (mm)</th>
<th>$K_t$</th>
<th>$F_m$</th>
<th>$R$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
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<td>40</td>
<td>$1.256 \times 10^3$</td>
<td>$1.524 \times 10^{-3}$</td>
<td>1.32</td>
<td>14</td>
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<td>2</td>
</tr>
<tr>
<td>2</td>
<td>{23,25}</td>
<td>15</td>
<td>50</td>
<td>$1.963 \times 10^3$</td>
<td>$1.524 \times 10^{-3}$</td>
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<td>26</td>
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<tr>
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<td>{2}</td>
<td>4</td>
<td>50</td>
<td>$1.963 \times 10^3$</td>
<td>$1.524 \times 10^{-3}$</td>
<td>1.32</td>
<td>8</td>
<td>0</td>
<td>2</td>
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<tr>
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<td>314.16</td>
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<tr>
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<tr>
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<td>{15,16}</td>
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<td>314.16</td>
<td>$1.524 \times 10^{-3}$</td>
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<td>50</td>
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### Table A.3: Physical Parameters for LAHXs - part I (c.f. Fig. 3.14)

<table>
<thead>
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<th>LAHX number</th>
<th>$D_	ext{hl} (m)$</th>
<th>$D_	ext{ha} (m)$</th>
<th>$L (m)$</th>
<th>$A_	ext{c} (m^2)$</th>
<th>$A_	ext{ca} (m^2)$</th>
<th>$A_	ext{s} (m^2)$</th>
<th>$A_	ext{sa} (m^2)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$9 \times 10^{-3}$</td>
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<td>$2 \times 10^{-3}$</td>
<td>1.5</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>$4 \times 10^{-3}$</td>
<td>$9 \times 10^{-3}$</td>
<td>2</td>
<td>$2 \times 10^{-3}$</td>
<td>1.5</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
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<td>$9 \times 10^{-3}$</td>
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<tr>
<td>5</td>
<td>$4 \times 10^{-3}$</td>
<td>$9 \times 10^{-3}$</td>
<td>2</td>
<td>$2 \times 10^{-3}$</td>
<td>1.5</td>
<td>20</td>
<td>50</td>
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<tr>
<td>6</td>
<td>$4 \times 10^{-3}$</td>
<td>$9 \times 10^{-3}$</td>
<td>2</td>
<td>$2 \times 10^{-3}$</td>
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### Table A.4: Physical Parameters for LAHXs - part II (c.f. Fig. 3.14)

<table>
<thead>
<tr>
<th>LAHX number</th>
<th>$F_	ext{fin-t} (kg)$</th>
<th>$F_	ext{fin-a} (kg)$</th>
<th>$m_w (kg)$</th>
<th>$c_w (kJ/kg - K)$</th>
<th>$t_	ext{fin-t} (m)$</th>
<th>$t_	ext{fin-a} (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6592</td>
<td>0.8539</td>
<td>100</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>0.6592</td>
<td>0.8539</td>
<td>100</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>0.6592</td>
<td>0.8539</td>
<td>125</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
</tr>
<tr>
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<td>0.6592</td>
<td>0.8539</td>
<td>125</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
</tr>
<tr>
<td>5</td>
<td>0.6592</td>
<td>0.8539</td>
<td>100</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>0.6592</td>
<td>0.8539</td>
<td>150</td>
<td>0.875</td>
<td>$1.524 \times 10^{-4}$</td>
<td>$1.016 \times 10^{-4}$</td>
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### Table A.5: Physical Parameters for LAHXs - part III (c.f. Fig. 3.14)

<table>
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<th>LAHX number</th>
<th>$L_	ext{fin-t} (m)$</th>
<th>$L_	ext{fin-a} (m)$</th>
<th>$t_	ext{wall} (m)$</th>
<th>$k_	ext{wall} (W/m - K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
<tr>
<td>2</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
<tr>
<td>3</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
<tr>
<td>4</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
<tr>
<td>5</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
<tr>
<td>6</td>
<td>$9.525 \times 10^{-4}$</td>
<td>$4.762 \times 10^{-3}$</td>
<td>$4.064 \times 10^{-4}$</td>
<td>177</td>
</tr>
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### Table A.6: Physical Parameters for chillers (c.f. Fig. 3.16)

<table>
<thead>
<tr>
<th>Chiller number</th>
<th>$D_	ext{h} (m)$</th>
<th>$L (m)$</th>
<th>$A_	ext{s} (m^2)$</th>
<th>$A_	ext{sa} (m^2)$</th>
<th>$m_w (kg)$</th>
<th>$c_w (kJ/kg - K)$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$2 \times 10^{-2}$</td>
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<td>$3.14 \times 10^{-4}$</td>
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<td>200</td>
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</tr>
<tr>
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<td>$1.5 \times 10^{-2}$</td>
<td>25</td>
<td>$1.76 \times 10^{-4}$</td>
<td>1.178</td>
<td>200</td>
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### Table A.7: Inputs corresponding to hydraulic components (c.f. section 4.4.2 and Fig. 4.6)

<table>
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<tr>
<th>Valve number, $v$</th>
<th>$A_{l,v} (m^2)$</th>
<th>Pump number, $p$</th>
<th>Corresponding pump</th>
<th>$\omega_p (rad/s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>1</td>
<td>Primary pump 1</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>2</td>
<td>Primary pump 2</td>
<td>700</td>
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<td>3</td>
<td>50</td>
<td>3</td>
<td>Secondary pump 1</td>
<td>950</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>4</td>
<td>Secondary pump 2</td>
<td>900</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
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<td></td>
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<td>85</td>
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<td>8</td>
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</table>
Table A.8: Inputs corresponding to thermal components (c.f. section 4.4.2 and Fig. 4.6)

<table>
<thead>
<tr>
<th>Source element (chiller) number, c</th>
<th>$Q_{ext,c}$ (kW)$^1$</th>
<th>Sink element (LAHX) number, h</th>
<th>$\dot{m}_{a-in,h}$ (kg/s)</th>
<th>$T_{a-in,h}$ ($^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1.987</td>
<td>35</td>
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<tr>
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<td>3</td>
<td>3.753</td>
<td>35</td>
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<td>3.15</td>
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<td>2.265</td>
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</table>

Table A.9: States for hydraulic components (c.f. section 4.4.1 and Fig. 4.6)

<table>
<thead>
<tr>
<th>Pipe number, l</th>
<th>Order of edges</th>
<th>$\dot{m}_l$ (kg/s)</th>
<th>Mass flow vertex number, j</th>
<th>$p_j$ (kPa)</th>
<th>$T_j$ ($^\circ$C)</th>
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</thead>
<tbody>
<tr>
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Table A.10: States for thermal components (c.f. section 4.4.1 and Fig. 4.6)

<table>
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<tr>
<th>Source element (chiller) number, c</th>
<th>$T_{L,c}$ ($^\circ$C)</th>
<th>$T_{w,c}$ ($^\circ$C)</th>
<th>Sink element (LAHX) number, h</th>
<th>$T_{L,h}$ ($^\circ$C)</th>
<th>$T_{w,c}$ ($^\circ$C)</th>
</tr>
</thead>
<tbody>
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</tr>
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</table>
Table A.11: System outputs (c.f. section 4.4.3 and Fig. 4.6)

<table>
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<th>Sink element (LAHX) number, h</th>
<th>$\dot{Q}_{out,h}$ (kW$^2$)</th>
</tr>
</thead>
<tbody>
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<tr>
<td>2</td>
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<tr>
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<td>-37.47</td>
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<tr>
<td>4</td>
<td>-33.49</td>
</tr>
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<td>5</td>
<td>-26.79</td>
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<tr>
<td>6</td>
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</tbody>
</table>

A.2 Derivation of Model Reduction Algorithm

Refer to Equation 4.17. Treating the hydraulic states, $m_l$ and $p_j$ as static, we can write:

$$
\begin{pmatrix}
A_1 & -A_3A^T \\
B_1A & [0]_{(n_j \times n_j)j}
\end{pmatrix}
\begin{pmatrix}
\dot{m}_l \\
p_j
\end{pmatrix} =
\begin{pmatrix}
A_4 & A_2 \\
[0]_{(n_j \times n_v)} & [0]_{(n_j \times n_p)}
\end{pmatrix}
\vec{v}
$$

(A.1)

Where, $\vec{v} = (A_{l,v} \omega_p)^T$

The above is a set of $n_l+n_j$ equations in the same number of variables. However, as noted in section 4.4.4, exactly one of these equations is redundant because the hydraulic circuit is closed. Also, one of the junction pressures $p_j$ can be treated as the reference pressure and assigned the value 0. Therefore, the solution for $m_l$ is obtained after deleting this redundant equation and is given by:

$$(\dot{m}_l) = Z_3\vec{v}$$

(A.2)

Next, treating $T_{L,i}$ and $T_{w,i}$ in Equation 4.17 as static, we obtain the following algebraic equations:

$$W_1C(\dot{m}_l) + W_2B_f(T_j) + W_2B_{tl}(T_{L,i}) = 0$$

(A.3)

$$Q_2EC(\dot{m}_l) + Q_1EB_f(T_j) + (Q_1EB_t + Q_3)(T_{L,i}) + Q_4(T_{w,i}) = 0$$

(A.4)
Solving Equations (A.3) and A.4 for \((T_j)\) and \((T_{L,i})\) in terms of the other quantities, we obtain the following:

\[
(T_{L,i}) = -Z_1^{-1}(Z_2Z_3\bar{v} + Q_4(T_{w,i})) \tag{A.5}
\]

\[
(T_j) = -Y_1Z_3\bar{v} + Y_2(T_{w,i}) \tag{A.6}
\]

In the above Equations \(Z_1\), \(Z_2\), \(Y_1\) and \(Y_2\) are as defined in section 4.5.2.

From Equation 4.17, we have:

\[
\frac{d}{dt}(T_{w,i}) = R_{1}\text{EC}(\dot{m}_l) + R_{2}\text{EB}_f(T_j) + (R_{2}\text{EB}_t + R_{5})(T_{L,i}) + R_6(T_{w,i})
\]

\[
+ R_7(\dot{Q}_{ext,c}) + R_8(\dot{m}_{a-in,h}) + R_4(T_{a-in,h}) \tag{A.7}
\]

Using Equations (A.2), (A.5) and (A.6) in (A.7), we obtain the reduced order state space Equation 4.19 with \(A_{ro}\) and \(B_{ro}\) given by Equations 4.25 and 4.26 respectively. On the other hand, Equation 4.20 trivially follows from Equation 4.18.

### A.3 MATLAB codes for reduced order state space modeling

The first step is to generate the Adjacency matrix for the graph representation of the system using the following m-file called ‘adjacency.m’. This matrix is written to the workspace as the variable \textit{Adjac}

```matlab
1 % adjacency.m
2 % PROGRAM FOR GENERATION OF THE ADJACENCY MATRIX IN SPARSE FORM
3 clear Nv i j s source dest l Adjac
4 Nv = input('Enter number of vertices');
5 i = [];
6 j = [];
7 s = [];
8 for l = 1:Nv
9    disp('Which vertices is are adjacent to vertex');
10    disp(l);
11    dest = input(' ');
```
Next, the following program, 'modeling.m' is used to generate the reduced order state space matrices for the system. It inputs the Adjacency matrix generated in the above program. The state space matrices are written to the workspace as the variables \( A_{ro}, B_{ro}, C_{ro} \) and \( D_{ro} \). Note that this program inputs the nominal operating conditions about which the linearization is performed. For this purpose, it generates Microsoft Excel spread-sheets for the user to fill in the appropriate linearization data.

```matlab
% cmpc.m
% IMPLEMENTATION OF C-MPC
function blah = cmpc(u)
global prev cnt N Nx Nu Ny alpha beta1 beta2 gammah A B C H Ac bc Ts
clear F;
xk = (u(1:Nx));
y0 = (u(Nx+1:Nx+Ny));
errorr = (u(Nx+Ny+1:Nx+Ny+Ny));
t = u(end)-5000;

options = optimset();
options.Display = 'off';
options.MaxIter = 20000000000;

if(t<cnt)
    blah = prev;
else
    F = zeros(1,Nu*N);
    for p = 1:Nu
        for r = 0:N-1
            summ = 0;
            dumm = 0;
```
humm = 0;
for i = r+1:N
    simsim = zeros(Nx,Nx);
    for s = 0:i-(r+1)
        simsim = simsim + Aˆ(s);
    end
    dimdim = zeros(Nx,Nx);
    for s = 0:i
        dimdim = dimdim + Aˆ(s);
    end
    for j = 1:Ny
        summ = summ + 2*alpha(j)*((C(j:j,1:Nx)ˆ(i)*xk - y0(j))*(C(j:j,1:Nx)ˆ((i-(r+1))))*B(1:Nx,p:p));
        humm = humm + 2*gammah(j)*((C(j:j,1:Nx)ˆ(i)*dimdim*xk*Ts - i*y0(j)*Ts+errorr(j)))*(C(j:j,1:Nx)ˆ(i)*simsim*B(1:Nx,p:p))*Ts;
    end
end
beta = [beta1 beta2];
dumm = beta(p);
F((p-1)*N+r+1) = summ+dumm+humm;
end
result = quadprog(H,F',Ac,bc,[],[],[],[],options);
blah = [result(1);result(N+1);result(2*N+1);result(3*N+1);result(4*N+1);result(5*N+1);result(6*N+1);result(7*N+1)];
cnt = cnt + 1;
end
prev = blah;

The user defined functions ‘excelin_mod.m’ and ‘ps2.m’ that the above program uses are as follows:

```matlab
% function excelin_mod.m
function [Afo,Bfo,Aro,Bro,Cro,Dro,EVertData,W1,W2,Z1,Z2,Z3,Y1,Y2,A1,A2,A3,A4, Afd,Bfd]= excelin_mod(l,k,j)

global pipe
global Ve
global Vm
```
% an array containing the letters of the alphabet for corresponding excel indexing

Alphabet = char('A' + (1:26) - 1);

% Function excellin() builds an excel spreadsheet to input the pump and hydraulic resistances, flow junction parameters, and chiller and heat exchanger parameters.

l = input('Please enter the number of pipes: ');
Ve = input('Enter the energy vertex array in ascending order');
k = length(Ve);

Excel = actxserver('Excel.Application'); % open an excel server
set(Excel, 'Visible', 1);

% excel workbook and sheet handles
Workbooks = Excel.Workbooks;
Workbook = invoke(Workbooks, 'Add');
Activesheet = Excel.Activesheet;

% excel workbook and sheet handles
Sheets = Excel.ActiveWorkbook.Sheets;
Sheets.Add;

% Sheet Handles
Labels = Sheets.Item(1);
Junctions = Sheets.Item(2);
HydRes = Sheets.Item(3);
ChilB = Sheets.Item(4);
LAHX = Sheets.Item(5);
(PV,chxlab,CON)=paramlab(l,pipe,Ve);

Labels.Name='Component Labels'; %rename worksheet 1

%Label sheet instruction comments
Labels.Range('A1').AddComment.Text('Enter your pump and valve numbers relating to these pipe numbers.');
Labels.Range('B1').AddComment.Text('This is the sequence of vertices of the corresponding pipe. Please label your schematic with the pipe numbers. ');
Labels.Range('C1').AddComment.Text('Enter the pump and valve numbers. If there is none in that pipe, leave the entry as zero.');
Labels.Range('F1').AddComment.Text('The energy vertex numbers');
Labels.Range('G1').AddComment.Text('Indicate whether the energy flow vertex is a chiller/boiler or heat exchanger by C or H respectively. ');
Labels.Range('I1').AddComment.Text('The following parameters will be assumed constant for your system and used in following calculations');
Labels.Range('I2').AddComment.Text('Fluid Bulk Modulus');
Labels.Range('I3').AddComment.Text('Fluid Density');
Labels.Range('I4').AddComment.Text('Specific Heat of Fluid');
Labels.Range('I5').AddComment.Text('Specific Heat of Air');

%change color of label cells
r=sprintf('A2:A%d', {l+1});
Labels.Range(r).Interior.ColorIndex=33;

r=sprintf('F2:F%d', {k+1});
Labels.Range(r).Interior.ColorIndex=33;

Labels.Range('I2:I5').Interior.ColorIndex=33;

%format dividing columns
Labels.Range('E:E').Interior.ColorIndex=16;
Labels.Range('E:E').Columns.ColumnWidth=0.5;
Labels.Range('H:H').Interior.ColorIndex=16;
Labels.Range('H:H').Columns.ColumnWidth=0.5;

%change horizontal alignment
Labels.Range('A:I').HorizontalAlignment=3;
%Section Borders
r=sprintf('A1:D%d',l+1);
Labels.Range(r).Border.LineStyle=1;
r=sprintf('F1:G%d',k+1);
Labels.Range(r).Border.LineStyle=1;
Labels.Range('I2:J5').Border.LineStyle=1;
Labels.Range('I1').Border.LineStyle=1;

r=sprintf('A1:D%d',(l+1));
ActiveSheetRange=get(Labels,'Range', r);
set(ActiveSheetRange,'Value',PV); %overlay the cell structure 'PV' into excel

r=sprintf('F1:G%d',(k+1));
ActiveSheetRange=get(Labels,'Range', r);
set(ActiveSheetRange,'Value',chxlab); %overlay the cell structure 'chxlab' into excel

ActiveSheetRange=get(Labels,'Range', 'I1:I5');
set(ActiveSheetRange,'Value',CON); %overlay the cell structure 'CON' into excel

%make the heading cells bold
Labels.Range('A1:I1').Font.Bold=1;

%auto fit all columns
Labels.Range('A:I').Columns.AutoFit;

%matlab will read these ranges from excel
f1=sprintf('C2:C%d',(l+1));
f2=sprintf('D2:D%d',(l+1));
f3=sprintf('G2:G%d',k+1);

response=menu('Enter your component labels. See comments for details. Push OK when finished.', 'OK');

if response==1
    temp=get(Labels,'Range',f1);
pumps=temp.value; %get the input pump numbers from the user and put in array 'pumps'
pumps=reshape([pumps;], size(pumps));

    temp=get(Labels,'Range',f2);
valves = temp.value; % get the input valve number from the user and put in array 'valves'

valves = reshape([valves{:}], size(valves));

temp = get(Labels, 'Range', f3);
label = temp.value;

temp = get(Labels, 'Range', 'J2');
beta = temp.value; % bulk modulus value

temp = get(Labels, 'Range', 'J3');
rho = temp.value; % fluid density

% label = reshape([label{:}], size(valves));

temp = get(Labels, 'Range', 'J4');
Cpl = temp.value; % specific heat of liquid

temp = get(Labels, 'Range', 'J5');
Cpair = temp.value; % specific heat of heat exchanger air

chlhx = zeros(k, 1);

for i = 1:k
    if strcmp(label(i), 'C') == 1
        chlhx(i) = 1;
    end
end

c = nnz(chlhx); % number of chillers
h = k - c;

% Junction Parameter Input Sheet
JT = junction(j, Vm, beta, rho);

Junctions.Name = 'Junctions';

r = sprintf('A1:G%d', (1 + j));
Range = get(Junctions, 'Range', r);
set(Range, 'Value', JT);

% Junction Sheet Style Formatting
% make the heading rows bold
Junctions.Range('A1:G1').Font.Bold=1;
%Color column pertaining to Mass Flow Vertex Numbers
r=sprintf('A2:A%d',1+j);
Junctions.Range(r).Interior.ColorIndex=33;
%Junction sheet instruction comments
Junctions.Range('A1').AddComment.Text('Enter the parameters corresponding to the mass flow vertices you indicated earlier.);
Junctions.Range('B1').AddComment.Text('Junction Temperature');
Junctions.Range('C1').AddComment.Text('Junction Pressure');
Junctions.Range('D1').AddComment.Text('Junction Volume');
Junctions.Range('E1').AddComment.Text('Calculation field for the coefficient b');
Junctions.Range('F1').AddComment.Text('The bulk modulus and fluid density entered on the previous page. Do not change these values');
%Autofit
Junctions.Range('A:G').Columns.AutoFit;
%Horizontal Alignment
Junctions.Range('A:G').HorizontalAlignment=3;
%Cell Borders
r=sprintf('A1:E%d',1+j);
Junctions.Range(r).Border.LineStyle=1;
Junctions.Range('F1:G2').Border.LineStyle=1;
invoke(Junctions,'Activate');
response2=menu('Enter your parameters. See cell comments. When finished, press OK','OK');
%Ranges to retrieve from input parameters
field1=sprintf('B2:B%d',j+1);
field2=sprintf('C2:C%d',j+1);
field3=sprintf('D2:D%d',j+1);
field4=sprintf('E2:E%d',j+1);
if response2==1
    temp=get(Junctions,'Range',field1);
    Tj=temp.value;
Tj = reshape(Tj{:}, size(Tj));

temp = get(Junctions, 'Range', field2);
Pj = temp.value;
Pj = reshape([Pj{:}], size(Pj));

temp = get(Junctions, 'Range', field3);
V = temp.value;
V = reshape([V{:}], size(V));

temp = get(Junctions, 'Range', field4);
b = temp.value;
b = reshape([b{:}], size(b));

B1 = diag(b); %coefficient matrix B1 for state space

clear temp

% the pressure difference across each pipe
Pdiff = (−Pj'*(D*C))';

% Hydraulic Resistance and Pump Calculation Sheet
p = nnz(pumps); % the number of pumps in the system
t = l−p; % the number of pipes without pumps

% Get Cell Field for Hydraulic Resistances from HYDR
HR = HYDR(t, l, pumps, valves, Pdiff);

% Get Cell Field for Pumps from PMP
PP = PMP(p);

% overlay HR and PP into excel on sheet 2
r = sprintf('A1:I%d', (1 + t));
Range = get(HydRes, 'Range', r);
set(Range, 'Value', HR);

r = sprintf('K1:O%d', (p + l));
```
Range = get(HydRes,'Range',r);
set(Range,'Value', PP);

% Hydraulic Resistance Sheet Style Formatting
% make the heading rows bold
HydRes.Range('A1:O1').Font.Bold = 1;

% color columns pertaining to pump and pipe numbers
r = sprintf('A2:A%d', (1+t));
r2 = sprintf('K2:K%d', (1+p));
HydRes.Range(r).Interior.ColorIndex = 33;
HydRes.Range(r2).Interior.ColorIndex = 33;

% format dividing column between pipe and pump parameters
HydRes.Range('J:J').Interior.ColorIndex = 16;
HydRes.Range('J:J').Columns.ColumnWidth = 0.5;

% auto fit all columns
HydRes.Range('A:P').Columns.AutoFit;

% center pump and pipe numbers
HydRes.Range('A:A').HorizontalAlignment = 3;
HydRes.Range('K:K').HorizontalAlignment = 3;

% format cell borders
r = sprintf('A1:I%d', 1+t);
HydRes.Range(r).Border.LineStyle = 1;
r = sprintf('K1:O%d', 1+p);
HydRes.Range(r).Border.LineStyle = 1;

% Attempt at freezepane
% HydRes.Window(1).SplitColumn = 0;
% HydRes.Window(1).SplitRow = 2;
% HydRes.Window(1).FreezePanes
% ActiveWindow.Split = 0;

% Add instruction comments
HydRes.Range('A1').AddComment.Text('Enter the parameters relating to these pipe numbers.');</n
HydRes.Range('B1').AddComment.Text('Mass Flow Rate');</n```

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HydRes.Range('C1').AddComment.Text('Pressure Difference Across Pipe');

HydRes.Range('D1').AddComment.Text('Cross-Sectional Area');

HydRes.Range('E1').AddComment.Text('Length');

HydRes.Range('F1').AddComment.Text('Isentropic Area of Valve');

HydRes.Range('G1').AddComment.Text('Calculated Coefficients a1-a4. Do not be concerned about the initial divide by zero error. These values will regenerate as you input your parameters.');

HydRes.Range('H1').AddComment.Text('Enter the coefficients a1-a4 relating to the pump numbers you entered on the previous page.');

HydRes.Name='Hydraulic Resistances'; %rename sheet 3

invoke(HydRes,'Activate'); %make sheet2 the active sheet in the excel module

response3=menu('Enter your system parameters. See cell comments. Push OK when finished.','OK');

%the cell ranges which will be retrieved from excel

field1=sprintf('G2:G%d',(t+1));

field3=sprintf('H2:H%d',(t+1));

field4=sprintf('I2:I%d',(t+1));

field5=sprintf('M2:M%d',(p+1));

field6=sprintf('N2:N%d',(p+1));

field7=sprintf('O2:O%d',(p+1));

field8=sprintf('P2:P%d',(p+1));

field9=sprintf('B2:B%d',(t+1));

field10=sprintf('L2:L%d',(p+1));

if response3==1
    %retrieve the calculated coefficients from excel
    temp = get(HydRes,'Range',field1);
    p1 = temp.value;
    p1 = reshape([p1{:}], size(p1));

    temp = get(HydRes,'Range',field3);
    p3 = temp.value;
    p3 = reshape([p3{:}], size(p3));

    temp = get(HydRes,'Range',field4);
    p4 = temp.value;
    p4 = reshape([p4{:}], size(p4));
temp = get(HydRes, 'Range', field5);
o1 = temp.value;

temp = get(HydRes, 'Range', field6);
o2 = temp.value;

temp = get(HydRes, 'Range', field7);
o3 = temp.value;

temp = get(HydRes, 'Range', field8);
o4 = temp.value;

temp = get(HydRes, 'Range', field9);
Pmdot1 = temp.value; % get the mass flow rates through the pipes w/o pumps
Pmdot1 = reshape([Pmdot1{:}], size(Pmdot1));

temp = get(HydRes, 'Range', field10);
Pmdot2 = temp.value; % get the mass flow rates through the pipes w/ pumps

if p̸= 1
    o1 = reshape([o1{:}], size(o1));
    o2 = reshape([o2{:}], size(o2));
    o3 = reshape([o3{:}], size(o3));
    o4 = reshape([o4{:}], size(o4));
    Pmdot2 = reshape([Pmdot2{:}], size(Pmdot2));
end

clear field1 field3 field4 field5 field6 field7 field8 field9 field10 temp

% create the a1, a2, a3, a4 arrays
q = 1;
a1 = zeros(1, 1);
a2 = zeros(1, 1);
a3 = zeros(1, 1);
a4 = zeros(1, 1);
Pmdot = zeros(1, 1); % pipe mass flow rates (overall)
for i = 1:1
    if pumps(i) == 0
        a1(i, 1) = p1(q);
        a3(i, 1) = p3(q);
        a4(i, 1) = p4(q);
Pmdot{i,1}=Pmdot1(q);
q=q+1;

else
    a1(i)=o1(pumps(i));
a2(i)=o2(pumps(i));
a3(i)=o3(pumps(i));
a4(i)=o4(pumps(i));
Pmdot{i,1}=Pmdot2(pumps(i));
end
end

clear o1 o2 o3 o4 p1 p3 p4 temp

[CH,HX,massedge]=chxparam(Pmdot,chlhx,c,k,h,Cpl,Cpair,rho);

%overlay Chillers into excel on sheet 4
r=sprintf('A1:U%d',1+c);
Range=get(ChilB,'Range',r);
set(Range,'Value',CH);

%chiller/boiler Sheet Style Formatting
%make the heading rows bold
ChilB.Range('A1:U1').Font.Bold=1;

%color columns pertaining to chiller numbers
r=sprintf('A2:A%d',1+c);
ChilB.Range(r).Interior.ColorIndex=33;

%highlight necessary inputs
r=sprintf('C2:C%d',1+c);
ChilB.Range(r).Interior.ColorIndex=6; %color T1 columns

r=sprintf('D2:D%d',c+1);
ChilB.Range(r).Interior.ColorIndex=6; %color Tw columns

r=sprintf('K2:U%d',c+1);
ChilB.Range(r).Interior.ColorIndex=6; %color Chiller coefficient columns

%auto fit all columns
ChilB.Range('A:J').Columns.AutoFit;
%center columns pertaining to chiller numbers
ChilB.Range('A:A').HorizontalAlignment=3;

%format cell borders
r=sprintf('A1:U%d',1+c);
ChilB.Range(r).Border.LineStyle=1;

%Add Instruction comments
ChilB.Range('A1').AddComment.Text('Enter the parameters relating to the chillers/boilers corresponding to these energy flow vertex numbers');
ChilB.Range('B1').AddComment.Text('Heat Source Flow');
ChilB.Range('C1').AddComment.Text('Liquid Temperature at operating point');
ChilB.Range('D1').AddComment.Text('Wall Temperature at operating point');
ChilB.Range('E1').AddComment.Text('Mass Flow of liquid (from previous input)');
ChilB.Range('F1').AddComment.Text('Cross-sectional area');
ChilB.Range('G1').AddComment.Text('Length');
ChilB.Range('H1').AddComment.Text('Wall Mass');
ChilB.Range('I1').AddComment.Text('Overall Heat Transfer Coefficient');
ChilB.Range('J1').AddComment.Text('Specific Heat Capacity');
ChilB.Range('K1').AddComment.Text('Chiller/Boiler Coefficients: q1–q4, r1–r7. All have units of (1/s).');

%Name the chiller parameter sheet
ChilB.Name='Chillers';
invoke(ChilB,'Activate');
response4=menu('Enter your system parameters. See cell comments. Push OK when finished.','OK');

q=11;
for i=3:13
    ChRange{i,1}={sprintf('%s2:%s%d',Alphabet(q),Alphabet(q),c+1)}; %the ranges
end

% retrieve cell ranges
CHrange=struct('rng',0,'val',0); % data structure which will hold all the ranges
% and coefficients to be pulled
% from the chiller/boiler calculation sheet

CHrange(1).rng=sprintf('%s2:%s%d',Alphabet(3),Alphabet(3),c+1); % the range to
% retrieve for Tl
CHrange(2).rng=sprintf('%s2:%s%d',Alphabet(4),Alphabet(4),c+1); % the range to
% retrieve for Tw

q=11; % counter starting with letter K
for i=3:13
    CHrange(i).rng=sprintf('%s2:%s%d',Alphabet(q),Alphabet(q),c+1); % the ranges
    for q1=q4 and r1=r7
        q=q+1;
    end
end
if response4==1
    for i=1:13 % pull the input and calculated information off the chiller
        temp=get(ChilB,'Range',CHrange(i).rng);
        temp1=temp.value;
        if length(temp1)̸=1
            temp1=reshape([temp1{1}],size(temp1));
        end
        CHrange(i).val=temp1;
        temp-get(ChilB,'Range',CHrange{i,1});
        CHrange{i,2}=temp.value;
        CHrange{i,2}=reshape(Chrange{i,2},size(Chrange{i,2}))
    end
    clear temp temp1
r=sprintf('A1:AD%d',1+h);
Range-get(LAHX,'Range',r);
set(Range,'Value',HX); % overlay the heat exchanger input sheet onto excel
% LAHX Sheet Style Formatting

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%make the heading rows bold
LAHX.Range('A1:AD1').Font.Bold=1;

%color columns pertaining to chiller numbers
r=sprintf('A2:A%d',(1+h));
LAHX.Range(r).Interior.ColorIndex=33;

%highlight columns of LAHX coefficients which require input
r=sprintf('O2:O%d',(1+h)); %q2
LAHX.Range(r).Interior.ColorIndex=6;

r=sprintf('R2:R%d',(1+h)); %r1
LAHX.Range(r).Interior.ColorIndex=6;

r=sprintf('T2:T%d',(1+h)); %r3
LAHX.Range(r).Interior.ColorIndex=6;

%auto fit all input
LAHX.Range('A:M').Columns.AutoFit;
LAHX.Range('AB1:AD2').Columns.AutoFit;

%center columns pertaining to HX numbers
LAHX.Range('A:AC').HorizontalAlignment=3;

%format cell borders
r=sprintf('A1:AA%d',1+h);
LAHX.Range(r).Border.LineStyle=1;
LAHX.Range('AB1:AD2').Border.LineStyle=1;

%Add Instruction comments
LAHX.Range('A1').AddComment.Text('Enter the parameters relating to the heat exchangers corresponding to these energy flow vertex numbers');
LAHX.Range('B1').AddComment.Text('Heat Sink Flow − Q_{dot\_sink} should be negative if analyzing a cooling system');
LAHX.Range('C1').AddComment.Text('Liquid Temperature at operating point');
LAHX.Range('D1').AddComment.Text('Wall Temperature at operating point');
LAHX.Range('E1').AddComment.Text('Air Inlet Temperature at operating point');
LAHX.Range('F1').AddComment.Text('Mass Flow of liquid (from previous input)');
LAHX.Range('G1').AddComment.Text('Mass Flow of Air at operating point');
LAHX.Range('H1').AddComment.Text('1−exp(NTU)');
LAHX.Range('I1').AddComment.Text('Cross-sectional area');
LAHX.Range('J1').AddComment.Text('Length');
LAHX.Range('K1').AddComment.Text('Wall Mass');
LAHX.Range('L1').AddComment.Text('Specific Heat Capacity');
LAHX.Range('M1').AddComment.Text('Overall Heat Transfer Coefficient');
LAHX.Range('N1').AddComment.Text('LAHX Coefficients: q1−q4, r1−r7, sl−s3. All have units of (1/s). Coefficients requiring input are colored yellow.');

% Name the LAHX parameter sheet
LAHX.Name='LAHX';
invoke(LAHX,'Activate');
response5=menu('Enter your system parameters. See cell comments. Push OK when finished.', 'OK');

HXrange=struct('rng',0,'val',0); % data structure which will hold all the ranges and coefficients that will be pulled from the LAHX calculation sheet

% HXrange=cell(16,2);

% HXrange(1,1)={sprintf('%s2:%s%d',Alphabet(3),Alphabet(4),h+1)};
% HXrange(2,1)={sprintf('%s2:%s%d',Alphabet(4),Alphabet(4),h+1)};
%
% q=14; % counter starting with letter N (the position of coefficient q1)
% for i=3:16
% HXrange(i,1)={sprintf('%s2:%s%d',Alphabet(q),Alphabet(q),h+1)};
% q=q+1;
% end

HXrange(1).rng=sprintf('%s2:%s%d',Alphabet(3),Alphabet(3),h+1); % range for Tl
HXrange(2).rng=sprintf('%s2:%s%d',Alphabet(4),Alphabet(4),h+1); % range for Tw

q=14; % counter starting with letter N (the position of coefficient q1)
for i=3:15
    HXrange(i).rng=sprintf('%s2:%s%d',Alphabet(q),Alphabet(q),h+1);
    q=q+1;
end

HXrange(16).rng=sprintf('%s2:%s%d',Alphabet(3),Alphabet(3),h+1);

if response5==1
for i=1:16
    temp=get(LAHX,'Range',HXrange(i).rng);
    templ=temp.value;
    if length(templ)ʻ1
        templ=reshape([templ{;}],size(templ));
    end
    HXrange(i).val=templ;
end
end
end
end

% MATRIX PROCESSING

%following code will need to be cleaned up — now working under assumption
%that user numbers chillers/LAHX's in prescribed order

%Since the coefficients are pulled off of the Excel file in left to right
%order, the EVertData data structure will keep the matrices pertaining to
%those coefficients in that same order. For reference —
%EVertData(1) :: Tl — needed for matrix W1, W2
%EVertData(2) :: Tw — needed for matrix W1, W2
%EVertData(3) :: Q1 ... EVertData(6) :: Q4
%EVertData(7) :: R1 ... EVertData(13):: R7
%EVertData(14):: S1 ... EVertData(16):: S3

%put Tl, Tw, q1-q4, r1-r7, and s1-s3 values for chillers and LAHX's into one array
EVertData=struct('mat',0);
for i=3:13
    temp1=CHrange(i).val;
    % display(temp1);
    temp2=HXrange(i).val;
    % display(temp2);
    tempcoeff=[temp1; temp2];
    EVertData(i).mat=diag(tempcoeff);
%use following code if the user has no restriction on chiller/LAHX

%number scheme
r=1; p=1;
for q=1:k
    if chlhx(q)==1
        tempcoeff(q)=temp1(r);
        r=r+1;
    else
        tempcoeff(q)=temp2(p);
        p=p+1;
    end
end

%all Chiller/LAHX matrices are the diag of their respective coefficient vectors except for R5, R6, R7, and S3, which requires further manipulation

%— the following code modifies the previously obtained diag matrices for R5, R3, R4, and S3

%modify R3
EVertData(9).mat=EVertData(9).mat(:,c+1:end);

%modify R4
EVertData(10).mat=EVertData(10).mat(:,(c+1):end);

%modify R7
EVertData(13).mat=EVertData(13).mat(:,1:c);

%create S1
EVertData(14).mat=diag(HXrange(14).val);

%create S2
EVertData(15).mat=diag(HXrange(15).val);

%create/modify S3
templ=zeros(h,1);
temp2=[templ;HXrange(16).val];
EVertData(16).mat=diag(temp2);
clear temp temp2 tempcoeff

Tl=[CHrange(1).val;HXrange(1).val];
display(Tl);

[W1,W2]=jncmtx(Tl,Tj,rho,V,j,1,massedge); % function to construct the junction matrices W1, W2

[A1,A2,A3,A4]=pvcoeff(a1,a2,a3,a4,pumps,values,1);
temp2=zeros(j);
temp3=zeros(j,k);

% Full-order state space representation
Afo=[A1, -A3*A', zeros(1,j), zeros(1,k), zeros(1,k); B1*A, temp2, temp2, temp3, temp3; W1*C, temp2, temp2, temp3, temp3; W2*Be, temp3; EVertData(4).mat*E*C, zeros(k,j), ... 
EVertData(3).mat*E*Bm, EVertData(3).mat*E*Be+EVertData(5).mat, EVertData(6).mat; ... 
EVertData(7).mat*E+C, zeros(k,j), EVertData(8).mat*E+Bm, EVertData(8).mat*E*Be+EVertData(11).mat, EVertData(12).mat];

Bfo=[A4, A2, zeros(1,c), zeros(1,j), zeros(1,h), zeros(1,h); zeros(k,nnz(valves)), ... 
zeros(k,p), EVertData(13).mat, EVertData(9).mat, EVertData(10).mat];
Bfo=sparse(Bfo);
clear temp temp2 temp3

% display(full(A3)); display(full(A1)); display(A);
% temp1=A3*A';
% display(temp1);
% temp=[A1, -A3*A', zeros(1,j), zeros(1,k), zeros(1,k)]
% tp=[B1*A, temp2, temp2, temp3, temp3]
% tp1=[W1*C, temp2, W2*Bm, W2*Be, temp3]
% tp2=[EVertData(4).mat*E*C, zeros(k,j), EVertData(3).mat*E+Bm, EVertData(3).mat*E*Be+EVertData(5).mat, EVertData(6).mat]
% tp3=[EVertData(7).mat*E+C, zeros(k,j), EVertData(8).mat*E+Bm, EVertData(8).mat*E*Be+EVertData(11).mat, EVertData(12).mat]
% temp1=full([A4, A2, zeros(1,c), zeros(1,j), zeros(1,h)])
% tp=full(temp)
% tp1=[zeros(k,nnz(valves)), zeros(k,p), EVertData(13).mat, EVertData(9).mat, EVertData(10).mat]
clear temp temp2 temp3
Reduced-order state space representation

\[ \text{tempmat} = \text{inv}(W2 \cdot Bm); \] % a repeated operation for Z1 and Z2

\[ Z1 = (EVertData(3).mat \cdot E - EVertData(3).mat \cdot E \cdot Bm \cdot \text{tempmat} \cdot W2) \cdot B + \text{EVertData(5).mat}; \]

\[ Z2 = (EVertData(4).mat \cdot E - EVertData(3).mat \cdot E \cdot Bm \cdot \text{tempmat} \cdot W1) \cdot C; \]

Afd = \[A1, -A3 \cdot A'; B1 \cdot A, zeros(j, j)\];

\% Afd=Afd(1:(j+1-p),1:(j+1-p));

Afd=Afd(1:(j+1-1),1:(j+1-1));

Bfd = \[A4, A2; zeros(j, nnz(valves)), zeros(j, p)\];

\% Bfd=Bfd(1:(l+j-1-p),:);

Bfd=Bfd(1:(l+j-1-1),:);

P = \[\text{eye}(1,1), zeros(1, j-1)\];

Z3 = \[-P \cdot \text{inv}(\text{Afd}) \cdot \text{Bfd}; \]

size(Z3)

Y1 = tempmat \cdot (W2 \cdot B + \text{inv}(Z1) \cdot Z2 - W1 \cdot C);

Y2 = tempmat \cdot W2 \cdot B + \text{inv}(Z1) \cdot EVertData(6).mat;

tempmat1 = EVertData(8).mat \cdot E \cdot Bm;

tempmat2 = EVertData(8).mat \cdot E \cdot Be;

\% test = \(EVertData(7).mat \cdot E \cdot C + \text{tempmat1} \cdot Y1 - (\text{tempmat2} + \text{EVertData(9).mat}) \cdot \text{inv}(Z1) \cdot Z2 \) \cdot Z3

Bro = \(\text{EVertData(7).mat} \cdot E \cdot C + \text{tempmat1} \cdot Y1 - (\text{tempmat2} + \text{EVertData(11).mat}) \cdot \text{inv}(Z1) \cdot Z2 \) \cdot Z3, EVertData(13).mat, EVertData(9).mat, EVertData(10).mat);

Aro = EVertData(12).mat + tempmat1 \cdot Y2 - (tempmat2 + EVertData(11).mat) \cdot \text{inv}(Z1) \cdot EVertData(6).mat;

Cro = EVertData(16).mat;

Dro = \[\text{zeros}(h, \text{nnz(valves)}), \text{zeros}(h, p), \text{zeros}(h, c), \text{EVertData(14).mat}, \text{EVertData(15).mat}\];

end

function [PV, chxlab, CON] = paramlab(l, pipe, Ve)

\%Pump and Valve Number Input Sheet

PV = cell(1+l, 1); %create a cell structure to be overlayed onto an excel spreadsheet

chxlab = cell(1+length(Ve), 2);

CON = cell(5, 2);

\%heading cells along row 1

PV(1, 1) = {"Pipe #"};
PV(1,2) = {'Pipe Sequence'};
PV(1,3) = {'Pump #'};
PV(1,4) = {'Valve #'};
chxlab(1,1) = {'Ve #'};
chxlab(1,2) = {'C or H?'};

CON(1,1) = {'System Constants'}; % better name?
CON(2,1) = {'Beta (kPa)'};
CON(3,1) = {'Rho (kg/m^3)'};
CON(4,1) = {'Cpl (kJ/kg.K)'};
CON(5,1) = {'Cpair (kJ/kg.K)'};

for i = 1:l
PV(1+i,1) = {i}; % the pipe numbers go along the first column
end

for i = 1:l
    temp = sprintf('%d ', pipe(i).seq);
    PV(1+i,2) = {temp};
end

for i = 1:l
    PV(1+i,3) = {0};
P(1+i,4) = {0};
end

for i = 1:length(Ve)
    chxlab((1+i),1) = {Ve(i)};
end

clear i
end

function JT = junction(j, Vm, beta, rho)

JT = cell(1+j, 7);

% heading labels
JT(1,1) = {'Mass Flow Vertex'};
JT(1,2)=\{Tj (deg C)\};
JT(1,3)=\{Pj (kPa)\};
JT(1,4)=\{V (m^3)\};
JT(1,5)=\{b (kPa/kg)\};
JT(1,6)=\{Beta (kPa)\};
JT(1,7)=\{Rho (kg/m^3)\};

%List Vm numbers
for i=1:j
    JT(1+i,1)=\{Vm(i)\};
end

for i=2:j+1
    b=sprintf('=($F$2/$G$2)*1/D%d',i);
    JT(i,5)=\{b\};
end

%Place the previously input constants rho and beta
JT(2,6)=\{beta\};
JT(2,7)=\{rho\};
end

function HR=HYDR(t,l,pumps,valves,Pdiff)

%Row 1: Hydraulic Resistance Parameter Labels
%Columns 2−6 and 8 are input fields for the user, columns 7,9, and 10 calculate
%coefficients a1, a3, a4. Columns 7−10 are returned as matrices to MATLAB.

HR=cell(1+t,9); %create a cell structure which contains the a1,a3,a4 computation information

%create heading labels in row 1
HR(1,1)=\{Pipe #\};
HR(1,2)=\{m_dot(kg/s)\};
HR(1,3)=\{Pin−Pout(kPa)\};
HR(1,4)=\{Ac (mm^2)\};
HR(1,5)=\{L (m)\};
HR(1,6)=\{AI (mm^2)\};
HR(1,7)=\{a1 (1/s)\};
HR(1,8)=\{a3 ((kg/s^2)/kPa)\};
HR(1,9)=\{a4 ((kg/s^2)/mm^2)\};
%Column 1: List Pipe Numbers
w=1;
for i=1:l
    if pumps(i)==0
        HR(1+w,1)={i};
        w=w+1;
    end
end
clear i w

%Column 3: List pressure differences across the pipes from input junction pressures on Junction input sheet
w=2;
for i=1:l
    if pumps(i)==0
        HR(w,3)={Pdiff(i)};
        w=w+1;
    end
end

%Coefficient Calculations
%Column 7: a1 Calculations
for i=2:(1+t)
a1=sprintf('=−2*(D%d/E%d)*(C%d/B%d)/1000',i, i, i, i);
HR(i,7)={a1};
end
clear a1 i

%Column 9: a3 Calculations
for i=2:(1+t)
a3=sprintf('=(D%d/E%d)/1000',i, i);
HR(i,8)={a3};
end
clear a3

%Column 10: a4 Calculations
n=2;
for i=1:l
    if pumps(i)==0

if valves(i)==0
    a4=0;
    HR(n,9)={a4};
    HR(n,6)={'No Valve'};
    n=n+1;
else
    a4=sprintf('=10^6*(D%d/E%d)*(B%d^2)/(F%d^3)/1048',n,n,n,n);
    HR(n,9)={a4};
    n=n+1;
end
end
clear a4

function PP=PMP(p)
%Pump Coefficient Inputs
%the user will input each coefficient, there are no calculations for
%a1,a2,a3,a4 for the pumps
PP=cell(1+p,6);
for i=1:p
    PP((1+i),1)={i};
end

function [CH,HX,massedge]=chxparam(Pmdot,chlx,c,k,h,Cpl,Cpair,rho) %ADD MASS FLOW RATES FROM HR PAGE
852
853 CH=cell((1+c),26);
854 HX=cell((1+h),29);
855
856 %chiller parameter headings  UPDATE UNITS!
857 CH(1,1)={'Chiller'};
858 CH(1,2)={'Q_{dot source} (W)'};
859 CH(1,3)={'T_{l^*} (deg\text{C})'};
860 CH(1,4)={'T_{w^*} (deg\text{C})'};
861 CH(1,5)={'m_{dot liq^*} (kg/s)'};
862 CH(1,6)={'A C (mm^2)'};
863 CH(1,7)={'L (m)'};
864 CH(1,8)={'Mw (kg)'};
865 CH(1,9)={'UA (kW/K)'}; %units
866 CH(1,10)={'Cw (kJ/kg.K)'};
867 CH(1,11)={'q1'};
868 CH(1,12)={'q2'};
869 CH(1,13)={'q3'};
870 CH(1,14)={'q4'};
871 CH(1,15)={'r1'};
872 CH(1,16)={'r2'};
873 CH(1,17)={'r3'};
874 CH(1,18)={'r4'};
875 CH(1,19)={'r5'};
876 CH(1,20)={'r6'};
877 CH(1,21)={'r7'};
878
879 q=2;
880 o=2;
881
882 %show the number of the energy flow vertices for the respective chiller or
883 %LAHX
884 for i=1:k
885    if chlhx(i)==1
886       CH(q,1)={Ve(i)};
887       q=q+1;
888    else
889       HX(o,1)={Ve(i)};
890       o=o+1;
891    end
massedge = C*Pmdot; % diagonal matrix containing the mass flow rates of each edge
mdotliq = nonzeros(E*massedge); % mass flow of liquid through the heat exchangers and chillers

o = 2;
q = 2;

for i = 1:length(mdotliq)
    if chlhx(i) == 1
        CH(o, 5) = [mdotliq(i)]; % display mass flow of liquid through chillers on respective calculation sheet
        o = o + 1;
    else
        HX(q, 6) = [mdotliq(i)];
        q = q + 1;
    end
end

% heat exchanger parameter headings
HX(1, 1) = {'Heat Exchanger'};
HX(1, 2) = {'Qdot_sink* (W)'};
HX(1, 3) = {'Tl* (deg_C)'}; % state: temperature of liquid
HX(1, 4) = {'Tw* (deg_C)'}; % state: temperature of wall
HX(1, 5) = {'Tair_in* (deg_C)'}; % system input: inlet temperature
HX(1, 6) = {'m_dot_liquid* (kg/s)'};
HX(1, 7) = {'m_dot_air* (kg/s)'};
HX(1, 8) = {'Fhx'}; % calculation(1-exp(NTU))
HX(1, 9) = {'Ac (mm^2)'}; % input: cross sectional area
HX(1, 10) = {'L (m)'}; % input: equivalent length of pipe
HX(1, 11) = {'Mw (kg)'}; % input: wall mass
HX(1, 12) = {'Cw (kJ/kg.K)'}; % input: specific heat capacity of LAMX
HX(1, 13) = {'UA (kW/K)'}; % overall heat transfer coefficient
HX(1, 14) = {'q1'};
HX(1, 15) = {'q2'};
HX(1, 16) = {'q3'};
HX(1, 17) = {'q4'};
HX(1, 18) = {'r1'}; % simulation/input by user
HX(1,19) = \{'r2\'}; \text{ % will be zero}
HX(1,20) = \{'r3\'}; \text{ % input by user}
HX(1,21) = \{'r4\'};
HX(1,22) = \{'r5\'};
HX(1,23) = \{'r6\'};
HX(1,24) = \{'r7\'}; \text{ % is zero for LAHX's}
HX(1,25) = \{'s1\'};
HX(1,26) = \{'s2\'};
HX(1,27) = \{'s3\'};
HX(1,28) = \{'Cpl \ (kJ/kg.K)\'};
HX(1,29) = \{'Cpair \ (kJ/kg.K)\'};
HX(1,30) = \{'Rho \ (kg/m^3)\'};

HX(2,28) = \{'Cpl\'}; \text{ % specific heat of working fluid (system constant)}
HX(2,29) = \{'Cpair\'}; \text{ % specific heat of air (system constant)}
HX(2,30) = \{'rho\'};

for i=2:h+1
    temp = sprintf('=F%d/(AD2*I%d*J%d*0.000001)', i, i, i); \text{ % calculate q1}
    HX(i,14) = \{temp\};
    temp = sprintf('=B%d/(C%d-D%d)', i, i, i); \text{ % calculate UA}
    HX(i,13) = \{temp\};
    temp = sprintf('=-1/(AD2*I%d*J%d*0.000001)*(F%d+M%d/AB2)', i, i, i, i); \text{ % calculate q3}
    HX(i,16) = \{temp\};
    temp = sprintf('=M%d/((AD2*I%d*J%d*0.000001)*AB2)', i, i, i); \text{ % calculate q4}
    HX(i,17) = \{temp\};
    HX(i,19) = \{0\}; \text{ % r2 is zero for LAHX's}
    temp = sprintf('=B%d/(G%d+AC2*(D%d-E%d))', i, i, i, i); \text{ % calculation of Fhx (1-exp(NTU))}
    HX(i,8) = \{temp\};
    temp = sprintf('=G%d+M%d+AC2/(K%d+L%d)', i, i, i, i); \text{ % calculate r4}
    HX(i,21) = \{temp\};
    temp = sprintf('=M%d/(K%d+L%d)', i, i, i); \text{ % calculate r5}
    HX(i,22) = \{temp\};
```
971  temp=sprintf('=−(U%d+V%d)',i,i); %calculate r6
972  HX(i,23)={temp};
973
974  HX(i,24)={0}; %is zero for LAHX's
975
976  temp=sprintf('=−K%d*L%d*T%d',i,i,i); %calculate s1
977  HX(i,25)={temp};
978
979  temp=sprintf('=−G%d*AC2*H%d',i,i); %calculate s2
980  HX(i,26)={temp};
981
982  temp=sprintf('=G%d*AC2*H%d',i,i); %calculate s3
983  HX(i,27)={temp};
984
985  end
986
987  end
988
989  function [W1,W2]=jncmtx(Tl,Tj,rho,V,j,l,massedge) %jncmtx calculates the matrices pertaining
to junctions
990
991  global C
992  global D
993  global Be
994  global Bm
995
996  massinc=D*diag(massedge); %incidence matrix of mass flow rates of junctions
997
998  temp=size(C,1); % # of edges
999  W2=zeros(j,temp);
1000  B=horzcat(Bm,Be);
1001  Ttemp=vertcat(Tj,Tl); %the temperature at the inlet of junctions and energy flow vertices
1002  Tedges=B*Ttemp;
1003  W1=D*diag(Tedges);
```
for i=1:j
  for o=1:temp
    W2(i,o)=(1/(rho*V(i)))*massinc(i,o);
    W1(i,o)=(1/(rho*V(i)))*W1(i,o);
  end
end

function [A1,A2,A3,A4]=pvcoeff(a1,a2,a3,a4,pumps,valves,l)
  A1=sparse(diag(a1));
  A3=sparse(diag(a3));

  %create A2
  s1=zeros([1 nnz(pumps)]); s2=zeros([1 nnz(pumps)]); s=zeros([1 nnz(pumps)]);
  z=1;
  for u=1:l
    if pumps(u) ≠ 0
      s1(z)=u;
      s2(z)=pumps(u);
      s(z)=a2(u);
      z=z+1;
    end
  end
  A2=sparse(s1,s2,s,l,nnz(pumps));
  display(full(A2));
  clear s s1 s2 u z

  %create A4
  s1=zeros([1 nnz(valves)]); s2=zeros([1 nnz(valves)]); s=zeros([1 nnz(valves)]);
  z=1;
  for u=1:l
    if valves(u) ≠ 0
      s1(z)=u;
      s2(z)=valves(u);
      s(z)=a4(u);
      z=z+1;
    end
  end
end
A.4 Reduced order model for the chilled water system

The reduced order model for the system, obtained using the modeling procedure described in chapter 5 is shown in Figure A.3, which is a snapshot of the MATLAB command window.
Figure A.3: State space matrices for the chilled water system
Appendix B

The MATLAB .m files used for the implementation of the various control schemes on the test system in chapter 7 are presented in this Appendix. The corresponding THERMOSYS model have not been presented here due to space considerations. However, they are made available in the digital media which accompanies this work.

B.1 Implementation of Traditional Control Schemes in MATLAB

The following user-defined function, ‘onoff.m’ was used for implementation of the on-off controller in THERMOSYS:

```matlab
1 % onoff.m
2 % IMPLEMENTATION OF ON-OFF CONTROL
3 function blah = onoff(u)
4 global umax umin prev Nx Ny Nu cnt
5 yk = zeros(6,1);  
6 y0 = zeros(6,1); 
7 yk = (u(1:Ny));  
8 y0 = (u(Ny+1:2*Ny));  
9 t = u(end)-5000;
10 theta = 0.004; %larger margin is needed than stipulated due to controller fighting
11 if(t<cnt)
12    blah = prev;
13 else
14
cnt = cnt+1;
```
16 if(sum(yk)>sum(y0)+theta*6)
17   u7 = umin(7);
18   u8 = umin(7);
19 else if(sum(yk)<sum(y0)-theta*6)
20   u7 = umax(7);
21   u8 = umax(8);
22 else
23   u7 = prev(7);
24   u8 = prev(8);
25 end

26 if(yk(1)>y0(1)+theta)
27   u1 = umax(1);
28 else if(yk(1)<y0(1)-theta)
29   u1 = umin(1);
30 else
31     u1 = prev(1);
32 end

33 if(yk(2)>y0(2)+theta)
34     u2 = umax(2);
35 else if(yk(2)<y0(2)-theta)
36     u2 = umin(2);
37 else
38       u2 = prev(2);
39 end

40 if(yk(3)>y0(3)+theta)
41     u3 = umax(3);
42 else if(yk(3)<y0(3)-theta)
43     u3 = umin(3);
44 else
45       u3 = prev(3);
46 end

47 if(yk(4)>y0(4)+theta)
48     u4 = umax(4);
49 else if(yk(4)<y0(4)-theta)
50     u4 = umin(4);
51 else
52   u4 = umax(4);
53 else if(yk(4)<y0(4)-theta)
54     u4 = umin(4);
55 else
\[ u_4 = \text{prev}(4); \]
\[ \text{end} \]

\[ \text{if}(y_k(5) > y_0(5) + \theta) \]
\[ u_5 = \text{umax}(5); \]
\[ \text{elseif} \ (y_k(5) < y_0(5) - \theta) \]
\[ u_5 = \text{umin}(5); \]
\[ \text{else} \]
\[ u_5 = \text{prev}(5); \]
\[ \text{end} \]

\[ \text{if}(y_k(6) > y_0(6) + \theta) \]
\[ u_6 = \text{umax}(6); \]
\[ \text{elseif} \ (y_k(6) < y_0(6) - \theta) \]
\[ u_6 = \text{umin}(6); \]
\[ \text{else} \]
\[ u_6 = \text{prev}(6); \]
\[ \text{end} \]
\[ \text{blah} = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8]^\prime; \]
\[ \text{end} \]
\[ \text{prev} = \text{blah}; \]

For implementing the decentralized PI scheme, the following code, ‘d-pi.m’ was used.

1 % d-pi.m
2 % Parameters for the D-PI control architecture
3
4 % system parameters
5
6 \text{w}_\text{nom} = 0;
7 \text{umax} = 25*[12.5 8.75 17.5 21.25 12.5 23.75 27 22]/20;
8 \text{umin} = -\text{umax};
9
10 \text{pow}_\text{nom} = 0;
11 \text{ppow}_\text{nom} = 0;
12 \text{cpow}_\text{nom} = 0;
13
14 %controller gains
B.2 Implementation of Centralized MPC Scheme in MATLAB

The parameters for centralized MPC are first generated using the following code, ’c-mpc-param.m’.

```matlab
% c-mpc-param.m
% PROGRAM FOR GENERATING THE RELEVANT PARAMETERS REQUIRED FOR C-MPC
%clear all
load ws % This is the workspace containing the continuous, reduced order state space model
clear prev cnt N Nx Nu Ny alpha beta gammah A B C umax umin dumax dumin Ts;
global prev cnt N Nx Nu Ny alpha beta1 beta2 gammah A B C umax umin dumax dumin Ts
warning off

%PART 1: Generation of State Space Matrices
Nx = 8;
Ny = 6;
Nu = 8;
prev = zeros(8,1);
cnt = 0;

%PART 2: Discretization of state space
Ts = (1/max(abs(eig(Acont))))/10; % Sampling time < (1/max(abs(eig(Acont))))/10
Ts = 1;
A = Acont*Ts + eye(Nx,Nx);
%A = exp(Acont*Ts);
B = Bcont*Ts;
C = Ccont;
%A = Ad;
B = Bd;
C = Cd;
```
Ts = 5;
[A,B,C,D] = c2dm(Acont,Bcont,Ccont,zeros(6,8),Ts,'zoh');
N = 15; %MPC control and cost horizon — assumed same

%MPC PARAMETERS
alpha1 = 1e5;
alpha2 = 5e3;
gam = 5;
pwork = [1.698e-3 2.359e-3 1.071e-3 2.291e-4 5.761e-4 1.804e-4];
beta1 = alpha1*pwork;
beta2 = 100*[1/3.0 1/2.0];
alpha = alpha2*[1 1 1 1 1 1];
gammah = gam*[1 1 1 1 1];
umax = 25*[12.5 8.75 17.5 21.25 12.5 23.75 27 22]/20;
umin = -umax;
dumax = 1*[1 1 1 1 1 1 1 1];
dumin = -dumax;
shy = le4*[1 1 1 1 1 1 1 1];

Next, the Hessian and constraint matrices are generated using the program 'c-mpc-matrices.m' below:

% c-mpc-matrices.m
% GENERATION OF HESSIAN AND CONSTRAINT MATRICES FOR C-MPC
global N Nx Nu Ny alpha gammah A B C umax umin H Ac bc dumax dumin Ts
warning off;

% Hessian Matrix Generation
H = zeros(Nu*N);
for p = 1:Nu
    for r = 0:N-1
        for q = 1:Nu
            for t = 0:N-1
                if((p==q) && (r==t))
                    summ = 0;
                    dumm = 0;
                    humm = 0;
                end
            end
        end
    end
end
for i = r+1:N
    simsim = zeros(Nx,Nx);
    for s = 0:i-(r+1)
        simsim = simsim + A^(s);
    end
    for j = 1:Ny
        summ = summ + alpha(j) * C(j:j,1:Nx) * (A^(i-(r+1))) * B(1:Nx,p:p) * C(j:j,1:Nx) * simsim * B(1:Nx,p:p)^2;
        humm = humm + gammah(j) * Ts^2 * C(j:j,1:Nx) * simsim * B(1:Nx,p:p) * C(j:j,1:Nx) * humm * simsim2 * B(1:Nx,1:Nx)^2;
    end
end
if((r==0)||(r==N-1))
    dumm = shy(p);
else
    dumm = 2*shy(p);
end
H((p-1)*N+r+1,(p-1)*N+r+1) = summ+dumm+humm;
else
    summ = 0;
    dumm = 0;
    humm = 0;
    for i = max(r,t)+1:N
        simsim1 = zeros(Nx,Nx);
        for s = 0:i-(r+1)
            simsim1 = simsim1 + A^(s);
        end
        simsim2 = zeros(Nx,Nx);
        for s = 0:i-(t+1)
            simsim2 = simsim2 + A^(s);
        end
        for j = 1:Ny
            summ = summ + alpha(j) * C(j:j,1:Nx) * (A^(i-(r+1))) * B(1:Nx,p:p) * C(j:j,1:Nx) * (A^(i-(t+1))) * B(1:Nx,q:q);
            humm = humm + gammah(j) * Ts^2 * (C(j:j,1:Nx) * simsim1 * B(1:Nx,p:p) * C(j:j,1:Nx) * simsim2 * B(1:Nx,q:q));
        end
    end
end
if((p==q) && (abs(t-r)==1))
    dumm = -shy(p);
else
The following user-defined function, ‘cmpec.m’ was used for implementation of the controller in THERMOSYS:

```matlab
% cmpec.m

% IMPLEMENTATION OF C-MPC

function blah = cmpec(u)
```

1 % cmpec.m
2
3 function blah = cmpec(u)
clear F;
xk = (u(1:Nx));
y0 = (u(Nx+1:Nx+Ny));
errorr = (u(Nx+Ny+1:Nx+Ny+Ny));
t = u(end)−5000;

options = optimset();
options.Display = 'off';
options.MaxIter = 20000000000;

if(t<cnt)
    blah = prev;
else
    F = zeros(1,Nu*N);
    for p = 1:Nu
        for r = 0:N−1
            summ = 0;
            dumm = 0;
            humm = 0;
            for i = r+1:N
                simsim = zeros(Nx,Nx);
                for s = 0:i−(r+1)
                    simsim = simsim + Aˆ(s);
                end
                dimdim = zeros(Nx,Nx);
                for s = 0:i
                    dimdim = dimdim + Aˆ(s);
                end
                for j = 1:Ny
                    summ = summ + 2*alpha(j)*(C(j:j,1:Nx)*(Aˆi)*xk − y0(j))*(C(j:j,1:Nx)*(Aˆ(1−(r +1))))*B(1:Nx,p:p));
                    humm = humm + 2*gammah(j)*(C(j:j,1:Nx)*dimdim*xk*Ts − i*y0(j)*Ts+errorr(j))*(C(j:j,1:Nx)*simsim*B(1:Nx,p:p))*Ts;
                end
            end
            beta = [beta1 beta2];
            dumm = beta(p);
            F((p−1)*N+r+1) = summ+dumm+humm;
        end
end
the relevant parameters are first generated using the following code, ‘d-mpc1-param.m’. 

```matlab
% d-mpc1-param.m
% PARAMETERS FOR D-MPC
%clear all
load ws
% PART 1: Generation of State Space Matrices
Nx = 8;
Ny = 6;
Nu = 8;
prev = zeros(8,1);
cnt = 0;
% PART 2: Discretization of state space
Ts = (1/max(abs(eig(Acont))))/10; % Sampling time < (1/max(abs(eig(Acont))))/10
Ts = 1;
```
Next, relevant matrices are generated using the program ‘d-mpc1-matrices.m’ below:

```matlab
% d-mpc1-matrices.m
% MATRICES FOR D-MPC1
global cnt prev N A B C umax umin dumax dumin alpha beta1 beta2 gammah shy Nx Nu Ny H Hmm     
       Hs1s1 Hs2s2 Hms1 Hms2 Am bm As1 bs1 As2 bs2 Ts

% Hessian Matrix Generation
H = zeros(Nu*N);
for p = 1:Nu
    for r = 0:N
        % Hessian matrix calculation
    end
end
```
for q = 1:Nu
    for t = 0:N-1
        if((p==q)&&(r==t))
            summ = 0;
            dumm = 0;
            humm = 0;
            for i = r+1:N
                simsim = zeros(Nx,Nx);
                for s = 0:i-(r+1)
                    simsim = simsim + A^(s);
                end
                for j = 1:Ny
                    summ = summ + alpha(j)*(C(j:j,1:Nx)*(A^(i-(r+1)))*B(1:Nx,p:p))^2;
                    humm = humm + gammah(j)*Ts^2*(C(j:j,1:Nx)*simsim*B(1:Nx,p:p))^2;
                end
            end
        end
        if((r==0)||(r==N-1))
            dumm = shy(p);
        else
            dumm = 2*shy(p);
        end
        H((p-1)*N+r+1,(p-1)*N+r+1) = summ+dumm+humm;
    else
        summ = 0;
        dumm = 0;
        humm = 0;
        for i = max(r,t)+1:N
            simsim1 = zeros(Nx,Nx);
            for s = 0:i-(r+1)
                simsim1 = simsim1 + A^(s);
            end
            simsim2 = zeros(Nx,Nx);
            for s = 0:i-(t+1)
                simsim2 = simsim2 + A^(s);
            end
            for j = 1:Ny
                summ = summ + alpha(j)*(C(j:j,1:Nx)*(A^(i-(r+1)))*B(1:Nx,p:p))*(
                    C(j:j,1:Nx)*(A^(i-(t+1)))*B(1:Nx,q:q));
humm = humm + gammah(j)*Ts^2*(C(j:j,1:Nx)*simsim1*B(1:Nx,p:p))*(C(j:j,1:Nx)*simsim2*B(1:Nx,q:q));

if((p==q) && (abs(t-r)==1))
    dumm = -shy(p);
else
    dumm=0;
end

H((p−1)*N+r+1,(q−1)*N+t+1) = summ+dumm+humm;

H = (H+H')/2;

%Local Hessians
Hmm = H(6*N+1:8*N,6*N+1:8*N);
Hs1s1 = H(1:3*N,1:3*N);
Hs2s2 = H(3*N+1:6*N,3*N+1:6*N);
Hms1 = H(1:3*N,6*N+1:8*N);
Hms2 = H(3*N+1:6*N,6*N+1:8*N);

%Constraints
A3 = zeros(N−1,N);
for i = 1:N−1
    A3(i,i) = −1;
    A3(i,i+1)= 1;
end

Am = [eye(2*N);−eye(2*N);blkdiag(A3,A3);−blkdiag(A3,A3)];

bm = [ones(N,1)*umax(7);ones(N,1)*umax(8);−ones(N,1)*umin(7);−ones(N,1)*umin(8);ones(N−1,1)*
dumax(7);ones(N−1,1)*dumax(8);−ones(N−1,1)*dumin(7);−ones(N−1,1)*dumin(8)];

As1 = [eye(3*N);−eye(3*N);blkdiag(A3,A3,A3);−blkdiag(A3,A3,A3)];
bs1 = [ones(N,1)*umax(1);ones(N,1)*umax(2);ones(N,1)*umax(3);−ones(N,1)*umin(1);−ones(N,1)*umin(2);−ones(N,1)*umin(3);ones(N−1,1)*dumax(1);ones(N−1,1)*dumax(2);ones(N−1,1)*dumax(3);−ones(N−1,1)*dumin(1);−ones(N−1,1)*dumin(2);−ones(N−1,1)*dumin(3)];

%Slave cluster 2 constraints
As2 = [eye(3*N);−eye(3*N);blkdiag(A3,A3,A3);−blkdiag(A3,A3,A3)];
bs2 = [ones(N,1)*umax(4);ones(N,1)*umax(5);ones(N,1)*umax(6);−ones(N,1)*umin(4);−ones(N,1)*umin(5);−ones(N,1)*umin(6);ones(N−1,1)*dumax(4);ones(N−1,1)*dumax(5);ones(N−1,1)*dumax(6);−ones(N−1,1)*dumin(4);−ones(N−1,1)*dumin(5);−ones(N−1,1)*dumin(6)];

The following user-defined function, ‘dmpc1.m’ was used for implementation of the controller in THERMOSYS:

% dmpc1.m
% IMPLEMENTATION OF D-MPC1
function blah = dmpc1(u)

global cnt prev N A B C umax umin dumax dumin alpha beta1 beta2 gammah shy Nx Nu Ny H Hmm Hs1s1 Hs2s2 Hms1 Hms2 Am bm As1 bs1 As2 bs2 Ts
xk = (u(1:Nx));
y0 = (u(Nx+1:Nx+Ny));
errorr = (u(Nx+Ny+1:Nx+Ny+Ny));
t = u(end)−5000;

options = optimset();
options.Display = 'off';
options.MaxIter = 20000000000;

if(t<cnt)
    blah = prev;
else
    F = zeros(1,Nu*N);
    for p = 1:Nu
        for r = 0:N−1
            summ = 0;
            dumm = 0;
            humm = 0;
            for i = r+1:N
                simsim = zeros(Nx,Nx);
                for s = 0:i−(r+1)
simsim = simsim + A^s;
end
dimdim = zeros(Nx,Nx);
for s = 0:i
dimdim = dimdim + A^s;
end
for j = 1:Ny
    summ = summ + 2*alpha(j)*(C(j:j,1:Nx)*(A^i)*xk - y0(j))*(C(j:j,1:Nx)*(A^(i-r+1))*B(1:Nx,p:p));
    humm = humm + 2*gammah(j)*(C(j:j,1:Nx)*dimdim*xk*Ts - i*y0(j)*Ts+errorr(j))*(C(j:j,1:Nx)*simsim*B(1:Nx,p:p))*Ts;
end
end
beta = [beta1 beta2];
dumm = beta(p);
F((p-1)*N+r+1) = summ+dumm+humm;
end
end

Fm = F(6*N+1:8*N);
Fs1 = F(1:3*N);
Fs2 = F(3*N+1:6*N);

% Initialization
us1 = [ones(N,1)*umin(1);ones(N,1)*umin(2);ones(N,1)*umin(3)];
us2 = [ones(N,1)*umin(4);ones(N,1)*umin(5);ones(N,1)*umin(6)];
um = zeros(2*N,1);

for iter = 1:3

% Master 1 Optimization
Fmm = Fm + 1*us1'*Hms1 + 1*us2'*Hms2;
um = quadprog(Hmm,Fmm',Am,bm,[],[],[],[],[],options);

% Cluster 1 Optimization
Fs1s1 = Fs1 + 1*um'*Hms1;
us1 = quadprog(Hs1s1,Fs1s1',As1,b1,[],[],[],[],[],options);

% Master 2 Optimization
Fmm = Fm + 1*us1'*Hms1 + 1*us2'*Hms2;
um = quadprog(Hmm,Fmm',Am,bm,[],[],[],[],[],options);

% Cluster 2 Optimization
Fs1s2 = Fs1 + 1*um'*Hms1;
us1 = quadprog(Hs1s1,Fs1s1',As1,b1,[],[],[],[],[],options);
%Cluster 2 Optimization
Fs2s2 = Fs2 + 1*um'*Hms2';
us2 = quadprog(Hs2s2,Fs2s2',As2,bs2,[],[],[],[],[],options);
end
blah = [us1(1);us1(N+1);us1(2*N+1);us2(1);us2(N+1);us2(2*N+1);um(1);um(N+1)];
cnt = cnt+1;
end
prev = blah;

B.3.2 D-MPC₂

The relevant parameters are first generated using the following code, ‘d-mpc2-param.m’.

% d-mpc2-param.m
% PARAMETERS FOR D-MPC2
%clear all
load ws
clear cnt prev N A B C umin dumax dumin alpha beta shy Nx Nu Ny H Hmm Hs1s1 Hs2s2 Hs3s3
Hs4s4 Hs5s5 Hs6s6 Hms1 Hms2 Hms3 Hms4 Hms5 Hms6 Am bm Aa1 bsa1 As2 bs2 Aa3 bs3 As4 bs4
Aa5 bs5 As6 bs6
global cnt prev N A B C umin dumax dumin alpha beta1 beta2 gammah shy Nx Nu Ny H Hmm
Hs1s1 Hs2s2 Hs3s3 Hs4s4 Hs5s5 Hs6s6 Hms1 Hms2 Hms3 Hms4 Hms5 Hms6 Am bm Aa1 bsa1 As2 bs2
Aa3 bs3 As4 bs4 As5 bs5 As6 bs6 Ts

% PART 1: Generation of State Space Matrices
Nx = 8;
Ny = 6;
Nu = 8;
prev = zeros(8,1);
cnt = 0;

%PART 2: Discretization of state space
% Ts = (1/max(abs(eig(Acont))))/10; % Sampling time < (1/max(abs(eig(Acont))))/10
% Ts = 1;
% A = Acont*Ts + eye(Nx,Nx);
% A = exp(Acont*Ts);
% B = Bcont*Ts;
% C = Ccont;
% A = Ad;
% B = Bd;
% C = Cd;
Ts = 5;
[A,B,C,D] = c2dm(Acont,Bcont,Ccont,zeros(6,8),Ts,'zoh');

N = 15; %MPC control and cost horizon - assumed same

%MPC PARAMETERS
alpha1 = 1e5;
alpha2 = 5e3;
gam = 5;
pwork = [1.698e−3 2.359e−3 1.071e−3 2.291e−4 5.761e−4 1.804e−4];
beta1 = alpha1*pwork;
beta2 = 100*[−1/3.0 −1/2.0];
alpha = alpha2*[1 1 1 1 1 1];
gammah = gam*[1 1 1 1 1 1];
umax = 25*[12.5 8.75 17.5 21.25 12.5 23.75 27 22]/20;
umin = −umax;
% dumax = Ts*[10*[1 1 1 1 1 1],0.02,0.02];
dumax = 1*[1 1 1 1 1 1 1];
dumin = −dumax;
shy = 1e4*[1 1 1 1 1 1 1 1];

Next, relevant matrices are generated using the program ‘d-mpc2-matrices.m’ below:
H = zeros(Nu*N);
for p = 1:Nu
    for r = 0:N-1
        for q = 1:Nu
            for t = 0:N-1
                if((p==q) && (r==t))
                    summ = 0;
                    dumm = 0;
                    humm = 0;
                    for i = r+1:N
                        simsim = zeros(Nx,Nx);
                        for s = 0:i-(r+1)
                            simsim = simsim + A^(s);
                        end
                        for j = 1:Ny
                            summ = summ + alpha(j)*(C(j:j,1:Nx)*(A^((i-(r+1)))*B(1:Nx,p:p)))*2;
                            humm = humm + gammah(j)*Ts^2*(C(j:j,1:Nx)*simsim*B(1:Nx,p:p))^2;
                        end
                    end
                    if((r==0) || (r==N-1))
                        dumm = shy(p);
                    else
                        dumm = 2*shy(p);
                    end
                    H((p-1)*N+r+1,(p-1)*N+r+1) = summ+dumm+humm;
                else
                    summ = 0;
                    dumm = 0;
                    humm = 0;
                    for i = max(r,t)+1:N
                        simsim1 = zeros(Nx,Nx);
                        for s = 0:i-(r+1)
                            simsim1 = simsim1 + A^(s);
                        end
                        simsim2 = zeros(Nx,Nx);
                        for s = 0:i-(t+1)
                            simsim2 = simsim2 + A^(s);
                        end
                        for j = 1:Ny
                            end
                        end
                    end
                end
            end
        end
    end
end
summ = summ + alpha(j)*(C(j:j,1:Nx)*(A^(i-(r+1)))\cdot B(1:Nx,p:p))\cdot (C(j:j,1:Nx)*(A^(i-(t+1)))\cdot B(1:Nx,q:q));

humm = humm + gammah(j)\cdot T_s^2\cdot (C(j:j,1:Nx)\cdot simsim1\cdot B(1:Nx,p:p))\cdot (C(j:j,1:Nx)\cdot simsim2\cdot B(1:Nx,q:q));

if((p==q)&&(abs(t-r)==1))
    dumm = -shy(p);
else
    dumm=0;
end

H((p-1)*N+r+1,(q-1)*N+t+1) = summ+dumm+humm;

H = (H+H')/2;

%Local Hessians

Hmm = H(6*N+1:8*N,6*N+1:8*N);
Hs1s1 = H(1:N,1:N);
Hs2s2 = H(N+1:2*N,N+1:2*N);
Hs3s3 = H(2*N+1:3*N,2*N+1:3*N);
Hs4s4 = H(3*N+1:4*N,3*N+1:4*N);
Hs5s5 = H(4*N+1:5*N,4*N+1:5*N);
Hs6s6 = H(5*N+1:6*N,5*N+1:6*N);

Hms1 = H(1:N,6*N+1:8*N);
Hms2 = H(N+1:2*N,6*N+1:8*N);
Hms3 = H(2*N+1:3*N,6*N+1:8*N);
Hms4 = H(3*N+1:4*N,6*N+1:8*N);
Hms5 = H(4*N+1:5*N,6*N+1:8*N);
Hms6 = H(5*N+1:6*N,6*N+1:8*N);

%Constraints

A3 = zeros(N-1,N);
for i = 1:N-1
    A3(i,i) = -1;
A3(i,i+1) = 1;

end

%Master Constraints
Am = [eye(2*N); -eye(2*N); blkdiag(A3,A3); -blkdiag(A3,A3)];

bm = [ones(N,1)*umax(7); ones(N,1)*umax(8); -ones(N,1)*umin(7); -ones(N,1)*umin(8); ones(N-1,1)*dumax(7); ones(N-1,1)*dumax(8); -ones(N-1,1)*dumin(7); -ones(N-1,1)*dumin(8)];

%Slave 1 constraints
As1 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs1 = [ones(N,1)*umax(1); -ones(N,1)*umin(1); ones(N-1,1)*dumax(1); -ones(N-1,1)*dumin(1)];

%Slave 2 constraints
As2 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs2 = [ones(N,1)*umax(2); -ones(N,1)*umin(2); ones(N-1,1)*dumax(2); -ones(N-1,1)*dumin(2)];

%Slave 3 constraints
As3 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs3 = [ones(N,1)*umax(3); -ones(N,1)*umin(3); ones(N-1,1)*dumax(3); -ones(N-1,1)*dumin(3)];

%Slave 4 constraints
As4 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs4 = [ones(N,1)*umax(4); -ones(N,1)*umin(4); ones(N-1,1)*dumax(4); -ones(N-1,1)*dumin(4)];

%Slave 5 constraints
As5 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs5 = [ones(N,1)*umax(5); -ones(N,1)*umin(5); ones(N-1,1)*dumax(5); -ones(N-1,1)*dumin(5)];

%Slave 6 constraints
As6 = [eye(N); -eye(N); blkdiag(A3); -blkdiag(A3)];
bs6 = [ones(N,1)*umax(6); -ones(N,1)*umin(6); ones(N-1,1)*dumax(6); -ones(N-1,1)*dumin(6)];

The following user-defined function, ‘dmpc2.m’ was used for implementation of the controller in THERMOSYS:

% dmpc2.m
% IMPLEMENTATION OF D-MPC2
function blah = dmpc2(u)
global cnt prev N A B C umax umin dumax dumin alpha beta1 beta2 gammah shy Nx Nu Ny H Hmm
Hs1s1 Hs2s2 Hs3s3 Hs4s4 Hs5s5 Hs6s6 Hms1 Hms2 Hms3 Hms4 Hms5 Hms6 Am bm As1 bs1 As2 bs2
As3 bs3 As4 bs4 As5 bs5 As6 bs6 Ts

xk = (u(1:Nx));
y0 = (u(Nx+1:Nx+Ny));
errorr = (u(Nx+Ny+1:Nx+Ny+Ny));
t = u(end)-5000;

options = optimset();
options.Display = 'off';
options.MaxIter = 20000000000;

if(t<cnt)
    blah = prev;
else
    F = zeros(1,Nu*N);
    for p = 1:Nu
        for r = 0:N-1
            summ = 0;
            dum = 0;
            humm = 0;
            for i = r+1:N
                simsim = zeros(Nx,Nx);
                for s = 0:i-(r+1)
                    simsim = simsim + A^(s);
                end
                dimdim = zeros(Nx,Nx);
                for s = 0:i
                    dimdim = dimdim + A^(s);
                end
                for j = 1:Ny
                    summ = summ + 2*alpha(j)*(C(j:j,1:Nx)*(A^i)*xk - y0(j))*(C(j:j,1:Nx)*(A^(i-(r +1)))*B(1:Nx,p:p));
                    humm = humm + 2*gammah(j)*(C(j:j,1:Nx)*dimdim*xk*Ts - i*y0(j)*Ts+errorr(j))*(C
(j:j,1:Nx)*simsim*B(1:Nx,p:p))*Ts;
                end
            end
            beta = [beta1 beta2];
            dum = beta(p);
            F((p-1)*N+r+1) = summ+dum+humm;
Fm = F(6*N+1:8*N);

Fs1 = F(1:N);
Fs2 = F(N+1:2*N);
Fs3 = F(2*N+1:3*N);
Fs4 = F(3*N+1:4*N);
Fs5 = F(4*N+1:5*N);
Fs6 = F(5*N+1:6*N);

% Initialization
us1 = 1*ones(N,1)*umin(1);
us2 = 1*ones(N,1)*umin(2);
us3 = 1*ones(N,1)*umin(3);
us4 = 1*ones(N,1)*umin(4);
us5 = 1*ones(N,1)*umin(5);
us6 = 1*ones(N,1)*umin(6);
um = zeros(2*N,1);

for iter = 1:3
  % Master 1 Optimization
  Fmm = Fm + 1*us1'*Hms1 + 1*us2'*Hms2 + 1*us3'*Hms3 + 1*us4'*Hms4 + 1*us5'*Hms5 + 1*us6'*Hms6;
um = quadprog(Hmm,Fmm',Am,bm,[],[],[],[],um,options);

  % Slave 1 Optimization
  Fs1s1 = Fs1 + 1*um'*Hms1';
  us1 = quadprog(Hs1s1,Fs1s1',As1,bs1,[],[],[],[],us1,options);

  % Slave 2 Optimization
  Fs2s2 = Fs2 + 1*um'*Hms2';
  us2 = quadprog(Hs2s2,Fs2s2',As2,bs2,[],[],[],[],us2,options);

  % Slave 3 Optimization
  Fs3s3 = Fs3 + 1*um'*Hms3';
  us3 = quadprog(Hs3s3,Fs3s3',As3,bs3,[],[],[],[],us3,options);

  % Slave 4 Optimization
Fs4s4 = Fs4 + 1*um'*Hms4';
us4 = quadprog(Hs4s4,Fs4s4',As4,bs4,[],[],[],[],us4,options);

%Slave 5 Optimization
Fs5s5 = Fs5 + 1*um'*Hms5';
us5 = quadprog(Hs5s5,Fs5s5',As5,bs5,[],[],[],[],us5,options);

%Slave 6 Optimization
Fs6s6 = Fs6 + 1*um'*Hms6';
us6 = quadprog(Hs6s6,Fs6s6',As6,bs6,[],[],[],[],us6,options);
end

blah = [us1(1);us2(1);us3(1);us4(1);us5(1);us6(1);um(1);um(N+1)];
cnt = cnt+1;
end
prev = blah;
References


[156] THERMOSYS. http://mr-roboto.mechse.illinois.edu/index.php?id=1161|THERMOSYS.


