EXPERIMENTAL STUDY OF LOW-ORDER MODELS OF HIGHLY-IRREGULAR ROUGHNESS AND THEIR IMPACT ON TURBULENT BOUNDARY LAYERS

BY

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DISSENRATION

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Abstract

The present effort explores the relative impact of various topographical scales present within irregular surface roughness on a turbulent boundary layer under both developing- and developed-flow conditions. Low-order representations of highly irregular surface roughness replicated from a turbine-blade damaged by deposition of foreign materials were generated using singular value decomposition to decompose the complex topography into a set of topographical basis functions of decreasing importance to the original “full” surface character. The low-order surface models were then formed by truncating the full set of basis functions at the first 5 and 16 modes (containing approximately 71% and 95% of the full surface content, respectively) so that only the most dominant and large-scale topographical features were included in the models, while the finer-scale surface details are excluded. Physical replications of the full surface and the two low-order models were created using rapid prototyping methods to generate short and long streamwise fetches of roughness, and 2-D particle-image velocimetry (PIV) was used to acquire ensembles of instantaneous velocity fields in the streamwise–wall-normal plane for developing- and developed-flow conditions at moderate Reynolds number followed by stereo PIV measurements in a wall-parallel plane deep in the roughness sublayer \((y = 0.047\delta)\). Comparison of both single- and multipoint statistics (mean velocity and Reynolds normal and shear stresses) as well as quadrant analysis of the instantaneous events contributing to the mean Reynolds shear stress from the 2-D PIV measurements indicates that a 16-mode model of the full surface faithfully reproduces the characteristics of flow over the full surface for both developing- and developed-flow conditions. For the latter scenario, both the 5- and 16-mode models reproduce the outer-layer characteristics for flow over the full surface in accordance with Townsend’s wall similarity hypothesis. However, neither low-order surface representation fully reproduces important details of the Reynolds-shear-stress-producing events within the roughness sublayer, particularly the contributions of the most intense ejection and sweep events.

The stereo-PIV measurements deep within the roughness sublayer at \(y = 0.047\delta\) reveal a wealth of information about roughness-induced effects, including the tendency of the roughness to promote ‘channeling’ of the flow in the form of low- and high-momentum pathways as noted in contour maps of the mean velocity defect. Similarly, enhanced turbulent and vortical activity is observed both between and along the spanwise
boundaries of these streamwise-elongated large-scale pathways. Taken together, these observations support the idea that these persistent low-momentum pathways might represent the statistical imprint of trains of hairpin vortex packets that are channeled along preferred paths over the roughness. Conditional averaging and two-point correlations of velocity further support these structural observations, particularly clear large-scale streamwise coherence of these motions. Of interest, while the \( M = 5 \) results show important differences from the full-surface results, the \( M = 16 \) results are virtually indistinguishable from those of the full surface, including in the single-point turbulence statistics as well as the analysis of the average spatial structure. This consistency is not simply qualitative but is indeed quantitative as the magnitudes of the \( M = 16 \) model single-point statistics mirror those of the full surface as do the spatial locations of the low- and high-momentum pathways identified in the mean velocity defect results as well as the enhanced turbulent and vortical activity along the spanwise boundaries of these large-scale motions. Hence, these observations provide significant evidence supporting the importance of the intermediate topographical scales in setting the flow conditions within the roughness sublayer, not only in a statistical sense but also in a structural sense.
To my father, who is still alive in my memory.

To my mother, for her love and support.
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<td>Biorthogonal Decomposition</td>
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<tr>
<td>ES</td>
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Latin symbols

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\( X \)  Spatial domain
\( y \)  Wall-normal coordinate
\( y^* \)  Viscous length scale
\( z \)  Spanwise coordinate

**Greek symbols**

\( \gamma \)  Arbitrary random variable
\( \delta \)  Boundary-layer thickness
\( \delta(\cdot) \)  Error in the estimation of \( (\cdot) \)
\( \delta_{ij} \)  Dirac delta function
\( \varepsilon \)  Origin offset of mean velocity profile
\( \varepsilon(\cdot) \)  Percentage error in \( (\cdot) \)
\( \eta \)  Surface elevation
\( \vartheta \)  Arbitrary random variable
\( \kappa \)  Von Kármán constant
\( \lambda_{ci} \)  Swirling strength
\( \xi \)  Arbitrary spatial coordinate
\( \Sigma \)  Diagonal matrix of singular values
\( \phi \)  Spatial eigenfunction
\( \Phi \)  Matrix of spatial eigenvectors
\( \psi \)  Temporal eigenfunction
\( \Psi \)  Matrix of temporal eigenvectors
\( \omega_y \)  Wall-normal vorticity
\( \Omega \)  Spatio-temporal domain

**Mathematical operators**

\( \langle \cdot \rangle \)  Ensemble average
\( (\cdot,\cdot) \)  Inner product
\( \overline{(\cdot)} \)  Complex conjugate of \( (\cdot) \)
Superscripts

′  Fluctuation

+  Normalization with inner scaling
Chapter 1

Introduction

It is well-known that when a stream of viscous fluid interacts with a solid surface, a region of reduced momentum forms near the wall. This region is termed the boundary layer, and its existence was first proposed by Prandtl (1904) (English translation: Prandtl, 2001). Boundary layers occur in, and in fact impact, many facets of our everyday lives, including in the atmospheric flow over the Earth’s surface, the flow over vehicles (including land-, sea- and air-based vehicles), within the vessels that deliver blood in the human body, within oil and natural gas pipelines, etc. From a philosophical viewpoint, the concept of the boundary layer is one the most important foundations of modern fluid dynamics since its identification helped resolve early disagreements between the purely mathematical field of hydrodynamics and the more empirical approach of hydraulics. Since the early work of L. Prandtl, the study of the boundary layer has evolved greatly in the last century. Indeed, a number of seminal works spanning the realms of theoretical, computational, and experimental mechanics have been compiled in the now classic text of Schlichting (1979) (see Schlichting and Gersten, 2000, for an updated and expanded version of the original text).

Despite these advances, the boundary layer is still one of the most challenging flows to study owing to the strong flow inhomogeneity that occurs in the direction normal to the boundary. In particular, the turbulent boundary layer (TBL) has been studied vigorously for decades with the hope that a basic understanding of its physics might lead to the development of robust prediction and control methods. Thus far, many in-roads have been made in uncovering the basic physical mechanisms that drive the TBL; however, robust models are not yet established. Therefore, we must still rely upon clever experimental and computational studies to provide insight into the fundamental nature of the TBL. The physics of the TBL is especially challenging to computational and experimental exploration due to the broad range of scales present within the flow, particularly at the high Reynolds numbers (Re) characteristic of most practical applications (Re $\sim 10^6$ in the atmospheric boundary layer and $\sim 10^7$ in the TBL on the surface of a submarine, for example.). The TBL can be further complicated in many practical applications by one or more non-canonical influences, including surface roughness, pressure gradients, severe thermal loads, multiple phases, etc., that render the near-surface flow physics extremely challenging to uncover and model. Thus, the TBL is still an area of
intense research focus and will continue to be for the foreseeable future as its control and prediction would provide unparalleled advances in a range of areas, from those of societal importance (enhanced fuel efficiency, for example) to those of importance to national security.

### 1.1 Fundamental characteristics of the boundary layer

The boundary layer is most conveniently described in terms of a flat plate exposed to an unbounded incoming flow with a uniform velocity distribution given by the free-stream velocity, $U_e$, as presented in figure 1.1. As soon as the flow reaches the leading edge of the plate, the fluid directly in contact with the solid wall loses its momentum since the viscous fluid cannot slip along the wall. Thus, this no-slip condition requires the fluid molecules directly in contact with the wall to decelerate immediately to zero velocity and, due to viscous effects, this momentum loss propagates in the wall-normal direction ($y$) as the fluid flows downstream of the leading edge of the plate. This growth continues for some length forming what is recognized as a laminar boundary layer as given by the Re based on the flow development length along the flat plate, $x$, as $Re_x = U_e x / \nu$, where $\nu$ is the kinematic velocity of the fluid. However, the competition between the momentum carried by the external flow and the viscous effects propagating through the boundary layer generates a vortical instability known as Tollmien–Schlichting waves (Baines et al., 1996; Schlichting and Gersten, 2000). Counter-intuitively, these instabilities tend to be amplified by viscosity as the boundary layer grows further. Finally, the boundary layer reaches a critical thickness in which the viscous forces are unable to overcome the inertial forces. In consequence, the vortical instabilities break down, in turn populating a broad range of wavelengths with energy and the boundary layer becomes turbulent. These stages of boundary-layer development and transition are schematically represented in figure 1.1 with $Re_x$ providing a guide as to when the flow begins transition ($Re_x \gtrsim 3.5 \times 10^5$) as well as when it reaches a turbulent state ($Re_x \gtrsim 4 \times 10^6$). Note, however, that while $Re_x$ provides a means of identifying the flow
regime of a given boundary layer, the Reynolds number based on the boundary-layer thickness, \( \delta \), is more often used to describe the local state of the boundary layer and is defined as \( \text{Re}_\delta \equiv \frac{U_e \delta}{\nu} \). Throughout, \( \delta \) is taken as the wall-normal location where the velocity reaches 99% of \( U_e \).

Once the boundary layer transitions to turbulence, it continues growing in thickness as the flow advects downstream of the leading edge of the plate. It should be noted that in the canonical case of a boundary layer with no streamwise pressure gradient (termed a zero-pressure-gradient (ZPG) TBL), the only external force acting on the flow is the shear stress imposed by the wall. Eventually, at some point far downstream of the transition the viscous dissipation compensates the wall shear stress and the boundary layer reaches equilibrium (Clauser, 2003). Under these circumstances, the mean streamwise velocity profile of the TBL attains a self-similar state with viscous and inertial effects confined at different distances from the wall. This redistribution of effects within the boundary layer is well documented in the classical text of Townsend (1976). In particular, one may infer that viscous effects dominate the near-wall region while inertial effects remain important away from the wall. Thus, one would expect the physics of these two distinct regions of the flow to adhere to different scalings. For example, the outer flow, where inertial effects dominate, is governed by the \( U_e \) and \( \delta \). In contrast, it is expected that flow structures close to the wall should scale with characteristic scales defined in terms of these viscous effects. Such scales can be derived by considering the important variables of interest in this region of the flow, namely the wall shear stress, \( \tau_w \) and the fluid properties (i.e., the kinematic viscosity, \( \nu \), and density, \( \rho \)). Using these variables, a characteristic velocity termed the friction velocity can be defined as \( u_\tau = \sqrt{\tau_w / \rho} \). Using \( u_\tau \), a characteristic length scale can also be defined as \( y^* = \nu / u_\tau \), where \( y^* \) is termed the viscous length scale. These two scales allow for an inner scaling of fluid behavior in the near-wall region of the flow that are commonly known as wall units and quantities scaled in this manner are labeled as \(( \cdot )^+\) throughout this text. With these two scalings, the separation of scales between the near-wall region and the outer region can be assessed via the Kármán number given as the ratio of \( \delta \) and \( y_* \) as \( \delta^+ = \delta / y^* \). It can be readily seen that the Kármán number is in fact a Reynolds number based on \( \delta \) and \( u_\tau \) since \( \delta^+ = \delta / y^* = \delta u_\tau / \nu = \text{Re}_\delta \). Therefore, when \( \delta^+ \) is sufficiently large an overlap region develops between the inner and the outer region when \( y^+ \equiv y / y^* \) is too large for viscous effects to be important and \( y / \delta \) is too small for the flow structures to scale with \( \delta \). This overlap region is known the logarithmic layer since the mean streamwise velocity profile in the overlap region follows a logarithmic law given by (Townsend, 1976)

\[
U^+ = \frac{1}{\kappa} \ln \left( y^+ \right) + A, \tag{1.1}
\]
where $\kappa$ is the von Kármán coefficient, first thought to be a universal constant though recent studies indicate it may vary with flow type (internal versus external flow) and in the presence of non-canonical influences (Nagib and Chauhan, 2008; Marusic et al., 2010), and $A$ is an additive constant. Despite these recent observations, the commonly accepted values of $\kappa \approx 0.41$ and $A \approx 5.0$ will be used throughout. This logarithmic representation of the mean velocity profile is illustrated in figure 1.2. It should be noted that the log layer sits above the buffer region where the peak in the streamwise contribution to the turbulent kinetic energy resides and is thus commonly referred to as the most active region of the TBL (Jimenez, 2004). The viscous sublayer and buffer layer together form the region of the TBL where viscous effects dominate.

### 1.2 Rough-wall TBLs

As mentioned above, many non-canonical influences can alter the canonical nature of the smooth-wall TBL, including surface roughness. Extensive research has been pursued to identify the impact of surface roughness on TBLs since most practical applications have flow surfaces that are inherently rough. Some key features of rough-wall turbulence are discussed herein and the reader is referred to the reviews of Raupach et al. (1991) and Jimenez (2004) for a more detailed treatment of the subject.

As was said previously, after the boundary layer experiences a transition to turbulence it will eventually reach a self-similar state under smooth-wall conditions as it advects downstream. However, this development
Figure 1.3: (a) Sudden transition of a self-similar TBL from smooth- to rough-wall conditions. (b) Self-similar TBL under rough-wall conditions; $5k/\delta$: roughness sublayer thickness.

can be altered when the flow encounters an abrupt change in surface conditions from smooth to rough like that illustrated in figure 1.3a. It is well documented that this sudden change in surface conditions will lead to the formation of an internal layer that grows in thickness with distance downstream (Smits, 1985). If these rough-wall conditions persist downstream, this internal layer will continue its growth until it eventually engulfs the entire boundary layer which will then attain a new self-similar state. Under these new conditions, the direct effects of roughness will often remain confined within a thin layer termed the *roughness sublayer*. This roughness sublayer typically occupies a region $3-5k$ in height away from the wall, where $k$ is a measure of the roughness height (Raupach et al., 1991) and is schematically illustrated in figure 1.3b. In the presence of roughness, the self-similar TBL experiences an additional momentum loss due to enhanced total drag at the surface which yields a downward shift in the mean velocity profile compared to smooth-wall flow.
that is termed the roughness function, $\Delta U^+$ (see figure 1.2). Under conditions of self-similarity, the log-law representation of the mean velocity profile remains unaltered save for the inclusion of $-\Delta U^+$ as an additive constant as

$$U^+ = \frac{1}{\kappa} \ln (y^+) + A - \Delta U^+, \quad (1.2)$$

where $\Delta U^+$ is a function of Re as well as $k$.

The effect of roughness on the flow can be equivalently quantified by means of the skin-friction coefficient, $C_f$ (or equivalently the friction factor, $f$, for internal flows). Nikuradse (1950) conducted exhaustive experiments to determine $f$ as a function of Re and $k_s$ in pipes roughened with various grain-sizes of sand (where $k_s$ represents the sand-grain diameter). Based on this extensive data, Nikuradse (1950) identified three regimes of rough-wall flow based on $k_s^+$. The first regime, termed hydraulically smooth, occurs for roughness that is so small that it is engulfed within the viscous sublayer ($k_s^+ \lesssim 5$) and therefore has no impact on the TBL. The second regime, termed transitionally rough, occurs for $5 \lesssim k_s^+ \lesssim 70$ whereby the skin friction depends upon both Re and $k_s$. The third regime, termed fully rough, occurs for $k_s^+ \gtrsim 70$ and is characterized by Reynolds-number independence of the skin friction, meaning that the flow behavior simply depends upon roughness height (Raupach et al., 1991). Note that the inner-scaled roughness height ($k^+ = k u_\tau/\nu$ or $k_s^+ = k_s u_\tau/\nu$) is equivalently a measure of the Reynolds number local to the roughness. Further, Nikuradse (1950) found a logarithmic law of the wall still held for the rough-wall flows, with a value of the von Kármán constant identical to the one for smooth-wall flows. He expressed this relationship in terms of $k_s$ as

$$U^+ = \frac{1}{\kappa} \ln (y/k_s) + 8.5. \quad (1.3)$$

This representation of the mean velocity profile can be used to derive an equation for $\Delta U^+$ as a function of $k_s^+$ by subtracting eq. (1.3) from (1.2), yielding

$$\Delta U^+ = \frac{1}{\kappa} \ln (k_s^+) + A - 8.5. \quad (1.4)$$

It should be mentioned that eq. (1.4) is valid only for the fully-rough regime where $\Delta U^+$ depends only on roughness height. Though Nikuradse’s experiments occurred over a half-century ago, they still remain the benchmark for rough-wall turbulence studies. Thus, it is common practice to characterize experimental data from non-sand-grain roughness in terms of the sand-grain roughness height from Nikuradse’s chart that yields the same friction factor (or equivalently roughness function) as the data at hand. It is in this regard that $k_s$ has become known as the equivalent sand-grain roughness height. Subsequent studies have
shown this relationship to hold for \( k\)\'-type roughness for which the roughness function scales with roughness height (Perry et al., 1969).

### 1.3 Realistic roughness

So far, all of the discussion has focused upon well-controlled laboratory investigations of rough-wall flows. However, roughness effects can also play a crucial role in a variety of practical engineering systems, from internal flows such as those through oil and gas pipelines to external flows like those over the surfaces of turbine blades and heat exchangers. In some instances, surface roughness occurs in isolated regions of a flow surface meaning that the flow will be intermittently perturbed by one or more step changes in surface condition from smooth to rough and vice-versa. Such transitions in surface quality can inhibit the flow from attaining a self-similar state (termed developed flow herein). On the other hand, there are other applications for which surface roughness occurs consistently along the entire length of a flow surface of interest, meaning that its development may eventually attain self-similarity since the flow is not intermittently perturbed by step changes in surface quality. Surface degradation of turbine blades and pump impellers are instances of the former case, because various damage mechanisms such as pitting, spallation, deposition of foreign materials, or cavitation may occur in different stages of such devices. Each damage mechanism induces a specific roughness pattern and, in consequence, the boundary layer is unlikely to reach self-similarity under the influence of a single roughness pattern. Bons et al. (2001) and Bons (2002a) have observed that surface conditions so generated induce important augmentation on skin friction \( C_f \) and heat transfer \( St \) coefficients, and in consequence, the overall energy conversion efficiency of turbines and pumps tend to decrease over time as surface degradation progresses. On the other hand, gradual accumulation of debris or corroded elements in oil or water supply pipes offer examples of systems in which surface conditions are likely to present a more consistent pattern for longer portions of the system. In such cases, surface degradation will increase drag by affecting primarily the roughness sublayer of fully-developed pipe flows. Henceforth, pumping power demand will increase gradually over time as the internal surface of the pipe degrades. Other examples of irregular roughness include accumulation of algae and barnacles on the exterior surfaces of submarines and ships (Karlsson, 1980) as well as cumulative erosion on the blades of wind turbines operating near the sea. Regardless of the roughness scenario, it is of significant interest to understand the impact of surface roughness for improved modeling, prediction and eventually control of practical flow systems in the presence of such effects.

Note that each one of the damage mechanisms mentioned above generates a very specific roughness
pattern that might be considered its fingerprint. However, in most cases these patterns are highly irregular and are composed by a wide range of topographical scales. Nevertheless, the vast majority of studies of flow over rough surfaces have been conducted with idealized roughness patterns such as sand grain, woven mesh, arrays of discrete elements, among others. The presence of few topographical scales, typically one, and their distribution in organized arrangements, are two fundamental factors that distinguish idealized roughness patterns from realistic ones. These important indicate that the effect of the latter on the flow might not be well-replicated by such simple topographies. In fact, these differences have been known for decades, most notably discrepancies in friction factor in the transitionally-rough regime between Nikuradse’s results for monodisperse sand-grain roughness (Nikuradse, 1950) and Colebrook’s relationship based on “industrial” roughness containing a broad range of topographical scales (Colebrook and White, 1937; Colebrook, 1939). More recently, Bons et al. (2001); Bons (2002a) and Bons and McClain (2004) showed that skin friction and heat transfer coefficients derived from idealized roughness patterns are not very representative of those due to highly irregular roughness. In addition, Itoh et al. (2006) found out that a simplified model of seal fur, containing only the dominant amplitude and wavelength of the original pattern, failed to reproduce turbulence statistics induced by the original surface on the flow. As such, an understanding of idealized roughness effects may not properly extrapolate to the more practical case of highly-irregular surface roughness. Consequently, it is doubtful that a simple parameter such as the equivalent sand-grain roughness height $k_s$ could be used as a representative length scale of the effect of the roughness on the flow. With this in mind, Napoli et al. (2008) proposed the effective slope, $ES$, as an alternative parameter to the classical equivalent sand-grain roughness height. $ES$ is defined as

$$ES = \frac{1}{L} \int_{L} \left| \frac{\partial \eta}{\partial x} \right| \, dx,$$

where $\eta$ is the surface elevation, and $L$ is the streamwise sampling length. In principle, $ES$ captures the relative importance of viscous and form drag based on the slope of roughness elements present in highly-irregular surfaces. Napoli et al. (2008) conducted direct numerical simulations (DNS) of turbulent channel flow in the presence of two-dimensional irregular roughness and successfully scaled $\Delta U^+$ with $ES$ in a region of mild to low surface gradients. For the case of steep gradients, Napoli et al. (2008) found $\Delta U^+$ to be relatively insensitive to $ES$ and instead more dependent on roughness height due to the presence of more favorable conditions for wake development downstream of roughness elements and hence a dominance of form drag over viscous drag (i.e., fully-rough conditions). Similarly, Schultz and Flack (2009) used $ES$ to characterize experimental data of TBLs in the presence of close-packed three-dimensional pyramid-shaped
roughness elements of varying slope and height. Their results showed good qualitative agreement with those of Napoli et al. (2008). Since this parameter seems to offer a promising alternative to $k_s$, a more detailed discussion of this parameter, particularly its use in the present effort, is given in Chapter 3.

### 1.4 Developing flow

As was mentioned above, a sudden change from smooth to rough surface conditions triggers the formation of an internal layer. Studies of idealized roughness indicate an overshoot in $\tau_w$ just downstream of this step change in surface conditions (Antonia and Luxton, 1971; Andreopoulos and Wood, 1982). In addition, a significant enhancement in the production of turbulence is noted in the immediate vicinity of the roughness which yields higher values of both Reynolds normal and shear stresses within the internal layer compared to the upstream smooth-wall flow (Antonia and Luxton, 1971; Andreopoulos and Wood, 1982; Schofield, 1975). In contrast, the flow outside this layer remains relatively undisturbed. Such roughness effects diffuse away from the wall with increasing downstream distance from the step change in surface conditions until the internal layer eventually engulfs the entire wall normal extent of the boundary layer and the flow approaches a self-similar (developed) state. Previous studies indicate that the precise growth in the thickness of the internal layer is tied to the details of the rough surface encountered (Andreopoulos and Wood, 1982). Further, roughness can also significantly reduce the spatial scales of the flow within the internal layer for developing flow (Antonia and Luxton, 1971). With regard to more irregular roughness, Wu and Christensen (2006) studied the impact of a short streamwise fetch of turbine-blade roughness due to spallation damage and marked by a broad range of topographical scales on incoming fully-developed, smooth-wall turbulent channel flow. This effort revealed both the formation of the expected internal layer upon transition from smooth- to rough-wall conditions as well as enhanced local Reynolds stresses within the internal layer of this developing flow predominantly due to the large-scale topographical features.

### 1.5 Developed flow and outer-layer similarity

For developed flow, wherein the internal layer has grown to engulf the entire boundary-layer thickness and the flow has attained a self-similar state, it is well-accepted that roughness governs the character of turbulence within the roughness sublayer (A region $3-5k$ immediately adjacent to the surface). Outside the roughness sublayer, however, roughness may not have a direct impact on the flow as many studies of idealized roughness report that the turbulence behaves similarly to that of smooth-wall flow when properly scaled (Grass, 1971; Raupach, 1981; Ligrani and Moffat, 1986; Bandyopadhyay and Watson, 1988; Perry and Li, 1990; Schultz
and Flack, 2003, 2005; Flack et al., 2005; Schultz and Flack, 2007; Volino et al., 2007). These observations are in accordance with Townsend (1976) who first hypothesized that at high Re, the turbulent motions in the outer layer are independent of surface conditions and viscosity except for their role in setting the wall shear stress, $\tau_w$, and the boundary-layer thickness, $\delta$. With respect to rough-wall flows, this hypothesis implies that if the characteristic roughness height, $k$, is sufficiently small compared to $\delta$ then the direct impact of roughness is confined within the roughness sublayer. Under such conditions, the turbulence in the outer layer is only indirectly influenced by roughness through its role in determining $u_\tau$ and $\delta$ (Raupach et al., 1991; Jimenez, 2004). Jimenez (2004) proposed a threshold of $\delta/k > 40 - 50$ for outer-layer similarity to exist, where $k$ is a geometric measure of the roughness height, while Flack et al. (2005) proposed a threshold based on $k_s$ of $\delta/k_s \gtrsim 40$.

Outer-layer similarity has also been reported for turbulent flow over more practical roughness. For example, Allen et al. (2007) studied turbulent pipe flow in the presence of a honed surface akin to the industrial-type roughness of Colebrook (1939) and reported smooth- and rough-wall mean velocity defect profiles, streamwise turbulence intensity profiles, and streamwise velocity spectra that collapsed in the outer layer in accordance with Townsend’s wall similarity hypothesis. Similarly, Wu and Christensen (2007) reported that the turbulence statistics outside the roughness sublayer remain unaffected by roughness replicated from a turbine blade damaged by deposition of foreign materials (compared to smooth-wall flow) when one accounts for the increased drag at the surface when scaling the statistics (using $u_\tau$ and $\delta$). This similarity was also found to extend to the average spatial structure of the flow through comparison of two-point velocity correlation coefficients outside the roughness sublayer. Thus, Townsend’s hypothesis provides a simple means of predicting outer-layer behavior for rough-wall flows based simply on knowledge of $u_\tau$ and $\delta$, though previous studies indicate that it may not be a universal characteristic of all developed rough-wall flows (Krogstad et al., 1992; Krogstad and Antonia, 1994; Keirsbulck et al., 2002; Bhaganagar et al., 2004). It should be noted, however, that many of these studies utilized roughness whose $\delta/k$ and/or $\delta/k_s$ did not satisfy the aforementioned criteria of Jimenez (2004) and Flack et al. (2005). In addition, recent studies suggest that outer-layer similarity should not be expected in the presence of two-dimensional roughness since such roughness generates structures that are much larger than $k$ due to the width of the roughness (Krogstad and Antonia, 1999; Keirsbulck et al., 2002; Lee and Sung, 2007; Volino et al., 2009). Further, while the outer layer may not be directly influenced by the details of the roughness topography in question, such details will undoubtedly have a defining impact on the local flow behavior within the roughness sublayer. Thus, given the crucial importance topographical details can play in both developing and developed flows, and the fact that idealized roughness characterizations generally embody a rather restricted distribution of topographical
scales, the effect of idealized roughness conditions upon wall-bounded turbulence may be insufficient for successful modeling and/or control of practical flows in the presence of irregular roughness.

1.6 The role of multiple roughness scales

Given the topographical complexity of realistic roughness, the relative impact that each topographical scale of an irregular surface has on the flow is certainly of interest. That is, are flows over irregular roughness predominantly governed by the impact of the largest roughness scales or do the finer surface features contribute in a meaningful way? Colebrook and White (1937) recognized the importance of this issue in their studies of industrial roughness and reported an enhancement in pipe-flow friction factor with the addition of sandgrain roughness to larger roughness protrusions compared to flow over the larger protrusions alone. Schultz and Flack (2005) compared the flow over uniform spheres and the same topography with the addition of finer-scale sandgrain roughness. They reported good agreement between the Reynolds stress profiles for flow over both surfaces throughout the boundary layer, indicating little effect of finer-scale roughness on the turbulence. However, as with the experiments of Colebrook and White (1937), it is not clear how this study of two roughness scales of substantially different size translates to the case of practical roughness which is marked by a broad spectrum of topographical scales. With respect to more irregular roughness, as was mentioned earlier, Itoh et al. (2006) measured turbulent flow over the fur surface of a seal which exhibited a riblet-like character, though both the amplitude and wavelength of these topographical features varied significantly in space. Itoh et al. (2006) also made measurements of turbulent flow over a model of the seal fur that consisted of ordered riblets manufactured with the dominant amplitude and wavelength of the seal fur. Comparison of these results indicated a lack of consistency in the turbulence statistics for flow over the real seal surface and the model. These differences highlight the importance of topographical characteristics beyond simply the dominant amplitude and wavelength of the real surface in determining its impact on the flow. Finally, Johnson and Christensen (2009) considered the development of low-order topographical models of roughness replicated from a turbine blade that contained deep recesses of varying size due to spallation damage. Model topographies were developed using singular value decomposition (SVD) and short fetches of the models were fabricated and tested in turbulent channel flow in which the upstream smooth-wall flow was fully-developed. Under these developing, internal-flow conditions, it was found that a model containing only the larger- and intermediate scales of the topography (10% of the total modes) adequately reproduced the single-point statistics of flow over the full surface within the internal layer formed by the abrupt transition from smooth to rough conditions.
1.7 Outer-layer structure of wall turbulence

1.7.1 Smooth-wall flow

Much has been uncovered regarding the outer-layer structure of smooth-wall turbulence (Robinson, 1989, 1991; Panton, 1997). In particular, it is well-documented that hairpin-like vortices populate the outer region and align coherently to form larger-scale structures termed hairpin vortex packets (Head and Bandyopadhyay, 1981; Smith, 1984; Smith et al., 1991; Zhou et al., 1999; Adrian et al., 2000b; Christensen and Adrian, 2001; Ganapathisubramani et al., 2003; Tomkins and Adrian, 2003; Wu and Moin, 2009). Adrian et al. (2000b) developed the conceptual picture of hairpin vortex packets and this structural model is illustrated in figure 1.4. The authors proposed that packets evolve from small vortical disturbances near the wall. These disturbances are initially of high vorticity and tend to arch in the streamwise direction taking the shape of horseshoes, with their legs aligned in the streamwise direction and pointing upstream. While some of these structures can form in a relatively symmetric manner whereby the legs develop in to a similar strength, it is more common for such structures to have one leg that is stronger than the other, resulting in asymmetric vortical patterns (Zhou et al., 1999; Delo et al., 2004). Due to their high vorticity, these disturbances induce strong upstream motions that promote the formation of new vortices at their upstream ends. This pattern of vortical birth continues, resulting in a streamwise-aligned train of hairpin-like structures that Adrian et al. (2000b) termed hairpin vortex packets. The particle-image velocimetry (PIV) measurements of Adrian et al. (2000b) in the streamwise–wall-normal plane revealed that these large-scale motions were
marked by an inclined shear layer formed by the heads of the streamwise-aligned hairpin heads beneath which a region of relatively uniform streamwise momentum deficit resides due to the collective induction of the vortices. In addition, each vortex was found to induce a strong ejection of low-speed fluid away from the wall that contributes heavily to the mean Reynolds shear stress. Further, these measurements indicated that such large-scale motions can occur in a hierarchy of scales across the boundary layer in a manner consistent with the mechanism of wall turbulence proposed by Perry and Chong (1982). Such structures also leave a definitive imprint in streamwise–spanwise measurement planes in the log layer (Tomkins and Adrian, 2003; Ganapathisubramani et al., 2003), particularly spanwise-alternating regions of low and high streamwise momentum, with the low-momentum regions (LMRs) likely associated with a wall-parallel cut through hairpin vortex packets as they are bounded by spanwise-separated counter-rotating wall-normal vortices (likely slices through the legs of the hairpins within the packet). Subsequent studies revealed that these structures play a pivotal role in outer-layer momentum and energy transport (Natrajan and Christensen, 2006) and their spatial characteristics leave definitive imprints on the spatial correlations (Christensen and Adrian, 2001; Marusic, 2001; Ganapathisubramani et al., 2005; Christensen and Wu, 2005; Wu and Christensen, 2010). For example, the streamwise extent and inclination angle of the two-point correlation coefficient of streamwise velocity ($\rho_{uu}$) are quite consistent with the spatial characteristics of hairpin vortex packets. In addition, the spatial extent of the two-point correlation of wall-normal velocity ($\rho_{uv}$) is comparable to the spatial extent of the heads of hairpin-like structures. These imprints provide not only an excellent measure of the average characteristics of such structures but also a glimpse at the consistency of their organization and persistence.

While hairpin vortex packets of streamwise length $1 - 3\delta$ play an important role in the overall dynamics of wall turbulence, recent studies indicate that even larger spatial scales can occur in wall turbulence. These very large scale motions (VLSMs) have been previously detected in smooth-wall studies based on velocity spectra computed from single-point time series acquired by hot-wire anemometry (Kim and Adrian, 1999; Guala et al., 2006; Balakumar and Adrian, 2007) as very energetic spatial scales that are several $\delta$ long in the streamwise direction. Recent experiments with a 10-wire hot-wire rake by Hutchins and Marusic (2007) illustrated that LMRs identified in $\delta$-scale PIV measurements can actually extend 10–20$\delta$ in the streamwise direction and meander significantly in the spanwise direction in smooth-wall flow. For their experiments, Hutchins and Marusic (2007) spaced hot-wires 0.1$\delta$ apart in the spanwise direction, long time traces of streamwise velocity were acquired and, using Taylor’s hypothesis, these time traces were converted to elongated streamwise fields of view. They interpreted these superstructures (as they termed them) as the energetic streamwise VLSMs observed by Kim and Adrian (1999), Guala et al. (2006) and Balakumar and Adrian (2007). Despite the clear evidence that VLSMs exist in wall turbulence, their spatial extent
is not directly reflected in the spatial statistics of velocity, particularly the two-point correlation of the streamwise velocity, $\rho_{uu}$, that has a characteristic streamwise extent of a few $\delta$ (Christensen et al., 2004; Ganapathisubramani et al., 2005). To explain this discrepancy, Hutchins and Marusic (2007) generated a VLSM in the form of an LMR bounded by two HMRs using a synthetic representation with a total streamwise extent of roughly $15\delta$. Without imposing any meandering to these motions, $\rho_{uu}$ calculated from this synthetic flow field faithfully captured the actual spatial extent of the VLSM. However, after imposing some spanwise meandering in the synthetic flow field with a given amplitude and wavelength, but without changing its spatial extent, the streamwise extent of the two-point correlation was reduced to less than half the length of the LMR. Thus, while VLSMs occur often in wall turbulence, their spanwise meandering motions appear to hide their true spatial extent in the spatial correlations of velocity.

1.7.2 Rough-wall flow

While the structure of smooth-wall turbulence is relatively well-understood, the impact of roughness on this structural paradigm is still a topic of intense interest. Many studies have considered the impact of roughness on the characteristics of coherent structures, but for rather idealized surface conditions (sandgrain, wire mesh, ordered arrays of elements, etc.). Krogstad and Antonia (1994), for example, computed two-point velocity correlations from cross-wire measurements in a mesh-roughened TBL and found a decrease in the streamwise extent of velocity and vorticity spatial correlations compared to flow over a smooth wall but found little difference in the spanwise extent of these correlations. In addition, an enhancement in the inclination angle of $\rho_{uu}$ was noted compared to smooth-wall flow. While somewhat weaker streamwise shortening of the velocity correlations has also been reported in other experimental studies (Sabot et al., 1977; Raupach et al., 1991; Nakagawa and Hanratty, 2001; Wu and Christensen, 2007; Volino et al., 2007; Wu and Christensen, 2010), these efforts also noted that this shortening diminished quickly with increasing distance from the wall until the rough-wall correlations eventually collapsed with those of smooth-wall flow. In addition, the PIV measurements reported by Volino et al. (2007) for idealized roughness and Wu and Christensen (2010) for irregular roughness revealed the presence of hairpin vortex packets in the outer layer of the rough-wall flows with similar spatial characteristics to those of smooth-wall turbulence. Indeed, the only difference reported by Wu and Christensen (2010) was a distinct streamwise shortening of the larger spatial scales of the flow as reflected in spatial correlations of the large-scale motions that were separated from the smaller spatial scales via low-pass filtering based upon proper orthogonal decomposition (POD). This reduction in the streamwise extent of the larger scales may be tied to enhanced spanwise meandering of these motions in the presence of roughness and not necessarily to a fundamental alteration of the structural paradigm of the
flow compared to smooth-wall turbulence given the consistent observations of outer-layer vortex organization in the rough-wall flow. In contrast, Wu and Christensen (2010) reported that the smaller scales showed little impact of roughness. This outer-layer structural consistency between smooth- and rough-wall flow is in accordance with Townsend’s wall similarity hypothesis, indicating that the outer-layer structure of rough-wall turbulence is not directly altered by roughness effects. It should be mentioned, however, that each of the aforementioned studies utilized roughness that satisfied the criterion $\delta/k_s \gtrsim 40$ as proposed by Flack et al. (2005). In contrast, while the roughness of Krogstad and Antonia (1994) gave $\delta/k \approx 50$, it also yielded $\delta/k_s \approx 15$ which does not satisfy the threshold of Flack et al. (2005) for the existence of wall similarity. Taken together, these results highlight the apparent importance of the ratio $\delta/k_s$ compared to $\delta/k$ which indicates that it is not necessarily just the geometric characteristics of the roughness that are important to the notion of wall similarity but rather the impact that the roughness has on the flow. This latter effect is implicitly captured in $k_s$ since its value is garnered directly from the roughness function which in turn implicitly captures the enhanced drag induced by the roughness in question.

Finally, recent computational and experimental studies of two-dimensional roughness effects in the TBL reveal that this particular class of roughness can have a dramatic impact on the structure of the outer layer. Volino et al. (2009) conducted experiments in a TBL roughened with bars periodically-spaced in the streamwise direction that spanned the entire test section ($\delta/k = 32$) and found the outer-layer structure to be directly altered by this two-dimensional roughness. It was concluded that this roughness generates flow structures much larger than $k$ due to the width of the roughness and that the growth of these motions into the outer layer enhanced the turbulent stresses and the integral length scales. Similar observations for two-dimensional roughness were reported by Krogstad and Antonia (1999), Keirsbulck et al. (2002) and Lee and Sung (2007), indicating that fundamental differences exist between the impact of two- and three-dimensional roughness on the TBL. It should be noted, however, that while $\delta/k$ in many of these studies was close to or exceeded the threshold put forth by Jimenez (2004) for the existence of outer-layer similarity, all of these studies reported $\delta/k_s$ values well below the threshold of Flack et al. (2005). Thus, as mentioned above, while $k_s$ does not necessarily capture the geometric characteristics of the roughness in question, it does seem to approximately capture the impact of the roughness on the flow.

1.8 The present work

Based upon this discussion, two main questions arise when considering the impact of a highly-irregular surface topography on a TBL. First, it is not clear if the effect of the surface on the flow is dominated exclusively by
the largest topographical scales or if the intermediate and smaller scales of the surface play any determinant role. Second, the degree of interaction of different topographical scales with the flow is likely to differ from one topography to another (i.e., a surface damaged by spallation versus a surface damaged by pitting). While both of these questions have garnered past research attention (see for example: Colebrook and White, 1937; Schultz and Flack, 2005; Allen et al., 2007; Johnson and Christensen, 2009; Wu et al., 2009; Wu, 2009; Mejia-Alvarez and Christensen, 2010), the present effort is devoted toward shedding some light on the first issue by studying the impact of a single roughness topography on a ZPG TBL. This work builds upon the initial efforts of Johnson and Christensen (2009) by considering the development of low-order topographical models of a different highly-irregular surface topography replicated from a turbine blade damaged by deposition of foreign materials. The ability of these models to reproduce the characteristics of flow over the original roughness was then assessed under both developing and developed flow conditions in a ZPG TBL. Singular value decomposition (SVD) was used to decompose the highly-inhomogeneous surface topography into a set of basis functions of decreasing contribution to the overall topography. Only the most dominant of these basis functions were used to reconstruct the surface topography, meaning that a substantial fraction of the larger-scale surface features were included in the low-order models while the finer topographical details were neglected. Short and long streamwise fetches of these low-order representations were then fabricated and tested in a ZPG TBL to assess how well they reproduce the flow modifications generated by the full surface topography under both developing- and developed-flow conditions. Such comparisons are meant to reveal the relative importance of finer-scale roughness features compared to the most dominant roughness scales in the context of highly irregular roughness in an external flow arrangement. In particular, the objectives of this work include:

- Developing low-order representations of a highly irregular rough surface replicated from a turbine blade damaged by deposition of foreign materials;
- Testing the original surface and its low-order models in a wind tunnel to identify the critical topographical scales that dominate the impact of the original surface on the flow;
- Assessing the relative role of large, intermediate, and small topographical scales of this irregular roughness on ZPG TBLs.

Chapter 2 provides background regarding biorthogonal decomposition, of which SVD is a subset, chapter 3 provides details of the roughness topography under consideration as well as a discussion of the low-order topographical models employed, and chapter 4 discusses the experiments undertaken. The results of these experiments are then discussed in chapters 5 and 6, particularly the statistical and structural differences
noted between the rough-wall conditions and smooth-wall flow and between the three rough-wall cases as well.
Chapter 2

The Biorthogonal Decomposition

2.1 Fundamentals

In general, a turbulent flow field is represented by its fluctuating velocity field, $u'(x, t)$, that contains finite kinetic energy and obeys conservation of momentum and mass. A direct consequence of the first condition is that such fields belong to the Hilbert space of square integrable functions $L^2[\Omega]$ (Doering, 2009), where $\Omega$ defines the spatio-temporal domain $\Omega = [X, T]$. It follows that it is possible to determine a decomposition for $u'(x, t)$ in terms of a unique orthonormal basis $B \in L^2[X, T]$ (Dunford and Schwartz, 1988). Note that $u'(x, t)$ as a turbulent flow field is simply a subspace of $L^2[X, T]$. As such, any property one determines for all of the members of $L^2[X, T]$ will be automatically inherited by any turbulent flow field $u'(x, t) \in L^2[X, T]$. Therefore, one may consider $u'(x, t)$ more generally as any complex-valued function contained in $L^2[X, T]$.

Henceforth, following Aubry (1991), this analysis makes no assumption of ergodicity or statistical stationarity for $u'(x, t)$. Moreover, the spatio-temporal domain $[X, T]$ is allowed to be either continuous or discrete which implies that $u'(x, t)$ may be taken as a signal collected at a finite number of spatial ($X$) and temporal ($T$) points (which is the case for experimental and numerical data). Further, the condition of orthonormality implies a decomposition in terms of two independent sets of basis functions: $\phi(x) \in X$ and $\psi(t) \in T$, with both contained in $L^2[X, T]$. It is in this regard that such a decomposition is termed biorthogonal decomposition. This chapter is devoted to the derivation of the biorthogonal decomposition of the signal $u'(x, t)$ (for a more detailed derivation, see Aubry et al., 1991).

As any Hilbert space is an inner product space that is also a complete metric with respect to the inner product, we may use the signal $u'(x, t)$ to construct a linear operator $U : L^2[\Omega] \rightarrow L^2[\Omega]$ as

$$(U \cdot)(\Omega) = \int_{\Omega} u'(x, t)(\cdot)d\Omega.$$  (2.1)

More specifically, when $U$ is applied to any spatial function in $L^2[X] \in L^2[\Omega]$, its action is $L^2[X] \rightarrow L^2[T]$, for $L^2[T] \in L^2[\Omega]$. Conversely, the action of its adjoint operator, $U^*$, to any temporal function in $L^2[T] \in L^2[\Omega]$
is \( L^2[T] \to L^2[X] \), for \( L^2[X] \in L^2[\Omega] \). Thus,

\[
(U\phi)(t) = \int_X u'(x,t)(\phi)dx \quad \forall \phi(x) \in L^2[X],
\] (2.2)

and

\[
(U^*\psi)(x) = \int_T \overline{u'(x,t)}(\psi)dt \quad \forall \psi(t) \in L^2[T],
\] (2.3)

where \( \overline{\cdot} \) denotes the complex conjugate. Note that if \( u' \in L^2[\Omega] \) is also a continuous function \( u' \in C[\Omega] \), which is typically the case, then \( U \) is a compact operator in the Hilbert space of square integrable functions \( L^2[\Omega] \) (Aubry et al., 1991). In general, compact operators in Hilbert spaces have only discrete spectra and may be diagonalized by the orthonormal basis of the operator. It follows that the spectral decomposition of \( U \) is of the form

\[
(U\cdot)(t) = \sum_{k=1}^{\infty} \sigma_k (\cdot, \phi_k(x)) \psi_k(t),
\] (2.4)

where \( (\cdot, \cdot) \) implies inner product. Equation (2.4) is the singular value decomposition (SVD) of \( U \), where \( \sigma_k \) are the singular values of \( U \). Note that the inner product \( (\cdot, \phi_k(x)) \) projects \( \cdot \) onto the spatial basis of \( U \) and \( \sigma_k \psi_k \) brings this projection to the temporal subspace of \( L^2[\Omega] \), which is consistent with the \( L^2[X] \to L^2[T] \) of eq. (2.2). Now, without loss of generality, if the spectral form of \( U \) is applied to an arbitrary spatial function \( \phi \in L^2[X] \), one obtains

\[
(U\phi)(t) = \sum_{k=1}^{\infty} \sigma_k (\phi, \phi_k(x)) \psi_k(t)
\]

\[
(U\phi)(t) = \sum_{k=1}^{\infty} \sigma_k \left[ \int_X \overline{\phi_k(x)}dx \right] \psi_k(t)
\] (2.5)

\[
(U\phi)(t) = \int_X \left[ \sum_{k=1}^{\infty} \sigma_k \overline{\phi_k(x)} \psi_k(t) \right] \phi dx.
\]

Comparison of eq. (2.2) with eq. (2.5) reveals that the signal \( u'(x,t) \) has a SVD of the form

\[
u'(x,t) = \sum_{k=1}^{\infty} \sigma_k \overline{\phi_k(x)} \psi_k(t).
\] (2.6)

An identical result is obtained if \( U^* \), in the form of its spectral decomposition, is applied to an arbitrary temporal function \( \psi \in L^2[T] \) and one compares the result with eq. (2.3). One may now develop a new operator \( L^2[\Omega] \to L^2[\Omega] \) that is compact with respect to its original subspace (that is, \( L^2[X] \to L^2[X] \) or \( L^2[T] \to L^2[T] \)) by applying \( U \) to its adjoint \( U^* \) (or vice versa). The present focus is on the case \( L^2[X] \to L^2[X] \), though the case \( L^2[T] \to L^2[T] \) is analogous. From eqs. (2.2) and (2.3), one may obtain the
operator $U^*U$ as
\[
[U^*U(\cdot)](x) = \int_T \overline{u'(x, t)} \left( \int_X u(x', t) \phi(\cdot) dx' \right) dt
\]
\[
[U^*U(\cdot)](x) = \int_X \left( \int_T u'(x', t) \overline{u'(x, t)} dt \right) \phi(\cdot) dx'
\]
\[
\Rightarrow [U^*U(\cdot)](x) = \int_X r(x, x') \phi(\cdot) dx', \quad (2.7)
\]
where
\[
r(x, x') = \int_T u'(x', t) \overline{u'(x, t)} dt
\]
is the spatial correlation function of the signal $u'(x, t)$. The operator $U^*U$ is non-negative and, as stated before, is compact in $L^2[X]$. Therefore, it can be shown that its spectral decomposition is of the form (see Aubry et al., 1991)
\[
[U^*U(\cdot)](x) = \sum_{k=1}^{\infty} \sigma_k^2 \phi_k(\cdot) \phi_k. \quad (2.9)
\]
Note that the orthogonality of $\phi$ implies that
\[
(\phi_j, \phi_k) = \delta_{jk}, \quad (2.10)
\]
where
\[
\delta_{jk} = \begin{cases} 1, & \text{if } j = k \\ 0, & \text{if } j \neq k \end{cases}. \quad (2.11)
\]
Thus, when the operator $U^*U$ is applied to any $\phi(X) \in L^2[X]$ equation (2.9) gives
\[
(U^*U)\phi_k = \sigma_k^2 \phi_k. \quad (2.12)
\]
After applying eq. (2.7) to $\phi \in L^2[X]$ and combining it with eq. (2.12), one obtains an eigenvalue problem of the form
\[
\sigma_k^2 \phi_k = \int_X r(x, x') \phi_k(x') dx'. \quad (2.13)
\]
If a similar procedure is followed for the operator $UU^* : L^2[T] \to L^2[T]$, one obtains the alternate eigenvalue problem
\[
\sigma_k^2 \psi_k = \int_T l(t, t') \psi_k(t') dt', \quad (2.14)
\]
where \( l(t, t') \) is the temporal correlation function of the signal \( u'(x, t) \) given by

\[
l(t, t') = \int_X u'(x, t')u(x, t)d x.
\]

Equations (2.13) and (2.14) are Fredholm integral equations of the second type. It follows from the Hilbert–Schmidt theorem that the eigenfunctions, \( \phi_k \) and \( \psi_k \), of these eigenvalue problems are orthogonal, their eigenvalues \( \sigma_k^2 \) are positive and the signal \( u'(x, t) \) can be fully reconstructed in terms of these eigenfunctions. It is this formulation that is known as the proper orthogonal decomposition (POD) (see Holmes et al., 1996; Cazemier et al., 1998). In the special case of a signal with no time coherence, i.e. a signal that was acquired at time intervals longer than the characteristic correlation time of the process, the temporal correlation in eq. (2.14) is effectively zero. In consequence, only the spatial eigenfunctions found in the eigenproblem (2.13) are used for reconstructing the original signal. This special case is typically known as snapshot POD (Sirovich, 1987). The above analysis clearly shows how POD is nothing more than a subdivision of SVD. Epps and Techet (2009) showed the same fact using matrix algebra.

On the other hand, it is possible to show that if the field is homogeneous, the POD eigenfunctions simply revert to Fourier modes (Holmes et al., 1996; George, 1999). One can illustrate this characteristic using the spatial correlation of the signal [eq. (2.8)]. First, it is recognized that the correlation depends only in the separation between the sampling points, \( x \) and \( x' \), and not on their specific locations when the process is homogeneous (i.e., translational invariance). That is, if one considers \( x' = x + \Delta x \), then the spatial correlation in a homogeneous field can be expressed as \( r(x, x') = r(\Delta x) \), which is a function solely of \( \Delta x \). Substituting these simplifications into eq. (2.13) gives

\[
\sigma^2 \phi(x) = \int_{-\infty}^{\infty} r(\Delta x)\phi(x + \Delta x) d(\Delta x).
\]

Multiplying both sides of this equation by the complex conjugate of \( \phi(x) \) eliminates the \( x \)-dependence on the left-hand side. Further, since \( \phi(x) \) depends only on \( x \), we can include it in the integral on the right-hand side. Thus, to satisfy the equality, the eigenfunctions inside the integral must eliminate the \( x \)-dependence on that side of the equation. It follows that the eigenfunctions should be of exponential form

\[
\phi(x) \sim e^{ikx}.
\]

Including the SVD of the signal [eq. (2.6)] in eq. (2.8) to express the spatial correlation in terms of the
spatial eigenfunctions gives

$$r(\Delta x) = \int_T \left[ \sum_{k=1}^{\infty} \sigma_k \phi_k(x + \Delta x) \psi_k(t) \right] \left[ \sum_{j=1}^{\infty} \sigma_j \phi_j(x) \psi_j(t) \right] \, dt, \quad (2.18)$$

or

$$r(\Delta x) = \sum_{j,k} \sigma_k \sigma_j \phi_k(x + \Delta x) \phi_j(x) \int_T \psi_k(t) \psi_j(t) \, dt. \quad (2.19)$$

The orthogonality condition

$$(\psi_k, \psi_j) = \int_T \psi_k(t) \psi_j(t) \, dt = \delta_{jk}, \quad (2.20)$$

therefore yields

$$r(\Delta x) = \sum_j \left[ \sigma_j^2 \phi_j(x + \Delta x) \phi_j(x) \right], \quad (2.21)$$

which, if eq. (2.17) is introduced in eq. (2.21), gives

$$r(\Delta x) = \sum_j \left[ \sigma_j^2 e^{-ik(x+\Delta x)} e^{ikx} \right] = \sum_j \sigma_j^2 e^{-ik\Delta x}. \quad (2.22)$$

Equation (2.22) is simply a Fourier decomposition of the spatial correlation. Hence, the spatial SVD eigenfunctions (and in consequence the POD eigenfunctions) are identical to Fourier modes when the process $u'$ is homogeneous.

### 2.2 Discrete formulation

It should be noted that experimental data is collected in a discretized manner and should therefore be considered as a collection of variables $\xi_i$, measured with a set of probes located discretely at a finite number of spatial positions that are sampled a finite number of times. There are no restrictions for the number of variables that can be sampled at a given spatial position. For instance, each grid-point in a stereo-PIV experiment can be considered as a probe that senses three different variables: the three velocity components.

With this in mind, an instantaneous realization of the $N$ variables of interest, measured at $P$ different positions at time $k$, can be expressed in vector form as

$$U_k = [\xi_1^{(1)}, \xi_1^{(2)}, \ldots, \xi_1^{(P)}, \xi_2^{(1)}, \xi_2^{(2)}, \ldots, \xi_2^{(P)}, \ldots, \xi_N^{(1)}, \xi_N^{(2)}, \ldots, \xi_N^{(P)}]. \quad (2.23)$$
Note that $U_k$ is simply a list of the values of the variables $\xi_i$ organized according to the location at which they are measured (expressed by the super-index). Note further that each variable may be measured at several locations and, even if the distribution of spatial locations is regular in space (i.e. a rectangular grid for PIV data), the shape of $U_k$ may still be a vector. All instantaneous realizations of $U_k$ may be organized into a matrix whose spatial distribution varies across columns and its temporal distribution varies across rows as

$$U = \begin{bmatrix}
(\xi_1^{(1)})_1 & \ldots & (\xi_1^{(P)})_1 & \ldots & (\xi_N^{(1)})_1 & \ldots & (\xi_N^{(P)})_1 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
(\xi_1^{(1)})_k & \ldots & (\xi_1^{(P)})_k & \ldots & (\xi_N^{(1)})_k & \ldots & (\xi_N^{(P)})_k \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
(\xi_1^{(1)})_T & \ldots & (\xi_1^{(P)})_T & \ldots & (\xi_N^{(1)})_T & \ldots & (\xi_N^{(P)})_T
\end{bmatrix}.$$  \hspace{2cm} (2.24)

Note that $U$ is a matrix of size $[T \times (P \cdot N)]$. To analyze experimental data arranged in this form, it is convenient to express the SVD in matrix form. From eq. (2.6), the SVD of $U$ may be expressed as (see Chatterjee, 2000; Epps and Techet, 2009)

$$U = \Psi \Sigma \Phi^T,$$  \hspace{2cm} (2.25)

where $\Psi$ is an orthonormal square matrix of size $[T \times T]$ whose columns are the temporal eigenvectors of $U$ (representing the temporal eigenfunctions $\psi_k$), $\Sigma$ is a rectangular diagonal matrix of size $[T \times (P \cdot N)]$ whose diagonal elements are the singular values of $U$ (representing $\sigma_k$) and $\Phi$ is an orthonormal square matrix of size $[(P \cdot N) \times (P \cdot N)]$ whose columns are the spatial eigenvectors of $U$ (representing the spatial eigenfunctions $\phi_k$). Note that matrices $\Psi$ and $\Phi$ are normalized such that the Euclidian norm of their columns is unity. Therefore, the temporal and spatial shapes of the modes are given by the columns of $\Psi$ and $\Phi$ while the magnitude of the modes is given by their respective singular values in $\Sigma$. The existence and uniqueness of the SVD of a matrix is guaranteed by the following theorem (see L.N.Trefethen and Bau, 1997; Golub and Van Loan, 1996, for a proof):

**Theorem.** Every matrix $A \in \mathbb{C}^{m \times n}$ has a singular value decomposition given by eq. (2.25). Furthermore, the singular values $\{\sigma_j\}$ are uniquely determined and, if $A$ is square and the $\sigma_j$ are distinct, the left and right singular vectors $\{\psi_j\}$ and $\{\phi_j\}$ are uniquely determined up to complex signs (i.e., complex scalar factors of absolute value 1).

Note that, in general, the type of experimental data under consideration can be expressed as a rectangular matrix since the number of realizations acquired is generally be different from the number of spatial positions.
sampled. This theorem ensures that one will always find a SVD for any given set of data—a powerful result. In this regard, it is of interest to explore the relationship between the SVD and the eigenvalue decomposition (EVE) which will prove to have important implications. Since the EVE strictly applies to square matrices, one can find a square matrix version of a data set by premultiplying it by its transpose as

\[ U^T U = \Phi \Sigma^T \Phi, \]  

(2.26)

which, due to orthonormality \( \Psi^T \Psi = I \), yields

\[ U^T U = \Phi \Sigma \Sigma \Phi^T. \]  

(2.27)

One can define a new matrix \( \Lambda \equiv \Sigma^T \Sigma \) which gives

\[ U^T U = \Phi \Lambda \Phi^T. \]  

(2.28)

Postmultiplying both sides of this result by \( \Phi \) gives the EVE of matrix \( U^T U \) as

\[ (U^T U) \Phi = \Phi \Lambda, \]  

(2.29)

whose matrix of eigenvalues is \( \Lambda \), where the \( k \)-th eigenvalue is \( \lambda_k = \sigma_k^2 \). Note that the matrix of eigenvectors of \( U^T U \) is the same as the matrix of spatial singular vectors of \( U \). In consequence, eq. (2.29) gives a direct link to the kinetic energy content of the SVD modes. It has been found in several studies that the eigenvalues of the eigenvalue problem (2.13) (when \( u' \) represents a turbulent flow field) represent the turbulent kinetic energy (TKE) per unit mass contained in the eigenvectors of the EVE (see for example Sirovich, 1987; Holmes et al., 1996; Epps and Techet, 2009). It is observed that eq. (2.29) is the matrix form analogy of eq. (2.13). Therefore, since the spatial singular vectors of \( U \) coincide with the eigenvectors of \( U^T U \), eq. (2.29) implies that the turbulent kinetic energy per unit mass contained in the SVD modes is given by the squares of the singular values as

\[ \text{TKE}_k = \frac{1}{2} \sigma_k^2. \]  

(2.30)

Note further that a given mode \( k \) is expressed as the matrix \( U_k \) as

\[ U_k = \Psi(1 : T,k) \cdot \Sigma(k,k) \cdot \Phi(k,1 : (P \cdot N))^T. \]  

(2.31)
Additionally, matrix $\Psi$ loses its meaning when the data is uncorrelated in time [as does eq. (2.14)] and the SVD simply reduces to snapshot POD. In other words, one can compute the snapshot POD by dropping $\Psi$ from the SVD (see Epps and Techet, 2009, for more details) to yield

$$\tilde{U} = \Sigma \Phi^T,$$

(2.32)

where the tilde is used to avoid confusion between eqs. (2.25) and (2.32). Therefore, eq. (2.32) captures the collection of spatial modes of the data set or, in other words, the spatial configuration of the flow. The PIV data collected in the present work is uncorrelated in time, meaning all of the modal analysis of this data is conducted by the snapshot POD, first by computing the SVD, then dropping matrix $\Psi$, and finally reconstructing the spatial configuration of the flow using eq. (2.32).

### 2.3 Low-order reconstructions

To retrieve a low-order reconstruction of signal $u'(x, t)$, one can simply truncate the series given in eq. (2.6) at $k = M$, where $M \ll T$ is finite and expresses the number of modes to be kept in the reconstruction. Thus, the low-order reconstruction of the signal will be

$$u'(x, t)_{\text{low}} = \sum_{k=1}^{M} \sigma_k \phi_k(x) \psi_k(t).$$

(2.33)

Additionally, the modes excluded from the reconstruction yield a residual signal of the form

$$u'(x, t)_{\text{residual}} = u'(x, t) - u'(x, t)_{\text{low}} = \sum_{k=M+1}^{\infty} \sigma_k \phi_k(x) \psi_k(t).$$

(2.34)

When the signal is decomposed using the matrix form of SVD, as is the case of discrete signals, a low-order model can be obtained by keeping non-zero only those diagonal elements of matrix $\Sigma$ corresponding to the modes that are wanted in the reconstruction. Thus, the matrix of singular values is

$$\Sigma_{\text{low}} = \begin{cases} \Sigma_{kk} = \sigma_k, & k \leq M \\ \Sigma_{kk} = 0, & k > M \\ \Sigma_{ij} = 0, & i \neq j \end{cases}$$

(2.35)
Hence, the low-order SVD is written as (see Chatterjee, 2000, for further details)

\[ U_{\text{low}} = \Psi \Sigma_{\text{low}} \Phi^T. \quad (2.36) \]

Similarly, the residual reconstruction of \( U \) can be found by defining the matrix of singular values as

\[ \Sigma_{\text{residual}} = \Sigma - \Sigma_{\text{low}} = \begin{cases} 
\Sigma_{kk} = 0 & k \leq M \\
\Sigma_{kk} = \sigma_k & k > M \\
\Sigma_{ij} = 0 & i \neq j 
\end{cases}, \quad (2.37) \]

and the residual SVD is

\[ U_{\text{residual}} = \Psi \Sigma_{\text{residual}} \Phi^T. \quad (2.38) \]

As was mentioned earlier, when the signal of interest is non time-resolved, snapshot POD [eq. (2.32)] is used to capture the spatial configuration of the signal. It should be noted that this equation captures only the overall spatial structure of the signal of interest, but is not sufficient for reconstructing an instantaneous realization of the signal. Therefore, to find the decomposition of an instantaneous realization \( \hat{u}' \), this particular realization should first be projected into the space of snapshot POD modes as

\[ \hat{\psi} = \hat{u} \Phi \Sigma^{-1}. \quad (2.39) \]

Following this, the low-order reconstruction and the reconstruction residual of the instantaneous realization \( \hat{\psi} \) is found using eq. (2.36) and (2.38) with matrix \( \Psi \) replaced with the vector representing the projection of the instantaneous realization of interest \( \hat{\psi} \) into the space of snapshot POD modes as

\[ \hat{\psi}_{\text{low}} = \hat{\psi} \Sigma_{\text{low}} \Phi^T, \quad (2.40) \]

and

\[ \hat{\psi}_{\text{residual}} = \hat{\psi} \Sigma_{\text{residual}} \Phi^T. \quad (2.41) \]

Finally, it should be mentioned that, despite the fact that this analysis was carried out based on a signal in a spatio-temporal domain, there is no restriction for exchanging the time coordinate with another spatial coordinate within \( X \). In other words, the signal may be expressed in the form \( u'(x_1, x_2) \), and the SVD of such a signal gives the spatial modes in each coordinate independently. That is, the orthonormal functions of the signal are \( \phi(x_1) \) and \( \psi(x_2) \). In matrix form, eq. (2.25) implies that matrix \( \Psi \) contains the spatial
eigenvectors in $x_2$ and matrix $\Phi$ contains the spatial eigenvectors in $x_1$. In fact, Chatterjee (2000) used this formulation to calculate the SVD of a surface and generated low-order reconstructions of it. This approach will be discussed further in Chapter 3 to show how the low-order models of surface roughness were developed in this study.
Chapter 3

Roughness Characterization

3.1 Background

As was mentioned previously, the focus of this effort is on the impact of highly irregular surface roughness on turbulent boundary layers—both within and outside the roughness sublayer. Moreover, with the purpose of following a realistic approach to roughness effects, it was decided to use roughness patterns replicated from an actual engineering system. With this in mind, this work is based on one of a series of roughness patterns measured from in-service turbine blades damaged by a variety of different mechanisms such as spallation, pitting, and deposition of foreign materials. Such roughness topographies were fully documented by Bons et al. (2001), Bons (2002a) and Bons and McClain (2004). In particular, two of these topographies (deposition of foreign materials and spallation) were graciously provided to our research group by Professor Jeffrey P. Bons of Ohio State University. Three-dimensional maps of these two roughness patterns are shown in figure 3.1, from which one can infer that the roughness topographies under consideration are not just highly irregular, but also embody a broad range of topographical scales. As can be seen in figure 3.2, the probability density functions (pdf) of surface elevation from the ‘deposition of foreign materials’ surface is closer to a Gaussian distribution than is the one of ‘spallation’. Moreover, the distribution of surface elevation for the ‘spallation’ surface presents a bi-modal behavior, with a primary peak centered at 0.6 mm and a wider secondary peak centered at −1.0 mm. Thus, these examples serve to highlight how different damage mechanisms can yield distinctly different distributions of surface elevation. Since this study is intended to address the question of how different topographical scales of a given highly irregular roughness pattern affect the statistical and structural character of wall turbulence, the deposition of foreign materials surface, given its broad range of topographical scales, was selected for study.

It should be noted that the original profilometry measurements of the turbine-blade surface damaged by deposition of foreign materials reported by Bons et al. (2001) yielded roughness heights on the order of tens to hundreds of microns. Therefore, in order to generate fully-rough conditions for the relatively thick boundary layers generated by the flow facility employed (δ ~ 100 mm) at the Re considered herein, the
Figure 3.1: Roughness patterns from turbine blades damaged by different mechanisms (Source: Bons et al., 2001).

Figure 3.2: Probability density functions of surface elevation for two different turbine-blade roughness patterns. A Gaussian distribution with a root-mean-square equivalent to that of the deposition surface is included for comparison.

original profilometry information was scaled up in all three dimensions to yield a topographical condition with $k = 4.25\text{ mm}$ (Following Bons (2002a), the characteristic roughness height, $k$, is taken to be the average peak-to-valley height.). The topographical features of this surface, shown in figure 3.3(a), are elliptical in shape, are generally aligned in the streamwise direction and are attributable to cumulative deposition of foreign materials on the blade surface. However, a broad range of topographical scales is also clearly evident in this surface. Of particular interest, the general characteristics of this surface are representative of surface roughness encountered in many practical flows of interest but are quite distinct from the “idealized” roughness typically studied in the laboratory, including sand grain, mesh and ordered arrays of elements. Given that the boundary layers under consideration have thicknesses of approximately 100 mm, the full-surface condition gives $\delta/k \sim 24 - 30$. Further, the root-mean-square (RMS) roughness height, $k_{\text{rms}}$, for this surface (hereafter termed the ‘full surface’) is 1.0 mm while its skewness and kurtosis are 0.16 and 2.27, respectively. For reference, a long fetch of this surface was studied by Wu and Christensen (2007) and
outer-layer similarity was observed in all single- and multi-point statistics.

3.2 Development of low-order topographical models

Low-order models of the full were generated using SVD to explore the relative impact of different roughness scales on the turbulent boundary layer. Building upon the discussion at the end of Chapter 2, one can define the elevation of this surface about the mean elevation as $\eta(x, z)$. However, since $\eta$ is defined in a discrete spatial domain, the data contained in $\eta(x, z)$ can be better expressed in the matrix form $\eta_{ij}$ (for $i = 1, 2, \ldots T$ and $j = 1, 2, \ldots N$) as

$$
\eta = \begin{bmatrix}
\eta_{11} & \eta_{12} & \cdots & \eta_{1j} & \cdots & \eta_{1N} \\
\eta_{21} & \eta_{22} & \cdots & \eta_{2j} & \cdots & \eta_{2N} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\eta_{i1} & \eta_{i2} & \cdots & \eta_{ij} & \cdots & \eta_{iN} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\eta_{T1} & \eta_{T2} & \cdots & \eta_{Tj} & \cdots & \eta_{TN}
\end{bmatrix}
$$

(3.1)

Note that the SVD of the matrix $\eta_{ij}$ is written as

$$
\eta = \Psi \Sigma \Phi^T
$$

(3.2)

and yields two independent sets of spatial orthogonal functions, $\Phi_k(x)$ and $\Psi_k(z)$, in the same form as in eq. (2.25). Additionally, analogous to eqs. (2.35)–(2.38), the low-order and residual reconstructions of $\eta_{ij}$ are constructed by keeping only those diagonal elements of the matrix of singular values corresponding to the order of the reconstruction (given by the highest mode number in the reconstruction, $M$) as

$$
\eta_{\text{low}} = \Psi \Sigma_{\text{low}} \Phi^T,
$$

(3.3)

and

$$
\eta_{\text{residual}} = \Psi \Sigma_{\text{residual}} \Phi^T.
$$

(3.4)

Further, the level of detail present in a low-order reconstruction of a surface can be quantified in terms of the fractional surface content (FSC) as defined by Johnson and Christensen (2009). As was mentioned
previously in Chapter 2, the singular value $\sigma_k$ represents the characteristic magnitude (amplitude) of the $k^{th}$ SVD mode. Therefore, it is natural to expect a relationship between the root-mean-square amplitude of the topography under consideration, $k_{rms}$, and its singular values as

$$
(k_{rms}^T)^2 \sim \sum_{k=1}^{T} \sigma_k^2,
$$

where $T$ is the total number of SVD modes. Note that, according to eq. (2.30) in the case of fluid flow, the turbulent kinetic energy per unit mass carried by a given mode is proportional to the square of its singular value. In consequence, the total turbulent kinetic energy contained in such a flow would be proportional to
the sum of the square of all its singular values. Equation (3.5) expresses such a relationship for the SVD of a surface. However, while the concept of ‘kinetic energy’ does not translate to the case of a surface, the same analogy can be used to define the result in eq. (3.5) as simply a measure of the surface content. Hence, the $FSC$ of a given low-order surface reconstruction is defined as the ratio between the surface content of a given low-order model and the surface content of the original ‘full’ surface:

$$FSC = \frac{\sum_{k=1}^{M} \sigma_k^2}{\sum_{k=1}^{T} \sigma_k^2},$$

(3.6)

where $M \ll T$ is the number of modes kept in the low-order reconstruction. Combining this result with eq. (3.5) gives

$$FSC = \frac{\sum_{k=1}^{M} \sigma_k^2}{\sum_{k=1}^{T} \sigma_k^2} = \frac{(k_{\text{rms}}^M)^2}{(k_{\text{rms}}^T)^2}.$$

(3.7)

Nevertheless, the influence of each topographical scale of a given surface on the flow is not known 
 priori. Some level of understanding in this regard would provide a direct means of at least approximating the level of detail that must be kept in a low-order surface reconstruction of a given surface. Moreover, even if one existed, it would not be simply a matter of linearly superimposing the effects of each individual mode on the flow to obtain the cumulative impact of a given surface on the flow since non-linear (inertial) effects dominate in the high-Re turbulent boundary layers under consideration. Thus, it is likely that one cannot effectively predict, at least in a simple manner, the effect of the surface on the flow based solely in the geometric characteristics of the roughness. Thus, the level of detail to include in the low-order models to be considered is instead determined using the $FSC$. That is, for a given surface, the same fluid-flow experiment is conducted over the full surface and various low-order models with different $FSC$. Subsequently, among all low-order models, it is looked for the one with the lowest $FSC$ (equivalently, lowest number of modes) that closely replicates the effect of the full surface on the flow. In a previous effort utilizing the aforementioned ‘spallation’ surface, Johnson and Christensen (2009) found out that an $FSC$ of 95% (using the first 20 SVD modes) was enough to nearly replicate single-point statistics induced by a short fetch of this roughness in turbulent channel flow. While the absolute number of modes is likely not an effective metric for determining a suitable low-order model since each surface considered will most likely have a different distribution of surface content as a function of mode number, it is hypothesized that $FSC$ is a more robust tool for making such an assessment. While $FSC$ provides an objective measure of surface content, it does not directly account for the morphology of the surface under consideration (for example, if the surface is mainly composed by
depressions or bulges or combinations of them, distribution of the surface’s main features, etc.). However, using $FSC$ provides a starting point when considering complex irregular surface roughness. If successful, this method could also benefit computational research, since meshing a reduced-order surface model that induces approximately the same effect on the flow as the full surface would be computationally less expensive than meshing the full surface itself.

The methodology utilized herein to develop the low-order surface models is identical to that described in Johnson and Christensen (2009). Two different $FSC$ values were selected for study: 95% [based on the findings of Johnson and Christensen (2009)] and 71% (to isolate the impact of only the largest topographical scales). In the 95% $FSC$ case, the first sixteen SVD modes (out of 383 total modes) were included in the low-order reconstruction while the 71% $FSC$ case required the inclusion of only the first five SVD modes. Figure 3.4 presents the $FSC$ versus number of modes included in the low-order model which highlights how quickly the decomposition converges toward $FSC = 1$ well prior to inclusion of all 383 modes derived from the SVD of the full surface. From hereon, each low-order model will be addressed by the highest SVD mode included in its reconstruction (i.e. $M = 16$ and $M = 5$). The original ‘full’ surface and its $M = 16$ and $M = 5$ low-order models are shown in figures 3.3(a)–(c), respectively. In addition, the residuals of these low-order models, obtained from eq. (2.38), are shown in figures 3.3(e) [$M = 16$] and 3.3(f) [$M = 5$].

![Figure 3.4: Fractional surface content ($FSC$) as a function of mode number (○) and $FSC$ as a function of the number of modes included in a low-order representation (△). Vertical dashed lines (—) demarcate the modal content of each low-order model. Only 100 modes (out of 383) are shown for clarity.](image-url)
is clear from these figures that the $M = 16$ model closely resembles the original ‘full’ surface, which is consistent with its $FSC$ of 95%, while its residual bears little resemblance to the full surface. Instead, this residual field is composed of small, high-frequency modes that account for just 5% of the $FSC$. On the other hand, the $M = 5$ model exhibits only a slight resemblance with the full surface, particularly in the largest topographical features, which is expected from its relatively low $FSC$ of 71%. In contrast, its residual field bears a striking resemblance to the full surface. Note that this residual contains the same modes as the residual of the $M = 16$ model with the addition of modes 6–16. In this regard, it is noted that modes 6–16 cumulatively account for 24% of the $FSC$ and this visual comparison indicates that these modes are likely important for capturing the overall characteristics of the full surface.

It is well-established that the lowest-order modes derived from SVD embody the largest fraction of content of the original signal (as they are ordered in terms of decreasing content) and subsequently represent the largest scales of the original signal. Thus, the $M = 5$ model constructed from the first five SVD modes captures only the largest topographical scales of the full surface. With the addition of the next 11 modes (modes 6–16), the $M = 16$ model, which has an $FSC$ of 95%, captures additional, more intermediate-scale detail of the full surface as embodied in modes 6–16 which add 24% to the $FSC$ compared to the $M = 5$ model. Therefore modes 6–16 are going to be referred as ‘intermediate-scale’ modes and modes beyond 16 as the ‘smaller’ scales of the full surface that are discarded in both low-order representations. Thus, if the $M = 16$ model, with an $FSC$ of 95%, is able to effectively reproduce the behavior of flow over the full surface then these smaller discarded scales have a negligible impact on the flow development.

It should be noted that the streamwise and spanwise positions of the streamwise–wall-normal PIV measurements were kept constant over each surface so that accurate comparisons of the flow characteristics could be made between flow over the full surface as well as the two low-order models. The location of this lightsheet position is demarcated by a yellow line in figures 3.3(a)–(c). A comparison of the streamwise surface-elevation profiles along this measurement location for the three surfaces is shown in figure 3.3(d). These profiles provide a direct means of assessing the level of detail of the full surface lost in the two low-order reconstructions. For example, the $M = 16$ model lacks the small-scale details of the full surface, but follows the main tendency of it quite consistently. In contrast, the $M = 5$ model only captures the lowest wavelength characteristics of the full surface.
Table 3.1: Characteristics of the rough surfaces under consideration.

<table>
<thead>
<tr>
<th>Surface</th>
<th>FSC</th>
<th>Sk</th>
<th>Ku</th>
<th>ES</th>
<th>$k_{\text{rms}}$</th>
<th>$k$</th>
<th>$k_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>1</td>
<td>0.16</td>
<td>2.27</td>
<td>0.29</td>
<td>1.0</td>
<td>4.25</td>
<td>2.09</td>
</tr>
<tr>
<td>$M = 16$</td>
<td>0.95</td>
<td>0.20</td>
<td>2.34</td>
<td>0.19</td>
<td>0.975</td>
<td>4.09</td>
<td>1.88</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>0.71</td>
<td>0.24</td>
<td>2.82</td>
<td>0.14</td>
<td>0.840</td>
<td>3.66</td>
<td>—</td>
</tr>
</tbody>
</table>

3.3 Roughness characteristics

As noted above, truncation of higher-order modes yields a loss of information and thus a reduction in the characteristic height of the roughness. Table 3.1 summarizes various geometric characteristics of the two low-order topographical models compared to those of the full surface. Of particular interest, based upon eq. (3.7), there exists a reduction in the rms roughness height from 1.0 mm in the full surface to 0.975 mm and 0.84 mm in the $M = 16$ and $M = 5$ models, respectively. In addition, there exists a rather dramatic shift in the kurtosis for the $M = 5$ model, which is close to 3 (indicating Gaussian behavior), compared to the value of 2.27 for the full surface. Interestingly, the $M = 16$ model yields a kurtosis value (2.34) quite close to the full surface, indicating that modes 6–16 of the SVD decomposition contribute heavily in this regard.

An additional quantitative comparison of the low-order representations to the full surface is presented in figure 3.5a which presents pdfs of the roughness amplitude about the mean elevation ($\eta$). Contrasting the pdf of the full surface with that of the $M = 16$ model, good agreement is found over the range of amplitudes presented, though the $M = 16$ model lacks the weak, local peaks observed in the full-surface pdf for the small and intermediate range of amplitudes. The loss of these local peaks in the $M = 16$ model is a reflection of the suppression of the small topographical scales embodied in higher order SVD modes that were discarded in the formulation of this low-order reconstruction. In this regard, it is likely that these discarded topographical scales account for the rather jagged edges of the larger-scale topographical features of the full surface [3.3(a)] that are notably absent in the $M = 16$ topographical map [3.3(b)]. The differences between the pdfs of roughness amplitude for the full-surface and $M = 5$ cases are more dramatic as the pdf of the latter suggests an important loss in the content at the intermediate scales. These trends are also apparent in pdfs of the residual roughness amplitude as shown in figure 3.5b, where the content of intermediate scales in the residual of the $M = 5$ model is reflected in the rather wide peak about zero while the pdf of residual amplitude for the $M = 16$ model is much narrower.

The smoothing effect of reducing the order of the surface reconstruction is also apparent in pdfs of the streamwise and spanwise spatial gradients, $\partial \eta / \partial X$ and $\partial \eta / \partial Z$, respectively, as these gradients provide a
Figure 3.5: Probability density functions (pdfs) of (a) roughness amplitude about the mean elevation ($\eta$), (b) residual roughness amplitude about the mean elevation, (c) streamwise surface gradient, $\partial \eta / \partial X$, and (d) spanwise surface gradient, $\partial \eta / \partial Z$.

measure of how sharp the edges of the roughness features are. Figures 3.5c and 3.5d indicate that the gradients along both the streamwise and spanwise coordinates of the full surface are larger than those of the low-order models. This difference is most notable for the $M = 5$ model though the $M = 16$ result shows important differences from the full-surface result as well. In particular, discarding the higher-order SVD modes, which embody smaller-scale topographical scales, seems to reduce the jagged edges of the larger-scale topographical features. As such, this apparent change in the upstream and downstream slopes of the larger-scale topographical features may have a direct impact on the conditions for flow separation and vortex shedding from the larger-scale topographical features present in the full surface compared to the $M = 5$ and $M = 16$ models.

Following up on the wake behavior of roughness elements, it is well known that the total drag exerted by the flow on a rough wall embodies contributions from two sources: viscous drag and form drag. Viscous drag occurs wherever there is no flow separation around the roughness elements and thus dominates the total drag when the flow is hydraulically smooth. This behavior is characteristic of low Re flows, as the Re local to the roughness elements is so low that flow separation is a negligible effect. However, with
increasing Re, flow separation becomes dominant and hence the form-drag component of the total drag increases dramatically while the viscous-drag contribution is greatly reduced. Fully-rough flow is defined as the flow regime where the total drag is dominated by form-drag effects. The conditions at which flow separation occurs on the downstream side of roughness elements is intimately tied to their aspect ratio, their upstream and downstream slopes as well as how jagged such elements might be. Hence, this complex dependence highlights why roughness height alone cannot provide a common scaling of roughness effects over vastly different roughness topographies. To overcome these difficulties, Napoli et al. (2008) recently proposed a parameter that captures the relative importance of viscous and form drag based on the slope of roughness elements within highly irregular surfaces. This parameter is termed the *effective slope* (ES) and is defined as

\[
ES = \frac{1}{L} \int_{L} \left| \frac{\partial \eta}{\partial x} \right| \, dx, \tag{3.8}
\]

where \( L \) is the streamwise sampling length. Values of \( ES \) for the full surface and the two low-order models are shown in table 3.1.

Napoli et al. (2008) conducted direct numerical simulations (DNS) of turbulent channel flow for several different irregular two-dimensional roughness patterns at a friction Reynolds number of \( Re_\tau = 395 \). For a fully-developed channel flow, the imposed pressure gradient perfectly balances the total drag exerted by

![Figure 3.6: Dependence of the form drag \((C_p)\) and the viscous drag \((C_f)\) on \( ES \) (Source: Napoli et al., 2008).]
the walls. Therefore, $C_p + C_f = |\Pi| = 1$, where $\Pi$ is the dimensionless streamwise pressure gradient, $C_p = D_p/\rho u^2$ is the form drag coefficient, $C_f = D_f/\rho u^2$ is the viscous drag coefficient, and $D_p$ and $D_f$ are the streamwise components of the form and viscous wall stresses, respectively. Of interest, Napoli et al. (2008) assessed both $C_p$ and $C_f$ as a function of $ES$ calculated from the various surfaces they considered and identified clear trends with increasing $ES$ for their surfaces. This result (reproduced in figure 3.6) reveals a transition from viscous-drag dominance of the total drag at low $ES$ to form-drag dominance at higher $ES$ with equal contributions from these two drag components near $ES \approx 0.15$. Further, for the surfaces under consideration, this value of $ES$ provided a clear division between transitionally-rough ($ES \lesssim 0.15$) and fully-rough flow ($ES \gtrsim 0.15$). Napoli et al. (2008) also investigated the dependence of the roughness function, $\Delta U^+$, on $ES$ and this result is presented in figure 3.7. It was found that $\Delta U^+$ varied at first linearly with $ES$ at low $ES$, followed by non-linear growth until it became approximately constant beyond $ES \approx 0.35$. Based on these trends, Napoli et al. (2008) demarcated the region $ES \lesssim 0.35$ as the ‘waviness’ or slope-dependent regime and the region $ES \gtrsim 0.35$ as the ‘roughness’ or height-dependent regime. While these trends were quite consistent for the specific surface roughness considered by Napoli et al. (2008), it appears that this classification holds for other surfaces as well. Data from the recent experiments of Schultz and Flack (2009) for close-packed pyramid-shaped roughness elements of varying slope and height is also included in figure 3.7 and it is noted that these results for three-dimensional roughness exhibit $ES$ trends that are consistent with those initially reported by Napoli et al. (2008) for two-dimensional roughness. Taken together, these results indicate that the regime where $\Delta U^+$ exhibits linear growth ($ES \lesssim 0.15$) is typical of wavy walls, in which the viscous drag dominates and the form drag has a minor effect. In this regime, the likelihood of flow separation in the wake of roughness elements is quite low. Thus, the total drag will vary significantly with $Re$ and the likelihood of reaching a fully-rough state is low. In contrast, in the regime of $ES \gtrsim 0.35$ where $\Delta U^+$ exhibits little dependence on $ES$, the roughness elements are steep enough that the probability of flow separation in the wake of roughness elements is much higher. Thus, the possibility of reaching a fully-rough state is quite high, and, in consequence, $\Delta U^+$ exhibits a strong dependence on the roughness height but little dependence on $ES$. Finally, the interval $0.15 \lesssim ES \lesssim 0.35$ can be considered a ‘transition’ regime, where the dependence of $\Delta U^+$ on $ES$ weakens as its dependence on the roughness height strengthens.

With this background in mind, values of $\Delta U^+$ and $ES$ for the present full surface and two low-order models are also included in figure 3.7 (the values for $\Delta U^+$ are taken from Chapter 5). It is interesting to note that the $M = 5$ surface falls below the $ES \approx 0.15$ boundary between transitionally and fully rough, indicating that it is likely a transitionally-rough flow (consistent with observations reported in Chapter 5.
Figure 3.7: Dependence of the roughness function ($\Delta U^+$) on $ES$ from various datasets. Re$_x$ =: 395 (■) (Napoli et al., 2008); 1 200 (−−−), 3 800 (○−), 6 800 (− △−), and 9 600 (− ⊲−) (Schultz and Flack, 2009); 4 580 (□, $M = 5$), 4 950 (△, $M = 16$), and 5 215 (◇, full surface) (this work). (−·−·−·): limit at which $C'_p = C_f$ for Napoli et al. (2008) DNS; (−−−−): limit between the Slope Dependent (SD) and the Height Dependent (HD) regimes for Schultz and Flack (2009) experiments.

Based on the equivalent sand-grain roughness). In contrast, the $M = 16$ case as well as the full surface sit above this boundary, indicating that both of these cases lie in the fully-rough regime based on this $ES$-based classification, though they both still sit within the slope-dependent regime of $ES \lesssim 0.35$ based on the results of Napoli et al. (2008) and Schultz and Flack (2009). Note, however, that these two cases exhibit only a weak variation with $ES$, suggesting that the transition to the HD regime may occur closer to $ES \approx 0.2$ for the present roughness. With this in mind, there might be propitious conditions for the flow over the $M = 16$ model and the full roughness to be fully rough (the flow classification for these surfaces is addressed in Chapter 5). There are several factors that could account for these differences. First, the present surfaces and the ones of Napoli et al. (2008) are highly irregular and contain multiple topographical scales while the surfaces of Schultz and Flack (2009) were idealized and contained a single roughness length scale. This difference accounts for the obvious inconsistency in the overall values of $\Delta U^+$ that tend to be lower in the case of the pyramids despite their relatively high Re. This effect of enhanced momentum loss by irregular surfaces is also consistent with the observations of Bons et al. (2001) and Bons (2002a). In addition, the surfaces used by Napoli et al. (2008) are two-dimensional in nature (no variation in the spanwise direction).
while the present surfaces and the ones used by Schultz and Flack (2009) are highly three-dimensional. Finally, the work of Napoli et al. (2008) involved an internal turbulent flow (channel flow) wherein there exists a slight favorable pressure gradient that is required to drive the flow whereas the results of Schultz and Flack (2009) and the present effort are for zero-pressure-gradient (ZPG) turbulent boundary layers. Thus, the level of shear is comparatively higher in the case of internal flows which accounts for the higher values of $\Delta U^+$ from Napoli et al. (2008) at relatively low Re compared to the higher-Re TBL studies. Despite these differences, however, $ES$ appears to provide a sound basis for evaluating the flow regime one might expect based upon effective slope considerations.

### 3.4 Fabrication of roughness tiles

Physical replicas of the full surface and its low-order reconstructions were fabricated using a rapid-prototyping, powder-deposition printer with a spatial resolution of $80 \mu m$ in the three directions. Over sixty individual tiles of each roughness case with a maximum footprint of $25 \times 30 \text{mm}^2$ and a mean thickness of $6 \text{mm}$ were fabricated and each contained two mirror images of the basic pattern. This mirroring was necessary since the original spatial footprint of the digitized topography was not sufficient to fill a large streamwise fetch in the wind tunnel. The tiles were then mounted to cast aluminum plates that were then laid along the downstream half of the wind tunnel. Accommodating these plates into the wind tunnel required raising the upstream half of the boundary-layer plate to make its surface coincide with the average height of the roughness laid on the downstream half. As will be discussed in Chapter 4, the boundary layer was allowed to reach self-similarity over smooth-wall conditions prior to reaching the leading edge of the roughness. Subsequently, in all experiments except those for developing flow (see figure 4.4a), the internal layer was allowed to grow for a streamwise distance of $\sim 25\delta$, engulfing the entire boundary layer to eventually reach a self-similar state over rough-wall conditions.
Chapter 4

Experiments

The experiments that form the basis of this work include particle image velocimetry (PIV) measurements in the streamwise–wall-normal ($x - y$) plane of turbulent boundary layers flowing over the full roughness and two low-order reconstructions of this surface (detailed in §3), as well as experiments in a streamwise–spanwise ($x - z$) plane very close to these surfaces. Smooth-wall measurements were also conducted for a baseline against which the rough-wall data can be compared.

4.1 Experimental facility

An open circuit Eiffel-type boundary layer wind tunnel was used in all experiments. The overall dimensions of the facility are 20.0 m long, 3.40 m wide, and 2.49 m tall. As can be seen in figure 4.1, it is composed by three main sections: the conditioning section, the test section and the exhausting section. Air enters the conditioning section through an elliptical inlet to avoid flow separation. It then travels through a series of meshes and a honeycomb in the settling chamber to break down external, large-scale disturbances, and obtain nearly isotropic turbulence. Subsequently, the turbulence intensity levels are reduced to about 0.16% (Meinhart, 1994) by means of a contraction with an area ratio of $CR = 10$. Following the conditioning section, the air enters the test section with a low-turbulence, homogeneous velocity profile. After the test section, the air enters the exhausting section through a long low-angle diffuser that transitions the cross-sectional area from the rectangular shape of the test section to the circular shape of the tunnel’s fan. The shape of this section is critical to avoid separation while the flow expands. At the end of the exhausting section, the fan discharges the air through an acoustic diffuser to damp aerodynamic noise.

The test section is 91.4 cm wide, 45.7 cm high and 609.6 cm long. It has transparent lateral and bottom windows with glass insets all along its length to facilitate optical access. There is a false ceiling that smoothly joins the exit of the contraction duct and continues through to the end of the test section. The height of this false ceiling can be adjusted along the streamwise length of the test section to set the desired streamwise pressure gradient condition. There are pressure taps located along the boundary layer plate at 30.5 cm
intervals that allow direct assessment of the streamwise pressure profile. Since the present work focused on zero-pressure-gradient (ZPG) conditions, the ceiling’s height was adjusted until all the streamwise-separated pressure taps gave the same pressure reading within the instrument error. The boundary layers under consideration do not form along the bottom of the tunnel. Rather, they form on a boundary-layer plate that consists of two 90 cm wide by 304.8 cm long plates that are suspended above the bottom wall and joined smoothly in the streamwise center of the test section. Cross flow through the edges of the boundary layer plate is eliminated by means of a PVC foam gasket. In addition, to eliminate corner vortices, two 2.54 cm radius wooden fillets run along the top corners between the side walls of the test section and the boundary-layer plate. The leading edge of the boundary-layer plate is elliptically shaped and the boundary layers are tripped approximately 25 cm downstream of this leading edge with a 4.7 mm rod. Hence, the transition to turbulence is both stabilized at a known streamwise location and forced to occur uniformly across the spanwise direction of the tunnel. For the experiments, the boundary layers reach a maximum thickness of about 10 cm at the end of the boundary layer plate. This thickness is roughly nine times smaller than the width of the test section, ensuring two-dimensionality in the experiments.

To conduct PIV experiments, the flow was seeded with olive-oil droplets generated by means of nine Laskin nozzles. When using a working pressure in the range 80–100 p.s.i, these nozzles generate a mist of olive-oil droplets with a narrow diameter distribution centered at \( \sim 1 \mu m \) (Meinhart, 1994). All of the nozzles are fixed in a common oil container and, when in operation, the oil mist exits the container through a common vertical pipe with multiple exits. This Laskin nozzle system was positioned upstream of the inlet to the wind tunnel at roughly spanwise center. When running experiments, a continuous feed of olive oil mist was maintained in the closed room to ensure it was nearly homogeneously mixed in the air prior to being drawn into the tunnel. To allow for further homogenization, the seeded air recirculate in the room for several minutes before beginning the measurements.

It was imperative to have accurate measurements of the free-stream velocity for repeatability of mea-
measurements for the different surface conditions. Thus, the free-stream velocity was measured independently in each set of experiments. In the case of the $x-y$ measurements, a second camera was added above the one used for the actual experiments [see figure 4.2a]. With this secondary camera PIV images in the free-stream region were acquired without disturbing the position (and therefore the calibration) of the primary camera. In the case of $x-z$ experiments, it was imperative that the flow conditions were consistent with their $x-y$ counterparts both to have comparable results and to set the laser light-sheet at the desired wall-normal position. Further, since the lightsheet was realigned parallel to the wall, it was not possible to probe the free-stream conditions directly via PIV. Thus, a pitot-static probe was utilized to measure and control the free-stream velocity. With this in mind, the secondary (top) camera shown in figure 4.2a was used to calibrate a pitot-static probe located in the free-stream region slightly downstream of the PIV measurement location. The free-stream velocity, $U_e$, was determined from the pitot-static probe measurement as

$$U_e = C_d \frac{\rho^{(s)}}{\rho} \sqrt{\frac{2 \Delta P}{\rho^{(s)}}},$$

(4.1)

where $C_d$ is the discharge coefficient, $\rho^{(s)} = 1.275 \text{kg/m}^3$ is the density of dry air for a IUPAC standard atmosphere ($P^{(s)} = 100 \text{kPa}, T^{(s)} = 273.15 \text{K}$) and $\rho \text{(kg/m}^3\text{)}$ is the density of humid air at current atmospheric conditions. Ten points were measured under different tunnel operating conditions as assessed by the frequency of the inverter controlling the tunnel fan. Using least-squares fitting, the discharge coefficient was
found to be $C_d = 0.934$. The resulting calibration curve is shown in figure 4.2b. It should be taken into account that this value of $C_d$ is valid only for ZPG conditions.

4.2 Streamwise–wall-normal plane ($x - y$) measurements

Over forty-five hundred statistically independent, two-dimensional velocity $(u, v)$ fields were acquired by PIV in the streamwise–wall-normal $(x, y)$ plane at the spanwise center of the tunnel for each surface condition to minimize sampling errors in the computed statistics. It should be noted that the streamwise and spanwise positions of this measurement plane were carefully maintained over the same roughness features for the developing- and developed-flow experiments, respectively, allowing a meaningful comparison of the flow statistics. The flow field was illuminated with a 700 µm-thick laser sheet generated by a pair of Nd:YAG lasers (200 mJ/pulse, 5 ns pulse duration; Quantel) and a combination of spherical and cylindrical lenses. A high-energy mirror directed the laser sheet into the wind tunnel such that it was normal to the flow boundary and parallel to the flow direction. Time-separated images of the scattered light from the particles were captured with a 4k × 2.8k, 12-bit frame-straddle CCD camera (TSI 11MP). The roughness at the measurement location was painted black to reduce reflections of laser light; however, the remaining unsuppressed reflections rendered measurements in the region $y \lesssim 0.06 - 0.1\delta$ impossible for the rough-wall cases. Figure 4.3 presents photos of the illumination and imaging arrangements for these streamwise–wall-normal experiments.

Measurements were conducted under both developing and developed flow conditions for flow over the full surface and the two low-order representations. Two different streamwise fetches of roughness were consid-
Figure 4.4: Schematics of the 2D-PIV experimental arrangement in the streamwise–wall-normal ($x$–$y$) plane for (a) developing and (b) developed flow experiments.

erred in this effort: 1 m (equivalently $\sim 10\delta$) and 3 m ($\sim 30\delta$) to facilitate the study of both developing and developed turbulent boundary layers. The boundary layers were tripped to initiate transition to turbulence at the leading edge of the suspended plate and grew over smooth-wall conditions for the first 3 m and 5 m for the developed and developing flow experiments, respectively, after which they encountered the roughness conditions considered herein. For reference, a 2 m-long smooth cast aluminum plate was laid along the downstream half of the boundary-layer plate to extend the upstream smooth-wall fetch an additional 2 m for the developing-flow cases, after which the flow encountered the 1 m-long patch of roughness. Since the initial boundary-layer development occurred over a smooth wall, this 1 m-long roughness scenario therefore represents the impact of short streamwise fetches of roughness on an incoming smooth-wall turbulent boundary layer. As such, one would expect the formation of an internal layer within the turbulent boundary layer wherein roughness effects are confined. In the 3 m-long roughness scenario, the flow is termed developed to reflect the fact that the internal roughness layer has grown to occupy the entire wall-normal extent of the turbulent boundary layer and a self-similar state is attained. The reproduction of over thirty individual roughness panels for each case was required to achieve a streamwise fetch of 1 m while an additional sixty panels per case were required to extend the streamwise fetch to 3 m. The panels were mounted to cast aluminum plates which were then laid along the downstream half of the wind tunnel. Figure 4.4 presents a schematic of both surface layout scenarios measurements were made 0.8 m ($\sim 8\delta$) and 2.5 m ($\sim 25\delta$) downstream of the leading edge of the roughness for the developing- and developed-flow cases, respectively. The fields of view were $1.15\delta \times 0.8\delta$ (streamwise by wall-normal) and $1.4\delta \times \delta$ for the developing and developed flow experiments, respectively. All relevant experimental parameters for the developing- and developed-flow
Table 4.1: Relevant experimental parameters for developing-flow experiments.

<table>
<thead>
<tr>
<th>Surface</th>
<th>( \text{Re}_\theta )</th>
<th>( U_e ) (m/s)</th>
<th>( \delta ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>15 970</td>
<td>16.9</td>
<td>125.8</td>
</tr>
<tr>
<td>( M = 16 )</td>
<td>15 540</td>
<td>16.8</td>
<td>120.9</td>
</tr>
<tr>
<td>( M = 5 )</td>
<td>15 400</td>
<td>16.9</td>
<td>126.2</td>
</tr>
</tbody>
</table>

Table 4.2: Relevant experimental parameters for developed-flow experiments.

<table>
<thead>
<tr>
<th>Surface</th>
<th>( \text{Re}_\theta )</th>
<th>( \delta^+ )</th>
<th>( \Delta U^+ )</th>
<th>( k^+ )</th>
<th>( k_s^+ )</th>
<th>( 5k/\delta )</th>
<th>( U_e ) (m/s)</th>
<th>( \delta ) (mm)</th>
<th>( u_\tau ) (m/s)</th>
<th>( y_* ) (( \mu m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>14 340</td>
<td>5 215</td>
<td>8.2</td>
<td>218</td>
<td>107</td>
<td>0.21</td>
<td>17.25</td>
<td>101.9</td>
<td>0.785</td>
<td>19.5</td>
</tr>
<tr>
<td>( M = 16 )</td>
<td>13 300</td>
<td>4 950</td>
<td>7.9</td>
<td>207</td>
<td>95</td>
<td>0.21</td>
<td>17.20</td>
<td>98.6</td>
<td>0.785</td>
<td>19.8</td>
</tr>
<tr>
<td>( M = 5 )</td>
<td>12 590</td>
<td>4 580</td>
<td>5.7</td>
<td>173</td>
<td>–</td>
<td>0.19</td>
<td>17.30</td>
<td>96.6</td>
<td>0.723</td>
<td>21.1</td>
</tr>
<tr>
<td>Smooth</td>
<td>11 400</td>
<td>3 350</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>16.77</td>
<td>96.1</td>
<td>0.543</td>
<td>28.7</td>
</tr>
</tbody>
</table>

\( x-y \) experiments are presented in tables 4.1 and 4.2. Note that \( \delta \) is taken as the wall-normal position where the mean streamwise velocity equals 99% of the free-stream velocity.

Finally, streamwise–wall-normal plane measurements were also conducted for smooth-wall flow to provide a baseline against which all rough-wall measurements were compared. These measurements were conducted at \( \text{Re}_\theta = 11409 \) which is equivalently \( \delta^+ = 3350 \). The relevant smooth-wall flow characteristics are summarized in table 4.2.

### 4.3 Streamwise–spanwise plane (\( x-z \)) measurements

Two different experiments were conducted in the \( x-z \) plane using the above mentioned facility. The first consisted of stereo PIV measurements for each rough-wall case as well as a smooth-wall baseline. Since the purpose of these measurements was to study the structure of the roughness sublayer very close to the wall, measurements were performed at a wall-parallel plane located at a wall-normal position of \( y = 0.047\delta \). It should be noted that the origin of the wall-normal coordinate under rough-wall conditions is defined by the flow characteristics near the wall (Perry and Li, 1990). Since this origin is located at a position that does not necessarily coincide with any roughness parameter (i.e. the roughness midplane), it is termed the virtual origin. The modified Clauser chart method was used to determine the virtual origin for each rough-wall case considered herein, a technique refined by various authors based on the early work of Clauser (1954, 1956) (see Clauser, 2003, for a reprint of the first one of these papers). There are different approaches to the modified Clauser chart method (see for example Perry and Li, 1990; Schultz and Flack, 2007; Dixit and...
Figure 4.5: Local roughness topography below the $y = 0.047\delta$ wall-parallel fields of view (demarcated by boxed regions) for the (a) $M = 5$ model, (b) $M = 16$ model and (c) full surface. The dashed lines represent the position of the $x - y$ PIV measurements. (d) Schematic illustrating stereo PIV view of the flow in the wall-parallel field of view above the roughness.

Ramesh, 2009), from which a concise and simple approach was developed utilizing the mean velocity profiles computed from the $x - y$ plane PIV measurements and is described in detail in Appendix B. The virtual origin of our rough-wall experiments was referenced to the midplane of the roughness, or equivalently to the upstream smooth-wall origin. The wall-normal distance from this midplane to the virtual origin above is called the origin offset and is defined as $\varepsilon$. Figure 4.5 presents contour maps of the roughness topographies directly beneath the $x - z$ measurement planes for each surface, providing a means of contrasting the local topographical characteristics below the stereo PIV measurements with the data to be presented. A schematic of the wall-normal position of the laser light-sheet relative to the rough walls is also presented. The origin offset, and other relevant experimental parameters, are presented in table 4.3.
Table 4.3: Relevant experimental parameters for streamwise–spanwise measurements.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$U_e$ (m/s)</th>
<th>$\overline{U}$ (m/s)</th>
<th>$\varepsilon$ (mm)</th>
<th>$0.047\delta$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>17.4</td>
<td>9.26</td>
<td>5.6</td>
<td>4.8</td>
</tr>
<tr>
<td>$M = 16$</td>
<td>17.5</td>
<td>9.68</td>
<td>5.6</td>
<td>4.6</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>17.1</td>
<td>10.43</td>
<td>3.9</td>
<td>4.5</td>
</tr>
<tr>
<td>Smooth</td>
<td>16.1</td>
<td>10.02</td>
<td>–</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Figure 4.6: Photo of the stereo imaging and illumination arrangement.

With regard to the stereo PIV experimental arrangement, two cameras, identical to that discussed above, were positioned approximately 0.76 m away from the measurement plane and imaged the flow through a transparent section in the wind-tunnel ceiling. The imaging paths of the cameras were rotated at $\pm 13.5^\circ$ with respect to the wall-normal ($y$) axis. The laser light sheets were formed in a manner identical to that described earlier but were introduced into the tunnel through a glass sidewall while their orientations were carefully adjusted to ensure they remained parallel to the wall at $y = 0.047\delta$. Figure 4.6 presents a photo of the illumination and imaging arrangements. Uniform image focus was ensured in both cameras across the entire field of view by satisfying the Scheimpflug condition. The pairs of images acquired by each camera were interrogated and validated independently in accordance with the methodology described in §4.4, resulting in pairs of instantaneous planar fields of two-dimensional particle displacements from the two cameras. These pairs of displacement fields were then recombined into three-component instantaneous velocity fields using a mapping function generated via calibration of the imaging system. This calibration employed a dual-plane (1-
mm separation) target containing white dots spaced at in-plane intervals of 10 mm over a 20 cm × 20 cm area. The use of this dual-plane target alleviated the need to physically translate the target in the depth direction through the lightsheet thickness. Instead, the target was carefully aligned in the field of view so that its mid-plane coincided with the laser lightsheet. With the target fixed at this location, an image was acquired from each camera, and these images were used to generate the mapping function via the least-squares method of Soloff et al. (1997) using third-order polynomials for the in-plane coordinates and a first-order polynomial for the out-of-plane coordinate. Figure 4.7 presents examples of the calibration images acquired by each camera in the stereo imaging arrangement. A drawback of this technique is that obtaining perfect alignment of the target with the laser lightsheet is virtually impossible. To overcome this issue, the self-calibration scheme originally proposed by Wieneke (2005) was used to optimize the mapping function. The final mapping function was then used to reconstruct three-dimensional velocity vectors on the measurement plane from the pairs of two-dimensional particle displacements. Figure 4.8 presents schematics of the calibration grids as viewed by the two cameras as well as the mapping of these grids onto a common reference frame using the derived mapping function. This latter comparison highlights the efficacy of the resulting mapping function in recombining the distinct views of the two cameras back to the physical coordinates defined by the location of the laser lightsheet.

In addition, a single high-speed 2D-PIV measurement was conducted for flow over the full surface at a wall-normal position of \( y = 0.065\delta \). The illumination source was a double-pulsed, Nd:YLF laser (10 mJ per pulse at 1 kHz pulse frequency) coupled with a single 1k × 1k pixel\(^2\) high-speed CMOS camera operating at a frame rate of 2 kHz. The laser pulses were frame-straddled across consecutive image pairs allowing time delays between images to be controllable down to 1 \( \mu \)s using a TSI synchronizer. Thus, the dual-head illumination source operating at 1 kHz and the camera running at 2 kHz allowed the acquisition of
velocity fields at a rate of 1 kHz. This camera has 2 GB of on-board memory on-board, meaning that a single run at a framing rate of 2 kHz lasted 1 s and yielded 1000 vector fields with a 1 ms time interval between consecutive fields (since the images were 8-bit). A field of view of $0.45\delta \times 0.46\delta$ was imaged and the PIV images were interrogated with $16 \times 16$ pixel$^2$ interrogation windows, yielding a vector grid spacing of approximately $400 \mu m (20y^*)$. For reference, the mean streamwise velocity at this wall-normal location was 9.2 m/s. This experiment was used to quantitatively study the modification of the structure of very large scale motions (VLSMs) within the roughness sublayer (Wu et al., 2009).

Figure 4.9 presents a schematic of both experimental arrangements. In all experiments, a smooth-wall self-similar turbulent boundary layer developed on the flat plate prior to the leading edge of the roughness. Upon reaching the roughness, an internal layer formed within this smooth-wall boundary layer that grew to eventually engulf the entire boundary layer over a $\sim 28\delta$ streamwise length of roughness prior to the measurement location. As such, all $x - z$ measurements were made under developed, self-similar turbulent boundary layer conditions.
4.4 PIV interrogation scheme

In all of the experiments described above the PIV image pairs were interrogated using a recursive, two-frame cross-correlation algorithm. The interrogation window size was chosen to have consistency in vector spacing when scaled in inner units (i.e., with $u_\tau$ and $\nu$) such that $\Delta x^+ = \Delta y^+ = \Delta^+ \approx 18$ (where $(\cdot)^+$ denotes normalization with respect to inner variables). To reduce bias errors due to loss of particle pairs, the second window of the first-pass cross-correlation was slightly larger than the first window and offset by the bulk displacement. During the second pass, the second window was offset by the estimated local particle displacement determined from the first-pass results. To validate each instantaneous velocity field, spurious vectors were removed by means of three different filters: absolute limits of displacement and median and mean neighbor comparison. To substitute eliminated spurious vectors for true displacement peaks, a Rohály-Hart analysis (Hart, 1998; Rohály et al., 2000) was performed. Finally, the remaining missing vectors were substituted by means of bilinear interpolation. Typically, no less that 98% of valid vectors were obtained in each instantaneous realization, minimizing the number of interpolated vectors. Finally, a narrow Gaussian low-pass filter was applied to each velocity field to reduce aliasing due to noise in frequencies higher than the spatial sampling frequency of the interrogation. Tables 4.4–4.7 summarize the specific interrogation and validation parameters utilized for each experiment undertaken.
Table 4.4: Interrogation parameters.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>1st pass</th>
<th>2nd pass</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x-offset</td>
<td>y-offset</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(pix)</td>
<td>(pix)</td>
</tr>
<tr>
<td>(x-y)</td>
<td>Full</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>(2D-PIV)</td>
<td>(M = 16)</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(M = 5)</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>(x-z)</td>
<td>Full</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>(TR-PIV)</td>
<td>(M = 16)</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(M = 5)</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.5: Filters to remove spurious vectors. Same values for both validation passes and for all experiments.

<table>
<thead>
<tr>
<th>(U_{\text{min}})</th>
<th>(U_{\text{max}})</th>
<th>(V_{\text{min}})</th>
<th>(V_{\text{max}})</th>
<th>Tol.</th>
<th>N.S.</th>
<th>Tol.</th>
<th>N.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(pix)</td>
<td>(pix)</td>
<td>(pix)</td>
<td>(pix)</td>
<td>(pix)</td>
<td>(G.P.)</td>
<td>(pix)</td>
<td>(G.P.)</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>-4</td>
<td>4</td>
<td>3</td>
<td>3 x 3</td>
<td>3</td>
<td>3 x 3</td>
</tr>
</tbody>
</table>

\(a\) Tolerance.
\(b\) Neighborhood Size.
\(c\) Grid Points.

4.5 Image pre-processing and reflection filtering

Performing stereo-PIV experiments deep within the roughness sublayer is a promising idea to study the structure of the boundary layer as modified by the different roughness configurations. However, there is an important challenge that must be overcome. In particular, as the laser lightsheet will be parallel to and within a few millimeters of the irregular surfaces, reflections from these irregularities will become an important, and in fact debilitating, issue. While smooth-wall measurements are not tainted by such effects, the complex nature of the surface roughness under consideration leads to intense reflections of laser light that will be scattered in all directions and thus will appear in the background of the PIV images. To illustrate this effect, a test experiment was run with the laser sheet located approximately 8 mm above the midplane of the \(M = 5\) model (approximately \(y = 0.047\delta\) from the virtual origin), using the same flow conditions given in table 4.3 for this surface condition. A sample PIV image is presented in figure 4.10 and the strong background reflections due to the largest roughness elements are obvious, particularly how much stronger these unwanted reflections are compared to the imaged scattered light from the tracer particles. These background reflections unfortunately corrupt the PIV interrogation and induce strong bias in single-point...
Table 4.6: Vector substitution parameters for 1st validation pass.

<table>
<thead>
<tr>
<th>Rohály-Hart&lt;sup&gt;a&lt;/sup&gt; (Median)</th>
<th>Rohály-Hart&lt;sup&gt;a&lt;/sup&gt; (Mean)</th>
<th>Bilinear Interpolation</th>
<th>Gaussian Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 3 4</td>
<td>1 3 2</td>
<td>3 4</td>
<td>3 1.1</td>
</tr>
</tbody>
</table>

<sup>a</sup> Rohály-Hart analysis was not used for time-resolved PIV experiments.

Table 4.7: Vector substitution parameters for 2nd validation pass.

<table>
<thead>
<tr>
<th>Rohály-Hart&lt;sup&gt;a&lt;/sup&gt; (Median)</th>
<th>Rohály-Hart&lt;sup&gt;a&lt;/sup&gt; (Mean)</th>
<th>Bilinear Interpolation</th>
<th>Gaussian Smoothing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 3 2</td>
<td>1 3 1</td>
<td>3 1</td>
<td>3 0.85</td>
</tr>
</tbody>
</table>

<sup>a</sup> Rohály-Hart analysis was not used for time-resolved PIV experiments.

Figure 4.10: Influence of background reflections on single-point statistics. (a) Raw PIV image; (b) wall-normal Reynolds normal stress, $\langle v'^2 \rangle^+$.  

statistics, as can be observed in the contour map of wall-normal Reynolds normal stress ($\langle v'^2 \rangle^+$ shown in figure 4.10(b)). Note that this single-point statistic is used as the ‘control’ result since it has been found to be the most sensitive to background reflections among all single-point turbulence statistics.

A solution to this type of background reflection problem was recently proposed by Deen et al. (2010). The basic premise of this method is to subtract frame $B$ from frame $A$ of pairs of PIV images which, in theory, should render the intensity of any fixed objects present in the images zero since they are likely to scatter similar light in both frames (compared to the random nature of the scattered light from the moving tracer particles). To compensate for intensity differences due to disparity in the power of the two laser pulses, Deen et al. (2010) introduced an intensity normalization based on local values of minimum and maximum intensity. To sample for extreme values and normalize locally, they used an image spot of size larger that the
average particle image diameter but smaller than the intended PIV window size. The local normalization is then carried out as

$$N(x) = \frac{I(x) - I_{\text{min}}(x)}{I_{\text{max}}(x) - I_{\text{min}}(x)},$$

(4.2)

where $I(x)$ is the sliding local intensity and $I_{\text{max}}(x)$ and $I_{\text{min}}(x)$ are its maximum and minimum values, respectively. After normalizing, since the intensity values are bounded between 0 and 1, the two PIV frames can be subtracted from each other. After subtracting frame $A$ from frame $B$, frame $A$ is recovered by extracting the positive values of intensity from the result while frame $B$ is recovered by extracting the negative values of intensity from the result and taking their absolute value. Figure 4.11 presents an example of this algorithm applied to the instantaneous PIV image in figure 4.10a. While this approach worked well for the application presented by Deen et al. (2010) [single-phase flow in spacer-filled channels] its use in the present case leaves a residual imprint of the normalization spot in the formerly bright regions of the PIV images. Since the local maximum intensity may change from one frame to another due to particle displacement, the relative intensity of the background will likely also be different after local normalization. Thus, as proposed, this correction method is found to be quite sensitive to extreme values.

To overcome this problem of extreme value sensitivity, this method was modified an initial local normalization based on the local median (instead of maximum) and minimum intensity values as

$$N(x) = \frac{I(x) - I_{\text{min}}(x)}{I_{\text{median}}(x) - I_{\text{min}}(x)}.$$ 

(4.3)

A frame subtraction is then applied followed by a local normalization according to eq. (4.2). Finally, the intensity values are stretched to a 16-bit intensity range to match their original form at the time of acquisition. The final result of this modified method is presented in figure 4.12 for the same PIV image wherein the effect of the normalization spot is difficult to discern. Hence this method can be used to effectively reduce bias associated with background reflections.

This approach was found to work well for $M = 16$ and full surface cases as is evident in figure 4.13 which presents contour maps of $\langle v'^2 \rangle^+$ wherein imprints of the background reflections from the roughness are not notable. However, despite the use of this modified image subtraction technique, the contour map of $\langle v'^2 \rangle^+$ for the $M = 5$ model did not show as much improvement. It is surmised that the reason for this behavior lies in the fact that the $M = 5$ model lacks the small- and intermediate-scale features that abound in the full surface and are present to some extent in the $M = 16$ model. Consequently, the light scatters from the $M = 5$ model in a more specular manner than from the other surfaces, increasing dramatically the background light intensity. Thus, it is then more likely to encounter clusters of saturated pixels in the
Figure 4.11: Image after fixed objects subtraction. (a) general view, (b) zoom in a former bright spot.

Figure 4.12: PIV image after modified fixed objects subtraction. (a) General view; (b) zoomed-in view of a former bright spot.

PIV images of flow over the $M = 5$ model than in the other surface cases. This condition is detrimental for the above mentioned background subtraction since it prevents the algorithm to recover any meaningful particle-image data from these saturated regions of the image.

To further reduce reflections, the $M = 5$ surface in the vicinity of the measurement location was painted with a fluorescent dye to alter the wavelength of the light leaving the surface and filter it out prior to reaching the imaging camera. The dye used was Sulforhodamine-101, which is useful for this application because of its large Stokes shift (Coppeta and Rogers, 1998; Natraj and Christensen, 2009) as evidenced by its absorption/emission spectra (see figure 4.14). Since this dye’s light absorptivity is relatively low ($\approx 25\%$) at the excitation wavelength (532 nm), some light will still be reflected from the surface without any modification in wavelength. To compensate for this effect, a relatively high concentration was employed. Preparation of
the dye solution involved dissolving it in tap water until it reached the saturation concentration at room temperature. It was poured on the surface and allowed to dry. Only three layers of solution were added to the surface since there was no significant further improvement in background attenuation with additional layers of dye. A bandpass filter with central wavelength of 532 nm and bandwidth of ±2 nm was placed upstream of the two cameras in the stereo-PIV imaging arrangement to block the light fluoresced from the dye at the surface while only collecting the 532 nm light scattered by the tracer particles. The result of this procedure was a significant attenuation of the background reflection, with a greatly-improved reduction of large clusters of saturated pixels in the resulting PIV images. All of the images acquired under these conditions were still pre-processed with the above mentioned image subtraction scheme to further reduce remaining background reflections. While a significant additional improvement was noted in $\langle v'^2 \rangle$ for the $M = 5$ surface (figure 4.15), a very weak residual bias remained in the out-of-plane velocity component of the $M = 5 x - z$ experiments. Nevertheless, this bias was only apparent in the ensemble average of wall-normal Reynolds normal stresses ($\langle v'^2 \rangle$). However, as can be seen in figure 4.15, this effect is so weak that the bias it induces is well within the spatial variations of $\langle v'^2 \rangle$.

### 4.6 Error analysis

Finally, it is important to assess the degree of uncertainty present in the above-mentioned experiments, particularly how these uncertainties propagate into the turbulence statistics to be presented in the coming chapters. According to Moffat (1988), experimental errors may be classified in three categories: fixed, random, and variable but deterministic. The first classification refers to errors that do not change along
Figure 4.14: Absorption and emission spectra of Sulforhodamine-101.

Table 4.8: Upper bounds of relative errors in the turbulence statistics.

<table>
<thead>
<tr>
<th>Error Term</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon \left( U_i^+ \right)$</td>
<td>5.03%</td>
</tr>
<tr>
<td>$\varepsilon \left( (u'_i u'_j)^+ \right)$</td>
<td>7.42%</td>
</tr>
<tr>
<td>$\varepsilon \left( \rho_{u'_i u'_j} \right)$</td>
<td>3.44%</td>
</tr>
<tr>
<td>$\varepsilon \left( \frac{U_i}{U_e} \right)$</td>
<td>0.01%</td>
</tr>
<tr>
<td>$\varepsilon \left( \frac{\langle u'_i u'_j \rangle}{U_e^2} \right)$</td>
<td>0.14%</td>
</tr>
</tbody>
</table>

The experiment. These ones are typically inherent to the sensors utilized. The second classification refers to errors that vary constantly in a stochastic manner. These errors are typically normally distributed or statistically stationary and may be due to either the nature of the phenomenon of interest or uncontrollable fluctuations in the data acquisition system. Finally, the third classification refers to errors that change throughout an experiment but in a predictable manner. These errors are usual in unsteady processes with slow transients, where the term “slow” implies “much slower than the sampling rate”.

For the present work, all experiments were conducted under statistical stationarity, meaning errors of the kind variable but deterministic are not considered. Additionally, spatial calibration targets were utilized to minimize fixed errors (see Wu, 2009, for detailed information about the calibration process). Therefore, fixed errors are also neglected in the error analysis. Hence, the predominant errors considered herein are
Figure 4.15: Residual bias in $\langle v'^2 \rangle$ after reflection filtering and background subtracting the PIV images of flow over the $M = 5$ model.

...those of a random nature. A detailed discussion of the specific random errors in the present experiments is presented in Appendix A, particularly how these errors propagate into the turbulence statistics presented herein. For brevity, only the largest error detected in each kind of turbulence statistic will be considered as a measure of the upper bound on the error. The most relevant of these uncertainty results are shown in table 4.8.
Chapter 5

Statistical Behavior of Developing and Developed Flow

This chapter presents analysis of the streamwise–wall-normal plane PIV datasets for smooth-wall flow as well as flow over the full surface and the two low-order models.

5.1 Analysis methodology

The efficacy of the two different low-order representations in reproducing the characteristics of flow over the full surface under both developing- and developed-flow scenarios is evaluated by directly comparing various single-point statistics, including the mean velocity, the Reynolds normal stresses ($\langle u'^2 \rangle$ and $\langle v'^2 \rangle$) and the Reynolds shear stress (RSS; $\langle u'v' \rangle$). In all cases, the mean velocity profile was formed by ensemble-averaging all velocity realizations for a given case followed by line-averaging in the streamwise direction. The turbulence statistics considered herein were then computed in a similar manner using fluctuating velocity fields derived from the aforementioned mean velocity profiles.

Probability density functions are also used to contrast the instantaneous contributions to the RSS for flow over the two low-order models and the full surface. In this regard, the instantaneous $u'v'$ events that are averaged to generate the mean RSS profile can be formed by different combinations of $u'$ and $v'$ depending upon which quadrant of the $u' - v'$ plane a given instantaneous RSS event resides. In particular, negative contributions to $\langle u'v' \rangle$ are attributable to ejection ($Q_2$: $u' < 0, v' > 0$) and sweep ($Q_4$: $u' > 0, v' < 0$) events while positive contributions are generated by inward ($Q_3$: $u' < 0, v' < 0$) and outward ($Q_1$: $u' > 0, v' > 0$) interactions. To further explore the efficacy of the low-order models in reproducing the character of these individual quadrant events for flow over the full surface, quadrant analysis, as first proposed by Lu and Willmarth (1973), is also applied to all cases. In quadrant analysis, the mean RSS at each wall-normal position is decomposed into contributions from four quadrants excluding a hyperbolic hole of size $H$ as

$$\langle u'v' \rangle_Q(y; H) = \frac{1}{P} \sum_{j=1}^{P} u'(x_j, y)v'(x_j, y)I_Q(x_j, y; H),$$

(5.1)
where $P$ is the total number of velocity vectors at each wall-normal position and $I_Q$ is the indicator function defined as

$$I_Q(x_j, y; H) = \begin{cases} 
1, & \text{when } |u'(x_j, y)v'(x_j, y)|_Q \geq T \\
0, & \text{otherwise}, 
\end{cases} \quad (5.2)$$

where $T$ is a threshold that allows one to consider various magnitudes of instantaneous RSS events that contribute to the mean RSS. For developing flow this threshold is taken to be $T = H|\langle u'^2 \rangle_{\text{full}}|$ (the maximum in the mean RSS for the full-surface case) while for developed flow the threshold is defined as $T = H\sigma_u(y)\sigma_v(y)$ where $\sigma_u \equiv \langle u'^2 \rangle^{1/2}$ and $\sigma_v \equiv \langle v'^2 \rangle^{1/2}$ are RMS streamwise and wall-normal velocities, respectively. Here, the value $H$ represents a threshold on the strength of the RSS-producing events considered in the analysis, with $H = 0$ allowing inclusion of all $u'v'$ events and increasing values of $H$ allowing inclusion of only increasingly intense RSS-producing events. Using this thresholding of $u'v'$ events, one can also document the fraction of space, $N_Q$, they occupy as

$$N_Q(y; H) = \frac{\sum I_Q(y; H)}{P}. \quad (5.3)$$

Finally, two-point velocity correlation coefficients are compared under developed flow conditions to discern the impact of the low-order surface models on the spatial structure of the flow compared to flow over the full surface as well as the smooth-wall baseline. In this regard, previous studies of wall turbulence have established a strong linkage between the average spatial characteristics of these correlations and the dominant spatial structure of the flow. For the present streamwise–wall-normal PIV measurements, two-point velocity correlation coefficients of the form

$$\rho_{ij}(\Delta x, y; y_{\text{ref}}) = \frac{\langle u'_i(x, y_{\text{ref}})u'_j(x + \Delta x, y) \rangle}{\sigma_i(y_{\text{ref}})\sigma_j(y)}, \quad (5.4)$$

where $\sigma_i$ and $\sigma_j$ are the root-mean-square (RMS) velocities of the $i^{\text{th}}$ and $j^{\text{th}}$ velocity components, are computed for the streamwise and wall-normal velocity components yielding three correlation coefficients $\rho_{uu}$, $\rho_{vv}$ and $\rho_{uv}$. For reference, previous studies indicate fair similarity in these correlation coefficients for smooth- and rough-wall flow outside the roughness sublayer, though a shortening in the streamwise extent of $\rho_{uu}$ has been reported for both idealized (Nakagawa and Hanratty, 2001; Volino et al., 2007) and irregular (Wu and Christensen, 2007) roughness that can extend well beyond the roughness sublayer. Nonetheless, such similarity is consistent with Townsend’s hypothesis and indicates relatively little impact of roughness on the spatial structure of the outer layer.
5.2 Results & discussion

5.2.1 Developing flow

This section presents analysis of the PIV datasets acquired 0.8 m (∼7δ) downstream of the leading edge of the roughness, meaning that roughness effects are confined within an internal layer initiated at the abrupt transition from smooth- to rough-wall conditions. Table 4.1 summarizes the relevant parameters of these experiments. Since the flow is far from achieving self-similarity for the experiments presented in this section, the free-stream velocity, $U_e$, is used for normalization of the velocity statistics.

5.2.1.1 Mean velocity

Figure 5.1 presents the mean streamwise velocity profiles for the various surfaces under developing flow conditions. A deficit in the mean velocity is evident for the rough-wall cases compared to the smooth-wall profile. The wall-normal extent of this deficit, which can be interpreted as an internal layer within which roughness directly impacts the flow, is surface-dependent, with the full surface showing the largest wall-normal extent of velocity deficit ($y \lesssim 0.6\delta$). This deficit increases with increasing modal content in the low-order representations and is largest for flow over the full surface. For example, the profile for the $M = 5$ model deviates least from the smooth-wall profile, indicating a reduced impact of roughness on the mean flow. In contrast, the mean profile for the $M = 16$ representation collapses quite well with the profile for
the full surface, indicating that the 16 most dominant topographical modes (4.2% of the 383 total modes but 95% of the FSC) are sufficient for capturing the impact of the full surface on the mean velocity for developing flow.

5.2.1.2 Reynolds stresses

Figure 5.2(a) presents profiles of the streamwise and wall-normal Reynolds normal stresses, \( \langle u'^2 \rangle \) and \( \langle v'^2 \rangle \), normalized by \( U_e^2 \) for developing flow. Enhancement of the Reynolds normal stresses is evident with increasing modal content, with both \( \langle u'^2 \rangle \) and \( \langle v'^2 \rangle \) enhanced by roughly 40% in the full-surface case relative to smooth-wall flow. With regard to the low-order representations, the \( M = 5 \) model shows only moderate enhancement of \( \langle v'^2 \rangle \) and \( \langle u'^2 \rangle \) compared to the smooth-wall baseline. Consistent with the trends in the mean velocity, the \( M = 16 \) result shows strong consistency with the full-surface results except near \( y = 0.1\delta \) where this low-order representation produces slightly lower values of \( \langle u'^2 \rangle \) compared to the full surface. This observation highlights the importance of even finer-scale topographical details in the local flow behavior in the near-wall region. The wall-normal extent of the internal layer formed by the roughness grows with increasing modal content, with the internal layer for flow over the full surface and the \( M = 16 \) model extending to roughly \( y \approx 0.55\delta \) for both \( \langle u'^2 \rangle \) and \( \langle v'^2 \rangle \) but to only \( y \approx 0.4\delta \) for the \( M = 5 \) model. Beyond these wall-normal locations, the rough-wall profiles collapse well with smooth-wall flow.

Interestingly, the \( M = 16 \) model reproduces the full-surface mean RSS, \( -\langle u'v' \rangle \), profile [figure 5.2(b)]
Figure 5.3: Probability density functions (pdfs) of instantaneous $u'v'$ events at (a) $y = 0.1\delta$, (b) $y = 0.15\delta$, (c) $y = 0.2\delta$ and (d) $y = 0.25\delta$ for developing flow. Not all data points shown for clarity. ⭕: Smooth; □: $M = 5$ model; △: $M = 16$ model; ♦: Full surface.

extremely well over the entire wall-normal range over which measurements were possible, including replication of the wall-normal extent of the internal layer for the full surface ($y \approx 0.5\delta$). This measure of the internal-layer thickness for both the $M = 16$ model and the full surface is slightly smaller than that noted from the mean velocity and Reynolds normal stress profiles. In contrast, the $M = 5$ model yields much lower values of mean RSS as well as a thinner internal layer ($y \approx 0.35\delta$), though the magnitude of the $M = 5$ mean RSS is still larger than that for the smooth wall. To further explore these trends, particularly the contributions of the various events that yield the overall mean RSS (ejections, sweeps, inward/outward interactions),
Figure 5.3 presents pdfs of instantaneous $u'v'$ events at various wall-normal locations for flow over all surface cases. Event magnitudes are normalized by the magnitude of the peak mean RSS for the full-surface case in order to directly compare the efficacy of the low-order surface models in reproducing the flow behavior of the full surface. Consistent with the trends noted in the mean RSS profiles, the $M = 5$ model does not reproduce the pdf of the full surface, most notably close to the wall. These differences are apparent in both the negative and positive tails of the pdfs, meaning that all four quadrant events are likely modified in the $M = 5$ model compared to the full surface. In contrast, the $M = 16$ surface model produces pdfs that are quite consistent with the character of their full-surface counterparts. Far from the wall, the pdfs from all three surfaces collapse toward the smooth-wall trends as the outer-edge of the internal roughness layer is approached.

### 5.2.1.3 Quadrant analysis

Extending the analysis afforded by pdfs of instantaneous RSS-producing events, quadrant analysis is employed to assess how consistent the various quadrant contributions to the mean RSS profiles are amongst all surface conditions. Figures 5.4(a) and 5.4(b) present contributions of the four quadrant events to the mean RSS for a threshold of $H = 0$ wherein all instantaneous $u'v'$ events are included in the decomposition. Consistent with the mean RSS profiles, ejections ($Q_2$) and sweeps ($Q_4$) dominate over inward ($Q_4$) and outward ($Q_1$) interactions for all cases. Of particular interest, while the $M = 5$ model fails to reproduce the quadrant contributions of the full-surface flow, all four quadrant contributions to the mean RSS for the $M = 16$ low-order representation are quite consistent with those for the full surface. Outside the internal layer generated by roughness ($y \simeq 0.4\delta$ and $0.5\delta$ for the $M = 5$ model and the $M = 16$ model/full surface, respectively), the quadrant contributions collapse well with the smooth-wall results. In contrast to the roughness-induced enhancement noted in the quadrant contributions to the mean RSS, the space fractions of these quadrant events [eq. (5.3)] are found to be similar for all surface cases [figures 5.4(c) and 5.4(d)]. Thus, roughness does not appear to produce more numerous RSS-producing events but instead tends to intensify the magnitude of these events compared to smooth-wall flow for $H = 0$.

Figures 5.5(a) and 5.5(b) present quadrant contributions for the three rough surfaces and the smooth-wall baseline for a threshold defined by $H = 4$ which allows only the most intense $u'v'$ events to be included in the decomposition. Note that only ejections ($Q_2$) and sweeps ($Q_4$) are presented since the contributions of inward and outward interactions are near zero for this threshold. Consistent with the $H = 0$ trends, the $M = 16$ surface model reproduces the full-surface trends quite well while the $M = 5$ model does not. Of interest, while the space fractions for the $H = 0$ threshold showed little difference in the number of
quadrant events generated by the two low-order models compared to the full-surface and smooth-wall flow, the space fractions for $H = 4$ [figures 5.5(c) and 5.5(d)], which includes only the most intense RSS-producing events, show clear differences from smooth-wall flow. In particular, the $M = 16$ model reproduces the space fractions of both ejections and sweeps from the full-surface flow quite well. In contrast, the $M = 5$ model fails to reproduce the quantitative behavior of the full-surface flow. Nevertheless, compared to smooth-wall flow, all of the rough-wall cases are found to generate more numerous intense ejection and sweep events by a factor of 3–6. Finally, outside the internal layer the rough-wall quadrant contributions and associated space
fractions again collapse well with the smooth-wall results as should be expected for developing flow.

5.2.2 Developed flow

This section presents results from the PIV measurements made $\sim 25\delta$ downstream of the leading edge of the roughness such that the rough-wall flows have attained self-similar states based on past measurements over such surfaces (Wu and Christensen, 2007). Table 4.2 summarizes relevant experiment parameters for these developed-flow measurements.
Figure 5.6 presents mean velocity profiles for flow over the two low-order models, the full surface and the smooth-wall baseline in both inner units (normalized by \( u_\tau \) and \( \nu \)) and in velocity defect form (normalized by \( u_\tau \) and \( \delta \)). The friction velocity, \( u_\tau \), for each case was determined using the total shear stress method which assumes a region of constant shear stress equal to the wall shear stress in the overlap and inner region of the boundary layer (Flack et al., 2005; Wu and Christensen, 2007). These values of \( u_\tau \) were then used to determine the virtual origin, \( y_0 \), and the roughness function, \( \Delta U^+ \), for the rough-wall cases by fitting the mean velocity profile to the expected logarithmic profile in the log layer given by

\[
U^+ = \frac{1}{\kappa} \ln(y^+ - y_0^+) + A - \Delta U^+, \tag{5.5}
\]

where \( \kappa = 0.41 \) and \( A = 5.3 \) are the log-law constants. As discussed in Chapter 1, knowledge of \( \Delta U^+ \) enables one to relate the roughness studied herein to the sand-grain experiments of Nikuradse (1950) via an equivalent sand-grain height, \( k_s^+ \), that yields the same \( \Delta U^+ \) as the present rough surfaces through the fully-rough asymptote given by

\[
\Delta U^+ = \frac{1}{\kappa} \ln(k_s^+) + A - 8.5. \tag{5.6}
\]

It should be noted that the constant stress method for determining \( u_\tau \) has an uncertainty of approximately 5\% (Schultz and Flack, 2005; Flack et al., 2005). According to the present error analysis (Appendix A),
this error dominates over the random error in the measurement itself when it propagates to the turbulence statistics. The uncertainty in each presented statistic herein is cited in the figure captions.

The inner-scaled profiles [figure 5.6(a)] reveal the downward shift of the rough-wall profiles relative to the smooth-wall profile due to the increased skin friction at the wall. Of interest, while the inner-scaled mean profile for the $M = 5$ case resides between the smooth and full-surface profiles ($\Delta U^+ = 5.7$ for the $M = 5$ model), the $M = 16$ case ($\Delta U^+ = 7.9$) sits just above, but nearly reproduces, the full-surface profile ($\Delta U^+ = 8.2$). One can therefore infer that the largest topographical scales of the full surface dominate $\Delta U^+$ for the roughness considered herein, though a slight difference does exist which could represent contributions of excluded topographical modes to $\Delta U^+$. However, this possibility cannot be confirmed as the difference between the 16-mode and full-surface $\Delta U^+$ is within the uncertainty in determining this quantity (3–5%). In contrast, the mean velocity profiles in defect form [figure 5.6(b)] show strong consistency between all surface cases outside the roughness sublayer ($y \gtrsim 0.2\delta$), consistent with Townsend’s wall similarity hypothesis, and also confirm the attainment of a self-similar state for all cases. Within the roughness sublayer, all three roughness cases sit below the smooth-wall case [consistent with many past studies of both idealized (Raupach, 1981; Ligrani and Moffat, 1986; Bandyopadhyay and Watson, 1988; Flack et al., 2005) and realistic roughness (Allen et al., 2007; Wu and Christensen, 2007)] yet both low-order representations reproduce the full-surface profile well. Finally, according to Nikuradse (1950), a flow would be considered fully rough for $k_+^s \gtrsim 70$. However, based on the fully-rough asymptote [eq. (5.6)], only the full surface and $M = 16$ model satisfy this condition based on the roughness-function values presented in table 4.2. Since the roughness function for the $M = 5$ model does not sit on the fully-rough asymptote, this implies that its behavior is transitionally rough (and so a $k_+^s$ value cannot be reported). This is a remarkable result since it means that the regime shifts from fully-rough to transitionally-rough when the FSC is reduced from 95% to 71% (or in the present case, when the number of modes in the reconstruction is reduced from 16 to 5). This result is in agreement with the discussion of the effective slope, $ES$, for the present roughness cases in Chapter 3.

5.2.2.2 Reynolds stresses

Figure 5.7 presents profiles of $\langle u'^2 \rangle^+, \langle v'^2 \rangle^+$ and $-\langle u'v' \rangle^+$ for all surface cases. Consistent with Townsend’s wall similarity hypothesis, the smooth, full-surface and model profiles collapse outside the roughness sublayer ($y \gtrsim 0.2\delta \approx 5k$). However, some differences are noted within the roughness sublayer. In the case of $\langle u'^2 \rangle^+$ [figure 5.7(a)], all three rough-wall cases sit below the smooth-wall profile which is a well-known effect of roughness in wall turbulence. However, while the $M = 5$ and $M = 16$ cases collapse with one-another,
they both yield slightly lower values of $\langle u'^2 \rangle^+$ compared to the full surface. This discrepancy is not entirely unexpected since the details of the roughness can have a significant impact on the local flow behavior close to the surface. As such, the higher-order topographical modes missing in the model surfaces appear to play a measurable role in $\langle u'^2 \rangle^+$ within the roughness sublayer. With regard to $\langle v'^2 \rangle^+$ [figure 5.7(a)], much better consistency is noted as both the 5- and 16-mode models reproduce the full-surface behavior well, even in the roughness sublayer. In contrast, both low-order surface models generate mean RSS profiles [figure 5.7(b)] that mimic the overall full-surface behavior well even within the roughness sublayer.

While the mean RSS profiles for the low-order surface models seem to reproduce the mean RSS behavior of flow over the full surface, this collapse need not require that the distributions of the instantaneous $u'v'$ events contributing to these mean profiles be identical for flow over the full surface and the two low-order models. To explore such issues, figure 5.8 presents pdfs of instantaneous $u'v'$ events contributing to the mean RSS profiles at $y/\delta = 0.1, 0.15, 0.2$ and 0.3 computed for all surface cases. While the pdfs of $u'v'$ events away from the wall [figures 5.8(d)] show strong similarity regardless of surface condition, the pdfs of $u'v'$ events within the roughness sublayer [figures 5.8(a)-(c)] indicate that flow over both the $M = 5$ and $M = 16$ surface models do not reproduce the number of intense and negative $u'v'$ events observed for flow over the full surface. As such, the low-order surface models appear to produce slightly fewer extremely intense ejection and/or sweep events compared to the full surface. In contrast, the positive tails of the pdfs in figure 5.8 show better collapse, even in the roughness sublayer, indicating that these positive contributions to $\langle u'v' \rangle^+$ are less affected by the finer-scale details of the surface topography.
Figure 5.8: Probability density functions (pdfs) of instantaneous inner-scaled $u'v'$ events at (a) $y = 0.1\delta$, (b) $y = 0.15\delta$, (c) $y = 0.2\delta$ and (d) $y = 0.3\delta$ for developed flow. Not all data points shown for clarity. ○: Smooth; □: $M = 5$ model; △: $M = 16$ model; ♦: Full surface.

5.2.2.3 Quadrant analysis

To study these trends in RSS-producing events further, quadrant analysis is employed to distinguish ejection from sweep contributions and inward- from outward-interaction contributions to the mean RSS profile for all surface cases. Figures 5.9(a) and 5.9(b) present quadrant contributions for a threshold given by $H = 0$, meaning that all contributions, weak through intense, are included in the decomposition. As should be expected from the behavior of the mean RSS profiles [figure 5.7(b)] and the pdfs of $u'v'$ events (figure 5.8), $Q_2$ and $Q_4$ contributions dominate over $Q_1$ and $Q_3$ contributions. In particular, the profiles of $Q_2$ and $Q_4$
Figure 5.9: (a,b) Quadrant contributions to the mean RSS, \( \langle u'v' \rangle_{Q_1} \), and (c,d) space fractions, \( N_{Q} \), for developed flow as a function of wall-normal position for \( H = 0 \). Lines as in figure 5.6(b) and not all data points shown for clarity. ⃣: Smooth; □: \( M = 5 \) model; △: \( M = 16 \) model; ♦: Full surface. Uncertainty in (a) and (b): 7.4%.

Contributions are consistently 3–4 times larger in magnitude than those of \( Q_1 \) and \( Q_3 \) events. Further, the rough-wall results show strong consistency with the smooth-wall baseline outside the roughness sublayer, again consistent with Townsend’s hypothesis. With regard to the consistency between the profiles of the low-order models and those of the full surface, while the \( Q_1 \), \( Q_3 \) and \( Q_4 \) profiles collapse regardless of surface, the contributions of ejections (\( Q_2 \)) show surface dependence close to the wall with both surface models yielding slightly weaker contributions of ejections to the mean RSS for the case of \( H = 0 \). This behavior is consistent with the surface-dependent behavior noted in the pdfs of \( u'v' \) events where the full surface was found to
produce slightly more numerous intense negative $u^\prime v^\prime$ events than the two low-order surface models. As such, this observation may indicate that it is more numerous ejection events in the full-surface case that are primarily responsible for the surface-dependent behavior noted in the pdfs of figure 5.8 within the roughness sublayer. Despite this difference, the fraction of space, $N_Q$, occupied by these various events [figures 5.9(a) and 5.9(b)] are quite similar for the different surface cases except near the boundary-layer edge where the weak magnitude of the velocity fluctuations can introduce increased uncertainty in quadrant decomposition.

Figure 5.10 presents quadrant contributions and space fractions for a threshold of $H = 4$, meaning that only the contributions of the most intense $u^\prime v^\prime$ events to the mean RSS are considered. While the $M = 5$ and $M = 16$ surface models clearly yield weaker contributions of intense $Q_2$ events compared to the full-surface
case (larger discrepancy but still qualitatively consistent with the $H = 0$ trends), these surfaces actually generate slightly higher contributions from $Q_4$ events compared to the full-surface result. As such, the finer-scale topographical details of the full surface excluded from the low-order surface models may actually contribute in a meaningful way to the generation of intense $Q_2$ and $Q_4$ events. Despite these near-wall differences, good consistency is noted between the smooth- and rough-wall results outside the roughness sublayer in accordance with outer-layer similarity.

5.2.2.4 Two-point velocity correlations

Figure 5.11 presents two-point correlation coefficients of streamwise velocity, $\rho_{uu}$, at $y/\delta = 0.1, 0.15, 0.2$ and 0.3 for flow over all surface conditions. While the correlations are computed within the two-dimensional measurement plane, they are presented in one-dimensional form herein to quantitatively assess their dependence on streamwise separation, $\Delta x$. Well within the roughness sublayer at $y = 0.1\delta$ [figure 5.11(a)], while the correlations at small $\Delta x$ collapse irrespective of surface condition, there exists a shortening in the streamwise extent of $\rho_{uu}$ at larger $\Delta x$ for the three rough-wall cases is evident compared to the smooth-wall baseline. These trends indicate that the larger spatial scales of the flow are more sensitive to roughness effects than are the smaller scales of the flow. Interestingly, the two low-order representations produce a slightly more pronounced streamwise shortening of $\rho_{uu}$ than the full surface. With increasing distance from the wall this difference between the full-surface and low-order-representation profiles diminishes quickly as the two low-order models collapse well with the full-surface result at $y = 0.15\delta$ [figure 5.11(b)] and $y = 0.2\delta$ [figure 5.11(c)], though all three rough-wall results still show shortening compared to the smooth-wall baseline. Outside the roughness sublayer at $y = 0.3\delta$ [figure 5.11(d)], however, the rough-wall correlations collapse well with the smooth-wall result which is consistent with Townsend’s wall similarity hypothesis. This collapse therefore suggests similarity in the average spatial structure of the smooth-wall flow with that of the rough-wall flows.

The two-point correlation coefficient of wall-normal velocity, $\rho_{vv}$, presented in figure 5.12 shows less influence of roughness, though a slight broadening of $\rho_{vv}$ is noted close to the wall at $y = 0.1\delta$ [figure 5.12(a)] and $y = 0.15\delta$ [figure 5.12(b)] for the three rough-wall cases compared to the smooth-wall baseline. However, in contrast to the trends noted at these wall-normal locations for $\rho_{uu}$, the low-order representations yield $\rho_{vv}$ profiles that match that of the full-surface flow very well. With increasing distance from the wall, $\rho_{vv}$ is found to collapse irrespective of surface condition. Comparing the streamwise extent of $\rho_{vv}$ to that of $\rho_{uu}$, it is clear that the latter is more representative of the larger spatial scales of the flow while the former is more strongly influenced by the smaller spatial scales. Finally, figure 5.13 presents streamwise profiles of the
Figure 5.11: Streamwise profiles of $\rho_{uu}$ at (a) $y = 0.1\delta$, (b) $y = 0.15\delta$, (c) $y = 0.2\delta$ and (d) $y = 0.3\delta$. Not all data points shown for clarity. ○: Smooth; □: $M = 5$ model; △: $M = 16$ model; ♦: Full surface. Uncertainty: 3.4%.

cross-correlation coefficient of the streamwise and wall-normal velocities, $\rho_{uv}$. Similar to $\rho_{vv}$, the streamwise extent of $\rho_{uv}$ is enhanced within the roughness sublayer at $y = 0.1\delta$ and $0.15\delta$ (figures 5.13(a) and 5.13(b), respectively) compared to smooth-wall flow, though the profiles for the low-order representations collapse well with the full-surface results. With increasing wall-normal position [figures 5.13(c) and 5.13(d)] these differences between the smooth- and rough-wall cases diminish indicating little impact of roughness on the average spatial structure embodied in $\rho_{uv}$ outside the roughness sublayer.
5.3 Summary

Low-order representations of highly-irregular surface roughness replicated from a damaged turbine blade were tested in a zero-pressure-gradient turbulent boundary layer under both developing- and developed flow scenarios. For the case of developing flow, a model embodying the first 16 of 383 modes (4.2% of the total modes) which captures 95% of the full surface content accurately reproduces the flow characteristics of flow over the full surface, including the mean velocity profile, Reynolds normal and shear stresses, pdfs of RSS-producing events and quadrant contributions to the mean RSS. In contrast, a 5-mode surface model fails to
reproduce the flow characteristics of the full surface despite containing 71% of the full-surface topographical content and having an RMS roughness height that is 84% that of the full surface.

For the case of developed flow, wherein the internal roughness layer has grown to engulf the entire boundary-layer thickness and a self-similar state is attained in all surface cases, both the 5- and 16-mode surface models faithfully reproduce the character of the full-surface flow outside the roughness sublayer, including both single- and multi-point statistics as well as important details of the instantaneous RSS-producing events as discerned from quadrant analysis. However, this consistency is a bit misleading due to the manner in which the flow statistics are scaled and compared (by $u_\tau$ and $\delta$). In fact, since these
flows have attained a self-similar state, this collapse is simply attributable to the existence of outer-layer similarity for the present surfaces in accordance with Townsend’s wall similarity hypothesis. In fairness, neither the 5- nor 16-mode model fully-reproduces the bulk flow characteristics of the full-surface flow as represented by the downward shift in the mean velocity profile via the roughness function, $\Delta U^+$, though the 16-mode model is quite close ($\Delta U^+ = 7.9$ compared to 8.2 for the full-surface flow). Nevertheless, this observation of outer-layer similarity for the two low-order representations, the full surface and the smooth wall is actually quite important because it highlights the relatively weak impact that the intermediate- and finer-scale topographical details of the roughness have on the outer-layer flow. As such, for the topography under study herein, knowledge of simply $u_\tau$ and $\delta$ would provide accurate prediction of the outer-layer behavior of the full-surface and low-order-model flows from smooth-wall statistics.

In contrast, important differences are noted between flow over the 5- and 16-mode models and the full surface within the roughness sublayer where the details of the roughness are expected to play a pivotal role in the flow development. In particular, both the 5- and 16-mode models yield lower values of $\langle u'^2 \rangle$ in the roughness sublayer compared to the full-surface case. Similarly, these models fail to reproduce the contributions of intense ejection and sweep events noted in the roughness sublayer of the full-surface flow. Further, both low-order surface representations produce an enhanced shortening in the streamwise extent of the two-point correlation of streamwise velocity compared to the full-surface flow. Taken together, these observations indicate that finer-scale topographical scales of the roughness play subtle, but measurable, roles in the overall flow development within the roughness sublayer for the topography considered. In this regard, it was noted that these finer-scale topographical details captured in the truncated higher-order SVD modes embody important details regarding the jagged edges of the large-scale protrusions. Thus, the observed differences within the roughness sublayer for the two surface models compared to that of the full-surface flow reflect the importance of these topographical details in the near-surface flow, possibly through modifications of the separation behavior downstream of large-scale protrusions and/or vortex shedding from these features.

Finally, the efficacy of the $M = 16$ surface model in reproducing many of the characteristics of flow over the full surface considered herein is quite interesting and provides some insight into the roughness characteristics that have the greatest impact on the flow. However, this particular level of modal content cannot be viewed as a universal requirement for constructing models of other irregular rough surfaces. Of interest, though, is the fact that the fractional surface content of 95% for the $M = 16$ surface model in this effort is consistent with that noted in the recent study of Johnson and Christensen (2009) wherein a very different realistic rough surface was reconstructed with 95% of the fractional surface content using the first
20 SVD modes of the topographical decomposition. Consistency was noted with flow over the full surface under developing-flow conditions except in the very near-wall region. Thus, while the number of SVD basis functions included in a topographical model will most certainly vary from surface to surface, these two efforts highlight the possibility that fractional surface content might provide at least partial guidance in determining the appropriate modal content for low-order models of irregular roughness. Further study of a range of realistic rough surfaces is needed to critically assess this possibility.
Chapter 6

The Structure of the Roughness Sublayer

6.1 Statistical signatures in the roughness sublayer

Given the observations discussed in the previous chapter, particularly the notable differences observed between flow over the low-order models and the full surface in the roughness sublayer, wall-parallel stereo-PIV measurements were performed deep inside the roughness sublayer as well as for a smooth-wall baseline. In order to have consistency between the different surface conditions, all measurements were made at $y = 0.047\delta$ which, for the rough-wall cases, was measured relative to the virtual origin determined from the $x$–$y$ measurements. All relevant experimental parameters are presented in table 4.3.

6.1.1 Single-point statistics

Figure 6.1 presents contour maps of the three roughness topographies coincident with the stereo-PIV field of view that sits at $y = 0.047\delta$ above. These views are presented so that the various single-point statistics presented in figures 6.2–6.10 at the end of this chapter can be contrasted with the local roughness topography that resides below the measurement plane to help identify any roughness-induced effects. Smooth-wall results are also presented in figures 6.2–6.10 as a baseline against which the rough-wall results are contrasted to further discern definitive roughness-induced heterogeneities. However, before discussing the details of these fields, table 6.1 presents the area-averaged values of the streamwise velocity defect as well as the streamwise and wall-normal Reynolds normal stresses and the RSS. These values are close, but not identical, to their counterparts from the $x$–$y$ experiments shown in figures 5.6 and 5.7. These small differences can be explained by recalling that the $x$–$y$ measurements were performed at a single spanwise location (demarcated in figure 4.5 by the dashed lines) while the values presented in figure 6.1 were averaged over an area spanning many different topographical details in the spanwise direction. Thus, the local details of the topography can lead to slight differences in the values of the single-point statistics.

It should be noted that the smooth-wall results are presented as a baseline for comparison as these statistics should be nearly homogeneous across the $\delta$-scale field of view. Thus, any variability noted in
Table 6.1: Ensemble- and area-averaged values of single-point statistics at $y = 0.047\delta$.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$(U_e - u)^+$</th>
<th>$\langle u'^2 \rangle^+$</th>
<th>$\langle v'^2 \rangle^+$</th>
<th>$(u'v')^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>10.7</td>
<td>4.6</td>
<td>1.4</td>
<td>-0.96</td>
</tr>
<tr>
<td>$M = 16$</td>
<td>10.3</td>
<td>4.8</td>
<td>1.4</td>
<td>-1.00</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>10.0</td>
<td>4.9</td>
<td>1.3</td>
<td>-0.98</td>
</tr>
<tr>
<td>Smooth</td>
<td>11.8</td>
<td>4.7</td>
<td>1.7</td>
<td>-0.88</td>
</tr>
</tbody>
</table>

these statistics represents a measure of their sampling error. Therefore, any spatial variability observed in the rough-wall statistics can be compared to the variability in the smooth-wall baseline as a means of discerning whether such differences are due to roughness effects or simply an artifact of sampling. The ensemble average of streamwise velocity in defect scaling, $(U_e - u)^+$, is shown in figure 6.2 for the smooth-wall baseline as well as the three rough-wall cases ($M = 5$, $M = 16$ and full surface). While the smooth-wall result is quite uniform, all rough-wall flows exhibit a spanwise-localized low-momentum pathway bounded by high-momentum pathways [It should be stressed again that the measurement plane is located a few millimeters above the crests of the roughness so the patterns presented throughout highlight the flow above the roughness, not within the roughness]. This result suggests that the roughness under consideration induces a “channeling” effect in the flow or the possible existence of persistent wakes generated by dominant roughness features in the case of the low-momentum pathway. These low- and high-momentum pathways are distinct from the LMRs and HMRs often identified in instantaneous realizations of wall turbulence because they appear in the ensemble-averaged field of the streamwise velocity. Thus, these patterns in the ensemble-averaged result can be interpreted as preferential paths for these low- and high-momentum motions in contrast to the instantaneous LMRs and HMRs present in smooth-wall turbulence that occur randomly in space and have clear spatial coherence. The low-momentum pathway in the rough-wall cases is located near $z \approx 0.3\delta$ for the $M = 5$ case (figure 6.2b), while the $M = 16$ and full-surface cases show a low-momentum pathway near $z \approx 0.5\delta$ (figures 6.2c and 6.2d, respectively). The location of the low-momentum pathway is demarcated by a dashed line (---), while the location of the high-momentum pathway is demarcated by a dash-dot line (----) in figures 6.2b–6.2c. These reference locations will be kept in all subsequent plots of single-point statistics for comparison. Further, it is observed that the peak values of the low- and high-momentum regions are higher in the $M = 5$ model than in the other two rough-wall cases. Thus, the roughness under consideration introduces large-scale heterogeneity in the mean streamwise velocity in the form of a spatial preference for the pathways of low- and high-momentum motions.

The ensemble-averaged wall-normal velocity, $\langle v \rangle^+$, is shown in figure 6.3. As one would expect, the smooth-wall result (figure 6.3a) is nearly zero across the field of view. It displays, however, few weak vertical
bands with a maximum deviation from the mean of about 10% of the maximum contour levels. These deviations are likely an artifact of the stereo-PIV calibration. Nevertheless, the rough-wall flows exhibit deviations from the mean that well exceed these weak artifacts in the smooth-wall result. Consequently, the signal recovered by the rough-wall flows is certainly a physical effect due to the roughness. In particular, while the streamwise velocity defect showed larger-scale heterogeneities due to the roughness under consideration, \( \langle v \rangle^+ \) exhibits more localized heterogeneities due to roughness. In particular, \( \langle v \rangle^+ \) is positive just upstream of large-scale crests in the roughness and negative just downstream of such surface features. Thus, as one might expect, the flow is diverted away and toward the wall by the roughness features that occur directly beneath the measurement plane. These vertical excursions of the flow toward and away from the wall are strongest for the \( M = 5 \) case (figure 6.3b) which only embodies the largest topographical scales of the full surface. Of interest, the \( M = 16 \) result (figure 6.3c) is remarkably consistent with the full-surface result (figure 6.3d), indicating that the intermediate topographical scales embodied in modes 6–16 that are missing in the \( M = 5 \) model play an important role in this mean vertical motion of the flow. As mentioned earlier, these intermediate topographical scales add to the jagged and irregular character of the largest roughness elements present in the topography. Note that the reference lines for the location of the low- and high-momentum pathways follow no tendency in the contours of \( \langle v \rangle^+ \). This fact emphasizes the localized heterogeneities in \( \langle v \rangle^+ \).

The results for ensemble-averaged Reynolds stresses, \( \langle u'_i u'_j \rangle^+ \), are presented in figures 6.4–6.10. As with the mean velocity components, the smooth-wall baseline results show a maximum variability across the field of view of about 10%, which is comparable to the experimental error of \( \varepsilon \left( \langle u'_i u'_j \rangle^+ \right) \approx 7.42 \) found for these second-order statistics (see table 4.8). Therefore, the smooth-wall flow can be considered as a measure of the error in these second-order statistics. In other words, all deviations away from the range exhibited by the smooth-wall flow can be interpreted as a consequence of the presence of rough-wall conditions. To aid in this comparison, the contour values falling within the range \( \pm 7.42\% \) (consistent with our error analysis) were omitted from the contour plots of these second-order statistics.

Of particular interest, the ensemble-averaged streamwise Reynolds normal stress, \( \langle u'^2 \rangle^+ \), for the \( M = 16 \) and full-surface results (figures 6.4c and 6.4d, respectively) display distinct streamwise-elongated regions of more intense \( \langle u'^2 \rangle^+ \) (one at \( z \approx 0.35\delta \) and the other at \( z \approx 0.65\delta \)) between which a region of less intense \( \langle u'^2 \rangle^+ \) is noted. Based on the aforementioned reference lines demarcating the spanwise locations of the low- and high-momentum pathways, this elongated region of weaker \( \langle u'^2 \rangle^+ \) is coincident with the low-momentum pathway noted in the \( M = 16 \) model and full-surface streamwise velocity defect results (figure 6.2) while the elongated regions of intense \( \langle u'^2 \rangle^+ \) are coincident with the spanwise boundaries of the low-momentum
pathway. A similar pattern is also notable in the $M = 5$ model $\langle u'^2 \rangle$ (figure 6.4b) where a low-momentum pathway was identified in the streamwise velocity defect near $z \approx 0.3\delta$. Similar results are observed in figures 6.5 and 6.6 for the other two Reynolds normal stresses, $\langle v'^2 \rangle^+$ and $\langle w'^2 \rangle^+$ respectively. However, the streamwise-elongated regions of intense $\langle v'^2 \rangle^+$ and $\langle w'^2 \rangle^+$ are somewhat weaker and less defined than their $\langle u'^2 \rangle^+$ counterpart. Of interest, since the Reynolds normal stresses embody the different components of the TKE ($\frac{1}{2} \langle q^2 \rangle \equiv \frac{1}{2} (\langle u'^2 + v'^2 + w'^2 \rangle)$), our results suggest that the roughness patterns under consideration tend to redistribute the TKE in a non-homogeneous localized manner within the roughness sublayer. This observation is consistent with the channeling of low- and high-momentum motions suggested by the ensemble average of streamwise velocity in defect scaling (figure 6.2).

An analogous behavior is also observed in the ensemble-averaged Reynolds shear stress, $\langle u'v' \rangle^+$ (figure 6.8), where streamwise-elongated regions of more intense $\langle u'v' \rangle^+$ sit near $z \approx 0.35\delta$ and $z \approx 0.65\delta$ in the $M = 16$ and full-surface results while a region of less intense $\langle u'v' \rangle^+$ is observed in-between. This region of weaker $\langle u'v' \rangle^+$ occurs spatially coincident to the low-momentum pathway noted in $\langle U_e - u \rangle^+$ while the two intense regions of $\langle u'v' \rangle^+$ lie along the boundaries of this low-momentum pathway. This imprint is also apparent in the ensemble-averaged Reynolds shear stress component $\langle u'w' \rangle^+$ as presented in figure 6.9. While the smooth-wall baseline (figure 6.9a) is nearly zero, as one would expect, $\langle u'w' \rangle^+$ is found to be intense in all three rough-wall cases with streamwise-elongated regions of positive and negative $\langle u'w' \rangle^+$ occurring along the boundaries of the low-momentum pathways identified in $\langle U_e - u \rangle^+$. For example, the $M = 16$ and full-surface $\langle u'w' \rangle^+$ are positive in a streamwise-elongated region near $z \approx 0.35\delta$ and negative near $z \approx 0.65\delta$. These streamwise-elongated regions occur spatially coincident to similar elongated regions noted in $\langle u'^2 \rangle^+$ and $\langle w'u' \rangle^+$. In comparison to $\langle u'v' \rangle^+$ and $\langle u'w' \rangle^+$, the intensity of $\langle u'w' \rangle^+$ tends to be much weaker. Nevertheless, its localized character is still evident, particularly in the flow over the $M = 5$ and $M = 16$ models, while its effect is practically masked by the sampling error in the flow over the full surface. Since Reynolds shear stresses play a defining role in the production of TKE from the mean flow, these results suggest that the roughness under consideration may promote generation of TKE in preferential regions within the roughness sublayer. In fact, this effect is notable in figure 6.7 where the smooth-wall result [6.7a] exhibits a quite homogeneous distribution of TKE whereas the rough-wall cases show heterogeneous intensified regions of TKE that bound the low-momentum pathway identified in figure 6.2 for each case.

### 6.1.2 Quadrant analysis

In the preceding sections, statistical signatures of large-scale heterogeneities in the roughness sublayer were identified, particularly indications of spatially-fixed low- and high-momentum pathways that appear to lead
Table 6.2: Definition of thresholds and hyperbolic hole size for quadrant analysis in the $x-z$ plane.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$\sigma_u$ (m/s)</th>
<th>$\sigma_v$ (m/s)</th>
<th>$H \cdot (\sigma_u \sigma_v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>1.63</td>
<td>0.89</td>
<td>6.98</td>
</tr>
<tr>
<td>$M = 16$</td>
<td>1.62</td>
<td>0.89</td>
<td>6.87</td>
</tr>
<tr>
<td>$M = 5$</td>
<td>1.49</td>
<td>0.78</td>
<td>4.64</td>
</tr>
<tr>
<td>Smooth</td>
<td>1.45</td>
<td>0.72</td>
<td>4.17</td>
</tr>
</tbody>
</table>

to a ‘channeling’ of the rough-wall flows over preferred topographical features compared to the random occurrence of such features in smooth-wall flow. Further, heterogeneities in the Reynolds normal and shear stresses were found to bound these low- and high-momentum pathways. Given these spatial signatures, it is of interest to understand how various quadrant events contribute to the patterns noted in the Reynolds shear stress $\langle u'v' \rangle^+$. To this end, a quadrant analysis of $u'v'$ events analogous to the one outlined in 5.1 will be used over the $x-z$ plane data. For the present case, the threshold for excluding events will be defined in terms of the area-based RMS of streamwise and wall-normal velocities, $\sigma_u$ and $\sigma_v$ respectively. These values, as well as the hyperbolic hole size for $H = 4$, are shown in table 6.2. Only ensemble averaging is performed, yielding contour maps of these contributions for each of the cases under consideration.

The results for quadrant analysis on $x-z$ plane experiments are presented in figures 6.11–6.16. In all figures, the smooth-wall baseline is included for comparison. For this case, the background roughness was not included for simplicity though the spanwise locations of low- and high-momentum pathways as deduced from figure 6.2 are included when necessary to highlight relevant physics. Figures 6.11 and 6.12 present outward ($Q_1; \langle u'v' \rangle^+_1$) and inward ($Q_3; \langle u'v' \rangle^+_3$) interactions respectively, considering events of all intensities ($H = 0$). Interestingly, no particular spatial signatures are notable in these fields as all are relatively homogeneous over the field of view. The overall intensity of the smooth-wall events, however, seems to be slightly higher than that of the rough-wall flows. According to figure 5.9, this difference holds some consistency with the results observed for the $x-y$ plane experiments, particularly for $Q_3$ events. However, the differences noted in figure 5.9 are too close to the error in this statistic (7.42%) to draw definitive conclusions of differing physics. In contrast, the contributions of ejections ($Q_2; \langle u'v' \rangle^+_2$) and sweeps ($Q_4; \langle u'v' \rangle^+_4$) (figures 6.13 and 6.14, respectively) when events of all intensities are included ($H = 0$) exhibits clear spatial heterogeneity in the rough-wall cases. While the smooth-wall results are nearly homogeneous, all rough-wall flows show streamwise-aligned regions of intensified $Q_2$ and $Q_4$ contributions. In the case of the $M = 16$ model and the full surface, this spatial heterogeneity tends to be outboard of the low-momentum pathway identified in the mean velocity defect (6.2). In the case of the $M = 5$ model, however, the heterogeneity is much weaker than the other two rough-wall cases and it only loosely coincides with one of the boundaries of
the dominant low-momentum pathway. Hence, in support of the FSC to characterize low-order models of highly-irregular rough surfaces, these differences highlight the efficacy of the $M = 16$ model in yielding ejections and sweeps similar to those of flow over the full surface. Finally, it should be noted that the overall intensity of ejections for smooth-wall flow (figure 6.13a) is stronger than that of the rough-wall flows (figures 6.13b–6.13d). This observation is in agreement with the quadrant analysis performed for the $x - y$ plane measurements (figure 5.9a). Similarly, the fact that the overall intensity of sweep contributions for smooth-wall flow is comparable to that for the rough-wall flows is also in agreement with the quadrant analysis of the $x - y$ plane measurements (figure 5.9b).

Contours of intense quadrant contributions corresponding to $H = 4$ are presented in figures 6.15 and 6.16. Only ejections and sweeps are included since very few inward and outward interactions meet this intense threshold and so the contributions of $Q_1$ and $Q_3$ events at this threshold are nearly zero. As with the $H = 0$ results, the contours of ejection and sweep contributions for the rough-wall flows exhibit clear heterogeneity in the form of localized regions of more intense events. In contrast, the smooth-wall results show a higher degree of homogeneity as one would expect. Further, the $M = 16$ model results seem to agree quite well with those of the full surface faithfully while the $M = 5$ model results differ considerably. Again, these observations support the role of intermediate topographical scales in generating turbulence in the roughness sublayer and hence the use of FSC to characterize low-order models of highly-irregular roughness. Furthermore, unlike the results observed in the $x - y$ plane quadrant analysis for $H = 4$ (figure 5.10), the overall intensities captured by the $x - z$ plane measurements do not exhibit significant differences. Thus, the local topographical features likely play a role in the observations of reduced ejection contributions and enhanced sweep contributions from the $x - y$ plane analysis. Of interest, the regions of intense ejections tend to bound the central location of the dominant low-momentum pathway, while the intense regions of sweeps tend to reside centered on the location of the dominant low-momentum pathway.

6.2 Spatial signatures in the roughness sublayer

6.2.1 Vortex identification

The results presented above provide evidence of significant spatial heterogeneity in the turbulence characteristics for the rough-wall flows under consideration. In particular, the existence of low- and high-momentum pathways in the mean velocity for the rough-wall flows indicates channeling of the flow along particular features of the roughness. However, it is not clear if these persistent large-scale pathways are accompanied by vortical activity, particularly the noted low-momentum pathways which could be the statistical imprint of
vortex packets that are channeled along certain paths over the roughness. With this in mind, the $x-z$ plane datasets are analyzed to identify any spatial heterogeneity in the distribution of vortices, particularly any relation to the identified large-scale low- and high-momentum pathways. For reference, such motions have been observed to occur randomly in the $x-z$ plane of smooth-wall flows, meaning that the vortical structures have no preferred spatial organization. To this end, Zhou et al. (1999) developed a frame-independent criterion for the eduction of vortices based on the imaginary part of the complex-conjugate eigenvalues of the velocity gradient tensor, $\nabla \mathbf{u}$. This idea stems from the fact that in a Cartesian coordinate system, the velocity gradient tensor can be decomposed as

$$ \nabla \mathbf{u} = [\mathbf{u}_r \mathbf{u}_{cr} \mathbf{u}_{ci}] \begin{bmatrix} \lambda_r & \lambda_{cr} & \lambda_{ci} \\ \lambda_{cr} & -\lambda_{ci} & \lambda_{cr} \\ -\lambda_{ci} & \lambda_{cr} & \lambda_{ci} \end{bmatrix} \begin{bmatrix} \mathbf{u}_r \mathbf{u}_{cr} \mathbf{u}_{ci} \end{bmatrix}^{-1}, \tag{6.1} $$

where $\lambda_r$ is the real eigenvalue of the decomposition with its corresponding eigenvector, $\mathbf{u}_r$, while $\lambda_{cr} \pm i\lambda_{ci}$ are the complex-conjugate eigenvalues associated with the complex eigenvectors $\mathbf{u}_{cr} \pm i\mathbf{u}_{ci}$. Note that while eq. (6.1) is written in a form of one real and two complex-conjugate eigenvalues, it is possible that all three eigenvalues are real, meaning simply that $\lambda_{ci} = 0$ which implies no local rotation. If a local curvilinear system of coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ is defined based on the eigenvectors of $\nabla \mathbf{u}$ [see figure 6.17], the local streamlines for a given instant in time $t$ can be expressed as

$$ \begin{align*}
  x(t) &= x(0)e^{\lambda_r t} \\
y(t) &= e^{\lambda_{cr} t} [y(0)\cos(\lambda_{ci} t) + z(0)\sin(\lambda_{ci} t)] \\
z(t) &= e^{\lambda_{cr} t} [z(0)\cos(\lambda_{ci} t) - y(0)\sin(\lambda_{ci} t)]
\end{align*} \tag{6.2} $$

From these equations, it is possible to deduce that the flow is locally either stretched or compressed along the axis spanned by the eigenvector $\mathbf{u}_r$, while simultaneously swirling around this axis in the plane spanned by the eigenvectors $\mathbf{u}_{cr}$ and $\mathbf{u}_{ci}$. In fact, the imaginary part of the complex conjugate eigenvalues, $\lambda_{ci}$, indicates the presence of rotation and quantifies its swirling strength. While this derivation is valid for flow fields for which the full local velocity gradient tensor is known, such is not the case for instantaneous PIV velocity fields that only resolve the velocity components on a measurement plane defined by the laser lightsheet. To overcome this issue, Adrian et al. (2000a) proposed a two-dimensional form of the velocity gradient tensor utilizing the planar velocity information. Under such a scenario, the velocity gradient tensor will either
have two real eigenvalues or a complex-conjugate pair. Thus, in-plane vortices can be effectively identified in instantaneous planar velocity fields using this methodology. Unfortunately, however, since the complex eigenvalues occur on conjugate pairs, the sense of rotation cannot be deduced from \( \lambda_{ci} \). Hence, the local sign of vorticity fluctuations is used herein to identify the sense of rotation of vortices educed by means of \( \lambda_{ci} \). Therefore, in this work, the following criterion for vortex identification based on the swirling strength is utilized:

\[
\begin{align*}
\lambda_{ci} \frac{\omega_y}{|\omega_y|} > 0 & \quad \text{positive vortex} \\
\lambda_{ci} \frac{\omega_y}{|\omega_y|} < 0 & \quad \text{negative vortex} \\
\lambda_{ci} \frac{\omega_y}{|\omega_y|} = 0 & \quad \text{no vortex}
\end{align*}
\]

where \( \omega_y \) is the fluctuating wall-normal vorticity.

### 6.2.2 Swirling-strength statistics

To study dominant vortex activity in the \( x-z \) measurements, both the ensemble average and RMS of signed swirling strength were considered. The RMS of swirling strength gives a measure of the average intensity of vortex activity. This result is shown in figure 6.18, including the spanwise positions of the dominant low- and high-momentum pathways identified in figure 6.2. As expected, the smooth-wall result shows clear homogeneity, consistent with no preferred locations for the existence of wall-normal vortices. However, all three rough-wall results display spatial heterogeneity. Analogous to the observations made in the single-point turbulence statistics, there exist intensified streamwise bands of \( \lambda_{ci} \)-RMS in the rough-wall flows that occur on either side of the dominant low-momentum pathway, particularly in the case of the \( M = 16 \) model and full surface results. This result suggests that wall-normal vortices tend to populate the boundaries of the dominant low-momentum pathway in these cases. However, since the RMS swirling strength is given by the sum of squares of \( \lambda_{ci} \), it does not retain a sign that would provide the sense of rotation in these regions of heterogeneity.

In contrast, the ensemble-averaged \( \lambda_{ci} \frac{\omega_y}{|\omega_y|} \), shown in figure 6.19, does retain the sense of the rotation via the sign of \( \lambda_{ci} \) given by the local wall-normal vorticity, \( \omega_y \). While the ensemble-averaged \( \lambda_{ci} \frac{\omega_y}{|\omega_y|} \) for smooth-wall flow is essentially zero, indicative of no preferential locations for clockwise- and counterclockwise-rotating wall-normal vortices, it displays clear preferential locations for such structures in all three rough-wall cases. Focusing on the \( M = 16 \) model and the full surface, recall that a clear low-momentum pathway existed near \( z \approx 0.5\delta \). The ensemble-averaged \( \lambda_{ci} \frac{\omega_y}{|\omega_y|} \) for these two cases display elongated
regions of $\lambda_{ci}\omega_y/|\omega_y| < 0$ above and $\lambda_{ci}\omega_y/|\omega_y| > 0$ below this location with a region of $\lambda_{ci}\omega_y/|\omega_y| \approx 0$ between. This pattern in both the $M = 16$ model and full surface is consistent with a preferential alignment of clockwise-rotating wall-normal vortices along the upper boundary of the low-momentum pathway as well as a preferential alignment of counter-clockwise-rotating wall-normal vortices below this pathway. The combined induction of such vortical alignments would generate flow against the mean-flow direction and thus is consistent with the existence of a low streamwise momentum region between these vortex paths. Recalling that the spatial signature of hairpin vortex packets in streamwise–spanwise measurement planes yields a qualitatively similar effect, particularly the existence of spanwise-separated counter-rotating wall-normal vortices between which a region of streamwise momentum deficit exists, these patterns in ensemble-averaged $\lambda_{ci}\omega_y/|\omega_y|$ may be indicative of a preferential alignment of such structures due to the roughness below the measurement plane or the generation of such structures directly by the roughness. Alternatively, this pattern could also be interpreted as a train of structures that are the result of unsteady shedding from dominant roughness elements either upstream or below the measurement plane. However, under this scenario it is not clear whether such ‘trains’ exhibit the same streamwise coherence as vortex packets. Regardless of the origin, this evidence supports that vortical structures are likely responsible for the streamwise-elongated character of the mean velocity defect as well as certain components of the Reynolds stresses deep within the roughness sublayer.

Finally, figure 6.20 presents spanwise profiles of mean velocity defect, $U_e^+ - U$, streamwise Reynolds normal stress, $\langle u'^2 \rangle^+$, Reynolds shear-stress components $\langle u'v' \rangle^+$ and $\langle u'u' \rangle^+$ as well as mean swirling strength, $\langle \lambda_{ci}\omega_y/|\omega_y| \rangle^+$ computed by streamwise-averaging the respective ensemble-averaged fields presented in figures 6.2, 6.4, 6.8, 6.9 and 6.19. Profiles are presented for all surfaces cases and vertical lines demarcating the spanwise positions of the low-momentum pathways noted in the rough-wall mean velocity defect results (figure 6.2) are included along with blue and red vertical lines denoting the spanwise positions of the bands of negative and positive vortices identified in the rough-wall ensemble-averaged swirling-strength results (figure 6.19). Thus, these plot provide a means of collectively assessing the consistencies discussed regarding enhanced turbulent activity along the boundaries of the identified low-momentum pathways as well as prominent vortical activity along these boundaries. While all three rough-wall flows show similar tendencies, this discussion will focus upon the $M = 16$ model and full-surface trends given their strong consistency. As noted earlier, a streamwise-elongated low-momentum pathway was apparent in the ensemble-averaged velocity defect near $z \approx 0.35\delta$ in both cases and the signature of this low-momentum pathway is also readily apparent in the streamwise-averaged, spanwise profiles of $U_e^+ - U^+$ in figure 6.20. Comparison of the spanwise profiles of $TKE$, $\langle u'v' \rangle^+$, $\langle u'u' \rangle^+$ and $\langle \lambda_{ci}\omega_y/|\omega_y| \rangle^+$ highlights the aforementioned consistencies of
enhanced turbulent and vortical activity in the vicinity of the low-momentum pathway in both surfaces cases. For example, a peak in $\langle \lambda_{ci} \omega_y/|\omega_y| \rangle^+$ is noted near $z \approx 0.65 \delta$ while a trough is noted near $z \approx 0.35 \delta$ with the former consistent with counter-clockwise rotating wall-normal vortices and the latter consistent with clockwise-rotating wall-normal vortices. It should be recalled that these spanwise positions are consistent with the spanwise boundaries of the low-momentum pathways identified in figure 6.2 for the $M = 16$ model and full-surface cases. Thus, this consistency further supports the notion that counter-rotating vortices populate the spanwise boundaries of the low-momentum pathways noted in the velocity defect and that the induction of these vortices is consistent with the existence of flow against the mean-flow direction and thus the formation of a low-momentum pathway.

Further, it is observed that both $TKE$ and $\langle u'w' \rangle^+$ show enhancement at these same spanwise positions that demarcate the spanwise boundaries of the identified low-momentum pathways. In the case of $\langle u'w' \rangle^+$, a strong peak is noted near $z \approx 0.35 \delta$ and a well-defined trough is observed near $z \approx 0.65 \delta$. The former occurs at the same spanwise location as the negative trough in $\langle \lambda_{ci} \omega_y/|\omega_y| \rangle^+$ while the latter occurs at the same spanwise position as the positive peak in $\langle \lambda_{ci} \omega_y/|\omega_y| \rangle^+$. The occurrence of enhanced $u'w'$ coincident with a strong mean velocity gradient in the spanwise direction, $\partial U/\partial z$, at the spanwise boundaries of the low-momentum pathways in the $M = 16$ and full-surface cases provides a possible explanation for the enhancement noted in the $TKE$ at these same spanwise locations since the products of the RSS components and the mean velocity gradients act as turbulence production terms in the $TKE$ equation. Finally, the spanwise profiles of $\langle u'v' \rangle^+$ reveal a region of intense negative RSS that resides between the spanwise boundaries of the identified low-momentum pathways in both the $M = 16$ model and full-surface cases. Such a region was a bit difficult to discern clearly in the ensemble-averaged $\langle u'v' \rangle^+$ plots in figure 6.8, likely due to sampling-error issues. However, the existence of a well-defined region of nearly constant, intense, negative RSS that occurs spatially coincident, in a spanwise sense, with the low-momentum pathways identified in the $M = 16$ model and full-surface velocity defect results indicates that the collective features of these patterns further support the notion that these imprints in the rough-wall statistics are compatible with the existence of hairpin vortex packets or trains of vortices shed from dominant roughness features.

6.2.3 Conditional averaging

So far, it has been observed that the highly irregular rough surfaces used in this study bias the spatial distribution of vortex activity in the roughness sublayer. Specifically, via statistical descriptors such as the ensemble-averaged and RMS swirling-strength, it was observed that vortices of similar rotational sense tend to populate common streamwise-aligned bands. Moreover, these bands tend to reside along the spanwise
boundaries of a region of reduced streamwise momentum. Hence, it was hypothesized above that this spatial redistribution of vortical activity is consistent with either a preferential “channeling” of hairpin vortex packets along these paths over the rough surfaces or possibly the generation of such structures preferentially at these spatial locations. To explore this issue further, conditional averaging was employed. Conditional statistics provide a means of recovering spatial structure from an apparently random ensemble of velocity fields. In fact, conditional statistics have been proven able to recover a rich arrangement of coherent structures even in isotropic turbulence; which, apart from some integral scales, appears featureless under the lens of unconditional statistics (Adrian, 1979).

The premise of conditional averaging in turbulence is to estimate a certain flow property generically given by \( g(x + \Delta x, t + \Delta t) \), given a set of specific events, \( E \), at points \([x_1, t], (x_2, t), \ldots (x_i, t), \ldots (x_N, t)\). Here, \((x + \Delta x, t + \Delta t)\) expresses a neighborhood of the \((x_i, t)\) points. It has been shown by Adrian (1979) and Adrian et al. (1989) that the best least-square estimate of \( g(x + \Delta x, t + \Delta t) \) is the conditional average given by

\[
\langle g(x + \Delta x, t + \Delta t)|E_1(x_1, t), E_2(x_2, t), \ldots E_i(x_i, t), \ldots E_N(x_N, t) \rangle. \tag{6.4}
\]

Though \( g \) and \( E \) can represent any sort of fluid flow property (velocity, pressure, components of the deformation tensor, etc), the present objective is to study the structure of velocity fields within the roughness sublayer. In this context, \( g \) will represent the velocity field induced by localized vortical events, \( E \). The swirling-strength criterion discussed in \( \S 6.2.1 \) is used for identifying such events. Subsequently, the velocity fields are conditioned to the existence of vortices (of a given rotation sense) at chosen points of space. Finally, all the velocity fields meeting such criteria will be ensemble averaged. These conditionally-averaged velocity fields are then scrutinized for features that are consistent with the characteristics of the statistics presented in the previous section, particularly the existence of specific low- and high-momentum regions bounded by vortical activity.

In a first exploration, the above described conditional average is conducted on several points, equally spaced along spanwise traces. According to the observations of \( \S 6.2.2 \), the vortical activity is strongly dependent on the spanwise position when compared with smooth-wall flow which shows no spatial preference for the existence of wall-normal vortices. Thus, it is expected that the results of conditional averaging will reflect these observations. Nonetheless, the way conditional average is conducted disregards any a priori knowledge of the statistical structure of the flow. Thus, it represents an objective means of elucidating additional structural characteristics of the flow, particularly via reinforcing the previously-observed statistical features of the rough-wall flows.

In the present analysis, the flow was conditioned on the existence of either clockwise or counter-clockwise
wall-normal vortices at different predefined locations. Therefore, \( \mathbf{g} \) expresses a fluctuating velocity field \( \mathbf{u}' \) conditioned according to eq. (6.3). Note that since the stereo-PIV realizations of this work are not time-resolved, the conditional average simply gives \( \mathbf{u}' \) as a spatially-distributed velocity field. As such, eq. (6.4) for the present case can be recast as

\[
\left\langle \mathbf{u}'(x + \Delta x, z + \Delta z) \left| \frac{\omega_y}{|\omega_y|} \lambda_{ci}(x, z) > 0 \right. \right\rangle, \quad \text{conditioned on a positive vortex}
\]

\[
\left\langle \mathbf{u}'(x + \Delta x, z + \Delta z) \left| \frac{\omega_y}{|\omega_y|} \lambda_{ci}(x, z) < 0 \right. \right\rangle, \quad \text{conditioned on a negative vortex}
\]

As indicated in eq. (6.5), two different conditions were independently applied for each spatial location in the velocity field. Three spanwise traces, equally spaced in the streamwise direction, were used for conditional averaging the flow based on clockwise and counter-clockwise wall-normal vortices. Twenty points, equally spaced along these spanwise traces, were sampled for clockwise and counterclockwise vortices. Since vortices occur intermittently rather than continuously in space, the swirling strength fields are predominantly zero except at grid-points where vortices reside. In consequence, it is necessary to use a sampling window, large enough to capture at least part of advecting vortices, but small enough to minimize the likelihood of capturing multiple vortices which might merge their effect. Based on trial and error, it was found that a sampling window of \( 4 \times 4 \) grid spacings in size would provide a robust means of identifying these structures. For each spatial location along the spanwise traces, the velocity field as a result of the vortical condition was unveiled. In addition, based on the vortex count, the probabilities of finding vortices of different sense of rotation, \( P[(\omega_y/|\omega_y|)\lambda_{ci}(x, z) > 0] \) and \( P[(\omega_y/|\omega_y|)\lambda_{ci}(x, z) < 0] \), were calculated for the same spatial locations. The findings along the three different spanwise traces for each rough-wall flow were equivalent. Therefore, only one representative spanwise trace for each experiment is presented.

The results for the above defined probability \( P \) are shown in figure 6.21. While the smooth-wall result shows only slight fluctuation about a clear mean probability, but with no clear spatial preference, for both clockwise (negative) and counter-clockwise (positive) wall-normal vortices, the rough-wall flows exhibit a much larger deviation from their average values. To assess the degree of spatial preference noted in the rough-wall cases, a dispersion band was calculated based on a 95% confidence interval using the smooth-wall flow result given that these result should provide an approximate value of the sampling error in this particular statistic. Thus, assuming a normal distribution, the half width of the dispersion band would be equal to one standard deviation, \( \sigma_P \), of \( P \). This dispersion band is presented in figure 6.21 using dash-dot-dot lines (---) for all cases. A comparison of the rough-wall trends with this dispersion band reveals excursions beyond the
dispersion band and thus are likely not attributable solely to sampling error. In addition, there is a clear
tendency in the rough-wall flows to form well-defined peaks that alternate in conditional sign. Of interest,
each large peak tends to coincide with the streamwise-aligned bands of swirling strength corresponding to
their sign [see figure 6.19]. For comparison, the spanwise positions of the dominant low- and high-momentum
pathways detected in figure 6.2 are represented by dashed- (--) and dash-dot- (⋅⋅⋅) lines respectively in
figure 6.21. In addition, for a more clear comparison with the tendencies observed in figure 6.19, shades
of red and blue, representing counter-clockwise and clockwise rotating vortices respectively, are superposed
on figures 6.21b–6.21d. Using the low-momentum pathway’s spanwise position as a reference, it should be
recalled that the ensemble-averaged swirling strength for the $M = 16$ model and the full surface displayed
a negative band indicative of clockwise vortices populating the upper boundary of this low-momentum
pathway near $z \approx 0.35\delta$ and a positive band indicative of counter-clockwise vortices populating its lower
boundary near $z \approx 0.65\delta$. Comparing these results to the probabilities in figure 6.21 reveals peaks in the
clockwise vortices for the $M = 16$ model and full surface near $z \approx 0.35\delta$ and simultaneous troughs in the
probability of counter-clockwise vortices at this spanwise position. Likewise, peaks in the counter-clockwise
vortices are evident near $z \approx 0.65\delta$ for the $M = 16$ model and the full surface along with simultaneous
troughs in the probability of clockwise vortices at this spanwise location. Similar trends are evident for
the $M = 5$ model, though the spanwise locations of these trends are shifted since the spanwise location
of the low-momentum pathway is also shifted relative to the other two rough-wall flows. Taken together,
these observations further support preferential paths for wall-normal vortices of opposing rotation bounding
the low-momentum pathways evident in the mean velocity defect for the rough-wall flows. Of interest,
the conditional probability of the flow structures given at these peaks is $P \approx 0.49$ for all cases. Such
large probability suggests that, whatever the flow structure evidenced by this conditional average, it is a
dominant localized feature of the rough-wall flows. In contrast, due to the narrow dispersion of its conditional
probability, the conditioned flow field of the smooth-wall flow appears oblivious to spatial localization.

Using eq. (6.5), the results for conditionally-averaged fluctuating velocity fields based on given clock-
wise (negative) and counter-clockwise (positive) vortices centered at the spanwise locations of maximum
probability as noted in figure 6.21 are shown in figures 6.22 and 6.23, respectively. The contour maps
of the conditionally-averaged fluctuating streamwise velocity are included in the background to reveal the
presence of low- and high-momentum pathways. In addition, horizontal lines demarcating the dominant
low-momentum pathway and high-momentum pathway detected in figure 6.2 for the rough-wall cases are
also included for reference. As before, the smooth-wall flow field is used as a baseline to study the effect
of roughness on the flow. It should be noted that since the conditional probabilities at the three different
spanwise traces discussed above revealed quite similar trends for all surface cases, the streamwise location for enforcing the conditional vortex was placed at the middle of the field to capture a larger extent of its effect in the streamwise direction. It is observed in all cases that the conditional vortex (whether it be a positive or negative vortex) induces spanwise alternating low- and high-momentum pathways whose streamwise extents are of order $\delta$. Of interest, they tend to coincide with the dominant low- and high-momentum pathways detected in figure 6.2 for the rough-wall flows and demarcated by the horizontal lines in figures 6.22 and 6.23. Focusing on the $M = 16$ model and full surface results, since they show strong consistency, the velocity field associated with a negative vortex at $z \approx 0.35\delta$ is marked by a clear clockwise-rotating swirling pattern beneath which a streamwise-elongated low-momentum pathway is evident and above which an elongated high-momentum pathway is revealed. The spanwise position of this low-momentum pathway is roughly coincident with the upper half of the low-momentum pathway noted in the mean velocity defect for both of these cases. Likewise, the velocity field associated with a positive vortex at $z \approx 0.65\delta$ yields a counter-clockwise-rotating swirling pattern above which a streamwise-elongated low-momentum pathway is evident and below which an elongated high-momentum pathway is evident. The spanwise position of this low-momentum pathway is roughly coincident with the lower half of the low-momentum pathway noted in the mean velocity defect. Thus, these conditional averages further support the strong linkage between the low- and high-momentum pathways noted in the rough-wall mean velocity defect results and the existence of wall-normal vortices along the boundaries of these pathways. In addition, these results indicate that these pathways display larger-scale coherence in the streamwise direction, meaning they are likely not simply a train of random structures that align to create the appearance of streamwise elongation. Instead, these streamwise-elongated pathways of low and high momentum appear more likely collectively-induced motions associated with streamwise-aligned vortical structures that act in a collective (and coherent) manner. Further, although the rough-wall flows show strong qualitative similarities in this regard, one important difference is observed: the streamwise extent of the low- and high-momentum pathways in the $M = 16$ and full-surface cases is reduced compared with the other cases. In contrast, their spanwise extent does not seem to be different between the surface cases. These observations, at least for $M = 16$ model and full roughness, are in good agreement with the shortening of the streamwise extent of $\rho_{uu}$ very close to the wall, presented in figure 5.11. Finally, it is remarkable how consistent the characteristics of the $M = 16$-model result mimics the character of the full-surface result. This consistency highlights the ability of the $M = 16$ model to reproduce the structural characteristics of the full-surface flow, even deep within the roughness sublayer.
6.2.4 Two-point correlation coefficients

To explore the spatial structure further, particularly large-scale coherence in the streamwise elongated low- and high-momentum pathways, two-point correlation coefficients of velocity are computed in the wall-parallel measurement plane for all surfaces cases. However, instead of assuming homogeneity in any of the spatial directions as was done in chapter 5 for the flow in the outer region, the results presented in this chapter support significant spatial heterogeneity for the rough-wall flows at this near-wall measurement location. Thus, inhomogeneous two-point correlation coefficients are computed at select reference locations as from

$$
ρ_{ij}(x, z; x_{ref}, z_{ref}) = \frac{⟨u_i'(x_{ref}, z_{ref})u_j'(x, z)⟩}{σ_i(x_{ref}, z_{ref})σ_j(x, z)},
$$

(6.6)

where \((x_{ref}, z_{ref})\) gives the spatial position of the reference location in the wall-parallel data. In consequence, while the correlations presented in chapter 5 were efficiently computed using Fast Fourier Transforms (FFTs) assuming homogeneity in the streamwise direction, the present inhomogeneous correlations must be computed directly in physical space which, in terms of computational time, is much more laborious. Thus, correlations were only computed for a few select locations in the fields of view based upon the heterogeneous observations discussed above with respect to the single-point statistics and the conditional-averaging results.

Figures 6.24–6.29 presents two-point correlation coefficients of the various fluctuating velocity components computed at a reference location of \((x_{ref}, z_{ref}) = (0.66δ, 0.40δ)\) for the smooth-wall flow, \((x_{ref}, z_{ref}) = (0.66δ, 0.34δ)\) for the \(M = 5\) model and \((x_{ref}, z_{ref}) = (0.66δ, 0.48δ)\) for the \(M = 16\) model and the full-surface cases. These reference locations were chosen so that they reside in the streamwise center of the field of view and are coincident with the spanwise centers of the low-momentum pathways noted in the rough-wall velocity defect results (figure 6.2). Of interest, all of the \(M = 16\) model and full-surface correlations shows quite strong consistency in their overall spatial characteristics. This consistency further supports the ability of the \(M = 16\) model to reproduce the behavior of flow over the full surface. In contrast, the \(M = 5\) model yields some correlation results that resemble those of the \(M = 16\) model and full-surfaces cases but other correlation results that are more consistent with smooth-wall flow. Thus, while the \(M = 5\) model embodies some of the roughness features of the full surface, the discarded topographical information in the \(M = 5\) model appears to play an important role in the spatial structure of the flow given this lack of universal consistency with the full-surface results.

As has been previously observed in both smooth- and rough-wall turbulence, \(ρ_{uu}\) is elongated in the streamwise direction and bounded by negative correlation regions in the spanwise direction indicative of the spanwise-alternating LMRs and HMRs that are characteristic of wall turbulence. Of interest, however, a
reduction in this streamwise extent is noted when comparing the rough-wall results to those of smooth-wall flow. This shortening of $\rho_{uu}$ in the presence of roughness has been previously observed in past studies of both idealized (Volino et al., 2007) and realistic (Wu and Christensen, 2010) roughness and could indicate some loss of streamwise coherence and therefore may support the increased occurrence of randomly-generated flow structures along the low-momentum pathway. Similar streamwise-elongation is notable in $\rho_{uv}$ and, to a lesser extent, in $\rho_{uw}$ and $\rho_{vw}$. In the case of $\rho_{uv}$, the streamwise coherence is significantly enhanced in the presence of roughness compared to the smooth-wall result, particularly in the $M = 16$ model and full-surfaces cases which show strong similarities. This streamwise coherence of $\rho_{uv}$ in the rough-wall flows at the same spanwise position as the low-momentum pathways identified in the velocity defect is consistent with the coherent induction of vortical aligned in the streamwise direction. Further, while $\rho_{vv}$ is quite compact for smooth-wall flow, which is consistent with previous observations, $\rho_{vv}$ in the rough-wall cases displays enhanced streamwise elongation, indicating patterns of streamwise-coherent $v'$ fluctuations that would be consistent, again, with streamwise-aligned trains of vortices collectively-inducing ejections of low-momentum fluid away from the wall. Thus, the enhanced streamwise coherence noted in $\rho_{uw}$ for the rough-wall flows is likely attributable to this enhanced streamwise coherence of the $v'$ fluctuations themselves. In contrast, $\rho_{ww}$ shows a slight enhancement in streamwise coherence in the $M = 5$ result compared to smooth-wall flow but a noted decrease in streamwise coherence for the $M = 16$ model and full-surface cases.

The two-point autocorrelation coefficients of swirling strength, $\rho_{\lambda\lambda}$, are presented in figure 6.30 reveal interesting tendencies with respect to structural organization. Both the smooth-wall and the $M = 5$ model results exhibit a wider correlation peak with a clear tendency toward a streamwise-elongated region of albeit weaker correlation. However, both the full surface and the $M = 16$ model present a visibly narrower correlation peak with a much weaker tendency of a streamwise-elongated correlation region. These tendencies suggest that the vortical structures channeled along the dominant low-momentum pathway in the full surface and $M = 16$ model are less correlated than the vortical structures of hairpin vortex packets observed in smooth-wall flows. Thus, it is possible that in the $M = 16$ model and full-surface cases the vortices that exist along this path are not coherently-connected in the sense of vortical packets but are rather less-correlated individual structures that could be the result of unsteady vortex shedding from dominant roughness elements.

Finally, it should be noted that two-point correlation coefficients computed at other spanwise locations, including at the edges of the low-momentum pathways identified in the mean velocity defect results as well as in identified regions of high-momentum pathways, display similar consistency between the $M = 16$ model and the full surface. As such, these results highlight the efficacy of the $M = 16$ model, containing the larger- and intermediate-scales of the full-surface topography, in reproducing the spatial structure of the full-surface
flow deep within the roughness sublayer where the impact of roughness is the most obvious. In contrast, while qualitative similarities are evident between the $M = 5$ correlations and those of the $M = 16$ model and full-surface cases, quantitative differences are notable. As such, the intermediate topographical scales not captured by the $M = 5$ model have a measurable impact on the flow within the roughness sublayer.

### 6.3 Evidence of very-large-scale motions (VLSMs) in rough-wall flow

As was mentioned in §1.7, the existence of large-scale ($\delta$-scale) and very-large-scale (> $\delta$-scale) motions (LSM and VLSM) in smooth-wall turbulence has been documented in previous studies (Kim and Adrian, 1999; Adrian et al., 2000b; Ganapathisubramani et al., 2003; Guala et al., 2006; Balakumar and Adrian, 2007; Hutchins and Marusic, 2007). However, the existence of the latter, in particular, is still an open question in rough-wall flows. In this section, the results obtained from the TR-PIV experiments described in §4.3 are presented. This data is specifically used to reconstruct streamwise-elongated fields of view using Taylor’s frozen-field hypothesis in order to study the possibility of the occurrence of VLSMs in rough-wall turbulence.

Figure 6.31 presents a representative time series of instantaneous velocity fields from the TR-PIV experiment. Of particular interest is the existence of a low-momentum region (LMR) located at roughly $z = 0.2 - 0.25\delta$ that is elongated across the streamwise field of view at $t = t_o$ [figure 6.31(a)]. This LMR has a spanwise width of approximately $0.2\delta$ and is angled slightly away from the streamwise direction. This orientation is consistent with observations of spanwise meandering of these motions (Hutchins and Marusic, 2007). After 1 ms [figure 6.31(b)], the LMR advects slightly in the streamwise direction and its upstream end (flow is left to right) appears to thin slightly in the spanwise direction. This LMR continues to advect through the fixed field of view in all of the frames presented. At $t = t_o + 5$ ms [figure 6.31(f)], this LMR is still readily apparent, though its spanwise width is reduced compared to the $t = t_o$ field and its spanwise position has shifted closer to $z = 0.15\delta$ compared to $z = 0.2 - 0.25\delta$ in the $t = t_o$ field. This spanwise shifting along with the angling of the LMR slightly away from the streamwise direction throughout the time series presented support the possibility of significant spanwise meandering of these elongated streamwise scales in the presence of roughness. Finally, in addition to this larger-scale LMR, several smaller-scale, wall-normal vortices are also well-resolved in these wall-parallel TR velocity fields. These wall-normal vortex cores, which tend to reside along the spanwise-separated boundaries of the LMR, are interpreted as slices through the legs/arches of hairpin-like vortices by the wall-parallel measurement plane. This pattern of counter-rotating wall-normal vortex cores bounding an LMR is consistent with a wall-parallel slice through a hairpin vortex.
packet, indicating that such structures still persist in the presence of roughness. These observations are consistent with previous observations of hairpin vortex packets in wall turbulence reported by Volino et al. (2007) and Wu and Christensen (2010). Nevertheless, while these results support the existence of hairpin vortex packets in flow over highly-irregular roughness, it is not clear whether these motions arrange themselves to form extremely elongated streamwise spatial scales ($5 - 10 \delta$) in rough-wall turbulence akin to the VLSMs reported by Balakumar and Adrian (2007) and Hutchins and Marusic (2007).

The use of Taylor’s frozen-field hypothesis to infer the spatial character of the flow from its temporal variation has been used for decades to infer details regarding the spatial distribution of turbulence from single-point time series acquired by hot-wire anemometry, for example. In short, Taylor’s hypothesis assumes that over a certain length of time (usually several eddy turnover times), the turbulence is predominantly advected by the mean flow in comparison to weak evolution due to pressure, viscous and turbulent stresses. Previous efforts have indicated that Taylor’s hypothesis can be used effectively in unidirectional turbulent flows with relatively low turbulence intensities (i.e., $\sqrt{u''^2}/U \lesssim 0.1 - 0.15$). Wall turbulence is one class of flows for which Taylor’s hypothesis has been widely used in this regard (more recently by Allen et al., 2007; Balakumar and Adrian, 2007; Hutchins and Marusic, 2007, amongst many others). However, it should be noted that Taylor’s hypothesis is not limited to single-point methods. In fact, this methodology can be applied to the TR measurements reported herein for qualitative studies of elongated streamwise spatial scales. However, instead of having time-resolved velocity data at a single, fixed point, as in hot-wire anemometry, TR-PIV yields time-resolved fields over a fixed area. As time passes, new fluid parcels enter the fixed field of view via advection by the mean flow while other fluid parcels exit the field of view under the same action of the mean. Figure 6.32 presents a schematic illustrating how Taylor’s hypothesis can be applied to the TR velocity fields described above. Note that only contours of negative streamwise velocity fluctuation ($u' < 0$) are plotted for illustration purposes since they mark regions occupied by LMRs. Given an instantaneous velocity field over a fixed field of view at time $t$ and another instantaneous field over the same fixed field of view at time $t + \Delta t$, there is a portion of the velocity field at $t + \Delta t$ that has advected into the fixed field over the time interval $\Delta t$ and the streamwise length of this patch is $\Delta x = -U_{	ext{oc}} \Delta t$, where $U_{	ext{oc}}$ is taken to be the area-averaged velocity of the instantaneous field at time $t + \Delta t$. Applying Taylor’s hypothesis to these two time-separated fields simply involves taking this patch of “new” data from the $t + \Delta t$ field and stitching it onto the upstream end of the field at $t$ to generate a composite velocity field elongated by an amount $\Delta x$ in the streamwise direction (shown schematically in figure 6.32). One simply continues this process by stitching the “new” portion of the $t + 2\Delta t$ field to the elongated field, then the “new” portion of the $t + 3\Delta t$ field, and so-on. However, due to the fact our data was acquired within the roughness sublayer, where the influence of
roughness renders the flow strongly inhomogeneous, the direct application of Taylor’s hypothesis might be questionable. Thus, rather than directly applying Taylor’s hypothesis to this data, we instead reconstructed a large field of view by minimizing the RMS error between consecutive fields. That is, the $t + n\Delta t$ field was shifted over the $t + (n-1)\Delta t$ field (where $n$ is an integer) and the RMS difference in streamwise velocity in the overlapped area between the two fields was evaluated as

$$
\varepsilon_{\text{RMS}} = \sqrt{\frac{1}{N \times M} \sum_{i,j}^{N,M} \left[ u_{i,j}(t+n\Delta t) - u_{i,j}(t+(n-1)\Delta t) \right]^2}
$$

within an overlapping region of size $N \times M$ grid points. For simplicity, the overlap between consecutive realizations is incremented in steps equal to the grid spacing. Therefore, the subtraction on the right-hand side of eq. (6.7) is carried out between coincident grid points. After repeating this calculation for several overlaps, the overlapping condition with the minimum $\varepsilon_{\text{RMS}}$ is selected as the optimal relative displacement between two consecutive instantaneous realizations. Since the focus of this analysis was to specifically capture LMRs, the velocity fields were normalized by the area-averaged mean velocity of the total ensemble of realizations. Then, all values larger than 1 were eliminated from the velocity fields, leaving only those values that correspond to LMRs. In consequence, the instantaneous area-averaged velocity $U_c$ of these modified fields plausibly corresponds to the instantaneous advection velocity of the LMRs. Thus, such a velocity would be the one used for an eventual reconstruction of LMRs based on Taylor’s hypothesis.

On the other hand, based on the time separation between consecutive instantaneous realizations in this particular experiment (1 ms), and the relative displacement found when minimizing $\varepsilon_{\text{RMS}}$, one can calculate the matching advection velocity $U_m$ directly. A comparison between $U_c$ and $U_m$ is presented in figure 6.33 where $U_m$ and $U_c$ show strong consistency. The discrete nature of $U_m$ is due to the fact that the fields are shifted discretely in the streamwise direction by an amount equal to the grid spacing. Nevertheless, these results indicate that $U_m$ follows $U_c$ closely, further supporting the use of Taylor’s hypothesis in the present application.

Using the TR-PIV fields acquired for flow over the rough surface, several $\sim 10\delta$-long velocity fields were reconstructed using the above mentioned stitching methodology. Figure 6.34 shows contour maps of negative streamwise velocity fluctuations ($u' < 0$). Plotting the fields in this manner is meant to reveal the streamwise extent of LMRs as well as any possible spanwise meandering as was observed by Hutchins and Marusic (2007) for smooth-wall flow. It is easy to identify the presence of long LMRs in figure 6.34. However, due to the limited spanwise extent of the field of view, coupled with the obvious meandering of the identified LMRs, it is difficult to capture entire LMRs within the reconstructed fields. Therefore, of all
reconstructions shown in figure 6.34, focus will be paid to (d) since it provides a quite complete view of a single LMR. In consequence, since the following analysis pertains to a single LMR, the conclusions will not be necessarily representative of the entire population of LMRs, but will at least give some qualitative insight as to their characteristics.

Figure 6.34(d) is reproduced independently in figure 6.35. Remarkably, the region of low streamwise momentum extends nearly the entire $10\delta$ domain in the streamwise direction. Further, this elongated feature displays significant waviness, consistent with the spanwise meandering reported for smooth-wall flow (Hutchins and Marusic, 2007). This preliminary result indicates that such structures may have a certain level of insensitivity to the presence of the rough surface considered herein. Figure 6.35 also presents a zoomed-in view of a portion of this elongated field. This zoomed-in view contains both gray-scale contours of $u' < 0$ as well as color contours highlighting the locations of clockwise and counter-clockwise rotating wall-normal vortex cores. Every fourth in-plane velocity vector is also plotted. As noted earlier, the vortex cores clearly populate the boundaries of the elongated LMR. This behavior is consistent with what one would expect if this LMR was generated by the collective induction of hairpin-like vortices aligned in the flow direction. Given this evidence, it appears that it is the hairpin packets themselves (or coherent trains of vortex packets) that are meandering in the spanwise direction. Finally, while quantitative analysis of this spanwise meandering has yet to be performed, comparing the result in figure 6.35 with the results of Hutchins and Marusic (2007) for smooth-wall flow indicates that the meandering may be more pronounced in the rough-wall case. If this is indeed the true, then this enhanced spanwise meandering would certainly account for the shortening of the streamwise extent of $\rho_{uu}$ (figure 5.11). Under such a scenario, the underlying structural foundation of this rough-wall flow would be identical to that of smooth-wall flow save for an enhanced meandering of the larger streamwise scales.

### 6.4 Summary

To study the differences noted in the roughness sublayer from the 2D-PIV measurements in the streamwise–wall-normal plane, stereo-PIV experiments were conducted in a streamwise–spanwise plane deep within the roughness sublayer ($y = 0.047\delta$). Interestingly, these measurements revealed a wealth of information regarding roughness-induced effects that were not discernable from the wall-normal plane measurements. In particular, contour maps of ensemble-averaged streamwise velocity deficit revealed the tendency of the roughness used in this study to promote ‘channeling’ of the flow in the form of low- and high-momentum pathways that occurred so consistently in the instantaneous fields that they persisted through the ensemble-
averaging performed. Similarly, enhanced turbulent and vortical activity was observed both between and
along the spanwise boundaries of these streamwise-elongated pathways. Finally, observations of vortical
activity via ensemble-averaged signed swirling strength revealed that wall-normal vortices of opposite sign
tend to organize on either side of these dominant low-momentum pathways in a manner consistent with the
collective induction of streamwise flow against the mean flow direction and therefore are likely the origination
mechanism of the low-momentum pathways observed in the mean velocity defect results for the rough-wall
flows. Taken together, these observations support the idea that these persistent low-momentum pathways
could represent the statistical imprint of trains of hairpin vortex packets that are channeled along preferred
paths over the roughness. Alternatively, these streamwise-elongated pathways could also be due, to some
extent, to vortical structures shed from dominant roughness features and could therefore have very little
streamwise coherence in the sense of the collectively-induced motions of hairpin vortex packets. However,
the notion of the occurrence of streamwise-correlated motions along the low-momentum pathways is further
supported by conditional averages of fluctuating velocity based on the presence of positive and negative
wall-normal vortices which contained well-defined low-momentum events induced by these vortical motions
that tend to occur spatially coincident to the central locations of the low-momentum pathways noted in the
mean velocity defect results. Inhomogeneous two-point correlation coefficients of velocity also support this
notion of preferred channeling of large-scale vortex packets rather than random, smaller-scale vortices shed
from the roughness that tend to advect along the preferred pathways noted in the mean streamwise velocity.

With regard to the fidelity of the low-order models in reproducing the characteristics of flow over the full
surface, both the $M = 5$ and $M = 16$ topographical models yield statistical and structural characteristics
that are qualitatively consistent with the full-surface flow at this near-wall measurement plane. However,
significant quantitative differences were noted when comparing the $M = 5$ model results with those of the
full surface, including different spanwise positions of the fixed low- and high-momentum pathways identified
in the mean velocity defect as well as slightly different results in terms of the average spatial structure as
educed from the conditional-averaging results and the two-point correlation coefficients. In contrast, the
$M = 16$ results are virtually indistinguishable from those of the full surface, including in the single-point
turbulence statistics as well as the analysis of the average spatial structure. This consistency is not simply
qualitative but is indeed quantitative as the magnitudes of the $M = 16$ model single-point statistics mirror
those of the full surface as do the spatial locations of the low- and high-momentum pathways identified in
the mean velocity defect results as well as the enhanced turbulent and vortical activity along the spanwise
boundaries of these large-scale motions. Hence, these observations provide significant evidence supporting
the importance of the intermediate topographical scales in setting the flow conditions within the roughness
sublayer, not only in a statistical sense but also in a structural sense. Connecting these observations with those discussed above regarding the flow-regime change between flow over the $M = 5$ model and the other two surfaces, it is quite possible that the wavy-wall behavior, and hence the lack of significant flow separation, in the case of the $M = 5$ model flow further manifests itself to yield the stark differences between its flow behavior in the roughness sublayer compared to the other two cases wherein flow separation likely plays an important role based on the analysis of Napoli et al. (2008).

Finally, though the existence of large- and very-large-scale motions (LSM and VLSM) in smooth-wall turbulence is well documented (Kim and Adrian, 1999; Adrian et al., 2000b; Ganapathisubramani et al., 2003; Guala et al., 2006; Hutchins and Marusic, 2007), their presence in rough-wall flows has remained an open question until the findings revealed in the present work. Specifically, elongated streamwise fields of view were reconstructed from TR-PIV data acquired in a wall-parallel plane within the roughness sublayer ($y = 0.0655$) for flow over the full surface. Using pattern reconstruction via Taylor’s frozen-field hypothesis, several $\sim 10\delta$-long streamwise fields of view were reconstructed, each of which contained low-momentum regions that extended nearly the entire field of view but meandered significantly in the spanwise direction. These observations are consistent with recent observations in smooth-wall flow (Hutchins and Marusic, 2007). Thus, despite the direct impact of roughness on the flow within the roughness sublayer, coherent motions on the order of $10\delta$ with similar characteristics to those noted in the near-wall region of smooth-wall turbulence still persist. However, further study of such motions in the presence of roughness is needed to assess the energy and Reynolds stress that they carry to determine how important they might be to the overall evolution of rough-wall turbulence.
Figure 6.1: Contour maps of the roughness directly beneath the streamwise-spanwise measurement plane for the (a) $M = 5$ model, (b) $M = 16$ model and (c) full surface. To highlight protrusions above the midplane of the roughness, negative roughness elevations are blended with the black background.
Figure 6.2: Contour maps of ensemble-averaged streamwise velocity in defect scaling, $(U_e - u)^+$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the identified high-momentum pathway (---) and low-momentum pathway (--).
Figure 6.3: Contour maps of ensemble-averaged wall-normal velocity, $\langle v \rangle^+$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (--) identified in figure 6.2.
Figure 6.4: Contour maps of ensemble-averaged streamwise Reynolds normal stress, $\langle u'^2 \rangle^+$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway ($\cdots$) and low-momentum pathway ($\cdots$) identified in figure 6.2.
Figure 6.5: Contour maps of ensemble-averaged wall-normal Reynolds normal stress, $\langle v'^2 \rangle^+$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (–·–) and low-momentum pathway (––) identified in figure 6.2.
Figure 6.6: Contour maps of ensemble-averaged spanwise Reynolds normal stress, $\langle w'^2 \rangle$, at $y = 0.047 \delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (-----) identified in figure 6.2.
Figure 6.7: Contour maps of the ensemble-averaged turbulent kinetic energy, $\frac{1}{2} \langle q^2 \rangle^+$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (----) identified in figure 6.2.
Fig. 6.8: Contour maps of ensemble-averaged Reynolds shear stress component \( \langle u'v' \rangle^+ \) at \( y = 0.047 \delta \). (a) Smooth; (b) \( M = 5 \) model; (c) \( M = 16 \) model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (----) identified in figure 6.2.
Figure 6.9: Contour maps of ensemble-averaged Reynolds shear stress component $\langle u'w' \rangle^+$ at $y = 0.047 \delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (– – –) and low-momentum pathway (– –) identified in figure 6.2.
Figure 6.10: Contour maps of ensemble-averaged Reynolds shear stress component $\langle v'w' \rangle^+$ at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (-- --) identified in figure 6.2.
Figure 6.11: Contour maps of outward-interaction contributions to the ensemble-averaged RSS, $\langle u'v' \rangle_1^+$, for $H = 0$. (a) Smooth wall; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface.
Figure 6.12: Contour maps of inward-interaction contributions to the ensemble-averaged RSS, $\langle u'v' \rangle^+_{3}$, for $H = 0$. (a) Smooth wall; (b) $M=5$ model; (c) $M=16$ model; (d) Full surface.
Figure 6.13: Contour maps of ejection contributions to the ensemble-averaged RSS, $\langle u'v' \rangle^+_2$, for $H = 0$. (a) Smooth wall; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (----) identified in figure 6.2.
Figure 6.14: Contour maps of sweep contributions to the ensemble-averaged RSS, $\langle u'v' \rangle^+_4$, for $H = 0$. (a) Smooth wall; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (-- --) identified in figure 6.2.
Figure 6.15: Contour maps of ejection contributions to the ensemble-averaged RSS, $\langle u'v' \rangle^+_2$, for $H = 4$. (a) Smooth wall; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (--) identified in figure 6.2.
Figure 6.16: Contour maps of sweep contributions to the ensemble-averaged RSS, $\langle u'v' \rangle^+$, for $H = 4$. (a) Smooth wall; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (--) identified in figure 6.2.
Figure 6.17: Local coordinate system defined by the eigenvectors of $\nabla u$ in the neighborhood of a vortex core, with a streamline pattern superposed (Source: Zhou et al., 1999).
Figure 6.18: Contour maps of RMS swirling strength $\lambda_{ci,RMS}^+$ at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (--) identified in figure 6.2.
Figure 6.19: Contour maps of ensemble-averaged swirling strength, $\langle \lambda_{ci} \omega_y/|\omega_y| \rangle^+ = 10^6 (\lambda_{ci}^+)$, at $y = 0.047\delta$. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (--) identified in figure 6.2. The vertical solid line (---) demarcates the spanwise trace (located at $X = 0.65\delta$) along which conditional vortices were sampled (see §6.2.3).
<table>
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<tr>
<th>Smooth</th>
<th>$M = 5$</th>
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Figure 6.20: Spanwise profiles of mean velocity defect, $U_e^+ - U$, turbulent kinetic energy, $TKE$, Reynolds shear-stress components $\langle u'v' \rangle^+$ and $\langle u'w' \rangle^+$ as well as mean swirling strength, $\langle \lambda_\omega \lambda_\omega^* \rangle^+$, computed by streamwise-averaging the ensemble-averaged fields presented previously. Results are shown for all four surface conditions and the vertical dashed lines demarcate the spanwise position of the low-momentum pathway identified in the rough-wall ensemble-averaged velocity defect results (figure 6.2) while the vertical blue and red lines demarcate the spanwise positions of the bands of negative and positive vortices noted in the rough-wall ensemble-averaged swirling strength results (figure 6.19).
Figure 6.21: Probability of the presence of wall-normal vortices at different spanwise positions for (a) smooth wall, (b) $M = 5$ model, (c) $M = 16$ model, and (d) full surface. Probability for: positive vortices (■ ■ ■); negative vortices (▲ ▲ ▲). The horizontal dash-dot-dot lines (····) demarcate the range of dispersion of the smooth-wall case based on a total bandwidth of $2\sigma_P$. The vertical lines demarcate the spanwise positions of: (−−) low-momentum pathways and (−−−) high-momentum pathways, as identified in figure 6.2. Shades of red and blue denote regions containing preferentially counter-clockwise- and clockwise-rotating vortices respectively as assessed by figure 6.19.
Figure 6.22: Velocity field conditioned to the existence of a clockwise-rotating vortex. The background contour map represents fluctuating streamwise velocity in inner units. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (---) and low-momentum pathway (---) identified in figure 6.2. The position of the reference negative vortex is demarcated by a green circle •.
Figure 6.23: Velocity field conditioned to the existence of a counter-clockwise-rotating vortex. The background contour map represents fluctuating streamwise velocity in inner units. (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway ($\cdots$) and low-momentum pathway ($\cdots$) identified in figure 6.2. The position of the reference vortex is demarcated by a green circle $\bullet$. 
Figure 6.24: Contour map of the inhomogeneous two-point correlation coefficient of streamwise velocity, $\rho_{uu}$, for (a) Smooth; (b) $M = 5$ model; (c) $M = 16$ model; (d) Full surface. Horizontal lines demarcate the locations of the high-momentum pathway (–·–) and low-momentum pathway (––) identified in figure 6.2. The reference location is demarcated with an ‘X’ and its spanwise position coincides with the center of the low-momentum pathways observed in the rough-wall velocity defect results (figure 6.2).
Figure 6.25: Same as figure 6.24, but for $\rho_{vv}$. 
Figure 6.26: Same as figure 6.24, but for $\rho_{ww}$.
Figure 6.27: Same as figure 6.24, but for $\rho_{uv}$. 
Figure 6.28: Same as figure 6.24, but for $\rho_{uw}$. 
Figure 6.29: Same as figure 6.24, but for $\rho_{vw}$.
Figure 6.30: Same as figure 6.24, but for $\rho_{\lambda \lambda}$. 
Figure 6.31: Time series of velocity fields in the streamwise–spanwise ($x-z$) plane at $y=0.065\delta$ for flow over the rough surface. Time separation between fields is 1 ms. Contours of streamwise velocity fluctuations are presented in the background to highlight low-momentum (LMR) and high-momentum (HMR) regions.
Additional portions of subsequent velocity fields \((t + 2\Delta t, t + 3\Delta t, \ldots)\) stitched together using Taylor’s hypothesis to generate elongated spatial velocity field in \(x\) from time-resolved fields.

Figure 6.32: Schematic illustrating the use of Taylor’s hypothesis in conjunction with TR velocity fields to generate an elongated velocity field in the streamwise direction. Contours represent regions of negative streamwise velocity fluctuations \((u' < 0)\).

Figure 6.33: Comparison between the instantaneous advection velocity of LMRs, \(U_c\) (—), and their matching advection velocity, \(U_m\) (—). The mean advection velocity of LMRs, \(\overline{U}_c\) (—), is also presented for reference. All velocities are normalized with respect to the mean velocity of the ensemble \(\overline{U}_{y=0.065d}\).
Figure 6.34: Reconstructions of $\sim 10\delta$-long velocity fields using TR-PIV data. Contours represent regions of negative streamwise velocity fluctuations ($u' < 0$).
Figure 6.35: Reconstructed velocity field in the streamwise–spanwise plane at $y = 0.065\delta$ revealing an elongated low-momentum region (LMR) via gray-scale contours of negative streamwise velocity fluctuations. The zoomed-in view includes gray-scale contours of the LMR and every fourth velocity vector is overlaid as well. The inset also includes red and blue contours highlighting the clockwise and counter-clockwise vortices. Flow is upward.
Chapter 7

Summary, Conclusions and Future Work

Turbulent boundary layers in the presence of roughness have been the subject of significant scrutiny due to both their complexity and their ubiquitous occurrence in technological applications. Despite this extensive effort, important open questions remain. Amongst these questions is the impact of highly-irregular roughness, reminiscent of the surface roughness encountered in practice, on TBLs. The specific focus of the present effort was on clarifying the role of large, intermediate and small topographical scales of irregular roughness on ZPG TBLs with the hope of identifying the critical scales of a topography replicated from a turbine blade damaged by deposition of foreign materials that dominate the impact of this irregular surface on the flow.

To this end, low-order representations of the aforementioned highly-irregular surface topography were generated using singular value decomposition which provides an optimal set of spatial basis functions for reconstructing inhomogeneous signals (like this irregular surface topography). Truncating this basis-function representation at low mode numbers effectively low-pass filters the surface and thus discards smaller scales while retaining larger scales. Two low-order topographical representations were created: one embodying the first 5 of the 383 total basis functions which embodied 71% of the full-surface content and the other constructed from the first 16 modes of the decomposition and containing 95% of the full-surface content. Turbulent boundary layers over these low-order models and the original, full surface were documented experimentally using 2D-PIV measurements in the streamwise–wall-normal plane for developing and developed flow conditions and stereo-PIV measurements in a streamwise–spanwise plane deep within the roughness sublayer for developed flow. Smooth-wall measurements at a similar Re provided a basis of comparison for the rough-wall cases. These tests were meant to reveal how much topographical detail of the original surface must be included in a low-order representation in order for it to replicate the effect of the original surface on the flow under developing- and developed-flow scenarios. The developing-flow results, wherein the internal layer formed at the abrupt transition from smooth- to rough-wall conditions had yet to engulf the entire boundary layer, revealed that a model embodying the first 16 of 383 modes (4.2% of the total modes) accurately reproduced the flow characteristics of flow over the full surface, including the mean velocity profile, Reynolds normal and shear stresses, pdfs of RSS-producing events and quadrant contributions to the mean
RSS. In contrast, a 5-mode surface model failed to reproduce the flow characteristics of the full surface despite containing 71% of the full-surface topographical content. These results therefore highlight how the intermediate topographical scales present in the 16-mode model and absent from the 5-mode model play an important role in the generation of turbulence in the first stages of growth of the internal layer. In contrast, the small-scale topographical content excluded from both low-order models appears to play a negligible role in the turbulence development of the internal layer.

For the case of developed flow, wherein the internal roughness layer grew to engulf the entire boundary-layer thickness and the rough-wall TBLs reached a self-similar state, three different kinds of experiments were carried out. In the first set, 2D-PIV measurements were conducted in the streamwise–wall-normal plane. The 16-mode model was found to again accurately reproduce the characteristics of flow over the full surface except very close to the wall (the roughness sublayer). Moreover, while the 5-mode surface model failed to reproduce characteristics of the full-surface developing flow, this model performed nearly as well as the 16-mode model in reproducing the turbulence statistics (except the inner-scaled mean velocity profile) for the case of developed flow where the outer-layer flow over all three surfaces was found to be in accordance with Townsend’s wall similarity hypothesis. However, despite the existence of outer-layer similarity, the low-order models failed to accurately capture the bulk effect of the full surface on the flow, particularly in terms of the roughness function, $\Delta U^+$. This difference is more dramatic in the $M = 5$ model, in which $\Delta U^+$ is only 70% of the full-surface value, whereas the $M = 16$ model yielded a $\Delta U^+$ that was 96% of the full-surface result. This observation suggests that intermediate scales do play an important role in the effect of the surface on the flow, since adding the intermediate topographical scales embodied in modes 6 to 16 to the $M = 5$ model brings the effect of the surface model remarkably close to that of the full surface. Of interest, it was found that flow over the $M = 5$ model was transitionally-rough and, based on the analysis of Napoli et al. (2008), might be considered more akin to a “wavy” surface wherein the total drag is predominantly due viscous drag. In contrast, the flows over the $M = 16$ model and the full surface both resided in the full-rough regime, indicating that the total drag is dominated by form-drag effects due to flow separation downstream of dominant roughness features. Applying the analysis of Napoli et al. (2008) to the $M = 16$ model and the full surface supports these observations of form-drag-dominated flow in the roughness sublayer. Thus, the intermediate scales absent in the $M = 5$ model but present in the $M = 16$ model may play a defining role in initiating flow separation around the larger-scale roughness features, particularly in enhancing the jaggedness of these features. Finally, important differences were observed in the roughness sublayer, which is directly exposed to the action of the roughness features. Among these differences, it is important to note that both low-order models induced a weaker streamwise Reynolds normal stress than the full surface,
intensified strong sweep events, attenuated intense ejection events and reduced the streamwise coherence of the large-scale motions as inferred from $\rho_{uu}$.

To study the differences noted in the roughness sublayer from the 2D-PIV measurements in the streamwise–wall-normal plane, stereo-PIV experiments were conducted in a streamwise–spanwise plane deep within the roughness sublayer ($y = 0.047\delta$). Interestingly, these measurements revealed a wealth of information regarding roughness-induced effects that were not discernable from the wall-normal plane measurements. In particular, contour maps of ensemble-averaged streamwise velocity deficit revealed the tendency of the roughness used in this study to promote ‘channeling’ of the flow in the form of low- and high-momentum pathways that occurred so consistently in the instantaneous fields that they persisted through the ensemble-averaging performed. Similarly, enhanced turbulent and vortical activity was observed both between and along the spanwise boundaries of these streamwise-elongated pathways. Finally, observations of vortical activity via ensemble-averaged signed swirling strength revealed that wall-normal vortices of opposite sign tend to organize on either side of these dominant low-momentum pathways in a manner consistent with the collective induction of streamwise flow against the mean flow direction and therefore are likely the origination mechanism of the low-momentum pathways observed in the mean velocity defect results for the rough-wall flows. Taken together, these observations support the idea that these persistent low-momentum pathways could represent the statistical imprint of trains of hairpin vortex packets that are channeled along preferred paths over the roughness. Alternatively, these streamwise-elongated pathways could also be due, to some extent, to vortical structures shed from dominant roughness features and could therefore have very little streamwise coherence in the sense of the collectively-induced motions of hairpin vortex packets. However, the notion of the occurrence of streamwise-correlated motions along the low-momentum pathways is further supported by conditional averages of fluctuating velocity based on the presence of positive and negative wall-normal vortices which contained well-defined low-momentum events induced by these vortical motions that tend to occur spatially coincident to the central locations of the low-momentum pathways noted in the mean velocity defect results. Inhomogeneous two-point correlation coefficients of velocity also support this notion of preferred channeling of large-scale vortex packets rather than random, smaller-scale vortices shed from the roughness that tend to advect along the preferred pathways noted in the mean streamwise velocity.

With regard to the fidelity of the low-order models in reproducing the characteristics of flow over the full surface, both the $M = 5$ and $M = 16$ topographical models yield statistical and structural characteristics that are qualitatively consistent with the full-surface flow at this near-wall measurement plane. However, significant quantitative differences were noted when comparing the $M = 5$ model results with those of the full surface, including different spanwise positions of the fixed low- and high-momentum pathways identified.
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the importance of the intermediate topographical scales in setting the flow conditions within the roughness
sublayer, not only in a statistical sense but also in a structural sense. Connecting these observations with
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in the case of the $M = 5$ model flow further manifests itself to yield the stark differences between its flow
behavior in the roughness sublayer compared to the other two cases wherein flow separation likely plays an
important role based on the analysis of Napoli et al. (2008).

From a practical viewpoint, these observations reported herein support the notion that an $FSC$ of 95%
yielded a low-order representation of the present full surface that quite closely reproduced the detailed
characteristics of the flow over the full surface. This conclusion is consistent with the recent work of Johnson
and Christensen (2009) who used a similar methodology to study low-order models of a turbine blade
damaged by spallation in a short-fetch arrangement in turbulent channel flow. They observed that a low-
order representation containing 95% of the full-surface content yielded turbulence statistics consistent with
those of flow over the full surface except in the immediate vicinity of the surface. Thus, while the absolute
number of modes required to reach an $FSC$ of 95% will certainly vary from surface to surface [16 of the 383
total modes for the deposition of foreign materials surface studied herein compared to 20 of the 216 total
modes for the spallation surface studied by Johnson and Christensen (2009)], it appears that this approximate
level of full-surface content provides a metric for the generation of low-order representations that produce
a similar impact on wall turbulence as the more complex full surface. Such a result might have important
implications in numerical simulations of wall turbulence in the presence of practical surface roughness, for
example, whereby a simulation could utilize a surface of reduced complexity, greatly reducing the gridding
requirements at the surface, while still yielding a simulation that captures the salient details of the flow over
the more complex, original surface. Further study of a range of realistic rough surfaces is needed to critically
assess this possibility. In addition, since no models with $FSC$ in the interval $71\% < FSC < 95\%$ were tested,
the possibility that a low-order model within this range might yield a similar reliability in reproducing the characteristics of flow over the full surface as the $M = 16$ model should not be discarded. With this in mind, $FSC = 95\%$ should be considered, for now, an upper limit for generating a reliable low-order model of highly-irregular roughness.

Finally, though the existence of large- and very-large-scale motions (LSM and VLSM) in smooth-wall turbulence is well documented (Kim and Adrian, 1999; Adrian et al., 2000b; Ganapathisubramani et al., 2003; Guala et al., 2006; Balakumar and Adrian, 2007; Hutchins and Marusic, 2007), their presence in rough-wall flows has remained an open question until the findings revealed in the present work. Specifically, elongated streamwise fields of view were reconstructed from TR-PIV data acquired in a wall-parallel plane within the roughness sublayer ($y = 0.065\delta$) for flow over the full surface. Using pattern reconstruction via Taylor’s frozen-field hypothesis, several $\sim 10\delta$-long streamwise fields of view were reconstructed, each of which contained low-momentum regions that extended nearly the entire field of view but meandered significantly in the spanwise direction. These observations are consistent with recent observations in smooth-wall flow (Hutchins and Marusic, 2007). Thus, despite the direct impact of roughness on the flow within the roughness sublayer, coherent motions on the order of $10\delta$ with similar characteristics to those noted in the near-wall region of smooth-wall turbulence still persist. However, further study of such motions in the presence of roughness is needed to assess the energy and Reynolds stress that they carry to determine how important they might be to the overall evolution of rough-wall turbulence.

While the present effort indicates that outer-layer similarity holds for highly irregular rough surfaces so long as the boundary layer remains thick compared with the characteristic roughness height, to date there remains conflicting observations regarding the precise conditions for which outer-layer similarity breaks down. The pathway to resolving this conflict is obscured by the fact that there is no consensus regarding how to define the characteristic length scale of a given surface. This issue is further complicated when considering surfaces that contain a broad range of topographical scales as is the case of the roughness encountered in practice (see: Jimenez, 2004, for a complete discussion in the subject). So far, the results of Nikuradse (1950) have been the main benchmark for characterizing rough-wall flows based on equivalent sand-grain height, though only with partial success, since recent findings revealed that correlations based on single-scale roughness fail to capture coefficients for skin friction and heat transfer for flow over realistic roughness patterns (see: Bons et al., 2001; Bons, 2002a,b; Bons and McClain, 2004). The effective slope ($ES$) defined by Napoli et al. (2008) offers an alternative to resolve this conflict, and it has proven effective in characterizing more complex roughness in which wake separation around roughness elements is not the dominant drag component. In fact, as was observed in the present work, $ES$ was effective in capturing the transitionally-
rough behavior of the $M = 5$ model flow. However, in the case of roughness in which separation dominates the total drag, $ES$ is not effective at characterizing the roughness function. In this regime, the roughness function depends mainly on the characteristic roughness height, and therefore, its characterization based purely on knowledge of the geometry of the surface (or possibly the equivalent sand-grain height) is still elusive for highly irregular roughness. Therefore, for future work, it will be necessary to devote some effort toward identifying an appropriate geometrical metric of roughness that consolidates that flow characteristics over a wide range of roughness types. The findings of Napoli et al. (2008) might encourage one to explore a characteristic height based on some integral scale of the roughness.

As discussed above, the present work as well as that of Johnson and Christensen (2009) supports the possibility of using the $FSC$ as an effective means of, at least approximately, predicting the topographical features that are relevant in the action of the surface on TBL. Aiding in this assessment is the fact that this metric provided a similar threshold despite the strong differences in the surfaces considered in these two efforts. However, there is certainly many issues that require further attention before declaring the $FSC$ as a general metric for assessing the quality of a low-order surface representation. First, all the studies involving the $FSC$ characterization to date have been conducted at moderate Reynolds numbers. Under such circumstances, the range of relevant length scales of the flow will change as the Re increases which could impact the effect of the small topographical scales of the surface as they become similar in size to the smaller scales of the flow. In this regard, as the Reynolds number increases, some of the ‘screened’ topographical scales of the surface may become active and actively participate in the production of turbulence. If that is the case, the $FSC$ would likely need to be larger than 95% to capture the effect of the surface on the flow. With this in mind, it is necessary to perform studies of the effect of the Reynolds number on $FSC$ to help clarify this potential issue. On the other hand, going back to the problem of characteristic heights, note that the $FSC$ is basically an optimization based on roughness height, and therefore it seems to offer a set of characteristic heights that describe the effect of the roughness on the flow. With this in mind, the $FSC$ seems to offer a golden opportunity to find what is in fact the characteristic height of an irregular roughness from geometrical properties alone. In other words, the $FSC$ filters out those scales that are irrelevant to the effect of the surface on the flow, and retains those that impose a significant effect. Therefore, it might be expected that the characteristic height of a given highly irregular surface is some statistical measure of the scales retained by the $FSC$. However, since the flow separation behavior around dominant roughness elements is strongly dependent on slope, as reflected in the effective slope ($ES$) criterion of Napoli et al. (2008), it would also be of interest to try an approach analogous to the $FSC$ but optimizing the slope instead of on the roughness elevation as was done herein. Doing so might provide a means of connecting these two
ideas and could aid in resolving the disconnect between the $ES$-dependent regime and the height-dependent regime.
Appendix A

Uncertainty Analysis

This Appendix is devoted to reporting a detailed uncertainty analysis for the measurements discussed herein. First, expressions for the magnitude of the random errors present in our system are derived. Then, the propagation of these errors through the measurement system employed is considered as well as how these errors eventually propagate into the turbulence statistics presented in Chapters 5 and 6. Representative values for the uncertainties present in these measurements are then documented. However, since the optical arrangement for 2D-PIV differs considerably from that of stereo-PIV, independent discussions are included for the uncertainty analysis of each measurement system.

A.1 Random error

Two sources of random error are considered in the analysis presented. The first of them is due to the inherent “random” fluctuations present in turbulent flows. The other is the result of inaccuracies induced by the digital sensors used in PIV. This analysis will begin with the first source of random error.

It is widely known that turbulent fluctuations delay convergence of statistics until “sufficient” samples are taken. For this reason, this kind of error is commonly termed the sampling error. Following Roberts (1979), the sampling error in estimating the mean value of a random variable is directly proportional to the population variability and inversely proportional to the square root of the random sample size. Therefore, the sampling error (or standard error of the mean) may be expressed as the ratio of the standard deviation of the population and the square root of the sample size. As the standard deviation of the population is rarely accessible, a more feasible approach to estimating the sampling error can be expressed as

\[ \delta_s(\vartheta) = \frac{S(\vartheta)}{\sqrt{n}}, \]

where \( \vartheta \) is a given random variable, \( \delta_s(\vartheta) \) represents the estimator of the standard error of the mean, \( n \) is the
sample size and $S(\vartheta)$ is the unbiased estimator of the standard deviation of the sample data defined as

$$S(\vartheta) = \sqrt{\frac{1}{n - 1} \sum_{j=1}^{n} (\vartheta_j - \bar{\vartheta})^2}. \quad (A.2)$$

If eq. (A.2) is translated to velocity, which is the random variable of interest in the present measurement, one obtains

$$S(u) = \sqrt{\frac{1}{n - 1} \sum_{j=1}^{n} (u_j - U)^2} = \sqrt{\frac{1}{n - 1} \sum_{j=1}^{n} (u_j')^2} \quad (A.3)$$

where Reynolds decomposition of the total velocity, $u$, into mean, $U$, and turbulent, $u'$, components as $u = u' - U$ is employed. Therefore, returning to eq. (A.1), the sampling error of a turbulent velocity signal can be expressed as

$$\delta_s(U) = \frac{1}{\sqrt{n}} \left( \frac{n}{n - 1} \langle u'^2 \rangle^{1/2} \right) = \frac{\langle u'^2 \rangle^{1/2}}{\sqrt{n - 1}}. \quad (A.4)$$

Thus, eq. (A.4) reveals that the sampling error of a turbulent velocity field depends on the turbulence intensity through $\langle u'^2 \rangle^{1/2}$ and the sample size, $n$. Since the turbulence intensity varies with the wall-normal coordinate $y$ in a turbulent boundary layer, the sampling error varies accordingly. In consequence, the sampling error in the experiments conducted in the streamwise–wall-normal ($x - y$) plane will depend on $y$. On the other hand, since the experiments in the streamwise–spanwise ($x - z$) plane were conducted at a fixed wall-normal location, one would expect the turbulence intensity to be constant across the field of view under smooth-wall conditions. However, since the roughness in question introduces spatial heterogeneities into these statistics within the $x - z$ field of view, the sampling error for the rough-wall measurements is expected to vary spatially in the $x - z$ plane. Nevertheless, for simplicity, it is proposed to identify representative (constant) values of uncertainty for the turbulence quantities derived from both the $x - y$ and $x - z$ measurements. To this end, the maximum value of the turbulence intensity is employed (rather than, for example, its average) to ascertain the upper bound on the sampling error and thus giving a conservative measure of uncertainty. Values for the maximum and average turbulence intensity in $x - y$ and $x - z$ plane experiments respectively, can easily be extracted from the profiles and contour plots of Reynolds normal stresses reported in Chapters 5 and 6. Note that the turbulence intensity of each velocity component is simply the square root of the Reynolds normal stress associated with a particular velocity component.

Two additional sources of uncertainty in PIV measurements include the peak-locking effect and the un-
uncertainty in the estimation of the sub-pixel particle displacement. Both of these uncertainties are intimately related to the particle-image diameter (Westerweel, 1997; Christensen, 2004). In this regard, peak-locking may play a role when the particle-image size is less than approximately two pixels. In such instances, estimation of the sub-pixel displacement of particles is rendered inaccurate. Apart from this potential bias, the random error in the estimation of the sub-pixel particle displacement is approximately 5% of the particle-image diameter (Prasad et al., 1992). Note that the particle-image diameter in the present experiments was typically 2–3 pixels, rendering peak-locking uncertainties negligible. Therefore, the dominant PIV uncertainty to be considered is due to randomness in the estimation of the sub-pixel displacement, whose upper bound would be approximately \( \delta_{sp}(\Delta \xi)_{max} = 0.15 \) pixels based on the above-mentioned dependence on particle-image diameter (taken here as 3 pixels), where \( \Delta \xi \) is the displacement from time 1 to time 2 derived from interrogation of the PIV images. Since this uncertainty is random in nature, it should decrease with the number of samples in the same manner as the sampling error [see eq. (A.1)], giving

\[
\delta_{sp}(\Delta \xi) = \frac{\delta_{sp}(\Delta \xi)_{max}}{\sqrt{n}}. 
\]

(A.5)

Once the displacement \( \Delta \xi \) is translated into velocity, the sub-pixel accuracy is expressed as

\[
\delta_{sp}(U) = \frac{\delta_{sp}(u)_{max}}{\sqrt{n}}. 
\]

(A.6)

Equations (A.4) and (A.6) thus provide a means for estimating the uncertainty in first-order turbulence statistics. That is, if we combine these two equations (using an odd preserving scheme as described in Moffat, 1988), we may use the result to calculate the random error in the estimation of mean and instantaneous velocities. Since the numerators of eq. (A.4) and (A.6) express the uncertainties in determining the true value of velocity in each instantaneous realization, we simply need to consider a single data point \( n = 1 \) to assess the uncertainty on an instantaneous basis. This particular calculation will be important for the time-resolved PIV experiments. Since the objective of those experiments is to reconstruct a large field of view based on instantaneous realizations, each grid point will be composed of a single data point. Nevertheless, it should be pointed out that, to remove the singularity from the estimation of instantaneous random error, one must relax the unbiased nature of the sampling error and use \( n \) instead of \( n - 1 \) as the denominator in eq. (A.4) to calculate the uncertainty of instantaneous realizations. Thus, the total random error is given by

\[
\delta(U) = \sqrt{[\delta_{sp}(U)]^2 + [\delta_e(U)]^2},
\]

(A.7)
which, if one considers the entire sample size, the random error in the mean is

$$\delta (U) = \sqrt{\left(\frac{\langle u''^2 \rangle^{1/2}}{\sqrt{n-1}}\right)^2 + \left(\frac{\delta_{sp} (u)_{max}}{\sqrt{n}}\right)^2}. \quad (A.8)$$

In contrast, if one considers a single data point, random error of an instantaneous realization is

$$\delta (u) = \sqrt{\left(\frac{\langle u''^2 \rangle^{1/2}}{\sqrt{n}}\right)^2 + \left[\delta_{sp} (u)_{max}\right]^2}. \quad (A.9)$$

Before calculating this uncertainty, the sample size, which is dependent upon the averaging method employed, must be determined. For the \(x - y\) experiments, the set of instantaneous realizations was first ensemble-averaged and then line-averaged in the streamwise direction, yielding profiles of turbulence statistics as a function of wall-normal direction, \(y\). Consequently, the sample size \(n\) for each of these experiments is the product between the number of instantaneous realizations (4500) and the number of grid points in the streamwise direction (343 points for \(M = 16\) and full surface and 235 for \(M = 5\)). In the case of the \(x - z\) experiments, since they were carried out inside the roughness sublayer where the rough-wall flow display a significant degree of heterogeneity, only ensemble averaging was performed to retain dependence of the statistics on \(x\) and \(z\). In consequence, the sample size for each grid point in the \(x - z\) measurements simply corresponds to the total number of instantaneous realizations acquired per experiment (4500).

Finally, the random error cannot be assessed until the uncertainty due to sub-pixel accuracy is converted from displacement to velocity units. This conversion is not the same for 2D-PIV and stereo-PIV, so this conversion is discussed separately below.

### A.1.1 Errors in 2D-PIV experiments

It should be emphasized that 2D-PIV does not measure velocity directly. Instead, it measures particle displacements over a fixed time interval and then the fluid velocity is estimating using a first-order estimate of the form

$$u = \frac{M}{\Delta t} \Delta \xi, \quad (A.10)$$

where \(\Delta \xi\) represents a displacement along a given coordinate axis \(\xi\), \(M\) is the magnification, and \(\Delta t\) is the time delay between the PIV image frames. Including the uncertainty associated with each variable in eq. (A.10), one obtains

$$u + \delta(u) = \frac{M}{\Delta t} [\Delta \xi + \delta(\Delta \xi)]$$
\[ M \frac{\Delta t}{\Delta t} \Delta \xi + M \frac{\Delta t}{\Delta t} \delta (\Delta \xi). \] (A.11)

After substituting eq. (A.10) into this result, it is found that the uncertainty transforms in the same way as the particle displacement:

\[ \delta (u) = M \frac{\Delta t}{\Delta t} \delta (\Delta \xi). \] (A.12)

This equation provides a means of calculating the uncertainty in the estimation of sub-pixel displacements in velocity units for 2D-PIV experiments in eqs. (A.8) and (A.9).

A.1.2 Errors in Stereo-PIV experiments

The geometrical transformations from machine displacements to physical displacements in stereo-PIV systems can be accomplished in two different configurations: the translation system and the angular displacement system (originally called Scheimpflug stereocamera by Prasad and Jensen, 1995), with the latter being the most widely used stereo PIV method employed because of its flexibility and accuracy. However, due to the complexity of its geometrical transformations, most uncertainty analyses in stereo-PIV systems consider the transformations of the translation system firstly presented by Lawson and Wu (1997) (see for example Willert, 1997; Prasad, 2000; van Doorne and Westerweel, 2007; Herpin et al., 2008). This estimation is possible since Zang and Prasad (1997) showed that the ratio between the out-of-plane error to the in-plane error follows an equivalent behavior in both systems. Therefore, the uncertainty analysis presented herein will be based on the translation system.

A general diagram for a stereo-PIV system is shown in figure A.1. Note that the system of coordinates in the diagram corresponds to the present \( x - z \) measurements (see figure 4.9 for reference). Based on this diagram, one can use geometric arguments to find the transformation of displacements from the machine world to the real world. This transformation is given by (Lawson and Wu, 1997)

\[ \Delta x = \frac{b_2 \Delta X_1 - b_1 \Delta X_2}{a_1 b_2 - a_2 b_1}, \] (A.13)

\[ \Delta z = -\frac{M_1 M_2}{M_1 + M_2} \{ \Delta Z_1 + \Delta Z_2 + z \left[ \left( \frac{1}{M_1 d_{x_1}} + \frac{1}{M_2 d_{x_2}} \right) \Delta y \cos \alpha \right. \}
\]

\[ \left. - \left( \frac{1}{M_2 d_{x_2}} - \frac{1}{M_1 d_{x_1}} \right) \Delta x \sin \alpha \} \}, \] (A.14)

\[ \Delta y = \frac{a_2 \Delta X_1 - a_1 \Delta X_2}{a_2 b_1 - a_1 b_2}, \] (A.15)
where

\[
\begin{align*}
  a_1 &= -\frac{\cos \alpha}{M_1} + \left( \frac{\sin \alpha}{M_1 d_{o1}} \right) \left[ y \sin \alpha - (x - h) \cos \alpha \right] \\
  a_2 &= -\frac{\cos \alpha}{M_2} + \left( \frac{\sin \alpha}{M_2 d_{o2}} \right) \left[ y \sin \alpha + (x + h) \cos \alpha \right] \\
  b_1 &= \frac{\sin \alpha}{M_1} + \left( \frac{\cos \alpha}{M_1 d_{o1}} \right) \left[ y \sin \alpha - (x - h) \cos \alpha \right] \\
  b_2 &= -\frac{\sin \alpha}{M_2} - \left( \frac{\cos \alpha}{M_2 d_{o2}} \right) \left[ y \sin \alpha + (x + h) \cos \alpha \right]
\end{align*}
\]

Here, \((\Delta x, \Delta y, \Delta z)\) are the displacements of a group of particles in the real-world coordinates, \((\Delta X_1, \Delta Z_1)\) and \((\Delta X_2, \Delta Z_2)\) are the projections of the displacements in the image planes of the PIV cameras (with the sub-indices corresponding to cameras 1 and 2 respectively), \(M_1\) and \(M_2\) are the magnifications of each camera (here defined as the ratio of the object-size to image-size \((M = H_{\text{obj}}/H_{\text{img}})\), \(d_{o1}\) and \(d_{o2}\) are the
object distances for each camera, and \( \alpha_1 \) and \( \alpha_2 \) are the angles between the optical axes of the cameras and the normal to the object plane. Since the cameras in the present experiments were located symmetrically with respect to the normal to the object plane, \( \alpha_1 = -\alpha \) and \( \alpha_2 = +\alpha \). Finally, \( h_1 \) and \( h_2 \) are the distances from the symmetry axis of the stereo-PIV system to the intersection of the optical axis of each camera with the object plane. However, since both cameras should be observing the same object, one can assume that their optical axes intersect the object plane at the same point, giving \( h_1 = h_2 = h = 0 \).

To study how uncertainties propagate from displacements in the machine world to displacements in the physical world, the root-sum square of the errors method is employed (Moffat, 1988). Consider a random variable \( \vartheta \) that depends on \( m \) random variables \( \vartheta(\gamma_1, \gamma_2, \ldots, \gamma_j, \ldots, \gamma_m) \). The total uncertainty in \( \vartheta \) introduced by the combined effect of the independent variables \( \gamma_j \) is given by

\[
\delta(\vartheta) = \left[ \sum_{j=1}^{m} \left( \frac{\partial \vartheta}{\partial \gamma_j} \delta(\gamma_j) \right)^2 \right]^{1/2}.
\] (A.17)

To estimate the uncertainty in stereo-PIV displacements, one may assume that the geometrical system is purely deterministic, which means that the only sources of error are the uncertainties introduced by the displacements in the image planes \((\Delta X_1, \Delta Z_1)\) and \((\Delta X_2, \Delta Z_2)\). The analysis can be simplified a bit more by considering that these uncertainties are of the same magnitude \( \delta(\Delta X_j) = \delta(\Delta Z_j) = \delta(\Delta \xi) \), and primarily due to the uncertainty in the estimation of sub-pixel displacements that was estimated in the previous section \( (\delta(\Delta \xi) = 0.15 \text{ pixel}) \). In consequence, after applying eq. (A.17) to eqs. (A.13)–(A.15), and taking into account that \( h = 0 \), the uncertainty in the displacements in physical coordinates is

\[
\begin{align*}
\delta_{sp}(\Delta x) &= \frac{\sqrt{b_1^2 + b_2^2}}{|a_1b_2 - a_2b_1|} \delta(\Delta \xi), \\
\delta_{sp}(\Delta y) &= \frac{\sqrt{a_1^2 + a_2^2}}{|a_1b_2 - a_2b_1|} \delta(\Delta \xi), \\
\delta_{sp}(\Delta z) &= \left( \sqrt{c_1^2 + c_2^2 + c_3^2} \right) \delta(\Delta \xi).
\end{align*}
\] (A.18)
where

\[ a_1 = -\frac{\cos \alpha}{M_1} + \left( \frac{\sin \alpha}{M_1 d_{x1}} \right) \left[ z \sin \alpha - x \cos \alpha \right] \]

\[ a_2 = -\frac{\cos \alpha}{M_2} + \left( \frac{\sin \alpha}{M_2 d_{x2}} \right) \left[ z \sin \alpha + x \cos \alpha \right] \]

\[ b_1 = \frac{\sin \alpha}{M_1} + \left( \frac{\cos \alpha}{M_1 d_{x1}} \right) \left[ z \sin \alpha - x \cos \alpha \right] \]

\[ b_2 = -\frac{\sin \alpha}{M_2} - \left( \frac{\cos \alpha}{M_2 d_{x2}} \right) \left[ z \sin \alpha + x \cos \alpha \right] \]

\[ c_1 = -\frac{2M_1 M_2}{M_1 + M_2} \]

\[ c_2 = -z \cos \alpha \frac{M_1 M_2}{M_1 + M_2} \left[ \left( \frac{1}{M_1 d_{x1}} + \frac{1}{M_2 d_{x2}} \right) \frac{\sqrt{a_1^2 + a_2^2}}{a_1 b_2 - a_2 b_1} \right] \]

\[ c_3 = -z \sin \alpha \frac{M_1 M_2}{M_1 + M_2} \left[ \left( \frac{1}{M_1 d_{x1}} - \frac{1}{M_2 d_{x2}} \right) \frac{\sqrt{b_1^2 + b_2^2}}{a_1 b_2 - a_2 b_1} \right] \]

Thus, the absolute uncertainties of stereo-PIV experiments in physical coordinates can be calculating using eqs. (A.18) and (A.19). These equations yield a three-dimensional field for the uncertainty, but since the interest herein is in estimating its magnitude rather than its spatial distribution, only its average value is reported. It should be mentioned though that the variability of the error around this average is ±2%. Note further that eqs. (A.18) and (A.19) give the uncertainties in dimensions of displacement (physical units), but these uncertainties need to be expressed in dimensions of velocity to add them the sampling error [recall eq. (A.7)]. Therefore, the values obtained for displacement uncertainties can be divided by the PIV time-delay (\(\Delta t = 47 \mu s\) for all experiments) as

\[ \delta_{sp} (u)_{max} = \frac{\delta_{sp} (\Delta x)}{\Delta t} \]

\[ \delta_{sp} (v)_{max} = \frac{\delta_{sp} (\Delta y)}{\Delta t} \]

\[ \delta_{sp} (w)_{max} = \frac{\delta_{sp} (\Delta z)}{\Delta t} \]

\[ \{ \text{.} \]  

(A.20)

### A.2 Propagation of uncertainties in the turbulence statistics

Now that a clear notion of how to estimate the magnitude of the random error for each one of the experiments has been developed, it must be determined how these uncertainties propagate into the turbulence statistics
presented herein. Note that eq. (A.8) already expresses the magnitude of the uncertainty in the estimation of mean velocities. Hence, uncertainty propagation into the other turbulence statistics presented, such as Reynolds stresses and two-point correlations, is considered. To do so, the theory of error analysis as described by Moffat (1988) will be employed. The general formulation for estimating the ensemble average of the product of two random variables is given by

$$\langle \vartheta \gamma \rangle = \frac{1}{n} \sum_{j=1}^{n} (\vartheta_j \gamma_j). \quad (A.21)$$

After applying eq. (A.17) to this ensemble average, the following result is obtained:

$$\delta (\langle \vartheta \gamma \rangle) = \left\{ \delta (\vartheta) \left( \frac{1}{n} \sum_{j=1}^{n} \gamma_j \right) \right\}^2 + \left\{ \delta (\gamma) \left( \frac{1}{n} \sum_{j=1}^{n} \vartheta_j \right) \right\}^2 \quad (A.22)$$

Note that the quantities between parenthesis represent averages of each one of the random variables of the product. Note further that in the study of turbulence, the ensemble average from which this result is obtained is typically used for determining the Reynolds stresses (and two-point statistics). Thus, the random variables of interest are velocity fluctuations in the present application. In consequence, since the average of velocity fluctuations is zero by definition, the quantities between parenthesis in eq. (A.22) are also zero, rendering the random error in the estimation of Reynolds stresses (and two-point statistics) identically zero. Even though this result suggests that there is no uncertainty in the estimation of Reynolds stresses (and two-point statistics), it should be taken into account that this result was obtained under the implicit condition of a large sample size \((n \gg 1)\). Therefore, if the sample size is large, the random error in the estimation of Reynolds stresses (and two-point statistics) tends to zero. The question that arises, however, is: how much has this uncertainty decayed for the sample size of the current experiments? The answer to this question can be determined by returning to the issue of sampling. In other words, the average value of the velocity fluctuations converges to zero according to eq. (A.1). Hence, one can reformulate eq. (A.22) by substituting the average of the fluctuating velocities by their sampling error, yielding

$$\delta (\langle \vartheta \gamma \rangle) = \left\{ \left[ \delta (\vartheta) \delta_s (\gamma) \right]^2 + [\delta (\gamma) \delta_s (\vartheta)]^2 \right\}^{1/2}, \quad (A.23)$$

which represents the uncertainty in the Reynolds stresses (and two-point statistics).

Finally, since all statistical descriptors are normalized for proper comparison, it is necessary to consider the uncertainty introduced in the turbulence statistics due to the uncertainty in the determination of the
given normalization factor. Consider first the ensemble average of a random variable \( \langle \vartheta \rangle \), and further consider a normalization factor \( \varrho \). The uncertainty introduced in the normalized statistical descriptor \( \langle \vartheta \rangle / \varrho \) due to the combination of the uncertainties of \( \langle \vartheta \rangle \) and \( \varrho \), is given by

\[
\delta \left( \frac{\langle \vartheta \rangle}{\varrho} \right) = \left\{ \frac{\partial}{\partial \langle \vartheta \rangle} \delta \langle \vartheta \rangle \right\}^2 + \left\{ \frac{\partial}{\partial \varrho} \delta \langle \vartheta \rangle \right\}^2 \right\}^{1/2},
\]

\[
\delta \left( \frac{\langle \vartheta \rangle}{\varrho} \right) = \frac{1}{\varrho} \left\{ \left[ \delta \langle \langle \vartheta \rangle \rangle \right]^2 + \left[ \langle \vartheta \rangle \delta \langle \vartheta \rangle \right]^2 \right\}^{1/2}. \tag{A.24}
\]

Now consider the ensemble average of the product of two random variables \( \langle \vartheta \varrho \rangle \) with another normalization constant \( \varpi \) introduced to have a more general analysis. The uncertainty introduced in the normalized statistical descriptor \( \langle \vartheta \varrho \rangle / \varrho \varpi \) due to the combination of the uncertainties in \( \langle \vartheta \rangle \), \( \varrho \), and \( \varpi \) is

\[
\delta \left( \frac{\langle \vartheta \rangle}{\varrho \varpi} \right) = \left\{ \frac{\partial}{\partial \langle \vartheta \rangle} \delta \langle \vartheta \rangle \right\}^2 + \left\{ \frac{\partial}{\partial \varrho} \delta \langle \vartheta \rangle \right\}^2 + \left\{ \frac{\partial}{\partial \varpi} \delta \langle \vartheta \rangle \right\}^2 \right\}^{1/2},
\]

\[
\delta \left( \frac{\langle \vartheta \rangle}{\varrho \varpi} \right) = \frac{1}{\varrho \varpi} \left\{ \left[ \delta \langle \langle \vartheta \rangle \rangle \right]^2 + \left[ \langle \vartheta \rangle \delta \langle \vartheta \rangle \right]^2 + \left[ \langle \vartheta \rangle \delta \langle \vartheta \rangle \right]^2 \right\}^{1/2}. \tag{A.25}
\]

The normalization factors employed herein are the free-stream velocity \( (U_c) \) for developing flow, the area mean velocity \( (\overline{U}) \) for the TR-PIV measurements, the friction velocity \( (u_f) \) for single-point statistics in developed flow, and the RMS velocity fluctuations \( (\sigma_u, \sigma_v, \sigma_w) \) for two-point statistics in developed flow. The uncertainty in the determination of \( U_c \) can be estimated with eq. (A.8) since it is derived directly from 2D-PIV measurements. On the other hand, according to Flack et al. (2005); Schultz and Flack (2005), \( u_f \) has an uncertainty of approximately \( \varepsilon(u_f) = 4 - 6\% \) when estimated using the constant stress method (as is done herein). Finally, the uncertainty in \( \sigma_u, \sigma_v, \) and \( \sigma_w \), can be found using a sensitivity analysis:

\[
\delta \langle \vartheta \rangle = \delta \left( \sqrt{\langle \vartheta^2 \rangle} \right) = \left\{ \left( \frac{\partial}{\partial \langle \vartheta \rangle} \left( \frac{1}{n} \sum_{j=1}^{n} \vartheta_j^2 \right) \right)^{1/2} \right\}^{1/2},
\]

\[
\delta \langle \sigma_\vartheta \rangle = \left( \frac{1}{n} \sum_{j=1}^{n} \vartheta_j^2 \right)^{-1/2} \left\{ \delta \langle \vartheta \rangle \left( \frac{1}{n} \sum_{j=1}^{n} \vartheta_j \right)^2 \right\}^{1/2}. \tag{A.26}
\]
The term inside curly brackets is analogous to eq. (A.22). Therefore, using arguments related to sampling error, one can simplify this term to achieve a result analogous to eq. (A.23). On the other hand, the term inside parenthesis is the ensemble average of the square of a random variable. Combining all of these simplifications, the uncertainty in the RMS of \( \vartheta \) is

\[
\delta (\sigma_\vartheta) = \frac{1}{\sqrt{\langle \vartheta^2 \rangle}} \left\{ [\delta(\vartheta)\delta_s(\vartheta)]^2 \right\}^{1/2} = \frac{\delta(\langle \vartheta^2 \rangle)}{\sqrt{\langle \vartheta^2 \rangle}}.
\]  

(A.27)

Finally, to compare the different uncertainties in a relative percentage sense, these uncertainties can be normalized by a characteristic value of the random variable of interest as

\[
\varepsilon(\vartheta) = \frac{\delta(\vartheta)}{\vartheta_c} \times 100.
\]  

(A.28)

The characteristic value, \( \vartheta_c \), is typically the mean of the random variable. In the present experiments, however, the only velocity component with a non-zero mean across the boundary layer is the streamwise velocity, though the wall-normal and spanwise velocities present a non-zero, yet small, mean in the rough-wall flows for the \( x-z \) measurements, where the surface defects induce strong non-homogeneities in the flow. Rigorously, since the ensemble average of these two velocity components is inhomogeneous, the percentage uncertainty varies across the domain of study. Nevertheless, for practical purposes it is more convenient to determine a single characteristic value for the uncertainty. In consequence, while the mean value of the streamwise velocity will be used to find its percentage uncertainty in the \( x-z \) experiments, the standard deviation of the mean \( (\sigma_V \text{ and } \sigma_W) \) will instead be used to find the percentage uncertainty of the other velocity components. The case of the \( x-y \) experiments is much more simple. Since these experiments were devoted primarily to study outer-layer similarity, and thus the spanwise and wall-normal velocity components are nearly zero in this region, the percentage uncertainty must only be determined for the streamwise velocity. The obvious choice of characteristic value \( \vartheta_c \) in this case is the free-stream velocity, \( U_c \). Finally, in the case of the Reynolds stresses and two-point correlations, \( \vartheta_c \) is defined as the product of the standard deviations of the velocity components of interest (i.e. \( \sigma_u^2 \) for \( \varepsilon((u')^2) \) and \( \varepsilon(\rho_{uu}) \), \( \sigma_u \sigma_v \) for \( \varepsilon((u'v') \) and \( \varepsilon(\rho_{uv}) \) ...). It should be taken into account that since eq. (A.28) will be used to calculate the percentage uncertainty of normalized quantities, the characteristic value \( \vartheta_c \) should be normalized accordingly. As such, \( \vartheta_c \) will be normalized by the free-stream velocity \( U_c \) in the \( x-y \) plane developing flow data, the area-mean velocity \( \langle \overline{U} \rangle \) in time-resolved experiments, and the friction velocity \( u_\tau \) for everything else.

Tables A.1 and A.2 present all relevant experimental parameters and velocity scalings that are needed for the uncertainty calculations detailed above. Tables A.3–A.5 report the basic uncertainties that propagate...
Table A.1: Relevant experimental parameters.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$M_1$ (µm/pixel)</th>
<th>$M_2$ (µm/pixel)</th>
<th>$\Delta t$ (µs)</th>
<th>$d_{o1}$ (mm)</th>
<th>$d_{o2}$ (mm)</th>
<th>$\alpha$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>42.24</td>
<td>—</td>
<td>35</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>41.97</td>
<td>—</td>
<td>35</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>41.96</td>
<td>—</td>
<td>35</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>45.42</td>
<td>—</td>
<td>35</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (TR-PIV)</td>
<td>Full</td>
<td>54.40</td>
<td>—</td>
<td>40</td>
<td>—</td>
<td>—</td>
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</tr>
</tbody>
</table>

Table A.2: Relevant velocity scalings.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$u_\tau$ (m/s)</th>
<th>$U_e$ (m/s)</th>
<th>$\overline{U}$ (m/s)</th>
<th>$\sigma_V$ (m/s)</th>
<th>$\sigma_W$ (m/s)</th>
<th>$(\sigma_{u_e})_e$ (m/s)</th>
<th>$(\sigma_{v_e})_e$ (m/s)</th>
<th>$(\sigma_{w_e})_e$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>0.79</td>
<td>17.15</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.70</td>
<td>0.90</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>0.79</td>
<td>17.19</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.66</td>
<td>0.91</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.72</td>
<td>17.34</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.52</td>
<td>0.83</td>
<td>—</td>
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<tr>
<td></td>
<td>Smooth</td>
<td>0.54</td>
<td>16.77</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1.24</td>
<td>0.62</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (TR-PIV)</td>
<td>Full</td>
<td>—</td>
<td>—</td>
<td>9.24</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
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<td>—</td>
<td>9.26</td>
<td>0.88</td>
<td>1.07</td>
<td>1.63</td>
<td>0.88</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.72</td>
<td>—</td>
<td>10.43</td>
<td>0.78</td>
<td>0.94</td>
<td>1.49</td>
<td>0.78</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
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<td>—</td>
<td>10.02</td>
<td>0.72</td>
<td>0.74</td>
<td>1.22</td>
<td>0.72</td>
<td>0.74</td>
</tr>
</tbody>
</table>

to the turbulence statistics according to the relations given above. Finally, all the results obtained using the present uncertainty analysis are included in tables A.6–A.9.
Table A.3: Uncertainties in velocity scalings.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$\delta (u_r)$</th>
<th>$\delta (U_e)$</th>
<th>$\delta (U)$</th>
<th>$\delta (\sigma_V)$</th>
<th>$\delta ((\sigma_u)_c)$</th>
<th>$\delta ((\sigma_v)_c)$</th>
<th>$\delta ((\sigma_w)_c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>0.0393</td>
<td>0.0014</td>
<td>—</td>
<td>—</td>
<td>0.0019</td>
<td>0.0010</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>0.0393</td>
<td>0.0013</td>
<td>—</td>
<td>—</td>
<td>0.0019</td>
<td>0.0011</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.0362</td>
<td>0.0015</td>
<td>—</td>
<td>—</td>
<td>0.0021</td>
<td>0.0012</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.0272</td>
<td>0.0012</td>
<td>—</td>
<td>—</td>
<td>0.0017</td>
<td>0.0009</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (TR-PIV)</td>
<td>Full</td>
<td>—</td>
<td>0.2040</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (Stereo-PIV)</td>
<td>$M = 16$</td>
<td>0.0393</td>
<td>—</td>
<td>0.0243</td>
<td>0.0141</td>
<td>0.0160</td>
<td>0.0344</td>
<td>0.0200</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.0362</td>
<td>—</td>
<td>0.0222</td>
<td>0.0127</td>
<td>0.0141</td>
<td>0.0315</td>
<td>0.0180</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.0272</td>
<td>—</td>
<td>0.0182</td>
<td>0.0119</td>
<td>0.0112</td>
<td>0.0258</td>
<td>0.0169</td>
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</table>

Table A.4: Random errors on an instantaneous basis.

<table>
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<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$\delta_{sp}(u)_{max}$</th>
<th>$\delta_{sp}(v)_{max}$</th>
<th>$\delta_{sp}(w)_{max}$</th>
<th>$\delta_s(u)$</th>
<th>$\delta_s(v)$</th>
<th>$\delta_s(w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>0.18</td>
<td>0.18</td>
<td>—</td>
<td>1.70</td>
<td>0.90</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>0.18</td>
<td>0.18</td>
<td>—</td>
<td>1.66</td>
<td>0.91</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.18</td>
<td>0.18</td>
<td>—</td>
<td>1.52</td>
<td>0.83</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.19</td>
<td>0.19</td>
<td>—</td>
<td>1.24</td>
<td>0.62</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (TR-PIV)</td>
<td>Full</td>
<td>0.20</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (Stereo-PIV)</td>
<td>Full</td>
<td>0.09</td>
<td>0.35</td>
<td>0.12</td>
<td>1.63</td>
<td>0.88</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>0.09</td>
<td>0.35</td>
<td>0.12</td>
<td>1.63</td>
<td>0.92</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.09</td>
<td>0.35</td>
<td>0.12</td>
<td>1.49</td>
<td>0.78</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.09</td>
<td>0.35</td>
<td>0.12</td>
<td>1.22</td>
<td>0.72</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Table A.5: Random errors on an ensemble basis.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$n$</th>
<th>$\delta_{sp}(U)$</th>
<th>$\delta_{sp}(V)$</th>
<th>$\delta_{sp}(W)$</th>
<th>$\delta_s(U)$</th>
<th>$\delta_s(V)$</th>
<th>$\delta_s(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>1 543 500</td>
<td>0.0001</td>
<td>0.0001</td>
<td>—</td>
<td>0.0014</td>
<td>0.0007</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>1543 500</td>
<td>0.0001</td>
<td>0.0001</td>
<td>—</td>
<td>0.0013</td>
<td>0.0007</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>1057 500</td>
<td>0.0002</td>
<td>0.0002</td>
<td>—</td>
<td>0.0015</td>
<td>0.0008</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>1057 500</td>
<td>0.0002</td>
<td>0.0002</td>
<td>—</td>
<td>0.0012</td>
<td>0.0006</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (TR-PIV)</td>
<td>Full</td>
<td>1 2040</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$ (Stereo-PIV)</td>
<td>Full</td>
<td>4500</td>
<td>0.0013</td>
<td>0.0052</td>
<td>0.0017</td>
<td>0.0243</td>
<td>0.0131</td>
<td>0.0160</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>4500</td>
<td>0.0013</td>
<td>0.0052</td>
<td>0.0017</td>
<td>0.0243</td>
<td>0.0137</td>
<td>0.0160</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>4500</td>
<td>0.0013</td>
<td>0.0052</td>
<td>0.0017</td>
<td>0.0222</td>
<td>0.0116</td>
<td>0.0140</td>
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<td>Smooth</td>
<td>4500</td>
<td>0.0013</td>
<td>0.0052</td>
<td>0.0017</td>
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<td>0.0107</td>
<td>0.0110</td>
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</table>
Table A.6: Percentage uncertainty for velocity ensembles.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$\varepsilon \left( \frac{U'}{U_e} \right)$</th>
<th>$\varepsilon \left( V' \right)$</th>
<th>$\varepsilon \left( W' \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>0.01</td>
<td>5.00</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 16$</td>
<td>0.01</td>
<td>5.00</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>$M = 5$</td>
<td>0.01</td>
<td>5.00</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.01</td>
<td>5.00</td>
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</table>

Table A.7: Percentage uncertainty of Reynolds stresses in inner variables.

<table>
<thead>
<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$\varepsilon \left( \langle u'^2 \rangle \right)$</th>
<th>$\varepsilon \left( \langle v'^2 \rangle \right)$</th>
<th>$\varepsilon \left( \langle w'^2 \rangle \right)$</th>
<th>$\varepsilon \left( \langle u'v' \rangle \right)$</th>
<th>$\varepsilon \left( \langle u'w' \rangle \right)$</th>
<th>$\varepsilon \left( \langle v'w' \rangle \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
<td>7.07</td>
<td>1.98</td>
<td>—</td>
<td>7.07</td>
<td>—</td>
<td>—</td>
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<td>$M = 16$</td>
<td>7.07</td>
<td>2.13</td>
<td>—</td>
<td>7.07</td>
<td>—</td>
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<td>—</td>
<td>7.07</td>
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</tr>
<tr>
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<td>Smooth</td>
<td>7.07</td>
<td>1.77</td>
<td>—</td>
<td>7.07</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$x - z$</td>
<td>Full</td>
<td>7.38</td>
<td>2.16</td>
<td>3.18</td>
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<td>7.38</td>
<td>7.40</td>
</tr>
<tr>
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<td>2.36</td>
<td>3.18</td>
<td>7.40</td>
<td>7.38</td>
<td>7.40</td>
</tr>
<tr>
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<td>7.41</td>
<td>7.38</td>
<td>7.41</td>
</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>7.38</td>
<td>2.39</td>
<td>2.72</td>
<td>7.42</td>
<td>7.38</td>
<td>7.42</td>
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</tbody>
</table>

Table A.8: Percentage uncertainty of Reynolds stresses in outer variables for developing flow experiments.

<table>
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<tr>
<th>PIV Plane</th>
<th>Surface</th>
<th>$\varepsilon \left( \frac{\langle u'^2 \rangle}{U_e^2} \right)$</th>
<th>$\varepsilon \left( \frac{\langle v'^2 \rangle}{U_e^2} \right)$</th>
<th>$\varepsilon \left( \frac{\langle u'v' \rangle}{U_e^2} \right)$</th>
</tr>
</thead>
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<td>(%)</td>
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<tr>
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<td>0.12</td>
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<tr>
<td></td>
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<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>$x - z$</td>
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<td>3.13</td>
<td>1.98</td>
</tr>
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<td>2.85</td>
<td>1.98</td>
</tr>
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<td>$M = 5$</td>
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</tr>
<tr>
<td></td>
<td>Smooth</td>
<td>0.73</td>
<td>2.31</td>
<td>1.99</td>
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Table A.9: Percentage uncertainty of correlation coefficients.

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<th>PIV Plane</th>
<th>Surface</th>
<th>$\varepsilon (\rho_{uu})$</th>
<th>$\varepsilon (\rho_{vv})$</th>
<th>$\varepsilon (\rho_{ww})$</th>
<th>$\varepsilon (\rho_{uw})$</th>
<th>$\varepsilon (\rho_{vw})$</th>
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<tr>
<td></td>
<td></td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>$x - y$</td>
<td>Full</td>
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<td>0.15</td>
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<td>0.04</td>
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<td>0.15</td>
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<tr>
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<td>0.02</td>
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<td>0.85</td>
<td>3.13</td>
<td>1.98</td>
<td>0.73</td>
<td>0.86</td>
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<td>1.98</td>
<td>0.76</td>
<td>0.86</td>
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<td>1.99</td>
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Appendix B

Modified Clauser Chart Method

The Clauser chart method was first developed by Clauser (1954, 1956) to deduce the friction velocity, $u_\tau$, in smooth-wall turbulent boundary layers assuming the existence of a logarithmic form of the mean velocity profile in the overlap region ($100 \lesssim y^+ \lesssim 0.15\delta^+$. This initial development was later extended to the case of rough-wall TBLs to estimate not only $u_\tau$ but also the origin offset, $\varepsilon$, and the roughness function, $\Delta U^+$, again with the assumption that the mean velocity profile follows a logarithmic relationship in the overlap region. Based on the work of Perry and Li (1990), Schultz and Flack (2007), and Dixit and Ramesh (2009), an approach was developed to apply the modified Clauser chart method (MCCM) to the datasets acquired herein.

The use of MCCM first must accommodate the fact that when the present experimental measurements were taken, the origin of coordinates of the instrument did not coincide with the virtual origin of the rough-wall flows. Thus, an origin error, $y_o$, is introduced in the wall-normal coordinate of the log law valid for rough-wall turbulence [eq. (5.5)] as

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \left( \frac{u_\tau}{\nu} (y + y_o) \right) + A - \Delta U^+.$$  \hfill (B.1)

The $u_\tau$ normalization of eq. (B.1) can be changed to a normalization instead with the free-stream velocity, $U_e$, as

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln \left( \frac{u_\tau}{\nu} (y + y_o) \right) + A - \Delta U^+.$$  \hfill (B.2)

which, after rearranging, gives

$$\frac{U + \Delta U}{U_e} = \frac{1}{\kappa} \frac{u_\tau}{U_e} \ln \left( \frac{U_e}{\nu} (y + y_o) \right) + \frac{u_\tau}{U_e} \left( \frac{1}{\kappa} \ln \left( \frac{u_\tau}{U_e} \right) + A \right).$$  \hfill (B.3)

A change of variables to $\tilde{y} = y + y_o$ and $\tilde{U} = U + \Delta U$, gives

$$\frac{\tilde{U}}{U_e} = \frac{1}{\kappa} \frac{u_\tau}{U_e} \ln \left( \frac{U_e}{\nu} \tilde{y} \right) + \frac{u_\tau}{U_e} \left( \frac{1}{\kappa} \ln \left( \frac{u_\tau}{U_e} \right) + A \right).$$  \hfill (B.4)
which defines a family of curves of the variables $U_e\tilde{y}/\nu$ and $\tilde{U}/U_e$, in which $u_\tau$ is the parameter of the family. An advantage of this family of curves is that the normalization factor is the free-stream velocity, $U_e$, which can be measured directly in contrast to the $u_\tau$. Therefore, no assumption must be made about the value of $u_\tau$ to normalize the velocity. Note that $\Delta U$ and $y_o$ in eq. (B.3) are basically corrections to $U$ and $y$. In other words, once the correct values of $\Delta U$ and $y_o$ are found, and the experimental data is corrected based on them, the logarithmic region of the corrected data should coincide with one of the curves defined by eq. (B.4). In fact, this is the principle that defines the MCCM: corrections to $U$ and $y$ are introduced until the logarithmic region of the data coincides with a curve of the above-defined family. Such corrections should be made methodically to obtain accurate results as will be discussed below.

All of the requisite steps necessary to carry out the MCCM are illustrated in figure B.1 using the mean velocity profile for the $M = 5$ model as determined from the $x - y$ experiments. The solid lines represent the family of curves defined by eq. (B.4) and it should be noted that the parameter $u_\tau a U_e$ increases upwards across the family of curves. To implement the MCCM, one should plot the raw data on top of the family of curves defined by eq. (B.4). The symbols define: ( ): raw data; (○): data after origin correction (1); (▲): data after velocity correction (2).
curves [open circles (○) in figure B.1]. Observe that, since the origin of the wall-normal coordinate does not coincide with the true virtual origin, the logarithmic region of the data does not follow a straight line in the semi-logarithmic plane $(U_e\tilde{y}/\nu-\tilde{U}/U_e)$. Therefore, the first step in implementing MCCM is to correct the origin until the number of data points following a straight line in the semi-logarithmic plane $(U_e\tilde{y}/\nu-\tilde{U}/U_e)$ is maximized. This step is illustrated by arrow (1) and the final result of this first correction is represented by the black circles (●) in figure B.1. In this particular case, 4490.25 was added to the horizontal coordinate to maximize the number of points following a straight line. Therefore, $U_e\gamma_o/\nu = 4490.25$. Following this first correction, the slope of the logarithmic region is found to be inconsistent with the slope of the surrounding lines belonging to the family $(U_e\tilde{y}/\nu-\tilde{U}/U_e)$. Therefore, the velocity $\tilde{U}/U_e$ must be adjusted until the slope of the logarithmic region of the data coincides with the slope of one of the lines of the family $(U_e\tilde{y}/\nu-\tilde{U}/U_e)$. This step is illustrated by arrow (2) and the final result is represented by the black triangles (▲) in figure B.1. For the present case, 0.2048 was added to the original velocity values to arrive at the correct slope. Hence, $\Delta U/U_e = 0.2048$. Additionally, it is observed that the final result coincides with the line for which $u_\tau/U_e = 0.0425$. Finally, once the correct value of the virtual origin is determined, it is compared with the geometrical reference position to find the origin offset. For the experiments, the geometrical reference position is taken as the midplane of the roughness since this height coincides with the elevation of the upstream smooth-wall conditions in all experiments. For this particular case of the $M = 5$ model, the distance between the midplane of the roughness and the virtual origin is 3.9 mm. Therefore, $\varepsilon = 3.9$ mm.

It should be noted that, while the example presented above utilized MCCM to determine $u_\tau$, $\Delta U^+$, and $\varepsilon$, the friction velocity could also be determined independently using the constant stress method (as discussed in section 5.2.2) and $\Delta U^+$ and $\varepsilon$ can be determined in a manner identical to that described herein. This is, in fact, how the values of $u_\tau$, $\Delta U^+$, and $\varepsilon$ reported herein were determined. A comparison of the friction velocity determination with these two methods indicates a roughly 5% difference for both smooth- and rough-wall flows which is consistent with the uncertainty associated with the constant stress method for determining $u_\tau$. 
References


Vita

Ricardo Mejia-Alvarez was born in May of 1974 in Medellin – Colombia. He obtained a BS degree in Mechanical Engineering from the National University of Colombia at Medellin in 2000. After graduation, he joined the Laboratory for Gas Combustion and Internal Combustion Engines and the Laboratory for Materials and Pyrometallurgical Research in the University of Antioquia as a research staff member. While working in those two labs, Mr. Mejia-Alvarez pursued a MS degree in Therm engineering from the University of Antioquia, which he received in 2004. The same year, he received a Fulbright fellowship to pursue doctoral studies in Theoretical and Applied Mechanics at the University of Illinois at Urbana-Champaign under the supervision of Prof. Kenneth T. Christensen. During his time at Illinois, Mr. Mejia-Alvarez has worked extensively in experimental research applied to bounded and unbounded turbulent flow. In his first three years at Illinois, Mr. Mejia-Alvarez worked in experimental studies of submerged turbulent impinging jets of dilute polymer solutions. As a result of that work, he received a MS degree in Theoretical and Applied Mechanics. For his doctoral work at Illinois, Mr. Mejia-Alvarez has dedicated the core of his research to study turbulent boundary layers over highly irregular rough walls. After graduating, Mr. Mejia-Alvarez will take a postdoctoral position with the Team of Extreme Fluids at Los Alamos National Laboratory in New Mexico.