MODELING CUMULATIVE DAMAGE TO FLOW SURFACES AND ASSESSING ITS IMPACT ON WALL TURBULENCE

BY

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THESIS

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Abstract

The present effort investigated the influence of cumulative damage of an initially smooth surface on the statistical characteristics of wall turbulence. Singular value decomposition was employed to decompose a highly-irregular surface topography replicated from a turbine blade damaged by spallation into topographical modes embodying successively smaller roughness length scales. These modes were then used to develop multiple reduced-order models of the original roughness that, at least in spirit, successively captured the evolution of the surface roughness from initial smaller-scale defects that would likely form just after deployment to the eventual cumulative formation of larger-scale roughness features. Particle-image velocimetry measurements of flow at a friction Reynolds number around 1825 over various topographical models were gathered in turbulent channel flow. The cumulative impact of these reduced-order models on wall turbulence was then assessed by comparing mean velocities, Reynolds normal stresses, and Reynolds shear stresses. It was observed that the turbulence statistics of the flow increase in magnitude as larger-scale features are introduced into the surface, leading up to the full, original surface. In addition, even weak surface damage that a practical flow surface might endure within the first fraction of its deployment lifetime can significantly enhance turbulence and therefore progressively degrade system performance. Thus, proper modeling of flow-system performance must account for the dynamic nature of flow surfaces, particularly those exposed to severe operating conditions.
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List of Symbols

Latin Symbols

- $a$: coefficient of SVD expansion
- $FSC$: fractional surface content, %
- $h$: channel half-height, m
- $H$: hyperbolic hole size for exclusion of quadrant events
- $k$: average peak-to-valley roughness height, m
- $k_s$: equivalent sand grain roughness, m
- $L$: total number of SVD basis functions
- $M_R$: number of residual SVD basis functions included in the reduced-order model
- $R$: universal gas constant, $J/molK$
- $Re$: Reynolds number
- $Re_\tau$: friction Reynolds number
- $SW$: smooth wall
- $u$: total streamwise velocity, $m/s$
- $u'$: fluctuating streamwise velocity, $m/s$
- $u'v'$: instantaneous Reynolds shear stress, $m^2/s^2$
- $u_\tau$: friction velocity, $m/s$
- $U$: mean streamwise velocity, $m/s$
- $U_c$: advection velocity, $m/s$
- $U_{CL}$: channel centerline velocity, $m/s$
- $v$: total wall-normal velocity, $m/s$
\( v' \) fluctuating wall-normal velocity, \( m/s \)
\( V \) mean wall-normal velocity, \( m/s \)
\( x \) streamwise direction, \( m \)
\( y \) wall-normal direction, \( m \)
\( y_s \) viscous length scale, \( m \)
\( z \) spanwise direction, \( m \)

**Greek**

\( \gamma \) arbitrary function
\( \eta \) fluctuating surface elevation, \( m \)
\( \lambda_{ci} \) swirling strength
\( \Lambda_{ci} \) unsigned swirling strength
\( \nu \) kinematic viscosity of fluid, \( m^2/s \)
\( \rho \) density of fluid, \( kg/m^3 \)
\( \Sigma \) diagonal matrix of singular values
\( \tau_w \) wall shear stress, \( Pa \)
\( \phi \) SVD basis function
\( \omega_z \) span-wise vorticity, \( 1/s \)

**Mathematical Operators**

\( \bar{\cdot} \) time average of \( \cdot \)
\( \langle \cdot \rangle \) ensemble and streamwise average

**Superscripts**

\( + \) normalization in inner units
\( SW \) smooth wall value
Subscripts

full  full surface
$j$  mode number
$Q$  quadrant
Chapter 1

Introduction

Most practical flow surfaces, such as those of heat exchangers, turbine blades, ship hulls, or windmill blades, begin aerodynamically (or hydrodynamically) smooth at first deployment, but can quickly become roughened due to various damage mechanisms (Bons, 2002; Karlsson, 1980). Many studies support the expected result that increased surface roughness, regardless of how roughness to the surface occurs, increases surface drag, thus, rendering higher heat loads, accelerated part degradation, and lower operating efficiencies (Bons, 2002). In the case of turbine blades, such damage is, in part, attributable to contaminants in the atmosphere, such as dust, sand, or pollutants caused by the combustion of fossil fuels rapidly traveling through the engine. Regardless of the particular application, damage to flow surfaces often occurs in a cumulative manner beginning with relatively small surface defects that can grow in both size and occurrence. Over time, such surfaces can become highly-irregular as they contain a broad range of topographical scales distributed in an intermittent manner. Unfortunately, these topographical characteristics are often counter to the roughness employed in laboratory investigations where ordered arrays of single-scale roughness elements are typically utilized. This chapter provides a brief overview of surface roughness encountered both in the field as well as in the laboratory, followed by structural details of turbulent flow over smooth and rough surfaces and the equations that describe them. Several previous studies pertaining to cumulative surface damage and its impact on wall turbulence are addressed to provide motivation for this present experimental effort.

1.1 Damage Mechanisms

Several factors may play a role in the development of roughness on a flow surface, such as location, temperature, and/or geometry. This damage can be the result of deposition of foreign materials,
pitting of the flow surface, erosion, and/or spallation of a thermal barrier coating (Bons, 2002).
Each of these damage mechanisms have unique damage characteristics that may alter the flow surface in a specific manner.

- Pitting can arise from the creation of a localized harsh or aggressively corrosive environment that breaks down the normally corrosion-resistant surface of the metal (Davis, 1999), leaving irregularly shaped and sized holes, or “pits”, in the surface. Figure 1.1 shows pitting that occurred on a high-pressure turbine blade. This damage, which leaves negative roughness elements (where zero indicates the original smooth wall), may be the result of an initial crack or defect in the material, subjected to a moist or even corrosive environment, leading to rust or corrosion that slowly dissolved the surface material. This damage is particularly abundant in steam turbine engines, when incorrect steam chemistry along with faulty valves or seals persist within the engine.

![Pitting on a high-pressure turbine blade](www.powerccl.co.uk)

- In high speed flows, where abrasive particulate approaching at relatively low angles of impact may be present, erosion can commonly be found due to these particles slowly transferring material throughout the surface (Hutchings, 1979). This is known to leave both positive and negative roughness elements throughout the affected surface. Figure 1.2 depicts erosion damage found on a low-pressure turbine blade, caused by extremely high-speed steam, along with micron-sized particulate being ingested through the engine for a long duration of time.
Notice the bands of debris that align themselves in the spanwise flow direction, and layer themselves in the streamwise flow direction, indicative of erosion-type damage.

- In certain applications, particulate within the flow may tend to adhere itself to the surface over which it comes in contact with. Known as deposition, this damage mechanism spawns positive roughness elements on the flow surface. In the case of turbine engines, flow surfaces near the aft of the engine, where temperatures are very high and may approach the softening temperature of certain particles such as sand, will experience deposition (Bons et al., 2008b). Ship hulls that are consistently submerged in water also experience deposition in the form of barnacle build-up. Figure 1.3 is a picture of a ship hull enduring significant barnacle build-up.
Thermal barrier coatings (TBC) are often used on gas turbine or aero-engine parts that operate at elevated temperatures as a form of exhaust heat management. These coatings, which typically use ceramic as the insulating material, serve to protect the metal engine components from extremely high and prolonged heat loads. Unfortunately, these coatings can chip or spall, due to the differing thermal expansion rates of both the ceramic outer layer and the bond just below the ceramic, each experiencing thermal cycling (Barber et al., 1999). Figure 1.4 shows spallation of a TBC on a turbine-blade pressure surface. From this picture, it becomes evident that spallation ultimately forms large, but shallow canyons within the surface, yielding negative surface elements.

Figure 1.4: Spallation of the thermal barrier coating on the pressure surface of a turbine blade. (Bons, 2010)

Several recent studies have been conducted in order to better understand exactly how surface damage progresses throughout the lifetime of a practical flow surface. In particular, Bons (Bons et al., 2008a) studied several turbine-blade coupons, each with different surface treatments, in an accelerated deposition facility, simulating flow conditions commonly experienced at the inlet to a first stage high-pressure turbine. The combustor exit flow was seeded with typical dust particulate found near large utility power plants, which, when accompanied by extreme heat and pressure, and sustained for several hours, represented the cumulative effect of several months of operation. Figure 1.5 presents the results of this study, exhibiting four measurements evenly spaced through the life cycle of a turbine-blade surface. Regardless of which sample is analyzed, it
is evident that the surface topography is highly-irregular, containing a vast array of topographical scales. Although these scales tend to increase in magnitude as the coupon is subjected to further operational environments, at no time does the surface damage resemble an ordered array or pattern-like structure. In fact, Bons et al. (2008a) noted that regardless of the surface treatment, all of the surfaces portrayed non-monotonic changes in roughness after repeated exposure in the accelerated deposition facility.

1.2 Irregular Roughness vs. Ideal Roughness

As discussed above, roughness encountered in typical engineering applications tends to be characterized by highly-irregular surface topographies, containing a broad range of topographical scales. Many studies have been conducted to observe the effects of surface roughness, but, unfortunately, most efforts employed ideal surface roughness elements, including woven mesh, sand grain, and patterned arrays of rough elements (Mejia-Alvarez and Christensen, 2010). Figure 1.6 depicts examples
of ideal surface roughness elements commonly used in laboratories for experimental investigations. Figure 1.6(a) shows an ordered array of cylindrical elements, while figure 1.6(b) illustrates a wire mesh arrangement. Both surfaces exemplify the characteristic single roughness scale, ordered arrangement that persist within such surfaces, and are absent of the rich topographical features encountered in practice. Although relatively easy to produce in laboratory settings, these ideal surfaces are often counter to what may be expected in practical flow systems. For example, figure 1.7 demonstrates typical roughness patterns most often encountered in practical flows. These surfaces, although damaged by distinct methods, contain a broad range of topographical scales.

For years, engineers and scientists have studied the effects of roughness on skin friction, drag, and heat transfer, contributing a vast array of correlations and design approaches that help address this inevitable phenomenon. Early studies, performed by Nikuradse (1933) in the 1930’s, were conducted on sand-roughened pipe walls and displayed different dependencies on Reynolds number (Re) and roughness for different flow regimes. A dimensionless roughness parameter $k_s^+ = k_s u_\tau/\nu$ was defined by Nikuradse (1933) using the measured friction velocity ($u_\tau$), kinematic viscosity ($\nu$), and actual sandgrain diameter ($k_s$). He found that for values of $k_s^+ > 70$, the pipe loss coefficient was only a function of $k_s$ and the flow was termed “fully” rough. For $5 < k_s^+ < 70$, the pipe
Figure 1.7: Irregular roughness samples garnered from profilometric scans of damaged turbine blades. (a) Deposition of foreign materials. (b) Spallation of thermal barrier coating. (c) Pitting of the surface. (Bons et al., 2001)

loss coefficient was a function of $k_s$ as well as $\text{Re}$ and the flow was termed “transitionally” rough. Values of $k^+ < 5$ were referred to as “smooth”, where roughness was found to have no impact on the pressure loss through the pipe due to the fact that the roughness peaks where entirely submerged within the laminar sublayer of the turbulent boundary layer. Shortly after the work of Nikuradse (1933), Schlichting (1936) proposed an equivalent sandgrain roughness approach, by which he experimentally determined $k_s$ for a variety of rough surfaces. He defined $k_s$ as the size of the sandgrain used in the experiments of Nikuradse (1933) that yields the same skin friction as that observed on a non-sandgrain rough surface. Although this approach has been utilized in practice for many years, limited skin friction data for more modern rough surfaces hinders the reliance of this technique in determining $k_s$, yielding potential errors of 73% in skin friction values, and errors anywhere from 26% to 555% for $k_s$ values (Taylor et al., 1985). Using resources from open literature, Bons (Bons, 2010) compiled a broad collection of equivalent sand grain roughness correlations for determining $k_s$ for gas turbine surface roughness. For the most part, these correlations convert measurable surface roughness parameters such as $Ra$ (arithmetical mean deviation), $Rq$ (root-mean-square), or $Rz$ (ten-point mean roughness) to equivalent sandgrain roughness, $k_s$. More recently, work has been done to predict turbulent rough-wall skin friction measurements using a discrete element approach (Taylor et al., 1985), in which surface roughness form drag and blockage
effects were included as part of the partial differential equations, and did not rely on a single-
length-scale concept such as equivalent sandgrain roughness.

As discussed in Section 1.1, there are many mechanisms responsible for damaging practical
flow surface, all of which yield highly-irregular surface topographies. This behavior raises the
question: How well does idealized roughness capture the effect an actual irregular surface may have
on the flow? Previous efforts have shown that outer-layer turbulence over an irregular roughness can
behave similarly to that over an idealized roughness (Allen et al., 2007; Wu and Christensen, 2007;
Mejia-Alvarez and Christensen, 2010). When appropriately scaled with $u_\tau$, this observed collapse of
the turbulent statistics supports Townsend’s wall similarity hypothesis (Townsend, 1976). On the
other hand, Bons (Bons, 2002) compared turbulent statistical data from irregular roughness and
ideal roughness and found that ideal roughness is fundamentally different from irregular roughness.
Bons (2002) was particularly interested in how well-accepted equivalent sandgrain ($k_s$) (where
$k_s$ is determined from a roughness shape/density parameter) correlations were able to predict
skin friction ($c_f$) and Stanton number ($St$, dimensionless number that measures the ratio of heat
transferred into a fluid to the thermal capacity of the fluid) values. Bons (2002) found that for
$k_s^+ > 70$, standard correlations provided a fair estimate on $c_f$, but overpredict $St$ by $10\%$. When
$k_s^+ < 70$, existing $c_f$ and $St$ correlations severely under-predicted the effects of irregular roughness,
all of which expose limitations in the common use of ordered arrays of roughness elements to model
practical rough flow surfaces.

The idea that an ideal roughness may not entirely duplicate the turbulent statistical results of
an irregular roughness is illustrated further through the recent work of Johnson and Christensen
(2009), Mejia-Alvarez and Christensen (2010) and Wu and Christensen (2006a) where it was found
that flow characteristics appear to be governed by the larger- and intermediate-scale features of
irregular surfaces. In each case, singular value decomposition (SVD) was used to decompose a
highly-irregular surface topography into spatial basis functions, where each basis function repre-
sented a unique scale of the surface ranging from the smallest- to the largest-scale topographical
features. Several low-order models of the surface topography were then constructed, each contain-
ing consecutively more small-scale features, beginning with the largest-scale mode. Particle-image
velocimetry (PIV) data taken over short streamwise fetches showed that the low-order models that
contained the intermediate- and large-scale features collapsed well with the full surface, indicating that these features of the full surface predominantly dictate its impact on the flow. Unfortunately, ideal surfaces such as woven mesh or sand grain rarely occupy more than one topographical scale, rendering them inadequate to reproduce practical rough wall damage.

1.3 Turbulent channel flow

At a rudimentary level, the nature of fluid flow can be subdivided into two distinct classes: free shear flows and bounded flows, the latter encompassing most practical turbulent flows (Pope, 2000), including the flow studied herein. Branching off from the class of bounded flows gives rise to internal flows, including flows through ducts and pipes, or external flows, such as flows around an aircraft or atmospheric boundary layers. A simple, yet imperative internal flow that has played a prominent role in the historical development of the study of turbulent flows is fully developed turbulent channel flow.

1.3.1 Description of flow

Figure 1.8 presents a schematic of channel flow, including the relevant directions and dimensions generally used. Channel flow is typically defined as being flow through a rectangular duct of height $H = 2h$ (where $h$ refers to the half-height of the channel) that is long ($L/h >> 1$) and has a large aspect ratio ($b/h >> 1$). The streamwise, wall-normal, and spanwise directions are referred to as $x$, $y$, and $z$, respectively. Similarly, the streamwise, wall-normal, and spanwise velocities are defined
as $u$, $v$, and $w$, and the velocity fluctuations are defined as $u'$, $v'$, and $w'$, respectively. In channel flow, the mean flow is entirely in the streamwise ($x$) direction with its strongest variation in the wall-normal ($y$) direction. The bottom wall is at $y = 0$ while the top wall is at $y = 2h$, placing the centerline of the channel at $y = h$. Because of its large aspect ratio, turbulent channel flow is statistically independent of $z$ when measured significantly distant from the end walls.

A flow development region forms at the entry of the channel ($x = 0$), then, after some distance in the streamwise direction, the flow becomes fully-developed (large $x$), marking the point at which velocity statistics no longer vary in the streamwise ($x$) direction (the attainment of statistical homogeneity in $x$). Experimentally, the flow is tripped at the entrance of the channel, accelerating the flow’s transition to turbulence. After some distance, the flow, as mentioned before, will constitute fully-developed, turbulent channel flow, and therefore be statistically one-dimensional, with the mean streamwise velocity only depending on $y$. When this fully-developed turbulent flow abruptly interacts with a patch of roughness, the formation of an internal roughness layer will occur, growing in thickness steadily as it proceeds downstream, as can be seen in figure 1.9. The flow within this layer generates a significant enhancement in the production of turbulence, whereas the flow outside of this layer remains relatively undisturbed by the presence of the roughness (Mejia-Alvarez and Christensen, 2010). This internal layer gradually grows with increasing downstream distance until it engulfs the entire channel, resulting in a fully-developed rough-wall flow.

Near the wall, the wall shear stress ($\tau_w$) plays an important role in determining the flow, but upon further investigation becomes clear that the viscosity ($\nu$) and the density ($\rho$) are also important parameters (Pope, 2000). Because of their rather influential properties near the wall, these quantities are used to define viscous scales that are appropriate length scales and velocity
scales in the near-wall region. The characteristic velocity is defined by

\[ u_\tau = \sqrt{\frac{\tau_w}{\rho}}, \]  

(1.1)

where \( u_\tau \), or sometimes denoted \( u_* \), is termed the friction velocity. The characteristic length scale is defined by

\[ y_* = \frac{u_*}{\nu}, \]  

(1.2)

where \( y_* \) is referred to as the viscous length scale. These scales are termed “inner scales” because they are able to properly scale the physics near the wall. Using these scales, dimensionless parameters such as velocity and spatial directions can then be composed, and are denoted with the plus sign notation,

\[ u_+ = \frac{u_i}{u_*}, \]  

(1.3)

and

\[ x_+^* = \frac{x_i}{y_*}, \]  

(1.4)

where \( u_i = (u_1, u_2, u_3) = (u, v, w) \) and \( x_i = (x_1, x_2, x_3) = (x, y, z) \). When attempting to characterize the flow with an appropriate Re, there are several options. First, the flow can be characterized by an Re based on the bulk fluid velocity, \( \bar{U} \), and the half-height of the channel, \( h \), where the bulk fluid velocity is defined as

\[ \bar{U} \equiv \frac{1}{2h} \int_0^{2h} U dy, \]  

(1.5)

giving a Reynolds number of the form

\[ \text{Re}_h = \frac{\bar{U}H}{\nu}. \]  

(1.6)

The other option, which is considered more meaningful in the realm of turbulent channel flow, is defined using the friction velocity, opposed to the bulk fluid velocity, and the half-height of the channel, and is denoted as

\[ \text{Re}_\tau = \frac{u_\tau h}{\nu}, \]  

(1.7)

and is denoted the friction Reynolds number. Here, \( \text{Re}_\tau \) can be rewritten to represent a ratio of the channel half-height to the viscous length scale and thus is a measure of the range of scales present
within the flow.

### 1.3.2 Description of turbulent structures

From afar, turbulence can appear as flow in a complete stage of disorder and chaos, manifesting rapid spatial and temporal variations in velocity and pressure. Although this is partly true, a better understanding of turbulent flow tells a different, more organized story. One that begins to catalogue coherent structures (Cantwell, 1981) that tend to persist for long times, promoting the idea that some of these motions may be thought of as individual entities, called eddies (Townsend, 1976). Early studies by Theodorsen began to paint a picture of what these coherent structures may actually look like in wall turbulence (Theodorsen, 1952, 1955), describing them as horseshoe-like vortices, the conceptual model of such shown in figure 1.10.

In his observations, Theodorsen described this structure as a thin filament extending in the spanwise direction of the mean flow, possessing a head or arch feature at its furthest position from the wall. Under this scenario, the head would experience a higher mean flow velocity than the rest of the structure, causing it to convect downstream faster than its subsequent parts. As the entire structure began to grow and evolve, the streamwise legs that connect the spanwise vortex would begin to expand and intensify, further advancing the head into higher mean flow velocity, resulting in even more leg expansion. These ideas were supported using smoke visualization, which unfortunately, failed to reveal structures in the interior of the flow where Theodorsen proposed
Figure 1.11: Conceptual scenario of hairpin vortex packets interacting with each other as they develop near the wall and proceed downstream (Adrian et al., 2000b).

them to occur first (Adrian, 2007).

Nearly a quarter-century later, after much debate over the exact nature of turbulent structures, the experimental studies of (Bandyopadhyay, 1980) and Head and Bandyopadhyay (1981) found that horseshoe vortices (also known as horseshoe eddies, hairpin vortices, and hairpin eddies) were actually a fundamental element of wall-bounded turbulent flows. With the use of smoke and inclined light sheets to visualize the flow, they found that there were copious numbers of hairpin structures over a large range of Re evolving downstream of transition.

As visualization techniques began to improve as well as methods for studying turbulence, like direct numerical simulation (DNS) and high-spatial-resolution particle-image velocimetry (PIV), the full nature of wall-bounded turbulent structures began to unfold. The hairpin vortices once alluded to were now consistently observed experimentally. Today, it is well-known that, when fluid flow is bounded by a wall, hairpin-like structures form and tend to align in the streamwise direction, arranging themselves into larger-scale structures typically referred to as hairpin vortex packets (Adrian et al., 2000a). Once a hairpin vortex is formed, it rapidly changes to appear more like an omega-shaped hairpin, where it will mature as this shape, continuing to grow in all
Figure 1.12: Instantaneous PIV velocity realization in the streamwise-wall-normal plane of a turbulent boundary layer (Min and Christensen, 2010).

directions. When this initial hairpin vortex is of sufficient strength, it can produce additional vortices both upstream and downstream of its own head (Adrian, 2007) which can eventually lead to the formation of a new hairpin packet. This formation process is termed auto-generation (Zhou et al., 1997, 1999), which is a nonlinear process whereby the production of new hairpin heads occurs only if the magnitude of the initial eddy exceeds a certain threshold. Hairpin vortices are seldom symmetric, usually exhibiting one leg longer than the other, or being distorted or warped by other turbulent features within the flow. Figure 1.11 presents the combined conceptual scenario of packets interacting with one-another as they develop downstream (Adrian et al., 2000b).

Recent studies using PIV have supported the idea of hairpin vortex packets within wall-bounded flow. From a PIV measurement plane in the $x-y$, or streamwise–wall-normal, plane, hairpin vortex packets can be identified by an inclined interface formed by clockwise-rotating spanwise vortex cores due to the hairpin heads and situated above regions of low momentum fluid due to the collective induction of these structures. Figure 1.12 presents an instantaneous PIV velocity field in the $x-y$
plane from a zero-pressure-gradient turbulent boundary layer (Min and Christensen, 2010). The red circles serve to indicate the location of the spanwise vortex cores, or hairpin heads, while the blue region indicates slower-moving, lower-momentum fluid, seen near the bottom of the figure. These observations are therefore consistent with the overall features one might expect to identify in the hairpin packet model of figure 1.11 in the \( x - y \) plane.

In contrast, when the PIV measurement plane is placed such that it captures motion of the fluid in the \( x - z \), or streamwise–spanwise, plane, particularly in the logarithmic region of the flow, hairpin vortex packets can be identified as alternating regions of high (HMR) and low (LMR) momentum regions. The LMRs identified in this plane are consistent with slicing the hairpin vortex packet in figure 1.11 in an \( x - z \) measurement plane and represent the region of the slower-moving fluid induced by hairpin packets. Figure 1.13 presents instantaneous PIV velocity fields...
Figure 1.14: Instantaneous PIV (a) fluctuating velocity field and (b) Reynolds shear stress field in the spanwise–wall-normal plane (Barros and Christensen, 2010).

as a temporal evolution over a short time period in this measurement plane. Figure 1.13 reveals the streaky behavior of HMRs and LMRs while also revealing the advancement of such regions downstream, both indicative of hairpin vortex packets.

Lastly, when the PIV measurement plane is positioned in the $y - z$, or wall-normal-spanwise plane, alternating regions of HMRs and LMRs are evident, but from a different perspective. Instead of identifying the streamwise length of such packets, as is portrayed in the $x - z$ plane, this point of view not only shows the spanwise distance between HMRs and LMRs, but also shows the wall-normal extent of these features. Figure 1.14 illustrates an instantaneous PIV fluctuating velocity field, where the large red and blue regions demarcate high and low momentum regions, respectively (Barros and Christensen, 2010), again, demonstrative of hairpin vortex packets.
Identifying these hairpin vortex packets within experimental and computational visualizations of instantaneous velocity fields was essential evidence for the above model of turbulent structures in wall-bounded flow. However, affirming that these packets leave an imprint on the statistical nature of the flow would further support the notion of the existence of such structures. Such work was carried out by Christensen and Adrian (2001), where they found, indeed, that the instantaneous structures observed prior, do in fact occur at a sufficient frequency, strength, and order to leave a signature on the flow (Christensen and Adrian, 2001; Natrajan and Christensen, 2007; Wu and Christensen, 2006b). This evidence, along with other statistical evidence, provides a foundation for the idea that the instantaneous structures witnessed within the flow are certainly not random occurrences. Other works have also shared this same conclusion, including studies on the geometrical modifications endured to hairpin vortex packets over rough-wall flows.

1.4 Turbulence equations

In turbulent flow analysis, acquiring quantitative information, other than being extremely challenging and computationally exhausting, can be rather useful in predicting velocity and/or temperature profiles and wall friction and/or heat transfer (White, 1991). One way of doing this is through direct numerical simulation or DNS, in which a supercomputer is required to solve the Navier–Stokes equations for a particular flow. Unfortunately, due to the wide range of flow scales persistent in turbulent flow, this technique is limited to very low Re. To simplify this complexity, Reynolds averaging can be employed. Reynolds averaging is a technique in which time-dependent properties are decomposed into a time-mean component and a fluctuation component. For convenience, Einstein’s indicial notation will be utilized throughout this section to expedite the nomenclature (See chapter 2, Batra (2006)).

1.4.1 Navier–Stokes equations

The Navier–Stokes equations, along with the continuity equation, describe the motion of an incompressible fluid. The Navier–Stokes equations can be derived by applying Newton’s second law to fluid motion, and, when solved, assuming a solution exists, provide a velocity field of the flow in question. The continuity equation and Navier–Stokes equations for an incompressible, Newtonian
fluid can be written in the form

$$\frac{\partial u_i}{\partial x_i} = 0$$  \hspace{1cm} (1.8)

and

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j},$$  \hspace{1cm} (1.9)

respectively, where $u_i$ is the $i$th component of the velocity vector, $x_j$ is the $j$th spatial direction, and $p$ is the pressure. The constants $\rho$ and $\nu$ are the density and kinematic viscosity of the fluid, respectively.

### 1.4.2 Reynolds-averaged equations

Reynolds was the first to recognize that the total flow variables in a turbulent flow could be recast in terms of mean and fluctuating components, with the latter embodying the turbulent aspects of the flow. In this regard, field variables within a flow can be decomposed as

$$u_i = \bar{u}_i + u'_i$$

$$p = \bar{p} + p'$$

$$T = \bar{T} + T'$$

where $\bar{\cdot}$ represents a time average of a variable in question and $(\cdot)'$ denotes a fluctuating quantity. This decomposition is referred to as Reynolds decomposition. Once these decomposed variables are substituted back into eq. (1.8) and (1.9), the equations are then time-averaged. The continuity equation becomes

$$\frac{\partial (\bar{u}_i + u'_i)}{\partial x_i} = 0,$$ \hspace{1cm} (1.10)

such that

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad \text{and} \quad \frac{\partial u'_i}{\partial x_i} = 0.$$ \hspace{1cm} (1.11)

Time-averaging the Navier–Stokes equations proves more challenging due to the nonlinear convective term on the left hand side (LHS). When averaged, eq. (1.9) takes the form

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j},$$ \hspace{1cm} (1.12)
where the equivalence of
\[ \frac{\partial (u_i u_j)}{\partial x_j} = u_i \frac{\partial u_j}{\partial x_j} + u_j \frac{\partial u_i}{\partial x_i}, \tag{1.13} \]
and recalling
\[ \frac{\partial u_i}{\partial x_i} = 0 \tag{1.14} \]
from continuity, has been utilized. When time-averaged, the nonlinear convective term takes the form
\[ \bar{u}_i \bar{u}_j = (\bar{u}_i + u'_i)(\bar{u}_j + u'_j) \tag{1.15} \]
or
\[ \bar{u}_i \bar{u}_j = \bar{u}_i \bar{u}_j + u'_i u'_j, \tag{1.16} \]
noting that \( \bar{u}'_i = 0 \). Plugging eq. (1.16) into eq. (1.12) then yields
\[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i \bar{u}_j + u'_i u'_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}, \tag{1.17} \]
and after expanding the derivative on the LHS, the final form of the Reynolds-averaged Navier–Stokes equations becomes
\[ \frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial (u'_i u'_j)}{\partial x_j}. \tag{1.18} \]

In comparing eq. (1.9) with eq. (1.18), an additional term on the right-hand side of eq. (1.18) is observed, which arises from the Reynolds decomposition of \( u_i \). As such, \( u'_i u'_j \) is termed the Reynolds stress tensor and represents an additional stress on the fluid caused by the turbulence. These stresses are called “apparent”, or Reynolds stresses, and constitute the main reason why the governing equations of turbulence are so difficult to solve. They introduce an additional unknown, without providing a supplementary governing equation and this issue is commonly referred to as the “closure” problem of turbulence since the mean flow cannot be solved without some knowledge of the turbulence through the Reynolds stress tensor.
1.5 Present effort

The present research is directed toward the chronological understanding of turbulent behavior when a flow encounters a surface experiencing cumulative damage. As previously mentioned, roughness models most commonly used in laboratory settings do not sufficiently capture the appropriate roughness scales present in practical applications. Thus, this research utilizes an actual surface topography taken from a profilimetric scan of a turbine blade which embodies highly-irregular surface features. In order to study the role of cumulative damage on the turbulent characteristics found within the flow, singular value decomposition was employed to decompose the surface topography into various topographical scales, essentially dividing the surface topography into a collection of new surfaces, each containing a relevant topographical length scale of the surface, gradually ranging from extremely small to the largest contained within the original surface. When the single-scaled surface topographies are appropriately appended, reduced-order models, beginning with models containing small-scale features and inevitably culminating to include all scales are fabricated. This reconstruction methodology is meant to mimic, at least in spirit, the evolution of a flow surface from only weakly rough via small-scale topographical damage in the early stages of deployment to cumulative formation of larger-scale topographical damage over time. The recent work of Bons et al. (2008a) lends support for this general evolutionary pattern of surface damage from smaller to larger topographical scales over time (see figure 1.5). A smooth-wall, the selected intermediate, reduced-order rough surfaces, and the original full surface are individually tested within a channel flow facility, subjecting them to fully-developed turbulent channel flow. Particle-image velocimetry measurements are made over each surface, and the resultant turbulent statistics are analyzed in order to explore the impact of cumulative surface damage on wall turbulence.
Chapter 2

Experimental Setup

The experiments reported herein were performed in the turbulent channel flow facility located in the Laboratory for Turbulence and Complex Flow. Originally designed to study smooth-wall, fully-developed turbulent channel flow, it has since been modified to accommodate short fetches of rough-wall models that are inserted as test panels in the test section of the channel (Wu and Christensen, 2006a). Two-dimensional PIV measurements were made over several rough-wall models in the streamwise–wall-normal (x–y) plane positioned nearly at the center of the channel to capture the turbulent enhancement within the internal roughness layer. This chapter elaborates on the details involved with the design and construction of the rough-wall test panels, the flow facility employed as well as a description of applicable PIV details and PIV measurements required for the current experimental work.

2.1 Surface roughness models

In order to model the evolution of a surface topography from initially smooth to eventually highly-irregular, without actually invoking artificial damage onto an existing smooth surface, singular value decomposition (SVD) was used to reconstruct several low-order surface roughness models. As a collection, these models, at least in spirit, represent the evolution of the surface topography desired from small-scale surface defects that might evolve into much larger topographical features over time. Once designed and fabricated, these surface roughness models were exposed to a chemical treatment in order to overcome particular dilemmas related to PIV experimentation.

The rough surface employed in this effort is the same as that studied by Wu and Christensen (2006a) as well as Johnson and Christensen (2009) and is a scaled version of a profilometric scan of a turbine blade damaged by spallation of the thermal barrier coating. This surface topography,
originally reported by Bons et al. (2001), was captured using a contact stylist profilometry system with a stylus tip radius of 1.5 µm. The extremely small size of this stylus tip enabled Bons et al. (2001) to resolve the surface topography down to just a few microns. Figure 2.1 depicts the surface topography map of the damaged turbine blade considered in the present effort. This topography map embodies highly-irregular surface features consisting of a broad range of topographical scales most commonly seen in practical flow surfaces. In particular, large-scale surface defects are accompanied by much finer-scale topographical features. These characteristics are counter to the “idealized” roughness typically encountered in the laboratory, as discussed in section 1.2. While the original surface scan yielded roughness heights in the tens of microns, the topography was scaled in all three dimensions to yield roughness applicable for study in the present work. This scaling, following both Wu and Christensen (2006a) and Johnson and Christensen (2009), yielded an average peak-to-valley height of this surface of $k_{\text{full}} = 1.35$ mm and a root-mean-square (RMS) roughness height (about the mean elevation) of $k_{\text{rms,full}} = 0.31$ mm. Since $k$ is usually taken as a representative measure of the roughness height (Bons, 2002), then the ratio of the channel half-height, $h = 25.4$ mm, to $k_{\text{full}}$ is $h/k_{\text{full}} = 18.8$ for the present study. This surface is hereafter referred to as the “full surface” upon which the reduced-order models are based.
2.1.1 Singular value decomposition

Singular value decomposition, which is the discrete form of proper orthogonal decomposition (POD), is a mathematical factorization commonly used in data analysis to provide an optimal basis for describing an inhomogeneous signal of interest. In general, POD has been used to obtain approximate, low-dimensional descriptions of turbulent fluid flow (Holmes et al., 1996), structural vibrations (Cusumano et al., 1994; Feeny and Kappagantu, 1998), and damage detection (Ruotolo and Surace, 1999), and is a very useful technique in signal processing, pattern recognition, and image compression (Chatterjee, 2000). Generally speaking, SVD is used to decompose a set of data, like a matrix, into orthonormal basis functions whereby an approximation of that data set can be reconstructed using a finite number of these basis functions. For instance, suppose the function \( \gamma(x, t) \) was approximated for some domain by a finite sum

\[
\gamma(x, t) \approx \sum_{n=1}^{N} a_n(t) \phi_n(x),
\]

where \( a_n(t) \) are the coefficient functions and \( \phi_n(x) \) are the orthonormal basis functions, with the reasonable assumption that the approximation becomes exact when \( N \to \infty \). This approximation is not unique, in that the functions \( \phi_n(x) \) can be determined using Legendre polynomials, Fourier series, Chebyshev polynomials, etc. Singular value decomposition is concerned with finding only one possible choice for the basis functions which yields the best possible approximation for a given modal basis \( N \). Therefore, when using SVD to obtain the basis functions \( \phi_n(x) \), any partial sum of these basis functions will yield the most effective reconstruction of the original signal compared to all other possible basis functions and so the SVD modes are termed optimal in this sense.

For the present study, the fluctuating surface elevation of the full surface, \( \eta_{full}(x, z) \), can be decomposed just as \( \gamma(x, t) \) was decomposed above, in the form

\[
\eta(x, z) = \sum_{j=1}^{L} a_j \phi_j(x, z)
\]

where \( a_j \) are the coefficients of the expansion, \( \phi_j \) represents orthogonal basis functions, and \( L \) represents the total number of basis functions. Using this decomposition, an approximation of the
fluctuating surface elevation can be assembled by retaining the first $M$ basis functions as

$$\eta_{\text{full}}(x, z) \approx \eta_M(x, z) = \sum_{j=1}^{M} a_j \phi_j(x, z)$$  \hspace{1cm} (2.3)$$

where $\eta_M(x, z)$ is referred to as the low-order representation of the full surface based on the first $M$ basis functions, or modes, of the decomposition. Thus, truncation of the original series at $M$ modes where $M \gg L$ effectively low-pass filters the original topography. Because modes $M + 1$ through $L$ have been discarded from the low-order representation, some detail of the original surface topography is lost in the reconstruction. However, these truncated modes $M + 1$ through $L$ can be used to construct a new series representation which embody the residual surface features of a given low-order representation by computing its residual topographical field, $\eta_{M_R}(x, z)$, as

$$\eta_{\text{full}}(x, z) \approx \eta_{M_R}(x, z) = \sum_{j=M_R}^{L} a_j \phi_j(x, z)$$  \hspace{1cm} (2.4)$$

where $M_R$ is the lowest-order mode contained in a given reduced-order model.

The optimal form of the basis functions and resulting coefficients of the expansion in eq. (2.2) are determined using SVD. In particular, this is accomplished by decomposing the topographical information stored in $\eta_{\text{full}}(x, z)$, which can be written in the form of a matrix $A$, as

$$A = U \Sigma V^T$$  \hspace{1cm} (2.5)$$

where $U$ and $V$ are orthogonal matrices, and $\Sigma$ is a diagonal matrix containing the singular values of $A$, $\alpha_n$, arranged by decreasing value. The squares of these singular values, $\lambda_n = \alpha_n^2$, represent the eigenvalues of $AA^T$ or $A^TA$, and the columns of $U$ and $V$ are related to the eigenvectors of $AA^T$ and $A^TA$, respectively. The matrices $U$, $\Sigma$, and $V$ for the present topography were calculated using the built-in SVD command within the MATLAB® software. In doing so, a total of $L = 216$ spatial basis functions were obtained in the decomposition of the complex and irregular full surface presented in figure 2.1. Consequently, the low-order representations of the full surface
topography can then be constructed via SVD as

$$
A_M = U \Sigma_M V^T
$$

(2.6)

where only the first $M$ eigenvalues of $\Sigma$ are embodied in the diagonal matrix $\Sigma_M$. When $M = L = 216$, the original full surface is recovered. Given the fact that the eigenvalues of $\Sigma$ are arranged in descending order, the first $M$ modes represent the most dominant modes of the surface topography, correlating to larger spatial scales contained within the surface elevation. Thus, mode 1 contains the largest-scale surface characteristics while mode 216 embodies only the smallest-scale surface details. Moreover, it can be shown that

$$
(k_{\text{rms}}^M)^2 = (k_{\text{rms}}^\text{full})^2 \frac{\sum_{n=1}^{M} \lambda_n}{\sum_{n=1}^{L} \lambda_n},
$$

(2.7)

where $k_{\text{rms}}^M$ and $k_{\text{rms}}^\text{full}$ represent the low-order representation containing the first $M$ modes and the
rms of the original full surface, respectively. The ratio of sums on the right hand side is referred to as the fractional surface content (FSC) and provides a measure for the amount of detail retained from the full surface in a given low-order representation. Furthermore, it can be shown that

\[
(k_{M_R}^\text{rms})^2 = (k_{\text{full}}^\text{rms})^2 \left(1 - \frac{\sum_{n=1}^{M} \lambda_n}{\sum_{n=1}^{L} \lambda_n}\right),
\]

(2.8)

where \(k_{M_R}^\text{rms}\) is the residual rms containing modes \(M + 1\) through \(L\), and the new factor on the right hand side is the FSC for the residual mode reconstructions, \(M_R\). It should now be clear that upon choosing a mode \(M\), a low-order representation \((1 \rightarrow M)\) or a residual mode representation \((M + 1 \rightarrow L)\) can be constructed. In order to model cumulative damage to a surface, the residual mode representation was invoked, whereby successive models containing an increasing degree of larger-scale surface detail were generated by beginning the mode summation with the highest-order modes followed by successive inclusion of modes containing more large-scale surface characteristics as represented in eq. (2.4). In other words, the evolution of the surface roughness is modeled herein by beginning with a smooth surface, and sequentially increasing the amount of larger-scale detail included until the full surface is recovered. Figure 2.2 presents the relationship between the residual FSC and the number of modes retained within the reconstruction. Considering the FSC trends from right to left, large \(M_R\) indicate very minute FSC, or detail, contained in the low-order representation. As \(M_R\) is decreased, the FSC begins to drastically increase, reconfirming the notion that low mode numbers embody the largest-scale details of the surface elevation. The model of \(M_R = 2\) only contains 70% of the full surface content, even though it is only lacking the first mode. Setting \(M_R = 1\) in eq. (2.4) recovers the full surface, and, referring to figure 2.2, yields, as expected, an FSC of unity. It should be noted that the SVD of the irregular full surface is much more convenient and practical than a Fourier decomposition due the inhomogeneity of the surface topography. In particular, SVD optimizes the surface detail in each basis function while minimizing the total number of basis functions needed to reconstruct the full surface.

Figure 2.3 shows topographical maps of the reduced-order surface models that represent the surfaces chosen for testing in the present study. For the remainder of this text, a reduced-order model will be referred to as a model containing modes \(M_R\) through 216, where \(1 \leq M_R \leq 216\). For example, figure 2.3(g) presents the surface with modes 36 through 216, or \(M_R = 36\). This
Figure 2.3: Topographical maps of the reduced-order models created by SVD. (a) $M_R = 1$ (Full Surface); (b) $M_R = 2$; (c) $M_R = 3$; (d) $M_R = 4$; (e) $M_R = 8$; (f) $M_R = 19$; (g) $M_R = 36$ ($M_R \equiv$ Modes $R - 216$; $FSC \equiv$ Fractional Surface Content (%))

surface contains only very small topographical details when compared to the full surface. Again, as $M_R$ is decreased to include additional modes, it can be seen in the actual surface elevations that the models begin to resemble the character of the full surface with the inclusion of additional larger-scale features, until $M_R = 1$, which is a perfect reconstruction of the full surface.

2.1.2 Roughness model fabrication

Replicas of the surfaces presented in figure 2.3, along with the full surface, were generated using a rapid-prototyping method based on powder deposition. The rapid-prototyping machine used was a ZCorp Spectrum Z510 3D, multi-color printer, operated under the assistance of the Visualization Laboratory within the Imaging Technology Group (ITG) at the Beckman Institute at Illinois. The 24-bit, high-definition printer has a spatial resolution of 80µm and can build up to 2 layers per minute. The machine uses the input of 3D computer data files to create objects using a plaster
and binder method. Working its way from the bottom of the object to the top, the software accompanying the printer decomposes the object into discrete thin layers. Layer by layer, the printer ejects binder from its printer head to create an appropriate 2D cross-section of the object, followed by spreading a thin layer of plaster over the binder. This is repeated for all layers of the object, and when complete, it yields a solid 3D model. The object is removed from the printing stage and air-dusted/brushed to remove excess plaster. The final step requires saturating the object in extremely strong adhesive in order to enhance the strength and rigidity of the final product. Figure 2.4 is a photograph of the rapid-prototyping printer employed in the present work.

The original surface elevation scan was 310 data points in the streamwise direction and 216 data points in the spanwise direction with a spatial data resolution of $\Delta x = 0.2529 \text{ mm}$ and $\Delta z = 0.2512 \text{ mm}$. In order to sufficiently fill the entire $10h$ by $20.25h$ roughness panel area under consideration with this data, a MATLAB code was developed to pattern the data accordingly. After being patterned, the roughness occupied the entire spanwise width of the channel but only extended over a short distance in the flow direction. As such, these experiments represent the abrupt interaction of fully-developed, smooth-wall turbulent channel flow with a short streamwise distance of roughness. Appendix A presents the detailed code used to create the appropriate data files for each roughness representation. The data were patched together by sequentially reflecting the topography across a streamwise axis, followed by a reflection across a spanwise axis, ensuring that the surface elevation was continuous at all data-data patch interfaces. This periodic extension
of the topography is shown more clearly in figure 2.5, where the orientation of the “R” represents the exact orientation employed in the roughness representation. In order to adequately fill the roughness panel with surface elevation data, multiple repetitions of this kind were invoked throughout the entire available surface area, apart from a smooth-wall border around the perimeter of the roughness panel.

These smooth leading and trailing edges of $0.38h$ in streamwise length of each model and positioned at the mean elevation of the roughness were manufactured directly into the roughness panels, allowing clean transition from smooth- to rough- to smooth-wall conditions. Once the appropriate data files were created using the MATLAB code, they then underwent multiple file-type manipulations via software provided by ITG en route to being usable by the printer software to manufacture the surface roughness tiles. Due to size restrictions of the printer, three separate roughness tiles were needed to assemble the full extent of each roughness panel (a single center tile and 2 end tiles). The three roughness tiles which comprise the roughness panel for a given reconstruction were carefully glued to a cast, $1/4''$ thick aluminum plate that was placed along the bottom wall of the channel within a special test section that allowed accurate adjustment of the
vertical position of the roughness relative to the upstream and downstream smooth walls. In this regard, the mean elevation of the roughness is adjusted to be consistent with the upstream and downstream smooth walls to within $\pm 25 \mu m$ (Wu and Christensen, 2006a) using the aforementioned built-in leading and trailing edges as references. Figure 2.6 demonstrates a single roughness tile and the accompanying roughness panel constructed from three separate roughness tiles glued side-by-side. Appendix B details the steps required to transform MATLAB data codes into printable models on the rapid-prototyping machine.

### 2.1.3 Mitigation of surface reflections

When laser light interacted with the roughness surface models, significant reflections off the surface arose. This was a major concern not only for the data accuracy, but also for the safety of the camera employed. Too intense incident laser light can destroy the CCD within the camera, causing streaks of over-saturated pixels. To reduce reflections of incident laser light, Rhodamine B was applied to each surface by means of an airbrushing procedure. Rhodamine B is a chemical compound and a dye, often used as a tracer dye or staining fluorescent dye. The red to violet powder has a maximum emission intensity from 550 – 570 nm when excited by laser light at 532 nm (Natrajan and Christensen, 2009). A solution of 25 g of pure Rhodamine B powder diluted in 1 L of water was
prepared by slowly adding Rhodamine B to a beaker of deionized water, continuously stirred by a magnetic stirrer. Once fully dissolved, the liquid mixture was applied to the roughness surfaces with an airbrush as a thin coat. This airbrushing technique was a particularly convenient method in that it allowed for an extremely thin layer to be applied that did not disrupt any fine-scale features that may have been present on the surface. Several coatings were applied, with ample time given between for drying. With the Rhodamine B now in place, any green laser light that interacted with the roughness excited the fluorescent dye to fluoresce at a higher wavelength than the incident light. A filter placed upstream of the imaging optics suppressed this fluoresced light while only allowing passage of the green scattered light from the PIV particles.

Figure 2.7 illustrates the advantage of Rhodamine B as an effective surface reflection retarder. Clearly, the reflections viewed in figure 2.7(a) compared to that in figure 2.7(b) provide evidence that the Rhodamine B acts to suppress incident laser light reflections on the roughness surfaces. Remaining reflections visible after filtering were contained to the near-wall surface region which was easily removed from the field of view during experiments. This methodology has been shown to provide higher vector yield in the vicinity of the surfaces and therefore improve the accuracy of the turbulence statistics derived from PIV ensembles (Palmer, 2009).
2.2 Particle-image velocimetry

2.2.1 A Brief History of PIV

The study of fluid flow and the forces which act on a fluid element date back to the Ancient Greeks, when Archimedes provided the fundamental principles of hydrostatics and buoyancy in his work “On Floating Bodies”, around 250 B.C. Since then, many brilliant philosophers, mathematicians, and scientists have advanced our knowledge of fluid mechanics through timeless insight and wisdom, providing the foundation and scaffolding of our knowledge-base today. It wasn’t until the late 19th century that scientists began designing carefully planned experiments to extract useful information about a flow using visualization techniques (Raffel et al., 1998). In 1904, Ludwig Prandtl (Anderson, 2005) became one of the first fluid mechanicians to use tracer particles seeded in the flow of a water tunnel as a method of flow visualization. The flow within the open channel water tunnel, which was manually driven by a rotating blade wheel, was visualized by distributing a suspension of mica particles on the surface. Several two-dimensional models like wings and cylinders were placed in the water tunnel which extended above the surface of the water. As the mica particles traveled downstream within the flow, they interacted with the bluff body, following the fluid motion as it traversed the object, whereby observations of the basic features of flow phenomena were made.

The technique used in Prandtl’s experiment provides the essence of particle-image velocimetry (PIV). Early versions of element displacement techniques, originally conceived for transparent solids but later adapted for fluid dynamic motions, were referred to as laser speckle velocimetry (LSV). LSV uses the granular appearance that diffusely reflecting and transmitting surfaces take on when illuminated by a laser beam, which arise due to constructive and destructive interference of coherent light scattered from a surface element whose roughness is large compared to the wavelength of the laser beam, to appropriately measure displacement fields (Krothapalli, 1991). This technique generally requires a very large particle density within the flow to acquire reliable measurements. In the 1980’s, experimentalists found it to be advantageous to reduce the concentration of seeding particles within a flow down to levels where individual particles could be followed. This novel approach of particle displacement tracking has progressed over the last 30 years to what is now used to measure quantitative, 2-dimensional, planar velocity fields in complicated flows. More
Figure 2.8: Typical experimental arrangement for particle-image velocimetry including laser, optics, camera, and seeded flow.

detail on the history of PIV, including the milestones that have enabled new and/or advanced measurements, as well as goals for future advancement, are presented in “Twenty Years of Particle-Image Velocimetry” by R. J. Adrian (Adrian, 2005).

2.2.2 PIV Fundamentals

As alluded to earlier, PIV is a quantitative fluid visualization technique whereby instantaneous velocity fields are captured within a flow. In most applications, tracer particles are added to the fluid which are generally assumed to faithfully follow the flow dynamics. Common particles which are used to seed the flow include atomized oil droplets, oil-based smokes, aluminum flakes, and glass spheres, among others. To capture the velocity of a particle, an image of the particle is taken at time $t_1$. Then, the particle is given a short time to move freely with the flow. Finally, another image of the particle is taken at time $t_2$. The distance the particle traveled in the short time allotted can be understood as it’s velocity by

$$\mathbf{v} = \frac{\Delta x}{t_2 - t_1}, \quad (2.9)$$
where $\Delta x$ is the short distance traveled by the particle. However, because tracking the velocities of micron-sized particles can become rather exhausting, typical applications employ sub-domains in which the field of interest is divided into small regions referred to as interrogation spots (anywhere from $16 \times 16$ to $32 \times 32$ size pixel windows). The average displacement of the group of particles within an interrogation spot, $\Delta X$, can then be used in conjunction with the time interval to cross-correlate a first-order velocity estimate

$$v = \frac{\Delta \bar{X}}{t_2 - t_1}. \quad (2.10)$$

Each interrogation spot within an image is analyzed separately to acquire the entire instantaneous velocity field of the flow. Illuminating the particles as they are imaged is required in order to directly visualize the particles location at that instant. To do this, a laser, which has been optically formed into a plane (light sheet), pulses synchronously with the camera taking the image. The laser sheet position and orientation within the flow is the direct plane in which velocity measurements are acquired. Figure 2.8 shows the experimental arrangement typically used for PIV. A more comprehensive review of PIV can be found in Raffel et al. (1998).

### 2.3 Channel-flow facility and PIV measurements

#### 2.3.1 Channel-flow facility

The channel-flow facility employed in the present effort, a schematic of which is shown in figure 2.9, uses air as the working fluid. It is driven by a 5 hp centrifugal blower, where the speed of the blower is altered by a transmission that is nested between the fan of the blower and the motor. Consequently, the gearing of the transmission provides control over the effective flowrate through the channel. Moreover, electrical power being supplied to the blower is passed through an inverter which provides extremely precise tuning of the flow speed by reducing the frequency of the electricity passed through the motor. Upon exiting the blower, the flow is directed toward a linear contraction section, followed by a series of perforated plates and screens, referred to as the flow-conditioning section. The linear contraction section is coupled to the flow-conditioning section by a flexible coupling which isolates the blower from the rest of the channel, necessary to prevent any significant
Figure 2.9: Schematic of channel-flow facility employed.
vibrations emanating from the blower from propagating downstream toward the test section. The flow-conditioning section ensures a high flow quality at the inlet of the channel by providing a spatially uniform mean velocity distribution across the inlet cross section, as well as a low turbulence level. Within the flow-conditioning section, the flow first passes through a perforated plate which is meant to eliminate any large-scale mean velocity nonuniformities which may lead to the generation of turbulence downstream. The flow is then directed into a 7.2 cm thick honeycomb to minimize lateral velocity fluctuations. The remaining part of the flow-conditioning section consists of three screens, each with a porosity of 62%, that further reduce the level of turbulence prior to entering the contraction section. The contraction section guides the flow smoothly into the development section of the channel, reducing the cross section at a ratio of 8.25 : 1.

As the flow enters the development section, it immediately encounters 36-grit sandpaper on the top and bottom walls of the channel which is used to trip the boundary layers. The sandpaper covers the entire width of the channel (20.25") and extends in the streamwise direction 9" into the development section. A boundary-layer trip is employed in order to “excite” the boundary layers into an immediate transition upon entering the channel to ensure a fully-developed flow at the test section. The development section of the channel flow facility is made of plexiglass, and is 216"
long (216h) with a cross section of 2'' × 20.25''. The uncertainty in the inner dimension is less than or equal to 0.001'' (0.03 mm). Once the flow traverses the development section, it then confronts the test section. The test section is 6' long (including an additional 3' development section) and contains glass inserts on all sides for optical measurements. The test section also houses a test panel stage where the roughness panels are inserted during experiments. Finally, the flow exits the test section where it is immediately directed into a return section, and sent back to the blower, completing the closed-loop system. Figure 2.10 presents a photograph of the 216h-long channel with the test section evident in the foreground. In particular, the dark region in the test section represents the spatial footprint of the short fetches of roughness tested in the present effort.

The closed-loop nature of the channel-flow facility allows for a steady-state temperature to eventually be achieved. This temperature was monitored using a T-type thermocouple inserted into the flow through the channel wall downstream of the test section, digitally output to an Omega Model DP462 thermocouple reader. This temperature, along with the atmospheric pressure, was used to determine the density of the air, \( \rho \), using the ideal gas law

\[
\rho = \frac{pM}{RT},
\]

where \( p \) is the pressure, \( M \) is the molar mass (\( M = 29 \text{ g/mol for air} \)), \( R \) is the universal gas constant, and \( T \) is the temperature. The kinematic viscosity, \( \nu \), is determined from Sutherland’s correlation

\[
\mu = \mu_0 \left( \frac{T}{T_0} + C \right) \left( \frac{T}{T_0} \right)^{3/2},
\]

where \( \mu_0 = 18.27 \times 10^{-6} \text{ Pa-s}, T_0 = 291.15 \text{ K}, C = 120 \text{ K}, \) and \( \nu = \mu/\rho \). Three static pressure taps, mounted along the length of the channel, were used to measure the streamwise pressure distribution within the channel. The taps were located upstream of the test section along the ceiling wall of the channel near the spanwise centerline, with 36'' spacing between them. The pressure taps were monitored using a Validyne Model DP45-20 pressure transducer coupled with a Validyne Model CD23-A-1-A-1-C digital voltage output, and recorded using LabVIEW. Once the pressure data was acquired, the streamwise pressure gradient, \( dp/dx \), was calculated by fitting a line to the pressure data profile. With this information, the wall shear stress, \( \tau_w \), of the smooth-wall flow upstream of
the test section was calculated by

\[ \tau_w = -\frac{dp}{dx}h. \]  

(2.13)

The upstream smooth-wall friction velocity was then calculated as

\[ u_{\tau}^{SW} \equiv \sqrt{\frac{\tau_w}{\rho}}. \]  

(2.14)

Measurements were made over each surface at approximately the same friction Reynolds number \( \text{Re}_\tau \) of 1825, where

\[ \text{Re}_\tau = \frac{u_{\tau}^{SW}h}{\nu} \]  

(2.15)

based on the friction velocity of the smooth-wall flow upstream of the roughness.

The channel-flow facility electrical inverter was initially adjusted to 50 Hz at the beginning of each experiment. Once the temperature within the channel reached an equilibrium, the pressure drop was calculated followed by \( \text{Re}_\tau \). If \( \text{Re}_\tau \) was significantly distant from the objective of 1825, the inverter frequency was adjusted, and the \( \text{Re}_\tau \) calculation process was repeated. After successfully obtaining the proper \( \text{Re}_\tau \), the viscous length scale was then calculated by

\[ y_* = \frac{\nu}{u_{\tau}}, \]  

(2.16)

which refers to the smallest eddy-scale persistent in the flow. An Excel spreadsheet was designed to aid in the organization and calculation of important constants and flow parameters for each case. Appendix C presents an example of the Channel-Flow Experimental Data spreadsheet employed. Table 2.1 summarizes the relevant parameters for each case under consideration.

### 2.3.2 PIV Setup and Measurements

Large ensembles of instantaneous two-dimensional velocity fields \((u, v)\) were acquired over a \(1.3 \times 1.0h^2\) field of view in the streamwise–wall-normal \((x-y)\) plane of the flow, situated approximately at the spanwise centerline of the channel. The air was seeded with \(1 \mu\text{m}\) droplets of olive oil generated by a Laskin nozzle (Meinhart, 1994; Kahler et al., 2002). This nozzle consists of a container of olive oil, a pressurized-air feed line, and an exit line for the atomized oil. The feed line was supplied
Table 2.1: Summary of parameters for all experiments.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Re&lt;sub&gt;r&lt;/sub&gt;</th>
<th>u&lt;sub&gt;SM&lt;/sub&gt;, m/s</th>
<th>y&lt;sub&gt;SM&lt;/sub&gt;, µm</th>
<th>FSC (%)</th>
<th>No. of realizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth (SW)</td>
<td>1821</td>
<td>1.20</td>
<td>14.0</td>
<td>-</td>
<td>4996</td>
</tr>
<tr>
<td>MR = 36</td>
<td>1822</td>
<td>1.18</td>
<td>13.9</td>
<td>3</td>
<td>6989</td>
</tr>
<tr>
<td>MR = 19</td>
<td>1823</td>
<td>1.16</td>
<td>13.9</td>
<td>7</td>
<td>6396</td>
</tr>
<tr>
<td>MR = 8</td>
<td>1817</td>
<td>1.18</td>
<td>14.0</td>
<td>21</td>
<td>6892</td>
</tr>
<tr>
<td>MR = 4</td>
<td>1823</td>
<td>1.17</td>
<td>13.9</td>
<td>41</td>
<td>6991</td>
</tr>
<tr>
<td>MR = 3</td>
<td>1819</td>
<td>1.16</td>
<td>14.0</td>
<td>49</td>
<td>6995</td>
</tr>
<tr>
<td>MR = 2</td>
<td>1823</td>
<td>1.18</td>
<td>13.9</td>
<td>70</td>
<td>6988</td>
</tr>
<tr>
<td>MR = 1 (Full)</td>
<td>1815</td>
<td>1.17</td>
<td>14.0</td>
<td>100</td>
<td>6988</td>
</tr>
</tbody>
</table>

with 80 psi of compressed air which was purged just below the surface of the olive oil through four small holes. This ejection of air atomizes the surrounding olive oil, forming an olive oil cloud above the liquid olive oil interface. The exit line supplies seeding to the channel just prior to the flow entering the blower. The supply pressure of 80 psi was found by trial and error until a proper seeding density could be maintained within the channel. Proper seeding density is paramount as PIV images with a low seeding density will lack sufficient particle displacement information within the entire flow field required to yield accurate velocity fields when cross-correlating image frames. In contrast, PIV images with a high seeding density fail to correlate individual particles from one frame to the next (Keane and Adrian, 1992). The olive-oil tracer particles were illuminated with a light sheet formed with a Quantel Big Sky dual-cavity Nd:YAG laser operating at 3 Hz with a 5 ns pulse duration. By incorporating several lenses in the optical path, the narrow, cylindrical laser beam profile exiting the laser head was optically modified to form a plane light sheet. Figure 2.11 shows a photograph of the optical setup employed in the present experiments. A spherical lens (SL) with a focal length of 80 cm, which thinned the light sheet, was followed by a cylindrical lens (CL) with a focal length of −30 mm, which fans out the beam profile to form the light sheet. A 90° right-angle prism situated above the channel-flow facility test section redirected the thin laser sheet, originally in the x–z plane, downward into the x–y plane, along the spanwise centerline of the channel.

Scattered light from the tracer particles were imaged with a TSI PIVCAM 13-8 12-bit, frame-straddle CCD camera with a 1280 × 1024 pixel array equipped with a Sigma 105 mm lens, visible in figure 2.10. Placement of the camera was chosen in order to position the field of view near
the downstream end of the surface roughness panel. A laboratory jack, as well as streamwise and spanwise slide rails, permitted adequate translation of the camera, ensuring proper placement. The laboratory jack provided wall-normal translation required to eliminate scattered light off the surface from entering the CCD of the camera by raising the field of view relative to the channel bottom until harsh reflections were eliminated. Before each experiment, the laser was turned on and the reflections were visualized by the camera, which was set to a very low aperture to avoid damage to the CCD. Adjustments to the camera height were carried out until manageable reflections were present. To determine the relative height of the field of view with respect to the channel bottom, as well as calculate the image magnification, a target was placed in the channel at the laser sheet location in the $x - y$ plane, then focused on by the camera.

Figure 2.12 presents a schematic of the target employed. Knowing the distance from a point
on the target to the channel bottom (\( \Delta y \)), the camera offset was calculated by subtracting the distance from this point to the bottom edge of the field of view (\( \delta \)) from \( \Delta y \),

\[
\text{Camera Offset} = \Delta y - \delta. \tag{2.17}
\]

This simple calculation was required to properly shift the acquired data to the appropriate wall-normal location. The magnification was calculated by again placing the target at the exact location of the laser sheet plane, then determining the pixel separation (\( \Delta \psi \)) between two dots on the target, for instance, between the dots offset from each other by 10 mm, in both the \( x \) and \( y \) directions. The magnification can then be determined by

\[
\text{Magnification} = \frac{10}{\Delta \psi} \left( \frac{\text{mm}}{\text{pixels}} \right), \tag{2.18}
\]

and can be averaged over both directions. To maintain consistency, these calculations were performed both before and after each experiment.

TSI's Insight 6 software was used for all image acquisition. Rough surface experiments typically
Table 2.2: Summary of PIV Processor and Validation settings within Insight 3G.

required near 7,000 image-pairs for data convergence, though only about 5,000 were required for
the smooth-wall experiments. Due to the lack of interrogating and validating options within Insight
6, Insight 3G was employed to carry out these actions. In order for Insight 3G to identify the image
files acquired by Insight 6, file names were adjusted using Bulk Rename Utility, a small program
downloaded off the internet which can quickly and easily alter file names in large quantities. Insight
3G was setup to run a two-pass cross-correlation, recursive scheme, employing a 50% window
overlap. The recursive scheme is meant to lower the RMS error typically acquired from cross-
correlation. Table 2.2 summarizes the specific parameters used to interrogate and validate the PIV images within Insight 3G. For a more rigorous explanation of PIV parameters and their subsequent usability, refer to Raffel et al. (1998).

2.3.3 Experimental procedure

The following section highlights the step-by-step procedure followed during all experiments.

1. The top panel of the test section was removed in order to clean all surfaces including the optical glass windows.

2. Prior to placing the roughness panel in the channel, the anodized aluminum smooth-wall panel was placed into the test panel stage, followed by accurately leveling the panel using the adjustment screws located underneath the channel.

3. The laser was turned on and the power was adjusted internally through Insight 6 to medium (Q-switch time set to $\approx 270 \mu s$).

4. A small mirror was placed on the aluminum smooth-wall panel at the location of the laser sheet. The laser was pulsed at approximately 3 Hz and the reflections were visualized. The reflection of the laser sheet should appear near the laser head pulse exit, confirming optical alignment and proper prism rotation. Adjustments were made if necessary.

5. With the laser pulsing, burn paper was placed on the smooth-wall panel at the location of the laser sheet in order to check the burn pattern and verify the laser sheet thickness ($\leq 1 \text{ mm}$ desired).

6. All optical lenses and prism were dusted with cotton swabs to eliminate debris.

7. A specially fabricated 1/4” thick, 20.25” spanwise width, aluminum insert was placed in the channel bottom. A line was marked on the panel that was parallel to the channel side-walls, and at the exact location where the laser sheet should be. When pulsed, the laser sheet was compared to the marked line, and adjusted if needed.
8. The aluminum smooth-wall panel was removed (unless being tested), and the roughness panel being tested was then placed in the test panel stage, then accurately leveled using adjustment screws.

9. The roughness panel was slid forward, in the upstream direction, to ensure no air gap existed between the bottom wall of the channel and the roughness panel.

10. Insight 6 software was opened, the camera was powered on, and the laser was pulsed.

11. The camera was translated in the positive wall-normal direction (toward the top of the channel) in order to remove the roughness from the field of view.

12. The camera’s lens cap was removed. Then, slowly, the camera was translated downward, in the wall-normal direction, until slight reflections due to incident laser light were identifiable in the field of view. The laser was then turned off.

13. The same aluminum insert was again placed in the channel, just downstream of the roughness panel. This time, the target shown in figure 2.12 was placed on top of the insert, facing the camera, adjusted to the location of the laser sheet.

14. Slight adjustments to the camera’s focus were performed in order to focus the target. An image was then captured to calculate and record the camera offset and magnification. The aluminum insert and target were then removed.

15. The top panel of the test section was then placed back on the channel, sealing the channel flow facility for testing.

16. The channel-flow facility was turned on and set to a frequency of 50 Hz, then allowed to run for approximately 3 hours. This allowed the air temperature within the channel to reach equilibrium. A frequency of 50 Hz, historically, produced a $Re_\tau$ of approximately 1825, so was the chosen starting frequency.

17. To check the $Re_\tau$, the temperature value on the digital thermocouple reader was recorded, then a pressure drop measurement was taken. Each pressure tap was sequentially plugged into the pressure transducer. For each pressure tap, LabVIEW recorded 30 s of pressure data.
(millivolts) at 1,000 Hz. This data was then averaged to acquire a single pressure reading. The 3 pressure readings were then fit to a linear curve in order to extract the pressure drop. The temperature, along with pressure drop, were used to find Re_τ. If significantly different from the desired 1825, the motor’s electric inverter was adjusted accordingly, by either speeding up or reducing the blower’s speed.

18. The channel was then given thirty minutes to reach a temperature equilibrium, and Re_τ was again calculated as before. This process was repeated until Re_τ ≈ 1825 was attained.

19. Compressed air (80 psi) was then supplied to the Laskin nozzle to commence the seeding of the flow with olive oil particles.

20. Insight 6 was used to monitor the seeding density, and any modifications to supply pressure and/or air flow rate were made at this time.

21. Insight 6 was programmed to capture the appropriate image quantity discussed in table 2.1, then executed.

22. Proceeding the conclusion of the experiment, the camera’s lens cap was returned, and the laser and camera were powered down.

23. The channel’s blower was then turned off.
Chapter 3

Results

In this chapter, the results of two-dimensional PIV experiments conducted in the streamwise-wall-normal \((x-y)\) plane, over the aforementioned rough surfaces are presented. For most plots in this chapter, coordinate directions are normalized by \(h (x_i/h)\). Other plots, such as mean velocity, have been normalized in the wall-normal direction by \(y^{SW}_s (y^+ = y/y^{SW}_s)\), in order to remain historically consistent with plots that report mean velocity data in the viscous sub-layer, the log-law region, and wake region, allowing for meaningful comparison. The 7,000 PIV images per roughness condition (5,000 for smooth-wall) were ensemble-averaged, denoted by angle brackets \(\langle \cdot \rangle\), whereby maps of \(u\) and \(v\) velocity components of the flow were extracted. Mean velocity components, velocity fluctuations, and vorticity magnitudes are all separately calculated and employed in further analyses which explore turbulent stress fields, swirling fields, and probability studies within the flow. Mean velocities and turbulent stresses (Reynolds stresses) presented in this chapter are normalized by the upstream, smooth-wall friction velocity, \(u^{SW}_\tau\), and bear a superscript + to make this inner scaling distinction. This velocity normalization is utilized since a measure of the drag on the short fetches of roughness was not available, meaning an accurate estimation of the rough-wall friction velocity could not be made.

3.1 Instantaneous velocity realizations

PIV data can be displayed as a vector field, yielding one vector per interrogation spot, comprising of both \(u\) and \(v\) velocity components. Figure 3.1 shows a random instantaneous velocity realization chosen from the set of 4,996 realizations acquired from the smooth-wall model experiment. A constant convection velocity, \(U_c\), is subtracted from the streamwise velocity in order to reveal vortex core structures advecting with the flow. Known as Galilean decomposition, this technique allows
Figure 3.1: Instantaneous PIV velocity realization of turbulent channel flow over the smooth-wall model with a constant convection velocity, $U_c = 0.83U_{CL}$, removed.

the viewer to “travel” with the vortex structures, exposing structural characteristics embedded in the flow previously unidentifiable (Adrian et al., 2000a). This advection velocity is typically taken as a fraction of the freestream or centerline velocity as

$$U_c = \alpha U_{CL}, \quad (3.1)$$

where $0 < \alpha \leq 1$ (Adrian et al., 2000a). Because vortex structures may advect at different speeds relative to one-another, $\alpha$ varies from field to field. For instance, in figure 3.1, the convection velocity was found to be $0.83U_{CL}$.

Several distinct characteristics, mentioned previously in section 1.3.2, can be identified in figure 3.1, where flow is from left to right:

- Several vortex cores, rotating in a clockwise fashion, are circled in blue. These vortex cores can be interpreted as a two-dimensional, streamwise–wall-normal cross-view of hairpin vortex heads, advecting together in a hairpin vortex packet in the streamwise direction (Zhou et al., 1997; Adrian et al., 2000b). Other smaller vortex cores are noticeable within this packet,
Figure 3.2: Instantaneous PIV velocity realization of turbulent channel flow over the 49% roughness model with a constant convection velocity, $U_c = 0.81 U_{CL}$, removed.

near the wall.

- The red line indicates the outer edge of the hairpin vortex packet. The angle of inclination of this line, relative to the wall, is approximately $18^\circ$, consistent with previous results (Zhou et al., 1999; Adrian et al., 2000b; Christensen, 2001).

- A region of slower-moving, low momentum fluid is apparent below the inclined interface, identifiable by negative-pointing velocity vectors. Further, each vortex core induces a strong ejection event (flow in the positive wall-normal, negative streamwise direction) just upstream and below its head which have been found to significantly contribute to the mean Reynolds shear stress (Ganapathisubramani et al., 2003; Wu and Christensen, 2010).

Figure 3.2 presents an instantaneous velocity realization chosen from the set of 6,995 realization acquired from the 49% FSC roughness model experiment. Although similar in terms of vortex core organization and evidence of low momentum regions near the wall, figure 3.2 differs from figure 3.1 in that the angle of inclination of the vortex heads is significantly more shallow ($\approx 12^\circ$) for the 49% case. This observation is consistent with past work done by Wu and Christensen (2005) and Wu
Figure 3.3: Instantaneous PIV velocity realization of turbulent channel flow over the 100% roughness model with a constant convection velocity, $U_c = 0.83U_{CL}$, removed.

and Christensen (2010), indicating that increased surface roughness tends to slightly decrease the inclination angle of vortex packets in wall turbulence.

Figure 3.3 presents an instantaneous velocity realization chosen from the 100% roughness model experiment. Again, similar characteristics of vortex organization, as well as regions of low momentum are evident. However, an even shallower inclination angle is present ($\approx 10^\circ$), due to the increase surface roughness content within the 100% model.

### 3.2 Mean velocity

Figure 3.4 presents profiles of mean streamwise velocity plotted in inner units ($U/u_{*}^{SW}$ versus $y/y_{*}^{SW}$) for all rough-wall cases as well as the smooth-wall result. All of the roughness models, regardless of surface content, show a deficit in the mean streamwise velocity compared to the smooth-wall baseline. The wall-normal extent of this deficit ($y^+ < 500$) can be interpreted as the internal layer, where within this layer, the fluid flow is retarded due to the rough surface. This mean streamwise velocity deficit increases as $FSC$ (residual fractional surface content) increases from 3%
Figure 3.4: Mean streamwise velocity profiles plotted in inner units (normalized by $u_{\tau}^{SW}$ and $y_{+}^{SW}$).

to 70%. In particular, the profiles for models 3% and 7%, 21% and 41%, and 49% and 70% collapse well with each-other, indicating that these model pairs produce very similar impacts on the mean velocity profile. The 100%, or full surface, profile shows a slightly lower deficit compared to the 49% and 70% profiles. These differences can be rationalized by considering the reconstruction process by which the full surface is decomposed and then reassembled to create subsequent reduced-order models. It should be noted that the average roughness height of each model was set as the global origin of the roughness panel, which was then aligned with the smooth wall in the channel when tested. The full surface model consists of all 216 modes created by SVD, whereas the 70% model consists of modes 2 – 216. Therefore, only mode 1 differentiates the 70% and 100% models. Mode 1 comprises the largest-scale, lowest-frequency topographical features, which, when utilized in the 100% reconstruction, skews the topography of the 70% model at a slight angle within the field of view. To gain a better understanding of how these surface topographies are interacting with
the mean velocity, two-dimensional contour maps were developed for both mean streamwise and wall-normal velocities. Figure 3.5(a)–(h) present contour plots of ensemble-averaged streamwise velocity normalized by $u_{r}^{SW}$ for all rough- and smooth-wall cases. The slight angle, or ramp, in the streamwise direction, is clearly evident in figure 3.5(h) where the topographies for each model have been included beneath the contour maps of the ensemble-averaged streamwise velocity. The difference in topographical features between the 49% [figure 3.5(f)] and 70% [figure 3.5(g)] models is meager, consistent with the similarity in their mean velocity profiles. Figure 3.6(a)–(h) present ensemble-averaged wall-normal velocity contour maps normalized by $u_{r}^{SW}$ for all rough-wall and smooth-wall cases. The 100% [figure 3.6(h)] case reveals a significant increase in mean wall-normal velocity above this ramp compared to the mean wall-normal velocity maps of the others, particularly the 70% model. This behavior is consistent with the inclination notable in the 100% surface due to the addition of mode 1 to the topography of the 70% case.

A consequence of this topographical skew that occurs when mode 1 is added to the 70% case is that within the field of view the local origin, or local average roughness height, is shifted downward in the 100% case compared to the global origin taken to be coincident with the upstream smooth wall. Thus, since the global origin was the baseline for all wall-normal measurements, the surface roughness on the 100% model appears to affect the flow less than the 49% and 70% models. Referring back to figures 3.5(f),(g), and (h), which present ensemble-averaged streamwise velocity contour maps for the 49%, 70%, and 100% models, respectively, the field of view for all three cases have a lower wall-normal extent of approximately $y/h = 0.05$. But, because of the local origin shift downward for the 100% model, the surface roughness is shifted downward relative to the field of view compared to the 49% and 70% models.

### 3.3 Reynolds stresses

#### 3.3.1 Reynolds normal stresses

Figures 3.7 and 3.8 present profiles of the streamwise and wall-normal Reynolds normal stresses, $\langle u'^{2} \rangle$ and $\langle v'^{2} \rangle$, respectively, normalized by $(u_{r}^{SW})^{2}$. Enhancement of the Reynolds normal stresses is clearly evident with increasing $FSC$, with an almost 50% increase in $\langle u'^{2} \rangle^+$ and nearly 40%
Figure 3.5: Contour plots of ensemble-averaged streamwise velocity normalized by $u_{r}^{SW}$. 
Figure 3.5: continued
Figure 3.5: continued
Figure 3.5: continued
Figure 3.6: Contour plots of ensemble-averaged wall-normal velocity normalized by $u_{\tau}^{SW}$. 
Figure 3.6: continued
Figure 3.6: continued
Figure 3.6: continued
Figure 3.7: Profiles of streamwise Reynolds normal stress normalized by $(u_{SW}^2)^2$.

Figure 3.8: Profiles of wall-normal Reynolds normal stress normalized by $(u_{r}^{SW})^2$. 
increase in \( \langle v'^2 \rangle^+ \) for the full surface relative to the smooth-wall baseline. Of interest, the 3% model shows a significant enhancement in both \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \) compared to the smooth-wall case. The average peak-to-valley height of this surface is approximately 200 \( \mu \text{m} \), which translates to approximately \( 14y_{SW}^* \). Thus, although the characteristic roughness height of the 3% surface is small compared to the other cases, it is not expected to behave aerodynamically smooth. As such, the observed enhancement of \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \) highlights how even weak surface defects can fundamentally alter the flow and therefore adversely impact performance. As was noted with the mean velocity in section 3.2, the similar topographical features embodied in the 49% and 70% models yield \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \) profiles that are comparable, along with the 41% model, which also possesses similar topographical features. The wall-normal extent of the internal layer formed by the roughness, measured from both \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \), is fairly consistent for all FSC cases, located at approximately \( y/h = 0.45 \). The wall-normal location of the peak stress value tends to shift in the direction toward the center of the channel as the FSC increases, shifting nearly 0.05\( h \) for \( \langle u'^2 \rangle^+ \) and 0.03\( h \) for \( \langle v'^2 \rangle^+ \).

To analyze the Reynolds normal stresses further, figures 3.9(a)–(h) and 3.10(a)–(h) present two-dimensional contour plots of streamwise and wall-normal Reynolds normal stresses, respectively, for all smooth- and rough-wall cases, normalized by \( (u'^{SW}_\tau)^2 \). Near the wall, in a region below \( y/h = 0.2 \), a strong increase in \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \) is again evident as FSC is increased from the SW to 100% case. An interesting trend evident in both \( \langle u'^2 \rangle^+ \) and \( \langle v'^2 \rangle^+ \) in the sequence of contour plots from the SW to 100% case is an apparent creep of higher Reynolds normal stress from upstream to downstream (left to right). In the 21% case, a region of high Reynolds normal stress begins to emerge, extending to \( x/h = 0.2 \) in the streamwise direction. However, for the full surface, this region of high Reynolds normal stress develops in the streamwise direction to approximately \( x/h = 0.9 \). Further, a shallow angle in this region of high Reynolds normal stress is faintly noticeable. This angle, which measures roughly 5° relative to the \( y = 0 \) line, is apparent in each of the rough-wall models but does not appear within the smooth-wall model.
Figure 3.9: Contour plots of ensemble-averaged streamwise Reynolds normal stress normalized by $(u''_{SW})^2$. 
Figure 3.9: continued
Figure 3.9: continued
Figure 3.9: continued
Figure 3.10: Contour plots of ensemble-averaged wall-normal Reynolds normal stress normalized by $(u_r^{SW})^2$. 
Figure 3.10: continued
Figure 3.10: continued
Figure 3.10: continued
3.3.2 Reynolds shear stress

Similar trends are observed in profiles of the Reynolds shear stress (RSS), $-\langle u'v' \rangle$, normalized by $(u_{rSW})^2$, shown in figure 3.11. The smooth-wall profile displays the characteristic linear profile in the outer region of the flow along with a distinct peak near $y/h = 0.1$. In contrast, the RSS profile for flow over the 100% model exhibits significant enhancement, having a peak value roughly 70% larger than the smooth-wall baseline. As with the Reynolds normal stresses, the wall-normal location of this peak value shifts away from the wall with increased FSC, beginning at $y/h = 0.1$ for the smooth-wall case and ceasing near $y/h = 0.125$ for the full-surface case. Farther from the wall, the RSS profiles for all the FSC models collapse well with the smooth-wall result for $y/h > 0.45$, consistent with the wall-normal location of collapse seen for all the FSC models in the Reynolds normal stresses and indicating the outer-most extent of the flow to which the roughness effects have propagated. Although the 70%, 49% and 41% models show a slight deviation in peak value compared to one-another, marginally further from the wall, these profiles exhibit similar behavior,
coinciding with the aforementioned topographical similarity between these model surfaces.

Two-dimensional contour plots of RSS, normalized by $(u'_{SW})^2$, are presented in figure 3.12(a)–(h) for all smooth- and rough-wall surface models. In a region below approximately $y/h = 0.3$, RSS shows a significant enhancement for much of the field of view in the streamwise direction as $FSC$ is increased from the smooth-wall to 100% case. As was noted with the contour plots of $\langle u'^2 \rangle^+$ and $\langle v'^2 \rangle^+$, the contour plots of $-\langle u'v' \rangle^+$ demonstrate a similar drift in a region of high RSS in the streamwise direction as $FSC$ is increased, beginning at $x/h = 0.22$ for the 7% case, where the region of high RSS first appears, progressing through the entire streamwise field of view for the full surface. In addition, the contour plots of $-\langle u'v' \rangle^+$ for the rough-wall cases, depict a similar angle ($\approx 5^\circ$) in the region of high RSS with respect to the wall, reported previously in the contour plots of $\langle u'^2 \rangle^+$ and $\langle v'^2 \rangle^+$.

To illustrate the origin of these observations, consistent in both Reynolds normal stresses and RSS, the rough-wall topography positioned just upstream of the field of view was analyzed. Figure 3.13 presents the RSS contour plot associated with the full surface model, along with the upstream surface topography experienced by the flow just prior to entering the field of view. Of interest, a large-scale topographical feature is evident just upstream of the field of view, the center of which is located at approximately $x/h = -0.2$. Furthermore, due to the nature of how the surface was designed, and due to the fact that this large-scale feature resides at the end of a patch of surface topography data, the large-scale feature is reflected upstream, producing two large-scale features in sequence. As such, this grouping of large-scale topographical features immediately preceding the field of view, provides an explanation for rationalizing the unique trends observed within the contour plots of $\langle u'^2 \rangle^+$, $\langle v'^2 \rangle^+$, and $-\langle u'v' \rangle^+$. These large-scale features are obviously most pronounced in the full surface, giving reason to why this surface was utilized in depicting their effect on Reynolds stresses. However, these features gradually reduce in size as $FSC$ is decreased from the 100% case down to the smooth-wall case, lending good reason to why a region of high Reynolds stress shows an apparent drift in the streamwise direction as $FSC$ is increased. This large-scale surface feature, which undoubtedly nudges the oncoming flow in the wall-normal direction, also provides an explanation for the slight angle at which this region of high Reynolds stresses resides.
Figure 3.12: Contour plots of ensemble-averaged Reynolds shear stress normalized by $(u_{SW}^{*})^2$. 

(a) 

(b)
Figure 3.12: continued
Figure 3.12: continued
Figure 3.12: continued
Figure 3.13: The upstream surface topography of the full surface experienced by the flow just prior to entering the field of view where a contour map of the full-surface RSS is presented.
3.3.3 Peak Reynolds stress trends

The Reynolds normal and shear stress profiles presented in section 3.3 clearly support the idea that increased FSC yields higher Reynolds stresses. Keeping with the theme of cumulative damage, which, at least in spirit, can be correlated to FSC, one might ask how peak Reynolds stresses are affected by an increase in the FSC. Therefore, observations can be made by comparing the peak Reynolds stress values \((u'\|^2, v'^2, \text{ and } u'v')\) accompanying each surface roughness, with the FSC of that particular roughness. As such, these trends might reveal the relative importance of the addition of FSC on the largest observable Reynolds stress values. Figure 3.14 presents the peak Reynolds stress values for each surface roughness normalized by the peak Reynolds stress values associated with the smooth-wall baseline. This relationship yields the relative enhancement in Reynolds stresses as FSC is increased with respect to the smooth-wall. Interestingly, a power-law trend, also included in the figure, is noticeable for each of the Reynolds stresses, whereby the peak value increases rapidly at low FSC, then begins to increase at a much slower rate at subsequent higher FSC. The one exception to this fit seems to be the 100% model, which, for all Reynolds
Figure 3.15: Maximum Reynolds stress values of each surface roughness normalized by the maximum Reynolds stress values for the full surface case.

stresses, shows a value higher than what is predicted by the power-law trend.

Figure 3.15 presents the dependence of the peak Reynolds stress values normalized by the peak Reynolds stress values associated with the full surface. Again, the data points, regardless of Reynolds stress orientation, reveal a trend closely fit by a power-law curve, included within the figure. Also, as was seen in the previous figure, the 100% model, for all Reynolds stresses, is slightly higher than anticipated by the power-law curve. However, one might expect a somewhat different curve-fit between the 70% and 100% models due to the process in which they were created by virtue of mode 1 of the SVD embodying only the largest-scale topographical features. The power-law trend observed in each figure is interesting in that it suggests that even weak surface damage that a practical flow surface might endure within the first fraction of its deployment lifetime can significantly enhance turbulence and therefore progressively impact system performance.
3.3.4 Probability density functions of Reynolds shear stress

A probability density function (PDF) is a statistical measure that defines a probability distribution for a random variable. In other words, it is a function that describes the relative likelihood for a specific instance of a random variable to occur at a given point in some pre-determined space. Figure 3.16(a)–(d) presents PDFs of the $u'v'$ events that contribute to the mean RSS, $\langle u'v' \rangle^+$, for flow over all of the smooth- and rough-wall surfaces considered at various wall-normal locations. As a means of assessing modifications to the intensity of the instantaneous $u'v'$ events for flow over the full surface, along with the flow over the smooth-wall and intermediate $FSC$ representations, $u'v'$ is normalized by $(u'^{SW})^2$. In general, the PDFs are prominently skewed toward negative values for all smooth- and rough-wall cases, primarily close to the wall. This observation is consistent with the sign (negative) of the mean RSS, demonstrative of the dominant contributions of ejections ($u' < 0, v' > 0$) and sweep events ($u' > 0, v' < 0$) over inward ($u' < 0, v' < 0$) and outward ($u' > 0, v' > 0$) interactions, to the overall Reynolds stresses (both in magnitude and frequency of occurrence). Interestingly, enhancement of both negative and positive $u'v'$ events is observed for the rough surfaces with respect to the smooth-wall baseline, both close to the wall at $y = 0.1h$ and further from the wall at $y = 0.2h$. At a wall-normal position of $y = h$, the centerline of the channel, the PDFs become symmetric, displaying the expected flow symmetry of turbulent channel flow.

Figure 3.16(a) presents PDFs at $y = 0.1h$, which, when referring to figure 3.11, corresponds to a wall-normal location where increased $FSC$ displays an enhancement to $-\langle u'v' \rangle^+$ compared with the smooth-wall baseline. Consistent with this tendency, the PDFs of $u'v'$ at $y = 0.1h$ show a progressive increase in the skew of the negative tails as $FSC$ content is increased from the smooth-wall case to the 100% case. As such, this represents the progressive increase of intense ejection and sweep events as $FSC$ is increased. Advancing in the wall-normal direction, the PDFs consistently show enhancement of $u'v'$ events as $FSC$ is increased. However, the magnitude of such events diminish as the centerline of the channel is approached, whereby the $u'v'$ PDFs for the various surfaces begin to show agreement at $y \geq 0.45h$, the location of profile collapse in figure 3.11 and hence the outer extent of the internal layer formed due to the abrupt transition from smooth- to rough-wall conditions.
Figure 3.16: Probability density functions of the $u'v'$ events that contribute to the mean RSS. (a) $y = 0.1h$; (b) $y = 0.2h$; (c) $y = 0.5h$; (d) $y = h$
Figure 3.16: continued
3.4 Quadrant analysis

To further evaluate the production of Reynolds-stress-producing events, and to identify the dominant contributors to the generation of RSS, the instantaneous $u'v'$ events were differentiated between which quadrant, $Q$, in the $u' - v'$ plane they resided in. It was observed in figures 3.16(a)–(d) that a progressive increase of $FSC$ was associated with an increase in the generation of negative $u'v'$ events. As mentioned earlier, four types of $u'v'$ interactions contribute to Reynolds shear stress:

- **Outward Interactions**
  \[ Q_1 : u' > 0, v' > 0 \]

- **Ejections**
  \[ Q_2 : u' < 0, v' > 0 \]

- **Inward Interactions**
  \[ Q_3 : u' < 0, v' < 0 \]

- **Sweeps**
  \[ Q_4 : u' > 0, v' < 0 \]

The method of quadrant analysis, first proposed by Wallace et al. (1972) and Lu and Willmarth (1973), is meant to separate the RSS-producing $u'v'$ events into each of the aforementioned quadrants. Clearly, the negative sign associated with the mean RSS indicates that ejections and sweeps must dominate over inward and outward interactions in contributing to the mean RSS. However, it remains unclear whether ejections, sweeps, or both, heavily contribute to the intense negative $u'v'$ events recognized in the PDFs of section 3.3.4 and the associated enhancement of negative mean RSS in figures 3.11 and 3.12. Quadrant analysis was performed on each of the smooth- and rough-wall surface models in order to understand the impact that an increase in $FSC$ may have on the specific generation and distribution of RSS events.

When employing quadrant analysis, the mean RSS, at a chosen wall-normal location, is decomposed into contributions from the above four quadrants ($Q = 1 - 4$), excluding a hyperbolic hole size $H$, as

\[
\langle u'v' \rangle_Q (y; H) = \frac{1}{P} \sum_{i=1}^{P} u'(x_i, y)v'(x_i, y)I_Q(x_i, y; H),
\]

where $P$ is the total number of grid points at each wall-normal position, and $I_Q$ is an indicator function defined as

\[
I_Q(x_i, y; H) = \begin{cases} 
1 & \text{when } |u'(x_i, y)v'(x_i, y)|_Q \geq H \langle |u'v'| \rangle_{SM}^{max} \\
0 & \text{otherwise}
\end{cases}
\]

In equation 3.3, the $H$ represents a nonzero threshold value, or hole size, used in determining
the relative strength of a particular $u'v'$ event. This function acts to exclude small magnitude
events and/or events composed of extreme $u'$ or $v'$ components, in order to examine contributions
from only more intense $u'v'$ events. That being said, a hole size of $H = 0$ allows all $u'v'$ events
to be considered in the analysis. The indicator function $I_Q$ is strategically based on the peak
magnitude of the smooth-wall RSS, $|\langle u'v' \rangle|_{SM}^{max}$, permitting relevant comparisons to the enhancement
of RSS-producing events as $F_{SC}$ is increased from the smooth-wall model through the full surface
representation, compared to the smooth-wall baseline. Furthermore, the presentation of data was
limited to a region beneath $y \leq 0.45h$, which is the wall-normal position that locates the upper boundary of the internal layer generated by the rough surfaces.

Quadrant analysis yields three distinct parameters that aid in the analysis process of RSS-
producing $u'v'$ events, particularly in the contributions of the four quadrant events to the mean RSS. Specifically, the RSS contributed by each of the four quadrant events for a given hole size is the first parameter, given by equation 3.2. The second parameter, which correlates the contribution of each quadrant event to the mean RSS for a given $H$ as a stress fraction, $S_Q$, can be represented by

$$S_Q(y; H) = \frac{\langle u'v' \rangle_Q(y; H)}{\langle u'v' \rangle(y)}.$$  \hspace{1cm} (3.4)$$

Finally, the fraction of space, $N_Q$, provides a measure of the population of each quadrant event, relative to all events taking place for a given $H$, and is defined as

$$N_Q(y; H) = \frac{\sum_{i=1}^{P} I_Q(x_i, y; H)}{P}.$$ \hspace{1cm} (3.5)$$

For the present analysis, each of these quantities were calculated for $Q = 1 - 4$, along with hole sizes of $H = 0$, which includes all events, and $H = 5$, which targets only the most intense quadrant events. A MATLAB code was developed in order to aid in the computation of the preceding quantities, which is available to review in Appendix D.

3.4.1  $H = 0$

Figure 3.17 presents $\langle u'v' \rangle_Q$ with a hole size of $H = 0$ for the smooth- and rough-wall surface representations. Because figures 3.17(a)–(d) have been plotted on the exact same scale, it becomes
evident, visually, that ejections (fig. 3.17b) and sweeps (fig. 3.17d) are the dominate effects contributing to the mean RSS. This fact is independent of $FSC$, prevailing from the smooth-wall up through the full surface model. However, an increase in $FSC$ tends to increase the magnitude at which ejections ($\langle u'v' \rangle_2$) and sweeps ($\langle u'v' \rangle_4$) occur within the flow for fixed $y$. Although outward ($\langle u'v' \rangle_1$) and inward ($\langle u'v' \rangle_3$) interactions are 4 – 5 times smaller in magnitude than ejections and sweeps, they still show a slight enhancement in magnitude for fixed $y$ as $FSC$ is increased. As was observed in the mean velocity and Reynolds normal stresses, the 41%, 49%, and 70% models exhibit similar behavior, differing by less than 10% for both $\langle u'v' \rangle_2$ and $\langle u'v' \rangle_4$ regardless of wall-normal position. In contrast to these observations, the stress fractions, $S_Q$, in each quadrant for $H = 0$ show little dependence on $FSC$, collapsing well with one another except for a very small region near the wall, presented in figure 3.18. In addition, the fraction of space, $N_Q$, occupied by each quadrant event for $H = 0$, presented in figure 3.19, demonstrates a similar independence of $FSC$ noted previously in $S_Q$, regardless of wall-normal position.

3.4.2 $H = 5$

As it stands, applying a threshold value of $H = 0$ reveals the strong dominance of ejections and sweeps, opposed to inward and outward interactions, in contributing to the mean RSS. Selecting a larger threshold value eliminates weak $u'v'$ contributions to the mean RSS, further quantifying the contributions to the mean RSS by only the more intense $u'v'$ events. Additionally, as weaker events are filtered out, a better understanding can be had of which events, ejections, sweeps, or both, serve as the principal features supplementing the mean RSS. Figure 3.20 presents $\langle u'v' \rangle_Q$ for a threshold value of $H = 5$. Due to the increase in threshold value, the magnitudes of $\langle u'v' \rangle_Q$ have diminished with respect to the $H = 0$ results. A consequence of this is that $\langle u'v' \rangle_1$ and $\langle u'v' \rangle_3$, which were already weak when computed with $H = 0$, are practically zero regardless of wall-normal location when $H = 5$. Therefore, results of $\langle u'v' \rangle_1$ and $\langle u'v' \rangle_3$ have not been presented in this text.

Despite an increase in the threshold value, $\langle u'v' \rangle_2$ and $\langle u'v' \rangle_4$ remain significant contributors to the mean RSS, particularly increasing in magnitude with the addition of $FSC$. Interestingly, ejections contribute approximately 1.5 times more than sweeps, regardless of $FSC$, to the mean RSS. Recall that ejections are defined as $u' < 0$ and $v' > 0$, which corresponds to slow moving
Figure 3.17: Quadrant contributions to the mean Reynolds shear stress for $H = 0$. (a) $Q_1$; (b) $Q_2$; (c) $Q_3$; (d) $Q_4$. 
Figure 3.17: continued
Figure 3.18: Stress fractions for $H = 0$. (a) $Q_1$ and $Q_2$; (b) $Q_3$ and $Q_4$. 
Figure 3.19: Space fractions for $H = 0$. (a) $Q_1$ and $Q_2$; (b) $Q_3$ and $Q_4$. 
Figure 3.20: Quadrant contributions to the mean Reynolds shear stress for $H = 5$. (a) $Q_2$; (b) $Q_4$. 

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Figure 3.21: Stress fractions for $H = 5$. (a) $Q_2$; (b) $Q_4$. 
Figure 3.22: Space fractions for $H = 5$. (a) $Q_2$; (b) $Q_4$. 

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fluid near the wall being lifted and pushed upstream. Thus, ejections appear to be a fundamental element contributing to the mean RSS for all surface topographies studied, including the smooth wall. Similar behavior is evident in the ejections and sweeps profiles of the stress fraction, \( S_2 \) and \( S_4 \), and fraction of space, \( N_2 \) and \( N_4 \). As FSC increases, the magnitude of such events significantly increase as well, counter to what was observed in the case when \( H = 0 \). However, ejection events continue to dominate over sweeps in terms of magnitude of stress and space fractions. Consistent with earlier observations, the 41%, 49%, and 70% models show similar results differing by less than 10% in \( \langle u'v' \rangle_2 \) and less than 15% in \( \langle u'v' \rangle_4 \), again, due to the topographical similarity between these surfaces.

### 3.5 Swirling strength analysis

A novel approach in identifying vortical structures within a flow is through a technique referred to as swirling strength. Swirling strength, \( \lambda_{ci} \), is the imaginary part of the complex eigenvalues of the local fluid velocity gradient tensor and is an unambiguous measure of rotation (Chong et al., 1990; Zhou et al., 1999). Unlike vorticity, which had been the primary measure in vortex identification, swirling strength does not address contributions from regions of shear to overall rotation, effectively distinguishing true vortex cores (Adrian et al., 2000a). Because the eigenvalues result in complex conjugate pairs, and the positive imaginary part is allocated to \( \lambda_{ci} \), the sign of swirling strength is inherently positive, \( \Lambda_{ci} \geq 0 \), meaning the rotation sense is not directly provided by \( \Lambda_{ci} \). To remedy this issue, the sign of vorticity (the spanwise component in the present effort) is locally used throughout the field, indicating regions of clockwise rotation \( (-\Lambda_{ci}) \) and counter-clockwise rotation \( (+\Lambda_{ci}) \), defined as

\[
\lambda_{ci} = \Lambda_{ci} \frac{\omega_z}{|\omega_z|},
\]

where \( \Lambda_{ci} \) is termed the unsigned swirling strength, and \( \omega_z \) is the local spanwise component of vorticity.

Figure 3.23 presents a typical instantaneous PIV velocity field in the streamwise–wall-normal plane for smooth-wall flow. A constant convection velocity has been removed from the field to reveal vortex structures advecting with the flow, as discussed in section 3.1. In addition, contours
Figure 3.23: Instantaneous velocity realization of smooth-wall flow, with a constant convection velocity, $U_c$, removed. Swirling strength contours overlaid to emphasize location of vortex cores (Blue: Clockwise rotation; red: Counter-clockwise rotation).

Figure 3.24: Instantaneous velocity realization of flow over the full surface, with a constant convection velocity, $U_c$, removed. Swirling strength contours overlaid to emphasize location of vortex cores (Blue: Clockwise rotation; red: Counter-clockwise rotation).
Figure 3.25: Ensemble-averaged rms swirling strength profiles for each roughness case. Solid data points represent the maximum value for each surface.

Of further interest, figure 3.25 presents ensemble-averaged rms swirling strength profiles for each topographical surface. All data has been normalized by the rms swirling strength of the smooth-wall in order to identify enhancement in swirling strength with respect to the smooth-wall.
case. Filled data points highlight the maximum value of rms swirling strength for each respective roughness. At wall-normal locations $y/h > 0.15$, an increase in $FSC$ yields greater magnitude in swirling strength with respect to the smooth-wall baseline. Below $y/h = 0.15$, and continuing until the maximum value, the 70% model shows a slight increase in swirling strength compared to the full surface. In addition, the 49% model begins to show a higher magnitude in swirling strength compared to the full surface at approximately $y/h = 0.08$, continuing this trend until the maximum value is attained. Strictly looking at maximum rms swirling strength values, as $FSC$ is increased from the smooth-wall case up to the 70% model, the maximum value also increases. However, the 100% model’s maximum value registers nearly 4% lower than the 70% model and almost 3% lower than the 49% model. This observation can again be reconciled with the same argument proposed for the mean velocities, section 3.2, where the addition of the largest-scale topographical features inherently lowers the local average surface roughness height with respect to the field of view. Furthermore, an increase in $FSC$ tends to advance the maximum rms swirling strength value in the positive wall-normal direction, toward the center of the channel. Again, the 70% model shows the greatest wall-normal advancement, positioned roughly 200% further away from the wall than the smooth-wall model.
Chapter 4

Conclusions and Future Work

A highly-irregular flow surface, replicated from a turbine blade that suffered spallation damage, was decomposed using singular value decomposition into topographical modes containing successively smaller roughness length scales. Multiple reduced-order models were then fabricated using these modes in a successive manner to, at least in spirit, capture the evolution of surface roughness from smaller- to larger-scale features over the deployment lifetime of a practical flow surface. Particle-image velocimetry measurements of turbulent channel flow at fixed Re of approximately 1825 were acquired over a smooth wall and multiple surfaces that embodied fractional surface content of 3%, 7%, 21%, 41%, 49%, 70%, and 100% (full surface). A review of the experimental results of the present study revealed that as $FSC$ increased, mean velocity deficit, Reynolds normal stress, and Reynolds shear stress all increased, apart from the full surface velocity deficit, which exhibited a slightly lower value than that of the 49% and 70% models. This slight ambiguity in the 100% model was attributed to the unique decomposition of the full surface whereby the addition of mode 1 in the 100% model significantly reduced the local average surface roughness height compared to the 49% and 70% models. A slight shift in a region of high Reynolds stress in the $x$ direction located very near the wall became evident in the two-dimensional contours, but was accredited to large surface topography features located just upstream of the field of view. Comparison of the peak RSS value was found to grow quickly with $FSC$ and display a power-law trend. Thus, even weak surface damage that a practical flow surface might endure within the first fraction of its deployment lifetime can significantly enhance turbulence and therefore progressively impact system performance over time.

Furthermore, a quadrant analysis of RSS-producing events revealed that ejections and sweeps, rather than inward- and outward-interactions, appear to be the fundamental contributors of RSS, regardless of $FSC$. Of these two dominant contributors, ejections ($u' < 0, v' > 0$) consistently
registered higher than sweeps in terms of magnitude of stress and space fractions. As $FSC$ was increased from the smooth wall to the full surface, the magnitude of RSS produced by each of the quadrants also increased, regardless of wall-normal location within the internal layer, consistent with the aforementioned analyses. In addition, rms swirling-strength statistics were computed for each surface representation whereby observations concluded that an increase in $FSC$ correlated to a greater magnitude in rms swirling strength at each wall-normal location within the flow, aside from a narrow region very close to the wall. Here, the 49% and 70% models displayed a slightly greater magnitude than that of the 100% model, again by virtue of the unique reconstruction technique employed.

Finally, instantaneous velocity fields taken from each of the tested surface roughness experiments manifested the geometrical dependence that hairpin vortex packets have on relative surface roughness damage. As $FSC$ was increased, the inclination angle of hairpin vortex heads within a packet decreased, ranging from approximately 18° for the smooth-wall case to roughly 10° for the full surface. The addition of swirling-strength fields superimposed on instantaneous velocity fields supports the existence of hairpin vortex packets traveling within the flow, as well as the dynamic response these packets portray over varying degrees of surface roughness.

Further studies of cumulative damage and its effect on turbulent flow seem irrefutable, bearing fruits in practically all aerodynamic and hydrodynamic related engineering problems, as well as many land-based flow-bearing mechanisms. The struggle to understand and reproduce the complexities of natural surface damage will only become more challenging as new, more unique composite materials find their way into the production of common flow-bearing devices. Practical future advances of the present study should explore the relationship between the statistics studied herein, to statistics collected over other surface roughness models, i.e. deposition of foreign materials, erosion, pitting, etc. In addition, future studies may employ different flow characteristics, for instance developed flow, whereby the internal roughness layer has grown to occupy the entire wall-normal extent of the flow, or flows bearing a pressure gradient, which most curved flow surfaces experience. These simple adjustments would further expand the already considerable set of experimental results, in hopes of more fully understanding the impact that surface roughness has on turbulent flows. As far as long-term considerations, effort should be directed toward develop-
ing/acquiring more accurate surface models representing cumulative damage. Collaborative studies could be arranged between laboratories investigating purely surface damage phenomena, which utilize rapid surface damage facilities, and those laboratories focused on the effective turbulent statistics produced by such surfaces. Further, operational flow surfaces, particularly those exposed to severe operating conditions, could be incrementally examined and scanned over the lifetime of the part in order to acquire a cumulative set of surface topography data.

Generally speaking, studying cumulative damage is extremely challenging, in that a universal definition of damage has yet to be adopted. Although there exists a vast array of distinct damage mechanisms, and the exact details and characteristics of cumulative surface damage are open to interpretation, future work will progress toward unifying surface damage metrics and its dynamic evolution, as well as exploring its effect on turbulent flow.
Appendix A

Creating Roughness Data Files - MATLAB Code

This code is used to create surface topography maps in the form of data files (.dat) for each of the surfaces tested. The full surface topography data is uploaded as a text file. The code then appropriately scales and arranges the data over the allotted streamwise-span-wise roughness area governed by the size of the channel-flow facility roughness panel stage. Output files are then employed in the rapid-prototyping printer at ITG in the Beckman Institute.

```matlab
format short;
A = load('SurfaceAEdited.txt');  % Load surface roughness topography data.
B = load('SurfaceBEdited.txt');
C = load('SurfaceCEdited.txt');
D = load('SurfaceDEdited.txt');

CombinedData = [A; C; B; D];

[Height Width] = size(CombinedData);

CutCombinedData = CombinedData(:, 14:(Width-13));
% Take columns 14 through 323 (original width was 1-326) for proper sizing

[DataHeight DataWidth] = size(CutCombinedData);

averageheight = sum(sum(CutCombinedData, 2))/(DataHeight*DataWidth);```

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Sums up all of the values in the matrix and divides by number of values

Fluctuations=CutCombinedData-averageheight;

ScaledData=0.3*Fluctuations; %Average height of ScaledData is zero.

[M,N,P]=svd(ScaledData); %Singular Value Decomposition performed on data.

n=diag(N);

totalenergyofsurface=sum(n);

modeapproximation=216; %This is user defined.

for k=1:216;
    %Full Surface
    z_full = M(:,1:k)*N(1:k,1:k)*P(:,1:k)';
end

for k=1:modeapproximation; %Transform data matrix into new matrix of selected modes.
    z = M(:,1:k)*N(1:k,1:k)*P(:,1:k)';
end

z_residual=z_full-z;

modalenergy=sum(n(1:modeapproximation));
modalenergyfraction=modalenergy/totalenergyofsurface;

Tile=[z_residual fliplr(z_residual);fliplr(rot90(rot90(z_residual)))
    rot90(rot90(z_residual))];
[Tile_rows, Tile_cols]=size(Tile);

End_Tiles=[zeros(67,Tile_cols+90);zeros(Tile_rows,90) Tile; zeros(Tile_rows, 90),
    Tile; zeros(67,Tile_cols+90)];
%Creates data for end surface roughness tiles/adds smooth wall perimeter.
Cent_Tiles=[zeros(67,Tile_cols); Tile; Tile; zeros(67,Tile_cols)];
%Creates center surface roughness tiles/adds smooth wall perimeter.
%contourf(Tile,20); %To view the constructed surface tiles

Ends=End_Tiles+9; %Add 9 mm to height for thickness of tiles
Center=Cent_Tiles+9;

[EndsRows EndsColumns]=size(Ends);
[CenterRows CenterColumns]=size(Center);

%Writes new files to be used in fabricating surface roughness panels.
FID=fopen('0percentFS_Center.dat','w');
for k=1:CenterRows;
    for l=1:CenterColumns;
        if l==CenterColumns;
            fprintf(FID,'%d
',Center(k,l));
        else
            fprintf(FID,'%d t',Center(k,l));
        end
    end
end
fclose(FID);

FID=fopen('0percentFS_Ends.dat','w');
for i=1:EndsRows;
    for j=1:EndsColumns;
        if j==EndsColumns;
            fprintf(FID,'%d
',Ends(i,j));
        else
            fprintf(FID,'%d t',Ends(i,j));
        end
    end
end
fclose(FID);
Appendix B

Producing Printable Models from Surface Roughness Data

The following steps should be executed when attempting to transform surface roughness data into printable models which can be printed by the powder deposition rapid-prototyping machine. The commands below are to be completed within the computer network in the Visualization Laboratory within the Imaging Technology Group (ITG) at the Beckman Institute UIUC.

• Open SSH Secure Shell → Secure Shell Client
  – Quick Connect
  – Host Name: zeus.itg.uiuc.edu
  – Username: “Enter username”
  – Password: “Enter password”

• Open conversion file in WordPad to adjust $\Delta x$ and $\Delta y$ values.

• Within the command window, type “name of conversion file” .py “name of file converting”.dat

• This will create a .obj file within the same directory.

• Open AutoDesk Maya 2008
  – Click on File → Import
  – Import .obj file
  – Directly export as .vrml2 file (If .vrml2 does not exist as an option, select Window → Settings/Preferences → Plug-in Manager → Check off .vrml2 at the bottom of the window)
  – Resultant file extension will be .wrl
- Open ZPrint 7.6

  - Open the .wrl. When prompted, enter:

    * units: mm

    * powder type: ZP131

    * YES

  - Click on Edit → Make Solid → Enter Thickness Below Lowest Point

  - Click on Transform → Justify → Left, Back, Bottom

- You can import more models if necessary following the same procedure.
Appendix C

Example Channel-Flow Experimental Data Sheet

This is an example Channel-Flow Experimental Data spreadsheet utilized in pre-experiment preparation. At the top, a “Things to check!” list serves as a reminder of some critical steps to follow prior to any experiment. Blue cells represent those cells which are required to be completed by the user. All other cells fill themselves. With regard to the pressure drop, $dp/dx$ is taken from the slope of the linear curve fit in the pressure curve.
CHANNEL-FLOW EXPERIMENTAL DATA

Anthony M. Licari

Experiment: 100% A

Things to check!
- Tile slid forward all the way
- Tile is level
- Laser sheet is perpendicular (check with mirror)
- Laser sheet is thin (1mm or less)
- Prism and lenses are free of dust/debris
- Laser sheet is parallel to sidewall
- Camera is horizontal with respect to channel

Magnification: 0.025781 mm/pix

Camera offset:
- $\delta = 762$ pixels
- 19.64512 mm
- Offset $= 1.194878$ mm
- 46.34723 pixels
($\delta$ is dist from center dot to bottom of FOV)

At: 10 $\mu$s

Temp: 32.2 $^\circ$C

305.35 K

<table>
<thead>
<tr>
<th>Constants</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ 0.029</td>
<td>Density $\rho$ 1.15739609 kg/m$^3$</td>
</tr>
<tr>
<td>$R$ 8.314472</td>
<td>Dynamic viscosity $\mu$ 1.8968E-05 kg/m s</td>
</tr>
<tr>
<td>$\mu_0$ 0.00001827</td>
<td>Kinematic viscosity $\nu$ 1.6388E-05 m$^2$/s</td>
</tr>
<tr>
<td>$T_w$ 291.15</td>
<td>Wall shear $\tau_w$ 1.5877286 Pa</td>
</tr>
<tr>
<td>$C$ 120</td>
<td>Friction velocity $u_z$ 1.17124332 m/s</td>
</tr>
<tr>
<td>$h$ 0.0254</td>
<td>Viscous length scale $y^*$ 13.9921664 microns</td>
</tr>
<tr>
<td>$P$ 101325</td>
<td>Reynolds number $Re_z$ 1815.30146</td>
</tr>
</tbody>
</table>
**Pressure:**

**Distance to:**

<table>
<thead>
<tr>
<th>Tap</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap 1</td>
<td>0.82</td>
</tr>
<tr>
<td>Tap 2</td>
<td>1.7344</td>
</tr>
<tr>
<td>Tap 3</td>
<td>2.6488</td>
</tr>
<tr>
<td>Tap 4</td>
<td>3.5632</td>
</tr>
</tbody>
</table>

**Avg. Pressure in Pascals:**

<table>
<thead>
<tr>
<th>Tap</th>
<th>DO</th>
<th>V</th>
<th>inH₂O</th>
<th>USE</th>
<th>Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap 2</td>
<td>2.96904876</td>
<td>1.067167</td>
<td>inH₂O</td>
<td>265.8036</td>
<td>Pa</td>
</tr>
<tr>
<td>Tap 3</td>
<td>2.3362972</td>
<td>0.845704</td>
<td>inH₂O</td>
<td>210.6429</td>
<td>Pa</td>
</tr>
<tr>
<td>Tap 4</td>
<td>1.65772475</td>
<td>0.608204</td>
<td>inH₂O</td>
<td>151.4877</td>
<td>Pa</td>
</tr>
</tbody>
</table>

![Pressure vs. Distance graph](image)

**y = -62.509x + 317.73**

**R² = 0.9996**

**dP/dx:** 62.509 Pa/m
Appendix D

Quadrant Analysis - MATLAB Code

This code was developed in order to compute the required quantities for Quadrant Analysis. The code requires the user to fill in the appropriate amount of fields that were acquired during an experiment, as well as the hole size $H$. The code then decomposes the $u'v'$ events into the four quadrants mentioned in section 3.4, followed by computing $\langle u'v' \rangle_Q$, $S_Q$, and $N_Q$. Finally, the code exports this data as .dat files to be used in Tecplot.

```matlab
%This program creates a data file which contains Quadrant Analysis
%information useful with TecPlot.
%By: Tony Licari March 2010

%Make sure to change the TEXTREAD FILE and DATA FILE NAME to the particular
%case you are calculating.

%Fill in fields and H

clear
tic

%Creates matrices to store number of and location of each quadrant event.
Q1 = zeros(127,159);
Q2 = zeros(127,159);
Q3 = zeros(127,159);
Q4 = zeros(127,159);

%Creates matrices to store all u’v’ values with hole size.
uv1 = zeros(127,159);
uv2 = zeros(127,159);
uv3 = zeros(127,159);
uv4 = zeros(127,159);

%Creates matrices to store all u'v' values for hole size zero calculation.

uv1_H0 = zeros(127,159);
uv2_H0 = zeros(127,159);
uv3_H0 = zeros(127,159);
uv4_H0 = zeros(127,159);

fields = 6892;    %# of realizations

H = 5;    %Hyperbolic hole size

for t = 1:fields;
    [x,y,z,ulist,vlist,wlist] =
        textread([’21_A-‘ num2str(t) ‘fluc.dat’], %One line
                 ’%f %f %f %f %f %f’, ‘headerlines’,1);

    u = zeros(127,159);
    v = zeros(127,159);

    for rows=1:127;
        for cols=1:159;
            u(rows,cols)=ulist(cols+159*(rows-1));
            %Makes matrix of u' for each realization.
        end
    end

    for rows=1:127;
        for cols=1:159;
            v(rows,cols)=vlist(cols+159*(rows-1));
            %Makes matrix of v' for each realization.
        end
    end
end
Determine which quadrant each value belongs in with hole size.

for m = 1:127;
    for k = 1:159;
        if u(m,k)>0 && v(m,k)>0 && abs(u(m,k)*v(m,k))≥H*0.17021;
            Q1(m,k) = Q1(m,k)+1;
            uv1(m,k) = uv1(m,k)+(u(m,k)*v(m,k));
        elseif u(m,k)<0 && v(m,k)>0 && abs(u(m,k)*v(m,k))≥H*0.17021;
            Q2(m,k) = Q2(m,k)+1;
            uv2(m,k) = uv2(m,k)+(u(m,k)*v(m,k));
        elseif u(m,k)<0 && v(m,k)<0 && abs(u(m,k)*v(m,k))≥H*0.17021;
            Q3(m,k) = Q3(m,k)+1;
            uv3(m,k) = uv3(m,k) + (u(m,k)*v(m,k));
        elseif abs(u(m,k)*v(m,k))≥H*0.17021;
            Q4(m,k)=Q4(m,k)+1;
            uv4(m,k) = uv4(m,k) + (u(m,k)*v(m,k));
        end
    end
end

% Determine which quadrant each value belongs in for H = 0 (this is for the totaluv).

for m = 1:127;
    for k = 1:159;
        if u(m,k)>0 && v(m,k)>0;
            uv1_H0(m,k) = uv1_H0(m,k)+(u(m,k)*v(m,k));
        elseif u(m,k)<0 && v(m,k)>0;

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\[ uv2_{H0}(m,k) = uv2_{H0}(m,k) + (u(m,k) \times v(m,k)) \]

\textbf{elseif} \ u(m,k) < 0 \&\& \ v(m,k) < 0; \\
\[ uv3_{H0}(m,k) = uv3_{H0}(m,k) + (u(m,k) \times v(m,k)) \]

\textbf{else} \\
\[ uv4_{H0}(m,k) = uv4_{H0}(m,k) + (u(m,k) \times v(m,k)) \]
\textbf{end}
\textbf{end}
\textbf{end}

\[ uvQ1 = uv1./fields; \quad \% \text{Divide by number of events at each location.} \]
\[ uvQ2 = uv2./fields; \]
\[ uvQ3 = uv3./fields; \]
\[ uvQ4 = uv4./fields; \]

\[ uvQ1lineavg = \text{mean}(uvQ1, 2); \quad \% \text{Line average.} \]
\[ uvQ2lineavg = \text{mean}(uvQ2, 2); \]
\[ uvQ3lineavg = \text{mean}(uvQ3, 2); \]
\[ uvQ4lineavg = \text{mean}(uvQ4, 2); \]

\[ \text{totaluv} = uvQ1lineavg + uvQ2lineavg + uvQ3lineavg + uvQ4lineavg; \]

\[ \text{TotalN} = Q1 + Q2 + Q3 + Q4; \]

\[ N1 = \text{mean}((Q1./fields), 2); \quad \% \text{Space fraction.} \]
\[ N2 = \text{mean}((Q2./fields), 2); \]
\[ N3 = \text{mean}((Q3./fields), 2); \]
\[ N4 = \text{mean}((Q4./fields), 2); \]
Calculations for H = 0 for stress fraction calculation.

% Divide by number of events at each location.

uvQ1_H0 = uv1_H0./fields;
uvQ2_H0 = uv2_H0./fields;
uvQ3_H0 = uv3_H0./fields;
uvQ4_H0 = uv4_H0./fields;

uvQ1lineavg_H0 = mean(uvQ1_H0, 2);  
% Line average.

uvQ2lineavg_H0 = mean(uvQ2_H0, 2);

uvQ3lineavg_H0 = mean(uvQ3_H0, 2);

uvQ4lineavg_H0 = mean(uvQ4_H0, 2);

totaluv_H0 = uvQ1lineavg_H0+uvQ2lineavg_H0+uvQ3lineavg_H0+uvQ4lineavg_H0;

% Stress fraction.

S1 = uvQ1lineavg./totaluv_H0;
S2 = uvQ2lineavg./totaluv_H0;
S3 = uvQ3lineavg./totaluv_H0;
S4 = uvQ4lineavg./totaluv_H0;

fid=fopen(‘QuadrantAnalysis_H=5_21A.dat’, ’w’);  
% Opens a file.

fprintf(fid, ’ZONE T = ’uvQ1_21”	I=%f	J=%f
’, 1, 127);

for l=1:127;
    fprintf(fid, ’%f
’,l,uvQ1lineavg(l));
end

fprintf(fid, ’ZONE T = ’uvQ2_21”	I=%f	J=%f
’, 1, 127);

for l=1:127;
    fprintf(fid, ’%f
’,l,uvQ2lineavg(l));
end
fprintf(fid, 'ZONE T = "uvQ3_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, uvQ3lineavg(l));
end

fprintf(fid, 'ZONE T = "uvQ4_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, uvQ4lineavg(l));
end

fprintf(fid, 'ZONE T = "N1_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, N1(l));
end

fprintf(fid, 'ZONE T = "N2_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, N2(l));
end

fprintf(fid, 'ZONE T = "N3_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, N3(l));
end

fprintf(fid, 'ZONE T = "N4_21" \t I=\t J=\t 1, 127);

for l=1:127;
    fprintf(fid, '\t %f \t %f', l, N4(l));
end

fprintf(fid, 'ZONE T = "S1_21"	 I=%f	 J=%f
', 1, 127);

for l=1:127;
    fprintf(fid, '
%ft
',l,S1(l));
end

fprintf(fid, 'ZONE T = "S2_21"	 I=%f	 J=%f
', 1, 127);

for l=1:127;
    fprintf(fid, '
%ft
',l,S2(l));
end

fprintf(fid, 'ZONE T = "S3_21"	 I=%f	 J=%f
', 1, 127);

for l=1:127;
    fprintf(fid, '
%ft
',l,S3(l));
end

fprintf(fid, 'ZONE T = "S4_21"	 I=%f	 J=%f
', 1, 127);

for l=1:127;
    fprintf(fid, '
%ft
',l,S4(l));
end

close(fid);  %Close the file.

tElapsed = toc;

Time_in_minutes = tElapsed/60
Appendix E

Swirling Strength (RMS) Analysis - MATLAB Code

This code aids in the computation of RMS swirling strength for each of the smooth- and rough-wall models tested. Because swirling strength is defined by an absolute magnitude, the vorticity is also computed in order to obtain the proper sign on swirling strength. The user is required to submit the data height and width, the distance between data points (pixels) and the number of realizations acquired for the particular model experiment. The code outputs .dat files, convenient when using Tecplot.

```matlab
%SWIRLING STRENGTH - Computes the RMS Swirling Strength affiliated with
%each surface model.
%By: Tony Licari April 2010

clear
tic
datawidth = 159;
dataheight = 127;
h = 8; %distance between data points
r = 6987; %Number of realizations

lambda = zeros(dataheight, datawidth, r);
vorticity = zeros(dataheight, datawidth);

for t = 1:r;
    [x,y,ulist,vlist,chclist] = textread(['70_A' num2str(t) '.vec'],
        %One line.
```
u = zeros(dataheight, datawidth);
v = zeros(dataheight, datawidth);
chc = zeros(dataheight, datawidth);

for rows=1:dataheight;
    for cols=1:datawidth;
        u(rows,cols)=ulist(cols+datawidth*(rows-1));
        \%Makes matrix of u' for each realization.
    end
end

for rows=1:dataheight;
    for cols=1:datawidth;
        v(rows,cols)=vlist(cols+datawidth*(rows-1));
        \%Makes matrix of v' for each realization.
    end
end

for rows=1:dataheight;
    for cols=1:datawidth;
        chc(rows,cols)=chclist(cols+datawidth*(rows-1));
        \%Makes matrix of chc for each realization.
        if chc(rows,cols) < 0;
            u(rows,cols) = NaN;
            v(rows,cols) = NaN;
        end
    end
end

u = flipud(u);
v = flipud(v);
chc = flipud(chc);
\texttt{u = fillmiss(u); \hspace{1em} \%Interpolate holes}
\texttt{v = fillmiss(v);} 
\texttt{

%Calculates the gradients of \(u\) and \(v\) matrices
\[ [\text{dudx, dudy}] = \text{gradient}(u,h); \]
\[ [\text{dvdx, dvdy}] = \text{gradient}(v,h); \]

\texttt{for rows=1:dataheight;}
\texttt{ \hspace{1em} for cols=1:datawidth;}

\texttt{ \hspace{2em} D = zeros(2,2);}

\texttt{ \hspace{2em} \%central difference}
\texttt{ \hspace{3em} \%dudx = \frac{(u(rows, cols+1) - u(rows, cols-1))}{2h};}
\texttt{ \hspace{3em} \%dvdx = \frac{(v(rows, cols+1) - v(rows, cols-1))}{2h};}
\texttt{ \hspace{3em} \%dudy = \frac{(u(rows-1, cols) - u(rows+1, cols))}{2h};}
\texttt{ \hspace{3em} \%dvdy = \frac{(v(rows-1, cols) - v(rows+1, cols))}{2h};}

\texttt{ \hspace{3em} D(1,1) = dudx(rows,cols);}  
\texttt{ \hspace{3em} D(1,2) = dvdx(rows,cols);}  
\texttt{ \hspace{3em} D(2,1) = dudy(rows,cols);}  
\texttt{ \hspace{3em} D(2,2) = dvdy(rows,cols);}  

\texttt{eigvals = eig(D);} 

\texttt{if imag(eigvals(1)) && imag(eigvals(2)) == 0;}
\texttt{ \hspace{4em} lambda(rows,cols,t) = 0;}
\texttt{else}
\texttt{ \hspace{5em} lambda(rows,cols,t) = abs(imag(eigvals(1)));}
\texttt{end}
\texttt{end}
\texttt{end}

\texttt{[curlz,cav]= curl(u,v); \hspace{1em} \%Vorticity}

\texttt{lambda(:, :, t) = lambda(:, :, t) .* (curlz./abs(curlz));}
vorticity = vorticity + curlz;

end

lambda_avg = nanmean(lambda,3);
%lambda = flipud(lambda);
vorticity = vorticity./r;
%vorticity = flipud(vorticity);

lambda_fluc = zeros(dataheight,datawidth);

for j = 1:r;
    lambda_fluc = (lambda(:,:,j) - lambda_avg).^2 + lambda_fluc;
end

lambda_fluc = fillmiss(lambda_fluc);
lambda_fluc_rms = sqrt(lambda_fluc./r);

%Line averaging...

lambda_lineavg = nanmean(lambda_fluc_rms,2);

fid=fopen('SwirlingStrength_2Drms_70A.dat', 'w'); %Opens a file.

fprintf(fid, 'VARIABLES="X", "Y", "lambda_fluc_rms",
   "vorticity" ZONE I=%f\t J=%f\t K=%f\n', datawidth, dataheight, 1);

for l=1:dataheight;
    for t=1:datawidth;
        fprintf(fid, '%f\t %f\t %f\t %f\n',t,l,
            lambda_fluc_rms(l,t),vorticity(l,t));
    end
end

fid=fopen('SwirlingStrength_lineavg_70A.dat', 'w'); %Opens a file.
fprintf(fid, 'VARIABLES="Y", "lambda_lineavg" ZONE I=%f\t J=%f\n', l, dataheight);

for l=1:dataheight;
    fprintf(fid, '%f %f\n', l, lambda_lineavg(l));
end
fclose(fid);  \%Close the file.

toc
References


