An Analytical Approach to Computing Joint Opening in Concrete Pavements

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ABSTRACT: Satisfactory performance of the transverse joints is crucial for achieving the intended service life of jointed plain concrete pavement. An accurate prediction of joint opening and movement is desired in order to quantify the effects of the environment, base type, concrete material constituents, and slab geometry on the concrete pavement responses. In this paper, an analytical model based on elasticity theory is presented to predict joint opening using a bilinear slab-subbase interfacial constraint assumption. The proposed model predicts the mean joint opening based on uniform temperature change and drying shrinkage through the slab thickness. To account for the temperature curling effect, a “correction” term to the joint opening is proposed using a closed-form solution derived from Westergaard’s temperature curling deflection equation. Initial model calculation using in-situ measured pavement temperature profile suggests that proposed analytical model generates reasonable joint opening during the monitoring period.

1 INTRODUCTION

Performance of the transverse joints is crucial for achieving the intended service life of jointed plain concrete pavement (JPCP). Research results show that most failures in JPCP are caused by failures at the joint rather than inadequate structural bearing capacity (Federal Highway Administration 1990). An accurate prediction of joint opening and movement is necessary in order to assess the effects of the environment, base type, concrete material constituents, and slab geometry on the concrete pavement response. In this study, the primary motivation was to develop a time-dependent, analytical model to predict joint opening and movement that accounts for climatic effects on "young" JPCP. Another motivation is to seek potential refinement of current joint opening prediction algorithm used in AASHTO pavement design guide (AASHTO 1993).

Advancement on a joint opening prediction algorithm will benefit the design of dowelled and non-dowelled JPCPs. For example, the use of dowels is not necessary in lower traffic volume facilities if the joint opening can be kept to a minimum. On airports, it is more common for all contraction joints to be dowelled whereas non-dowelled joints could be used if more consideration was given in designing the maximum joint opening. Furthermore, most transverse joints need sealing in order to minimize the ingress of water and incompressible materials into the joint and pavement structure, which reduces the moisture-related distresses such as pumping and faulting and pressure-related distresses such as spalling and blowups (Federal Highway Administration 1990). One important criterion in selecting and installing appropriate joint sealant is the maximum joint opening over the entire pavement service life (Biel & Lee 1997, Huang 2004). Currently, the joint opening is approximated using Equation 1 from AASHTO pavement design guide (AASHTO 1993)

$$\Delta L = CL(\alpha \cdot \Delta T + \varepsilon) \tag{1}$$

where $\Delta L =$ joint opening due to temperature and moisture changes in concrete pavement (mm); $L =$ slab length (mm); $\alpha =$ thermal coefficient of expansion/contraction of Portland cement concrete (PCC) (strain/°C); $\Delta T =$ temperature change (°C); $\varepsilon =$ PCC coefficient of drying shrinkage (mm/mm); $C =$ adjustment coefficient to account for the slab-base frictional restraint (0.65 for stabilized bases and 0.8 for granular bases).

This equation is intended to give an approximation of the mean joint opening over a daily or yearly time interval, keeping in mind that the adjustment coefficient, C, was derived using limited field testing data (Minkarah et al. 1982). As a result, comparing measured field data with calculated values from Equation 1 can result in significant discrepancy. Morian et al. (1999) came up with the same conclusion based on data collected for the Long-Term Pavement Performance Program.

To capture the "dynamic" feature of joint opening due to temperature and moisture changes in PCC slab and slab-subbase frictional restraint, a mechanistic model is preferred. From a detailed literature review, an existing elasticity-based model was modified and implemented to account for the main contributing factors to the joint opening.

2 MODEL DEVELOPMENT

2.1 *SLAB-SUBBASE INTERFACIAL RESTRAINT*

To easily present the underlying analytical model, let x be the direction along PCC slab length, z be the direction along PCC slab thickness. z is measured positive downward and $z = 0$ is at the mid-depth of slab. The ends of slab are located at $x = 0$ and $x = L$. For symmetry of the problem, only half of the slab ($0 \le x < L/2$) is analyzed. The coordinate system is shown in Figure 1.

Slab-subbase interaction serves as a restraint to joint opening, and proper characterization of this friction is critical for accurately predicting the joint opening in JPCP. A full contact condition between the slab and subbase is assumed in this paper. Field push-off test results suggest that the stress-slippage behavior of a slab-base interface can be satisfactorily approximated by a bilinear function as presented in Equation (2) below (Rasmussen & Rozycki 2001; Wimsatt et al. 1987).

$$\tau(x) = \begin{cases} \frac{\tau_0}{\delta_0} u(x) & \text{if } |u(x)| \le \delta_0 \\ \tau_0 & \text{if } u(x) > \delta_0 > 0 \\ -\tau_0 & \text{if } u(x) < -\delta_0 < 0 \end{cases} \quad (2)$$

where $\tau(x)$ = slab-subbase interfacial friction at x (MPa), and stress sign convention is applied (Timoshenko & Goodier 1970); τ_0 = steady-state friction (MPa); δ_0 = slippage or displacement (mm) corresponding to the friction of τ_0; $u(x)$ = average displacement through PCC slab thickness (mm), $u(x)$ > 0 means that slab contracts and $u(x)$ < 0 means slab expansion for $0 \le x < L/2$.

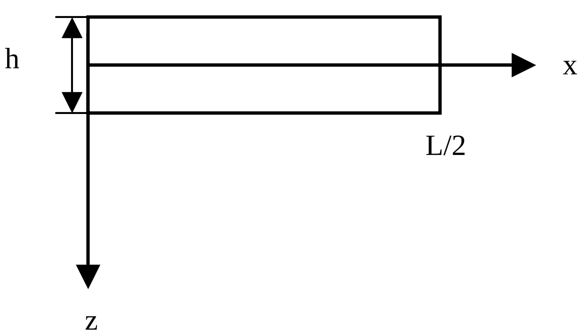

Figure1. Coordinate system used in this model

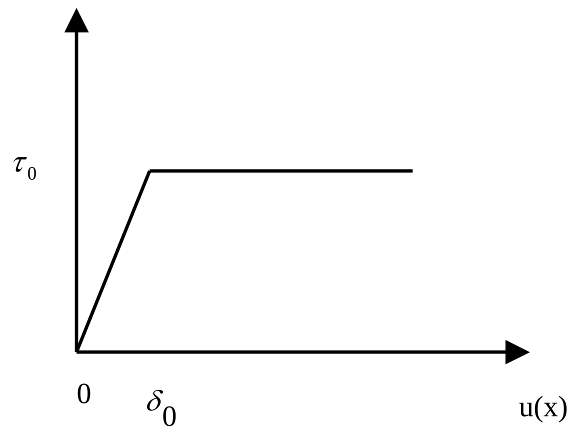

Figure 2. Bilinear Slab-subbase restraint mo

Equation (2) is plotted in Figure 2. Table 1 lists some typical values of τ_0, δ_0 for different subbase types, and indicates that larger slab-subbase restraint and smaller threshold displacement δ_0 values exist in cement stabilized subbase compared to those in other types of subbase. This is one of the main reasons why JPCP with semi-rigid subbase are susceptible to environment-induced cracking at early ages.

Table1. Typical slab-subbase frictional restraint values for different types of subbase (after Rasmussen & Rozycki 2001)

Subbase Type	τ_0 (MPa)	δ_0 (mm)
Dense-Graded HMA (Rough)	0.069	0.25
Dense-Graded HMA (Smooth)	0.035	0.51
Cement Stabilized	0.103	0.025
Lime Treated Clay	0.010	0.76
Natural clay	0.007	1.00
Granular	0.014	0.51

2.2 *1-D analytical model to computing the displacement field in PCC slab*

Zhang and Li (2001) developed a one-dimensional (1-D) analytical model for predicting the shrinkage-induced stress in PCC slab using Equation 2 to describe slab-subbase restraint. In this paper, this model is modified to predict joint opening based on drying shrinkage and temperature changes (uniform and differential) in the concrete slab. The following summarizes the main results of this analytical model.

The governing equilibrium equation is

$$\frac{d^2u}{dx^2} - \frac{\tau}{Eh} = 0 \tag{3}$$

where $\tau(x)$, $u(x)$ are defined in equation (2); E = elastic modulus of concrete (MPa) and h = the height of slab (mm).

Equation (3) plus appropriate boundary conditions (BCs) constitute a boundary value problem (BVP) for the unknown variable u. Rasmussen & Rozycki (2001) used the same equation along with appropriate BCs to calculate the thermal- and moisture-induced slab stresses using a finite difference scheme. Xin et al. (1992) also used the same equilibrium equation to approximate the structural response of continuously reinforced concrete pavement. Once the displacement field u is determined for the given BCs, the joint opening is simply equal to twice the $u(0)$.

Defining the appropriate BCs for the problem is required to derive the closed-form solution for the displacement u. There are two cases involved and their solutions are given as follows (Zhang & Li 2001).

Case 1: $|u(0)| < \delta_0$

The corresponding BVP for the displacement u is

$$\frac{d^2u}{dx^2} - \frac{\tau_0}{Eh\delta_0} u = 0 \tag{4}$$

subjected to two BCs

$$u\left(\frac{L}{2}\right) = 0, \quad \frac{du}{dx}(0) = \frac{\sigma_0}{E} + \varepsilon_e \tag{5}$$

where σ_0 = average axial stress at $x = 0$ (MPa), usually taken to be zero at the joint face; ε_e = environment-induced strain due to uniform temperature and drying shrinkage changes (mm/mm) through the slab thickness.

The final result for u in this case is

$$u(x) = -\frac{1}{\beta}\left(\frac{\sigma_0}{E} + \varepsilon_e\right)\frac{e^{-\beta x} - e^{-\beta(L-x)}}{1 + e^{-\beta L}} \tag{6}$$

where $\beta = \sqrt{\dfrac{\tau_0}{Eh\delta_0}}$.

Case 2: $u(0) > \delta_0$ and $u(x_0) = \delta_0$ for $x_0 \in (0, L/2)$

In this case, only solutions for the slab contraction (i.e. $\varepsilon_e < 0$) are presented, and similar arguments can be extended for slab expansion.

(1) For $x \in (0, x_0)$

$$\frac{d^2u_1}{dx^2} - \frac{\tau_0}{Eh} = 0 \tag{7}$$

subjected to two BCs

$$u_1(x_0) = \delta_0\,,\ \frac{du_1}{dx}(0) = \frac{\sigma_0}{E} + \varepsilon_e \tag{8}$$

The solution for this BVP is

$$u_1(x) = \delta_0 + \frac{1}{2}\beta^2\delta_0\left(x^2 - x_0^2\right) + \left(\frac{\sigma_0}{E} + \varepsilon_e\right)\left(x - x_0\right) \tag{9}$$

where β is defined in Case 1.

(2) For $x \in (x_0, L/2)$

$$\frac{d^2u_2}{dx^2} - \frac{\tau_0}{Eh\delta_0}u_2 = 0 \tag{10}$$

subjected to two BCs

$$u_2(\frac{L}{2}) = 0\,,\ \frac{du_2}{dx}(x_0) = \frac{\sigma_{00}}{E} + \varepsilon_e \tag{11}$$

where σ_{00} = average axial stress at x_0.

The solution for this BVP is

$$u_2(x) = -\frac{1}{\beta}\left(\frac{\sigma_{00}}{E} + \varepsilon_e\right)\frac{e^{-\beta x} - e^{-\beta(L-x)}}{e^{-\beta x_0} + e^{-\beta(L-x_0)}} \tag{12}$$

where β is defined in Case 1.

In order to determine $u(0)$, x_0 must be known, which can be obtained by applying the continuity condition of displacement at x_0, i.e., $u_1(x_0) = u_2(x_0)$, one obtains

$$\delta_0 = -\frac{1}{\beta}\left[\beta^2\delta_0 x_0 + \frac{\sigma_0}{E} + \varepsilon_e\right]\frac{e^{-\beta x_0} - e^{-\beta(L-x_0)}}{e^{-\beta x_0} + e^{-\beta(L-x_0)}}\,. \tag{13}$$

x_0 can then be numerically solved by using a nonlinear equation solver, such as Newton-Raphson iterative method (Burden & Faires 2001).

The environment-induced strain, ε_e defined in Equation 5 consists of two components, i.e. $\varepsilon_e(t) = \varepsilon_T(t) + \varepsilon_{dry}(t)$, where ε_T = strain due to uniform temperature change through concrete slab thickness (mm/mm); and ε_{dry} = uniform drying shrinkage strain of concrete slab (mm/mm). The next two subsections focus on methods for predicting the thermal strain, ε_T and drying shrinkage strain, ε_{dry}.

2.3 *Thermal strain,* $\varepsilon_T(t)$

Thermal strain due to uniform change in temperature through the slab thickness at time t can be calculated using Equation 14

$$\varepsilon_T(t) = \alpha \cdot \Delta T(t) = \alpha \cdot \left(T_c(t) - T_{ref}\right) \tag{14}$$

where α is defined in Equation 1; $\Delta T(t)$ = uniform concrete slab temperature change (°C); $T_c(t)$ = constant temperature through slab thickness (°C); T_{ref} = reference temperature which may be equal to the final set or zero-stress temperature of PCC slab (°C). Here, a positive value of $\varepsilon_T(t)$ means expansion while negative means contraction.

The measured and predicted temperature profile data through the slab thickness typically is nonlinear (Liang & Niu 1998, Wang et al. 2007). Any arbitrary nonlinear temperature profile can be decomposed into three components: a constant temperature, an equivalent linear temperature, and a nonlinear temperature component. Since the nonlinear strain causing component "does not precipitate either expansion or bending, but only produces thermal strains, which tend to distort the plate cross section" (Ioannides & Khazanovich, 1998), the effects of nonlinear temperature component on the joint opening is ignored in this paper.

The constant temperature, $T_c(t)$ and equivalent linear temperature component, $T_L(z,t)$ can be extracted from the measured pavement temperature profile, $T(z,t)$ using Equation 15 and 16 respectively, which are derived using the concepts of equivalent resultant force and moment, respectively (Ioannides & Khazanovich, 1998)

$$T_c(t) = \frac{1}{h}\int_{-h/2}^{h/2} T(z,t)dz \tag{15}$$

$$T_L(z,t) = T_{ref} + \frac{12z}{h^3}\int_{-h/2}^{h/2} zT(z,t)dz \tag{16}$$

2.4 *Drying shrinkage,* $\varepsilon_{dry}(t)$

This type of shrinkage is associated with the hardened concrete. The driving force to cause the drying shrinkage is the loss of water from the hardened concrete. In this paper, uniform drying shrinkage $\varepsilon_{dry}(t)$ through the thickness of the slab is assumed and calculated using Equation 17, which is proposed by American Concrete Institute (ACI) committee 209 (Mindess et al. 2003) for concrete's free drying shrinkage

$$\varepsilon_{dry}(t) = \frac{t}{35+t}\varepsilon_u \tag{17}$$

Where t is measured in days; ε_u = ultimate drying shrinkage strain value.

2.5 *Effect of temperature curling strain on the joint opening*

To calculate the joint opening due to the curling strain caused by the equivalent linear temperature component defined in Equation 16, a formula based on Westergaard's curling deflection formulation is proposed in this paper.

Referring to Westergaard (1926), let y be the direction along concrete slab width, the deflection $Z(x, y)$ for a rectangular panel due to temperature curling is

$$Z(x,y) = f(y-\frac{B}{2}) + F(x-\frac{L}{2}) \quad (18)$$

where f ,is the deflection function for the panel with infinite length and finite width B, and F is the deflection function for the panel with infinite width and finite length L. The explicit form for f is given in Equation 18 on page 208 in Westergaard (1926), and F can be analogously expressed as

$$F(x) = -z_0 \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}[(-\tan\lambda + \tanh\lambda)\cos\frac{x}{l\sqrt{2}}\cosh\frac{x}{l\sqrt{2}} + (\tan\lambda + \tanh\lambda)\sin\frac{x}{l\sqrt{2}}\sinh\frac{x}{l\sqrt{2}}] \quad (19)$$

where the radius of relative stiffness, $l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$, μ = Poisson's ratio of concrete, k = the modulus of subgrade reaction; $\lambda = \frac{L}{l\sqrt{8}}$; $z_0 = \frac{(1+\mu)l^2\alpha\Delta T}{h}$, $\Delta T = T_L(-\frac{h}{2},t) - T_L(\frac{h}{2},t)$, linear temperature component difference between the top and bottom slab, and $T_L(z,t)$ is given in Equation 16.

The temperature curling along the slab length direction is the main contributor to the joint opening thus the curling along slab width direction is ignored in this paper. The joint opening that is related to temperature curling, CW_{curl} can be approximated as

$$CW_{curl}(t) = \begin{cases} h\left|F'\left(\frac{L}{2}\right)\right| & \text{if } T_L\left(-\frac{h}{2},t\right) < T_L\left(\frac{h}{2},t\right) \\ -h\left|F'\left(\frac{L}{2}\right)\right| & \text{if } T_L\left(-\frac{h}{2},t\right) > T_L\left(\frac{h}{2},t\right) \end{cases} \quad (20)$$

where $F'\left(\frac{L}{2}\right)$ is the first derivative of F evaluated at $x = L$.

2.6 *Steps involved in joint opening prediction*

This section summarizes the main steps involved in the proposed analytical model to calculate the time-dependent joint opening.

Step 1: Calculate $T_c(t)$ and $T_L(z,t)$ from Equations 15 and 16 respectively, then calculate $\varepsilon_T(t)$ from Equation 14.

Step 2: Calculate $\varepsilon_{dry}(t)$ from Equation 17.

Step 3: Calculate $u(0)$ from Equation 6 by setting $x = 0$, $\varepsilon_e(t) = \varepsilon_T(t) + \varepsilon_{dry}(t)$ and $\sigma_0 = 0$.

Step 4: If $|u(0)| < \delta_0$, then go to Step 5; if $|u(0)| > \delta_0$, firstly calculate x_0 from Equation 13 by setting $\varepsilon_e(t) = \varepsilon_T(t) + \varepsilon_{dry}(t)$ and $\sigma_0 = 0$, then calculate $u_1(0)$ from Equation 9 by setting $x = 0$.

Step 5: Calculate $CW_{curl}(t)$ from Equation 20.

Step 6: The joint opening, $CW(t)$ can be calculated as

$$CW(t) = \begin{cases} 2u(0) + CW_{curl} & \text{if } |\mathrm{u}(0)| < \delta_0 \\ 2u_1(0) + CW_{curl} & \text{if } |\mathrm{u}(0)| > \delta_0 \end{cases} \quad (21)$$

3 APPLICATION OF MODEL

In this model calculation, the measured temperature data is taken from a field test section of a 254 mm (10 in.) thick slab with 4.57 m (15 ft) slab length and 3.66 m (12 ft) width, cast on June 22, 2006 in Rantoul, Illinois, USA. The subbase is 406.4 mm (16 in.) thick Hot-Mix Asphalt. The pavement temperatures are measured at different depths of slab, i.e., surface, 1, 3, 5, 7 and 9 inches at 15-minute intervals. The model testing period used in this paper started at 12:08 a.m. on July 01, 2006, ended at 12:38 p.m. on July 13, 2006 (1203 time steps). The slab was constructed on June 22, 2006 and therefore the predicted opening on July 1, 2006 will be greater than zero. The assumed material parameters in the model calculation are given in Table 2.

Table 2: Material parameters used in the joint opening prediction

Parameter	Value
Concrete setting temperature, T_{ref} (°C)	45
Coefficient of thermal expansion of concrete, α (1/°C)	10.35 x 10^{-6}
28-day Elastic modulus of concrete E_{28} (GPa)	25.9
Steady-State slab-subbase friction τ_0 (MPa)	0.052
Slab slippage δ_0 corresponding to τ_0 (mm)	0.38
Poisson's ratio	0.2
Ultimate drying shrinkage strain	7.8 x 10^{-4}
Composite modulus of subgrade reaction k (pci)	450

Also the time-dependent elastic modulus of concrete, E_c is approximated by (Mosley & Bungey 1990)

$$E_c = E_{28}\left[0.52 + 0.15 \ln t\right] \quad \text{for } t \le 28 \quad (22)$$

where E_c is the elastic modulus at time t (in days).

The measured pavement temperature at 25.4 mm (1 in.), 127 mm (5 in.), 228.6 mm (9 in.) and the calculated constant strain causing (mean) temperature are plotted versus time in Figure 3. Figure 4 presents the predicted joint opening values with (red) and without (black) temperature curling correction along with the measured ones (blue), respectively. Figure 3 indicates that the mean temperature varied from 22.5 °C to 36 °C approximately. Figure 4 shows that the predicted joint opening changed between 1.02 mm (0.04 in.) and 2.29 mm (0.09 in.) approximately under the above-mentioned varying mean temperature and increasing drying shrinkage given in Equation 17, while the measured openings vary between 0.76 mm (0.03 in.) and 1.9 mm (0.075 in.) during the monitoring period.

4 DISCUSSION

There are many factors influencing joint opening prediction, such as the slab's temperature profile, concrete setting temperature, the drying shrinkage over the first few weeks and months, the slab-subbase interfacial restraint and concrete time-dependent material properties (e.g. elastic modulus, creep and stress relaxation). During the first 220 hours of the model calculation (Figure 4), the predicted maximum joint opening matches well with the measured values for each daily cycle. However the measured daily joint movement, i.e. difference between maximum and minimum joint opening, was under-predicted by the model. The main reason for this discrepancy is the drying shrinkage model does not allow for temperature-dependent reversible shrinkage strain. It is the reversible part that tends to decrease joint opening especially during the day. This reversible shrinkage strain has been studied for concrete and is referred to as the hygrothermal strain (Grasley 2006). The current free drying shrinkage model used in this paper does not adequately capture the hygrothermal strain or the concrete's tensile creep, which are both currently under investigation.

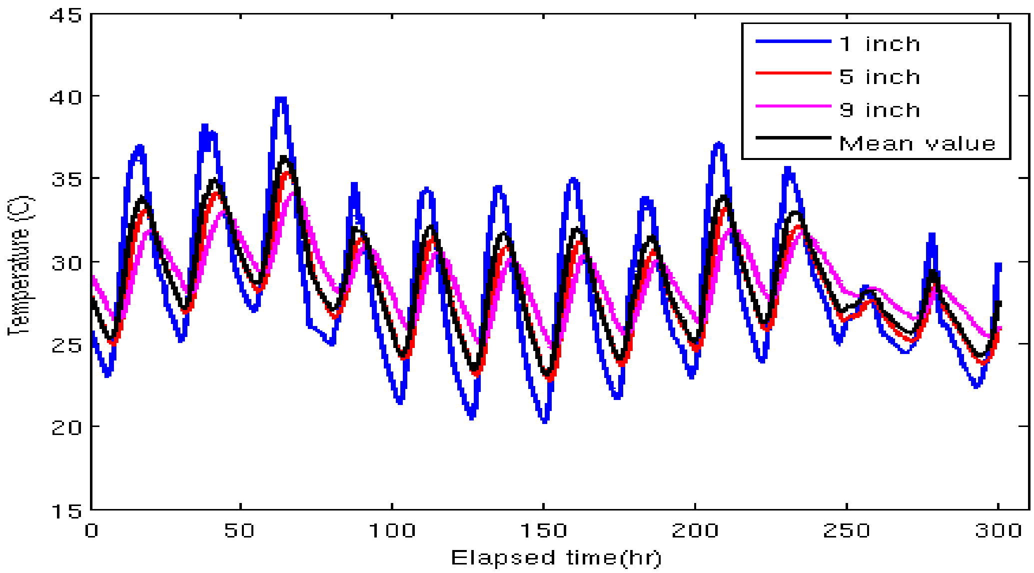

Figure 3. Measured pavement temperature at different slab depths and calculated mean temperature

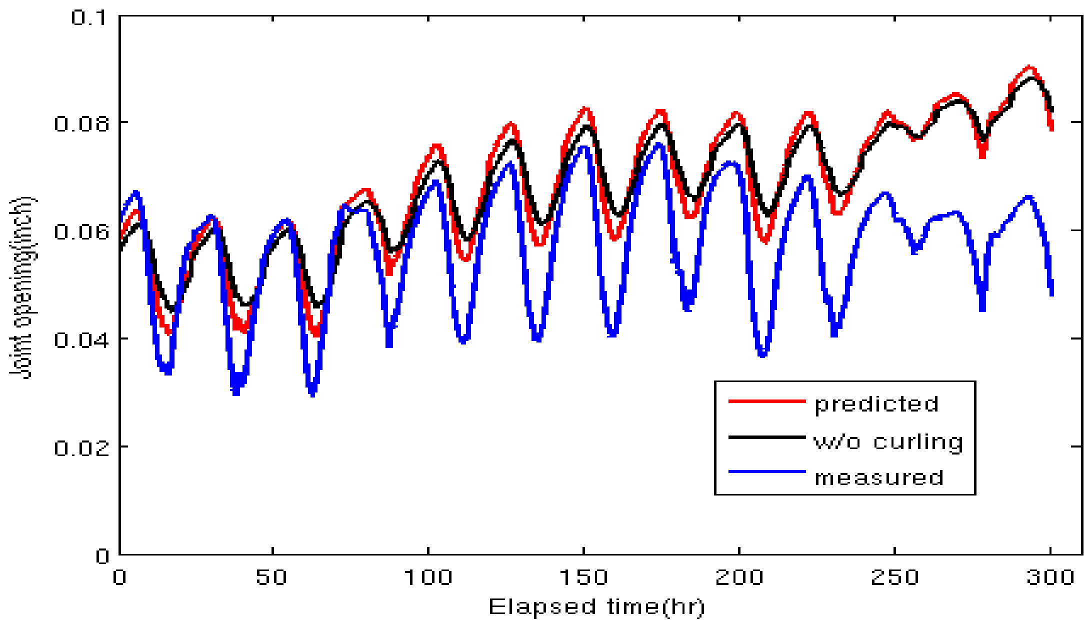

Figure 4. Measured and predicted joint opening values using the proposed model

5 CONCLUSION

A rational approach to predicting the time-dependent joint opening in JPCP is systematically presented in this paper. The proposed analytical model for joint opening takes the effects of the environment, subbase type, concrete material constituents, and slab geometry into consideration. In particular, a closed-form solution to correct the joint opening value due to temperature curling is also proposed. Initial model calculations using measured slab temperature profiles and assumed material parameters suggest that the proposed analytical model generate reasonable joint opening value for JPCP.

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