American Association for the Advancement of Science Annual Meeting, Student Symposium, Boston, MA (2008)

ELECTRON BEAM FOCUSING FOR THE INTERNATIONAL LINEAR COLLIDER

Forrest Iandola
Illinois Mathematics and Science Academy
Aurora, IL 60506
forrest@imsa.edu

Michael Syphers
Fermi National Accelerator Laboratory
Batavia, IL, 60510
syphers@fnal.gov

ABSTRACT

The International Linear Collider (ILC) will be a pair of linear accelerators (LINACs) that accelerate electron beams and increase their energy to 250 giga-electron volts (GeV) and facilitate the collision of the two beams. The design parameters include the number of focusing magnets and their focal length. The investigation discussed here involves the construction of a computer model of the ILC, and its use as a tool for mathematically determining optimal values of these parameters. Misaligned focusing magnets cause electrons to lose energy through synchrotron radiation and increase the beam size, which is detrimental to the operation of the collider. Surveyors can feasibly install magnets in the tunnel environment to an alignment tolerance of 0.25mm, so this investigation considers solutions to compensate for the misalignment. A practical solution for correcting the focusing magnet alignment error is to use additional correction magnets, although this method causes more synchrotron radiation. By using the computer model, the beam focusing system can be optimized; the results of the analysis will be presented.

Key Words: High-energy physics, Particle accelerator, Computer modeling,
Scientific computing, Statistical analysis, Quadrapole magnet

1. INTRODUCTION AND HYPOTHESIS

This study seeks to determine the necessary tolerances for the quadrapole focusing magnets of a 250 GeV LINAC. The final beam size for the LINAC should be at or below 5 microns, and this should be accurate to a 90% probability. As a basis of comparison, Tevatrons magnets have a tolerance of approximately 0.25 mm. In other words, the Tevatron could still function with a misalignment of as much as 0.25 mm. However, we expect the ILC will need more specific tolerances than the Tevatron, because electrons are much more sensitive to steering than protons, due to their lesser mass. Preliminary calculations of beam dispersion predict that the tolerance will be below 0.1 mm. Since surveyors can align the focusing magnets to a minimum tolerance of 0.25 mm in the tunnel environment, dipole corrector magnets are considered in the design if the required tolerance is less than 0.25 mm.

2. REVIEW OF LITERATURE

One of the most important tools in particle physics research is the particle collider. This section will discuss the basics of how colliders work, a brief history of colliders, and some specific accelerator physics principles that will be used in this investigation.

2.1 General Collider Design and History

Colliders, also called accelerators, use a series of electrically charged cavities to accelerate electrons or protons to high energies in opposite directions. Once a beam of particles reaches the prescribed energy, the beam is directed into a collision with the beam traveling in the opposite direction. When the collision occurs, the particles break down into subatomic particles, which are analyzed by physicists. Fermilab is host to the largest collider in the world, the Tevatron. The Tevatron is a synchrotron collider, which accelerates proton beams through many revolutions of circular path before the protons are sent into collision.

Synchrotron accelerators are in many ways more efficient as colliders in comparison to high-energy linear accelerators (LINACs), because circulating the beams allows the particles to be used over and over again for collisions. (While groups of billions of particles may pass through each other in a collider, only a few particles actually "interact" during each passage.) However, the circular motion of a synchrotron accelerator can limit the energy that a particle can accumulate because of synchrotron radiation. Synchrotron radiation is a charged particles loss of energy caused by its acceleration. Just as electrons oscillating up and down in a radio antenna produce radio waves, electrons circulating in an accelerator produce synchrotron radiation.

Circular accelerators can have a maximum particle energy that cannot be exceeded. This maximum energy occurs when all of the energy the acceleration cavities added to the particles is lost in synchrotron radiation. To change the velocity vector of a particle in a circular collider, steering magnets must be used. The Tevatron has quadrapole magnets placed in approximately 100-foot intervals, which keep the particle beam focused along its circular path. Linear accelerators (LINACs) also require quadrapole magnets to focus the beam. However, LINACs only need to make minor corrections to their beam trajectory rather than forcing the beam into a circular path. LINACs lose only a small amount of energy to synchrotron radiation, because they do not force the electron beam into a circular path [6].

As the construction of the CERN synchrotron accelerator, the Large Hadron Collider (LHC), proceeds in Switzerland, the Tevatrons reign as the highest energy synchrotron collider draws to a close. Fermilab is already busy carving itself a new niche: electron collision analysis. Unlike protons, which have a mass of $1.67 * 10^{-27}$ kg; electrons have an even smaller mass of $9.11 * 10^{-31}$ kg. Due to their low mass, electrons are more susceptible to synchrotron radiation than protons. Because of synchrotron radiation, a synchrotron collider like the Tevatron cannot be used to accelerate electrons to a high enough energy to produce useful electron collisions.

LINACs are necessary for studying electron collisions, because their lack of circular motion creates an environment with much less synchrotron radiation than in a synchrotron collider. The new LINAC is much less powerful than the Tevatron, which is rated for over 2000 GeV [5]. However, much more length would be required to increase the electrons power to 2000 GeV, and

250 GeV is sufficient for the next set of experiments involving electron collisions at near-light speed. This is due, in part, because electrons are fundamental particles, whereas protons are made up of quarks and gluons. Since a proton is made up of three quarks and several gluons, the protons energy is divided amongst these particles.

The Stanford Linear Accelerator Center (SLAC) has a 50 GeV, 3 km LINAC [6]. When the SLAC accelerator was first built in 1966, it ran at a cutting edge 20 GeV [4]. 20 GeV is not an especially high energy by todays terms, but it was an immense amount of power when compared with the sub-100,000 eV accelerators of the 1940s [7]. Advances in electromagnetic technology have greatly increased the power and compactness of accelerating cavities since 1966. These advances include superconducting and cryogenic (supercooled) cavities. The SLAC accelerator is a world leader in LINAC technology, yet it is much smaller and less powerful than the proposed International Linear Collider (ILC).

To study electron collision, Fermilab is positioning itself to host an underground LINAC, which, due to its low synchrotron radiation energy loss, is the perfect tool for studying high-energy electrons. The ILC will be capable of accelerating electrons to 250 GeV.

2.2 Physics Principles Pertaining to this Investigation

Each individual electron does not begin its journey down the accelerator at the *exact* same time, or from the *exact* same point. These small time and location differences cause the individual electrons in the beam to have slightly different amounts of energy and slightly different trajectories at any given time.

Quadrapole focusing magnets are employed in LINACs to focus electrons into a more homogenous beam. As shown in Figure 1, these magnets have four poles with alternating fields. These poles surround a hole at the center of the magnet, which the beam passes through. The fields created by these poles cancel in the center of magnets hole. Thus, a particle that passes precisely through the center of the magnet will see no field, but a particle that passes through another point in the magnet will see a field. The magnet does not affect a particle if the particle passes through the center of the magnet. [3]

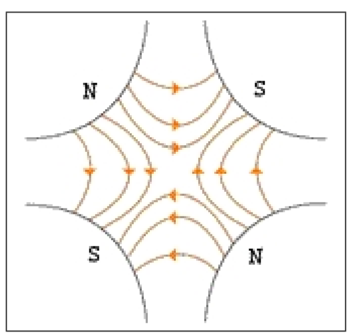

Figure 1: Field lines of a quadrapole magnet.

However, if the particle is displaced from the magnets center, the particle will be steered. The field of a quadrapole magnet increases from the center outward. Therefore, if a particle is displaced from the center of the magnet by a large amount, it will be steered more than a particle of identical energy that is only displaced by a small amount, acting much like a thin lens that focuses light. If x is equal to the particles displacement from the center of the magnet, p is equal to the momentum, q is the charge, l is the length of the magnet, and B is the magnetic field gradient, the angle at which a particle is steered ($\Delta x'$) can be approximated with Equation 1.

$$\Delta x' = \frac{Blx}{p/q} \tag{1}$$

Each quadrapole magnet focuses in one dimension and defocuses in the other dimension. For example, if a particle travels through an x- (horizontal) plane focusing magnet and it is displaced in the x-dimension, the particle will be steered toward the center of the magnetic field in the x-dimension. However, if the particle is displaced in the y- (vertical) plane, the x-plane focusing magnet will steer the particle away from the center of the field in the y-dimension. The focusing and defocusing magnets are arranged in an alternating fashion, so that alternating dimensions of the beam are focused as the electrons speed down the LINAC. Over the course of only a few meters of acceleration cavity, the electrons accelerate from rest to near the speed of light. However, once the particle beam approaches the speed of light, its momentum continues to increase, but its velocity remains constant. By studying the relativity equation

$$p = \frac{mv}{\sqrt{1-(v/c)^2}} \tag{2}$$

it is apparent that, when the beam velocity nears the speed of light, v/c asymptotically approaches 1. Here, m is the rest mass of the particle. The denominator of this function nears 0, which causes the momentum p to near infinity. However, after the velocity is virtually equal to the speed of light, the beam momentum continues to increase as the accelerator cavities add energy to the beam. This results in a virtual increase in the mass of high-energy particles.

Considering $F = ma$, we see that, if the mass of the particle m increases and the force from the magnetic field F does not change, the force of the field will cause a low energy electron to accelerate more than a high-energy electron. If a quadrapole focusing magnet exerts a force on a low energy, low mass electron and then exerts the same force on a high energy, high-mass electron, the low-energy electron will be forced to steer more than the high energy electron. In short, when a particle travels near the speed of light, an increase in the particles energy causes the particle to be less affected by a quadrapole magnets magnetic field.

As was mentioned in the introductory paragraph of this section, each particle in an electron beam has a slightly different energy. As a result, a quadrapole magnet affects each particle slightly differently, and this effect creates a unique trajectory for each particle. However, the particles radiate energy each time they are steered by a quadrapole magnet. This loss of energy effectively reduces a particles mass, which creates a new ideal trajectory for the particle. Although the ideal trajectory changes, the particle remains on the trajectory for its previous energy. This trajectory discrepancy, called dispersion, causes the particle to oscillate around its new ideal trajectory. When most of the particles in the accelerator are oscillating from this effect, the beam width begins to grow.

In order to calculate how much beam dispersion results from the energy loss, the dispersion function can be employed. The heart of the dispersion function is a comparison between the trajectories of particles with different energies as they are steered by a field. By comparing two trajectories for slightly different initial energies, the dispersion function (Equation 3 can determine how much a beam would disperse (D) with various quadrapole magnet displacements.

$$D = \frac{x(E + \Delta E) - x(E)}{(\Delta E/E)} \tag{3}$$

Then, as particles radiate while being steered by a quadrapole magnet of focal length F_q, the spread in the trajectories, and hence the total beam width along the LINAC, can be estimated by using Equation 4. [6]

$$\Delta < x^2 >= C_x (E/E_0)^5 |(x - d)/F_q|^3 D^2 \tag{4}$$

Using this technique, the dispersion function can determine how much a beam would disperse with various quadrapole magnet displacements. Then, as particles radiate while being steered by a quadrapole magnet, the spread in the trajectories, and hence the total beam width along the LINAC, can be estimated.

3. METHODS AND MATERIALS

Now that quadrapole magnets, synchrotron radiation, high momentum particles, and beam dispersion have been discussed, enough information is present to create a computer program to model the dispersion and subsequent beam size growth due to misaligned focusing magnets in a high-energy LINAC. The computer model will track particles of different initial energies in order to ascertain the dispersion function, and then equations for synchrotron radiation [6] will be used to determine the increase in beam size at the end of the LINAC. Effects of alignment error tolerance, distance between focusing magnets, and focal strengths on the final beam size can be explored using the model.

In order to create a computer model, software to write in a programming language must be procured. Since this study emphasizes the statistical probability that the accelerator will deliver a beam with the proper characteristics, the ideal programming language would have built-in statistical tools. R [2] is a piece of free, open-source software that meets these qualifications and runs on Windows, Mac OS, and Linux. Because of its features and flexibility, R is the language of choice for this computer model.

The first step for creating a linear collider simulation with R is to write a simple code to track an electron through a quadrapole magnet, through an acceleration cavity, and to the next magnet. The function $\Delta x' = \frac{Blx}{p/q}$ that was discussed in the Review of Literature section can be used to find the angle at which the quadrapole magnet steers a particle. The particle tracking process can be applied for a long series of magnets, effectively modeling the path of an electron through the accelerator.

The particle-tracking loop can be expanded to track numerous individual particles of different energies through an accelerator with misaligned magnets. The computer generates a series of random numbers; each of these numbers corresponds with a magnets displacement. The program is then revised to move the values from the particle tracking loop into the dispersion function, and the dispersion function gives the dispersion for each magnet. Then, an outer loop is created so that the program can be re- run thousands of times. Each time the program runs, the random number generator creates new magnet misalignment values, so that a new accelerator with random misalignment values is built in the program.

After the program executes, the user can analyze the dispersion at the accelerators final magnet. In order to meet the physicists specification for the beam width (90% probability of a 5-micron beam), the root mean squared (RMS) value of the randomly generated misalignment numbers can be adjusted. The `rnorm` function of the R language is used to generate a distribution of random numbers from a normal (bell-shaped) distribution. Since the RMS is the typical misalignment value, the RMS is equal to the alignment tolerance of the accelerator. For example, if the user gave an RMS value of 0.1, 68.27% of the magnet offset values would be between 0.1 and 0.1. In order to achieve a 90% chance that the accelerator has the proper beam specifications, the built-in R analysis tools can be used to find the RMS value at which 90% (almost two standard deviations) of the accelerators that the program builds can deliver a 5-micron beam. A practical way to analyze the data from the program is to track the beam dispersion results as the RMS value is adjusted.

Since the magnet tolerance was hypothesized to be below 0.25 mm, some form of correction may need to be installed. One possibility for correcting the results of a misaligned magnet is to install another magnet next to each misaligned magnet. The correction magnets would be variable-energy dipole magnets, which steer in one plane but do not steer in the other. The magnets also contribute to synchrotron radiation; the beam radiates from the misaligned focusing magnet and from the correction magnet. For example, if a quadrapole magnet is displaced by 0.1 mm vertically, it steers the beam downward. A dipole corrector, arranged to steer vertically at that location, can be energized to re-steer the beam into its intended path. The beam radiates energy at the quadrapole magnet and at the dipole magnet, so the dipole correcting magnets contribute to beam size growth. However, the dispersion in this case is reduced, and so helps to mitigate the overall effect.

The ILC simulation program was modified to have corrector magnets at each focusing magnet. The user controls the percentage of correction induced by the dipole magnets in proportion to the individual quadrapole misalignments. For example, if a quadrapole magnet is misaligned by 0.5 mm, and the dipole magnets correct 50% of the quadrapole error, the overall effect is like a quadrapole misaligned by 0.25 mm. For the quadrapole magnet with an offset of 0.5 mm with 0.25 mm of correction, the summation of these magnets would cause a similar amount of synchrotron radiation to an uncorrected quadrapole misaligned by 0.75 mm.

4. RESULTS

After running sets of 1000 trials with varied RMS displacement values, it is apparent that the magnets must be aligned to within .05 mm of their optimum location in order to attain a 5-micron beam width for 90% of the electrons in the beam. As demonstrated in Figure 2, if RMS quadrapole magnet displacement is greater than .05 mm, the beam will be too broad, and fewer collisions will occur between the two opposing beams.

Unfortunately, the required tolerance is much lower then 0.25 mm, the lowest feasible tolerance in the accelerator tunnel environment, so dipole magnets are employed in the calculation. Dipole magnets that correct 95% of each quadrapole magnet error allow the quadrapole tolerance to be relaxed to 0.25 mm (Figure 3). To ensure that the beam location is inside the accelerator, the offset from the center of the accelerator at the end of the LINAC was also considered in relation to quadrapole misalignment (Figure 4) and dipole correction (Figure 5).

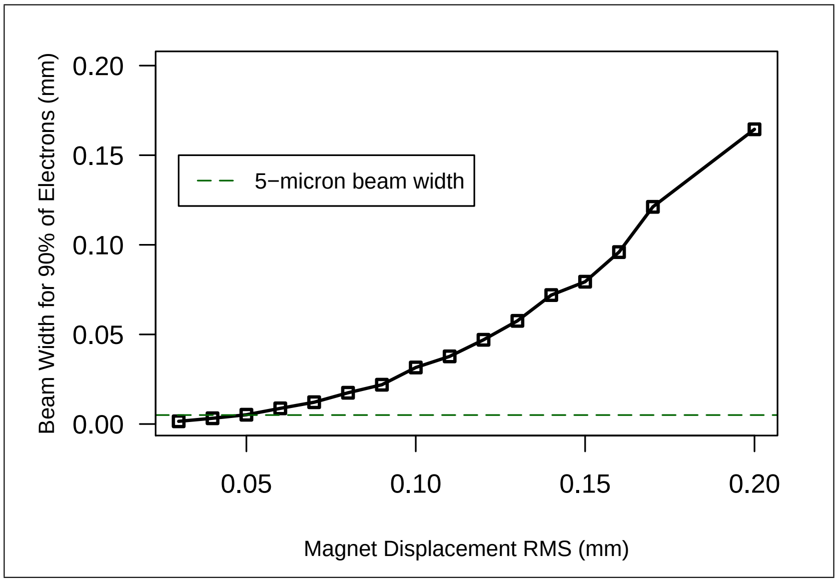

Figure 2: RMS of Beam Displacement vs. Beam Width.

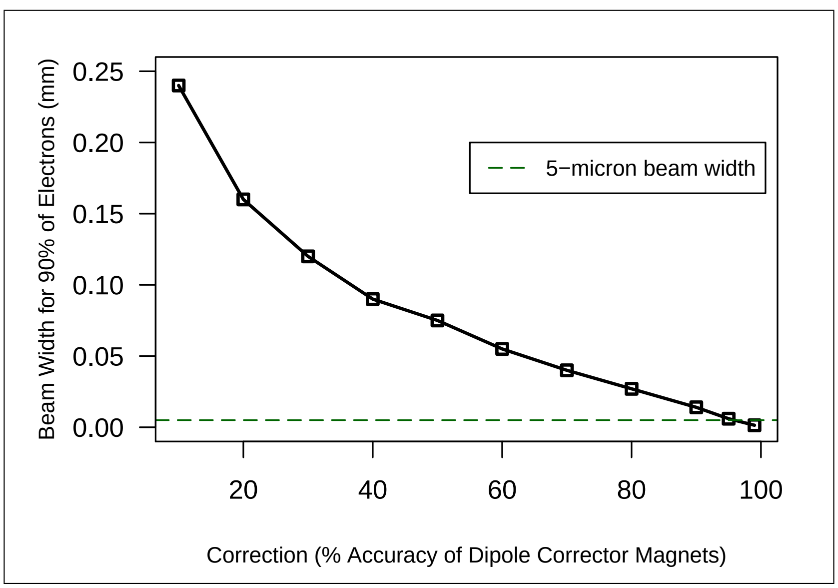

Figure 3: Correction vs. Beam Width; Magnet Displacement RMS = 0.25

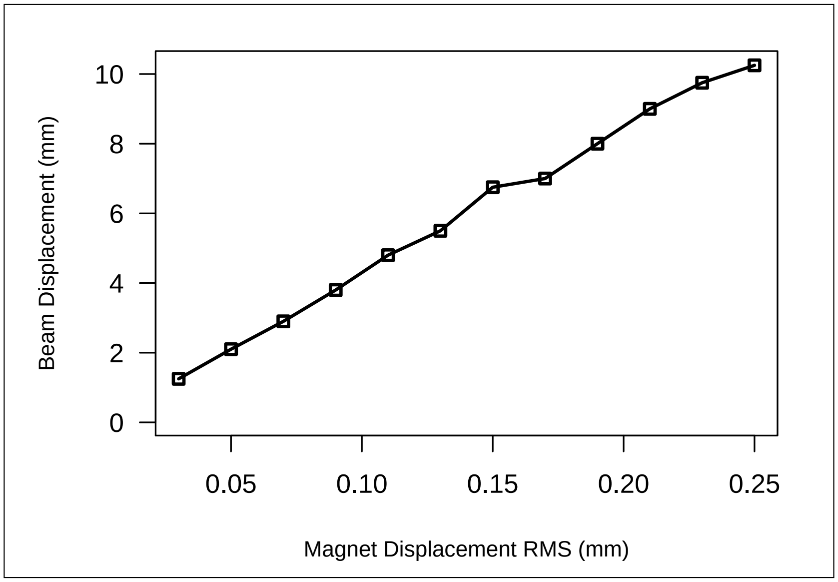

Figure 4: RMS of Beam Displacement vs. Beam Displacement.

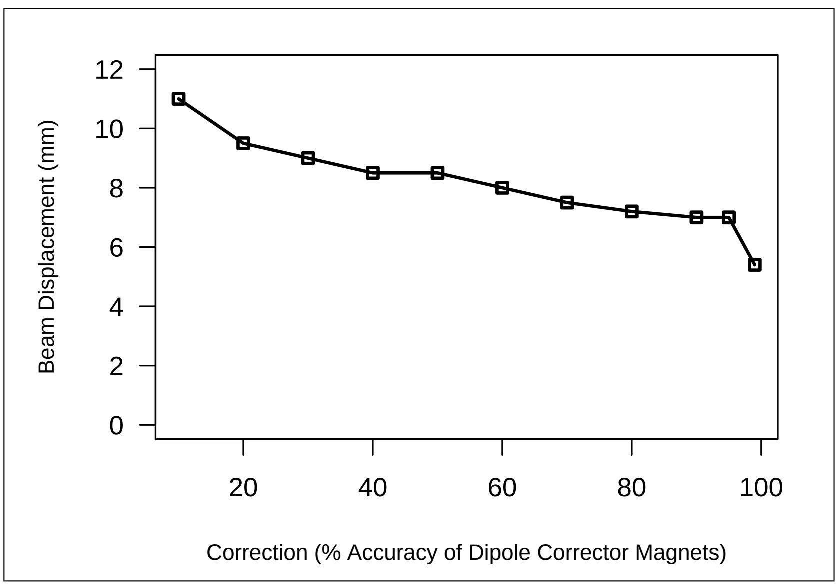

Figure 5: Correction vs. Beam Displacement; Magnet Displacement RMS = 0.25

5. CONCLUSIONS

Since this simulation was run thousands of times for each RMS input value, and the average final beam size is the data used in the study, the margin of error involved in this calculation is fairly low. From this investigation, it is concluded that, without extra correction, the quadrapole magnet tolerance of the International Linear Collider is 0.05 mm. With a minimum of 95%

correction from dipole magnets, a 0.25 quadrapole tolerance may be used. Many other effects may contribute to the beam size growth, such as magnet and cavity imperfection, but these effects would need to be examined in future studies.

ACKNOWLEDGEMENTS

This work was supported by Dr. Judith Scheppler and the Illinois Mathematics and Science Academy Office of Scientific Inquiry and Research.

REFERENCES

1. D.A. Edwards. Notes on Relativity, `http://www-ap.fnal.gov/~syphers/uspas/RelativityNotes.pdf` (2005).

2. R. Ihaka and R. Gentlemen. R-Project, `http://www.r-project.org` (2006).

3. R.B. Neal *The Stanford Two-Mile Accelerator*. W.A. Benjamin Corporation, New York, NY, USA, (1968).

4. W.K.H. Panofsky and R.B. Neal. Electrons Accelerated to the 10- to 20-Gev Range. *Science.*, **152**, pp. 1353-1355, (1966).

5. A.L. Robinson. Fermilab Installing Superconducting Magnets. *Science.*, **217**, pp. 814-817, (1982).

6. M.J. Syphers and D.A. Edwards. *An Introduction to the Physics of High Energy Accelerators.* John Wiley and Sons, Inc, New York, NY, USA (1993).

7. M.S. Livingston. *Particle Accelerators: A Brief History.* Harvard University Press, Cambridge, MA, USA (1969).

8. W.M. Mitchell. SLAC Feels the Thrill of the Chase. *Science.*, **244**, pp. 771-773, (1989).