TWO-DIMENSIONAL SUPERCONDUCTING STATES 
NEAR ZERO TEMPERATURE QUANTUM PHASE TRANSITIONS 

BY 
LUKAS URBAN 

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Doctoral Committee: 
Professor John Stack, Chair 
Professor Ali Yazdani, Director of Research 
Professor S. Lance Cooper 
Professor Paul Debevec
Abstract

The competition between localization and superconductivity in two dimensions has puzzled physicists for decades. In two dimensions, only two electronic phases are predicted to exist at zero temperature: superconducting and insulating. Contrary to this theoretical expectation, previous transport measurements on 2-dimensional (2D) thin films have found evidence for metallic resistivity down to extremely low temperatures when using either disorder or magnetic field to tune between the superconducting and insulating phases. In this thesis we further investigate the mechanism for the superconductor to insulator transition at zero temperature. We study the destruction of superconductivity due to the application of sufficient magnetic field in 2D films of MoGe and InOx.

Unlike previous works, which concentrated on four point resistivity measurements, we focus primarily on measuring the AC conductivity of the 2D films, which probes the superfluid response. Using a contactless technique, we have been able to measure the superconducting transition as a function of temperature and demonstrate the existence of a Kosterlitz-Thouless (KT) transition in zero magnetic field. Applying a magnetic field to the sample, we have been able to observe the suppression of the superconducting response. Temperature sweeps in a magnetic field showed a similar discontinuity as observed in the zero field KT transition, suggesting a similar process for the destruction of superconductivity in both the zero and non-zero magnetic field cases. Analysis of the change in the critical temperature as a function of magnetic field suggests a quantum phase transition at zero temperature.
Dedicated to my parents
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Chapter 1

Introduction

1.1 Introduction of Thesis Problem

The development of the BCS theory explained the mechanisms responsible for many of the properties observed in many superconductors. While in three dimensions (3D) the destruction of the superconducting state is well understood, the physics in two dimensional (2D) superconductors still poses many open questions. What is the nature of the superconducting state? What happens when a magnetic field is applied or the disorder changed? What happens during the transition out of the superconducting state?

In 2D superconductors, disorder in the phase of the order parameter plays a much stronger role than it does in 3D. As a result 2D films are an ideal system to test the predictions of various theories involving phase fluctuations. Of particular interest is the destruction of superconductivity at the zero temperature limit. 2D films are expected to exist in only one of two states at zero temperature, superconductor or insulator. Unlike in 3D, a metallic state should not exist in 2D in the zero temperature limit because all electron states are expected to localize. This leads to an expected zero temperature transition in 2D superconducting films that can be tuned with a tuning parameter such as disorder or magnetic field from superconductor with zero resistance to an insulator with infinite resistance. Because at zero temperature, there are no thermal fluctuations, the mechanism responsible for the suppression of superconductivity is expected to be quantum mechanical. As a result 2D superconductors at zero temperature serve as a good system to investigate these quantum phase transitions (QPT).
Another reason for investigating 2D superconductors is that the physics is different from the 3D systems. New physics in the 2D case applies not only to superconducting thin films, but also to other systems that are of interest today including one of the most interesting topics in superconductivity in the last 20 years, the exotic high-temperature superconductors. These materials have a critical temperature \( T_c \) that is much higher than that of the conventional superconductors. While they have been studied for more than two decades, they are still not well understood. Because the high \( T_c \) superconductors are layered, in some ways they act like stacks of 2D superconductors. For this reason understanding 2D superconducting films can lead to a greater understanding of this class of materials.

In this thesis we will discuss measurements we have performed on the magnetic field tuned transition out of the superconducting state in MoGe and InOx films which in the zero temperature limit is expected to be a quantum phase transition. While in the past most measurements on these systems were done using transport techniques, we have employed a new contactless tool that allows us to extract the complex conductivity and superfluid density of our samples as it changes as a function of both magnetic field and temperature. While resistive transport measurements can measure insulators and metals until their resistance vanishes to the noise floor, we measure inside the superconducting state and are sensitive to changes in the superfluid density as well as resistances below the noise floor of transport measurements. In combination with 4-point transport measurements we are able to measure both inside and outside of the superconducting state. Probing the superfluid in zero magnetic field, we observe a Kosterlitz-Thouless transition. With the application of a perpendicular magnetic field, we find similar behavior to the zero field transition suggesting loss of phase coherence is responsible for the destruction of superconductivity in both zero and non-zero magnetic field. Analysis of the non-zero field data points to a quantum phase transition at zero temperature.
1.2 Introduction to Superconductivity

Before continuing with discussions about the superconducting-insulating transition in thin films, it is necessary to give a basic introduction to superconductivity and some of its properties that will be relevant to the theories and data presented throughout the thesis. While a superconductor is typically experimentally characterized as having zero resistance to the flow of an electrical current, we exploit another property in our measurements - the Meissner effect. This occurs when a superconductor is exposed to a magnetic field, and in response shielding currents flow near the surface in order to cancel out the field inside the bulk. In 3D the bulk of the sample is fully shielded from the magnetic field. In 2D, however, the situation changes. In 2D the characteristic length scales of the material are longer than the thickness of the sample. Both the superconducting coherence length $\xi$ and the magnetic penetration depth $\lambda$ are much longer than the thickness of our samples. Because $\lambda \gg \xi$ our samples are type II superconductors. This classification comes from the way superconductors respond to an applied magnetic field. Those of type I set up currents at the surface in order to completely shield the applied magnetic field from the interior of the sample. Once the field is increased to a critical field $H_c$, the whole sample becomes normal. Type II superconductors on the other hand have two critical fields. At the first critical field $H_{c1}$, the field begins to penetrate the material in solenoid like tubes called vortices. A vortex is a core of normal material surrounded by a shielding supercurrent and carries one quantum of flux. As the field is increased past $H_{c1}$ more vortices enter the sample until a field $H_{c2}$ is reached, at which point the sample cannot have any more vortices and becomes normal.

While many properties of superconductors have been observed since the discovery of zero resistance by Onnes in 1911, it wasn’t until 1957 that a microscopic theory was developed. The BCS theory postulates that the superconducting carriers are pairs of electrons that act as bosons. These pairs, known as Cooper pairs, are formed between two electrons of opposite spin. While typically electrons should repel, it turns out that due to a rearrangement of and
screening by the positive nuclei of atoms, an attractive potential between electrons is possible. The existence of a macroscopic fraction of electrons bound into Cooper pairs is an obvious requirement for the existence of superconductivity. However, for resistance to become zero another condition is placed on the system. The Cooper pairs must be phase coherent with each other throughout the sample. This leads to the two ways in which superconductivity can be destroyed. Either the Cooper pairs can be broken apart or they remain paired but lose phase coherence. These two possibilities are often considered in existing theories of 2D superconductors.

Because the films studied in this thesis are type II superconductors, a brief introduction to vortices is necessary. As mentioned earlier a vortex is a magnetic flux tube surrounded by a circulating supercurrent. At the center of the supercurrent is a normal core with a radius approximately the size of $\xi$, the superconducting coherence length, through which most of the magnetic field passes. The supercurrents, which extend out to a radius of $\lambda$, the magnetic penetration length, shield the magnetic field in the core from the rest of the superconductor. Figure 1.1 shows a drawing of the normal core along with the shielding

Figure 1.1: Illustration of a vortex showing the normal core (red region) and the circulating shielding currents.
currents. The first vortex appears in the sample at $H_{c1}$, when the Gibbs free energy of creating the vortex core and the surrounding shielding currents becomes equal to the energy required to keep the magnetic field out of the sample. As field is increased more and more vortices enter the sample until the field approaches $H_{c2}$, when vortex cores begin to overlap and the sample becomes normal. Because of the presence of vortices and their dynamics, the field range between $H_{c1}$ and $H_{c2}$ is interesting to study as it affects the superconducting state. A supercurrent can flow around vortices without dissipation as long as the vortices do not move. However, such a current $J$ leads to a Lorentz force on a vortex, $f = J \times \frac{\Phi_0}{c}$, perpendicular to the current flow. If the vortex moves a voltage is generated opposite to the current flow and a resistance is measured. However, because real world samples are not homogenous and have a certain amount of disorder, in many cases the vortices do not move in response to the supercurrent flowing around them because of the local variations of the strength of superconductivity. Because the creation of a vortex requires that the vortex core becomes normal, this is energetically more favorable to happen in a local region of weak superconductivity. These regions pin vortices and prevent them from moving unless the Lorentz force exceeds the energy required to move the vortex out of this local depression of the superconducting order parameter. As a result a sample between $H_{c1}$ and $H_{c2}$ should exhibit no dissipation, unless the Lorentz force exceeds the pinning force at which point the vortex will move and resist the supercurrent.

1.3 Phase Diagram

The basic description of type II superconducting films seems to have two different states inside the superconducting regime. One is the Meissner state which exists for fields less than $H_{c1}(T)$ and the other is a vortex state which exists between $H_{c1}(T)$ and $H_{c2}(T)$. The Meissner state does not contain any vortices and is a true superconducting state. Cooper pairs have phase coherence and the DC resistance is zero. The vortex state, however, can
exist in several forms discussed below some of which are also superconducting with zero resistance. Figure 1.2 shows the expected possible states in type II superconductors discussed below. Even though Fisher et al suggest the diagram in the inset applies to 3D type II superconductors, similar states could exist in the 2D case despite the preclusion of long range order in 2D[6].

The most ordered vortex state is the vortex lattice. Because vortices of the same flux direction repel each other to minimize energy, they arrange themselves in a two dimensional lattice which is typically triangular, but a square one is also possible. In a perfect sample with no disorder the lattice is exact but exhibits dissipation when subjected to a current because vortices in a current experience a force perpendicular to the current. With no disorder the vortex lattice is free to move when this Lorentz force is applied. As a vortex moves across the width of a sample the phase changes by $2\pi$ leading to dissipation. However, real life samples have disorder. If the disorder potential is sufficiently strong to pin the vortices in the lattice, the lattice will not move when subjected to a current, and the measured resistance will be zero ohms.

Disorder in the sample can lead to another vortex state, the vortex glass. The vortex glass is similar to the vortex lattice but lacks global order. Instead of being in a perfect lattice, vortices arrange themselves in a perturbed lattice to minimize their energy near pinning sites created by disorder. Instead of one ground state arrangement, there exist many possible ground states due to the disordered energy landscape for vortices. Due to thermal fluctuations above zero temperature, the vortices move among these ground states by collectively jumping among pinning sites which is known as flux creep. Without an external force this creep occurs in random directions, but when a current is applied, the Lorentz force causes flux creep in a direction perpendicular to the current and dissipation occurs.

In contrast to the lattice and glass state, is the vortex liquid. The vortex liquid is made up of vortices that are free to move in response to external forces. As a result transport
Figure 1.2: General phase diagram for type II superconductors. Inset shows the predicted phase diagram for 3D samples. Figure from [6].
measurements would measure dissipation in response to the transport current. The vortex liquid state should exist in two parts of the phase diagram. First, just above \( H_{c1} \) the vortex density is low and hence the vortex spacing is large. Because the repulsive force between vortices weakens with distance, at low vortex densities and nonzero temperatures the energy of thermal fluctuations can be larger than the binding energy of the lattice and the lattice melts. However, in most real world samples with disorder, the vortex liquid near \( H_{c1} \) does not occur because the vortices are pinned. The second area of the phase diagram where a vortex liquid can exist is at large enough temperatures just below \( H_{c2} \) where the vortex density is high such that the vortices overlap. At this point the thermal fluctuations are again larger than the lattice energy and the lattice melts. In this region even in the case of disorder the high vortex density breaks the pinning, and a vortex liquid is possible.

In two dimensions the situation is somewhat different. Figure 1.3 shows the phase diagram proposed by Fisher for 2D films as function of magnetic field \( B \), temperature \( T \), and disorder \( \Delta \). A normal sample cooled along the \( T \)-axis in zero magnetic field and zero disorder will first cross the mean field transition temperature \( T_{c0} \) at which Cooper pairs form. However, below \( T_{c0} \) the sample is still resistive, because of the presence of thermally excited free vortices which destroy phase coherence among the Cooper pairs. These vortices pair up at a temperature \( T_c \) and a zero resistance state exists. As temperature decreases towards zero, these bound thermally excited pairs of vortices slowly disappear with the reduction in \( k_B T \).

Moving away from the \( T \)-axis in the \( T-\Delta \) plane the same thing happens except \( T_c \) and \( T_{c0} \) are reduced with increasing disorder. At zero temperature \( \Delta_c \) is the critical point between superconductor and insulator. Because this happens at zero temperature, the vortices are governed by quantum mechanics and can be seen as Bose condensing above \( \Delta_c \) while the Cooper pairs are localized in what is known as the electron glass.

Looking at the other plane where \( \delta = 0 \) something fundamentally different should happen as the field is increased from zero. A non-zero field creates vortices of one direction inside
Figure 1.3: Suggested phase diagram for 2D films studied in this thesis. Modified figure from [7].

the sample. Below a melting field $B_M(T)$ these vortices are arranged in a vortex lattice as in 3D. Above $B_M(T)$ the lattice melts and the vortices are free due to the presence of thermal fluctuations that exceed the lattice energy. The presence of free vortices disrupts the phase of the cooper pairs and the sample resistance becomes non-zero. In non-zero disorder and non-zero temperature, however, the vortices in a lattice are expected to creep. Moving vortices will destroy the phase coherence and once again resistance becomes non-zero. Only at zero temperature will the creeping vortices freeze out into a vortex glass with zero resistance.

The above mentioned states are not the only ones possible, but some have been experimentally verified and provide a good background for what we can expect to see in our measurements. More complex theories of other phases will be discussed later in this chapter.
1.4 Thermal Phase Transitions

Given a basic understanding of a possible phase diagram, it is good to understand the known thermal transitions of 2D superconductors in more detail before discussing the field tuned transition at zero temperature. The ability to measure and verify the known transitions is useful to help us develop methods for understanding the mechanism for the transitions at zero temperature in our data.

In 3D in zero magnetic field, the typical way in which superconductivity is destroyed is by the breaking of Cooper pairs. This happens when temperature is increased to the point where the energy of thermal fluctuations is comparable to the binding energy of the Cooper pairs. While the same case does occur in 2D, another mechanism is also possible for the loss of the superconducting ground state. Instead of breaking the Cooper pairs, it is possible for them to lose phase coherence and the resistance will become non-zero. One such transition mentioned earlier occurs when the phase fluctuations are caused by free vortices. This transition was proposed by Kosterlitz and Thouless[16] for helium films and applied to superconducting films by Beasley et al[1]. We will now look at why it occurs in more detail.

A Kosterlitz-Thouless transition happens in zero magnetic field out of the superconducting phase when cooper pairs lose phase coherence at a temperature $T_{KT}$ which is less than $T_c$, the BCS mean field transition temperature. This transition is marked in Figure 1.3 in the $T$-$\Delta$ plane as $T_c$. The reason for the loss of phase coherence is the presence of free vortices even at zero magnetic field at non-zero temperatures. To explain this we look at the change in free energy $\Delta F = E - TS$ for the addition of a free vortex, where $E$ and $S$ are the energy and entropy of an isolated vortex, respectively. The energy of a single vortex is $E = \frac{\pi \hbar^2 \rho}{2m} \ln \frac{A}{A_0}$, where $\rho$ is the superfluid density, $m$ is the particle mass, $A$ is the area of the sample, and $A_0$ is an area scale. The entropy is $S = k_B \ln \frac{A}{A_0}$. Because both the energy $E$ and entropy $S$ of a single vortex scale as the logarithm of the area of the system, we can see that $\Delta F$ will be dominated by $E$ at low temperatures and $S$ at high temperatures. We
can see that $\Delta F$ is zero at a temperature $T_{KT}$ when

$$k_B T_{KT} = \frac{\pi \hbar^2 \rho}{2m}.$$

Above this temperature $\Delta F$ is negative, and it becomes favorable for free vortices to enter the sample destroying global phase coherence among the Cooper pairs. A more in depth calculation uses renormalization theory[16].

As shown above, minimization of the free energy precludes the existence of single vortices at low temperatures. However, vortex pairs of opposite vorticity can exist because the energy of such a pair is finite due to the cancellation of currents far away from the vortex-anti-vortex pair, while the entropy scales as the logarithm of the area. Vortex-anti-vortex pairs contribute nothing to the net magnetic field through the sample, because each carries a flux quantum of the opposite sign. This allows vortex pairs to form, and increase in both number and size as the temperature increases from zero up to $T_{KT}$ at which point the pairs unbind because free vortices are energetically possible as shown above. This unbinding happens starting with the largest pairs due to screening by smaller pairs. When the largest radius pairs unbind, the presence of free vortices destroys DC superconductivity through the destruction of phase coherence among Cooper pairs. AC techniques can be sensitive to the unbinding of smaller pairs because they measure on shorter length scales and as a result measure a higher $T_{KT}$.

Kosterlitz and Thouless predicted $T_{KT}$ to be on a universal line as a function of the superfluid density. While this was initially theorized to only occur in non-charged superfluids, Beasley et al[1] showed the same could be applied to 2D superconducting films. They have taken the above result for $T_{KT}$ and showed it to be $k_B T_{KT} = \frac{1}{2} \pi \hbar^2 n_s^{2D}/m^*$, where $n_s^{2D}$ and $m^*$ are the 2D superfluid density and effective particle mass, respectively, for the 2D superconductors. They then calculated the relationship between the perpendicular penetration
length $\lambda_\perp$ and $T_{KT}$ to be

$$\lambda_\perp(T_{KT}) = \frac{\Phi_0^2}{8\pi\mu_0 k_B T_{KT}}$$

where $\Phi_0$ is the magnetic flux quantum. This describes the linear relationship between $T_{KT}$ and $\frac{1}{\lambda_\perp}$ known as the universal jump line

$$\frac{1}{\lambda_\perp(T_{KT})} = \frac{8\pi\mu_0}{\Phi_0^2} k_B T_{KT}.$$  

As temperature is increased, superconductivity is destroyed when $\frac{1}{\lambda_\perp(T)}$ falls below this line and vortex-anti-vortex pairs unbind.

Experimentally, the KT transition is expected to give a non-linear current response near
the transition[6]. Transport measurements should observe an electric field that scales as the cube of the applied current. As a function of frequency we expect a $1/\omega$ dependence of the both the real and imaginary parts of the conductivity as we approach the transition from the superconducting side. Figure 1.4 shows the inverse inductance $L^{-1}$ as a function of temperature $T$ measured by Hebard and Fiory on a superconducting aluminum film at different frequencies[10]. The inverse inductance is an experimentally measurable quantity which is proportional to the superfluid density. The plot shows the mean field $T_c$ as well as the Kosterlitz-Thouless transition temperature $T_{KT}$. In the plot the highest frequency signal persists to beyond the mean field $T_c$, while the lowest frequency one drops sharply towards zero at $T_{KT}$. This frequency dependence is because the length scale of the measurement is set by the frequency. Low frequency measurements are sensitive to the unbinding of vortex-anti-vortex pairs at large separations which occurs at lower temperatures, whereas high frequency measurements are sensitive to the unbinding of pairs of small pair separations. Figure 1.5 shows the I-V plot for an Indium oxide film for different temperatures[11]. Curve d shows the predicted cubic dependence of the voltage on the current for a Kosterlitz-Thouless transition.

Another observed thermal phase transition in 2D is an analogue to the above mentioned case in non-zero magnetic field. As was discussed earlier in the presence of a perpendicular magnetic field, vortices enter the sample and form a lattice. In the case of sufficient disorder this lattice is pinned and does not move when subjected to an electric current, leading to a true superconducting state. However, at non-zero temperature, the lattice isn’t perfect and dislocations in the lattice exist[13]. Energetically the dislocations behave similarly to the vortex anti-vortex pairs in the zero field case. Both the energy $E$ and entropy $S$ of adding a single dislocation scale as the logarithm of the sample. However, the energy of a dislocation anti-dislocation pair is finite. This means that bound dislocation pairs can form at non-zero temperature. Following the same process as at zero field, there exists a temperature $T_M$ at which $E = TS$ and the creation of a free vortex becomes energetically favorable leading to the unbinding of dislocation pairs. As a result of free dislocation, the vortex lattice melts
Figure 1.5: Graph showing the voltage as a function of transport current for an Indium oxide film taken at different temperatures ranging from 1.939 K(a) to 1.460 K(m). Curve d at 1.903 K shows the predicted cubic power law dependence for a Kosterlitz-Thouless transition[11].
giving rise to non-zero resistance. Because of the dependence of $T_M$ on the pinning strength of vortices, this transition can be tuned by changing the disorder in the film as well as the magnetic field. The phase diagram in Figure 1.3 shows the field dependent vortex lattice melting line $B_M$. Evidence of the vortex lattice melting transition has been shown in MoGe samples by Yazdani et al[30, 28]. In their experiments they measured samples of varying disorder for different values of the perpendicular magnetic field. Figure 1.6 shows the real and imaginary parts of the conductance of two of these films as a function of temperature. The thicker film shows a discontinuous jump in the imaginary part at the predicted melting temperature $T_M$. The thinner film which is more disordered does not show such a jump but instead is continuous through the predicted $T_M$ all the way to zero. The experimenters showed that the melting of the vortex lattice occurred for less disordered films but was suppressed by increasing disorder. When probed on different length scales the samples showed two different behaviors. For short length scales the KT lattice melting was present, but on longer length scales the transition appeared to be vortex creep driven. The more disordered samples only showed vortex creep behavior consistent with the vortex glass phase and a disorder-tuned quantum phase transition at zero temperature.

1.5 Quantum Phase Transition

While the previous mentioned transitions occur at non-zero temperatures, at zero temperature the system changes its ground state via a quantum phase transition (QPT). A quantum phase transition is a transition which occurs not as a function of temperature, but instead as a function of some parameter in the Hamiltonian of the system, such as the disorder or magnetic field. Unlike classical phase transitions that occur when the fluctuations of size $k_B T$ become of the order of the relevant energy scale of the system, quantum phase transitions occur at zero temperature as a result of quantum mechanical zero point motion. However, because we cannot measure at zero temperature, we must extrapolate and analyze data at
Figure 1.6: Temperature sweeps showing the real and imaginary parts of the conductance of two MoGe films of different thickness. The less disordered 3000 Å film shows a discontinuous jump in $\omega G_I$ near the predicted melting temperature $T_M$ (indicated by arrow). However, the more disordered 300 Å film goes to zero continuously[28].
non-zero temperatures to verify and learn about a quantum phase transition. To do this we must understand how the transition changes as temperature increases from zero.

Theoretically at zero temperature a quantum phase transition in \( d \) dimensions has an analogue in a classical phase transition of \( d + 1 \) dimensions, where the added dimension is an imaginary time dimension[22]. As a result, we can define two diverging length scales near the phase transition. The first is a physical length scale which diverges as a power of the parameter in the Hamiltonian which is responsible for the phase transition. In the case of our system this parameter is the field \( B \), and we can define the distance to criticality as \( \delta = B - B_c \). The physical diverging length scale then becomes \( \xi \sim |\delta|^{-\nu} \). The second length scale is for the additional time dimension and by convention is related to the physical length scale as \( \xi_\tau \sim \xi^z \). While the quantum phase transition occurs only at zero temperature, certain QPTs can also be indirectly observed at non-zero temperature. In this case close enough to the \( T = 0 \) quantum critical point, the physical observables of the system will scale as a function of the two length scales, \( \xi \) and \( \xi_\tau \).

Figure 1.7 shows a schematic phase diagram for a 2D Josephson junction array near a quantum phase transition that is analogous to the 2D superconducting films we study, only in our case the coupling constant \( K \) is replaced with the magnetic field \( B \) or the disorder parameter \( \Delta \). As the temperature decreases towards zero there are two transitions that approach the quantum critical point. As \( B \) or \( \Delta \) is varied from zero to infinity at non-zero temperatures, the resistance is expected to change gradually from zero to infinity as the transitions shown by the solid and dashed lines are crossed. Only at zero temperature is resistance expected to be discontinuous. At non-zero temperature we must look at the scaling behavior of the observables along these two lines to learn about the quantum critical point. In particular, in our experiments we can only measure near the superfluid regime and hence can only observe the solid line.
Figure 1.7: Schematic of a quantum phase transition at $T = 0$ as a function of the coupling constant $K$ for a 2D Josephson junction array that may also be applicable to the 2D MoGe and InOx films studied in this thesis. Studying the thermally driven transitions at non-zero temperature may allow insight of the $T = 0$ QPT. Figure from [22].
1.6 Probing the Quantum Phase Transition

While the zero field KT transition has been proposed and tested in superconducting thin films, there have been other experiments\[9, 14, 18, 12, 29, 2, 5\] exploring the phase diagram of 2D superconductors looking for the proposed quantum phase transition at zero temperature. These experiments study the transition from superconductor to insulator as a function of increasing disorder or magnetic field and attempt to scale the data measured at non-zero temperatures to infer the existence of the QPT at $T = 0$. Most of these previous experiments have only measured the transport resistance of a sample as it changes when the tuning parameter is varied. In contrast to the previous experiments described below, we study the superfluid density to provide new information about the phase diagram and the proposed QPT.

Disorder tuned experiments looking for the quantum critical point $\Delta_c$ shown in Figure 1.3 are typically done by varying the film thickness. By incrementally depositing a superconducting film in situ and performing resistance measurements at each thickness as a function of temperature, a map of the resistance can be made in the temperature-disorder plane. Increasing the thickness reduces the amount of disorder and the film transitions from insulator to superconductor. Early experiments on films of various materials including Bi, Ga, Pb and others showed the expected transition from insulator to superconductor as the thickness was incrementally increased\[9, 14\]. To verify the existence of the quantum phase transition, scaling analysis was performed on a Bi film to check for power law behavior near the critical point\[18\]. The researchers were able to collapse the low temperature conductivity onto two curves, one for the insulating curves and the other for the superconducting ones by scaling the temperature dependence by $1/T_0^S$ and $1/T_0^I$ for the superconducting and insulating sides of the transition, respectively. Figure 1.8 shows the conductance $G$ of different thickness Bi films plotted against $\ln(T)$ along with the above mentioned data collapse plotted in the inset. The scaling parameters showed divergent behavior and power law dependence on both
sides of the transition. This type of divergent behavior in the conductance is consistent with the existence of a quantum critical point $\Delta_c$ at zero temperature between the insulating and superconducting states. In these experiments the value of the critical resistance was close to the theoretical prediction$^{[8]}$ of a universal value equal to the quantum of resistance for Cooper pairs, $h/4e^2 = 6453 \Omega$.

Similarly, experiments on the magnetic field tuned superconductor-insulator transition looking for the quantum critical point along the $B$ axis in Figure 1.3 showed the same type of scaling behavior as the disorder tuned ones. Scaling by Hebard and Paalanen$^{[12]}$ on InOx films and Yazdani and Kapitulnik$^{[29]}$ on MoGe confirmed the existence of the
phase transition from superconductor to insulator. Figure 1.9 shows the 2D resistance $R_{\square}$ plotted against the scaling parameter $\frac{|B-Bc|}{T^{1/\nu}}$. The critical exponents $z$ and $\nu$ were determined experimentally and showed agreement with the predictions by Fisher[7]. As in the disorder tuned case, in the magnetic field tuned transition the scaled insulating data collapses on one curve and the superconducting data falls on a second curve. The resistance shows the expected divergent behavior at the critical resistance. In contrast to some of the earlier experiments, the critical resistance did not match the predicted universal value $\hbar/4e^2$ and varied significantly between samples.

Despite the scaling evidence for a direct phase transition from superconductor to insulator at zero temperature, later experiments indicated the presence of an unexpected metallic resistance state in the transition region that extrapolated to zero temperature [14, 2]. Figure 1.10 shows temperature sweeps for different thicknesses of a Ga film. Curves for low thickness films turn towards infinite resistance as the temperature decreases, while thicker films show zero resistance as the temperature approaches zero. However, the plot also shows an unexpected flattening of resistance at low temperatures between the superconducting and insulating curves which implies the existence of a metallic state in the transition region.

Transport measurements performed on thin films while applying a perpendicular mag-
Figure 1.10: Resistance plotted against temperature for varying thicknesses of a Ga film. There are several curves between superconducting and insulating that show a flattening of resistance near zero temperature[2].
Figure 1.11: Resistance plotted against the inverse of the temperature for various magnetic fields. There is a crossover from a temperature dependent regime into a flat regime similar to that of the disorder tuned transition[5].

netic field showed similar results to those in the disorder-tuned transition[5]. Inside the transition region, $R_\square$ flattened as the temperature approached zero. Figure 1.11 shows the resistance of a MoGe film plotted against the inverse of the temperature for several values of magnetic field. For low values of magnetic field, the resistance crosses from a temperature dependent activated regime into a flat temperature independent region at lower temperatures.

However, the metallic state found in the above experiments posed a theoretical problem because it should not exist in two dimensions at zero temperature due to the predicted localization of electrons. Given these unexpected experimental results, new theories[4, 3] have been developed to explain the existence of the metallic state. Because in 2D unpaired electrons localize at zero temperature due to the presence of disorder, Cooper pairs lacking phase coherence were proposed to be the charge carriers in this new metallic state called a Bose Metal. Theorists propose that a Bose Metal can form between the superconductor where Cooper pairs are coherent across the whole sample and a Bose insulator where the Cooper pairs are localized. In the Bose Metal the bosons (Cooper pairs) fail to condense
into the lowest ground state, but do not localize and are able to carry a non-superconducting current.

Das and Doniach suggest that the Bose Metal occurs between the Bose Insulator and the superconductor[4]. The superconductor is characterized by a phase order parameter, as opposed to the charge order parameter characterizing the Bose Insulator. They argue that the Bose metal occurs when both order parameters are zero. Phase coherence has been lost due to the existence of free vortices, but charge order has not yet established because these vortices have not Bose condensed. Instead the vortices are in a dissipative liquid.

Dalidovich and Phillips proposed a different candidate for this metal, the Phase Glass[3]. In this case the superconducting parameter does not vanish. Instead the phase varies from site to site. The superconductor gives way to the metallic state when the average of the order parameter over disorder becomes zero, but its thermal average remains nonzero. The glassy nature of the phase prevents the bosons from condensing into a superconductor and a nonzero resistance exists. The insulator follows once the thermal average of the order parameter becomes zero.

Along with the proposed theories there exists a concern over possible experimental error in the measurements which observed a flattening of resistance near $T = 0$ due to the way in which transport is measured. Because transport measurements are performed by attaching wires to the sample and flowing a current, it becomes difficult to reach low sample and electron temperatures. Experimentally the temperature measured on a thermometer next to the sample, may not be the temperature of the electrons in the sample or even the sample itself. This could lead to an apparent flattening of resistance when the thermometer keeps cooling while the sample does not.

An important experimental feature observed in the magnetic field tuned transition that could help theorists understand the superconductor-insulator transition is shown in Figure 1.12. The plot shows the resistance as a function of magnetic field as it is swept through zero in two directions. While sweeping the field, Mason and Kapitulnik observed possible
hysteretic behavior, which is consistent with the presence of metastable states characteristic of glassiness[19]. If verified this discovery could help further in developing a theory for the suppression of superconductivity in these films.

### 1.7 Open Questions about the SI Transition

The above mentioned experiments raised several questions about the superconductor-insulator transition. Is the zero temperature transition from superconductor direct into the insulating state or does a metallic state exist between the two? How does disorder affect the available states in these films? Is a metallic state possible at zero temperature or is it an experimental artifact? What is the mechanism by which superconductivity is lost in the presence of a magnetic field—loss of phase coherence or breaking of Cooper pairs?

Despite the existing experiments and theories, a full understanding of the zero temperature superconductor insulator transition has not been developed. While it has been shown that at zero temperature these 2D superconductors become insulators at large enough fields, how this happens is not clear. Experimentally, there has also been evidence of similarities
between the high $T_c$ superconductors and InOx thin films in large applied magnetic fields[23]. Another study on $La_{2-x}Sr_xCuO_4$ at low temperatures have revealed a quantum vortex liquid which is responsible for the phase fluctuations in the superconducting order parameter at magnetic fields less than the Cooper pair depairing field[17]. These experiments suggest that increased understanding of the mechanisms for the destruction of superconductivity in thin films may help in developing the theories of high temperature superconductivity. New experiments are needed to help answer these and other questions raised by the previous experiments and proposed theories.

To address this, our experiments look at the superconductor-insulator transition in different way from previous studies. Instead of transport measurements of the resistance outside of the superconducting state, we measure the complex conductance and impedance of the superconductor. In the remainder of this thesis, we will concentrate on one part of the phase diagram, the magnetic field tuned superconductor-insulator transition. In Chapter 3 we will look at the KT transition in zero magnetic field. In Chapter 4 we will look at how the transition changes as an increasing perpendicular magnetic field is applied to the sample. But first, in the next chapter we will look at our measurement technique in more detail and explain how it works.
Chapter 2

Experimental Techniques

Most of the previous measurements of the properties of 2D superconductors and their superconductor-insulator phase transition were performed using traditional transport techniques. Transport allows one to measure the resistance of a film down to the noise floor which is typically no better than $0.1\,\Omega$. A resistance dropping below the noise floor is the only evidence of superconductivity from transport measurements. On the other hand, transport can measure very large resistances so it is well suited for measuring insulators and metals but yields little detailed information about the superconducting state. This is one of the inherent disadvantages of transport when looking at superconducting samples. Another drawback is the possible heating due to wires being connected to the sample which has been considered as a possible explanation for the flattening of resistance mentioned in the previous chapter. If the charge carriers in the sample are not at the same temperature as indicated by the sample thermometer, it is possible to measure a flattening of resistance as a function of the thermometer instead of the actual carrier temperature. A lot of filtering and thermal anchoring of the sample leads is required to minimize these heating issues.

The above mentioned disadvantages naturally lead to a need for a technique that can provide more information about superconductors. We employ one such method in our measurements which does not require electrical contacts to the sample and hence should minimize heating effects of the leads. It also allows us to measure the conductance of the sample instead of the resistance. In effect our signal gets larger as the sample becomes more conducting which makes this technique ideally suited for a 2D superconductor, where the conductivity is non-zero but finite leading to a measureable signal. In contrast to transport, the resistance
range we measure is typically from 10 µΩ to 100 Ω. We can use this technique as a natural extension of transport measurements into the low temperature state with superconducting correlations.

2.1 Measurement theory

Figure 2.1: A sketch showing the driving AC magnetic field $B_D$ along with the shielding current response (red circular arrow) in the sample and the associated magnetic field $B_S$. $B_D$ induces an electric field in the sample which creates shielding currents which can be calculated from measurements of the amplitude $A$ and phase $\phi$ of $B_S$. The inset shows a photo of the actual coil probe next to a penny showing the bottom of the coils against which the samples are mounted.
Our experimental probe is similar to the one reported by Jeanerette et al[15]. It consists of two sets of coils, and measurements are performed using a lock-in amplifier. Figure 2.1 shows a sketch of the principle behind the measurements. Applying an AC current to the outer drive coil creates an alternating magnetic field $B_D$ above the sample, which induces a circular electric field in the sample and screening currents are generated in response shown by the circular red arrow in the figure. These currents create a separate alternating magnetic field $B_S$ at the same frequency as the drive, inducing an AC voltage on the pickup coil. Using a lock-in amplifier we measure the magnitude $A$ and phase $\phi$ of this signal with respect to the drive current. $A$ and $\phi$ are then used to calculate the shielding currents. Knowing the shielding currents and the electric field induced by $B_D$ allows us to extract the complex conductivity of the sample. This calculation is explained in detail below while the details of the construction of the coil can be found in Appendix C.

The calculation uses the coulomb gauge $\nabla \cdot A = 0$. It is best to use cylindrical coordinates placing the sample in the $z = 0$ plane and the $z$-axis along the cylindrical axis of the coils. To make the calculation easier the coils are modeled as rings. This gives a current distribution $j_D$ for a current $I_D$ in the drive coil as

$$j_D(\rho, z) = I_D \delta(\rho - R_D) \sum_{n=0}^{N_D-1} \delta(z - h_D - n\delta h_D)$$

given the drive coil radius $R_D$, number of turns $N_D$, distance to sample $h_D$, and the coil spacing $\delta h_D$. This can then be plugged into Ampere’s law

$$-\nabla^2 A(\rho, z) = \mu_0[K_S(\rho)\delta(z) + j_D(\rho, z)]$$

where the sheet current in the sample

$$K_S(\rho) = G(\omega)E(\rho, z = 0) = -i\omega G(\omega)A(\rho, z = 0)$$
when we assume an \( e^{i\omega t} \) time dependence. Solving for \( K_S \) we get

\[
K_S(\rho) = I_D R_D \int_0^\infty dx \frac{xe^{-xh_D}}{1 + \left( \frac{2}{\mu_0} \right) \left( \frac{1}{i\omega G} \right) x} J_1(xR_D)J_1(x\rho) \frac{1 - e^{xN_D\delta h_D}}{1 - e^{x\delta h_D}} \phi.
\]

From the current \( K_S \) we can calculate \( A \) as above but with \( j_D = 0 \) since its contribution to the induced voltage is cancelled by the two counter-wound sections of the pickup coil. From \( A \) we can get the electric field \( E \), and integrating \( E \) along the pickup coil yields the pickup voltage

\[
\delta V = i\omega I_D \int_0^\infty dx \frac{M(x)}{1 + \left( \frac{2}{\mu_0 h} \right) \left( \frac{1}{i\omega G} \right) x}
\]

where

\[
M(x) = \pi \mu_0 h \alpha \beta J_1(\alpha x)J_1(\beta x)e^{-x} \left[ \frac{1 - e^{-N_D\gamma x}}{1 - e^{-\gamma x}} \right] \left[ \frac{1 - e^{-N_R\delta x}}{1 - e^{-\delta x}} \right].
\]

\( M(x) \) is entirely dependent on the coil geometry, with \( \alpha, \beta, \gamma, \) and \( \delta \), equal to \( R_D, R_R, \delta h_D, \) and \( \delta h_R \), divided by \( h = h_R + h_D \), respectively. \( J_1 \) is the first Bessel function. The above equation holds for a single layer pickup coil. To get the equation for a multi-layer pickup coil, \( M(x) \) is replaced with the sum of \( M_i(x) \) for each layer \( i \) of the coil. In effect this replaces \( \alpha J_1(\alpha x) \) with \( \sum_i \alpha_i J_i(\alpha_i x) \).

The process of extracting the complex conductivity \( G(T,\omega,B) \) from the voltage \( \delta V \) involves the use of a look up table since simple inversion of the above integral is not possible. Using a Matlab program, which is described in Appendix B, we calculate a table of \( Gs \) from a grid of \( \delta Vs \). The generated table is used to interpolate a \( G \) for each \( \delta V \) measurement. The resistance \( R \) and inductance \( L \) of the sample can be extracted from the impedance \( Z \) using

\[
\frac{1}{G} = Z = R + i\omega L.
\]

An example of the resulting data for a MoGe film is shown in Figure 2.2. Figure 2.2a shows the real and imaginary parts of the conductivity multiplied by \( \omega \) plotted against temperature along with the resistance measured using transport. When the transport resistance drops to the noise floor the conductance measured by the coils becomes measurable. Figure 2.2b shows the calculated resistance \( R \) from the complex conductivity.
as well as the transport resistance plotted against temperature. The two methods are in reasonable agreement, but the coil measurement can measure much lower resistances than transport and is better suited to measure superconductors.

### 2.2 Four lead resistance measurements

Another measurement we can perform during the contactless coil measurement is a four wire resistance measurement. To do this we make contacts to all four corners, pressing 0.001” diameter gold wire between the sample and the coil probe. Measuring the voltage between two of the contacts on one side of the sample while flowing current through the other two, gives us a value $R_{xx}$. Doing the same measurement in a perpendicular direction to $R_{xx}$ gives us $R_{yy}$. Using these two values we can solve for $R_{\square}$ using the van der Pauw equation\[26\] for a square sample $e^{-\frac{R_{xx}}{R_{\square}}} + e^{-\frac{R_{yy}}{R_{\square}}} = 1$. This provides a comparison to $R_{\square}$.
calculated from the contactless measurement of the complex conductance. The van der Pauw method works without a need to pattern the sample and has the advantage of allowing us to perform transport and inductance measurements on the same film simultaneously. $R_{\square}$ data obtained in this way was shown in Figure 2.2. The noise from these types of measurements is typically larger than transport on patterned samples, but patterning would not allow us to make this measurement simultaneously with the coil measurements because they require a large sample.

### 2.3 Samples

Previous experiments on the superconductor-insulator transition have been performed on many different materials including Pb, Bi, Ga, MoGe, and InOx. Some of these such as Ga form superconducting grains when deposited in thin films. As a result these films are not usually homogeneous on atomic scales. To minimize granularity, such films are typically deposited in situ at low temperatures and cannot be warmed much above 20K. Because these restrictions are very impractical, we have chosen our sample materials carefully in order to have homogenous films that could be studied repeatedly.

The first material we chose is an amorphous composition of molybdenum and germanium. Amorphous MoGe samples can be deposited by magnetron sputtering and are homogenous on atomic length scales. The concentration of MoGe affects the superconducting transition temperature of bulk samples with increased germanium adding disorder to the superconducting molybdenum. We have sputtered our own samples with an atomic ratio of Mo:Ge equal to 43:57 which gives a bulk $T_c$ about 1.05 K. We used the thickness of our samples to control the superconducting transition temperature. The thickness of the films effectively modifies the disorder because the boundary scattering from the two surfaces of the sample becomes more important as the thickness is decreased. Typically we made our samples about 100 Å thick to have a critical temperature about 500 mK. The specifics of the growth
of our samples are discussed in Appendix A. As has been mentioned previously applying a perpendicular magnetic field to these samples introduces vortices and eventually kills superconductivity. Experimentally MoGe samples have been the easiest to work with because they provide consistent data regardless of cool down and do not significantly degrade in quality over time.

The second set of films that we have chosen is InOx. Like the MoGe films, these films are also amorphous and homogeneous on atomic length scales but are much more disordered. While increased disorder lowers our signal, it can make the effects of phase fluctuations more significant. The InOx films were grown in the lab of Sambandmurthy Ganapathy at SUNY Buffalo by Minsoo Kim and are nominally 100 Å thick. Unlike changing the thickness in MoGe, the superconductivity of these samples is tuned by changing the oxygen concentration, with more oxygen increasing the disorder in the superconducting indium and lowering its $T_c$. In our experience these films are not as robust as the MoGe samples. When the samples are left out in air for longer than a few hours, the $T_c$ decreases or they become insulating, possibly due to absorption of water or oxygen.

All of the above mentioned samples can be studied simultaneously with our coil probe and four wire transport mentioned above. The probe is attached to the insert of a top loading dilution refrigerator from Oxford Instruments with a working temperature range from about 1.5 K to less than 20 mK. In this system the probe and sample are directly cooled by being immersed inside the helium mixture. The cryostat also has a superconducting magnet which can be used to apply a perpendicular magnetic field to the sample of up to 8 T. Data obtained on the above samples using this setup will be shown throughout the rest of this thesis.
Chapter 3

Thermally Driven Kosterlitz-Thouless Transition in Zero Field

Having shown the details of our measurements in the previous chapter, we will now proceed with the investigation of the magnetic field tuned quantum phase transition in our 2D superconducting films. In this chapter, we will look at the Kosterlitz-Thouless transition in zero magnetic field and demonstrate the effectiveness of our technique at measuring this phase driven transition. In the next chapter we will show what happens when we apply a perpendicular magnetic field and see how the superconducting transition changes as the field is increased and $T_c$ is suppressed to the quantum critical point at $T = 0$. We will start this chapter with an explanation of the quantities that can be extracted from our measurements and a discussion of what we expect to see followed by a comparison of our zero field data to a prediction made by BCS, and then show our measurement of the Kosterlitz-Thouless transition.

3.1 Complex Conductance

As we discussed in Chapter 2, the coil probe allows us to measure the complex conductance $G(\omega)$ or its inverse, the complex impedance $Z(\omega)$ of the sample. The complex impedance is related to the complex AC penetration length $\lambda_{ac}$ as $Z(\omega) = R(\omega) + i\omega L(\omega) = \frac{i\omega\mu_0\lambda_{ac}^2}{d}$, where $\mu_0$ is the permeability constant, $d$ is the thickness of the film, and $R(\omega)$ and $L(\omega)$ are frequency dependent resistance and inductance, respectively.

In zero magnetic field, the relationship between $\lambda_{ac}$ and the superfluid density $n_s$ yields
a linear relationship between the inverse of $L(\omega)$ and $n_s$.

$$\text{Re} \left( \lambda_{ac}^2 \right)^{-1} = \frac{\mu_0 n_s e^2}{dm_e} \implies L^{-1} = \frac{d}{\mu_0 \text{Re} \left( \lambda_{ac}^2 \right)} = \frac{n_s e^2}{m}$$

This means that we can probe the superfluid density of a sample using our technique at zero field.

Because we measure and plot $L^{-1}$ and not $\frac{1}{\lambda_\perp}$ we shall now convert the prediction of the universal jump line we saw in 1. Beasley et al[1] showed

$$\frac{1}{\lambda_\perp(T_{KT})} = \frac{8\pi \mu_0}{\Phi_0^2} k_B T_{KT}$$

for 2D superconductors. Because $\lambda_\perp = \frac{\lambda^2}{d}$ and $L = \mu_0 \frac{\lambda^2}{d}$ we can calculate the universal line for the size of the KT jump in the inverse inductance to be

$$L^{-1} = \frac{1}{\mu_0 \lambda_\perp} = \frac{8\pi}{\Phi_0^2} k_B T_{KT} = 0.081 T_{KT}(K) \text{nH}^{-1}.$$ 

When $L^{-1}$ drops below this line as the temperature is increased, pairs are predicted to unbind, and $L^{-1}$ should exhibit a sudden discontinuous drop to zero.

### 3.2 Zero Field Measurements

The first measurements to characterize our samples and see how they fit with respect to previous work involve sweeping the temperature in zero magnetic field. Figure 3.1 shows such a temperature sweep for a sample with a normal state resistance around 600$\Omega$ as measured using a four point contact technique. The plot shows the four point resistance, measured using the van der Pauw method described in the previous chapter, dropping steeply to the noise floor as the temperature decreases from 550 mK to 520 mK. The resistance measured by the contactless coil technique decreases with temperature between 490 mK and 400 mK.
Outside of this temperature range it disappears into the noise floor of our measurement. Even though the resistance measured by the contact technique disappears into noise of a few ohms and the contactless technique doesn’t measure any resistance until it drops below 0.1 Ω, the plot shows that the two measurements are in reasonable agreement on the superconducting transition despite not having an overlapping region. Figure 3.1 also shows the inductive response $L^{-1}$ measured using our coil measurement. $L^{-1}$ decreases monotonically with increasing temperature until about 500 mK, at which point there is a discontinuity, above which the signal settles to zero. The discontinuity appears to be a Kosterlitz-Thouless type transition which will be analyzed further in a later section. From the figure we see that while the coil measurement cannot measure anything if the resistance of the sample is measureable by transport, it does give us information about the resistance and superfluid density of the superconductor in a temperature range where transport measures only noise around zero. We will later use this ability to probe the superconducting regime in an applied magnetic field.

At low temperatures far from the transition temperature the superfluid density in these samples and hence $L^{-1}$ are expected to follow the mean field prediction from BCS theory, which is explained below. Because this prediction only considers the suppression of the order parameter, it is expected to break down when phase fluctuations become important near the transition region. Kosterlitz-Thouless theory predicts when phase fluctuations should become important and consequently suppress superconductivity in zero magnetic field. To check against the prediction from BCS theory and to extract the mean field transition temperature $T_{c0}$, we can plot the normalized $L^{-1}$ data against the following mean field calculation.

For a BCS superconductor in the dirty limit, it can be shown that

$$\frac{\lambda^2(0)}{\lambda^2(T)} = \frac{\Delta(T)}{\Delta(0)} \tanh \frac{\Delta(T)}{2k_B T}$$
Figure 3.1: Plot of the inverse inductance $L^{-1}$ (blue) and resistance per square $R_{\square}$ (red) as a function of temperature $T$. For comparison $R_{\square}$ is shown as measured by a 4 point contact technique (x) as well as the contactless coil technique (+). The two do not cover the same resistance range but do agree reasonably well on the location of the superconducting transition.

in terms of the gap parameter $\Delta[24]$. Using the BCS result $\Delta(0) = 1.76k_B T_c$ and

$$
\left( \frac{\Delta(T)}{\Delta(0)} \right)^2 = \cos \frac{\pi t^2}{2}
$$

from Sheahen[21] where $t = T/T_{CO}$ we get

$$
\frac{\lambda^2(0)}{\lambda^2(T)} = \left( \cos \frac{\pi t^2}{2} \right)^{\frac{3}{2}} \tanh \left[ \frac{1.76}{2t} \left( \cos \frac{\pi t^2}{2} \right)^{\frac{1}{2}} \right].
$$

Because $L^{-1} \propto \frac{1}{\lambda^2}$ this yields

$$
\frac{L^{-1}(T)}{L^{-1}(0)} = \left( \cos \frac{\pi t^2}{2} \right)^{\frac{3}{2}} \tanh \left[ \frac{1.76}{2t} \left( \cos \frac{\pi t^2}{2} \right)^{\frac{1}{2}} \right].
$$
Figure 3.2 shows the measured inductive response as a function of temperature plotted along with the above mean field equation. To fit the data we set $L^{-1}(0) = 0.52 \text{nH}^{-1}$ so that the zero temperature value matches the data. Then we found $T_{c0} = 0.514 \text{K}$ to get agreement with the data near $T_c$. The fit deviates from the data between 0.2 and 0.85$T_{c0}$. In the figure we observe a deviation from the fit at temperatures below 0.96$T_{c0}$, at which point we observe a sudden drop to zero in the inverse inductance. This drop is believed to be due to a Kosterlitz-Thouless transition which was described in an earlier chapter. Such a deviation from the mean-field value between 0.85$T_{c0}$ and 0.96$T_{c0}$ had been previously attributed to classical longitudinal and transverse phase fluctuations[25]. The former account for a linear deviation from the mean-field value, while the latter cause a higher order suppression of $L^{-1}$. The Kosterlitz-Thouless theory predicts the magnetic penetration length $\lambda$ for a particular temperature at which vortex-anti-vortex pairs unbind and suppress superconductivity through phase fluctuations. This prediction of $\lambda$ can be converted into a prediction on $L^{-1}$ which is the dashed line plotted along with the BCS fit. As predicted the KT jump occurs when $L^{-1}$ decreases below this line.

3.3 KT Transition

We attribute the discontinuous deviation from mean field behavior near $T_c$ to phase fluctuations caused by the presence of free vortices above the Kosterlitz-Thouless transition. As was mentioned in Chapter 1, Kosterlitz and Thouless predict that vortex-anti-vortex pairs unbind and destroy superconductivity at a temperature $T_{KT}$ which is lower than the mean field critical temperature $T_{c0}$. As temperature increases from zero, DC resistance is first measureable when vortex-anti-vortex pairs begin to unbind. However, because our technique measures at kHz frequencies, we expect to see a frequency dependent signal near $T_{KT}$. Figure 3.3 shows $L^{-1}$ plotted for five different frequencies near the KT transition. Below 430 mK away from $T_{KT}$, we observe $L^{-1}$ to be frequency independent, which is consistent
Figure 3.2: Plot of the normalized measured inverse inductance $L^{-1}$ along with the predicted BCS fit as a function of the reduced temperature $T/T_c$. The line shows the size of the predicted KT jump as a function of temperature. The deviation from the fit near $T_c$ attributed to classical phase fluctuations has been observed previously[25]. The KT jump occurs when the data crosses the predicted jump line. The inset shows a zoom near the transition. For this film $L^{-1} = 0.520\, \text{nH}^{-1}$ and $T_c = 0.514\, \text{K}$.

with the predicted $1/\omega$ behavior for the conductance $G[6]$. Above 430 mK $L^{-1}$ is suppressed with decreasing frequency. Suppression of $L^{-1}$ has previously been postulated to be due to classical phase fluctuations[25]. Longitudinal phase fluctuations are expected to cause a linear suppression of the ratio between the measured and mean-field predicted inverse inductances[20]. Transverse ones caused by vortices are predicted to cause a higher order curvature in this ratio. We believe the frequency dependence is caused by vortices unbinding. The frequency of our measurements sets a length scale at which we probe the sample with lower frequencies corresponding to longer length scales. Because vortex-anti-vortex pairs with large separations are the first to unbind as temperature increases, we expect a larger
suppression of $L^{-1}$ at lower frequencies.

Figure 3.3: Plot of the inverse inductance $L^{-1}$ as a function of temperature $T$ for five different frequencies near the Kosterlitz-Thouless transition. $L^{-1}$ is frequency independent below 430 mK, but decreases with decreasing frequency above this temperature.

Figure 3.4 shows $L^{-1}$ plotted against $T$ on a log-log scale for three MoGe samples of different thickness. The log-log scale makes it easier to compare KT jumps occurring at different temperatures. These samples had $T_c$'s of approximately 300, 500, and 850 mK. The 300 and 800 mK samples were measured at 10 kHz, while the 500 mK sample was measured at 20 kHz. The inset shows a zoom of the KT jumps on a linear scale. All three samples exhibit KT like behavior. The thinnest one, however, appears to have a jump size larger than expected from KT theory. This could be due to the noise in the measurement because the measured signal decreases with decreasing thickness and as a result the signal to noise ratio decreases. Another possibility is that the data may not have been in linear response in the transition region because it was measured at a drive current that led to a higher
current density in the sample than in the two thicker films. The KT transition is inherently a non-linear phenomenon because larger measurement currents lead to larger Lorenz forces on vortices. Because each vortex in a vortex-anti-vortex pair experiences a Lorenz force in the opposite direction, a larger measurement current leads to earlier unbinding and moves the jump to a lower temperature and a higher inverse inductance. The 300 mK and 850 mK films were both measured when we were still perfecting our technique using ten times the drive current of the 500 mK sample. The large current is more noticeable on the thinnest film because it leads to the largest current density due to the decreased thickness. The bulk of our data presented here has been measured on the 500 mK sample because it provided a good compromise between a low enough \( T_c \) easily measured in a dilution refrigerator and a large enough signal to measure with our electronics. We also measured some InOx samples that will be shown in the next chapter.

### 3.4 Resistance

As was shown in Figure 3.1, we can extract the resistance per square of the sample from our coil measurement. Kosterlitz-Thouless theory predicts \( R = R_0 e^{-b_\pi(T-T_{KT})^{-1/2}} \) for the form of the resistance above \( T_{KT} \). Using our technique we cannot measure the resistance high enough above the transition to verify this prediction, but we can measure resistance below \( T_{KT} \). Figure 3.5 shows \( R_\square \) on a log scale plotted against \( T \) for five different frequencies. We can see that in the absence of a magnetic field the resistance drops sharply over three decades in a 60 mK range below the observed discontinuity in \( L^{-1} \). The resistance we measure below \( T_{KT} \) is frequency dependent with lower resistance measured at lower frequency. Such a frequency dependence suggests that non-zero resistance measured below \( T_{KT} \) could be caused by the AC nature of our measurement. At DC, the resistance should be zero below \( T_{KT} \) suggesting a true superconducting state exists in the zero field case.
Figure 3.4: Plot of the inverse inductance $L^{-1}$ as a function of temperature $T$ for three different MoGe films. Inset shows the zoom of the data near the KT transition. The two thickest films exhibit the correct jump size while the thinnest film does not. This could be due to noise or nonlinear effects.

3.5 Conclusions

In this chapter we showed that MoGe films exhibit a KT transition in zero magnetic field. We observed a limited agreement between the measured inverse inductance and a BCS fit. The KT jump size followed the universal jump line for two of three films of different thickness, with the excepting film’s jump resolution possibly limited by noise and measurement limitations. In zero magnetic field our resistance data suggests a true superconducting state with zero resistance below the transition temperature. In the next chapter we will investigate the $T = 0$ quantum phase transition in an applied perpendicular magnetic field.
Figure 3.5: Plot of the resistance $R_{\square}$ against $T$ measured at five frequencies in zero magnetic field. The frequency dependence is consistent with zero resistance at zero frequency.
Chapter 4

Applying a Perpendicular Magnetic Field

In the previous chapter, we measured the thermally driven zero field Kosterlitz-Thouless transition in 2D MoGe. In this chapter we will use the same technique, to probe the magnetic field tuned superconductor insulator transition down to $T = 0$ in search of the predicted quantum phase transition. We want to understand how the application of a perpendicular magnetic field destroys the superconducting state and makes the sample become insulating. Because the Kosterlitz-Thouless transition occurs only in zero field we expect a different mechanism for the destruction of superconductivity in non-zero magnetic field. Phenomenologically we expect that applying an increasing perpendicular magnetic field to the sample creates vortices of one direction until the field penetrates the entire sample and superconductivity gives way to insulating behavior. At finite temperature we expect the resistance of the sample to change continuously through this transition. However, at zero temperature 2D films are expected to have a discontinuous quantum phase transition between the superconducting and insulating states at some critical field $B_c$. We will first look at what happens to the superfluid as a magnetic field is applied at zero temperature. After this we will show temperature sweeps in increasing magnetic field to see how the transition changes as the $T_c(B)$ goes to zero.
4.1 Suppression of the Superfluid with Increasing Magnetic Field

The application of a magnetic field in 2D superconductors leads to the formation of vortices above $H_{c1}$ which is very small for our thin samples. In Chapter 1 we described some of the possible states for the vortices. In low disorder we expect a vortex lattice, that melts into a vortex liquid as field or temperature are increased. However, our films are highly disordered and we expect a vortex glass which is expected to exhibit resistance above $T = 0$. Only at $T = 0$ should the vortex glass freeze and a true superconducting state exist.

Figure 4.1 shows the inverse inductance $L^{-1}$ and resistance $R$ of a MoGe film as a function of the magnetic field $B$ measured at 10mK. The measurement was done at two different frequencies, 50 and 250 kHz. We observe a sharp drop in $L^{-1}$, and the corresponding
superfluid density, between 0 and about 60 mT as the first vortices enter the sample. As field increases further $L^{-1}$ drops monotonically to zero at a critical field $B_c = 1$ T. From the two frequencies it appears $L^{-1}$ is frequency independent. Turning our attention to the resistance we notice that steeply rises from the noise floor over about two decades as soon as the field is increased between 0 and 50 mT, the same field range when $L^{-1}$ exhibits a sharp drop. Above this range the resistance increases at a slower rate, about two decades between 50 and 900 mT. Above this range the resistance turns up again before disappearing out of our resolution window above 1.05 T. Comparing the two frequencies we see an obvious frequency dependence in the resistance. Naively we might guess $R$ to be linear in $\omega$, because $R$ is the real part of the inverse of the conductance $G$ which was shown in the previous chapter to go as $1/\omega$ in zero field. However, upon closer inspection $R$ goes as $\omega^\alpha$ with $\alpha = .78 \pm 0.1$. Extrapolating this behavior to zero frequency, the resistance goes to zero below $B_c$.

We can compare this behavior to our transport measurements on the same film measured using the van der Pauw technique. Figure 4.2 shows the longitudinal transport resistance $R_{xx}$ measured at different temperatures as a function of magnetic field. $R_{xx}$ is one component necessary to calculate $R_{\square}$ using the van der Pauw method and shows the qualitative shape of the $R_{\square}$ curve. We can see that the lowest temperature measurement at 10 mK shows the resistance drops below the noise floor as the field decreases below about 1 T. This behavior is consistent with the resistance measured with the coil technique. Because of the resolution of conventional transport, we cannot confirm whether or not the DC resistance is truly zero below 1 T. We can use this plot to get a more accurate estimate of the $T = 0$ critical field. Because all the temperature curves cross at one field, shown in the inset, we expect the discontinuous $T = 0$ quantum phase transition to occur at this field which is about 1.41 T. We expect that if we could make measurements at lower temperatures using our coil technique, we would observe a non-zero $L^{-1}$ up to about this value of magnetic field.
Figure 4.2: Plot of the longitudinal resistance $R_{xx}$ of a MoGe film as a function of magnetic field $B$ measured at different temperatures. The zoom of the crossing of the curves shown in the inset can be used to extract the critical field $B_c$ from transport measurements.
4.2 Temperature Evolution of the Magnetic Field

Tuned Transition

In this section we will attempt to learn about the quantum phase transition from superconductor to insulator as we apply a perpendicular magnetic field to these films. Because we cannot measure at $T = 0$, to observe the QPT we must look at the non-zero temperature transition and see how it changes as we approach the QPT by increasing field and lowering temperature. Experimentally, it is easier for us to do temperature sweeps at a set magnetic field than to do field sweeps at a set temperature, because we can keep the field much more stable than the temperature over long time scales and temperature sweeps take less time than field sweeps. Figure 4.3 shows the inverse inductance $L^{-1}$ measured from temperature sweeps taken at increasing magnetic field values. The fields plotted are 0, 10, 100, 200, 400, 600, 800, 925 mT along with the dashed KT line from the previous chapter. To reduce noise, the data plotted for each curve is an average of six to twelve temperature sweeps taken at the given applied magnetic field. To our surprise, the non-zero field curves are very similar in shape to the zero field data. As the field increases, each $L^{-1}$ curve shifts more and more towards zero temperature and zero inverse inductance. We do not expect to see a discontinuous jump in the superfluid density, because the KT transition does not occur at non-zero fields and this sample is too disordered for a vortex lattice melting transition to occur. Even more surprising is that the size of this KT-like jump follows the universal jump line towards zero as the field is increased suggesting phase fluctuations are responsible for the destruction of superconductivity in both the zero and non-zero field transitions.

We can also see the discontinuous jump in magnetic field sweeps. Figure 4.4 shows $L^{-1}$ plotted against $B$ for different temperatures. We can see the discontinuous jump in the zoomed view shown in the inset. The green +’s are near each jump correspond to the predicted $L^{-1}$ for the temperature of the field sweep using the universal Kosterlitz-Thouless line. Again there is reasonable agreement between the predicted and measured size of the
Figure 4.3: Plot of the inverse inductance $L^{-1}$ against temperature $T$ for a MoGe film measured at different values of the applied perpendicular magnetic field. All curves exhibit a discontinuous feature that follows the predicted Kosterlitz-Thouless line.
We can also look at how the resistance curves change as a magnetic field is applied. As shown before, we can measure the resistance in two ways, using the coil or putting contacts on the sample and using the van der Pauw method. The data plotted in Figure 4.5 shows the resistance plotted against temperature for many magnetic fields ranging from 0 T to 1.5 T. The ×’s and □’s were measured using the contact technique where as the +’s and ◦’s were measured with the coil. We can see that as the field is applied the transition widens and shifts towards zero temperature.

Turning our attention back to the resistance measured using the coil measurement, Figure 4.6 shows $R_{□}$ plotted against the temperature for several fields from 0 to 925 mT. We can see
Figure 4.5: Plot showing the resistance per square of a MoGe film as a function of temperature for different magnetic field values from 0 to 1.5 T. □’s and ×’s were measured using transport, while ◦’s and +’s were measured with the coil. The dashed lines do not represent data but connect same-field points measured during field sweeps at a fixed temperature. Field values plotted from right to left are 0, 1, 2, 5, 10, 20, 50, 100, 200, 500, 600, 700, 800, 900, 1000, 1050, 1100, 1200, and 1500 mT.
a drastic change from the zero field behavior as soon as the applied field becomes non-zero. Instead of the exponential drop with temperature we observed in zero field, the resistance decreases at a much slower rate and remains within our resolution window of 0.1 to $1 \times 10^{-6} \Omega$ for most of the temperature range below $T_c$. Again because we measure at kHz frequencies, our measurements do not directly compare to transport so we can only attempt to infer what happens to the resistance in the DC case by measuring data at different frequencies. Figure 4.7 shows a plot of $R_{\square}$ versus $T$ measured in a number of fields at different frequencies. Data measured at 20 kHz and 175 kHz are plotted for 100, 700, 850, and 950 mT. For 100 mT, curves measured at 40 kHz and 75 kHz are also included. In the figure we see that the measured resistance decreases with decreasing frequency as it did in the 10 mK field sweep shown in 4.2 suggesting that in the zero temperature and zero frequency limit the resistance is zero.
Figure 4.7: Resistance of a MoGe film plotted against temperature for different values of magnetic field measured at different frequencies. The resistance decreases with frequency for a given field value suggesting zero resistance in the zero frequency and zero temperature limit.
Figure 4.8: Plot of the inverse inductance $L^{-1}$ of an InOx film as a function of temperature $T$ for several magnetic field values. As with MoGe the InOx sample exhibits a discontinuity which follows the Kosterlitz-Thouless line.

For comparison to the MoGe films we have also measured some InOx samples, which exhibited similar behavior. Because the quality of InOx samples changes over time, it was difficult to find samples that were easily measureable in the temperature range accessible to us with a dilution refrigerator. Figure 4.8 shows the inverse inductance plotted against temperature for several magnetic fields of one such sample. We were unable to measure the transition at zero field because the critical temperature was too high for our dilution refrigerator. However, the data looks similar to that of MoGe. As the field is increased both the measured $T_c$ and the magnitude of $L^{-1}(T)$ decrease towards zero. Also note that for 3 decades of field the KT-like jump tracks the universal jump line similarly to the MoGe sample.

Figure 4.9 shows the inverse inductance as a function of temperature of another InOx sample. In this case we were able to measure from zero field all the way until our signal
Figure 4.9: Plot of the inverse inductance $L^{-1}$ of a different InOx film as a function of temperature $T$ for several magnetic field values. Again a discontinuity is present which follows the Kosterlitz-Thouless line for all non-zero fields. The zero field curve shows a discontinuity which is much smaller than expected.

disappeared into the noise. While for non-zero fields the data is similar to the previous two samples and tracks the KT jump line well, the zero field curve exhibits a much smaller jump than predicted, which we cannot explain.

4.3 KT-like Transition in Non-Zero Field

From our measurements of the inverse inductance, it appears that the way in which superconductivity is destroyed as a function of temperature is similar in zero and non-zero magnetic fields. Our proposed explanation involves similar vortex-anti-vortex pair unbinding as in the zero field case. Because Kosterlitz-Thouless theory only applies to zero magnetic field, we must look at what changes when a field is applied. The theory requires that both the energy $E$ of a free vortex and the entropy $S$ are proportional to the logarithm of the area
such that minimizing the free energy \( F = E - TS \) leads to a critical temperature \( T_{KT} \) when \( E = T_{KT}S \) for the addition of a free vortex. In non-zero magnetic field the energy of a free vortex is no longer proportional to the logarithm of the area so at first glance vortex-anti-vortex pair separation should not be responsible for the observed transition. However, the only requirement of the theory is that both the entropy and energy scale in the same way for a critical temperature \( T^* \) to exist where \( E = T^*S \). Something that meets this requirement can nucleate at \( T^* \) and be responsible for the destruction of superconductivity due to phase fluctuations. Even though we do not know whether this something is a vortex dislocation or something else, it should have the same form for the energy and entropy and hence be able to nucleate at \( T^* \). This is suggested by Figure 4.3 which shows a KT-like jump in the inverse inductance for increasing magnetic fields with the jump size following the predicted universal line. The resistance plots in Figures 4.1 and 4.7 show that the superconducting phase in a non-zero field exhibits dissipation at our measurements frequencies, but when extrapolated to zero frequency should be resistanceless. The existence of finite resistance in our measurements supports the view that due to large disorder, the pinned vortices do not form a lattice, but instead a vortex glass state forms below \( T^* \). Figures 4.8 and 4.9 show that the same mechanism applies to InOx films as MoGe films.

### 4.4 Zero Temperature Quantum Phase Transition

Even though we cannot directly measure the quantum phase transition expected to occur at zero temperature, we can attempt to show its existence using our data. In order to do this we look at the data plotted in Figure 4.3 and extract the critical temperature \( T^* \), where \( L^{-1} \) shows a jump to zero, for each magnetic field \( B \) shown. To do this we will choose the temperature at which \( L^{-1} \) appears to flatten before the KT-like discontinuity. While the choice of \( T^* \) is somewhat arbitrary, other choices of the jump temperature exhibit the same behavior. This array of points is plotted in Figure 4.10. Notice that for higher magnetic
Figure 4.10: Plot of the critical temperature $T^*$ extracted from temperature sweeps at magnetic field $B$ plotted on a log axis. For higher fields the points fall on a line suggesting a $-\log(B)$ dependence of $T^*$. The field values $T^*$ falls on a line when the field is plotted on a log scale. This means that $T^*$ is proportional to $-\log(B)$. From this plot we then extrapolate this line to $T^*=0$ where it crosses the B axis and find the critical field $B^* = 1.2 \pm 0.1 \, T$. Notice that the critical field $B^*$ extracted from the coil measurement is close to but less than the critical field $B_c = 1.41 \, T$ extracted from the transport measurements.

With the extracted critical field $B^*$, we can now look at $T^*$ as a function of the normalized distance from criticality $\delta = \frac{B^*-B}{B^*}$. Figure 4.11 shows $T^*$ as a function of $\delta$ plotted on a log-log plot. From the figure we see that as $\delta$ approaches zero the points fall on the black line which has a slope of 1.3. This result gives $T^* \propto \delta^{\nu z}$ as expected, with $\nu z = 1.3 \pm 0.3$. 
Figure 4.11: Log-log plot of the critical temperature $T^*$ as a function of the normalized distance from criticality $1 - B/B^*$. With increasing field the points approach a line with slope 1.3.
Chapter 5

Conclusions

In this dissertation we described measurements performed on 2D films of MoGe and InOx. These measurements were done using a novel AC technique involving two sets of coils which measured the sheet impedance of the samples without making electrical contact to the sample. We employed this technique in both zero and non-zero applied magnetic fields inside a dilution refrigerator at temperatures as low as 15 mK. We will now summarize the results from these measurements and discuss future directions these experiments can take.

5.1 Results

We have shown that there exists a Kosterlitz-Thouless like transition even in the presence of an applied perpendicular magnetic field. This transition manifests itself in the same way as in zero field with a discontinuity in the superfluid density, the size of which follows the universal jump line towards zero temperature as the field is increased, suggesting that loss of phase coherence is responsible for the destruction of superconductivity in both the zero and non-zero perpendicular magnetic field case. We did not observe any evidence of an exotic metallic state inside the magnetic field tuned transition from superconductor to insulator. Even though most of the data shown was measured on MoGe films, InOx films showed similar behavior.

Our measurements have yielded valuable insight into the superconductor insulator transition. The observed KT-like jump suggests that fluctuations in the phase of the superconducting order parameter, not the suppression of its magnitude, are responsible for the
destruction of superconductivity in both the zero and non-zero magnetic field cases. These results agree with those of Steiner et al suggesting Cooper pairs exist in the dissipative state of InOx films and some high $T_c$ superconductors[23] as well as Li et al who observed loss of phase coherence due to the existence of a quantum vortex liquid in $La_{2-x}Sr_xCuO_4$ at low temperatures and high magnetic fields[17]. Because of the similarities between high $T_c$ superconductors and superconducting thin films, our experiments can aid in the understanding of the mechanisms for high temperature superconductivity.

5.2 Future Experiments

The above observations are only a small part of the possible data that can be obtained using this inductive technique. Because all measurements are made at discrete frequencies and either discrete temperatures or magnetic fields, with more time finer measurements can be made along each of these three parameter axes. In particular the frequency axis can also be expanded with a higher frequency lock-in amplifier and changes to the probe wiring. With more data further scaling analysis can be performed near the quantum critical point.

This technique can also be applied to study the destruction of superconductivity in other transitions. One particular experiment that we are working on is measuring the superconductor insulator transition in aluminum films. In these films the magnetic field tuned SI transition changes from second order to first order when the applied field is within a few degrees of parallel to the sample because of the spin-paramagnetic effect[27]. Measuring the inverse inductance and by extension the superfluid density as a function of magnetic field magnitude and angle, the coil provides a great tool to study the destruction of superconductivity in these samples. To this end we have been constructing a rotator for the coil assembly with the intent to measure and compare the first and second order SI transitions in these samples.
Chapter 6

References

Apr 1979.


[3] Denis Dalidovich and Philip Phillips. Phase glass is a Bose Metal: A new conducting

1275, Jul 1999.

dissipation in the vortex state of a highly disordered superconducting thin film. *Phys.


Appendix A

Film Growth

As was mentioned earlier in the thesis, the films were sputtered onto an insulating substrate. We have tried sapphire, silicon, and silicon with SiN or SiO$_2$ insulating top layer. Even though the only requirement is that the substrates are insulating at low temperatures, it’s helpful that they are insulating at room temperature if the films need to be patterned in order to check the continuity of the pattern. The substrates are diced to the appropriate size, in this case about 0.5” squares. The squares are then cleaned using the following procedure.

1. Measure out equal amounts of H$_2$O$_2$ and H$_2$SO$_4$
2. Pour the H$_2$O$_2$ into a dish and place the substrates growth side up in the dish.
3. Now add the H$_2$SO$_4$ into the dish and sonicate for 30 minutes
4. Rinse substrates with DI water and sonicate in acetone for 10 minutes
5. Boil some methanol and wash substrates in methanol
6. Blow dry the substrates with pressurized nitrogen

After cleaning the substrates we would sputter the films. The magnetron sputtering system was a custom ATC 2000, a four gun system made by AJA International (www.ajaint.com). The system was equipped with a nitrogen flow-through cold trap in order to trap unwanted contaminants. It was equipped with both RF and DC power supplies. The targets were purchased from Super Conductor Materials, Inc. (www.scm-inc.com). The MoGe target was made to a 43:57 atomic ratio of Mo:Ge and guaranteed to be of 99.99% purity. The Ge target was of the same purity. Both targets were circular, 2” in diameter and 1/8” thick, attached to a copper backing plate of the same dimensions.

To sputter, first the cleaned substrates were attached to the substrate holder using
double-sided carbon tape. The holder was then put into the load lock, and the load lock was pumped down to within a decade of the main chamber pressure. Then the samples were transferred to the main chamber. For sputtering the chamber pressure was about $2 \times 10^{-7}$ Torr without the nitrogen cold trap running and about $9 \times 10^{-8}$ Torr after it was running for a few minutes. The MoGe target was usually presputtered for at least 10 minutes to clean the surface of the target as well as to coat the walls of the chamber in the hope that other materials wont come off the walls onto the substrate during sputtering. If the target hasnt been used before or not in a long time a longer presputter time was used ($\sim 30$ min). First a 30 Å layer of Ge was deposited on the substrate to even out the surface. This was followed by a MoGe layer of the required thickness, and another 30 Å layer of Ge was sputtered to cap the film and prevent surface oxidization. The thicknesses were measured using a crystal monitor. In order to sputter 4 mTorr of Argon were introduced into the sputtering chamber. Usually the power supply was set to 150 W for either target. While the MoGe target can be sputtered with a DC supply, the Ge target required RF sputtering. In this geometry this was a rate of about 1 Å/s for MoGe (DC) and 0.5 Å/s for Ge (RF). A higher power setting gives a higher rate of sputter, but the target is more likely to crack. Changing the power of the supply in either direction should be done slowly to minimize the risk of cracking the target. To promote even growth the substrates were rotated at 300 RPM during sputtering. Once sputtering was complete the samples were taken out through the load lock and stored in a desiccator.
Appendix B

Converting Voltage To Conductance

What we measure is a voltage across the pickup coils. This voltage must be converted to conductance before the data can be analyzed. The equation relating the pickup voltage $\delta V$ and $\omega G$ is

$$\delta V = i\omega I_D \int_0^\infty dx \frac{M(x)}{1 + \left(\frac{2\mu_0 h}{\omega G}\right)x}$$

where

$$M(x) = \pi \mu_0 h_0 \alpha \beta J_1(\alpha x)J_1(\beta x)e^{-x} \left[\frac{1 - e^{-N_D \gamma x}}{1 - e^{-\gamma x}}\right] \left[\frac{1 - e^{-N_R \delta x}}{1 - e^{-\delta x}}\right].$$

Unfortunately, the integral cannot be inverted analytically. To get around this problem we have written a Matlab program that creates a lookup table between $\delta V$ and $\omega G$. A short description of the program follows along with instructions on how it is to be used. The actual code will be included at the end.

The program is relatively simple. First it creates a vector $mags$ of magnitudes of $\omega G$ from 0.01 to 1000 nH$^{-1}$ and a vector $angs$ of the angles of $\omega G$ from 0 to 2$\pi$. From these vectors we create matrices $X$ and $Y$ such that the matrix $wG = X.*exp(i*Y)$ covers all values of $\omega G$. A matrix $gmat$ proportional to $1./wG$ is calculated. Each value in the matrix is then plugged into the integral and numerically integrated to generate matrix $Vout2$. The integral is split into two parts because the integrand changes much more in the interval 0 to 1.5 than from 1.5 to infinity. Without splitting, the numerical integration would yield an incorrect value. The matrices $wG$ and $Vout2$ are then used for the conversion of all voltage data into conductance.

Since no two coils are identical each probe requires its own $Vout2$. To calculate $Vout2$
for a new coil first the geometrical parameters of the coil are entered into the program. The only parameter left to vary is the distance $XX$ from the coils to the sample. This depends on how close the coils were sanded. To calibrate this number, a measurement must be made on a thick superconductor. The voltage obtained is then assumed to correspond to infinite inductance of the sample. The calibration involves varying the distance to the coils and integrating $M(x)$ such that the result equals the voltage measured for a thick superconductor divided by $i\omega I_D$. Once the parameter is set the program can calculate the entire $V_{out2}$ matrix.
function [M2out]=calctable(mflag)

% Function calctable calculates the Voltage(matrix Vout) measured
% corresponding to a conductance value(matrix wGgrid). Vout is actually
% the voltage divided by i*w*Id.
% The only input argument is mflag.
% If mflag=0 prog_2 returns only the expected signal in nV divided by
% (2*pi*f(Hz)*I(A)) for a superconductor with infinite inductance and does
% not calculate any of the above mentioned matrices.
% Any other value of mflag calculates and saves Vout and wGrid in
% tables3.mat along with mags and angs, the magnitude and angle of wG,
% respectively.

global PI U0 RD RR DHR DHD NR ND XX HD HR H a b1 b2 b3 b4 c d I2 I1 g1 g2

PI = 3.141592653589793;
U0 = 12.56637e-7;
RD = 2.00;  %radius of drive coil
RR = 1.0414; %radius of pickup coil
DHR = 0.0635; %pickup coil pitch (0.020"/(NR+1) in mm)
DHD = 0.235; %drive coil pitch
NR = 7;    %number of turns in pickup coil
ND = 25;   %number of turns in drive coil
XX = 0.22958;  %fudge factor
HD = 0.1082+XX; %distance to first drive coil from sample
HR = 0.0508+XX; %distance to first pickup coil from sample
H = HD+HR; %HD+HR
a = RD / H; %dimensionless radius of drive coil

%dimensionless radius of each layer of pickup coil
b1 = (RR + 0)/ H;
b2 = (RR + 0.050) / H;
b3 = (RR + 0.100) / H;
b4 = (RR + 0.150) / H;

%dimensionless coil spacing for the drive and pickup coils
c = DHD / H;
d = DHR / H;

g1=0.0;
g2=0.0;

mags = 0.01.*ones(1,1158).*1.01.^(0:1157); %create mag(wG) vector
angs = pi*(0:0.25:360)/180; %create ang(wG) vector

[magsmat,angsmat] = meshgrid(mags,angs); %make matrices of vectors

I1=@A1;
I2=@A;

%calculates Vout for perfect superconductor for calibration
if mflag==0
    mm=1e6*(quadv(I1,1e-14,0.1)+quadv(I1,0.1,100)+quadv(I1,100,1e17))
\[
M_{2out} = \text{mm}; \quad \% \text{result should match calibration signal (nV)/(2\pi f(\text{Hz})I(\text{A}))}
\]

\text{return}

end

[Vout \text{ wGgrid}] = \text{GtoV(magsmat,angsmat)};

\text{save tables3 Vout mags angs wGgrid}

M_{2out} = \text{Vout};

\% \text{function } M(x) \text{ in the numerator of the integral}
\text{function } M = fM(x)
\text{global PI U0 NR ND H a b1 b2 b3 b4 c d;}
M = PI * U0 * H * a .* besselj(1,a.*x) .* (b1*besselj(1,b1.*x)
 + b2*besselj(1,b2.*x)+b3*besselj(1,b3.*x)+b4*besselj(1,b4.*x))
.* exp(-x) .* ((1 - exp(-ND * c .* x)) ./ (1 - exp(-c .* x)))
.* ((1 - exp(-NR * d .* x)) ./ (1 - exp(-d .* x)))

\text{function } fA1 = A1(x) \% \text{imag part of function inside integral}
\text{global g1 g2;}
fA1 = fM(x) .* ((1.0 + g1 .* x) ./ (1.0 + 2.0 .* g1 .* x + (g1.^2 + g2.^2)
 .* (x.^2)));

\text{function } fA = A(x) \% \text{function inside integral}
\text{global g PI U0 NR ND H a b1 b2 b3 b4 c d;}
fA = (1.0e6 * PI * U0 * H * a .* besselj(1,a.*x) .* (b1*besselj(1,b1.*x)
 + b2*besselj(1,b2.*x)+b3*besselj(1,b3.*x)+b4*besselj(1,b4.*x))
.* exp(-x) .* ((1 - exp(-ND * c .* x)) ./ (1 - exp(-c .* x))));
.* ((1 - exp(-NR * d .* x)) ./ (1 - exp(-d .* x))))/(1+x*g);

%GtoV calculates table
function [vgrid wGgrid] = GtoV(X,Y)
global U0 H I2 g;

wG=X.*exp(i*Y);
gmat=2./(i*U0*1e9*(H./1000).*wG);
[rows, columns]=size(gmat);
Vout2=zeros(rows,columns);
for lct = 1:rows
    (lct/rows) %displays how far along the program is from 0 to 1
    for lct2 = 1:columns
        g = gmat(lct,lct2);
        Vout2(lct,lct2)=quadv(I2,1e-14,0.1)+quadv(I2,0.1,100)
        +quadv(I2,100,1e17);
    end
end
wGgrid=wG;
vgrid=Vout2;
Appendix C

Coil Construction and Measurement Techniques

Making the probe is a relatively quick process, with the exception of epoxy curing times. First the two coils are wound separately on machined pieces of clear plastic, in this case either lexan or plexiglass. The drive coil is wound using a 0.005” diameter superconducting wire in threads machined on a 4 mm diameter hollow cylinder. The result is about a 6 mm long coil containing 23 turns. The pickup coil is made using 0.002” diameter copper wire wound in two grooves machined 5 mm apart in a plastic cylinder. The two sections are wound in opposite directions to eliminate pickup from the drive coil. Each section is a 0.5 mm long coil with a 2 mm inner diameter consisting of about 32 turns. The two pieces are then inserted together and aligned to cancel out the mutual inductance as well as possible. This assembly is then epoxied together with Stycast 1266 epoxy and inserted into a machined piece of garolite-10 (G10) and epoxied into place. The leads of the pickup coil are then soldered to SMA connectors, while the drive coil wires are soldered to a simple 2-pin connector. The bottom of the probe is then carefully sanded down as close to the coils as is reasonably possible. A schematic of the coils and sample is shown in Figure C.1.

To mount the sample, it is first affixed to a plastic disk with the aid of vacuum grease. The disk is then attached to the probe by four screws such that the film is pressed up against the bottom of the probe. A ruthenium-oxide thermometer is mounted between the probe and the plastic disk next to the sample.

To perform a measurement we use a Perkin Elmer 7265 lock-in amplifier. The drive coil is connected to the oscillator output of the lock-in in series with a large resistor, usually 10 kΩ or 100 kΩ, in order to limit the current in the coil and keep it constant. The two leads
from the pickup coil are connected to the input of the lock-in. Most of the measurements are
done at frequencies from 10 kHz to 250 kHz. Other frequencies are possible but are usually
impractical due to noise or high frequency signal loss.

Figure C.1: Diagram depicting the coil geometry. The geometry of the coils is used to
calculate the function $M(x)$ in order to convert the measured voltage to conductance of the
film.

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Appendix D

Data Analysis Techniques

As had been mentioned previously, our measurement consists of flowing an AC current through the drive coil and measuring the signal on the pickup coils using a lock-in amplifier. Hence the signal that we measure is an in phase and out of phase voltage. We will now describe how this data is converted into the complex impedance that is reported throughout this thesis.

The first step to any data analysis is to have a temperature sweep in zero applied magnetic field from above the transition down to the lowest temperature. Because the coils are not perfect, the counterwound pickup coils do not cancel the driving field completely and have an offset. This constant offset is easily seen in the raw data above the transition. Hence the first step is to find this offset from the data and subtract it. The offset is frequency dependent, so it should be done at each frequency. The next step is rotating the phase of the data such that the low temperature saturated signal is purely inductive. This rotation angle is also frequency dependent and hence should be determined for each frequency separately. Once the data has been offset and rotated, we put in the frequency and drive current into our conversion program which uses the tables explained in Appendix B. This program then returns the complex impedance and conductance, from which we calculate the relevant information such as $L^{-1}$ and $R$. Similarly, in the case of field sweeps at the lowest temperature, we can use the same technique, getting the offsets from data above the critical field and the rotation angle from the data at zero field.

In the case of temperature sweeps at non-zero fields or field sweeps at elevated temperatures, we use the offsets and rotation angles from data taken at the same frequency during
temperature sweeps at zero field or field sweeps at the lowest temperature. As is typically
the case during any experimental measurements, there are often sources of noise outside
our control that can complicate matters somewhat when trying to determine the offsets and
rotation angles for the data.