IMPROVED NANOTOPOGRAPHY SENSING VIA TEMPERATURE CONTROL OF A HEATED
ATOMIC FORCE MICROSCOPE CANTILEVER

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ABSTRACT

This thesis reports thermal nanotopography sensing using a heated atomic force microscope cantilever with a sensitivity as high as 4.68 mV/nm, which is two orders of magnitude higher than previously published results for heated cantilevers. The sensitivity improvement arises from closed-loop control of cantilever temperature during the topography sensing. The cantilever temperature is controlled by maintaining constant electrical resistance, current, power, or voltage across either the entire electrical circuit or individual components of the circuit. A model that links the cantilever heat flow and temperature-dependent cantilever properties to the circuit behavior in order to predict and then optimize the cantilever topography sensitivity was developed. Topography measurements on a 100 nm tall silicon grating show cantilever sensitivity ranging 0.047 to 4.68 mV/nm, depending on the control scheme. The application of closed loop control yields a topography sensitivity that is 100X increased over previously published work on heated cantilevers.
This thesis is dedicated to my family, colleagues and friends.
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CHAPTER 1: INTRODUCTION

The atomic force microscope (AFM) [1] is widely used for nanometer-scale sensing. In typical operation, an AFM measures the topography of a surface by tracking the deflection of a cantilever using a laser-photo-detector setup as the cantilever tip scans the surface [2]. Although very robust and easy to implement, this technique cannot be conveniently scaled up to multiple parallel cantilevers in an array. AFM using large cantilever arrays requires each cantilever to have its own independent sensor [3-4].

Several strategies have been demonstrated for integrating a tip height position sensor into an AFM cantilever. The tip position can be sensed by tunneling current [1], mechanical strain via embedded piezoresistors [5], or the flow of heat using a heater-thermometer [6]. Silicon AFM cantilevers with integrated heater-thermometers, originally developed for high density data storage [3], have been used to sense topography features on a substrate at the nanometer-scale [7-9].

Nanotopography sensing with heated cantilevers is achieved by detecting changes in heat flow from the cantilever to the substrate [7, 9-11]. The cantilever electrical resistance is a function of temperature, thus the thermal conductance from the cantilever can be measured in order to measure topography. The nanotopography sensitivity, or the magnitude of change in the heat flow from the cantilever for a given change in topography in the substrate, depends upon cantilever temperature and the cantilever temperature-dependent properties. Previous research has involved extensive modeling of the dynamics of thermal mapping of topography through analytical solutions, numerical simulations, as well as a systems approach [10, 12]. The control of a microheater sensor temperature via feedback on the sensed electrical parameters, such as power,
or electrical resistance, has also been demonstrated [13]. Cantilevers with a single integrated microheater have also been shown to be capable of high speed multi-scale thermal mapping of surfaces [14]. However, the focus of previous work has mostly been on studying cantilever sensing using a constant voltage applied to the circuit [7, 9, 12]. Moreover, a comparative study of the effect of different cantilever temperature control schemes on the topography sensitivity has not been performed.

This thesis explores the use of cantilever temperature control schemes, including control of electrical resistance, power, voltage and current over different parts of the electrical circuit. A model is constructed to predict the topography sensitivities and then to explore how various cantilever and circuit design parameters affect topography sensitivity. Topographies of a 100 nm tall silicon grating were acquired using each of the control schemes, and the results compared well with those obtained from the laser-deflection based signal. Both experimental and theoretical topography sensitivities are found to be up to 100 times greater via control of current to the cantilever compared to the conventional method of voltage control across the circuit.
CHAPTER 2: SYSTEM DESCRIPTION

2.1 Doped Silicon Microcantilever

Figure 2.1 shows the heated microcantilever used in this experiment. The design, fabrication, and operation of this type of cantilever is well understood [15] and only briefly summarized here. The cantilever is doped single crystal silicon with legs that are 1 µm thick, 135 µm long, and 15 µm wide. The cantilever free end has a sharp tip of height 1 µm and a radius of curvature approximately 20 nm. The cantilever used in this study has a spring constant of 0.53 N/m and the resonant frequency of the first mode is 69.4 kHz. The cantilever electrical resistance is 2.33 kΩ, at 25 °C. When current is passed through the cantilever, the heater region near the cantilever tip dissipates 95% of the power, resulting in a temperature rise at the cantilever free end. The cantilever has a thermal time constant of 300 µsec [15].

Figure 2.1. Scanning Electron Microscope (SEM) image of the microcantilever with the integrated resistive heater. The single crystal silicon cantilever has high phosphorous doped leg regions and a low phosphorous doped heater region near the tip.
Figure 2.2(a) shows the cantilever electrical resistance as a function of temperature. This temperature calibration was performed using Raman spectroscopy [15-16]. Figure 2.2(b) shows the electrical resistance of the cantilever with respect to voltage applied across the circuit. These cantilever responses are typical for doped silicon heated cantilevers.
2.2 Experimental Setup

Figure 2.3 shows the experimental setup used to obtain the thermal topography signal. The cantilever was mounted in a commercial AFM (Asylum Research MFP-3D SA) to acquire the topography of a silicon substrate having 100 nm tall gratings from both the laser-deflection signal and the thermal signal. The cantilever lateral speed was maintained at 10 µm/sec while 1024 data points were collected for each line of the scan. Since the length of each scan line is 10 µm, the data was sampled at 1024 Hz. The cantilever was an arm of a balanced Wheatstone bridge where $R_S$ and $R_C$ are the resistances of the bridge “sense” resistors and the cantilever. $V_S$ and $V_C$ are the voltage across the sense resistor in series with the cantilever and the voltage across the cantilever, and $V_T$ is the total voltage applied to the circuit. A National Instruments Data Acquisition Board (NI-DAQ USB 6259) in conjunction with a computer running Labview was used maintain constant cantilever temperature.

![Figure 2.3 Schematic of the experimental setup used to acquire the thermal signal, $V_C$, in order to sense the topography. $V_T$, $V_S$, and $V_C$ are the total voltage applied to the circuit, the voltage across the sense resistor, and the voltage across the cantilever respectively. $R_S$ is the resistance of the sense resistor, and $R_C$ is the cantilever electrical resistance. A computer controlled data-acquisition board (DAQ) applies the appropriate $V_T$ to the circuit depending on the cantilever temperature control mechanism and the sensed $V_S$. The corresponding $V_C$ is also calculated and sent to the AFM controller as the thermal map of the topography.](image)
2.3 Cantilever Temperature Control

Seven different control schemes were used to maintain constant cantilever temperature. These schemes are characterized by two control variables. The first variable is the component of the external electrical circuit over which control is exercised: cantilever, the arm of the Wheatstone-bridge containing the cantilever, or the entire circuit. The second variable is the electrical parameter that is held constant: electrical resistance, current, power, or bias applied. The seven control schemes chosen for this study are Cantilever Resistance control, Cantilever Power control, Cantilever Current control, Cantilever-arm Power control, Circuit Voltage control, Circuit Power control, and Circuit Current control. These control schemes hold the cantilever temperature constant by modulating $V_T$ based on the measured $V_S$, as described in the following equations:

- **Cantilever Resistance control:**  
  \[ V_T = V_S \left( 1 + \frac{R_C^*}{R_S} \right) \]  
  (1)

- **Cantilever Power control:**  
  \[ V_T = V_S + P_C^* \frac{R_S}{V_S} \]  
  (2)

- **Cantilever Current control:**  
  \[ V_T = I_C^* (R_C + R_S) \]  
  (3)

- **Cantilever – arm Power control:**  
  \[ V_T = P_{C-A}^* \frac{R_S}{V_S} \]  
  (4)

- **Circuit Power control:**  
  \[ V_T = \frac{P_T^*}{I_T} \]  
  (5)

- **Circuit Voltage control:**  
  \[ V_T = V_T^* \]  
  (6)

- **Circuit Current control:**  
  \[ V_T = I_T^* R_T \]  
  (7)

where the superscript ‘*’ denotes the parameter that was kept constant. $P_C$ and $R_C$ are the power supplied, and the electrical resistance of the cantilever, while $I_T$ and $R_T$ are the current supplied, and the total resistance of the circuit. $P_{C-A}$ and $P_T$ are the power supplied to the arm of
the Wheatstone bridge containing the cantilever and the power supplied to the entire circuit respectively. The aforementioned six values are calculated using $V_T$, $V_S$, $R_s$, and the control parameter. $V_C$ was calculated as the difference between $V_T$ and $V_S$ and was supplied via the DAQ to the AFM controller as the thermal signal. The temperature control feedback loops had a time constant of 2 msec.

2.4 Thermal Sensing of Topography

It is important to understand how and why the thermal map of the topography varies among the control schemes, since the control of different electrical parameters has different consequences. Figure 2.4 shows the concept of thermal nanotopography sensing with a heated cantilever. Since most of the heat generated in the cantilever flows into the substrate, the thermal conductance from the cantilever depends on the distance between the cantilever and the substrate [7]. The heat flow decreases when the cantilever tip follows a topography feature that moves the cantilever further away from the substrate and it increases when the cantilever comes closer to the substrate.
Figure 2.4 Working principle for thermal topography sensing. The heat flow from the heater region of the cantilever through the legs is $q_{\text{leg}}$, and the heat flow from cantilever across the air gap to the substrate is $q_{\text{air}}$. Eventually, most of the heat generated in the heater region of the cantilever is conducted to the substrate. The thermal conductance from the cantilever is inversely proportional to the thickness of the cantilever-substrate air gap. The surface topography can be measured by measuring the changes in the cantilever voltage that arise due to changes in the thermal conductance of the cantilever.

When maintaining constant cantilever electrical resistance and operating the cantilever in a region of positive temperature coefficient of resistance, a decrease in heat flow reduces the power required to keep the cantilever at the same temperature set point, thereby decreasing the voltage drop across the cantilever. However, when holding any other electrical parameter constant, a decrease in the heat flow causes the cantilever temperature, resistance and voltage drop to rise. The topography of a substrate can therefore be measured by monitoring the changes in the voltage drop across the cantilever. All of the temperature control schemes are investigated over the range 202°C to 486°C.
CHAPTER 3: TOPOGRAPHY SENSITIVITY MODELLING

The cantilever topography sensitivity can be defined

\[ S = \frac{\Delta V_C}{\Delta z} \]  \hspace{1cm} (8)

where \( \Delta V_C \) is amplitude of the thermal topography signal for a given topography change, \( \Delta z \), in the sample. In order to compute \( \Delta V_C \), it is necessary to solve the governing equations pertaining to the electrical and thermal aspects of the thermal mapping of topography.

The cantilever-substrate heat transfer has been rigorously studied in previous publications [10, 17]; therefore, lumped models are used for the cantilever and the substrate, where there is a linear relationship between \( P_C \) and cantilever temperature \( T_C \). Operating the cantilever in a regime of linear temperature coefficient of resistance, the cantilever power is

\[ P_C = -G (\beta R_C + T_0 - T_{\text{substrate}}) = C_0 + C_1 R_C \] \hspace{1cm} (9)

The constants \( C_0 \), and \( C_1 \) are determined experimentally by measuring the electrical power dissipation from the cantilever for various temperature set-points when the cantilever is in contact with the substrate. \( G \) is the thermal conductance from the cantilever to the substrate and \( \beta \) is the rate at which the cantilever temperature rises with respect to the resistance. In the same way, changes in the cantilever resistance, \( \Delta R_C \), and the power dissipation, \( \Delta P_C \), that occur due to a change in topography can also be derived from the heat conduction equation as follows:

\[ \Delta P_C = C_2 + C_3 \Delta R_C + C_4 R_C \] \hspace{1cm} (10)

Again, \( C_2 \), \( C_3 \), and \( C_4 \) are constants determined experimentally by noting the \( \Delta P_C \) and \( \Delta R_C \) for the cantilever resting on the top and bottom portions of the grating in the sample. Clearly, \( \Delta R_C \)
is zero when the cantilever resistance is held constant and $\Delta P_C$ is zero when cantilever power dissipation is the control variable.

The solution for the three unknowns - $\Delta V_C$, $\Delta P_C$, and $\Delta R_C$ requires two additional equations besides (10). These equations are derived from Ohm’s Law for each control scheme by expressing the $\Delta R_C$, $\Delta P_C$, or $\Delta V_C$ required to keep the control variable constant for changes in the topography. As an example, the following two equations are derived in the case of Cantilever Current control:

$$I_C^* = \frac{V_C}{R_C} = \frac{V_C + \Delta V_C}{R_C + \Delta R_C} \quad (11)$$

$$I_C^* = \frac{P_C}{V_C} = \frac{P_C + \Delta P_C}{V_C + \Delta V_C} \quad (12)$$

Equations (10-12) provide a solution for $\Delta V_C$ which is required to calculate the topography sensitivity in (8). Supplementary equations can be found in Appendix A1.
CHAPTER 4: RESULTS

4.1 Thermal Topography Maps

Figure 4.1 shows topography images of a 100 nm tall silicon grating generated from the laser-deflection signal and the cantilever voltages corresponding to Cantilever Current control, Cantilever Resistance control and Circuit Current control. The thermal signals corresponding to all control schemes generate topographies that are very similar to that provided by the laser-deflection signal. However, the Cantilever Resistance control method actually results in an inverted topography because $V_C$ decreases as the cantilever comes closer to the substrate. Although each of the temperature control methods provides the same qualitative topography, there is a large variation in the amplitude of the thermal signal, $\Delta V_C$. For a given topography, large $\Delta V_C$ translates to high topography sensitivity. Large $\Delta V_C$ generally results in an improved signal-to-noise ratio, since the noise in the thermal signals is approximately the same for a given cantilever temperature.
Figure 4.1 Three dimensional topography images of a 100 nm tall silicon grating comparing the laser-deflection based measurement with the thermally derived signals. The Cantilever Current control signal is obtained at a cantilever temperature of 324°C, while the Cantilever Resistance control and the Circuit Current control signals are obtained at a cantilever temperature of 486°C. Note that the thermal signals appear qualitatively similar to the laser-deflection signal. The Cantilever Resistance control signal is inverted due to the opposite relation between the power dissipation and the cantilever-substrate separation distance.

4.2 Topography Sensitivities

Figure 4.2 shows the measured and predicted nanotopography sensitivity for a 100 nm grating for each of the control schemes as a function of cantilever temperature. Overall, there is excellent agreement between model and experiment. As expected, the sensitivity increases with temperature for all control schemes. Furthermore, the control schemes with a constant circuit parameter show considerably lower sensitivities than their constant cantilever parameter counterparts. Cantilever Current control is the highest sensitivity control scheme, but it can only be employed for a small temperature range due to the small linear regime for the cantilever
current against the cantilever resistance as seen in Figure 2.2(b). At high cantilever temperatures, Cantilever-arm Power control shows the highest sensitivity.

![Graph showing sensitivity vs. cantilever temperature](image)

**Figure 4.2** - Experimental and predicted nanotopography sensitivity of the cantilever at various temperature set points using different cantilever temperature control mechanisms for a 100 nm tall silicon grating. *RC*, *PC*, *PA*, *PT*, *IC*, *IT*, and *VT* correspond to cantilever resistance control, cantilever power control, cantilever-arm power control, circuit power control, cantilever current control, circuit current control, and circuit voltage control.

Table 4.1 shows the highest achieved sensitivities for each control scheme and the relative improvement over the traditional Circuit Voltage control scheme, which shows the lowest sensitivity among all schemes. Previous research has shown a maximum sensitivity of 0.2 mV/nm using Circuit Voltage control [7, 9]. The difference in the maximum achieved sensitivity between published results [7] and this experiment can be attributed to the differences in the cantilever
properties and the resistance of the sense resistor. Since sensitivity increases with a rise in cantilever temperature, the improvements in sensitivity would further increase as the temperature is increased up to the thermal runaway point of the cantilever. Topography sensitivity of heated cantilevers via Circuit Voltage control was demonstrated to be 100 times greater than that achieved by similarly sized piezoresistive cantilevers [11]. Therefore, if Cantilever Current control were used in place of the conventional Circuit Voltage control thermal sensing of topography can be showed to be $10^4$ more sensitive than the piezoresistive method.

Table 4.1. Maximum topography sensitivities achieved for each control mechanism.

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>Sensitivity (mV/nm)</th>
<th>Sensitivity Improvement (multiple of VT)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>0.33</td>
<td>6.94</td>
<td>Inverted topography. Easiest feedback scheme to implement</td>
</tr>
<tr>
<td>PC</td>
<td>0.36</td>
<td>7.56</td>
<td></td>
</tr>
<tr>
<td>PA</td>
<td>0.47</td>
<td>10.08</td>
<td>Highest sensitivity at very high cantilever temperatures. Most accurate topography.</td>
</tr>
<tr>
<td>PT</td>
<td>0.09</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>VT</td>
<td>0.05</td>
<td>1.00</td>
<td>Conventional method. Simplest to implement.</td>
</tr>
<tr>
<td>IC</td>
<td>4.68</td>
<td>99.54</td>
<td>Highest recorded sensitivity. Best at lower temperatures.</td>
</tr>
<tr>
<td>IT</td>
<td>0.14</td>
<td>2.87</td>
<td></td>
</tr>
</tbody>
</table>

4.3 Improving Sensitivity by Alteration of System Parameters

The model was used to study the effects of circuit resistances and thermal conductance on the nanotopography sensitivity of future cantilever designs. These system parameters were modulated by appropriate changes to the constants $C_0$, $C_1$, $C_2$, $C_3$, and $C_4$.

Figure 4.3 shows nanotopography sensitivity predictions for various values of $R_S$. An $R_S$ value of 1kΩ is the reference and the results from all other $R_S$ cases are expressed as percentage
improvement in sensitivity over the reference case. Three main trends can be seen in topography sensitivity for a given cantilever temperature. First, control schemes wherein resistance, power, or current supply to the cantilever is kept constant (RC, PC, and IC) are not included in the plot due to the independence from $R_S$ in the governing equations for these schemes. Second, when control is exercised over the entire circuit as in the control of power, current, or voltage supply to the entire circuit (PT, IT, and VT), a direct relation between the sensitivity and is $R_S$ observed. Finally, an inverse relation between the sensitivity and is $R_S$ can be noted when the power supply to the arm of the Wheatstone bridge containing the cantilever (PA) is controlled. Furthermore, the sensitivity improvement increases marginally for a rise in temperature for all control schemes. Overall, one could improve the sensitivity by increasing $R_S$ if the power, current, or voltage were kept constant across the entire circuit; however, this would come at the cost of increased power requirements. Conversely, decreasing $R_S$ when using Cantilever-arm Power control would not only increase the sensitivity but also lower the power consumption.
Another approach to improve topography sensitivity would be to alter the cantilever design parameters to increase the thermal conductance from the cantilever to the substrate. Increasing this thermal conductance increases the heat flow, which results in improved resolution and sensitivity [10] and can be achieved by changing the medium between the cantilever and the substrate, or altering the cantilever dimensions or thermal properties. In figure 4.4, the results from changed values of conductance are normalized as percentage improvements in sensitivity with respect to that of the reference conductance case of 22.8μW/K. This plot shows that a marginal increase in the cantilever conductance resulted in a linear rise in topography sensitivity for all control schemes over all cantilever temperatures.
Figure 4.4 Effect of changing cantilever thermal conductance on the topography sensitivity. Percentage change in sensitivity for increased conductance over sensitivity for a reference case with conductance of 22.8 µW/K.

Overall, there are many ways to improve topography sensitivity; however, the most convenient way is to simply change the sense resistance value as it does not require a redesign of the cantilever. Another option would be to change the medium of heat conduction, although the convenience of air operation is a main feature of thermal topography reading. Finally, for improved cantilever sensitivity it would be ideal for a cantilever to be designed with a decreased tip height, an increased heater region area, or a decreased heater region impurity doping level [10]. A combination of these techniques could be implemented to achieve the best sensitivity using the control schemes.
CHAPTER 5: CONCLUSIONS

5.1 Applications

The temperature control schemes shown in this thesis could be used in applications other than thermal topography sensing. Such temperature control schemes for these heated probes can be used to maintain specific probe temperatures during nano-manufacturing [18-22]. Temperature-dependent nanotribology can also be studied in detail using such temperature control schemes [23-24]. Other applications include the ability to make far more accurate fundamental heat transfer and material property measurements [25].

5.2 Future work

I plan to fabricate arrays of cantilevers with independent tip-height actuation capabilities in addition to integrated sensors such as doped silicon heaters at the tips. Signals from the cantilever temperature feedback circuit could drive the cantilever height control feedback circuit as shown in figure 5.1.

Figure 5.1 Schematic of interdependent feedback loops for performing atomic force microscopy using a single cantilever. The sense voltage signal - $V_{S}$ serves as an input signal to both the cantilever temperature control and cantilever height control signals.
The improved topography sensitivity achieved using the various schemes discussed in this thesis can be used to provide better sensitivity to topography changes and thereby improve the accuracy of the cantilever tip height actuation. The symbiotic relationships delineated in Figure 5.1 can help achieve even higher sensitivity to changes in topography.

5.3 Summary

This work demonstrates temperature control of a heated cantilever to improve nanotopography sensitivity for a doped silicon heated AFM cantilever. The topographies of a silicon grating obtained from the thermal signals corresponding to each control scheme compare well with that from the laser-deflection signal. The topography sensitivities improved by nearly two orders of magnitude from 0.047 mV/nm, as achieved using the conventional Circuit Voltage Control to 4.68 mV/nm via Cantilever Current control. Overall, the sensitivity of the heated cantilever is now $10^4$ times greater than that achieved by similarly sized piezoresistive cantilevers. A mathematical model was used to accurately explain and predict the sensitivity using different control schemes. Methods to further improve the sensitivity by varying experimental parameters were explored. Since the thermally-sensed imaging technique obviates the need for optical detection of the cantilever movement, the results obtained in this study will enable parallel displacement monitoring for cantilevers in arrays with significantly higher sensitivity. Furthermore, devices incorporating such temperature control for applications such as high density data storage would consume considerably less power while providing significantly higher sensitivity.
APPENDIX

A1 Supporting Equations for Sensitivity Modeling

A1.1 Equations from Heat Conduction

Expressing $P_c$ as a function of $R_c$

$$T(R_c) = T_0 + \beta R_c$$

$$P_{\text{cant}} = -G (T_c - T_{\text{sample}}) = -G(\beta R_c + T_0 - T_{\text{sample}})$$

$$P_{\text{cant}} = (-G\beta)R_c + \left(-G(T_0 - T_{\text{sample}})\right)$$

$$P_{\text{cant}} = C_1 \ast R_c + C_2$$

Expressing $\Delta P_c$ as a linear function of $\Delta R_c$ and $R_c$:

![Diagram showing change in V, P, and R for a given change in topography of the substrate.]

Figure A.1 Change in the electrical resistance, power dissipation and the voltage across the cantilever for a given change in the topography of the substrate.
Assumptions:

- No spatial variation in sample temperature
- Cantilever oversimplified as a lumped, large, flat plate parallel to the substrate with effective thermal conductivity \( k_{\text{eff}} \), area \( A \), with tip height \( g \).

From figure A.1 the change in electrical power supplied to the cantilever can be related to the change in the air-gap thickness equal to the change in vertical topography \( z \) using the following equations:

\[
P_C - \Delta P_C = -k_{\text{eff}} A \frac{T_C + \Delta T_C - T_{\text{sample}}}{g + z}; \quad P_C = -k_{\text{eff}} A \frac{T_C - T_{\text{sample}}}{g}
\]

\[
\Delta P_C = -k_{\text{eff}} A \left[ \frac{T_C - T_{\text{sample}}}{g} - \frac{T_C + \Delta T_C - T_{\text{sample}}}{g + z} \right]
\]

\[
\Delta P_C = -k_{\text{eff}} A \left[ (T_C - T_{\text{sample}}) \left( \frac{1}{g} - \frac{1}{g + z} \right) - \frac{\Delta T_C}{g + z} \right]
\]

\[
T_C = T_0 + \beta R_C; \quad \Delta T_C = \beta \Delta R_{\text{cant}}
\]

\[
\Delta P_C = -k_{\text{eff}} A \left[ (T_0 + \beta R_C - T_{\text{sample}}) \left( \frac{1}{g} - \frac{1}{g + z} \right) - \frac{\beta \Delta R_C}{g + z} \right]
\]

\[
\Delta P_C = -k_{\text{eff}} A \left[ (T_0 - T_{\text{sample}}) \left( \frac{1}{g} - \frac{1}{g + z} \right) + \beta R_C \left( \frac{1}{g} - \frac{1}{g + z} \right) - \frac{\beta \Delta R_C}{g + z} \right]
\]

\[
C_3 = \frac{\beta k_{\text{eff}} A}{g + z} \quad C_4 = -\beta k_{\text{eff}} A \left( \frac{1}{g} - \frac{1}{g + z} \right) \quad C_5 = -k_{\text{eff}} A (T_0 - T_{\text{sample}}) \left( \frac{1}{g} - \frac{1}{g + z} \right)
\]

\[
\Delta P_C = C_3 \Delta R_C + C_4 R_C + C_5
\]
A1.2 Expressing \( \Delta V_C \) as a function of \( R_C \) and \( \Delta R_C \):

1. (RC) Cantilever Resistance Control:

\[
\bar{R}_C = \frac{(V_C)^2}{P_C} = \frac{(V_C + \Delta V_C)^2}{P_C + \Delta P_C}
\]

\[
\Delta V_C = V_C \left( \sqrt{1 + \frac{\Delta P_C}{P_C}} - 1 \right)
\]

2. (PC) Cantilever Power Control:

\[
\bar{P}_C = \frac{(V_C + \Delta V_C)^2}{R_C + \Delta R_C}
\]

\[
\Delta V_C = V_C \left( \sqrt{1 + \frac{\Delta R_C}{R_C}} - 1 \right)
\]

3. (PA) Cantilever Arm Power Control:

\[
\bar{P}_A = \frac{(V_A + \Delta V_A)^2}{R_A + \Delta R_A}
\]

\[
\Delta V_A = V_A \left( \sqrt{1 + \frac{\Delta R_A}{R_A}} - 1 \right) = \Delta V_T = V_T \left( \sqrt{1 + \frac{\Delta R_C}{R_A}} - 1 \right)
\]

4. (PT) Circuit Power Control:

\[
\bar{P}_T = \frac{(V_T + \Delta V_T)^2}{R_T + \Delta R_T}
\]

\[
\Delta V_T = V_T \left( \sqrt{1 + \frac{\Delta R_T}{R_T}} - 1 \right)
\]
5. (IC) Cantilever Current Control:

\[ I_C = \frac{V_C + \Delta V_C}{R_C + \Delta R_C} \]

\[ \Delta V_C = \frac{\Delta R_C V_C}{R_C} \]

6. (IT) Circuit Current Control:

\[ I_T = \frac{V_T + \Delta V_T}{R_T + \Delta R_T} \]

\[ \Delta V_T = \frac{\Delta R_T V_T}{R_T} \]

7. (VT) Circuit Voltage Control

\[ V_C = \frac{R_C}{R_S + R_C} V_T \]

\[ V_T = \frac{R_C + R_S}{R_C} V_C \]

\[ V_T = \frac{R_C + R_S}{R_C} V_C = \frac{R_C + \Delta R_C + R_S}{R_C + \Delta R_C} (V_C + \Delta V_C) \]

\[ \Delta V_C = V_C \frac{\Delta R_C * R_S}{R_C (R_C + \Delta R_C + R_S)} \]

**A1.3 Expressing \( \Delta P_C \) as a function of \( R_C \) and \( \Delta R_C \)**

1. (RC) Resistance Control: Already expressed in previous section

2. (PC) Cantilever Power Control: The fact that \( \Delta P_C = 0 \) simplifies the heat transfer equation, thus not requiring a second equation for power dissipation from the cantilever as a function of temperature change.
3. (PA) Cantilever Arm Power Control:

\[ P_A = P_C + \frac{V_S^2}{R_S} \]

\[ \Delta P_A = 0 = \Delta P_C + \left( \frac{2V_S \Delta V_S + \Delta V_S^2}{R_S} \right) \]

\[ \Delta P_C = -\left( \frac{2V_S \Delta V_S + \Delta V_S^2}{R_S} \right) \]

4. (PT) Circuit Power Control:

\[ \Delta P_T = 0 \rightarrow \Delta P_C = -\left( \frac{2V_T \Delta V_T + \Delta V_T^2}{2R_S} + \frac{2V_S \Delta V_S + \Delta V_S^2}{R_S} \right) \]

5. (IC) Cantilever Current control:

\[ I_C = \frac{P_C}{V_C} = \frac{P_C + \Delta P_C}{V_C + \Delta V_C} \]

\[ \Delta P_C = \frac{P_C \Delta V_C}{V_C} = \left( \frac{V_C}{R_C} \right)^2 \Delta R_C \]

6. (IT) Circuit Current Control:

\[ I_T = \frac{P_T}{V_T} = \frac{P_T + \Delta P_T}{V_T + \Delta V_T} \]

\[ \Delta P_T = \frac{P_T \Delta R_T}{R_T} = \left( \frac{V_T}{R_T} \right)^2 \Delta R_T \]

7. (VT) Circuit Voltage Control:
\[ V_T = P_T R_T = (P_T + \Delta P_T)(R_T + \Delta R_T) \]

\[ \Delta P_T = -P_T \frac{\Delta R_T}{R_T + \Delta R_T} \]

### A1.4 Supporting Equations

Total to Cantilever conversions:

1. **Resistance:**

\[ R_T = \frac{2R_S \cdot (R_S + R_C)}{3R_S + R_C} \]

\[ R_T + \Delta R_T = \frac{2R_S \cdot (R_S + R_C + \Delta R_C)}{3R_S + R_C + \Delta R_C} \]

2. **Voltage:**

\[ V_S = \frac{R_S}{R_S + R_C} V_T \]

\[ V_C = V_T - V_S = \frac{R_C}{R_S + R_C} V_T \]

3. **\( \Delta V_C \):**

\[ V_C = \frac{R_C}{R_C + R_S} V_T \]

\[ V_C + \Delta V_C = \frac{R_C + \Delta R_C}{R_C + \Delta R_C + R_S} (V_T + \Delta V_T) \]

\[ \Delta V_T = \frac{R_C + \Delta R_C + R_S}{R_C + \Delta R_C} (V_C + \Delta V_C) - V_T = \frac{R_C + \Delta R_C + R_S}{R_C + \Delta R_C} \left( \frac{R_C}{R_C + R_S} V_T + \Delta V_C \right) - V_T \]
\[ \Delta V_T = \Delta V_C \left( 1 + \frac{R_S}{R_C + \Delta R_C} \right) - V_T \left( \frac{\Delta R_C R_S}{(R_C + R_S)(R_C + \Delta R_C)} \right) \]

4. \( \Delta V_S \):

\[ V_S + \Delta V_S = \frac{R_S}{R_C + \Delta R_C + R_S} (V_T + \Delta V_T) \]

\[ \Delta V_S = \frac{R_S}{R_C + \Delta R_C + R_S} (V_T + \Delta V_T) - \frac{R_S}{R_C + R_S} V_T \]

5. Power: Power for the entire circuit is just a summation of power dissipation at all resistors:

\[ P_T = \Sigma P_{R_i} \]

\[ P_T = P_{\text{bot}} + P_{\text{top}} \]

\[ P_T = 2 \frac{V_T^2}{R_S} + \frac{V_S^2}{R_S} + P_c \]

\[ \Delta P_T = \Delta P_c + \frac{2V_T \Delta V_T + \Delta V_T^2}{2R_S} + \frac{2V_S \Delta V_S + \Delta V_S^2}{R_S} \]
REFERENCES


