ANALYSIS AND APPLICATIONS OF COUPLED LEAKY-MODE, IMPLANT-DEFINED SURFACE-EMITTING LASER ARRAYS

BY

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DISSERTATION
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ABSTRACT

Anti-guided vertical-cavity surface-emitting laser (VCSEL) arrays have been designed, fabricated, and characterized using a number of methods. Two-dimensional coherent arrays are useful for biological and atmospheric sensing, free-space and fiber-based optical links, high-power laser pumps, and optical imaging. Coherently coupled arrays exhibit desirable characteristics such as low beam divergence and high brightness. However, the fabrication procedures necessary for such designs are typically complicated and expensive. This work demonstrates and explores a new and much simpler anti-guided VCSEL array design using ion implantation and photonic crystal confinement. The origin of anti-guiding in these laser arrays is described in detail, and design rules for optimizing performance are discussed. A complete description of the means to achieve optical coupling in surface-emitting laser arrays, as well as the coherence in these arrays, is presented through both theoretical and experimental investigations. These lasers are shown to be capable of producing highly coherent, single-mode, in-phase output beams. The application of such arrays as low-divergence and steerable sources is demonstrated experimentally.
To my parents
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1.1 Motivation

Coherently coupled laser arrays are useful to create both high-power and steerable laser sources, which have potential applications in imaging, sensing, and communications. Vertical-cavity surface-emitting laser (VCSEL) arrays are of particular interest because of their low cost, high yield, manufacturing ease, and two-dimensional configurability. For numerous applications, it is desirable to have a single coherent array mode that produces an on-axis, angularly narrow beam.

The coherent addition of the fields emitted from multiple sources on a plane with the same phase results in large optical powers concentrated in an angularly narrow area perpendicular to the source plane. By increasing the number of coherent sources, the total output power increases and the angular distribution of optical intensity decreases. As a result, a coherent collection of emitters, such as a two-dimensional VCSEL array, is useful for applications such as targeting and imaging that require high-power, narrow-divergence beams.

By adjusting the relative phase between emitters, the angular location of the beam can be adjusted. In VCSEL arrays, altering of the phase can be accomplished by changing the relative current injected to individual emitters. This results in a steerable laser source without mechanical parts. A steerable source such as this would be useful for sensing or imaging applications in which robust, reliable, and fast scanning is required.
1.2 Previous Work

Coherent coupling of semiconductor edge-emitting laser and VCSEL arrays has been achieved using a variety of approaches [1–17] and has been studied using many different theoretical formulations [18–27]. Side and facet views of an edge-emitting array and a VCSEL array are illustrated in Fig. 1.1. Experimentally, coherently coupled VCSEL arrays have been designed using mirror etching [5, 7], metal grids [6, 8, 11], phase-adjusting layers [9], regrown anti-guides [10, 12, 14], photonic crystals [13, 15, 16], and ion implantation [17]. Theoretically, laser arrays have been treated using coupled mode theory [18, 19, 21, 22, 25], Bloch function analysis [24], rate equation analysis [20], and coherence theory [26, 27].

Figure 1.1: Side and facet views of (a)-(b) an edge-emitter array and (c)-(d) a VCSEL array.

A common disadvantage of many of the coupling approaches used is that the out-of-phase mode, with an on-axis null in the far zone (as shown in
Fig. 1.2), is favored to lase. Only in phase-adjusted [9], regrown anti-guided (leaky-mode) [10, 12, 14], and ion-implanted arrays [17] has this difficulty been consistently overcome. However, the phase-adjusted and anti-guided arrays require complicated etching and material regrowth procedures, and the ion-implanted arrays (which will be shown to be leaky-mode) often have unstable mode behavior. VCSELs and VCSEL arrays similar to these have been used for beam steering [28–33]. However, these approaches often exhibit discontinuous steering, incoherent fields, or complicated mechanical parts.

![Far-field radiation patterns](image)

(a) (b)

Figure 1.2: Measured far-field radiation patterns of (a) out-of-phase and (b) in-phase two-element VCSEL arrays.

In the theoretical treatments of coupled laser arrays, many approaches considering either the modes or coherence of the array are used. In the cases of coupled mode theory [21, 22, 25] and Bloch-function analysis [24], the modes and modal properties are accurately predicted. However, no deduction of the coherence of the array can be made from these analyses. Conversely, the coherence theory developed to date has failed to incorporate the ability to predict the modal or spectral properties of coupled arrays [26, 27]. Thus, the theoretical framework that has been developed only captures portions of the full picture of coherent laser array behavior.

1.3 Scope of Dissertation

This work focuses on the theoretical treatment and experimental realization of coherent single-mode, ion-implanted VCSEL arrays. The goal of this work is to overcome the pitfalls of the theoretical and experimental methods in designing VCSEL arrays described in the previous section. Additionally,
this work will elucidate some of the challenges to creating large, high-power VCSEL arrays using ion implantation.

Chapter 2 presents theoretical methods useful for designing anti-guided ion-implanted arrays. Chapter 3 describes the fabrication procedure for the various types of implanted arrays. Chapter 4 introduces a modal coherence theory of laser arrays that describes the origins of partial coherence and predicts the coherence properties of fabricated arrays. Chapter 5 describes the application of ion-implanted arrays for single-mode, low-divergence lasers, large-area coherent lasers with increased power, and steerable sources. This work is summarized in Chapter 6.
CHAPTER 2

DESIGN OF ANTI-GUIDED VCSEL ARRAYS

2.1 Introduction

For the bulk of this work, two-dimensional photonic crystal (PhC) implant-defined VCSEL arrays will be investigated. In some cases, designs without the photonic crystal pattern will be considered. However, the merit of using proton implantation to define the elements of the laser array is the central thesis of this investigation. In particular, it will be shown that the implantation confinement creates an anti-guided, also known as leaky-mode, array, which creates strong, uniform optical coupling between all elements of the array. Therefore, all VCSEL array designs in this work will incorporate implant-defined laser apertures.

First, a complex index model of photonic crystals for mode control in single-element VCSELs is described. This will motivate the use of photonic crystals for index confinement and suppression of unwanted modes in VCSEL arrays. Following this, two models to help design anti-guided laser arrays will be introduced. The first is a transfer matrix algorithm that is useful for creating step-index arrays. The second is a model of the thermal- and carrier-induced index profile in ion-implanted arrays. These two models, used either separately or in conjunction, will be shown to be useful tools for designing coherently coupled VCSEL arrays. Additionally, the models will be used to elucidate some of the most significant challenges to realizing arrays with a large number of elements.
2.2 Complex-Index Photonic Crystal Confinement

Stable mode control and suppression of unwanted modes, such as the out-of-phase supermode, are desirable in VCSEL arrays. Photonic crystals have been shown to be useful for maintaining single-mode operation in single-element VCSELs [34–39]. Additionally, two-dimensional VCSEL arrays have been defined using multiple-defect photonic crystals [15, 16]. In order to appropriately utilize photonic crystals, it is necessary to understand their roles in index and loss guiding. This section will discuss an approximate waveguide model of single-element PhC VCSELs that incorporates both the index step and loss introduced by the photonic crystal [40, 41].

The complex index PhC model employs cylindrical waveguide analysis. The effect of loss on the transverse confinement in single-emitter VCSELs is investigated using finite difference calculations. A complex index is used for the lossy photonic crystal cladding region. The loss is classified broadly as scattering and diffraction loss from the photonic crystal holes etched partially through the top distributed Bragg reflector (DBR). Lasers operating in both single and multiple modes are modeled. The single-mode condition is defined as a measured >30 dB side mode suppression ratio for all modes of higher order than the fundamental. The results of calculations are compared to experimental spectral measurements from fabricated photonic crystal ion-implanted VCSELs. The comparison between theory and experiment reveals the validity of modeling a photonic crystal as a lossy structure.

This method is a semi-empirical approach that uses experimental data as an input. Calculations using the lossy model are developed from the experimental spectral splitting between the fundamental and first-order modes, and the results are then used to predict higher-order mode wavelengths. An analysis of the calculated normalized frequency, $V_{\text{eff}}$, and modal loss of studied lasers is presented as a more complete characterization of single-mode lasing operation and can enable accurate design of single-mode photonic crystal VCSELs.

2.2.1 Photonic Crystal VCSEL Design

Figure 2.1 shows a cross-section of the VCSEL design, which incorporates ion implantation and a photonic crystal to provide electrical and optical
confinement, respectively. A large implant aperture size is chosen so that the thermal lens created is essentially uniform across the much smaller photonic crystal defect aperture. The photonic crystal pattern is a hexagonal array of circular holes, which is illustrated in Fig. 2.2. A single missing hole in the center of the pattern is a crystal defect that forms the optical aperture. Several parameters of the photonic crystal region are varied in this study such that a total of 46 VCSEL designs are characterized. These parameters are labeled in Fig. 2.2(b). The hole period (labeled $a$) varies from 2 to 7 µm in 0.5-µm steps. The ratio of the hole diameter (labeled $b$) to the hole period ($b/a$) is fixed at 0.6 and 0.7. Finally, the depths of the photonic crystal holes are varied using three different etch times. All of these parameters serve to alter the waveguiding properties of the photonic crystal.

Figure 2.1: Cross-section schematic of a single-element PhC VCSEL.

Figure 2.2: (a) A lasing PhC VCSEL and (b) a schematic top view with PhC parameters labeled.
2.2.2 Complex-Index Photonic Crystal Model

To find the properties of a cylindrical, step-index VCSEL, an effective index approach is used [42]. In previous analyses of photonic crystal VCSELs [38], an effective waveguide approach outlined in Fig. 2.3 was used to identify the number of lasing modes. The plane-wave expansion method is used to find the effective refractive index for each of the DBR layers penetrated by the photonic crystal, and then these indices are used in a transmission matrix calculation to find the resonances and effective indices for the entire DBR structure. The VCSEL is modeled as a cylindrical step-index optical fiber. With the core and cladding refractive indices, it is possible to apply optical fiber waveguide analysis to find the mode cutoff, determined by the normalized frequency parameter [43]:

\[
V_{\text{eff}} = \frac{2\pi r}{\lambda_0} \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2},
\]

where \( r \) is the core radius, \( \lambda_0 \) is the lasing wavelength, and \( n_{\text{core}} \) and \( n_{\text{clad}} \) are the refractive indices of the core and cladding regions, respectively. The single-mode condition is \( V_{\text{eff}} < 2.405 \), which is the first zero-crossing of the lowest-order Bessel function.

![Diagram of photonic crystal model](image)

Figure 2.3: Procedure used for finding the real part of the effective photonic crystal refractive index. Each individual high and low index layer penetrated by the photonic crystal is replaced with a layer with an effective index. These new values are inserted into a transfer matrix calculation to determine the index difference between the core and cladding regions.

It is often observed that a laser cavity can support multiple modes but
only lases in one. This occurs because modes experience too much loss or not enough gain to reach threshold. Since it is desirable to design single-mode lasers, even for multimode cavities, loss-induced single-mode operation is an important concept. Such loss can be introduced using an etched photonic crystal. In modeling lossy photonic crystal lasers, a complex refractive index for the cladding is used [44]. In a complex-index model, the traditional $V_{eff}$ method cannot be applied. Thus, a finite difference approach is used to solve for the modes using the scalar Helmholtz equation given by

$$\nabla^2 U (r, \phi, z) + n^2 (r) k_0^2 U (r, \phi, z) = 0,$$

(2.2)

where $U$ is the field profile, $n$ is the refractive index profile, and $k_0$ is the wavenumber. The solutions are of the form

$$U (r, \phi, z) = u(r) e^{-im\phi} e^{-i\beta z},$$

(2.3)

where $m$ is an integer and $\beta$ is the propagation constant assumed to be equal to $2\pi/L$ ($L$ is the longitudinal cavity length). The resulting differential equation is

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} + \left[ n^2 (r) k_0^2 - \beta^2 - \frac{m^2}{r^2} \right] u (r) = 0.$$

(2.4)

Taking a finite differences approach transforms Eqn. 2.4 into an eigenvalue problem with eigenvalues (mode resonances) given by $k_0$ and eigenvectors (mode profiles) given by $u$. The finite difference equations are

$$\frac{u_{j+1} - 2u_j + u_{j-1}}{(\Delta r)^2} + \frac{1}{r_j} \frac{u_{j+1} - u_{j-1}}{2\Delta r} - \left( \beta^2 + \frac{m^2}{r_j^2} \right) u_j = -n_j^2 k_0^2 u_j,$$

(2.5)

where $j$ is an index associated with a point in space and $\Delta r$ is the grid spacing.

The variable parameters that determine the solutions to Eqn. 2.4 are the core and cladding refractive indices. The core refractive index is taken as entirely real (no loss) and fixed at the value 3.5, which is an approximation of the expected effective index of the DBR structure for an 850 nm VCSEL without a photonic crystal. For the photonic crystal cladding region, the real part of the refractive index is calculated as discussed above (see Fig. 2.3).
The inclusion of a complex refractive index, \( n_{clad} = n'_{PhC} + in''_{PhC} \), implies that the field eigenvector and the wavenumber in Eqn. 2.4 also will be complex. Since the aim of the calculation is to obtain the resonant wavelengths, the complex wavenumber is of interest. The resonant wavelength can be extracted from the wavenumber:

\[
k_0 = \frac{\omega_0}{c} = \frac{2\pi}{\lambda_0},
\]

(2.6)

where \( \omega_0 \) is the angular frequency and \( c \) is the speed of light. The real-valued resonance must be found by taking

\[
\Re \{ \lambda_0 \} = \frac{2\pi c}{\Re \{ \omega_0 \}} = \frac{2\pi}{\Re \{ k_0 \}},
\]

(2.7)

and the loss (in inverse distance) can be found from the imaginary part of the wavenumber, i.e.

\[
\alpha_i = \Im \{ k_0 \},
\]

(2.8)

where \( \alpha_i \) is defined to be the field amplitude loss (this factor is multiplied by 2 for intensity-based loss). In this model, a complex refractive index is used only for the cladding region. Thus, differences of modal gain from the active region are neglected. This is justified because the gain cross-sectional area for the lasers defined by implantation is much larger than the optical cavity defined by the photonic crystal defect.

Figure 2.4 shows the calculated mode spectra for a photonic crystal VCSEL emitting nominally at 850 nm. The particular design considered is a single-mode waveguide (\( a = 4.5 \) µm, \( b/a = 0.6 \), \( V_{eff} = 1.585 \)) when loss is not included. As is apparent in Fig. 2.4, as the imaginary component of the refractive index in the cladding region becomes nonzero, higher-order modes subsequently appear. All resonances shift to shorter wavelengths (although at different rates), consistent with increased confinement induced by optical loss of the photonic crystal region [44]. The increased loss also leads to greater mode intensity in the core region, thus altering the radial mode profile \( u \).

Figure 2.5 shows the calculated spectral splitting between the fundamental and first higher-order modes for lossless and lossy cases. For this calculation, a constant cladding index of 3.495 (an estimate appropriate for typical pho-
tonic crystal patterns) is used, and an imaginary part of 0.05 is included for the lossy case. Figure 2.5 illustrates that for small apertures the introduction of loss into the cladding has a significant effect on the spectral splitting. This size dependence is expected since a smaller cavity diameter implies that the modes overlap more with the photonic crystal cladding region. In summary, Figs. 2.4 and 2.5 demonstrate that using a complex refractive index for the photonic crystal cladding region significantly changes the modal properties of the laser due to the effective confinement induced by loss. The imaginary component of the cladding refractive index can thus be extracted from the spectral characteristics of the laser cavity. This procedure is a general result that is not specific to a photonic crystal. Therefore, it should be expected that this approach for extracting the loss from VCSELs also can be used for other device structures (e.g. oxide-confined).

2.2.3 Experimental Results

The modal loss is extracted using the measured fundamental mode resonance wavelength and the spectral splitting between the modes. In the experimental procedure, the threshold current, slope efficiency, and cold-cavity spectra of the VCSELs are determined. Spectra are measured below threshold cur-
rent (approximately 0.9 times threshold) in order to avoid thermal effects. As previously mentioned, this analysis can only be performed on cavities supporting more than one mode, and it is important to clarify that all tested devices have cavities that support multiple modes. As seen from Fig. 2.4, loss confinement can create higher-order modes in lasers expected to be single-mode. However, devices referred to as single-mode lasers are devices that lase only in the fundamental mode with power at least 30 dB above any higher-order modes up to maximum output power. Therefore, single-mode lasers may have a laser cavity that supports multiple modes but only the fundamental mode lases.

Figure 2.6 shows a typical example of a comparison between theory and experiment for a particular photonic crystal VCSEL spectrum. The laser tested has a photonic crystal with $a = 6.5\mu m$, $b/a = 0.7$, and an etch depth of $2.94\mu m$. Using the method outlined above with a core refractive index of 3.5, the cladding refractive index is extracted to be approximately $3.497 + i0.05$. In Fig. 2.6, the open squares are the points used to fit the experiment, and the open circles are the calculated resonances. This figure demonstrates that resonant wavelengths predicted from the model for higher-order modes agree well with experimental data.

Another device parameter to evaluate loss is the laser slope efficiency.
Figure 2.6: Optical spectrum of a photonic crystal VCSEL showing fit points to the lowest two modes used for determining the loss (squares) and the solutions found for higher-order modes using the lossy model (circles).

Optical loss is related to the slope efficiency by the proportionality

$$\eta \propto \frac{\alpha_m}{\alpha_m + \alpha_i}, \quad (2.9)$$

where $\alpha_m$ is the mirror loss and $\alpha_i$ is defined in Eqn. 2.8. Figure 2.7 shows the measured slope efficiencies plotted against the extracted modal losses for the fundamental mode of various photonic crystal VCSEL structures. Also shown in Fig. 2.7 is the expected slope efficiency obtained using Eqn. 2.9, where a constant value of $\alpha_m$ fit to a single data point is used to determine the curve. Figure 2.7 reveals that the experimental measurements of slope efficiency follow the trend indicated by the model. As the fundamental mode loss increases, the slope efficiency decreases, even for multimode lasers. The distribution of the data around the line is the result of differences in mirror reflectivity, injection efficiency, and contributions from higher-order modes.

Finally, single- and multimode operation of these photonic crystal devices is investigated using the lossy model. In past work, the number of modes has been correlated to the $V_{eff}$ parameter, which is dependent on the resonance wavelength, the core size, and the core/cladding index difference [38]. Here, this correlation is augmented by using the calculated $V_{eff}$ as well as the extracted modal loss. Figure 2.8 illustrates the single-mode and mul-
Figure 2.7: The slope efficiency plotted against the fundamental mode loss. Single- and multimode devices are distinguished by diamonds and triangles, respectively.

timode devices and their calculated $V_{\text{eff}}$ (using only the real part of the refractive index) and the modal loss difference between the fundamental and first higher-order modes. This modal loss difference is significant since it is related to whether the higher-order mode experiences the gain necessary to achieve lasing. From Fig. 2.8, it can be seen that several single-mode devices lie above $V_{\text{eff}} = 2.405$ and several multimode devices lie below this cutoff. However, for sufficiently large values of modal loss, the photonic crystal VCSELs are single-mode regardless of the $V_{\text{eff}}$ values. For a particular value of modal loss difference (approximately 5 cm$^{-1}$), there is a clear division between single-mode and multimode operation with few exceptions. This empirically deduced modal loss is comparable to the typical value of optical loss found in 850 nm VCSELs. Thus, loss plays a significant role in determining the number of lasing modes of a photonic crystal VCSEL device. Moreover, the lossy model enables a better prediction of the modal characteristics for the design of single-mode VCSELs.
Figure 2.8: $V_{eff}$ and modal loss difference between the fundamental and first higher-order modes for various photonic crystal VCSELs. Single- and multimode operation is indicated by diamonds and triangles, respectively. The solid line indicates the single-mode cutoff condition for $V_{eff} = 2.405$, and the dotted line indicates an empirical modal loss difference cutoff.

2.3 Transfer Matrix Design of Anti-Guided Arrays

Design rules for creating anti-guided laser arrays have been well known for many years [24, 45, 46]. However, the simplicity of the models used is achieved at the expense of rigor and versatility. In particular, these models have not been used to design arrays with more than two index variations and are unable to treat more complicated structures such as those with a cladding layer. As a result, the design of anti-guided arrays has been limited to only the simplest of structures.

In this section, a more robust method of designing anti-guided arrays utilizing the transfer matrix approach is introduced. It is shown that it is possible to re-derive the leaky-mode array design rules for simple arrays. Moreover, design rules for arrays with a cladding are also detailed. Finally, it is shown that inherent in the formulation is the possibility of creating more complicated resonant anti-guided array structures.
2.3.1 Transfer Matrix Formulation

The fields traveling through a layered medium of different index, such as that illustrated in Fig. 2.9, can be calculated using a transfer matrix formulation. Specifically, the amplitudes of forward-traveling and backward-traveling waves in two adjacent media are related by [47]

\[
\begin{bmatrix}
A_j \\
B_j
\end{bmatrix} = B_{j(j+1)} \begin{bmatrix}
A_{j+1} \\
B_{j+1}
\end{bmatrix}.
\]  

(2.10)

In the equation above, the backward-propagation matrix is

\[
B_{j(j+1)} = \frac{1}{2} \begin{bmatrix}
(1 + P_{j(j+1)}) e^{-ik_{j(j+1)} x_{d_{j+1}}} & (1 - P_{j(j+1)}) e^{ik_{j(j+1)} x_{d_{j+1}}} \\
(1 - P_{j(j+1)}) e^{-ik_{j(j+1)} x_{d_{j+1}}} & (1 + P_{j(j+1)}) e^{ik_{j(j+1)} x_{d_{j+1}}}
\end{bmatrix},
\]

(2.11)

where

\[
P_{j(j+1)} = \frac{k_{j+1} x}{k_{jx}},
\]

(2.12)

\(k_{jx}\) is the transverse wavenumber in the \(j^{th}\) layer, and \(d_j\) is the thickness of the \(j^{th}\) layer. For a sequence with \(N\) interfaces, the amplitudes of the incident and reflected fields are found from the transmitted field using

\[
\begin{bmatrix}
E_0 \\
\rho E_0
\end{bmatrix} = B_{01} B_{12} B_{23} \ldots B_{(N-1)N} \begin{bmatrix}
tE_0 \\
0
\end{bmatrix}
\]

(2.13)

\[
= \begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix} \begin{bmatrix}
tE_0 \\
0
\end{bmatrix}.
\]

From this equation, the transmission coefficient is

\[
t = \frac{1}{b_{11}},
\]

(2.14)

and the reflection coefficient is

\[
r = \frac{b_{21}}{b_{11}}.
\]

(2.15)
2.3.2 Array Structure

In this analysis, a waveguide model of the laser arrays is employed. As illustrated in Fig. 2.10, the structure has four constitutive sections: a high-index anti-guide, a core of lower index, an edge guide of arbitrary index, and a low-index cladding. The arrays are symmetric about some center axis, although the analysis can also be performed on asymmetric structures. Though it is not incorporated in the model, the core sections are assumed to be the only regions with gain and therefore are the sections that must have large overlap with the field. The anti-guide sections are present to provide leaky-mode coupling between the different array cores. The cladding provides confinement around the array so there is no edge radiation loss from the array. Finally, the edge waveguides provide a means of phase matching to compensate for the reflection phase at the cladding interface; this will be discussed in more detail in the next section.
2.3.3 Resonant Array Design

For strong, uniform lateral coupling, it is desirable to have an array in which field is equally shared among all the array elements. This can be accomplished by creating a design that has perfect lateral transmission through all the layers of the array. A perfectly transmissive structure can be made by recognizing that

\[
B_{j(j+1)}B_{(j+1)(j+2)} = \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \pm I,
\]

when \( k_{jx} = k_{(j+2)x} \) and the layers are a multiple of a half-wavelength thick. Resultantly, a structure consisting of alternating layers with a multiple of half-wavelength thickness represents one in which there is strong and uniform coupling across the entire array.

In order for there to be a stable waveguide mode, the field must satisfy the condition that it repeat itself after one full roundtrip. As in a resonant cavity, this condition is expressed as

\[
r_1r_2e^{2\pi d} = 1,
\]

where \( r_1 \) and \( r_2 \) are the reflection coefficients calculated using the transfer matrix formulation, \( k \) is the wavenumber in the layer between the two reflectors, and \( d \) is the thickness of that layer. In previous work, designs in which the array is terminated with layers that allow propagation out of the structure were created [4, 12, 48]. The obvious disadvantage of this is that there is radiative loss, and therefore gain is necessary to have a stable mode. Moreover, it is clear from the above equations that if the array is not appropriately designed, this loss can be very large. In the worst case, the entire array is perfectly transmissive and all power is lost at the edges of the array.

In this work, the array is terminated with cladding layers that do not allow radiation from the edges of the array. Figure 2.10 shows this design, and also labels the reflectivities, \( r_1 \) and \( r_2 \), used in the resonance condition. The first reflectivity is simply that of the cladding layer given by

\[
r_1 = \frac{k_x - i\alpha}{k_x + i\alpha},
\]
where $k_e$ is the wavenumber in the edge waveguide and $\alpha$ is the decay constant in the cladding. The second reflectivity can be calculated using the transfer matrix approach. The matrix for the entire structure, after taking into account all the identity matrices resulting from perfect transmission, is

$$ \mathbf{B} = \pm \mathbf{B}_{el} \mathbf{B}_{le} \mathbf{B}_{ec}, $$

where

$$ \mathbf{B}_{el} \mathbf{B}_{le} = \begin{bmatrix} e^{-ik_e d_e} & 0 \\ 0 & e^{ik_e d_e} \end{bmatrix}, $$

$$ \mathbf{B}_{ec} = \frac{1}{2} \begin{bmatrix} k_e + i\alpha & k_e - i\alpha \\ k_e - i\alpha & k_e + i\alpha \end{bmatrix}. $$

Equation 2.20 indicates that this structure has perfect transmission from edge to edge since the magnitude of the $b_{11}$ element of the matrix is unity. The final result for the transfer matrix is

$$ \mathbf{B} = \pm \frac{1}{2} \begin{bmatrix} (k_e + i\alpha) e^{-ik_e d_e} & (k_e - i\alpha) e^{-ik_e d_e} \\ (k_e - i\alpha) e^{ik_e d_e} & (k_e + i\alpha) e^{ik_e d_e} \end{bmatrix}, $$

and therefore

$$ r_2 = \frac{k_e - i\alpha}{k_e + i\alpha} e^{i2k_e d_e}. $$

Using the reflectivities in the roundtrip equation yields

$$ \left( \frac{k_e - i\alpha}{k_e + i\alpha} \right)^2 e^{i4k_e d_e} = 1, $$

$$ e^{i(4k_e d_e - 2\phi_{ec})} = 1, $$

where

$$ \phi_{ec} = \tan^{-1} \left( \frac{\alpha}{k_e} \right) $$

is the reflection phase shift from the cladding. For a fixed $k_e$, the above
condition requires
\[ d_e = \frac{m_e \pi + \phi_{ec}}{2k_e}, \] (2.26) where \( m_e \) (and all future \( m \) terms) is an integer.

2.3.4 Resonant Modes

The array will now be treated as a set of slab waveguides in order to determine the modes. In this case, the wavenumbers must obey the dispersion relations
\[ k_j = k_0 \sqrt{n_j^2 - n_{eff}^2}, \] (2.27)
\[ \alpha = k_0 \sqrt{n_{eff}^2 - n_e^2}, \] (2.28)
where \( k_0 \) is the free-space wavenumber, \( n_j \) is the refractive index of the \( j^{th} \) layer, and \( n_{eff} \) is the effective index of the propagation constant. In order to realize a resonant array, the half-wavelength requirements,
\[ d_j = m_j \frac{\lambda_j}{2}, \] (2.29)
\[ = m_j \frac{\lambda_0}{2\sqrt{n_j^2 - n_{eff}^2}}, \]
and Eqn. 2.26 must be satisfied. Choosing a fixed value of \( d_j \) then sets the effective index and the remainder of the widths for resonance. For the
lowest-order fundamental mode in the low-index core, \( m_l = 1 \) and then

\[
\begin{align*}
    n_{\text{eff}} &= \sqrt{n_l^2 - \left(\frac{\lambda_0}{2d_l}\right)^2}, \quad (2.30) \\
    d_h &= \frac{m_h\lambda_0}{2\sqrt{n_h^2 - n_l^2 + \left(\frac{\lambda_0}{2d_l}\right)^2}}, \quad (2.31) \\
    d_e &= \frac{m_e\pi + \phi_{\text{ec}}}{2k_0\sqrt{n_e^2 - n_l^2 + \left(\frac{\lambda_0}{2d_l}\right)^2}}, \quad (2.32) \\
    \phi_{\text{ec}} &= 2\tan^{-1}\left(\sqrt{\frac{n_l^2 - \left(\frac{\lambda_0}{2d_l}\right)^2 - n_c^2}{n_e^2 - n_l^2 + \left(\frac{\lambda_0}{2d_l}\right)^2}}\right). \quad (2.33)
\end{align*}
\]

Equations 2.30 and 2.31 were previously derived using a different approximate approach for arrays with radiative loss at the edges [45]. However, the introduction of a cladding to prevent loss while maintaining the resonant mode is a new result. The advantage of such a structure is that it reduces the gain needed to reach threshold. Moreover, as will be seen, the transfer matrix approach is a useful and robust tool for designing resonant anti-guided arrays and investigating the modes.

Using the above design rules, it is possible to create a resonant array of arbitrary size. Here, a two-element array is considered, and an illustration of the index profile is shown in Fig. 2.11. The values of the refractive indices are fixed for all calculations and are \( n_h = n_e = 3.42, n_l = 3.415, \) and \( n_c = 3.4, \) which are chosen as typical values for an ion-implanted array. The core width is set to be \( d_l = 7 \) µm. For simplicity of calculation, the mode field profiles are found using the finite differences approach.

First, it is important to recognize the effects of designing arrays with different integer values of \( m_e \) and \( m_h, \) which change the widths of \( d_e \) and \( d_h, \) respectively. The influence of \( m_e \) on the mode profile is especially significant. Figure 2.12 shows the mode intensities when \( m_e = 1 \) and \( m_e = 2. \) The peaks and nulls of the two modes occur at exactly opposite positions. Specifically, for \( m_e = 1 \) (and all odd values), the peaks are aligned with the waveguide centers and the nulls are at the waveguide edges. Conversely, for \( m_e = 2 \) (and all even values), the nulls are aligned with the waveguide centers and
Figure 2.11: Drawing of index profile used for calculation of resonant modes. The widths of $d_h$ and $d_e$ are changed to investigate their effects on the mode.

Figure 2.12: Modes of a two-element array with $m_e = 1$ (solid) and $m_e = 2$ (dashed). In general, $m_e$ odd is desirable since the peaks of the mode align with the centers of the guides.

The influence of $m_h$ must also be considered. A plot of the mode intensities for $m_h = 1$, $m_h = 2$, and $m_h = 3$ (with $m_e = 1$) is given in Fig. 2.13. Here, it is illustrated that the value of $m_h$ determines the number of subsidiary lobes between the dominant peaks in the field. These small lobes exist entirely in the high-index anti-guide regions, while the dominant peaks are contained in the low-index cores. Zero-crossings of the field occur at the interfaces between the core and anti-guide regions. As a result, it is important to
recognize that the value of $m_h$ influences the phase relationship between the dominant peaks. In particular, the peaks are in-phase for $m_h$ odd and out-of-phase for $m_h$ even.

![Figure 2.13: Modes of a two-element array with $m_h = 1$ (solid), $m_h = 2$ (dashed), and $m_h = 3$ (dotted). It is clearly seen that the value of $m_h$ equals the number of subsidiary lobes between the field peaks.](image)

Thus, the transfer matrix formalism can be used to design resonant anti-guided laser arrays. From recognizing the conditions for perfect field transmission, simple design rules are established. Although a fairly specific design has been considered here, it is important to note that this approach can be used to create a variety of different or more complicated arrays. In particular, designs can include more or fewer index variations, have different layer ordering, incorporate varying layer thicknesses, etc. The important result is that perfect transmission is realized with an appropriate lateral index profile, and a resonant mode can be created by making edge waveguides of an appropriate thickness to compensate for reflection phases from the cladding interfaces.

### 2.3.5 Non-Resonant Modes

Once a resonant array is designed, it is possible to solve for all the leaky modes of the array using the transfer matrix formalism. This involves finding all values of $n_{\text{eff}}$ for which the roundtrip condition is satisfied (Eqn. 2.17). As with a typical waveguide problem, this must be done either graphically or
using a non-linear solver.

The two-element array example already investigated and illustrated in Fig. 2.11 is considered here. The solution to the roundtrip equation is plotted in Fig. 2.14, where the zero-crossings of the roundtrip phase angle correspond to the effective indices of the waveguide modes. It can be seen that there are thirteen confined leaky-wave modes in total (note that the evanescently coupled modes are not found using the transfer matrix method). Thus, the one desired resonant mode for this design is in competition with twelve other unwanted modes.

Figure 2.14: Graphical solution of the modes of a two-element anti-guided array. The values for which there are zero crossings are the $n_{\text{eff}}$ for the modes. There are 13 confined anti-guided modes total.

In order to determine the discrimination between modes, it is illustrative to consider the mode intensity overlap with the waveguides, which is defined as the confinement factor. The confinement factor is calculated by solving for the mode intensity profiles using the finite differences with the one-dimensional Helmholtz equation. The mode overlap with the waveguides having gain (the low-index cores) and the regions with loss (the anti-guides, edge waveguides, and cladding layers) is plotted in Fig. 2.15. The first three modes in Fig. 2.15 are the evanescently coupled modes that are confined by the high-index anti-guide waveguides. The small confinement factor in the gain regions arises because only evanescent fields reach these areas, and therefore the evanescent modes are unimportant in this type of design. The remaining thirteen modes are the leaky-wave modes found in Fig. 2.14. The
resonant mode is mode number 4. It has the largest overlap with the cores, approximately 96%, which is nearly 7% more than any other mode.

Figure 2.15: The mode intensity overlap with the gain regions (low-index cores) and loss regions (all other areas) in a two-element array. The resonant mode is mode number 4.

2.3.6 Comparison with a Ray Tracing Approach

In the ray tracing approximation, for there to be a standing wave pattern across the entire waveguide structure that does not change with propagation, a roundtrip condition must be satisfied [47, 49]:

\[ q_d d_k + q_h d_h k_k + 2d_e k_e - \phi_{ec} = m_t \pi, \tag{2.34} \]

where the lateral wavenumber in the \( j^{th} \) region is

\[ k_j = k_0 \sqrt{n_j^2 - n_{eff}^2}. \tag{2.35} \]

In the above equations, \( k_0 \) is the free-space wavenumber, \( n_{eff} \) is the effective index of the propagation constant, \( q_j \) is the number of sections, \( \phi_{ec} \) is the reflection phase at the cladding interface, and \( m_t \) is an integer. Note that Eqn. 2.35 is an approximation based on the assumption that reflections at the interfaces on the interior of the array are negligible. As already shown, resonance implies that there should be no reflection (i.e. perfect transmission through the structure between the cladding layers), and so this is a
valid assumption for that mode. In each individual waveguide section, a similar roundtrip condition must be satisfied. For the $j^{th}$ section (adjacent to sections $i$ and $k$), this is

$$2d_jk_j - \phi_{ji} - \phi_{jk} = 2m_j\pi, \quad (2.36)$$

where the $\phi$ terms again are reflection phases. In general, for such a multilayered structure, the $\phi$ terms are non-trivial to calculate since reflections from other layers must be taken into account. However, on resonance, $\phi_{ji} + \phi_{jk}$ is a multiple of $2\pi$ in the interior of the array. Then, for the core and anti-guide sections, Eqn. 2.36 reduces to

$$d_jk_j = m_j\pi, \quad (2.37)$$

and at the edge waveguide

$$2d_ek_e = 2m_e\pi + \phi_{en} + \phi_{ec}. \quad (2.38)$$

Inserting these expressions into Eqn. 2.34 yields

$$qlm_l\pi + qhm_h\pi + m_e\pi + \phi_{en} = m_t\pi. \quad (2.39)$$

If $\phi_{en}$ is a multiple of $\pi$, then the roundtrip equation is exactly satisfied. In order to produce such a structure, the layer widths must be appropriately set. It can be shown that these widths are exactly the same as those already derived using the transfer matrix approach.

In order to approximately solve for all the leaky-wave modes of the array, one only needs to solve Eqn. 2.34, which is computationally simpler than the transfer matrix approach. A comparison of the effective index solutions found by the two methods is shown in Fig. 2.16. It can be seen that the solutions are close, although they are not identical. Moreover, it is possible for the ray tracing approach to find spurious modes or fail to find a valid mode.

The utility of the ray tracing method is derived from the simplicity of its analytical solutions. In particular, the waveguide cutoff condition can be
Figure 2.16: Comparison of the mode effective indices as found by the transfer matrix method (solid) and the ray tracing method (dotted).

found to be

\[ q_d k_0 \sqrt{n_l^2 - n_c^2} + q_h d_h k_0 \sqrt{n_h^2 - n_c^2} + 2d_e k_0 \sqrt{n_e^2 - n_c^2} = m_t \pi, \]  

(2.40)

where the number of modes increases with increasing \( m_t \). Equation 2.40 provides some insight into how the number of modes scales with the array dimensions. To reduce the number of modes, it is apparent that it is necessary to have low index contrast and narrow waveguides. Additionally, the number of modes increases with the number of waveguides. Therefore, there are inevitably a large number of modes for large arrays.

2.3.7 Two-Dimensional Modes

To solve for the two-dimensional modes, it is assumed that the Helmholtz equation is approximately separable as presented in [23]. In particular, the dielectric function is assumed to be of the form

\[ \epsilon(x, y) = \epsilon_r - \Delta \epsilon_x(x) - \Delta \epsilon_y(y), \]  

(2.41)

where \( \epsilon_r = n_r^2 \) is the background permittivity. Under this assumption, the two-dimensional Helmholtz equation can be separated into two one-
dimensional equations:

\[
\frac{\partial \psi_i}{\partial x} + k_0^2 \left[ (\epsilon_r - \Delta \epsilon_x(x)) - \bar{\epsilon}_i \right] \psi_i(x) = 0, \quad (2.42)
\]

\[
\frac{\partial \psi_j}{\partial y} + k_0^2 \left[ (\epsilon_r - \Delta \epsilon_y(y)) - \bar{\epsilon}_j \right] \psi_j(x) = 0, \quad (2.43)
\]

where the total field profile is \(E(x, y) = \psi_i(x)\psi_j(y)\). These one-dimensional equations are identical in form to the Helmholtz equation whose solution was found in the previous section. Thus, the design guidelines for making a two-dimensional array are approximately identical to those of a one-dimensional array.

The total effective index for the two-dimensional array is given by

\[
\textit{n}_{\text{eff}} = \sqrt{\epsilon_i + \epsilon_j - \epsilon_r}. \quad (2.44)
\]

For the resonant modes,

\[
\textit{n}_{\text{eff}} = \sqrt{n^2_{lx} - \left( \frac{\lambda_0}{2d_{lx}} \right)^2 + n^2_{ly} - \left( \frac{\lambda_0}{2d_{ly}} \right)^2 - n^2_r}, \quad (2.45)
\]

and in the special case of an array and mode that are symmetrical in the two orthogonal directions \((i = j)\)

\[
\textit{n}_{\text{eff}} = \sqrt{2n_i^2 - 2 \left( \frac{\lambda_0}{2d_i} \right)^2 - n_r^2}. \quad (2.46)
\]

The modal discrimination can be approximated by the difference in the imaginary part of \(n_{\text{eff}}^2\) for two modes. The result here is the same as previously shown [23]:

\[
\Delta n_{\text{eff}}^2 = \Delta \tau_i + \Delta \tau_j, \quad (2.47)
\]

and for \(i = j\),

\[
\Delta n_{\text{eff}}^2 = 2\Delta \tau. \quad (2.48)
\]

Thus, the mode discrimination for a symmetric two-dimensional array is twice that of a comparable one-dimensional array. This highlights one of
the significant advantages of surface-emitting laser arrays over edge-emitting arrays: the ability to create two-dimensional arrays allows for better mode selectivity.

2.4 Thermal and Carrier Index Model

In this section, it is shown that the interplay of thermal- and carrier-induced shifts in the refractive index profile for an implant-defined array create an anti-guiding condition [50]. The anti-guiding effect is very similar to that already described in implanted edge-emitting laser arrays [3]. Since anti-guided arrays couple via leaky modes, design parameters such as element separation are important in determining the dominant array mode. Using a simple thermal and carrier model, the conditions for anti-guiding can be reasonably determined and the dominant mode can be predicted. It is shown that comparisons between this theory and experiments are in good agreement.

2.4.1 Index Anti-Guiding Analysis

Since the focus of this work is on the dynamics in the coupling region between laser elements (i.e. where there are no etched holes), the effects of the photonic crystal holes are ignored in the theoretical treatment in order to simplify the problem. Thus, the index model employed incorporates both thermally induced and carrier-induced index shifts superimposed onto a constant index background. Many of the assumptions are taken from the analysis of thermal effects in VCSELs in [51]. In particular, the medium is assumed to be two-dimensional (infinite in the longitudinal direction) with conductivity $\sigma = 0.14 \text{ W/cm-K}$, where the change in index due to carrier concentration, $N$, is $\partial n / \partial N = -10^{-21} \text{ cm}^3$ and the index change due to temperature, $T$, is $\partial n / \partial T = 4 \times 10^{-4} \text{ K}^{-1}$. The carrier concentration is modeled as an abrupt step with the same width as the aperture, and the carrier injection is assumed to be $\Delta N = 5 \times 10^{18} \text{ cm}^{-3}$. This carrier distribution is justified by the observation of luminescence only inside the implant apertures.

The heat sources are assumed to be Gaussian, i.e. of the form $e^{-(r-r_c)^2/a^2}$ where $r^2 = x^2 + y^2$, $r_c$ is the center of a particular aperture, and $a$ is the
aperture radius. The apertures are set to the nominal radius of 3.5 \(\mu m\). The amount of power dissipated as heat in each aperture is assumed to be around \(Q = 7.5\) mW (15 mW total), which is estimated from the current-voltage characteristics above threshold for representative 1x2 arrays [50]. All measurements are performed at approximately threshold current. At that current, the majority of power is dissipated as heat rather than infrared emission, and this dissipated power can readily be estimated using the current-voltage characteristic. It is worth noting that the series resistance is relatively high because the electrical contacts are deposited on top of the implanted regions [52]. The power is assumed to be dissipated in a volume of \(5\pi a^2\) \(\mu m^3\) (i.e. 5 \(\mu m\) in the longitudinal direction and the aperture area in the transverse directions). This yields the diffusion equation

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{Q}{\sigma} \left( e^{-(r-r_c)^2/a^2} + e^{-(r+r_c)^2/a^2} \right). \tag{2.49}
\]

The diffusion equation is solved numerically using the finite element method. Dirichlet boundary conditions are applied 20 \(\mu m\) away from the four sides of the array. This effectively simulates heat sinking that occurs far from the array elements. Figure 2.17 shows a temperature profile found by solving Eqn. 2.49. The highest temperature is in the center of each aperture with a sharp drop in temperature outside of the array. However, a significant feature is that there is only a very slight drop in temperature between the array elements.

![Temperature profile](image)

Figure 2.17: The temperature profile above ambient temperature in degrees Celsius for a two-element array.
Using the index parameters given above and assuming a background refractive index of 3.38 at room temperature, the net refractive index profile can be calculated. Figure 2.18 shows a calculated index profile across a 1x2 array that takes into account both thermal and carrier effects. This figure illustrates that the region between the array elements, which is assumed to be subject only to thermal effects, has a higher index than the adjacent aperture regions. The small index shift introduced by the temperature gradient can be overcome by the index suppression created by the implant-confined carriers. The net effect is to create a region of anti-guiding in the coupling region with approximate index step $\Delta n_c$ between the elements, as can be seen in Fig. 2.18.

Figure 2.18: The calculated index profile for a two-element array along the direction of coupling. The different index regions and the index steps, $\Delta n_e$ and $\Delta n_c$ at the edge and center of the coupling region, respectively, are labeled.

Since the index step is dependent on the temperature profile between the implant apertures, it is expected that it will change with both separation and heating. Figure 2.19 illustrates the change in the index step at the edge and center of the coupling region in Fig. 2.18. It is evident that the index step decreases as both the separation and dissipated power increase, which corresponds to the thermally induced index shift eventually overcoming the carrier-induced shift. Nevertheless, for the chosen parameters, the index anti-guiding condition is met for a wide range of separations and powers. This implies that it is possible to design these anti-guided implant arrays in a variety of configurations that should be able to operate over a wide range of currents.
Using the index profiles found via the methods described in the previous section, it is possible to calculate the array modes. The approach used is similar to that in [23]. In particular, a two-dimensional Helmholtz equation is assumed to be approximately separable so that the problem can be solved for a one-dimensional cut of the index profile, as illustrated in Fig. 2.18. The carrier concentration inside each aperture provides material gain, so it is assumed that this gain perturbs the index with an imaginary part of 0.002. It is also assumed that the other regions of the array have negligible gain or loss.

The gain discrimination between modes is of primary interest. Following the arguments made by Hadley [23], the two-dimensional system can be reduced to a pair of one-dimensional problems. Assuming separability and solutions of the form $H_{ij}(x, y) = \psi_i(x)\psi_j(y)$ in the Helmholtz equation

\[
\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = k_0^2 [\epsilon - \epsilon(x, y)] H, \tag{2.50}
\]

a one-dimensional equation results [23]:

\[
\frac{\partial^2 \psi_i}{\partial x^2} = k_0^2 [\epsilon_i - \epsilon_r - \Delta \epsilon(x)] \psi_i, \tag{2.51}
\]

where $\psi_i$ is the mode profile along one direction, $k_0$ is the free-space wavenum-
ber, $\varepsilon_r$ is the eigenvalue, $\varepsilon_r$ is the background index, and $\Delta \varepsilon(x)$ is the index variation along one direction. If the modes along one direction are identical (in this case, along the direction perpendicular to the coupling), then $\Delta \varepsilon_j = 0$ and the total modal gain discrimination between the $m$th and $n$th modes is equal to that of the other direction, given by

$$\Delta \varepsilon^{2D} = \Delta \varepsilon_i = \varepsilon_i^{(m)} - \varepsilon_i^{(n)}.$$

The one-dimensional Helmholtz eigenvalue equation (Eqn. 2.51) is solved using the finite differences approach. The imaginary parts of the eigenvalues are the modal gains, and the eigenvectors give the corresponding mode profiles. The modal gain discrimination for the dominant mode (i.e. the difference in gain for the two modes experiencing the most gain) is calculated as a function of element separation and power dissipated as heat in Fig. 2.20. As is expected for anti-guided arrays, the dominant mode changes with the inter-element separation due to the change in lateral resonance [53]. Specifically, the number of fringes between the two major lobes of the mode increases as the separation increases. This is significant because the number of fringes changes the mode between being in-phase and out-of-phase (i.e. one fringe corresponds to in-phase with an on-axis maximum in the far field, two fringes corresponds to out-of-phase with an on-axis null in the far field, three fringes corresponds to in-phase, etc.). Interestingly, the dominant mode also can change as the power dissipated as heat increases, which again can be attributed to a change in lateral resonance from a change in inter-element index. However, it is evident that this change only occurs for elevated heating, and otherwise the mode is very stable.

2.4.3 Experimental Results

A set of 1x2 VCSEL arrays were tested and compared with the results found using the theory of the previous section. These arrays have elements of varying center-to-center separations from 9 µm to 14.5 µm in 0.5 µm steps. The implant apertures are designed to be circular with a radius of 3.5 µm, although it is expected that the actual apertures are smaller than this. Moreover, the elements of an array are designed to be identical, but factors such as misalignment during photolithography steps or non-uniformity in the resist
Figure 2.20: The calculated modal gain discrimination as a function of (a) element separation for 15 mW of heat and (b) power dissipated as heat for a separation of 9.5 µm. The insets show the dominant mode intensity profile associated with each set of points (i.e. each shape corresponds to the indicated near-field mode profile).

Implant mask can cause variations. These variations can result in different aperture sizes, resistances, thresholds, etc. that create asymmetry in the array. These arrays exhibit reproducible modal characteristics over the injection current range from threshold to maximum output power, with a single, stable mode lasing at threshold current. Measurements were taken at approximately threshold current. In order to verify the anti-guiding hypothesis, the near-field mode profiles of the different VCSEL arrays were measured using a CCD camera. An image of a tested array with two inter-element fringes while lasing is shown in Fig. 2.21.

![Image](image_url)

Figure 2.21: A near-zone image of the top facet of a 1x2 implant array with two fringes.

As predicted and previously observed in other anti-guided VCSEL arrays
the mode at threshold changes with the separation between the two array elements. The results are summarized in Table 2.1, which notes the number of central lobes between the dominant lobes of the mode profile. The predicted behavior matches closely with the experimental observations. One exception is the 14 µm separation, where three lobes are predicted but only two are observed. Referring to Fig. 2.20, it is evident that at this separation the modal gain discrimination is very low, so small variations in the assumed parameters could explain the disagreement.

<table>
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<tr>
<th>Separation</th>
<th>Experiment</th>
<th>Theory</th>
<th>Separation</th>
<th>Experiment</th>
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</tr>
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<td>1</td>
<td>12 µm</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>12.5 µm</td>
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<td>2</td>
<td>14.5 µm</td>
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</tbody>
</table>

In addition to the good agreement with the modal trends in Table 2.1, there is also good qualitative agreement between the profiles of the calculated and measured array modes. Figure 2.22 shows the two measured modes for separations of 9 µm and 11 µm. Comparing these profiles with those in the insets of Fig. 2.20 reveals that the modes have nearly identical features. In particular, in both cases there are two major lobes that are contained within the implant apertures. Between these lobes, there are either one or two subsidiary lobes that are nearly an order of magnitude lower in intensity. For an even number of subsidiary lobes, such as observed with 11 µm separation, the far field shows an out-of-phase profile, while for an odd number of lobes, such as observed with 9 µm, an in-phase far-field pattern is observed. This is illustrated in Fig. 2.23 where the far-field intensity profiles are shown for separations of 10 µm, 11 µm, and 14.5 µm. The transition from in-phase to out-of-phase and back to in-phase with increasing separation is apparent, which is characteristic of anti-guided arrays [45].
Figure 2.22: The measured near-zone modes of two laser arrays with separations of 9 µm (solid) and 11 µm (dashed). The change in the number of central fringes is characteristic of anti-guided arrays.

Figure 2.23: The far-field intensity profiles for separations of (a) 10 µm, (b) 11 µm, and (c) 14.5 µm.

2.4.4 Analysis of Larger Arrays

In order to reach higher power and lower beam divergence, it is necessary to fabricate larger laser arrays. In the analysis of larger arrays, approximate separability again is assumed, and therefore only one-dimensional arrays are considered. Since the solution to the larger array problem can be expressed as a superposition of solutions for individual elements, it is expected that the overlap of temperature profiles being maximum near the center of the array will result in the highest temperature being localized in the center elements. If the temperature gradient from the center to the edge is great enough, an array with uniform aperture sizes will unfortunately have a non-uniform index distribution. As a consequence, a resonant array mode across
the extent of the array cannot be realized.

Figure 2.24 shows the index profile and favored mode of a uniformly sized four-element array. As expected, the index profile is not uniform across the array. As a result, the favored mode only has significant field intensity in the two center apertures. This suggests that it would be expected that in an experiment the outer elements either will not lase or, if all elements turn on, the laser will operate in multiple modes.

\[\text{Figure 2.24: The index profile (dashed) and corresponding mode (solid) of a four-element array with uniform aperture sizes.}\]

To circumvent this problem, the array elements can be adjusted in size to compensate for the non-uniform heating effects. In this example, it is assumed that the dissipated heat and carrier concentration are the same for all array elements. By reducing the size of the inner elements, the temperature profile is made more uniform, and, resultantly, the index profile is nearly uniform across the array. Figure 2.25 shows the index profile of an array with outer elements larger than the inner elements. For comparison, the index profile of a resonant array design found using the transfer matrix approach is shown. The step-index resonant array design is well approximated by the adjusted thermal- and carrier-influenced index profile.

The favored modes for these index profiles are shown in Fig. 2.26. The ideal resonant mode is shown as a dashed line, and, as expected, there is uniform field intensity distributed amongst all elements of the array. The mode found for the thermal and carrier model is very similar to the ideal case. Deviations from the ideal are likely a result of the non-step-like transitions in
refractive index, particularly at the cladding layers. The cladding index shift could be made more step-like by introducing a large, stable index variation using etched features, such as a photonic crystal. Nevertheless, the mode approximates the resonant array mode well, and therefore the associated implant array design represents one that shows promise for making a four-element, single-mode laser array.

This is one approach for overcoming the problems of non-uniform heating
in larger ion-implanted VCSEL arrays. The method should be extendable to even larger array sizes, requiring only that the thermal and carrier model be utilized to find an appropriate design. Other means for avoiding the heating problems also exist. One method could be to bond the array to a heat sink that draws away enough heat from all the elements to make only carrier effects significant. Another method might be to engineer the thermal profile by changing the thermal conductivity in certain areas using etched features. Other means could be to help eliminate the heating by using a pulsed current source to drive the lasers or reducing the electrical resistance with different device or epitaxial structures. Regardless of what approach is taken, it is evident that appropriate heat management must be performed in order to realize a large, single-mode VCSEL array.

2.5 Conclusion

A lossy model for photonic crystal waveguides incorporated in VCSELs has been developed. Loss can have a significant role in the modal characteristics of etched photonic crystal VCSELs and, in fact, can be a primary mechanism for maintaining single-mode operation. Optical loss values for the photonic crystal are extracted from the transverse mode splitting of fabricated devices. Comparisons of these modal losses with higher-order mode splitting, slope efficiency, and single- or multimode operation serve to verify the model. This semi-empirical approach is useful for demonstrating the potential significance of photonic crystals for providing mode discrimination needed in coherent laser arrays.

Two models have been developed for modeling such arrays. The first model uses a transfer matrix approach to design ideal anti-guided arrays. Simple design rules have been derived using this approach. The model also has been shown to be useful for solving for all the anti-guided modes of a design. Moreover, the confinement factors of arrays designed using the transfer matrix approach can be found. Thus, the transfer matrix approach is useful as a design guide for making single-mode arrays.

The second model considers the effects of heat diffusion and carrier concentration on the index profile of arrays. The model illustrates the origin of index anti-guiding and demonstrates that the anti-guiding can be stable for a
variety of designs and over a wide range of operating conditions. Experimental evidence agrees well with the theoretical treatment and serves to verify the anti-guiding hypothesis. It is also demonstrated that careful design must be carried out in order to maintain single-mode operation. For larger arrays, it is shown that non-uniform heating can be problematic. Potential solutions have been presented for making a uniform thermal profile or eliminating the heat sources.
CHAPTER 3

FABRICATION OF IMPLANTED VCSEL ARRAYS

3.1 Introduction

Methods for fabricating ion-implanted and photonic crystal VCSELs have been well known for many years [34, 52]. Additionally, photonic crystal VCSEL arrays [13, 15, 16] and ion-implanted VCSEL arrays [17] have been successfully demonstrated. However, prior to this work, VCSEL arrays incorporating both a photonic crystal and ion implantation had not been demonstrated.

This chapter will describe the design and fabrication of top-emitting ion-implanted VCSEL arrays with a photonic crystal. The different array designs and their purposes will be explained. The various methods of fabricating the arrays will also be presented. Finally, the remaining challenges to improving and perfecting the fabrication process will be explored.

3.2 Mask Designs

In the course of this work, several photolithography masks incorporating many different array designs have been created. In all cases, four mask levels are required for a complete device fabrication: top metal contacts, photonic crystal patterns, implant apertures, and device isolation structures. Designs exist both for target applications, such as large, high-power arrays and beam steering, and for empirical probing of array behavior, such as coupling and coherence properties. The most significant examples of PhC VCSEL array designs are shown in Fig. 3.1. In this figure, the top metal is shown in yellow, the photonic crystal is shown as white circles, and the unimplanted laser apertures are shown in blue. The isolation structure is not shown,
but is a pattern that covers the entire array and has the same shape as the perimeter of the metal.

Figure 3.1: Examples of PhC VCSEL designs: (a) 1x2 array on a hexagonal lattice, (b) 1x2 array design for continuously varying aperture separation, (c) 2x2 array on a square lattice, (d) 2x2 array with separated contacts for beam steering experiments, (e) 3x3 array with larger aperture size for the center element, and (f) 3x3 array with metal runners.

The top metal is patterned into a ring contact or a variation on one, as can be seen in Fig. 3.1. The variations on the original ring contact design are necessary for larger arrays and beam steering devices. In the case of arrays larger than 2x2, lateral resistance causes current to be favorably injected into the outer array elements, which is similar to what has been observed in large-area implanted VCSELs [54]. In order to circumvent this problem, metal runners, such as those illustrated in Fig. 3.1(f), are incorporated. For beam steering arrays, current must be injected independently into the different array elements in order to adjust relative phase. Thus, separations in the top contact, as seen in Fig. 3.1(d), must exist. Although this can be done post-processing using a focused ion beam (FIB), it is much simpler to include the separations in the top contact mask layer. Note that most
VCSEL structures have a highly conductive top epitaxial layer, so to achieve adequate isolation the separation must also be included in the device isolation mask so the top layer can be selectively etched away.

The photonic crystal designs are based on research performed on single-element PhC VCSELs [38, 40, 41]. Photonic crystals that were shown to produce single-mode VCSELs with one element are used in this work on arrays of laser elements. The designs include both square and hexagonal lattices, and the holes are on the order of 1 µm or slightly larger in diameter with a diameter-to-period ratio of 0.6 or 0.7. These dimensions allow for optical photolithographic patterning. The holes nearest the implant apertures are also varied in size in order to adjust and investigate the effects of the photonic crystal. Representative designs of different lattices and hole sizes are shown in Figs. 3.1(a) and 3.1(c). Based on the evidence of previous work [40], the photonic crystal is introduced to provide stable index guiding and selective loss to undesired modes. Thus, the photonic crystal is designed to help maintain single-mode, highly coherent operation of the VCSEL array.

The implant apertures are designed to be either circular or square for hexagonal or square photonic crystal lattices, respectively. The aperture sizes are made to be approximately the same as those that produced single-mode operation in single-element VCSELs. Typically, the apertures are between 3 µm to 9 µm in diameter or edge length. Appropriate separation between apertures is critical to ensure the desired array mode turns on. The designed edge-to-edge separations for most devices is between 1 µm and 3 µm, although smaller and larger separations are used in special designs, such as that in Fig. 3.1(b). In some cases with arrays larger than 2x2 elements, the center apertures are larger than the outer ones (as in Fig. 3.1(e)) in order to help with non-uniform current injection. This was found to be unsuccessful, and the use of metal runners is a much better solution for uniform current injection in top-emitting arrays.

Finally, the isolation structures are used to define electrical isolation between different laser arrays. The isolation can be performed either using an etch or implantation, and the mask is useful for both approaches. The design is simply a structure that covers the entirety of each array. There are no variations of this design, nor is there any expected effect on performance from any variation.
3.3 Fabrication

A cross-sectional sketch and a top view of a 1x2 photonic crystal implant-defined VCSEL array are shown in Fig. 3.2. For the majority of the lasers studied there are 27 p-type top distributed Bragg reflector (DBR) periods and 35 n-type bottom DBR periods on an n-type GaAs substrate. Between the top and bottom DBR mirrors are three GaAs quantum wells that emit nominally at 850 nm. Fabrication begins with n-type backside metal (AuGe/Ni/Au) deposition on the substrate. Following this, a number of fabrication processes can be followed depending on the masks used and desired device designs. The typical fabrication procedure is illustrated in Fig. 3.3, which shows the essential aspects of VCSEL array fabrication. A process follower is shown in the appendix.

Figure 3.2: (a) Cross-sectional sketch of a photonic crystal implant-defined VCSEL array and (b) a top-view near field of a 1x2 array.

A flowchart detailing all the employed fabrication steps and sequences is shown in Fig. 3.4. There are two types of arrays that must be fabricated in different manners: those with and without an overlap between the top metal and the photonic crystal. When there is an overlap between the two, the photonic crystal must be etched prior to top metal deposition, since the etching step cannot penetrate metal. In this case, only the metal alignment marks are deposited, followed by the etch. However, when there is no overlap, it does not significantly matter in which order the metal deposition and the etch are performed. Therefore, because it is easiest to deposit the metal alignment marks at the same time as the top contacts since they are on the same mask, metal deposition is completed first. If there is no photonic crystal
at all, aperture implant is immediately done after the metal deposition and
device isolation can be achieved using either an etch or implantation. If the
photonic crystal is incorporated, the etch is performed, followed by aperture
and isolation implantation steps.

For the top metal step, p-type metal (Ti/Au) is deposited on patterned
photoresist. After the metal is deposited, acetone is used to remove the pho-
toresist and thereby lift off the unwanted metal. An image of the deposited
top contact prior to any further fabrication is shown in Fig. 3.5.

Using plasma-enhanced chemical vapor deposition (PECVD), SiO$_2$ is de-
posited on the top surface to be used as an etch mask. The photonic crystal
patterns are defined using photolithography and transferred into the SiO$_2$ us-
ing a freon reactive ion etch (RIE) with CF$_4$. Then, an inductively coupled plasma reactive-ion etch (ICP-RIE) with SiCl$_4$ and Ar gas is used to etch the photonic crystal patterns into the top DBR, where the target depth of the photonic crystal holes is approximately 75% through the top mirror. Figure 3.6 shows top and angled views from a scanning electron microscope (SEM) of a photonic crystal hole. The variations in color in the angled view along the sidewall of the hole result from the variations in material composition of the periods of the DBR. From this, the depth of the photonic crystal can be estimated. A rough sidewall near the top of the hole resulting from mask erosion is also evident. However, since the holes primarily serve to stabilize the modes, the sidewall roughness is not of significant concern.

The gain apertures of the array are patterned using thick photoresist and defined by proton implantation performed at a 7° tilt and a dose and energy of $4 \times 10^{14}$ cm$^{-2}$ and 340 keV, respectively. A Transport of Ions in Matter (TRIM) simulation (using the SRIM, Stopping and Range of Ions in Matter, software package [55]) of the ion concentration profile using these parameters in GaAs is shown in Fig. 3.7. The implant photolithography is performed after the photonic crystal etch so that the implant apertures can be aligned to the photonic crystal apertures. An example SEM image of the photoresist mask used for the aperture implantation is shown in Fig. 3.8. In order to block the ions, the photoresist must be over 6.5 µm thick (as indicated in Fig. 3.7), and typically thicker resist is desired for easier removal after
Figure 3.6: (a) SEM top view and (b) angled view of an etched PhC hole. The layers of the top DBR are apparent in the angled view.

implantation. This means that a very high aspect ratio is needed for the implant aperture designs in this work, which is evident in Fig. 3.8.

Figure 3.7: Concentration profiles for ion implantation at a 7° tilt, a dose of $4 \times 10^{14}$ cm$^{-2}$, and energy of 340 keV in different materials.

After the aperture implantation, additional fabrication steps are performed to electrically isolate the devices from each other. In some cases, another thick resist step is performed and a multiple-implantation of protons is performed. Then, a short ICP-RIE step is utilized to etch through the high-conductivity top contact layer. In other cases, a long ICP-RIE step is used to etch a mesa to a depth beyond the active region. As another alternative, a photoresist mask and an isotropic chemical etch in a hydrogen peroxide
Figure 3.8: (a) SEM top view and (b) angled view of the photoresist implant mask for a 1x2 array.

and phosphoric acid solution can be used. Finally, the SiO$_2$ is removed using freon RIE.

### 3.4 Challenges

The primary challenge for fabricating photonic crystal implant-defined VCSEL arrays is in producing implant apertures of uniform size and shape. For the array elements to coherently couple, the individual waveguides must be nearly identical. Since thick photoresist is used as a mask, it is difficult to realize this type of uniformity, especially for large arrays. For example, the SEM in Fig. 3.8 shows that the photoresist pillars are somewhat cone-shaped and of different shapes and sizes. These variations could lead to poor coupling.

A suggestion for circumventing this problem is to use a different implant mask material. Using either silicon nitride or gold as a mask can significantly reduce the needed mask thickness (as shown in Fig. 3.7), and therefore decrease the aspect ratio. Based on TRIM calculations, for the implantation parameters listed in the previous section, approximately 2.3 μm of nitride or 1.9 μm of gold is needed to block the ions. Moreover, these materials can be patterned using either anisotropic reactive ion etching or electroplating. Thus, the thinner masks and different materials could make it easier to create pillars with vertical sidewalls and consistent geometries. The result could be the improved uniformity necessary to fabricate large coherent arrays.
3.5 Conclusion

The design and fabrication of photonic crystal implant-defined VCSEL arrays have been described. Various masks have been designed both for demonstration of applications and empirical studies of array behavior. Fabrication can follow several different procedures, but all require the four steps of contact metal deposition, photonic crystal etching, aperture implant definition, and device isolation. Finally, challenges to perfecting the fabrication have been explained and suggestions for improvement have been made.
CHAPTER 4

COHERENCE THEORY OF VCSEL ARRAYS

4.1 Introduction

To analyze semiconductor laser array properties, a variety of approaches have been taken based on the manner of the coupling [23, 56, 57]. For evanescently coupled lasers, coupled mode theory has been a valuable tool and has successfully predicted the observed spatial modes [21, 22]. However, VCSEL arrays have demonstrated partially coherent behavior that is not explained within the deterministic coupled mode theory [58]. To address this issue, a stochastic harmonic oscillator model was recently developed [27]. Although this model well describes partially coherent arrays, it is unable to predict, \textit{ab initio}, the spectra and coupling strength.

A stochastic coupled mode formalism is developed below [59, 60]. The coherence matrices in the time and frequency domains are derived. This new model predicts the observed partially coherent operation of the array as well as the spectra and coupling strength obtained from deterministic coupled mode theory. The coherence of implant-defined VCSEL arrays is analyzed using this theory. A theoretical analog to describe approximately the leaky-mode behavior of the coherent VCSEL arrays is developed. This model is based on the observed near-field intensity of the array modes. Using this with the stochastic theory, the array coherence is predicted. A far-field intensity profiler and an optical tabletop imaging spectrometer are used to experimentally characterize the properties of 1x2 implant-defined VCSEL arrays. These measurements are in agreement with the theory and reveal properties of the optical coupling. This analysis is valuable for the design of coherently coupled VCSEL arrays.
4.2 Formulation

In this work, the laser array is modeled as a pair of coupled waveguides terminated by the top and bottom reflectors forming a simple Fabry-Perot cavity. A schematic representation of the problem under investigation is shown in Fig. 4.1. This formulation is applicable to a wide variety of devices such as VCSEL arrays, edge-emitting laser arrays, and laser amplifier arrays. For VCSELs, an effective mirror model can be used to reduce the structure to the one investigated here [61].

Figure 4.1: A sketch of the Fabry-Perot cavity and waveguide array system under investigation.

Following the analysis given in [47], the field solutions for two coupled waveguides, \( a \) and \( b \), are given by

\[
U(x, y, z) = a(z)U^{(a)}(x, y) + b(z)U^{(b)}(x, y),
\]

where \( U^{(a)} \) and \( U^{(b)} \) are the unperturbed transverse mode profiles of the waveguides with complex propagation constants \( \beta_a \) and \( \beta_b \), respectively, and all quantities should be understood to depend on frequency. In [62], an approximate theory of coupled Fabry-Perot devices is derived, giving the transmitted field at the output mirror located at \( z = z_0 \):

\[
\begin{bmatrix}
a(z_0) \\
b(z_0)
\end{bmatrix} = \mathbf{VFV}^{-1} \begin{bmatrix}
a(0) \\
b(0)
\end{bmatrix},
\]

(4.2)
where

\[
\mathbf{V} = \begin{bmatrix}
K & K \\
\Delta \beta + \psi & \Delta \beta - \psi
\end{bmatrix},
\]

and

\[
\mathbf{F} = \begin{bmatrix}
\xi_+(\omega) & 0 \\
0 & \xi_-(\omega)
\end{bmatrix}.
\]

(4.3) (4.4)

The terms used here are defined in Table 4.1, and all have dimensions of inverse length.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_{a,b})</td>
<td></td>
<td>Propagation constant in (a, b)</td>
</tr>
<tr>
<td>(K)</td>
<td></td>
<td>Coupling strength</td>
</tr>
<tr>
<td>(\bar{\beta})</td>
<td>(\frac{\beta_+ + \beta_-}{2})</td>
<td>Average propagation constant</td>
</tr>
<tr>
<td>(\Delta \beta)</td>
<td>(\frac{\beta_+ - \beta_-}{2})</td>
<td>Propagation constant difference</td>
</tr>
<tr>
<td>(\psi)</td>
<td>([\Delta \beta^2 +</td>
<td>K</td>
</tr>
<tr>
<td>(\beta_{\pm})</td>
<td>(\bar{\beta} \pm \psi)</td>
<td>Propagation constant of (\pm) mode</td>
</tr>
</tbody>
</table>

Table 4.1: Definitions of terms (units: \(\mu\text{m}^{-1}\))

The diagonal terms of \(\mathbf{F}\) are the responses of the Fabry-Perot cavity for the propagation constants, \(\beta_+\) and \(\beta_-\), of the \(+\) and \(-\) mode solutions of the coupled eigenvalue problem. The model does not account for effects of gain saturation, mode competition, gain clamping, etc. Assuming that these effects manifest themselves simply as changes in the overall spectral intensities of the two coupled modes, the mode expressions for a cavity of length \(L\) with mirrors of amplitude reflectivity \(R_1 = R_2 = R\) are

\[
\xi_+(\omega) = \sigma_+ \frac{(1 - R^2)e^{-i\beta_+ L}}{1 - R^2e^{-i2\beta_+ L}},
\]

(4.5)

\[
\xi_-(\omega) = \sigma_- \frac{(1 - R^2)e^{-i\beta_- L}}{1 - R^2e^{-i2\beta_- L}}.
\]

It also is assumed that the propagation constants and coupling constant vary linearly with frequency:

\[
\beta_p = \frac{\omega}{c} n_p,
\]

(4.6)

\[
K = \frac{\omega}{c} \kappa.
\]

(4.7)
where the subscript \( p \) is \( a, b, +, \) or \( - \); \( n_p \) is the frequency-independent effective refractive index; \( \kappa \) is a unitless coupling term; \( c \) is the vacuum speed of light. The first restriction put on the propagation constants assumes that the waveguides are operating in a linear regime of the dispersion curve. Additionally, in general, \( \kappa \) is frequency-dependent, but it is assumed that, over the narrow frequency range of interest, this dependence is negligible and \( \kappa \) may be assumed to be constant.

In terms of the variables defined in Table 4.2, the matrix \( \mathbf{V} \) is given by

\[
\mathbf{V} = \begin{pmatrix}
\kappa & \kappa \\
\Delta n + \eta & \Delta n - \eta
\end{pmatrix},
\]

while the matrix \( \mathbf{F} \) remains unchanged. Since the lasers of interest in this work have only a single longitudinal mode, the expressions of Eqn. 4.5 are simplified to Lorentzians:

\[
\xi_+(\omega) = \frac{\alpha}{\alpha + i(\omega - \omega_+)};
\]

\[
\xi_-(\omega) = \frac{\alpha}{\alpha + i(\omega - \omega_-)},
\]

where \( \omega_+ \) and \( \omega_- \) are the resonances of the two modes and \( \alpha \) is the total cavity loss.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{a,b} )</td>
<td></td>
<td>Effective index of ( a, b )</td>
</tr>
<tr>
<td>( \kappa )</td>
<td></td>
<td>Coupling strength</td>
</tr>
<tr>
<td>( \bar{n} )</td>
<td>( \frac{n_b + n_a}{2} )</td>
<td>Average effective index</td>
</tr>
<tr>
<td>( \Delta n )</td>
<td>( \frac{n_b - n_a}{2} )</td>
<td>Effective index difference</td>
</tr>
<tr>
<td>( \eta )</td>
<td>( [\Delta n^2 +</td>
<td>\kappa</td>
</tr>
<tr>
<td>( n_{\pm} )</td>
<td>( \bar{n} \pm \eta )</td>
<td>Effective index of ( \pm ) mode</td>
</tr>
<tr>
<td>( \omega_{\pm} )</td>
<td>( \frac{N\pi c}{n_{\pm} L} )</td>
<td>Resonance of ( \pm ) mode ( (N \text{ integer}) )</td>
</tr>
</tbody>
</table>

The theory presented in the following sections is general and does not depend on the specific form of \( \xi_+ \) and \( \xi_- \). These assumptions are made for clarity of presentation, i.e. an explicit frequency dependence of the propagation constants is necessary for numerical calculations.
4.2.1 Deterministic Analysis

Before addressing the stochastic problem, it is illustrative to review the deterministic coupled mode problem. Explicitly carrying out the matrix and vector operations of Eqn. 4.2 yields the solution

\[
\begin{bmatrix}
a(z_0) \\
b(z_0)
\end{bmatrix} = \frac{1}{2\eta} \left[ \begin{array}{c} A_a^a a(0) + A_b^b b(0) \\ A_a^b a(0) + A_b^a b(0) \end{array} \right],
\]

(4.10)

where

\[
\begin{align*}
A_a^a & = \xi_+(\omega)(\eta - \Delta n) + \xi_-(\omega)(\eta + \Delta n), \\
A_a^b & = \xi_+(\omega)\kappa - \xi_-(\omega)\kappa, \\
A_b^a & = \xi_+(\omega)\kappa^* - \xi_-(\omega)\kappa^*, \\
A_b^b & = \xi_+(\omega)(\eta + \Delta n) + \xi_-(\omega)(\eta - \Delta n).
\end{align*}
\]

(4.11)

For comparison to the stochastic case, it is useful to calculate the products of the amplitudes, \( a \) and \( b \). For an ergodic, stationary, random field, these products form the cross-spectral density matrix, the diagonal elements being the power spectral densities. The cross-spectral density is simply related by Fourier transform to the time domain correlation and cross-correlation functions, i.e. the Wiener-Khintchine-Einstein theorem [63–65] applies. In computing the deterministic analogue of the cross-spectral density, some care must be taken. No simple relationship exists between the products of the coefficients of the fields in the frequency domain at a single frequency and the products of coefficients of the fields in the time domain. With this caveat, we refer to the matrix of Hermitian products of coefficients as the deterministic cross-spectral density. The deterministic cross-spectral density matrix of the output field is given by

\[
W = \begin{bmatrix}
\alpha^*(z_0) a(z_0) & a^*(z_0) b(z_0) \\
b^*(z_0) a(z_0) & b^*(z_0) b(z_0)
\end{bmatrix}
\]

(4.12)

\[
= \frac{1}{|2\eta|^2} \begin{bmatrix}
W_{aa} & W_{ab} \\
W_{ba} & W_{bb}
\end{bmatrix}.
\]

Solving for the matrix in Eqn. 4.12, it is found that the off-diagonal element
is expressible in terms of the diagonal terms, viz.

\[ W_{ab} = [W_{aa}W_{bb}]^{1/2} e^{i\phi}, \]  

(4.13)

where \( \phi \) is real, and therefore the magnitude of the spectral degree of coherence, defined as

\[ |\mu(\omega)| = \left| \frac{W_{ab}}{[W_{aa}W_{bb}]^{1/2}} \right|, \]  

(4.14)

is found to be equal to unity, implying that the fields in the two waveguides are completely coherent with each other as must be expected.

Moreover, the coherent mode decomposition is computed via the eigenvalue problem

\[ \mathbf{W} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \lambda \begin{bmatrix} V_a \\ V_b \end{bmatrix}. \]  

(4.15)

There is only one coherent mode with a nonzero eigenvalue, given by

\[ \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \frac{1}{2\eta^*} \begin{bmatrix} W_{aa}^{1/2} e^{i\phi} \\ W_{bb}^{1/2} \end{bmatrix} = \frac{1}{2\eta^*} \begin{bmatrix} A_a^b a(0) + A_b^b b(0) \\ A_b^a a(0) + A_a^b b(0) \end{bmatrix}^*, \]  

(4.16)

with eigenvalue \((W_{aa} + W_{bb})/(2|\eta|^2)\), as one would expect for a spectrally fully coherent field. Again, some care must be taken in the comparison here as the deterministic field is not statistically stationary.

### 4.2.2 Coupled Modes from Incoherent Sources

In order to treat stochastic fields, the boundary conditions are taken to be random, stationary, and ergodic. Moreover, it is supposed that the boundary
fields in waveguide $a$ and waveguide $b$ are uncorrelated,

\begin{align}
\langle a^*(0)a(0) \rangle &= S_a^{(0)}, \\
\langle b^*(0)b(0) \rangle &= S_b^{(0)}, \\
\langle a^*(0)b(0) \rangle &= 0.
\end{align}

The modal cross-correlation at the output mirror is defined by the matrix

\begin{equation}
W = \begin{bmatrix}
\langle a^*(z_0)a(z_0) \rangle & \langle a^*(z_0)b(z_0) \rangle \\
\langle b^*(z_0)a(z_0) \rangle & \langle b^*(z_0)b(z_0) \rangle 
\end{bmatrix}
\tag{4.18}
\end{equation}

In terms of the definitions given in Eqns. 4.11 and 4.17, the elements of the cross-correlation matrix are

\begin{align}
W_{aa} &= |A_a|^2 S_a^{(0)} + |A_b|^2 S_b^{(0)}, \\
W_{bb} &= |A_a|^2 S_a^{(0)} + |A_b|^2 S_b^{(0)}, \\
W_{ab} &= A_a^a A_b^a S_a^{(0)} + A_a^b A_b^b S_b^{(0)}.
\end{align}

One can solve for the coherent modes of this cross-correlation matrix, although a general expression is particularly complicated. However, two limiting cases can provide insight: when the two spectral terms ($\xi_+$ and $\xi_-$) are equal and when only one spectral term is nonzero. For $\xi_+(\omega) = \xi_-(\omega) = \xi(\omega)$, the coherent modes are given by

\begin{align}
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix} &= \begin{bmatrix} 1 \\
0
\end{bmatrix}, \\
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix} &= \begin{bmatrix} 0 \\
1
\end{bmatrix},
\end{align}

with eigenvalues $|\xi(\omega)|^2 S_a^{(0)}$ and $|\xi(\omega)|^2 S_b^{(0)}$, respectively. It can be seen here that there are two coherent modes, the intensity of each localized in one waveguide or the other. Additionally, the magnitude of the spectral degree of coherence is zero since $W_{ab} = 0$.

In the limit where $\xi_+(\omega) = 0$, there is only one mode with a nonzero
eigenvalue,

\[
\begin{bmatrix}
V_a \\
V_b
\end{bmatrix} = \begin{bmatrix}
\kappa \\
\Delta n \pm \eta
\end{bmatrix}^*,
\]

with the eigenvalue \((W_{aa} + W_{bb})/|2\eta|^2\). Respectively, these two modes can be identified as the + and − modes as defined in [47]. Since these two cases represent single-mode states, the magnitude of the complex spectral degree of coherence is unity.

Thus, in these two limiting case, the two extremes of complete coherence and incoherence are observed. For conditions lying between these two limits, partial spectral coherence can be observed. Therefore, the stochastic coupled mode formalism is capable of predicting partially coherent behavior that was previously inaccessible through deterministic methods.

As a final note, it is important to point out that results using deterministic theory can be recovered using the stochastic theory. That is, when the random seeding fields of the stochastic theory, \(a(0)\) and \(b(0)\), are completely mutually coherent, the output is single-mode and the component mode amplitudes are the same as the deterministic field amplitudes. Thus, the two theories agree for the calculation of observables dependent on the mutual coherence such as the interference pattern produced in the far zone. However, with the stochastic theory, the fields are stationary and ergodic, so it is possible to compute quantities that depend not just on the mutual coherence, but the degree of coherence such as the power spectra or the autocorrelation. Thus, the stochastic coupled mode theory is capable of making predictions that are in agreement with previously investigated deterministic approaches as well as ones that are inaccessible to the deterministic approach.

4.2.3 Numerical Spectral Analysis of Coupled Lasers

The results of the previous sections can best be illustrated through numerical calculations. Two particular cases are treated: symmetric and asymmetric waveguides. Calculations are performed for both deterministic and random boundary conditions to demonstrate the significance of the statistical nature of the fields seeding the coupled system.

The device being modeled here is a two-element VCSEL array. The pa-
arameters used for the calculations are presented in Table 4.3. The VCSELs are assumed to operate at 850 nm (angular frequency of $2.218 \times 10^{15}$ rad/sec) and have a full-width at half-maximum linewidth of approximately 0.95 nm ($2.4 \times 10^{12}$ rad/sec). This corresponds to VCSELs with cavity loss of about 40 cm$^{-1}$. This value of loss is larger than a typical value for a VCSEL, but it is used primarily for illustration.

Table 4.3: Parameters used for symmetric calculation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{a,b}$</td>
<td>Effective index of $a$, $b$</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Coupling strength</td>
<td>$5 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Cavity loss</td>
<td>$1.2 \times 10^{12}$</td>
<td>rad/sec</td>
</tr>
<tr>
<td>$L$</td>
<td>Cavity length</td>
<td>0.243</td>
<td>(\mu m)</td>
</tr>
</tbody>
</table>

The deterministic boundary conditions are $a(0) = b(0) = 1$, and the random conditions are specified by $\langle a^*(0)a(0) \rangle = \langle b^*(0)b(0) \rangle = 1$ and $\langle a^*(0)b(0) \rangle = 0$. In other words, the sources seeding the two waveguides are of equal intensity. Figure 4.2 shows the unperturbed spectra for the two guides when no coupling is present. Note that since the waveguides are symmetric, the spectra for guides $a$ and $b$ are identical.

![Figure 4.2: The unperturbed power spectra for no detuning between the propagation constants of guides $a$ and $b$. The spectrum from $a$ is shown with a solid line, and the spectrum from $b$ is shown with a dashed line. The spectra are identical.](image)

The spectra for guides $a$ and $b$, respectively represented by $W_{aa}$ and $W_{bb}$,
with deterministic boundary conditions are shown in Fig. 4.3(a). In this plot, it can be seen that the spectra from the two guides are identical. Moreover, it is apparent from the single peak that only one coupled mode is excited, specifically the $+$ mode. The coupling between the guides causes a frequency shift of the modes, which is apparent in Fig. 4.3(a) since the peak is at a lower frequency than in Fig. 4.2. This illustrates that, for the deterministic problem, the boundary conditions entirely determine the mode or admixture of modes that is excited, just as here $a(0) = b(0)$ excites the $+$ mode.

From the analysis of the previous section, it is expected that both coupled modes will turn on with equal intensity when stochastic boundary conditions are used. Figure 4.3(b) shows the power spectra for guides $a$ and $b$ in the case that the boundary conditions are random. Again, the spectra from the two guides are identical. However, as a result of the stochastic boundary conditions, now both the $+$ and $-$ modes are excited. Thus, unlike the deterministic case, the random boundary conditions equally excite both modes. This implies that one would expect both modes to turn on in a symmetric, coupled laser array (note that this analysis neglects mode competition, gain saturation, hole burning, etc. which would impose asymmetry in the array). In typical experiments, one coupled mode is preferentially excited and this mode usually dominates.

![Figure 4.3](image)

**Figure 4.3:** The (a) deterministic and (b) stochastic coupled power spectra for no detuning between the propagation constants of guides $a$ and $b$. The spectrum from $a$ is shown with a solid line, and the spectrum from $b$ is shown with a dashed line. The spectra from the two guides exactly overlap.

As mentioned above, the deterministic approach cannot predict partial
spectral coherence regardless of the spectra or the coupling strength. However, this is not the case for stochastic boundary conditions, as is illustrated by Fig. 4.4. In this figure, the maximum spectral degree of coherence is plotted as a function of the coupling strength, $\kappa$. The frequency of maximum coherence changes with the value of $\kappa$, but it is typically at or near the resonances of the $+$ and $-$ modes. The plot shows that the maximum spectral degree of coherence increases as the coupling strength increases. This comes as a result of a decrease in the overlap between the lineshapes of the two modes.

![Figure 4.4: Maximum complex degree of coherence plotted as a function of the coupling strength $\kappa$ for random boundary conditions and no detuning.](image)

For the asymmetric calculations, the same definitions as in Table 4.3 are used except that $n_b = 3.495$. This example represents a two-element VCSEL array with some asymmetry between the array waveguides, such as a difference in aperture geometry or core index. Figure 4.5 shows the unperturbed spectra of the two uncoupled guides. As a result of the asymmetry, there is a noticeable splitting between the unperturbed resonances of the two guides.

The coupled spectra for deterministic boundary conditions are shown in Fig. 4.6(a). In this case, there is one mode that is dominant, again the $+$ mode. However, it is apparent that there is some power in the $-$ mode. This comes as a result of the detuning altering the $+$ and $-$ modes such that the boundary conditions excite an admixture of them. Despite this, the admixture represents a single coherent mode, and the spectral degree of coherence remains unity. Thus, the deterministic problem is shown to not
Figure 4.5: The unperturbed power spectra for a small detuning between the propagation constants of guides $a$ and $b$. The spectrum from $a$ is shown with a solid line, and the spectrum from $b$ is shown with a dashed line.

allow for any partial spectral coherence between the fields of the two guides, even when more than one mode is present.

The random boundary conditions again provide an equal total excitation of the $+$ and $-$ modes as shown in Fig. 4.6(b). Now, however, the $+$ mode is more localized in guide $a$, and the $-$ mode is more present in guide $b$. Thus, random boundary conditions still cause both modes to turn on with equal total intensity, but the detuning serves to redistribute the modal power between the guides. In experiments, this redistribution of modal power can break the symmetry that allows mode competition to select only one mode, and thus two incoherent modes can simultaneously lase.

Considering again the spectral degree of coherence for random boundary conditions, it is found that the trend is very similar to that seen in Fig. 4.4. The increase in coherence with $\kappa$ is slightly slower with detuning, which suggests that the coupling strength has more influence on the degree of coherence than the detuning.

4.2.4 Time-Domain Analysis of Stochastic Coupled Lasers

Photodetectors provide a signal that is proportional to a time integral of the intensity falling on the detector. Therefore, the time-domain correlation functions are of primary importance. The time-domain correlation matrix is found by taking the Fourier transform of the frequency-domain matrix and
Figure 4.6: The (a) deterministic and (b) stochastic coupled power spectra for a small detuning between the propagation constants of guides $a$ and $b$ and $\kappa = 0.005$. The spectrum from $a$ is shown with a solid line, and the spectrum from $b$ is shown with a dashed line.

is expressed

$$\Gamma = \frac{1}{|2\eta|^2} \alpha \left( \frac{\pi}{2} \right)^{1/2} \begin{bmatrix} \Gamma_{aa} & \Gamma_{ab} \\ \Gamma_{ba} & \Gamma_{bb} \end{bmatrix},$$

where the $\Gamma$ terms are the time-domain autocorrelations and cross-correlations.

In order to directly measure the time-domain cross-correlations, it is assumed that a pinhole is placed at the output facet over each waveguide such that the single pinhole emissions are $\Gamma_{aa}(0)$ and $\Gamma_{bb}(0)$ from guides $a$ and $b$, respectively. The far-field intensity produced by two pinholes is then (up to a multiplicative factor) [66]

$$I_{FF} = \Gamma_{aa}(0) + \Gamma_{bb}(0) + 2 \Re \{ \Gamma_{ab}(\tau) \},$$

where $\tau$ is the time offset between the signals from the two pinholes.

If the seeding fields are of equal intensity ($S_a^{(0)} = S_b^{(0)} = S^{(0)}$) and there is equal gain or loss in the two guides ($\Delta n$ is real), the far field then becomes

$$I_{FF} = I_a^+ + I_b^+ + I_a^- + I_b^- + 2 \left[ I_a^+ I_b^+ \right]^{1/2} e^{-\alpha|\tau|} \cos(\omega_+ \tau + \phi)$$

$$- 2 \left[ I_a^- I_b^- \right]^{1/2} e^{-\alpha|\tau|} \cos(\omega_- \tau + \phi),$$
where

\[
I^+_a = \sigma^*_a S(0) [(\eta - \Delta n)^2 + \kappa^2], \\
I^-_a = \sigma^2 S(0) [(\eta + \Delta n)^2 + \kappa^2], \\
I^+_b = \sigma^*_b S(0) [(\eta + \Delta n)^2 + \kappa^2], \\
I^-_b = \sigma^2 S(0) [(\eta - \Delta n)^2 + \kappa^2].
\]

In terms of the average frequency and the frequency difference

\[
\bar{\omega} = \frac{\omega_- + \omega_+}{2}, \\
\Delta \omega = \frac{\omega_- - \omega_+}{2},
\]

and for sufficiently small \(\Delta \omega \tau\) and \(\alpha |\tau|\), Eqn. 4.25 can be approximated as

\[
I_{FF} \approx I^+_a + I^+_b + I^-_a + I^-_b \\
+ 2([I^+_a I^+_b]^{1/2} - [I^-_a I^-_b]^{1/2}) \cos(\bar{\omega} \tau + \phi).
\]

From this expression we can identify the temporal degree of coherence, \(\gamma\), as defined in [66]:

\[
|\gamma| = \left| \frac{[I^+_a I^+_b]^{1/2} - [I^-_a I^-_b]^{1/2}}{[(I^+_a + I^-_a)(I^+_b + I^-_b)]^{1/2}} \right|.
\]

The visibility of the far-field fringe pattern is

\[
V = 2 \frac{[I^+_a I^+_b]^{1/2} - [I^-_a I^-_b]^{1/2}}{I^+_a + I^+_b + I^-_a + I^-_b}.
\]

Thus, using this analysis, it is possible to calculate the degree of coherence and visibility from the mode intensities in the two waveguides. Alternatively, partial coherence comes as a result of the existence of more than one coupled mode. It is proposed here that Eqns. 4.29 and 4.30 are general expressions that can be used to experimentally determine the degree of coherence of a laser array from measurement of the mode intensities present in the two guides. Note that unlike in previous work [58], the visibility and degree of coherence are known exactly from the modal intensities, and a direct measurement of the visibility is unnecessary. In other words, this theory
directly predicts the visibility.

Considering the eigenmode solutions of the coupled mode theory allows for further modification of Eqn. 4.29 and a better understanding of the applicability of this result. In this work and in general, the vector representations of the eigenmode solutions are

\[
v_+ = N \begin{bmatrix} \kappa \\ \Delta n + \eta \end{bmatrix} = N' \begin{bmatrix} \rho \\ 1 \end{bmatrix}, \quad (4.31)
\]

\[
v_- = N \begin{bmatrix} \Delta n + \eta \\ -\kappa^* \end{bmatrix} = N' \begin{bmatrix} 1 \\ -\rho^* \end{bmatrix}, \quad (4.32)
\]

where \(N\) and \(N'\) are normalization factors and

\[
\rho = \frac{K_{ab}}{\Delta + \psi}. \quad (4.33)
\]

\(|\rho|\) is then a measure of the mode asymmetry (i.e. if \(|\rho| = 1\) then the intensity profiles of the in-phase and out-of-phase modes are perfectly symmetric and anti-symmetric, respectively).

The measured mode intensities yield the weighting on each mode of Eqns. 4.31 and 4.32. In other words, if the measured total intensities of the two modes are \(I_+\) and \(I_-\), then the corresponding eigenmodes are \(\sqrt{I_+}v_+\) and \(\sqrt{I_-}v_-\). Using the definitions above, the intensities in the individual waveguides for the two modes are the measured intensity times the square of the corresponding vector element or

\[
I_a^+ = I_+ N'^2 |\rho|^2,
\]

\[
I_b^+ = I_+ N'^2,
\]

\[
I_a^- = I_- N'^2,
\]

\[
I_b^- = I_- N'^2 |\rho|^2.
\]

It can be seen that \(I_a^+/I_b^+ = |\rho|^2\) and \(I_a^-/I_b^- = 1/|\rho|^2\), and Eqn. 4.29 then can be rewritten as

\[
|\gamma| = \left| \frac{|\rho| (1 - I_-/I_+)}{\left[ (|\rho|^2 + I_-/I_+) (1 + I_-/I_+ |\rho|^2) \right]^{1/2}} \right|. \quad (4.35)
\]

When the coupling strength is great enough or the array asymmetry is small
enough (i.e. $\kappa \gg \Delta n$ and $|\rho| \to 1$), there is a fundamental limit to the coherence given by

$$|\gamma| = \frac{|I_+ - I_-|}{|I_+ + I_-|}.$$  \hspace{1cm} (4.36)

Equation 4.35 clearly illustrates the change in the degree of coherence as the mode asymmetry ($|\rho|$) and mode intensities ($I_+$ and $I_-$) change. The form of Eqn. 4.35 makes it easier to see the limits of high coherence. The degree of coherence is plotted for different values of the mode ratios in Fig. 4.7. Even for $|\rho| = 0.75$, the degree of coherence deviates little from the maximum limit. Thus, it is evident that there is a wide range of mode asymmetries for which the array fields have high coherence.

![Figure 4.7](image.png)

Figure 4.7: The degree of coherence as a function of mode intensities for different array mode asymmetries, $|\rho|$.

The conventional coupled mode theory that is the basis of this analysis is generally applicable only to evanescently coupled fields [53]. The implant-defined coherent VCSEL arrays that are analyzed, however, have been demonstrated to be leaky-mode coupled, i.e. light is shared between elements with propagating fields rather than evanescently decaying fields [50]. However, since the stochastic theory is based on the modes, the representation of the coupled modes is more important than the coupling mechanism. In short, if the behavior of the modes is reasonably well approximated by coupled mode theory, then the stochastic theory should be applicable.
4.3 Experiment

In order to apply the stochastic coupled mode theory to the leaky-mode arrays, the array modes must behave as described in the previous section. Specifically, the system must consist of two modes that approximately follow the form in Eqns. 4.31 and 4.32. In this work, arrays with the lowest order out-of-phase (no central fringe) and in-phase (one central fringe) modes operating are investigated. If it is possible to neglect the central fringe of the in-phase mode (e.g. if the central lobe intensity is negligible in comparison to the two outer waveguide intensities) then it can be possible for the modes to be approximated by two-element vectors that behave as those in coupled mode theory. In this case, one can directly apply the stochastic coupled mode analysis. It will be shown that the arrays studied satisfy these criteria.

Experiments are performed on 1x2 implant-defined VCSEL arrays similar to those already described in the last chapter [17]. However, there is no photonic crystal pattern in the arrays considered here since multimode operation is desired. Multiple measurements of the near- and far-field profiles are taken at different injection currents, since the modes change with current. At threshold, only one array mode is present, but a second array mode begins to turn on as the current is increased.

Coherence experiments are performed by directly measuring the fringe visibility [58] and by measuring the mode intensities. Using a grating spectrometer (setup shown in Fig. 4.8(a)), spectrally resolved images of the near-field modes of a 2x1 implant-defined VCSEL array are collected (Fig. 4.8(b)) [67]. These measurements provide the mode intensities needed in Eqn. 4.29. Using a goniometric radiometer, the far-field radiation pattern is also imaged, from which the coherence can be directly measured [58]. A far-field profile corresponding to the spectrometer data in Fig. 4.8(b) is shown in Fig. 4.9.

As already discussed, the leaky modes must satisfy the conditions in Eqns. 4.31 and 4.32 in order to be treated using the stochastic coupled mode theory. In particular, it must be found that \[ \left( I_a^+ / I_b^+ \right)^{1/2} = \left( I_b^- / I_a^- \right)^{1/2} = |\rho| \] for the theory to be applicable. A plot of \( \left( I_a^+ / I_b^+ \right)^{1/2} \) and \( \left( I_b^- / I_a^- \right)^{1/2} \) as a function of the ratio of the total mode intensities (which changes with injection current) is given in Fig. 4.10. Ideally, the points in the plot would overlap for every value of \( I_- / I_+ \) (which experimentally corresponds to different injection currents), and this would give the unambiguous value of \( |\rho| \). In general,
Figure 4.8: (a) The tabletop imaging spectrometer setup, and (b) spectrometer data showing the two array supermodes.

Figure 4.9: A far-field radiation profile taken from the implant-defined VCSEL array tested.

this condition is approximately satisfied for all data points. Discrepancies can be attributed to experimental error arising as a result of limitations of the spectrometer and charge-coupled device (CCD) camera sensitivities and resolutions. Therefore, these data suggest that the stochastic coupled mode theory can be suitably applied to the implant-defined VCSEL arrays under investigation. Moreover, the value of $|\rho|$ tends to be above 0.7, which is in the range of the upper limit of coherence as illustrated in Fig. 4.7.

Using near-field spectrally resolved measurements and Eqn. 4.29 as well as the far-field measurements [58], the degree of coherence is extracted. Figure 4.11 shows the degree of coherence measured using these two approaches as a function of the ratio of the mode intensities. Excellent agreement is
found between the two sets of data, which serves to verify the validity of the approximations as well as that of the stochastic coupled mode theory [59]. In particular, this result reveals that the predicted reduction in coherence of a coupled laser array is a direct result of the transition from single-mode to multimode operation.

The theoretical high-coherence limit, given by Eqn. 4.36, is also indicated by the dotted line in Fig. 4.11. The measured coherence is observed to lie along this limit, which suggests that the implanted VCSEL arrays operate in a high-coherence regime, consistent with the results of Fig. 4.10.
coherence is in fact expected for leaky-mode laser arrays [53], and the analysis presented here accurately characterizes the coupling properties of the implanted coherent VCSEL arrays.

4.4 Conclusion

The coupled mode formalism with stochastic boundary conditions has been used to predict and investigate partial coherence in coupled semiconductor laser arrays. This model is an improvement over previous approaches as it is directly applicable to partially coherent coupled laser systems. In particular, the spectra and coupling can be calculated from the physical laser structure \textit{ab initio}. Calculations reveal that there is a strong connection between the spectral and temporal coherence and the number of coupled modes. For asymmetric systems (real devices are generally asymmetric to some degree), the degree of coherence scales with the level of asymmetry.

Using this stochastic coupled mode theory, implant-defined coherent VCSEL arrays are analyzed. Despite the fact that the arrays support leaky modes, it is shown that the coupled mode theory provides an adequate approximation to the coherence behavior of the arrays. Measurements are performed using a tabletop imaging spectrometer and a far-field profiler. These measurements serve to verify the predictions of the stochastic theory that coherence is determined by the number and intensities of the array modes. In addition, the analysis reveals that the arrays are operating in a strong coupling, small asymmetry regime, as is to be expected for leaky-mode arrays.

The approach presented here can be particularly useful for the design of single-mode coupled laser arrays. It has been clearly demonstrated that more strongly coupled arrays (which are inherently more symmetrical) are more likely to exhibit high coherence. Moreover, this work has shown that the stochastic coupled mode theory is generally applicable to a variety of systems. Not only should the theory be useful for the evanescently coupled arrays for which it was designed, but it also has shown itself to be useful for leaky-mode arrays. This suggests that the theory could be useful for a wider class of laser arrays than originally intended. Moreover, the theory yields the important result that the critical determinants of array coherence are uniformity among the elements and control of the array modes. This
provides guidelines by which larger laser arrays can be designed for single-mode, highly coherent emission at high output powers.
CHAPTER 5

APPLICATIONS OF IMPLANT-DEFINED VCSEL ARRAYS

5.1 Introduction

In this chapter, two potential applications of leaky-mode, implant-defined VCSEL arrays are considered. The first is for a single-mode, low-divergence laser source. The design is potentially useful to increase the single-mode output power of VCSELs as well as to create a high-brightness beam that does not require external optics. Prospects for increasing the array size for this application are considered. The second application is for electronically controlled beam steering. The beam steering is shown to be consistent and controllable. Moreover, it is shown to have significant advantages over other approaches, including simple fabrication, no need for moving parts, and continuous steering. Both of these applications of the leaky-mode arrays could be useful for imagining, sensing, or communications systems.

5.2 Single-Mode VCSEL Arrays

Coherently coupled index-guided VCSEL arrays typically operate in an out-of-phase mode as a result of inter-element loss introduced by mirror etching [5, 13] or metal contacts [6, 8, 11] in the coupling region. In order to circumvent this problem, phase-adjusted [9], regrown anti-guided (leaky-mode) [10, 12, 14], and implant-defined [17] arrays have been demonstrated. The phase-adjusted and leaky-mode arrays, however, typically require complicated fabrication procedures, while implant-defined arrays tend not to have stable mode control above threshold.

An alternative method for defining two-dimensional arrays has been to use a photonic crystal with a modified pattern in the coupling region [15, 16].
This approach relies exclusively on the photonic crystal to provide optical confinement between elements. Resultantly, the etched holes typically introduce inter-element loss, which causes preferential excitation of the out-of-phase mode. For this reason, in-phase operation is achieved only with limited consistency [16]. In this work, VCSEL arrays incorporate a photonic crystal etched into the top distributed Bragg reflector (DBR) to provide stable index confinement and selective loss to higher-order modes [41] combined with proton implantation used to pixelate the gain region. The implant and photonic crystal are defined so as to minimize the amount of loss introduced in the coupling region between the elements of the laser array. Resultantly, single-mode, in-phase operation of a 2x2 VCSEL is realized [68].

5.2.1 Design

By combining a photonic crystal with an implant-defined structure, stable in-phase operation of the array is realized [68]. The fabricated devices are 2x2 laser arrays defined by a square photonic crystal lattice. The photonic crystal hole pattern provides stable index guiding around the array and helps to ensure only one supermode lases [15, 16]. The periods of the photonic crystals are either 5.5 μm or 6 μm, and the diameter-to-period ratios are 0.6 or 0.7 for both photonic crystal periods. The defects are defined such that the coupling region directly between adjacent apertures is not occupied by an etched hole, as can be seen in Fig. 5.1. The lack of an air hole in the coupling region reduces the amount of loss introduced into this region by the photonic crystal, which allows sufficient gain for the in-phase mode to lase. Simultaneously, the photonic crystal provides stable index guiding that suppresses higher supermodes from turning on. To further reduce the inter-element loss, the photonic crystal holes in the coupling region also can be reduced in size as shown in the device pictured in Fig. 5.1. The implant aperture is designed to be approximately the same size as the photonic crystal aperture. The implantation serves to pixelate the gain as well as create the anti-guiding conditions described in previous chapters.
5.2.2 Performance

Multiple 2x2 arrays with these design parameters, particularly those with a reduced coupling-hole size, tend to lase in a single in-phase mode. A typical device with good single-mode characteristics is presented in Fig. 5.2. This device has a photonic crystal period of 6 µm, a diameter-to-period ratio of 0.6, and a coupling hole ratio reduced to 0.3. The unimplanted regions are square apertures of approximate side lengths 6.5 µm and nearest neighbor center-to-center separation of 8.5 µm. Figure 5.1 shows a near-field image of this device while lasing. Between the array elements, there are single fringes in the near field, which have been correlated with an on-axis maximum in the far field [17].

Threshold current of this array is 7.1 mA, and the output at 12 mA reaches a maximum power of 1.4 mW. The lasers operate at a noticeably higher voltage and series resistance due to lateral resistance introduced by the implant damage near the surface of the laser [52]. Kinks are apparent in the light-intensity (LI) plot in Fig. 5.2(a). However, their origin is not obvious, and they are not a result of higher-order modes turning on. Polarization data (total powers are estimated to compensate for polarizer loss) are included in the LI plot, showing also that one polarization dominates. In this example, the polarizations are along the diagonals of the square formed by the four elements, although polarization rotated 45° from that is also observed. At this point, it is unclear as to the mechanism of polarization control, although it has previously been suggested that the square-lattice photonic crystal has an influence [36]. The laser spectra are shown in Fig. 5.2(b) and, at each measurement, only one mode is evident. Thus, from threshold to maximum
power, this array operates in a single mode.

![Graph showing LIV characteristic and single-mode spectra](image)

Figure 5.2: (a) LIV characteristic of a single-mode, in-phase 2x2 array with polarization data, and (b) single-mode spectra taken at 7.1 to 12.1 mA in 1 mA steps.

5.2.3 Performance Merits and Issues

The low loss in the coupling region between elements allows this device to operate in-phase, as can be seen from the on-axis maximum in the far-field beam pattern shown in the inset of Fig. 5.3. At all currents, the same general radiation pattern is apparent, with a central peak and eight subsidiary lobes. The distinguishing characteristic between lower and higher current injections is that the power in the outer lobes decreases as the injection current increases, as seen in Fig. 5.3. A significant advantage of the photonic crystal VCSEL arrays is that a large percentage of the output power is contained in the central far-field lobe at higher currents. The power in the central lobe at high currents is approximately 20% of the total, which corresponds to 1.5 times more power in this lobe than in any other subsidiary lobe. Since this lobe is not limited by the aperture diffraction, the angular divergence is much less than that of a conventional VCSEL. In the inset of Fig. 5.3, for example, the full-width half-maximum (FWHM) along a vertical cut of the central lobe is $2.7^\circ$. In general for these 2x2 coupled arrays, the FWHM of the central lobe is between $2.5^\circ$ and $3.5^\circ$. In comparison, regrown leaky-mode 2x2 VCSEL arrays produced approximately a $3^\circ$ divergence [69]. Thus, the implanted photonic crystal laser arrays could be useful as low-cost, easily
manufacturable single-mode laser sources with a very small angular divergence.

Figure 5.3: The percentage of total power that is contained in the central lobe of the far field for different injection currents. The inset shows a 3D view of the far-field radiation pattern taken at rollover.

5.3 Arrays with a Metal Grid and Moving to Larger Arrays

As mentioned in the fabrication chapter, high lateral resistance resulting from the ion implantation confinement could result in non-uniform current injection to the elements of arrays larger than 2x2. In order to circumvent this problem, a metal grid that runs between all elements of the array could be used to ensure that equal current is directly injected to all array elements. In many array designs, optical loss introduced by the metal grid forces the array to operate in an out-of-phase mode [6, 8, 11]. However, metal grids have been used in anti-guided arrays in which the in-phase mode still dominates [12, 14].

For implant-defined VCSEL arrays to maintain in-phase operation, it is necessary that the gain discrimination provided by the anti-guiding is greater than the loss introduced by the metal to the in-phase mode. Since the anti-guiding effect in the implant arrays is much weaker than that of conventional regrown structures, it is not obvious that this will be the case. However, it is experimentally demonstrated that the in-phase mode still lases even with
metal runners between elements. Figure 5.4 shows near-field and far-field images of a 2x2 VCSEL array with metal between the elements operating at threshold current. The on-axis peak in the far field characteristic of the in-phase mode is apparent.

![Near-field image and far-field radiation pattern of a 2x2 array with metal runners.](image)

Figure 5.4: (a) Near-field image and (b) far-field radiation pattern of a 2x2 array with metal runners.

The light-current-voltage (LIV) characteristic and spectra of this array are shown in Fig. 5.5. From both of these data, it is evident that the array lases in multiple higher-order modes at higher currents. This could be a result of a number of factors, including the lack of a photonic crystal to help with mode control, the use of large implant apertures, and decreased mode discrimination from loss introduced by the metal runners. However, for up to nearly 1 mW of output optical power the array lases in a single, in-phase mode.

Despite the possible improvement in the uniformity of current injection across the array, realizing larger coherent arrays is not trivial. Of further concern are non-uniformities in implant aperture size and geometry defined during fabrication and in heat profiles. Evidence of this can be found in 3x3 arrays. Figure 5.6 shows near-field images and spectra from a 3x3 array under continuous wave (CW) and pulsed operation. Pulsing is performed using a voltage source with a 100 ns pulse width and a 10% duty cycle.

In Fig. 5.6(a) it is apparent that not all the elements are lasing. This image shows the maximum number of elements that ever turn on during CW
operation. Thus, several of the elements of the array do not ever experience threshold gain. From the spectrum in Fig. 5.6(b), it is also apparent that the array operates in multiple modes. This suggests that the different waveguides support modes at different wavelengths, which could be a result of both non-uniform heating and fabrication imperfections.

The near field of the same array under pulsed operation is shown in Fig. 5.6(c). In this case, all the elements of the array reach threshold and lase, although they do so at different current injection levels. Since the pulsed operation of the array should significantly reduce or eliminate the non-uniform heating, it is likely that fabrication non-uniformities are still problematic. The spectrum in Fig. 5.6(d) shows the multimode behavior of the array, which further suggests that the waveguide modes operate at different wavelengths, implying that the waveguides themselves are significantly different. Although this difference could be due in part to non-uniform heat or current injection, these problems also could be traced back to fabrication issues.

The behavior of larger arrays suggests that the primary roadblock to successful coherent operation results from flaws in fabrication. As mentioned in the modeling chapter, CW operation makes it difficult to design around issues created by non-uniform heating. However, as has been shown, this problem appears to be mitigated by pulsing the lasers. Additionally, placing a heat sink near the active region could further aid in eliminating heating problems. Nevertheless, further improvements in the fabrication procedure are likely needed before larger coherent VCSEL arrays are realized. Sug-
Figure 5.6: (a) Near-field image and (b) spectrum of a 3x3 array under CW operation. (c) Near-field image and (d) spectrum of the same array under pulsed operation with pulse width of 100 ns and 10% duty cycle.

Suggestions for overcoming these problems have been offered in the chapter on fabrication.

5.4 Steerable VCSEL Arrays

VCSEL array structures have demonstrated beam steering capabilities, but these approaches often exhibit discontinuous steering, incoherent fields, or complicated and unreliable mechanical parts [28–31]. An alternative electronic steering method utilizes phase tuning via separate electrical injection to the array elements [32]. This approach is an optical analog of a phased
array radio-frequency beam steering source [70], and it has the potential benefits of system robustness, radiation hardening, reduced steering time, greatly reduced system weight and size, and relatively low operation power. Recently, in-phase [17] and steerable ion-implanted VCSEL arrays [33] have been demonstrated, but unstable mode control causes the array output to become incoherent and the steering to be unpredictable at higher injection currents. In this work, single-mode in-phase 2x2 photonic crystal VCSEL arrays are realized, and it is demonstrated that the electronic beam steering in two dimensions is predictable and controllable [71].

5.4.1 Design

The design and fabrication of the steerable VCSEL arrays are nearly identical to that presented in the previous section. However, an additional focused ion beam (FIB) etch through the top ring contact is used to provide electrical isolation to allow for separate current injection to the array elements. A scanning electron micrograph (SEM) of the VCSEL array with separated contacts is shown in Fig. 5.7.

Figure 5.7: Scanning electron micrograph of photonic crystal VCSEL array with separated contacts.
5.4.2 Performance

In order to deflect the beam from an on-axis position, the in-phase array mode must be used. In addition, to maintain high coherence and continuous steering, it is necessary to have only a single mode lasing. This is achieved in the same manner as with the single-mode arrays described in the previous section. The resulting coherent in-phase far-field intensity pattern of a 2x2 photonic crystal VCSEL array is shown in Fig. 5.8.

Figure 5.8: The in-phase far-field intensity pattern emitted by a 2x2 photonic crystal VCSEL array.

For reference, a light-current-voltage characteristic of a similar single-contact array (i.e. no FIB etch of the top contact) is shown in Fig. 5.9. The approximately 7 mA threshold current is typical for the 2x2 arrays tested. In general, the peak of the far-field radiation pattern is not necessarily directly on-axis. This appears to be a result of fabrication imperfections that cause the current injected into the array elements to be non-uniform, leading to small phase differences between array elements. By independently adjusting the current injected into the four laser elements in a separated-contact device, the location of the central peak can be varied, as is expected when relative phase tuning between the elements of the coupled array occurs [32]. The far field in Fig. 5.8 results from adjusting the relative injection currents above threshold to the four separate contacts so the central peak lies along the optical axis. The values of current to each contact are: top = 1.9 mA,
bottom = 2.3 mA, right = 2.6 mA, and left = 2.1 mA.

Figure 5.9: The LIV characteristic of a single-contact 2x2 coherent VCSEL array.

The changes made in the current injected into the four contacts (percent difference from the baseline values given for on-axis far field) are shown in Table 5.1. In each row of Table 5.1, only one current level is changed; the bold entry for each combination is the current that changes from the previous setting with a maximum variation of 10.5%. The angular location that the central lobe of the in-phase mode steers to for each combination in Table 5.1 is mapped in Fig. 5.10. It is apparent from Fig. 5.10 that the steering angle of the peak of emission changes as the current to the different contacts is altered. It is also evident that this is well-controlled steering, as changes in current to the left/right contacts deflect the beam left/right and changes to the top/bottom contacts steer the beam up/down. The 2x2 VCSEL array studied here is capable of steering the beam in two dimensions over a full angle of approximately 1°. An increased maximum steering angle is expected by increasing the electrical isolation between the contacts and decreasing the separation between the array elements. Although this is a smaller steering range than previously reported [33], this steering occurs for the in-phase mode, and thus the emitted field is highly coherent and the beam deflection is predictable and controllable.
Table 5.1: Percent change of current from baseline values

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<th>Lft</th>
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<td>3.8</td>
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5.4.3 Performance Merits and Issues

For practical beam steering applications, it is important that the central peak maintain its high visibility (and thus narrow angular beam profile) and high power. Figure 5.11 demonstrates that the visibility is typically very high along all directions of the radiation pattern, and only at a few points does it diminish. It has been shown in edge emitting lasers that the preferred lasing mode can be changed by selectively tuning the current injected into different array elements [72]. Thus, the reduction in visibility can be attributed to another array supermode turning on as it becomes excited by a more favorable gain distribution. However, this photonic crystal VCSEL array provides very good mode control and typically maintains single-mode operation. Additionally, Fig. 5.12 illustrates that the percentage of total power in the central lobe is relatively constant as the beam is steered. The power in the central lobe is 19–23% of the total, which corresponds to approximately 1.5 to 2 times as much power as in any of the subsidiary lobes of the field pattern. The small decrease in the central lobe power with increasing steering angle can be attributed to the central peak deviating from on-axis and thus power becomes redistributed from that lobe to subsidiary lobes that move closer to on-axis. In analogy with phased antennas, this power redistribution is
Figure 5.10: The angular locations of the far-field maximum for different current injections as described in Table 5.1. The dots indicate that the current is varied between the left/right contacts, and crosses indicate the result when the top/bottom currents change. The circle represents 0.5° from the surface normal.

A result of shifting of the peaks in the far-field envelope. Overall, the array maintains a stable, narrow-divergence beam that is desirable for steering applications.

Figure 5.11: Measured visibility along four directions (see inset) for the steering points specified in Fig. 5.10 and described in Table 5.1.
5.5 Conclusion

2x2 coherent VCSEL arrays using hybrid ion-implantation and photonic crystal confinement have been described. These laser arrays operate in-phase, producing a low-divergence, on-axis beam in the far field. Moreover, these coupled lasers operate in this single mode from threshold to maximum power. With this knowledge and by addressing the issue of uniform current injection, it is possible to scale these arrays to larger sizes.

Implanted arrays with a metal grid are introduced. It is shown that despite the loss introduced by the metal, the in-phase mode still lases. This is attributed to the large mode discrimination provided by the anti-guiding resonance effect. An example of a coherent, in-phase 2x2 array is presented. The metal grid should be useful to provide uniform current injection when moving to arrays with a larger number of elements. However, issues related to heating and fabrication problems prevent coherent operation of larger arrays.

Finally, the first realization of electronic beam steering in a VCSEL array utilizing only the in-phase array mode has been demonstrated. Since only the in-phase mode is used, the steering is highly coherent and controllable. Moreover, unlike other beam steering approaches, this work exhibits continuous electronic steering achieved without moving parts or complicated fabrication steps. In order to increase the controllability and steering extent, better electrical isolation between array elements and different array geometries can be investigated.
Theoretical and experimental analyses of ion-implanted vertical-cavity surface-emitting laser (VCSEL) arrays have been presented. Design and performance modeling have been developed, and the application of the VCSEL arrays as single-mode, increased-power, and beam steering sources has been demonstrated.

Coherently coupled arrays of VCSELs are fabricated using ion implantation to define electrical and optical apertures for the array elements. As a result of the current confinement, a carrier-induced suppression of the refractive index leads to anti-guiding coupling between neighboring emitters. This is theoretically demonstrated by modeling the effects of heat and carrier concentration on the refractive index profile, and experimental results agree well with the theory. A transfer matrix model can be used to develop simple design rules that take advantage of the anti-guiding conditions.

A coupled mode coherence theory is developed to describe partial coherence in laser arrays. It is shown that operation of multiple array modes is the source of reduced coherence of the array emission. The theory suggests that highest coherence is achieved for an array with uniform intensity to all array elements that are operating in a single mode. Experiments using an imaging spectrometer and a far-field profiler agree with the predictions made by the theory. Moreover, the experiment demonstrates that the theory is applicable not only to the evanescently coupled arrays for which it was designed, but also to leaky-mode arrays such as those investigated in this work.

Fabricated implant-defined arrays are shown to be useful as low-divergence single-mode and beam steering lasers. 2x2 arrays are shown to operate in a single, in-phase mode from threshold current to maximum output power. The far-zone divergence of the central peak of the array emission is shown to be significantly smaller than that from a single emitter. By separating the top contact of a 2x2 array, the individual laser elements can be separately
addressed. By selectively changing the current distribution to the elements, relative phase tuning of the emission is achieved and beam steering results. Steering of only the in-phase mode that is consistent, continuous, and controllable is demonstrated. Finally, issues related to providing uniform current injection to all array elements is addressed by using a metal grid. Despite the loss introduced by the metal, the in-phase mode is still shown to lase. However, further problems with regulating heat and fabricating uniform arrays still prevent the realization of arrays larger than 2x2.

In the course of this work, several significant challenges to creating larger arrays have been recognized. Of utmost importance is the problem of non-uniform heating, which results in a non-uniform index profile. Resultantly, elements in different areas of the array support different modes, and therefore multimode operation is expected. As revealed by the coupled mode coherence theory, the multimode operation leads to low coherence and, thus, an array whose output is not useful for the target applications. However, the framework set up in this research can be used to develop improved designs aimed at overcoming these issues.

Moreover, issues related to fabrication have been uncovered. The current method of using photoresist as an implantation mask seems to fail to provide the aperture uniformity necessary for coherent coupling in large arrays. The behavior of a 3x3 array under pulsed operation supports this supposition. Suggestions for using different mask materials, such as silicon nitride or gold, have been made, and preliminary calculations show promise for reducing the needed mask thickness.

The research presented here represents a step forward in the understanding of ion-implanted VCSEL arrays. The theoretical and experimental foundations developed can lead to improved array designs and performance. New developments, such as the realization of single-mode, in-phase arrays and continuous electronic steering, have been achieved. Further research into ion-implanted VCSEL arrays guided by this work should lead to larger arrays with higher output powers and lower divergence for improved performance as targeting, imaging, or sensing system components.
# APPENDIX

## PROCESS FOLLOWER

Process Sheet: PhC VCSEL Arrays

**Sample Name:**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>Cleave</td>
</tr>
<tr>
<td></td>
<td>□ Cleave, label backside, clean</td>
</tr>
<tr>
<td>1.</td>
<td>Backside Contact (N)</td>
</tr>
<tr>
<td></td>
<td>□ Degrease</td>
</tr>
<tr>
<td></td>
<td>□ 400 Å Au-Ge/200 Å Ni/1500 Å Au</td>
</tr>
<tr>
<td>2.</td>
<td>Top Contact</td>
</tr>
<tr>
<td></td>
<td>□ Degrease</td>
</tr>
<tr>
<td></td>
<td>□ Dehydration bake</td>
</tr>
<tr>
<td></td>
<td>(110 °C for 3 min)</td>
</tr>
<tr>
<td></td>
<td>□ AZ4330 spin</td>
</tr>
<tr>
<td></td>
<td>(3 sec 500 rpm, 30 sec 5000 rpm)</td>
</tr>
<tr>
<td></td>
<td>□ Bake (95 °C for 90 sec)</td>
</tr>
<tr>
<td></td>
<td>□ Edge bead removal</td>
</tr>
<tr>
<td></td>
<td>(25 sec (C); 35 sec AZ400K)</td>
</tr>
<tr>
<td></td>
<td>□ Expose: 27 sec (A) or 10 sec (C)</td>
</tr>
<tr>
<td></td>
<td>□ Aligner: ______; Time: ______sec</td>
</tr>
<tr>
<td></td>
<td>□ Develop AZ400K: ______sec</td>
</tr>
<tr>
<td>3.</td>
<td>Top Contact (P)</td>
</tr>
<tr>
<td></td>
<td>□ O₂ Plasma (300W - 3 min)</td>
</tr>
<tr>
<td></td>
<td>□ DI Rinse (10 min)</td>
</tr>
<tr>
<td></td>
<td>□ 1:10 NH4OH:DI dip (15 sec)</td>
</tr>
</tbody>
</table>
4. _____ Metal Liftoff
   □ Boiling acetone (40 °C for 5 min)
   □ Squirt gun
   □ Repeat until clean

5. _____ SiO₂ Deposition
   □ Degrease
   □ 4000 Å - Time: _____ min
   (18 min @ 220 Å/min)
   □ Thickness: ______

6. _____ Photonic Crystal
   Photolithography
   □ Degrease
   □ Dehydration bake
   (125 °C for 3 min)
   □ AZ5214 spin
   (3 sec 500 rpm, 30 sec 4000 rpm)
   □ Bake (110 °C for 45 sec)
   □ Edge bead removal
   □ Expose: 25 sec (A)
   □ Aligner: ______; Time: ______ sec
   □ Develop AZ327MIF: ______ sec
   □ Bake (110 °C for 60 sec)

7. _____ SiO₂ Etch
   □ Freon 14 (CF₄) for 4000 Å (22 min)
   □ Time: ______ min
   □ REMOVE PR MASK
8. Clean ICP
   □ ICP SiCl₄ recipe
   (P=1.7 mT, SiCl₄=2.0 sccm,
   Ar=2.5 sccm, RF1=35 W,
   RF2=105 W, V_{DC} ≈ 120 V)
   □ Time: ______min
   □ SEM: ______pairs

9. Aperture Implant
   □ Degrease
   □ Photolithography
   □ Dehydration bake
   (125 °C for 3 min)
   □ AZ9260 spin (30 sec 7000 rpm)
   □ Bake (110 °C for 6.0 min)
   □ AZ9260 spin (30 sec 7000 rpm)
   □ Bake (110 °C for 6.0 min)
   □ Edge bead removal
   (4 min on C; 1.5 min in AZ421K)
   □ Expose: 60 sec (C)
   □ Aligner: ______; Time: ______sec
   □ Develop AZ421K: ______sec (75 sec)
   □ Thickness: ______ (> 7 µm)
   □ UV Harden (Aligner A for 5+ min)
   □ Send for implantation

10. Resist Removal
    □ Boiling acetone (40 °C for 5 min)
    □ Squirt gun
    □ O₂ Plasma (700W - 5 min)
    □ Repeat until clean

11. Isolation Implant
    □ Degrease
Photolithography

- Dehydration bake (125 °C for 3 min)
- AZ9260 spin (30 sec 7000 rpm) (reduce speed if not thick enough)
- Bake (110 °C for 6.0 min)
- AZ9260 spin (30 sec 7000 rpm) (reduce speed if not thick enough)
- Bake (110 °C for 6.0 min)
- Edge bead removal (4 min on C, 1.5 min in AZ421K)
- Expose: 30 sec (C)
- Aligner: _____; Time: ______ sec
- Develop AZ421K: ______ sec (75 sec)
- Thickness: ______ (> 10 µm)
- UV Harden (Aligner A for 5+ min)
- Send for implantation

12. _____ Cap Layer Etch

- Freon 14 (CF₄) for 4000 Å (22 min)
- Time: ______ min
- ICP RIE (4 min)

13. _____ Resist Removal

- Boiling acetone (40 °C for 5 min)
- Squirt gun
- O₂ Plasma (700W - 5 min)
- Repeat until clean

14. _____ SiO₂ Removal

- Freon 14 (CF₄) for < 4000 Å (15 min)
- Time: ______ min

15. _____ Test

- Make sure conducting
16. _____ SiO₂ Removal
   □ CF₄ RIE etches in 2 or 3 minute increments until devices are conducting

17. _____ Contact Anneal
   □ Oxide furnace w/N₂ only ( 10 min)
REFERENCES


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AUTHOR’S BIOGRAPHY

Dominic F. Siriani received his B.S. and M.S. degrees in electrical engineering from the University of Illinois at Urbana-Champaign in 2006 and 2007, respectively. His research interests include photonic crystal vertical-cavity surface-emitting lasers (VCSELs), VCSEL arrays, beam combining and beam steering, coupled mode theory, and coherence theory. Mr. Siriani is a student member of the IEEE/Photonics Society and the Optical Society of America. Over the course of his graduate work, he has been awarded a National Science Foundation Graduate Research Fellowship (NSF GRF) and a National Defense Science and Engineering Graduate (NDSEG) Fellowship. He has also been awarded a Distinguished Fellowship and the 2011 P. D. Coleman Outstanding Research Award from the Department of Electrical and Computer Engineering at the University of Illinois.